Energy
Volume 102, 1 May 2016, Pages 231-243
https://doi.org/10.1016/j.energy.2016.02.025

Modelling the UK Electricity Price Distributions using Quantile Regression

Lars Ivar Hagfors[a] Derek Bunn[b] Eline Kristoffersen[c]
Tiril Toftdahl Staver[c] Sjur Westgaard[c]

[a]Corresponding author. E-mail: lars.i.hagfors@iot.ntnu.no. Department of Industrial Economics and Technology Management, Norwegian University of Science and Technology, Alfred Getz vei 3, 7041 Trondheim, Norway.

[b]Department of Management Science and Operations, London Business School, Sussex Place, Regent's Park, NW1 4SA, London, UK.

[c]Department of Industrial Economics and Technology Management, Norwegian University of Science and Technology, Alfred Getz vei 3, 7041 Trondheim, Norway.

Abstract

In this paper we develop fundamental quantile regression models for the UK electricity price in each trading period. Intraday properties of price risk, as represented by the predictive distribution rather than expected values, have previously not been fully analysed. The sample covers half hourly data from 2005 to 2012. From our analysis we are able to show how the sensitivity towards different fundamental factors changes across quantiles and time of day. In the UK the supply of electricity is to a large extent generated from coal and gas plants, thus the price of gas and coal, as well as the carbon emission price, are included as fundamental factors in our model. We also include the electricity price lagged by one day, as well as demand and margin forecasts. We find that the sensitivities vary across the price distribution. Our findings also suggest that the sensitivity to fundamental factors exhibit intraday variation. We find that the sensitivity to gas relative to coal is higher in high quantiles and lower in low quantiles. We have demonstrated a scenario analysis based on the quantile regression models, showing how changes in the values of the fundamentals influence the electricity price distribution.

Keywords: Electricity markets; Quantile regression; Prices; Risk

1 Introduction and Literature Review

In an economy heavily reliant on electricity, and where the market structure is becoming increasingly complex, considerable time and energy is devoted towards understanding the electricity price formation. Electricity prices are characterized by complicated non-linear relationships to fundamental variables (Chen et al. 2010), and the relationships are challenging to model. Bunn et al. (2016) introduced quantile regression for modelling the electricity price. Whilst they demonstrated the value of quantile models for Value-at-Risk forecasting, compared to the benchmarks of GARCH and CAViAR methods, that study was a methodological comparison and did not address in detail the distinctly different intraday characteristics hour by hour of the price risk distribution. Specifically, in that study, only a single time series of prices at period 38 (GMT 18:30-19:00) was analysed. Yet it is well known that price formation (and hence risk) varies systematically throughout the day with different models generally being specified for peak, off-peak and mid-peak hours to reflect the dynamics of load following and the various technologies setting the marginal prices. With this in mind, it is an open question how the determinants of risk vary on an intraday basis. In this study we address that question through the application of multifactor, quantile regression on all 48 half hourly prices from the GB market, across the range of quantiles from 1% to 99%, estimated over the period 2005-2012. This represents a more complete analysis of the intraday price risks and their separate drivers than has so far been undertaken.

For several agents in the energy market, such as consumers, suppliers, traders and regulators, modelling the tails of price distributions is often more important than formulating central expectations. Due to the sparseness of data in the tails and the extreme sensitivity of the results to misspecification in the functional form of the distribution, modelling can be difficult. Thus, robust parametric methods for specifying predictive distributions (e.g. Guermat and Harris, 2001, 2002), regime switching models (eg. Dias and Ramos, 2014; Arvesen et. al., 2013) as well as semi-parametric formulations for estimating specific quantiles (e.g. Engle and Manganelli, 2004; Gerlach et al., 2011), have characterized recent research.

Quantile regression, introduced by Koenker and Basset in 1978, offers a semi-parametric formulation of the predictive distribution so that the quantiles of the distribution can be estimated with distinct regressions. This makes it possible to estimate different coefficient values for the fundamental factors at different quantile levels. As electricity prices are likely to have different sensitivities to fundamental variables across the price distribution, due to the non-linear properties of the merit order curve, quantile regression is well suited for modelling the electricity prices. Many different models have been developed to capture different price formation processes for normal and extreme events. Karakatsani and Bunn (2008a) applied a Markov regime-switching model, while Chen et al. (2010) used a smooth transition logistic regression model.

With quantile regression we are able to model the quantiles directly, without any assumptions about the distribution of the residuals. Electricity prices are characterized by high volatility, skewness, volatility clustering and large spikes. This highly non-normal behavior of electricity prices makes a semi-parametric technique, such as quantile regression, even more appealing.

By looking at each pre-defined trading period as an individual market, we are able to reveal intraday variation in the price sensitivity towards fundamental factors, as well as varying sensitivities across the price distribution. We model the electricity price as a function of the fundamental factors, resulting in a simple and parsimonious model. The explanatory variables are the price of the main input factors in production, gas and coal, the carbon emission price, the lagged price as well as forecasts of demand and reserve margin. By running separate models for each year, we find that the sensitivities to the fundamental factors are stable over time, implying that our model withstands time-varying structural changes.

Further, we demonstrate a scenario analysis that market participants can use as an example to plan for a range of scenarios concerning the distribution of the price given different input ranges for the fundamental variables. We can create a conditional distribution of the electricity price by changing each fundamental factor, ranging from its minimum to maximum value of our data set. This enables us to detect the main risk drivers at different parts of the day as well as at different parts of the price distribution. This information can be utilized by all market participants in order to reduce risk and make better trading and bidding strategies.

This paper has the following structure: section 2 presents the background of the GB electricity market and section 3 describes the data set we use for our analysis. Section 4 contains a description of the models we apply and the results are reported in section 5. In section 6 you find our scenario analysis. In section 7 we present our conclusion.

2 Electricity Market Fundamentals

2.1 The GB electricity market

Since April 2005, under the British Electricity Trading and Transmission Arrangements (BETTA), the electricity systems of England, Wales and Scotland have been integrated. The transmission system is also linked to continental Europe through interconnectors to France and the Netherlands. Six major retail suppliers, British Gas, SSE, Npower, Scottish Power, E.On and EDF, cover most of the integrated generation market. However, different suppliers operate at different times of the day, thus implying a less competitive environment especially at times of scarcity. When reserve margin is low, the competition will decrease and generators with market power may create market prices substantially above short-term marginal costs.

Electricity is a flow commodity and is sold and consumed continuously and instantaneously. Traded products are therefore defined and sold in the form of metered contracts for the constant delivery of a certain amount of power over a specific period of time. In GB the specified time period is half an hour, giving 48 periods each day. Period 1 corresponds to GMT 00:00-00:30, period 2 corresponds to GMT 00:30-01:00 and so on, ending with period 48, corresponding to GMT 23:30-24:00. The APX (formerly UKPX) is the spot market where power contracts are traded. Members submit their bids electronically up to two days ahead of delivery, and the market is cleared.

In the short run consumers are inelastic (Karakatsani and Bunn, 2008b) and prices are thus a function of demand, competition and costs. The electricity supply curve is a merit order curve, where each plant's spot on the curve represents the cost and capacity of the plant. The difference between costs is mainly due to technology and fuels used in production. The plants with the lowest marginal costs, enter at the lowest level of the curve. These are renewables and nuclear plants. Coal fired plants follow, and together they cover base load, operating most of the time. At the right end of the curve, natural gas enters through CCGT plants, which are fired up to cover peaks in demand. CCGT plants are mostly powered using natural gas, but they can also be fueled using coal and biomass, making them very flexible.

2.2 Electricity price formation

We model the electricity price as a function of the fundamental variables influencing the price.[1] [2] Naturally, the electricity price will to a large extent depend on the price of the main fuels used in production. In 2012 the electricity in Britain was generated from coal (39%), gas (28%), nuclear (19%), renewables (11%) and other sources (3%) (Macleay and Annut, 2013). Gas and coal are the two largest fuel sources and are thus considered fundamental factors in our model. We have chosen not to include renewables. The share was only 4% in 2005, and even though it increased towards the end of our data set, ending at 11% in 2012 (Macleay and Annut, 2013), the share was still not sufficiently high for it to have a large impact on prices in the whole timespan we are studying.[3] Coal is the fuel that emits the most carbon, hence the carbon emission price acts as an add-on to the coal price. For period 38 (GMT 18:30-19:00), Bunn et al. (2016) found that the carbon emission price did not significantly affect the electricity price. This might, however, be different for other periods when coal comprises a larger share of the fuels used in production. Further, we believe it is still important to include the carbon emission price in the model, because it is intended to affect the dynamics between coal and gas based electricity generation.

The market clearing price is set at the level where demand equals supply, thus demand has a crucial role in the price formation process and should be a part of our model. Further, we include the reserve margin forecast, as it reflects the level of scarcity in the market. With inelastic demand the level of scarcity will be crucial for determining the price. We include the demand forecast and reserve margin forecast made by the system operator. These forecasts are available the previous day and may be used by market participants when submitting

[1]The explanatory variables used in this kind of model needs to be specifically adapted to the market under investigation, as well as the period of the day that is being modelled. If the input mix changes dramatically over time one should also allow for time varying coefficients. (Paraschiv et al., 2014)

[2]The model is built on observable variables for some fundamentals that might influence the price formation. We acknowledge that there can be certain unobservable factors that we are unable to include in the model, and therefore we are not able to fully explain the whole price formation.

[3]The share of renewables, particularly wind, has increased significantly after 2012 in the UK. There is therefore a need to investigate specifically how the wind production influences the energy price formation, similar to Hagfors et al. (2016) who investigates the influence on renewables on the price formation in the german energy market.

their bids. We expect the lagged price, fuel prices and demand to have a positive effect on the electricity price, whereas the margin level is expected to have a negative effect. The sensitivity of the electricity price to each fundamental variable, both across time and across quantiles, is elaborated in the sections below.

Lagged price

High prices have a tendency to be followed with high prices. (Bunn et al., 2016) Also, as prices approach marginal cost, we expect them to stabilize at a certain level depending on the degree of market power in the market. Market power can allow producers to keep prices high enough to make a profit, but at the same time keep them sufficiently low to prevent other producers, with technologies higher on the merit order curve, to enter. Market power opens up possibilities for repeated gaming (Rothkopf, 1999), such as signaling between the producers to keep prices above what can be explained by marginal costs. This can be seen as a form of behavioral adaption and is reflected in a high sensitivity to yesterday's price. Because high prices are associated with situations with a strong degree of market power, we expect to see sensitivity to lagged price increasing with higher quantiles as well.

Gas price

We expect the sensitivity to changes in the gas price to be higher in high demand periods, because gas is the main fuel used to cover peak load (demand in excess of base load). Also, we expect electricity prices to be more sensitive to changes in all fuel prices during periods with high demand. This being because higher demand gives producers greater capability to exercise market power, and thus allow changes in fuel prices to be more directly reflected in electricity prices. Sensitivity to the gas price should increase with quantiles.

Coal price

Unlike gas, coal is mainly used to cover base load, and thus the use of coal is relatively constant for the entire 24-hours. However, as for gas, we expect electricity prices to be more sensitive to changes in all fuel prices during periods with high demand. This also means that sensitivity to the coal price is not expected to increase across quantiles in the same way as gas price sensitivity.

Because coal comprises a larger share in base load production than gas, the electricity price should be more sensitive to changes in coal price than gas price at low quantiles and in periods when demand is low. Likewise, prices should be more sensitive to changes in gas price than coal price for high quantiles and in periods when demand is high, since gas plants are fired up to cover demand in excess of base load. We note that this to some extent is determined by the relative price levels.

Carbon emission price

Because coal emits more carbon than gas when utilized in power production, the carbon emission fee will have a higher incremental effect on the coal price compared to the gas price. The intention is that in times when the coal price lies below the gas price, the carbon emissions cost will rise to prevent substitution from gas to coal. Therefore, we expect the variation in sensitivities to carbon emission prices across periods and quantiles to follow the same trend as sensitivity to the coal price. However, in our data the carbon emissions price is very low, mainly due to too many issued quotas, and thus it is unclear how it actually effects the electricity price.

Demand forecast
We expect demand to have the largest effect on electricity prices during the day and early evening, when the demand is higher and the margin levels are lower. Hence, prices should also be more sensitive to demand at higher quantiles. As the supply function is convex, we expect this sensitivity to increase non-linearly with higher quantiles. High prices are likely to coincide with low margin levels, making the price very sensitive to changes in demand. Also, an increase in demand above normal levels implies firing up additional plants higher on the merit order curve, thus pushing prices up.

Reserve margin forecast
A reduction in reserve margin will push prices upwards. We expect prices to be more sensitive to margin levels in high demand periods and for the higher quantiles for each period. These situations are likely to represent times of scarcity. Since demand is inelastic and producers have more capacity to exercise market power at times of scarcity, changes in margin is expected to cause large price changes.

3 Data

3.1 Variable description

Our data set spans from 22.04.2005 until 28.06.2012. Two events make 2005 a natural starting point for our analysis. Firstly, Scotland was included in the British wholesale electricity market April 1, 2005. Secondly, the EU Emission Trading Scheme was established on January 1, 2005, allowing carbon emission trading to commence at the beginning of 2005. For electricity prices we have data for the same period, although there are some observations missing. For these periods we have interpolated linearly by taking the average of the price the previous and next day, within the same period. For fuel prices we have daily prices, weekends not included. By the same principle as for electricity prices we have interpolated using the prices quoted for Friday and Monday.

[Table 1 about here.]

Gas, coal and carbon emission prices are all lagged by one day in the model. Demand forecasts and margin forecasts are both made the previous day. This means that all variables used in the analysis are known to the market before the power exchange closes for the trading period concerned. This is done to ensure exogeneity of the explanatory variables.

Power price
UKPX (now APX) is the day-ahead and on-the-day power exchange, allowing high frequency trading up to an hour before real time. Every day consists of 48 periods of 30 minutes each. Prices are quoted in £/MWh and represents the volume weighted prices for each period as cleared on the exchange in the preceding 48 hours.

Demand forecast
This forecast is made available the previous day by the System Operator for each half-hourly trading period. It reflects available market information and avoids the endogeneity issues concerning simultaneity, which might be a problem when using actual demand, since it is released the day before. The basis of which the demand forecast is calculated, however, is not known to us. This means that other endogeneity issues such as omitted variables and measurement errors might still be a problem. However, because demand is such an important price driver, we still choose to include it in our model.

Reserve margin forecast
The System Operator makes forecasts of the available reserve margin for each half-hourly trading period. This is defined as the difference between the sum of the maximum available output capacities, as initially nominated by each generator prior to each trading period, and the demand forecast described above.

Gas price
We use the daily UK natural gas spot price from the National Balance Point (NBP). The price is quoted in £/MMBtu(MM British Thermal Unit).

Coal price
We use the daily HWWI world index coal price. The price is quoted in $/ton. We have translated it into £/ton, taking into account the $/£rate.

Carbon emission price
We use the EEX-EU carbon emissions allowance daily spot price. The price is quoted in €/ton. We have translated it into £/ton, taking into account the €/£rate.

3.2 Organization of data

In order to give an overview of the differences across periods in our analysis we chose to divide the 48 periods into six groups each describing a certain time of the day. This allows us to capture similar features such as sensitivities to the different exogenous variables and price characteristics for a specific time of the day, thus providing relevant information to different market participants. A representative period for each group has been chosen to present a comparison of differences throughout the day. These are outlined in Table 2 below.

[Table 2 about here.]

We further define period 10 as the anti-peak, period 25 as the day-peak and period 35 as the super-peak for each day. From Table 3 we see that of all the periods, period 35 has the highest average price of the day and exhibits the highest volatility, skewness and kurtosis in our data set. Period 10 has the lowest average price, and exhibits the lowest volatility, skewness and kurtosis.

3.3 The data series

Figure 1-4 shows the evolution of the power prices, the spot prices of gas, coal and carbon emission, as well as the day ahead demand and reserve margin forecasts. Due to the large data set we have chosen only to show the data series of the representative periods for electricity prices, demand and margin forecasts. The price series reveal typical spot electricity features such as spikes, mean reversion, seasonality and high, time varying volatility. Figure 1 also shows clear signs that the price dynamics vary between the different time periods.

[Figure 1 about here.]

[Figure 2 about here.]

[Figure 3 about here.]

[Figure 4 about here.]

3.4 Descriptive statistics

In Table 3 we present a summary of descriptive statistics for the representative periods, confirming what we observed in Figure 1, that is a high standard deviation and substantial skewness and kurtosis. A more extensive analysis of the descriptive statistics for all 48 periods has also been performed. All skew coefficients are positive. This effect is anticipated for electricity markets at that time and reveals that extreme price outliers occur on the upside of the average. Extreme prices are also common in electricity markets. We also see that there is high correlation between the period's mean price and the standard deviation, skewness and kurtosis levels. We detect severe serial correlation in the data. However, it drops substantially from lag 1 to 2. For some periods we detect prominent autocorrelation in lag 7.

[Table 3 about here.]

As a benchmark we ran OLS regressions and performed various residual tests, revealing that the residuals are non-normal, heteroscedastic, serial correlated and have ARCH-effects. Results for our representative periods are summarized in Table 4. We also performed an ADF test for stationarity in the series. The results are reported in Table 5 and 6. Electricity price and margin forecasts appear stationary, so does the demand forecast for the most part. We cannot, however, reject the unit root null hypothesis for gas, coal and carbon emission prices.

[Table 4 about here.]

[Table 5 about here.]

[Table 6 about here.]

4 Models

4.1 Linear quantile regression

Linear quantile regression was introduced by Koenker and Basset in 1978, and seeks to compute a set of regression functions, each corresponding to a different quantile of the conditional distribution of the price. The difference between quantile regression and OLS is that while OLS estimates the regression coefficients so that the regression line run through the average of the data set, quantile regression lines will pass through different quantiles of the distributions. For lower quantiles the majority of the data set will lie above the quantile regression line. For higher quantiles the majority of the data set will lie below the quantile regression line (Alexander, 2009). The advantage of quantile regression is that we are able to investigate the relationship between the dependent and independent variables across the entire distribution, and thus build up a more complete picture of how fundamental factors affect the electricity price in various price ranges.

From a risk perspective we want to be able to estimate the tail dependencies accurately and quantile regression works well for this purpose. Quantile regression is closely related to value at risk in estimating the price at extreme quantiles. For traders and risk managers it is thus a useful tool for assessing price risk and developing hedging strategies.

The quantile regression model is semi parametric, thus we do not make any assumptions about the distribution of our data or about the residuals. Due to the highly non-normal behavior of the electricity price, as well as time-varying volatility, this is an advantage for our research. Significance testing of quantile regression is still very much in the exploratory stage, and no single approach has yet gained widespread support. For further discussion, see Volgushev et al(2013).

We let $q\epsilon(0,1)$ be quantile $1\%, 5\%, 10\%, 25\%, 50\%, 75\%, 90\%, 95\%, 99\%$. Our linear quantile regression model will then be given by:

$$\begin{aligned} Q_q(\ln\mathrm{P}_{i,t}) =& \alpha_i^q + \beta_{i,1}^q \ln\mathrm{P}_{i,t-1} + \beta_{i,2}^q \ln\mathrm{GAS}_{t-1} + \beta_{i,3}^q \ln\mathrm{COAL}_{t-1} + \beta_{i,4}^q \ln\mathrm{CO2}_{t-1} + \\ & \beta_{i,5}^q \ln\mathrm{DF}_{i,t} + \beta_{i,6}^q \ln\mathrm{MF}_{i,t} \end{aligned}$$

Where $i = 1, ..., 48$

Further we let $\boldsymbol{X}_{i,t}$ be the 6-dimensional vector, representing the six independent variables in Section 3.1. We can then rewrite the model as:

$$Q_q(\ln\mathrm{P}_{i,t}|\boldsymbol{X}_{i,t}) = \alpha_i^q + \boldsymbol{X}_{i,t}\boldsymbol{\beta}_i^q$$

We find the q quantile regression coefficients for period i, $\hat{\alpha}_i^q$ and $\hat{\boldsymbol{\beta}}_i^q$, as the solution to the following minimization problem:

$$\min_{\alpha_i^q, \boldsymbol{\beta}_i^q} \quad \sum_{t=1}^{T} (q - \mathbf{1}_{\ln \mathrm{P}_{i,t} \leq \alpha_i^q + \boldsymbol{X}_{i,t}\boldsymbol{\beta}_i^q})(\ln \mathrm{P}_{i,t} - (\alpha_i^q + \boldsymbol{X}_{i,t}\boldsymbol{\beta}_i^q))$$

$$\text{Where} \quad \mathbf{1}_{\ln \mathrm{P}_{i,t} \leq \alpha_i^q + \boldsymbol{X}_{i,t}\boldsymbol{\beta}_i^q} = \begin{cases} 1 & \text{if } \ln \mathrm{P}_{i,t} \leq \alpha_i^q + \boldsymbol{X}_{i,t}\boldsymbol{\beta}_i^q \\ 0 & \text{otherwise} \end{cases}$$

We run the quantile regression in EViews, obtaining 432 models (48x9). The associated standard errors are obtained using the Huber Sandwich method. This method is robust when the residuals are heteroscedastic (Koenker, 2005). The use of natural logarithms implies that our coefficients will be interpreted as elasticities, i.e. how sensitive the electricity price is towards a change in the fundamental factors, measured in relative terms.

5 Quantile Regression Results

In Table 7 we present the comprehensive quantile regression results. We show the quantile regression results for the six representative periods, with its associated pseudo R-squared (Koenker and Machado, 1999).

[Table 7 about here.]

In general, coefficients are significant, and we find that lagged price, fuel prices and demand forecast have a positive effect on electricity prices, while margin forecast has a negative effect. At night, when market activity is low, lagged price is by far the variable in our model that affect prices most. As activity increases during the day, demand, margin and fuel prices affect the electricity price with increased strength, while the lagged price affects electricity prices less. Carbon emission prices have little or no effect on electricity prices.

We use Pseudo R-square to measure the goodness-of-fit of the model for each associated period and quantile. In general the Pseudo R-square is quite stable over the whole 24 hours, and at a level ranging from 0.42 to 0.76. We note that the pseudo R-squares are slightly higher for off-peak than peak periods. This suggests that during times of scarcity, electricity spot prices are not completely determined by fundamental factors, but partly influenced by the exertion of market power by producers.

Lagged price
For the trading periods from the late morning until midnight (period 16-48), sensitivity is generally increasing with higher quantiles, as we expected. However, for the periods with the lowest activity, during the second half of the night (period 7-12), sensitivities are instead decreasing with higher quantiles. For the first half of the night and the early morning (period 1-6 and 13-15), sensitivities are larger for the middle quantiles. These are all periods with moderate demand. For the periods with the least demand, in the second half of the night, the lowest quantiles probably represent prices very close to marginal cost and behavioral adaption might thus explain why these prices are similar to the ones observed on the previous day. During the night, demand is neither high nor low, and under such circumstances it seems

reasonable that the middle quantiles, representing "normal" prices, will be consistent with the corresponding prices the day before.

[Figure 5 about here.]

Gas price

For nearly all periods, sensitivity is higher for periods with high demand than for periods with low demand. The highest sensitivities are found during the day and early evening, while the lowest sensitivities are found during the night and late evening. This is according to expectations.

During the night, early morning and first half of the late morning (period 1-19), sensitivities are generally increasing with higher quantiles. From period 12 to 13, when we approach daytime, the coefficient makes a positive jump, larger for the lowest quantiles. This underlines the fact that prices are much more sensitive to changes in gas price when demand is high. In the second half of the late morning, afternoon and early evening (period 20 to 38), coefficients are more equal across quantiles (except the 95% and 99% quantile), with most coefficients within the 0.25-0.35 range. Towards the end of this period, the coefficients start to decrease with higher quantiles, a trend that continues for the rest of the day.

[Figure 6 about here.]

Coal price

As expected, sensitivity is higher for periods with high demand than for periods with low demand. The highest sensitivities are found during the day and evening, while the lowest sensitivities are found during the night. However, we can see that the variation is smaller than for gas. This is according to expectations, because coal mainly is used for base load production.

In the late morning, afternoon, early evening and late evening (period 15-46) the coefficient is higher for low quantiles than high quantiles. During several evening periods, the coefficients are so small that they even are insignificant for the 95% and 99% quantiles. At night, sensitivities are generally highest for the extreme quantiles. The reason for the increased sensitivity at high quantiles might be that during nighttime few gas plants are operating and thus coal plants will cover peaks in demand caused by e.g. extreme weather. High prices during the night will therefore be very sensitive to changes in the coal price.

[Figure 7 about here.]

Carbon emission price

As suspected, the carbon emission price has little effect on electricity prices. The coefficient is generally close to zero and often insignificant. It is worth noticing that the carbon price was equal to zero for quite some time in our data set.

The carbon emissions price does, however, tend to follow the sensitivity pattern of coal. It is generally increasing with higher quantiles during the night, and decreasing with higher quantiles during the day and evening. During the night the coefficient is mainly insignificant for low quantiles, while during the day and night it is often insignificant for high quantiles.

[Figure 8 about here.]

Demand forecast

[Figure 9 about here.]

Looking at the results for the night, early morning and late morning (period 45-22) as well as the early evening (period 31-38) we observe that sensitivities for the middle quantiles generally are higher when the period's demand level is higher. For the extreme quantiles there is greater variation.

In the late morning (period 15-22), the sensitivity is slightly increasing with quantiles. In the night and early morning (period 45-14) as well as tge early evening (period 31-38) sensitivities are generally highest for the extreme quantiles. The difference across quantiles is clearly largest during the night.

We also notice some slightly negative coefficients around the beginning of the afternoon and the beginning of the late evening (periods 23 and 39). This is when the demand is dropping and marginal technolgoies may be reluctant to be called off. Hence their offers become more competitive. With higher demand and mor expensive plant being called, this effect is likely to be more pronounced.

Reserve margin forecast

Margin levels affect the price according to expectations, with sensitivity increasing in periods with higher demand and with higher quantiles. During the periods of night with the lowest demand levels (period 7-12), the coefficient gets so low that margin forecast has an insignificant effect on prices for the 1%-10% quantiles. During peak hour periods, the effects from reserve margin get higher (in absolute terms) with higher quantiles.

[Figure 10 about here.]

A comparison of sensitivities to the prices of gas and coal

Looking at Figure 11 we clearly see that the coal coefficient generally is larger than the gas coefficient at low quantiles, whilst the opposite is the case at high quantiles. This is according to expectations.

The coal price affects the electricity price more for the 5% quantile as compared to the median, while the gas price has more effect for the 95% quantile as compared to the median. This indicates that producers are generally more vulnerable to coal price volatility, while consumers are more exposed to gas price volatility. Not surprisingly, electricity price is more sensitive to demand and margin forecasts at the 95% quantile than at the 5% quantile.

[Figure 11 about here.]

6 Scenario Analysis Based on the Quantile Regression Model

In this section we present an example that demonstrates how the models can be used to perform a scenario analysis, showing the effect of changes in the fundamental variables on the electricity price distribution. Starting with a base scenario, we can introduce shocks to one or more fundamental variable and obtain the resulting price distribution.

Our base scenario was formed by applying the values of the fundamental variables from the last day of our data set, 28.06.2012, to the associated quantile regression models for period $10, 14, 19, 25, 35$ and 43. The actual values on this date are reported in Table 8. By looking at ranges of values for the fundamentals, we are able to construct scenarios of distributions for the electricity price. In our example we investigate the effect of shocks of varying magnitude to the reserve margin forecast. In a similar way, we can also analyze the effects on the price distribution from changing other fundamental variables, individually or jointly. Hence we can directly investigate how a change in one or more of the independent variables affect the different value at risk estimates for the different time periods.

[Table 8 about here.]

6.1 Scenario analysis example - Reserve margin forecast

[Figure 12 about here.]

We applied a set of margin forecasts ranging from 1500MW to 40925MW, which is equal to the minimum and maximum margin forecasts in our data set. The results can be seen in Figure 12. For all periods and parts of the distribution, a decrease in reserve margin will lead to an increase in the electricity price. However, the effect on the electricity price is rapidly decreasing with higher levels of reserve margins. As soon as the reserve margin reaches a threshold level the effect converges for all quantiles and approach zero. When there is no scarcity in the market prices will simply not respond to changes in the margin forecast. On the other hand, if margin levels fall below the threshold, prices will respond to this by increasing exponentially as the margin levels drop further. Changes in margin below the threshold affect the electricity prices more than any other fundamental variable. This implies that both producers and buyers should monitor the threshold level carefully, and take into account whether margin levels are expected to fall below or rise above it when placing their bids. The threshold level is different for each period and quantile. During nighttime (period 10 and 43) the effect is small and almost equal to zero for levels of reserve margin above 1500MW. During daytime the effect on the price of changes in reserve margin is larger. For low margin levels the conditional price distribution has a long right tail. The thresholds levels are higher during day than night, and increasing with higher quantiles during daytime. Market participants should pay close attention when margin levels drop below 20000MW. Above this level effects on the price will be minor. With the extreme impacts of forecasted scarcity on the electricity price, producers will have incentives to under-report the production capacity of their plants. This underlines the importance of strong regulation and surveillance of the reporting procedures.

7 Conclusions

Using quantile regression, we have characterized the non-linear effects of fundamental factors on the wholesale electricity price for each delivery period in the UK electricity market. The complex market dynamics were confirmed as we found that the sensitivity to the different factors vary substantially both across the day and across the price distribution. We have paid special attention to the tails, both in our regression analysis and in the scenario analysis.

We demonstrated how lagged prices, prices of gas, coal and carbon, and forecasts of demand and reserve margin influence the price distribution in each of the 48 periods in rather intuitive ways. In general, we find positive elasticities for the underlying fuel commodities. It was revealed how the sensitivity to gas relative to coal is increasing with the demand level throughout the day. We found, that for our data set, carbon emission prices generally had no significant effect on electricity prices. The sensitivity to changes in demand is generally positive, but the way its impact on prices develops over quantiles varies with the time of day. The elasticity of reserve margin is negative, with increased impact on higher quantiles and in periods with high demand. We confirm the positive sensitivity to lagged price and how it is decreasing with the demand level. We found that the model explained more of the variation in electricity prices, as measured by the adjusted R-squared, in off-peak than in peak periods. This is likely because periods with low margin allow producers to exercise market power more effectively, pushing prices above what is explained by the fundamental variables included in our model.

By performing an example scenario analysis, we have demonstrated how scenario analysis can be used to illustrate the actual magnitude changes in the fundamental variables have on the electricity price distribution. The effect of previous prices, as captured by the lagged price, represents the main risk factor for producers, in terms of large price drops. Additionally, producers face risk if the price of the fuel used in production increases, and they cannot recover the extra cost through a sufficient increase in the electricity price. The main risk drivers for buyers and consumers are a high lagged price and low levels of reserve margin. In general, the main risk is carried by the consumer side.

We believe that our findings have important implications for market participants in both the spot and financial electricity market. Our paper provides a deeper understanding of the price formation process and reveals insight on the main risk drivers. Based on this market participants can fine tune their bids and reduce their exposure to risk. An advantage of quantile regression is that it is easy to apply compared to alternatives such as regime switching models or CaViaR based models. This gives it a widespread appeal, and increase the probability that it will be implemented by market participants.

The next natural step is to do forecasts based on this model and test its forecasting ability. Further research can extend the quantile regression analysis to include more explanatory variables. Our model can then serve as a point of reference. Renewables have over the time span of our data set become a much more influential fuel source, and will have a natural place in future electricity market modelling, when the share of electricity produced from renewables has stabilized at a sufficient level. For some periods the model might benefit from including lag 7 of the endogenous variable in order to capture weekday effects. Also, a proxy for market power could be included for the peak periods. By comparing the goodness-of-fit and forecasting performance to our model, one can evaluate whether these modifications are successful.

References

Alexander, C. *Market Risk Analysis: Practical Financial Econometrics*, vol. 2. John Wiley & Sons, 2009.

Arvesen, Ø., Medbø, V., Fleten, S.-E., Tomasgard, A., and Westgaard, S. Linepack storage valuation under price uncertainty. *Energy 52* (2013), 155–164.

Bunn, D., Andresen, A., Chen, D., and Westgaard, S. Analysis and forecasting of electricity price risks with quantile factor models. *The Energy Journal 37*, 2 (Forthcomming 2016).

Chen, D., and Bunn, D. W. Analysis of the nonlinear response of electricity prices to fundamental and strategic factors. *IEEE Transactions on Power Systems 25*, 2 (2010), 595–606.

Dias, J. G., and Ramos, S. B. Energy price dynamics in the us market. insights from a heterogeneous multi-regime framework. *Energy 68* (2014), 327–336.

Engle, R. F., and Manganelli, S. Caviar: Conditional autoregressive value at risk by regression quantiles. *Journal of Business & Economic Statistics 22*, 4 (2004), 367–381.

Gerlach, R. H., Chen, C. W., and Chan, N. Y. Bayesian time-varying quantile forecasting for value-at-risk in financial markets. *Journal of Business & Economic Statistics 29*, 4 (2011), 481–292.

Guermat, C., and Harris, R. D. Robust conditional variance estimation and value-at-risk. *Journal of Risk 4*, 2 (2001), 25–42.

Guermat, C., and Harris, R. D. Forecasting value at risk allowing for time variation in the variance and kurtosis of portfolio returns. *International Journal of Forecasting 18*, 3 (2002), 409–419.

Hagfors, L. I., Molnar, P., Paraschiv, F., and Westgaard, S. Using quantile regression to analyze the effect of renewables on eex price formation. *Journal of Physics, Conference Series* (Forthcoming 2016).

KARAKATSANI, N. V., AND BUNN, D. W. Forecasting electricity prices: The impact of fundamentals and time-varying coefficients. *International Journal of Forecasting 24*, 4 (2008a), 764–785.

KARAKATSANI, N. V., AND BUNN, D. W. Intra-day and regime-switching dynamics in electricity price formation. *Energy Economics 30*, 4 (2008b), 1776–1797.

KOENKER, R. *Quantile regression*. Cambridge university press, 2005.

KOENKER, R., AND BASSETT JR, G. Regression quantiles. *Econometrica: journal of the Econometric Society* (1978), 33–50.

KOENKER, R., AND MACHADO, J. A. Goodness of fit and related inference processes for quantile regression. *Journal of the american statistical association 94*, 448 (1999), 1296–1310.

MACLEAY, I., HARRIS, K., AND ANNUT, A. *Digest of United Kingdom Energy Statistics 2013*. Department of Energy and Climate Change, 2013.

PARASCHIV, F., ERNI, D., AND PIETSCH, R. The impact of renewable energies on eex day-ahead electricity prices. *Energy Policy 73* (2014), 196–210.

ROTHKOPF, M. H. Daily repetition: A neglected factor in the analysis of electricity auctions. *The Electricity Journal 12*, 3 (1999), 60–70.

VOLGUSHEV, S., BIRKE, M., DETTE, H., NEUMEYER, N., ET AL. Significance testing in quantile regression. *Electronic Journal of Statistics 7* (2013), 105–145.

Test

A Appendix A: List of figures and tables

List of Figures

List of Tables

Figures

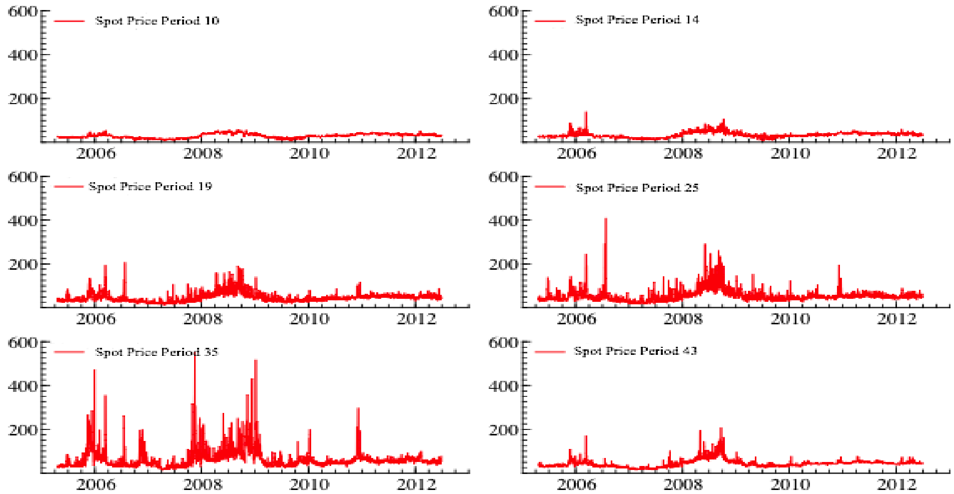


Figure 1: The actual series of the power price for period 10, 14, 19, 25, 35 and 43 respectively. The data spans from 22.04.2005 to 28.06.2012.

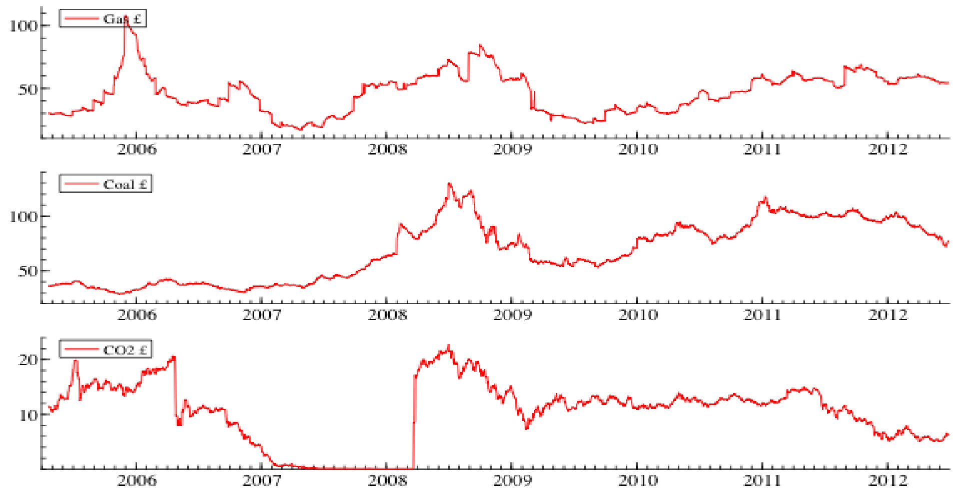


Figure 2: The actual series of the daily UK natural gas spot price from the National Balance Point (NBP) (quoted in £/MMBtu), the daily HWWI world index coal price(translated into £/ton) and the EEX-EU carbon emissions allowance daily spot price(translated into £/ton), respectively. The data spans from 22.04.2005 to 28.06.2012.

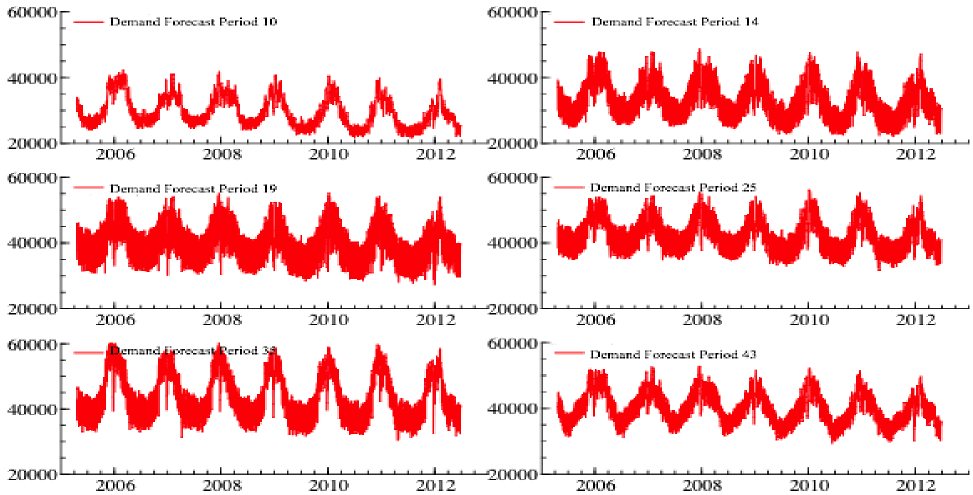


Figure 3: The actual series of the UK national demand forecast from the system operator for period 10, 14, 19, 25, 35 and 43 respectively (quoted in MW). The data spans from 22.04.2005 to 28.06.2012.

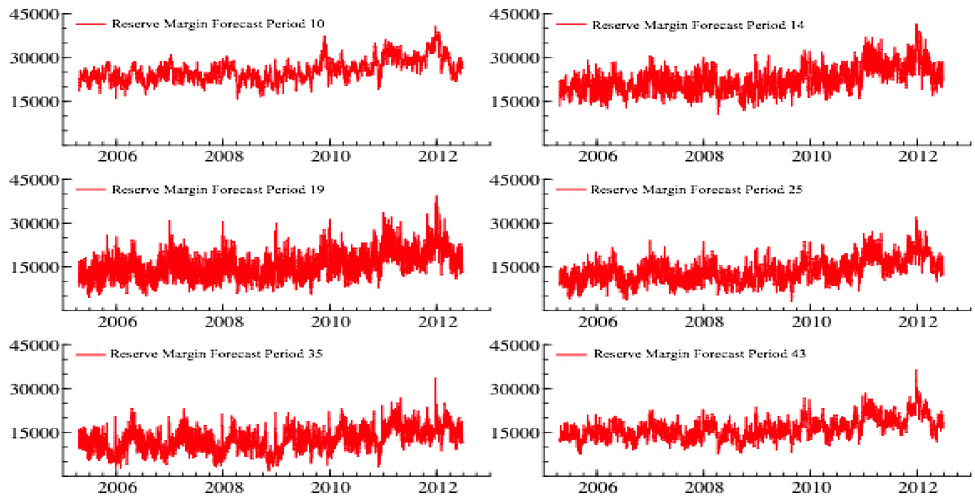


Figure 4: The actual series of the UK national forecast of reserve margin from the system operator for period 10, 14, 19, 25, 35 and 43 respectively (quoted in MW). The data spans from 22.04.2005 to 28.06.2012.

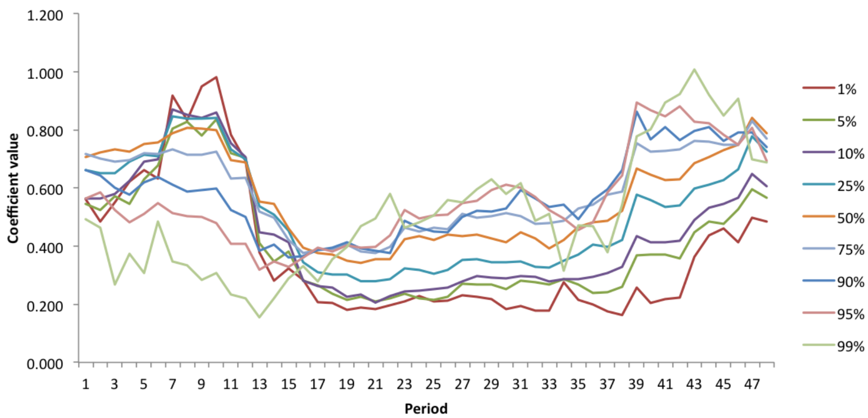


Figure 5: The graphs show the development in the lagged price coefficient value associated with each quantile, across all 48 periods, found from quantile regression.

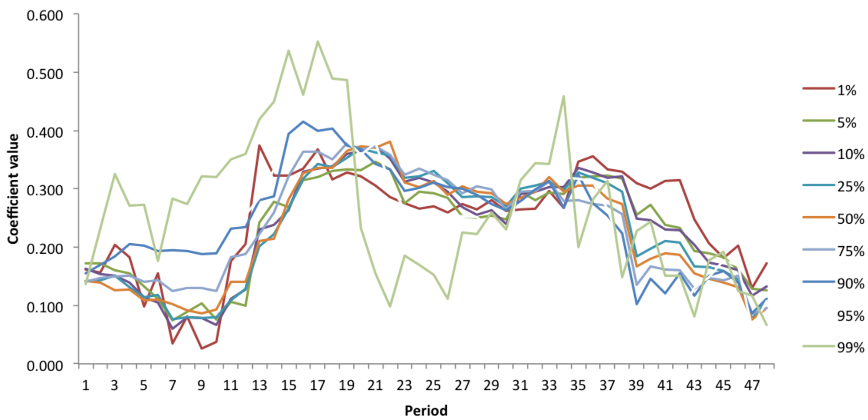


Figure 6: The graphs show the development in the gas price coefficient value associated with each quantile, across all 48 periods, found from quantile regression.

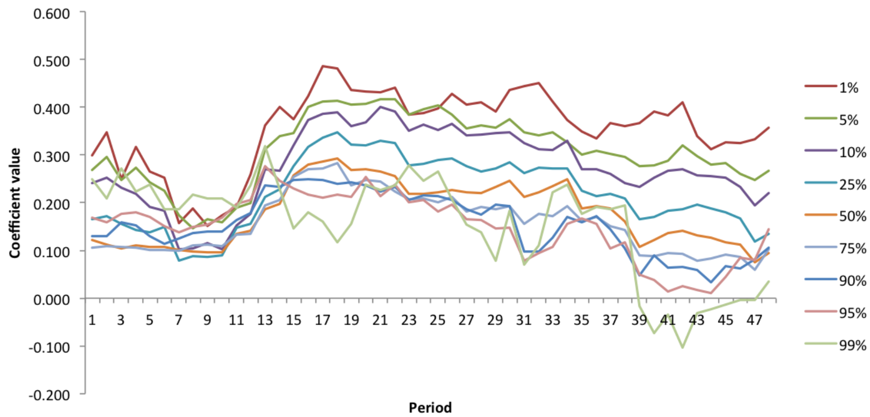


Figure 7: The graphs show the development in the coal price coefficient value associated with each quantile, across all 48 periods, found from quantile regression.

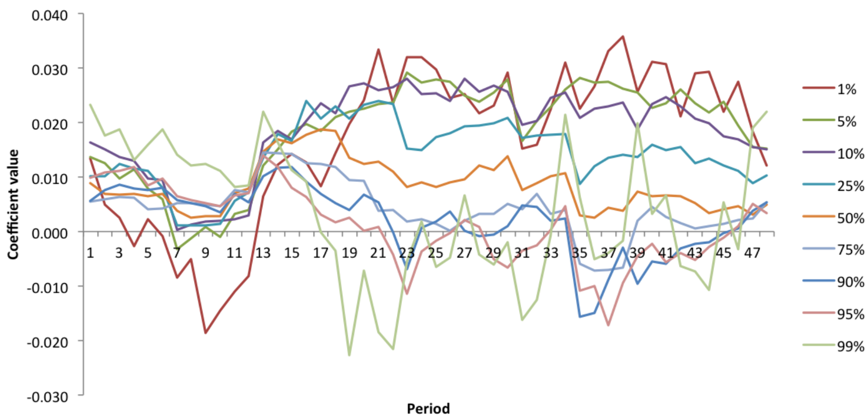


Figure 8: The graphs show the development in the carbon emission price coefficient value associated with each quantile, across all 48 periods, found from quantile regression.

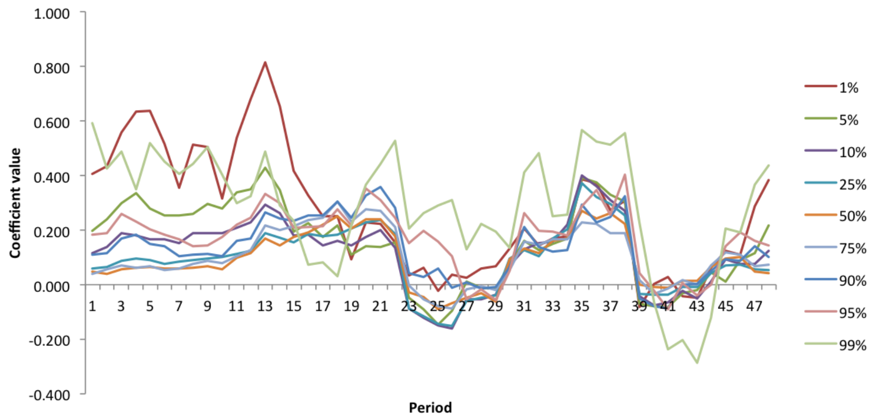


Figure 9: The graphs show the development in the demand forecast coefficient value associated with each quantile, across all 48 periods, found from quantile regression.

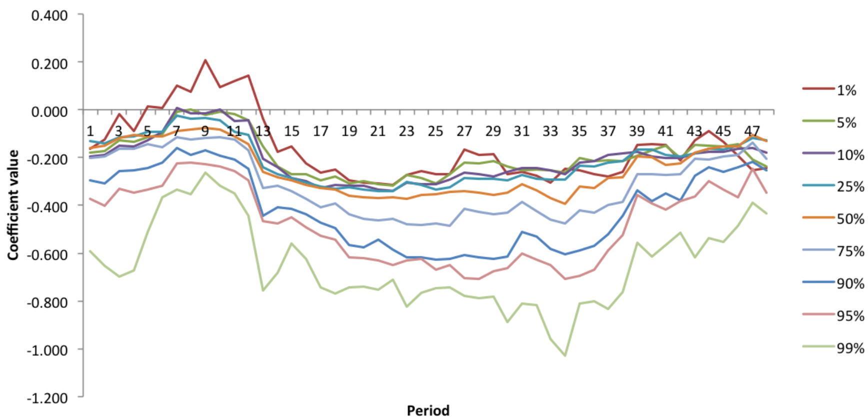
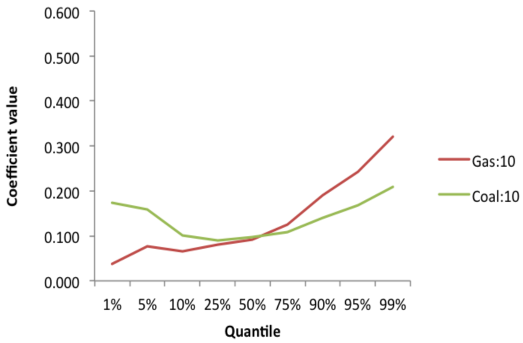
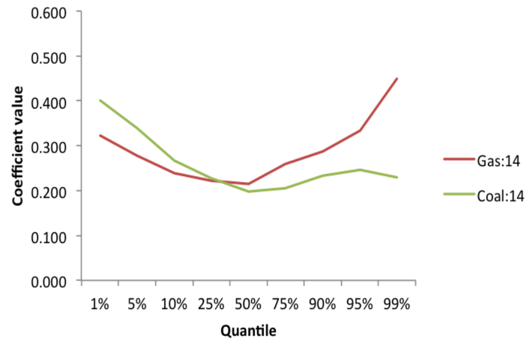


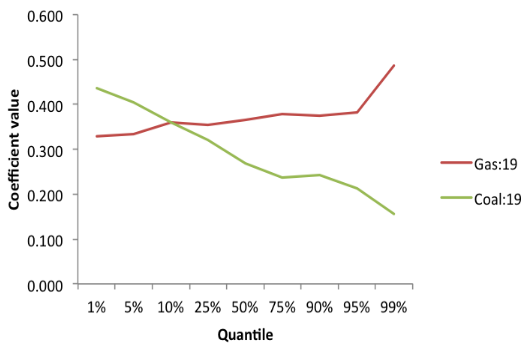
Figure 10: The graphs show the development in the reserve margin forecast coefficient value associated with each quantile, across all 48 periods, found from quantile regression.



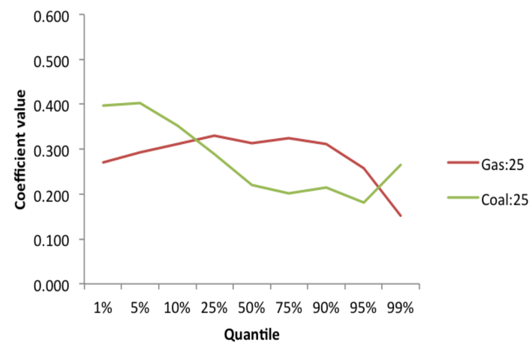
Period 10



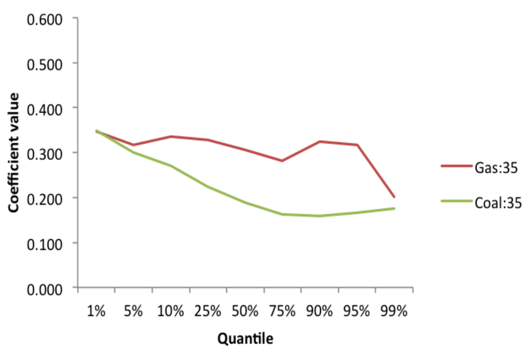
Period 14



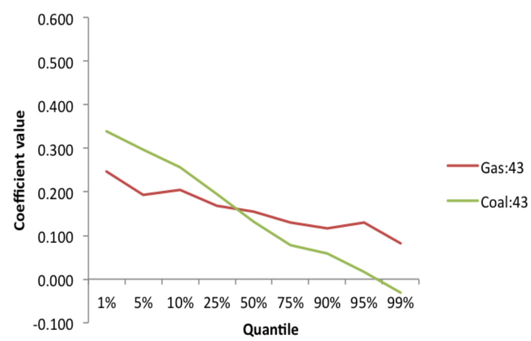
Period 19



Period 25



Period 35



Period 43

Figure 11: The graphs show the development in the gas price coefficient value relative to the coal price coefficient value across quantiles for period 10, 14, 19, 25, 35 and 43, found from quantile regression.

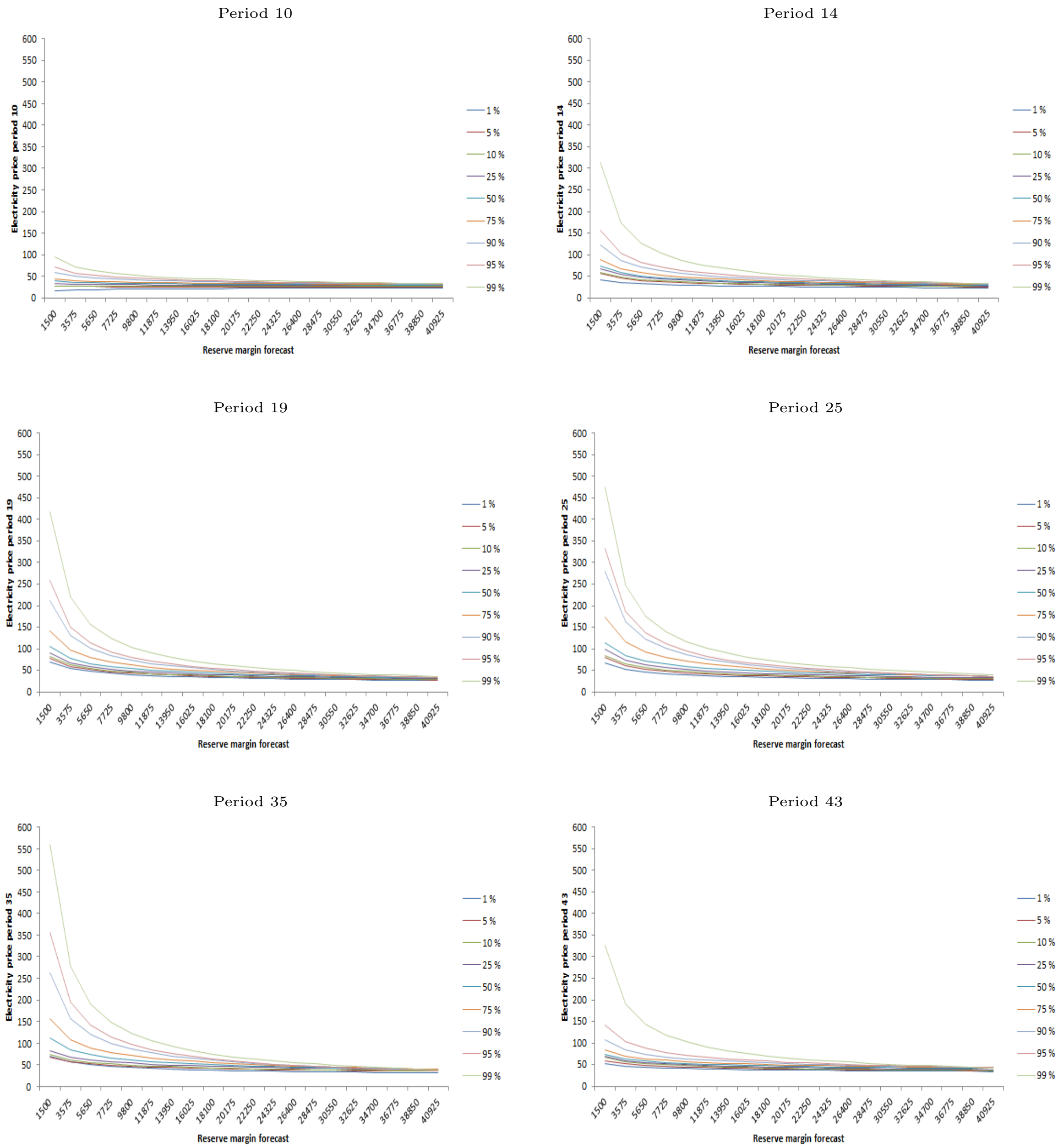


Figure 12: Scenario analysis of the power electricity price for period 10, 14, 19, 25, 35 and 43, when the reserve margin forecast varies from 1500MW to 40925MW. The base scenario is calculated by applying data from the last day of the data set, 28.06.2012, to the different quantile regression models. The reserve margin forecast on 28.06.2012 was 26442MW, 22005MW, 13392MW, 12068MW, 11812MW and 18260MW for period 10, 14, 19, 25, 35 and 43, respectively.

Tables

Table 1: Data granularity of the explanatory variables in our model.

Variable	**Half-hourly**	**Daily**
Power Prices	X	
Gas Prices		X
Coal Prices		X
Carbon Emission Prices		X
Demand Forecasts	X	
Reserve Margin Forecasts	X	

Table 2: We have chosen 6 periods; 10, 14, 19, 25, 35 and 43, each representing a certain time period of the day, that we will focus on in our analysis.

Time of day	Period	Time period	Representative Period
Night	47-12	23.00-06.00	10 (04.30-05.00)
Early Morning	13-15	06.00-07.30	14 (06.30-07.00)
Late Morning	16-22	07.30-11.00	19 (09.00-09.30)
Afternoon	23-31	11.00-15.30	25 (12.00-12.30)
Early Evening	32-38	15.30-19.00	35 (17.00-17.30)
Late Evening	39-46	19.00-23.00	43 (21.00-21.30)

Table 3: Descriptive Statistics of the power price for period 10, 14, 19, 25, 35 and 43.

Period	**Mean**	**Median**	**Maximum**	**Minimum**	**Volatility**	**Skewness**	**Kurtosis**
10	29.51	29.10	57.62	6.38	9.64	0.18	2.31
14	34.74	33.50	139.36	5.27	12.42	1.11	6.55
19	47.22	43.71	206.14	13.79	21.49	2.16	10.95
25	54.20	48.38	409.66	15.89	28.43	3.38	23.90
35	61.94	51.78	553.30	13.22	42.08	3.93	28.98
43	43.97	41.91	208.46	15.98	17.94	2.38	13.92

Table 4: Test statistics from the Jarque Bera normality test, Breuch-Godfrey LM test, White's Heteroscedasticity test and ARCH LM test. *** indicates that we reject the respective null hypothesis at the 1% level.

Period	**Jarque Bera Test**	**Breuch-Godfrey Test**	**White's test**	**ARCH LM test**
10	7659.96***	44.45***	523.75***	169.63***
14	13955.5***	284.06***	331.18***	145.39***
19	1365.8***	125.69***	311.5***	57.96***
25	517.69***	31.8***	431.75***	107.04***
35	1278.07***	46.55***	402.16***	61.02***
43	3451.77***	55.39***	63.72***	63.72***

Table 5: ADF test for stationarity in the power Price, the UK national demand forecast and the UK national margin forecast, for period 10, 14, 19, 25, 35 and 43. The data spans from 22.04.2005 to 28.06.2012. We have chosen 5 lags in the ADF test. *, ** and *** indicates that we reject the null hypothesis and find stationarity at the 10%, 5% and 1% level respectively.

	10	**14**	**19**	**25**	**35**	**43**
Price, t-ADF	-3.446***	-4.217***	-6.855***	-8.558***	-8.858***	-4.983***
Demand, t-ADF	-2.665*	-4.197***	-7.227***	-5.149***	-3.270**	-2.804*
Margin, t-ADF	-6.234***	-5.758***	-7.458***	-5.892***	-6.241***	-5.644***

Table 6: ADF test of the daily UK natural gas spot price, the daily HWWI world index coal price(translated into £/ton) and the EEX-EU carbon emissions allowance daily spot price(translated into £/ton). The data spans from 22.04.2005 to 28.06.2012. We have chosen 5 lags in the ADF test. *, ** and *** indicates that we reject the null hypothesis and find stationarity at the 10%, 5% and 1% level respectively.

	Gas price	**Coal price**	**Carbon emission price**
t-ADF	-2.284	-1.367	-2.24

Table 7: Quantile regression results for period 10, 14, 19, 25, 35 and 43. Numbers in italic represent coefficients that are insignificant at a 5% level assuming a t-distribution. R-squared is a Koenker and Machado (1999) goodness-of-fit measure (pseudo R-squared).

	1%	**5%**	**10%**	**25%**	**50%**	**75%**	**90%**	**95%**	**99%**
Period 10									
const	-5.278	-3.333	-2.261	-0.771	*0.198*	*0.375*	1.009	*0.874*	*-0.456*
lprice	0.981	0.836	0.861	0.840	0.798	0.724	0.599	0.479	0.307
gasprice	*0.037*	0.076	0.066	0.080	0.092	0.125	0.190	0.243	0.321
coalprice	0.174	0.159	0.102	0.089	0.097	0.109	0.140	0.169	0.209
carbon	-0.015	*-0.001*	*0.002*	*0.001*	0.003	*0.005*	0.003	0.005	0.011
demand	0.316	0.278	0.187	0.100	0.055	0.079	0.105	0.175	0.403
margin	*0.094*	*-0.009*	*0.001*	-0.044	-0.083	-0.113	-0.192	-0.238	-0.319
R-squared	0.611	0.682	0.718	0.749	0.752	0.716	0.659	0.627	0.571
Period 14									
const	-5.788	-1.636	*-0.572*	0.729	1.246	1.039	1.704	*1.799*	4.141
lprice	0.281	0.346	0.440	0.508	0.545	0.497	0.404	0.348	0.220
gasprice	0.322	0.278	0.239	0.222	0.215	0.259	0.288	0.333	0.450
coalprice	0.401	0.339	0.267	0.228	0.198	0.205	0.233	0.246	0.229
carbon	0.012	0.015	0.019	0.018	0.017	0.014	0.012	0.012	0.016
demand	0.654	0.348	0.262	0.172	0.142	0.201	0.241	0.300	0.291
margin	-0.175	-0.239	-0.239	-0.269	-0.281	-0.317	-0.408	-0.476	-0.682
R-squared	0.580	0.637	0.671	0.684	0.677	0.641	0.611	0.596	0.565
Period 19									
const	1.513	1.520	1.325	*0.722*	1.197	1.634	2.819	3.821	*5.110*
lprice	0.181	0.216	0.226	0.303	0.350	0.406	0.414	0.403	0.399
gasprice	0.329	0.334	0.360	0.354	0.365	0.378	0.374	0.382	0.486
coalprice	0.436	0.405	0.360	0.321	0.268	0.237	0.242	0.212	0.156
carbon	0.020	0.022	0.027	0.021	0.014	0.009	*0.004*	*0.003*	*-0.023*
demand	0.092	0.110	0.145	0.205	0.204	0.230	0.244	0.217	*0.217*
margin	-0.295	-0.310	-0.319	-0.325	-0.361	-0.436	-0.565	-0.617	-0.743
R-squared	0.596	0.616	0.608	0.583	0.549	0.528	0.545	0.542	0.505
Period 25									
const	2.738	4.386	4.480	4.773	4.408	5.504	5.664	5.240	4.822
lprice	0.210	0.214	0.252	0.305	0.422	0.464	0.451	0.506	0.506
gasprice	0.270	0.292	0.311	0.330	0.314	0.324	0.311	0.258	0.152
coalprice	0.397	0.403	0.353	0.289	0.221	0.201	0.214	0.181	0.265
carbon	0.030	0.028	0.025	0.017	0.008	*0.001*	*0.002*	*-0.002*	*-0.007*
demand	*-0.023*	-0.147	-0.149	-0.143	*-0.090*	*-0.078*	*0.060*	*0.159*	0.291
margin	-0.269	-0.308	-0.310	-0.336	-0.353	-0.476	-0.627	-0.669	-0.746
R-squared	0.538	0.518	0.494	0.449	0.421	0.431	0.470	0.489	0.530
Period 35									
const	-1.795	-2.174	-2.038	-1.611	*0.276*	1.776	2.846	4.122	*2.781*
lprice	0.216	0.268	0.288	0.372	0.467	0.528	0.492	0.455	0.471
gasprice	0.346	0.318	0.336	0.329	0.306	0.281	0.324	0.318	0.201
coalprice	0.349	0.301	0.270	0.224	0.189	0.162	0.159	0.166	*0.176*
carbon	0.023	0.028	0.021	0.009	*0.003*	*-0.006*	-0.016	*-0.011*	*0.006*
demand	0.385	0.391	0.400	0.373	0.270	0.227	0.292	0.286	0.565
margin	-0.253	-0.202	-0.220	-0.236	-0.321	-0.420	-0.590	-0.695	-0.810
R-squared	0.624	0.597	0.577	0.569	0.556	0.560	0.573	0.577	0.561
Period 43									
const	*1.450*	1.514	2.184	1.806	1.606	2.118	2.861	4.241	9.207
lprice	0.364	0.449	0.490	0.597	0.684	0.762	0.795	0.827	1.006
gasprice	0.247	0.194	0.204	0.168	0.155	0.129	0.116	0.130	*0.082*
coalprice	0.340	0.297	0.257	0.195	0.131	0.078	0.059	*0.017*	*-0.031*
carbon	0.029	0.024	0.021	0.013	0.005	*0.000*	*-0.002*	*-0.005*	*-0.007*
demand	*-0.047*	*-0.019*	-0.051	*-0.008*	*0.014*	*0.000*	*0.002*	*-0.041*	-0.286
margin	-0.128	-0.148	-0.182	-0.182	-0.178	-0.205	-0.276	-0.362	-0.616
R-squared	0.660	0.692	0.695	0.682	0.665	0.637	0.633	0.620	0.589

Table 8: Base scenario. Actual value of each fundamental variable on 28.06.2012.

Period	Gas (-1)	Coal (-1)	Carbon Emission (-1)	Price(-1)	Demand Forecast	Margin Forecast
Period 10	54.363	77.197	6.318	30.100	25161.000	26442.000
Period 14	54.363	77.197	6.318	32.570	30296.000	22005.000
Period 19	54.363	77.197	6.318	41.100	39454.000	13392.000
Period 25	54.363	77.197	6.318	48.250	41094.000	12068.000
Period 35	54.363	77.197	6.318	55.200	40874.000	11812.000
Period 43	54.363	77.197	6.318	48.260	34956.000	18260.000