Using operational data to estimate the running resistance of trains. Estimation of the resistance in a set of Norwegian tunnels.

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Abstract

Two approaches to estimate the running resistance from operational data have been studied: A direct approach based on a measured/estimated acceleration to obtain a resistance time-series, and a velocity-fitting approach based on fitting a predicted to a measured velocity time-series. Two data sets have been considered: The first consists of a velocity time-series, extracted from the train event recorder. The second is logged from the vehicle control unit and includes a time-series of energy consumption and velocity. Velocity-fitting has been shown to be more robust with respect to the data resolution and is the recommended approach. The inclusion of energy consumption (or traction) data is recommended, resulting in more precise estimates. The approach is considered feasible in terms of data storage and transfer requirements, allowing a routinely collection of data from trains in operation.

The additional tunnel resistance has been evaluated for a set of 10 tunnels in Norway, giving resistance coefficients ranging from 2.4-16.0 kg/m for tunnels with an aerodynamic cross section of 87-27 m² respectively. The comparison to a set of predictions showed that there is a large spread in the predicted additional tunnel resistance. This demonstrates that the choice of prediction method and corresponding coefficients must be taken carefully.

Keywords: tunnel resistance, aerodynamic drag, running resistance, operational data, energy consumption

1. Introduction

The running resistance of a train directly effects journey times and energy consumption, two key indicators for train operation. With increasing speed in the railway sector, the running resistance is getting more and more important for these indicators. This is both a result of the increased aerodynamic resistance at higher velocities, but also a result of an increase in the amount of tunnels for modern railroad tracks, where the running resistance is substantially higher than in open air. Being a mountainous country, the latter is a typical issue for the Norwegian railway network.

As a consequence, it is important to develop appropriate methods to estimate the running resistance and in particular the additional tunnel resistance. From our experience, there is a lack of clarity on the effect a tunnel has on the train resistance and subsequentially on the journey times and energy consumption. This is a paradox, since this effect has been the subject of numerous studies the last century and a large number of trains run through tunnels every day, where these effects are present and may be observed.

Typical full-scale train resistance measurements are performed with separate test runs, often with the use of special equipment and personnel, and are costly to perform and organize[1]. A method to utilize the operational data from trains in regular service will give access to a large set of measurements at a low cost, typically limited to logging equipment, data transfer and storage. An aim of this study was to develop methodology to estimate the running resistance from trains in regular service based on operational data that is (or can be) logged. This has the advantages that (i) there is no need to perform separate and cost-intensive test runs; (ii) the amount of available data for analysis is increased substanially and (iii) real-life conditions may be observed, such as variable weather conditions and interference with other trains. The main disadvantages are the lack of control of the test conditions and that the data logging possibilities may be limited.

A number of predictive equations have been proposed for the additional tunnel resistance. However, NSB experiences that in several cases the choice of equations and corresponding parameters is not given enough attention, leading to resistance predictions that do not correspond to reality. This study was initiated to document the tunnel resistance observed in a set of Norwegian railroad tunnels as well as to evaluate the precision of the predictive equations.

The main objectives include:

- (i) Develop methodology to estimate the running resistance from trains in operation
- (ii) Estimate the tunnel resistance for a set of norwegian railway tunnels
- (iii) Compare these estimates to a set of predictions.

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2. Literature Review

The running resistance of a train, and in particular the additonal resistance in a tunnel, has been addressed in numerous studies the last two centuries. Early measurements of the running resistance was performed already in 1818 by Stephenson and Wood [2] but was investigated in more detail in the 1840's by Harding[3] and Gooch[4, 5]. The latter studies lead to early predictive equations for the running resistance[2]. These early measurements reached velocities up to about 100 km/h, and during the next half-century the velocity range for resistance measurements was expanded, reaching up to 210 km/h in 1904[6, 7]. The running resistance is commonly determined by employing traction measurements, typically at constant speed[3–5, 8] or by coasting or run-down measurements[8]. Recent studies primarily focus on the latter technique[9–16]. The aerodynamic resistance may also be determined by wind-tunnel tests at reduced scale, however these require a correction to account for the difference in scale and the difference between experimental and full-scale conditions[17–19].

Formulas for the running resistance were developed by numerous authors, among others Harding[3], Clark[2], Barbier[20], Leitzmann and von Borries[21, 22], Frank[23], Schmidt[24] and Strahl[25] (see Clark[2], Sanzin[8] or Sachs[26] for a historical overview), all based on a formula describing the running resistance as a quadratic function of the velocity, in the general form as

$$R = A + Bv + Cv^2 \qquad , \tag{1}$$

where R is the running resistance, v the velocity and A, B, C are coefficients determined from theoretical considerations or measurements. This equation is commonly known as the Davis equation[12, 27]. The coefficients of the Davis equation are related to different resistance contributions, where A can be related to the rolling resistance, B to other mechanical resistance as well as drag associated with ingested air and C to the aerodynamic resistance[19, 28].

Due to the confined space, the aerodynamics for a train passing through a tunnel is different from the aerodynamics in open-air, which leads to increased running resistance. This increase can be related to the increase in skin friction drag and pressure drag[29, 30]. The additonal resistance in a tunnel was estimated theoretically by Stix[31] already in 1906, followed by wind tunnel experiments by Tollmien and Langer[32, 33] in 1927 and full-scale measurements in two Swiss tunnels by Sutter in 1930[34]. The additional tunnel resistance can be obtained with the same methods as for the resistance in open air[11, 14], but may also be derived from pressure measurements[34–38]. Early studies of the additional tunnel resistance assumed (quasi)-steady flow conditions to deduce global resistance coefficients for complete trains[29], typically on the form

$$R_{at} = C_{at}v^2 \tag{2}$$

where R_{at} is the additional running resistance inside a tunnel and C_{at} a coefficient. The value of C_{at} will in principle depend on a large number of factors, among others the train and tunnel length and aerodynamic cross section (including the blockage ratio), the friction along the train and the tunnel wall, the train and tunnel design, the local weather and wind conditions as well as possible interferences with other trains[39, 40].

The introduction of numerical simulations made it possible to study local effects and the air flow in more detail[29, 41]. In addition the effects of the tunnel design could be studied, including the effect of compensating features such as entry hoods, tunnel branches and pressure-relief shafts[40, 42, 43]. Modern numerical simulations enable the detailed study of the air flow around a train[1, 19, 39, 44–52] allowing the prediction of effects such as pressure waves and slipstreams. Numerical simulations give a powerful tool to predict the aerodynamic properties within a train tunnel, which is important for the design of new train tunnels[44, 53]. Still, numerical simulations require a careful choice of method depending on the topic to be studied[1, 19].

Even with detailed numerical simulation methods available, it is in many cases sufficient to use analytical methods [40]. This is examplified by the level of detail in state-of-the-art planning tools for railway operations, where simple global equations (such as Equation 2) are used to predict the effect that a tunnel has on the energy consumption and running time of a train. Possible reasons for such a simplification are that: (i) tools used for planning and operational purposes rely on simple run time calculators, and do not have the necessary algorithms / computational power for a numerical simulation; (ii) a detailed numerical simulation requires detailed information of the tunnel and train design, which are rarely easily available; (iii) these global equations give satisfactory precision, thus not creating a demand for a more advanced computational procedure. However, the use of simple global equations requires a proper choice of coefficients in order to get realistic results.

Various predictive equations have been proposed for the additional tunnel resistance, and a subset of these have been chosen for comparison in this work. The tool Togkjør, used by the Norwegian Railroad Administration (Jernbaneverket)[54, 55], is based on the work of Gackenholz[56] and predicts the total resistance inside a train tunnel as

$$R_t = Bv + Cv^2 \qquad , \tag{3}$$

where R_t is the total resistance inside the tunnel and the coefficients B and C depend on the tunnel and train length,

the train type (cargo or passenger) and the number of tracks. The equation is based on a typical aerodynamic cross section for a single or double track tunnel, and is adopted to a variable cross section as a perturbation from the reference resistance in a single or double track tunnel[54]. To be able to compare this equation to predictions and results of the same functional form as Equation 2, an effective C_{at} is obtained by subtracting the open air contribution R_o and dividing by v^2 as

$$C_{at}(v) = \frac{R_t - R_o}{v^2} \qquad . \tag{4}$$

Viriato 6 [57] and Opentrack [58] use as standard an additional tunnel resistance defined as in Equation 2 where the C_{at} -coefficient is chosen from a set of predefined values for single and double track, based on train type (cargo or passenger) and the roughness of the tunnel walls[59]. Version 8 of Viriato[60] uses a predictive equation developed by SBB[61] on the format

$$R_{at} = \left[\frac{(2+n_e)}{3}\frac{a_e}{Q^{b_e}} + n_w \frac{a_w}{Q^{b_w}}\right]v^2 \qquad , \tag{5}$$

where Q is the aerodynamic cross section of the tunnel in m^2 , a_e , b_e and a_w , b_w are coefficients for engines and wagons respectively and n_e and n_w are the number of engines and wagons. Typical values of the coefficients are $b_e = 1.48$, $b_w = 1.75$, $a_e = 1140$ [kg m^{2b_e-1}], $a_w = 662$ [kg m^{2b_w-1}] [61].

As an example of a recent tunnel resistance prediction, the Norwegian high speed rail assessment study also discussed the resistance in high-speed tunnels[62–66]. Two sets of tunnel resistance calculations are reported: (i) The additional tunnel resistance is discussed for corridor west[62], and based on the equilibrium velocity in the reported velocity time-series, together with the train mass and an idealized traction curve for an ICE3, coefficients $C_{at}(60 \text{ m}^2) = 35 \text{ kg/m}$ and $C_{at}(72 \text{ m}^2) = 30 \text{ kg/m}$ may be derived; (ii) The additional tunnel resistance is used in the energy consumption estimation[63, 64], where the additional tunnel resistance is modelled by scaling the open air C-coefficient. For an AGV-train travelling through a single-track tunnel with a length of 10 km a coefficient $C_{at} = 9.0 \text{ kg/m}$ may be derived.

3. Methodology

3.1. Data sources and tunnel details

The data for analysis has been collected from a set of train runs with NSB Class 73 (from ADtranz/Bombardier) and Class 74/75 (from Stadler AG) as described in Table I, and relevant characteristics of the train classes are shown in Table II. The runs T_{73} and $T_{74/75}$ are regular passenger service runs, where data has been collected from the train event recorders installed on these units. The driver has in these runs been instructed to use maximal power for acceleration. The test runs $E_{74/75}$ are test runs without passengers where special logging equipment has been used to log data from the vehicle control unit (VCU). In principle, these data could also be logged for passenger service runs, if the necessary logging equipment would be present.

Table II: Relevant characteristics of the train classes used in this work. The number of coaches includes traction units.

Class	Length [m]	Number of coaches	Cross section $[m^2]$
73	106.6	4	??
74 /75	105.5	5	??

Five tunnels situated between Drammen and Eidsvoll in the greater Oslo area and five tunnels situated between Stavanger and Kristiansand in the southern and south-western part of Norway have been studied. The tunnel details are presented in Table III. These may all be considered as long tunnels and range from a length of 3.1-14.6 km. The tunnels between Stavanger and Kristiansand are all single-track, built in the 1940's and have a relatively small cross section and a relatively rough inside. The tunnels between Drammen and Eidsvoll are all double-track, relatively modern with a large cross section and a relatively smooth inside, with the exception of Lieråsen from the 1970's which is more narrow and rough.

ecorder (TER) or the vehicle control unit (VCU)[67]. The resolution in time, velocity and energy of the	t no restriction in the recorded precision has been observed. m_{tara} is defined as the mass of the empty	nber of passengers n_{psg} and a mass of 80 kg per passenger gives the total mass m_{tot} .
le I: Overview of train runs analyzed. The data has been collected from the train event recorder (TER) or the	rded data (t_{res} , v_{res} and E_{res} respectively) is displayed. A resolution of 0 indicates that no restriction in the	ı including full operational supplements and staff [68, 69], which combined with the number of passengers n

C ¹	Prain number,	Train	Data	Data	t_{res}	vres	$\mathbf{E}_{\mathbf{res}}$	$\mathbf{m_{tara}}$	Section	n_{psg}	mțot	Relevant
	date	class	source	registered	[s]	[km/h]	[kJ]	$[10^{3}kg]$			$[10^{3} kg]$	tunnels
К.	28, 21 Nov 2014	73	TER	v(t)	ca 3-4	ca 5		231.5	Stavanger-Kristiansand	202 [Sira]	247.7	Kvineshei, Sira, Tronås, Hægebostad, Gyland
÷.	20, 17 Sep 2009	73	TER	v(t)	ca 3-4	ca 5	'	231.5	Eidsvoll-Oslo	188 [Oslo S]	246.5	Romeriksporten
	17, 29 Sep 2009	73	TER	v(t)	ca 3-4	ca 5	,	231.5	Oslo-Moss	105 [Moss]	239.9	
1	20, 17 Sep 2009	73	TER	v(t)	ca 3-4	ca 5	'	231.5	Moss-Oslo	87 [Moss]	238.5	
- 4	30-21 Sep 2012	74	VCU	E(t), v(t)	0.2	0	0	218.4	Oslo-Eidsvoll	0	218.4	Romeriksporten
	21 Sep 2012	74	VCU	E(t), v(t)	0.2	1	0	218.4	Eidsvoll-Oslo	0	218.4	${ m Romeriksporten}$
	31 Aug 2012	74	VCU	E(t), v(t)	0.2	1	0	218.4	Sandvika-Asker	0	218.4	Tanum, Skaugum
	7 Sep 2012	74	VCU	E(t), v(t)	0.2	1	0	218.4	Asker-Sandvika	0	218.4	${\operatorname{Tanum}},{\operatorname{Skaugum}}$
	7 Sep 2012	74	VCU	E(t), v(t)	0.2	1	0	218.4	Asker-Drammen (nonstop)	0	218.4	Lieråsen
	7 Sep 2012	74	VCU	E(t), v(t)	0.2	1	0	218.4	Drammen-Asker (nonstop)	0	218.4	Lieråsen
0)	0-31 Aug 2012	74	VCU	E(t), v(t)	0.2	1	0	218.4	Drammen-Asker (incl. stops)	0	218.4	Lieråsen
1									Drammen-Asker	156	230.9	Lieråsen
									Asker-Sandvika	202	234.6	Tanum, Skaugum
									Sandvika-Lysaker	210	235.2	Bærum
ŋ	10, 5 Dec 2013	75	TER	v(t)	ca 0.2-0.3	3.8	,	218.4	Oslo-Lillestrøm	62	223.4	Romeriksporten
									Lillestrøm-Gardermoen	55	222.8	
									Gardermoen-Eidsvoll verk	17	219.8	
									Eidsvoll verk-Eidsvoll	12	219.4	
									Eidsvoll-Eidsvoll verk	221	220.2	
ЪĊ,	15. 5 Dec 2013	7.5	TER.	v(t)	ca. 0.2-0.3	8.6	,	218.4	Eidsvoll Verk-Gardermoen	43^{1}	221.8	
,		-		(2)2		0			Gardermoen-Lillestrøm	93^{1}	225.8	
									Lillestrøm-Oslo	113^{1}	227.4	${ m Romeriksporten}$
1									Lysaker-Sandvika	125^{1}	228.4	Bærum
ŋ	17, 5 Dec 2013	75	TER	v(t)	ca 0.2-0.3	3.8	'	218.4	Sandvika-Asker	107^{1}	227.0	Tanum, Skaugum
									Asker-Drammen	871	225.4	Lieråsen

¹ Actual train not counted, n_{psg} is estimated from counted trains in the period 1 Oct 2013- 1 Dec 2013

Table III: Tunnel properties together with the maximum allowed velocity v_{max} in the tunnel [70]. All tunnels are single-tube without cross vents and air-shafts.

Tunnel	Year of	Aerodynamic cross	Length	v_{max}
	Completion	section $[m^2]$	[km]	[km/h]
Romeriksporten	1999	87	14.6	200
Bærumstunnelen	2011	87	5.4	160
Tanumtunnelen	2005	79	3.5	160
Skaugumtunnelen	2005	79	3.8	160
Lieråsentunnelen	1973	58	10.7	130
Kvinesheitunnelen	1944	27	9.1	160
Siratunnelen	1943	27	3.1	130
Tronåstunnelen	1943	27	3.2	130
Hægebostadtunnelen	1943	27	8.5	160
Gylandtunnelen	1943	27	5.7	130

3.2. Applied Approaches

Two different approaches to estimate the train resistance have been studied, a direct approach using the acceleration to calculate the train resistance as a time-series and an indirect approach based on the fitting of a predicted to the measured velocity time-series, where the train resistance is implicit in the velocity prediction. Two variants of the direct approach have been studied, one where the acceleration is calculated or measured and stored as a separate time-series (D1), and one where the acceleration is obtained from a stored velocity time-series at the given resolution (D2). Due to the unavailability of acceleration measurements, only acceleration obtained from high-resolution velocity difference has been evaluated in this work. However, this is believed to give more accurate results than using acceleration measurements because of lower levels of noise.

The studied approaches are summarized in Table IV.

Table IV: The approaches used to estimate the train resistance

Approach	Description	Input Data
D1	Direct approach using a stored acceleration (measured or calculated from high-resolution velocity difference)	$a(t), v(t), F_{tr}$
D2	Direct approach using acceleration obtained with velocity difference using stored velocity data	$v(t), F_{tr}$
VF	Velocity-fitting	$v(t), F_{tr},$ functional form of R

3.2.1. The Direct Approach

In the direct approach, a time-series of the train resistance is obtained by using a force balance and adding or subtracting terms to get the resistance. The total force acting on a train is represented by

$$F_{tot} = F_{tr} + F_g - R_{tot} \qquad , \tag{6}$$

where F_{tot} is the total force, F_{tr} is the traction force, F_g is the gradient force and R_{tot} is the total resistance acting on the train and is defined as the force acting in the opposite direction of train motion. The curve resistance is here neglected. Equation 6 can be rewritten as

$$R_{tot} = F_{tr} + F_g - F_{tot} \qquad . \tag{7}$$

To obtain R_{tot} we require the traction force, the gradient force and the total force. Assuming the train is a point mass, the gradient force F_g can be expressed as

$$F_g = -mg\sin(\alpha) \approx -mgp \qquad , \tag{8}$$

where m, g, α, p is the mass, the gravitational constant, the angle of inclination and the gradient, which is defined as positive for a train running uphill, respectively. Newton's second law of motion gives the total force as

$$F_{tot} = m\rho a \qquad , \tag{9}$$

where a and ρ is the acceleration of the train and the mass factor respectively. The latter models the effect of accelerating rotational components of the train as the train itself accelerates. The mass factor is in principle velocity-

dependent but is in practice taken as a constant parameter with typical values in the range 1.05-1.15[63, 71]. The resistance can then be calculated by

$$R_{tot} = F_{tr} - mgp - m\rhoa \qquad . \tag{10}$$

The mass and mass factor of the train are assumed to be known and the gradient is known for a given infrastructure. The remaining unknown terms in Equation 10 are the traction force (F_{tr}) and the acceleration (a), and the variants of the direct approach are distinguished by different ways to obtain the acceleration. Equation 10 can be applied to any time point of a train run to create a time-series for R_{tot} . The resistance coefficients may then be obtained by fitting the Davis formula to the set of resistance-velocity pairs from the resistance and velocity time-series .

3.2.2. The velocity-fitting approach

In the velocity-fitting approach the resistance is obtained by minimising the deviation between the measured velocity time-series and a velocity prediction (where a resistance formula is implicit), using the coefficients of the resistance formula as variables.

Equation 10 can be rearranged to obtain

$$a = \frac{F_{tr} - mgp - R_{tot}}{m\rho} \qquad . \tag{11}$$

In principle, we can obtain a predicted velocity (v_{pre}) by integrating the acceleration from a given initial time t_0 and velocity v_0 as shown in Equation 12.

$$v_{pre}(t) = v_0 + \int_{t_0}^t a(t')dt'$$
(12)

This is the same integral as is solved in a common run-time calculation. To be able to solve this integral an expression for R_{tot} is needed, and the resistance is commonly assumed to be a function of the velocity and a set of coefficients, *e.g.* the Davis formula (Equation 1). The coefficients of $v_{pre}(t)$, can be obtained by a fit to the observed velocity time-series v_{obs} as

$$min \qquad ||v_{pre}(v_0, A, B, C) - v_{obs}|| \qquad w.r.t. \quad v_0, A, B, C \qquad .$$
(13)

As opposed to the direct approach, this approach will automatically give the resistance as a function of velocity and there is no need for post-processing to obtain the resistance coefficients.

3.3. Obtaining the traction force

An expression for the traction force is needed to estimate the resistance (Equation 10 and 11) from a train in operation. Historically the traction force for resistance measurements was obtained by the reading of steam pressure [2, 8]. On modern trains, the traction force may be possible to obtain for different phases of a train run, depending on the availability of logged data and the action of the driver.

If the energy consumption is logged, it may be possible to obtain the traction force for all but the braking phases using

$$F_{tr} = \eta \frac{P_{tr}}{v} \qquad , \tag{14}$$

where P_{tr} is the power applied for traction and η is a traction-chain efficiency factor that accounts for the energy loss between the applied power and the resulting mechanical traction. To obtain the power applied for traction, the power used for auxiliary systems must be subtracted from the total power as

$$P_{tr} = P_{tot} - P_{aux} \qquad . \tag{15}$$

If no (direct or indirect) information of the applied traction is logged, only certain situations allow the determination of the traction force: Maximum traction phase: A train unit has a well defined maximum (on-wheel) traction force $F_{tr,max}(v)$ and assuming the driver applies all possible traction and that there are no limiting factors (such as insufficient power from the catenary system, bad adhesion or additional inefficiencies in the traction chain) one may assume that $F_{tr} = F_{tr,max}(v)$. Coasting phase: The coasting phase is defined as the phase where no traction is applied, thus $F_{tr} = 0$. It may be possible to use both the maximal traction phases and the coasting phases for the estimation of the train resistance. The analysis of the coasting phase is in principle expected to give more precise results as the question of data quality and accuracy for the traction force F_{tr} is eliminated. Test runs are commonly set up as run-down or coasting tests to create a long coasting phase for analysis which span a large velocity range[9, 10, 12–16]. However, for trains in regular service, the construction of long coasting phases is commonly not possible as the train has to follow its predefined time table path. The analysis of the maximal traction phases of a regular run may then be advantageous of the following reasons: (i) a broad velocity range is typically covered; (ii) the phases are easily recognizable in the logged velocity time-series; (iii) a simple instruction can be given to the driver to obtain $F_{tr} = F_{tr,max}(v)$; (iv) the possibly altered driving behaviour because of (iii) reduces the journey time, thus is not expected to induce delays.

4. Computational details

The Davis equation (Equation 1) was used to model the resistance in both open-air and inside a tunnel, with the A-coefficient given as

$$A = \mu g m_{tot} \qquad , \tag{16}$$

where μ is a friction coefficient, g the gravitational constant and m_{tot} the total train mass, B = 0 and optimising the C-coefficient only. Attempts to let more variables optimise freely (A and/or B) gave values out of the expected (physical) range, without improving the quality of the fit substantially. This effect is also observed by Kim et al[14]. A value of $\mu = 10^{-3}$ has been used in this work.

The C-coefficients in the D1/D2-approach were obtained by minimising the sum of weighted square deviations between the resistance-velocity pairs of the resistance and velocity time-series and the Davis equation. The C-coefficients in the VF-approach were obtained by minimising the sum of weighted square deviations between the measured and predicted velocity time-series (Equation 13) using the leastsq routine from SciPy[72]. The weights w were set as a function of the number of data points at a given velocity as

$$w(v_i) = \frac{n_{max}}{n_{tot}p(v_i) + n_{max}} \qquad , \tag{17}$$

where $p(v_i)$ is the density of data points at velocity v_i , n_{tot} the total number of data points and n_{max} a parameter. This equation is constructed as to compensate for the fact that the sum of unweighted square deviations will be dominated by the most frequent data points, i.e. the velocity that is sampled the most. The parameter n_{max} has been introduced to limit the weight for the initial data points, in order to make the algorithm less vulnerable to error in data with a few samples. A value of $n_{max} = 30$ has been used in this work. The density $p(v_i)$ has been estimated by a kernel density estimation using the gaussian_kde routine from SciPy[72, 73] on a grid with resolution 10 km/h. A time step of 0.01 s has been used in the velocity integration in the VF-approach.

If a velocity segment in the VF-approach is not well described by the fit, an error in the fitted velocity time-series may arise. Since the fitting algorithm seeks to minimise the deviation to the measured time-series, the algorithm will seek to compensate this error by leading the predicted velocity back to the measured. To limit this effect, a maximum time span t_{max} is selected and longer sections are divided into subsections of equal length. The number of subsections is taken as the lowest number that gives subsections of length shorter than t_{max} . In this work $t_{max} = 300$ s has been used. If there are enough sections may be identified, and sorted out if justified, based on the quality of the fit on these sections. In this work, multiple sections with deviating velocity time-series were identified. These led to the identification of sections where the necessary assumptions were not valid (see Chapter 3.3) as well as the identification of inconsistencies in infrastructure data and train location with respect to infrastructure data. The time windows of the analysis were subsequently adjusted and inconcistensies were corrected. This demonstrates that the use of operational data requires a procedure to ensure the correctness of the data used for analysis.

The power consumption was obtained by using a difference in the energy consumption over a period of 2 s. Shorter values were tested but gave higher amount of noise in the resulting power consumption. P_{aux} and η has been obtained by analysing the energy consumption data[67].

The variance for the method comparison has been estimated using the Moving-Block-Bootstrapping algorithm [74], using a block length of 60 s and 100 samples. For the VF-approach a new parameter v_0 is given at the start of each block as well as for each chosen subsection of the complete data set. The variance for the tunnel resistance was estimated using 50 samples and a block length of 300s for class 74/75 and 200s for class 73.

The mass factor has been implemented as a function of the empty weight only such that the mass of passengers does not contribute to the rotational mass. If not reported explicitly, a mass factor of $\rho = 1.1$ has been used.

The effective C_{at} from Togkjør (Equation 4) has been estimated using $R_o = \mu g m_{tot} + C_o v^2$, v = 200 km/h, $\mu = 10^{-3}$, $C_o = 6.8$ kg/m and $m_{tot} = 220 \cdot 10^3$ kg and $440 \cdot 10^3$ kg for a train of length 100 and 200 m respectively.

5. Results

5.1. Method comparison

Operational data may consist of a conglomeration of data with various resolution and accuracy. It is therefore an advantage to have a method for resistance estimation that is robust with respect to the data quality. To evaluate the robustness with respect to the velocity and time resolution, the approaches proposed in Table IV have been



Figure 1: The open-air C-coefficients estimated from the sample $E_{74/75,1} + E_{74/75,2}$ together with bootstrap variance $\hat{\sigma}^2$ for the VF,D1 and D2 approaches. In (a) the velocity resolution is fixed to 1 km/h and in (b) the time resolution is fixed to 1s.

applied on the same data set $(E_{74/75,1} + E_{74/75,2})$, using different time and velocity resolution. Since the acceleration is not measured in this data set, this acceleration in the D1 approach is estimated by velocity difference using a time resolution of 1 s and a velocity resolution of 1 km/h.

Figure 1 shows the estimated C-coefficients together with the bootstrap variance. At a high time and velocity resolution all methods give similar results. The VF-approach give stable values for the C-coefficient and a slowly increasing variance in the studied resolution range (1-10 s). The direct approaches (D1,D2) are sensitive to the time resolution, indicated by the increase in variance at a time resolution of 5-10 s. In addition, the reduced time resolution seems to introduce a bias in the estimated coefficients by these approaches.

All methods are less sensitive to a reduction in the velocity resolution. The D2-approach shows a higher variance, which is likely to be related to the reduction in resolution used for determining the acceleration from velocity difference.

The VF-approach shows more robust properties than the direct approaches, both in terms of low time and velocity resolution. In addition the approach does only require velocity measurements (together with a model for the applied traction) which are more readily available than acceleration measurements. The velocity-fitting approach is therefore the recommended method for the estimation of running resistance from trains in operation.

5.2. Evaluation of data sources

In this work, two primary data sources have been used (Table I): the velocity time-series from the train event recorder (TER) $(T_{74/75,1-3})$ and the velocity time-series and energy consumption time-series from the VCU $(E_{74/75,1-7})$. These both differ in terms of the amount of data, the availability of energy consumption data, the data resolution as well as the inherent measurement/logging accuracy. Figure 2 shows the *C*-coefficient and estimated variance, estimated from these different data sources. The *C*-coefficient values estimated from the TER data are higher than the estimates from the VCU data, but within the estimated statistical error. The VCU estimates benefit from a higher data resolution of energy consumption of 10s is sufficient for the VF-approach, however the energy consumption data is in this work analysed using a resolution of 2s. A time resolution of at least 2s is therefore recommended. Figure 1b suggests that a velocity precision of 5 km/h is sufficient for the VF-approach, however the energy consumption data is in this work analysed with floating-point precision for the velocity. A lower precision than floating-point precision for the velocity can therefore not be recommended on the basis of this work. To summarize, the recommended approach is to log velocity data with floating-point precision at a time resolution of at least 2s and to include energy (or traction) measurements if possible.



Figure 2: The open-air coefficient $C_o^{74/75}$ calculated from VCU-data $(E_{74/75,1-7})$ and TER-data $(T_{74/75,1-3})$ and the combined set. Error bars indicate the estimated standard deviation

In terms of data storage and transfer requirements, the VF-approach requires maximum four floating-point numbers: time, velocity, energy consumption and distance (the latter is required to obtain the gradient). Using a time resolution of 1s, 32-bit floating-point precision and assuming that a train unit logs 50 % of a day, this gives a transfer/storage demand of less than 1 MB raw data per train unit and day and about 240 MB raw data per train unit and year. This amount of data is easy to handle with modern data storage and communication facilities and a routinely collection of the necessary data for the VF-approach would therefore be feasible. In addition, the application of data compression software is expected to give an even lower storage and transfer demand than described above. A large-scale collection of data from trains in operation will allow more detailed analyses than presented in this work, *e.g.* a more detailed study of the effect of train and tunnel design and configuration, an analysis of the natural variance in the train resistance in a tunnel as well as further refinement of the estimation method itself (including its assumptions and parameters).

5.3. Mass factor influence

The value of the mass factor in run-time calculations is often taken based on earlier values or crude estimates and is rarely calculated for a given train set individually. In this work the coefficients of the Davis equation are optimised given a specific mass factor value. As shown in Figure 3, the choice of mass factor influences the value of the optimised *C*-coefficient, especially if only acceleration phases are analysed. This is natural, as an increased mass factor will reduce acceleration and therefore lower the contribution that the method assigns to the resistance formula, resulting in lower resistance coefficients. This demonstrates that care has to be taken when using an approximate mass factor for run-time and energy calculations, as the chosen mass factor may not be compatible with the resistance coefficients. Since the resistance coefficients in the VF-approach are optimised to reproduce a velocity time-series using a specific mass factor, realistic run times may be obtained even with an inaccurate mass factor, if this is compatible to the resistance coefficients. This is generally not the case for energy consumption estimates which are more sensitive to a shift in the balance between the mass factor and the resistance.



Figure 3: The open-air coefficient $C_o^{74/75}$ calculated with different values for the mass factor using the acceleration and cruising phase only or the combination of the two. The dataset used consists of all VCU-data $E_{74/75,1-7}$

By comparing the energy consumption during acceleration phases (where the mass-factor is active) and phases of constant speed (where the mass factor is inactive), one may deduct a more correct value for the mass factor. Figure 3 shows that the C-estimate from an acceleration phase depends on the mass factor while a C-estimate from a cruising phase (approximately constant speed) is nearly independent. Since one would expect the C-estimate from an acceleration phase and a cruising phase to be the same, a more correct mass factor estimate can be found by taking the value where these two C-estimates intercept, in this case $\rho \approx 1.1075$.



Figure 4: The C-coefficients for open-air, inside the tunnel and the additional tunnel resistance (C_o , C_t and C_{at} respectively) are shown as function of the mass-factor ρ for Romeriksporten (a) and the set of tunnels Hægebostad/Kvineshei/Gyland/Tronås/Siratunnelen (b)

Since both the C-coefficients estimated in open-air and inside a tunnel show a similar dependence on the mass factor, the additional tunnel resistance is less dependent due to error-cancellation (Figure 4). The additional tunnel resistance shows a slow increase as the mass factor increases. However, considering the magnitude of this increase, one may conclude that the additional tunnel resistance (determined from velocity-fitting) is practically independent of the exact value of the mass factor (in the examined range 1.075-1.125). The additional tunnel resistance in the following chapters is therefore calculated using a constant mass factor ($\rho = 1.1$).

5.4. Estimation of additional tunnel resistance

Using all available data sets for each tunnel, the additional tunnel resistance is shown in Table V. Each tunnel is treated individually, and grouped together based on their aerodynamic cross section. It should be noted that the data set used is not extensive enough to investigate the natural variation of the resistance inside the tunnel, with varying weather conditions (*e.g.* draft, temperature, humidity) or influence from other trains (*e.g.* in the same or opposite direction). Also, no separation between train classes is made. One of the tunnels (Lieråsen) is also equipped with a gate to prevent cold air to enter the tunnel during the winter, which is likely to influence the aerodynamics inside the tunnel. These effects are not included in the presented error estimates, which are only displaying the estimated statistical error within the (limited) data set and are primarily used to analyse the error in the estimation method rather than the expected natural variance of the resistance inside a tunnel. Some data sets produced outliers in the tunnel resistance, *e.g.* Lieråsen and Tanum/Skaugumtunnelen, which indicate that different conditions may give different resistance values.

The additional tunnel resistance observed in this study is displayed as a function of the aerodynamic cross section in Figure 5 together with a set of predictions used in previous studies [54, 57, 61, 62]. It is noticeable that some of the predictions clearly overestimate the tunnel resistance. The predictions from Viriato 6/Opentrack [57, 59] for tunnels with a rough surface give a tunnel resistance more than 2 times as high as the results from this study. In the case of a modern double-track tunnel with an aerodynamic cross section of 87 m², the additional tunnel resistance is overestimated with a factor 8 by Viriato 6/Opentrack. The predictions for smooth tunnels are closer to the results from this study, but still overestimate substantially with the exception of the Lieråsen tunnel.

The additional tunnel resistance for single-track tunnels in the Norwegian high speed rail assessment study has been used to estimate travel times through tunnels in corridor west[62] and to estimate the additional energy consumption running in tunnels[63, 64]. It should be noted that these resistance predictions encompass higher velocities than observed in this work (up to 330 km/t), but since the predictions are velocity-dependent they are expected to also reproduce the resistance at lower velocities. The estimates for corridor west are based on a tunnel of length 30 km and an aerodynamic cross section of 60 and 72 m² respectively. Both these estimates overshoot the observed resistance by at least a factor 5. Although the tunnels in this study are significantly shorter (3-15 km), the additional tunnel length is not expected to give such a high difference and it is likely that the effect that a tunnel has on the maximal possible velocity is significantly smaller than claimed in the Norwegian high speed rail assessment study[62]. The additional tunnel resistance used for the energy consumption estimates[63, 64] is plotted against the recommended aerodynamic cross section [66] in Figure 5. This prediction is more in agreement with the observed resistances in this work and at the same level as the additional resistance observed in the Lieråsen tunnel. The recommended aerodynamic cross section is however larger than in the Lieråsen tunnel, which indicates that the additional resistance is smaller than predicted. However, more observations are needed to be more conclusive and to quantify such an overestimate.

The equations used in Togkjør[54, 55] give reasonable predictions for both the narrow single-track tunnels and the narrow double-track tunnel (Lieråsen) but overestimates the resistance for tunnels with a larger aerodynamic cross section. The predictive equations from SBB[61] fit very well to the results from this study, except for Lieråsen (narrow double-track tunnel). However the equations agree well with the outlier in the Lieråsen data set.

Table V: The coefficient for additional tunnel resistance obtained with velocity-fitting using all available data sets. The estimated standard deviation is shown as \pm , and estimates from data sets identified as outliers are shown in parentheses

Tunnel	Aerodynamic cross section [m ²]	Individual C_{at} [kg/m]	Grouped C_{at} [kg/m]
Romeriksporten	87	2.4	24 ± 05
Bærumstunnelen	87	5.3	2.4 ± 0.0
Tanumtunnelen	79	3.5	$26 \pm 0.2(20)$
Skaugumtunnelen	79	3.8	$3.0\pm0.3(-2.0)$
Lieråsentunnelen	58	8.8	$8.8 \pm 0.5 (3.5)$
Hægebostadtunnelen	27	14.8	
Kvinesheitunnelen	27	16.0	
Gylandtunnelen	27	14.3	15.2
Tronåstunnelen	27	15.8	
Siratunnelen	27	12.1	



Figure 5: The additional tunnel resistance for the studied tunnels as a function of their aerodynamic cross section. For comparison, a set of predictions are displayed: (---) Togkjør[54, 55], for a tunnel length of 10 km where low and high values are for a train of length 100 and 200 m respectively; (···) SBB[61], where low and high values are for a train with 3 and 7 wagons (length \approx 100 and 200 m) respectively; (---) Viriato 6/Opentrack[57, 59], the low and high values are for a tunnel with smooth and rough walls, respectively; (-) Norwegian high speed rail assessment study, for a tunnel in corridor west[62] (length 30 km, 60 and 72 m²) and for energy consumption estimation[63, 64] (length 10 km, plotted against the recommended cross section of 65 m²[66]).

6. Conclusion

The velocity-fitting approach has been shown to be most stable with respect to the data resolution (time and velocity resolution) and is therefore the recommended approach for the determination of running resistance from trains in operation. As a consequence, acceleration measurements are not considered necessary. In the velocity-fitting procedure, the best results were obtained by optimising the quadratic coefficient only, confirming the effect reported by Kim *et al.*[14].

Variations in the predefined mass factor will in general influence the determined resistance coefficients, however the coefficients for the additional tunnel resistance have been shown to, for practical purposes, not depend on the exact value of the mass factor (in the examined range 1.075-1.125).

Both data from the train event recorder (TER) as well as from the vehicle control unit (VCU) have been used for the estimation of running resistance. The VCU data was logged at a higher resolution and included energy consumption data which gave access to the applied traction and made a larger fraction of the train run available for analysis. This resulted in more precise resistance estimates. The recommended data collection approach is therefore to log velocity data with floating-point precision at a time resolution of at least 2s and to include energy consumption (or traction) measurements if possible. This is considered feasible for the routinely collection of data both in terms of data storage and transfer.

The additional tunnel resistance has been evaluated for a set of 10 tunnels on the Norwegian rail network, giving quadratic coefficients ranging from 2.4-16.0 kg/m for tunnels with an aerodynamic cross section of 87-27 m² respectively, for a train unit of length ≈ 105 m. It has been shown that tunnels of similar aerodynamic cross section give similar values for the additional tunnel resistance. It should be noted that the additional tunnel resistance has only been determined for one train class per tunnel, and that the data set is not extensive enough to investigate the variance in the tunnel resistance due to weather conditions nor the influence of other trains. In addition, the additional tunnel resistance has been estimated for a specific combination of a tunnel and a train, which limits the predictive ability of these results. However, this work outlines a procedure to evaluate the running resistance from trains in operation, which will give access to a large amount of measurements and facilitate more exensive analyses of the tunnel resistance in the future and the possible development of more accurate predictive equations.

The additional tunnel resistance observed in this work has been compared to a set of predictions, and the comparison showed that there is large difference in their predictive quality. The predictions by Viriato 6/Opentrack[57, 59] as well as the resistance used for corridor west of the Norwegian high speed rail assessment study[62] are substantially higher than observed. The predictions used in the energy consumption estimates in the latter study are in better agreement with the observed resistances, but overestimate the additional resistance slightly[63, 64]. The best agreement to the observed resistances is found by the method from SBB[61].

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