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An Experimental Study on the Wake Development Behind a Yawed Model Wind Turbine

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NTNU
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MASTER THESIS

for

Student Mari Grønning Vatn

Spring 2017

Wind turbine wakes in yawed condition

Clustering wind turbines as a wind farm to share the infrastructure is an effective strategy to reduce the cost of energy. This leads to aerodynamic interaction among the turbines. As a turbine extracts energy from the wind, a speed deficit and increased turbulence occur in it's wake. Downstream wind turbines located in such a wake produce less power and experience altered structural loading. This couples the power production and loading of upstream and downstream turbines. The significance of such coupling depends on the topology of the wind farm, wind direction and wake recovery time. Since traditional control strategies aim to maximize the power production of the individual wind turbines, the total wind farm power production is suboptimal. Control strategies that take into account the wake effects are capable of mitigating this problem.

A recent approach is to redirect the wake of upstream turbines away from downstream turbines using yaw-misalignment. As the literature shows, the yaw-misalignment has the potential to increase the power output of a wind farm. While the loads of upstream turbines may benefit from yaw-misalignment, downstream turbines can potentially experience increased loads due to partial wake overlap. To decrease the cost of wind energy, it is important to increase the power production without significantly reducing the lifetime of a wind turbine. Therefore, any application of control algorithms should not negatively impact the wind turbine loads.

In this study we will perform experiments on model wind turbines in yawed condition in the NTNU wind tunnel and evaluate/design analytical methods for the gross characteristics of the wakes as function of downwind distance.

A useful entrance to relevant literature:
"Wind farm multi-objective wake redirection for optimizing power production and loads", Mike T. van Dijk, Jan-Willem van Wingerden, Turaj Ashuri, Yaoyu Li; *Energy* 121 (2017) 561-569

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When the thesis is evaluated, emphasis is put on processing of the results, and that they are presented in tabular and/or graphic form in a clear manner, and that they are analyzed carefully.

The thesis should be formulated as a research report with summary both in English and Norwegian, conclusion, literature references, table of contents etc. During the preparation of the text, the candidate should make an effort to produce a well-structured and easily readable report. In order to ease the evaluation of the thesis, it is important that the cross-references are correct. In the making of the report, strong emphasis should be placed on both a thorough discussion of the results and an orderly presentation.

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The final report is to be submitted digitally in DAIM. An executive summary of the thesis including title, student's name, supervisor's name, year, department name, and NTNU's logo and name, shall be submitted to the department as a separate pdf file. Based on an agreement with the supervisor, the final report and other material and documents may be given to the supervisor in digital format.

X Work to be done in lab (Water power lab, Fluids engineering lab, Thermal engineering lab)
☐ Field work

Department of Energy and Process Engineering, 27 February 2017

Lars Sætran

Lars Sætran
Academic Supervisor

Research Advisor: Franz Mühle

Sammendrag

I dag er vindturbiner vanligvis optimalisert på individuell basis, og hele vindparken er ikke tatt i betrakting i sin helhet. Ved å rotere en oppstrøms turbin, slik av vaken med vilje avbøyes til siden, kan den totale mengen energi som produseres i vindparken økes.

I denne studien har eksperimenter i vindtunnelen ved NTNU blitt utført. Vaken bak en vindturbin med en rotordiameter på $0.45m$ har blitt undersøkt, både når vinden treffer normalt på turbinen og med turbinen rotert med en vinkel på $30°$. En sammenligning av utviklingen av de to vakene har blitt utført, så vel som en kvantitativ analyse av den avbøyde vaken.

Målingene viser at vaken bak turbinen når vinden treffer normalt på rotoren, får en sirkulær form. Vaken bak den roterte turbinen får en heller c-lignende form. Resultatene viser at den minimale hastigheten som oppstår i vaken, har en lik utvikling med henyn til avstand nedstrøms, i tillegg til at verdien er relativt lik.

Tre metoder som anslår banen som vaken følger blir brukt med målingene fra vaken for den roterte turbinen. Resultatene viser at grad av avbøyning av vaken er høyst avhengig av metoden som velges.

Til slutt ble en analytisk modell implementert og sammenlignet med resultatene som ble gitt av metodene. Modellen ser ut til å anslå en større avbøyning en den metoden som ga mest lignende resultater som tidligere studier har påvist.

Summary

Nowadays, wind turbines are usually optimized on an individual basis, and the entire wind farm is not considered as a whole. By yawing the upstream turbine, such that the wake is intentionally deflected away from the downstream turbine, the total amount of energy produced in the wind farm could be increased.

In this study, experiments in the wind tunnel at the Norwegian University of Science and Technology have been conducted. The wakes behind a model wind turbine with a rotor diameter of $0.45m$ have been investigated in both non-yawed condition and with a yaw angle of $30°$. A comparison of the development of the two wakes is performed, as well as quantification of the deflected wake.

It is shown that the wake behind the turbine in a non-yaw condition adopts a circular shape, while the wake behind the turbine in yawed condition has a more curled shape. The development of the velocity deficit of the two wakes is very similar.

Three methods that estimates the wake center trajectory are applied to the measurements obtained from the experiment with the turbine in yaw. The results show that the quantification of the wake deflection is strongly dependent of the method used.

At last, an analytical model was implemented and compared with the methods. The model seems to over predict the deflection compared to one of the models that gives the most similar results to prior studies.

An Experimental Study on the Wake Development Behind a Yawed Model Wind Turbine

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July 9, 2017

Abstract. Nowadays, wind turbines are usually optimized on an individual basis, and not considering the entire wind farm as a whole. By yawing the upstream turbine, such that the wake is intentionally deflected away from the downstream turbine, the total amount of energy produced from the wind farm could be increased. Experiments with a model wind turbine in both non-yawed and yawed condition were performed in the wind tunnel at the Norwegian University of Science and Technology, in order to compare the development of the two wakes as well as to describe the deflected wake. The results show that the minimum velocity development of the yawed wake is very similar to the non-yawed wake. Additionally, the wake behind the turbine in yaw, is deflected up to 0.67 D at 15 D. However, the quantification of the wake deflection is strongly dependent on the method used. The analytical model of predicting the deflection of the wake coincide well with one of the methods, while over predicts the trajectory compared to the other method.

1. Introduction

Over the past decades, there has been an increase in the demand of energy, and especially of renewable energy. Consequently, it is necessary to improve the efficiency of existing and future wind farms. Wind turbines are usually placed in clusters due to limited space and infrastructure, and most turbines are interfering with the wakes from upstream turbines. Therefore, it is necessary to understand the development of wakes. In the last years, it has been suggested that an intentional deflection of the wake by yawing the upstream turbine, could reduce the interference between the single turbines in a wind farm. Several studies have been conducted within this topic, but it is not yet fully understood.

A wake is normally divided into two sections; the near and the far wake [1]. Previous studies have concluded that the far wake typically begins at approximately $4-5$ D downstream [2]–[4], and as the downstream spacing of $6-10$ rotors diameters between each turbine is common [1], the far wake is of most interest in this work. However, the whole wake is considered in order to investigate its development. It is commonly known that, in a wake, the velocity is reduced and the turbulence intensity is increased. As the wake develops downstream, it will gradually recover and eventually reach the same characteristics as the free stream. During this recovering process, a mixing between the air outside and inside of the wake creates a shear layer, and the wake will expand due to momentum being transferred into the wake [1].

Previous studies have shown that when the incoming wind is normal to the turbine, the wake will get an approximate circular shape and be axisymmetric [5]. Additionally, the wake

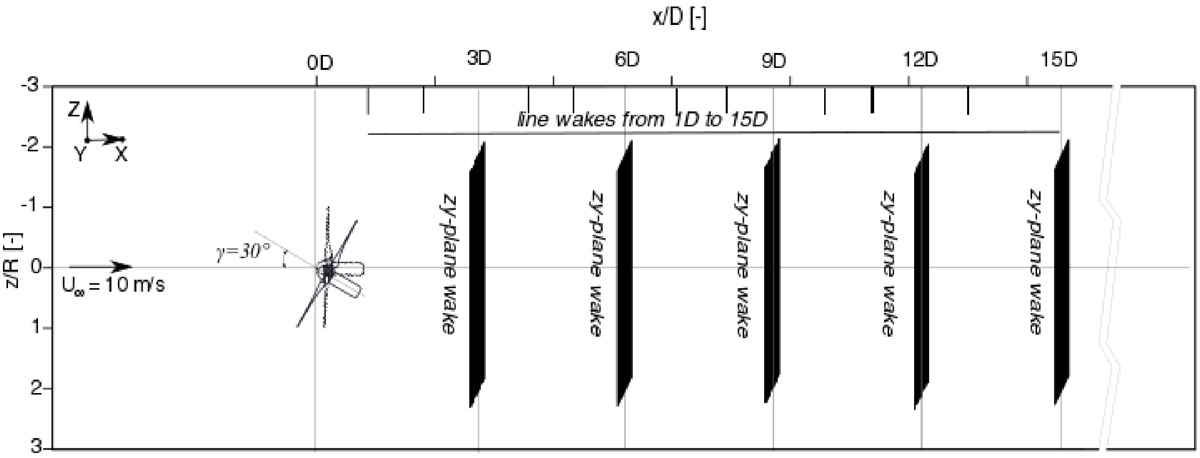

Figure 1: A simple sketch of the composition of the experiments performed in the wind tunnel. The yaw angle is defined positive in the clockwise direction.

trajectory will appear as an approximately straight line in the streamwise direction [6]. On the contrary, a wake behind a yawed turbine, will skew to one side and is investigated in prior work, e.g [7]–[9]. It is also shown that the wake no longer has an approximate symmetric shape, but rather adopts a more curled shape, thus becomes asymmetric [10]. The lateral deflection of the wake behind a yawed turbine is of importance in order to estimate the wake center trajectory, and to what extent the wake will interfere with downstream turbines. It exists several methods that calculate the trajectory based on measurements, as well as some analytical models. Schottler et al. [8] used a method that estimates the wake center by calculating the power of a potential downstream turbine. They found, for a rotor with a diameter of $0.90m$ in turbulent inflow conditions and a yaw angle of $-30°$, that the wake was deflected $0.19D$ at 3 x/D and $0.32D$ at 6 x/D. Howland et al. [11] used a porous disk with a diameter of $0.03m$ in uniform inflow conditions, and concluded that the wake deflection was $0.45 - 0.6D$ at $x/D = 8$ when the disk was yawed with an angle of 30°. The variance in the deflection of the latter is due uncertainties and that they measured with both a Pitot probe and a hot-wire probe.

In addition to finding the wake trajectory, the width of the wake also impacts how much the wake from an upstream turbine will affect the downstream turbine. Methods of estimating the wake width have been developed, such as the 'half-width', which is where the velocity deficit is half of the maximum velocity deficit [10].

In the present work, wind tunnel experiments have been conducted. The wake of a horizontal axis model wind turbine in uniform flow is investigated. Measurements with the turbine in a non-yawed condition, as well as in a yaw condition of 30° are performed. Further, the measurements are evaluated and compared, and existing methods for finding the center wake trajectory are applied. These results are compared with an existing analytical model. Additionally, the wake width of the two measured cases are compared and evaluated.

2. Experimental Setup

The experiments in this study were conducted in the closed-loop wind tunnel at the Department of Energy and Process Engineering at the Norwegian University of technology and Science (NTNU). A presentation of the setup of the experiment is shown in figure 1.

The cross-sectional area (zy-plane) of the wind tunnel has the dimensions 2.7 m × 1.8 m and a length (x-direction) of 11 m. The airflow in the wind tunnel has uniform flow conditions with a zero pressure gradient in the streamwise direction. The turbulence intensity (TI) of the empty tunnel was measured to 0.23%, consequently, the experiments were performed in a low turbulent flow. The model wind turbine, with a rotor diameter (D) of 0.45 m, was located 2.71

D downstream from the inlet of the tunnel. In order to locate the turbine at the center of the cross-section, the hub height was adjusted to 1.98 D (0.89 m). The inlet velocity (U_∞) was kept constant at 10 m/s throughout the experiment, and this was ensured by measuring the pressure difference at the contracted duct at the inlet of the wind tunnel for every measurement. A Laser Doppler Velocimetry (LDV) probe was used to measure the velocities in the x and y direction of the flow. The probe was mounted onto a traverse in order to move the probe to measure at different locations in the wind tunnel. Small particles with a size of 0.5-2.0 μm were generated by a smoke machine in order for the LDV probe to detect the airflow. The smoke machine was located far back in the wind tunnel such that there was no interference with the measurements as well as making sure the particles were evenly distributed in the closed-loop wind tunnel. For each measurement, 50 000 samples were collected by the LDV probe.

2.1. Model turbine

The rotor diameter of the horizontal axis model turbine is 0.45 m. With this diameter, the wind tunnel blockage ratio is only 3.3%, which is considerable smaller than the suggested maximum limit of 10%. This higher limit is suggested to avoid interference between the measurements and the walls of the wind tunnel [12]. Therefore, it is assumed that the wake measured is not affected by the wind tunnel walls. In the experiments conducted, the tip speed ratio (TSR) was obtained at $TSR = 3.5$ throughout all measurements, as this TSR gives maximum coefficient of power, and consequently giving the optimum TSR. Figure 2 presents curves of the coefficient of power (C_P) and the coefficient of thrust (C_T) of the turbine, respectively, with respect to TSR. The C_P and C_T at the optimal TSR is marked by an asterisk (*). As the experiments were performed with an inlet velocity (U_∞) of 10 m/s at optimal TSR, the Reynolds number at the tip resulted in $Re_{tip} = 60000$. The rather low Re_{tip} is assumed not to affect the measurements too much, due to low dependence of the Reynolds number in the far wake. A more detailed description of the turbine is found in [5].

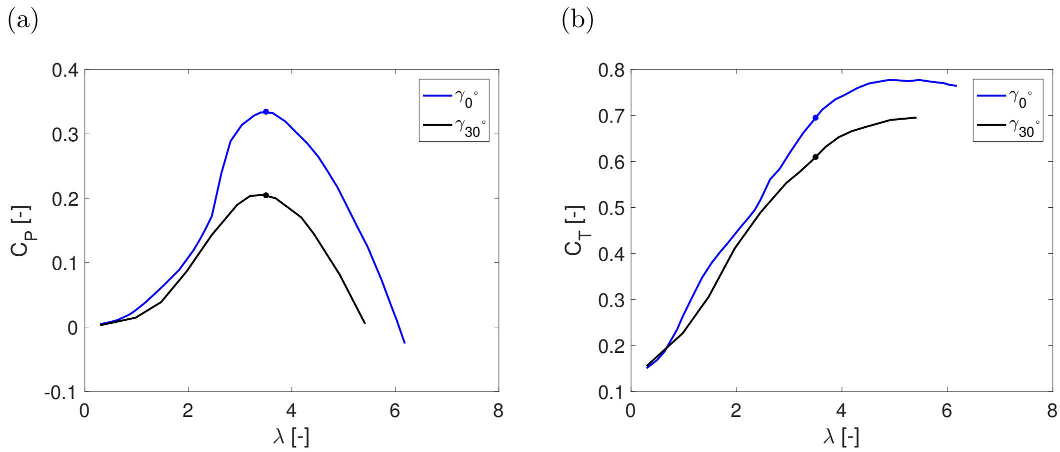

Figure 2: The turbine performance is presented by the curves of (a) the coefficient of power, C_P, and (b) the coefficient of thrust, C_T, both with respect to TSR. The optimal TSR ($\lambda = 3.5$), which was used in the experiments, is indicated on the curve by an asterisk (*).

2.2. Measurements
The wake measurements behind the turbine were performed with the turbine at two different yaw angles; 0° (non-yaw) and 30°. The yaw angle (γ) is defined as positive in the clockwise direction. Line measurements were performed to get an overview of the streamwise development of the wake, while cross sections of the wake were measured to further analyze the wake in detail. The grid size, downstream distances and number of wakes measured are not equal for the two yaw cases, and a description of each situation is presented below:

2.2.1. Non-Yawed Three full cross-sectional (zy-plane) wake measurements were conducted at different distances downstream of the turbine; at 6 D, 12 D and 18 D. The grid of the measured wakes ranged from -1.33 D to 1.33 D in z-direction, and from -1.07 D to 1.07 D in y-direction, locating the origin at the hub. An increment of 0.13 D between each step in both direction was chosen. Additional line measurements were measured at distances ranging from 1 D to 18 D, including every distance D in between. These measurements were measured over the same distance in z-direction, -1.33 D to 1.33 D, as well as obtaining the same increment size of 0.13 D. The vertical position of the line measurements was chosen to be at $y = -0.22D$, as a downshift of the wake center was observed in the cross sectional wake measurement.

2.2.2. Yaw angle of 30° Five full cross-sectional wake measurements were measured at different downstream distances behind the turbine; 3 D, 6 D, 9 D, 12 D and 15 D. Measuring two more wakes compared with the non-yawed measurements was chosen due to the assumption of a more asymmetric wake. Letting the hub be the origin, the grid measured was ranging from -1.2 D to 1 D in z-direction and from -0.8 D to 0.8 D in y-direction, giving 391 measured points in total. The increment size was set to 0.1 D in both directions. A finer grid compared to the non-yawed measurements was chosen as the wake was expected to be more asymmetric.Additionally, line measurements in z-direction at hub height ($y = 0$) were measured at every distance D downstream of the turbine, ranging from 1 D to 15 D in x-direction. The increment size was obtained the same as for the cross sectional wake measurements, 0.1 D. However, the range of the z-direction was changed to -1.5 D to 1 D, in order to ensure that the deflected wake was captured.

3. Assessment of Wake Deflection

3.1. Estimation of Wake Deflection from Experimental Data
The center of a wake can be described as the location of where the largest velocity deficit is found in the wake at a certain distance downstream. The wake center trajectory is described as the path of the wake center downstream of the turbine. For a turbine in non-yawed condition, the wake has an approximately circular and symmetric shape, and the wake center trajectory will follow a relatively straight path in the streamwise direction [11]. However, when a turbine is in yawed condition, the wake will develop an asymmetric shape and it will skew sideways due to transverse pressure gradients [7]. In order to predict how the wake develops, it is necessary to have methods and models that describe the wake development quite well. Prior work have suggested several methods [8], [11] and analytical models [9], [10] that estimate the wake center trajectory. Below, three methods for finding the wake center are presented. These are used to identify the wake center trajectory of the wake measured in the conducted experiment when the turbine is yawed.

3.1.1. U_{min} Method As the wake center is commonly described as the location of where the highest velocity deficit is found, a simple method based on this definition is used, and is in this paper referred to as the *U_{min} Method*. The method expresses the location of the where the lowest velocity is found by evaluating the measured velocities in z-direction at hub height at various location downstream. One single point is obtained for every distance D downstream of the turbine, and by assessing these points, the wake center trajectory of the wake is found. The U_{min} *Method* is expressed as:

$$u_{min}(z, y) = min(u(z, y)) \tag{1}$$

where u is the velocity vector at certain distance downstream, z is the position in the lateral direction, and $y = 0$ (hub height).

3.1.2. Available Power Method Another method that can be used to estimate the wake center trajectory is a method which is in this article referred to as the *Available Power Method*. As it is of interest to evaluate the impact the wake of an upstream turbine has on a downstream turbine, this method estimates the wake center trajectory by placing an imaginary turbine in the cross-section of the wake measured, at various distances downstream. The imaginary turbine is divided into ten circular segments such that the sum of them equals the front area of a circle with a diameter of $0.45m$. In each segment, the velocities are averaged. By calculating the potential power extracted by an imaginary turbine in the wake at different locations in the lateral direction at a certain distance downstream, the wake center is defined as the position of the imaginary turbine of which the turbine extracts the least amount of power. This is due to that the extracted power by a wind turbine is proportional to the incoming wind speed cubed, and consequently, the location of the least power also implies the lowest velocity.

Similar approaches have been used in prior work [13], [14], and the exact same approach was used in [8], where also a more detailed description of the method can be found.

The expression used in the *Available Power Method* is:

$$P = \sum_{i=1}^{10} \rho A_i \langle u_i(t) \rangle^3_{A_i,t} \tag{2}$$

where P is the potential power at a certain z-location of the imaginary turbine, i is the number of segments the area is divided into, A_i is the area of the $i'th$ segment, and $\langle u_i(t) \rangle^3_{A_i,t}$ is the averaged velocity in the $i'th$ segment quadric.

3.1.3. Gaussian Fit Method A third method is to estimate the wake center trajectory by using a Gaussian fit on the measurements, which in this paper will be referred to as the *Gaussian Fit Method*. At a specified downstream distance x, the cross section of the measured wake is evaluated by fitting the velocities measured along the z-direction of the grid for all y-values. Further, the minimum velocity found along the Gaussian fitted curve for each y-value, are then fitted into a new curve, and thus, by finding the minimum of this second curve, the location of the wake center is found. Applying this method to several cross sectional measured wakes, the wake center trajectory is estimated. Differing from the two previous methods, the *Gaussian Fit Method* is not limited to $y = 0$.

3.2. Prediction Models for Wake Deflection

To predict the wake deflection, analytical models are great tools. There exist some analytical models that have been developed to compute the skew angle of the wake behind a turbine in yawed condition, and furthermore making it possible to calculate the wake center trajectory. However, the number of models is limited. In this article, one model is implemented and evaluated based on how well it fits with the methods that are applied to the measurements obtained from the experiments. The model investigated is developed by Jiménez et al. [9] and is based on the forces that are exerted by the turbine on the air flow. By simplifying the expressions of the forces, the model developed has become a simple analytical model that only requires two input parameters; C_T and the yaw angle (γ) of the turbine. It is assumed that the wake cross-section increased linearly with downstream distance x. The model calculates the skew angle of the wake at a certain distance downstream. By calculating the skew angle for several distances downstream, the wake center trajectory can be found. The skew angle is expressed as [9]:

$$\alpha = \frac{\cos^2\gamma \sin\gamma \frac{C_T}{2}}{\left(1 + \beta \frac{x}{D}\right)^2} \tag{3}$$

where α is the skew angle, γ is the yaw angle, C_T is the coefficient of thrust, β is a constant, x is the downstream distance, and D is the diameter of the turbine. β represents the wake's growth rate and is in this experiment ($\gamma = 30°$) set to 0.125 [9].

4. Results

The results obtained from the experiments in the wind tunnel and the implementation of the methods estimating the wake center trajectory are described in this section.

4.1. The Shape of the Wake

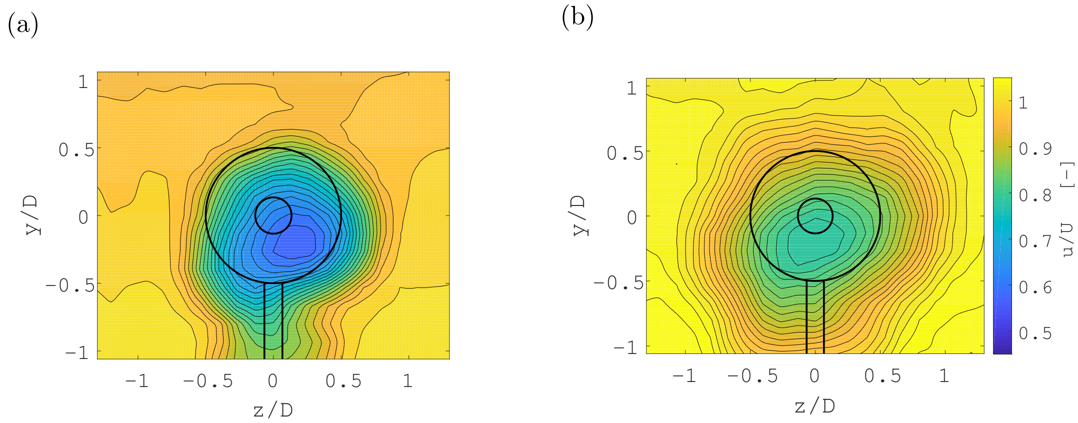

Figure 3: Normalized velocity distribution of the cross section of the wake in non-yawed condition, measured at distances (a) 6D and (b) 12 D downstream of the turbine. The black solid lines illustrate the outline of the non-yawed turbine.

4.1.1. 0° Yaw Figure 3 presents the normalized velocity distribution of the cross-section of the wake, at distances 6 D and 12 D downstream the turbine in non-yawed condition. The shape of the wakes are approximately circular, which was expected as prior studies have shown such shape for turbines with no yaw [1], [6]. A downshift of the center of the wake can be observed and this is assumable due to interference with the tower [15]. The measured cross-section at 18 D is not considered due to a possible interference with the traverse in the wind tunnel and this is described in more detail in [5].

4.1.2. 30° Yaw Figure 4 presents the normalized velocity profile, seen from above, in the streamwise direction of the wake with a yaw angle of 30°. The lateral deflection can be seen clearly. Additionally, it can be observed that the velocity deficit gradually decays with increasing distance downstream.

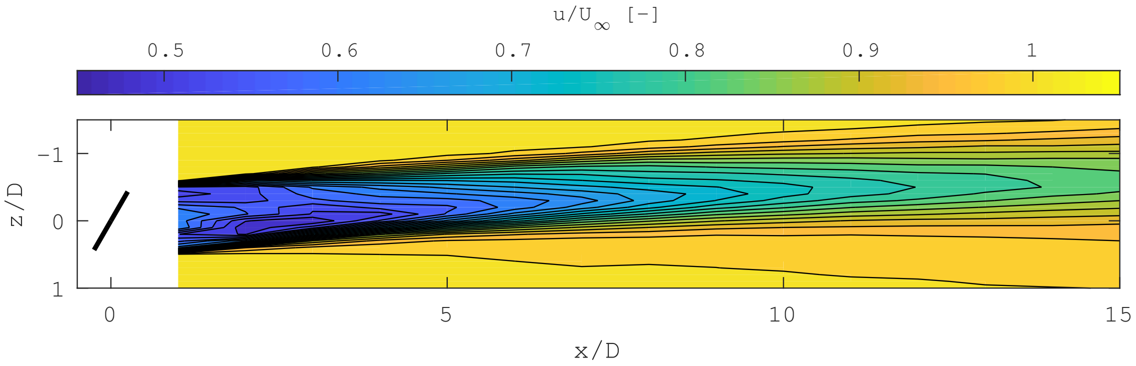

Figure 4: Normalized velocity distribution of the wake with the turbine at yaw 30°. The wake is seen from above and the solid black line illustrate the yawed turbine.

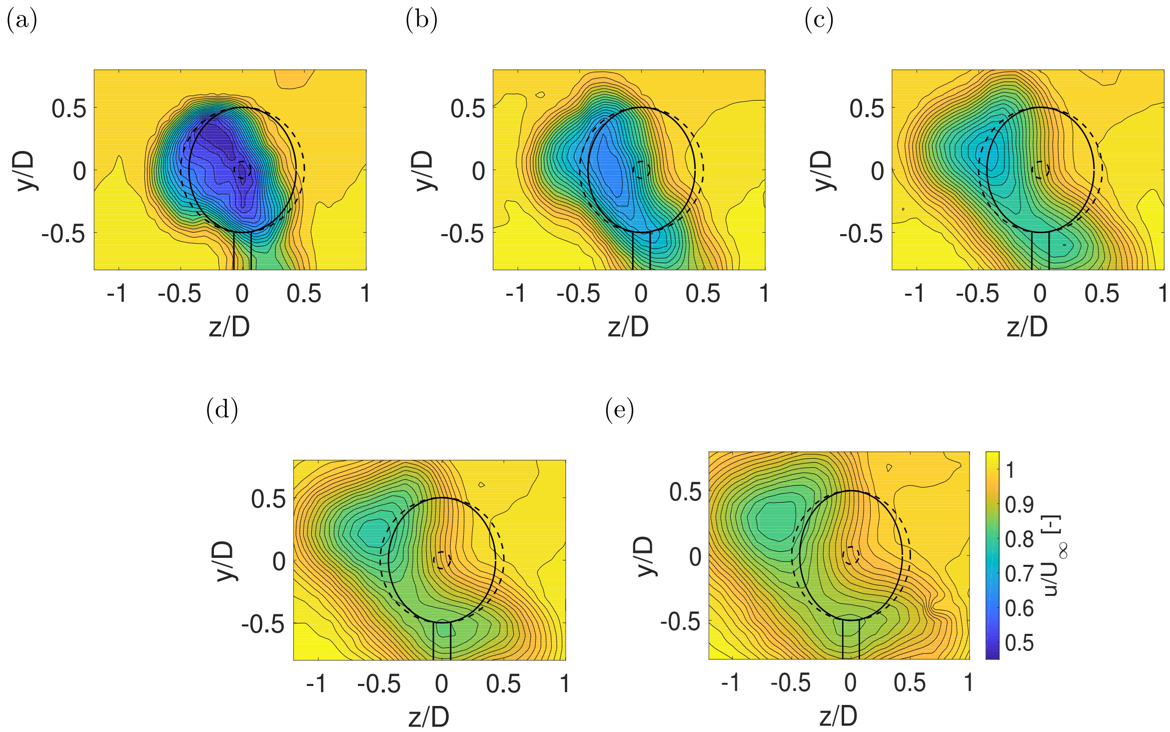

Figure 5: Normalized velocity distribution of the cross-section of the wake when the turbine in a 30° of yaw. The distances shown are (a) 3D, (b) 6D, (c) 9D, (d) 12D, and (e) 15D downstream from the turbine. The solid line illustrates the outline of the yawed turbine, while the dotted line illustrates the outline of an imaginary non-yawed turbine.

The normalized velocity distribution of the cross-section of the wake is shown in figure 5. The development of the wake is observed by considering the wake at five different distances downstream; at 3 D, 6 D, 9 D, 12 D and 15 D. As expected [7], [10], [11], the wake is clearly asymmetric and a curled shape is detected. The entire wake deflects to the side, in the opposite direction of the yaw rotation. Furthermore, it is observed that the center of the wake is deflected more than the top and the bottom of the wake. This is due to the spanwise velocity and that vorticity is generated on the top and bottom of the rotor leading to velocities in the opposite direction than in the center of the wake [11]. A slight deflection in the vertical direction can also be observed. Additionally, the tower effects can be seen in the lower part of the wake [15], making the wake slightly divide into two parts.

4.2. Velocity deficit

The velocity deficit is used to illustrate the velocity decay in the wake profile. The normalized velocity deficit is expressed as [16]:

$$\frac{\Delta U}{U_\infty} = \frac{U_\infty - U_w}{U_\infty}, \tag{4}$$

where U_∞ is the free stream wind velocity and U_w is the streamwise velocity in the wake.

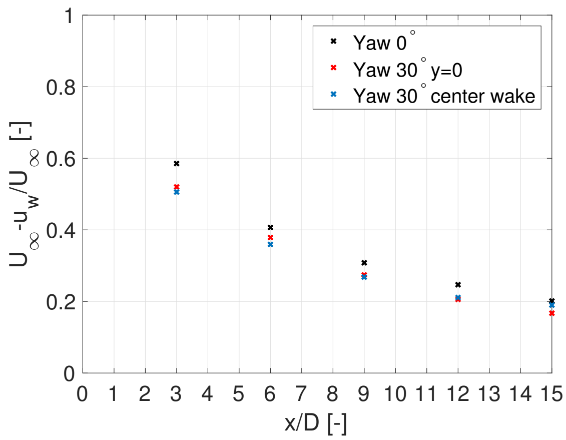

Figure 6: The maximum normalized velocity deficit of the wake at several distances downstream. The values from the turbine in non-yawed condition (black), $\gamma = 30°$ with measurements at $y = 0$ (red) and $\gamma = 30°$ with velocities in the wake center (blue) are shown.

Figure 6 shows the maximum normalized velocity deficit of the experimental results of the wake at several distances downstream. The measurements form the turbine in non-yawed condition are evaluated at $y = -0.22D$, while the measurements from the turbine in yaw ($\gamma = 30°$) are evaluated for both at $y = 0$ and in the center of the wake. For all distances downstream, the case of where $\gamma = 30°$, the velocity deficit is slightly lower than with the turbine in non-yawed condition. Furthermore, the maximal velocity deficit of the two cases with yaw are very coincidental.

4.3. Wake trajectory path

An implementation of the different methods described in section 3 is performed with the measurements obtained from the experiment with the turbine in yaw ($\gamma = 30°$). The result is shown in figure 7. Both the *Gaussian Fit Method* and *Available Power Method* trace a smooth wake center trajectory curve. The wake deflects at a higher rate in the beginning of the wake, and then the deflection rate decreases for increasing distance downstream. Even though the *Gaussian Fit Method* and *Available Power Method* give a similar path, the *Gaussian Fit Method* estimates a slightly higher deflection than the *Available Power Method*. This might be explained by the fact that the *Gaussian Fit Method* evaluate the measurements at all values in y-direction, while the *Available Power Method* is limited to obtain the center at $y = 0$. As

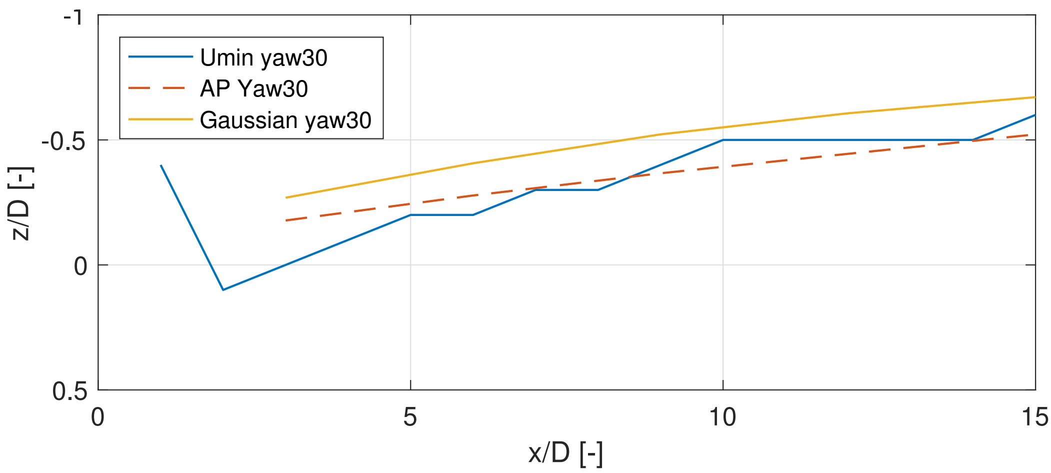

Figure 7: The wake center line trajectory is presented by using three different methods; the U_{min} *Method* (blue line), the *Available Power Method* (orange dotted line), and the *Gaussian Fit Method* (yellow line).

stated in section 4.1.2, the wake center can be observed to deflect in the lateral direction as well as slightly in the vertical direction, thus, the wake center is not exactly at hub height. The U_{min} *Method* on the other hand, fluctuates more than the other two methods, especially in the near wake. This might be explained by the fact that the method uses the extreme values, and in the near wake the lowest velocity is not necessarily found at the center of the wake [1]. It can be discussed how reliable the U_{min} *Method* is due to the extreme values taken into account, and hence, the U_{min} *Method* will no longer be considered.

4.4. Evaluation of the Width of the Wake

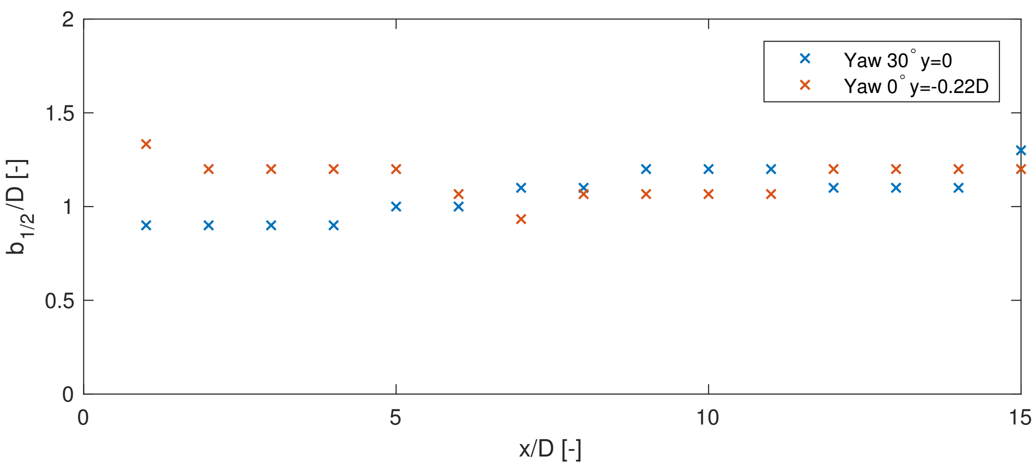

Figure 8: The normalized half-width development of the wake with the turbine in both non-yawed condition (orange) and yaw angle of 30° (blue).

The half-width ($b_{1/2}$) is defined as the width of the wake where the velocity deficit is half of the maximum velocity deficit [17]. Figure 8 presents the width of the wake in terms of the normalized half-width with respect to downstream distance. The normalized half-width is calculated from the velocities obtained from the line measurements for both cases; the turbine in non-yawed condition and the turbine with 30° yaw. In the near wake, approximately $1 - 5$ D, the width of the wakes differ significantly. However, in the far wake, from about 6 D and further downstream, the width of the wakes coincides more. Though, a comparison of the two wake widths should be done carefully, as the curled shape appearing in the wake with the turbine in $\gamma = 30°$ results in an asymmetric shape. The method for finding the half-width is widely used for axisymmetric wakes, while the application of this method on asymmetric wakes needs more investigation. Additionally, it can be discussed whether the half-width method is applicable in the near wake of the non-yaw case, as the wake is not fully developed in this region [1].

5. Discussion

A further evaluation of the measurements is performed by comparing the applied methods and the analytical model by Jiménez et al. [9]. Figure 9 presents the wake center trajectory estimated by the two methods *Gaussian Fit Method* and *Available Power Method* along with the analytical model developed by Jiménez et al. [9]. The analytical model coincide very well with the *Gaussian Fit Method* at distances between 3 D and 12 D downstream, with only a deviation of 0.02 at $x/D = 3$ and 0.06 at $x/D = 12$. At the distances even further downstream, it can be observed that the deflection rate of the analytical model decreases more when compared to the *Gaussian Fit Method*. After the downstream distance 12 D, the analytical model approaches the trajectory estimated by the *Available Power Method*. At $x/D = 15$, the analytical model estimates a deflection approximately in the middle of the two methods, with $z/D = -0.60$, where the deflection is $z/D = -0.67$ and $z/D = -0.52$ for the *Gaussian Fit Method* and the *Available Power Method*, respectively.

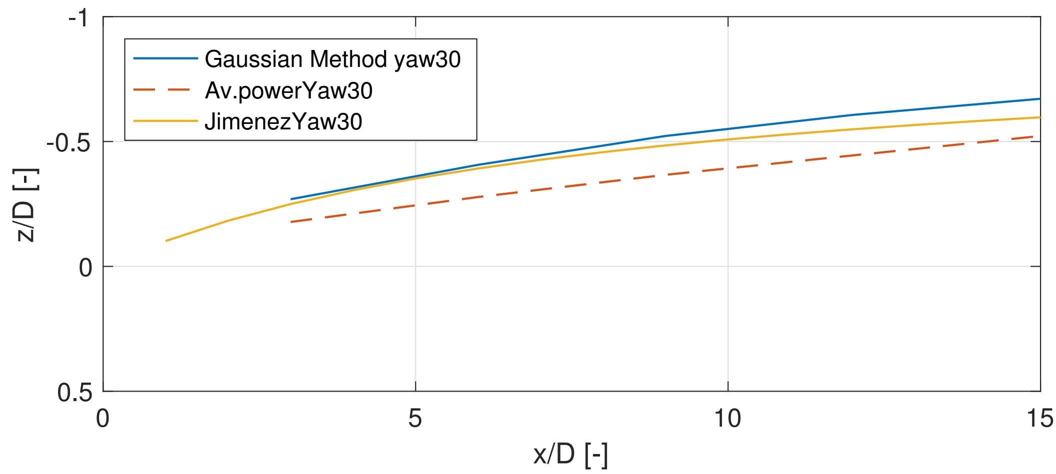

Figure 9: Center wake trajectory estimated by the Gaussian Method (blue), the Available Power method (orange) and the analytical model by Jimenez (yellow). The evaluation is performed with optimal TSR and yaw angle of $\gamma = 30°$.

Additionally, the skew angle of the deflected wake at several distances downstream calculated from the different methods and model are presented in table 1. The skew angles show, which also can be seen from figure 9, that the *Gaussian Fit Method* estimates a more deflected wake than the other two cases. Furthermore, it can be noted that in all three cases, the wake deflection seems to be approaching a constant value, thus being asymptotic.

Table 1: The skew angles of the wake center trajectory at several distances downstream from the turbine estimated by the methods and model.

	3D	6D	9D	12D	15D
AP method	3.4°	2.7°	2.3°	2.1°	2.0°
Gaussian method	5.1°	3.9°	3.3°	2.9°	2.7°
Jiménez model	4.8°	3.7°	3.1°	2.6°	2.3°

According to the findings, the *Gaussian Fit Method* and the analytical model coincide very well, while, in comparison, the *Available Power Method* under predicts the wake center trajectory. On the contrary, previous studies have concluded that the analytical model by Jiménez et al. [9] over predicts the wake center trajectory [10] and that the *Available Power Method* seems to predict the wake center trajectory quite well [8]. Additionally, previous studies have show similar results as the *Available Power Method* when evaluating the wake center trajectory for a turbine in $\gamma = 30°$. Vollmer et al. [13] found a wake deflection of $z/D = 0.3-0.4$ at $x/D = 8$, Bastankhah et al. [10] found a wake deflection of $z/D = 0.45 - 0.5$ at $x/D = 12$, and Howland found a wake deflection of $z/D = 0.45 - 0.6$ at $x/D = 8$. Consequently, it can be discussed whether the *Available Power Method* is a better method and that *Gaussian Fit Method* and the analytical model by Jiménez et al. are over predicting the wake center trajectory.

6. Conclusion

An experimental study of the wake behind a model wind turbine in both non-yaw and $\gamma = 30°$ was conducted. It was found that the minimum velocity development in the wake with $\gamma = 30°$ was very similar to the non-yawed wake. The wake behind the model wind turbine in yaw was investigated and methods were applied to find the wake center trajectory. Results from the application of the models indicates that 3D effects of the wake need to be considered when estimating the wake center trajectory. Further, the methods were compared to an analytical model developed by Jiménez et al.. The *Gaussian Fit Method* and *Available Power Method* show a similar path, however, the *Gaussian Fit Method* estimates the wake center trajectory to deflect $z/D = 0.09 - 0.17$ more than the estimations from *Available Power Method*. The deflection at $x/D = 15$ is $z/D = -0.67$ and $z/D = -0.52$ with the *Gaussian Fit Method* and the *Available Power Method*, respectively. It is showed that quantification of the wake deflection is strongly dependent on the method used for the estimation. Additionally, it seems that the prediction model by Jiménez et al over predicts the deflection compared to the *Available Power Method* and prior studies.

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