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Evaluation of static hedging strategies for hydropower producers in the Nordic market

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In this paper we develop an optimization model to derive static hedge positions for hydropower producers with different risk characteristics. Previous research has primarily considered dynamic hedging; however, static hedging is the common choice among hydropower producers because of its simplicity. Our contribution is to evaluate such hedging out of sample. The hedging strategies we analyze include a natural hedge, which means no hedging, and output from an optimization model that we develop ourselves. The results show that, although optimized positions vary over time, hedging with use of forward contracts significantly reduces the risk in terms of value-at-risk, conditional value-at-risk and standard deviation of the revenue. Furthermore, this improvement results in only a minor reduction in mean revenue.

1 INTRODUCTION

The liberalization of the Nordic power market in the early 1990s dramatically changed the competitive environment for hydropower producers. Before the liberalization, the

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electricity price was regulated by the governments. Consequently, producers did not have any incentives to hedge the electricity price. However, after deregulation, control of the electricity price was removed and, as a result, price variation has increased (Knittel and Roberts (2005)). This has led to the development of a market for electricity derivatives. As a result, Nord Pool,[1] the power exchange for the Nordic countries, was established in 1993. At Nord Pool, standardized derivatives, such as forwards/futures and options, are traded and provide a way for producers to manage and handle their risk exposure to the electricity price. However, the task of managing the risk with respect to the electricity price is not an easy one. As mentioned above, the electricity price is highly volatile and may have spikes of several orders of magnitude within a short time. This is mainly caused by the fact that there are very limited storage options for electricity. Hydropower producers can, to some extent, store energy indirectly in water reservoirs. However, consumers cannot buy electricity for storage. This implies that the cost-of-carry relationship between spot prices and forward prices breaks down. In other words, the relationship between the spot price and the forward prices is weaker than for other commodities. The electricity price therefore also experiences strong seasonality.

Over the last decade there has been increasing interest, both among practitioners and in academia, in risk management for electricity producers. These producers have had to adapt to the new environment that the abovementioned liberalization has caused and, in one way or another, to employ methods that aim to manage the new risk exposures. For a hydropower producer, the electricity price and the inflow (the volume of water that flows into the reservoirs) are the two most significant determinants of revenue. As both price and inflow experience large variation, they are also the two most important risk factors for future revenue. Previous research has primarily considered dynamic hedging strategies. Fleten *et al* (2002) use stochastic programming to find the optimal integrated production schedule and financial hedging plan for a hydropower producer. Kettunen *et al* (2010) use a similar approach but take the production plan as given and focus on finding the optimal financial hedging plan. Less dynamic, but not quite static, is the two-stage stochastic programming approach, as explained in Conejo *et al* (2008). On the other hand, Näsäkkälä and Keppo (2005) use a static hedging strategy with forward contracts. This strategy is derived by minimizing the variance of the portfolio at the horizon, ie, it is assumed that the risk-adjusted expected value of the portfolio is maximized when the portfolio variance is minimized. Mean-variance approaches to energy portfolios began with Haurie *et al* (1992). Oum *et al* (2006) use the framework of Brown and Toft (2002) to derive optimal static hedging functions for electricity companies facing both quantity and price uncertainty. Woo *et al* (2004) and Huisman *et al* (2007) devise models for static hedging in forward contracts for a

[1] URL: www.nordpool.com.

retailer or end user of electricity. In this paper we will present an optimization model for deriving static hedging strategies. However, instead of minimizing the portfolio variance, the hedge positions are derived by maximizing the expected revenue subject to constraints on the portfolio variance and value-at-risk (VaR). The static strategies will be evaluated and compared with a benchmark: the natural hedging strategy. This strategy, which basically means no hedging of the electricity price, benefits from the fact that the inflow and price are negatively correlated and thereby inherently provide a "natural hedging" of the revenue. The static hedging strategies can be explained in simple terms as using forward/future contracts to sell some percentage of the expected future production. These strategies can, of course, include options and other derivatives, but they are static in the sense that the positions are not changed as new market information becomes available. The natural hedging strategy and the static strategies will be evaluated by empirical tests on historical data and on predicted price and production scenarios. The tests aim to show which of the two approaches yields the best result from the point of view of a typical hydropower producer in the Norwegian market.

The paper is structured as follows. In Section 2 we will discuss the purpose and goal of risk management from the point of view of a hydropower producer. In Section 3 we present the risk measures that will be used to evaluate the hedging strategies. In Section 4 we present and discuss the natural hedging strategy and the static hedging strategies. In Section 5 we present and discuss the results from the empirical tests. Section 6 gives our conclusions.

2 RISK MANAGEMENT FOR HYDROPOWER PRODUCERS

In this section we will discuss different considerations that have to be taken into account when managing risk. This will be from the point of view of a hydropower producer. To start with, we need a definition of risk management. According to Krapels (2000), risk management can be defined as the control and limitation of the risks faced by an organization due to its exposure to changes in financial and commodity markets.

In order to employ risk management properly, this means that an organization first has to identify the risk factors that it faces and what the exposure to these risk factors is. When the risks are identified and the amount of exposure the organization has to each of them is measured, there is a need to prioritize and to decide how the risks should be handled and controlled. Depending on the organization's goal and its attitude towards risk, some risks should be eliminated, some should be limited and some should be left as they are or increased. It is important to note that risk management does not imply that all risks should be eliminated, because without any risk exposure the return will be limited. However, the key to proper risk management is to be aware of all the risks that the organization faces and to continuously measure, control and handle them in a way that is consistent with the organization's goal and risk attitude.

Section 2.1 briefly presents the price and inflow risk, Section 2.2 considers the purpose of risk management and risk premiums and Section 2.3 presents a historical analysis of the electricity price and of a hydropower producer's inflow (production volume).

2.1 Review of the risk aspects for a hydropower producer

As mentioned above, the first step when doing risk management is to identify the risks that the organization faces and to evaluate the level of exposure to these risks. In this section we will present the electricity price risk and inflow risk, which are the two most important risks that a hydropower producer faces (Kristiansen (2006)).

Price risk is risk that stems from changes in the value of spot positions due to changes in the electricity spot price. For instance, if a producer has decided to sell 50% of this year's production on the spot market, the value of that 50% will change as the electricity spot price changes. The electricity spot price has high volatility (Benth *et al* (2008)) and will therefore have significant impact on the value of the production. The price risk is therefore one of the most important risks for a hydropower producer.

Inflow risk is risk that stems from the fact that precipitation and inflow to the water reservoirs may vary a lot from year to year. Because the production volume depends on the inflow to the reservoirs, this variation consequently causes variation and uncertainty in the future revenue. We consider inflow risk to be the same as uncertainty in production volume.

2.2 Purpose of risk management and risk premium

An important consideration when deciding how risk is to be managed is the hydropower producer's attitude towards risk. For a risk-averse producer that wants good predictability of future revenue and needs to ensure that the revenue will be higher than a certain level, a risk management program with extensive use of hedging is suitable. On the other hand, for a less risk-averse producer that can handle a greater standard deviation in the revenue and is able to survive a period with unusually low prices and/or production, a risk management program with less hedging is needed.

Another important issue when considering hedging strategies is the risk premium that is embedded into the derivatives. At first, one might think that a producer should have to pay a premium (which reduces the expected revenue) if derivatives are used to reduce the variability and downside risk of future revenue. However, in its most general form, this argument could also be applied to the consumer side and one would get the opposite result, because the derivatives payout is a zero-sum game. As a consequence, deciding what the risk premium should be is not easy. Some previous research has been done on this topic. For instance, Bolinger *et al* (2002) show that natural gas swap prices in the US have a negative risk premium, which means that the swap prices are an overestimate of their corresponding spot prices. Bessembinder and Lemmon (2002) find that electricity prices in the US have a negative risk premium if expected demand is low and demand variance is moderate, and a positive risk premium when expected demand is high and demand variance is high. Geman and Vasicek (2001) find evidence of a negative risk premium in the Pennsylvania–New Jersey–Maryland electricity market for forward contracts with a short time to delivery. For contracts with long time to delivery, the risk premium becomes positive. These results are supported by Longstaff and Wang (2004), who find evidence of a significant negative risk premium for contracts with a short time to delivery. Benth *et al* (2008) also find evidence of a negative risk premium at Nord Pool for contracts with a short time to delivery and a positive risk premium for contracts with a long time to delivery. Furió and Meneu (2010) analyze the Spanish power market and find the presence of a negative risk premium. This is a result of higher flexibility on the supply side that leaves the demand side with higher incentives of hedging under normal market conditions.

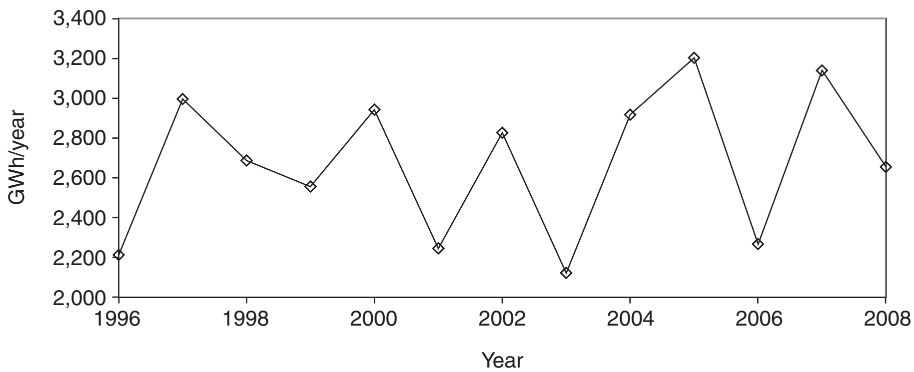
Krapels (2000) suggests that the positive skewness, due to spikes, in the electricity price may lead to a negative risk premium. Generally, price spikes give the consumer an incentive to pay a premium for hedging the price, while the producer wants to receive a premium because it will not benefit from the spikes if the price is hedged. Krapels (2000) supports this with an anecdote about pricing of electricity options:

> It is common knowledge, however, that traders in many [over-the-counter] electricity options markets have become so fearful of being physically "net short" (having agreed to deliver electricity in the future at an earlier agreed-upon price) when one of the price spikes occurs that they place extremely high standard deviation assumptions into the pricing of OTC electricity call options.

2.3 View of historical price risk and inflow risk

In this section we will discuss the properties of the historical data on the spot price and the production volume. The distributions of the historical data can be computed analytically, by estimating them with a certain probability distribution, or they can be estimated empirically. We have chosen to focus on the last method. We will discuss the statistical properties and show the empirical distributions in order to give a quantitative overview of the two main risks that the hydropower producer faces. For production volume, the inflow is assumed to be the only varying factor, which mainly depends on the weather. It should therefore repeat itself, and historical data should consequently be representative of the future. The same can be true for prices if the circumstances are believed to be stable. However, as the historical data is only based on past events,

FIGURE 1 The historical annual production for the hydropower producer.



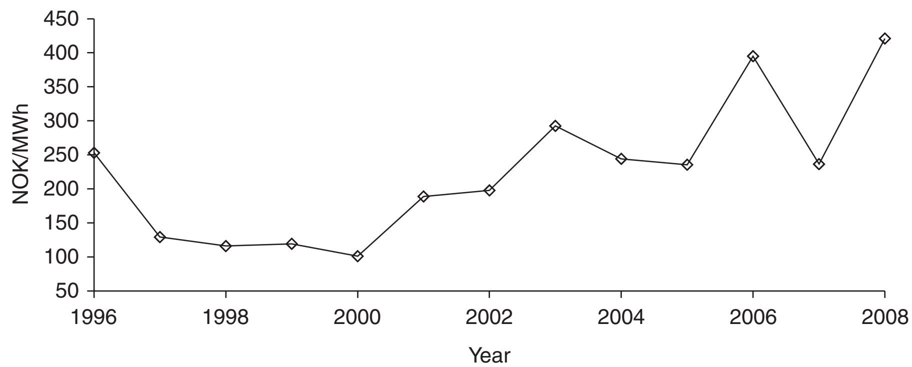
The average and standard deviation are 2,674 and 369 gigawatt-hours per year, respectively.

they will lack events that are yet to be seen. It should also be noted for the historical data that the statistical measurements are only calculated based on 13 observations. Lack of data may therefore be a source of noise. In the application of our optimization model, which is presented in Section 4.3, we will base our calculations on the price and production data presented in this section.

Figure 1 and Figure 2 on the facing page show the annual production for a Norwegian hydropower producer and the annual average spot price, respectively, in the period 1996–2008. The production is adjusted for reservoirs that were acquired during the period. Both the production volume and the spot price have a high standard deviation and are considered to be the two most important risk factors for a hydropower producer's future revenue. From Figure 2 on the facing page it can be seen that the price has followed a strong upward moving trend in the period 2000–2008. As this trend is unlikely to continue in the long run, it implies that the historical data may not be representative of the future, and may be considered as a special case.

Table 1 on the facing page shows statistics for the production volume and the spot price for the historical data. As we can see, there is a significant difference between the maximum and minimum values for both the production volume and the spot price. Furthermore, the standard deviation is high for the production volume, and particularly high for the spot price. It can also be seen in the table that the correlation between the spot price and production volume is negative. This gives a decrease in the standard deviation for the annual revenue and will be investigated further in Section 4.2.

FIGURE 2 The historical average annual spot price for the hydropower producer.



The average and standard deviation are 225 NOK/MWh and 101 NOK/MWh, respectively.

TABLE 1 Descriptive statistics on an annual basis for the historical data.

Statistics	Production (GWh/year)	Spot price (NOK/MWh)	Spot revenue (million NOK)
Mean	2,674	225	586
Standard deviation	369	101	237
Skewness	−0.2	0.7	0.5
Kurtosis	−1.4	−0.1	−0.5
Minimum	2,122	101	295
Maximum	3,202	421	1,064
Correlation	−0.33	—	—

3 RISK MEASUREMENT

In this section we will describe how the hydropower producer can measure its risk using standard statistical tools. These risk measurements will be used to reach an optimal hedging strategy with respect to the hydropower producer's risk aversion. The risk measurements are based on the end-of-year revenue and are conclusive and straightforward to interpret with regard to the risk profile that the hydropower producer is seeking. Note that the reliability of the risk measurements will depend on the reliability of the estimated revenue distribution. The risk measurements are calculated from empirical distributions because the profit from the electricity market is hard to model analytically. This stems from the fact that they consist of price spikes that will violate the normality assumptions, which are often used for stocks and other underlying assets. It should be noted that these distributions consist of market risk as well as the specific hydropower producer's risk. Using derivatives from Nord Pool will therefore only secure the market risk, while the specific business risk will still be present. By measuring the end revenue, both risks will be taken into account in our analysis. In Section 3.1 we will introduce the VaR technique and a modification of VaR, the conditional value-at-risk (CVaR), in preparation for a further explanation of the downside risk presented in Section 3.2. Additionally, cash flow at risk, as described by Guth and Sepetys (2001), could have been used. However, we have chosen not to use this, since cash flow at risk is just an alternative measure of VaR and will therefore provide no further information.

3.1 Value-at-risk

We have chosen a 10% VaR of the end-of-year revenue to define an acceptable threshold. The threshold is chosen by the hydropower producer, and the time horizon is based on what we believe the hydropower producer will have most benefit from focusing on, as it is coherent with the time period of most budgets and balance sheets. Additionally, we believe monthly and quarterly fluctuations will be of less importance than the end-of-year revenue, as the demand for liquidity on a shorter term will be of less importance than the liquidity on an annual term. As the 10% VaR sets the minimum possible value that the revenue can obtain in a 90% interval, we believe this value is of more interest for the hydropower producer and will stress this value in the testing. This value is crucial for defining a threshold limit, which, if violated, could lead to capital structure crisis and therefore higher debt yield. Even though VaR may be one of the most popular risk measurements, we believe it will be insufficient for our analysis. As the distribution of the revenue has a spiky behavior and the shape of the left tail may be thick and not monotonically decreasing, we believe the VaR should be evaluated in the context of other risk measurements as well. This is supported by Unger and Lüthi (2002). We will therefore investigate the VaR in combination with the CVaR as described in the next section.

3.2 Conditional value-at-risk

Conditional value-at-risk was proposed by Rockafellar and Uryasev (2002) as a measure that combines features from expected shortfall and VaR. It follows from its definition that CVaR will be at least as low as VaR. It will represent the expected value given that we are below the VaR limit. Therefore, when solving an optimization model of the revenue, with restrictions on VaR, the CVaR may not be optimal in the view of the hydropower producer. Even though VaR is a popular statistical measurement regarding the risk taken by companies, it could easily be misleading, especially in the case of heavy tails. Major shortfalls may be possible even though the 10% VaR shows a high value. The CVaR method allows us to further investigate the potential shortfalls by giving us an impression of the length of the downside tail. In other words, to get an impression of what happens if the revenue is known to be below VaR, we will use CVaR. We will also use it to compare two strategies with very similar VaR.

4 SUGGESTED RISK MANAGEMENT STRATEGIES

In this section we introduce the natural hedging strategy and present an optimization model for deriving static strategies. We will start by explaining the derivatives at Nord Pool, which are the cornerstones of the hedging strategies, in Section 4.1. We give an introduction to the natural hedging strategy in Section 4.2, then, finally, in Section 4.3 we describe the model that we use to derive the static hedging positions.

4.1 Securities market

There are four main types of derivatives available at Nord Pool: future contracts, forward contracts, options and contracts for difference. The future and forward contracts are different from traditional future and forward contracts in the financial markets. The main difference is that they have a delivery period. The underlying is delivered not at a fixed point in time, but over a period in which the payout of the contract is calculated as the hourly difference between the forward/future price and the spot price. In this sense, the Nord Pool future/forward contracts correspond to the textbook definition of swaps. The future contracts are marked-to-market each day prior to the delivery period. In the delivery period the payout is calculated as the difference between the spot price and the future price on the last trading day. The future contracts have either a daily or a weekly delivery period. At any time there are between 1 and 7 daily future contracts and 6 weekly future contracts available. The forward contracts are settled in the same manner as the future contracts, albeit without the mark-to-market settlement prior to the delivery period. Forward contracts are available with monthly, quarterly and annual delivery periods. At any one time there are 6 monthly and 5 annual contracts available. The number of quarterly contracts will be between 8 and 11 contracts, reaching ahead two years from the current year. The liquidity is high for both future and forward contracts, except for the daily contracts and the annual contracts with three, four and five years to delivery.

European-style call and put options with quarterly or annual forward contracts as underlying are also available. However, the liquidity of these contracts is low. Ideally, option contracts would provide very efficient hedging strategies (see, for example, Krapels (2000)), but the low liquidity makes it hard to use them for risk management purposes in practice and would result in high transaction costs because of the bid–ask spreads.

The fourth main type of contract on Nord Pool is contracts for difference. These contracts are made for hedging the difference between the system price and the local-area price. The forward and future contracts are settled against the system price while the hydropower producer gets the local-area price when selling the electricity generated. This local-area price is only equal to the system price when there is no congestion on the transmission grid. However, in reality, there is often a difference between the system price and the local-area price. Therefore, the forward/future contracts will not eliminate all the price risk as in the case of a perfect hedge. Contracts for difference can be used to eliminate this difference and, if used in combination with the forward/future contracts, a perfect hedge of the price is achievable. However, the liquidity is low for these contracts as well.

As a result of liquidity and time horizon we will use quarterly and annual forward contracts in our further analysis. This is also the common choice for risk management among hydropower producers in the Nordic market. We have left out call and put options in the electricity market, because of low liquidity, and derivatives for commodities in related markets as a consequence of low correlation with the Norwegian electricity market (see, for example, Gjølberg (2001)).

4.2 Natural hedging strategy

The natural hedging strategy can be seen as the maximum degree of risk that the hydropower producer is able to undertake, under the assumption that it is not speculating. This is a result of the fact that a natural hedge is the same as not hedging at all. The strategy leads to the highest uncertainty in future revenue and the highest possible shortfalls, but also the highest upside potential. The strategy will therefore be best suited for producers with a low degree of risk aversion. We saw in Figure 1 on page 6 and Figure 2 on page 7 that prices and volume are both very volatile, but negative correlation between them may lead to an acceptable standard deviation for the future revenue. A negative correlation should be stable and significant in order for this to be true.

The main reason for the negative correlation between price and hydropower production in the Norwegian market is that the market is regional, and 99% of the electricity production comes from hydropower. For hydropower production, the most important factor for the production volume is the inflow to the reservoirs, which again depends on precipitation. Because local precipitation is correlated with national precipitation, water shortage is often national and not just local. Additionally, most of the residential heating is powered by electricity. This means that when the temperature is low, the electricity demand will increase. However, when the temperature is low, there is more likely to be less precipitation and inflow. Consequently, when the demand is high, the supply and production volume is likely to be limited and the electricity price rises. In

TABLE 2 Correlation (annual granularity) between price and production volume in the respective time periods.

Period	Correlation	Period	Correlation
1996–1999	−0.78	1997–2008	−0.33
1996–2000	−**0.81**	1998–2008	−0.28
1996–2001	−**0.82**	1999–2008	−0.30
1996–2002	−0.67	2000–2008	−0.39
1996–2003	−**0.78**	2001–2008	−0.32
1996–2004	−**0.58**	2002–2008	−0.59
1996–2005	−0.36	2003–2008	−0.61
1996–2006	−0.47	2004–2008	−**0.86**
1996–2007	−0.40	2005–2008	−**0.89**
1996–2008	−0.33		

Values in bold are significantly different from zero based on t-tests.

years with high precipitation it is the other way around. Supply increases due to the high precipitation and demand decreases due to higher temperature. This again leads to a lower electricity price. To investigate this empirically, we have estimated the correlation in Table 2. The estimations are made on an annual basis to avoid seasonal effects, and measured during different time periods to investigate stability.

Table 2 shows that the correlation is high for most time intervals. t-tests on the significance show that 7 out of 19 coefficients are significantly different from zero. It is important to note that the robustness of the t-tests is limited due to the low number of data points. However, if, in addition to these results, we consider the fundamental properties of hydropower production (which intuitively imply a negative correlation), it is tempting to conclude that a negative correlation is present under normal market conditions.

The negative correlation will reduce the standard deviation of the revenue compared with the high standard deviation in price and volume, and is the basis for the natural hedging strategy. When investing in forward/future contracts this correlation effect will be lost, but since the price is locked for the given period, the standard deviation will only stem from the risk in volume. The standard deviation will therefore still be lower. The natural hedging strategy may, however, still be a good choice (depending on the risk aversion of the hydropower producer) as a result of one of the following: no transaction costs; no loss of revenue in means of hedging costs; utilization of negative correlation between price and inflow. However, it should also be noted that, even though correlation has been negative historically and there are reasons to believe that the correlation, under normal circumstances, will be negative in the future, there could be special events in the future that would cause both production volume and price to collapse: a crisis in the global economy for instance. An example is the current financial crisis, which has resulted in a reduction of both price and production. In this case, the natural hedging strategy will give no protection.

The natural hedging strategy can also be used as a benchmark for other hedging strategies. In our empirical tests we will therefore compare our static hedging strategies with the natural hedging strategy. This enables an evaluation of the hedging costs compared with the standard deviation and the shortfalls that the producer will have in the case of this no-hedging method.

4.3 Static hedging strategy

We will define a static hedge as a strategy in which the positions are fixed for a period of time according to a predetermined scheme. Consequently, the positions are not adjusted as new market information becomes available. A static hedging strategy can use all types of derivatives, and the strategy is defined by the proportions held by each of the derivatives and the derivatives' time horizons. Among producers in the Nordic market it is common to use contracts with quarterly and annual time horizons.

4.3.1 Model introduction

The two main goals of a static strategy are to reduce the standard deviation of future revenue for better decision and budgeting support, and to insure against major shortfalls. For a static strategy, the degree of standard deviation reduction and the protection against shortfalls are determined by the proportion of the production shorted on forward contracts and by the time horizon of these contracts. The main issue when designing a static hedging strategy, therefore, is what the proportions and time horizons should be in order to meet the hydropower producer's risk preferences. In order to determine the optimal proportions and time horizons of the contracts, we chose to develop an optimization model that determines (based on input on spot price, production volume and forward prices) what the best proportions and time horizons are. When the model finds the proportion in the form of weights of expected production, the purpose of the producer's risk management and the properties of their risk aversion are taken into account. Additionally, the model implicitly finds the optimal time horizons from the set of available forward contracts. Historical data can be used as input to determine what would have historically been the best strategy. One could also use predicted data for future years as input and, in this way, determine the best possible strategy for the future.

4.3.2 The model

The model aims to determine the optimal weights for a given set of forward contracts. This is done by maximizing profit, subject to a set of CVaR constraints and a set of trading constraints. Because hydropower production costs are constant with respect to the choice of hedging strategy, maximizing the revenue is equivalent to maximizing profit. The revenue consists of three parts. The first part is spot revenue; that is, revenue from sale of the production at spot price. This part is independent of the weights and can therefore be omitted from the problem formulation. The second and third parts are profit and loss from annual and quarterly forward contracts, respectively. The profit and loss over a given time period is calculated from the forward contracts that had delivery during that time period, ie, the contracts are not marked-to-market prior to delivery. This corresponds with how the payout of these contracts is settled at Nord Pool. The CVaR constraint is a measure of the producer's aversion to shortfalls. Additionally, we impose the trading constraints that ensure that only the producer can be short in the contracts, because long positions are considered as speculation (Kettunen *et al* (2010)). For the out-of-sample test in Section 5, where the strategy is a rolling intrinsic strategy, we have also implemented restrictions that ensure that previous trades of the forward contracts are taken into account.

4.3.3 Definitions

$\pi_{i,t}(s)$: profit from position in quarterly contract $i = Q$ and yearly contract $i = Y$ in scenario s for time period t.

$\pi_i(s)$: cumulative profit from position in quarterly contract $i = Q$ and yearly contract $i = Y$ in scenario s.

prob(s): probability of scenario s occurring.

$X_{i,t}$: weights of the short position that should be traded for quarterly and yearly contracts ($i = Q, Y$) for time period t.

$XP_{i,t}$: previously shorted positions, before optimization is done, in quarterly and yearly contracts ($i = Q, Y$) for time period t.

$P_{i,t}(s)$: spot price in quarter or year ($i = Q, Y$) for time period t in scenario s.

$F_{i,t}$: forward price for quarterly/yearly contracts ($i = Q, Y$), t time steps ahead.

$\mathrm{FP}_{i,t}$: previous forward prices for quarterly/yearly contracts ($i = Q, Y$), t time steps ahead.

N_i: number of quarterly contracts, yearly contracts or scenarios, where $i = Q, Y$ and S, respectively.

$\mathrm{EP}_{i,t}$: expected production for quarter or year ($i = Q, Y$) for time period t.

$\mathrm{CVaR}(\alpha)$: given $\mathrm{CVaR}(\alpha)$ limit for the cumulative revenue of a chosen future period.

CumRevYR1(s): cumulative revenue for the chosen future period in the CVaR limit.

$k(s)$, VaRinOpt, CVaRinOpt: support variables used for CVaR restriction.

4.3.4 Problem formulation

Maximize:

$$\sum_{s=1}^{N_s} \text{prob}(s)[\pi_Q(s) + \pi_Y(s)] + \text{CVaRinOpt} \cdot \varepsilon$$

subject to:

$$\pi_i(s) = \sum_{t=1}^{N_i} \pi_{i,t}(s), \quad i = Y, Q \tag{1}$$

$$\pi_{Q,t}(s) = [F_{Q,t} - P_{Q,t}(s)]X_{Q,t} + [\text{FP}_{Q,t} - P_{Q,t}(s)]XP_{Q,t} \quad \text{for all } t = 1, \dots, N_Q \tag{2}$$

$$\pi_{Y,0}(s) = [\text{FP}_{Y,0} - P(s)]XP_{Y,0} \tag{3}$$

$$\pi_{Y,t}(s) = [F_{Y,t} - P(s)]X_{Y,t} + [\text{FP}_{Y,t} - P(s)]XP_{Y,y} \quad \text{for all } t = 1, \dots, N_Y \tag{4}$$

$$0 \leqslant X_{i,t} + XP_{i,t} \leqslant \text{EP}_{i,t}, \quad i = Y, Q \text{ for all } t = 1, \dots, N_i \tag{5}$$

$$XP_0 + \sum_{t=1}^{N_Q-8} [X_{Q,t} + XP_{Q,t}] \leqslant \text{EP}_{Y,0} \tag{6}$$

$$X_{Y,1} + XP_{Y,1} + \sum_{t=1}^{4} [X_{Q,t+(NQ-8)} + XP_{Q,t+(NQ-8)}] \leqslant \text{EP}_{Y,1} \tag{7}$$

$$X_{Y,2} + XP_{Y,2} + \sum_{q=5}^{8} [X_{Q,t+(NQ-8)} + XP_{Q,t+(NQ-8)}] \leqslant \text{EP}_{Y,2} \tag{8}$$

$$\text{CVaRinOpt} \geqslant \text{CVaR}_\alpha \tag{9}$$

$$\text{CVaRinOpt} = \text{VaRinOpt} - \frac{1}{\alpha}\sum_{s=1}^{N_S} k(s) \tag{10}$$

$$k(s) \geqslant \text{Prob}(s)[\text{VaRinOpt} - \text{CumRevYR1}(s)] \tag{11}$$

$$k(s) \geqslant 0 \tag{12}$$

4.3.5 Explanation of the problem

The problem is to maximize expected cumulative profit. The constraints are as follows:

- Equation (1) calculates the profit from quarterly and yearly contracts for all the time that the contracts can be traded. This is done for each predicted scenario.
- Equations (2)–(4) calculate the profit from the quarterly and yearly contracts for each future time period in each scenario.
- Equation (5) shows that no total long position in the market is allowed for each contract and no total short position should exceed the expected production.
- Equations (6)–(8) are special restrictions for no total short position exceeding expected production. This is because the quarterly and yearly contracts are overlapping for the current year and the subsequent two years.
- Equations (9)–(12) restrict the CVaR revenue for the cumulative revenue of a given future time period to be not lower than a given CVaR_α, where α is the probability of occurring in the lower tail. The restrictions are set up as proposed by Rockafellar and Uryasev (2002).

Solving this model will return the weights Y_i and Q_i, which completely specify the static hedging strategy by denoting the amount that should be traded to find the short position in annual and quarterly forward contracts at any point in time from when the optimization is run. It should be noted that limitations to standard deviation, VaR and/or other risk measures can easily be incorporated into this model by adding additional restrictions.

4.3.6 Model evaluation

Because the model is an optimization model, it will return the strategy that gives the highest revenue, subject to the constraints and the input data. In this way the model can be used to determine a producer's hedging strategy once their risk aversion in terms of risk measurement restrictions is identified. However, it is important to note that the strategy is optimal with respect to the input data for spot price, production volume and forward prices. For instance, if the model is run on historical data, the model finds the strategy that has historically been the best. If one thinks that the historical data gives a good prediction, the strategy might be a good choice. However, if the future is expected to differ a lot from the past, using the best historical strategy could obviously lead to poor results. Running the model on historical data is therefore best as a performance measure, and the hydropower producer's current hedging strategy can be compared with the strategies from the optimization model, which can be considered as theoretical upper limits for the years under consideration. Running the model on predicted data will possibly give a strategy that is close to optimal for the future, as long as the predictions are accurate. However, as with historical data, using the strategy may lead to poor results if the predictions do not turn out to be accurate. Deriving a strategy based on predicted data and testing it on historical data may therefore be a good way of stress-testing the strategy and evaluating its robustness as long as the future has similar properties to the past.

Finally, it is important to be aware of what Smith and Winkler (2006) call the "optimizer's curse". This is a statistical phenomenon which states that when decision makers make a choice among different alternatives, they are in danger of overestimating the value of the chosen alternative. The chosen alternative is therefore likely not to be optimal. In our model this may lead to an upward bias in the performance of the chosen strategy. However, the fact that the different strategies are positively correlated with respect to the input parameters will reduce the problem of the optimizer's curse. The spot price may lead to some optimization bias if the estimation errors of the spot price in different time periods are not correlated, ie, if Q1 spot prices are overestimated while Q2 spot prices are underestimated, a bias towards Q2 contracts will occur. This effect will be reduced by diversifying the weights of the contracts.

5 EMPIRICAL TESTS

In this section we will investigate the performance of different static hedging strategies and the natural hedging strategy, and compare them based on the risk measurements. In Section 5.1 we give an evaluation of the data set. Section 5.2 gives an explanation of the different strategies, how their weights were derived and a brief overview of the test methods. The strategies' performances will be evaluated in Section 5.3 and, finally, in Section 5.4 we will compare the strategies with respect to the results from the historical and out-of-sample test in combination.

5.1 Data evaluation

Actual production data is collected from the hydropower producer, while actual data for the price is collected from Nord Pool. Because the forward prices at Nord Pool are denominated in euros (EUR), we convert them into Norwegian krone (NOK) by using the spot NOK/EUR exchange rate for the day that the forward price was collected, ie, we have ignored the interest parity of the exchange rate, as this is of minor importance. We have used the system price (in NOK) to calculate both the payout of the forward contracts and revenue from spot sale of production. In reality, when selling electricity, a producer does not get the system price but rather a local-area price, which might be different from the system price. However, if we use the local-area price for spot-revenue calculation, this difference will influence and reduce the generality of the test results.

The data used for the tests falls into two main categories. Firstly, historical data was collected for the period 1998–2008 and consists of weekly production volume, weekly spot prices and historical forward prices. Secondly, predicted scenarios that are used by a rolling intrinsic strategy for the period from January 2007 to April 2009.

The predicted data consists of 70 equiprobable scenarios, where each scenario consists of a weekly production level with a corresponding weekly price. The price predictions are made from the bottom-up multi-area power scheduling electricity sector model. This is an equilibrium model that is frequently used for price forecasting in Scandinavia. The model was developed by SINTEF Energy Research and is described in Botnen *et al* (1992) and Egeland *et al* (1982). The production scenarios are made from a generation-planning tool, the one-area power-market simulator, which was developed by SINTEF Energy Research. The one-area power-market simulator model takes predicted spot market prices generated from multi-area power scheduling as input and provides a production schedule based on stochastic calculations on incremental water values in an aggregate-reservoir model. In other words, it finds the best production plan based on different scenarios of inflow and spot price.

For the static strategies we have used quarterly and annual contracts, as these are long-term contracts that suit the time perspective of the hydropower producer. Additionally, these contracts have high liquidity. For the historical test, the forward prices are collected on 7 September each year, while the data for the out-of-sample test is collected on the first date in each month, corresponding to the date that the prediction was made. The choice of dates should be of minor importance given an efficient market assumption, and because we want to focus on the weights in the strategies and not the timing of the sale, we have chosen to only use these dates. The weights have also been calculated on other dates, with minor differences, and they are therefore left out in the remainder of the paper.

5.2 Derivation of hedging strategies

For the historical test we have derived two strategies by running the optimization model on the historical data. These strategies are referred to as H1 and H2 and have 10% VaR constraints at 340 million and 350 million Norwegian krone, respectively. These bounds are chosen because the hydropower producer had a similar 10% VaR during the test period, and this makes a direct comparison between H1, H2 and the current strategy of the hydropower producer possible. The weights derived for H1, H2 and the hydropower producer's current strategy can be found in Table 3 on the next page.

For the out-of-sample test we have also derived two different strategies. The strategies are now derived by running the model on predicted data, and are run in a semi-static behavior by using a rolling hedge strategy. That is, a static strategy is derived

TABLE 3 Optimized weights for historical data.

Strategy	H1	H2	Current strategy
Y1	0.20	0.31	0.5
Y2	0.03	0.10	0.4
Y3	—	—	0.3
Q1	0.80	0.69	—
Q2	0.37	—	—
Q3	—	—	—
Q4	—	0.23	—
Q5	—	—	—
Q6	—	—	—
Q7	—	—	—
Q8	—	0.23	—
Amount of expected production hedged:	0.49	0.54	0.50

each month as new predictions are available and the weights are thereby adjusted. The first strategy is called no restrictions (NR) and is simply derived by optimizing the profit subject only to the restriction of not being long in the contracts, which is the no speculation restriction. The second strategy is called conditional value-at-risk increase (VI) and is derived by increasing the CVaR for the cumulative revenue of the future 12 months of the natural hedging strategy by as much as possible, up to a maximum increase of 10%. The model is run each month from January 2007 to April 2009 and takes previous hedge positions into account. That is to say, if it hedged 75% of the expected production for the next quarter in January 2007, it is only allowed to additionally hedge 25% of the next quarter's production in February. The weights for the VI strategy are summarized in Table 4 on the facing page.

As can be seen from Table 3 and Table 4 on the facing page, the model primarily chooses to hedge the positions in quarterly contracts. For the historical data, the hedged percentage of the expected production is low in yearly contracts compared with the quarterly positions, and in the out-of-sample test the yearly contracts are only held for a short period of time. For the historical data, the contracts with short times to maturity are preferred. For the intrinsic rolling hedge strategy we can see that the positions of most of the contracts are shifting. Considering this table in general, there are no signs of a stable static strategy in which some contracts are preferred.

TABLE 4 Weights for the CVaR increase strategy derived in the out-of-sample test.

	Quarterly contracts											Yearly contracts			
Date	**Q1**	**Q2**	**Q3**	**Q4**	**Q5**	**Q6**	**Q7**	**Q8**	**Q9**	**Q10**	**Q11**	**Y1**	**Y2**	**Y3**	**Y4**
01/2007	—	321	—	—	—	461	—	—	526	481	—	—	2,162	—	—
02/2007	—	177	—	—	—	13	—	—	−526	−3	—	—	−2,162	—	—
03/2007	487	74	—	750	658	−6	—	—	542	15	—	104	2,100	3,180	3,157
04/2007	25	393	−750	−658	11	—	—	10	−4	—	—	−104	−2,100	−3,180	−3,157
05/2007	24	−393	—	—	24	—	—	−1	−3	—	—	2,483	—	—	—
06/2007	−180	—	—	—	57	—	—	−550	−486	—	—	−2,483	—	—	—
07/2007	—	—	637	−61	—	—	548	489	—	—	—	2,135	—	—	—
08/2007	730	—	−219	−474	—	972	24	−489	—	—	—	−2,135	—	—	—
09/2007	25	1,114	−418	687	—	−23	−6	−0	—	—	—	—	—	—	—
10/2007	−538	—	−55	—	−949	22	—	—	—	—	—	—	—	—	—
11/2007	625	660	−658	—	1,094	−25	605	—	—	—	—	1,554	966	—	—
12/2007	−1,201	−35	674	—	−1,094	−1	−23	—	—	—	—	−160	−966	—	—
01/2008	−10	−674	—	0	−561	−35	—	—	553	—	—	—	—	—	—
02/2008	−615	−0	—	—	−0	−5	—	967	−51	—	—	—	—	—	—

TABLE 4 Continued.

Date	Quarterly contracts											Yearly contracts			
	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Y1	Y2	Y3	Y4
03/2008	463	517	—	—	–0	92	—	–48	136	—	—	—	—	—	—
04/2008	–517	—	119	—	–634	—	–42	–19	—	—	—	—	—	—	—
05/2008	0	188	–119	—	0	—	–877	–34	—	—	—	—	—	—	—
06/2008	—	–188	191	620	—	—	964	2	—	—	—	—	—	—	—
07/2008	—	511	–534	521	—	–964	15	—	—	—	—	—	—	—	—
08/2008	510	322	–86	–521	821	1,085	70	692	—	—	—	1,391	—	—	—
09/2008	–486	–114	—	682	–821	–1,085	–33	–692	—	—	—	–599	—	—	—
10/2008	–910	—	–370	768	1,103	–5	752	—	—	—	—	–792	—	—	—
11/2008	—	—	–312	–3	–1,103	–635	–752	—	—	—	—	—	—	—	—
12/2008	—	—	633	1	0	654	—	954	—	—	—	1,586	—	—	3,065
01/2009	608	–633	–619	6	–654	—	23	934	635	—	—	—	—	–3,065	—
02/2009	–608	—	676	–6	–0	674	–54	–60	42	—	—	—	—	—	—
03/2009	—	55	4	0	—	–674	0	8	–677	—	—	2,460	—	—	—
04/2009	–55	–211	–0	—	698	55	–39	674	—	—	—	–747	1,765	—	—

The weights show the amount of production that should be sold on forward contracts in the respective month. For instance, the weight Q2 in 02/2007 means the sale of 177 gigawatt-hours on the quarterly contract for the period from July 2007 to September 2007 in February 2007.

TABLE 5 Statistics on an annual basis of the different strategies using the historical data.

Strategy	Mean	Standard deviation	10% VaR	Cost
Natural hedging	636	245	295	0
H1	649	295	340	−13
H2	630	279	350	6
Current strategy	591	221	347	45

All numbers are in million NOK.

5.3 Results of the empirical tests

This section will show results from the historical test and the out-of-sample test. The different strategies will be evaluated with respect to VaR, standard deviation and mean revenue/hedging cost.[2] Additionally, we will calculate CVaR for the out-of-sample test and compare the expected mean revenue with the actual revenue. The CVaR is not calculated for the historical data because there are only 10 data points. The hedging cost shows how much it will cost the hydropower producer, in terms of lost revenue, to reduce volatility and to secure against shortfalls.

5.3.1 Historical test

Table 5 shows the mean, standard deviation and 10% VaR of the annual revenue, as well as the annual hedging cost during the test period, 1999–2008. Table 6 on the next page shows the ranking of the different strategies for each of the measurements from Table 5. There is a clear relationship between the risk measures (standard deviation and VaR) and the mean revenue: the more risky the strategy, the higher the mean revenue. The current strategy performs well during the test period with the second highest VaR and the lowest standard deviation. However, it should be noted that this strategy clearly has the highest hedging costs, which results in the lowest mean revenue. The H1 and H2 strategies have values for VaR and standard deviation that are similar to the current strategy, but with much lower hedging costs. This is due to the fact that these strategies are optimized based on the data in the test period and, consequently, as emphasized in Section 4.3.6, can be considered as theoretical upper limits on the mean revenue for the given VaR values.

There is a clear relationship between the measurements and the amount of production that is hedged. From Table 3 on page 18 we can see that for the H1 strategy 49% of expected production is hedged, while for the H2 strategy 54% of expected production is hedged. In other words, the less risky the strategy, the higher the amount of

[2] The hedging cost of a strategy S is defined as the difference between the mean revenue of the natural hedging strategy and the mean revenue of S.

TABLE 6 Ranking of the strategies with respect to the statistics for the historical data.

Mean		Standard deviation		10% VaR	
H1	649	Current strategy	221	H2	350
Natural hedging	636	Natural hedging	245	Current strategy	347
H2	630	H2	279	H1	340
Current strategy	591	H1	295	Natural hedging	295

All numbers are in million NOK.

hedged production. We also see that no hedging is done three years prior to delivery: that is, Y_3 is equal to 0. One possible explanation is that the spot price has increased a lot during the test period (see Figure 2 on page 7) and this increase has probably not been anticipated in the forward curve, especially not for maturities going far ahead. In general, the strategy will therefore benefit by using the contracts with maturity close ahead. Another explanation is that, in general, the forward price of annual contracts tends to increase as the time to delivery decreases (see, for example, Benth *et al* (2008)). A producer therefore benefits by choosing the contracts that are closest to delivery. For the quarterly contracts, we see that the Q1 contract is used most. As is the case with annual contracts, the forward price for quarterly contracts tends to increase as the delivery date approaches (Benth *et al* (2008)).

We also see that the hedging costs are consistent with previous research on risk premiums. Strategies that mainly use contracts with short time to delivery, for instance H1 and H2, have either a hedging profit or a small loss, while the strategies that use contracts with longer time to delivery have higher hedging losses. This is consistent with the findings of Benth *et al* (2008), who show that there has historically been a negative risk premium for contracts with a short time to delivery and a positive risk premium for contracts with a long time to delivery. Consequently, the producer benefits by using the quarterly contracts with shortest time to delivery.

5.3.2 Out-of-sample test

Table 7 on the facing page shows the expected values of annual mean revenue, VaR, CVaR and standard deviation, and the actual mean revenue for NR, VI and the natural hedging strategy. The natural hedging strategy has slightly better expected mean revenue than VI, while NR has the highest. The reason that NR only has a slightly higher expected mean revenue than the natural hedging strategy, despite no restrictions, is that the revenue depends not only on contracts traded at the present time, but also on contracts that have previously been traded. The optimized value with respect to mean profit may therefore also be below the natural hedging value because of the loss from

TABLE 7 Results from out-of-sample tests.

(a) Expected results

	Natural hedging strategy				**VI**				**NR**			
	Revenue	**CVaR 1 year**	**VaR 1 year**	**Std. dev. 1 year**	**Revenue**	**CVaR 1 year**	**VaR 1 year**	**Std. dev. 1 year**	**Revenue**	**CVaR 1 year**	**VaR 1 year**	**Std. dev. 1 year**
01/2007	987	818	864	90	995	819	869	91	998	807	854	99
02/2007	940	785	836	80	954	793	835	82	970	810	852	82
03/2007	831	707	737	72	888	756	784	69	898	747	783	78
04/2007	998	841	881	90	1,019	867	912	79	1,031	832	875	113
05/2007	1,006	847	898	91	1,022	855	885	94	1,018	823	850	113
06/2007	1,070	903	943	105	1,067	917	953	91	1,035	868	908	105
07/2007	1,069	878	904	115	1,080	794	847	186	1030	744	797	186
08/2007	1,180	638	767	348	1,112	702	802	270	1,067	525	655	348
09/2007	1,160	627	712	366	1,098	855	900	181	1,049	807	852	181
10/2007	1,301	731	794	420	1,219	804	863	314	1,163	594	656	420
11/2007	1,407	790	935	391	1,313	975	1,049	206	1,312	974	1,048	206
12/2007	1,382	912	1,002	349	1,267	1,003	1,080	179	1,269	799	889	349
01/2008	1,361	798	951	344	1,337	910	982	279	1,733	1,216	1,352	325
02/2008	1,283	779	931	375	1,334	901	1,033	325	1,688	1,183	1,336	375

TABLE 7 Continued.

(a) Expected results

	Natural hedging strategy				VI				NR			
	Revenue	CVaR 1 year	VaR 1 year	Std. dev. 1 year	Revenue	CVaR 1 year	VaR 1 year	Std. dev. 1 year	Revenue	CVaR 1 year	VaR 1 year	Std. dev. 1 year
03/2008	1,136	439	597	514	1,263	707	792	392	1,548	1,072	1,133	341
04/2008	1,317	654	759	497	1,262	719	810	417	1,505	842	947	497
05/2008	1,438	692	821	495	1,342	761	847	410	1,626	880	1,009	495
06/2008	1,756	909	1,035	602	1,560	1,000	1,056	402	1,944	1,097	1,224	602
07/2008	2 101	1,149	1,325	644	1,854	1,264	1,404	394	2,157	1,205	1,381	644
08/2008	1,623	830	904	648	1,596	1,156	1,293	316	1,712	1,272	1,409	316
09/2008	1,853	944	1,077	619	1,547	1,038	1,133	316	1,716	807	939	619
10/2008	1,575	773	955	609	1,443	851	982	426	1,327	677	804	473
11/2008	1,396	659	785	578	1,311	724	826	454	1,179	442	568	578
12/2008	1,128	559	618	458	1,118	768	913	200	924	574	719	200
01/2009	1,191	580	734	523	1,094	638	778	341	985	527	667	342
02/2009	1,132	588	645	461	1,088	759	829	206	978	648	718	206
03/2009	887	424	552	300	977	765	849	132	866	655	719	132
04/2009	976	444	533	460	960	675	748	212	814	529	602	212
Average	1,267	739	839	380	1,219	849	930	252	1,269	820	912	308

The numbers show the cumulative values for the future 12 months' revenue in each respective month.

TABLE 7 Continued.

(b) Actual results

	Natural revenue	VI revenue	NR revenue
01/2007	953	975	977
02/2007	969	992	992
03/2007	996	1,028	1,017
04/2007	1,043	1,081	1,043
05/2007	1,033	1,102	1,026
06/2007	1,045	1,115	1,014
07/2007	1,061	1,094	999
08/2007	1,113	1,097	1,018
09/2007	1,115	1,048	989
10/2007	1,126	1,026	964
11/2007	1,141	1,033	964
12/2007	1,180	1,069	989
01/2008	1,159	1,051	939
02/2008	1,158	1,053	915
03/2008	1,150	1,043	888
04/2008	1,130	996	876
Average	1,086	1,050	976

previous hedge positions, which stems from differences between the predictions and the actual values or from changes in the predictions. Of all the strategies, the natural hedging strategy has the highest actual revenue drawn from the period from January 2007 to April 2009. The actual revenues of VI and NR are 12% and 22% lower, respectively.

For the risk measurements, VI has the best values. The CVaR is 10% higher than the VAR of the natural hedging strategy and it is 12% higher than the VaR of NR. For CVaR, the corresponding numbers are 13% and 3%. In addition to higher VaR and CVaR, VI also has the lowest standard deviation: 34% and 18% lower than the natural hedging strategy and NR, respectively.

Overall, the results from the out-of-sample test suggest that there are clear benefits to using static hedging strategies with forward contracts. The VI strategy significantly decreases the risk in terms of VaR, CVaR and standard deviation, both when compared with the natural hedging strategy and with NR. Furthermore, this risk reduction comes at the cost of just a minor decrease in revenue when compared with the natural hedging strategy.

5.4 Performance summary

In this section we will compare the results from the historical test and the out-of-sample test. The results have shown that static hedging is able to increase VaR and CVaR, and is able to reduce the standard deviation of the revenue. The reduction of standard deviation by using forward contracts is also reported by Näsäkkäla and Keppo (2005). The risk reduction comes with only a minor decrease in mean revenue. The minor reduction in mean revenue is in line with empirical evidence on the risk premium in the electricity market, where it is shown that there is an expected gain associated with being on the short end of these contracts.

For the historical test we have shown that quarterly contracts are the preferred choice of contracts. This is also the case for the out-of-sample test. For the yearly contracts, the contracts with shorter time to maturity are the preferred choice. This is in accordance with the results of Näsäkkäla and Keppo (2005), who show that a hydropower producer with high load uncertainty can postpone hedging to obtain better estimates. However, in our results, the reason for the preference of contracts with a shorter time to maturity stems from the risk premiums.

In a practical setting, it is normal to take new marked information into account. This is simulated in the out-of-sample test, where the model is rerun monthly and the weights are adjusted accordingly. Such a reoptimization is similar to the method proposed by Bjerksund *et al* (2008) and allows the strategy to incorporate new information and to capture changes in the distributions of the input parameters. However, we have seen that the optimized weights do not stabilize around certain contracts and are more or less random. The only general fact for the weights is that quarterly contracts seem to be preferred over yearly contracts. This indicates that it is hard to find a static strategy with certain risk characteristics based on the same weights at all times. A more dynamic optimization should therefore be considered for the weights of the strategy to adjust to the risk premiums at the given optimization time and possible outcomes. A more suitable optimization model would be to make a model that incorporates a strategy that is subject to future changes in weights and is not so heavily dependent on the assumption that the weights should be retained for the whole time period.

6 CONCLUSION

In this paper we have developed an optimization model for deriving static hedging positions. We have used this model to propose strategies with different risk characteristics for a hydropower producer and have run it in a completely static manner on historical data, and in a semi-static manner in an out-of-sample test. The derived strategies were tested and compared with the natural hedge on historical and predicted data. The results show that hedging with use of forward contracts significantly reduces the risk in terms of VaR, CVaR and standard deviation. This improvement results in only a minor reduction in the mean revenue. However, it has been shown that a static position is hard to derive for a longer period of time because of the rapid shift in characteristics of the forward contracts. This suggests that a model that incorporates possible future shifts in the weights during optimization may yield better results. In other words, a static strategy may be beneficial but we would recommend further research on dynamic positions.

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