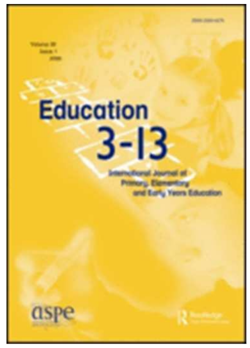

Collaborative Talk in Mathematics – Contrasting Examples from Third Graders

Journal:	*Education 3-13*
Manuscript ID	RETT-2017-0030.R1
Manuscript Type:	Original Article
Keywords:	mathematical problem solving, use of tools, representations, exploratory talk, collaborative work

Collaborative Talk in Mathematics – Contrasting Examples from Third Graders

There is a substantial body of knowledge on the importance of language for learning in general, and for learning mathematics in particular. Hence, language skills and collaborative learning are emphasised in the Norwegian curriculum. Even so, we have few studies on what supports and what impedes mathematical progress in authentic learning situations. In this article, we investigate contrasting dialogues between two pairs of eight-year-old pupils solving mathematical tasks. The analysis in our video-based study shows that both communication skills and use of tools have a profound impact on third graders' potential to solve tasks as a joint enterprise.

Keywords: mathematical problem solving; use of tools; representations; exploratory talk; collaborative work

Introduction

This article is based on classroom observations in Norway where pupils in the third grade are working on multiplication tasks. The study is part of a larger research and development project entitled Language Use and Development in the Mathematics Classroom (LaUDiM). The main objective of the project is to develop deeper knowledge of the learning environment's significance for developing young learners' mathematical thinking and understanding, as well as to develop their ability to express mathematical concepts and ideas. One of the research questions is aimed at understanding more about how young pupils collaborate on solving mathematical tasks.

Both theory (Vygotsky [1934] 1987) and researchers (Mercer and Sams 2006) point out the importance of language and social interaction in learning mathematics. The Norwegian national curriculum for primary school (KD 2006) also clearly states this. However, some researchers call for caution, arguing that just putting pupils together will not always work, as the talk may often be uncooperative, off-task, inequitable and ultimately unproductive (Mercer and Sams 2006). Sfard and Kieran (2001) concluded that 'interaction with others, with the numerous demand on one's attention, can often be counterproductive. Indeed, it is very difficult to keep a well-focused conversation going when also trying to solve problems and be creative about them' (70). They argue that strong motivation is necessary to engage in mathematical conversations and make them work, and a prerequisite for a productive mathematical discourse is the effectiveness of the communication between the interlocutors. Van Oers (2013) claims that we need to find out more about what productive dialogues that support mathematical thinking and learning entail.

In this article, we present, analyse and discuss two dialogues between two pairs of eight-year old pupils, two girls and two boys. The dialogue between the two girls ends with the exclamation 'Yes, we did it' which we took as preliminary evidence of successful collaborative talk. The boys' dialogue, on the other hand, shows little enthusiasm and gives the immediate impression of unproductive competition. Thus, the research question for this paper is: What stimulates and what impedes mathematical progress in the collaborative process of solving tasks?

Theoretical Framework

Our point of departure is sociocultural theory as developed by Vygotsky ([1934] 1987) and his followers. Two important features of this theory are particularly relevant for our study. The first is the claim that higher mental functioning in the individual, such as reasoning and problem solving, derives from social life. Second, higher mental functioning and human action in general are mediated by tools and signs. Vygotsky's accounts of mediation provide the bridge that connects the external with the internal and thus the social with the individual (Wertsch and Stone 1985). He considered language to be the most important tool, both for the development and sharing of knowledge between people and for structuring the process and content of individual thought. From a sociocultural perspective, it is particularly interesting to study talk in educational settings and identify the ways in which humans learn to handle and use cultural tools effectively to solve problems.

Exploratory talk is a typification of a way of using language effectively for joint, explicit, collaborative reasoning (Barnes and Todd 1977; Littleton and Mercer 2010). In exploratory talk knowledge is made publicly accountable and reasoning is visible. It represents a form of co-reasoning where speakers share knowledge, challenge ideas, evaluate evidence and consider options in a reasoned way. Explanations are compared and joint decisions reached. 'It is a speech situation in which everyone is free to express their views and in which the most reasonable views gain acceptance' (Littleton and Mercer 2010, 279). According to Barnes and Todd (1977), exploratory talk depends on learners who share the same idea of what is relevant to the discussion and have a joint conception of what one is trying to achieve through it. Two other kinds of talk are presented by Littleton and Mercer (2010). In cumulative talk, speakers build positively but uncritically on what the others have said. It is characterized by shared information, joint decisions, repetitions, confirmations and elaborations, but there are no critical considerations of ideas. Disputational talk is characterized by disagreement and individualized decision making with few attempts to combine resources, offer constructive criticism or make suggestions. There can be an interchanging of these three types of talk.

To explore and communicate mathematical ideas, the use of tools in the form of different representations is indispensable. This is due to the abstract nature of mathematical objects. Duval (2006) claims that all mathematical activity involves substituting some semiotic representation for another, and he classifies semiotic representation into four registers: natural language, symbolic systems, iconic and non-iconic drawings, and diagram and graphs. The classification is based on the possibilities each system holds for performing mathematical processes. Like Vygotsky, Duval considers natural language to hold a special position, as it can be used not only for processing mathematics, but also for communication, awareness, imagination and so on. Transformations between representations within the same semiotic system are denoted by Duval as treatments, and transformations between different registers are denoted as conversions. Duval claims that conversions are more complex transformations than treatments 'because any change of register first requires recognition of the same represented object between two representations whose content have very often nothing in common' (112). Hence, the ability to perform successful conversions is often a critical threshold for making progress in problem solving.

The two dialogues presented in this article have been taken from a teaching sequence where the mathematical aim was to give the pupils experiences with different multiplicative situations. Steffe (1994) characterizes a multiplicative situation as one where 'it is necessary to at least coordinate two composite units in such a way that one of the composite units is distributed over the elements of the other composite unit' (19). Depending on the situation, four different multiplicative structures can be distinguished: equal groups, rectangular area, multiplicative comparison and Cartesian product (Greer 1992). The tasks explored in this article concern the first two structures. In an equal group situation, the multiplicator counts how many groups are involved, while the multiplicand tells the number of objects in each group. Such situations are asymmetric problem situations, meaning that the role of the factors cannot be interchanged without reinterpreting the situation. The first task is of this type, asking how many eggs one needs to bake 12 portions of muffins, given that there are four eggs in one portion. Here the composite unit 'four eggs' is distributed over the elements of the '12-portion' unit, corresponding to the multiplication task 12·4. In a rectangular area situation, the elements are arranged in, as the name suggests, a rectangular shaped array, where the convention is that the first factor of the corresponding multiplication task counts the number of rows, while the second factor counts the number of elements in each row.

These situations are symmetric in their nature because the role of the factors can be interchanged by just rotating the array. The second task involved in this article asks how many muffins can be placed on a baking tray if there is room for five rows of muffins on the tray and each row has room for seven muffins. This is clearly an example of a rectangular area situation, corresponding to the multiplication task 5·7.

Methodology

LaUDiM is an intervention project where two teachers from different primary schools and researchers from the field of mathematics education and pedagogy plan and set goals for the teaching of mathematics, which subsequently are carried out and followed by the teachers. In the classroom, whole class discussions and dialogues between selected groups of third graders have been video-recorded. Parts of these video-recordings, together with the pupils' written work, are discussed by researchers and teachers in a joint session. When interesting sequences are identified, this represents the first step in analysing the data material. The presented dialogues have been chosen from video-recordings of six collaborating pairs of eight-year-old pupils working on a set of multiplication tasks. By carefully viewing all the recordings we became aware of two rather contrasting dialogues. The first dialogue was chosen due to the task-focused content, and to the engagement and passion we could see between the two girls. Moreover, as mentioned above, the session ended with the exclamation 'Yes, we did it' which we understood as preliminary evidence of successful collaborative talk. The dialogue between the two boys contrasts with the first, characterized by a lack of both enthusiasm and the willingness to share.

Thus, the empirical data for this article consists of two video-recorded and transcribed dialogues, one seven-minute long dialogue between two girls working on one task and one 14-and-a-half-minutes-long dialogue between two boys working on another task. The pupils' discussions have been planned as a collaborative effort to solve a mathematical problem.

Introducing the tasks, the teacher reminds the pupils that the next step, after solving the problem, is for them to explain their strategies to another pair of pupils.

To answer the research question, we started by conducting a conversational analysis. Keywords from Littleton and Mercer's (2010) characteristics of different types of talk served as guidelines in this process. Examples of questions asked about the material are how do the pupils respond to each other; how do they share ideas; how do they give reasons; and how do they build upon each other's ideas. Due to the video-based design of the study we were able to identify not only their oral talk, but also the use of gestures and other mediational tools. As both the cognitive challenge made by peers that is the catalyst for the co-construction of understanding and the resolution of the constructive conflict might take the form of action rather than verbal exchange, such non-verbal interaction needs to be included in research on dialogues between young children (Patterson 2016).

The next step was to identify shifts of focus in the dialogues. This helped us to divide them into sequences which were analysed further with respect to the mathematical content. In this process, use and shifts of representations became visible. This turned our attention to Duval's (2006) work on this issue. In the third step, we analysed and interpreted each sequence more thoroughly by combining the two analytical perspectives. As we find it important to show how the pairs build, or do not build, upon each other, we present and analyse the dialogues as they unfold, omitting a few utterances we find unnecessary for the purpose here.

Ethical care has been addressed through the processes of informed consent (Bogdan and Biklen 2003) and by anonymizing the participants. Each of the authors has analysed the dialogues on their own, comparing their findings before reaching a joint understanding. To further strengthen the credibility of the study, we have discussed our findings with the collaborative two primary teachers in LaUDiM who know the pupils well. They find our analysis reasonable.

Research has shown that the use of ground rules for talk increases the incidence of exploratory talk (Mercer and Sams 2006; Rojas-Drumond and Zapata 2004). However, the pupils involved in this study did not receive any specific education in communication skills beforehand, nor were such ground rules implemented.

Analysis of the Girls' Dialogue

The girls' dialogue has been taken from their work on the task:

The 3rd grade is going to have a party at school. The day before the party they are baking muffins. Anne is going to the store to buy eggs for the muffins. In the recipe it says that they need four eggs for one portion. The children have decided that they are going to bake twelve portions of muffins. How many eggs does Anne need to buy?

The dialogue starts by Kate reading the word problem out loud, until Beth interrupts her:[1]

(G1) B: I'll draw four eggs?

(G2) K: Wait, wait (continues to read the task out loud).

(…)

(G7) B: I'll just draw some circles (starts to draw a row of small circles).

(G8) K: Draw four circles. There you are. Good. And then we should..., and then we have twelve…, just write twelve, no, forget it.

While Kate is still reading the word problem, Beth suggests a conversion from the problem stated in natural language to an iconic representation (G1, G7). Kate supports this transformation by monitoring and evaluating Beth's action (G8). The girls are unsure of the

1 We have numbered both the girls' and the boys' utterances consecutively from one and up, using a G for the girls' dialogue and a B for the boys' dialogue.

role of the number 12, and it is not likely that they recognize the problem as multiplication. Kate then goes back to the written task, and after some thinking time, the conversation continues.

(G13) B: This is an addition problem.

(G14) K: No, (whispers) it is 12 times 4.

(G15) B: Oh, yes.

(G16) K: No, it's 4 times 12.

(G17) B: (Laughs) Yes, that's the same.

(G18) K: It's 4 times 12, …no, it's not the same. For if we take 12 times 4, then we take 12 four times.[2]

G19 B: Yes.

G20 K: And that doesn't work here.

At this point it seems as if the girls have given up pursuing an iconic representation, instead they try to find a number sentence that fits the word problem, indicating a conversion from natural language to mathematical symbols. As Beth is not given the chance to explain her thinking (G13), it is not clear whether she makes a successful conversion, suggesting repeated addition of 4s. Eagerness to explain the difference between 12·4 and 4·12 (G18) is taken as an account showing it is important to Kate that Beth follows her reasoning. Recognizing the situation as multiplicative gives Kate some new input on how the problem situation can be modelled, and so the problem-solving moves on.

(G22) K: (Points at the four eggs) So that means four…, we should get to… we are going to have twelve. (Takes the paper from Beth.) If I draw twelve.

2 Kate is aware of the difference between 4·12 and 12·4, but her interpretation does not follow the usual convention.

(…)

(G25) B: Just do it there (points right beneath the four eggs).

(G26) K: I'll draw twelve muffins[3] (starts to draw bigger circles, stops to count).

(G27) B: That's funny looking muffins.

(G28) K: I know, but we can see, we can see what it is anyway (completes the drawing of twelve muffins; two rows with six circles in each row).

(G29) B: Now you have twelve.

(G30) K: Here we have twelve muffins, and then there should be four in each muffin (points at the eggs Beth has drawn at the top of the paper).

(G31) B: (Points at the four eggs) Then we put these down here, these four in one, then we have to… (points from the four eggs to the twelve muffins).

Kate identifies that the muffins are the essential units to start with in the iconic representation, and she makes the crucial connection between the muffins and the eggs by pointing at Beth's drawing of four circles (G30). This shows that she has grasped the multiplicative structure of the problem, one unit distributed over the other, and is thus a mathematical breakthrough. The gesture also serves as an acknowledgement of Beth's contribution. Beth actively monitors Kate as she draws the muffins (G29), and by suggesting to 'put down' the eggs (G31), she lets Kate know that she both understands the structure of the problem and approves her representation of it, and the girls are ready to proceed.

(G34) K: Because in this, if we add them together we get eight. (Points to the first muffin in each row, writes the number 8). Because in each there is eight.

(G35) B: Here, just read from here again. Slowly.

(…)

3 There is some confusion between muffins and portions, but that is not important for the solution.

(G39) B: Stop. We need four eggs in a portion, right?

(G40) K: Yes, because one portion, that is one muffin for us then (points at herself). So, that means that in this one there are four (points at the first of the muffins).

(G41) B: (Points at the four eggs) all of these circles here, just draw a line down to… (points at the first of the muffins).

(G42) K: In one there are four, and in that one there are four, so if we add them, we get eight.

(G43) B: I'll take four of them in here (draws four small circles inside the first muffin).

(G44) K: No, just... I'll… (takes the pencil from Beth). Eight plus four, we do it like this, four, four, four (writes the number 4 above each muffin).

(G45) B: Can I do the last ones?

(G46) K: Yes, you can do these four.

(G47) B: Oh no (draws a sloppy looking 4).

(G48) K: That's fine, that's fine, we make see it out anyway.

Kate seems ready to use the representation of the twelve muffins to start calculating, keeping the number of eggs in each muffin as a mental image (G34). Beth, on the other hand, needs a more concrete representation of the eggs (G41, G43). They compromise by writing "4" over each muffin (G44), see Figure 1. After confirming that they share a common understanding of the new representation, they continue.

[Figure 1 here]

(G52) K: No, look here, do you know what, wait, we have to do it again now, because…, if we take… (points to and counts the six muffins in the first row) this is six, right (writes 4+4+4+ on a line below the drawing of the muffins). Now I have taken these three (puts a mark after the first three muffins, counts as she writes more +4s) 1-2-3-4-5-6-7-8.

(G53) B: (Counts the muffins silently.) Just take twelve of those. Ok, I'll just read (takes the problem sheet, reads to herself, following the text with her finger).

(G54) K: (Counts out loud, finishes to write +4) 9-10-11-12. Ok, here I made a plus-problem with all these (points to the muffins). Then we have twelve fours, just that…, here we have the answer (writes =___ below the row of +4s).

Both girls are able to use the drawing of the muffins, combined with the rows of 4s, to start a process of repeated addition, but face some challenges keeping track of the preliminary calculations. Kate takes the lead in transforming them into a more structured symbolic representation (G52), thinking out loud to ensure that they agree. Beth is not passive in this process, she monitors Kate's work, and checks once again that the representation they have come up with is in line with the written task (G53). After some negotiation over the notation, the girls are ready to perform the needed calculations.

(G63) B: It's 16 (points).

(…)

(G66) K: Ok, ok I believe you. Plus four, 16… (draws more vertical lines and writes 16), and here we have four.

(G67) B: 16

(G68) K/B: (both counting on their fingers) 17-18-19-20 (Kate writes 20).

(G69) B: 24 (Kate writes 24), 28 (Kate writes 28)

(G70) K: (counting on her fingers) 29-30-31-32 (writes 32)

(G71) B: (counting on her fingers) 33-34-35-36 (Kate writes 36)

(G72) K/B: (counting on their fingers) 37-38-39-40 (Kate writes 40), 41-42-43-44 (Kate writes 44)

(G73) K: Oh, that one, that one we could have done right away.

(G74) B: 48 … I think.

(G75) K: Yes, it's 48.

(G76) B: Yes, it's 48. (Kate writes 48 behind =). So, we have to buy 48. Yes, we did it!

The new representation works for the calculation and the girls share the strategy taking turns counting in fours. They use their fingers as support, but the counting is rhythmic, so they might be capable of using an internal count. Kate keeps control over the number of 4s they have added by inserting vertical lines in the calculation expression, see Figure 2. When there are only a few more fours to add, they turn to a choral count, indicating that they are enthusiastic as they approach an answer. Beth's 'Yes, we did it' shows pride in having fulfilled their common project.

[Figure 2 here]

To summarize, the dialogue between Kate and Beth is characterized by a willingness to involve oneself and the wish to involve the other. Every new idea is stated out loud, every drawing is accompanied with an explanation of what is being drawn and whenever one of them is writing, the other monitors the work. We find multiple examples of common explanatory terms and phrases like 'I think', 'because', 'if', 'so' (Knight and Mercer 2014) in the dialogue (G18, G22, G34, G40, G42, G52, G74, G76), showing that the reasoning is shared publicly. There seems to be an atmosphere of trust and acknowledgement between the girls, visible for instance when Kate gives positive feedback on Beth's drawing (G8), when the girls do not mind that their drawings are not perfect (G28, G48) and when Kate trusts Beth's calculation (G66). The repeated use of 'we' instead of 'I' indicates that the girls share the responsibility for the project.

Another prominent feature of the girls' dialogue is their extensive use of tools in the form of drawings and other semiotic representations. The different representations are used to communicate ideas, but also as a tool for thinking. Whenever stuck, one of them takes the initiative to make a shift in representation, eventually leading to the answer to the problem.

Analysis of the Boys' Dialogue

The boys' dialogue has been taken from the work on the task:

To bake the muffins, they are put on a baking sheet. The baking sheet takes five rows of muffins with seven muffins in each row. How many muffins are there room for on the baking sheet?

After reading parts of the task out loud, Fred withdraws. Noah continues the reading:

(B1) N: To bake the muffins (…) How many muffins are there room for on the baking sheet?

(B2) F: (Whispers while Noah seems to reread the task silently) Eh, 5 times 5? Doesn't that work?

(B3) N: Yes. Then we just have to find 7+7=14. And 14+14…

(B4) F: (Whispers out into the air) Well, we could use the five times table?

(B5) N: No, we take fourteen, because that makes…, no, no, it's much simpler using the five times table, yes.

(B6) F: Yes.

Accepting Fred's suggestion, Noah starts calculating by writing down the five times table. He writes 5-10-2 …, but has trouble because he has forgotten 15. Fred then says out loud:

(B9) F: We should have 35, no 45 … 25. I have no idea!

(B10) N: What? (Fills in his number series so he now has 5-10-15-20)

(B11) F: (Still looking out in the air) 30

(B12) N: (Stops writing) Yes, it's 30.

Fred and Noah understand the multiplicative structure in the situation. Noah stays with the number sentence 5·7 related to the situation in the task, and suggests the calculation strategy 'doubling' (B3). He is interrupted by Fred who suggests calculating 7·5 (B4). Accepting that Fred has found a simpler strategy, Noah starts writing down the five times table. When he has trouble, he gets no help from Fred who from the very start takes on the role of a whispering side-lines commentator (B2, B4, B9). Well on the way to getting the five times table correct, he is interrupted by Fred a second time, now suggesting 30 to be the answer (B11). Suggesting four different answers (B9, B12), Fred is obviously unsure. But he offers no explanation and Noah accepts Fred's final suggestion, 30, without further question (B12).

Noah and Fred proceed to the next task (seven minutes, utterances B13-B59 omitted in this article). A reminder from an observing researcher (OR) brings them back to task; remember, the teacher wanted them to prepare to explain their strategies to another pair of pupils. Their dialogue on the baking-sheet task then continues:

(B60) N: (Starts drawing) Then we have to have five rows of muffins.

(B61) F: (Whispering:) 5 times 7.

(B62) N: Like this and like this (draws a rectangle with five columns, as shown in Figure 3)[4].

(B63) F: (Leans over the drawing) Wait a minute!

(B64) N/F: (Both counting the columns out loud, going in each their direction) 1-2-3-4-5!

(B65) N: We counted in each our own direction!

(B66) N: (Draws two horizontal lines in the rectangle, stops to recount the number of squares in one row). 1-2-3-4-5. No, this is all wrong! (He crosses out his drawing)

4 As the boys seem to agree, we have chosen not to problematize that Noah breaks the convention of horizontal rows.

Here Fred monitors Noah's drawing; reminding him of how many muffins to draw in the five rows (B61) and counting the number of columns out loud together with him (B64). Both mix up rows and columns. When Noah recounts the squares in the first row (B66), he seems to expect there to be seven, and finding only five, he rejects the drawing. Fred then draws back. Noah's second attempt to produce a sufficient illustration occurs without involvement from Fred. As shown in Figure 3, Noah quickly gives up his second attempt.

[Figure 3 here]

The dialogue then continues:

(B71) N: (Fred numbers the different tasks on the sheet where Noah so far has done all the writing and drawing) What are you doing?

(B72) F: Writes which task is which (draws lines between the different writings and drawings concerning the same task).

(B73) N: What are you doing?

(B74) F: That one and that one and that one (points with his pencil).

(B75) F: (Noah crosses out Fred's numbering of the third task) Thank you Noah, for crossing it out!

(B76) N: I do like this (writes 'task 3' below Fred's former numbering).

(B77) F: You write the answers. Good luck, do it yourself!

(…)

(B80) F: Those two are related (starts drawing lines again)

(B81) N: Oh, yes. No, you – you – don't write on my project!

(B82) F: (Laughs) It's not only your project!

(B83) N: No, it's yours too.

In this sequence, we see a dispute between the two boys concerning the ownership of the answering sheet. Typically, the pronouns in use are 'I', 'my', 'you', 'yours' and 'yourself'.

Once again, the observing researcher (OR) reminds them that they chose to illustrate their calculation and she motivates Fred to help Noah. Claiming that he feels able to draw the baking sheet (B91), Fred grabs for the pencil but is pushed back by Noah. Being dismissed, Fred pulls back making side-line comments like; 'one, two, three, four, five, six, seven' (B94) and twice 'five times seven' (B103, B105), comments that might support Noah's drawing. But he also gives an evaluating comment; 'you don't have a clue [, do you]?' (B101). Noah never asks for help or takes any initiative to involve Fred. He keeps working individually, rereading the task, wondering out loud about the number of vertical and horizontal lines, showing signs of increasing frustration at not being able to make the drawing. At one point, Fred says:

(B110) F: You're supposed to have five rows with seven in each, draw that. Draw five …, four lines. Do you have five rows?

(B111) N: (Draws four vertical lines in the rectangle) Yes, five rows.

(B112) F: Then you draw seven in each.

(B113) N: It's like this, seven down here (draws 6 horizontal lines in the rectangle, completing a 7·5 grid).

(B114) F: In all of them.

(B115) N: No, it should be six down here. Because then it's 1-2-3-4-5 (counts squares horizontally). No, no, it's only five (draws another vertical line in the grid). 1-2-3-4-5-6. Yuck! (draws a sixth vertical line and ends up with a 7·7 grid. Fred watches and laughs and Noah crosses out his third attempt.)

(…)

(B117) F: You fail every time, Noah!

Noah's third attempt to draw a 5·7 grid then fails, he ends up with the first of two 7·7 grids (see Figure 4).

In this sequence, Fred makes a second attempt to be more directly involved in the drawing. He literally instructs Noah in how to draw a 5·7 grid. But when Noah once again mixes up rows and columns and adds two more vertical lines (B115), Fred does not stop him or argue to help him understand the idea of the 5·7 grid. He only gives another evaluating comment showing that he takes no responsibility for the result; 'you fail every time, Noah' (B117). What happens next is that Noah struggles on with the drawing. Working alone, he rereads the task several times. Fred goes back to monitoring Noah's work giving unanswered side-line comments; '7 times 7?' (B125), 'On one side there are to be seven and five on the other - probably? (B134) and 'I think you should leave more space between them' (B154). This shows that most of the time he is monitoring Noah' work.

[Figure 4 near here]

As shown in Figure 4, it takes a fourth and a fifth attempt to arrive at a correct drawing that Noah accepts. Having the iconic representation in place (see Figure 4), Noah starts counting each square in the grid:

(B169) N: 1-2-3-4 ... 21, (counts silently). Or – it's 30 because you just take 5 times 7 (writes =30 behind the grid).

(B170) F: Hem, or 7 times 5.

(B171) OR: How did you find out that 5 times 7 makes 30?

(B172) F: 30

(B173) N: Because, I thought that if you take seven five times, it makes 30.

(B174) F: 5…35 (counts in fives, using his fingers to keep control over the number of fives).

(B175) N: 35?

(B176) F: Yes

(B177) N: Fred! (Expressed like an accusation. Both boys smiling.)

In this sequence, we can see that Noah starts to count the squares in the correct representation. Nearly there, he stops counting and returns to their former agreed answer, 'it's 30 because you just take 5 times 7' (B169). Our interpretation is that the competitive climate makes it difficult for Noah to use the tools that are available to him. He does not seem to trust his own counting or the representation he has made enough to oppose Fred. Fred seems to monitor Noah's work but offers no help to understand the drawing as an representation of five times seven. On the contrary, he keeps confusing Noah, pointing out that the drawing might just as well represent '7 times 5' (B170). It is only later, either motivated by the observing researcher's question (B171), or possibly by the drawing itself, that Fred rethinks and corrects his answer (B174). He is still relying on his mental calculation with the support of finger counting and neglects to involve Noah in his thinking.

To summarize, the lack of reasoning is prominent in the boys' dialogue, neither Noah nor Fred argue for their own ideas or ask for arguments from the other. Except for a few examples of 'because' (B5, B115, B169, B173), we find no common explanatory terms and phrases like 'I think', 'if', 'so' (Knight and Mercer 2014) in the dialogue. There are also few signs of acknowledgement of each other's contributions or willingness to adapt to the other's needs. On the contrary, Noah stops Fred's initiatives to take an active part in the drawing

(B91) and only reluctantly agrees that Fred has a part in the written product (B83). Fred, on the other hand, offers his help through indirect 'side-line comments' (e.g. B61, B94, B103, B105), and by suggesting answers (B11, B174) and by instructing what to draw (B110-B114). When they reach their first answer, 30, both Noah and Fred use the pronoun 'we'. Later the choice of pronouns indicates that Noah and Fred have no common project, using the pronouns 'I', 'mine', 'you' and 'yours' much more frequently than 'we' and 'ours' (e.g. B73-B82). The lack of a common project is highlighted by Fred's pointing out Noah's lack of competence (B101) and failure (B117) and by Noah's reaction to Fred's attempt to write on the answering sheet; 'No, you – you – don't write on my project!' (B81).

Discussion

What stimulates and what impedes the mathematical progress in the collaborative process of solving the tasks? To be able to answer this we first identify what comprises mathematical progress in the dialogues. We then show how combining our two analytical perspectives contributes to answering the research question.

According to Blum and Niss (1991), a mathematical problem is a situation that challenges somebody intellectually who is not in immediate possession of direct procedures sufficient to answer the question. This is the case for the girls, as a solution process is not straightforward for either of them. Anghileri (1989) claims that multiplication differs significantly from addition in complexity because there are three pieces of information to coordinate: the number of sets; the number of elements in each set; and the procedure for executing the product. The mathematical progress in the girls' dialogue can be described in two steps. The first step is the mathematical breakthrough that occurs when the girls identify the multiplicative structure of the problem situation (G30, G40-G43). They recognize that the group of eggs constitutes a composite unit that is to be distributed over the muffins. The task can then be solved by repeated addition of fours. The girls' actual calculation constitutes the second step of the mathematical progress. This, of course, leads them to the final answer, but the identification of the multiplicative structure was crucial to starting the calculation.

The analysis shows that when the girls are stuck in the process of solving the task they use two strategies to make progress: they either re-read the task, or they perform a shift of representation (Duval, 2006). By constantly going back to the written problem the girls check that they have a joint conception of what they are trying to achieve (Barnes & Todd 1977), while the change of representation serves as a tool that helps them to realize what is important in the task, and to structure and communicate their thoughts.

Reading the task, both Noah and Fred reveal that they immediately understand the situation as multiplicative. They also show that they know relevant calculation strategies such as doubling (B3) and the five-times table (B5, B10). Disregarding Fred's miscalculation, solving the problem does not seem to be an intellectual challenge to them, and the making of a representation is not a problem for them either. Even if Noah mixes up the drawing, the analysis shows they both know how to produce iconic representations and number series that can support their calculations. As they think they have arrived at the answer, they do not need a written representation to solve the mathematical problem and they seem to go on drawing because the observing researcher asks them to. Lack of motivation might also explain why they do not use the available representations in a more efficient way. They seem to lack the strong motivation needed (Sfard and Kieran 2001) to keep up the conversation during the drawing and for using the representation for controlling and arguing. Noah seems happy to obtain a quick answer and does not challenge Fred's mental calculation capacity. Even so, the monitoring of Noah's drawing and square counting might play an active role when Fred eventually corrects his answer (B174).

What first and foremost stimulates the girls' mathematical progress is the fact that they have such a common goal in solving the task (Sfard and Kieran 2001). The repeated use of ‘we’ instead of ‘I’ indicates that they share the responsibility for the project. The girls create a positive collaborative atmosphere by giving each other positive feedback and by acknowledging each other’s contributions (e.g. G8, G30, G31). This mutual acceptance is a necessary condition for co-reasoning, as it creates a space within which the girls dare to share ideas.

Two characteristics of the girls’ communication seem especially important for stimulating mathematical progress; the girls’ ability to think out loud, and their eagerness to actively involve themselves in each other’s reasoning. Accompanying their written work with verbal explanations and gestures means making their thinking public. This makes it possible to follow each other’s reasoning, to evaluate it and build on it. One example of this is when Beth draws four eggs and gives Kate a chance to follow her thinking by saying out loud what she is drawing. Kate then tries to build on Beth’s work, but is unsure of the role of the number 12 (G1-G8). This shows that the conversion of the problem from written text to drawing is challenging (Duval, 2006). Nevertheless, this initiated change of representation is the first step in realizing the structure of the problem. It is striking that whenever a change of representation is performed, the girls very carefully explain their actions. We see this again when Kate makes the drawing of twelve muffins (G22-G30), and later when she turns the problem into a repeated addition problem (G52-G54). As the use and shift of representations is the dominant tool, a shared understanding is crucial for keeping the solution process a common project in which they are able to support each other and continue the co-reasoning.

Thinking out loud not only helps the listener, it also enables the one sharing her idea to think it through more thoroughly, leading to a deeper insight (Vygotsky, [1934] 1987). An example of this is when Kate explains the difference between 4·12 and 12·4 (G18). Almost immediately it seems like she sees the connection between the pair of numbers and an iconic representation of the problem. The drawing should, as the multiplication sentence, show twelve groups of four (G22-G30). This represents the turning point in the solution process as it reveals the multiplicative structure of the task. The girls' need for a model of the problem situation as a tool for thinking is in line with previous research on young children's pre-instructional multiplicative strategies (Kouba 1989).

We find the shared use of spoken language that creates meaning and common goals in the girls' dialogue to be an example of 'inter-thinking' (Littleton and Mercer 2013). Involvement in each other's reasoning is important for mathematical progress because it ensures that the reasoning is supported and understood by both participants, and hence serves as a green light to continue. The analysis shows that the girls are constantly involved, either by monitoring each other's actions, as when Beth confirms that Kate has drawn exactly 12 muffins (G29), or by actively participating in the other's construction of a new representation (G45). In 'ideal' exploratory talk, ideas are often challenged or questioned. This does not happen often – if at all – in the dialogue between Kate and Beth. As shown, this does not mean that they passively accept each other's ideas. As the two girls are using language effectively for joint, explicit collaborative reasoning, we claim that the conversation contains features of exploratory talk (Littleton and Mercer 2010).

The boys' working climate is the opposite of the girls' and the repeated use of 'I', 'mine', 'you', 'yours' and 'yourself' indicates that they do not share the common responsibility for the project as we see in the girls' case. There is an atmosphere of competition and unwillingness to discuss common solutions. After a positive start talking about the number of rows and the number of muffins in each row and counting out loud together to control the number of rows (B60-B65), the collaboration breaks down. Without consulting Fred, Noah crosses out what they produced together. And even though he has obvious problems for a long time, he shuts Fred out from writing on the answering sheet (B81, B91), using the opportunity provided by that to follow his own ideas without much argument (e.g. B71-B76, B115). There are a few examples where Noah explains his calculation strategy, doubling (B3), or thinks out loud when drawing (B60, B113). But mostly Noah does not share his thoughts.

Fred makes very weak initiatives to help. He monitors Noah's work and shares his ideas out loud, but without reasoning, his instructions and comments (e.g. B61, B94, B103, B105) prove to be insufficient. Reasoning for what he from the start seems to be convinced is the correct iconic representation might have disclosed their miscalculation at a much earlier stage. Twice they have the correct 7·5 grid in place; the first attempt (Figure 3) and the third attempt (Figure 4). When Noah mixes this up, instead of arguing that they should stay with what they already have, he draws back and lets Noah reject the correct representations (B66, B115).

Fred's communication actually prevents Noah from completing his calculations based on his own doubling strategy (B3-B4) or based on written representation that could have led him to the correct answer. This happens first when based on mental calculation (or guessing) Fred suggests an answer and stops Noah's initiative to write down the number series that will help him see the correct answer (B11). Second, this occurs indirectly when Noah stops counting the squares in the 5·7 grid at 21, remembering Fred's suggestion that five times seven makes 30 (B169). For a long time, they are unable to build on their total competence producing an iconic representation of their idea.

We find that the boys' dialogue can be categorized as disputational talk, defined by Littleton and Mercer (2010) as communication characterized by disagreement and individualized decision making with few attempts to combine resources, offer constructive criticism or make suggestions. Fred's comments on Noah's lack of competence (B101) and failure (B117) escalates the competitive climate. We saw that the ability to think out loud, and the eagerness to actively involve themselves in each other's reasoning were productive in the girls' dialogue. But these features are just about completely absent from the boys' dialogue. The tools that helped the girls to make progress are also available to the boys. Noah rereads the task several times when he struggles with the illustration. He is also able to shift between representations when he is calculating (B1-B10). But this is never a joint activity that ensures that they share the same idea of what is relevant to the discussion or that they have a joint conception of what they are trying to achieve through it.

Concluding Remarks

As shown, the dialogue between the girls is productive, while the boys' talk, though on task, is rather counterproductive. According to the teacher, the boys' skills in mathematics exceed those of the girls. Thus, the reason for the boys' relatively greater difficulties in solving the task may lie elsewhere. As argued by Sfard and Kieran (2001), strong motivation is necessary to engage in mathematical conversations and make them work. Due to differences in intellectual challenges, this condition seems present in one of our cases while absent in the other, having strong influence on the quality of the dialogue. Our study thus adds to the field showing how the quality of the communication is closely connected to the experience of intellectual challenge. To engage pupils in collaborative work, teachers must provide tasks pupils regard as intellectually challenging. An implication of our study is that from the early years in school, teachers must strive to broaden the pupils' understanding of mathematics. Mathematics is more than finding answers. It is just as much about explaining your thinking and arguing why your answer is reasonable. Understanding this to be the true nature of mathematics, the task might have motivated the boys in our study in a different way and subsequently enhanced the quality of the collaboration. To solve the mathematical task, the girls are involved in what we call collaborative tool-mediated talk. Thus, our study also adds to the field by shedding light on how semiotic representations are used as mediational means in third-graders' co-reasoning. An implication of this is that early learning of mathematics

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must equip pupils with a variety of semiotic representations. This can be achieved by encouraging children to develop their own tools and by creating arenas for sharing these.

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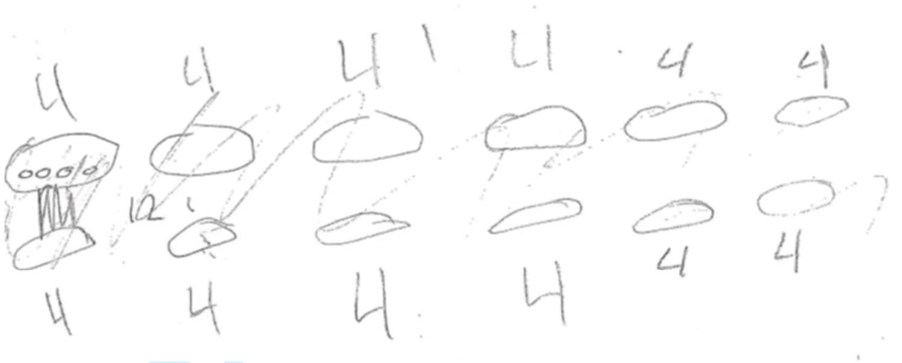
Figure 1. Beth's and Kate's representation of twelve muffins each containing four eggs.

Figure 2. The representation Beth and Kate use for repeated addition of fours.

Figure 3. Noah's first and second attempts to illustrate 35 muffins distributed in five rows of seven.

Figure 4. Noah's third, fourth and fifth attempts to illustrate 35 muffins distributed in five rows of seven.



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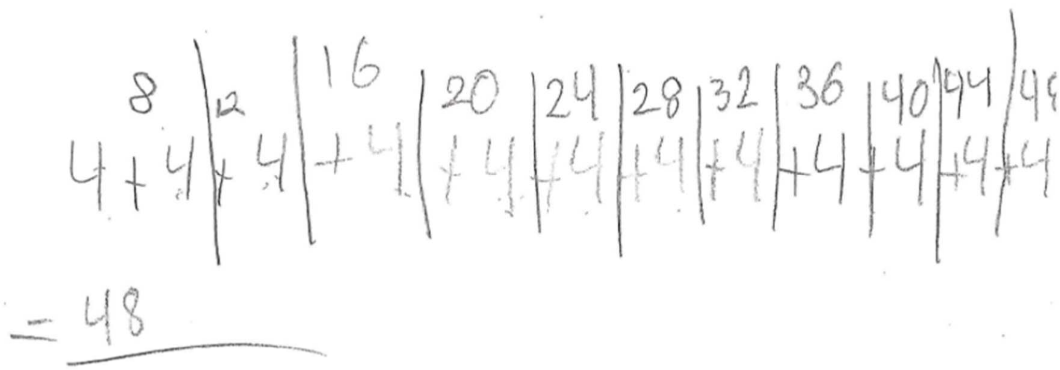

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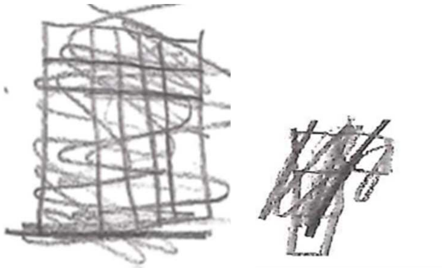

Figure 3: Noah's first and second attempts to illustrate 35 muffins distributed in five rows of seven.

Figure 4: Noah's third, fourth and fifth attempts to illustrate 35 muffins distributed in five rows of seven.