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# Short Sea Shipping of Fresh Salmon

## Inventory Routing with Continuous Time Formulation for a Perishable Product

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## INTRODUCTION

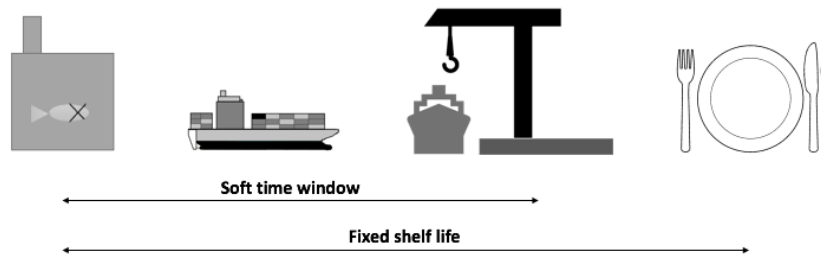
In 2015 Norway produced 1,31 million tonnes of farmed salmon, and this is expected to grow to 5 million tonnes in 2050. Every day around 120 semi-trailers are on the road, just with the mission of transporting fresh salmon from Norway to markets in Europe. The road network has an overall high load and Norwegian roads are not dimensioned for the heavy transport the fish transportation generates. To handle future growth in the seafood industry it is therefore important to come up with new sustainable logistic solutions, and short sea shipping can be a solution to this problem.

**Objective and Scope of Work:** The aim of this thesis is to utilize optimization to gain insight into the problem of seaborne transportation of fresh salmon from Norway to Europe. The problem is modelled as an inventory routing problem, with perishable considerations.

## PROBLEM DESCRIPTION

Short sea shipping does not usually bring the goods all the way from the producer to the supplier, and it usually has to cooperate with other land-based modes. This thesis has a main focus on seaborne transportation and distribution to and from the ports will not be included.

Slaughter facilities often work three shifts per day, and the production rate is therefore assumed to be constant per hour. Fresh fish has a fixed shelf life from being slaughtered until being consumed. The time from being slaughtered until it is at the unloading port is in this thesis not fixed, and soft time windows can be applied. The longer transportation time the shorter remaining durability of the salmon for transportation further, and an extra cost will therefore be added when delivering happens after a given time.



## MATHEMATICAL MODEL

**Sets:** Every port, either a production port or consumption port is represented by an index  $i$  and the set of ports is given by  $N$ . Number of available ships to be routed and scheduled is given by  $V$ , indexed by  $v$ . Every port can be visited several times during the planning period and  $M_i$  represents possible visits at port  $i$ . The visit number is represented by an index  $m$ , and the last possible visit at port  $i$  is  $|M_i|$ . The set of nodes in the flow network represents the set of port visits, and each port visit is specified by  $(i, m)$ ,  $i \in N$ ,  $m \in M_i$ . The set  $A$  contains the set of visits  $(i, m)$  where  $i \in N$  and  $m \in M_i$ .  $A_v$  is the set of feasible arcs for ship  $v$  including the dummy node  $d(v)$ .

**Parameters:**  $C_{ij}$  is the sailing cost from port  $i$  to port  $j$ . Capacity for ship  $v$  is  $Q_v$ .  $T_{ij}$  is the sailing time from port  $i$  to port  $j$ .  $T_i^Q$  is the unit loading or unloading time at port  $i$ , and  $T_i^B$  is the minimum time between ships at port  $i$ . The length of the planning horizon is  $T^{MAX}$ .  $S_i^0$  represents the initial inventory level at port  $i$ ,  $S_i^{MAX}$  and  $S_i^{MIN}$  the maximum and minimum inventory level, and  $S_i^{LOW}$  is the lower safety stock. The parameter  $I_i$  is equal to 1 for loading port and -1 for unloading port, and  $R_i$  is the production/consumption rate. It is positive in loading ports, and negative in unloading ports.  $D_1^{MAX}$  is the longest time since the oldest salmon was slaughtered to being delivered without being penalized, while  $D_2^{MAX}$  is the maximum allowable time since the oldest salmon was slaughtered to it has to be delivered.  $P_i^{TIME}$  represents the penalty for delivery between  $D_1^{MAX}$  and  $D_2^{MAX}$ .  $P_i^{DEL}$  is the extra cost for not deliver everything that is produced and  $P_i^{LOW}$  is the penalty cost for each tonnes of lower safety stock shortfall.

**Decision Variables:**  $x_{imjnv}$  is 1 if ship  $v$  sails from visit  $(i, m)$  to visit  $(j, n)$ , and 0 otherwise. The binary variable  $w_{im}$  equals 1 if no ship  $v$  visit port call  $(i, m)$  and 0 otherwise. The variable  $q_{imv}$  represents the quantity loaded or unloaded at port visit  $(i, m)$  done by ship  $v$ . The variable  $l_{imv}$  represents the total load on board ship  $v$  just after service is completed at visit  $(i, m)$ .  $s_{im}^S$  and  $s_{im}^E$  are the amounts of salmon in stock at the start and end of visit  $(i, m)$ , respectively.  $t_{im}^S$  and  $t_{im}^E$  represent the times at which service begin and end for visit  $(i, m)$ .  $t_{im}^{SL}$  is a variable that consider the time since the salmon was slaughtered when leaving visit  $(i, m)$ .  $s_{im}^{LOW}$  is the lower safety stock shortfall at port visit  $(i, m)$ .

$$\min f = \sum_{v \in V} \sum_{(i, m, j, n) \in A_v} C_{ij} x_{imjnv} + \sum_{i \in N} P_i^{DEL} \left( 1 - \frac{\sum_{m \in M_i} \sum_{v \in V} q_{imv}}{R_i T^{MAX} + S_i^0} \right) \quad (1)$$

$$\sum_{(i, m) \in A_v} P_i^{TIME} y_{im} + \sum_{(i, m) \in A_v} P_i^{LOW} s_{im}^{LOW} \quad (2)$$

$$\sum_{(j, n) \in A_v} \sum_{v \in V} x_{imjnv} + w_{im} = 1 \quad \forall (i, m) \in A \quad (3)$$

$$\sum_{(j, n) \in A_v} x_{o(v)jnv} = 1 \quad \forall v \in V \quad (4)$$

$$\sum_{(j, n) \in A} x_{jnimv} - \sum_{(j, n) \in A_v} x_{imjnv} = 0 \quad \forall (i, m) \in A \setminus \{o(v)\}, v \in V \quad (5)$$

$$\sum_{(i, m) \in A} x_{imd(v)v} = 1 \quad \forall v \in V \quad (6)$$

$$w_{im} - w_{i(m-1)} \geq 0 \quad \forall (i, m) \in A | m > 1 \quad (7)$$

$$L_v^0 + I_i q_{o(v)v} = l_{o(v)v} \quad \forall v \in V \quad (8)$$

$$x_{imjnv} (l_{imv} + I_j q_{jnv} - l_{jnv}) = 0 \quad \forall (i, m, j, n) \in A, v \in V \quad (9)$$

$$q_{imv} \leq l_{imv} \leq \sum_{(j, n) \in A_v} Q_v x_{imjnv} \quad \forall (i, m) \in A, v \in V | I_i = 1 \quad (10)$$

$$l_{imv} \leq \sum_{(j, n) \in A_v} Q_v x_{imjnv} - q_{imv} \quad \forall (i, m) \in A, v \in V | I_i = -1 \quad (11)$$

$$l_{imv} x_{imjnv} = 0 \quad \forall (i, m, j, n) \in A, v \in V | I_i = -1, I_j = 1 \quad (12)$$

$$l_{imv} x_{imd(v)v} = 0 \quad \forall (i, m) \in A, v \in V \quad (13)$$

$$x_{imjnv} (t_{im}^E + T_{ij} - t_{jn}^S) \leq 0 \quad \forall (i, m, j, n) \in A, v \in V \quad (14)$$

$$t_{im}^S + \sum_{v \in V} \frac{q_{imv}}{T_i^Q} = t_{im}^E \quad \forall (i, m) \in A, v \in V \quad (15)$$

$$t_{im}^E \leq T^{MAX} \quad \forall (i, m) \in A \quad (16)$$

$$w_{im} (t_{im}^S - t_{i(m-1)}^E) = 0 \quad \forall (i, m) \in A | m > 1 \quad (17)$$

$$t_{im}^S - t_{i(m-1)}^E + T_i^B w_{im} \geq T_i^B \quad \forall (i, m) \in A | m > 1 \quad (18)$$

$$t_{im}^{SL} = \frac{s_{im}^S}{R_i} + \frac{q_{imv}}{T_i^Q} \quad \forall (i, m) \in A, v \in V | I_i = 1 \quad (19)$$

$$t_{jn}^{SL} \geq \left( \frac{s_{jn}^S}{R_j} + \frac{q_{jnv}}{T_j^Q} \right) x_{imjnv} \quad \forall (i, m, j, n) \in A, v \in V | I_i = 1, I_j = 1 \quad (20)$$

$$t_{jn}^{SL} \geq (t_{im}^{SL} - t_{jn}^E - t_{im}^E) x_{imjnv} \quad \forall (i, m, j, n) \in A, v \in V | I_i = 1, I_j = 1 \quad (21)$$

$$x_{imjnv} (t_{im}^{SL} + t_{jn}^E - t_{im}^E - t_{jn}^{SL}) = 0 \quad \forall (i, m, j, n) \in A, v \in V | I_j = -1 \quad (22)$$

$$t_{im}^{SL} \leq D_2^{MAX} \quad \forall (i, m) \in A \quad (23)$$

$$y_{im} \geq t_{im}^{SL} - D_1^{MAX} \quad \forall (i, m) \in A | I_j = -1 \quad (24)$$

$$S_i^0 + R_i t_{im}^S = s_{im}^S \quad \forall (i, m) \in A | m = 1 \quad (25)$$

$$s_{i(m-1)}^E + R_i (t_{im}^S - t_{i(m-1)}^E) = s_{im}^S \quad \forall (i, m) \in A | m > 1 \quad (26)$$

$$s_{im}^S + R_i (t_{im}^E - t_{im}^S) - I_i \sum_{v \in V} q_{imv} = s_{im}^E \quad \forall (i, m) \in A \quad (27)$$

$$s_{im}^E + R_i (T^{MAX} - t_{im}^E) \leq S_i^{MAX} \quad \forall (i, m) \in A | m = |M|, I_i = 1 \quad (28)$$

$$s_{im}^E + R_i (T^{MAX} - t_{im}^E) \geq S_i^{LOW} \quad \forall (i, m) \in A | m = |M|, I_i = -1 \quad (29)$$

$$q_{imv} \leq \sum_{(j, n) \in A_v} S_i^{MAX} x_{imjnv} \quad \forall (i, m) \in A, v \in V | I_i = -1 \quad (30)$$

$$s_{im}^S + s_{im}^{LOW} \geq S_i^{LOW} \quad \forall (i, m) \in A | I_i = -1 \quad (31)$$

The objective function minimizes the total cost. Constraints (3)-(7) describe the flow on the route. Constraints (8)-(13) ensure consistency in load on board. Constraints (14)- (18) make sure that the time is correct. To make sure that the salmon delivered is fresh, some constraints on the perishability are made. Constraint (19) tracks the time the salmon was first slaughtered in a loading port. When the ship sails between two loading ports it is important to track the oldest product. This is ensured with constraints (20) and (21). Constraint (22) keeps track of time when sailing to an unloading port. Constraint (23) sets the maximum time the salmon is allowed to use from it first was slaughtered to being unloaded at an unloading port. To penalize delivery time over the given value  $D_1^{MAX}$  in the objective function, constraint (24) is made.  $y_{im}$  is not allowed to be less than zero, and is therefore only given a value when  $t_{im}^{SL}$  is larger than  $D_1^{MAX}$ . Constraints (25)-(31) ensure consistency in inventory levels, make sure they does not exceed the maximum and minimum levels, and update them after service. In addition constraints for maximum and minimum values for the variables  $s_{im}^S$  and  $s_{im}^E$  have to be included.

## RESULTS

The model will be tested with different hypothetical scenarios and running of the model in the commercial software Mosel Xpress Optimizer. The results will contribute to discussion regarding seaborne transportation of fresh salmon.

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