

Electricity Capacity Expansion in a Cournot Duopoly

Helene K. Brøndbo, Axel Storebø, Stein-Erik Fleten, Trine K. Boomsma

Abstract—This paper adopts a real options approach to analyze marginal investments in power markets with heterogeneous technologies and time-varying demand. We compare the investment behavior of two firms in a Cournot duopoly to a central planner's when two categories of power plants are available; base and peak load power plants. We find that producers exercise market power and the prices increase. Furthermore, the peak load plants become relatively more valuable and the share of installed peak load capacity exceeds the peak load share in a perfectly competitive market. In a numerical example, we show that this results in welfare losses above 10 %, and significantly larger reduction in the consumer surplus. Further, we examine the effect of analyzing power markets without time-varying demand and find that this underestimates investments in peak load capacity.

Keywords—Capacity expansion, duopoly, real options, social welfare.

I. INTRODUCTION

Expansion of capacity in power systems is on the agenda, both in developing countries, where demand is growing, and in industrialized countries, where concerns about climate change is a driving force. Following the deregulation of European power markets in the last decades, the authorities' focus on maximizing social welfare has been replaced by the companies' aim to maximize their profits. Several mergers and acquisitions have resulted in markets with few suppliers having significant market shares. Despite the clear need for a better understanding of capacity expansions in power markets with actors in possession of market power, there is a limited amount of academic research addressing this.

Investments in power equipment are capital intensive, and the equipment is difficult to sell once it is installed, particularly when considering the whole industry at the same time. Hence, we assume investments to be irreversible. Furthermore, capacity expansions are rarely now-or-never decisions. The investment can be delayed until the company has more information about the uncertain demand. These assumptions suit real options problems well. Treating capacity expansions in the power sector with a real options approach provides flexibility to the investor because it takes the value of waiting into account while the investment is considered irreversible.

We take as a starting point the set-up of [1]. This paper introduces a real options capacity expansion model for power generation under perfect competition. The combination of real options and a social welfare perspective is also found in [3] and [4]. The framework of [1] includes heterogeneous technologies, and power is treated as a differentiated product by dividing the year into load segments, where the power demand is different in each segment. On this basis, we develop a real options capacity expansion problem under a Cournot duopoly. This makes us able to compute social welfare losses

in settings with market power relative to a market governed by a central planner.

This paper aims to contribute to the literature by implementing the particularities described below. A number of articles consider capacity expansion by real options, e.g. [2], [13], [14] and [15]. However, these approaches consider only one technology or heterogeneous technologies. We study capacity expansions for electricity technologies that may differ in both operational and investment costs. For instance, peak load plants typically have higher operational costs but lower investment costs than base load plants.

We cast the capacity expansion problem as a canonical real options problem. Such models fit well into a stochastic setting while allowing for an endogenous electricity price when the level of capacity is held constant. Canonical real options models consider a sequence of marginal capacity expansions instead of a single. Furthermore, the value of the capacity expansion and the optimal expansion path are determined simultaneously. Canonical real options theory is mainly used in markets with monopoly and perfect competition due to assumptions about homogeneous companies and symmetric technologies. By assuming myopia, however, we apply it to a diverse portfolio of technologies.

Myopia implies that each investment in incremental capacity is the last one over the time horizon, and holds for electricity capacity expansions with one technology or several technologies with identical cost characteristics. Although myopia does not necessarily hold for our capacity expansion problem, we use this as an assumption to facilitate a solution. We argue that myopia is an acceptable approximation because of the way profit maximizing firms act. In deciding whether the next investment is attractive, this is assumed to be the last one. As time passes and the electricity demand increases, a new investment might be undertaken, despite the earlier belief that the previous investment was the last one.

Electricity is treated as a differentiated product both between years and within each year. Pindyck [12] argues that long-term development of electricity prices follows a geometric Brownian motion. This view is supported in [9], [10] and [11] among others. Hence, we model the long-term fluctuations in electricity demand as a geometric Brownian motion. Additionally, our real options approach for electricity capacity expansion considers the fluctuations in short-term demand by dividing each year into a set of load segments, where the electricity demand differs between each segment. Short-term fluctuations of the electricity price in the load segment is modeled by an inverse demand function. This way, the electricity price depend on the dispatch in the market and is thus endogenous as argued in e.g. [5] and [13].

According to [5], [6], [7] and [8], power markets may be considered Cournot oligopolies. Hence, we use a partial equi-

librium model to describe the market structure in the capacity expansion model. Here, the firms extract market power and their investment decisions depend on actions of the competitors as well as economic variables. At each point in time, the firms decide their production and investments simultaneously with a view to long-term market shares and profitability. For simplicity, we assume a power market consisting of two firms in this paper.

The paper is structured as follows. Section 2 introduces a framework for capacity expansion in duopolies. Section 3 presents a numerical example and studies capacity investments, surpluses and welfare losses in a Cournot duopoly compared to a perfectly competitive market. The section also looks at the effect of modeling the power market with and without time-varying demand. Section 4 concludes.

II. CAPACITY EXPANSION MODEL

A. Instantaneous Profit of a Duopolistic Firm

We model the electricity demand shock process Y_t as a geometric Brownian motion

$$dY_t = \mu Y_t dt + \sigma Y_t dz_t, \quad (1)$$

in which μ is the deterministic drift, $\sigma > 0$ is the standard deviation and dz_t is the increment of a Wiener process. Electricity cannot be stored, and is thus a differentiated product. As a result, the electricity price is time-varying. This is modelled by dividing each year into $d(\mathbf{L})$ load segments with different electricity demand. Power generation depends on the load, which is the energy demand per unit of time. We use linear inverse demand functions $D_l(Q_l)$ to find the electricity prices, in which Q_l is the total dispatch in each load segment $l \in \mathbf{L}$.

Hence, we let the electricity price depend on both the inverse demand function $D_l(Q_l)$ and the exogenous and multiplicative shock process Y_t

$$P_l = Y_t D_l(Q_l) = Y_t (A_l - b_l Q_l), \quad \forall l \in \mathbf{L}. \quad (2)$$

We assume a Cournot duopoly consisting of two firms, Firm 1 and Firm 2. Firm 1 is in possession of power plants using technology 1, and Firm 2 is in possession power plants using technology 2. Due to Cournot assumptions, the investment approach is symmetric for both firms. Hence, we only show the investment approach of Firm 1. The generators available to Firm 1 have a capacity K_1 . The produced electricity by firm 1 in load segment l is $q_{1,l}$. Hence, K_1 is the maximal value of $q_{1,l}$. Operational and maintenance costs of technology 1 are given in terms of the installed capacity K_1 . Hence, OMC_1 is the operational and maintenance cost per unit of installed capacity of technology 1 for Firm 1. The unit production cost for each technology 1 is denoted c_1 . The cost of investing in one additional capacity unit of technology 1 is denoted I_1 . We assume that the cost occurs instantaneously after an investment decision and that the additional capacity is available immediately after the investment. The revenues in load segment l are given as the product of the price function in (2) and the amount of sold electricity by Firm 1 in each

load segment $l \in \mathbf{L}$. Thus, Firm 1 finds its instantaneous profit from the optimization problem

$$\pi_1(Y_t, K_1, K_2) = \max_{q_{1,l}} \sum_{l=1}^{d(\mathbf{L})} \tau_l \left[P_l(Y_t, Q_l) q_{1,l} - c_1 q_{1,l} \right] - OMC_1 K_1 \quad (3)$$

s.t.

$$q_{1,l} \geq 0, \quad \forall l \in \mathbf{L} \quad (4)$$

$$q_{1,l} \leq K_1, \quad \forall l \in \mathbf{L} \quad (5)$$

$$Q_l = q_{1,l} + q_{2,l}, \quad \forall l \in \mathbf{L} \quad (6)$$

where τ_l is the duration of load segment l . (4) and (5) constrain the electricity produced by firm 1 $q_{1,l}$ not to exceed its upper limit K_1 or fall below its lower limit 0. (6) states that in each load segment the total dispatch Q_l is the sum of the dispatches of Firm 1 and Firm 2. Due to a downward sloping inverse demand curve, (3) is concave. Combined with linear constraints makes the problem convex. Thus, the problem can easily be solved numerically and has a solution.

The inverse demand function $D_l(Q_l)$ proves the profits from each technology 1 and 2 to be non-additively separable. When the dispatch $q_{1,l}$ increases, the inverse demand function $D_l(Q_l)$ decreases. By holding Y_t fixed, a larger dispatch results in reduced electricity prices. Holding the inverse demand function $D_l(Q_l)$ fixed, an increase in Y_t results in a larger instantaneous profit $\pi_1(Y_t, K_1, K_2)$. Changes in K_1 and K_2 also effect the profit flow $\pi_1(Y_t, K_1, K_2)$ through setting an upper limit on the electricity generation.

We find the welfare losses in the duopoly by comparing the welfare in the duopoly with the welfare under perfect competition. Social welfare is the sum of the producer and the consumer surplus, $\psi(Y_t, K_1, K_2) = \pi(Y_t, K_1, K_2) + cs(Y_t, q_{1,l}, q_{2,l})$. The producer surplus equals the profit of the producers $\pi(Y_t, K_1, K_2) = \pi_1(Y_t, K_1, K_2) + \pi_2(Y_t, K_1, K_2)$. The consumer surplus cs is given by

$$cs(Y_t, K_1, K_2) = \sum_{l=1}^{d(\mathbf{L})} \tau_l \left\{ \int_0^{Q_l} P_l(Y_t, x_l) dx_l - P_l(Y_t, Q_l)(Q_l) \right\}. \quad (7)$$

Thus, social welfare is given by

$$\psi(Y_t, K_1, K_2) = \sum_{l=1}^{d(\mathbf{L})} \tau_l \left\{ \int_0^{Q_l} P_l(Y_t, x_l) dx_l - \sum_{k \in \mathbf{K}} c_k q_k \right\} - \sum_{k \in \mathbf{K}} OMC_k K_k. \quad (8)$$

The social welfare is maximized when solving (3)-(6) using (8) as the objective function.

B. Value of Capacity Expansion

Investments are assumed irreversible and incremental over an infinite time horizon. Future cash flows are discounted with the exogenous annual rate ρ . In year zero, the demand shock is Y_0 and the installed capacity of Firm 1 is $K_{1,0}$. For every demand shock in each time interval Y_t , Firm 1 expands its capacity to $K_{1,t}$ at the per unit investment cost I_1 that maximizes its expected value. This implies that at each point in time, Firm 1 adapts its capacity to the demand. $F_1(Y, K_1, K_2)$ represents the value of all optimal capacity expansions of Firm 1. When Firm 1 has no other assets except from its generation capacity, $F_1(Y, K_1, K_2)$ is equivalent to the value of Firm 1. $K_{1,t}$ is the installed capacity of technology 1 at time t so that $K_{1,t} \leq K_{1,t+dt}$. Thus, the value of capacity expansion is

$$F_1(Y_t, K_1, K_2) = \max_{K_{1,t}} \mathbf{E} \left[\int_0^\infty \pi_1(Y_t, K_{1,t}, K_{2,t}) e^{-\rho t} dt - \int_0^\infty I_1 e^{-\rho t} dK_{1,t} \right]. \quad (9)$$

Firm 1 invests in new capacity to maximize its expected value over an infinite time horizon. The first term on the right-hand side of the equality represents all future expected discounted profits of Firm 1. The second term on the right-hand side is the total expected discounted investment costs from capacity investments. Hence, we integrate over every point in time t to find the value of installed capacities $F_1(Y, K_1, K_2)$.

C. Optimal Stopping Problem

If the properties of myopia hold, the stochastic control problem can be converted to an optimal stopping problem. We propose a regression $\bar{\pi}_1$ in (10) to express the instantaneous profit analytically as a function of Y_t , K_1 and K_2 . This particular expression is chosen for the real options problem to have an analytical solution.

$$\bar{\pi}_1(Y, K_1, K_2) = \sum_{i,j=1}^{d(\gamma), d(\alpha)} b_{1,ij} Y^{\gamma_i} K_1^{\alpha_j} + \sum_{i,j,l=1}^{d(\gamma), d(\lambda), d(\lambda)} c_{12,ijl}(\gamma_i) Y^{\gamma_i} K_1^{\lambda_j} K_2^{\lambda_l} - OMC_1 K_1. \quad (10)$$

The first term of the regression shows the profit flow from technology 1. The regression coefficients $b_{1,ij}$ describe how changes in the capacity of technology 1 effect the instantaneous profit flow for a given shock process Y_t . Since new installed capacity has a positive effect on the profits, $b_{k,ij} \geq 0$. Both synergies between technologies and the impact of the other firm's capacity are captured in the regression coefficients $c_{12,ijl}$. The coefficients are positive if the technology synergies outweigh the lower price caused by the other players installed capacity, and negative otherwise. Negative coefficients may cause several roots of (18). γ , α and λ are positive base vectors of dimensions $d(\gamma)$, $d(\alpha)$ and $d(\lambda)$ used to describe changes in $\bar{\pi}(Y, K_1, K_2)$ with respect to Y , K_1 and K_2 . We

constrain the base vectors in the regression like [1]. We set γ_i by $0 < \gamma_i < \beta_1 \forall i$, where β_1 represents the positive solution of the fundamental quadratic equation $\beta_1 = (\frac{1}{2} - \frac{\mu}{\sigma^2}) + \sqrt{(\frac{1}{2} - \frac{\mu}{\sigma^2})^2 + \frac{2\rho}{\sigma^2}}$. This is done to increase the likelihood of getting a unique investment trigger. To ensure concavity and non-increasing return to scale, we establish $0 < \alpha_j < 1, \forall j$, $0 < \lambda_j < 1 \forall j$ and $\lambda_i + \lambda_j \leq 1$ when $i \neq j$.

We convert the stochastic control problem to an optimal stopping problem and introduce the convenience yield $\delta = \rho - \mu$ to simplify. The convenience yield of electricity is interpreted as the relative benefit of delivering the commodity earlier rather than later, according to [16]. Then the Bellman equation of a marginal capacity of Firm 1 is stated

$$\frac{1}{2} \sigma^2 Y^2 \frac{\partial^3 F_1(Y, K_1, K_2)}{\partial K_1 \partial Y^2} + (\rho - \delta) Y_1 \frac{\partial^2 F_1(Y, K_1, K_2)}{\partial K_1 \partial Y_1} - \rho \frac{\partial F_1(Y, K_1, K_2)}{\partial K_1} + \frac{\partial \bar{\pi}_1(Y, K_1, K_2)}{\partial K_1} = 0, \quad (11)$$

with boundary conditions

$$\frac{\partial F_1(0, K_1, K_2)}{\partial K_1} = 0, \quad (12)$$

$$\frac{\partial F_1(Y_1^*, K_1, K_2)}{\partial K_1} = I_1, \quad (13)$$

$$\frac{\partial^2 F_1(Y_1^*, K_1, K_2)}{\partial K_1 \partial Y_1} = 0. \quad (14)$$

(12) ensures that the value of Firm 1's value of the option to invest in new capacity is zero when the demand shock equals zero. (13) and (14) are respectively the value matching and the smooth pasting conditions for an incremental investment in new capacity. The solution to 11 is given by

$$F_1(Y, K_1, K_2) = A_1 Y^{\beta_1} + \sum_{i,j=1}^{d(\gamma), d(\alpha)} \bar{b}_{1,ij} Y^{\gamma_i} K_1^{\alpha_j} + \sum_{i,j,l=1}^{d(\gamma), d(\lambda), d(\lambda)} \bar{c}_{12,ijl}(\gamma_i) Y^{\gamma_i} K_1^{\lambda_j} K_2^{\lambda_l} - \frac{OMC_1 K_1}{\rho}. \quad (15)$$

where β_1 is given by the positive root of the quadratic equation and $\bar{b}_{1,ij}$ and $\bar{c}_{12,ijl}$ are given by

$$\bar{b}_{1,ij}(\gamma) = \frac{b_{1,ij}}{\rho - \mu \gamma_i - \frac{1}{2} \sigma^2 + \gamma_i(\gamma_i - 1)}, \quad (16)$$

$$\bar{c}_{12,ijl}(\gamma) = \frac{c_{12,ijl}}{\rho - \mu \gamma_i - \frac{1}{2} \sigma^2 + \gamma_i(\gamma_i - 1)}. \quad (17)$$

By using (12)-(14), the myopic investment trigger for Firm 1 is therefore the solution of (18) with respect to Y_1^*

$$\sum_{i=1}^{d(\gamma)} Y_1^* \gamma_i \left(\frac{\beta_1 - \gamma_i}{\beta_1} \right) \left\{ \sum_{j=1}^{d(\alpha)} \alpha_j \bar{b}_{1,ij}(\gamma_i) K_1^{\alpha_j - 1} + \sum_{u=1, u \neq k}^{d(K)} \sum_{j,k=1}^{d(\lambda), d(\lambda)} \bar{c}_{12,ijl}(\gamma_i) K_2^{\lambda_j} K_1^{\lambda_l - 1} \right\} = I_1 + \frac{OMC_1}{\rho}. \quad (18)$$

It is optimal to invest when $Y_t > Y_1^*$, and Firm 1 thus invests until Y_1^* reaches Y_t at each point in time. The identical procedure is completed for Firm 2. Firm 2 finds its trigger Y_2^* and invests until Y_2^* reaches Y_t in every time step. We emphasize that represents a simultaneous Cournot duopoly capacity expansion game where two firms are investing in new capacity in order to maximize their value over an infinite time horizon.

III. RESULTS

A. Base Case

We demonstrate our approach by presenting an illustrative example. We examine a Cournot duopoly where one firm is in possession of base load power plants and the other firm is in possession of peak load power plants. The demand is split into six load segments such that $d(\mathbf{L}) = 6$. All parameters are defined in Appendix A, table II to IV. We compare the duopoly to a market governed by a central planner in possession of both base and peak load power plants. Thus, we investigate the welfare effect of imperfect competition.

TABLE I. SOCIAL WELFARE

	Perfect competition	Duopoly
Discounted social welfare [M€]	282 450	193 410
Discounted producer surplus [M€]	121 940	128 130
Discounted consumer surplus [M€]	160 500	65 587
Value of the firm, F [M€]	-	$F_{base} = 99\ 860$ $F_{peak} = 21\ 958$
Discounted social welfare net of investment costs [M€]	209 220	184 930
Percentage loss in discounted social welfare subtracted for investment costs	-	11.6 %

Table I presents the discounted total surplus, the discounted producer surplus and the discounted consumer surplus. As expected, the discounted social welfare and consumer surplus of the central planner exceeds that of the Cournot duopoly. This is due to central planner's aim to maximize the social welfare. Firms of the duopoly consider only the producer surplus when deciding to invest and consequently have significantly larger producer surpluses than the central planner. The firms hold back capacity to increase prices. Hence, the social planner invests in more capacity.

The central planner has flexibility to invest in both base and peak load capacity as well as to choose the amount of electricity generated by each technology. The duopolistic firms, on the other hand, have to invest in and generate electricity by

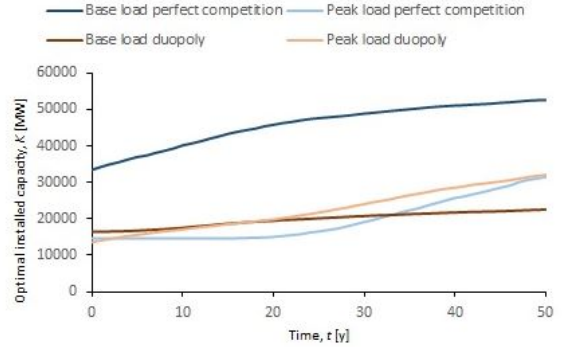


Fig. 1. Optimal investments in base and peak load capacity

the one technology that is available to them. Additionally, both of the duopolistic firms aim to maximize their own profit. As illustrated in Figure 1, the duopolistic firm possessing peak load capacity has a higher rate of investment than the firm possessing base load capacity. In spite of this, the value of the firm in the position of base load capacity exceeds the value of the firm in the position of peak load capacity. This is a result of the lower operational costs provided by the base load capacity which leads to a contribution margin that outweighs the lower investment cost provided by the peak load capacity.

When subtracting the discounted investment costs from the discounted social surplus, the difference between the discounted social welfare in perfect competition and a duopoly is reduced. This indicates high investment costs to be a major investment barrier for firms operating under market power. The last line in Table I presents the percentage losses in total discounted surplus adjusted for investment costs. The welfare loss is 11.6 % in the duopoly. Although significant, the social losses are modest compared to the losses for the consumers of 59.1 %. This demonstrates that consumers are the ones who suffer from producers exercising market power.

B. Capacity Expansions and Time-Varying Demand

In actual power markets, the demand varies over the year. Traditional real options models do not capture this. To quantify the effect of a modeling different load segments, we compare peak load investments when the electricity demand is time-varying and when it is fixed throughout the year. We use weighted averages of the variables in Table IV to compute new values of A and b for the new load segments in the inverse demand function, presented in Table V.

Figure 2 illustrates the peak load investments with 1, 2 and 6 load segments. We observe that peak load investments increase with the number of load segments. An economical interpretation is as follows. In periods with high demand, the high electricity price results in a high contribution margin on peak load generation. Due to the minor investment costs on peak load capacity, it is sufficient with short periods of high demand for peak load to be profitable. This effect is not captured when demand is assumed constant throughout the year, i.e. when $d(\mathbf{L}) = 1$. When each year contains

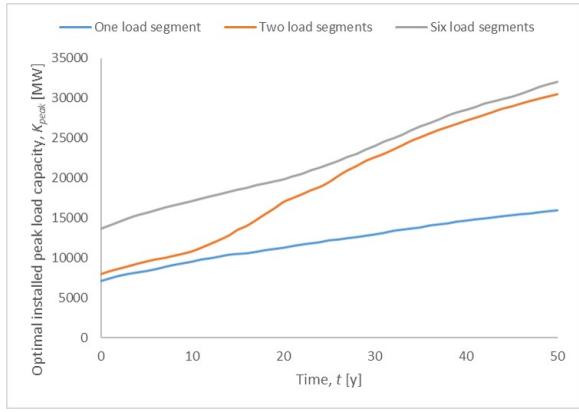


Fig. 2. Investments in peak load capacity with one, two and six load segments

2 load segments, the effect of the peak load contribution margin is captured to some extent. We witness additional peak load investments compared to when it is 1 load segment, but the peak load investments are not as high as with 6 load segments and take some time to catch up. Notice that the firms always invest in additional peak load capacity due to a positive contribution margin.

IV. CONCLUSION

We have adopted a real options approach to analyze marginal investments in peak and base load generation capacity. We study capacity expansion within a Cournot duopoly and a market governed by a central planner, and we compare optimal capacity installations for base and peak load power plants. Our approach considers several features of the real world power markets, including heterogeneous technologies, endogenous electricity prices, time-varying electricity demand, and markets with imperfect competition. We find that with imperfect competition the installed capacity increases with the number of firms in the market. In particular, imperfect competition may boost peak load investments at the expense of a loss in social welfare, explained mainly by a substantial loss in consumer surplus. We also observe that fluctuations in the electricity demand over the year enhance peak load investments.

Our capacity expansion framework may provide decision support to both policymakers and private investors. It is important for policymakers to ensure a certain capacity and flexibility to cover the electricity demand and a certain number of firms in order to avoid market power. We show how increased competition leads to a higher installed capacity and lower electricity prices, which result in smaller welfare losses.

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APPENDIX A INPUT DATA FOR EXAMPLE

TABLE II. POWER PLANT PROPERTIES

Technology index, k	Base	Peak
Marginal cost, c [€/MWh]	5	65
O. & M. cost, OMC [€/MWy]	100 000	20 000
Investment cost, I [€/MW]	3 000 000	80 000
Initial capacities $K_{t=0}$ [MW]	15 000	5 000

TABLE III. INPUT PARAMETERS

Y_0	μ	σ	ρ	β_1	N	$d(\Omega)$	T	Δt	$\Delta \kappa$
1	0.02	0.03	0.1	4.62	50	50	50y	1y	500 MW

TABLE IV. DEMAND DATA

Load Segment l	1	2	3	4	5	6
Duration, τ [h]	10	40	310	4400	3000	1000
Max. demand, A_l	900	180	165	120	90	60
Slope, b_l	0.007	0.0014	0.0014	0.0014	0.0015	0.0020

TABLE V. DEMAND DATA, ONE AND TWO LOAD SEGMENTS

Load Segment l	One Load segment		Two load segments	
	1	2	1	2
Duration, τ [h]	8760	360	8400	
Max. demand, A_l	105.63	187.08	1102.14	
Slope, b_l	0.00151	0.00156	0.00151	

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