Master thesis in financial economics

# The profitability of the BEER in the Norwegian market

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# Preface

This paper marks the end of two year master program in financial economics at the Department of Economics at the Norwegian University of Science and Technology. The work has been both demanding and personally fulfilling. I want to thank my supervisor Gunnar Bårdsen for valuable insight and comments. The work is done by myself, and any mistakes or shortcomings are therefore of my own doing, of which I am solely responsible.

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Ruben Rolandsen.

# Abstract

The thesis sets out to examine if the properties of the Bond Equity Earnings Yield ratio (BEER), and the Norwegian economy's relation to the oil market, in combination with a suitable econometric model, can be used to derive a trading rules for guiding investment decisions. The empirical modeling of the bond equity earnings ratio leads to the definition of two sets of trading rules. The first trading on the realizations of the oil price over 3 month period. The second bases investment decisions on a derived econometric model forecasting the directions of the BEER-series. The accumulated returns of following both strategies are then compared to a passive buy-and-hold strategy holding the wide market index (OSEBX). The paper finds that both trading rules generate a higher level of accumulated return, compared to the benchmark index, over the sample period from 1997 to the end of 2016.

The success can both be attributed to the non-linear LSTR model that determines the threshold values of oil price changes that are then used to make investment decisions, and the linear specification that provide forecasts of the BEER movements of which are traded on. It is however important to emphasize that trading on the changes in the oil price alone and the forecasted values of the BEER, do not categorically produce higher excess return to risk performances when compared to the benchmark index, and does come with a higher level of risk.

The higher level of accumulated return found in the paper is in alignment with some of the some of the previous research of the BEER, raising the concern for market efficiency. The paper wishes to provide an indication for whether of not the market is efficient, using a modified version of the CAPM. On the basis of these results, the authors view is that the returns generated from following both strategies are not sufficient to disprove the hypothesis of weak market efficiency. It is important to emphasize that the examination of market efficiency is limited, and should not be taken as an definitive result.

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# 1 Introduction

The Bond Equity Earnings Ratio (BEER) is a metric measure that can be used to provide a signal for determining whether bonds or equities are over valued relative to each other. The metric is defined as the income yield between long term government bonds and the yield of equities. The power of the ratio is claimed by Clare, Thomas, and Wickens (1994) to be a useful investment decision making tool when combined with an econometric model. On the basis of this, and other publications of the BEER as an investment decision making tool, the paper seeks to determine if a suitable econometric model can provide profitable investment decisions in the Norwegian bond and equity market. The paper also seeks to make a new contribution to the existing literature by determining if the Norwegian economy's dependence of the oil market can be utilized to determine the directions of BEER movements. Specifically by using the price of oil as a main source of information along with other macro economic variables to determine the future movements of the BEER.

The reason for using this particular approach is based on the extensive research of the predictive power of macroeconomic factors of the future movements of the equity market. The work of Cochrane (1991), Fama and French (1989), Ferson and Harvey (1993) and Campbell and Hamao (1992) explores the predictive power of several macroeconomic factors over the equity market. Furthermore, Black, Fraser, and MacDonald (1997) and Fama and French (1989), also explores the predictive power of macroeconomic variables over the bond markets. The results of the aforementioned literature and the fact that the largest company of the OSEBX market index is an oil company (Statoil), which amounts for roughly 15% of the market capitalization of the market portfolio(Oslo Børs, 2017), gives a strong motivation for using the price of oil as a basis for a trading rule.

With regard to econometric modeling, the developments in the field of non-linear modeling has been of great usage in the modeling of financial time-series. In previous research of the BEER, such as Brooks and Persand (2001), McMillan (2012), and Levin and Wright (1998), non-linear models like *Smooth Transition* models and the *Markov switching* model are used and compared to linear models. The argument for using non-linear models arises from the examination of non-linear behavior in both financial and macroeconomic data (for example DeLong and Summers (1986), or Martens, Kofman, and Vorst (1998),). The general conclusions of the research has been that non-linear models provide superior results in terms of statistical criteria, and forecasting performances. However, there is research that finds the opposite with respect to forecasting performance (Bradley & Jansen, 2004). In regard to the forecasting procedure, the paper also seeks to evaluate how well the model specification performs in terms of predicting the correct directions of the BEER movements, in form of a formal test. The motivation begin that, from an investor perspective, the models ability to predict the correct buy-or-sell signals is arguably more important for making investment decision than the statistical properties.

# 2 The Bond and Equity market

According to basic financial theory, both equity markets and bond markets are impossible to predict as both are subject to numerous uncertain economic factors that cannot be forecasted or predicted without a level of uncertainty. In the equity market, particular returns are highly affected by levels of expectations of future outcomes. One way of illustrating how these beliefs affect the price of both equities and bonds is to illustrate the theoretical price of both assets:

$$P_e = \sum_{t=1}^{T-1} \frac{FCF_t}{(1+r_e)^t} + \frac{FCF_T}{r_e - g} \tag{1}$$

$$P_b = \sum_{i=1}^{T} \frac{C}{(1+r_b)^i} + \frac{F}{(1+r)^T} \tag{2}$$

As can be seen from (1), future values of the free cash flow $(FCF_t)$[1] and the terminal value of the free cash flow, $(FCF_T)$[2] and the growth rate $(g)$ are unobservable (Note that $r_e$ is different from $r_b$ in (2) due to the risk of investing in equities). On the basis of these expectations individual equities are traded on the stock exchange (Oslo Børs). The determination of these prices are largely based on demand and supply. In the sense that buy-and-sell orders are placed on the stock exchange and market clearing prices are determined accordingly. Usually there are some discrepancies between the observed buy and sell price. The explanation for this in economical terms is that the difference between the two is a reflection of the markets transaction costs.

A wide market index serves a twofold purpose. The first is as an investable asset. And secondly as a measure of the general return from equity investment. The market index consists of a representative sample of all the stock market listed companies. The index is revised bi-annually and dividend adjusted. The strength of a market index portfolio is mainly diversification. The power of having a diversified portfolio eliminates the unsystematic risk tied to a particular asset or holding several assets in the same industry. The downside is that the elimination of risk generally yields a lower return compared to investing in riskier portfolios.

[1]The free cash flow is equal to the cash flow subtracted by the investment.
[2]The terminal value is equal to the expected future ”stable” free cash flow levels

The bond market can be described as a lending and borrowing market for both commercial entities and the public sector, like the Norwegian government. The interest earned by the lender is largely determined by the risk of default, and to any possible default settlements in such an event. Government bonds generally pay the lowest levels of interest rates, due to the probability of default is largely considered to be highly implausible (especially for Norwegian government bonds). The effective interest rate of a particular government bond is synthesized using a weighted average of long and short term bonds(Norges Bank, 2017a). The reason for issuing government bonds (in Norway) is mostly due to the purpose of liquidity and refinancing short term debt. Although several developed economies have a wast amount of wealth, the management of wealth ties it to investments in various assets. The bond market therefore provides liquidity that is used for managing the short term government expenses. In the Norwegian economy the magnitude of government debt is under the responsibility of the ministry of finance, but the operational activity is under the administration of the monetary authorities (Norges Bank). In order to ensure the liquidity of government bonds, Norges Bank has a primary trade agreement with four Norwegian and Nordic banks. The agreement also stipulates that the banks are required to give the same buy and sell prices as Oslo Børs, these bonds can also be sold in the secondary market (Norges Bank, 2017b).

As an alternative to making any predictions of the future returns of equity or bond returns, there are alternative methods for guiding investment decisions. The most common term to define these methods is often called technical trading rules. These rules often consist of measuring buy and sell orders of a particular asset, or a variety of other metrics. The usefulness of technical trading has been highly controversial between academic and applied finance. Since the paper of Fama and Blume (1966) the general academic consensus has been that the profitability of technical trading is very small. The argument has been that a resilient and transaction cost adjusted trading rule will be picked up by all market participants, and will in it self become so predictable that excess profits will disintegrate.

In terms of trading rules, the Bond Equity Earnings Ratio (also known as the Gilt Equity Yield Ratio or the Bond Equity Yield Ratio) as a trading rule is rather simple. Since the only tools for investment decisions are the yield from equities and bonds, the BEER utilizes the competitiveness of the two asset classes to determine whether the one is undervalued relative to the other. Thus the investor can better determine which asset provides the best return relative to it's price. It is however important to note that the ratio has lost some popularity over the recent years. This may largely be because of the growing complexity of the financial markets and increasing digitalization of these markets. All of which make prices and returns more observable.

# 3 The BEER

As implied by the name, it is a simple ratio between the respective assets where the numerator is the bond yield from a benchmark bond. This benchmark is typically a five or ten year government bond. The denominator is a measure of earnings yield of an equity market benchmark, typically a stock market index. In mathematical terms this can be expressed as:

$$BEER = \frac{\frac{d_b}{P_b}}{\frac{d_e}{P_e}} \tag{3}$$

The ratio provides a way of measuring the relative price of the assets. Under the (fairly valid) assumption that bonds and equities are competing assets, the BEER provides a signal for whether or not the one asset is expensive relative to the other. The discussion becomes more clear by inverting the denominator and the numerator. Given an equity earnings yield of say 5%, the inverse of this is equal to the P/E ratio of 20. Conversely if bonds have the same yield, the bond will have the same P/E ratio, and the BEER will have a theoretical value of 1. So if the observed BEER is less than one, it sends a signal that equities are cheap relative to bonds and by implication that one should buy equities. Furthermore the competitiveness of bonds and equities ensures that investors will in such an event want to buy equities rather than bonds. By using the BEER to make investment decisions the investor will not make higher period-by-period percentage returns than the asset signaled by the BEER. However, if the BEER correctly signals when equities are overvalued, the investor will over time avoid losses in the equity market. As a result, the investor will theoretically be left with a higher level of accumulated return. It should be added that there are extensions of the BEER that build in other macroeconomic variables. Such as the model by Levin and Wright (1998):

$$BEER = \frac{(r_f + \pi)[1 + (\overline{P}_e - P_e)(r_f - g_e + \rho)/d_e]}{(r_f - g_e + \rho)[1 + (\overline{P}_b - P_b)(r_f + \pi)]/d_b} \tag{4}$$

Where $r_f$ is the risk free rate, $\overline{P}$ is the theoretical price of the asset (the subscript refers to the asset class b = bonds, e = equities), P is the observed price of the asset, $d$ is the dividend/cupon from the asset, $\rho$ is the equity risk premium, and $\pi$ is the inflation rate.

The model is extended to better account for such factors as inflation or changes in the market risk premium. Note that Modigliani and Cohn (1979) and Fama (1981) showed empirical evidence that equity prices may not respond fully to increased inflation. The market risk premium, which is a function of equity market volatility, is added to account for variations in volatility. The argument being that in periods of tranquility in the equity market the risk premium is lower, hence lowering equity returns (Ferson & Harvey, 1991).

Although the model in (4) gives a more advanced model estimation of the BEER, the paper will only use the simpler form of the BEER (3). The reason is partly because it is a more parsimonious representation, and easier to interpret, and partly because of access to data. On the basis of the discussion above the following trading rules are used:

## 3.1 Trading-rules

- Decreasing BEER: If the BEER-value is downward sloping, the signal is that equities are overvalued, and that the price of equities is dropping. The strategy is therefore to sell equities and buy bonds. Since the data has a quarterly frequency, the strategy is therefore to buy and hold bonds for 12 weeks, or 3 months.

- Increasing BEER: If the BEER-value is increasing, the signal is that equities are undervalued and the price will therefore have to increase. The strategy is therefor to buy equities and sell bonds.

The qualities of the BEER has previously been investigated by Mills (1991, 1998). Mills treats the BEER as a ”confidence factor” where he argues that a close monitoring of the BEER might provide information of possible movements in the equity market. Levin and Wright (1998) find that the BEER alone is unlikely to provide profitable asset investment decision. They do however find that the BEER is likely to provide profitable criteria for asset investment decisions, even during periods of changing inflation expectations, provided that the changes in the variables in (4) are also added into the estimation of the threshold values of the BEER. The work by Brooks and Persand (2001) finds that the BEER coupled with a Markov-Switching model provides forecasts that generate investment decision with superior risk-return characteristics, compared to a buy-and-hold strategy in the UK equity market (The approach also provides slightly better returns in the US and German markets as well). Notably Brooks and Persand (2001) opted for the more parsimonious version of the BEER (3), hence contradicting the results of Levin and Wright (1998). The results also support the conclusions of Clare et al. (1994), thereby giving support to the notion that the simple BEER metric (3) might be sufficient for making investment decisions.

In more recent publications Tangjitprom (2012) finds results in support of Brooks and Persand (2001), as well as Clare et al. (1994), in the emerging market Thailand. The conclusions of Tangjitprom (2012) and Clare et al. (1994) raises important concerns of the efficiency of the equity markets. Basic finance theory advocates that the adjustment speeds of equity market would make any trading rule incapable of providing abnormal returns, thus the results in the aforementioned literature stand in contradictions of the underlying hypothesis of the adjustment speeds under market efficiency. Fama (1970) defines the three following terms for capital market efficiency.

- Weak-form efficiency: States that no market participant can earn excess returns by developing trading rules based of historical information, such as price or return information. Which implicitly means the historical information of returns and prices have no predictive power over future returns and prices:
- Semi-strong-form efficiency: States that no market participant can earn excess returns by developing trading rules based on any publicly known, or available, information. These types of publicly available known information may for example be annual reports of a company, or investment advisory data.
- Strong-form efficiency: States that no market participant can earn excess returns by using any public or non-public information what so ever.

Obviously the latter form of efficiency is quite strong, especially regarding non-public information. The terms of efficiency have been subject to numerous empirical experiments (for example Cohen, Black, and Scholes (1972), Lehmann (1988) and Jensen (1978)). In terms of empirical modeling, the terms of market efficiency imply that future equity prices and returns can only be modeled by a random walk, since they are subject to random outcomes. In the event of evidence against market efficiency, the paper will make little effort to provide an absolute answer as to why it is possible to construct empirical models, who's performance give such implications, since it merely recognizes these implications.

# 4 Modeling and evaluation

As can be seen from the graph of the BEER/GEYR(Gilt-Equity-Earnings-Ratio)-series below, the underlying data suggest that a linear model might not be a sufficient tool for modeling the series. However, before embarking on non-linear modeling the researcher needs to assess if the usage of non-linear models are: 1) necessary, and 2) if non-linear modeling provide any improvement in describing the underlying series. The reason is for simplicity. If there is no increased gain in the from of explanatory power or forecasting performance from using a non-linear model, as opposed to a linear specification, then it is not worthwhile since working with non-linear models is more cumbersome. It is therefore necessary to test if the BEER-series has non-linear behavior such that a non-linear model is of use. The next subsection will therefore examine the necessity of non-linear models, and how to test for non-linear behavior.

## 4.1 Non-linear modeling

There are several benefits of using non-linear modeling compared to linear modeling. However there are several models that fall under the non-liner model category. In the previous research the non-linear modeling is mainly limited to two models: 1). The general smooth transition (STR) model and 2). The Markov-switching (MS) model. The latter model does however present some issues in this case. The problem with the MS model is that it treats the underlying driving forces of the series in question as latent, in the sense that they are not explicitly modeled in as explanatory variables. This can be illustrated by giving a brief summery of how the MS-model works.

Introduced by Hamilton (1989, 1990), the model assumes assumes N states of nature, or regimes, with a mean $\mu_N$ and variance $\sigma_N^2$. Using financial data, specially in the case of modeling the BEER, the number of states ($s_t$) can be restricted to two: *high* and *low* states. The approach assumes the state variable of interest, in this case the BEER, to follow a first order Markov process with transition probabilities:

$$p(s_t = 1|s_{t-1} = 1) = p_{11}$$

$$p(s_t = 2|s_{t-1} = 1) = 1 - p_{11}$$

$$p(s_t = 2|s_{t-1} = 2) = p_{22}$$

$$p(s_t = 1|s_{t-1} = 2) = 1 - p_{22}$$

Thus implied by the assumptions above, the model requires the given parameter vector $\phi = (\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, p_{11}, p_{12}, p_{22}, p_{21})$. The parameter vector can be estimated using a local function optimum.

The observed BEER series can be expressed as $y_t, (t = 1, ....., T)$ the conditional likelihood function of each state can be expressed as:

$$p(y_t|s_t, s_{t-1}, ...., s_1, y_{t-1}, y_{t-2}, ...., y_1; \phi) = \frac{1}{(2\pi|\Omega_{s_{(t)}})^{1/2}} exp[\frac{-[y_t - \mu_{s(t)}]\Omega_{s(t)}^{-1}[y_t - \mu_{s(t)}]}{2}]$$

Under these assumptions the Markov-switching model generates the smoothed probabilities of the states $p(s_t|y_T, y_{T-1}, ...., y_1; \phi)$. Moreover these "smoothed probabilities" are used to calculate the probability of the dependent variable $(y_t)$ being in a particular state at any given point in time. The MS-model therefore puts usage of the oil price as a driving force mechanism to explain the BEER-series at a serious disadvantage, because the variable itself does not enter in the estimation process. Keeping in line with the initial motivation, the paper will therefore concern the non-linear modeling to using the STR-model.

### 4.1.1 STR-modelling

The general STR model is given as:

$$y_t = \beta_0 + \Sigma_{i=1}^{k} \beta_i x_{i,t-n} + \theta[\gamma_0 + \Sigma_{i=1}^{k} \gamma_i x_{i,t-n}] + \epsilon_t \tag{5}$$

What characterizes the STR-model is the form of the parameter $\theta$. Keeping in line with not exploring a variety of non-linear econometric models, only two more "well known" STR-models are considered, namely: The logistic STR-model (LSTR) and the exponential STR-model (ESTR). In the LSTR model the parameter $\theta$ is given as:

$$\theta = [1 + exp^{-\gamma(s_{t-d}-c)}]^{-1} \tag{6}$$

Where $s_t$ denotes the transition variable, with the subscript $d$ denoting the number of lags. $c$ is the critical value. $\gamma$ is the smoothness parameter which determines the speed of transition.

In ESTR model the parameter $\theta$ is given as:

$$\theta = [1 - exp^{(-\gamma(S_{t-d}-C)^2)}] \tag{7}$$

Some existing literature shows that the usage of STR models provide improved forecasts (for exampleMcMillan (2003), Bredin and Hyde (2008)). In addition STR-models have the ability of capturing different types of transitions/adjustments, depending on the form of transition variable, as shown above.

However the test procedure for LSTAR or ESTAR behavior is not so straightforward. This can be showed by looking at a simple STR-model:

$$y_t = x_t'\beta + (x_t\varphi)G(\gamma, c; s_t) + u_t \tag{8}$$

Where $x_t'$ is the vector of explanatory variables, $\beta$ and $\varphi$ are parameter vectors, G is dummy variable which takes the value of 1 if $s_t > c$ and 0 for $s_t < c$. Testing for linearity would therefore be equivalent to testing the null hypothesis of $H_0 : \gamma = 0$, or the alternative null hypothesis $H_0 : \varphi = 0$. However, since the two methods are equivalent to each other, a problem arises. The problem is that the model is identified under the alternative hypothesis of $\gamma > 0$, *or* $\varphi \neq 0$ but not under the null hypothesis. This in turn leads to the problem of not knowing which distribution the null hypothesis model is subject to. The solution provided by Teräsvirta (1998) is therefore to take a third order Taylor expansion of (8), which yields the following:

$$y_t = x_t'\beta_0 + x_t'\beta_1 S_t + x_t'\beta_2 S_t^2 + x_t'\beta_3 S_t^3 + v_t \tag{9}$$

The non-linear model is now linearized by the Taylor expansion, and the vectors containing the explanatory variables and the respective parameter vectors are also multiplied by the transition variable (see (14) for specification of the transition variable) to create interaction terms. If the model is linear it will be equivalent to the acceptance of the null hypothesis:

$$\beta_1 = \beta_2 = \beta_3 = 0 \tag{10}$$

Yet there is a more efficient way to conduct the test for non-linearity, and at the same time determine the specific form of the function (i.e distinguish between ESTR or LSTR).

Following Bårdsen, Hurn, and McHugh (2010), one can employ the *Automatic Model Selection (Autometrics)* function available in the program Oxmetrics developed by Doornik (2009) using the following procedure:

1. Model specification. Builds on the same approach as in the joint hypothesis test in (9) and (10), but only includes the cubic interaction terms. This is to test specifically for LSTR-behavior.

$$y_t = x_t'\beta_0 + x_t'\beta_3 S_t^3 + v_t \tag{11}$$

Autometrics then performs a specification search of (11) and gives the chosen specification of the model. If the model chosen by the Autometrics contains a non-zero element in the $\beta_3$ vector, the null-hypothesis in (10) is rejected. Moreover, the non-zero parameters in the $\beta_3$ vector provides a useful indication of the variables that enter non-linearly in the LSTR model. The variables corresponding to the $\beta_3$ vector will enter non-linearly in the LSTR-model. The reason being their significant interaction with the cubic specification of the transition variable $S_t$, which itself is a non-linear specification.

The next step will then be to estimate LSTR model that corresponds to the aforementioned procedure:

$$BEER = \beta' X_t + \varphi' X_{t,n} G_t(\gamma, c; S_t) + \epsilon_t \tag{12}$$

$$G_t = [(1 + exp\{-\gamma(s_t - c)\})]^{-1} \tag{13}$$

$$S_t = \frac{dlog(Oilprice_{t-1})}{\sqrt{var(dlog(Oilprice_{t-1}))}} \tag{14}$$

The transition variable is the differenced logarithm of first lagged value of the oil price, divided by the standard deviation of itself. The reason for choosing such a specification of $S_t$ is to scale the smoothness parameter in an appropriate fashion. Intuitively this can be explained by thinking of the changes in the oil price. If the oil price changes are sufficiently high and abrupt, the smoothness parameter will take on high values so that transition to the non-linear part of the model will be abrupt. If the transition process mimics a vertical line (the transition process will be vertical for $\gamma \rightarrow 25$) rather than smooth line, the second derivative of the transition function will approach infinity. Hence, the smoothness parameter (and its variance) will be hard to pin down, and subsequently the critical value c will be subject to the same problems (Teräsvirta, 1994). As previously mentioned the $X_t$ variable is the variable vector that contains the chosen explanatory variables from the automatic model selection procedure. By implication $\beta'$ is the parameter vector containing the associated parameters for the variables in the $X_t$ vector. The $X_{t,n}$ variable contains the explanatory variables who show significant LSTR-behavior from the LSTR test procedure. Finally, the parameter $\varphi$ is then the associated parameter vector for the non-linear variables.

2. The next step in case of non-zero values for the $\beta_3'$ vector is to estimate the model:

$$BEER = X_{t,0}'\alpha_0 + X_{3,t}'\phi_3\hat{G}(\hat{\gamma}, \hat{c}S_t) + \epsilon_t \tag{15}$$

   Letting $\alpha_0$ and $\phi_3$ correspond to the non-zero elements in $\beta$ and $\varphi$ in (12) we can estimate the equation above as an LSTR model. Remembering the G-function for the LSTR-model from (6). Again the parameters $\gamma$, c and $s_t$ respectively serve as the steepness-parameter, critical value, and the value of transition variable.

3. Finally the encompassing is conducted. Encompassing in this context is a test of the LSTR model in (12) explaining the same as a linear specification, while the revers does not. Given the estimated values for the steepness parameter $\hat{\gamma}$ and the critical value $\hat{c}$ and the estimated G-function. By expanding the general linear model given in (11) thereby obtaining the following model:

$$BEER = \delta_0' X_t + \delta_1' X_t S_t^3 + \eta_3 X_{3,t}\hat{G}(\hat{\gamma}, \hat{c}; s_t) + \epsilon_t \tag{16}$$

   Testing the joint hypothesis of:

$$H_0 : \alpha_0 = \delta_0,\ \delta_1 = 0,\ \eta_3 = \phi_3 \tag{17}$$

Using a F-test to evaluate the hypothesis. However there is an easier approach: Following Bårdsen et al. (2010) a more simple test for encompassing is applied. By letting *Autometrics* evaluate (16) and see if the resulting model is the estimated from (15), in which case the test statistic is the F-test of omitted variables in the final model specification. Note that the test is conditional on $\hat{G}(\hat{\gamma}, \hat{c}; s_t)$

## 4.2 Forecasting

One of the advantages using econometric models is the ability of forecasting. Forecasting may be done either for 1-step ahead, or h-steps ahead. The distinction between the two procedures is that the former is a static procedure, meaning that any lagged information required to form the forecasts are based on observed values. The latter procedure is known as dynamic forecasting where the forecasted values, from the previous period, are re-used to make forecasts for the next period. The distinction may be summarized by considering a simple regression model:

Static forecasting:

$$\begin{aligned}
y_t &= \beta_1 y_{t-1} + \beta_2 y_{t-2} + \beta_3 z_t + v_t \\
\hat{y}_{t+1} &= \hat{\beta}_1 y_t + \hat{\beta}_2 y_{t-1} + \hat{\beta}_3 z_{t+1} \\
\hat{y}_{t+2} &= \hat{\beta}_1 y_{t+1} + \hat{\beta}_2 y_t + \hat{\beta}_3 z_{t+2} \\
\hat{y}_{t+3} &= \hat{\beta}_1 y_{t+2} + \hat{\beta}_2 y_{t+1} + \hat{\beta}_3 z_{t+2}
\end{aligned}$$

Dynamic forecasting:

$$y_t = \beta_1 y_{t-1} + \beta_2 y_{t-2} + \beta_3 z_t + v_t$$
$$\hat{y}_{t+1} = \hat{\beta}_1 \hat{y}_t + \hat{\beta}_2 y_{t-1} + \hat{\beta}_3 z_{t+1}$$
$$\hat{y}_{t+1} = \hat{\beta}_1 \hat{y}_{t+1} + \hat{\beta}_2 y_t + \hat{\beta}_3 z_{t+2}$$
$$\hat{y}_{t+1} = \hat{\beta}_1 \hat{y}_{t+2} + \hat{\beta}_2 \hat{y}_{t+1} + \hat{\beta}_3 z_{t+3}$$

The similarity between the two is that they both need previous data up to the beginning of the forecasting point $t$ in order to estimate the coefficients. Additionally, both methods do not update the coefficient estimates over the forecast period. Furthermore, static forecasting requires information of $z$ for $t = t+1, t+2, ...., t+h$ and $y$ up to $t+h-1$, while dynamic forecasting only requires information about $z$ for $t = t+1, t+2, ...., t+h$. In practice its possible to update the coefficient values in the 1-step ahead procedure, which would imply running a 1 step-forecast and re-estimating the model up to the forecast point and repeat the procedure over the forecast period of interest. The problem with static forecasting is that it requires the future values of $z$ to be known. The assumption of knowing these future values in advance is however unrealistic, and a static forecasting procedure can therefore be thought of as a measure of coefficient constancy.

In terms of the actual estimation I have chosen to employ the h-step/dynamic forecasting procedure. The reason is partly to investigate if the model specification of the BEER can be used over longer forecasting horizons. And partly to see if trading based on the forecasted values produce higher returns than the market index.

Furthermore, there are some important procedures used to make the forecasts. Given that the forecasts generally come with some level of uncertainty (represented by the standard deviation), a level of error is bound to happen. If the uncertainty level is sufficiently high, the forecast procedure runs the risk of systematically over/underestimating the values of the BEER. A high uncertainty level can forecast the BEER in wrong directions. If the model forecasts an upward sloping BEER, and the actual series is downward sloping, the investor will run the risk of taking losses in the equity market. The uncertainty can however be utilized to an advantage. Oxmetrics allows level mean forecast, which means taking the exponential of the logarithm of the BEER (thereby converting it to level form) and making the forecast as the average of the initial (level) forecast $+/-$ 2 times the standard deviation. In the empirical results this approach has been highly successful, giving higher accuracy and increased the models ability to predict the direction of the BEER.

The forecasting strategy has been to estimate the model from the first quarter of year 1998 up to the first quarter of year 2002. The forecasts are then made for 8-quarters ahead, thus ending the forecast period in the first quarter of 2004. The model is then re-estimated up to the last forecast period and the procedure of 8-forecasts is then repeated until the end of the sample. The model is then evaluated both qualitatively and by the generated returns. These qualitative measures will be discussed in the following subsection.

### 4.2.1 Forecasting evaluation

There are several measures for evaluating the accuracy of the forecasts. In order to restrict a long discussion of different measures, the paper uses two main measures for evaluating the forecasts, namely: The Root Mean Squared Error (RMSE), and the Mean Absolute Percentage Error (MAPE). The respective statistics are given as:

$$RMSE_{e_{t+h_t}} = \sqrt{\frac{1}{T}\sum_{t=0}^{T} e^2_{t+h_t}}$$

$$MAPE = \frac{100}{n}\sum_{t=1}^{n}\left|\frac{A_t - F_t}{A_t}\right|$$

Where the:

- t and T denotes time and total number of observations
- $e_{t+h,t}$ denotes the forecast error
- $A_t$ denotes the actual value

- $F_t$ denotes the forecasted value
- n denotes the number of fitted values.

The reasons for using two measures of forecasting errors are due to their respective properties. The widely used RMSE provides a unit measure of the forecast error between the forecasted and observed value. The problem with this is that the RMSE provides a unit measure of the forecasting error, which in some cases might not be as informative as a unit free measure such as the MAPE. The MAPE is included to provide this unit free measure, and to give information about the percentage error of the forecasts. Although both measures have good attributes, there are however some drawbacks to using both measures: The RMSE measures the mean of all the squared forecasting errors, therefore abnormally large errors will penalize the measure harder. This may, to some extent, cause misleading results in the case of outlier values. To circumvent this potential problem the MAPE is included. The short comings of the MAPE are researched by Hyndman and Koehler (2006). In short terms, the problem is mainly that if $A_t$ is equal to zero the measure is not defined.

Although both measures provide information of the forecasting performance of the model they do not provide an explicit measure of the models ability to forecast the correct direction of the BEER movements. As previously mentioned, practitioners may care more about the direction of BEER, as it provides a buy or sell signal, rather than the actual quantity of the movements. Hence, if the model forecasts a declining BEER when the actual movement is inclining, the investment decision will be guided in the wrong direction. Moreover, this puts the investor at risk of potential losses, or missing out on potential winnings. A measure of the directional forecasting ability of the model is provided by the (non-parametric) Pesaran-Timmermann test (Pesaran & Timmermann, 1992):

$$S_n = \frac{(\hat{P} - \hat{P}_*)}{\sqrt{[\hat{V}(\hat{P}) - \hat{V}(\hat{P}_*)]}}$$

$$\hat{P} = n^{-1} \sum_{t=1}^{n} I(y_t x_t)$$
$$P_* = \hat{P}_y \hat{P}_x + (1 - \hat{P}_y)(1 - \hat{P}_x)$$

$$\hat{V}(\hat{P}) = n^{-1} \hat{P}_* (1 - \hat{P}_*)$$
$$\hat{V}(\hat{P}_*) = n^{-1}(2\hat{P}_y - 1)^2 \hat{P}_x (1 - \hat{P}_x)$$
$$+ n^{-1}(2\hat{P}_x - 1)^2 \hat{P}_y (1 - \hat{P}_y)$$
$$+ 4n^{-2} \hat{P}_y \hat{P}_x (1 - \hat{P}_y)(1 - \hat{P}_x)$$

$$\hat{P}_y = n^{-1} \sum_{t=1}^{n} I(y_t)$$
$$\hat{P}_x = n^{-1} \sum_{t=1}^{n} I(x_t)$$
$$I = \begin{cases} 1 & if\ x_t or\ y_t > 0 \\ 0 & otherwise \end{cases}$$

The test has the null hypothesis that the forecasts x from the econometric model, have no ability to forecast the correct direction of the actual series y over the n-observations. And assumes that:

- The test statistic $S_n$ is asymptotically normally distributed as N(0,1)
- $y_t$ and $x_t$ are independently distributed (which is effectively the same as saying that $x_t$ has no predictive power of $y_t$).
- $n\hat{P}$ is assumed to follow a binominal distribution with mean $nP_*$.
- The probability of the change in the direction of $y_t$ and $x_t$ is time-invariant, and

does not take on the extreme value of 1. If the joint distribution of $y_t$ and $x_t$ is continuous and stationary the requirement is satisfied.

- Stationarity is however not a requirement for the test to be valid. The test can be utilized for non-stationary distributions given that $y_t$ and $x_t$ are symmetrically distributed around zero.

It therefore calculates the probability of an increase in the BEER and the forecasted value (P). In effect this measures the probability of the forecast making correct prediction of an increase in the BEER. $P_x$ and $P_y$ measures the respective probability of an increase in the forecasted and realized value of the BEER. The interpretation of $P_*$ can found as:

$$P_* = Pr(\overline{Z} = 1) = Pr(x_t, y_t > 0)$$
$$= Pr(y_t > 0, x_t > 0) + Pr(y_t < 0, x_t < 0)$$
$$= P_y P_x + (1 - P_y)(1 - P_x)$$

Since $P_x$ and $P_y$ are known, the interpretation of $P_*$ can be thought of as the predictive failure test based on the standardized binomial variate. $\hat{V}(P)$ can be shown as the variance of $P$ and $P_*$.

# 5 Data

## 5.1 Macroeconomic indicators

When choosing the data to explain the BEER-series, macroeconomic variables are chosen, since they are meant to reflect the state of the economy, and by extension are assumed to have explanatory power of both the equity and bond market. The data is provided by *Thomson Reuters Datastream*, *Thomson Reuter Eikon* and the monetary authorities in Norway (Norges Bank). Broadly speaking the data can be separated into different groups depending on their respective characteristics:

- Oil related indicators: The variables chosen are *Norwegian Oil Exports, World Oil Production*, and the *(spot)price of Crude Oil*. The reason for choosing these variables is to provide an estimate of the activity level in the oil sector, both nationally and globally.

- Production related indicators: The variables chosen are the *Norwegian Mainland GDP* [3] *Growth*, and the *Unemployment Rate*. The variables are selected to 1.) Describe the activity in the Norwegian production, excluding oil production. 2.) Describe the activity through the rate of unemployment.

- Monetary indicators: Perhaps the most important group of explanatory variables to explain both movements in both the bond/- and equity market. The variables selected are: *Inflation, (using the CPI* [4] *as a proxy)*, *The Norwegian Inter Bank 3-month Interest Rate*, *Term structure*, and *The Norwegian Krone / US Dollar Exchange Rate*.

- With regard to the notation in estimation transcripts, most of the variables used are reported using their respective names in order to make them more understandable. There are however a few points that deserve comment. Firstly, the lagged values of a variable are reported with the variable name and underlined by the number associated with the number of lags. For example the first lag of the term structure is reported as Term_structure_1. Secondly the log (logarithmic) differenced variables are reported as Dlog. Thirdly the *The Norwegian Krone, Dollar Exchange Rate* is reported as: NOK/$. The *The Norwegian Inter Bank 3-month Interest Rate* is reported as: NWINTER3. The percentage change int inflation is reported as Dlog_CPI.

  Regarding some of the monetary indicators its important to make a few remarks: First, in terms price movements in the bond and equity markets, inflation has a peculiar effect: In response to increased inflation, the price of bonds and equities

[3]GDP is the gross domestic product
[4]CPI is the consume price index

act asymmetrically. The reason is that increased inflation increases the nominal interest rate that the cash-flow of bonds are discounted with, thereby lowering the present value of future bond payments, subsequently decreasing the price of bonds. The same applies for the present value of the future cash-flows related to businesses, and by extension the value of the firms equity. However, a business has the power of adjusting prices, thereby increasing its cash-flows. This in turn might lead to an upward sloping BEER. Second, the stock market index is composed of many Norwegian companies, including ones subject to international trade and operations. By implication this gives the potential for losses and winnings in terms of currency fluctuations, which in turn may also affect their respective cash-flows and firm values. 3.) Finally, the term structure has been calculated as the difference between the yield of a 10-year and 3-year government bond.

### 5.1.1 Unit root testing

In order to ensure that the model becomes stationary, certain explanatory variables have to be modified/transformed. In order to determine which of the explanatory variables need to be modified in order to achieve stationarity, the *Dickey Fuller* (DF) and the *Augmented Dickey-Fuller* (ADF) tests are used (Dickey & Fuller, 1979). Using a simple regression model, the test procedure can be shown as:

$$
\begin{aligned}
y &= \phi y_{t-1} + u_t \\
\Delta y_t &= \psi y_{t-1} + u_t \\
\psi &= (\phi - 1) \\
test\ statistic &= \frac{\hat{\psi}}{\hat{se}(\hat{\psi})}
\end{aligned}
$$

The underlying principle of the test is to test if the parameter $\phi$ is equal to 1 (unit root), in which case shocks to the y-variable would have infinite persistence. The alternative hypothesis is therefore $\phi < 1$, implying no unit root (and be extension a stationarity process). In short, the procedure tests the null hypothesis $H_0 : \psi = 0$ against the alternative hypothesis $H_a : \psi < 0$.

Under the null hypothesis the normal t-statistics are not valid since series is assumed to be non-stationary. To circumvent the problem, new critical values have been derived trough simulation experiments by Dickey and Fuller (1979). The main difference between these simulated critical values (DF-values) and standard normal distributions is that the DF values are much larger in absolute terms (more negative) than the latter distributions. The procedure might seem more cumbersome than examining the autocorrelation function (ACF) of the series, to see if it was decaying. The issue with employing such a procedure is if the unit root exits in the series, the ACF of that particular series can be seen to decay very slowly, even if the shocks have infinite persistence. This might in turn lead to a false rejection of the null hypothesis. Moreover, the inclusion of a non-stationary variable in an econometric model might lead to a *spurious regression* (Granger & Newbold, 1974). A spurious regression generates high explanatory power ($R^2$) with significant t-statistics, but the results have no, or limited, economic meaning. As such, the results may seem to be all good and well, but the least-squares estimates are not consistent, and the model will fail diagnostic tests.

One important extension of the Dickey-Fuller test:

$$\Delta y_t = \psi y_{t-1} + \Sigma_{i=1}^{q} \alpha_i \Delta y_{t-i} + u_t$$

The test procedure is called a *Augmented Dickey Fuller* test (ADF), and is an extension of the original DF test (Note that the DF-test can be interpreted as a ADF(0) test). To ensure that the error term $u_t$ follows a white-nose process the term $\Sigma_{i=1}^{q} \alpha_i \Delta y_{t-i}$ is added. The lags of $\Delta y_t$ capture any dynamic structure in the dependent variable, thereby avoiding the problem of autocorrelated errors. The particular lag structure of the augmentation term can however be hard to determine, and usual trial and error may be required. There is although a general rule of thumb: Using the frequency of the data to determine the lag length. Meaning that in the case of monthly data the lag length can be set to twelve lags. In this case one could set the lags to 4 (alternatively the information criterion could be used).

## 5.2 The BEER-series

As previously discussed the BEER is calculated as the dividend yield ratio between the benchmark assets of equities and bonds (see (1)). Roughly following the same method as Brooks and Persand (2001), using a 10-year Norwegian government bond and the index values from the OSEBX to calculate the BEER for a given period (3-months). It is important to add that the returns from both assets has been calculated on growth form in order to avoid negative value of the BEER. The approach may deviate somewhat from the work of Levin and Wright (1998) and others, giving a lower BEER-value in this paper. The argument for using such an approach is that the paper aims to determine the directions of the BEER rather than the quantities. In addition the BEER series has been transformed by taking the logarithm. The notion is that by taking the logarithm, the variations in the BEER will become more compatible with the log differenced explanatory variables.

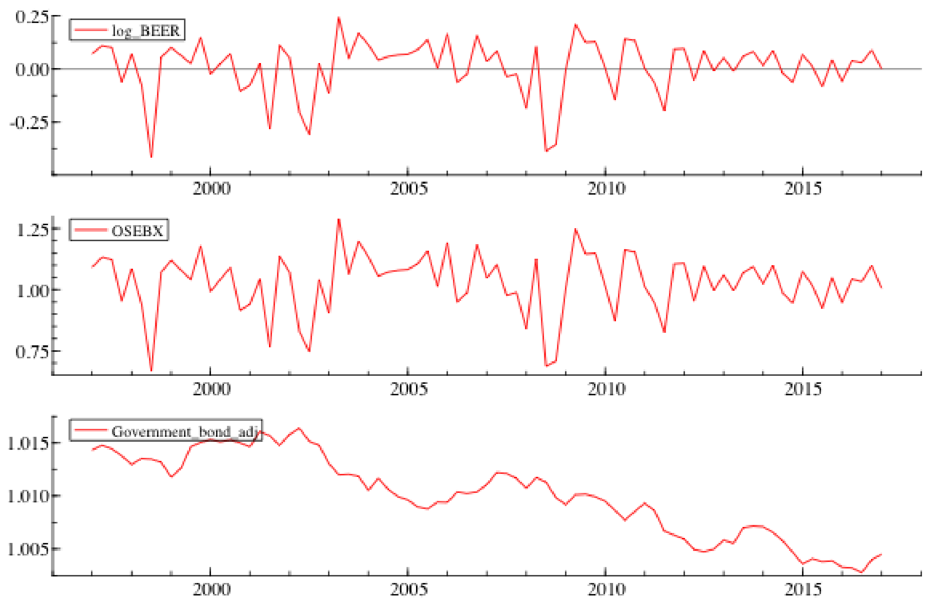



Figure 1: BEER-series

The series has a mean of 0.0103 with a standard deviation of 0.0427, with a maximum of 0.243 and a minimum of -0.42 (see table 17 in appendix). By taking the exponential of the mean logarithm of the BEER, the results fit comfortably with the initial discussion of the BEER having a theoretical value close to 1. In the table the observations of the BEER around the time of the financial crisis of 2008, the dot.com bubble around 2000, and the Asian financial crisis of 1997/1998 also fit comfortably with the observed declines in the equity market. Overall the BEER appears to be relatively stable, excluding the events of the Asian financial crisis of , the dot.com bubble and the financial crisis. As can be seen from the chart, sharp declines in the observed values (*low* state) are followed by sharp increases in the BEER, implicitly the changes in the states where the logarithm of the BEER is below zero are more abrupt than in the states where it is above zero, which may indicate that there is some form of market correction.

In more recent times with long declining oil prices from the period of around 2015 to present time, the BEER-value does not appear to be declining. This seems somewhat surprising for a small oil dependent economy like Norway. The peculiarity can possibly be explained by the lowering of interest rates, subsequently leading to lower bond returns. Lower bond returns therefore increases the attractiveness of equities, thus increasing the demand, and subsequently the price of equities, and offsetting the effect of declining oil prices.

# 6 Empirical Results

The empirical results are obtained using the approaches discussed in the previous section, using the associated enhanced modeling cycle. The specification procedure of the STR-model as discussed in section 4.1 gives the following results:

## 6.1 Spesification

Table 1: LSTR-test

| | Coefficient | std. deviation | t-value | t-prob |
|---|---|---|---|---|
| Norway_GDP_Growth_Mainland_3 | 0.029 | 0.008 | 3.43 | 0.0012 |
| Dlog_World_Production_Crude_OIL_1 | 2.647 | 0.804 | 3.29 | 0.0018 |
| Dlog_cpi_1 | -4.00 | 1.091 | -3.68 | 0.0006 |
| Dlog_NOK/$_1 | 0.49 | 0.173 | 2.84 | 0.0064 |
| Dlog_NOK/$_3 | 0.61 | 0.153 | 3.96 | 0.0002 |
| Dlog_Unemployment_rate | -0.28 | 0.078 | -3.66 | 0.0006 |
| Dlog_Unemployment_rate_1 | -0.23 | 0.077 | -2.94 | 0.0048 |
| s3 | -0.03 | 0.009 | -3.25 | 0.0020 |
| Dlog_NWINTER3_s3 | 0.20 | 0.049 | 4.17 | 0.0001 |
| Dlog_NWINTER3_1_s3 | -0.397 | 0.098 | -4.04 | 0.0002 |
| Dlog_NWINTER3_4_s3 | -0.88 | 0.105 | -8.37 | 0.0000 |
| Dlog_NOK/$_2_s3 | 0.466 | 0.116 | 4.01 | 0.0002 |
| Dlog_NOK/$_4_s3 | 0.522 | 0.133 | 3.94 | 0.0002 |
| Dlog_Unemployment_rate_1_s3 | -0.62 | 0.1241 | -4.99 | 0.0000 |
| Dlog_Unemployment_rate_2_s3 | -0.092 | 0.038 | -2.44 | 0.0179 |
| Dlog_Unemployment_rate_4_s3 | -0.52 | 0.070 | -7.42 | 0.0000 |
| log_BEER_3_s3 | 0.17 | 0.037 | 4.62 | 0.0000 |
| Term_structure_1_s3 | 6.43 | 1.315 | 4.89 | 0.0000 |
| dlog_OILPRICE | 0.20 | 0.054 | 3.80 | 0.0004 |
| dlog_OILPRICE_1 | -0.164 | 0.067 | -2.44 | 0.0182 |
| dlog_OILPRICE_s3 | 0.105 | 0.025 | 4.13 | 0.0001 |
| Diagnostics: | | | | |
| $\sigma$ : | 0.0600878 | no. of observations | 74 | |
| log-likelihood | 115.432 | RSS: | 0.191358786 | |
| mean(log_GEYR) | 0.00612135 | se(log_GEYR) | 0.132188 | |

As can be seen from the table, there are several economic variables/indicators that in interaction with the defined transition variable in (14) are statistically significant at level of less than 2.5%, and testing the null of joint hypothesis of no significant cubic terms is soundly rejected. Hence, the null hypothesis of LSTR-behavior cannot be rejected. Furthermore the interpretation of the log differenced variables can be somewhat cumbersome. To make the interpretation easier it is beneficial to write the expression of the equation:

$$log(y) = \beta \Delta log x_t$$

$$log(y) = \beta(log x_t - log x_{t-1})$$

$$\%\Delta y = \beta(\Delta log x_t - \Delta log x_{t-1})$$

The interpretation is that if the percentage changes in the x variable increases by 1%, (implying that the x-series becomes more volatile), the increase in the y variable will increase by the coefficient ($\beta$) times the percentage change in the percentage change of the explanatory variable ($\beta\%\Delta\%\Delta x$). The interpretation of other explanatory variables is more straight forward. Since both the term structure and mainland GPD growth are already given in percentage form, they can be considered in the same way as non-differenced log variables. Meaning that if one of them increases by 1% BEER increases by $\beta\%\Delta x$. The same applies for the autoregressive term of the logarithm of the BEER.

Regarding the model selected by the automatic selection function, the explanatory variables reflect both the developments in the real economy as well as the financial sector. In addition the function selects several lagged values of the explanatory variable (especially the unemployment rate and the short term interest rate). This is interpreted as an indication of shocks to explanatory variables may have long-run effects on the BEER. Having established the presence of LSTR behavior, the next step will therefore be to estimate a LSTR-model using the results in table 1.

## 6.2 LSTR-modeling

Several of the explanatory variables have LSTR-behavior in interaction with the oil price. The model is therefore specified accordingly:

1. Variables chosen by *Autometrics* who have both a linear and non-linear effect on the BEER will enter in both the linear and non-linear part of the model.

2. Variables chosen by *Autometrics* that have no significant interaction effect with the oil price will only enter in the linear part of the model.

3. Variables chosen by *Autometrics* that are only significant in interaction with the oil price will only enter in the non-linear part of the model.

Table 2: LSTR specification

| Variable name | coeff estimate | std deviation | t-value |
|---|---|---|---|
| Constant | 0.04037 | 0.0187 | 2.159 |
| dlog_OILPRICE | 0.3337 | 0.06948 | 4.80 |
| dlog_OILPRICE_1 | 0.1293 | 0.091 | 1.413 |
| Dlog_Unemployment_rate | -0.165 | 0.105 | -1.58 |
| Dlog_Unemployment_rate_1 | -0.2 | 0.115 | -1.723 |
| Norway_GDP_Growth_Mainland_3 | 0.02 | 0.014 | 1.411 |
| Dlog_World_Production_Crude_OIL_1 | 0.693 | 1.075 | 0.644 |
| Dlog_cpi_1 | -3.55 | 1.655 | -2.147 |
| Dlog_NOK/$_1 | 0.499 | 0.2101 | 2.377 |
| Dlog_NOK/$_3 | 0.455 | 0.195 | 2.327 |
| | | | |
| $\gamma$ | 3383 | .NaN | .NaN |
| C | 0.0507 | .NaN | .NaN |
| | | | |
| Non-linear parameters: | | | |
| Constant | -0.12 | 0.03489 | -3.439 |
| dlog_OILPRICE | -0.08485 | 0.1259 | -0.6741 |
| log_BEER_3 | 0.1701 | 0.1595 | 1.067 |
| Term_structure_1 | 10.28 | 3.252 | 3.16 |
| Dlog_Unemployment_rate_1 | 0.332 | 0.2538 | 1.308 |
| Dlog_Unemployment_rate_2 | -0.5284 | 0.1558 | -3.391 |
| Dlog_Unemployment_rate_4 | -0.3146 | 0.1575 | -1.998 |
| Dlog_NOK/$_2 | -0.1754 | 0.2802 | -0.6258 |
| Dlog_NOK/$_4 | -0.1435 | 0.264 | -0.5436 |
| Dlog_NWINTER3 | -0.8745 | 0.1758 | -4.974 |
| Dlog_NWINTER3_1 | 0.0513 | 0.1867 | 0.2747 |
| Dlog_NWINTER3_4 | -0.1362 | 0.1174 | -1.159 |
| Diagnostics: | | | |
| SSR | 0.3215892 | AIC | -4.7899 |
| SBIC | -4.04263 | 1000*Residual variance | 6.43178 |
| $R^2$ | 0.74789 | Std error of residuals | 0.0307445 |

The results in the model above show that several of the explanatory variables chosen by *Autometrics* become statistically insignificant under the LSTR specification. Moreover, the transitions process, reflected by the transition parameter $\gamma$, implies highly abrupt transitions. A visual imagination of the transition process is in this case equivalent to a vertical line. This provides an empirical evidence supporting the initial discussion of problems concerning the estimation of the smoothness parameter and the critical value. In an effort to see if a more parsimonious version of the model yields more encouraging results, the model is tested down by removing the insignificant explanatory variables.

Table 3: LSTR-specification 2

| Variable name | coeff estimate | std deviation | t-value |
|---|---|---|---|
| Constant | 0.033 | 0.014 | 2.4 |
| Dlog_OILPRICE | 0.283 | 0.05 | 5.175 |
| Dlog_Unemployment_rate_1 | -0.27 | 0.113 | -2.37 |
| Dlog_Unemployment_rate | -0.26 | 0.1 | -2.65 |
| Dlog_NOK/$_3 | 0.42 | 0.20 | 2.094 |
| | | | |
| $\gamma$ | 208.9 | 15.8 | 13.22 |
| C | 0.0525 | .NaN | .NaN |
| | | | |
| Non-linear parameters: | | | |
| Constant | -0.1034 | 0.025 | -4.152 |
| Dlog_Unemployment_rate_1 | 0.73 | 0.234 | 3.116 |
| Dlog_Unemployment_rate_2 | -0.55 | 0.144 | -3.825 |
| Dlog_NWINTER3 | -0.1 | 0.15 | -6.6 |
| log_BEER_3 | 0.47 | 0.136 | 3.48 |
| Term_structure_1 | 9.144 | 2.63 | 3.47 |
| Diagnostics: | | | |
| SSR | 0.45437 | AIC | -4.74155 |
| SBIC | -4.33678 | 1000*Residual variance | 7.44872 |
| $R^2$ | 0.583 | Std error of residuals | 0.0863 |

The results in the table above are not directly encouraging in terms of the transition parameter. Although the parameter value of the transition parameter has decreased drastically, the transition process is still so abrupt that visually it is equivalent to a vertical line. Subsequently, the problems of the initial STR-estimation are still present. In addition, the coefficient values for the current lagged unemployment rate are close to equal in the linear part of the model, meaning that for an equal increase in both variables the marginal effect is equal. To further simplify the model the two variables can be written as $\beta_i(Dlog\_Unemployment\_rate + Dlog\_Unemployment\_rate\_1)$ (the re-estimation of the model under this specification can be found in the appendix in table 16). The model is although more parsimonious and favored by the AIC (Akaike, 1974) information criterion but, not by the SBIC (Schwarz et al., 1978). The reason can be examined by looking at the respective models for the information criterion:

$$AIC = T \times ln(SSR) + 2n$$

$$SBIC = T \times ln(SSR) + n \times ln(T)$$

n = number of parameters estimated

T = number of observations

Since the last term of the SBC is written as $n \times ln(T)$ rather than $2n$, the SBIC will generally pick more parsimonious models as T increases (provided T>2) than the AIC. However, since the number of observations is not particularly high, the preferred model will be the more parsimonious one.

Although the model specification does not appear to be ideal, it does provide some useful information. The latter LSTR-model sets the critical value for the change in oil price to 5.25%. Implying that if the price of oil increases by more than the critical value, the non-linear part of the model kicks in. Under the assumption that this is correct, using the defined critical values to define a dummy variable (or a step function), the problems concerning the estimation of the transition process can be circumvented, under a linear model specification. Utilizing the defined critical value to specify a linear model, building on the defined LSTR-model in table 3, consider a general linear (threshold) specification:

$$BEER = \Sigma_{i=1}^{n} \rho_{1,i} X_{it} + \Sigma_{i=1}^{n} \rho_{2,i} X_{i,t} I_t + \epsilon_t \tag{18}$$

with

$$I = 1 \; if \; S_t > C$$

$$I = 0 \; if \; S_t < C$$

By multiplying the dummy variable $I$ with the variables in the non-linear part of the model and keeping the linear variables in the same model, the LSTR model is in effect replicated by a linear specification. It is in effect replicated because the dummy specification, using the defined critical value, has the same vertical transition process as the estimated transition parameter in the non-linear specification. Hence, the problem with estimating the second derivative of the transition parameter (which discussion can be found in the last paragraph on page 9) is avoided. On the basis of this discussion the next steps will therefore be to estimate the linear threshold model, and subsequently determine if the linear specification has the same statistical properties as the LSTR specification (hence, making it the preferred model), by performing the encompassing test.

The model can then be estimated using standard OLS and evaluated using standard qualitative tests:

Table 4: Threshold specification

| | Coefficient | Std.Error | t-value | t-prob |
|---|---|---|---|---|
| Constant | 0.032 | 0.013 | 2.43 | 0.0179 |
| Dlog_Unemployment_rate | -0.257 | 0.097 | -2.64 | 0.0106 |
| Dlog_Unemployment_rate_1 | -0.267 | 0.11 | -2.4 | 0.0194 |
| Dlog_NOK_DOLLAR_3 | 0.417 | 0.192 | 2.17 | 0.0334 |
| Dlog_OILPRICE | 0.28 | 0.0536 | 5.23 | 0.0000 |
| I | -0.1033 | 0.0247 | -4.19 | 0.0001 |
| log_BEER_3_I | 0.414 | 0.136 | 3.04 | 0.0035 |
| Dlog_Unemployment_rate_1_I | 0.70 | 0.23 | 3.04 | 0.0034 |
| Dlog_Unemployment_rate_2_I | -0.51 | 0.144 | -3.54 | 0.0008 |
| Dlog_NWINTER3_I | -0.88 | 0.156 | -5.65 | 0.0000 |
| Term_structure_1_I | 9.56 | 2.66 | 3.59 | 0.0006 |
| Diagnostics: | | | | |
| $\sigma$ | 0.0850728 | RSS | 0.456 | |
| $R^2$ | 0.642552 | F(10,63) | 11.32 [0.000]** | |
| SBIC | -1.6118 | AIC | -1.9543 | |
| mean(log_GEYR) | 0.00612135 | se(log_GEYR) | 0.132188 | |

The model above appears to be more favorable in terms of both the information criterion, and small increase in explanatory power. Moreover, it gives support to the notion that a correctly specified linear model can be used to achieve the same objectives as the non-liner model. The next subsection will therefore examine if the threshold model is a valid simplification of the LSTR model.

## 6.3 Encompassing test

As discussed in section 4.1.1, the encompassing test evaluates if a linear specification of the underlying data has the same properties as a non-linear specification. The model provided by the automatic model selection is given as:

Table 5: Encompassing test

| | Coefficient | Std.Error | t-value | t-prob |
|---|---|---|---|---|
| Constant | 0.035 | 0.0134 | 2.59 | 0.0117 |
| Dlog_Unemployment_rate | -0.23 | 0.098 | -2.37 | 0.0207 |
| Dlog_NOK_DOLLAR_3 | 0.37 | 0.193 | 1.90 | 0.0614 |
| dlog_OILPRICE | 0.264 | 0.054 | 4.87 | 0.0000 |
| I | -0.105 | 0.025 | -4.18 | 0.0001 |
| log_BEER_3_I | 0.404 | 0.140 | 2.89 | 0.0053 |
| Dlog_Unemployment_rate_1_I | 0.437 | 0.209 | 2.09 | 0.0404 |
| Dlog_Unemployment_rate_2_I | -0.50 | 0.148 | -3.37 | 0.0013 |
| Dlog_NWINTER3_I | -0.87 | 0.160 | -5.46 | 0.0000 |
| Term_structure_1_I | 9.61 | 2.735 | 3.52 | 0.0008 |
| Diagnostics: | | | | |
| $\sigma$ | 0.0875 | RSS | 0.4977 | |
| $R^2$ | 0.612 | AIC | -1.9107 | |
| mean(log_GEYR) | 0.0072 | se(log_GEYR) | 0.1316 | |

The final model chosen by *Autometrics* clearly shows that the linear specification is preferable to the linearized LSTR-model. It is clear because the model chosen does not include any of the cubic interaction terms from the test for LSTR-behavior in section 4.1.1. In short terms, the encompassing test concludes that the linear specification has the same explanatory power as the starting model. Furthermore, the model chosen by the procedure gives a more parsimonious model description by excluding the first lag of the log differenced unemployment rate. The reason for this in econometric terms is most likely due to the coefficient values having the unequal marginal effects under the specification of the encompassing test (16). Additionally, the threshold specification does not fail important diagnostic tests, such as, autoregression in the residuals, ARCH (Auto Regressive Conditional Heteroscedasticity) effects, or any of the specific heteroscedasticity test or the RESET test for misspecification (which can be found in table 17 in the appendix).

The results suggest that in periods of low volatility in the oil price, the BEER is mainly dependent of shocks to the current price of oil, the exchange rate, and the unemployment rate. Moreover the autoregressive nature of the unemployment rate in the model appears to have peculiar structure (with an over all negative effect), under changes in the oil price above the critical value. The positive coefficient value of the one period lag of the unemployment rate implies a positive relation between it and the BEER, while the current and second lag implies the contrary. Some economic reasoning is provided by Boyd, Hu, and Jagannathan (2005), who find that rising unemployment affects equity markets positively during periods of economic expansion, and negatively during contraction. In addition, they also point out the information content of the unemployment rate. In short terms, news of the unemployment rate provides three types of information: 1) Information about future interest rates. 2) The equity risk premium. 3) Information of future cash flows and dividend payments. Note that the information content and interpretation of these types can change over time.

The positive coefficient value of the term structure implies a relationship with the BEER value. In economical terms this seems to be counterintuitive since a higher term structure (which is calculated as the difference between interest rate of a 10/- and 3-year government bond) should generally increase the attractiveness of bonds relative to equities. However, a graphical description of the returns from the OSEBX index and the term structure shows that the two appear to co-vary.

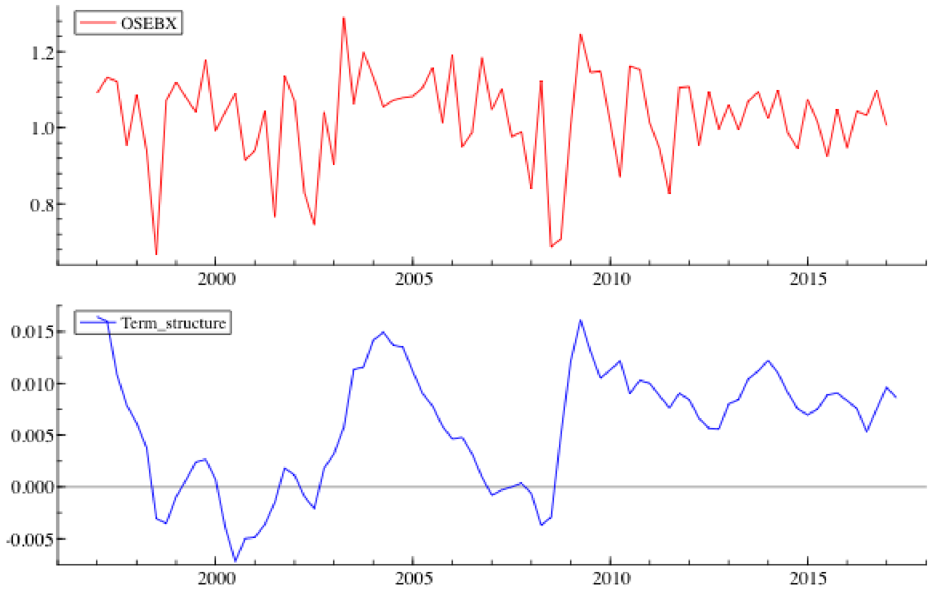



Figure 2: OSEBX, Term structure

Estimating the correlation between the two gives a correlation of 0.35. Although the competitiveness of the two assets might suggest a negative relationship between the two, there are some economical explanations for the positive relationship. In times of economic contraction the returns of equities decrease. In response to economic downturn the monetary authorities act by decreasing the interest rates in order to make capital more accessible and help the economy. This in turn implies lower interest rates for bonds as well. And by extension supporting the positive relationship between the term structure and the BEER. Furthermore if the demand for bonds increases sufficiently, and the available amount of bonds is finite, the excess demand of bonds at a given rate will have to be offset by lowering the returns of bonds, subsequently making equities more attractive.

Finally the exchange rate between NOK and US Dollar (USD) is chosen as an explanatory variable. Although the connection between the currency price and the BEER might be spurious or time dependent, the economic reason might be due to international trade denominated in dollars. Since a firms value is in effect the value of its future cash flows, (which for oil related firm's is often denominated in USD) a higher NOK price of USD can be translated to a higher cash-flow. However there are some issues with the logic. Namely that firms who receive payments in dollars often apply some form of hedging strategy against fluctuation in NOK/Dollar currency prices. To amend the argument Børsum and Ødegaard (2005) provides the following definition for the value of firms with a hedging strategy:

Change in cash flows = Exposure × Change in exchange rate

Change in company's value = Exposure × Change in exchange rate

The equation states that the change in firm value positively depends on the exchange rate. The model states that the BEER is also positively dependent of the NOK/Dollar price. As such, the result fit comfortably with economic reasoning.

Although the approach of modeling the BEER series using the LSTR-specification has led to a steepness parameter with some undesirable qualities, it has produced information leading to a useful threshold model, with favorable econometric properties. Although these properties are well and good, there has to be a practical usage in order give a thorough evaluation of the BEER's ability to guide investment decisions. The next step will therefore be to simulate the profitability of using the BEER forecasts and the changes in the oil price as investment decision making tools.

# 7 Trading profitability

## 7.1 Post realization trading

In examining the trading profitability of the BEER there are two selected strategies. 1) Trading based on the defined critical value of the change in the oil price, where the strategy is to hold equities for changes in the oil price below 5.25% and sell equities and buy bonds for changes above the critical value. 2) Trading on the forecasted values BEER series (which is discussed later on). It is important to point out that this strategy is employed regardless of the realized values of the BEER. The reasoning is that only the oil price changes are used to dictate the states of the model. Therefore, if the dummy variable takes the value of 1 it is interpreted as a signal for overvalued equities. Furthermore the trading rules are based on *post hoc* realizations, meaning that the decisions are not made until the realizations of the oil price are observed at the end of the quarter. The performance of the trading rule is presented by the chart below:

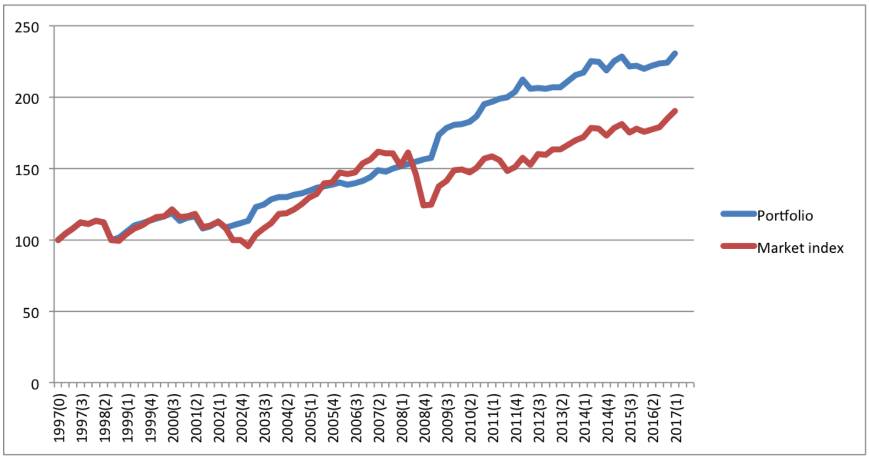



Figure 3: Oil price trading returns

The returns are based on a 100 Norwegian Kroner investment in a portfolio which follows the trading rule, and a portfolio replicating the market index. As can be seen from the chart, the trading rule does not unequivocally "outperform" the market index over the time period from 1997 to the end of 2016. This is shown particularly in the period from 2005 to 2007. However, from 2008 and throughout the rest of the sample, the accumulated return from the trading rule guided portfolio clearly surpasses the market index portfolio. The profitability may be largely attributed to the decision of moving into bonds at the beginning of the financial in 2008/2009.

To give more analytical perspective of the performance of the trading rule, the return and the associated risk (represented by the standard deviation) are reported. In addition, the Sharpe-ratio for periods of two years are also reported in order to measure the excess return to risk of the portfolio.

Table 6: post hoc performance

| | Trading rule constructed portfolio | Market index portfolio | risk free asset |
|---|---|---|---|
| Investment | 100 | 100 | 100 |
| Total return | 230.93 | 190.14 | 180.61 |
| Risk (std.dev) | 44.02 | 26.93 | |

Table 7: post hoc Sharpe

| Period | Sharpe Portfolio | Sharpe Market |
|---|---|---|
| 1997(1)-1998(4) | -1.03 | -1.42 |
| 1999(1)-2000(4) | 2.45 | 4.00 |
| 2001(1)-2002(4) | -2.30 | -5.16 |
| 2003(1)-2004(4) | 5.04 | 5.39 |
| 2005(1)-2006(4) | 0.33 | 7.48 |
| 2007(1)-2008(4) | 4.11 | -3.84 |
| 2009(1)-2010(4) | 6.00 | 6.02 |
| 2011(1)-2012(4) | -0.21 | -1.35 |
| 2013(1)-2014(4) | 0.07 | 1.31 |
| 2015(1)-2017(1) | -0.30 | 1.86 |

The table shows that the increased returns from the portfolio constructed using the critical values of the oil price changes, does carry added risk. The explanation for the added risk is largely related to the changes in the government bond yields. Although government bonds are largely considered to be a risk free investment, there is substantial risk to the value of the bond when selling the bond in the secondary market. As previously discussed, if the interest yield of the bond increases or decreases, the value of the bond changes with it. In this perspective government bonds are not expected, or in reality, held to the maturity. Thus adding to the risk of the portfolio. However, when the excess return of the portfolios are measured in terms of the Sharpe-ratio, the results in this case do show some mixed results. The excess return from the market index is both higher and more negative than the trading rule guided portfolio. The Sharpe-ratio developed by Sharpe (1964) is given as:

$$S_p = \frac{R_p - R_f}{\sigma_p} \tag{19}$$

Where $R_p$ and $\sigma_p$ is the return and standard deviation of the portfolio, $R_f$ is the risk free interest rate. In short, the ratio measures the excess return over the risk free rate per unit of volatility (measured by the standard deviation of the portfolio). The ratio therefore gives a real sense of the excess return of the portfolio when compared to the market index. It is important to give some comment to the measure of the risk free rate and the market index performance. A measure of the risk free rate is found in Bernt Arne Ødegaard's home pages(Ødegaard, 2017). The risk free rate is a proxy synthesized from government securities and the Norwegian Interbank Offered Rate (NIBOR). The average of these risk free rates is roughly 3%, which is what is used as a proxy for the risk free rate. Moreover, the table shows that the market index barely outperforms the risk free asset. This is largely due to the fact that the sample period includes three large economic downturns during the Asian financial crisis, the dot.com bubble, and the financial crisis. As a result, the market index portfolio struggles to regain the losses from these periods, subsequently giving lower Sharpe-ratios.

The results are quite interesting since the return from trading on oil price changes gives a substantially higher return than the market index. It is important to emphasize that the excess return is largely generated around the financial crisis, when the trading rule dictates to move into bonds rather than holding equities, thereby avoiding serious losses in the equity market. Furthermore, the portfolio is not adjusted for transaction costs, which will affect the results. The size of these cost are however debatable. Normally market indexes have low transaction costs since there is no active management of these portfolios, the transaction costs usually vary between 0.2 and 0.3 percent of the returns(Morning Star, 2017). In the estimation of the transaction cost adjusted portfolio the paper makes some assumptions: 1) By buying in to the market index the investor pays a fee of 0.2% of the returns. 2) The bid-ask-spread of selling bonds in the secondary market is equal to 0.2%. Under these assumptions the portfolio returns generated from following the trading rule are:

Table 8: Transaction cost adjusted post hoc performance

| | Trading rule constructed portfolio adjusted | Market index portfolio | risk free asset |
|---|---|---|---|
| Investment | 100 | 100 | 100 |
| Total return | 222.45 | 190,14 | 180,61 |
| Risk (std.dev) | 44.02 | 26.94 | |

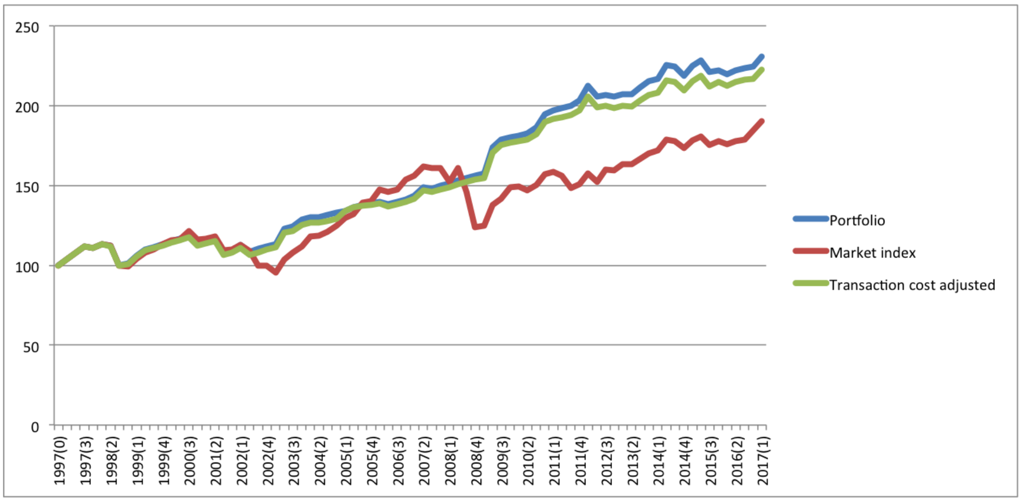


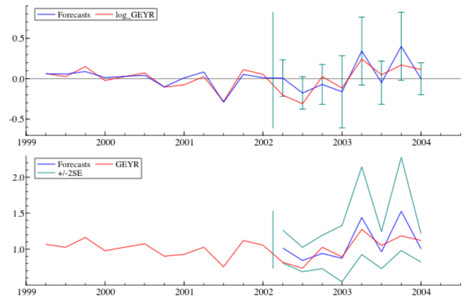

Figure 4: Oil price trading returns. Adjusted for transaction costs

As can be seen from the figure and the total returns, the transaction cost does not affect the overall return enough to give a lower return than the market index. As previously mentioned the transaction costs are quite low since the market portfolio does not require any active management, and replications of it is offered by several financial institutions. Notice that the transaction cost have a lower impact on the return of the portfolio in the beginning of the sample period. Again since the costs are so low, it takes several "switching points" between the market index and bonds for the costs to accumulate in such a manner that they affect the overall return. It is also important to add that the transaction costs for bonds is a general estimate of the bid-ask spread, which in turn may be subject to fluctuations. The general notion from the adjustment is however that the level of transaction costs would have to be substantially higher before having such an impact on the total return of the portfolio to give a lower return than the market index. Although the results give a good indication of the models ability to recognize profitable switching (or trading) points in the equity market, the empirical usage of the model is not fully explored. The next section will therefore consider the models ability to predict the future movements of BEER-series.

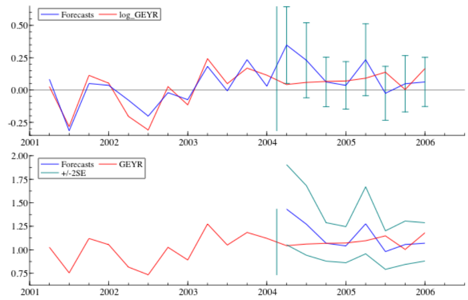
## 7.2 Forecasting performance

As previously mentioned, econometric models can provide important information that can be utilized for forecasting purposes. If the investor is provided with the information of the future movements of the BEER, the investor is allowed to make investment decisions at an earlier stage than in the previous section. Again, the final model from the encompassing test is used to make the forecasts. The strategy is to estimate the model up to the first quarter of 2002 and make an 8-step ahead dynamic forecast. The model is then re-estimated up to 2004, thus giving re-estimated coefficients, and repeating the procedure. In short terms the procedure can be called a rolling forecast procedure. By using a rolling forecast procedure, the relative importance of the explanatory variables and their respective information content is allowed to vary between periods in explaining the variations in the BEER. The model generates the following forecasts: (the explicit forecast values are presented in the appendix in table 18)

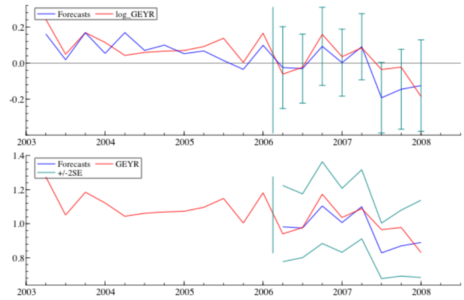
Figure 5: Forecasts



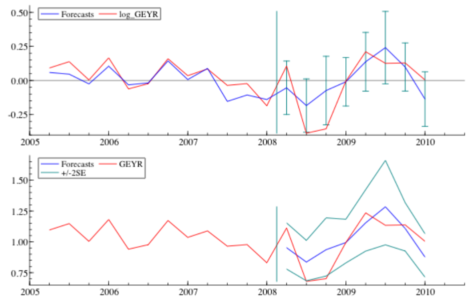

(a) [2002(1)-2004(1)]



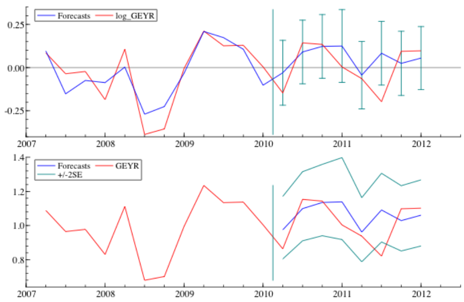

(b) [2004(1)-2006(1)]



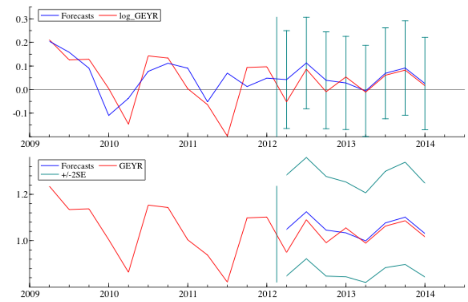

(c) [2006(1)-2008(1)]



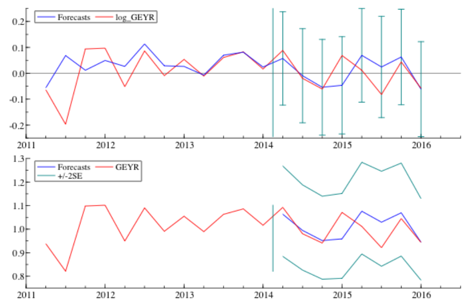

(d) [2008(1)-2010(1)]




(e) [2010(1)-2012(1)]




(f) [2012(1)-2014(1)]




(g) [2014(1)-2016(1)]


(h) [2016(1)-2016(3)]

The charts above do show some mixed results. In short terms, they show the estimation process up the vertical line indicating in the beginning the forecast period. The results are more accurate forecasts in periods where the BEER is more volatile, whereas the model struggles to provide more accurate forecasts in the more tranquil periods. The forecasts for the periods 2002(1)-2004(4) and 2006(1)-2008(4) appear to predict the directions and the timing quite well, giving support to the models predictive power. The returns generated from trading on the movements on the BEER gives the following returns and risk:

Table 9: Returns, using forecasts

| | Portfolio return | Portfolio return, adjusted | Market index return | Risk free return |
|---|---|---|---|---|
| Investment | 100 | 100 | 100 | 100 |
| Return | 208 | 193 | 163.5 | 151.26 |
| $\sigma$ | 30.9 | 26.8 | 21.07 | |

Table 10: Forecasts, Sharpe ratio

| Period | Portfolio Sharpe | Market index Sharpe |
|---|---|---|
| 2002(1)-2003(4) | 4.76 | -1.33 |
| 2004(1)-2005(4) | 4.52 | 9.90 |
| 2006(1)-2007(1) | 5.75 | 3.81 |
| 2008(1)-2009(4) | -0.01 | -0.05 |
| 2010(1)-2011(4) | -1.12 | -1.74 |
| 2012(1)-2013(4) | 6.11 | 2.64 |
| 2014(1)-2016(4) | 0.71 | -0.26 |

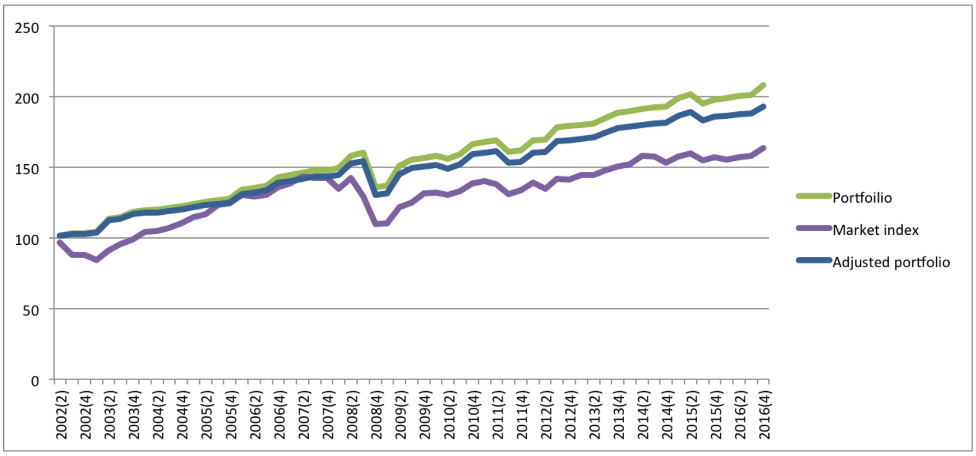



Figure 6: Forecast returns

Clearly the accumulated profits by trading on the forecasted values of BEER, gives superior total returns to the market index and the risk free rate. The results are quite similar to the ones obtained by trading on the oil price movements. In addition these results are generated over a shorter period. When compared to the strategy of trading on oil price movements it gives a lower total return (trading on oil price movements gives a return of 221 over the same period). However, the portfolio trading on the forecasted values of the BEER produces higher Sharpe-ratios than the market index in 6 out 7 periods. Whereas the portfolio trading on oil price changes generates a higher Sharpe-ratio in 4 out 10 periods. The positive returns give further support that the model has predictive power of the future movements in the BEER. As shown figure 6, the portfolio guided by the forecasted values of the BEER-series (green line) show that it generally produces higher accumulated returns than the market index (blue line) over the available period. Again it is important to emphasize that the turmoil from important economic events are relevant in explaining the low returns and Sharpe-ratios of both the market index and the portfolio guided by the forecasted BEER-series. The next procedure will be to formally examine the predictive power of future BEER movements by employing the Pesaran-Timmermann (PT) test.

## 7.3 Predictive power

As discussed in section 4.2.1, the modeling and evaluation the PT-test allows for a non-parametric procedure to evaluate the predictive power of the model. The following results are obtained from estimating the PT-test:

Table 11: PT-test

| | |
|---|---|
| n | 59 |
| P | 0.695 |
| $P_x$ | 0.5085 |
| $P_y$ | 0.4746 |
| $P_*$ | 0.4995 |
| V(P) | 0.00424 |
| $V(P_*)$ | 8.37785E-05 |
| Sn | 3.031 |
| tail-probability | 0.004 |

The test shows that the null hypothesis of the model having no predictive power over the future movements of the BEER is rejected at a 0.4% level, meaning that there is a 0.4% chance of falsely rejecting the null hypothesis. The results give a strong support to the argument that the returns generated from trading on the forecasted values of the BEER are not merely coincidental.

# 8 Strategy evaluation

The results in section 6 and 9 show that both strategies have qualities that produce higher levels of accumulated return than the benchmark index. The question is therefore which strategy is best from an investor point of view. The answer to that is that it depends. Although trading on the oil price changes produces higher levels of accumulated returns compared to the forecasting strategy, the associated risk is higher. As shown by the Sharpe-ratios in table 7 and 10, the number of periods where the trading-rule guided portfolios have higher Sharpe-ratios than the market index are more frequent for the portfolio guided by the forecasted value of the BEER. Assuming a risk averse investor, the optimal investment allocation would be to follow the portfolio guided by the forecasted values of the BEER. A more risk willing investor might however be more prone to favor the added return over the associated increase in risk, depending on the level of risk aversion. In conclusion, the answer to the optimal portfolio in effect comes down to the investors level of risk aversion.

# 9 The results and market efficiency

As discussed in section 3.1, the defined terms of Fama (1970) states that in a weak-form of efficiency, no market participant can earn excess returns by developing trading rules based on historical information. As such the results in the thesis stand in contradiction to the defined term. Although there will be given little effort to provide one definitive answer to solving these implications, it is important to give some reflections. First, since the terms of efficiency were published in 1970 there has been vast improvements in econometric modeling techniques, especially in non-linear modeling. These improvements have in turn led to both increased understanding of financial time-series, and the interaction of macroeconomic data. More important, the developments can also be argued to have led to further improvements in the accuracy of forecasting financial time-series data. Support to the predictability of equity returns are given by Cochrane (2000) who argues that the predicability of equity returns stems from the time variations in required return rates. Ferson and Harvey (1993) argue that economic variables, partly explaining equity returns, are in them selves predictable.

Support for market efficiency is given by Chan, Gup, and Pan (1997) who test for market efficiency by performing a co-integration test of stock prices in several equity markets, including the Norwegian equity market. In short terms, the co-integration test examines if two non-stationary time-series (integrated of order 1), have a linear combination that makes them stationary. If the linear combination of the two underlying series is stationary it is possible to construct an equilibrium correction model with predictive power, giving arbitrage opportunities. In summation, the aforementioned literature revealed the Norwegian market to have no long run co-movements thus, implying weak efficiency.

As previously mentioned the paper will make little effort to provide an absolute answer to the efficiency question. However, it will make an effort to provide an indication of how efficient the market is. The following results should only be taken as an indication: Using more historic methods there are more direct ways to test for market efficiency, like basing the test on a model for expected returns. On the basis that all market participants recognize patterns in either equity or bond market that can be used to give excess returns. If all the participants use these patterns for investment purposes, the investment behavior would become so predictable that the excess returns would eventually disappear. Using the method found in (Copeland, Weston, Shastri, et al., 2014, p. 368-369), describing theory of behavioral prices in the financial time-series is the *fair-game-model*:

$$\epsilon_{j,t+1} = \frac{P_{j,t+1} - P_{j,t}}{P_{j,t}} - \frac{E(P_{j,t+1}|\eta_t) - P_{j,t}}{P_{j,t}} = 0$$
$$\epsilon_{j,t+1} = \frac{P_{j,t+1} - E(P_{j,t+1}|\eta_t)}{P_{j,t}} = 0$$

- $P_{j,t+1}$ = the actual price of the asset j in the next period

- $E(P_{j,t+1}|\eta_t)$ = the predicted end-of-period price of the asset j given the current information structure $\eta_t$.

- $\epsilon_{t+1}$ = the error between actual and predicted returns.

By combining the fair game with the CAPM (Capita Asset Pricing Model) by Sharpe (1964), Treynor (1961), Mossin (1966), and Lintner (1965) the CAPM can be expressed as:

$$\epsilon_{j,t} = R_{j,t} - E(R_{j,t}|\hat{\beta}_{j,t})$$
$$E(R_{j,t}|\hat{\beta}_{j,t}) = R_{f,t} + \hat{\beta}_{j,t}[E(R_{m,t}|\hat{\beta}_{m,t}) - R_{f,t}]$$
$$E(\epsilon_{j,t}) = 0$$

- $E(R_{j,t}|\hat{\beta}_{j,t})$ = The expected return of the asset $j$ during time period $t$, given an estimate of the systematic risk $\hat{\beta}_{j,t}$

- $\hat{\beta}_{j,t} = \frac{COV(R_j,R_m)}{VAR(R_m)}$

- $R_{f,t}$ = The risk free rate in the time period t

- $E(R_{m,t}|\hat{\beta}_{m,t})$ = The excess market return given the estimated systematic risk $\hat{\beta}_{m,t}$

- $\hat{\beta}_{m,t} = \frac{COV(R_m,R_m)}{VAR(R_m)} = 1$

- $\hat{\beta}_{j,t}$ = The estimated systematic risk of the asset j based on the last period's information structure $\eta_{t-1}$

If the CAPM is true, and the market is efficient, then the expected return of every underlying asset should fall on the security market line (the graphical representation of the CAPM), and the error term $\epsilon_{j,t}$ will be equal to zero. Any discrepancies from zero are therefore interpreted as anomalies, and can be taken as evidence against market efficiency if (and only if) the CAPM is correct. Under the model the only relevant coefficient is the $\beta$ is the systematic risk of the underlying asset. The CAPM does however come with some strong assumptions:

- All investors are single period expected utility of wealth maximizers whose utility functions are based on the mean and variance of return.

- All investors can borrow or lend an infinite amount of at the risk-free rate. Additionally there are no restrictions on short-sales.

- All investor have homogenous expectations of the end-period joint distributions of returns.

- Security markets are frictionless and perfectly competitive.

Although the general CAPM has good qualities for measuring the expected returns of equities, there are some important issues with using the basic CAPM in this case. Since bonds and equities are traded in separate markets, and both portfolios switches the entire investment allocation between these markets, the conventional measure of the beta value for both portfolios is not appropriate. In order for the CAPM to be an appropriate measure for the expected returns, it has to account for the change in volatility between various shifts. The author has therefore chosen to specify the CAPM by distinguishing between the beta of the market portfolio, and the beta of government bond in relation to the equity market. The approach draws inspiration from (but is NOT based on) the basic CAPM above. To make the discussion more clear: Since government bonds are generally perceived as risk-free, holding them lowers the associated risk levels of the portfolio, relative to the market index. The beta value of the portfolio will therefore be lower in periods where it holds bonds, compared to periods where it holds the market index, and by extension lowering the (CAPM) expected return of the portfolio. Additionally, by including the beta value of the bond, the expected return of the bond, and the portfolio, is allowed to vary relative to the risk free rate between different periods. The reason for modeling the bond return this way is to capture the risk related to the variations in the bond interest rates. It is important to emphasize that this procedure might not capture the full level risk tied to changes in the bond interest rates. The beta of the portfolio holding bonds in relation to the market index is equal to:

$$\beta_b = \frac{COV(R_b, R_m)}{VAR(R_m)} \tag{20}$$

Over a two year period (with eight quarters) the expected return of the portfolio will therefore be equal to:

$$E(R_p) = \sum_{1}^{8} I(R_f + \beta_b \times (R_m - R_f)) + \sum_{1}^{8} (1 - I)(R_f + \beta_e(R_m - R_f)) \tag{21}$$

With

$$I = \begin{cases} 1 & for\ holding\ bonds \\ 0 & for\ holding\ equities \end{cases}$$

$\beta_p$, $\beta_b$, $\beta_e$ denotes the beta value of the portfolio, bond, and market index. $R_b$ $R_m$ and $R_f$ denotes the bond return, market return, and the risk free return.

Of previous work, estimated beta values of various bond classes[5] have been provided by Schaefer and Strebulaev (2009) who estimate the following bond betas:

Table 12: Bond beta

| Rating | A and above | BBB | BB | B | CCC |
|---|---|---|---|---|---|
| Average $\beta$ | <0.05 | 0.10 | 0.17 | 0.26 | 0.31 |

Since Norwegian government bonds have a very low risk of default, the associated rating is AAA.[6] By extension the expected beta values for bonds in this case is very low. This means that under the CAPM the expected return of the periods where the portfolios hold bonds will be close to the risk free rate.

Table 13: CAPM, based on oil price trading

| Period | $\beta$ Bond | Sum portfolio return | Sum CAPM adjusted | Error |
|---|---|---|---|---|
| 1997(1)-1998(4) | 0.0 | 0.02 | 0.02 | 0.00 |
| 1999(1)-2000(4) | -0.02 | 0.12 | 0.08 | 0.04 |
| 2001(1)-2002(4) | 0.00 | -0.01 | -0.04 | 0.03 |
| 2003(1)-2004(4) | -0.01 | 0.18 | 0.16 | 0.02 |
| 2005(1)-2006(4) | -0.01 | 0.06 | 0.05 | 0.01 |
| 2007(1)-2008(4) | 0.01 | 0.10 | 0.08 | 0.02 |
| 2009(1)-2010(4) | 0.01 | 0.23 | 0.22 | 0.01 |
| 2011(1)-2012(4) | -0.01 | 0.06 | 0.06 | 0.00 |
| 2013(1)-2014(4) | 0.03 | 0.06 | 0.06 | 0.00 |
| 2015(1)-2017(1) | 0.01 | 0.06 | 0.06 | -0.01 |
| $\sum$ Error = 0.12 | | | | |

Table 14: CAPM, based on forecasted values

| Period | $\beta$ bond | Sum portfolio return | Sum market return | Error |
|---|---|---|---|---|
| 2002(1)-2003(4) | -0.02 | 0.17 | 0.15 | 0.02 |
| 2004(1)-2005(4) | -0.01 | 0.08 | 0.06 | 0.01 |
| 2006(1)-2007(4) | -0.02 | 0.15 | 0.21 | -0.06 |
| 2008(1)-2009(4) | 0.00 | 0.08 | 0.07 | 0.01 |
| 2010(1)-2011(4) | 0.01 | 0.03 | 0.03 | 0.00 |
| 2012(1)-2013(4) | 0.01 | 0.16 | 0.16 | 0.00 |
| 2014(1)-2016(4) | 0.01 | 0.10 | 0.11 | -0.01 |
| $\sum$ Error -0.03 | | | | |

[5] Bonds are generally rated according to the risk of default. The highest class is AAA+.
[6] https://tradingeconomics.com/norway/rating

As can be seen from the tables above, the specified model generally performs well in predicting the returns of both portfolios. As a whole, the portfolio trading on the realized oil prices has higher realized returns than the CAPM expected returns. However, when excess returns are broken down, period-by-period, the sizes are not particularly large over the majority of periods. Consider that the model may not be entirely appropriate, my personal opinion is that there should be some small room for error between the expected and realized return, before taking the results as evidence against market efficiency. In the case of the portfolio trading on the forecasted values of the BEER the negative total error is largely due to the negative error (6 percent) in the period from 2006(1)-2007(4). Hence, the total level of error between the expected and realized returns should not be emphasized so heavily, before considering the period-by-period size of the errors.

With regard to the returns of the both trading strategies, it is important to recognize that the estimation is done over a relatively short period of time, so the true long run error between the CAPM and the realized returns are not fully captured. In what has become known as Roll's critique (Roll, 1977) the approach of testing market efficiency by using the CAPM approach is criticized for some potential shortcomings:

1. If there is a single testable hypothesis of the CAPM the market portfolio is mean-variance efficient.

2. Concerning the implications of the model, the most known being the linear relationship between expected return and the beta, follow from the market portfolio's efficiency and are therefore not independently testable.

3. In any sample of observations there will be an infinite number of observed (ex. post) mean-variance efficient portfolios using the sample period returns and covariances, as opposed to expected values of the same variables. If the resulting beta values are calculated against these portfolios they will satisfy the SML (security market line) relation exactly, autonomously of whether or not the market portfolio is mean-variance efficient.

4. The CAPM is not testable unless the exact composition of the true market portfolio is known, and used in the test. The implication of this is that all available assets must be included in the market portfolio

5. Using the OSEBX as a proxy for the market portfolio carries two difficulties: 1) The proxy can itself be mean-variance inefficient, even if the true market portfolio is not. Conversely the revers may cause the same problem. 2) Different market proxies can be highly correlated, and the usage of different proxies may lead to unequal results. The problem is known as the benchmark error.

Stressing that the results above should only be taken as an indication, and not a definitive result. Additionally, there are other ways to test for market efficiency, like basing the test for market efficiency on how the market reacts to new information (see Fama (1998)), which may yield different results. Since the general examination of market inefficiency is so limited, a general conclusion should not be made without considering the results of other tests for market efficiency. The general notion from the excess period-by-period returns over the model in (21), generated by both trading rule guided portfolios, is that they are interpreted as anomalies (represented by the error). These anomalies might over a longer timespan (or under a higher data frequency) sum to zero, supporting the argument against market inefficiency. Additionally, if the efficient market hypothesis is to be rejected, the alternative hypothesis will be a form of vague market inefficiency that will have to fulfill the daunting task of explaining the performance of both portfolios.

In light of this discussion, and the fact that the accumulated returns over the market index from both trading rule guided portfolios carry added risk. My personal view is that the results are not sufficient evidence to disregard the efficient market hypothesis. Especially in light of the 2 year period Sharpe-ratios in table 7 and 10, which show that the market index provides better excess risk returns over certain periods.

# 10 Conclusion

To the question of whether or not the BEER has properties that can be utilized for allocating investment, my immediate answer is yes. Furthermore, the paper finds that the uniqueness of the Norwegian economy allow using the oil price as an aid for basing investment decisions. This is to the extent that the trading on oil price realization provide a higher level of accumulated return. The excess return to risk of basing investment decisions on these realizations are however questionable. Although the portfolio trading on the realizations of oil prices produces higher accumulated returns, it does not provide higher excess return to risk measures over the majority of estimated periods (as shown in table 7). Basing investment decisions on the forecasted values of the BEER also provides a higher level of accumulated returns, and higher levels of excess risk to return, over the majority of periods. These results broadly agree with the existing literature such as: Brooks and Persand (2001), Clare et al. (1994), and Levin and Wright (1998), in terms of the predictive power and profitability of the BEER. The paper also finds that the specified linear model of the BEER has significant predictive power, proven by the Pesaran-Timmermann test.

The impression of the results are initially incompatible with the assumptions of weak-form efficiency by Fama (1998). Levin and Wright (1998) argue that the variations in the BEER might signal profitable investment opportunities as a result of equity mis-pricing, or changes in the equilibrium BEER value. If the argument holds, then the implications will be that the predictability of the BEER will be a statistical anomaly. In the papers investigation of the efficiency, the general indication is that the returns generated by both trading rules do not contradict the terms of efficiency. There are three main reasons for this: 1) By using both trading rules to guide investment decisions, the investor takes on the added risk of changes to the interest rate from bonds. 2) Both trading rule guided portfolios do not give consistently higher excess return to risk measures than the market index. 3) Using the modified CAPM in (29) to examine market efficiency, gives the overall indication that the market efficiency should not disregarded. It is important to emphasize that the results of the modified CAPM is only an indication, and not a definitive result.

In retrospect, there are several possible directions of empirical modeling, and strategy, approaches that have not been explored in this paper. One possible direction of further research could be to derive the actual probabilities, rather than the smoothened probabilities in the Markov switching model, for the future directions of the BEER value. These probabilities can in turn be used for deriving filtering rules. In addition other econometric models, like equilibrium correction models might be useful for capturing the features of the BEER-series.

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# A Appendices

## A.1 Unit root test

| GEYR: DF tests | | |
|---|---|---|
| D-lag | t-adf | $\beta Y_1$ |
| 3 | -3.315* | 0.39244 |
| 2 | -3.697** | 0.39005 |
| 1 | -5.097** | 0.28157 |
| 0 | -6.457** | 0.28290 |
| Norway_GDP_Growth_Mainland: DF tests | | |
| D-lag | t-adf | $\beta$ |
| 3 | -2.855 | 0.39142 |
| 2 | -3.521** | 0.29463 |
| 1 | -5.371** | 0.074751 |
| 0 | -9.151** | -0.073273 |
| dlog_OILPRICE: DF tests | | |
| D-lag | t-adf | $\beta$ |
| 3 | -4.405** | -0.013842 |
| 2 | -5.491** | -0.057313 |
| 1 | -6.299** | 0.022525 |
| 0 | -7.719** | 0.11057 |
| Dlog_World_Production_Crude_OIL: DF tests | | |
| D-lag | t-adf | $\beta$ |
| 3 | -4.554** | -0.0054890 |
| 2 | -4.988** | 0.052332 |
| 1 | -7.043** | -0.038910 |
| 0 | -7.429** | 0.14530 |

| D_log_cpi: DF tests | | |
|---|---|---|
| D-lag | t-adf | $\beta$ |
| 3 | -4.443** | -0.33945 |
| 2 | -5.099** | -0.27823 |
| 1 | -8.753** | -0.49368 |
| 0 | -9.977** | -0.14900 |
| Dlog _NWINTER3: DF tests | | |
| D-lag | t-adf | $\beta$ |
| 3 | -3.698** | 0.50051 |
| 2 | -4.494** | 0.46805 |
| 1 | -4.195** | 0.54979 |
| 0 | -4.274** | 0.59112 |
| Dlog_NOK_DOLLAR: DF tests | | |
| D-lag | t-adf | $\beta$ |
| 3 | -4.895** | -0.11436 |
| 2 | -5.143** | -0.0039595 |
| 1 | -5.995** | 0.056955 |
| 0 | -7.574** | 0.11275 |
| Dlog_Unemployment_rate: DF tests | | |
| D-lag | t-adf | $\beta$ |
| 3 | -2.789 | 0.33638 |
| 2 | -7.067** | -0.67156 |
| 1 | -7.878** | -0.40015 |
| 0 | -10.23** | -0.18107 |

| Dlog_NW_Oil_export: DF tests | | |
|---|---|---|
| D-lag | t-adf | $\beta$ |
| 3 | -4.253** | 0.23232 |
| 2 | -4.531** | 0.27899 |
| 1 | -5.393** | 0.27916 |
| 0 | -5.954** | 0.35325 |

## A.2 ADF(1) test

Term_structure: ADF tests

| | Coefficient | Std.Error | $t - value$ |
|---|---|---|---|
| Term_structure_1 | -0.13522 | 0.042484 | -3.1828 |
| Constant | 0.00076256 | 0.00032976 | 2.3125 |
| $\Delta$ Term_structure_1 | 0.50036 | 0.098768 | 5.0660 |
| Diagnostics: | | | |
| Critical values used in ADF test: | $5\% = -2.899$, $1\% = -3.515$ | | |
| ADF-Term_structure $= -3.183*$ | | | |

## A.3 LSTR-specification

Table 16: LSTR-specification

| | | | |
|---|---|---|---|
| Constant | 0.03449 | 0.01306 | 2.641 |
| Dlog(Unemployment_rate+_Unemployment_rate_1) | -0.2696 | 0.07629 | -3.535 |
| dlog_OILPRICE | 0.2796 | 0.05239 | 5.336 |
| Dlog_NOK_DOLLAR_3 | 0.4345 | 0.1875 | 2.317 |
| $\gamma$ | 210.6 | 16.41 | 12.83 |
| c | 0.05254 | .NaN | .NaN |
| Non-linear parameters: | | | |
| Constant | -0.1045 | 0.02413 | -4.33 |
| log_GEYR_3 | 0.4774 | 0.1324 | 3.605 |
| Dlog_Unemployment_rate_2 | -0.5565 | 0.1399 | -3.976 |
| Dlog_Unemployment_rate_1 | 0.731 | 0.2057 | 3.553 |
| Dlog_NWINTER3 | -0.9978 | 0.1455 | -6.859 |
| Term_structure_1 | 9.106 | 2.569 | 3.544 |
| Diagnostics: Std error of residuals | 0.0844629 | RSS | 0.456575 |
| $R^2$ | 0.64507 | AIC | -4.79895 |
| mean(log_GEYR) | 0.0072363 | se(log_GEYR) | 0.131647 |

## A.4 Encompassing test, further diagnostics

Table 17: Encompassing test, further diagnostics

| | |
|---|---|
| AR 1-5 test: | F(5,60) = 1.2580 [0.2939] |
| ARCH 1-4 test: | F(4,67) = 0.18951 [0.9431] |
| Normality test: | $Chi^2(2)$ = 3.5097 [0.1729] |
| Hetero test: | F(17,57) = 0.65081 [0.8356] |
| Hetero-X test: | F(41,33) = 0.71006 [0.8518] |
| RESET23 test: | F(2,63) = 2.2631 [0.1124] |

## A.5 Dynamic forecast results

Table 18: Dynamic forecast results

| Horizon | Forecast | $\sigma$ | Actual | Error | t-value | -2$\sigma$ | +2$\sigma$ |
|---|---|---|---|---|---|---|---|
| 2002(2) | 1.01436 | 0.1151 | 0.815693 | -0.19867 | -1.725 | 0.80378 | 1.2638 |
| 2002(3) | 0.842501 | 0.08505 | 0.73405 | -0.10845 | -1.275 | 0.68534 | 1.0253 |
| 2002(4) | 0.938937 | 0.1164 | 1.02633 | 0.087398 | 0.751 | 0.72788 | 1.1929 |
| 2003(1) | 0.871892 | 0.197 | 0.89132 | 0.019428 | 0.099 | 0.54435 | 1.3287 |
| 2003(2) | 1.44037 | 0.306 | 1.27469 | -0.16568 | -0.541 | 0.92547 | 2.1449 |
| 2003(3) | 0.961131 | 0.1292 | 1.05033 | 0.089198 | 0.691 | 0.72894 | 1.2448 |
| 2003(4) | 1.52772 | 0.3254 | 1.18449 | -0.34322 | -1.055 | 0.98051 | 2.277 |
| 2004(1) | 1.00418 | 0.09964 | 1.12153 | 0.11734 | 1.178 | 0.8198 | 1.218 |
| RMSE | 0.16816 | | | | | | |
| MAPE | 13.844 | | | | | | |
| 2004(2) | 1.43208 | 0.2136 | 1.04345 | -0.38863 | -1.82 | 1.0529 | 1.9055 |
| 2004(3) | 1.27225 | 0.1854 | 1.06093 | -0.21133 | -1.14 | 0.94218 | 1.6823 |
| 2004(4) | 1.0693 | 0.1024 | 1.06869 | -0.0006067 | -0.006 | 0.87924 | 1.2886 |
| 2005(1) | 1.041 | 0.09601 | 1.07237 | 0.031374 | 0.327 | 0.86233 | 1.2461 |
| 2005(2) | 1.27634 | 0.1784 | 1.09604 | -0.18029 | -1.01 | 0.95703 | 1.6695 |
| 2005(3) | 0.980498 | 0.1025 | 1.14785 | 0.16735 | 1.632 | 0.79159 | 1.2013 |
| 2005(4) | 1.05605 | 0.1154 | 1.00379 | -0.052257 | -0.453 | 0.84418 | 1.3055 |
| 2006(1) | 1.06956 | 0.1019 | 1.18076 | 0.1112 | 1.091 | 0.88035 | 1.2877 |
| RMSE | 0.18449 | | | | | | |
| MAPE | 13.225 | | | | | | |
| 2006(2) | 0.981467 | 0.1119 | 0.940143 | -0.041324 | -0.369 | 0.77683 | 1.2241 |
| 2006(3) | 0.974326 | 0.09367 | 0.977106 | 0.0027793 | 0.03 | 0.80056 | 1.175 |
| 2006(4) | 1.10399 | 0.1201 | 1.1729 | 0.068908 | 0.574 | 0.88343 | 1.3635 |
| 2007(1) | 1.00662 | 0.09397 | 1.03608 | 0.029464 | 0.314 | 0.8319 | 1.2075 |
| 2007(2) | 1.09963 | 0.1013 | 1.08847 | -0.011163 | -0.11 | 0.91103 | 1.3161 |
| 2007(3) | 0.828519 | 0.08148 | 0.964792 | 0.13627 | 1.672 | 0.67764 | 1.0033 |
| 2007(4) | 0.86981 | 0.09694 | 0.977609 | 0.1078 | 1.112 | 0.69221 | 1.0796 |
| 2008(1) | 0.889319 | 0.1136 | 0.830467 | -0.058851 | -0.518 | 0.68397 | 1.1377 |
| RMSE | 0.071686 | | | | | | |
| MAPE | 5.8328 | | | | | | |
| 2008(2) | 0.95237 | 0.09367 | 1.11185 | 0.15948 | 1.703 | 0.77892 | 1.1533 |
| 2008(3) | 0.836005 | 0.08201 | 0.679314 | -0.15669 | -1.911 | 0.68411 | 1.0119 |
| 2008(4) | 0.936372 | 0.1181 | 0.701314 | -0.23506 | -1.99 | 0.72255 | 1.1945 |
| 2009(1) | 0.994511 | 0.08857 | 0.99468 | 0.00016827 | 0.002 | 0.82927 | 1.1833 |
| 2009(2) | 1.15404 | 0.1248 | 1.23496 | 0.080921 | 0.648 | 0.9248 | 1.4234 |
| 2009(3) | 1.28411 | 0.1714 | 1.13409 | -0.15001 | -0.875 | 0.97571 | 1.6604 |
| 2009(4) | 1.10858 | 0.09807 | 1.13741 | 0.028824 | 0.294 | 0.92553 | 1.3175 |
| 2010(1) | 0.876789 | 0.08813 | 1.0043 | 0.12751 | 1.447 | 0.71389 | 1.0661 |
| RMSE | 0.13756 | | | | | | |
| MAPE | 13.244 | | | | | | |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 2010(2) | 0.974614 | 0.09165 | 0.863597 | -0.11102 | -1.211 | 0.8043 | 1.1706 |
| 2010(3) | 1.09905 | 0.1013 | 1.15354 | 0.05449 | 0.538 | 0.91046 | 1.3155 |
| 2010(4) | 1.13502 | 0.1046 | 1.14307 | 0.0080434 | 0.077 | 0.94027 | 1.3586 |
| 2011(1) | 1.13896 | 0.1202 | 1.0036 | -0.13537 | -1.126 | 0.91773 | 1.398 |
| 2011(2) | 0.961621 | 0.09407 | 0.937875 | -0.023746 | -0.252 | 0.78734 | 1.1633 |
| 2011(3) | 1.09115 | 0.1007 | 0.821126 | -0.27002 | -2.681 | 0.90374 | 1.3063 |
| 2011(4) | 1.02899 | 0.09569 | 1.09836 | 0.069373 | 0.725 | 0.85102 | 1.2335 |
| 2012(1) | 1.06059 | 0.09693 | 1.10168 | 0.04109 | 0.424 | 0.88009 | 1.2675 |
| RMSE | 0.1192 | | | | | | |
| MAPE | 9.6542 | | | | | | |
| 2012(2) | 1.04891 | 0.109 | 0.949539 | -0.099371 | -0.911 | 0.84791 | 1.2837 |
| 2012(3) | 1.12463 | 0.1094 | 1.09022 | -0.034419 | -0.315 | 0.9219 | 1.3591 |
| 2012(4) | 1.04562 | 0.1077 | 0.990949 | -0.054667 | -0.508 | 0.84696 | 1.2773 |
| 2013(1) | 1.03365 | 0.1024 | 1.05492 | 0.021261 | 0.208 | 0.84418 | 1.2534 |
| 2013(2) | 0.998971 | 0.09681 | 0.989396 | -0.009575 | -0.099 | 0.8195 | 1.2064 |
| 2013(3) | 1.07661 | 0.1041 | 1.06287 | -0.013739 | -0.132 | 0.88351 | 1.2997 |
| 2013(4) | 1.1014 | 0.1105 | 1.0861 | -0.015301 | -0.138 | 0.89712 | 1.3387 |
| 2014(1) | 1.03074 | 0.1013 | 1.01663 | -0.01411 | -0.139 | 0.84318 | 1.248 |
| RMSE | 0.043608 | | | | | | |
| MAPE | 3.2764 | | | | | | |
| 2014(2) | 1,06327 0 | 0.09598 | 1.09236 | 0.029089 | 0.303 | 0.88436 | 1.268 |
| 2014(3) | 0.994685 | 0.09056 | 0.980582 | -0.014103 | -0.156 | 0.82599 | 1.188 |
| 2014(4) | 0.951402 | 0.08815 | 0.940879 | -0.010523 | -0.119 | 0.78741 | 1.1398 |
| 2015(1) | 0.958781 | 0.09019 | 1.07109 | 0.11231 | 1.245 | 0.79119 | 1.1517 |
| 2015(2) | 1.07609 | 0.09743 | 1.01187 | -0.064222 | -0.659 | 0.89453 | 1.284 |
| 2015(3) | 1.0295 | 0.101 | 0.921278 | -0.10822 | -1.072 | 0.84249 | 1.246 |
| 2015(4) | 1.0694 | 0.09885 | 1.04489 | -0.024516 | -0.248 | 0.88548 | 1.2806 |
| 2016(1) | 0.944529 | 0.08671 | 0.94363 | -0.0008984 | -0.01 | 0.78312 | 1.1297 |
| RMSE | 0.061448 | | | | | | |
| MAPE | 4.53 | | | | | | |
| 2016(2) | 1.17801 | 0.1049 | 1.04012 | -0.13789 | -1.314 | 0.98227 | 1.4016 |
| 2016(3) | 0.843113 | 0.07633 | 1.03005 | 0.18693 | 2.449 | 0.70087 | 1.006 |
| 2016(4) | 1.09539 | 0.09735 | 1.09389 | -0.0015002 | -0.015 | 0.91373 | 1.3029 |
| RMSE | 0.13412 | | | | | | |
| MAPE | 10.514 | | | | | | |

# B Descriptive statistics

## B.1 Mean, Standard deviation, max, min

Table 19: Descriptive statistics

| Variable name | Observations | Mean | Std.Devn. | Minimum | Maximum |
|---|---|---|---|---|---|
| log BEER | 78 | 0.0086204 | 0.12927 | -0.41856 | 0.24270 |
| Norway GDP Growth Mainland | 78 | 0.66026 | 0.96108 | -2.3000 | 4.3000 |
| OILPRICE | 78 | 58.978 | 34.710 | 9.8700 | 128.32 |
| Unemployment rate | 78 | 0.036179 | 0.0067401 | 0.021000 | 0.049000 |
| Dlog NW Oil export | 78 | 0.0044721 | 0.12603 | -0.36712 | 0.32052 |
| Term Structure | 78 | 0.0056389 | 0.0057103 | -0.0072000 | 0.016133 |
| NOK DOLLAR | 78 | 6.9275 | 1.1140 | 5.0800 | 9.3001 |
| NWINTER3 | 78 | 0.038076 | 0.021391 | 0.0099000 | 0.080067 |
| Dlog World Production Crude OIL | 78 | 0.0026247 | 0.010368 | -0.030688 | 0.029879 |
| Dlog cpi | 78 | 0.0051553 | 0.0066312 | -0.016099 | 0.026761 |

## B.2 Residual density, final model

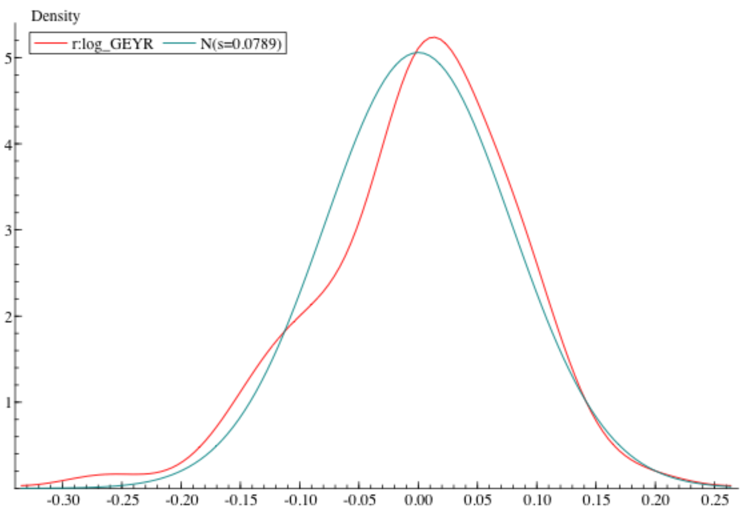


Figure 7: Residual density, final model

## B.3 Correlation matrix

Table 20: Correlation matrix

| | log GEYR | NW GDP Growth | OILPRICE | Unemp. | Dlog NW Oil_exp | Term_struc | NOK/$ | NWINTER3 | Dlog WP_C_OIL | log_CPI |
|---|---|---|---|---|---|---|---|---|---|---|
| log GEYR | 1.0000 | 0.25069 | 0.027490 | 0.18484 | 0.26450 | 0.37135 | -0.091568 | -0.35199 | 0.093771 | -0.050726 |
| NW GDP Growth | 0.25069 | 1.0000 | -0.013182 | 0.12143 | 0.17873 | 0.11824 | -0.10968 | -0.094193 | -0.037362 | 0.010717 |
| OILPRICE | 0.027490 | -0.013182 | 1.0000 | -0.32455 | 0.063343 | 0.25634 | -0.75562 | -0.46710 | 0.023052 | -0.0015934 |
| Unemployment rate | 0.18484 | 0.12143 | -0.32455 | 1.0000 | 0.040759 | 0.43441 | 0.36241 | -0.43138 | 0.033641 | -0.0081297 |
| Dlog NW Oil export | 0.26450 | 0.17873 | 0.063343 | 0.040759 | 1.0000 | -0.041771 | -0.10165 | 0.052342 | 0.15566 | 0.12111 |
| Term structure | 0.37135 | 0.11824 | 0.25634 | 0.43441 | -0.041771 | 1.0000 | -0.37408 | -0.83480 | 0.15291 | -0.11902 |
| NOK DOLLAR | -0.091568 | -0.10968 | -0.75562 | 0.36241 | -0.10165 | -0.37408 | 1.0000 | 0.38747 | -0.067977 | 0.080601 |
| NWINTER3 | -0.35199 | -0.094193 | -0.46710 | -0.43138 | 0.052342 | -0.83480 | 0.38747 | 1.0000 | -0.12586 | 0.087137 |
| Dlog WP C_OIL | 0.093771 | -0.037362 | 0.023052 | 0.033641 | 0.15566 | 0.15291 | -0.067977 | -0.12586 | 1.0000 | 0.081439 |
| D log cpi | -0.050726 | 0.010717 | -0.0015934 | -0.0081297 | 0.12111 | -0.11902 | 0.080601 | 0.087137 | 0.081439 | 1.0000 |