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Empirical Tests of Multifactor Capital Asset Pricing Models and Business Cycles. U.S. Stock Market Evidence Before, During and After the Great Recession

Alla Kursenko

Thesis Supervisor: Prof. Gunnar Bårdsen

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Preface

This thesis is made as a completion of my Master's degree in Financial Economics at the Department of Economics at the Norwegian University of Science and Technology (NTNU). I was engaged in researching and writing it from January till May 2017. The work was quite challenging and extensive, especially regarding the need to process all the stock return and accounting information, necessary to create the database for my research. The testing part of this work was quite demanding as well, as I had to master a completely new to me software package - EViews. Therefore I would like to express my gratitude to several persons who have contributed academically, practically and with support to make the creation of this thesis possible. First of all, I would like to thank Professor Snorre Lindset who drew my attention to the problems associated with empirical tests of the CAPM in his course "Capital Markets and Uncertainty", and who provided me with relevant advice concerning the choice of topic for my thesis. Secondly, I am very grateful to Professor Bjarne Strøm who introduced me to Econometrics during the first year of my Master's program and therefore made it possible for to carry out empirical tests of the multifactor capital asset pricing models in practice and interpret the results. At last but not least, I would like to thank my thesis supervisor Professor Gunnar Bårdsen for introducing me to EViews, as well as for his guidance, support and constructive feedback during my work on this thesis.

Trondheim, May 22, 2017.

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Alla Kursenko

Abstract

I estimate and perform empirical tests on the three most commonly used multifactor capital asset pricing models - Fama and French three-factor, Carhart four-factor and Fama and French four-factor models - in the U.S. stock market before, during and after the Great Recession. I prove that the critique directed at each of these models is fair, and none of the models is able to deliver persistent results. I demonstrate that the Fama and French three-factor model has a better performance compared to the four- and five-factor models. RMW and CMA factors are shown to be statistically insignificant in the Fama and French five-factor model, what reduces it to the traditional Fama and French three-factor model. I conjecture that the reason for that so many researchers have failed to recreate Fama and French's results might lie in that Fama and French omit some steps when describing how they construct the models' factors. Both the Fama and French three-factor and Carhart four-factor models demonstrate worse results when applied to the U.S. stock portfolio returns during the Great Recession than otherwise, which is first of all observed in the increased redundant variable problem. Surprisingly, it is the relative distress variable, HML, that tends to be redundant in the recession state of economy. Based on the achieved results, I point out the need for a more reliable asset pricing model, a model that would demonstrate robust results at explaining stock portfolio returns both in normal times and in crisis times in the economy.

Keywords: asset pricing, multifactor models, business cycles, "recession beta", performance measurement, the Great Recession in the United States

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1. Introduction

1.1. Background

"When Harry Markowitz's paper "Portfolio Selection" appeared in the Journal of Finance's March 1952 edition, it arrived in a vacuum. Investment research of one type or another may be as old as civilization, but mathematically rigorous portfolio analysis is younger than Bill Clinton." Picerno (2010)

The issue of estimating the expected return on portfolios, cost of equity on individual securities, as well as making optimal portfolio choice has bothered investors and asset managers through many generations. The future return on individual securities and portfolios of individual securities is subject to uncertainty and therefore - subject to risk. In 1950's the modern portfolio theory pointed out the relationship between the expected return and risk, and in the middle of 1960's Sharpe and Lintner introduced their capital asset pricing model (CAPM) which was supposed to solve the problem of asset pricing and optimal portfolio choice. The model said that investors are compensated for all additional systematic risk they take by additional return and was revolutionary for the portfolio theory. However this Nobel prize-winning model has caused a proliferation of empirical studies testing its validity. A steadily growing number of studies show that systematic risk alone isn't sufficient to explain the variation in expected returns. Therefore a variety of multifactor models have been developed to predict asset returns, amongst them: the Fama and French three-factor model, the Carhart four-factor model and the Fama and French five-factor model. The ongoing empirical testing of these models show different results, and there is no consensus in the economic literature as to what an appropriate measure of risk is, and therefore, as to what is an appropriate measure for estimating risk-adjusted portfolio performance.

In the time of globalization, when risk sharing intensifies and local economic crises easily spread to other economies, it is crucial to find a robust model that prices risk and therefore predicts returns adequately. This is a key prerequisite for a proper risk management. The global financial crisis in 2008-2009, which started as the Great Recession in the United States in December 2007[1], was evidence of that the financial economists still haven't learned to price assets correctly.

1.2. Problem Discussion

The problem that will be addressed in this paper is which of the three multifactor capital asset pricing models - Fama and French three-factor model, Carhart four-factor model or Fama and French five-factor model - has best performance when it comes to explaining the variation in the expected stock returns. In this thesis the research will be focused on the time period before, during and after the Great Recession in the United States, and the best model will be chosen based on its performance not only in normal times, but also in crisis times. Due to the dominating in the empirical world rejection of the CAPM, the short-time character of the Recession and only one risk factor in the model, the CAPM will not be included in the tests.

During my work on this paper I discovered quite little research comparing performance of these models in different states of economy, while I find it to be highly important that investors, economists and asset managers use the asset pricing model that gives robust results independently of the stage of a business cycle.

1.3. Purpose

The purpose of this thesis boils down to answering five specific questions:

- Is the Fama and French three-factor model a valid model for explaining the variation in the expected stock returns?
- Is the Carhart four-factor model a valid model for explaining the variation in the expected stock returns?
- Is the Fama and French five-factor model a valid model for explaining the variation in the expected stock returns?
- Is there a difference in the models' performance depending on the state of economy?
- Which model is the most optimal when it comes to explaining the variation in the expected stock returns?

[1] Business Cycle Dating Committee, National Bureau of Economic Research (2008): Determination of the December 2007 Peak in Economic Activity. Available at: http://www.nber.org/cycles/dec2008.html. Last Access: 15.02.2017.

1.4. Methodology

Literature review has shown that a great number of researchers have failed to recreate Fama and French's estimation results which made me question the validity of the model factors posted by Kenneth R. French on his website[2]. In order to conduct an independent research I construct all portfolios and regression factors on my own by following the guidelines of the models' authors. As the basis for my database construction I use historical returns and accounting data from the software product Eikon by Thomson Reuters[3] and the U.S. Department of Treasury website[4]. Each model is estimated by the ordinary least squares (OLS) method in the statistical package EViews, and the estimation results are checked for validity, corrected where it was necessary, and tested with Wald test.

1.5. Limitations

Due to limited the time factor and software capacity, as well as the quality of the available financial information, the dataset was reduced to 548 companies, while Fama and French's research (1993, 1995, 1996, 2015, 2016a) is based on stocks of all companies listed on NYSE, NYSE MKT (the former AMEX) and Nasdaq stock exchanges (around 4256 companies in total). The smaller sample size made it reasonable to construct 9 portfolios instead of the 25 portfolios typically constructed by Fama and French, the value-weighted monthly returns of which were used as the left-hand side variable in the regressions.

Unlike Fama and French (1996, 2012, 2015, 2016a, 2016b), who tend to use quite long estimation periods - 15 to 52 years of monthly data - in their research, I had to divide my 14,6-year-long sample into three subsamples in order to meet the purpose of this thesis. While most researchers would agree that the "before" and "after" the Great Recession subperiods had a satisfactory length, the length of the Recession itself constituted only 18 months, or 1,5 year, with the typically recommended in the empirical world time series length of 36 months. This may eventually lead to inadequate estimation results, especially for the four- and five-factor models. However, Garson (2008) argues that as long as the number of observations exceeds the number of dependent variables in a regression, the sample size can be considered as adequate. Kline (1979) suggests that the number of obervations should at least exceed the

[2] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

[3] Available at the Economics and Management Library at NTNU Business School. Last access: 30.03.2017.

[4] https://www.treasury.gov/resource-center/data-chart-center/interest-rates/Pages/TextView.aspx?data=billratesAll.

number of dependent variables by a factor of 2. Armstrong (2001) points out that even 12-months-long time series can be used for making up to 5-year ahead forecasts, with these forecasts having the same precision as the ones that have a 5- or 10-year base. Based on this information I make an assumption that my "Great Recession" sample satisfies at least minimum requirements for a multiple regression.

Furthermore, because of the limited access to data the following simplifications were made when constructing the risk factors:

- Total book equity was used as a proxy for Fama and French's book equity which also required deduction of book value of preferred stock.
- Operating profitability was computed as operating profit divided by the total book equity instead of using Fama and French's book equity.

However, these simplifications shouldn't have a big impact on my results since Fama and French (1993, 1996) point out themselves the difficulties with finding information on the book value of preferred stock.

1.6. Outline

Section 2 of this thesis provides an overview over the development of modern portfolio theory, the most commonly used capital asset pricing models and the controversies associated with these models.

Section 3 provides information about the nature of business cycles and addresses their influence on the performance of the multifactor capital asset pricing models. It focuses on the Great Recession in the United States as an example of a contraction stage in a business cycle.

Section 4 outlines the methodology used to construct the factors for the Fama and French three-factor, Carhart four-factor and Fama and French five-factor models. It describes estimation method and tests applied to these models, as well as the criteria used to differentiate between them.

Section 5 provides analysis of the estimation and test results.

Section 6 summarizes the results of the conducted research.

2. Modern Portfolio Theory and Capital Asset Pricing Models

2.1. Foundations of the Modern Portfolio Theory

The main aspect of security analysis is its valuation through the relationship between the security return and the associated risk. Economic models that solve this problem can, according to Galagedera (2007), be divided into five categories: the single factor model, multifactor models, CAPM with higher order systematic co-movements, CAPM conditional on market movements and time-varying volatility models. To better understand why and how these models can be useful at pricing securities and risk management it's wise first to get aquainted with the main theories that might have contributed to their development.

The foundations of the capital asset pricing were set by Markowitz (1952), Roy (1952) and Tobin (1958). Early theories associated risk of an individual security with volatility of its returns, that is usually measured by standard deviation, so higher standard deviation of returns was interpreted as higher risk of a security. Both Markowitz and Roy assumed that investors are risk averse and, therefore, demand additional return on the security, a risk premium, to compensate for the additional risk they are exposing themselves to by undertaking an investment.

Galagedera (2007) argues that Markowitz was the first to develop a specific measure of portfolio risk and derive the expected return and risk of portfolio. In his article "Portfolio Selection" Markowitz (1952) associates optimal portfolio choice with risk diversification and makes the following conclusions:

- The hypothesis that the investor seeks to maximize the expected return has to be rejected since it implies that investor will invest all his wealth in the security with the highest present value, while it actually exists a well-diversified portfolio which is preferable to all non-diversified portfolios.
- The expected return and variance of portfolios of securities are the core driving forces in optimal portfolio choice.
- Given that the investor beliefs about securities follow the same probability rules as random variables, the expected portfolio return equals to a weighted average of the expected returns on individual securities. The variance of the portfolio return is then represented by a function of the return variances of individual securities, the covariances between them and weights of these securities in the portfolio.

- If two risky assets have equal variance, then the variance of the resulting portfolio will be smaller than the variance of either risky asset, provided that asset returns are not perfectly positively correlated.

The model developed by Markowitz generates an efficient frontier of portfolios, and investors are expected to choose portfolios that are appropriate for them from this efficient set.

Tobin (1958) suggested a course of action to select an appropriate portfolio from the efficient set. His model is based on the Expected Utility theory and is referred to by Markowitz (1999) as the first capital asset pricing model (CAPM). Tobin divides investors into two categories: risk-lovers, willing to accept high risk for a possibility of an unusually high return (with negatively-sloped indifference curves), and risk-averters, willing to accept higher risk only for a promise of a higher return. When it comes to risk-averters, Tobin distinguishes between diversifiers, who have indifference curves that are concave upward, and plungers, who have either linear or convex upward sloping indifference curves. An investor in his optimal portfolio choice (a) estimates subjective probability distributions of capital gain or loss in holdings of assets, (b) evaluates his prospective increase in wealth in terms of a cardinal utility function and (c) ranks alternative prospects according to expected value of utility.

Unlike Markowitz and Tobin, Roy (1952) suggests a way of choosing a specific portfolio from the efficient set. According to Roy, the investor's optimal portfolio choice boils down to the Safety First principle that implies minimizing the probability of "disasters". "Disasters" are defined as a net loss from investor's activity or an income lower than the income that could have been reached if the capital had been differently allocated. The investor will therefore choose the portfolio that maximizes the portfolio $(E-d)/\sigma$, where d is a fixed disastrous return, E is expected return and σ is the standard deviation of return. In addition, Roy's model allows for both positive and negative investments.

Markowitz's approach to computation of risk reduction is rather involved. In 1994 a more efficient method was introduced by Sharpe, the single index model, where return on an individual security is related to the return on a common index. The common index may be represented by any variable of dominant influence on stock returns. This single index model can be extended to stock portfolios as well. Galagedera (2007), though, points out that when analyzing the risk of an individual security, it's crucial to evaluate how much risk it adds to the portfolio as well.

In the total amount of risk faced by investors it's important to distinguish between diversifiable (independent, or unsystematic) and non-diversifiable (common, market, or systematic) risk. Risks that share no correlation belong to unsystematic component of the total risk and can be eliminated by creating a well-diversified portfolio. If this is the case, adding extra individual securities to the portfolio will lead to reduction of the total risk. On the other hand, risks that are perfectly correlated are referred to as systematic and are associated with the overall movements in the economy. The systematic component of the total risk can't be eliminated through diversification[5].

2.2. The Capital Asset Pricing Model

The Capital Asset Pricing Model (CAPM) developed by Sharpe (1964) and Lintner (1965) relates the expected rate of return of an individual security to a measure of its systematic risk and has been widely used in finance for estimation of cost of capital, portfolio performance, portfolio diversification, valuation of investments and optimal portfolio choice.

The CAPM is developed in a hypothetical world where the following assumptions are made[6]:

- All investors are risk averse individuals who maximize their expected utility of returns over a one-period horizon.
- Investors are able to make their portfolio decisions solely based on the mean and standard deviation of the rates of return associated with the alternative portfolios.
- Investors are price-takers and have homogeneous expectations about asset returns that have a joint normal distribution. Consequently, the market portfolio is the efficient portfolio.
- There exists a risk-free asset such that investors can borrow or lend unlimited amounts at a risk-free rate. All investors have therefore an unlimited access to the risk-free borrowing and lending.
- The quantities of assets are fixed. All assets are marketable and perfectly divisible.
- There're no frictions in the asset markets, the information is costless and available to all investors at the same point of time.
- There're no market imperfections such as regulations, taxes, transaction costs or restrictions on short-selling.

[5] See Berk and DeMarzo (2014) or Galagedera (2007) for more details.

[6] Based on Copeland w. oth. (2014), Francis and Kim (2013), Lintner (1965) and Sharpe (1964).

Although not all of the listed assumptions are close to reality, they're necessary for the derivation of the CAPM, and most of them can be relaxed in the extensions to the CAPM.

In the academic literature the CAPM is usually represented by the following equation:

$$\mathrm{E}(\mathrm{R_i}) = \mathrm{R_f} + \beta_\mathrm{i}\mathrm{E}\left[\mathrm{R_m} - \mathrm{R_f}\right], \tag{1}$$

where the required rate of return on any asset, $E(R_i)$, is equal to to the risk-free rate of return, R_f, plus a risk premium $\beta_i[E(R_m) - R_f]$. The risk premium is the price of risk, expressed as a difference between expected rate of return on the market portfolio, $E(R_m)$, and the risk-free rate, multiplied by the sensitivity of asset *i*'s return to the market risk. This sensitivity to the market risk is often referred to as beta, β_i:

$$\beta_\mathrm{i} = \frac{\sigma_\mathrm{im}}{\sigma^2_\mathrm{m}} = \frac{\mathrm{Cov}(\mathrm{R_i}, \mathrm{R_m})}{\mathrm{Var}(\mathrm{R_m})}. \tag{2}$$

From equation (2) it follows that β_i is the covariance between the returns on risky asset *i* and market portfolio, divided by the variance of the market portfolio. Beta measures the sensitivity of returns of an individual security to the market risk[7].

The steadily growing popularity of the single-factor capital asset pricing model raised the question of its validity in the real world, and the model has been a subject to empirical tests for a number of researchers. Their main findings are described in the next subsection.

2.2.1. Testability of the Capital Asset Pricing Model

The major empirical tests of the CAPM were published by Friend and Blume (1970), Black w. oth. (1972), Miller and Scholes (1972), Blume and Friend (1973), Blume and Husick (1973), Fama and Macbeth (1973), Basu (1977), Reinganum (1981), Litzenberger and Ramaswamy (1979), Banz (1981), Gibbons (1982), Stambaugh (1982), Shanken (1985), Harvey (1989), Fama and French (1992) and Kothari w. oth. (1995).

Copeland w. oth. (2014) summarizes their findings and recommends, as the most commonly used strategy for performing an empirical test of the CAPM, transforming equation (1) into:

$$\mathrm{R}'_\mathrm{pt} = \gamma_0 + \gamma_1\beta_\mathrm{p} + \varepsilon_\mathrm{pt}, \tag{3}$$

where R'_{pt} is the excess return on the risky portfolio *p* at time *t*, computed as the difference between R_{pt} and R_{ft}, γ_1 is the excess return on the market portfolio, computed as the difference

[7] See for instance Berk and DeMarzo (2014) or Copeland w. oth. (2014).

between R_{mt} and R_{ft}, β_p measures sensitivity of portfolio *p*'s returns to the market risk and ε_{pt} is the error term of the model.

The CAPM formulation like in equation (3) allows to perform empirical tests on the following predictions made by the model:

- $\gamma_0 = 0$: The constant term of the model, γ_0, should be statistically insignificant (not different from zero). A constant that is significantly different from zero would indicate that the excess return on the market portfolio, $R_{mt} - R_{ft}$, fails to explain all variation in the portfolio returns. The unexplained variation, due to the absence of other explanatory variables in the model, is in that case "picked up" by the intercept.
- $\gamma_1 = R_{mt} - R_{ft}$: The slope of the model, γ_1, that measures the time t effect of portfolio *p*'s sensitivity to the market risk on portfolio *p*'s excess return, should be equal to the time t excess return on the market portfolio.
- $\gamma_1 > 0$: For long estimation periods the rate of return on the market portfolio should exceed the risk-free rate.
- The model should be linear in β_p, which is, according to Wooldridge (2013), a standard assumption for any model estimated by OLS.
- β_p should be the only factor that explains the rate of return on risky asset.

The numerous tests of the capital asset pricing model have delivered various results, but most of the rearchers have achieved agreement in the points that are critical for the CAPM:

- $\gamma_0 \neq 0$: The constant term in the model, γ_0, is statistically significant, so the excess return on the market portfolio might be not the only factor that explains portfolio returns.
- $\gamma_1 < R_{mt} - R_{ft}$: The excess return on the market portfolio exceeds the size of the beta coefficient, γ_1. This means that the CAPM underestimates returns on low-beta securities and overestimates returns on high-beta securities.
- $\gamma_1 > 0$: In the long-run the return on the market portfolio exceeds the risk-free rate.
- The model is linear in β_p.
- β_p is a more efficient measure of risk compared to the beta-squared or unsystematic risk terms used in the extensions to the CAPM.
- β_p fails to capture a number of significant factors that help explain stock and portfolio returns. For instance, Basu (1977) discovered that the CAPM underestimates returns on stock portfolios with low price-to-earnings ratios. Banz (1981) and Reinganum (1981) demonstrated

that the CAPM neither captures the tendency of smaller firms to have abnormally high returns, nor the tendency of large firms to have extremely low returns. It was also discovered a positive correlation between such factors as - leverage, book-to-market equity ratio and dividend yield - and the average rate of return which the single-factor CAPM misses out[8]. Not to mention the seasonality of stock returns reported by Keim (1983 and 1985).

Given these empirical results the CAPM is being rejected and there's been an ongoing debate concerning its empirical validity. Some studies - like for instance the ones by Fama and French (1992) and Roll and Ross (1980) - suggest that the CAPM is misspecified and requires the addition of factors other than beta to explain security returns. Other focus on the potential errors in the execution and design of the empirical tests[9], frictions in capital markets[10] or irrational behaviour[11]. Finally, there's a number of studies - like the ones by Jagannathan and Wang (1996), Scruggs (1998) and Ferson and Harvey (1999) - that discuss the possibility of the market risk premium and betas being time-dependent. Roll (1977) looked at the CAPM from a completely different standpoint. He argued that even though it should be possible to test the CAPM in principle, "no correct and unambiguous test of the theory has appeared" and that "there is practically no possibility that such a test can be accomplished in the future". He pointed out that there's only one potentially testable hypothesis in CAPM - the hypothesis that the true market portfolio is the mean-variance efficient portfolio - and it can't be tested because the market portfolio is unobservable.

2.3. Multifactor Models of Capital Asset Pricing

2.3.1. The Arbitrage Pricing Theory

Ross (1976) presented the arbitrage pricing theory (APT) that offers a model that, unlike the CAPM, has been proved to be testable. In contrast to the CAPM where security rate of returns are predicted based on the single factor - the excess return on the market portfolio, the APT assumes that security returns can be explained by a linear combination of k macroeconomic risk factors:

$$\tilde{R}_i = E(\tilde{R}_i) + b_{i1}\tilde{F}_1 + \dots + b_{ik}\tilde{F}_k + \tilde{\varepsilon}_i, \qquad (4)$$

[8] See Bhandari (1988), Stattman (1980), Rosenberg w. oth. (1985), Litzenberger and Ramaswamy (1979) and Keim (1983, 1985) for further information on the named anomalies in stock returns.

[9] See for instance Kothari w. oth. (1995).

[10] See for instance Amihud and Mendelson (1986).

[11] See for instance Lakonishok w. oth. (1994).

where $\tilde{R}_i$ is the random rate of return on asset *i*, $E(\tilde{R}_i)$ is the expected rate of return on asset *i*, $\tilde{F}_k$ is the k^{th} zero-mean systematic risk factor that is common to all asset returns, b_{ik} is the sensitivity of the asset *i*'s rate of return to factor *k*, and $\tilde{\varepsilon}_i$ is the error term in regression *i*.

The foundations of the APT lie in creating arbitrage portfolios, or portfolios that (a) are self-financing (require no change in wealth) and (b) contain no risk. The assumption is that a riskless portfolio earns zero return on average. The arbitrage pricing theory gives us the rate of return we can use to price an asset. If deviations from equilibrium occur, the arbitrage trading will make sure of restoring the equilibrium.

Copeland w. oth. (2014) show that that the expected return vector is a linear combination of the constant vector and coefficient vectors:

$$E(\tilde{R}_i) = \lambda_0 + \lambda_1 b_{i1} + ... + \lambda_k b_{ik}, \tag{5}$$

where $\lambda_0 = R_f$ if there's a riskless asset with riskless return, and λ_k represents the risk premium for factor *k*: $\lambda_k = \bar{\delta}_k - R_f$ with $\bar{\delta}_k$ standing for the expected return on a portfolio that is sensitive to factor *k*, but has no sensitivity to all other factors.

Therefore the APT can be more generally written as:

$$E(R_i) - R_f = \left[\bar{\delta}_1 - R_f\right] b_{i1} + ... + \left[\bar{\delta}_k - R_f\right] b_{ik}, \tag{6}$$

where the coefficients b_{ik} are computed in the same way as in the CAPM:

$$b_{ik} = \frac{Cov(R_i, \delta_k)}{Var(\delta_k)}, \tag{7}$$

where $Cov(R_i, \delta_k)$ is the covariance between asset *i*'s returns and the linear transformation of factor *k*, and $Var(\delta_k)$ is the variance of the linear transformation of factor *k*[12].

The APT allows us measure risk in multiple dimensions and is superior to the CAPM for a number of reasons. However there's still no complete agreement in the academic world on the choice of factors to be included in the model[13].

[12] The linear transformation here implies transforming the macroeconomic risk factors $\bar{\delta}_1, \bar{\delta}_2, ..., \bar{\delta}_k$ in a way that their vectors become both mutually orthogonal and normalized. See Ross (1976) for details.
[13] See for instance Chen (1986), McElroy and Burmeister (1988), Poon and Taylor (1991), Fama and French (1993, 1996) or Jagannathan and Wang (1996).

2.3.2. The Fama and French Three-Factor Model

Fama and French (1992) conclude that stock risks are multidimensional provided that assets are priced rationally. One dimension of risk is proxied by market capitalization (a measure of size) of a firm, another - by book-to-market equity ratio. These two proxies combined were shown to absorb the roles of leverage and price/earnings ratio, and their explanatory power in the cross-section of average stock returns turned out to be better than the one of beta.

In 1993 Fama and French introduced their three-factor model as a specification of the APT, further described in 1996:

$$\mathrm{E(R_i)} - \mathrm{R_f} = \mathrm{b_i}\left[\mathrm{E(R_m)} - \mathrm{R_f}\right] + \mathrm{s_i E(SMB)} + \mathrm{h_i E(HML)}. \qquad (8)$$

The model above says that the excess expected return on portfolio *i*, $[E(R_i)-R_f]$, is explained by sensitivity of its return to three factors:

1) Excess return on the market portfolio $[E(R_m)-R_f]$.

2) The difference between the return on a portfolio of small stocks and the return on a portfolio of large stocks (SMB, or small minus big).

3) The difference between the return on a portfolio of stocks with high book-to-market equity and stocks with low book-to-market equity (HML, or high minus low).

$E(R_i)-R_f$, $E(R_m)-R_f$, E(SMB) and E(HML) can be interpreted as expected risk premiums. The factor sensitivities - b_i, s_i and h_i - are slopes in the regression:

$$\mathrm{R_{it}} - \mathrm{R_{ft}} = \alpha_\mathrm{i} + \mathrm{b_i}\left[\mathrm{R_{mt}} - \mathrm{R_{ft}}\right] + \mathrm{s_i SMB_t} + \mathrm{h_i HML_t} + \varepsilon_\mathrm{it}. \qquad (9)$$

Here R_{it}, R_{ft} and R_{mt} are historical rates of return on risky asset (portfolio) *i*, riskless asset (represented by Treasury bill) and the market portfolio respectively in period t. α_i is the constant term of the regression, also known as Jensen's alpha. It is used to determine abnormal return of security or portfolio of securities over the expected equilibrium return, that can't be explained by the other factors in the model[14]. Finally, ε_{it} is interpreted as the error term of the model.

Fama and French (1996) argue that their three-factor model takes care of most of the anomalies the CAPM is exposed to. In 1995 they showed that book-to-market equity ratio and slopes on HML are proxy for relative distress. Weak firms with low earnings tend to have high book-to-market equity ratio and positive slopes on HML, while strong firms with high

[14] See Jensen (1968) for more details.

earnings have low book-to-market equity ratio and negative slopes on HML. Therefore using HML helps explain covariation in returns related to distress that the CAPM fails to capture. Accordingly, using SMB helps explain covariation in the returns on small stocks that is not captured by the market return and is compensated in average returns. Finally, the main role of the market factor, according to Fama and French (1993), lies in explaining the large difference between the rate of return on stocks and the rate of return on the riskless asset.

Fama and French (1993) claim that their model also explains the strong patterns in returns observed when portfolios are formed based on earnings/price ratio, cash flow/price ratio, sales growth, and captures reversion of the long-term returns. However, they point out themselves in their paper from 1996 that (8) has its drawbacks, among which - not being able to account for the continuation patterns in the short-term stock returns. Fama and French (1993) mention that (8) should be considered only as a model, and we shouldn't expect it to deliver valid results for all securities and portfolios. Besides they admit to failing at explaining some of the results they had gotten in their research.

After the Fama and French three-factor model was introduced to the academic world, it has been continuously tested by scientists all over the world. A great part of them received results confirming the empirical validity of the model[15]. On the other hand, there're academic papers - for instance the ones by Daniel and Titman (1997) and Grauer and Janmaat (2010) - where the model is rejected. Some researchers, like Tauscher and Wallmeier (2016), argue that the traditional model is biased, and therefore factor-mimicking portfolios and test portfolios should be constructed from different samples, at least for small markets. Others show that the results of the empirical tests of the model vary depending on the sector of economy, estimated time horizon or specific country[16]. Titman w. oth. (2004) and Novy-Marx (2013) show that the three factors in the model miss much of the variation in average returns related to profitability and investment.

[15] See for instance Connor and Sehgal (2001), Faff (2001), Gaunt (2004), Doganay (2006), Guzeldere and Sarioglu (2012), Dolinar (2013), Walkshausl and Lobe (2014) or Boutabba (2015).
[16] See for instance Aleati w. oth. (2000), Griffin (2002), Chandra (2015) or Vo (2015).

2.3.3. The Carhart Four-Factor Model

Inspired by Fama and French's (1993) three-factor model and Jegadeesh and Titman's (1993) research on the one-year momentum anomaly, Carhart (1997) in his paper "On Persistence in Mutual Fund Performance" introduced the four-factor model:

$$R_{it} - R_{ft} = \alpha_i + b_i [R_{mt} - R_{ft}] + s_i SMB_t + h_i HML_t + p_i PR1YR_t + \varepsilon_{it}, \qquad (10)$$

where R_{it} is the historical rate of return on portfolio *i* in period t; R_{ft} is Treasury bill return in period t; R_{mt} is the return on the market portfolio in period t; SMB_t, HML_t and $PR1YR_t$ are returns on value-weighted, zero-investment, factor-mimicking portfolios for size, book-to-market ratio and and one-year momentum in portfolio or stock returns in period t; α_i is the constant term, or Jensen's alpha; the factor sensitivities b_i, s_i, h_i and p_i are slopes and ε_{it} is the error term in the model.

Carhart (1997) argued that his model is consistent with a model of market equilibrium with four risk factors. The core difference from Fama and French model lies in the fourth factor, PR1YR. The fourth factor is supposed to account for the momentum effect, or a tendency of well-performing stocks (winners) and poorly performing stocks (losers) to persist over several months, which Fama and French (1996) admitted to not being able to capture by their model.

Jegadeesh and Titman (1993) found that investment strategies which consist in buying winners and selling losers, generate significant positive returns over 3- to 12-months holding periods. In the following two years the momentum effect, however, fades away, and such portfolio loses over 50% of its return as the previous periods' losers start outperforming the past periods' winners.

Carhart (1997) showed that this one-year anomaly in stock returns can be captured by including the difference between the equal-weight average of firms with the highest 30% 11-months returns lagged one month and the equal-weight average of firms with the lowest 30% 11-months returns lagged one month (winners minus losers), which is represented by the PR1YR-factor, in the model.

According to Carhart (1997), accounting for the momentum effect in stock returns improves the explanatory power of the model by 15%, what was among others confirmed by Cakici w. oth. (2013) performing empirical test on 18 emerging stock markets and Lutzenberger (2015) applying the model to the European market and comparing it with other seven multifactor models. On the other hand, Chen and Fang (2009) showed that Fama and French three-factor model outperformed Carhart's model in multiple tests on seven Pacific Basin stock markets. It was also found that the four-factor model captures returns mainly on small size winner-portfolio[17]. Due to the controversial results in various papers, implying possible redundancy of the momentum factor in the given specification, the applicability of the model still remains under question.

2.3.4. The Fama and French Five-Factor Model

Motivated by the critique of their three-factor model, Fama and French (2015) attempted to extend the original model with additional factors in to increase its explanatory power and ability to capture the core anomalies in stock returns. Their starting point was the evidence provided by Titman w. oth. (2004), Novy-Marx (2013) and the dividend discount model:

$$m_t = \sum_{\tau=1}^{\infty} E\left(d_{t+\tau}\right) / \left(1+r\right)^{\tau}, \tag{11}$$

where m_t is the share price at time *t*, $d_{t+\tau}$ is the expected dividend per share for period $t+\tau$, and r is the long-term average expected stock return.

Equation (11) says that the market value of a share of a stock equals the discounted value of expected dividends per share. Therefore, if stocks of different firms have the same expected dividends but different prices, the cheapest stock must in the long-run have the highest expected return and, consequently, the highest risk.

In order to explain the relation between the expected return and expected profitability, as well as the expected investment and book-to-market equity ratio, Fama and French (2015) refer to Miller and Modigliani's extension of equation (11) saying that the time t total market value of a stock of a firm (M_t) is:

$$M_t = \sum_{\tau=1}^{\infty} E\left(Y_{t+\tau} - dB_{t+\tau}\right) / \left(1+r\right)^{\tau}, \tag{12}$$

where $Y_{t+\tau}$ is total equity earnings for period $t+\tau$, and $dB_{t+\tau} = B_{t+\tau} - B_{t+\tau-1}$ is the change in total book equity. Dividing by time t book equity Fama and French get the equation:

$$M_t/B_t = \sum_{\tau=1}^{\infty} E\left(Y_{t+\tau} - dB_{t+\tau}\right)/\left(B_t\left(1+r\right)^{\tau}\right), \tag{13}$$

which allow them to make three statements about expected stock returns:

[17] See for instance Balakrishnan (2016).

- Higher book-to-market equity ratio, B_t/M_t, implies higher expected return.
- Higher expected earnings, $E(Y_{t+\tau})$, imply higher expected return.
- Higher expected growth in book equity, $E(dB_{t+\tau})$, or in other words an increase in investment, implies lower expected return.

On the basis of these results Fama and French (2015) add profitability and investment factors into their model from 1993, and introduce the five factor model directed on capturing size, value, profitability and investment patterns in average stock returns:

$$R_{it} - R_{ft} = \alpha_i + b_i [R_{mt} - R_{ft}] + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t + \varepsilon_{it}, \quad (14)$$

where RMW_t is the difference between returns on diversified portfolios of stocks with robust and weak profitability at time t, CMA_t is the difference between returns on diversified portfolios of stocks of low and high investment firms, or conservative and aggressive firms, r_i and c_i are sensitivities of portfolio *i* to RMW_t and CMA_t factors respectively. Other parameters have the same interpretation as in Fama and French three-factor model.

Fama and French showed that their five-factor model explains more of the average stock returns than the three-factor model. Besides, Fama and French (2016a) argue that it helps solve more of the average return anomalies of the CAPM:

(1) Positive exposures to RMW_t and CMA_t, which mean that stock returns behave like those of conservatively investing profitable firms, help explain the high average returns associated with low market beta, share repurchases and low stock return volatility.

(2) Negative exposures to RMW_t and CMA_t capture the low average returns associated with high market beta, large stock issues and high volatility of returns.

They also suggest that in order to avoid overspecification, HML_t can be dropped out from equation (14) without reducing the explanatory power of the model. From equation (13) it follows book-to-market equity ratio approximately corresponds to forecasts of investment and earnings which are already used as variables. Fama and French (2015, 2016b) point out, however, that the five-factor model is the best choice in cases when sensitivity of portfolio returns to changes in value are of particular interest. In later research they find that performance of equation (14) improves significantly with addition of the momentum factor in the model[18].

[18] See Fama and French (2016a).

One problem with the model, which Fama and French (2015, 2016b) describe as its failure, is that it doesn't fully capture the low average returns of small stocks whose returns behave like returns of firms that invest aggressively despite low profitability. Among other problems a number of economists mention its overspecification, failure to capture the momentum effect and lack of evidence that supports choice of the two new factors[19].

3. Capital Asset Pricing Models and Business Cycles

3.1. Business Cycles and Stock Market Cycles

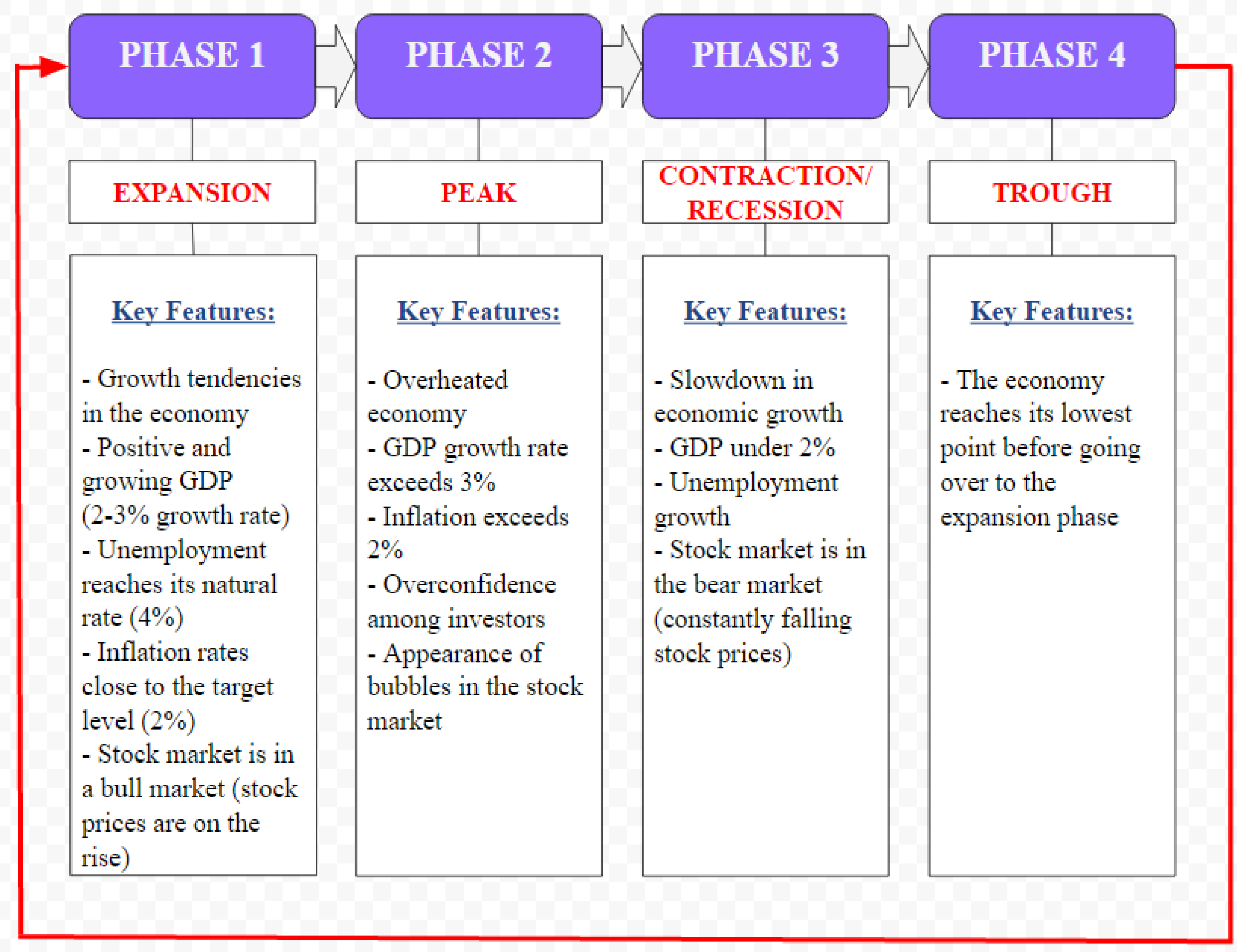
One of the today's most used definitions of business cycles was provided by Burns and Mitchell (1946):

"Business cycles are a type of fluctuation found in the aggregate economic activity of nations that organize their work mainly in business enterprises: a cycle consists of expansions occurring at about the same time in many economic activities, followed by similarly general recessions, contractions, and revivals which merge into the expansion phase of the next cycle; in duration, business cycles vary from more than one year to ten or twelve years; they are not divisible into shorter cycles of similar characteristics with amplitudes approximating their own."

In the United States the Business Cycle Dating Committee of the National Bureau of Economic Research (NBER) provides official time frames for business cycles. It's worth mentioning that the Business Cycle Dating Committee doesn't determine the exact dates, but the starting and ending month for each business cycle. In its approach NBER distinguishes between four phases of a business cycle - expansion, peak, contraction or recession, and trough. Amadeo (2017) gives a good overview over what kind of economic processes characterize each of the phases. Her comments on the key features of each phase are summarized in Figure 1.

[19] See for instance Racicot and Rentz (2015), Blitz w. oth. (2016), Racicot and Rentz (2016) and Walkshausl (2016).

Figure 1. Phases of a Business Cycle



NBER (2003) describes business cycle phases as: “Recession is a significant decline in economic activity spread across the economy, lasting more than a few months, normally visible in real GDP, real income, employment, industrial production, and wholesale-retail sales. A recession begins just after the economy reaches a peak of activity and ends as the economy reaches its trough. Between trough and peak, the economy is in an expansion. Expansion is the normal state of the economy; most recessions are brief and they have been rare in recent decades”. NBER considers real GDP to be the most efficient indicator of the overall economic activity, so this factor gets the largest weight in determining the time frames of the phases of a business cycle.

NBER (2012) documented that during 1945-2009 there were 33 business cycles in the U.S. economy with an average length of around 56 months (4,7 years). The average length of the expansion phase was over two times larger than the one of the contraction phase and corresponded to almost 39 months (3,3 years). However, as seen in Table 1, the cycles’ length, time distribution between their phases and their duration weren’t constant.

Table 1. U.S. Business Cycle Average Frequency and Duration in 1854-2009 [20]

Reference Dates and Corresponding Number of Cycles	Number of cycles per year	Duration in Months			
		Expansion	Contraction / Recession	Cycle	
				From Peak to Peak	From Trough to Trough
1854-1919 (16 cycles)	0.25	26.6	21.6	48.2	48.9*
1919-1945 (6 cycles)	0.23	35.0	18.2	53.2	53
1945-2009 (11 cycles)	0.17	58.4	11.1	69.5	68.5
1854-2009 (33 cycles)	0.21	38.7	17.5	56.2	56.4**

* 15 cycles
** 32 cycles

From Table 1 we can see that in 1945-2009 the frequency of cycles declined by 0,06 and 0,08 compared to 1919-1945 and 1854-1919 respectively, while the average cycle length increased by almost 16 months (1,3 year) and 20 months (1,6 year). The cycle length increased due to a significant increase in the duration of the expansion phase - over 58 months (4.9 years) - which was 1,7 times longer than in 1919-1945 and 2,2 times longer than 1854-1919. At the same time, the duration of the contraction phase was roughly half of the one in the two previous periods and corresponded to only 11,1 months.

Despite the tight relationship between business cycles and the stock market, business cycles and stock market cycles don't always coincide. Stock market cycles have the same phases as business cycles, but peak and trough phases are determined by the Bureau of Economic Analysis on the basis of monthly average stock returns. Rea and Marcis (1996) point out that during 1944-1995 only 8 out of 14 stock market cycles were associated with business cycles. However, they also found that it was only the shortest fluctuations in the U.S. stock market that weren't associated with the overall situation in the economy. But regardless of whether a contraction in the stock market is a part of a downturn in the overall economy or is caused by other factors, its consequences are normally quite severe. Findings of Reinhart and Rogoff (2009) show that "asset market collapses are deep and prolonged". By examining the largest systemic crises since 1899 they find that during the contraction phase equity prices decrease by whole 55% on average.

[20] Constructed on the basis of data from the National Bureau of Economic Research (2012).

3.2. Relation between Business Cycles and Capital Asset Pricing Models

As shown in Section 3.1, stock market suffers in the recession phase of a business cycle and prospers during the expansion period. Fama and French (1989) found that stock returns contain a term (or maturity) premium that exhibits an obvious business-cycle pattern. In the recession phase stock prices fall dramatically, the rates of return decline and can become negative. Cyclical processes in the economy lead to abnormal fluctuations in stock returns, which, according to Picerno (2010), should be empirically captured by the "recession beta". One of examples are stocks of small-sized, high book-to-market equity firms that are the most exposed to recessions. Small firms are normally credit constrained and have cash-flow problems, which tighten even further in an economic downturn. Fama and French (1992) state: "...for small firms, the 1980-1982 recession turns into a prolonged earnings depression. For some reason, small firms don't participate in the economic boom of the middle and late 1980's". Stocks of high book-to-market equity firms in the recession times normally get replaced by more liquid stocks of better quality (investors "flight to quality" or "flight to liquidity").

Certainly, the one-factor CAPM couldn't account for such variations in stock returns. Such factors as SMB and HML, according to Picerno (2010), solve the problem and are (combined) an appropriate proxy for the "recession beta". Liew and Vassalou (2000) tested Carhart PR1YR factor's ability to predict the future economic growth, but came back with negative results. Yu (2012) shows that it is efficient to include PR1YR in the model only to explain stock returns in the expansion phase and only on non-volatile markets. Picerno (2010) mentions interest rates as an alternative proxy for the "recession beta". Generally speaking, there's as much confusion in the academic world concerning the choice of this proxy as concerning the choice of the best-performing asset pricing model.

3.3. The Great Recession in the United States in 2007-2009

The Great Recession, that hit the U.S. economy in the end of 2007, was an unexpected event to most investors and financial institutions, and therefore turned into the most severe crisis since the Great Depression[21]. Among events that led to this recession most researchers name the collapse of the housing bubble in the middle of 2007, the real estate market correction and the subprime mortgage crisis in the United States. According to the Housing Finance

[21] See Eigner and Umlauft (2015).

Network[22], in 2003 the Federal Reserve reduced the the target rate to 1%. With 2-3% inflation rate it became more beneficial to lend money than save, with a significant share of loans provided by the unregulated shadow banking sector. Securitization, the new trend on the global financial markets, that allowed to pool risky loans into AAA-rated tranches and sell them to other actors as safe loans, motivated the financial institutions to reduce their lending requirements. Denning (2011) writes that 83% of subprime loans, provided by shadow banks in 2006, were directed to households with low or medium income. As demand for housing grew, housing prices also increased while opportunities for refinancing the loans shrinked. Eventually, the housing bubble exploded. People weren't able to repay their mortgages, lenders became exposed to bank runs and the system of shadow banking started to collapse, followed by other financial institutions. Gertler and Kiyotaki (2015) argue that the asset fire sales induced by the runs amplified the overall distress in financial markets, raising credit costs which led to a sharp contraction in the overall economic activity. Such factors as neglecting possibility of a nationwide drop in the U.S. house prices combined with securitization, irresponsibility of credit rating agencies and the financial multiplier effect, are among the factors that made this recession as severe and unexpected as it was. According to Gennaioli w. oth. (2013), securitization made interconnected not only financial institutions inside the United States, but all over the world, so once the recession broke out there, it spread out rapidly to other countries. The time frames of the Great recession in the United States were determined by NBER (2012) and can be examined in Table 2.

Table 2. Timing of the 2007-2009 Great Recession in the United States in Terms of a Business Cycle[23]

	Before the Great Recession		**During the Great Recession**		**After the Great Recession**
Business Cycle Phase	Expansion	Peak	Contraction / Recession	Trough	Expansion
Time Frames	December 2001 - November 2007	December 2007	January 2008 - May 2009	June 2009	July 2009 - Today
Duration, months	72	1	17	1	92+

22 http://www.housing-finance-network.org/index.php?id=330.
23 Constructed on the basis of data from the National Bureau of Economic Research (2012)

It's worth mentioning that NBER determined the peak date in December 2007 and the trough date in June 2009 only 11 and 15 months respectively after these dates passed.

4. Methodology

4.1. Data Sources and Characteristics

In order to create my database I use daily stock returns and accounting data of U.S. firms, listed on NYSE, NYSE MKT LLC and Nasdaq stock exchanges, from Thomson Reuters Eikon software package[24]. Data on the one-month Treasury bill rate is extracted from the U.S. Department of Treasury website[25]. Due to the specifics of the models' factors construction, which will be described later in this section, the sample period for the stock returns is from July 1, 2000 to July 1, 2016. The sample period for the accounting data is from December, 1994 to July, 2016.

Since I need to estimate and test the asset pricing models based on monthly data, I use the following formula to convert daily returns into monthly returns:

$$R_m = \left(\prod_{d=1}^{n}\left(\frac{R_d}{100} + 1\right) - 1\right) * 100, \tag{15}$$

where R_m is monthly return on a stock, R_d is daily return and n stands for the number of days in the particular month.

My original database consisted of 750 stocks. In order to reduce potential biases, only firms which had their Thomson Reuters Eikon revenue data for six years before the beginning of the estimation period, were included in the estimations. Stocks missing a lot of data on returns, market capitalization and book-to-market equity also were excluded from the database. In addition, like in Fama and French papers (1993, 1996) only stocks of firms with ordinary common equity were considered. Accounting for all these factors left me with stocks of 548 companies (see Appendix A). It's worth pointing out that my data isn't affected by the survivor bias because Thomson Reuters Eikon includes not only active firms, but also dead firms.

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[24] Available at the Economics and Management Library at NTNU Business School. Last access: 30.03.2017.
[25] https://www.treasury.gov/resource-center/data-chart-center/interest-rates/Pages/TextView.aspx?data=billratesAll.

Based on the final list of companies I construct dependent and explanatory variables for my time-series regressions. All data preparation is performed by following Fama and French's (1993, 1996, 2012, 2015, 2016a and 2016b) and Carhart's (1997) research.

4.2. Construction of Variables

4.2.1. The Fama and French Three-Factor Model

4.2.1.1. Dependent Variables

There is one dependent variable in the model - the excess portfolio return, (R_{it} - R_{ft}). In order to form a proxy for the return on a risky portfolio of stocks, R_{it}, I form 9 portfolios of stocks.

Firstly, in the end of June of each year τ I sort NYSE stocks by size and book-to-market equity (independently). The size is calculated as number of stocks outstanding in the end of June of year τ, times price per share. Top 30% of NYSE stocks are ranked as big (B), middle 40% as medium (M) and bottom 30% as small (S). For ranking by book-to-market equity the book-to-market equity ratio is computed as book common equity for the fiscal year ending in calendar year τ-1, divided by market equity in the corresponding month of year τ-1. Based on book-to-market equity NYSE stocks also get split into three groups: high (H) for the top 30% of stocks, medium (M) for the middle 40% and low (L) for the bottom 30%.

Secondly, the breakpoints for size and book-to-market equity ratio for NYSE stocks are used to allocate NYSE, NYSE MKT LLC and Nasdaq stocks to three size-based and three book-to-market-based portfolios (see Appendix B). Fama and French (1996) explain that the use of NYSE breakpoints helps reduce the influence of the numerous but less important small stocks in the equally-weighted portfolios.

At last, on the intersection of the size-based and book-to-market-based portfolios I construct 9 portfolios of stocks and compute their value-weighted monthly returns from July in year τ-1 to June in year τ (see Appendix C). It's worth mentioning that stocks of firms with negative values of book equity aren't included to the portfolios (see Appendix D).

The proxy for the riskless rate, R_{ft}, is the one-month U.S. Treasury bill rate observed in the beginning of the month t+1. Monthly excess returns are, consequently, computed as returns in excess of the one-month Treasury bill rate.

4.2.1.2. Explanatory Variables

The explanatory returns R_{mt}, $SMBb_t$[26] and HML_t are formed as follows. First, in the end of June each year τ I rank NYSE, NYSE MKT LLC and Nasdaq stocks by size. The median size of NYSE is used to split stocks into two groups - big (B) and small (S) - depending on whether their size is above or below the median. Then I break my original sample of NYSE, NYSE MKT LLC and Nasdaq stocks into three categories based on the book-to-market equity ratio: high (H) for the top 30% of stocks, medium (M) for the middle 40% and low (L) for the bottom 30%. Based on the intersection of the two size-based groups and three book-to-market-based groups, six stock portfolios are constructed: B/H, B/M, B/L, S/H, S/M and S/L, and I calculate monthly value-weighted returns for each portfolio from July in year τ-1 to June in year τ. Here stocks of firms with negative values of book equity are also not included to the portfolios.

Return on SMBb portfolio, meant to reflect size-related risk in returns, is computed as the difference, each month t, between arithmetic average of the returns on the three small-stock portfolios (S/L, S/M and S/H) and arithmetic average of the returns on the three big-stock portfolios (B/L, B/M and B/H):

$$\mathrm{SMBb_t} = \frac{(\mathrm{S/L + S/M + S/H})_\mathrm{t}}{3} - \frac{(\mathrm{B/L + B/M + B/H})_\mathrm{t}}{3}, \tag{16}$$

where $SMBb_t$, $(S/L)_t$, $(S/M)_t$, $(S/H)_t$, $(B/L)_t$, $(B/M)_t$ and $(B/H)_t$ are the rates of return on respective portfolios in month t.

Return on HML portfolio, meant to reflect the risk in returns related to book-to-market equity, is computed as the difference, each month t, between arithmetic average of the returns on the two portfolios with high book-to-market ratio (S/H and B/H) and arithmetic average of the returns on the two portfolios with low book-to-market ratio (S/L and B/L):

$$\mathrm{HML_t} = \frac{(\mathrm{S/H + B/H})_\mathrm{t}}{2} - \frac{(\mathrm{S/L + B/L})_\mathrm{t}}{2}, \tag{17}$$

where HML_t, $(S/H)_t$, $(B/H)_t$, $(S/L)_t$ and $(B/L)_t$ are the rates of return on respective portfolios in month t.

[26] I introduce the $SMBb_t$ notation in order to differentiate between the size factors in the Fama and French three-factor, Carhart four-factor and Fama and French five-factor models. Since the size factors in the Fama and French three-factor and Carhart four-factor models are constructed in the same way, they are denoted by $SMBb_t$, while the size factor in the Fama and French five-factor model is denoted by SMB_t.

Finally, I construct the proxy for the return on the market portfolio in month t, R_{mt}, as the value-weighted return on all stocks included in the portfolios, plus stocks of firms with negative book equity values.

4.2.2. The Carhart Four-Factor Model

4.2.2.1. Dependent Variables

Following Carhart (1997) and Fama and French (2012), in the end of June of 2001 I sort NYSE stocks by size and lagged momentum (independently) in a similar way to the one described in 4.2.1.1, except that the size-momentum portfolios are re-formed every month. The lagged momentum return in month t is calculated as a stock's cumulative return from t-11 to t-1. Based on the breakpoints for size and momentum for NYSE stocks I allocate NYSE, NYSE MKT LLC and Nasdaq stocks to three size-based and three momentum-based portfolios. On the intersection of the 3x3 size and momentum groups I construct 9 value-weighted portfolios which are used as proxies for risky portfolios in the regressions (see Appendix C). Proxy for the risk-free rate, R_{ft}, is the same as in the Fama and French three-factor model.

4.2.2.2. Explanatory Variables

For the Carhart four-factor model I use the same proxies for the rate of return on market portfolio, R_{mt}, and size and book-to-market equity risk-mimicking factors, $SMBb_t$ and HML_t, as in Fama and French three-factor model.

In order to construct $PR1YR_t$, each month I sort my database of NYSE, NYSE MKT LLC and Nasdaq stocks by size and lagged momentum (independently). The median size of NYSE is used to split stocks into two groups - big (B) and small (S). Then I break the original sample of NYSE, NYSE MKT LLC and Nasdaq stocks into three categories based on the lagged momentum: W that indicates winners (top 30%), N that indicates neutral (middle 40%) and L that indicates losers (bottom 30%). Based on the intersection of the two size-based groups and three momentum-based groups, I construct six portfolios: B/W, B/N, B/L, S/W, S/N and S/L, and calculate monthly value-weighted return for each portfolio. Finally, I construct the momentum factor, $PR1YR_t$, as an equal-weight average of winner-minus-loser returns on small and big stocks:

$$\mathrm{PR1YR_t} = \frac{(\mathrm{S/W} - \mathrm{S/L})_t + (\mathrm{B/W} - \mathrm{B/L})_t}{2}, \tag{18}$$

where $(S/W\text{-}S/L)_t$ is the winner-minus loser return on small stocks in month t and $(B/W\text{-}B/L)_t$ is the winner-minus loser return on big stocks in month t.

4.2.3. The Fama and French Five-Factor Model

4.2.3.1. Dependent Variables

In the end of June each year τ I construct 9 size-book-to-market equity, 9 size-operating profitability and 9 size-investment portfolios, which will be used as proxies for risky asset in the asset pricing regressions. Here I use the same portfolio construction principle as in 4.2.1.1 with 30 and 70 percent NYSE breakpoints both for size, book-to-market equity, profitability and investment (see Appendix C). As before the excess return, R_{it}-R_{ft}, is calculated as return on the risky asset in excess of the one-month U.S. Treasury bill rate.

4.2.3.2. Explanatory Variables

Proxies for the rate of return on market portfolio, R_{mt}, and book-to-market equity factor, HML_t, are also the same as in Fama and French three-factor model.

Following Fama and French (2015, 2016a, 2016b), in the end of June each year τ I allocate NYSE, NYSE MKT LLC and Nasdaq stocks into 2x3 size and book-to-market equity groups using the breakpoints for NYSE stocks like in 4.3.1.2, form six portfolios (B/H, B/M, B/L, S/H, S/M and S/L) and calculate monthly returns for each portfolio from July of year τ-1 to June of year τ. The book-to-market-based size factor, SMB_{Bt}, is the difference between the arithmetic average of returns on the three small stock portfolios and the arithmetic average of returns on the three big stock portfolios:

$$\mathrm{SMB_{Bt}} = \frac{(\mathrm{S/L + S/M + S/H})_t}{3} - \frac{(\mathrm{B/L + B/M + B/H})_t}{3}. \tag{19}$$

In order to construct the profitability factor, RMW_t, I start over again and in the end of June each year τ sort NYSE, NYSE MKT LLC and Nasdaq stocks by size and operating profitability into 2x3 groups. Operating profitability in the sort for June of year τ is measured with accounting data from the fiscal year ending in year τ-1 as operating profit, divided by book equity. The median size of NYSE is used to split stocks into two groups: big (B) and small (S). 30th and 70th percentile breakpoints for profitability of NYSE stocks allow me allocate NYSE, NYSE MKT LLC and Nasdaq stocks to three operating profitability groups: robust (R), medium (M) and weak (W). Based on the intersection of the 2x3 size and profitability groups, six portfolios are constructed: B/R, B/M, B/W, S/R, S/M and S/W, and I calculate monthly value-weighted return for each portfolio from July in year τ-1 to June in year τ. Finally, I construct the RMW_t factor as the difference between the arithmetic average of returns on small and big stocks with robust profitability and the difference between the arithmetic average of returns on small and big stocks with weak profitability:

$$\mathrm{RMW_t} = \frac{(\mathrm{S/R + B/R})_t}{2} - \frac{(\mathrm{S/W + B/W})_t}{2}. \tag{20}$$

To construct the investment factor, CMA_t, I sort NYSE, NYSE MKT LLC and Nasdaq stocks into 2x3 groups by size and investment. Investment is computed as the change in total assets from the fiscal year ending in τ-2 to the fiscal year ending in τ-1, divided by τ-2 total assets. Using the same breakpoints as before, I get two size groups - B and S - and three investment groups - aggressive (A), neutral (N) and conservative (C). On the intersection of these 2x3 groups I construct six portfolios: B/A, B/N, B/C, S/A, S/N and S/C, and I calculate monthly value-weighted return for each portfolio from July in year τ-1 to June in year τ. CMA_t is constructed as the difference between the arithmetic average of returns on conservatively investing small and big stocks and the difference between the arithmetic average of returns on aggressively investing small and big stocks:

$$\mathrm{CMA_t} = \frac{(\mathrm{S/C + B/C})_t}{2} - \frac{(\mathrm{S/A + B/A})_t}{2}. \tag{21}$$

The 2x3 sorts used to construct RMW_t and CMA_t produce two additional size factors: profit-based SMB, or SMB_{Pt}, and investment-based SMB, or SMB_{It}. The overall size factor for Fama and French five-factor model is calculated as:

$$\mathrm{SMB_t} = \frac{\mathrm{SMB_{Bt} + SMB_{Pt} + SMB_{It}}}{3}, \tag{22}$$

where SMB_{Bt}, SMB_{Pt} and SMB_{It} are size factors based on book-to-market, profitability and investment sort respectively.

4.3. Estimation and Differentiating between the Models

4.3.1. Estimation Period

The estimation period is from December 2001 to June 2016, what covers the expansion phase before the Great Recession, peak phase, the Great Recession itself, trough and a part of the expansion phase after the Great Recession. In order to meet the purpose of this thesis I split the estimation sample into three subsamples:

1) “Before the Great Recession” sample (from December 2001 to December 2007) that includes the expansion and peak phases of the considered business cycle;

2) "During the Great Recession" sample (from January 2008 to June 2009) that includes the contraction and trough phases;
3) "After the Great Recession" sample (from July 2009 to June 2016) that covers a part of the expansion phase of the ongoing business cycle.
This division is different from the one represented in Table 2 because NBER doesn't provide the exact dates for peak and trough phases, and it seemed logical not to include the peak month to the sample describing the Great Recession, while the trough month, that identifies the very bottom of the economic development at that time, from my point of view, rather belongs to the recession period than to the period of economic growth (expansion).

4.3.2. Model Estimation Method and Validity Diagnostics

Since the initial specification of the Fama and French three-factor, Carhart four-factor and Fama and French five-factor models implies that these are linear multifactor models I decided to carry out their estimation by the most usual method applied to this kind of models - the ordinary least squares (OLS). The core of the OLS lies in minimizing the sum of squared residuals (errors) or, in other words, minimizing the sum of squared differences between the actual values of the dependent variable and its predicted values. This method is practically easy to implement, and the estimation results delivered by it are easy to interpret, which is of great significance concerning the number of regressions to be estimated. However, in order to meet the purpose of this thesis' research I had to ensure the validity of the OLS assumptions listed in Table 3.

Table 3. Assumptions about the OLS Estimators

Assumption	Description
1. $E(\varepsilon_t \mid F_{1t},F_{2t},\ldots,F_{kt}) = 0$	The error ε has an expected value of zero given any values of the explanatory variables $F_{1t},F_{2t},\ldots,F_{kt}$
1.1. $E(\varepsilon_t) = 0$	The expected value of error is zero
1.2. $Cov(\varepsilon_t,F_{it}) = 0$ for i = 1, 2,...,k	The error is uncorrelated with any of the explanatory variables
2. No Perfect Collinearity	No exact linear relationships among the explanatory variables
3. $Cov(\varepsilon_{t-l},\varepsilon_{t-m} \mid F_{1t},F_{2t},\ldots,F_{kt}) = 0$ for $l \neq m$	No serial correlation between error terms given any values of the explanatory variables
4. $Var(\varepsilon_t \mid F_{1t},F_{2t},\ldots,F_{kt}) = \sigma^2$	The error term is homoscedastic (has a constant variance) for all combinations of the outcomes of the explanatory variables
5. $\varepsilon_t \sim N(0,\sigma^2)$	The error term follows a normal distribution

According to Wooldridge (2013), assumptions 1 to 3 from the table above are crucial because under these assumptions the OLS estimators are unbiased ($E(\tilde{\beta}_j) = \beta_j$) and consistent (the probability distribution of $\tilde{\beta}_j$ becomes more and more tightly distributed around β_j as the sample size increases). Assumptions 1 to 4 ensure that the OLS delivers the best linear unbiased estimator (BLUE). BLUE means that the estimator β_j has the smallest variance, can be expressed as a linear function of the data on the dependent variable and has its expected value equal to the true value. Both assumption 4 and 5 are important for conducting the inference.

While the value of the error term in the OLS estimation is always zero on average as long as an intercept is included in the model, I found it necessary to conduct validity check of assumptions 2 to 5. All estimations and tests were performed using build-in tools in the statistical package EViews.

4.3.3. Hypothesis Testing

The hypothesis testing consisted in testing the significance of the regressions' intercept and slopes. The desired outcome was:

1) <u>For all models:</u> $\alpha_i = 0$ (the true intercepts are zero for all left-hand-side assets, what means that the slopes and explanatory returns capture all variation in the expected returns);

2) <u>For three-factor model:</u> $b_i \neq 0$, $s_i \neq 0$, $h_i \neq 0$ (all slopes are jointly statistically significant, what means that R_{mt}, $SMBb_t$ and HML_t help explain variation in the returns, and the model isn't overspecified);

3) <u>For four-factor model:</u> $b_i \neq 0$, $s_i \neq 0$, $h_i \neq 0$, $p_i \neq 0$ (all slopes are jointly statistically significant, what means that R_{mt}, $SMBb_t$, HML_t and $PR1YR_t$ help explain variation in returns, and the model isn't overspecified);

4) <u>For five-factor model:</u> $b_i \neq 0$, $s_i \neq 0$, $h_i \neq 0$, $r_i \neq 0$, $c_i \neq 0$ (all slopes are jointly statistically significant, what means that R_{mt}, SMB_t, HML_t, RMW_t and CMA_t help explain variation in the returns, and the model isn't overspecified).

All tests were performed using Wald test for coefficient restrictions that computes a test statistics based on the unrestricted regression. According to Harell (2001) and EViews User's

Guide[27], given that $\tilde{\beta}_k$ is an estimator of k parameters (kx1 vector) that follows a normal distribution with covariance matrix Ω, the test of m restrictions on the k parameters is expressed by the difference between mxk matrix A and mx1 vector a:

H_0: $A\beta$ - a = 0, against the alternative hypothesis

H_A: $A\beta$ - a $\neq$ 0.

The Wald statistics is then:

$$W = (A\tilde{\beta}_k - a)'(A(\tilde{\Omega}_k/n)A')^{-1}(A\tilde{\beta}_k - a) \sim X_m^2. \quad (23)$$

Under the assumption that errors ε are independently and identically normally distributed the finite sample F-statistics is:

$$F = \frac{W}{m} = \frac{(\bar{\varepsilon}'\bar{\varepsilon} - \varepsilon'\varepsilon)/m}{\varepsilon'\varepsilon/(N-k)}, \quad (24)$$

where $\bar{\varepsilon}$ is the vector of residuals from the restricted regression, and N is the total number of observations. Following EViews User's Guide, I reject the null hypothesis provided that F-statistics has a probability lower than 0,05.

4.3.4. Choice of the Best Model

Among typical requirements to an econometric model is the goodness of fit (which implies using many parameters and minimizing the residual sum of squares) and at the same time - parsimony (keeping model simple, using as few parameters as possible). Therefore except for making sure of the validity of the assumptions from Table 3 and checking the models for overspesification, I use three criteria to differentiate between the regressions: adjusted R-squared (or $\bar{R}^2$), Akaike Information Criterion (AIC) and Schwartz Bayesian Information Criterion (SBC).

According to Wooldridge (2013) the adjusted R-squared can be expressed by the following formula:

$$\bar{R}^2 = 1 - \frac{SSR/(n-k-1)}{SST/(n-1)} \text{ with } 0 \leq \bar{R}^2 \leq 1, \quad (25)$$

where SSR denotes the residual sum of squares, SST denotes the total sum of squares, n stands for sample size and k for number of explanatory variables in a regression.

[27]Available at: http://www.eviews.com/help/helpintro.html#page/content%2Fpreface.html%23wwconnect_header. Last Access: 18.04.2017.

Like the determination coefficient, R^2, the adjusted R-squared is interpreted as the proportion of the sample variation in the dependent variable explained by the independent variables in the regression. However while R^2 increases with each additional variable in a regression, the adjusted R-squared penalizes adding additional explanatory variables to a model and for this reason can be useful when choosing between three-, four and five-factor models.

AIC and SBC are typically used to identify the model that is both parsimonious and has a minimal SSR. EViews reports these criteria, according to Enders (2015), using the formulas:

$$\mathrm{AIC} = -\frac{2\ln(\mathrm{L})}{\mathrm{T}} + \frac{2\mathrm{n}}{\mathrm{T}}, \tag{26}$$

$$\mathrm{SBC} = -\frac{2\ln(\mathrm{L})}{\mathrm{T}} + \frac{\mathrm{n}\ln(\mathrm{T})}{\mathrm{T}}, \tag{27}$$

where T is the number of observations, n is the number of estimated parameters and L is the maximized value of the log of the likelihood function. For a normal distribution the term -2ln(L) can be expressed as sum of $\mathrm{T}\ln(2\pi)$, $\mathrm{T}\ln(\sigma^2)$ and $(1/\sigma^2)(\mathrm{SSR})$ with σ^2 denoting variance of the error term.

When choosing between models, the goal is to get the model that has both smallest value of AIC and SBC. However, there’s a difference between the two criteria: SBC will always choose a more parsimonious model than AIC since ln(T)>2, while AIC can be a more reliable criterion in small samples. Also, comparing models over different estimation periods it’s necessary to take into account that reduction of sample size reduces the values of the information criteria.

It’s worth mentioning that the adjusted R-squared, AIC and SBC would normally require that the compared models have the same left-hand side. Although this is not exactly the case for the three-, four- and five-factor models, the dependent variables there are of the same type, are represented in the same measurement units and belong to the same time horizon, so I assume that it’s acceptable to use $\bar{R}^2$, AIC and SBC to determine which model is best.

5. Estimation and Empirical Tests Results

5.1. Series Distribution

5.1.1. Series Distribution before the Great Recession

The sample is represented by returns data from December 2001 to December 2007. Each series within the sample includes 73 observations.

From the descriptive statistics (Appendix E) I can see that the mean excess return was negative for all risky portfolios in this period. The mean excess return on the market portfolio was also negative (and corresponded to -1,67%). However, for SMB (both the one belonging to the three-, four- and five-factor model), HML and PR1YR the mean return positive. The maximum excess return varied from 5,89% for big aggressively investing companies to 25,79% for medium-size loser companies. The highest maximum excess return was registered among medium and small companies with high and low book-to-market equity, losers, medium and small conservatively investing firms, and companies with weak profitability. Variation in minimum return was also large among portfolios: from -19,11% for small-size losers to -8,27% for big neutral portfolios of firms based on size and momentum. Firms with high book-to-market equity and conservatively investing firms exhibited lowest minimum excess returns. Average standard deviation was rather small and corresponded to 4,55%.

Average skewness for the risky portfolios was 0,16 which is close to zero and indicates a symmetrical distribution. However, portfolios of big and medium losers, as well as medium-size firms with weak profitability had a distribution with a long right tail, while PR1YR's distribution had an indication of a long left-side tail. Kurtosis was 3,4 on average which is close to the normal distribution standard of 3. For big and medium losers, as well as HML and PR1YR there was the strongest indication of a peaked distribution. Based on the Jarque-Bera test results, I reject the null hypothesis of that the excess returns of medium firms with high book-to-market equity, big and medium losers, big and medium conservatively investing firms and medium firms with weak profitability follow a normal distribution at 5% significance level. HML and PR1YR fail normality test at all significance levels, likely because of skewness and kurtosis.

5.1.2. Series Distribution during the Great Recession

The sample is represented by returns data from January 2008 to June 2009. Each series within the sample consists of 18 observations.

The descriptive statistics in Appendix F shows that the mean excess return was negative for all risky portfolios, except for small firms with high book-to-market equity and weak profitability. The mean excess return on the market portfolio was -2%. Unlike the period preceding the Great Recession, PR1YR had a negative mean return, while RMW - positive. The maximum excess return was also higher than in the previous sample and varied from 5,86% for big winners to 52,28% for small losers. Companies with high book-to-market equity and losers tended to have highest maximum excess return.Variation in the minimum excess return among portfolios was even larger than in the previous period: from -29,98% for small losers to -11,77% for big conservatively investing firms. Average standard deviation almost doubled compared to the period before the Recession and corresponded to 9,67%.

Skewness for the risky portfolios was 0,43 on average which is a bit further away from zero than in 5.1. Excess returns on portfolios of medium and small losers, neutrally and conservatively investing small firms as well as small firms with robust weak profitability had signs of a long right tail in their distribution. So did also both returns on SMB and HML portfolios. PR1YR’s distribution had an indication of a long left tail. Kurtosis was 3,62 on average which is close to the normal distribution standard of 3. For medium and small firms with high book-to-market equity, medium and small losers and small firms with weak profitability there was the strongest indication of a peaked distribution. The same was valid for both SMB portfolios, HML and PR1YR. CMA exhibited signs of a flat distribution. Excess returns of medium and small firms with high book-to-market equity, medium and small losers and small firms with weak profitability fail the Jarque-Bera normality test at 5% significance level. The same applies to both SMB portfolios, HML and PR1YR, which isn’t surprising given their skewness and kurtosis values.

5.1.3. Series Distribution after the Great Recession

The sample is represented by returns data from July 2009 to June 2016. Each series within the sample contains 84 observations.

From the descriptive statistics (Appendix G) I can see that, unlike 5.1.1 and 5.1.2, the mean excess return was positive for all risky portfolios (1,5% on average) which indicates a general increase in the level of risky returns relative to the one-months Treasury bill rate. Among the factor portfolios only RMW exhibited a negative average return of -0,13%. The maximum excess return varied from 9,07% for big conservatively investing companies to 20,86% for small firms with robust profitability, which is a much smaller variation than before and during the Recession. High values of the maximum return were especially observed among losers. Variation in minimum return among portfolios was also smaller than in the previous periods: from -13,28% for small winners to -6,29% for big conservatively investing firms. Average standard deviation was quite small and corresponded to 4,6%.

Average skewness for the risky portfolios was 0,08 which is close to zero and indicates a symmetrical distribution. Big losers and small companies with robust profitability had an indication of a long right tail in the distribution. The same applied to HML and CMA, while PR1YR and RMW had signs of a long left tail. Kurtosis was 3,24 on average which is close to the normal distribution standard. For big losers, as well as RMW and CMA there was the strongest indication of a peaked distribution. Based on the Jarque-Bera test I reject null hypothesis of that the excess returns of big losers, as well as the excess returns of the factor portfolios RMW and CMA, follow a normal distribution at 5% significance level.

5.2. Estimation Results and Validity Tests

5.2.1. Introduction

Because of having to run three regressions (one for each estimation subperiod) for each risky portfolio I estimated 135 regressions in total: 27 regressions related to the Fama and French three-factor model, 27 regressions related to the Carhart four-factor model and 81 regression related to the Fama and French five-factor model. The preliminary analysis of the estimators and their p-values has shown that intercept, α_i, wasn't significantly different from zero in most cases, independently of the state of economy. This is an indication of the models' good explanatory power. The influence of the excess return on the market portfolio, R_m-R_f, on the risky portfolio excess returns was very strong and statistically significant for all models. On average 1% increase in R_m-R_f led to 1,02% growth in the excess returns of the risky portfolios. However the other variables didn't exhibit any clear pattern both concerning the sign of their correlation with the dependent variables and their significance. From this standpoint the Fama and French five-factor model exhibited the most unstable results. Despite receiving the desired results on α_i and the high values of the $\bar{R}^2$, these estimation results couldn't be trusted before performing the validity tests, described in the next section.

5.2.2. Validity Tests of the Estimated Models

The validity tests, or validity diagnostics, of the estimated models consisted in checking whether 2 to 5 assumptions about the OLS estimators from Table 3 were true, and included the following steps:

Step 1. In order to check for the presence of the perfect collinearity I first plot the correlation matrices of the explanatory variables represented in Table 4.

Table 4. Correlation Matrices for the Before, During and After the Great Recession Samples

Before the Great Recession:							
	RM-Rf	SMBb	HML	PR1YR	SMB	RMW	CMA
RM-Rf	1,00	0,31	-0,04	-0,36	0,28	-0,27	0,08
SMBb	0,31	1,00	0,10	0,14	0,97	-0,35	0,13
HML	-0,04	0,10	1,00	0,16	0,28	-0,08	0,33
PR1YR	-0,36	0,14	0,16	1,00	0,17	0,32	-0,33
SMB	0,28	0,97	0,28	0,17	1,00	-0,30	0,15
RMW	-0,27	-0,35	-0,08	0,32	-0,30	1,00	-0,45
CMA	0,08	0,13	0,33	-0,33	0,15	-0,45	1,00
During the Great Recession:							
	RM-Rf	SMBb	HML	PR1YR	SMB	RMW	CMA
RM-Rf	1,00	0,67	0,52	-0,63	0,68	-0,41	-0,23
SMBb	0,67	1,00	0,75	-0,86	0,99	-0,81	0,17
HML	0,52	0,75	1,00	-0,88	0,83	-0,67	0,24
PR1YR	-0,63	-0,86	-0,88	1,00	-0,89	0,82	-0,32
SMB	0,68	0,99	0,83	-0,89	1,00	-0,79	0,16
RMW	-0,41	-0,81	-0,67	0,82	-0,79	1,00	-0,60
CMA	-0,23	0,17	0,24	-0,32	0,16	-0,60	1,00
After the Great Recession:							
	RM-Rf	SMBb	HML	PR1YR	SMB	RMW	CMA
RM-Rf	1,00	0,47	0,17	-0,13	0,46	-0,34	0,08
SMBb	0,47	1,00	0,24	-0,01	0,99	-0,34	0,11
HML	0,17	0,24	1,00	-0,44	0,32	-0,67	0,35
PR1YR	-0,13	-0,01	-0,44	1,00	-0,04	0,25	-0,06
SMB	0,46	0,99	0,32	-0,04	1,00	-0,37	0,11
RMW	-0,34	-0,34	-0,67	0,25	-0,37	1,00	-0,49
CMA	0,08	0,11	0,35	-0,06	0,11	-0,49	1,00

From the table above I can make the following conclusions:

1) In the sample covering the months preceding the Great Recession there seemed to be no signs of perfect correlation between the explanatory variables. The strongest correlation was between the returns on RMW and CMA portfolios which tended to move in opposite directions.
2) In the "during the Great Recession" sample the correlation between the explanatory variables got stronger, and the correlation signs changed. In particular Corr(SMBb,HML), Corr(SMBb,PR1YR), Corr(HML,PR1YR), Corr(HML,SMB) and Corr(SMB,RMW) could potentially be of concern.
3) The sample covering the post-recession period had lower correlation between the variables than the "during the Great Recession" sample, but still higher than in the time period preceding the Recession. The correlation signs were in most cases the same compared to the Recession period. The strongest correlation was between the returns on HML and RMW portfolios and equaled to -0,67, which shouldn't be of concern.

In order to ensure that there's no perfect collinearity problem, I then take a look at the Coefficient Variance Decomposition for each model (see Appendix H). According to the EViews User's Guide[28], the eigenvalues shouldn't have a condition number value smaller than 0,001, which would be a sign of multicollinearity. The variance decomposition proportions in column "1" (the column related to the eigenvalue with the smallest condition number) also provide information about presence a strong collinearity: two or more variables with variance decomposition proportion larger than 0,5 would indicate multicollinearity. From Appendix H I can see that both conditions are fulfilled for all three time period samples, so I can conclude that there's no exact linear relationships between the variables in the three-, four- or five-factor models.

Step 2. In order to check for serial correlation, or correlation of the residuals with their own lagged values, I perform Breusch-Godfrey test for serial correlation for each of the regressions using five lags in the residuals. As an example in Table 5 I present the results of this test for the first regression.

[28] Available at: http://www.eviews.com/help/helpintro.html#page/content%2Fpreface.html%23wwconnect_header. Last Access: 18.04.2017.

Table 5. Results of Breusch-Godfrey Serial Correlation LM Test for the Fama and French Three-Factor Model with the Excess Return on B/H portfolio as a Regressand before the Great Recession

Breusch-Godfrey Serial Correlation LM Test:

F-statistic	2.914309	Prob. F(5,64)	0.0197
Obs*R-squared	13.53827	Prob. Chi-Square(5)	0.0188

Test Equation:
Dependent Variable: RESID
Method: Least Squares
Date: 04/25/17 Time: 00:13
Sample: 2001M12 2007M12
Included observations: 73
Presample missing value lagged residuals set to zero.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.045464	0.240830	-0.188779	0.8509
RM_RF	-0.011022	0.063496	-0.173581	0.8627
SMBB	0.021232	0.100813	0.210606	0.8339
HML	0.051098	0.095944	0.532587	0.5962
RESID(-1)	0.484097	0.133270	3.632458	0.0006
RESID(-2)	-0.249030	0.146863	-1.695663	0.0948
RESID(-3)	0.051082	0.151840	0.336422	0.7377
RESID(-4)	0.032006	0.153245	0.208854	0.8352
RESID(-5)	-0.008675	0.132357	-0.065539	0.9479

R-squared	0.185456	Mean dependent var	3.16E-16
Adjusted R-squared	0.083638	S.D. dependent var	1.703080
S.E. of regression	1.630304	Akaike info criterion	3.930409
Sum squared resid	170.1050	Schwarz criterion	4.212794
Log likelihood	-134.4599	Hannan-Quinn criter.	4.042944
F-statistic	1.821443	Durbin-Watson stat	1.918067
Prob(F-statistic)	0.089212		

In the top section of Table 5 I can see that F-statistics has a probability lower than 0,05, so I reject the null hypothesis of no serial correlation at 5% significance level. In the middle section of the table I see that there's a strong significant correlation between the current residual and the residual in the preceding month. Therefore the regression needs to be corrected before proceeding to the heteroskedasticity test, otherwise I'll get false results.

Step 3. EViews Tutorials[29] suggest to use AR(p), MA(q) or ARMA(p,q) models to correct for serial correlation. By trying out different model specifications I've found out that in this case

[29] Available at: http://www.eviews.com/Learning/. Last Access: 25.04.2017.

the error term is highly likely to follow AR(1) process and the estimation results are presented in Table 6.

Table 6. Estimation Results for the Fama and French Three-Factor Model with the Excess Return on B/H portfolio as a Regressand and ε_t Following AR(1) Process before the Great Recession

Dependent Variable: B_H_SB_RF
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 05/22/17 Time: 22:15
Sample: 2001M12 2007M12
Included observations: 73
Convergence achieved after 9 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.421907	0.446622	0.944663	0.3482
RM_RF	1.128619	0.052460	21.51370	0.0000
SMBB	-0.190728	0.095642	-1.994190	0.0502
HML	0.671991	0.087864	7.648076	0.0000
AR(1)	0.439249	0.122638	3.581686	0.0006
SIGMASQ	2.407956	0.364336	6.609164	0.0000

R-squared	0.862843	Mean dependent var	-1.244625
Adjusted R-squared	0.852607	S.D. dependent var	4.219010
S.E. of regression	1.619751	Akaike info criterion	3.883975
Sum squared resid	175.7808	Schwarz criterion	4.072232
Log likelihood	-135.7651	Hannan-Quinn criter.	3.958999
F-statistic	84.29829	Durbin-Watson stat	1.773296
Prob(F-statistic)	0.000000		

Inverted AR Roots	.44

From the table above I can see that influence of AR(1) on the excess portfolio return is strong and significant at 5% significance level. Besides, the roots of the inverted characteristical polynomial lie inside unit circle, so the process is stationary.

In order to check whether this helped to eliminate serial correlation in the residuals I re-estimate the regression using ARMA conditional least squares method and perform Breusch-Godfrey test for serial correlation one more time. The results are shown in Table 7.

Table 7. Results of Breusch-Godfrey Serial Correlation LM Test for the Fama and French Three-Factor Model with the Excess Return on B/H portfolio as a Regressand and ε_t Following AR(1) Process before the Great Recession

Breusch-Godfrey Serial Correlation LM Test:

F-statistic	1.303611	Prob. F(5,62)	0.2741
Obs*R-squared	6.849287	Prob. Chi-Square(5)	0.2321

Based on the probability value for the F-statistics in the top part of the table I can't reject the null hypothesis about no serial correlation, so the regression modification was successful and now I'm able to proceed to step 3.

Step 4. In order to detect heteroskedasticity I performed White test for most of the regressions, except for the five-factor model in the period of the Great Recession when the estimation period was too short to use this test, so Breusch-Pagan-Godfrey test was used instead. Continuing to work with the regression corrected for serial correlation in Step 2 for illustration, I present the results of White test for heteroskedasticity in Table 8.

Table 8. Results of White test for Heteroskedasticity for the Fama and French Three-Factor Model with the Excess Return on B/H portfolio as a Regressand and ε_t Following AR(1) Process before the Great Recession

Heteroskedasticity Test: White

F-statistic	9.45E+22	Prob. F(27,45)	0.0000
Obs*R-squared	73.00000	Prob. Chi-Square(27)	0.0000
Scaled explained SS	90.83536	Prob. Chi-Square(27)	0.0000

Based on the zero probability for the F-statistic I reject the null hypothesis of homoskedasticity.

In order to get more correct standard errors and therefore valid results when testing the hypotheses about variables in the regressions this model needs to be corrected for heteroskedasticity as well. This leads med to the next step.

Step 5. Both Wooldridge (2013) and EViews User's Guide[30] suggest to use heteroskedasticity-robust, or Huber-White, standard errors which is a built-in option in

[30] Available at: http://www.eviews.com/help/helpintro.html#page/content%2Fpreface.html%23wwconnect_header. Last Access: 18.04.2017.

EViews. I apply this option to the regressions that were originally free for serial correlation. On the other hand, for the regressions with presence of both serial correlation and heteroskedasticity I use heteroskedasticity and autocorrelation consistent, or HAC (Newey-West), standard errors which are also available in EViews.

Since the regression, considered in the previous steps belongs to the second case, I return to its original form and re-estimate it using HAC standard errors. The estimation results are presented in Table 9.

Table 9. Estimation Results for the Fama and French Three-Factor Model with the Excess Return on B/H portfolio as a Regressand using HAC Standard Errors before the Great Recession

Dependent Variable: B_H_SB_RF
Method: Least Squares
Date: 05/22/17 Time: 22:36
Sample: 2001M12 2007M12
Included observations: 73
HAC standard errors & covariance (Bartlett kernel, Newey-West fixed bandwidth = 4.0000)

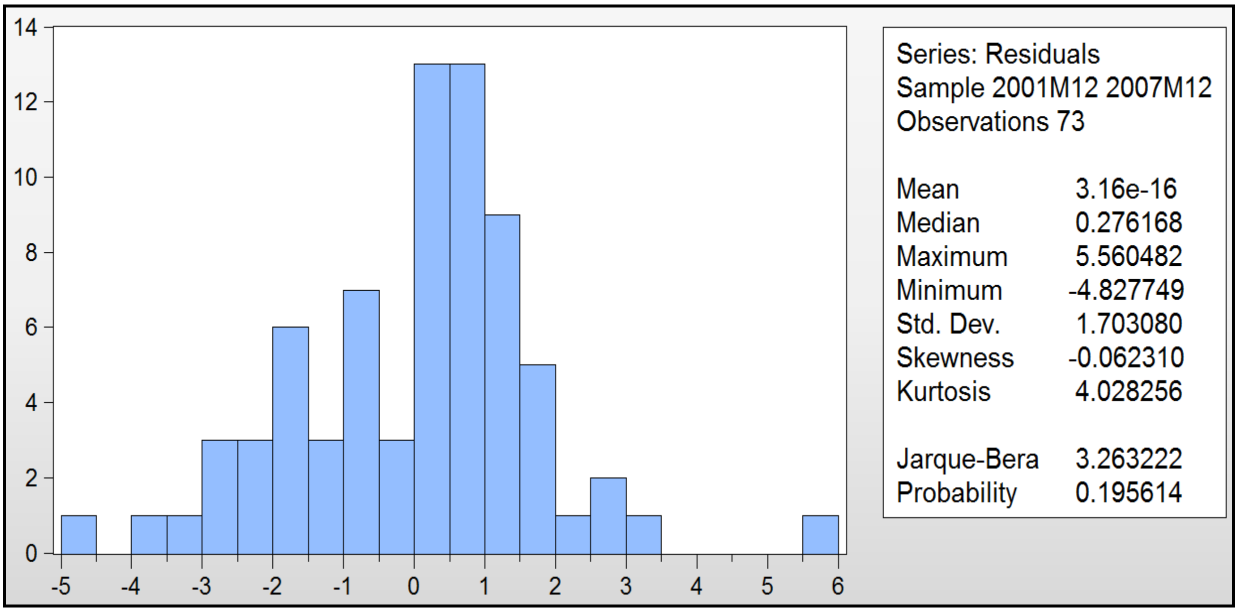
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.429325	0.206581	2.078244	0.0414
RM_RF	1.106684	0.057519	19.24020	0.0000
SMBB	-0.189360	0.083755	-2.260883	0.0269
HML	0.525259	0.106371	4.937997	0.0000

R-squared	0.837052	Mean dependent var	-1.244625
Adjusted R-squared	0.829967	S.D. dependent var	4.219010
S.E. of regression	1.739709	Akaike info criterion	3.998549
Sum squared resid	208.8345	Schwarz criterion	4.124054
Log likelihood	-141.9470	Hannan-Quinn criter.	4.048565
F-statistic	118.1494	Durbin-Watson stat	1.264931
Prob(F-statistic)	0.000000	Wald F-statistic	135.2604
Prob(Wald F-statistic)	0.000000		

In the table above I get standard errors and therefore t-statistic that are both robust to heteroskedasticity and serial correlation, while the coefficients remain unchanged. However this still doesn't allow me to test hypotheses about joint significance of variables before going through the next step.

Step 6. Whether the error term follows a normal distribution is checked using a histogram with Jarque-Bera test statistics. The results of this test for the HAC-modified in step 5 model are presented in Figure 2.

Figure 2. Normality Test of the Residuals for the Fama and French Three-factor Model with the Excess Return on B/H portfolio as a Regressand and HAC Standard Errors before the Great Recession



From the figure above I see that the probability of Jarque-Bera test statistics is much higher than 0,05, so I can't reject the null hypothesis of the normal distribution of the error terms. This means that now I can test the hypotheses about joint significance of the variables in this regression using Wald test and will get valid results.

In total non-normal distribution of residuals was a problem in 19 out of 135 regressions: 9 regressions based on the "before the Great Recession" sample, only 1 regression based on the "during the Great Recession" sample and 9 regressions based on the "after the Great Recession" sample. According to Wooldridge (2013) the "before" and "after the Great Recession" samples should be big enough to use the central limit theorem to conclude that the OLS estimators in the respective regressions are approximately normally distributed. When it comes to the only one problematic regression from the "during the Great Recession" sample, the number of observations there is smaller than 30 which some econometricians consider to be satisfactory, so I decided to exclude this regression from the further analysis.

5.2.3. Final Estimation and Empirical Tests Results

The final estimation and empirical tests results are presented in Appendix I. The models there are already corrected for serial correlation and heteroskedasticity where it was necessary, and the the variables, including the intercepts, that were shown to be statistically insignificant by the Wald test are listed. For the convenience of the analysis of my results, with respect to identifying the best performing model and evaluating the models' performance in different states of economy, I also provide statistical tables in Appendices J and K which summarize the main findings.

In the whole estimation period (from December 2001 til June 2016) the Fama and French five-factor model with the excess return on the stock portfolio constructed based on size and investment as the left hand side variable had the smallest AIC, SBC and the largest explanatory power expressed by $\bar{R}^2$ on average. The next two best models were size- and book-to-market equity-based and size- and operating profitability-based five-factor models. The Fama and French three-factor and Carhart four-factor models showed the worst result.

However it's obvious from Appendix 10 that out of 81 regressions estimated for the Fama and French five-factor model there wasn't any where all explanatory variables would be statistically different from zero, independently of the state of economy. Wald test has shown that CMA was statistically insignificant in 74% of cases, followed by HML and RMW (70% each). Exclusion of HML from the model had no influence on significance of RMW and CMA. Although this doesn't lead to bias, all of the five-factor models are overspecified. As a consequence of this, the statistically significant variables get larger variances and the precision of their estimators gets reduced. As mentioned before, my goal is to get the most parsimonious model with the highest explanatory power, and the Fama and French five-factor model fails the parsimony requirement. In addition the intercept α_i was statistically different from zero in 14% of the regressions, what normally would be a sign of that the model's factors don't completely explain the excess return on the risky portfolios.

The Carhart four-factor model performed a bit better. For six out of 27 regressions all explanatory variables were statistically significant, with three of these regressions being based on the time period preceding the Great Recession and three based on the period after the Great Recession. The model showed the best results in explaining returns on portfolios of the medium and small winners and small losers. Although the momentum effect factor, PR1YR, turned out to be significant in 21 regression, SMBb and HML factors didn't explain the excess return on the size- and momentum-based risky portfolio in 37% and 59% of cases respectively. This leads to that the Carhart four-factor model faces the same problem as the five-factor model, and therefore the precision of the estimators can't be trusted. However SMBb had mostly negative effect on the returns of the big stock portfolios and strong positive effect on the returns of the small stock portfolios which is the desired result according to Fama and French (2015). PR1YR also had the expected effect on the portfolio returns: positive for winners and negative for losers.When it comes to the intercept, like in the five factor model, it was statistically significant in 14% of regressions. In addition the model was proved to be inefficient at explaining the portfolio returns in the recession state of economy: all regressions that were estimated for that time period turned out to be overspecified.

The Fama and French three-factor model delivered the best results concerning avoiding overspecification - in 16 of 27 regressions all explanatory variables were statistically significant and seems to be most reliable. For six of these regressions December 2001 - December 2007 was used as the estimation period, four of them belonged to the recession period of January 2008 - June 2009, and six to July 2009 - June 2016. The intercept α_i was statistically significant only in the very first regression which is related to the period before the Great Recession. This means that in the Fama and French three-factor model the excess return on the market portfolio, SMBb and HML factors don't fully explain the excess return on the portfolio of big stocks with high book-to-market equity in the period preceding the Great Recession and some significant variable, related to this kind of stocks and their behaviour in the expansion period, might be missing. In addition, the Fama and French three-factor model outperformed the Carhart four-factor model based on AIC, SBC and the adjusted R-squared.

Concerning the explanatory variables in the three-factor model, the impact of SMBb on the excess return on the risky portfolio was statistically insignificant in 15% of cases, while the impact of HML - in 33% of cases. In the sample corresponding to the Great Recession period the number of regressions with statistically insignificant SMBb and HML is larger than otherwise. Although for the "before the Recession" sample the model has delivered more regressions with insignificant variables than for the "after the Recession" sample. Like in Fama and French's findings (1995), for portfolios of firms with high book-to-market equity ratio, or weak firms, the distress factor HML had negative effect on returns, independently of the state of economy. For portfolios of strong firms the effect was negative. The size factor SMBb had mostly negative effect (in regressions where it was positive it was also very small) on returns on portfolios of big stocks, while the effect of SMBb on returns on small stock portfolios was positive and quite large. This also corresponds to Fama and French's findings.

6. Conclusion

The conducted research has proved that there's a clear basis for all the critique directed at the Fama and French three-factor, Carhart four-factor and Fama and French five-factor models. In order to achieve true results I followed the instructions, provided by each model's creators, as precisely as it was practically possible when constructing the factors for each respective model. Despite that, none of the models has shown steady results and satisfied the criteria listed in 4.3.3 for all portfolios of stocks.

The only factor that turned out to be statistically significant in explaining the excess returns on all portfolios in all three models, was the excess return on the market portfolio, R_m-R_f. The effect of this factor was approximately of the same strength and had the right sign in all 135 cases. However both findings of the CAPM opponents, mentioned in Section 2.2.1 of this thesis, and my own findings, showing that there're other factors correlated with return on the market portfolio and with significant influence on stock returns, indicate that R_m-R_f can't be used as a single factor for explaining portfolio returns.

The purpose of this thesis though wasn't to find or suggest a perfect capital asset pricing model, but to check whether the Fama and French three-factor, Carhart four-factor and Fama and French five-factor models are actually valid, which of them has the best performance for my estimation sample and whether there's a difference the models' performance depending on the state of economy. When pursuing this purpose I've made the following important findings:

<u>1. Findings about the Fama and French three-factor model:</u>

- The Fama and French three-factor model turned out to be the best performing model.
- Its estimation and empirical test results have confirmed that risks influencing stock returns are multidimensional. The size of the firm and the relationship between its

book and market value have in most cases been shown to be significant at explaining portfolio returns in addition to the return on the market portfolio.

- Zero values of the intercept α_i in all regressions, except for one, indicate that there's no unexplained variation in portfolio returns that is left out.
- Despite the presence of the distress factor HML, which was expected to partially proxy the "recession beta", the model showed a slightly weaker performance when estimating portfolio returns during the Great Recession than otherwise. In fact, exactly this factor turned out to be insignificant 2,5 times more often compared to the periods preceding and after the Great Recession.

2. Findings about the Carhart four-factor model:

- The Carhart four-factor model is the second-best model, although it has gotten this title exclusively because of the extremely poor performance of the five-factor model. It has shown quite unstable results depending on the left-hand side portfolio excess returns and economy state which doesn't allow me to conclude that this model is valid in it original formulation.
- PR1YR has been found to be statistically significant in the majority of the regressions so from the first point of view it seems that it's quite an important factor to consider when trying to explain the variation in stock returns. However I have a strong suspicion that its high significance can be explained by the way the left-hand side variables in the Carhart model are created, namely that the ranking by the one-year momentum effect is used both for constructing the dependent variable, R_i, and the explanatory variable, PR1YR. This could also potentially explain the weak ability of SMBb and HML factors to explain the excess return in this model.
- Despite the claims of the supporters of the Carhart four-factor model about the large positive effect of PR1YR on the model's ability to explain the variation in returns, inclusion of this factor into the three-factor model has actually led to reduction of the explanatory power of the regressors, measured by the adjusted R-squared.
- The model has shown the worst performance when estimating portfolio returns during the Great Recession - there was no regression with all variables being statistically significant. The distress factor HML was redundant even in more cases than for the Fama and French three-factor model what makes me question its actual ability to capture the sensitivity of stock returns to the economic distress.

3. Findings about the Fama and French five-factor model:

- The use of the three different dependent variables in the Fama and French five-factor model has led to equally bad results. The preliminary expectation, based on Fama and French's recent papers, was that: a) with the excess returns on portfolios ranked by size and book-to-market equity at least the HML factor would be statistically significant; b) with the excess returns on portfolios ranked by size and profitability at least the RMW factor would be statistically significant; c) with the excess returns on portfolios ranked by size and investment at least the CMA factor would be statistically significant. These expectations have failed completely. Moreover, exclusion of HML based Fama and French's suggestion (2015) haven't improved the model. The empirical tests of the model have shown that neither RMW or CMA help explain the variation in the portfolio returns. Since without these two factors I end up with the traditional three-factor model, I conclude that the Fama and French five-factor model isn't a valid model.

In conclusion I would like to point out that even though the Fama and French three-factor model turned out to be the most reliable out of the three considered models, it delivered quite nonpersistent results, especially when trying to explain portfolio returns in the recession state of economy which are of big interest for a wide range of market players. This confirms the need for further search for a more reliable model. In addition, I find it surprising that when Fama and French provide the empirical test results of their three- and five-factor models, the models always turn out to be valid. However, when other researchers, including me, try to recreate their steps, they often fail at receiving the same results. This enhances my conjecture that Fama and French might use some additional adjustments when constructing the models' factors which they don't describe in their papers.

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Appendices

Appendix A

List of Companies in the Database by RIC and Exchange

NYSE Consolidated												NYSE MKT	Nasdaq		
GE	XOM	ADM	BAX	OLN	DDR	NNN	OCN	SWC	FCH	ORCL.K	SO	CCF	AAPL.O	DOX.O	MNRO.O
VZ	WMT	DDS	MAS	REG	TSS	BIO	BYD	CVA	BRO	ARNC.K	WY	ESTE.K	NVDA.O	RGLD.O	BRKS.O
IP	JNJ	RCL	PHM	IPG	GEO	HLS	KEX	UVV	IEX	SPGI.K	DY	PTN	MSFT.O	ABMD.O	SCHN.O
HD	BMY	OKE	HSY	TFX	AEO	USG	DNR	TEN	FDP	KATE.K	FE	FRD	CSCO.O	MYGN.O	HIBB.O
PG	PFE	CLX	HOG	EME	SCG	OGE	CAA	FCN	HRB	BRKa	F	LEU	QCOM.O	IDTI.O	RGEN.O
GS	CVX	HCN	HAR	AEE	IRM	KBH	VAL	MMS	NCR	EVHC.K	M	SKY	SBUX.O	NDSN.O	WDFC.O
BA	IBM	NBL	HRS	CNX	EGN	TTC	SUI	CBL	CSL	AGCO.K	K	VISI.K	BIIB.O	CREE.O	GK.O
LB	TJX	CAG	CHD	COO	MTD	DIN	JLL	RHP	INT	SCCO.K	O	VSR	COST.O	VIAV.O	PLXS.O
KO	NKE	COG	TXT	IFF	VAR	TKR	UGI	BSX	DDD	CLGX.K	A	MSN	AMAT.O	MSCC.O	PSMT.O
CL	DIS	DHI	HCP	BWA	LPX	THO	ELS	HAL	PNM	DECK.K	R	TRT	MCHP.O	PLCE.O	IMGN.O
MS	DOW	AME	BXP	ALK	EAT	CCK	ATO	BEN	DBD	FCEa	X	JOB	ATVI.O	VSAT.O	LANC.O
MO	FCX	ABC	PVH	VNO	CHS	TRN	WST	NSC	HLX	INGR.K	T	UQM	WBA.O	MRCY.O	ACIW.O
DE	MCD	HRL	EQT	MSI	CTB	SCI	RPM	RTN	AAN	HUBB.K	D	MOC	INTU.O	SONC.O	RAVN.O
FL	MRK	PEG	BCR	MUR	PBI	AVP	CPT	GIS	YUM	WWW		VII	ADBE.O	IIVI.O	BOBE.O
KR	SLB	CHK	MHK	SEE	CPE	HRC	HRG	DVN	TIF	CWEI.K		SIF	ADI.O	WETF.O	EPAY.O
LH	CAT	APA	BLL	DTE	TER	AVY	LSI	TOL	LUK	BXMT.K			LRCX.O	UNFI.O	CRMT.O
DD	LOW	UTX	KMX	RAD	CMC	MTN	AIV	ROK	CBS	TLRD.K			EA.O	TVTY.O	WRLD.O
PX	UNP	JWN	GGP	FMC	OHI	ASH	WRI	TAP	CLF	MMM			ROST.O	JKHY.O	QSII.O
EL	CVS	LUV	OMC	UHS	SKX	LII	CLH	APD	MLM	GWW			SYMC.O	WWD.O	SPLS.O
ED	PEP	APC	CAH	PKG	RHI	ATW	DRE	EFX	ARW	MMC			PAYX.O	SANM.O	COHR.O
RL	STZ	EMR	WHR	SNA	SLG	CVG	JBL	CSC	BBY	CIEN.K			WFM.O	TELL.O	SAFM.O
SM	TWX	WMB	ETR	AES	BDC	UDR	MDU	EQR	NOC	TGNA.K			ADSK.O	TECH.O	JJSF.O
PH	PXD	MGM	LEN	CMS	PII	BID	PKI	KNX	MRO	RRC			XLNX.O	AMAG.O	RMBS.O
HP	UPS	LMT	BHI	NUS	TEX	TUP	DRQ	XRX	NFX	NUE			NTAP.O	TTEK.O	ACXM.O
ES	ABT	NWL	SYY	PWR	SAM	EV	BMS	NHI	HST	CUZ			LLTC.O	AMWD.O	CPRT.O
WM	HON	VLO	DOV	WAT	VMI	AJG	GVA	DCI	JCP	CMI			QRVO.O	UHAL.O	ICON.O
LM	HPQ	SHW	PCG	JEC	LNT	PNW	BFb	NVR	MCK	DHR			AMTD.O	CALM.O	
NI	COP	AZO	BDX	CNP	CXW	AOS	SON	WDR	HES	AKS			CAKE.O	EGHT.O	
RS	TGT	NEM	AVB	KIM	SMG	CDE	TDS	NFG	CCL	PPG			CBRL.O	FINL.O	
HL	KMB	AXP	KSU	ALB	RMD	ATI	WCC	ALE	KRC	GLW			HOLX.O	APOG.O	
DO	OXY	NEE	SYK	ARE	OSK	GRA	GWR	LAD	EXC	EIX			CRUS.O	TECD.O	
IT	EOG	VFC	DVA	THC	GXP	MAC	HIW	OMI	TSO	WSO			KLAC.O	ASNA.O	
BC	LLY	SPG	GPC	MAA	MTZ	RJF	LPT	OFC	ITW	BLK			URBN.O	IMMU.O	
DF	DRI	URI	MKC	WSM	FRT	LEG	DYN	RGR	AEP	SWN			BBBY.O	FRED.O	
AN	FLS	DUK	HFC	ESS	BIG	AVT	KMT	MDR	PSA	CTL			PDCO.O	MLHR.O	
FR	APH	VMC	WAB	EMN	OII	TCO	FDS	KEM	GPS	CW			BEAV.O	FIZZ.O	
OA	SRE	KSS	SWK	DGX	MSM	ANF	MDP	GGG	ECL	OI			CASY.O	OSIS.O	
GD	RSG	TSN	ROP	AMG	RES	MD	HXL	WGL	TMO	CR			PRXL.O	PRGS.O	

Appendix B

NYSE Breakpoints for the Three- and Five-Factor Models in 2001-2016[1]

Breakpoints by Criterion	Size			Book/Market Equity		Investment		Profitability	
	70 %	Median	30 %	70 %	30 %	70 %	30 %	70 %	30 %
2001	6657259	2341013	1224260	0,616404	0,282712	0,030862	0,185495	0,197051	0,342725
2002	6966464	2810121	1339108	0,627467	0,31266	-0,002311	0,134076	0,144841	0,29045
2003	6798033	2723996	1341466	0,646682	0,334539	0,002305	0,129637	0,139763	0,28689
2004	8167752	3677122	1879062	0,5342	0,302618	0,038834	0,134789	0,150613	0,28497
2005	10743399	4331261	2261417	0,493387	0,278334	0,047873	0,159446	0,16766	0,298351
2006	11318383	4655564	2732474	0,469242	0,263845	0,017314	0,156877	0,175463	0,332749
2007	13156229	5613380	3470448	0,454648	0,257367	0,027010	0,147155	0,179156	0,338827
2008	10803653	5064869	2663938	0,515513	0,27567	0,028860	0,152845	0,18008	0,33049
2009	7739974	3220039	1779296	0,815978	0,429971	-0,026093	0,088393	0,136727	0,334502
2010	9880118	4301180	2435150	0,611557	0,357357	-0,023502	0,061957	0,093015	0,254139
2011	12650063	5764407	3377792	0,571507	0,317792	0,013614	0,104840	0,136979	0,270954
2012	12642878	5475658	3190744	0,630236	0,33044	0,01824	0,108389	0,152363	0,309791
2013	15616202	6906464	4171113	0,571207	0,31211	0,019758	0,104326	0,143652	0,307229
2014	18406154	8438193	5401706	0,507054	0,261813	0,008894	0,084763	0,152716	0,288857
2015	17123089	8694227	4714658	0,482474	0,245179	-0,004736	0,081324	0,145626	0,305201
2016	17899421	8313609	4853907	0,525419	0,247701	-0,040443	0,051744	0,121417	0,298676

<u>Interpretation of the Notation:</u>

[1]NYSE Breakpoints for the four-factor model aren't presented here due to their large amount as size and momentum portfolios are reformed on a monthly basis

Appendix C

Construction of Portfolios of Risky Returns for the Three-, Four- and Five-Factor Models by Size and Groups of Return

Groups of Return		Size		
		Small	**Medium**	**Big**
Book/Market Equity	**Low**	S/L	M/L	B/L
	Medium	S/M	M/M	B/M
	High	S/H	M/H	B/H
Momentum	**Losers**	S/L	M/L	B/L
	Neutral	S/N	M/N	B/N
	Winners	S/W	M/W	B/W
Investment	**Agressive**	S/A	M/A	B/A
	Neutral	S/N	M/N	B/N
	Conservative	S/C	M/C	B/C
Profitability	**Robust**	S/R	M/R	B/R
	Medium	S/M	M/M	B/M
	Weak	S/W	M/W	B/W

Appendix D

List of Companies with Negative Equity in 2001-2016

2000	2001	2002	2003	2004	2005	2006	2007	2008
AVP	AVP	AES	AES	GRA	CCK	CCK	GRA	BA
CRUS.O	FCX	AVP	AKS	HLS	CLX	CLX	HLS	CCK
FCX	GRA	CCK	ELS	HXL	GRA	F	LLTC.O	CLX
GIS	HLS	FCX	GRA	MDR	HLS	GRA	SONC.O	CPE
GRA	HXL	GRA	HLS	RAD	IMMU.O	HLS	TCO	F
YUM	IT	HLS	HXL		MDR	IMMU.O		GRA
	MOC	HXL	MDR		RAD	OI		HLS
	RAD	IT	RAD		USG			IMMU.O
	WETF.O	MDR	WETF.O					IT
		RAD						LLTC.O
		TEN						PBI
		WETF.O						SONC.O
								SUI
								TCO
								TEN
								URI
								YUM
2009	2010	2011	2012	2013	2014	2015	2016	
AZO	AZO	AZO	AKS	AKS	AKS	AKS	AZO	
BXMT.K	BXMT.K	BXMT.K	AZO	AZO	AZO	AVP	EAT	
CCK	CCK	CCK	CCK	CIEN.K	CIEN.K	AZO	HPQ	
CLX	F	CLX	CIEN.K	FCH	CLF	CHK	IMGN.O	
CPE	GRA	DF	CLX	KATE.K	FCH	CL	IMMU.O	
F	HLS	KATE.K	FCH	LB	LB	CLF	LB	
GRA	PBI	KATE.K	KATE.K	LEU	RAD	EAT	PTN	
HLS	RAD	PBI	LEU	RAD		FCH	SONC.O	
LLTC.O	SUI	RAD	RAD	TCO		IMMU.O	TELL.O	
PBI	TCO	SUI	TCO			IT	TLRD.K	
RAD	TEN	TCO				KMB		
SONC.O	URI					LEU		
SUI						MAS		
TCO						MSI		
TEN								
URI								

Appendix E

Descriptive Statistics for December 2001 – December 2007 Sample

Variable[1]	Mean	Median	Maximum	Minimum	Std. Dev.	Skewness	Kurtosis	Jarque-Bera	Probability	Obs.
B/H_sb-Rf	-1,2446	-1,2728	8,9832	-13,7678	4,2190	0,0160	3,6880	1,4429	0,4860	73
B/M_sb-Rf	-1,5979	-1,5707	8,3092	-10,1185	3,6857	0,2024	3,1708	0,5873	0,7455	73
B/L_sb-Rf	-1,8993	-1,6263	9,3392	-11,5520	3,4194	0,1923	4,1092	4,1925	0,1229	73
M/H_sb-Rf	-1,0065	-1,1953	16,6242	-14,8489	5,0956	0,4052	4,6112	9,8934	0,0071	73
M/M_sb-Rf	-1,2519	-0,9031	8,5859	-10,4262	4,2472	0,0299	2,4983	0,7764	0,6783	73
M/L_sb-Rf	-1,1905	-1,5372	11,2317	-11,4989	4,6039	0,2131	3,0317	0,5556	0,7574	73
S/H_sb-Rf	-0,5509	-1,1941	12,7000	-14,7454	5,3039	0,2674	3,1318	0,9230	0,6303	73
S/M_sb-Rf	-0,6308	-0,2717	7,5725	-13,7006	4,4970	-0,3694	2,8651	1,7158	0,4241	73
S/L_ sb-Rf	-0,7832	-2,0442	11,4642	-11,4040	5,6373	0,3820	2,2502	3,4855	0,1750	73
B/W_sm-Rf	-1,2979	-1,1278	8,9968	-9,8672	4,1092	0,1449	3,0830	0,2764	0,8709	73
B/N_sm-Rf	-1,7186	-2,1186	6,0437	-8,2730	3,1717	0,0696	2,8921	0,0944	0,9539	73
B/L_sm-Rf	-1,8046	-2,0548	12,2591	-13,5108	4,4220	0,8537	5,6078	29,5522	0,0000	73
M/W_sm-Rf	-1,0330	-0,4602	11,0201	-12,1073	4,5347	-0,1237	2,9343	0,1992	0,9052	73
M/N_sm-Rf	-1,4999	-1,5607	6,8381	-9,2814	3,9262	0,0232	2,3665	1,2272	0,5414	73
M/L_sm-Rf	-1,3060	-1,5513	25,7861	-16,6285	6,0610	1,0974	7,6709	81,0133	0,0000	73
S/W_sm-Rf	-0,5586	-0,1813	10,7520	-15,4278	5,1872	-0,2157	3,1374	0,6234	0,7322	73
S/N_sm-Rf	-1,6190	-2,0779	8,3009	-15,0372	4,4445	-0,1671	3,1574	0,4152	0,8125	73
S/L_sm-Rf	-1,0086	-1,1322	17,4637	-19,1135	6,5653	0,3242	3,8557	3,5061	0,1732	73
B/A_si-Rf	-1,6146	-0,9744	5,8929	-9,5528	3,4144	-0,3560	2,7454	1,7393	0,4191	73
B/N_si-Rf	-1,8020	-2,3680	9,2746	-11,4183	3,4881	0,2728	3,7246	2,5025	0,2862	73
B/C_si-Rf	-1,7950	-2,3395	9,2021	-12,1983	4,0537	0,5581	3,9595	6,5908	0,0371	73
M/A_si-Rf	-0,9886	-0,5936	9,2702	-12,1677	4,7445	-0,0937	2,5548	0,7095	0,7013	73
M/N_si-Rf	-1,3214	-1,5909	7,3505	-9,1661	3,9343	0,1310	2,6410	0,6008	0,7405	73
M/C_si-Rf	-1,1624	-1,4686	18,4347	-13,1781	5,1526	0,6431	4,9362	16,4346	0,0003	73
S/A_si-Rf	-0,5877	-1,4582	10,6899	-13,4606	5,1963	0,1314	2,4290	1,2018	0,5483	73
S/N_si-Rf	-0,7575	-0,6893	9,2918	-11,2376	4,6436	-0,0728	2,4910	0,8526	0,6529	73
S/C_si-Rf	-0,5574	-1,2304	12,6690	-17,6690	5,4227	0,0022	3,6066	1,1193	0,5714	73
B/R_sp-Rf	-1,7961	-1,8260	8,7573	-11,3622	3,3934	0,1695	3,6402	1,5963	0,4502	73
B/M_sp-Rf	-1,8149	-1,5084	6,9008	-11,2676	3,5593	-0,1665	3,6399	1,5828	0,4532	73
B/W_sp-Rf	-1,5691	-1,9982	11,0239	-10,8683	4,3362	0,5457	3,7866	5,5056	0,0637	73
M/R_sp-Rf	-1,1064	-1,1308	9,9758	-12,3883	4,4322	-0,0714	2,8806	0,1054	0,9487	73
M/M_sp-Rf	-1,2224	-1,4601	7,4208	-11,3807	3,9728	-0,0779	2,7861	0,2129	0,8990	73
M/W_sp-Rf	-1,1940	-1,9125	19,6678	-11,8062	5,7352	0,7727	4,3128	12,5068	0,0019	73
S/R_sp-Rf	-0,0912	-0,0165	11,4400	-12,9233	5,3838	-0,2266	2,7043	0,8905	0,6407	73
S/M_sp-Rf	-0,7853	-1,3563	8,0190	-13,1945	4,3957	-0,0036	2,9453	0,0093	0,9954	73
S/W_sp-Rf	-0,7498	-1,5861	13,0385	-13,6681	5,5734	0,3018	2,5880	1,6245	0,4439	73
Rm-Rf	-1,6680	-1,9020	7,4870	-10,5825	3,4746	0,1181	3,1832	0,2718	0,8729	73
SMBb	0,5894	0,6525	4,7929	-4,0149	2,1196	-0,1018	2,3081	1,5821	0,4534	73
HML	0,5400	0,5182	8,6399	-6,3532	2,2245	0,2180	5,9622	27,2669	0,0000	73
PR1YR	0,3635	0,3920	8,2507	-16,1437	4,0722	-1,1045	5,9165	40,7134	0,0000	73
SMB	0,7163	0,4396	5,2888	-4,8591	2,2150	-0,2157	2,5737	1,1190	0,5715	73
RMW	-0,1940	0,0625	5,5359	-5,9601	2,2869	-0,2369	3,1541	0,7548	0,6857	73
CMA	-0,0497	-0,4193	4,7826	-3,8703	2,0501	0,3085	2,3107	2,6030	0,2721	73

Interpretation of the Notation:

[1]The two capital letters, separated by the slash line, stand for the name of a portfolio like in Appendix 3. The two small letters stand for the ranking criteria used create that portfolio with s indicating size, b – book-to-market equity, m – one-year momentum effect, i – investment, and p – profitability. Ex.: B/H_sb-Rf is the excess return on the portfolio og big (B) stocks with high (H) book-to-market equity which was created by ranking the stocks by size (s) and book-to-market equity (b).

Appendix F

Descriptive Statistics for January 2008 – June 2009 Sample

Variable[1]	Mean	Median	Maximum	Minimum	Std. Dev.	Skewness	Kurtosis	Jarque-Bera	Probability	Obs.
B/H_ sb-Rf	-1,2213	-1,1435	15,7124	-17,9257	7,7555	0,2349	3,3456	0,2551	0,8802	18
B/M_ sb-Rf	-2,3051	-1,9215	10,1102	-13,2008	6,5788	0,0928	2,4294	0,2700	0,8737	18
B/L_sb-Rf	-2,1019	-1,3801	7,5302	-13,6880	5,7582	-0,1655	2,2229	0,5350	0,7653	18
M/H_sb-Rf	-1,2232	-1,9714	35,0623	-24,8402	12,3496	1,1063	5,6526	8,9485	0,0114	18
M/M_sb-Rf	-1,6701	-1,4153	20,8068	-22,4398	9,6908	0,1643	3,4788	0,2529	0,8812	18
M/L_sb-Rf	-2,2750	-2,7073	16,6987	-20,5169	8,3759	0,0666	3,3606	0,1108	0,9461	18
S/H_sb-Rf	0,7815	0,7330	40,4644	-21,0682	13,1935	1,2836	5,8339	10,9661	0,0042	18
S/M_sb-Rf	-1,7668	-1,3630	23,0507	-19,0578	10,1491	0,6188	3,2952	1,2141	0,5450	18
S/L_sb-Rf	-1,3330	-0,2774	25,8491	-16,6782	10,7578	0,6747	3,2356	1,4073	0,4948	18
B/W_sm-Rf	-2,6363	-1,4679	5,8624	-12,5453	5,2857	-0,2584	2,2808	0,5883	0,7452	18
B/N_sm-Rf	-1,9256	-1,8326	9,3637	-13,9275	6,0712	0,0343	2,5912	0,1289	0,9376	18
B/L_sm-Rf	-1,0310	-1,4069	27,1164	-15,5964	11,0252	0,9216	3,5013	2,7367	0,2545	18
M/W_sm-Rf	-3,3216	-2,9888	8,1407	-19,9976	7,7420	-0,2921	2,4610	0,4739	0,7890	18
M/N_sm-Rf	-1,8632	-0,7247	18,8738	-23,1672	9,4162	0,0000	3,5122	0,1968	0,9063	18
M/L_sm-Rf	-0,0973	-0,3179	51,7693	-27,2717	16,4024	1,5620	6,8367	18,3597	0,0001	18
S/W_sm-Rf	-3,2247	-1,8657	12,6839	-18,4252	8,3247	0,1065	2,4229	0,2838	0,8677	18
S/N_sm-Rf	-2,5368	-1,6471	26,6146	-27,6498	12,0179	0,2810	3,7025	0,6070	0,7382	18
S/L_sm-Rf	-0,0499	-0,2865	52,2821	-29,9833	17,8744	1,1391	5,3425	8,0079	0,0182	18
B/A_si-Rf	-1,6930	-0,7620	10,9391	-16,3736	7,2467	-0,0426	2,5015	0,1918	0,9085	18
B/N_si-Rf	-2,3800	-1,8319	6,9396	-13,8480	5,6589	-0,1209	2,4322	0,2857	0,8669	18
B/C_si-Rf	-2,1352	-1,7882	7,4761	-11,7694	5,8592	0,1028	1,8972	0,9439	0,6238	18
M/A_si-Rf	-1,4710	-1,8973	19,7515	-23,1952	9,7710	0,0329	3,3005	0,0709	0,9652	18
M/N_si-Rf	-1,7127	-1,1557	23,4057	-21,9180	9,5488	0,5003	4,4271	2,2784	0,3201	18
M/C_si-Rf	-2,3391	-1,6358	28,0625	-21,5154	10,8029	0,9438	4,8071	5,1215	0,0772	18
S/A_si-Rf	-0,8925	-1,0222	27,5322	-21,7011	11,0632	0,6245	3,8439	1,7040	0,4266	18
S/N_si-Rf	-0,6862	-0,2974	29,5019	-16,2504	10,9266	1,0103	4,2813	4,2937	0,1169	18
S/C_si-Rf	-0,4941	-0,7121	35,2205	-22,0806	12,7614	0,9360	4,7225	4,8538	0,0883	18
B/R_sp-Rf	-1,8263	-1,4589	7,4429	-14,3295	5,7444	-0,1680	2,5916	0,2097	0,9005	18
B/M_sp-Rf	-2,3711	-0,7494	9,6261	-14,4358	6,7888	-0,0009	2,2149	0,4623	0,7936	18
B/W_sp-Rf	-2,3668	-2,5022	9,0921	-12,3490	6,3861	0,1552	2,2761	0,4653	0,7924	18
M/R_sp-Rf	-1,7526	-2,3659	22,1003	-19,7780	9,2586	0,6135	3,9515	1,8080	0,4049	18
M/M_sp-Rf	-1,9226	-2,5027	17,0867	-22,1482	8,6786	-0,1267	3,5919	0,3109	0,8560	18
M/W_sp-Rf	-1,6573	-1,5482	35,1600	-25,8984	13,2431	0,8629	4,7029	4,4089	0,1103	18
S/R_sp-Rf	-0,3613	-2,5241	35,3645	-21,0392	12,9033	1,0447	4,4306	4,8090	0,0903	18
S/M_sp-Rf	-1,4068	-0,4288	23,2167	-17,0931	9,6892	0,6660	3,5071	1,5233	0,4669	18
S/W_sp-Rf	0,0558	-0,9091	38,4647	-20,9439	13,0927	1,2002	5,3027	8,2979	0,0158	18
Rm-Rf	-2,0090	-1,6720	10,5155	-15,1273	6,5924	0,0478	2,5292	0,1731	0,9171	18
SMBb	0,6809	-0,0668	14,2154	-6,0048	4,5900	1,4337	5,3701	10,3797	0,0056	18
HML	1,2182	0,5285	14,9351	-4,3808	4,2392	1,8899	7,2026	23,9620	0,0000	18
PR1YR	-2,5551	0,4512	9,8397	-35,0753	10,2590	-1,7604	6,6447	19,2600	0,0001	18
SMB	0,9465	-0,0068	17,3909	-6,9113	5,1814	1,7218	6,8530	20,0281	0,0000	18
RMW	0,0438	0,4458	4,2288	-6,7629	2,8611	-0,5367	2,8786	0,8750	0,6456	18
CMA	-0,1886	-0,1185	3,9380	-4,0753	2,7552	0,0726	1,5499	1,5929	0,4509	18

Interpretation of the Notation:

[1]The two capital letters, separated by the slash line, stand for the name of a portfolio like in Appendix 3. The two small letters stand for the ranking criteria used create that portfolio with s indicating size, b – book-to-market equity, m – one-year momentum effect, i – investment, and p – profitability. Ex.: B/H_sb-Rf is the excess return on the portfolio og big (B) stocks with high (H) book-to-market equity which was created by ranking the stocks by size (s) and book-to-market equity (b).

Appendix G

Descriptive Statistics for July 2009 – June 2016 Sample

Variable	Mean	Median	Maximum	Minimum	Std. Dev.	Skewness	Kurtosis	Jarque-Bera	Probability	Obs.
B/H_sb-Rf	1,1590	1,2523	10,7438	-8,7032	3,8942	0,0452	2,7220	0,2991	0,8611	84
B/M_sb-Rf	1,0551	1,3857	10,6251	-7,8852	3,5769	0,0449	3,1173	0,0763	0,9626	84
B/L_sb-Rf	1,2231	1,2464	9,3331	-8,1475	3,4779	-0,0729	2,9374	0,0881	0,9569	84
M/H_sb-Rf	1,6264	1,6436	13,4639	-8,8512	4,7207	0,0524	2,8981	0,0748	0,9633	84
M/M_sb-Rf	1,6172	1,6360	16,4422	-9,7059	4,5765	0,1136	3,8144	2,5017	0,2863	84
M/L_sb-Rf	1,3777	1,7037	14,9304	-8,3826	4,4321	0,0234	3,3540	0,4463	0,8000	84
S/H_sb-Rf	2,1895	2,3394	17,2401	-10,7871	6,2326	0,1023	2,6940	0,4741	0,7889	84
S/M_sb-Rf	1,6384	1,5582	17,4190	-10,4598	4,9619	0,2521	3,5901	2,1086	0,3484	84
S/L_sb-Rf	1,7854	2,0457	15,6441	-10,0772	4,9938	-0,0024	2,9932	0,0002	0,9999	84
B/W_sm-Rf	1,3490	1,6822	10,1626	-8,2970	3,9872	-0,2527	2,6047	1,4407	0,4866	84
B/N_sm-Rf	1,0780	1,2154	9,4883	-9,8488	3,4960	-0,2294	3,3324	1,1236	0,5702	84
B/L_sm-Rf	1,3403	0,8675	18,4270	-7,9003	4,2792	0,8902	5,1207	26,8352	0,0000	84
M/W_sm-Rf	1,5286	2,0679	13,7626	-11,3640	4,8608	-0,3008	3,0947	1,2979	0,5226	84
M/N_sm-Rf	1,4822	1,2298	14,5104	-9,8725	4,4004	0,1333	3,4552	0,9738	0,6145	84
M/L_sm-Rf	1,4479	1,5811	18,8078	-10,0577	5,4039	0,3861	3,5385	3,1020	0,2120	84
S/W_sm-Rf	1,3054	2,0064	13,6323	-13,2840	5,4193	-0,2149	2,8446	0,7314	0,6937	84
S/N_sm-Rf	1,3121	1,4726	15,4045	-10,9042	4,6697	0,0482	3,3040	0,3559	0,8370	84
S/L_sm-Rf	1,4869	1,4042	20,4762	-11,3839	6,1038	0,3322	3,7176	3,3473	0,1876	84
B/A_si-Rf	1,2453	1,2942	10,8334	-7,7327	3,8704	0,0269	2,7897	0,1649	0,9209	84
B/N_si-Rf	1,0298	1,1528	11,2878	-8,9609	3,5205	-0,0121	3,5293	0,9825	0,6119	84
B/C_si-Rf	1,2512	1,3011	9,0664	-6,2928	3,3872	-0,0329	2,5539	0,7115	0,7006	84
M/A_si-Rf	1,4261	1,4563	16,0833	-10,4011	4,6613	-0,0232	3,5333	1,0031	0,6056	84
M/N_si-Rf	1,4807	1,5481	14,4281	-8,4383	4,1929	0,1204	3,3236	0,5695	0,7522	84
M/C_si-Rf	1,7386	1,7824	13,9653	-9,6223	4,9094	0,0302	3,1446	0,0859	0,9580	84
S/A_si-Rf	1,7084	1,8574	17,4972	-10,2261	4,9262	0,0202	3,3577	0,4534	0,7971	84
S/N_si-Rf	1,8110	1,9355	14,7011	-9,6016	5,0414	0,1183	2,8848	0,2425	0,8858	84
S/C_si-Rf	2,1074	1,9542	18,2005	-12,0288	6,1949	0,3003	3,2944	1,5661	0,4570	84
B/R_sp-Rf	1,1161	1,2738	8,3489	-7,0184	3,2463	-0,0528	2,7340	0,2867	0,8664	84
B/M_sp-Rf	1,1236	1,3112	12,1766	-8,9242	3,9239	-0,0477	3,2421	0,2370	0,8882	84
B/W_sp-Rf	1,1900	1,4501	12,2538	-8,9233	3,8197	0,1887	3,4593	1,2367	0,5388	84
M/R_sp-Rf	1,4884	1,5350	12,2209	-7,9208	4,2831	0,0143	2,8034	0,1382	0,9332	84
M/M_sp-Rf	1,4079	1,6186	15,4825	-8,1453	4,2919	0,1694	3,5963	1,6465	0,4390	84
M/W_sp-Rf	1,7528	2,3967	16,9273	-12,6209	5,3159	-0,0651	3,5093	0,9671	0,6166	84
S/R_sp-Rf	2,1692	2,0650	20,8574	-8,8127	5,5760	0,4194	3,5856	3,6624	0,1602	84
S/M_sp-Rf	1,4412	1,8986	15,3079	-8,8436	4,5752	0,1048	2,9778	0,1555	0,9252	84
S/W_sp-Rf	2,3393	2,3230	19,4936	-13,2972	6,5547	0,1609	3,2114	0,5189	0,7715	84
Rm-Rf	1,2307	1,6433	10,9566	-8,1097	3,5953	-0,0140	3,1241	0,0566	0,9721	84
SMBb	0,5100	0,6397	4,9683	-4,4526	2,1541	0,0712	2,4816	1,0116	0,6030	84
HML	0,2321	-0,1518	5,6720	-4,3674	2,0454	0,4462	2,9160	2,8117	0,2452	84
PR1YR	0,0572	0,1871	7,5440	-8,0015	3,1822	-0,4102	3,3577	2,8036	0,2462	84
SMB	0,5469	0,5548	5,5216	-4,6157	2,2246	0,1808	2,5516	1,1614	0,5595	84
RMW	-0,1267	-0,0448	3,8507	-6,5472	1,8375	-0,4946	4,2638	9,0154	0,0110	84
CMA	0,2206	0,0147	6,0778	-3,6367	1,6339	0,6729	4,2086	11,4522	0,0033	84

Interpretation of the Notation:

(1)The two capital letters, separated by the slash line, stand for the name of a portfolio like in Appendix 3. The two small letters stand for the ranking criteria used create that portfolio with s indicating size, b – book-to-market equity, m – one-year momentum effect, i – investment, and p – profitability. Ex.: B/H_sb-Rf is the excess return on the portfolio og big (B) stocks with high (H) book-to-market equity which was created by ranking the stocks by size (s) and book-to-market equity (b).

Appendix H

Coefficient Variance Decomposition for the Three-, Four- and Five-Factor Models before, during and after the Great Recession

		Before the Great Recession					During The Great Recession					After the Great Recession				
Three-factor Model	Eigenvalues	0,0634	0,0106	0,0070	0,0027		0,1607	0,0269	0,0059	0,0018		0,0135	0,0041	0,0028	0,0007	
	Condition	0,0431	0,2585	0,3896	1,0000		0,0113	0,0673	0,3068	1,0000		0,0540	0,1804	0,2585	1,0000	
	Variance Decomposition Proportions															
	Associated Eigenvalue															
	Variable	1	2	3	4		1	2	3	4		1	2	3	4	
	C	0,9948	0,0021	0,0025	0,0006		0,9994	0,0000	0,0006	0,0000		0,9995	0,0000	0,0000	0,0004	
	Rm-Rf	0,2617	0,0551	0,0028	0,6805		0,2463	0,1171	0,4067	0,2299		0,0655	0,2108	0,2108	0,5130	
	SMBb	0,1696	0,5470	0,2807	0,0027		0,0293	0,8749	0,0715	0,0244		0,0066	0,8328	0,1335	0,0271	
	HML	0,0208	0,5216	0,4532	0,0044		0,0638	0,7498	0,1697	0,0168		0,0019	0,3032	0,6895	0,0053	
Four-factor Model	Eigenvalues	0,0326	0,0052	0,0037	0,0019	0,0009	0,0551	0,0121	0,0087	0,0012	0,0002	0,0189	0,0069	0,0044	0,0016	0,0010
	Condition	0,0287	0,1782	0,2521	0,4922	1,0000	0,0045	0,0203	0,0282	0,1998	1,0000	0,0520	0,1426	0,2245	0,6266	1,0000
	Variance Decomposition Proportions															
	Associated Eigenvalue															
	Variable	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
	C	0,9922	0,0036	0,0036	0,0004	0,0001	0,9996	0,0001	0,0001	0,0002	0,0000	0,9985	0,0008	0,0002	0,0002	0,0003
	Rm-Rf	0,3073	0,0740	0,0079	0,4416	0,1692	0,2561	0,0076	0,1196	0,5985	0,0181	0,0726	0,0995	0,3049	0,0753	0,4478
	SMBb	0,2067	0,5801	0,1822	0,0309	0,0001	0,0050	0,0872	0,9043	0,0001	0,0033	0,0028	0,5257	0,4423	0,0068	0,0224
	HML	0,0293	0,4071	0,5585	0,0016	0,0035	0,0054	0,8903	0,0969	0,0050	0,0023	0,0105	0,6672	0,2876	0,0275	0,0072
	PR1YR	0,0564	0,0287	0,1217	0,4530	0,3401	0,0194	0,7878	0,0911	0,0455	0,0561	0,0115	0,2787	0,0790	0,5968	0,0339

Five-factor Model	Before the Great Recession						During The Great Recession						After the Great Recession					
Eigenvalues	0,0522	0,0171	0,0085	0,0072	0,0037	0,0021	0,0891	0,0715	0,0150	0,0064	0,0020	0,0007	0,0146	0,0098	0,0057	0,0035	0,0018	0,0006
Condition	0,0410	0,1250	0,2525	0,2983	0,5767	1,0000	0,0079	0,0098	0,0469	0,1092	0,3466	1,0000	0,0440	0,0658	0,1123	0,1820	0,3642	1,0000
Variance Decomposition Proportions																		
Associated Eigenvalue																		
Variable	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6
C	0,9942	0,0009	0,0026	0,0007	0,0011	0,0005	0,8531	0,1437	0,0007	0,0023	0,0002	0,0000	0,7818	0,2141	0,0036	0,0001	0,0001	0,0003
Rm-Rf	0,2615	0,0058	0,0482	0,0467	0,0212	0,6165	0,2118	0,1584	0,1484	0,0655	0,2985	0,1174	0,1179	0,0041	0,0189	0,2129	0,2437	0,4024
SMB	0,1224	0,2437	0,2836	0,2254	0,1221	0,0026	0,3015	0,3290	0,3300	0,0098	0,0176	0,0123	0,0122	0,0033	0,0772	0,8594	0,0199	0,0280
HML	0,0119	0,3438	0,3939	0,1245	0,1243	0,0016	0,0005	0,1430	0,7060	0,0938	0,0459	0,0108	0,1419	0,3633	0,3377	0,0059	0,1461	0,0050
RMW	0,0284	0,4884	0,1239	0,2597	0,0906	0,0090	0,2448	0,7094	0,0126	0,0291	0,0032	0,0008	0,3429	0,5781	0,0000	0,0190	0,0555	0,0045
CMA	0,0261	0,6638	0,1115	0,1404	0,0568	0,0015	0,0383	0,7523	0,0510	0,1493	0,0091	0,0000	0,1433	0,0983	0,6543	0,0617	0,0412	0,0011

Appendix I

Estimation and Test Results on the Three-, Four- and Five-Factor Models before, during and after the Great Recession(1)

Regressand(2)	α	Rm-Rf	SMBb	SMB	HML	PR1YR	RMW	CMA	$\bar{R}^2$	AIC	SBC	Redundant Variable(3)	Remarks
B/H_sb-Rf	0,429	1,107	-0,189		0,525				0,830	3,999	4,124		*, **, ***
B/H_sb-Rf	0,305	1,089	-0,614		0,886				0,967	3,722	3,920		
B/H_sb-Rf	-0,088	0,996	-0,245		0,628				0,934	2,884	3,000		****
B/M_sb-Rf	-0,192	0,988	0,084		0,358				0,927	2,882	3,007	SMBb	
B/M_sb-Rf	-0,248	0,985	0,051		-0,092				0,955	3,701	3,899	SMBb, HML	
B/M_sb-Rf	-0,138	0,984	-0,083		0,103				0,955	2,343	2,458	SMBb	
B/L_sb-Rf	-0,028	0,976	-0,180		-0,254				0,971	1,810	1,936		
B/L_sb-Rf	0,194	0,983	-0,140		-0,185				0,990	1,907	2,105		
B/L_sb-Rf	0,142	1,002	-0,152		-0,321				0,968	1,947	2,063		
M/H_sb-Rf	0,285	1,167	0,656		0,497				0,881	4,015	4,141		***, ****
M/H_sb-Rf	-0,304	1,118	0,841		0,619				0,979	4,216	4,414		
M/H_sb-Rf	0,006	1,009	0,500		0,534				0,943	3,124	3,239		
M/M_sb-Rf	-0,273	0,944	0,769		0,264				0,945	2,885	3,011		
M/M_sb-Rf	-0,205	1,031	0,701		0,105				0,970	4,073	4,271	HML	
M/M_sb-Rf	0,027	1,001	0,674		0,062				0,964	2,595	2,711	HML	***
M/L_sb-Rf	0,098	1,035	0,782		-0,043				0,909	3,552	3,677	HML	
M/L_sb-Rf	-0,406	1,044	0,405		-0,039				0,936	4,529	4,727	SMBb, HML	
M/L_sb-Rf	-0,127	1,014	0,606		-0,228				0,957	2,709	2,825		
S/H_sb-Rf	0,021	0,995	1,187		0,717				0,941	3,401	3,527		
S/H_sb-Rf	1,054	1,022	0,891		0,964				0,970	4,669	4,866		
S/H_sb-Rf	0,086	1,058	1,232		0,749				0,956	3,417	3,533		
S/M_sb-Rf	0,093	0,890	0,970		0,350				0,908	3,515	3,641		
S/M_sb-Rf	-0,532	0,955	1,032		-0,016				0,980	3,741	3,938	HML	
S/M_sb-Rf	-0,074	0,974	0,991		0,078				0,954	3,068	3,271	HML	**, ****
S/L_sb-Rf	0,411	1,098	1,219		-0,150				0,854	4,427	4,553	HML	****
S/L_sb-Rf	0,303	1,030	1,251		-0,342				0,949	4,799	4,997	HML	
S/L_sb-Rf	0,126	1,010	0,900		-0,182				0,915	3,641	3,757		
B/W_sm-Rf	0,203	1,118	0,117		0,135	0,612			0,913	3,291	3,448	SMBb	
B/W_sm-Rf	0,044	0,995	-0,055		-0,047	0,229			0,976	2,672	2,920	SMBb, HML	
B/W_sm-Rf	0,061	1,061	-0,049		-0,066	0,396			0,907	3,284	3,429	SMBb, HML	
B/N_sm-Rf	-0,189	0,938	-0,092		0,091	0,111			0,933	2,514	2,671	HML	***
B/N_sm-Rf	-0,396	0,844	-0,104		-0,080	-0,109			0,943	3,884	4,230	SMBb, HML, PR1YR	**
B/N_sm-Rf	-0,081	0,988	-0,122		0,013	0,037			0,959	2,194	2,339	SMBb, HML, PR1YR	***
B/L_sm-Rf	0,041	0,994	-0,092		0,033	-0,418			0,951	2,862	3,019	SMBb, HML	***, ****
B/L_sm-Rf	-0,325	1,001	-0,125		-0,220	-0,649			0,946	4,948	5,196	SMBb, HML	
B/L_sm-Rf	0,136	1,025	-0,030		-0,052	-0,511			0,943	2,944	3,089	SMBb, HML	***
M/W_sm-Rf	0,041	1,073	0,661		0,310	0,439			0,924	3,350	3,507		***
M/W_sm-Rf	-0,455	1,177	0,676		0,631	0,678			0,862	5,184	5,431	SMBb, HML	
M/W_sm-Rf	-0,164	0,978	0,806		0,219	0,478			0,947	4,861	3,121		
M/N_sm-Rf	-0,579	0,865	0,669		0,262	-0,037			0,924	3,069	3,226	PR1YR	*, **, ***
M/N_sm-Rf	0,221	1,189	0,642		0,322	0,206			0,990	2,938	3,186	HML	***
M/N_sm-Rf	-0,042	1,009	0,551		0,000	0,015			0,957	2,713	2,858	HML, PR1YR	

Regressand[(2)]	α	Rm-Rf	SMBb	SMB	HML	PR1YR	RMW	CMA	$\bar{R}^2$	AIC	SBC	Redundant Variable[(3)]	Remarks
M/L_sm-Rf	0,068	1,072	0,952		0,131	-0,598			0,906	4,143	4,300	HML	***, ****
M/L_sm-Rf	0,092	1,263	0,498		0,770	-0,407			0,984	4,637	4,983	SMBb	**
M/L_sm-Rf	-0,204	1,103	0,580		0,128	-0,551			0,953	3,202	3,346	HML	
S/W_sm-Rf	0,216	1,058	1,046		0,400	0,431			0,902	3,872	4,028		
S/W_sm-Rf	-1,149	1,012	0,859		-0,051	0,222			0,903	4,969	5,216	HML	
S/W_sm-Rf	-0,577	1,076	0,920		0,267	0,482			0,936	3,533	3,678		*, ****
S/N_sm-Rf	-1,110	0,809	1,040		0,458	-0,056			0,913	3,447	3,604	PR1YR	*
S/N_sm-Rf	-0,863	1,188	1,341		0,276	0,210			0,979	4,198	4,445	HML	***
S/N_sm-Rf	-0,284	0,952	0,782		0,107	0,016			0,931	3,304	3,449	HML, PR1YR	
S/L_sm-Rf	0,118	1,243	1,201		0,730	-0,427			0,949	3,696	3,853		
S/L_sm-Rf	0,139	1,357	0,523		0,365	-0,680			0,973	5,232	5,479	HML	
S/L_sm-Rf	-0,380	0,996	1,166		0,332	-0,526			0,940	3,697	3,842		*
B/H_sb-Rf	0,275	1,093		-0,187	0,687		-0,219	-0,484	0,864	3,803	3,991	SMB, RMW	***
B/H_sb-Rf	-0,095	0,988		-0,572	1,086		-0,061	-0,433	0,978	3,374	3,671	RMW	
B/H_sb-Rf	-0,060	0,989		-0,232	0,667		-0,036	-0,137	0,933	2,917	3,091	RMW, CMA	****
B/M_sb-Rf	-0,195	0,987		0,104	0,325		0,025	0,046	0,926	2,917	3,105	SMB, RMW, CMA	***
B/M_sb-Rf	0,139	1,077		-0,043	-0,145		-0,030	0,327	0,969	3,404	3,701	SMB, HML, RMW, CMA	
B/M_sb-Rf	-0,160	0,980		-0,083	0,052		-0,015	0,185	0,960	2,225	2,399	HML, RMW	
B/L_sb-Rf	0,010	0,974		-0,189	-0,257		0,025	0,129	0,977	1,616	1,804	RMW	***
B/L_sb-Rf	0,125	0,966		-0,143	-0,136		-0,020	-0,086	0,990	1,987	2,284	RMW, CMA	
B/L_sb-Rf	0,146	1,009		-0,147	-0,248		0,063	-0,084	0,970	1,907	2,080	RMW	*
M/H_sb-Rf	0,222	1,142		0,596	0,383		-0,220	-0,018	0,887	3,996	4,184	RMW, CMA	***, ****
M/H_sb-Rf	-0,851	1,010		1,201	0,394		0,446	-0,107	0,988	3,677	3,974	RMW, CMA	
M/H_sb-Rf	-0,009	1,001		0,494	0,400		-0,081	0,139	0,946	3,098	3,271	RMW, CMA	
M/M_sb-Rf	-0,300	0,937		0,747	0,115		-0,073	0,034	0,948	2,854	3,042	RMW, CMA	
M/M_sb-Rf	-0,530	0,926		0,685	-0,003		-0,319	-0,474	0,975	3,936	4,232	HML, RMW, CMA	
M/M_sb-Rf	0,062	0,983		0,661	-0,089		-0,201	-0,099	0,968	2,491	2,665	HML, RMW, CMA	***
M/L_sb-Rf	0,007	1,024		0,775	-0,147		-0,136	-0,154	0,921	3,435	3,623	HML, RMW	
M/L_sb-Rf	-1,147	0,899		0,803	-0,125		0,529	-0,257	0,970	3,855	4,152	HML, RMW, CMA	*
M/L_sb-Rf	-0,118	1,014		0,597	-0,293		-0,049	-0,076	0,957	2,738	2,912	RMW, CMA	
S/H_sb-Rf	0,009	0,977		1,096	0,474		-0,181	0,131	0,940	3,441	3,629	CMA	
S/H_sb-Rf	1,691	1,134		0,579	0,730		-0,544	0,234	0,978	4,432	4,729	RMW, CMA	*, ***
S/H_sb-Rf	0,014	1,085		1,242	0,712		0,216	0,109	0,959	3,373	3,547	RMW, CMA	
S/M_sb-Rf	0,058	0,912		1,016	0,217		0,098	-0,144	0,927	3,306	3,495	RMW, CMA	***
S/M_sb-Rf	-0,180	0,996		0,775	-0,247		-0,571	-0,033	0,983	3,645	3,942	HML, CMA	
S/M_sb-Rf	-0,029	0,939		0,992	-0,058		-0,117	-0,148	0,953	3,046	3,220	HML, RMW, CMA	
S/L_sb-Rf	0,236	1,070		1,166	-0,283		-0,344	-0,330	0,879	4,263	4,452	RMW, CMA	***
S/L_sb-Rf	0,407	1,006		1,102	-0,637		-0,548	-0,206	0,950	4,860	5,156	RMW, CMA	
S/L_sb-Rf	0,107	1,000		0,898	-0,366		-0,099	0,130	0,919	3,611	3,784	RMW, CMA	
B/A_si-Rf	-0,150	0,933		-0,099	0,238		-0,023	-0,609	0,922	2,826	3,015	SMB, RMW	**, ***
B/A_si-Rf	0,448	1,075		-0,191	0,103		-0,146	-0,419	0,964	3,734	4,031	SMB, HML, RMW, CMA	
B/A_si-Rf	0,092	1,064		-0,104	0,051		0,048	-0,476	0,927	2,995	3,169	SMB, HML, RMW	****
B/N_si-Rf	0,027	1,011		-0,125	-0,070		0,064	0,048	0,966	2,035	2,223	HML, RMW, CMA	****

Regressand(2)	α	Rm-Rf	SMBb	SMB	HML	PR1YR	RMW	CMA	$\bar{R}^2$	AIC	SBC	Redundant Variable(3)	Remarks
B/N_si-Rf	-0,319	0,950		-0,132	-0,006		0,094	0,130	0,980	2,658	2,955	SMB, HML, RMW, CMA	
B/N_si-Rf	-0,135	0,992		-0,144	-0,014		-0,043	0,091	0,968	1,970	2,144	HML, RMW	
B/C_si-Rf	0,150	1,077		-0,164	-0,043		-0,089	0,509	0,945	2,817	3,005	HML, RMW	
B/C_si-Rf	0,302	1,010		-0,186	-0,138		-0,136	0,312	0,945	3,736	4,033	SMB, HML, RMW, CMA	
B/C_si-Rf	0,099	0,950		-0,199	0,023		0,156	0,483	0,935	2,620	2,793	HML, RMW	
M/A_si-Rf	0,154	1,076		0,821	0,082		-0,007	-0,375	0,923	3,464	3,652	HML, RMW	
M/A_si-Rf	-0,580	0,933		0,683	0,135		0,113	-0,884	0,984	3,547	3,844	HML, RMW	
M/A_si-Rf	-0,119	1,018		0,657	-0,069		-0,163	-0,324	0,965	2,646	2,820	HML, RMW	
M/N_si-Rf	-0,432	0,870		0,624	0,173		-0,124	0,042	0,943	2,785	2,973	CMA	*
M/N_si-Rf	-0,700	0,948		0,896	0,023		0,348	0,002	0,984	3,496	3,793	HML, RMW, CMA	****
M/N_si-Rf	0,057	0,932		0,525	-0,052		-0,170	-0,089	0,962	2,514	2,687	HML, RMW, CMA	
M/C_si-Rf	0,205	1,139		0,728	-0,045		-0,240	0,218	0,924	3,613	3,801	HML, RMW	***, ****
M/C_si-Rf	-1,395	0,960		1,203	-0,092		0,279	0,287	0,973	4,245	4,542	HML, RMW, CMA	*
M/C_si-Rf	-0,032	1,065		0,660	0,052		0,008	0,401	0,965	2,727	2,901	HML, RMW	
S/A_si-Rf	0,460	1,106		0,959	0,066		-0,231	-0,594	0,931	3,533	3,721	HML	*
S/A_si-Rf	0,177	0,961		0,948	-0,105		-0,436	-0,589	0,974	4,246	4,543	HML, RMW, CMA	
S/A_si-Rf	0,089	0,960		0,942	0,091		-0,014	-0,455	0,958	2,926	3,099	HML, CMA	
S/N_si-Rf	-0,256	0,846		1,157	0,198		0,101	0,118	0,913	3,543	3,731	RMW, CMA	
S/N_si-Rf	0,844	1,051		1,033	-0,255		-0,175	0,421	0,977	4,113	4,410	HML, RMW, CMA	
S/N_si-Rf	0,025	0,979		1,017	0,164		0,197	0,052	0,961	2,890	3,063	HML, RMW, CMA	***, ****
S/C_si-Rf	0,008	0,974		1,132	0,407		-0,201	0,224	0,926	3,698	3,886	RMW	***
S/C_si-Rf	0,939	1,176		0,143	0,716		-1,259	0,126	0,988	3,749	4,046	SMB, CMA	*
S/C_si-Rf	-0,043	1,075		1,175	0,217		-0,162	0,516	0,950	3,564	3,737	RMW	
B/R_sp-Rf	0,106	1,007		-0,207	-0,024		0,330	-0,068	0,952	2,332	2,520	HML, CMA	
B/R_sp-Rf	0,114	0,968		-0,028	0,024		0,603	0,131	0,984	2,435	2,732	SMB, HML, CMA	
B/R_sp-Rf	0,108	0,963		-0,222	-0,132		0,153	-0,024	0,956	2,145	2,319	CMA	****
B/M_sp-Rf	-0,167	0,979		-0,051	0,045		-0,012	0,104	0,907	3,080	3,269	SMB, HML, RMW, CMA	***
B/M_sp-Rf	0,076	1,054		-0,191	-0,122		-0,526	-0,123	0,976	3,182	3,479	SMB, HML, RMW, CMA	
B/M_sp-Rf	-0,222	1,069		0,009	0,144		0,081	0,011	0,965	2,302	2,476	SMB, HML, RMW, CMA	*
B/W_sp-Rf	0,066	1,023		-0,096	0,008		-0,685	-0,067	0,914	3,394	3,582	SMB, HML, CMA	***
B/W_sp-Rf	0,023	1,022		-0,437	0,097		-0,677	0,059	0,952	3,777	4,074	HML, CMA	
B/W_sp-Rf	0,075	0,967		-0,279	0,036		-0,566	-0,012	0,934	2,863	3,037	HML, CMA	****
M/R_sp-Rf	-0,018	1,017		0,801	0,116		0,166	-0,088	0,924	3,313	3,502	HML, CMA	
M/R_sp-Rf	-0,793	0,933		0,949	-0,007		0,565	-0,001	0,987	3,221	3,518	HML, CMA	*
M/R_sp-Rf	-0,050	1,010		0,588	0,073		0,364	0,013	0,959	2,617	2,790	HML, CMA	
M/M_sp-Rf	-0,231	0,926		0,654	0,185		0,080	0,016	0,948	2,718	2,906	RMW, CMA	
M/M_sp-Rf	-0,807	0,921		0,442	0,167		0,036	-0,591	0,967	4,011	4,308	HML, CMA	
M/M_sp-Rf	-0,065	0,951		0,577	-0,030		-0,078	-0,073	0,957	2,668	2,842	HML, RMW, CMA	***
M/W_sp-Rf	0,137	1,155		0,737	-0,152		-0,752	-0,078	0,914	3,961	4,150	HML, CMA	***, ****
M/W_sp-Rf	-1,022	0,985		1,492	-0,085		-0,139	-0,218	0,965	4,905	5,201	HML, RMW, CMA	

Regressand(2)	α	Rm-Rf	SMBb	SMB	HML	PR1YR	RMW	CMA	$\bar{R}^2$	AIC	SBC	Redundant Variable(3)	Remarks
M/W_sp-Rf	0,041	1,058		0,677	-0,133		-0,688	-0,074	0,958	3,090	3,263	HML, CMA	
S/R_sp-Rf	0,666	1,069		1,282	0,413		0,616	-0,058	0,898	3,997	4,186	CMA	*
S/R_sp-Rf	0,700	1,160		0,531	0,602		-0,641	-0,323	0,967	4,813	5,110	RMW, CMA	
S/R_sp-Rf	0,185	1,098		1,067	0,265		0,567	0,269	0,891	4,131	4,304	HML, CMA	***, ****
S/M_sp-Rf	-0,087	0,875		0,968	0,133		0,015	0,008	0,914	3,429	3,617	HML, RMW, CMA	**, ***
S/M_sp-Rf	0,186	0,983		0,767	-0,255		-0,390	0,094	0,973	4,034	4,331	HML, RMW, CMA	***
S/M_sp-Rf	-0,181	0,926		0,881	0,098		0,229	0,031	0,955	2,841	3,014	HML, RMW, CMA	
S/W_sp-Rf	0,073	1,060		1,035	0,143		-0,597	-0,218	0,944	3,469	3,657	HML, CMA	****
S/W_sp-Rf	1,310	1,137		0,828	0,251		-0,700	0,153	0,980	4,348	4,645	HML, CMA	*, ***
S/W_sp-Rf	0,216	1,072		1,298	0,124		-0,620	-0,062	0,958	3,487	3,661	HML, CMA	

Interpretation of the Notation:

(1)All lines in the table are placed in both logical and chronological order: from the first lines with the estimation results for Fama and French three-factor model to the last lines for Fama and French five-factor model with the risky return portfolio constructed on the basis of size and profitability. There're three lines describing each regressand: the first one is related to the time period before the Great Recession, the second one – the period during the Great Recession and the third one – the period after the Great Recession.

(2)The notation in column Regressand indicates excess return on risky portfolios. The two capital letters, separated by the slash line, stand for the name of a portfolio like in Appendix 3. The two small letters stand for the ranking criteria used create that portfolio with s indicating size, b – book-to-market equity, m – one-year momentum effect, i – investment, and p – profitability. Ex.: B/H_sb-Rf is the excess return on the portfolio og big (B) stocks with high (H) book-to-market equity which was created by ranking the stocks by size (s) and book-to-market equity (b).

(3)The column Redundant Variable contains the variables that, as a result of empirical tests, were found to be statistically insignificant in the respective regressions at 5% significance level.

* Indicates regressions where α is significantly different from zero.
** Indicates regressions that were corrected for serial correlation.
*** Indicates regressions that were corrected for heteroskedasticity.
**** Indicates regressions where residuals failed normality test.

Appendix J

Statistical Analysis of the Estimation and Empirical Test Results for the Three-, Four- and Five-Factor Models in December 2001 – June 2016

Criteria[1]	**Three-Factor Model**	**Four-Factor Model**	**Five-Factor Factor Model**		
			Size and Book-to-Market Equity Based	**Size and Investment Based**	**Size and Profitability Based**
Average AIC	3,3915	3,6533	3,2669	3,2107	3,3542
Average SBC	3,5411	3,7135	3,4865	3,4303	3,5738
Average Adj. R^2	0,9410	0,9386	0,9487	0,9538	0,9481
Total number of regressions	**27**	**27**	**27**	**27**	**27**
Number of regressions with:					
1. All statistically significant explanatory variables	16	6	0	0	0
2. Statistically significant alpha	1	4	3	4	4
3. Statistically insignificant SMBb	4	10	3	6	5
4. Statistically insignificant HML	9	16	12	22	23
5. Statistically insignificant PR1YR		6			
6. Statistically insignificant RMW			24	22	11
7. Statistically insignificant CMA			19	15	26

<u>Interpretation of the Notation:</u>

[1]Average AIC, SBC and $\bar{R}^2$ were computed as an arithmetic average for all regressions for each model over three sample periods: before, during and after the Great Recession.

Appendix K

Statistical Analysis of the Estimation and Empirical Test Results by Capital Asset Pricing Model and Estimation Period

	Criterion[1]	Three-Factor Model	Four-Factor Model	Five-Factor Model		
				Size and BME Based	Size and Investment Based	Size and Profitability Based
Before Crisis	Average AIC	3,3874	3,3605	3,2922	3,1459	3,2993
	Average SBC	3,5129	3,5174	3,4805	3,3342	3,4875
	Average Adj. R^2	0,9073	0,9237	0,9187	0,9326	0,9239
	Total number of regressions	**9**	**9**	**9**	**9**	**9**
	Number of regressions with:					
	1. All statistically significant explanatory variables	6	3	1	0	0
	2. Statistically significant alpha	1	2	0	2	1
	3. Statistically insignificant SMBb	1	3	2	1	2
	4. Statistically insignificant HML	2	3	1	5	7
	5. Statistically insignificant PR1YR		2			
	6. Statistically insignificant RMW			7	7	3
	7. Statistically insignificant CMA			4	4	9
During Crisis	Average AIC	3,9285	4,2958	3,6856	3,7250	3,8585
	Average SBC	4,1264	4,5651	3,9824	4,0218	4,1553
	Average Adj. R^2	0,9663	0,9506	0,9757	0,9743	0,9723
	Total number of regressions	**9**	**9**	**9**	**9**	**9**
	Number of regressions with:					
	1. All statistically significant explanatory variables	4	0	0	0	0
	2. Statistically significant alpha	0	0	2	2	2
	3. Statistically insignificant SMBb	2	5	1	4	2
	4. Statistically insignificant HML	5	7	4	8	8
	5. Statistically insignificant PR1YR		1			
	6. Statistically insignificant RMW			7	8	4
	7. Statistically insignificant CMA			8	8	9
After Crisis	Average AIC	2,8587	3,3036	2,8229	2,7612	2,9048
	Average SBC	2,9841	3,2389	2,9965	2,9348	3,0784
	Average Adj. R^2	0,9494	0,9415	0,9517	0,9545	0,9481

Criterion[1]	Three-Factor Model	Four-Factor Model	Five-Factor Model		
			Size and BME Based	Size and Investment Based	Size and Profitability Based
Total number of regressions	**9**	**9**	**9**	**9**	**9**
Number of regressions with:					
1. All statistically significant explanatory variables	6	3	0	0	0
2. Statistically significant alpha	0	2	1	0	1
3. Statistically insignificant SMBb	0	2	0	1	1
4. Statistically insignificant HML	2	5	3	7	8
5. Statistically insignificant PR1YR		3			
6. Statistically insignificant RMW			8	7	4
7. Statistically insignificant CMA			7	3	9

Interpretation of the Notation:

[1]Average AIC, SBC and $\bar{R}^2$ were computed as an arithmetic average for all regressions for each model for each sample period: before, during and after the Great Recession.