# Adaptive Approximation-Based Estimation of Downhole Pressure in Managed Pressure Drilling

Lars Imsland and Glenn-Ole Kaasa

***Abstract*— Low-complexity multi-model algorithms for adaptive bottomhole pressure estimation in drilling is derived, and theoretical and practical conditions for convergence (persistency of excitation) discussed, assuming a) topside measurements only, b) topside measurements and bottomhole pressure measurement, and c) topside measurements and delayed bottomhole pressure measurement.**

## I. Introduction

The use of hydrocarbons is ubiquitous in modern society. When extracting hydrocarbons from underground geological formations it is usually necessary to create a well by drilling a wellbore. During drilling, a mud circulation system is used to transport cuttings from the drilling out of the wellbore, for bit lubrication, and for controlling the downhole pressure in the well.

The mud is pumped downhole inside the drill string and through the drill bit, and returns to the top through the annulus surrounding the drill string. The downhole pressure needs to be controlled within its margins: above the reservoir pore pressure and wellbore collapse pressure, but below the wellbore fracture pressure. In many cases, this margin is quite wide and the pressure can safely be manually controlled, but as oil and gas reserves begin to be depleted, reservoirs with narrower margins are being drilled, demanding automated pressure control [5], [6]. The downhole pressure is usually measured, but with conventional equipment this measurement has low bandwidth, is delayed, and is unreliable. Good pressure control therefore requires pressure estimation using also topside measurements [7], [3].

To infer downhole pressure from topside pressure measurements, one needs to know the hydrodynamic (friction) and hydrostatic (gravity) pressure drops in the mud flow path. The hydrostatic pressure drop is usually fairly well known, assuming reasonably accurate knowledge of the density of the mud. However, the hydrodynamic pressure drop is much harder to model beforehand, since the (temperature and pressure-dependent) viscosity properties of the non-Newtonian mud are not well known, and there is a significant dependence on a somewhat uncertain well geometry.

The purpose of this paper is to look into low complexity adaptation laws using multi-model function approximation techniques for modeling hydrodynamic friction loss, for some common measurement configurations. Furthermore, we want to study persistency-of-excitation (PE) requirements for these laws, and take measurement-delays into account.

In the approach proposed here, we assume that the flow of mud in the well is known/measured, while in reality it typically has to be calculated/estimated from other measurements. These measurements typically include measurements of the flow into, and possibly out of, the well, so it is not an unreasonable assumption to make. The reason for making this assumption here is that it simplifies analysis considerably (compare the work in [7], which estimates friction not using a multi-model approach, but where the analysis does not rely on knowledge of flow), and it facilitates PE-insights in the multi-model case. It is the authors' opinion that the PE analysis of the approach considered herein can give insight into PE requirements of other schemes for friction estimation in drilling, also schemes using methods not based on 'local learning'.

We start by describing the class of multi-model approximators, and discuss modeling and problem description in Section III. The adaptation laws are derived and analyzed in Section IV, which ends with a simulation example. In Section V we apply the adaptation laws to actual data from a drilling operation, and we compare the estimated bottomhole pressure to the measured bottomhole pressure that is logged in the bit (and not available real-time).

## II. Function approximation

The approach to adaptive function approximation we will employ, is a 'local learning/neuro/fuzzy'-type approach [1]. We approximate (an unknown) function $h(x)$ with a normalized weighted average of $N$ local approximators $\hat{h}_k(x)$. That is, the approximator $\hat{h}(x)$ is given as

$$\hat{h}(x) = \sum_{k=1}^{N} \phi_k(x)\hat{h}_k(x) = \Phi(x)\begin{pmatrix}\hat{h}_1(x)\\ \hat{h}_2(x)\\ \vdots \\ \hat{h}_N(x)\end{pmatrix},$$

where $\hat{h}_i(x)$ are local approximators, and we use 'basis functions' $\phi_i(x)$ that (for each $x$) forms a 'partition of unity',

$$\phi_i(x) = \frac{\omega_i(x)}{\sum_{k=1}^{n_\theta}\omega_k(x)}.$$

This work was in part supported by NFR project 210432/E30 Intelligent Drilling.

L. Imsland is with the Department of Engineering Cybernetics, Norwegian University of Science and Technology, Trondheim, Norway. lars.imsland@itk.ntnu.no

G.-O. Kaasa is both with Statoil AS, Research Centre Porsgrunn, Norway, and the Department of Engineering Cybernetics, Norwegian University of Science and Technology, Trondheim, Norway. gkaa@statoil.com

The function $\omega_k(x)$ is the local weighting function, or local support. Examples of such functions can be

$$\omega_i(x) = \begin{cases} \left(1 - \left(\frac{|x-c_i|}{\mu_i}\right)^2\right)^2, & \text{if } |x - c_i| < \mu_i, \\ 0, & \text{otherwise.} \end{cases}$$

Here, $c_i$ is the center of the 'local support' of the local approximator $\hat{h}_i(x)$, while $\mu_i$ describes the area (radius).

Conditions for when this class of approximators is a universal approximator for a class of functions, can be found [1].

We will assume that the local approximators are parameterized, $\hat{h}_k(x) = \hat{h}_k(x, \theta_k)$, where $\theta_k$ is a vector of parameters for the local approximator, and design update laws for all these local parameter vectors, $\theta = (\theta_1, \theta_2, \ldots, \theta_N)$.

## III. Problem description and modeling assumptions

### A. Modeling

In drilling operations, one usually measures the standpipe pressure, the pressure of the mud pumped into the drillpipe going down the wellbore, denoted $p_p$. The pressure of the mud exiting from the wellbore through the drillpipe annulus is conventionally atmospheric, but in cases where the bottomhole pressure is of significant concern, one often employs what is called Managed Pressure Drilling (MPD). This implies that a choke is placed in the mud return line and the pressure is measured upstream this choke. This is assumed here, and we call this the pressure measurement the choke pressure, and denote it $p_c$.

One may then reasonably assume that the bottomhole pressure $p_b$ is related to these two measurements as [3]

$$p_b = p_p - F_d(l, q) + G_d(h)$$
$$p_b = p_c + F_a(l, q) + G_a(h)$$

where $q$ is an average flow in the mud circulation system, assumed known. The drillstring (annulus) hydrodynamic pressure is denoted $F_d(l, q)$ $(F_a(l, q))$ and the hydrostatic pressure $G_d(h)$ $(G_a(h))$. The length of the well is denoted $l$, and the true vertical depth of the well is denoted $h$. Both of these are in general time-varying.

A reasonable model for the flow $q$ can be found from a momentum balance at the bit [3],

$$M(l)\dot{q} = p_p - p_c - F_d(l, q) - F_a(l, q) + G_d(h) - G_a(h),$$

where $M(l)$ is a parameter that depends on well geometry (and fluid density). We parametrize the unknown friction as

$$F_d(l, q) = f_d^\top(l, q)\theta^d, \quad F_a(l, q) = f_a^\top(l, q)\theta^a,$$

where the vector functions $f_d^\top(l, q)$ and $f_a^\top(l, q)$ are based on nominal friction models (see next section), and $\theta^d$ and $\theta^a$ are unknown parameter vectors. For simplicity, we introduce $s(h) = G_d(h) - G_a(h)$ ("difference in static head"), and use notation that suppress dependence on $h$ and $l$ in the following. We assume that parameters variations due to these are so slow (compared to the flow dynamics) that that we can assume them constant.

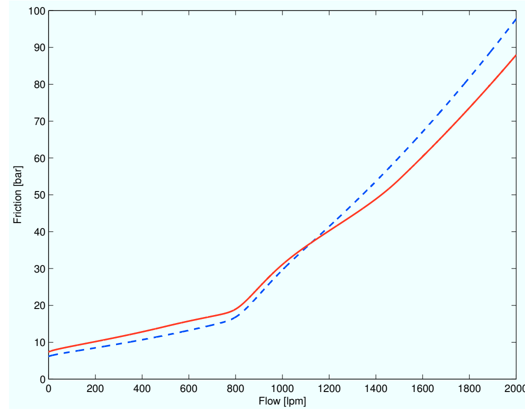



Fig. 1. A model of drillstring friction (dashed line) with a two-parameter approximation (solid line). The approximation is for $\theta_1^d = 1.2$ and $\theta_2^d = 0.9$.

### B. Friction approximation

For a nominal friction model $F_i(q)$, $i \in \{a, d\}$, let us parameterize model error with a parameter vector $\theta^i \in \mathbb{R}^{n_\theta}$ and accompanying basis functions[1], $\Phi(q) = \left(\phi_1(q), \phi_2(q), \ldots, \phi_{n_\theta}(q)\right)^\top$, such that we can use

$$F_i(q)\Phi(q)^\top\theta^i, \quad i \in \{a, d\}$$

as an approximation to the real friction. Compared to Section II, this means (loosely) that we use $\hat{h}_k(q, \theta_k) = F_i(q)\theta_k^i$, and $N = n_\theta$. We assume (for sake of analysis) $n_\theta$ large enough such that we can use this to construct a universal approximator to the real friction.

Let us use $f_i(q)$ as a shorthand for the vector function $F_i(q)\Phi(q)$. Specifically, we will assume the annulus friction is $f_a^\top(q)\theta^a$ and drillstring friction is $f_d^\top(q)\theta^d$, for some (unknown) paramaters $\theta^a$ and $\theta^d$, that we want to adapt.

As an example of the friction approximation, we plot a nominal model of drillstring friction together with an approximation with $n_\theta = 2$, in Figure 1.

## IV. Simple adaptive schemes for downhole pressure estimation

In this section, we derive adaptation laws using surface measurements only (Subsection IV-A), surface measurements and bottomhole pressure measurement (Subsection IV-B), and finally a scheme taking delay in bottomhole pressure measurement into account (Subsection IV-C. The two first approaches are illustrated in simulations in Subsection IV-D. We then apply the final adaptation law, including delayed bottomhole pressure measurement, to data from an actual drilling operation in Section V.

### A. Information from surface measurements only

Based on the previous section, we use the following model for design:

$$M\dot{q} = p_p - p_c - f_d^\top(q)\theta^d - f_a^\top(q)\theta^a + s, \tag{1}$$

[1] We assume for simplicity that we use the same basis functions both for annulus and drillstring

and assume that $q$, $p_p$, $p_c$ and $s$ are measured and bounded.

Even though we assume $q$ measured, we introduce an observer for $q$,

$$M\dot{\hat{q}} = p_p - p_c - f_d^\mathsf{T}(q)\hat{\theta}^d - f_a^\mathsf{T}(q)\hat{\theta}^a + s + k\,(q-\hat{q})\,, \quad (2)$$

where we aim (with reservations regarding PE) to find update laws for $\hat{\theta} = \left((\hat{\theta}^a)^\mathsf{T}, (\hat{\theta}^d)^\mathsf{T}\right)^\mathsf{T}$ such that $\hat{\theta} \to \theta$.

Define error variables $\tilde{q} = q - \hat{q}$, $\tilde{\theta} = \theta - \hat{\theta}$, and consider the Lyapunov-like function $V(\tilde{q},\tilde{\theta}) = \frac{1}{2}\left(M\tilde{q}^2 + (\tilde{\theta}^a)^\mathsf{T}\Gamma_a^{-1}\tilde{\theta}^a + (\tilde{\theta}^d)^\mathsf{T}\Gamma_d^{-1}\tilde{\theta}^d\right)$ where $\Gamma_a$ and $\Gamma_d$ are positive definite. The derivative of this function is

$$\begin{aligned}\dot{V} &= \tilde{q}\dot{\tilde{q}} + (\tilde{\theta}^a)^\mathsf{T}\Gamma_a^{-1}\dot{\tilde{\theta}}_a + (\tilde{\theta}^d)^\mathsf{T}\Gamma_d^{-1}\dot{\tilde{\theta}}_d \\ &= -k\tilde{q}^2 - \tilde{q}f_d^\mathsf{T}(q)\tilde{\theta}^d - \tilde{q}f_a^\mathsf{T}(q)\tilde{\theta}^a \\ &\qquad + (\dot{\hat{\theta}}^d)^\mathsf{T}\Gamma_d^{-1}\tilde{\theta}^d + (\dot{\hat{\theta}}^a)^\mathsf{T}\Gamma_a^{-1}\tilde{\theta}^a.\end{aligned}$$

Selecting parameter update laws

$$\begin{aligned}\dot{\hat{\theta}}^a &= -\Gamma_a f_a(q)\,(q-\hat{q})\,, \\ \dot{\hat{\theta}}^d &= -\Gamma_d f_d(q)\,(q-\hat{q})\,,\end{aligned}$$

we obtain, under the assumption that the 'real' parameters are constant,

$$\dot{V} = -k\tilde{q}^2,$$

and hence we can conclude $\tilde{q} \to 0$ and boundedness of parameter estimates by standard arguments (Barbalat's lemma) under some technical assumptions on $q$ and the friction functions [4].

However, we want more than boundedness; we want parameter convergence for the parameters that have local support. For this, we need to examine the information content in the data; i.e., whether the data is persistently exciting (PE).

Note that the parameters that do not have local support (that is, the $\theta_k$ for which $\phi_k(q) = 0$) will not be updated. Therefore, a PE-analysis must take this into account.

*1) PE analysis:* PE analysis of the above estimation problem is straightforward as the error system is essentially LTV and we can resort to standard theory (e.g. [4], [2]). The closed loop error system can be written as

$$\begin{pmatrix}\dot{\tilde{q}} \\ \dot{\tilde{\vartheta}}\end{pmatrix} = \begin{pmatrix} A & B\phi(t)^\mathsf{T} \\ -\phi(t)C^\mathsf{T} & 0\end{pmatrix}\begin{pmatrix}\tilde{q} \\ \tilde{\vartheta}\end{pmatrix}$$

where

$$\tilde{\vartheta} = \begin{pmatrix}\sqrt{\Gamma_a^{-1}}\tilde{\theta}^a \\ \sqrt{\Gamma_d^{-1}}\tilde{\theta}^d\end{pmatrix}, \quad \phi(t) = \begin{pmatrix}-\sqrt{\Gamma_a}f_a(q) \\ -\sqrt{\Gamma_d}f_d(q)\end{pmatrix},$$

and $A = -k$ and $B = C = 1$. Since $(A,B,C)$ is Strictly Positive Real (SPR), this system is (globally) exponentially stable if we have PE, that is, if there exists $\alpha$ such that at all time instants $t$, there exists a $T$ such that

$$\int_t^{t+T} \phi(\tau)\phi^\mathsf{T}(\tau)d\tau \geq \alpha I.$$

If $q$ varies within a small window, then only two parameters, one in each of $\theta^a$ and $\theta^d$, are being estimated. The interesting question is: Is there enough variation to estimate both of these? Assume, as an approximation to this case, that $n_{\theta^a} = n_{\theta^d} = 1$, that is $f_a(q)$ and $f_d(q)$ are scalars. The question is then, does $f_a(q)$ and $f_d(q)$ vary "sufficiently differently", that is, will the matrix

$$\int_t^{t+T} \begin{pmatrix} f_a(q) \\ f_d(q)\end{pmatrix}\begin{pmatrix} f_a(q) \\ f_d(q)\end{pmatrix}^\mathsf{T} d\tau$$

become non-singular? Since both signals depend in a similar manner on $q$, this requires significant variations in $q$ that really excite the nonlinearities in $f_a(q)$ and $f_d(q)$ differently.

*2) PE in practice:* If we look at typical drilling operations, this is unlikely to happen apart from short time-periods (pipe connections), however, these time-periods are likely too short to obtain robust parameter convergence. This means that in practice there is only information to estimate one parameter, and this parameter will only converge to its true value provided the other parameter is set correctly.

However, it is easy to see that the total friction is estimated correctly in steady state. Since $\tilde{q} \to 0$, when $\dot{\hat{q}} = 0$ (in steady state), then, from (2),

$$f_d^\mathsf{T}(q)\hat{\theta}^d + f_a^\mathsf{T}(q)\hat{\theta}^a = p_p - p_c + s.$$

Since the expression on the right hand side is the total pressure loss from standpipe to choke due to friction, the overall friction will be estimated correctly.

In conclusion, we claim it is only realistic to update one part of the friction, and assume the other known, during a drilling operation when using topside measurements only. That is, one may either assume friction in the drillpipe known, and estimate the parameters related to annulus friction, or the other way around. The fact that annulus friction in general is the 'most unknown' friction, speaks in favor of estimating annulus friction. On the other hand, the following issues,

- drillpipe friction often is about an order of magnitude larger,
- annulus friction influences the main variable of interest (bottomhole pressure) directly, and hence we should be careful with updating annulus friction without using measurements of bottomhole friction,

points towards estimating only drillpipe friction as long as bottomhole pressure is not available.

### *B. Surface measurements and bottomhole pressure measurement*

Consider the system (1), but assume now (somewhat unrealistically) that we have available the additional measurement $p_b$,

$$p_b = p_c + f_a^\mathsf{T}(q)\theta^a + G_a. \quad (3)$$

We incorporate this information by extending the observer (2) for $q$ in the following manner:

$$\begin{aligned}\dot{\hat{q}} = p_p - p_c - f_d^\mathsf{T}(q)\hat{\theta}^d - f_a^\mathsf{T}(q)\hat{\theta}^a + s \\ + k_q\,(q-\hat{q}) + k_p\,(p_b - \hat{p}_b)\,, \quad (4)\end{aligned}$$

where

$$\hat{p}_b = p_c + f_a^{\mathsf{T}}(q)\hat{\theta}^a + G_a.$$

This gives the following error dynamics:

$$\dot{\tilde{q}} = -f_d^{\mathsf{T}}(q)\tilde{\theta}^d - f_a^{\mathsf{T}}(q)\tilde{\theta}^a - k_q\tilde{q} - k_p\tilde{p}_b.$$

We first note that since $\tilde{p}_b = -f_a^{\mathsf{T}}(q)\tilde{\theta}^a$, it is straightforward to extend the design in Section IV-A to obtain the parameter update law

$$\dot{\hat{\theta}}^a = \Gamma_a(1-k_p)f_a(q)\,(q-\hat{q})\,.$$

The information in the bottomhole pressure is now used only indirectly (through $\hat{q}$) to affect the parameter estimates (in $\theta^a$). It is of interest to see if we can obtain parameter update laws that use the information more explicitly.

We therefore propose

$$\begin{aligned}\dot{\hat{\theta}}^a &= -\Gamma_a(1-k_p)f_a(q)\,(q-\hat{q}) + K_a(q)\,(p_b - \hat{p}_b)\,,\\ \dot{\hat{\theta}}^d &= -\Gamma_d f_d(q)\,(q-\hat{q})\,,\end{aligned}$$

where the matrix $K_a(q)$ has a specific form, $K_a(q) = \Gamma_a f_a(q)k_a$. This gives the tuning parameters $\Gamma_a$, $\Gamma_b$, $k_q$, $k_p$, and $k_a$.

Considering the same Lyapunov function as before, we obtain

$$\begin{aligned}\dot{V} = -k_q\tilde{q}^2 - \tilde{q}f_d^{\mathsf{T}}(q)\tilde{\theta}^d - \tilde{q}f_a^{\mathsf{T}}(q)\tilde{\theta}^a\\ - k_p\tilde{q}\tilde{p}_b + (\tilde{\theta}^a)^{\mathsf{T}}\Gamma_a^{-1}\dot{\hat{\theta}}_a + (\tilde{\theta}^d)^{\mathsf{T}}\Gamma_d^{-1}\dot{\hat{\theta}}_d\end{aligned}$$

Inserting the parameter update laws and using $\tilde{p}_b = -f_a^{\mathsf{T}}(q)\tilde{\theta}^a$, we get

$$\dot{V} = -k_q\tilde{q}^2 - (\tilde{\theta}^a)^{\mathsf{T}}\Gamma_a^{-1}K_a(q)f_a^{\mathsf{T}}(q)\tilde{\theta}^a.$$

Now, since $K_a(q) = \Gamma_a f_a(q)k_a$,

$$\dot{V} = -k_q\tilde{q}^2 - k_a\left(f_a^{\mathsf{T}}(q)\tilde{\theta}^a\right)^2,$$

allowing us to conclude that $\tilde{q} \to 0$ and $f_a^{\mathsf{T}}(q)\tilde{\theta}^a \to 0$ (using Barbalat's lemma, provided the derivatives of $q$ and $f_a(q)$ are bounded).

*1) PE analysis:* Since we now have a measurement equation involving (some of) the unknown parameters, the analysis is not entirely standard anymore. Define

$$\mathscr{A}(t) = \begin{pmatrix} -k_q & -(1-k_p)f_a^{\mathsf{T}}(q) & -f_d^{\mathsf{T}}(q)\\ (1-k_p)\Gamma_a f_a^{\mathsf{T}}(q) & -k_a\Gamma_a f_a(q)f_a^{\mathsf{T}}(q) & 0\\ \Gamma_d f_d^{\mathsf{T}}(q) & 0 & 0\end{pmatrix},$$
$$\mathscr{C}(t) = \begin{pmatrix}\sqrt{k_q} & 0 & 0\\ 0 & \sqrt{k_a}f_a^{\mathsf{T}}(q) & 0\end{pmatrix}.$$

Then, the closed loop system is given by $\dot{x} = \mathscr{A}(t)x$, and the derivative of the Lyapunov function is $\dot{V}(x) = -x^{\mathsf{T}}\mathscr{C}^{\mathsf{T}}(t)\mathscr{C}(t)x$. We see that compared to Section IV-A.1, we have obtained a 'stabilizing' (2,2)-element, which should improve the stability properties, and an increased output dimension (larger $\mathscr{C}$-matrix), which should relieve some of the excitation requirements.

Since the system is LTV, we will have exponential stability of $x=0$ if the pair $(\mathscr{A}(t),\mathscr{C}(t))$ is uniformly observable [4]. Uniform observability of $(\mathscr{A}(t),\mathscr{C}(t))$ is equivalent to uniform observability of $(\mathscr{A}(t)-\mathscr{K}(t)\mathscr{C}(t),\mathscr{C}(t))$. Choose

$$\mathscr{K}(t) = \begin{pmatrix} -\sqrt{k_q} & -\frac{(1-k_p)}{\sqrt{k_a}}\\ \frac{1-k_p}{\sqrt{k_q}}\Gamma_a f_a^{\mathsf{T}}(q) & 0\\ \frac{1}{\sqrt{k_q}}\Gamma_d f_d^{\mathsf{T}}(q) & 0\end{pmatrix}, \tag{5}$$

then

$$\mathscr{A}(t)-\mathscr{K}(t)\mathscr{C}(t) = \begin{pmatrix}0 & 0 & -f_d^{\mathsf{T}}(q)\\ 0 & -k_a\Gamma_a f_a(q)f_a^{\mathsf{T}}(q) & 0\\ 0 & 0 & 0\end{pmatrix}.$$

Due to the decoupled structure of the system $(\mathscr{A}(t)-\mathscr{K}\mathscr{C}(t),\mathscr{C}(t))$, we can divide the problem in two. That is, the pair $(\mathscr{A}(t),\mathscr{C}(t))$ is uniformly observable if the systems

$$\dot{\xi}_1 = \begin{pmatrix}0 & -f_d^{\mathsf{T}}(q)\\ 0 & 0\end{pmatrix}\xi_1,\ \zeta_1 = \begin{pmatrix}\sqrt{k_q} & 0\\ 0 & 0\end{pmatrix}\xi_1, \tag{6a}$$
$$\dot{\xi}_2 = -k_a\Gamma_a f_a(q)f_a^{\mathsf{T}}(q)\xi_2,\ \zeta_2 = \sqrt{k_a}f_a^{\mathsf{T}}(q)\xi_2, \tag{6b}$$

are uniformly observable.

- Uniform observability of (6a) is implied by $f_d(q)^{\mathsf{T}}$ being PE, that is, by the existence of $T$ and $\alpha$ such that for all $t$,
$$\int_t^{t+T} f_d(q)f_d(q)^{\mathsf{T}}d\tau \geq \alpha I.$$
- By looking at the measurement equation for (6b), it is clear that the corresponding condition,
$$\int_t^{t+T} f_a(q)f_a(q)^{\mathsf{T}}d\tau \geq \alpha I$$
is a conservative[2] condition for uniform observability of (6b).

In the case of $n_\theta = 1$ (or, analysis within a local support), this reduces to that (the scalar) $f_a(q)$ and $f_d(q)$ must be positive.

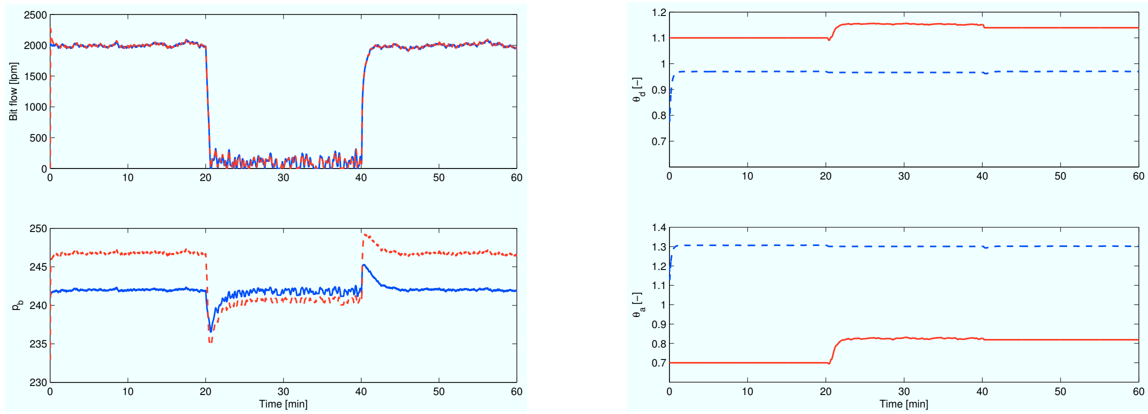
*2) PE in practice:* That is, as long as the model predicts friction, we have enough information to obtain estimates of both annulus and drillpipe friction friction (within a local support). As we basically estimate two parameters from two equations ((1) and (3)), this is not surprising. We have less information when there is little friction (for small flows), which is not surprising either, since then the parameters do not influence the equations.

### C. Time-delayed bottomhole pressure measurement

Assume now that the bottomhole pressure measurement is $T$ time-units delayed, that is,

$$p_b(t) = p_c(t-T) + f_a^{\mathsf{T}}(q(t-T))\theta^a + s_a.$$

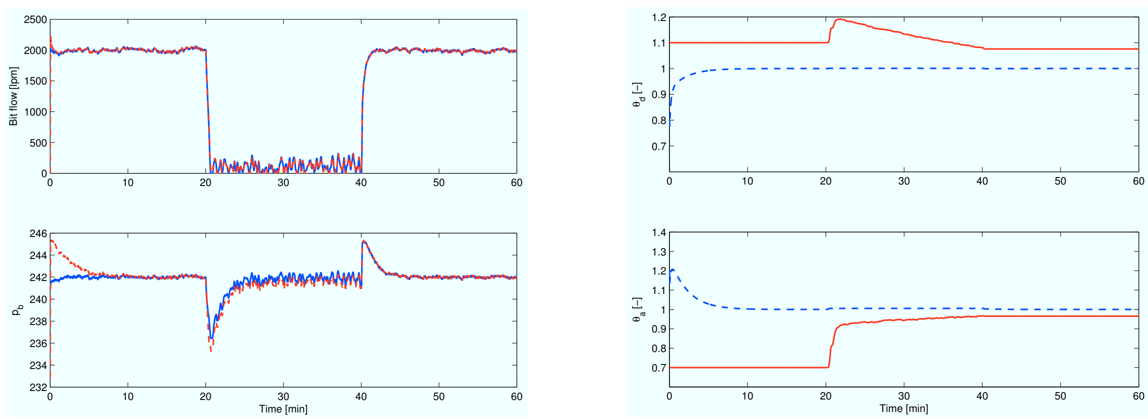
[2]In this case, it is likely that less conservative conditions exist, in contrast to for the first system.




(a) Top: simulated (whole) and estimated (dashed) bit flow; bottom: simulated (whole) and estimated (dashed) bottom-hole pressure.

(b) Top: drillstring parameters; bottom: annulus parameters. Red (whole): parameters valid for large flows, blue (dashed): parameters valid for small flows.

Fig. 2. Simulation of adaptive law using topside measurements only, with $\Gamma_d = \mathrm{diag}([0.2, 0.02])$, $\Gamma_a = \mathrm{diag}([0.5, 0.2])$, $k_q = .001$.




(a) Top: simulated (whole) and estimated (dashed) bit flow; bottom: simulated (whole) and estimated (dashed) bottom-hole pressure.

(b) Top: drillstring parameters; bottom: annulus parameters. Red (whole): parameters valid for large flows, blue (dashed): parameters valid for small flows.

Fig. 3. Simulation of adaptive law using topside measurements and bottomhole pressure measurement, with $\Gamma_d = \mathrm{diag}([0.2, 0.02])$, $\Gamma_a = \mathrm{diag}([0.5, 0.2])$, $k_q = .001$, $k_p = 2 \cdot 10^{-4}$, $k_a = 2 \cdot 10^{-4}$.

Using the same setup as in Section IV-B except for setting $k_p = 0$, that is, we use the observer (4) (with $k_p = 0$) and the parameter update laws

$$\dot{\hat{\theta}}_a = -\Gamma_a f_a(q)\,(q(t) - \hat{q}(t)) + K_a(q(t-T))\,(p_b(t) - \hat{p}_b(t)),$$
$$\dot{\hat{\theta}}_d = -\Gamma_d f_d(q(t))\,(q(t) - \hat{q}(t)),$$

with $K_a(q(t-T)) = \Gamma_a f_a(q(t-T))k_a$ and

$$\hat{p}_b(t) = p_c(t-T) + f_a^{\mathsf{T}}(q(t-T))\hat{\theta}^a(t) + s_a.$$

Noting that now, $\tilde{p}_b(t) = -f_a^{\mathsf{T}}(q(t-T))\tilde{\theta}^a$, we use the same Lyapunov function as above to arrive at

$$\dot{V} = -k_q\tilde{q}^2 - k_a\left(f_a^{\mathsf{T}}(q(t-T))\tilde{\theta}^a\right)^2.$$

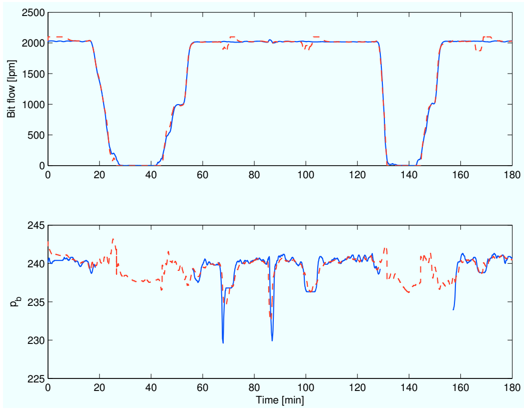
Note that even though we have a time-delayed measurement, the system we analyze ($\tilde{q}$, $\tilde{\theta}^a$, $\tilde{\theta}^d$) is not a time-delay system, it merely involves a time-delayed signal, and therefore an analysis based on Barbalat's lemma can be readily invoked. That is, using Barbalat's lemma, we can conclude that $\dot{V} \to 0$ and hence $\tilde{q} \to 0$ and $f_a^{\mathsf{T}}(q(t-T))\tilde{\theta}^a \to 0$.

For PE analysis, set the $(2,1)$-element of $\mathscr{K}(t)$ in (5) to zero to get

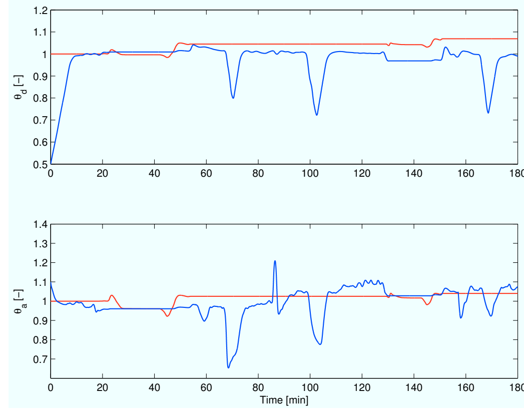
$$\mathscr{A}(t) - \mathscr{K}(t)\mathscr{C}(t) = \begin{pmatrix} 0 & -f_a^{\mathsf{T}}(q(t)) & -f_d^{\mathsf{T}}(q(t)) \\ 0 & -k_a\Gamma_a f_a(q(t-T))f_a^{\mathsf{T}}(q(t-T)) & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Decoupling the system as in Section IV-B.1, we see that (6b) is as before (and therefore observable under the same condition), while the state of (6b) becomes an input to system (6a). Due to linearity, this input does not influence observability, and we can conclude uniform observability under the same conditions.

It is notable that the way we have included time-delay in this section is considerably less complex than by state augmentation, as is a much-used way to include time-delays in an extended Kalman filter.

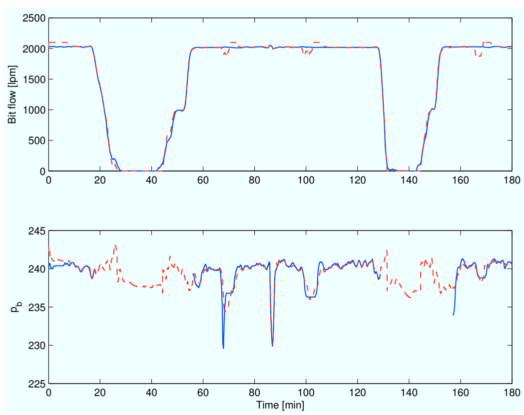



(a) Top: simulated/measured (whole) and estimated (dashed) bit flow; bottom: measured (whole) and estimated (dashed) bottomhole pressure.

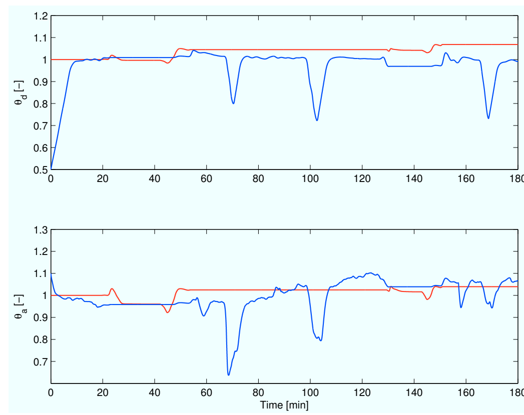



(b) Top: drillstring parameters; bottom: annulus parameters. Red lines are parameters valid for small flows, blue lines parameters valid for large flows.

Fig. 4. The algorithm in Section IV-B with $\Gamma_d = 5 \cdot 10^{-3}\,\mathrm{diag}([2,1])$, $\Gamma_a = 3 \cdot 10^{-2}\,\mathrm{diag}([2,1])$, $k_q = .001$, $k_p = 0$, $k_a = 2 \cdot 10^{-3}$.




(a) Top: simulated/measured (whole) and estimated (dashed) bit flow; bottom: measured (whole) and estimated (dashed) bottomhole pressure. Note that estimated bottomhole pressure is shifted to facilitate comparison with measurement.




(b) Top: drillstring parameters; bottom: annulus parameters. Red lines are parameters valid for small flows, blue lines parameters valid for large flows.

Fig. 5. The algorithm in Section IV-C with $\Gamma_d = 5 \cdot 10^{-3}\,\mathrm{diag}([2,1])$, $\Gamma_a = 3 \cdot 10^{-2}\,\mathrm{diag}([2,1])$, $k_q = .001$, $k_p = 0$, $k_a = 2 \cdot 10^{-3}$.

### *D. Simulations*

We test the approach on a simulator based on the model in [3], with pressure loss (hydrodynamic and hydrostatic) taken from a model of a real well. To make things slightly more realistic, we add noise to the pump flow input, and use estimated bit flow in friction calculation for the adaptive estimation (the latter improves adaption since it gives more excitation).

The scenario is initially full circulation, then rapid transition to low circulation (not zero flow, so not a pipe connection), then rapid transition to full circulation again.

*1) Simulation using topside measurements only:* See Figure 2. As predicted by theory, the estimated flow converges, and the estimated parameters are bounded, but does not converge to their 'true' values (which in the simulation is 1 for all parameters). This gives large errors in predicted downhole pressure.

If we fix one of the parameter sets to its true value, then the other will converge to the true value for full circulation (not shown). However, for low circulation, low gain in the adaptation loop gives convergence problems.

*2) Simulation using topside measurements and bottomhole pressure measurement:* See Figure 3. Now all parameters converge, as predicted by theory. For small flows, convergence of $\theta^d$ is slow. This seems rather generic, increasing the gain gives faster convergence but larger 'overshoot' (and hence probably less robustness).

## V. Testing using measurements from actual drilling operation

We test the algorithm using actual measurements from on offshore well in the North Sea. During the period of data it is drilled horizontally, with two pipe-connections. Being real data, there are factors complicating the picture:

- "Downlink"-procedures (topside communication with bottomhole assemply through pulsing in mud) causes

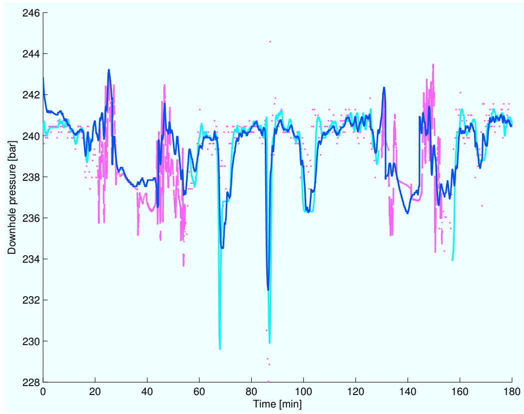



Fig. 6. Adaptation based (blue, whole), measurement transferred to topside (cyan, dashed) and tool logged (magenta, whole/dotted) annulus bottomhole pressure for the algorithm in Section IV-B.

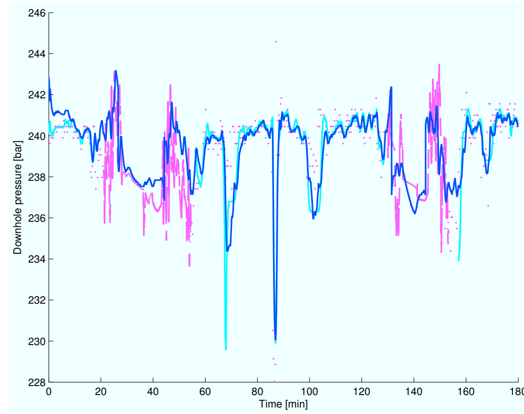



Fig. 7. Adaptation based (blue, whole), measurement transferred to topside (cyan, dashed) and tool logged (magenta, whole/dotted) annulus bottomhole pressure for the algorithm in Section IV-C. Note that estimated bottomhole pressure has been shifted to facilitate comparison with topside measurement.

large disturbances to the stand-pipe/bottomhole pressure measurements at ca. 65 min., 100 min., and 170 min..

- At ca. 90 min. the main choke become plugged, whereby the operators commands a large opening to the second choke, giving a dip in choke pressure which propagates to downhole and standpipe pressure.

We use $n_\theta = 2$. The default adaptation algorithm is the one in Section IV-B. During pipe connections (when flow is stopped), the bottomhole pressure measurement is lost, and the adaptation algorithm used is the one in Section IV-A. For offline analysis, the bottomhole pressure measurement is logged in the bottomhole assembly (bit) during periods with no communication, and is therefore available for comparison in this study (but not used in the adaptation/estimation algorithms).

We first try without correcting for time-delay in the pressure measurement, then with.

### A. Assuming no time-delay

Figure 4 shows flow, pressure, and estimated parameters for a specific tuning of the adaptation algorithm. Figure 6 plots the calculated downhole pressure (again) together with the measurement transferred to topside (when available) and measurements logged in the bit (at other times).

We see that the pressure follows fairly well, but:

- The downlink events severely affect parameter adaptation, best seen in the plot of $\theta^d$ (Figure 4b). Probably, one should turn off estimation during these events. This would allow lower gains.
- The parameters corresponding to friction at small flows are seldom updated. This follows since the flow is zero most of the time with small flows, and the model predicts that friction does not affect $q$ or $p_b$ at zero flow.

### B. Including time-delay in measurement

Lastly, the estimation algorithm is run with the same parameters, but accounting for time-delay in the bottomhole pressure measurements. We assume the delay is 30s. See Figure 5, and especially 7 and 8. It is seen that by taking delay in pressure measurement into account, we more accurately capture the dynamic changes in the bottomhole pressure. This could be important in demanding control applications relying on this pressure estimate.

## VI. Concluding Remarks

A low-complexity multi-model friction estimation algorithm is proposed, analysed, and tested in simulations and using actual data. The tests are fairly successful, but it seems that the way a typical pipe-connection procedure is performed makes it challenging to get convergence in the friction model for the parameters that cover small flows.

The derivation of the algorithm is based on a simplifying assumption of measured bit flow. However, it is believed the PE analysis give insights that to some degree is general (in "single-model" cases, and for cases with no assumption of measured bit flow).

Taking delays in the pressure measurement into account improved pressure estimation significantly. This observation is expected to also hold true for other adaptive estimation algorithms using bottomhole pressure measurement.

## Acknowledgments

Statoil AS is acknowledged (Glenn-Ole Kaasa, Alexey Pavlov and John-Morten Godhavn) for a rewarding long-time cooperation on drilling automation, and for use of the data in Section V.

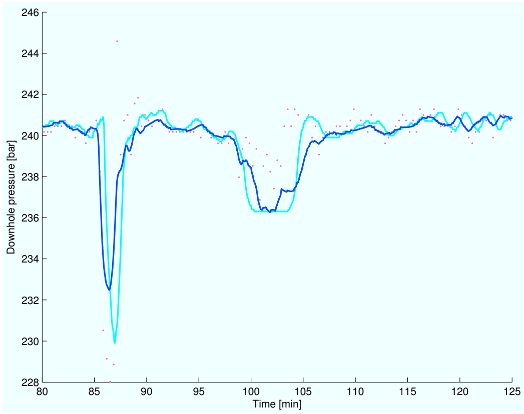
## References

[1] J. A. Farrell and M. M. Polycarpou, *Adaptive Approximation Based Control: Unifying Neural, Fuzzy and Traditional Adaptive Approximation Approaches*. Wiley-Interscience, 2006.

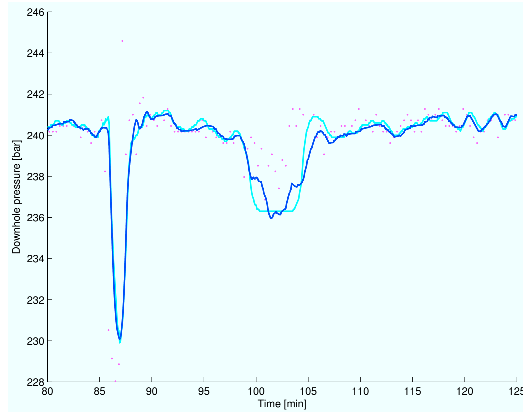
[2] P. A. Ioannou and J. Sun, *Robust Adaptive Control*. Upper Saddle River, NJ 07458, USA: Prentice-Hall PTR, 1995.

[3] G.-O. Kaasa, Ø. N. Stamnes, L. Imsland, and O. M. Aamo, "Simplified hydraulic model used for intelligent estimation of downhole pressure for an mpd control system," *SPE Drilling & Completion*, 2012, accepted.

[4] H. K. Khalil, *Nonlinear Systems*, 3rd ed. Upper Saddle River, NJ: Prentice Hall, 2002.




(a) A zoom on Figure 6.




(b) A zoom on Figure 7. Note that estimated bottomhole pressure has been shifted to facilitate comparison with measurement.

Fig. 8. A comparison of Figure 6 and 7.

[5] G. Nygaard, L. Imsland, and E. Johannessen, "Using NMPC based on a low-order model for controlling pressure during oil well drilling," in *Proceedings of 8th International Symposium on Dynamics and Control of Process Systems (DYCOPS)*, Cancun, Mexico, 2007.

[6] A. Pavlov, G.-O. Kaasa, and L. Imsland, "Experimental disturbance rejection on a full-scale drilling rig," in *Proc. of IFAC NOLCOS 2010, Bologna, Italy*, 2010.

[7] Ø. N. Stamnes, G.-O. Kaasa, and O. M. Aamo, "Adaptive estimation of downhole pressure for managed pressure drilling operations," in *Proc. of IEEE Multi-Conference on Systems and Control*, 2011.