Master's thesis

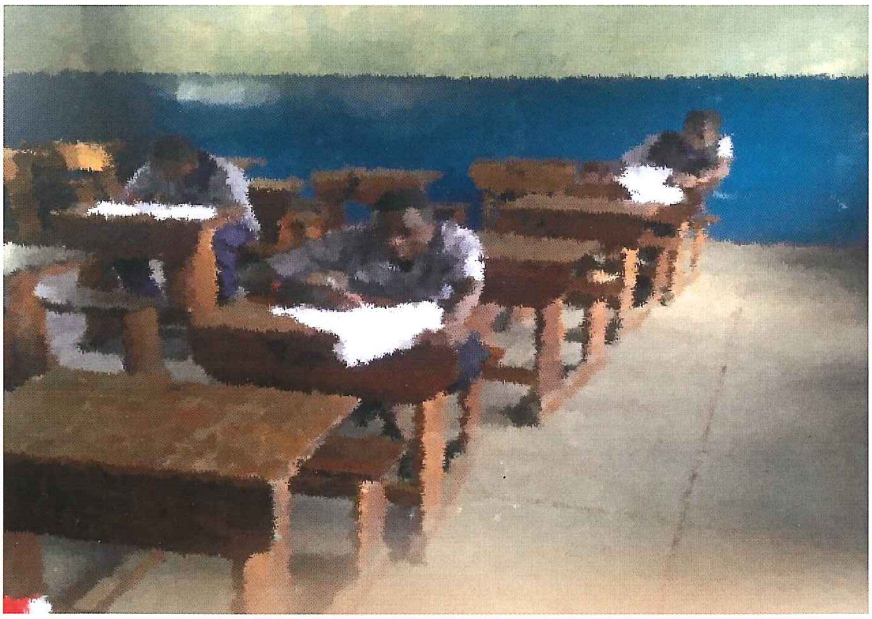
NTNU
Norwegian University of Science and Technology
Department of Teacher Education

Stephen Imbuwa Munalula

Computational strategies in school and out-of-school

A qualitative study of Grade 8 students' computational strategies in school and out-of-school settings

Master's thesis in Mathematics Didactics
Supervisor: Liping Ding
Trondheim, August 2017



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DEDICATION

I dedicate this work to my late parents whose interest in my education has never ceased to inspire me. I also dedicate this work to my siblings and to my wife who have supported me in every way possible all my life.

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First and foremost, praises and thanks to God, the Almighty, for His showers of blessings throughout my research work up to its successful completion.

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ABSTRACT

Students' poor performance in national examination remains a major concern worldwide and Zambia in particular. Teachers, students, parents, curriculum developers and the public have tended to blame one another for the poor performance in Mathematics in schools at all levels. In an attempt to respond to this problem, the Zambian Mathematics scholars have carried out many studies in Mathematics education. Despite these studies, students' performance has continued to remain poor. This means that the main reason for this poor performance has not been established yet. The reason could probably be the gap between mathematics learning and practices in school and out of school as described in some studies. The gap could be narrowed by incorporating some of the out-of-school practices into the daily classroom practices so that they can build on and complement each other. In this way, students can bear their mathematical knowledge gained in out-of-school experiences on their school mathematics. Likewise, students can use their school mathematics in solving problems that occur in everyday situations. The aim of the study was to find out what computational strategies Grade 8 students use in school and out-of-school settings and also to compare the strategies in both settings in Mongu District.

The study employed a qualitative research and focused on four Grade 8 students who were followed up in school and out-of-school. Data collection methods in this study included observations, semi-structured interviews and testing. The findings revealed that students' use of semantically-based mental computational strategies was more predominant in out-of-school settings than in school settings whereas written school-like computational strategies were used more frequently. The students' use memorised mathematics facts was common to both settings.

Based on the findings, the study recommended that teachers should bridge the gap between school mathematics and everyday mathematics such that students can bear their mathematical knowledge gained in out-of-school experiences on their school mathematics. Likewise, students can use their school mathematics in solving problems that occur in everyday situations. In this way, the performance of pupils in mathematics will be improved.

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CHAPTER 1
INTRODUCTION

1.1. Introduction

This chapter presents the background to the study, statement of the problem, purpose of the study, research questions, significance of the study, definitions of terms and organization of the thesis remaining chapters.

1.2. Background of the study

Mathematics education researchers have for several years investigated out-of- school mathematics and how it can enhance the learning of mathematics in schools. This is the mathematics that enables unschooled and sometimes illiterate people to practice crafts and trades, conduct business transactions and make their living in a variety of ways. Out-of-school mathematics has been called differently by several researchers, ‘informal’ mathematics (Ginsburg, 1988), ‘everyday’ mathematics (Lave, 1988) ‘ethnomathematics’ (D’Ambrosio, 1992), or even ‘street’ mathematics (Nunes et al., 1993). A number of studies have investigated the arithmetic problem-solving behaviour of schooled and unschooled subjects in out-of-school contexts in different countries and cultures.

In Zambia, the overall performance of students in mathematics has been very poor in the recent years and therefore, there is need to come up with strategies that will address the poor performance as mathematics is cardinal to national development. One of the strategies by the Zambian government to improve student performance in mathematics is to embark on capacity building in teaching staff at colleges of education by sending them for further studies so that they are equipped with appropriate teaching strategies which they will in turn transfer to student teachers who will implement them in various schools. It was in this good cause that Mongu College of Education on behalf of the Zambian government entered into partnership with the Norwegian University of Science and Technology (NTNU) that facilitated my study in Mathematics Education in Norway.

Various studies have indicated that poor performance in mathematics in schools is due to the gap in mathematics learning and practices in school and out of school. Knowledge acquired in school is believed to grow out of a transmission paradigm of instruction as such it is said to lack context, relevance and specific goal. Resnick (1989) has argued that schools place too much emphasis on the transmission of syntax (procedures) rather than on the teaching of semantics (meaning) and this "discourages children from bringing their intuitions to bear on school learning tasks" (Resnick,1989, 166). Students need in-school mathematical experiences to build on and formalise their mathematical knowledge gained in out-of-school situations. An important part of mathematical experience in school is the guidance and structure that can be provided by a teacher to help students make connections among mathematical ideas. The existing gap can be narrowed by incorporating some of the out-of-school practices into the daily classroom practices so that they can build on and complement each other. All students bring to school mathematical knowledge from everyday situations they have experienced. This knowledge is often hidden and unused by the students in school as they learn to use the mathematical procedures that the teacher demonstrates and evaluates. Just as the mathematics practice of everyday activity is ignored by teachers in school, mathematics practice in schools is likewise devalued by students because of the lack of use of it in out-of-school situations. For instance, the uses of out-of-school mathematics in school, whether finger counting strategies to solve an arithmetic problem presented in a grade one classroom or sophisticated regrouping strategies to solve multidigit problems later, could be viewed as the intrusion of inappropriate and primitive strategies that should be adaptively replaced by the formal mathematics of the classroom. By building upon the mathematical knowledge students bring to school from their everyday experiences, teachers can encourage students to make connections between these two worlds in a manner that will help formalise the students' informal mathematical knowledge, and learn mathematics in a more meaningful, relevant way. My study intends to find out what computational strategies students apply when solving mathematical problems in out-of-school and in school setting and also to compare the strategies.

1.3. Statement of the problem

Despite the various interventions being put by the Ministry of Education in Zambia to improve the performance of students in mathematics, the performance still does not seem to improve. The problem is the gap that exist between the mathematics learnt at school and the mathematics that is generated and used outside school setting. It is not clear as to what extent teachers build on the mathematics knowledge which the students come with to school from outside school. Hence my study.

1.4. Purpose of the study

The purpose of this study was to find out what computational strategies are used by Grade 8 students in school and out-of-school settings and also to ascertain to whether there are any similarities and differences in the use of strategies used in school setting and out-of-school setting. My interest lies in working to close up the gap between doing mathematics in school and doing mathematics in out-of-school situations.

1.5. Research questions

The research questions of the study were:

1. What computational strategies do Grade 8 students use when solving mathematical problems in school and out-of-school settings?
2. What are the similarities and differences between the computational strategies used by Grade 8 students in school and out-of-school settings?

1.6. Significance of the study

The results of the study would help teachers know and understand the various computational strategies students use outside school to be able to profitably take account of the teaching as many children can compute mentally before they learn the relevant formal written algorithms at school. The results will also help teachers of mathematics to incorporate mathematics into the real world and to encourage them investigate mathematical ideas and practices of their students. Curriculum developers can also use the results of this study to incorporate some elements of the sociocultural environment of the students into the curriculum. It is important that the mathematics curriculum in schools incorporate elements belonging to the sociocultural environment of the students and teachers, in such a way that they facilitate the acquisition of knowledge, understanding, and compatibilisation of known and current popular practices, because "cognitive power, learning capabilities and attitudes towards learning are enhanced by keeping the learning ambiance related to cultural background (D'Ambrosio, 1995). In this way the motivation, interest and curiosity of the students will be increased and the attitude towards mathematics for both students and teachers will be changed and this in turn will improve their performance.

1.7. Definition of terms

School mathematics: refers to problem-solving strategies taught and used in schools.

Out-of-school mathematics: refers to problem-solving strategies not taught in schools and used in out-of-school settings.

Mental computation strategies: the ability to calculate exact numerical answers without the aid of calculating or recording devices.

Algorithm: refers to a set of step-by-step procedures that provide the correct answer to a numerical problem.

1.8. Layout of the Thesis

The second chapter deals with literature review and theoretical framework. It has attempted to describe and analyse what has been done by other researchers and presented my theoretical framework for this study.

Chapter three discusses the research methodology used in the study. The chapter is divided into sections subsumed under the following headings: research design, population and sample, data collection methods, data collection procedures, validity and reliability, ethical considerations and methods for data analysis.

Chapter four deals with presentations of my data findings. It presents data for each of the four students who took part in the study under the following subheadings: Case for student 1(S1), Case of student 2(S2), Case of student 3(S3) and Case of student 4(S4).

Chapter five deals with analysis of my data findings of the study while chapter six deals with discussion, conclusion and recommendations. This chapter ends with my ending remarks in which I have presented suggestions for further research. The subsequent pages consist of the references and appendices.

CHAPTER 2
LITERATURE REVIEW AND THEORETICAL FRAMEWORK

2.1. Introduction

This chapter presents relevant literature on the studies done in school mathematics and out-of-school mathematics and also presents the theoretical framework for my study. I will begin by looking at the general perspective of mathematics practice in out-of-school and school settings and then review some earlier studies on out-of-school and school mathematics. I will also discuss Situated Cognition theory which is the theory that guided my study and later on present the theoretical framework for my study.

2.2. Mathematics practice in out-of-school and school setting

Mathematics practice in school and out-of-school settings is different in several ways. Lave (1988) has found evidence that mathematics practice in everyday settings differs from school mathematics. People look efficacious as they deal with their daily complex tasks to meet the demand of their everyday lives through many socio-economic activities. They easily deal with the tasks because mathematics practice is structured in relation to ongoing activity and setting. In everyday settings, people have more than sufficient mathematical knowledge to deal with problems and mathematics practice is nearly always correct because people are able to estimate using the available mental strategies. Since problems in everyday setting are created by the problem solver, they are either transformed to make them easier to work with or abandoned if they prove to be difficult to solve. People invent procedures on the spot as needed rather than depending on pre-formulated algorithms which may not work in some instances. Resnick (1987) discussed the differences of mathematics learning in-school and out-of-school situations. He noted that whereas school learning emphasises individual cognition, pure mentation, symbol manipulation and generalized learning, everyday practices rely on shared cognition, tool manipulation, contextualized reasoning and situation specific competencies. The differences that exist between the two types of mathematics could be attributed to the variations in the goals of each of the types. There are at least two goals for school mathematics: to prepare students to deal with novel problems and to help students acquire the concepts and skills that are useful to solve many of the sorts of routine dilemmas that people encounter in life. In order to achieve the second goal, it is important that students work with concepts and procedures that they can generalise. In out-of-school mathematics practice, persons may generalise procedures within a context but may not be able to generalise to another context since the problems tend to be context specific. However, knowing and using students' out-of-school mathematics practice is important in school situations because it provides contexts in which students can make connections.

2.3. LITERATURE REVIEW

2.3.1. Studies on out-of-school and school mathematics

It is a common belief that children learn mathematics in school contexts. This belief has been increasingly questioned by educators who have joined ranks in the study of the kind of knowledge children acquire outside school. Several researchers have investigated how people use mathematics in out-of-school situations to solve their problems and achieve their goals and found out that children and adults construct complex strategies to address arithmetical problems that emerge in everyday commercial transactions (Carraher, Carraher and Schliemann, 1985; Saxe, 1982) and work activities such as tailoring (Lave, 1977).

Lave's early study of the use of arithmetic by Liberian tailors is in many ways a typical example of studies of arithmetic in other cultures. Lave studied 63 tailors who were given arithmetic problems, half involving numbers and problems actually encountered in their daily work ('tailoring problems'), half involving problems of equal difficulty that were rarely encountered in their work ('nontailoring problems'). According to Lave, tailors with no schooling solved on average 91% of the former and 70% of the latter, while tailors with 5–10 years of schooling averaged 95% and 91% respectively. One message of this was clear: don't presume that all arithmetic procedures used in everyday life originate in school; some arithmetic procedures can be picked up outside schools. Through observation and participation in the activities of the tailor shop, they found that tailors must be able to read off inches on the tape measure; measure various lengths; and quarter, halve, double, and quadruple measurements as they cut out the four main pieces of a pair of trousers before assembling them into a finished product. Multiplying and dividing by 2 and 4 are therefore by far the most common operations. (Reed and Lave, 1979). It is observed that tailors encounter a range of arithmetic operations.

The Adult Mathematics Project (Lave, 1988) explored the use of school mathematics by U.S. grocery shoppers in solving real life practical problems that are encountered in the supermarket. Participants were asked to choose the best value for money when the choice was from several articles of the same type and quality with varied weights and prices. The task was set both in form of a test, like a school test and performed in the home, when the best buy was correctly identified in 59 per cent of the cases, and it was also carried out in the supermarket when the success rate was a remarkable 98 per cent. She concluded that adult shoppers used a gap closing procedure to solve problems, which turned out to yield a higher rate of correct answers than were achieved when the adults solved a similar problem in formal testing situations using the tools of school mathematics. The obvious reason is that mathematics knowledge is linked to the situation in which it is used. Clearly, the calculation of a best buy in the supermarket matters a great deal to the purchaser, whereas a de-contextualised test question does not.

Lave collaborated with de la Rocha to carry out this investigation of Weight Watcher dieters' use of arithmetic. They observed that dieters employed a variety of 'calculating devices': They measure a precise amount of food and transfer the measured amount to a glass or bowl. Once the measured food is in its container, they note its position relative to some feature of the container such as a decorative pattern. By using the same glass or bowl over and over, and always filling it to the same position, they eliminate the need for continual measuring. Dieters do not employ manufactured measuring devices for every measurement they have to do and try to avoid repeated measurements by 'storing' the appropriate amounts in purpose-specific devices. These rather uncontroversial observations are, however, seen to point to two different kinds of procedures, 'universal' or 'formal' ones and 'everyday' or 'informal' ones. de la Rocha (1986) observed that many problems that might have been solved by quantitative means were solved in non-quantitative ways that accomplished the same end. For example, an old cracked coffee cup became 'my rice cup' and replaced the standard measuring cup in the preparation of rice, a circumstance leading to the disappearance of numbers from the preparation process. (de la Rocha, 1986). This study shows that dieters have ways of customising their measurements to make them more efficient or convenient. The study exhibits that these different ways of measuring measure standard amounts and that dieters are aware of their equivalence.

Nunes et al (1993) set out to determine the similarities and differences between school mathematics and street mathematics or rather to provide “systematic comparisons of informal mathematics and formal mathematics"(Nunes et al, 1993, 5). They undertook an interesting study in which they studied the problem- solving behaviour of vendors in the market and in a school-like setting in Brazil. Their goal was to establish connections among three types of mathematics: one constructed by children outside of school, one embedded in everyday cultural practices, and another that school aims to teach. They intended to determine the effectiveness of traditional mathematics instruction in the elementary school versus mathematics learned informally through working. Much of the researcher’s time was spent shopping in the street markets undertaking the same transactions at each stall; this was designed to measure particular arithmetical skills which were being taught in school. These same questions became part of a paper and pencil exercise undertaken with the same children in school. Problems presented and addressed orally in the streets were more easily solved than those included in the more formal test in which pencil and paper were available. The study gives many similar examples from other children who worked as market traders showing the interesting situation where children could calculate when the mathematics was presented in a real-life situation which they could relate to (street mathematics) but not when presented in a standard arithmetic form. Thus, the term ‘street mathematics’ seemed appropriate, and this mathematics was compared with school mathematics. There was a purpose to the street mathematics, where the question makes sense and has meaning, in direct contrast to the standard symbolic approach taken in the elementary school.

Saxe (1982) documented perhaps an extreme instance of the invention of a system of mathematics among unschooled Oksapmin adults in Papua New Guinea. The Oksapmin are adapting their traditional 27 body-part counting system to solve new problems that arise in commercial transactions introduced by money economy. Saxe (1991) showed that Brazilian candy sellers, with little or no schooling, can develop through their selling experiences arithmetic practices that differ from those taught in schools and normally are associated with a high success rate. Saxe (1998), argues that mathematics learning ‘is not limited to acquisition of formal algorithmic procedures passed down by mathematicians to individuals via school. Mathematics learning occurs as well during participation in cultural practices as children and adults attempt to accomplish pragmatic goals’ (Saxe, 1988, 14-15).

2.3.2. Situated Cognition Theory

This study was based on Situated Cognition theory. The theoretical foundations of the theory of situated cognition lie in both sociocultural approaches to education and in anthropology. Situated Cognition theory is centered around the idea that knowing is “inseparable” from actually doing and highlights the importance of learning within context (Brown et al, 1989). Often, proponents of the view that learning is basically situated have been influenced by the work of Vygotsky. Vygotsky (1962) proposes that the cognitive functioning of the individual come as a result of internalizing social activity. This internalisation is enhanced by the introduction of the child to the society’s tools and practices in the child’s zone of proximal development through the interaction with adults or collaboration with more capable peers. One of the most influential proponents of a strong form of situated cognition has been Jean Lave.

Lave (1988), developed an ethnographic critique of traditional theories of problem solving and learning transfer and elaborated a theory of cognition in practice. Based on several empirical studies on problem solving in mundane settings and everyday activities, she argues that cognition is not within the mind but stretches over mind, body, activity, and culturally organised settings, hence always involving other actors. Lave (1988) argues that individual cognition is inseparable from the social-cultural context reflected in everyday activity. Her project was in fact twofold: It was criticism of schooling and what she called the knowledge transfer assumption. The assumption that the skills and knowledge acquired in schools are widely applicable in other arenas of life as well. Second, it is an in-depth analysis and critique of the lack of ecological validity in laboratory-type cognitive research. She says that both schooling practices and laboratory-type cognitive research ignore discontinuities between situations. Lave (1988) has shown how arithmetic activity in the real world does not reflect the formal procedures taught in the classroom. Lave’s study records two things that cannot be challenged: firstly, that some arithmetic competence can be learned without schooling; secondly, that those who attend school learn some arithmetic competences that are not so easily picked up in out-of-school situations (Greiffenhagen and Sharrock, 2008).

The situated cognition theory (Lave, 1988; Lave and Wenger, 1991) argues that knowledge is situated in the context in which it is acquired and proposes that learning is a process of participation in communities of practice. This approach develops an understanding of learning as emergent and social, and discusses issues of identity, context, and transfer. The situated cognition perspective challenges the conventional belief which assumes the separation between learning and doing, where mathematical knowledge learned in school is expected to be automatically transferred into other contexts in a straightforward manner. Instead, the situated cognition approach argues that learning and cognition are fundamentally situated. Therefore, social and cultural contexts of learning should be taken into account in the mathematics teaching process.

The theories of situated cognition advocate that knowledge is not independent but, rather, fundamentally "situated," being a product of the activity, context, and culture in which it is developed. This is contrally to many methods of didactic education that assume and treat knowledge as an integral, self-sufficient substance, theoretically independent of the situation in which it is learned and used and thus separating knowing and doing (Brown et al, 1989). Our school mathematics curriculum is really based on the assumption that mathematics is a formal body of knowledge, a self-contained subject domain which contains mathematical objects with meaning which do not have to be applied outside the subject. This mathematics, it is assumed, can therefore be completely detached from the experiential world and can studied purely for its own sake, although we do also intend our pupils to use them in later life (Orton, 2004). This idea has led to situations where students fail to use school learnt strategies in out-of-school settings. According to situated cognition theories, knowledge transfer in out-of-school situations is dependent on the learner's participation in a social and material context, such as an apprenticeship or 'guided participation'. This implies that students learn mathematics in out-of-school settings by observing and taking part in the sociocultural activities while receiving guidance from the elders. In my study students applied computational strategies they acquired from their parents in the process of helping them conduct business transactions. Through practice participation, children construct and operate on mathematical problems that are influenced by artifacts of culture and social interactions like assistance provided clerks. Children learn many skills, trades, and crafts by working alongside a master and perhaps other apprentices. The apprenticeship system often involves a group of novices, students, who serve as resources for each other in exploring the new domain and aiding and challenging one another. The expert or teacher is relatively more skilled than the novices, with a broader vision of the important features of the activity. Moreover, the way these social processes influence children's construction of knowledge determines the problem-solving strategies they will adopt.

Other major contributors to the situated cognition debate are Nunes, Schliemann and Carraher. Cultural tools mediation is a theme highlighted in studies which falls under the broad title of 'everyday cognition' approach (Carraher et al., 1985; Nunes, 1993). These studies argue that human thinking is embedded in social and cultural activities. They investigated the mediation role of sociocultural tools, such as different cultural systems of signs and skills, on cognition and how people in different cultures can develop certain mathematical procedures to deal with their everyday mathematical aspects. According to this view, learning mathematics in school and outside-school can involve different procedures which are considered as one type of cultural tools. Therefore, schools should not teach mathematics as an abstract context-free subject but rather to seek ways of incorporating mathematical concepts learned in school with real contexts and meaningful problems to the learners. Nunes et al (1993) have addressed further issues such as which aspects of different contexts could account for the observed differences in performance. They have suggested one important difference concerns the social relations between researcher and subject, for example, whether the customer is known to the researcher. In one study, keeping the context constant (testing in school), and comparing the three situations of simulated store problems, word problems and computational exercises, they were able to show that oral calculations were done correctly more often than written ones, and that when the procedure was controlled, the difference in performance across situations disappeared (Carraher et al.,1987). Furthermore, Nunes et al. (1993) have separated out and researched different levels of transfer, namely application to problems with unfamiliar parameters, reversibility and transfer across situations.

2.4. THEORETICAL FRAMEWORK FOR MY STUDY

The analysis for my study will be mainly based on Nunes, Schliemann and Carraher (1993) study in which they studied the problem-solving behaviour of vendors in the market and in a school-like setting in Brazil. One of the examples included in the study is the purchase of four coconuts which cost 35 cruzeiros (Cr$) each. The twelve-year-old boy replied: 'There will be one hundred five, plus thirty, that's one thirty-five . . . one coconut is thirty-five . . . that is . . . one forty' (Nunes et al., 1993: 24). When facing the question in the market setting, the boy began by breaking the problem up into simpler ones based on his prior knowledge which was that three coconuts cost Cr$105. Then, to add on the cost of the fourth coconut, he first rounded the cost of a coconut to Cr$30 and added that amount to give Cr$135 and added in the correction factor to give the answer Cr$140. However, when facing the same question in the school situation his response was 'Four times five is twenty, carry the two; two plus three is five, times four is twenty.' He then wrote down '200' as his answer. Here he has applied a formal algorithm for column multiplication, although as he was able to maintain the positions of the places the respective numbers would occupy, he was unable to apply the necessary carrying rule resulting in a much larger price. While he was able to answer this question in the real setting, he did not apply this knowledge or an appreciation of the magnitude of the anticipated answer in the school setting.

The authors illustrated very clearly the remarkable differences in success when comparing a child's performance during the informal test with his or her performance during the formal school-like test. In the informal test children relied on mental calculations or oral arithmetic practices where they used a variety of strategies, in the formal test they used school taught routines for addition and multiplication. Oral computation procedures involved the use of two identifiable routines: Decomposition and repeated grouping.

2.4.1. Computational strategies in transactions

According to Nunes et al (1993), computations in out-of-school settings are in many cases done mentally. Mental computation has been defined as 'the ability to calculate exact numerical answers without the aid of calculating or recording devices' (Reys,1984). According to Nunes et (1993), children used mental calculations to solve mathematical problems in the out-of-school settings. Mental strategies do not usually involve the use of written symbol systems to produce mathematical computations but rely, instead on invented procedures that may include mentally regrouping terms to arrive at sums or manipulating objects in computations. Good mental calculations are characterised by having many strategies, which can be applied flexibly to meet the task at hand. Sometimes, particular numbers suit particular strategies. Sometimes strategy selection is guided by personal preferences, or special number combinations that a student happens to spot. Mental strategies are not carried out like written algorithms, in a standard way. Instead, students need to choose them and adjust them to suit the calculation.

Mental strategies are intimately connected with objects and events; children often use the objects and events directly in their reasoning, without necessarily using symbols to represent them (Resnick, 1987). This is termed as quantity manipulation as opposed to symbol manipulation (Lave, 1991). Nunes et al (1993) argue that unschooled participants had competent ways of performing the calculations necessary to their chosen professions, even though they had not formally been taught mathematics in a classroom. Mental strategies can also be referred to as heuristics. A heuristic is a rule or method that is applied to solve problems without necessarily computing. Heuristics are more commonly used in everyday life tasks such as finding the price of marked down merchandise, figuring out which size of item provides the best value for their money and accurately doubling or halving cooking recipes. Specific cultural activities such as buying and selling promote the development of mathematical ideas that were previously thought to be only acquired through formal instruction. The following mental strategies will be considered for this study: decomposition, counting-up, compensation and repeated addition.

2.4.1.1. Decomposition

Decomposition or breaking apart (place value), also known as "Separating"is a mental computational strategy mainly used for addition. Carraher et al. (1987) have defined decomposition as 'working with quantities smaller than those mentioned in the problem' (Carraher et al, 1987, 91). It consists of two principles: the first being that a number is composed of parts that can be separated without changing the value of the number; and the second being that addition can be carried out on these parts, and the final result will not be affected. Nunes et al. (1993) established that these principles correspond to the property of associativity of addition. Decomposition involves knowledge of the number system where students need to construct sums through their own mental actions of putting numbers into relationships. When they do that, they remember a coherent network of relationships much better than isolated bits of numbers. Rearranging numbers into simpler forms involve breaking them in a way that makes use of the base ten structure. It can be done by breaking both numbers down to place value and add each, starting with the largest. It can also be done by keeping one number intact and only break second number down by place value and adding each place.

2.4.1.2. Counting-Up or Adding-Up

Jurdak and Shanin (1999) counting up as a mental strategy used for subtraction. Counting-up or Adding-up strategy is a powerful and rapid way of finding answers to take-away problems. It is a common strategy among children when dealing with subtraction, they start with a number being subtracted and count up or add up to the landmark number, from the landmark they add up to get the target number. The answer is the number of counts to the target number or the sum of the numbers used in the adding up process. Counting-up subtraction is similar to the process of making change in transactions: in both processes, the problem solver counts up from the lesser number to the greater number. The person making a purchase counts up from the amount due to the amount tendered. The person using the counting-up strategy counts up from the subtrahend (the lesser number) to the minuend (the greater number), records each count-up amount, and then totals all the count-up amounts to find the difference between the minuend and the subtrahend. There is a special form of counting or adding up known as subtracting across zeros. This operates on the same principles as that in the counting up or adding up, only that the minuend should have zeros. This is a good strategy taking into consideration that generally students have a great deal of difficulty subtracting across the zeros.

2.4.1.3. Compensation

Compensation in subtraction works by transforming or changing one of the numbers in a subtraction problem in order to make it easier to work with, then compensate. This means that one needs to subtract the number which was added from the difference or add the number which was subtracted to the difference in order not to change the value of the original number before the subtraction. In other words, compensation can be done by either adjusting one of the numbers and then adjust the answer or adjusting both numbers. Then it’s not necessary to adjust the answer.

2.4.1.4. Repeated Addition

Repeated addition is a strategy used for multiplication. It works on the principle of successive additions. It builds upon the already established understanding children have about addition but extends this from adding the contents of a grouping to adding the contents of one group and then using this to add the contents of several equally-sized groups. Carraher et al. (1987) maintained that repeated grouping involved 'working toward the solution in a stepwise fashion with quantities equal to or larger than those mentioned in the problem' (Carraher et al, 1987, 91). Repeated grouping mainly involves two basic ideas: the first that a number can be decomposed into parts without changing its value; and the second, these parts can be multiplied by the same number and the products then added, resulting in a value that is the same as what would be obtained if the two numbers were directly multiplied.

2.5. Computational strategies in school

According to Nunes et al (1993), computations in school settings are characterised by the use of school-learned procedures. The school-learned strategies have been termed symbol manipulation, in contrast to manipulation of quantities (Reed & Lave, 1981). School-learned strategies are mostly symbol based which makes them lose the connections to the events and objects symbolised. Symbol manipulations are divorced from reality. Students sometimes find difficulty using their school mathematics to solve the contextual problems because they fail to keep the meaning of the problem in mind and concentrate instead on the numbers, sometimes arriving at absurd solutions (Nunes et al, 1993). School-learned strategies for solving computation exercises make use of two sorts of resources: memorisation of mathematics facts and algorithms.

2.5.1. Memorisation of Mathematics Facts

Memorisation is the ability to retrieve facts quickly, accurately and effortlessly. Three levels of expertise have been identified by Klapp et al. (as cited in Barrouillet and Fayol, 1998) and these are: the novice stage, the automatised stage and beyond automaticity. In the novice stage, the may need to revert to a step-by-step process that they have learnt to obtain the answer. The automatised stage, here, the repetitive part may still play a role, but the student is both far more rapid with their response and not so easily distracted. Beyond automaticity, at this stage the response is even more rapid, and the student will not experience any interference as a result of doing some other task simultaneously. This really what we want, because students need to recall basic facts without it interfering with their focus on larger mathematical problem.

Nunes et al (1993) in their conclusion of their study mentioned that children use memorised basic mathematics facts to solve problems in the school settings. Memorisation of mathematics facts is a teaching and learning strategy where teachers encourage learners to master mathematical operations of addition, subtraction and multiplication in the early years of elementary school. Although recall of facts is important and expected; however, recall of basic facts developed from memorisation alone does not help students to develop the number sense that is required to solve problems. According to Nunes et al (1993), children with restricted schooling master arithmetical operations, properties of integers and of the decimal system, and proportional relations often without much attention to conceptual understanding. To have true mastery and robust recall of basic facts, students need to have efficient strategies. If facts learned through memorisation are forgotten, students have no strategies to compute a result because memorisation doesn't lead to number sense. Although rote memorisation can lead to recall for some students, for many students it leads to anxiety and/or a dislike of mathematics. This is especially true when recall of basic facts is timed. O'Connell and SanGiovannii (2011) argue that, asking students to memorise dozens of number facts can be discouraging and confusing when students view them simply as pairs of numbers. Students who simply memorise mathematics facts miss a prime opportunity to expand their problem-solving skills in mathematics education.

2.5.2. Algorithms

An algorithm is a step-by-step procedure designed to achieve a certain objective in a finite time, often with several steps that repeat or "loop" as many times as necessary. Knuth (1977) defines an algorithm as a set of rules for getting a specific output from a specific input, the steps being so precisely defined that they could be executed by a machine. Algorithms can be considered as tools, as Lave (1984) points out, that can be taken out of a bag and applied to several situations without being changed. Algorithms were originally born as part of mathematics – the word "algorithm" comes from the Arabic writer Muḥammad ibn Mūsā al-Khwārizmī. The original purpose of algorithms in the previous centuries was for clerks to be able to carry out a large number of calculations in a short period of time. The most familiar algorithms are the elementary school procedures for adding, subtracting, multiplying, and dividing, but there are many other algorithms in mathematics. Computational algorithms have been a common feature in mathematics education for a long time. Written algorithms have been designed for efficient calculation using pencil-and-paper technology, and are not so suitable for mental computation.

In school like settings, students tend ‘to produce without question, algorithmic, place holding, school-learned techniques for solving problems even when they could remember them well enough to solve problems successfully’ (Lave, 1985, 173). Algorithms taught in school mathematics tend to suppress children’s natural problem-solving strategies. Hiebert (1984) writes, “Most children enter school with reasonably good problem-solving strategies. However, after several years many children abandon their analytic approach and solve problems by selecting a memorized algorithm based on a relatively superficial reading of the problem.” (Hiebert, 1984 as cited by O’Connell and SanGiovannii, 2011). By third or fourth grade, according to Hiebert, “many students see little connection between the procedures they use and the understandings that support them. If taught properly, with understanding but without demands for “mastery” by all students by some fixed time, paper-and-pencil algorithms can reinforce students’ understanding of our number system and of the operations themselves. Exploring algorithms can also build estimation and mental arithmetic skills and help students see mathematics as a meaningful and creative subject.

CHAPTER 3
METHODOLOGY

3.1. Introduction

This chapter presents the research methodology, discussing how data was collected and analysed. The other elements to be disused under this chapter include research design, population and sample, data collection methods, validity and reliability, data collection procedure, methods of data analysis and ethical considerations.

3.2. Research Design

A research design is a detailed outline of how an investigation will take place. Gay (1996) defines the design of a study as "basically the overall approach used to investigate the problem of interest, i.e.; to answer the question of interest. It includes the method of data collection and related specific strategies" (Gay, 1996, 218). A research design will typically include how data is to be collected, what instruments will be employed, how the instruments will be used and the intended means for analysing data collected. I decided to use a qualitative research design in this study.

3.2.1. Qualitative Research Design

A qualitative research design was preferred. There are different interpretations and definitions for qualitative research in all educational research. It is alternatively called naturalistic inquiry, field study, case study, participant observation and ethnography (Bryman, 2008; Merriam, 1998; Yin, 2003). According to Maxwell (2012), there are five purposes for qualitative research: to understand the meaning of the event, situations and actions involved; to understand the context within which the participants act; to identify unanticipated phenomenon and to generate new grounded theories; to understand the process by which events and actions take place and; to develop casual explanations. Primarily, this study intends to investigate computational strategies students apply in mathematical problem-solving in school and out-of-school settings.

Qualitative research provides depth and detail through direct quotations and descriptions of situations, events, interactions and observed behaviours (Labuschagne, 2003). In my study, I collected data by observing, interviewing and testing the students both in schools and out-of-school settings such as markets where they conducted their transactions. Creswell (2008), argues that qualitative researchers collect data in the field site implying that they do not bring individuals in a contrived situation. The qualitative research design will enable me to understand the context within which the students act for them to apply a particular strategy.

3.3. Population and Sample

A population is a group of elements or cases, whether individuals, objects or events that conform to particular criteria and to which we intend to generalise the results of research (McMillian & Schumacher 2001: 169). A population can also be defined as a group of individuals, objects or items from which samples are taken for measurement. A sample is a portion, piece, or segment that is representative of a whole. The school under this study has a population of about 1000 pupils. The sample consisted of four grade 8 students, all aged 13 years. The students included in the sample were selected mainly on the criteria that they came from homes where their parents conducted some form of business transactions to help them raise income to meet their daily needs. In this study, the business transactions identified were selling commodities in various places such as streets, markets and shops. It is common for sons and daughters to help their parents in their businesses. For instance, there could be situations when parents are busy with a customer, the child is expected help by attending to other customers or when the parents are away on some business errand. In their work, these children must solve many mathematical problems usually without recourse to paper and pencil.

3.4. Data Collection Methods

Cohen et al (2011) argues that qualitative research ‘uses rigorous procedures and multiple methods for data collection (Cohen et al, 2011, 226). In qualitative research, interviewing, observations and document analysis are the major source of the qualitative data for understanding the phenomenon under study (Drew, Hardman, and Hosp, 2008; Fontana and Frey, 2005). Data collection methods in this study included observations, semi-structured interviews and testing. Additionally, secondary sources of data were also used in this research. The combination of these different data collection methods according to Matthews and Ross (2010) enable the researcher to get a holistic picture of the subject matter under consideration. Limb and Dywer (2001) argue that these qualitative research approaches help researchers to understand life experiences and the collective meanings of the everyday lives of people. The combination of the various data collection methods enabled me to identify the computational strategies students apply in mathematics problem-solving in school and out-of-school settings.

3.5. Data collection procedure

I had a meeting with the school management at the target school where I explained the objectives of my study and how I intended to carry it out. I was granted permission to go ahead and meet the students so that I could pick the desired students for my study. I first observed the students in their classrooms where I was introduced by their teacher as someone who was interested in how children solve arithmetic problems. The observations in the classroom were important for me to familiarise myself with them and then hold informal discussions to get a broad picture of the nature of their daily activities that have aspects of mathematics and the nature and extent of their knowledge of everyday mathematics, and to get an initial understanding of the variation among children of out-of-school mathematical knowledge, as well as involvement in economic activity. I identified four students who took part in the study and gave out written consent forms to the students for their parents/guardian in the presence of their teachers of mathematics.

3.5.1. Observations in transaction places

All the four students identified in the study helped their parents sell various items ranging from food stuffs, groceries to Airtime. The first approach I undertook was observation in the transaction places, where the students interacted with customers as they conducted their business transactions. Observations involve collecting qualitative information about human actions and behaviours in social activities and events in a real social environment (Cohen et al., 2011; Neuman, 2007). They also involve the use of the human senses such as "sight, touch, smell, hearing and taste" to collect data (Mathews and Ross, 2010, 255). Through observation, I learnt more about the activities of the students under the study in their natural settings as I could check for non-verbal expression of feelings, determine who interacted with whom, grasp how participants communicated with each other, and check for how much time was spent on various activities.

There are two main observation strategies: participant observation and non-participant observation (Bryman, 2008; Cohen et al., 2011; Johnson & Christensen, 2012). Participant observation is when the researcher becomes part of the group under study and participates in everyday social activities of that social system to obtain the actual feelings and experiences of the phenomena while at the same time taking notes of the actions and behaviours of the participants. The observer as a participant can inform the participants of the study about their participation in the social activity (Bryman, 2008; Cohen, Manion& Morrison, 2011). In contrast, a non-participant observation technique involves the researcher sitting or standing on the side while social activities are taking place (Bryman, 2008; Cohen et al., 2011). I was also able to record videos of the students' interactions with customers during my role as a non-participant observer. Video recording can offer a more unfiltered observational record than human observation (Simpson and Tuson, 2003: 51).

Video recording was chosen because the use of mechanical recording devices usually gives greater flexibility than observations done by hand (Smith, 1981). While I could have used audio tape to capture pupils' talk, this, on its own, would not have been enough to answer a research question focused on students' computational strategies in and outside-school, since it is important to recognise the potential ambiguities introduced by an analysis based upon words alone (Edwards and Westgate, 1987). The video served to provide context, together with the opportunity to search for meaning in the dialogue and the actions before and after any specific utterance. Video recording enables several playbacks to be conducted, to scrutinise the data more fully.

3.5.2. Interviews in transaction places

I decided to use interviews for data collection to investigate ideas and beliefs of students further and to gather data which may not have been obtained by other methods such as observation in the field (Cohen et al., 2007; Shaughnessy, 2007). I used interviews also to try to verify some of the observational data and partly to add breadth by obtaining information which could not be collected reliably through observation. The interview is a flexible tool for data collection, enabling multi-sensory channels to be used: verbal, non-verbal, spoken and heard (Cohen et al, 2011). According to Matthews & Ross (2010), interviews enable the interviewer to get the experiences and views of the person being interviewed through a dialogue (Matthews & Ross, 2010). Important aspects in interviews include maintaining a relaxed manner, asking clear questions, note-taking, appropriate use of follow-up question or probes, establishing trust, and keeping track of responses (Cohen, Manion and Morrison, 2000; Drew et al, 2007). In this study, interviews ranged from informal conversations to semi-structured interviews.

I used informal conversations when I first met the students who took part in the study for being familiar with them. These informal conversations consisted of general and open-ended questions which helped me obtain their bio-data and specific information about the students' lifestyles since they could talk about their lives freely.

I conducted semi-structured interviews with the students in the various transaction places place where they conducted their business transactions. The researcher decided to use the semi-structured interviews because it is flexible, allowing new questions to be brought up during the interview as a result of what the interviewee says. Semi-structured interview can be defined as "a two-person conversation initiated by the interviewer, for the specific purpose of obtaining research, relevant information as specified by research objectives of systematic description, or explanation (Cohen and Manion, 1997 as cited in Muzumara, 1998: 51). The researcher in a semi-structured interview generally has a framework of themes to be explored. However, the specific topic or topics that the researcher wants to explore during the interview should be thought about well in advance. It is generally beneficial for the researcher to have an interview guide prepared which is informal '' grouping of topics and questions that the researcher can ask in different ways for different participants. The interview guide helps the researchers to focus on an interview on the topic at hand without constraining them to a particular format. This freedom can help the researchers to tailor their questions to the interview context or situation to the people they are interviewing.

The semi-structured interviews were administered in the students' native language, and they were video recorded along with students' explanations of the procedures used for obtaining the answer. The use of the students' native language was to ensure that they do not lose ground conceptually as the study was intended to collect data as natural as possible and allowing students to express their views freely in their own terms. I posed questions to do with the prices of the commodities and the procedures they used to transact. The questions used in the interviews varied from student to student because they were involved in different activities but what guided the researcher was to ask questions to do with addition, subtraction and multiplication. The flow was driven by the students' responses.

3.5.3. Testing

I used formal tests in my study to collect data about the computational strategies of the students in school. Formal testing, as the name implies, are formal ways of finding out how much a student has learnt or improved during the instructional period. I administered formal tests to the students on a separate occasion a few days after the informal test was conducted. Informal testing was used to collect data about the students' computational strategies in their work places that is the places where they conducted their transactions and it was done through observations and interviews as discussed earlier. The informal test items were presented during normal sales transactions where I posed as a customer and sometimes carried out some purchases and in the process, I asked the students to perform calculations on how they were making transactions. I prepared the items for the formal tests for each student based on the problems that they solved during the informal test. Each problem solved in the informal test was mathematically represented according to the students' problem-solving routine. Paper and pencil were available on the desk, but the students were free to use whatever procedure they wanted when solving problems. Testing enabled me as a researcher to identify students' behaviours and documented their performance. The tests were conducted in a classroom as shown in the photo below:

Students writing a formal test

3.6. Validity and Reliability

The credibility of any research relies on its validity and reliability. Since I decided to use qualitative research methods which are regarded subjective, it was important to ensure that my study is valid and reliable. For instance, the field notes are purely subjective because they are my opinion about what I gathered during the observations and interviews.

3.6.1. Validity

Validity of a qualitative design refers to the degree to which the interpretations have mutual meanings for the participant and the researcher. The validity of this study is whether I will get a true picture of the students' computational strategies in-school and out-of-school settings. Validity for this study was ensured by using multiple data collection methods, some of which were in the natural setting, that was in the transaction places such as markets and shops. Validity was also ensured by me taking part as a participant-observer as I became part of the group under study and participated in their activities to obtain the actual feelings and experiences of the phenomena while at the same time I took notes of the actions and behaviours of the participants. The data under this study included descriptive and reflective field notes, transcribed video recorded interviews as well as problem solutions. This ensures validity of the study.

3.6.2. Reliability

Reliability refers to the consistency between independent measurements of the same phenomenon (Muzumara,1998, 49). In ensuring that my study is reliable, I was consistent in how I was handling my data. For instance, I had specific questions which were answered by the students since I used semi-structured interviews and also in my field notes I was making specific observations. Reliability implies that, the same methods used by different researchers at different times under similar conditions should yield the same results.

3.7. Limitations

There was less time allocated for data collection which made it difficult to follow up as many students as possible. The nature of the study was to follow up the students to their work places for observations and interviews. Due to limited time, only four students from one school took part in the study implying that generalisability of the results is limited. Patton says, “while one cannot generalise from a single case or a small sample, one can learn from them and learn a great deal, often opening up new territories for further research” (Patton, 2002, 46). I feel the study would have been more representative if many students took part and many schools were involved.

3.8. Ethical Considerations

Ethical Considerations is one of the most important parts of any research, hence, in this section I will discuss how I addressed ethical considerations aspect of my study. I secured permission to carry out my study in the school from the Ministry of Education through the school administration. After I identified the four students, I asked them if they were willing to take part in the study to which they consented and I gave out written consent forms to the students for their parents/guardian in the presence of their mathematics teachers. According to Cohen et al (2007), informed consent is “the procedures in which individuals choose whether to participate in an investigation after being informed of facts that would be likely to influence their decisions” (Cohen et al., 2007, 52). I avoided the use of offensive, discriminatory, or other unacceptable language in the formulation of Interview schedules since one of the data collection methods I used was the semi-structured interviews. In my study, I have used pseudonyms to represent the students who took part in the study to ensure their privacy and anonymity. I have also acknowledged works of other authors used in any part of the study.

3.9. Methods of data analysis

Qualitative data analysis involves organizing, accounting and explaining the data; in short, making sense of data in terms of the participants’ definitions of the situation, noting patterns, themes, categories and regularities (Cohen, Manion, & Morrison, 2011, 537). Qualitative research uses analytical categories to describe and explain social phenomena. In my study, I will derive these categories inductively that is as they will emerge from the data. Qualitative modes of data analysis provide ways of discerning, examining, comparing and contrasting, and interpreting meaningful patterns or themes. Meaningfulness is determined by the goals and objectives of the study.

The data in my study includes descriptive and reflective field notes, transcribed video interviews as well as problem solutions for the students. Transcripts and notes will provide a descriptive record of my study since they are raw data. I will analyse the computational strategies applied in school and out-of-school by each of the four students who took part in the study. The strategies in school will be analysed by carefully reading and rereading the problem solutions by the students identifying the common strategies used. The out-of-school strategies will be analysed by using a combination of the field notes taken during observations and interviews, transcribed video recordings and playing back the videos taken during the study.

CHAPTER 4

PRESENTATION OF MY DATA FINDINGS

4.1. Introduction

This chapter presents the findings of the study according to the research questions. The interactions with the students both in out-of-school and school settings revealed that a variety of computational strategies were used in solving mathematical problems. My data was obtained from descriptive and reflective field notes, transcribed video interviews as well as problem solutions of the students. The data for the in-school strategies was obtained from the formal tests and students' answer scripts and the data for the out-of-school strategies was obtained from the transcribed video interviews field notes taken during observations. The following pseudonyms: Student 1(S1), Student 2(S2), Student 3(S3) and Student 4(S4) will be used in the presentation of my data for the four students who were involved in the study. The use of the pseudonyms was in order to protect the identity of the students and the school.

4.2. Case of Student 1(S1)

Student 1(S1) helped his parents sell various commodities at a stand in the market. The commodities that he sold ranged from tomatoes, kapenta, eggs, onion, cooking oil and beans. During the interview, S1 was confident and quick in giving out responses.

Student 1(S1) selling various commodities at the market

Dialogue 1:

The dialogue below is based on the first interview with student 1(S1) in the field:

Me: How much does one egg cost?

S1: It costs K1.50

Me: How much should a customer pay for three eggs?

S1: A customer should pay K4.50

Me: How did you get that answer (K4.50)?

S1: I first separated K1.50 into K1 and 50n and then added the K1s separately and also 50 ngwees separately and then added the sums I got together.

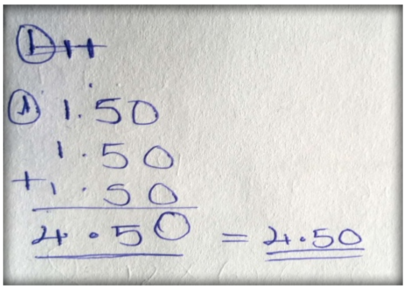
Me: Is there any method other than the one you used to find the K4.50?

S1: When I added together the cost of three eggs it gave me K4.50

Me: Can multiplication be used to get the same answer?

S1: No, I just know how to add as I said earlier on

In line with the first dialogue, Student 1(S1) was asked to find the total cost of three eggs at K1.50 in the formal test. The solution of the student is shown below:



Vertical addition with regrouping

Dialogue 2:

The dialogue below is based on the second interview with student 1(S1) in the field:

Me: A customer buys a bottle of cooking oil worth K15 from a K100 note, how much should you give them back as change?

S1: I will give them back K85

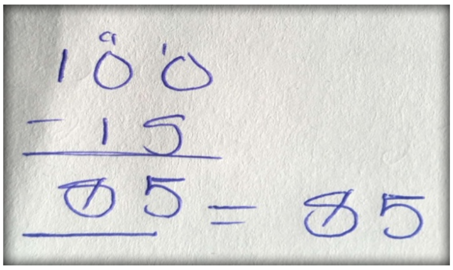
Me: How did you come up with K85?

S1: I started counting from K15 up to K100 to get K85

Me: How did you do that?

S1: I started counting from K15 using K10, that is 25, 35, 45,...95, then added 5 to get to 100.

In line with the second dialogue, Student 1(S1) was asked to find the change for a customer who bought a bottle of cooking oil at K15 from K100. The solution for S1 is shown below:



Vertical subtraction with regrouping

Dialogue 3:

The dialogue below is based on the third interview with student 1(S1) in the field:

Me: Three customers, each of them buys Kapenta worth K6 each. How much should they pay altogether?

S1: They are supposed to give me K12. Sorry you said each one of them bought three packets?

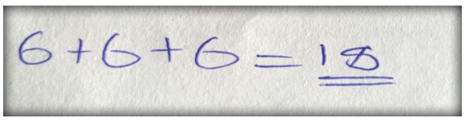
Me: Yes

S1: Ok then they should give me K18

Me: How did you get the K18?

S1: I added 6 plus 6 plus 6 to get K18

In line with the third dialogue, Student 1(S1) was asked to find change of a customer who bought a bottle of cooking oil at K15 from K100. The solution is shown below:



Sums algorithm (horizontal

4.3. Case of Student 2 (S2)

Student 2 (S2) helped his parents sell popcorns outside a small store at the market. When S2 was interviewed at the stand where he was selling popcorns was quick in giving out responses.

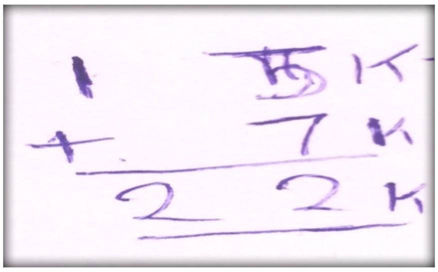
Student 2 (S2) selling Popcorns at the market

Dialogue 1:

The dialogue below is based on the first interview with student 2(S2) in the field:

> Me: Two customers buy packets of popcorn, the first buys for K15, and the other one buys for K7. How much are they supposed to pay you?
>
> S2: They are supposed to give me K22 altogether
>
> Me: How did you get the K22?
>
> S2: I first added 15 and 5 to get 20 and then added 2 to the 20 which gave me 22

In line with the first dialogue, Student 2(S2) was asked to find the total cost of two customers who buy popcorns worth K15 and K7 respectively. The solution is shown below:



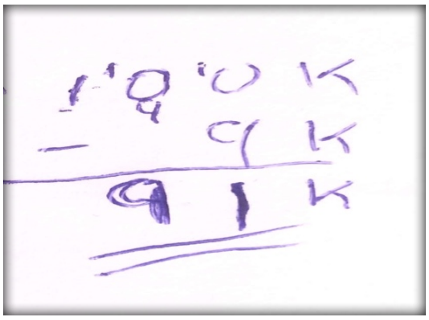
Vertical addition with regrouping

Dialogue 2:

The dialogue below is based on the second interview with student 2(S2) in the field:

> Me: If a customer buys packets of popcorn worth K9 from a K100 note, how much will they get back as change?
>
> S2: They would get back K91.
>
> Me: How did you find the answer?
>
> S2: I first added 1 to 9 to make it 10, then I subtracted 10 from 100 to get 90. I then added the 1 added to 9 to the 90 to get K91.

In line with the second dialogue 2, Student 2(S2) was asked in the formal test to find the change for a customer who buys popcorns worth K9 from K100. The solution is shown below:



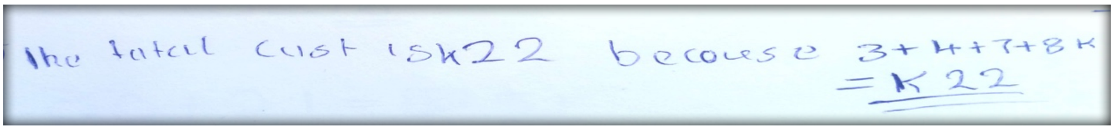
Vertical subtraction with regrouping

Dialogue 3:

The dialogue below is based on the third interview with student 2(S2) in the field:

> Me: Four customers come to buy packets of pop corns, the first buys for K3, the second buys for K4, the third buys for K7 and the fourth buys for K8. How much are they supposed to pay?
> S2: They are supposed to pay K22 altogether
> Me: How did you do that?
> S2: I added 3 + 4 + 7 + 8
> Me: Explain to me how you added
> S2: I just added in my head

In line with the third dialogue, Student 2(S2) was asked in the formal test to find the total cost of four customers who buy popcorns for K3, K4, K7 and K8 respectively. The solution is shown below:



4.4. Case of student 3 (S3)

Student 3(S3) helped her parents sell various goods in a small shop at the market. S3 lacked confidence and took a bit of some time to give out answers when he was being interviewed.

Student 3 (S3) selling various items in a shop

Dialogue 1:

The dialogue below is based on the first interview with student 3(S3) in the field:

Me: How much does two bottles of Super Shake Maheu cost at K7 each?

S3: It will be K14

Me: How did you get hat K14?

S3: I multiplied 2 by 7

Me: Which other way would you have used to arrive at the same answer?

S3: I would have used addition

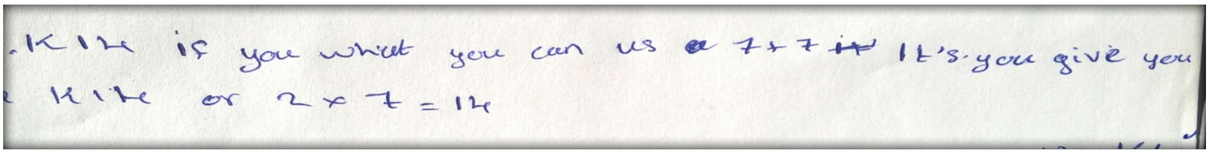
Me: What would you have added?

S3: I would have added 7 and 7 to get K14

Me: Why didn't you use multiplication to solve the first question of pens?

S3: It cannot work because the cost of one pen is K1

In line with the first dialogue, Student 3(S3) was asked in the formal test to find the total cost of a customer who buys two bottles of Super Shake Maheu drink each costing K7. The solution is shown below:


K14 is you what you can us 7+7 It's you give you K14 or 2×7 = 14

Dialogue 2:

The dialogue below is based on the second interview with student 3(S3) in the field:

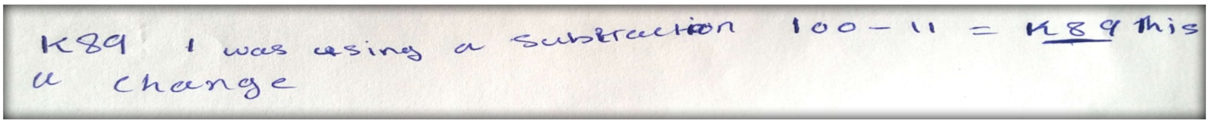
Me: If one buys a packet of Boom Washing powder at K11 from a K100. How much would the customer get back?

S3: The customer would get back K89

Me: How did you get that?

S3: I first subtracted 1 from 11 to make it 10 and then subtracted the 10 from 100 to get 90. Since I subtracted 1 from 11, I had to subtract 1 from the 90 to get K89.

In line with the second dialogue, Student 3(S3) was asked in the formal test to find change for a customer who bought a packet of Boom Washing Soap at K11 from K100. The solution is shown below:


K89 I was using a subtraction 100 − 11 = K89 this a change

Dialogue 3:

The dialogue below is based on the third interview with student 3(S3) in the field:

Me: If a customer buys three soft drinks at K3 each from a K50, how much will be their change?

S3: They are supposed to get K41 back

Me: How did you get that?

S3: I subtracted K9 from K50

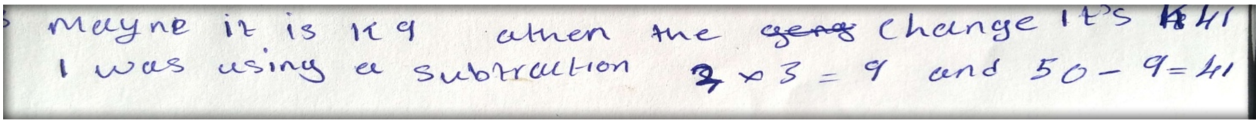
Me: Where has the K9 come from?

S3: To get the K9, I multiplied 3 by 3 which gave me 9 and then subtracted it from K50.

Me: How did you subtract the 9 from 50?

S3: I added 1 to 9 and subtracted 10 from 50

In line with the third dialogue, Student 3(S3) was asked in the formal test to find change for a customer who bought three Coca cola drinks at K3 each from K50. The solution is shown below:


Mayne it is K9 athen the change it's K41
I was using a subtraction 3 x 3 = 9 and 50 - 9 = 41

4.5. Case of student 4 (S4)

Student 4(S4) helped her parents sell scones during break time at school. S4 was slow in responding when interviewed.

Student 4 (S4) selling Scones and Freezits at school during break

Dialogue 1:

The dialogue below is based on the first interview with student 4(S4) in the field:

Me: Each of the three customers buys scones worth K5. How much are they supposed to buy altogether?

S4: They are supposed to pay K15 altogether

Me: How did you find that?

S4: I added 5 plus 5 plus 5 to get 15

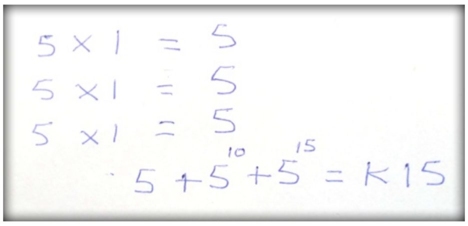
Me: Why did add 5 to itself three times?

S4: Since there are three customers who bought equal amounts of K5 each

Me: Is there any other method that could have been used to find the answer?

S4: I could have used multiplication

In line with the first dialogue, Student 4(S4) was asked in the formal test to find the total cost for three customers who buy scones worth K5 each. The solution is shown below:



Sums Algorithm (horizontal)

Dialogue 2:

The dialogue below is based on the second interview with student 4(S4) in the field:

Me: If a customer buys scones for K3 from K20. How much will they get back as their change?

S4: They will get back K17 as change

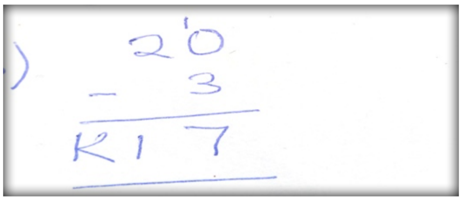
Me: How did you get that?

S4: I subtracted K3 from K20

Me: How did you do the subtraction?

S4: I counted from 3 to 20 and I got 17

In line with the second dialogue, Student 4(S4) was asked in the formal test to find change for a customer who bought scones for K3 from K20. The solution is shown below:



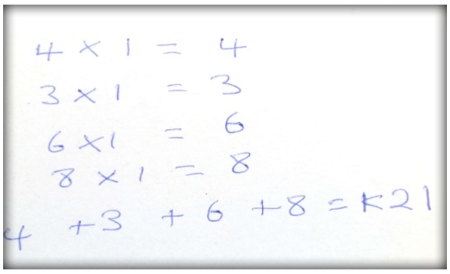
Vertical Subtraction with Grouping

Dialogue 3:

The dialogue below is based on the third interview with student 4(S4) in the field:

> Me: Four customers come to buy scones. The first buys for K4, the second buys for K3, the third buys for K6 and the fourth buys for K8. What will be their total bill?
> S4: It will be K21
> Me: How did you calculate that?
> S4: I added 4 + 3 + 6 + 8 and I got K21

In line with the third dialogue, Student 4(S4) was asked in the formal test to find the total cost of four customers who bought scones for K3, K4, K6 and K8 respectively. The solution is shown below:



Sums Algorithm (horizontal)

CHAPTER 5

DATA ANALYSIS

5.1. Introduction

This chapter presents the analysis of my data findings. The data will be analysed by describing and comparing the computational strategies of the Grade 8 students in school and out-of-school settings. The purpose of the present study is to describe and compare the computational strategies used by grade 8 students while solving similar problems in school and out-of-school settings. After careful scanning of my data, I came up with three categories which will be used for my data analysis. The categories were mental computational strategies, algorithms and memorisation of mathematics facts. This classification was based on my observations of the students interacting with customers in the market places, the interviews I had with the students while conducting their business transactions and also their calculations as exhibited on the answer scripts obtained from the formal test which was administered in the school setting.

In analysing my data findings, I will maintain the pseudonyms I used in the previous chapter where I presented my data and these are Student 1(S1), Student 2(S2), Student 3(S3) and Student 4(S4). It is important to note that each of the students in the study hard three interviews dubbed as dialogue 1, dialogue 2 and dialogue 3 in the data findings chapter (Chapter 4) and each dialogue was followed by a question which was posed in the formal test followed by the solution by the student for that particular question.

5.2. Mental computational strategies

Mental computational strategies in mathematics refer to problem-solving strategies which involve doing calculations mentally or all in one's head. Mental mathematics is an extremely common and practical skill which makes it easy for people to complete ordinary daily tasks. Most students in the study used mental computational strategies to solve problems in the out-of-school settings. Computations in the natural situation of the informal test were in all cases carried out mentally, without recourse to external memory aids for partial results or intermediary steps. This is in line with Nunes et al (1993) who stated that problems presented and addressed orally in the streets were more easily solved than those included in the more formal test in which pencil and paper were available. Students made calculations in their minds without the guidance of pencil and paper, calculators or other aids. Mental mathematics is often used as a way to calculate and estimate quickly, using mathematics facts that a student has committed to memory. It was initially difficult to classify into categories the mental strategies the students used but what was common was that the students transformed the problems into simpler ways in which they could manipulate easily. However, after careful scanning of the strategies, the following types of computational strategies were identified: decomposition, counting-up, compensation, repeated addition and memorisation of mathematics facts.

5.2.1. Decomposition

Students used decomposition strategy which is sometimes known as break up strategy to solve addition problems especially when regrouping was required. It involves working with quantities smaller than those mentioned in the problem. The students' reasoning when using decomposition was that a number is composed of parts that can be separated without changing its value and that addition can be carried out on the parts and the final result will not be affected. When using decomposition, one of the addends is broken up into its expanded form and added in parts to the other addend. The students demonstrated knowledge of the number system where they seemed to construct sums through their own mental actions of putting numbers into relationships.

My findings show that student 1(S1) and student 2(2) used decomposition method to the addition problems in the out-of-school settings as shown in the dialogues. To find the cost of three eggs at K1.50 each, according to dialogue 1of S1, S1 decomposed K1.50 into K1 and 50 ngwees and added K1 + K1 + K1 separately and got K3 and 50 ngwee + 50 ngwee to get K1. Then added the K3 and K1 to get K4 and finally added the remaining 50 ngwee to the K4 to get the total cost of K4.50. According to dialogue 1 of S2, S2 decomposed 7 into 5 and 2 and decided to maintain the 15 because it was easy to add 5 to 15 to get 20 and then add the 2 to 20 to get 22. As seen from both cases of S1 and S2, they both avoided the idea of regrouping.

5.2.2. Counting-up or Adding-up

Students used counting-up or adding-up strategy to approach subtraction problems in the out-of-school settings. They started counting with smaller addend and counted up to larger ones. In order to determine the change, they counted the number of counts and that was the amount of change which was given. Although this strategy could be confusing especially when dealing with large numbers, the students demonstrated their ability to use their number sense as we will see in some of the examples which will be given. My findings showed that student 1(S1) and student 4(S4) used counting-up strategy to solve subtraction problems. This finding is in agreement with the study of Nunes et al (1993). According to dialogue 2 of S1, S1 first added 5 to 15 to get 20 because it was easier to count up from 20 using multiples of 10. Then he counted up from 20 to 100 and realised that he had 8 counts which was 80. To get the final answer, he added the 5 which was added to 15 to 80 to make 85 which was the change. As can be seen from the explanation, S1 had a basic understanding of number sense because he was able to bring in the idea of multiples and compensation. According to dialogue 2 of S4, S4 easily counted up from 3 to 20 and had 17 counts which was the customer's change. As earlier mentioned when the numbers involved in the subtraction problem are small, the strategy works so easily as seen in the case of S4.

5.2.3. Compensation

Students used compensation method to solve subtraction problems. They transformed one of the numbers in the subtraction problem to make it easier work with by adding or subtracting the same number from or to the number. The goal of adding or subtracting a number was to make one or more of the numbers easier to work with. My findings show that Student 2(S2) and Student 3(S3) used compensation method to solve subtraction problems. According to dialogue 2 of S2, S2 first added 1 to 9 to get 10 and then subtracted 10 from 100 to get 90. S2 decided to work with 10 because it was easier to work with than the 9. The 1 which was added to 9 was compensated by adding it to 90 which was the difference. According to dialogue 2 of S3, S3 first subtracted 1 from 11 to make 10 and then subtracted the 10 from 100 since it was easy to work with 10. In order to get the final answer, S3 compensated the 1 by subtracting it from 90 to make it K89. It can be seen in both cases for S2 and S3, the idea was to avoid regrouping which is difficult to apply when using mental strategies.

5.2.4. Repeated addition

Students used repeated addition as an alternative method to multiplication in out-of-school settings. This is in agreement with Nunes et al (1993) study in which they stated that students used repeated addition instead of multiplication. Multiplication is defined as meaning that you have a certain number of groups of the same size. It is this property that makes it possible to use repeated addition in place of multiplication. Repeated addition means adding the same number over and over again or in other words successive additions. My findings show that S1and S4 used repeated addition to calculate instead of using multiplication. According to dialogue 3 of S1, S1 added 6 to itself 3 times to get 18 because there were three customers buying the same amount of Kapenta. According to dialogue 1 of S4, S4 added 5 to itself three times to get the 15.

5.2.5. Memorisation of mathematics facts

Students used memorised mathematics facts to solve some mathematical problems in the out-of-school settings though memorisation of mathematics facts is a believed to be a school learnt computational strategy. The use of memorised mathematics facts was demonstrated by the students' failure to explain the procedures they used to compute some mathematical problems in the out-of-school settings as can be seen in the following interviews with:

My findings show that S2 and S3 used memorised mathematics facts to solve mathematical problems in the out-of-school settings. S2 used memorised addition facts to find the total cost of the four customers. This was evident in dialogue 3 of S2 when he was asked to explain how he did the addition and said he just added. S3 used two ways to calculate the cost of two bottles of Maheu. According to dialogue 1 of S3, S3 mentioned to have used two basic mathematics facts to calculate the cost of the Maheu. He first used multiplication facts to multiply 2 by 7 to get 14 and then later used addition facts to add 7 and 7 to get the same14. When S3 was further asked to explain how they calculated using any of the two methods, he kept on switching between the two operations suggesting he had an idea of repeated addition in mind. It can be seen from both examples that the students did not explain the procedure properly how they arrived at the answers. All they could do was to state the mathematical operation which they used without explaining the procedure used.

5.3. Algorithms

An algorithm specifies a series of steps that perform a particular computation or task. When the problem solutions of the students which were used in the formal test were carefully scanned, written algorithms and came out to be the common strategies that were used by the students in the school settings. The school-linked algorithms were mainly used to solve addition and subtraction problems in the formal test.

5.3.1. Addition Partial – Sums Algorithm (horizontal)

Students used horizontal partial sums algorithms to add numbers horizontally. This strategy works when working with numbers which have the same place value especially small numbers. S1, S2 and S4 use horizontal partial sums algorithms to solve addition problems as can be seen from the solution under dialogue 3 of S1, dialogue 3 of S2 and dialogue 1 of S4 respectively.

5.3.2. Place Value Column Addition

Students used place value column addition to solve addition problems in the school settings. This strategy works on the principle that the numbers to be added have to be written according to their place values in such a way that place values fall under the same column. My findings show that S1 and S2 used place value column addition with trading as can be seen from the solution under dialogue 1 of S1 and dialogue 1 of S2 respectively. To find the total cost of three eggs at K1.50 each, S1 started by adding numbers in the hundredth-place value that is 0 + 0 + 0 and got 0. Then added 5 + 5 + 5 in the tenth-place value and 15 tenth which is 1 one and 5 tenth. S1 carried over the 1 one added it to the numbers in the ones as 1 + 1 + 1 + 1 and got 4. That's how S1 got K4.50 as the total cost of the three eggs. To find the total bill of the two customers S2 added 5 + 7 to get 12, since 12 is equal to 1 ten and 2 ones, then took 1 ten to the tens column and left 2 ones in its unit column. S2 carried over 1 ten and added to the 1 ten in tens column to get 2 tens. The addition in both cases involved regrouping (trading). Regrouping comes in when the sum of the digits in the given column goes beyond that place value.

5.3.3. Place Value Column Subtraction

Students used place value column subtraction to solve subtraction problems in the school settings. This strategy works on the same principle as that in the addition where the numbers to be subtracted have to be written according to their place values in such a way that place values fall under the same column. My findings show that S1, S2 and S4 used vertical subtraction with regrouping as can be seen the solutions under dialogue 2 of S1, dialogue 2 of S2 and dialogue 2 of S4 respectively. S1 used place value column subtraction to find the change for a customer. Since in the first column, the subtrahend is greater than the minuend and that there are no groups of ten in the tens, S1 took one group of hundreds from the hundreds and added it to 0. That made it ten groups of ten, and further took one group of tens from the tens and added it to 0, then it made ten ones. That's how it was possible for S1 to subtract 5 from 10 to get 5 and then subtracted 1 from 9 to get 8 since there were nine groups of tens remaining in the tens. S2 also used place value column subtraction with trading because it was not feasible to take away 9 from 0. So S2 traded tens with hundreds and ones with tens. S2 traded 1 hundred to the tens and then 1 ten to the ones. S2 subtracted 9 ones from 10 ones to get 1 one, while 0 ten from 9 tens to get 9 tens. S2 got K91 to be the change for the customers. Just like S1 and S2, S4 used place value vertical subtraction with trading because it was not feasible to take 3 from 0. S4 traded ones with tens by taking 1 ten from the tens and added it to 0 one and got 10 ones and then subtracted 3 ones from the10 ones to get 7. Since there was nothing remaining in the tens for the subtrahend, S4 subtracted 0 from 1 to get 1.

5.4. Memorisation of mathematics facts

Memorisation of mathematics facts is a teaching and learning strategy where teachers encourage learners to master mathematical operations of addition, subtraction and multiplication in the early years of elementary school. Students used memorised mathematics facts to solve problems in the school setting. Memorisation of mathematics facts were used to solve addition, subtraction and multiplication problems in the formal test. This could be seen by the students' failure to show proper step by step procedures in solving problems which were given in the formal test. Student 3(S3) used both addition, subtraction and multiplication facts to solve problems in the formal test as seen in dialogue 1, dialogue 2 and dialogue 3 of S3. To find the total cost of two bottles of Maheu drink, S3 wrote 7 + 7 =14 and also 7 x 2 = 14 without showing the proper procedures used to get the answers suggesting that memorised mathematical facts were used. In the other instance, to find change for the customer who bought a packet of Boom washing soap at K11 from a K100, S3 used memorised subtraction facts as S3 just wrote K100 – 11 = K89 without showing the step by step procedure used to find the answer. S3 to find the change for a customer bought three soft drinks at K3 each from a K50, he first used multiplication facts and then subtraction facts. S3 first showed that 3 x 3 = 9 and then wrote K50 – K9 = K41. S3 did not show the procedure meaning that he calculated mentally.

CHAPTER SIX

DISCUSSION, CONCLUSION, RECOMMENDATIONS AND ENDING REMARKS

6.1. Introduction

This chapter presents the discussion of my findings, my concluding remarks, recommendations and my ending remarks.

6.2. Discussion

The findings of my study indicate that students employed different computational strategies in out-of-school setting and when solving problems in a school-like formal setting. They used mental computational strategies in out-of-school settings and used conventional written algorithms in school settings. The findings also point out that students used memorised mathematics facts in both out-of-school and school settings.

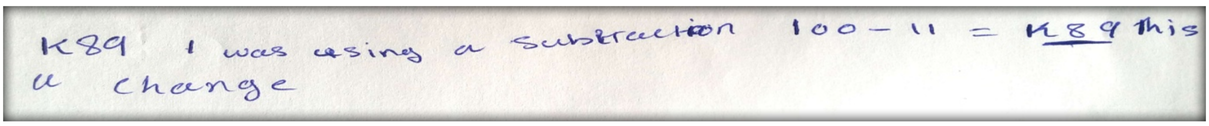
My findings confirm the claim by situated cognition theories that knowledge is used in the context in which it is acquired, it could be in out-of-school or in school setting. The problems of the lack of transfer of school-learned concepts, methods and skills to other areas of activity, together with the propensity for individuals to invent or construct their own ways of solving the problems they encounter in life, have together led to the theoretical standpoint that all learning is essentially situated. Situated cognition theories advocate that knowledge is not independent but, rather, fundamentally "situated," being a product of the activity, context, and culture in which it is developed (Brown et al, 1989). This implies a view toward knowledge construction and use that is related to that of the constructivists (Duffy & Jonassen, 1992). Tools as resources, discourse, and interaction all play a role in producing the dynamic knowledge of situated cognition. Learning and cognition are viewed as being linked to arena and setting, to activity and situation in such a way that they can be said to coproduce each other. Concepts and knowledge are fully known in use, in actual communities of practice, and cannot be understood in any abstract way. Authentic activity are the ordinary activities of a culture (Brown et al., 1989). School activity is seen as inauthentic because it is implicitly framed by one culture, that of the school, but is attributed to another culture, that of a community of practice. Students are exposed to the tools of many academic cultures, but this is done within the all-embracing presence of the school culture. The subtleties of what constitutes authentic and inauthentic activity probably are not as important as the fact that the situation within which activity occurs is a powerful cultural system which coproduces knowledge.

Memorisation of mathematics facts is believed to be a teaching and learning strategy used by teachers to teach mathematics in the early years of elementary school by encouraging learners to master mathematical operations of addition, subtraction and multiplication. The use of memorised mathematics facts in out-of-school settings could have been because the students under my study learnt the skills of memorisation in elementary school since they were in their eighth at the time of the study. The examples show the use of memorised mathematics facts in both settings:

Example 1: The dialogue below is based on the third interview with student 2(S2) in the field:

> Me: Four customers come to buy packets of pop corns, the first buys for K3, the second buys for K4, the third buys for K7 and the fourth buys for K8. How much are they supposed to pay?
> S2: They are supposed to pay K22 altogether
> Me: How did you do that?
> S2: I added 3 + 4 + 7 + 8
> Me: Explain to me how you added
> S2: I just added in my head

Example 2: In line with the second dialogue, Student 3(S3) was asked in the formal test to find change for a customer who bought a packet of Boom Washing Soap at K11 from K100. The solution is shown below:


K89 I was using a subtraction 100 – 11 = K89 this
a change

Example 1 shows that S3 used memorised addition facts because when asked further to explain how 22 was got as the total cost of the four customers, S3 said he just added in the head. It can be seen from example 2 that Student 3(S3) used memorised subtraction facts in the school setting as demonstrated in the working shown. S3 did not show any evidence of having used any strategy to work out the problem, all that he did was to write down that he used subtraction and showed it that he subtracted K89 from K100. The use of memorised mathematics facts in both settings was exhibited by the students' failure to explain how they got the answers in the out-of-school setting and to demonstrate proper step by step algorithms in the school settings.

6.2.1. Comparing mental computations and written algorithms

Comparisons of computational strategies used by the students in out-of-school and school-like settings will be discussed by looking at some of the prominent features of the strategies in both settings. The findings point out that the choice of computational strategies by the students was influenced by the context in which they were acquired just as the situated theories claim. The way social processes influence children's construction of knowledge determines the problem-solving strategies they will adopt.

Knowledge transfer outside school according to situated cognition theories takes place through the learner's participation in a social and material context, such as an apprenticeship or 'guided participation'. This implies that students learn mathematics orally in out-of-school settings by observing and taking part in the sociocultural activities while receiving guidance from the elders. Some of the elders who pass on this kind of knowledge to their children may not have passed through 'corridors' of education as such they depend on their experience that they have gained in a long time dealing with the various socio-cultural activities to meet their daily needs. This mode of knowledge transfer is intimately connected with objects and events; children often use the objects and events directly in their reasoning, without necessarily using symbols to represent them (Resnick, 1987). This is termed as quantity manipulation as opposed to symbol manipulation (Lave, 1991). The use of mental strategies in the out-of-school settings could be attributed to the way knowledge is transferred in that context.

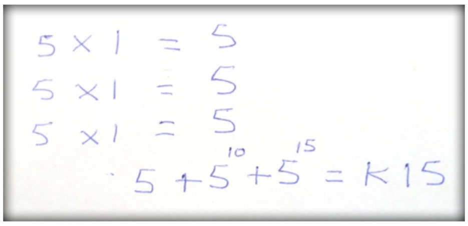
Traditionally, much of the focus of school mathematics has been on teaching algorithms for arithmetic calculation because the original purpose of algorithms in the previous centuries was for clerks to be able to carry out a large number of calculations in a short period of time. Thinking was not the focus, but rather quick and reliable answers. Technology has changed the relative importance of algorithms; some become more important, some less important. Most clerks today, given a large number of calculations, would use either a calculator or pre-prepared spread sheet to carry out these calculations. The use of algorithms in the schools in Zambia by the students could be as a result of the school curriculum which promotes the teaching of mathematics through algorithms. School mathematics is linked to the place-value structure of our notational system for number and to associated procedures for computation such as carrying and borrowing algorithms

The teaching of algorithms in schools dominate the curriculum with concerning effects on both student understanding and self-confidence. The ways in which algorithms are traditionally taught discourage the application of number sense by estimating first or assessing the reasonableness of the answer afterwards. Algorithms tend to blind acceptance of results and over-zealous applications. Given the focus on procedures that require little thinking, children often use an algorithm when it is not necessary as can be seen in the example of Student 4(S4) solving a similar question in both out-of-school and school setting:

The dialogue below is based on the first interview with student 4(S4) in the field:

> Me: Each of the three customers buys scones worth K5. How much are they supposed to buy altogether?
> S4: They are supposed to pay K15 altogether
> Me: How did you find that?
> S4: I added 5 plus 5 plus 5 to get 15
> Me: Why did add 5 to itself three times?
> S4: Since there are three customers who bought equal amounts of K5 each
> Me: Is there any other method that could have been used to find the answer?
> S4: I could have used multiplication

In line with the first dialogue, Student 4(S4) was asked in the formal test to find the total cost for three customers who buy scones worth K5 each. The solution is shown below:



Sums Algorithm (horizontal)

From the example, it can be seen that S4 used mental strategies by simply adding 5 to itself three times but when it came to the formal test, S4 had to show unnecessary steps to justify how she got the answer as shown in the working. This shows that the student lacked confidence by not working it out mentally but instead used an algorithm.

Teachers usually encourage students to master algorithms without them having proper understanding of the underlying basis of the algorithm and students proceed to apply them in solving mathematics problems. The use of mental computation strategies demand individual understanding of place value, number sense and understanding of the meaning of the arithmetic operation and its properties. Algorithms do not correspond to the way people tend to think about numbers; for example, in the context of most conventional algorithms, the '3' in the number 537 is treated as 3 not '30'.

Algorithms are powerful in solving classes of problems, particularly where the computation involves many numbers, where memory maybe overloaded. Mental computations are limited in terms of their capacity to deal with large numbers and certain types of numbers. My findings indicate that students had difficulties to use mental strategies to solve problems when many numbers were involved and found it easy when it came to solving the same problems using paper and pencil in the formal test as shown in the example below:

Dialogue 3:

The dialogue below is based on the third interview with student 1(S1) in the field:

Me: Three customers, each of them buys Kapenta worth K6 each. How much should they pay altogether?

S1: They are supposed to give me K12. Sorry you said each one of them bought three packets?

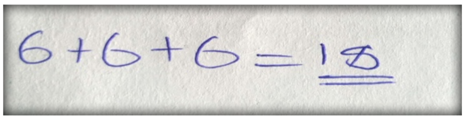
Me: Yes

S1: Ok then they should give me K18

Me: How did you get the K18?

S1: I added 6 plus 6 plus 6 to get K18

In line with the third dialogue, Student 1(S1) was asked to find change of a customer who bought a bottle of cooking oil at K15 from K100. The solution is shown below:



Sums Algorithm (horizontal

From the example, it can be seen that the student was confused at first and gave the answer to be 12 and later corrected it and gave out the correct answer. The confusion could be attributed to the failure by the student to deal with many numbers mentally and did not find a problem when it came to the formal test. In the formal test, the student showed that he used addition and got the answer without difficulties.

Algorithms provide a written record of computation, enabling teachers and students to locate any errors in the algorithm while mental computation strategies are performed orally making them difficult to manage and assess as compared to algorithms. Students can be increasingly encouraged to record the various steps in their calculations, in ways that make sense to them. The danger is not so much with the written form, but the imposition of the teacher’s method for recording, as in algorithms, can have unfortunate consequences. In this way, students are developing gradually refining their own invented algorithms, in conversations with their peers and the teacher.

Students used mental computation strategies to eliminate the need for ‘borrowing and carrying’ which in many cases confuse students by forgetting to reduce the number they have borrowed from or increase the number they have carried to. The ‘borrowing and carrying challenges were not common in my findings because I used Grade 8 students who could have had a better understanding but they are common in primary school students where students begin to learn algorithms.

Students used mental strategies which involved traditional front-end approaches, that is they proceeded from left-to-right order when solving addition and subtraction problems rather than the usual right-to-left order taught in many algorithms in schools. The left-to-right order used by the students in the out-of-school setting, demonstrated students’ understanding of the place-value, number sense and their understanding of the meaning of the arithmetic operations and their properties. The following example demonstrates that student 1(S1) used decomposition in out-of-school setting and used an addition algorithm in the school setting:

Dialogue 1:

The dialogue below is based on the first interview with student 1(S1) in the field:

Me: How much does one egg cost?

S1: It costs K1.50

Me: How much should a customer pay for three eggs?

S1: A customer should pay K4.50

Me: How did you get that answer (K4.50)?

S1: I first separated K1.50 into K1 and 50n and then added the K1s separately and also 50 ngwees separately and then added the sums I got together.

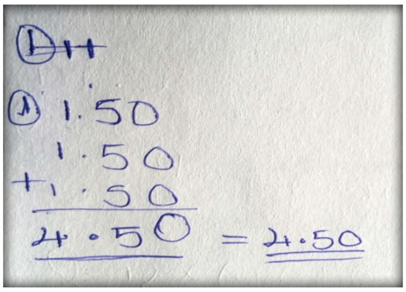
Me: Is there any method other than the one you used to find the K4.50?

S1: When I added together the cost of three eggs it gave me K4.50

Me: Can multiplication be used to get the same answer?

S1: No, I just know how to add as I said earlier on

In line with the first dialogue, Student 1(S1) was asked to find the total cost of three eggs at K1.50 in the formal test. The solution of the student is shown below:



Vertical addition with regrouping

It can be seen from the example that in the out-of-school setting, S1 separated the cost of cost of egg which was K1.50 into K1 and 50ngwee, then started by adding the kwachas (K1+K1+K1) which is the big unit and then added ngwees (50n + 50n + 50n) in a left to right order. In the school setting, S4 used vertical addition with grouping algorithm which operates in a right to left order. When S1 when was asked whether there was any other method that could have been used apart from the decomposition method used to solve the problem, he said there was nothing. This is the effect of encouraging students to use only one method to solve problems, they lose some of their capacity for flexible and creative thought.

6.3. CONCLUSION

The findings from this study make very interesting reading and reinforce important issues for mathematics teaching in Zambia. The study has revealed that the problem-solving routines used were different in the two situations. In the natural situations that is in the out-of-school situations children tended to use mental calculations and used conventional written algorithms in the school settings. There was also evidence by the students that they used memorised mathematics facts in both the out-of-school setting and the school setting. The study has revealed that most students became 'slaves' of algorithms by continuously using them even when it was not necessary because of the association of school mathematics with algorithms. We should be cautious of the fact that teaching of algorithms may result in children giving up their own numerical thinking and becoming dependent on others. Kamii and Dominick (1997) note that "when we try to teach children to make relationships between numbers (logico-mathematical knowledge) by teaching them algorithms (social-conventional knowledge), we redirect their attention from trying to make sense of numbers to remembering procedures" (Kamii and Dominick, 1997, 59).

It is believed that by encouraging students to use only one method (algorithmic) to solve problems, they lose some of their capacity for flexible and creative thought. They become less willing to attempt problems in alternative ways, and they become afraid to take risks. Narode et al. (1993) says, there is a high probability that the students will lose conceptual knowledge in the process of gaining procedural knowledge (Narode, Board and Davenport, 1993). It is important to note that many children are able to do mental computations before they are taught written computations because of their capacity to invent and refine their own strategies of calculation in a classroom where emphasis is put on appropriate number activities. Research over the last several decades has shown that students who are encouraged to use efficient mental computation strategies develop deeper understanding of number relationships. It is important, then, that children learn to apply efficient mental computation strategies.

Reducing the emphasis on complicated paper-and-pencil computations does not mean that paper- and-pencil arithmetic should be eliminated from the school curriculum. Paper-and-pencil skills are practical in certain situations, are not necessarily hard to acquire, and are widely expected as an outcome of elementary education. If taught properly, with understanding but without demands for "mastery" by all students by some fixed time, paper-and-pencil algorithms can reinforce students' understanding of our number system and of the operations themselves. Exploring algorithms can also build estimation and mental arithmetic skills and help students see mathematics as a meaningful and creative subject.

The learner should never be told directly how to perform any operation in arithmetic… Nothing gives scholars so much confidence in their own powers and stimulates them so much to use their efforts as to allow them to pursue their own methods and encourage them in them (Colburn, 1912, 463).

6.4. RECOMMENDATIONS

Based on the findings from this study, the following recommendations were made:

1. The curriculum should include a wide variety of rich problems that build upon the mathematical understanding students have from their everyday experiences, and engage students in doing mathematics in ways that are similar to doing mathematics in out-of-school situations. In most cases, students are not encouraged to and maybe even discouraged from making connections between how they do mathematics in school and how they do mathematics out of school. Students often do not see school mathematics as connected to the real world, and as a result they often do not evaluate solutions to school exercises to see if they make sense. By giving arithmetic a problem-solving focus, and by providing a whole range of problems for children to solve preferably in story contexts of interest to children, from the task of remembering what to do and in what order to do it, to a problem of figuring out why arithmetic rules make sense in the first place.

2. Teachers should also learn more about how parents teach their children at home, and provide parents with more information about how mathematics is taught in school. Teachers should learn more about the importance of children's participation in different social interactions, the different cultural tools they used in their everyday life activities, and the values which they associate with certain cultural tools in certain social contexts. Teachers and school administrators should provide more opportunities for parents who belong to different social, cultural, educational backgrounds to be involved in school-centred parental involvement activities (e.g. communication, decision-making, volunteering etc.) and also recognise and build upon home-centred parental involvement activities such as utilising parents' close relationships with their children and their shared engagement in authentic out-of-school practices.

3. The Ministry of Education in Zambia should work more on developing policies and initiatives which encourage stronger home-school relationships. They also should encourage more collaborative or action research work in schools in order to learn

more about the students' and their families' funds of knowledge and utilise this knowledge to extend and enrich children's learning in school.

4. The Ministry of Education should restructure the curriculum of teacher colleges such that it encompasses techniques that will promote teaching of mental computation strategies rather than entirely promoting teaching of algorithms in schools. The Ministry of Education should also organise more in-service and pre-service training for their teachers in the area of home-school relationships and in learning in different contexts.

5. Mental computation should not be delayed until after formal written algorithms have been mastered. In fact, delaying it until that time encourages students to mentally use the algorithms meant only for pencil-and-paper calculations.

6. In order to incorporate out-of-school mathematics into school mathematics teachers should seek parents' help and support and build upon parents' experiences and knowledge. Parents of all different social, cultural, and educational backgrounds should be involved in their children's education and they should be seen as holding different experiences and knowledge and not deficient. Their knowledge and experiences should be identified, acknowledged and utilised in school learning processes. Teachers should exchange their knowledge with parents' knowledge in a two-way mutual manner. Teachers should also learn more about their children's characteristics, out-of-school activities, and their social, cultural and educational backgrounds. Schools should encourage hard-to-reach parents to be more involved through identifying their needs and concerns and building upon their resources instead of viewing them as deficient.

ENDING REMARKS

The place of algorithms in school mathematics is changing. One reason is the widespread availability of calculators and computers outside of school. Before such machines were invented, the preparation of workers who could carry out complicated computations by hand was an important goal of school mathematics. Employers want workers who can think mathematically. How the school mathematics curriculum should adapt to this new reality is an open question, but it is clear that proficiency at complicated paper-and-pencil computations is far less important outside of school today than in the past. It is also clear that the time saved by reducing attention to such computations in school can be put to better use on such topics as problem solving, estimation, mental arithmetic, geometry, and data analysis (NCTM, 1989).

Another reason the role of algorithms is changing is that researchers have identified a number of serious problems with the traditional approach to teaching computation. One problem is that the traditional approach fails with a large number of students. Despite heavy emphasis on paper-and-pencil computation, many students never become proficient in carrying out algorithms for the basic operations. A principal cause for such failures is an overemphasis on procedural proficiency with insufficient attention to the conceptual basis for the procedures. This unbalanced approach produces students who are plagued by “bugs,” such as always taking the smaller digit from the larger in subtraction, because they are trying to carry out imperfectly understood procedures.

Further research on the pedagogical importance of using linkages between out-of-school mathematics and school mathematics as means of strengthening children’s mathematical intuitions should be done and recommendations of appropriate classroom techniques to facilitate and build on these linkages should be made in Zambia. Further research on children’s and adult’s out-of-school of mathematics, should also examine how we can better make school mathematics more readily accessible and transparent to children as they approach and pursue problems in the course of their everyday out-of-school activities.
The country would clearly benefit from systematic empirical work exploring ways which may help children to use what they know to decipher the mathematics of school instruction.

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APPENDICES

A. CONSENT LETTER

Dear Parent or Guardian,

Parent/Guardian consent letter

I am a third-year student at Norwegian University of Science and Technology (NTNU). I intend to carry out a study to find out and compare Grade 8 students' computational strategies used to solve mathematical problems in out-of-school and school settings. Your child has been selected to take part in this study and I would like to request for permission to allow him/her to participate in the study. I intend to observe and interview your child at the place where they sell commodities. Your child will also be requested to take part in a formal test that will be formulated using information gathered from the interview. Participation in this study is voluntary and will not affect your child's attendance in class or any other school activities in any way. All information collected will be considered confidential to ensure your child's privacy. The results of this study will help the school implement teaching and learning strategies that will improve performance of students in mathematics.

Please indicate on the attached form whether you permit your child to take part in this study. Your cooperation will be very much appreciated. If you have any questions or would like more information, please contact me by phone at +260977444746 or by e-mail: munalulastephen@yahoo.com.

Thank you for your consideration. Yours sincerely,

Stephen Munalula

B. CONSENT FORM

Parent/Guardian Consent Form

I agree/disagree that my child: ______________________________ participate
(son/daughter's name)

In the test: ☐

In the interview: ☐

Parent's/Guardian's signature: _________________Date: ______________________