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Degradation Modelling for Predictive Maintenance

An Application to High Voltage Rotary Machines

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Reliability, Availability, Maintainability and Safety (RAMS)
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This is dedicated to my parents Abul Faiz Chowdhury & Shamsun Naher- the purest sources of my motivation all along.

Summary

Large high voltage rotary machines are commonly utilized in gas processing plants for operations such as dewatering and compression. The availability of these machines are very critical as the operation down times are generally associated with expensive production loss. Therefore this is no surprise that industries put a lot of effort in ensuring the maximum availability of these machines. However accurate failure prediction of such machines is challenging due to the complexity associated with technicality, data collection, testing and condition monitoring, etc.

This project addresses such an issue regarding the high voltage motors in Kollsnes gas processing plant that are currently in operation. It is operated by Gassco and Statoil serves as technical service provider. Karsten Moholt AS conducts the condition monitoring and ABB conducts the assessment of the conditions of these motors and claims to predict time to failure of an individual machine with certain confidence level. However, this prediction method is under the copyright of ABB and how the process works is not known by any other party. Therefore it leaves some room for further investigations regarding estimation of remaining useful life and in addition the current prognostics practice is limited to unit level.

In this situation, Statoil is interested in estimating remaining useful life of the motors due to ageing in order to reduce uncertainty regarding operation outage and to support overall maintenance decisions. They are further interested in extending the boundary of unit level prognostic to system level prognostic because the demand for motor operation varies depending on the two seasonal periods- summer and winter. In addition, they would like to explore the possibility of developing a simulator that is capable of estimate remaining useful life of a motor (or possibly the system) under given current health condition, previous history and future probable usage profile of the machine in order to further facilitate maintenance decision making process.

Various approaches have been taken by researchers to address the issues in high voltage rotary machine prognostic but there are still remaining many challenges that are making the whole prognostic process complicated. The main focus of this thesis is to develop a degradation model for the rotary machines in order to estimate remaining useful life under the given current health condition and make a possible transition from unit level prognostic to system level prognostic. The required preliminary task of prognostic estimation involves finding a good indicator that describes the health condition of a motor reasonably well.

During the process, it's been observed that, failure due to ageing process in stator winding insulation is the most critical failure mechanism in high voltage rotary machines and the health of a motor basically depends on the condition of the stator winding insulation. It's been noticed that, ageing processes can be influenced by multiple stresses acting in synergistic fashion which makes any sort of life modeling or degradation modeling very difficult. It's been further noticed that regardless of the stress acting most dominantly on a failure process, the final failure usually occurs due to electrical ageing. Further progress in the study leads to the conclusion that, partial discharge test is currently the most acceptable

testing method for health condition indication of an insulation system among the available methods.

Under the assumption that condition monitoring data is available, statistical approach based on non-homogeneous Gamma process has been employed for the degradation modeling in order to estimate remaining useful life of a given rotary machine. Important properties of Gamma process has been discussed in correlation with the rotary machine prognostic. Associated parameters have been calculated with a 95% confidence interval. Quality of parameter estimation has been discussed for several inspection strategies. In case of prognostic, current condition (actual degradation level) has been incorporated with remaining useful life estimation. This is due to the fact that, condition-based prognostic tends to be more accurate than traditional age-based prediction. Some relevant insights have been discussed and a demonstration have been provided regarding possible transition from unit level prognostic to system level prognostic.

Based on expert opinion provided by Statoil and literature surveys, non-homogeneous Gamma process appears to be the most appropriate for degradation modeling of winding insulation system utilizing partial discharge information. However reminding of the famous quote by *George E. P. Box*, *"All models are wrong but some are useful"*; proposed model requires to go through some validation process with the help of useful field data. Nevertheless the proposed model is full of possibilities for making transition from a theoretical model to a more practical model as more information becomes available. In addition, application of such degradation modeling is not only limited to this specific case. Gamma process is already a popular choice for this purpose and non-homogeneous gamma process have significant implications for civil engineering applications.

This thesis proposes an initial framework for prognostics of remaining useful life of high voltage rotary machines under the assumption of non-linear degradation increment of insulation system. It shows potential for further research leading to some interesting and useful outcomes in this particular area of research.

Preface

This thesis is submitted as a partial requirement for the degree in Master's of Science in Reliability, Availability, Maintainability and Safty (RAMS) under the department of Mechanical and Industrial Engineering at Norwegian University of Science and Technology (NTNU) in the year of 2017. The research has been done under the direct supervision of Professor Anne Barros in the department of Mechanical and Industrial Engineering, NTNU. The thesis is co-supervised by Mr. Erling Lunde, Principal Researcher at Statoil.

The thesis is an extension of the Specialization Project conducted in autumn semester 2016 at NTNU and motivated by the topic provided by Statoil. The original topic is related to the prognostics of stochastically ageing rotary machines in one of the important gas processing plant in Norway. The thesis revolves around the main topic with a deeper focus in degradation modeling based on non-homogeneous Gamma process and prognostics of remaining useful life.

The main approach employed here is related to probability and statistics and the readers of this thesis are assumed to have some prior knowledge in probability, statistics, reliability analysis and stochastic process. Information regarding Kollsnes gas processing plant and it's associated compression systems are collected from the meeting and email correspondence with representative of Statoil and relevant websites.

Trondheim, 2017-05-31
Abu MD Ariful Islam

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I would like to thank the following persons for their co-operation during this Masters thesis.

First I would like to thank Mr. Erling Lunde, representative of Statoil and my co-supervisor, for proposing this interesting topic for Masters project. Information and feedback that were provided during the thesis work was very relevant and useful in order to conduct this research.

I would like to specially thank my main supervisor Professor Anne Barros for her very encouraging guidance and constructive feedback during this project which helped me utilizing my strengths and overcoming some weaknesses.

A.M.A.I.

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Acronyms

ANN	=	Artificial Neural Network
CBM	=	Condition Based Maintenance
CDF	=	Cumulative Distribution Function
CM	=	Condition Monitoring
EIS	=	Electrical Insulation System
FTA	=	Fault Tree Analysis
FPT	=	First Passage Time
HMM	=	Hidden Markov Model
HSMM	=	Hidden Semi Markov Model
HV	=	High Voltage
IR	=	Insulation Resistance
MLE	=	Maximum Likelihood Estimate
MTTF	=	Mean Time To Failure
MRL	=	Mean Residual Life
NGL	=	Natural Gas Liquid
NHGP	=	Non-homogeneous Gamma Process
OLPD	=	On-Line Partial Discharge
PD	=	Partial Discharge
PDF	=	Probability Density Function
PI	=	Polarization Index
RUL	=	Remaining Useful Life
VSD	=	Variable Speed Drive

Chapter 1

Introduction

This chapter starts with underlying motivation behind this thesis along with the specific objectives that are targeted to achieve. Then it describes the scope and limitations associated with the achievement of the objectives. An overview of the approach is briefly discussed further by acknowledging the associated limitations. Finally it provides a blue print of rest of the report structure.

1.1 Motivation

Regardless of the types of industrial sectors, maintenance cost has a growing trend due to the expansion of capital inventory, increasing requirements for the functioning of system and outsourcing of maintenance (Dekker and Scarf, 1998). In oil and gas industry, a need of corrective maintenance may incur a huge cost as production loss is very expensive along with high unplanned maintenance cost. Therefore, maintenance optimization plays a great role in reducing cost by planning minimum maintenance action ahead of an actual breakdown. In addition, quite often the maintenance decisions are taken under uncertainty in terms of time to failure (lifetime) and/or actual deterioration (Van Noortwijk, 2009). This leads to a possibility that the maintenance action is conducted either too early (and/or more often than necessary) or too late.

Traditional reliability approach is one way to help maintenance planning and decision but this approach is rather a general reliability estimation of identical units and mostly useful to manufacturers of mass production (Heng et al., 2009). In addition to that, in reality, data for employing such approach may be unavailable or non-existing for some systems or units which make traditional reliability approach inappropriate and leaves a room for better alternatives.

This thesis is motivated by the optimal maintenance planning and decision making regarding the High Voltage (HV) compressor units in Kollsnes gas processing plant that depict a similar scenario, mentioned above. The problem statement of this thesis is proposed by Statoil, the technical service provider of the plant. These compressors fall under the category of HV rotary machines and reliability of these machines are usually quite high.

However failure of these machines during a demand period would result in a huge opportunity cost due to expensive production loss. In addition, the maintenance inspection or condition monitoring is not readily available and generally quite expensive. For example, some of the tests are destructive to the unit being tested which itself degrades the condition of the unit. Furthermore, the machines have some unique and customized features and the operating conditions vary depending on many other factors related to operating environment and usage profile. Therefore, utilizing failure data from similar machines has not been proven to be an useful option as well. All of these factors constitute a very complex problem leading the maintenance decisions to be taken under great uncertainty. Statoil is interested to know remaining operational life of these compression unit in order to support maintenance decisions regarding repair and replacement of the machines. They identified the ageing of HV motor's insulation system as the most critical factor for the production availability and this thesis will limit it's discussions accordingly.

Given the scenario, in order to aid optimal maintenance decision, transition from traditional reliability approach towards Condition Based Maintenance (CBM) is necessary which basically utilizes current health condition of an unit to predict the future condition. This approach is more customized to the units and it is not entirely dependant on historical failure data. Remaining Useful Life (RUL) estimation is a key factor in CBM and it is generally defined as the period of time remaining until the termination of the product/component. The concept of RUL has been used widely in operational research, reliability and statistical literature. In case of Kollsnes gas processing plant where there are no failure data of the machines are available, estimation of RUL based on the current condition of the component and the future usage profile can be of great interest. This thesis approaches this problem of machine prognostics with the purpose of understanding the associated challenges and look for possible solutions and/or useful recommendations for further research.

1.2 Objectives

In order to estimate RUL accurately, it is inevitable to understand the degradation mechanism of the unit and/or system in order to find an appropriate condition indicator that can reasonably describe the degradation process. Therefore the first part of the this thesis discusses about relevant degradation mechanisms and prognostic condition indicator and the second part discusses regarding quantitative implications. Following tasks have been identified in order to approach the problem.

- Literature review on the degradation mechanism of HV motor insulation system and connect it to the discussion of a condition indicator for prognostics.

- Literature review on the prognostics and degradation modeling in relevance with rotary machines prognostics. This objective sets the boundary of this thesis.

- Development of a degradation model and discussion of it's behavior under different conditions. In addition, establishment of parameter estimation from training dataset.

- Estimation of unit level RUL with test data-set by incorporating the known degradation level at current time and discussions of the results. In addition, providing a demonstration of unit level prognostics to system level prognostics.

- Connecting the study of unit level RUL prognostics with the problem under consideration in form of recommendation, future research, data requirement and transition towards system level prognostics.

1.3 Scopes and Limitations

The scope of this thesis is limited to the study of prognostics based on statistical analysis. Actual physical feature of the component has not been strictly considered due to couple of reasons- limited accessibility to such information and the stochastic degradation modeling being the main point of interest for the author. Unavailability of any sort of actual field data was the biggest obstacle towards directly approaching the problem. Therefore only simulation has been used under some assumptions developed based on the communication with the representative of Statoil regarding the problem statement. The discussions of this thesis is rather more general pointing towards similar problems. Some of the justification requires more rigorous studies in order to validate the applicability of the proposed degradation model to the given problem. However, the study is useful in terms of Non-homogeneous Gamma Process (NHGP) degradation modeling which has practical implications in other disciplines. Finally the main focus of this thesis is unit level RUL estimation and the discussion of system level RUL estimation is limited to simple demonstration.

1.4 Approach

The main building block of this thesis is the literature review. In order to develop a degradation model, available modelings under similar assumptions are studied in order to establish and propose an appropriate model. In the absence of actual field data, simulations are implemented for the demonstrations and illustrations of the modeling results. Regarding literature search, NTNU's digital subscription facility in major journal database provided by Oria.no, is extensively utilized.

1.5 Structure of the Report

Rest of the chapters are structured as follows. Chapter 2 provides problem overview and set up the background for rotary machine prognostics. Chapter 3 first discusses degradation modeling in general for the engineering system and later becomes more specific to HV rotary machines. Based on the selected degradation model, chapter 4 describes the mathematical background, development and estimation process of required parameters. In addition, it describes the simulation process and training data-set. Chapter 5 utilizes simulation data to estimate RUL to study the behavior under different parameter values, inspection frequency etc and discusses the interpretations of the results. Chapter 6 concludes with additional remarks, future research and associated challenges.

Chapter 2

Overview of the Problem

In 2015 Norway sold 115 Billion Sm^3 gas that hit a new record and in 2016 gas sales were at the same record level as 2015[1]. In order to process and distribute the gas received from offshore producers, there are quite a few onshore facilities in Norwegian shelf from Kårstø in the south to Melkøya in the north. Kollsnes is one of those many gas processing plants located in Øygarden municipality in Hordaland, which is in north-west of Bergen operated by Gassco and Statoil is technical service provider[2]. This chapter will provide a system overview and relevant background information necessary for the study of rotary machine prognostics.

2.1 Kollsnes Gas Processing Plant

Kollsnes is an important hub facility that receives gas from offshore producers and distribute the gas to different customers. Rich gas from Troll, Kvitebjørn and Visund in the North Sea is sent to Kollsnes where it gets separated into gas, natural gas liquids (NGL) and condensate. During the process, it goes through two major processings- dewatering and compression before being exported by pipelines to the relevant customers. The gas is then transported to United Kingdom and continental Europe via Sleipner gas field[3]and Draupner S and E platforms[4]. While on the other hand, NGL is transported to Mongstad via the vestprosess pipeline[5]. Figure 2.1[6] shows simplified receiving and distribution channel of Kollsnes gas processing plant.

[1]Norwegian Petroleum website (`http://www.norskpetroleum.no/en/production-and-exports/oil-and-gas-production/`)

[2]Gassco website (`https://www.gassco.no/en/our-activities/processing-plants/Kollsnes-processing-plant/`)

[3]Operated by Statoil that consists of a riser platform for gas export- (`http://www.statoil.com/en/OurOperations/ExplorationProd/ncs/sleipner/Pages/default.aspx`)

[4]Operated by Gassco and it is a key hub of submarine gas pipeline network in Norway- (`https://www.gassco.no/en/our-activities/pipelines-and-platforms/draupner-SE/`)

[5]Norwegian Petroleum website (`http://www.norskpetroleum.no/en/production-and-exports/onshore-facilites/`

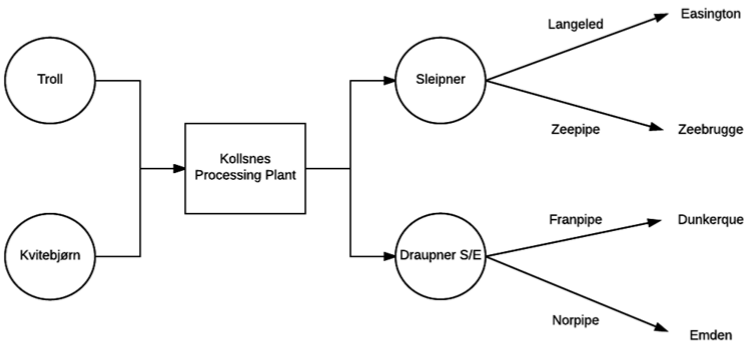


Figure 2.1: Receiving and distribution network of Kollsnes gas processing plant

2.2 System Overview

Gas compression system in Kollsnes is carried out by 6 parallel compression trains and each compressor's power consumption rating is upto 42MW. A variable speed drive (VSD), a motor, a gearbox and a compressor are the main basic units for each compression train set as shown in figure (2.2).

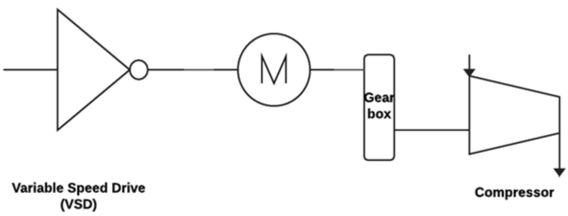


Figure 2.2: Main components of one compression train (source: Statoil)

Kollsnes gas processing plant is divided into two time periods based on the production demand- winter and summer. During the winter season the capacity requirement is full and all 6 compressor trains are required to function. If any of the compression train fails during the demand period, significant production loss is expected. On the other hand, during summer, only part of the total capacity is required and a reduced portion of the compression trains can fulfill the demand.

Statoil is convinced that, the most critical component of one compressor train is the motor's condition that contributes most to the uncertainty about its availability. It can be assumed that, all motors are identical where the first 5 motors were commissioned in 1996 and approaching their predicted end life. Whereas the 6th motor is relatively new which

[6] Gassco website- (`https://www.gassco.no/static/transport-2.0/`)

was commissioned in 2006 and thus expected to have a relatively longer remaining life. Figure 2.3 shows the overall schematic of the system.

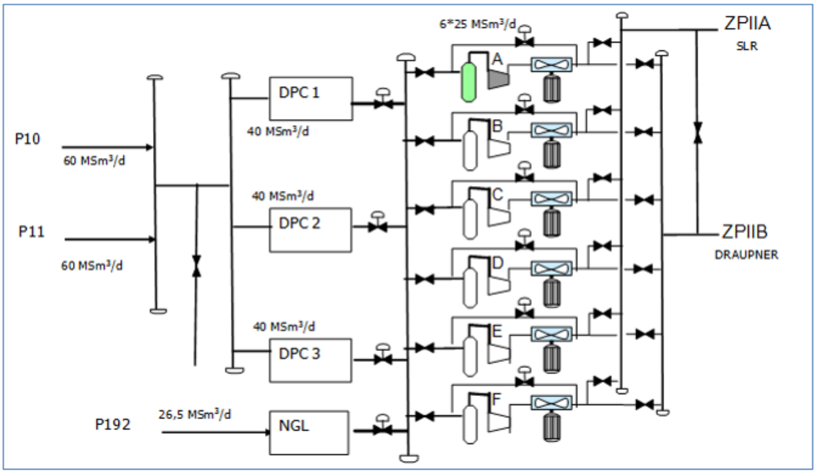


Figure 2.3: Kollsnes gas processing plant consisting of 6 compression trains (source: Statoil)

The motors in question are HV large synchronous motors. Several statistical surveys dealing with failure causes of HV rotating machines agree that the breakdown of winding insulation is a major failure cause ((Bruetsch et al., 2008)). Statoil identifies the ageing of stator winding insulation as the most critical degradation mechanism and rotor winding insulation condition may also be significant. Due to enough degradation of motor winding insulation exceeding a threshold point, winding short circuits may occur which can cause an immediate motor breakdown and put it out of the operation.

Main components of a synchronous motor are main frame, stator and rotor. Stator is the stationary part that creates a rotating magnetic field and the rotor is the rotary part that consists of an electromagnet for developing a rotor magnetic field by dc excitation. Then it rotates at the synchronous speed with the rotating magnetic field of stator. Figure 2.4[7] shows the main components of a synchronous motor. Stator and rotor conduct the magnetic field through the winding coils and the insulation material isolates high voltage from ground and conductors, thus reducing the chance of short circuits and winding failures. It also conducts heat from the conductors for rotating machines.

Critical Factor for Motor Health Condition

Isolation quality between conducting materials is crucial for the proper functioning of the motor system. It basically physically and electrically separates two conducting parts of an electrical device to prevent shock hazards and ground loops. It is also used for separating high voltage and low voltage circuits. Electrical Insulation System (EIS) is a means of achieving isolation in electrical machines and it is composed of insulating

[7]Image source- (http://www.bitlanders.com/blogs/synchronous-motor/201606)

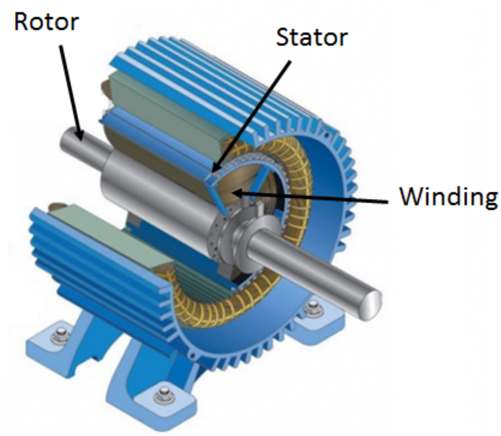


Figure 2.4: Main components of a synchronous motor

materials and conductors. Insulation materials have very small electrical conductivity to serve the purpose of isolation.

One justification for the EIS to get more attention is due to the fact that, organic materials are primary constituent in winding insulation system. Mechanical strength of organic materials are significantly lower than the stator components that are generally made of copper or steel (Stone, 2005). Therefore, health condition of stator winding insulation defines the life of a stator winding more than the stator core or conduit.

2.3 Degradation Mechanism

Based on the literature review and referring to IEC60505 (2011) standard, primarily it is observed that, ageing in EIS is not dominated by a single stress rather multiple stresses influence during the process. All stresses can eventually lead to electrical ageing although it is not necessary that electrical stress is the most dominating one. For instance, Bruetsch et al. (2008) shows that, electrical stress does not dominate ageing of mica (polymeric material) insulation alone rather it's a combination of different stresses where thermal and mechanical stresses are the most important ones. In addition, it should be mentioned that, contaminants and defects in the insulation material is always playing it's part in deteriorating the condition of EIS besides the above mentioned stresses. Some of the important ageing processes are briefly discussed below-

2.3.1 Ageing Process of Stator Winding Insulation

According to IEC60505 (2011), *"ageing (intrinsic and extrinsic) is irreversible changes of the properties of an EIS due to action by one or more stresses"*. Different stresses contribute to the aging process of winding insulation that eventually lead to degradation of an EIS. These significant stresses are thermal, electrical, mechanical and environmental (ambient) and they are broadly classified as TEAM stresses. Ageing of an EIS can be influenced by a single stress or by a combination of stresses. Even in the case of one dominant stress factor, other factors can play vital role in a synergistic fashion. For example, polymeric materials in EIS undergo thermal and electrical degradation with strong synergistic effects (Montanari et al., 2002). This section provides a general overview of ageing mechanisms influenced by TEAM stresses according to IEC60505 (2011) standard and relevant literature.

Thermal Ageing

According to IEC60505 (2011), thermal ageing process involves chemical reaction in insulation system, changes in material, thermo-mechanical stress (due to thermal expansion and/or contraction) and modification of electric stress. These ageing processes may eventually result in EIS failure[8]. For example, due to physical and chemical changes in material or thermo-mechanical stress, delamination and crack progression may occur in EIS. Further it can result in loss of mechanical strength or external contaminants can penetrate to cause insulation failure. Alternatively, loss of mechanical strength or external contaminant can also contribute to electrical ageing leading to insulation failure.

Thermal stress due to the operating temperature of winding, is the most commonly acknowledged cause of gradual deterioration of insulation. High operating temperature above a threshold limit causes chemical reaction in modern day insulation which contributes in insulation brittlement and delamination for all types of insulation (Stone, 2005).

For large machines, thermo-mechanical stress is an important issue that results from thermal (or load) cycling. It arises from the variation of thermal stress due to the quick change in operating temperature of windings as a result of sudden or rapid change of machine loads. Insulation materials have lower coefficient of thermal expansion than copper which implies a relatively slower expansion of insulation materials. It results in shear stress between the conductors and insulation and after a number of thermal load cycles, the bond may collapse in stator winding (Stone, 2005).

Electrical Ageing

It is observed from the description of ageing process in IEC60505 (2011), electrical breakdown due to electrical ageing is usually the final failure of an EIS. This general observation is also consistent in terms of HV rotary machines as Bruetsch et al. (2008) shows that, although most dominating stresses associated with ageing in a HV rotary machine with mica insulation are thermal and mechanical stresses but electrical ageing is usually the final failure.

Electrical ageing involves the effects of Partial Discharge (PD), electrical tracking, electrical treeing etc. PDs can directly lead to EIS failure or can turn into *tracking* or *treeing*. It is observed as symptoms for most of the degradation mechanisms found in stator winding insulation in motors and generators rated 6 kV or above (Tetrault et al., 1999).

[8] Here *failure* is defined by end-point criterion which is according to IEC60505 (2011)- *moment when a system is no longer able to fulfil its service purposes*

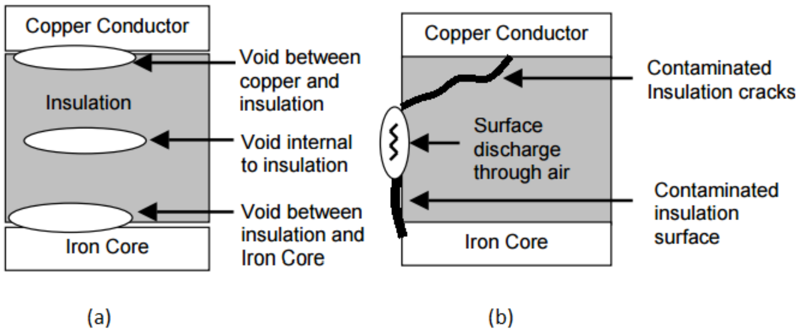


Figure 2.5: (a) PD locations in an EIS (b) PD leading to tracking (Paoletti and Golubev, 1999)

Paoletti and Golubev (1999) defines PD as *"an electrical pulse or discharge in a gas-filled void or on a dielectric surface of a solid or liquid insulation system"*. Within the insulation system these PD charges can occur in any void such as in between copper conductor and insulation wall, grounded motor frame and outer insulation wall or within itself (see figure 2.5). *Electrical tracking* is the formation of continuous conducting paths across the surface of the insulation resulting from surface erosion under voltage application (Kamaraju, 2009). In such situation, current leakage occurs between two insulated points and can give rise to a flashover (arc) along the completed tracking pathway. Surface contamination influences electrical tracking and can occur in the end-windings of a rotating machine (Kamaraju, 2009). *Electrical treeing* is one of the main causes of degradation and breakdown of insulating materials (Danikas and Tanaka, 2009). It is an initiation of volume failure of the insulation material which progresses by producing many fine dendritic degradation paths (Wiley, 2011). The breakdown path of *tracking* occurs on the surface while *treeing* occurs through the surface.

Mechanical and Ambient Ageing

Based on IEC60505 (2011) stator coil or stator bar can be subjected to mechanical stress due to the mechanical vibration when the coils are loose in the stator slot. It causes groundwall insulation to abrade (Stone, 2005).

Table 2.1: Degradation mechanisms due to mechanical stress (IEC60505, 2011)

Degradation Mechanism	Cause
Failure of insulation components	Large number of low-level stress cycles
Thermo-mechanical effects	Thermal expansion and/or contraction
Rupture	High level of mechanical stress
Abrasive wear	Relative motion between components
Insulation creep	Electrical, thermal or mechanical stress

Several ambient stresses have the potential to influence rotor and stator winding insulation deterioration such as condensed moisture on the windings, oil, humidity, abrasive particles, dirt and debris, etc. Even if ambient stresses may not be responsible for ageing of insulation directly, however they have synergistic potential to accelerate another type of stress to accelerate the ageing process (IEC60505, 2011).

2.4 Remaining Useful Life

The concept of RUL is the basis for this thesis and therefore it is introduced here briefly. The word 'useful' is rather an economic aspect as the technical lifetime of an industrial machine is often much longer than it's economic lifetime (Ahmadzadeh and Lundberg, 2014). Si et al. (2011) defines RUL as *"the useful life on an asset at a particular time of operation"* where RUL is a random variable that depends on the asset's current age, the operating environment and the observed condition monitoring or health information. Furthermore future usage profile of an equipment should also be taken into account in prognostic of failure prediction.

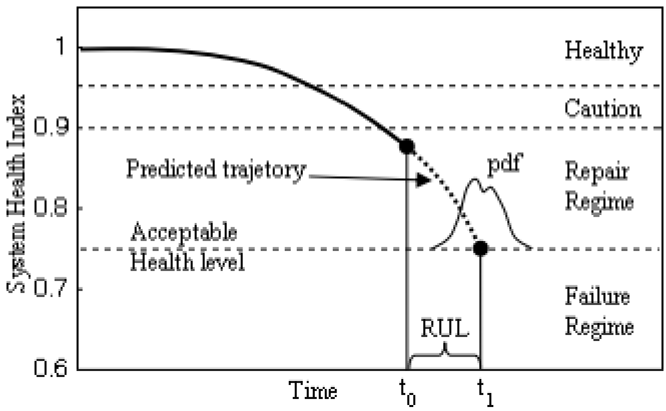


Figure 2.6: Illustration of RUL definition (Xiongzi et al., 2011)

RUL can be considered as a time dependant random variable as it depends on the condition of an equipment at time t. A formal definition of *RUL(t)* corresponding to the RUL at time t can be given as:

$$RUL(t) = inf\{h : X(t+h) \in S_L \mid X(t) \notin S_L\} \tag{2.1}$$

where:

$X(t)$ = condition indicator of the unit at time t

$X(t+h)$ = prognosis of the unit for an additional time unit h at any time after t

S_L = set of failed or unacceptable states of the unit

X(t) can be depended on both past operational conditions and future usage profile and from the definition, condition indicator X(t) and unacceptable states (S_L) must be defined in order to estimate $RUL(t)$ reasonably. Figure 2.6 illustrates the RUL definition with current condition t_0 and when system reaches a certain unacceptable level at t_1.

Reaching to a failed state is generally defined by setting up a threshold level. When the degradation process first exceeds this pre-defined threshold, it's called First Passage Time (FPT) and can be defined with respect to a predefined failure threshold L and time to failure T as follows-

$$T_L = inf\{t > 0 : X(t) \geq L\} \tag{2.2}$$

Degradation process along with threshold determines the probability density function (PDF) of RUL. However, statistical models for prognostics of RUL independent of threshold are also possible if the failure data are available (Xu and Wang, 2012). In this thesis, it's been assumed that, a deterministic failure threshold level is known.

2.5 Prognostic Condition Indicator

Estimation of RUL requires a defined set of failed states (S_L) and information about current health condition. However there are numerous ways for an EIS can fail. For example, stator winding insulation itself has more than 20 failure processes (Stone and Culbert, 2010). A single test or method can not be sensitive for all the degradation mechanisms alone and therefore identifying the most relevant degradation mechanism(s) is crucial in order to estimate RUL. Condition indicators or diagnostic parameters then can be chosen that are characterizing the ageing process for monitoring the condition. For instance, hot-spot temperature is considered as one of the most important diagnostic parameters for transformers to calculate RUL, while in case of rotating machines, a single parameter is not enough but dissipation factor ($tan\delta$) is one of the most important one (Trnka et al., 2014).

As discussed earlier, all the ageing processes described above usually ends up with electrical failures. More specifically in HV large rotary machines, severe insulation failure commonly occur due to PD formation which is considered as one of the most important symptoms of insulation failure (Younsi et al., 2010). There are different condition indicators or diagnostic parameters that gives information about insulation health. For example, PD pulse voltage at rated voltage is capable of detecting PD (Stone, 2005). On the other hand, capacitance between the stator conductor and the grounded core can indicate possible thermal deterioration, moisture absorption and end winding contamination while measure of dielectric losses can provide indication of general condition of stator insulation (Younsi et al., 2010).

Realizing the absence of a clear established guidelines for selecting a prognostic condition indicator for HV rotary machines, this section is dedicated to discussions regarding the selection of a possible useful prognostic condition indicator that are available.

2.5.1 Diagnostic vs. Prognostic Condition Indicator

Often the concept of *diagnostics* and *prognostics* are confused and the concepts are not clearly distinguished in most of the literature. Lee et al. (2014) attempts to provide a clear definition and underlines the differences as follows-

"Diagnostics is conducted to investigate or analyze the cause or nature of a condition, situation, or problem, whereas prognostics is concerned with calculating or predicting the

future as a result of rational study and analysis of available pertinent data. In terms of the relationship between prognostics and diagnostics, diagnostics is the process of detecting and identifying a failure mode within a system or sub-system; while prognostics is the process of generating a rational estimation of the remaining useful life and/or remaining performance life until complete failure occurs. Prognostic, in its simplest form, is to monitor and detect the initial indications of degradation in a component, and be able to consistently make accurate predictions."

2.5.2 Measurement of HV Motor's Health Condition

Several off-line tests and on-line monitoring techniques are available to measure the health condition of an EIS. Off-line tests refer to the measurements that require taking the test subjects out of operation while on-line monitoring is possible during the normal operation (Stone et al., 2004). Insulation Resistance (IR), Polarization Index (PI), High-pot and PD tests etc. are some common diagnostic tests for winding insulation of HV rotary machines. Paoletti and Golubev (1999) studies the comparison between the above mentioned traditional tests and on-line PD monitoring in terms of evaluating the insulation condition.

Table 2.2 summarizes the comparison and shows that, when original insulation condition is good and marginal, all the test readings are indifferent. When insulation condition is dry and delaminated, PD test detects the presence of insulation voids while traditional tests provide a false result. Further insulation deterioration can be correctly detected by both traditional and PD test but traditional tests fail to distinguish between *poor* and *unacceptable* condition and the regions of insulation voids. Only *near-failure* condition is more accurately evaluated by traditional tests because PD intensity doesn't continue to increase until failure in stator windings, rather it tends to level off (Stone, 2012).

Table 2.2: Comparison between PD test and traditional test results

Real condition of insulation	**Insulation Resistance (IR)**	**Polarization Index (PI)**	**Hi-Pot Test**	**On-line PD monitoring**
Perfect	High	Good	Linear leakage current vs. Voltage is minimal	Unmeasurable PD activity
Minimal void formation	Fair	Fair	Linear leakage current vs. Voltage is stable	Minimal PD activity observed
Dry but insulation delaminated	False fair	False fair	False linear leakage current vs. Voltage	PD observed
Poor	Low	Poor	High leakage current.	High positive polarity discharges.
Unacceptable	Low	Poor	High leakage current	High negative polarity discharges.
Near failure condition	Very low	Very low	High leakage current.	Minimal PD activity

A single individual test is not sufficient to facilitate RUL estimation of a component (Stone and Culbert, 2010) and there is no straightforward path to choose the most appropriate test. Above mentioned off-line tests are most widely accepted but each test is effective for diagnosing certain types of insulation problems (Lee et al., 2005). Although off-line tests can detect many associated problems with the winding insulation, however, it requires operation outage. Therefore frequent off-line tests are not economical as operation outage in oil and gas industry is subjected to huge cost. Furthermore, applied stresses during the tests are not subjected to actual operating stresses (Younsi et al., 2010) and thus estimated RUL might differ from the actual value. Another important limitation of off-line tests is that, they are not frequently conducted and the test condition of each tests are not identical to each other which makes assessing a motor's present condition or predicting RUL very difficult (Younsi et al., 2010).

As oppose to off-line test, on-line monitoring provides more accurate and reliable diagnostic information regarding the insulation condition (Younsi et al., 2010) due to the fact that, the measurements can be obtained under actual operating conditions without an outage (Lee et al., 2005).

Based on 50 years of experience in relevant field, Stone and Warren (2006) claims that, PD testing is capable of giving reasonably early warning about the likelihood of winding failure. On-line PD monitoring is a powerful tool to assess the insulation condition in form-wound stators and very similar to off-line PD test (Stone, 2005). In last 10 years, periodic PD measurement has been replaced with continuous PD monitoring during normal motor and generator operations (Stone, 2013). PD testing and monitoring receives quite an attention in maintaining high voltage rotary machines. Trend in PD magnitude has been adopted as primary method to determine whether insulation maintenance is necessary or not but it's been claimed that in severely deteriorated insulation, PD will not increase indefinitely until the insulation fails (Stone, 2013). However, despite the limitations, both off-line PD testing and on-line monitoring seems to be widely accepted as an effective tool specially in recent days.

2.5.3 A Potential Candidate- PD

PDs are measured as voltage pulses and during positive waveform cycle, a discharge (partial short-circuit) results in negative polarity pulse ($-Q_m$) and during the negative waveform cycle a PD results in positive polarity pulse ($+Q_m$) (Paoletti and Golubev, 1999). They further describe the relationships between the positive and negative polarity PD pulses and the probable root causes that are summarized in table 2.3.

The pick magnitude Q_m denotes the severity of deterioration in the worst spot of winding and total PD activity (number of pulses per second) denotes how widespread the deterioration of winding is. However, Q_m is more indicative about how close the winding failure is (Warren and Power, 2003).

Table 2.3: Interpretation of PD test results (Paoletti and Golubev, 1999)

PD test result	Probable root cause
Positive polarity pulses prevalent	Voids between insulation & iron core/winding end turns or surface tracking
Negative polarity pulses prevalent	Voids between the copper conductor & insulation
Equally prevalent	Voids within the insulation material itself

This is promising that, peak PD magnitude (Q_m) shows some correlation with insulation health and can probably be a good candidate as a condition indicator for RUL estimation. However this claim raises several other questions. PD is rather a cause or a symptom. It's not an intrinsic property of a material and there is no absolute PD magnitude that accurately describes the condition of a stator winding insulation (Zhu et al., 2001). Therefore to define a set of unacceptable states, it is essential to know what is an acceptable PD level in relation with insulation health condition.

What is an Acceptable Level of PD?

In order to support industries to plan appropriate maintenance by observing the gradual deterioration of stator winding, a paper[9] with comparison of thousands of PD test results was presented in 1998 and updated annually in Iris Rotating Machines Conference (IRMC) and this database was established by keeping the following significant parameters constant (Warren and Power, 2003):

- Test instrument bandwidth and noise separation techniques
- Types of sensors
- Operating voltages
- Operating gas coolant (if applicable)
- Quality of design, manufacturing and installation

On the other hand, type of insulation system, machine type and winding type was found to be insignificant. For example, Stone and Warren (2006) analyzed the database to determine the effect on Q_m by operating voltage for air-cooled stators that used 80-pF sensor for PD monitoring (table 2.4). The table can be interpreted as, for a 13-15 kV air-cooled stator, 25% of tests had a Q_m value below 44 mV and 90% of tests had Q_m value below 508 mV. Therefore, if a similar machine is tested and yielded a Q_m value of 500 mV, it means the machine is likely to have a deteriorated stator as it's PD level is higher than 90% of the similar machines.

Table 2.4: Distribution of Q_m for air-cooled stators

	Operating voltage				
	2-4kV	6-8kV	10-12kV	13-15kV	>16kV
25%	7mV[10]	17mV	35mV	44mV	37mV
50%	27	42	88	123	69
75%	100	116	214	246	195
90%	242	247	454	508	615[11]

[9]Original paper- V. Warren, "How much PD is too much PD?" Iris Rotating Machinery Conference, USA, March 1998 is not found by the author of this report)

[10]mV- Millivolt

[11]strong influences by few manufacturers

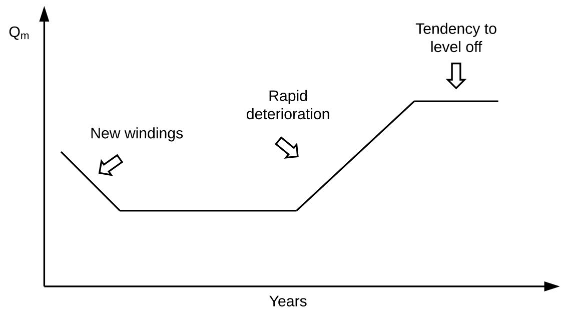


Figure 2.7: Typical trend in PD magnitude of stator windings (Stone, 2012)

In contrast, PD activity trend of two identical motors operating at the same site had been studied by Zhu et al. (2001). One motor (A) had lower PD magnitude compare to other motor (B) at a certain time. However, the PD magnitude of motor A increased significantly over a short period of time. They demonstrated that, motor A failed before motor B despite having a lower PD magnitude. They further explained that, stable PD activity indicates that deterioration didn't progress much and therefore motor B survived longer.

The purpose of presenting these examples is to show that, the result of the database can be compared with actual test result to get an overview of PD activity in the similar machines. However trending of PD activity over time for a given machine is considered more reliable for diagnosis of severe insulation deterioration (Warren and Power, 2003). In addition, saturation of PD trend (see figure 2.7) after a strong increase for a period of time requires some attention. If the initial PD data is not available, then stable PD test reading can lead to misleading interpretation as the winding can be either in good condition or in the saturated phase where it's about to fail soon (Stone, 2005).

In summary, there is no absolute level of PD magnitude value associated with the risk of failure and meaningful interpretation is only possible by the means of trending and comparing a machine with identical and/or similar machines using the same monitoring method and keeping the previously mentioned significant parameters constant (Stone, 2005).

In this thesis, PD measurement has been considered as the condition indicator of insulation health despite of it's limitations due to the lack of better alternatives.

PD Measurement and Interpretation

Stone (2005) describes PD monitoring principle in detail. PDs create small current pulses that propagate through stator winding. After that, Fourier transform is used to generate high frequencies up to several hundred megahertz. Sensitive devices to high frequencies can detect these PD electrical pulses which are then processed by PD monitoring system.

PD measuring instruments measure the number, magnitude and phase position with respect to the 60 Hz AC cycle. The pick positive $(+Q_m)$ and the pick negative $(-Q_m)$

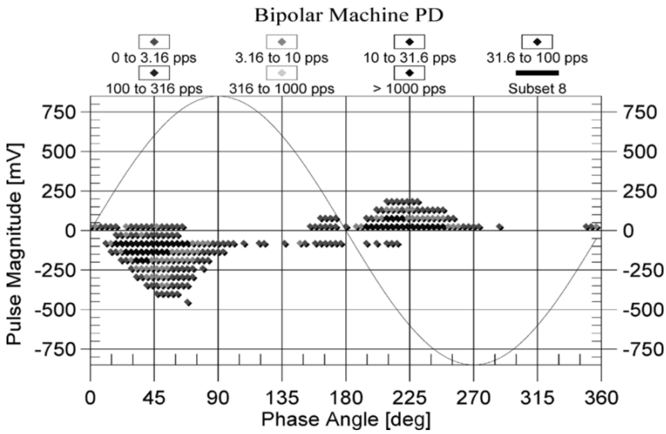


Figure 2.8: Typical PD measurement w.r.t. a 60 Hz AC cycle

represents the highest PD pulses with a minimum PD repetition rate[12]of 10 pulses per second. Q_m is considered to be a reasonable predictor of insulation condition because higher value of Q_m indicates more deteriorated winding compare to another winding with a lower value (Stone and Warren, 2006). Figure 2.8 shows typical PD data from one phase with respect to the 60 Hz AC cycle where vertical axis represents $+Q_m$ and $-Q_m$ in millivolts and horizontal axis represents phase angle. Dots represent the number of the occurrences of PD in a particular magnitude and phase position. Peak PD magnitude (Q_m) for this phase is -400 mV and +200 mV (Stone and Warren, 2006) .

[12]number of partial discharge pulses during one cycle of an AC waveform (Paoletti and Golubev, 1999)

Chapter 3

Overview of Prognostics and Degradation Modeling

There have been much research done and still going on in the field of prognosis and degradation modeling in close connection with CBM of engineering assets which is evident from numerous publications. There are some useful review papers that organize these vast amount of works in different categories. Jardine et al. (2006) reviews developments in diagnostics and prognostics implementing CBM and focuses the increasing trend of using multiple sensors in CM. This paper is however more general to the mechanical systems. Ahmadzadeh and Lundberg (2014) attempts to generally describes typical approaches of RUL estimation and discusses their advantages and disadvantages. Heng et al. (2009) review prognostic techniques based only on rotating machines and discuss about incomplete trending data and effects of maintenance actions. Si et al. (2011) addresses modeling development of RUL estimation focusing on statistical data driven approaches.

3.1 Literature Review on Prognostics

In machinery prognosis, the remaining operational (useful) life, future condition or probability of reliable operation of an equipment are forecast based on observed condition monitoring data in order to reduce downtime, spares inventory, maintenance costs and safety hazards (Heng et al., 2009). Compared to the past, the machinery design and construction has become more complex that require better maintenance strategies with increasing cost. In addition maintenance decisions are generally taken under uncertainties and therefore ineffective maintenance decisions can cost a huge expenditure for the industries. Therefore the maintenance practices shifted from a corrective maintenance strategy towards preventive maintenance and from their the transition is towards CBM in recent days. CBM provides a health assessment of machinery based on collected condition monitoring or inspection data without interrupting normal operation of machine to determine required maintenance action prior to any predicted failure (Grall et al., 2002).

In the category of machinery prognostics, there have been much research on the specific field of rotary machine prognostics as well and Heng et al. (2009) covers a wide range of prognostic techniques and identify their merits and weaknesses and discusses associated challenges. Ma (2007) discusses the need of paradigm shift from the condition monitoring being used only as a *maintenance alarm* tool to more holistic future prediction in order to achieve total engineering asset management. George Vachtsevanos (2006) discusses intelligent fault prognostic techniques such as model-based. probability-based and data-driven techniques etc. for engineering systems with some examples.

Heng et al. (2009) groups the existing approaches of rotary machine prognostics as traditional reliability approaches, prognostic approaches and integrated approaches. In traditional reliability approaches, predictions are based on the event records of identical units and estimate population characteristics such as Mean Time To Failure (MTTF) and reliability of the unit. Parametric failure models like Poisson, Weibull, exponential etc. have been in use for modeling reliability. In reality when the historical event records for similar machines are not available, this approach is not very useful. In contrast, prognostic approaches use condition monitoring data to predict future reliability and health of the unit. This approach is the main topic of this thesis and therefore it will be discussed in more detail in the upcoming section. Integrated approach combines both above mentioned approaches where condition monitoring data is complemented with the reliability data for prognostic purpose.

3.1.1 Condition Based Prognostic Approaches

George Vachtsevanos (2006) broadly categorizes condition based prognostic approaches into following categories that are briefly described below-

Model-based Approach

It involves developing mathematical models in order to describe the relation between physics of the system and failure mode progression (Heng et al., 2009). This approach is very customized to the specific component and operating conditions. It is quite useful for components where the model remains constant across the system and for the critical components that has high demand for functioning (Heng et al., 2009). On the downside, it's expensive as it is very much component specific. Physics-based fatigue models, crack growth model, ARMA model, particle filtering etc. are some examples of this approach.

Data-driven Approach

This approach derives model from routinely collected condition monitoring data directly and uses simple projection models such as exponential smoothing and auto-regressive model (Heng et al., 2009). This approach is advantageous in terms of simplicity of calculations but projection relies a great deal on past degradation patterns and therefore may lead to some inaccuracy if the future degradation deviates from the expected path. Artificial Neural Network (ANN) is most commonly used data-driven prognostic technique (Heng et al., 2009) and Bayesian-related methods, Hidden Markov Models (HMM) etc. are some others.

Probability-based Approach (Statistical)

If collected condition monitoring data and/or historical data from similar class of machinery takes some statistical forms that can be transformed into some probabilistic distribution then probability-based approach or statistical data driven approach can be employed for prognostics. These methods generally provide confidence limits of the predictions that are useful in terms of accuracy and precision of the predictions (George Vachtsevanos, 2006). It should be noted that, data driven and statistical approach is not distinguished clearly from each other in literature which makes this classification rather confusing given the fact that, probability-based approach can also be considered as data-driven as well. Therefore in this document, data driven and probability-based approach are not critically distinguished and considered both in general under the term statistical data driven approach.

3.1.2 Types of Condition Monitoring Data

Statistical data-driven approach is clearly dependent on the data availability and the nature of the data. Based on Wang and Christer (2000), the available CM data is divided into *direct CM* and *indirect CM* data. Direct CM data describes underlying state of the system directly so that the prediction of CM data to reach a predefined threshold limit is same as the prediction of RUL. While on the other hand, indirect CM data can only partially indicate the underlying state of the system and additional failure event data may be required for RUL estimation (Si et al., 2011). For instance, wear and crack size are typical direct CM data whereas vibration monitoring data falls under the category of indirect CM data.

PD magnitude has been discussed as a prognostic condition indicator in the previous chapter. Given the definitions above, it can be stated that, PD magnitude data is rather fall in the category of indirect CM data as the PD is not an intrinsic property of a material and there is no absolute PD magnitude that accurately describes the condition of a stator winding insulation (Zhu et al., 2001). Therefore to set up a threshold limit to define a critical state of the system may require some expert's judgment in case of the unavailability of failure data.

3.2 Classification of Statistical Data Driven Approaches

It's been previously discussed that, traditional reliability approach of failure time analysis may not be always practical. Failure event data of an unit or a system may not be available. For example, in the case of compressors in Kollsnes gas processing plant, the machines have some unique design features and they are not directly and simply comparable to a similar machine in a similar operating environment. Therefore the failure event data is basically non-existent which makes the traditional reliability approach irrelevant.

Therefore prognostics based on CM data makes more sense practically but the path is not straightforward. RUL is basically an estimation of degradation path reaching a predefined threshold level. Therefore if the threshold is known and CM data is available, predictions can be made about the RUL. However using indirect CM data, determining a threshold level is also not straightforward.

Statistical approaches for describing CM data to model RUL is discussed based on Si et al. (2011). They divided statistical data-driven approaches in four categories as *regression-based, Brownian motion, Gamma process* and *Markovian-based* models. First three approaches assume state evolution as continuous process and Markovian approach assumes discrete state evolution process. These four approaches are briefly described below mostly based on Si et al. (2011).

3.2.1 Regression-based Models

This approach is popular in industries mostly due to it's simplicity for trending CM path. First the health condition is mapped by some key CM variables and then RUL is estimated by monitoring, trending or predicting the CM variables against a fixed threshold. *Machine Learning* and *Random Coefficient Regression* are two commonly used methods. Machine Learning technique does not have any probabilistic orientation and thus do not provide any PDF of RUL which is very essential for risk analysis and maintenance decision making (Wang and Christer, 2000) and therefore have little significance in this thesis. In contrast, Random Coefficient Regression methods depict CM path from CM data to estimate lifetime distribution and able to provide PDF of RUL but except for some special cases, closed form of such PDF is not available. The biggest limitation of these models are incapability of modeling the temporal variability in RUL estimation (Pandey et al., 2009).

3.2.2 Brownian Motion with Drift (Wiener Process)

Wiener processes are suitable for such cases where degradation processes move bidirectionally over time with Gaussian noises. It is well known from literature that, the FPT follows inverse Gaussian distribution and can be formulated analytically. There are two limitations of the Wiener process that make degradation modeling questionable with this approach. Degradation of most of the engineering assets is monotonic but Wiener process progresses bidirectionally over time. It implies an unit is self healing during the process of degradation, which is not very practical in most occasions. Therefore monotonic degradation processes are not very suitable for modeling with this approach. In addition, it's a time homogeneous process, which is again may not be true for cases like fatigue crack growth or even for PD accumulation over time.

3.2.3 Gamma Process

Gamma process is a natural choice where degradation processes that are monotonic and unidirectional with tiny positive increment over time. Mathematical calculations for RUL estimation is comparatively straightforward and it's capable of considering temporal variability in the degradation model. In addition, Gamma process is not limited to be time homogeneous process. Realizing these advantages, Non-homogeneous Gamma process is utilized in this thesis for degradation modeling. Therefore Gamma process will be discussed in detail in upcoming sections.

3.2.4 Markovian-based Model

Based on the assumptions that future degradation state of an unit depends only on current state (memoryless property) and the unit's state can be known directly by condition monitoring; the degradation is assumed to be evolved on a finite state space. This approach is widely used in industries for RUL estimation and to support maintenance decisions due to their comprehensiveness as the states can be defined as *good, bad, repair required* etc. However there are also some critical limitations. The above mentioned assumptions are quite strong and often do not represent the reality. In addition, transition rates between states are required to be calculated from a large number of data sets, that are quite often unavailable.

Based on the George Vachtsevanos (2006) and Si et al. (2011) a detailed and updated taxonomy of machinery prognostics is proposed as in figure 3.1. Data driven and probability based approach are put inside stippled rectangle box to indicate the generalization of both of these approaches under a common approach termed as *statistical data driven approach.*

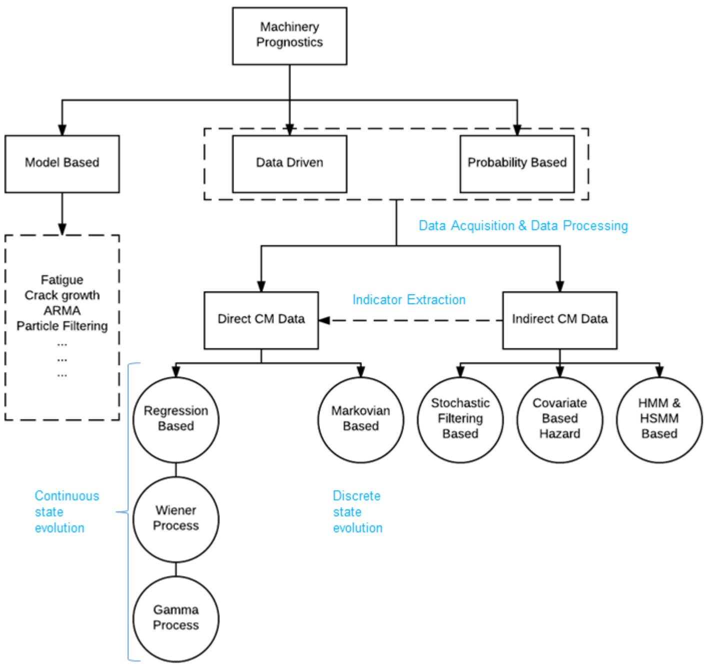


Figure 3.1: Taxonomy of machinery prognostics

3.3 Literature Review on Gamma Process Based Model

For the modeling of continuous degradation process such as wear, corrosion and crack growth with positive increments, the Gamma process is widely used (Xu and Wang, 2012). It's been proven to be useful for supporting optimal inspection and maintenance decision (Pandey et al., 2009). Van Noortwijk (2009) provides an excellent review paper that covers theoretical aspects and successful maintenance applications of Gamma process. To name a few, Singpurwalla (1995) applied Gamma process model in dynamic operating environment. Wang et al. (2000) utilized Gamma process for predicting the distribution of RUL of individual pumps in a large soft drink manufacturing plant. van Noortwijk et al. (2007) provide method to combine Gamma process for modeling degradation and Poisson process for modeling load.

Although most of the works are based on homogeneous Gamma process but there are several evidences for employing NHGP as well. For example, Cinlar et al. (1977) use NHGP for modeling concrete deterioration process due to creep. Van Noortwijk (2009) states that, according to empirical studies the expected deterioration at time t is often proportional to power law and it's been utilized in this thesis for modeling the NHGP degradation process and therefore in upcoming chapter mathematical foundation and parameter estimation process of Gamma process is described more in detail.

3.4 Justification of NHGP Assumption

Based on the discussion regarding the behavior of PD activity in chapter 2, it is evident that, the degradation process of EIS does not represent a time homogeneous property. Although figure 2.7 is an overly simplified representation of the behavior of PD magnitude but it provides a broad picture of how it progresses over time. From a stable progression, PD magnitude tends to increase rapidly and the increase is monotonic and non-decreasing. In addition, based on the expert knowledge from Statoil and indication from the literature regarding PD activity, NHGP assumption tends to be the most practical in the given situation.

Chapter 4

Gamma Process Modeling

This chapter provides formal definitions of both homogeneous and non-homogeneous Gamma process along with the parameter estimation process. NHGP parameter estimation is further incorporated with power law assumption for shape function and the number of sample size. Then the simulation process according to NHGP is described and the behavior of the model is discussed under different parameter values.

4.1 Homogeneous Gamma Process

The probability density function of gamma distribution is defined as

$$f_{a,b}(x) = \frac{1}{\Gamma(a)} b^a x^{a-1} e^{-bx} \boldsymbol{I_{x>0}} \tag{4.1}$$

where $I_{A(x)}$ is the indicator function and $\Gamma(a) = \int_0^\infty u^{a-1} e^{-u}\, du$ is the corresponding Gamma function for shape parameter $a > 0$ and scale parameter $b > 0$.

In general, a stochastic process X_t is time homogeneous if *the transition probability between two given state values at any two times depends only on the difference between those times*.[1] For homogeneous Gamma process, the shape parameter a is a linear function at when $a > 0$ and can be formally defined as follows:

Definition: For $a, b > 0$, a continuous-time stochastic process $X_{t\geq 0}$ is a homogeneous Gamma process, such that:

1. $X_0 = 0$
2. $X_{t\geq 0}$ has independent increments
3. Increments are Gamma distributed. Therefore, for $0 \leq s < t$, the distribution of $X_t - X_s$ follows Gamma distribution $\Gamma(a(t-s), b)$ and thus only depends on $t - s$

[1] Glossary of Statistical Terms, OECD (`https://stats.oecd.org/glossary/detail.asp?ID=3674`)

Expectation and variance of a Gamma distribution $\Gamma(a, b)$ for $\forall t \geq 0$ can be derived as $\mathbb{E}(X_t) = \frac{a}{b}t$ and $var(X_t) = \frac{a}{b^2}t$ respectively.

4.1.1 Properties of Interest

Homogeneous Gamma process has some interesting properties as direct consequences of the definition as follows:

1. For $0 \leq s < t$ and $X_t - X_s \geq 0, X_{t\geq 0}$ follows non-decreasing trajectories. This is a suitable property to model degradation process which is normally the trend in engineering system given that there is no maintenance action.

2. $\mathbb{E}(X_t)$ and $var(X_t)$ are both linear functions which implies that Gamma process is a suitable choice to model degradation process with linear tendency.

3. $X_{t\geq 0}$ is a pure jump process with Markov property such as for $0 \leq u < s < t$, the distribution of $X_t - X_s$ is not dependent of X_u (memory-less property).

4.1.2 Parameter Estimation

Moments estimation and the Maximum Likelihood Estimation (MLE) are often mentioned for the parameter estimation of Gamma process. The moments approach is more straightforward to implement and less time consuming than MLE method but MLE is asymptotically unbiased meaning that it converges to true values as the number of observations approaches to infinity (Grall-Maes et al., 2014). Therefore MLE method is adopted for parameter estimation in this document.

Let us consider a Gamma process with n values of process increments where time increments are disjoint. For $1 \leq i < n$, the observations can be denoted as follows:

$$\Delta t_i = t_i - t_{i-1} \tag{4.2}$$

and the observed deterioration increments,

$$\delta_i = x_i - x_{i-1} \tag{4.3}$$

Time increments can be either equal or different and results of the parameter estimation of a Gamma process vary accordingly. Therefore, depending on the assumptions of the nature of time increments, parameter estimation based on MLE will be discussed separately.

Uniform Time Increments

When all time increments are equal then $\Delta t_i = \Delta t$ for all $i \geq 1$ and observed increments are independent and identically distributed (i.i.d.) random variables with $\Gamma(a\Delta t, b)$

Therefore, the likelihood function can be written as follows:

$$\begin{aligned}\mathcal{L}(a,b) &= \prod_{i=1}^{n} f_{(a\Delta t,b)}(\delta_i) \\ &= \prod_{i=1}^{n} \frac{1}{\Gamma(a\Delta t)} b^{a\Delta t} (\delta_i)^{a\Delta t-1} e^{-b\delta_i}\end{aligned} \tag{4.4}$$

Therefore the log-likelihood,

$$\mathcal{L}(a,b) = l(a,b) = \sum_{i=1}^{n} (a\Delta t\ ln(b) - ln(\Gamma(a\Delta t)) + (a\Delta t - 1)ln(\delta_i) - b\delta_i) \tag{4.5}$$

Taking derivative with respect to a and b respectively and maximizing the likelihood function afterwards,

$$\begin{aligned}\frac{\partial}{\partial a} ln(a,b) &= \sum_{i=1}^{n} (\Delta t\ ln(b) - \Delta t\ \psi(a\Delta t) + \Delta t\ ln(\delta_i)) = 0 \\ \frac{\partial}{\partial b} ln(a,b) &= \sum_{i=1}^{n} \left(\frac{a\Delta t}{b} - \delta_i \right) = 0\end{aligned} \tag{4.6}$$

where ψ is known as digamma function, defined as follows:

$$\psi(x) = ln\Gamma(x) = \frac{\Gamma'(x)}{\Gamma(x)} \tag{4.7}$$

Further mathematical operation of equation 4.6 can be presented as:

$$\hat{b} = \hat{a} \frac{n\Delta t}{\sum_{i=1}^{n} \delta_i} \tag{4.8}$$

and

$$ln \left(\hat{a} \frac{n\Delta t}{\sum_{i=1}^{n} \delta_i} \right) + \Delta t\ \sum_{i=1}^{n} (ln(\delta_i) - \psi(\hat{a}\Delta t)) = 0 \tag{4.9}$$

Likelihood estimator $\hat{a}$ and $\hat{b}$ can be obtained by solving equation Equation 4.8 and Equation 4.9.

Non-uniform Time Increments

In this case, although δ_i are independent random variables but they are not identically distributed. However, MLE can be used to find the estimates a and b in similar fashion as uniform time increment case by maximizing following likelihood function:

$$\begin{aligned}\mathcal{L}(a,b) &= \prod_{i=1}^{n} f_{(a\Delta t_i,b)}(\delta_i) \\ &= \prod_{i=1}^{n} \frac{1}{\Gamma(a\Delta t_i)} b^{a\Delta t_i} (\delta_i)^{a\Delta t_i-1} e^{-b\delta_i}\end{aligned} \tag{4.10}$$

Similarly for uniform time increment case, the following estimates can be obtained:

$$\hat{b} = \hat{a}\frac{\sum_{i=1}^{n}\Delta t_i}{\sum_{i=1}^{n}\delta_i} \tag{4.11}$$

and

$$\left(\sum_{i=1}^{n}\Delta t_i\right) ln\left(\hat{a}\frac{\sum_{i=1}^{n}\Delta t_i}{\sum_{i=1}^{n}\delta_i}\right) + \sum_{i=1}^{n}\Delta t_i(ln(\delta_i) - \psi(\hat{a}\Delta t_i)) = 0 \tag{4.12}$$

4.2 Non-homogeneous Gamma Process

Unlike homogeneous gamma process, some degradation processes do not follow the linear trajectories. In that situation, assumption of homogeneous Gamma process is limited and non-homogeneous Gamma process is naturally more suitable for such degradation modeling and is formally defined as:

Definition: Let $A(t)$ a non-decreasing, real valued shape function with $t \geq 0$ and scale parameter is b; a stochastic process X_t is a NHGP, such that:

1. $X_0 = 0$
2. $X_{t\geq 0}$ has independent increments
3. Increments are Gamma distributed. Therefore, for $0 \leq s < t$, the distribution of $X_t - X_s$ follows Gamma distribution $\Gamma(A(t) - A(s), b)$

When $A(t) = at$, the process turns into a homogeneous Gamma process for $a > 0$. Expectation and variance of a non-homogeneous Gamma process $\Gamma(A(t), b)$ for $\forall t \geq 0$ can be derived as $\mathbb{E}(X_t) = \frac{A(t)}{b}$ and $var(X_t) = \frac{A(t)}{b^2}$ respectively.

4.2.1 Properties of Interest

Some interesting properties as the direct consequences of the definition of NHGP are as follows:

1. $X_{t\geq 0}$ follows non-decreasing trajectories which is as previously mentioned, suitable for modeling degradation process
2. Unlike homogeneous Gamma process, $\mathbb{E}(X_t)$ and $var(X_t)$ are adjustable. This property allows modeling of degradation with non-linear trajectories by defining a proper shape function $A(t)$
3. Again $X_{t\geq 0}$ is a pure jump process with Markov property

When a deterioration model with Gamma process takes temporal variability into account, empirical studies show that the expected deterioration increase over time t is often proportional to a power law (Van Noortwijk, 2009) :

$$E(X(t)) = \frac{A(t)}{b} = \frac{ct^u}{b}$$
$$Var(X(t)) = \frac{A(t)}{b^2} = \frac{ct^u}{b^2} \tag{4.13}$$

Here, $c, u > 0$ and when $u = 1$ deterioration increase over time is linear which represents a homogeneous Gamma process. Often shape of the expected deterioration is known in terms of the parameter u (Van Noortwijk, 2009). Mahmoodian and Alani (2013) summarizes a set of typical values of u from literature which represents different types of deterioration of concrete. Figure 4.1 illustrates time-dependent degradation model based on different values of exponential parameter u including a graphical representation of deterioration of concrete according to Mahmoodian and Alani (2013).

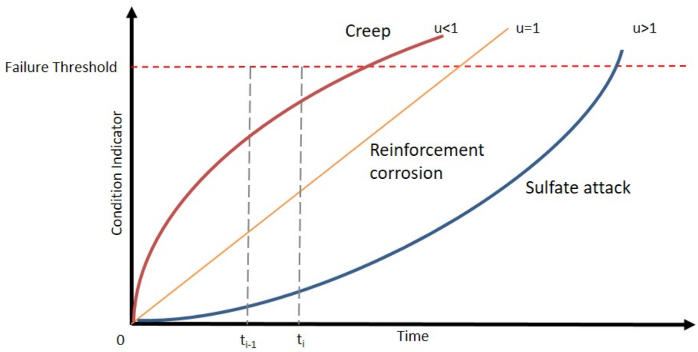


Figure 4.1: Shape of expected deterioration path in terms of parameter u

4.2.2 Parameter Estimation

By maximizing the logarithm of the likelihood function of the observed deterioration increments, MLEs of c and b can be estimated. The likelihood function is,

$$\begin{aligned}\mathcal{L}(\delta_i|c,b) &= \prod_{i=1}^{n} f_{X(t_i)-X(t_{i-1})}(\delta_i) \\ &= \prod_{i=1}^{n} \frac{b^{c(t_i^u - t_{i-1}^u)}}{\Gamma[c(t_i^u - t_{i-1}^u)]} \delta_i^{c[t_i^u - t_{i-1}^u]-1} e^{-b\delta_i}\end{aligned} \tag{4.14}$$

After taking logarithm of the likelihood function and setting the derivatives to zero, the likelihood function can be maximized and estimates $\hat{c}$ and $\hat{b}$ can be solved as follows (Van Noortwijk, 2009):

$$\hat{b} = \frac{\hat{c}t_n^u}{x_n} \tag{4.15}$$

and

$$\sum_{i=1}^{n}[t_i^u - t_{i-1}^u]\{\psi(\hat{c}[t_i^u - t_{i-1}^u]) - log\delta_i\} = t_n^u log\left(\frac{\hat{c}t_n^u}{x_n}\right) \tag{4.16}$$

All the equations 4.9, 4.12 and 4.16 can be solved with the help of numerical method such as Newton-Raphson method (refer to appendix A.2.1). The MLE method can be extended to estimate the power-law parameter u which can be determined by numerically maximizing the likelihood function 4.14 (Van Noortwijk, 2009).

Here, it should be noted that, expected deterioration at time t can be obtained by using maximum likelihood estimator $(\hat{b})$ (equation 4.15) in equation 4.13 as follows-

$$E(X(t)) = x_n\left[\frac{t}{t_n}\right]^u \tag{4.17}$$

It implies that, at the last inspection at time t_n, the expected deterioration equals x_n meaning that last inspection contains the most information (Van Noortwijk, 2009).

Now, continuing with the estimation process, if $j = 1, 2, 3......, m$ is a set of m components of interest then $X_{i,j}$ denotes the degradation path of jth component at ith observation and $\delta_{i,j}$ denotes the associated increments. Therefore recalling the pdf of Gamma process:

$$f_{c(t_i^u - t_{i-1}^u),b}(\delta_{i,j}) = \frac{b^{c(t_i^u - t_{i-1}^u)}}{\Gamma(c(t_i^u - t_{i-1}^u))}\delta_{i,j}^{c(t_i^u - t_{i-1}^u)-1}e^{-b\delta_{i,j}} \tag{4.18}$$

Likelihood function of jth component-

$$\begin{aligned}\mathcal{L}(\delta_{i,j}|c,b) &= \prod_{i=1}^{n} f_{X(t_i)-X(t_{i-1})}(\delta_{i,j}) \\ &= \prod_{i=1}^{n}\frac{b^{c(t_i^u - t_{i-1}^u)}}{\Gamma[c(t_i^u - t_{i-1}^u)]}\delta_{i,j}^{c[t_i^u - t_{i-1}^u]-1}e^{-b\delta_{i,j}}\end{aligned} \tag{4.19}$$

Log-likelihood function of jth components-

$$\begin{aligned}l(\delta_{i,j}|c,b) &= ln\left(\prod_{i=1}^{n}\frac{b^{c(t_i^u - t_{i-1}^u)}}{\Gamma[c(t_i^u - t_{i-1}^u)]}\delta_{i,j}^{c[t_i^u - t_{i-1}^u]-1}e^{-b\delta_{i,j}}\right) \\ &= \sum_{i=1}^{n} ln\left(\frac{b^{c(t_i^u - t_{i-1}^u)}}{\Gamma[c(t_i^u - t_{i-1}^u)]}\delta_{i,j}^{c[t_i^u - t_{i-1}^u]-1}e^{-b\delta_{i,j}}\right)\end{aligned}$$

For all components-

$$\begin{aligned}&= \sum_{j=1}^{m}\sum_{i=1}^{n} ln\left(\frac{b^{c(t_i^u - t_{i-1}^u)}}{\Gamma[c(t_i^u - t_{i-1}^u)]}\delta_{i,j}^{c[t_i^u - t_{i-1}^u]-1}e^{-b\delta_{i,j}}\right) \\ &= \sum_{j=1}^{m}\sum_{i=1}^{n}(c(t_i^u - t_{i-1}^u)ln(b) - ln(\Gamma[c(t_i^u - t_{i-1}^u)]) \\ &+ (c(t_i^u - t_{i-1}^u) - 1)ln(\delta_{i,j}) - b\delta_{i,j})\end{aligned} \tag{4.20}$$

Taking partial derivative with respect to c and b-

$$\begin{aligned}\frac{\partial}{\partial c}[l(\delta_{i,j}|c,b)] &= \sum_{j=1}^{m}\sum_{i=1}^{n}(([t_i^u - t_{i-1}^u])ln(b) - (t_i^u - t_{i-1}^u)\psi(c(t_i^u - t_{i-1}^u)) \\ &\quad + (t_i^u - t_{i-1}^u)ln(\delta_{i,j})) = 0 \\ \frac{\partial}{\partial b}[l(\delta_{i,j}|c,b)] &= \sum_{j=1}^{m}\sum_{i=1}^{n}\left(\frac{c[t_i^u - t_{i-1}^u]}{b} - \delta_{i,j}\right) = 0\end{aligned} \tag{4.21}$$

Estimated parameters are obtained by solving equation 4.21 as follows:

$$\hat{b} = \frac{m\hat{c}t_n^u}{\sum_{j=1}^{m} x_{n,j}} \tag{4.22}$$

and

$$\sum_{i=1}^{n}[t_i^u - t_{i-1}^u]\psi(\hat{c}[t_i^u - t_{i-1}^u]) - \frac{\sum_{j=1}^{m}\sum_{i=1}^{n}[t_i^u - t_{i-1}^u]ln(\delta_{i,j})}{m} = t_n^u ln\left(\frac{m\hat{c}t_n^u}{\sum_{j=1}^{m} x_{n,j}}\right) \tag{4.23}$$

Here $\hat{c}$ must be computed numerically (Van Noortwijk and Pandey, 2004) and Newton-Raphson method is a powerful technique to find the root of a non-linear function and it is well accepted due to it's speed and efficiency compare to other methods (Akram and ul Ann, 2015). Therefore, Newton-Raphson method have been utilized to approximate the parameter values in this study and a short description of this method is provided in the appendix A.2.1.

Approximation of $\hat{c}$ and $\hat{b}$

Primary function $f(c)$ and it's derivative $f'(c)$ for approximating $\hat{c}$ are-

$$\begin{aligned}f(c) = &\sum_{i=1}^{n}[t_i^u - t_{i-1}^u]\psi_0(c[t_i^u - t_{i-1}^u]) \\ &- \frac{\sum_{j=1}^{m}\sum_{i=1}^{n}[t_i^u - t_{i-1}^u]ln(\delta_{i,j})}{m} - t_n^u ln\left(\frac{mct_n^u}{\sum_{j=1}^{m} x_{n,j}}\right)\end{aligned} \tag{4.24}$$

$$f'(c) = \sum_{i=1}^{n}[t_i^u - t_{i-1}^u])^2\{\psi_1(c[t_i^u - t_{i-1}^u])\} - \frac{t_n^u}{c} \tag{4.25}$$

According to Newton-Raphson method, a better approximation of c after n iterations can be obtained by $c_{n+1} = c_n - \frac{f(c_n)}{f'(c_n)}$. The process c_1 is first initiated by guessing a root of the function c_0 that is close to the root value. After that the process gets repeated until a sufficiently accurate value is obtained. In order to guess a c_0 value, a plot of $f(c)$ is drawn (figure: 4.2) within a range $c \in [1, 10]$ and it can be graphically observed that the solution of $f(c)$ lies within the range of $c \in [3.9, 4.1]$.

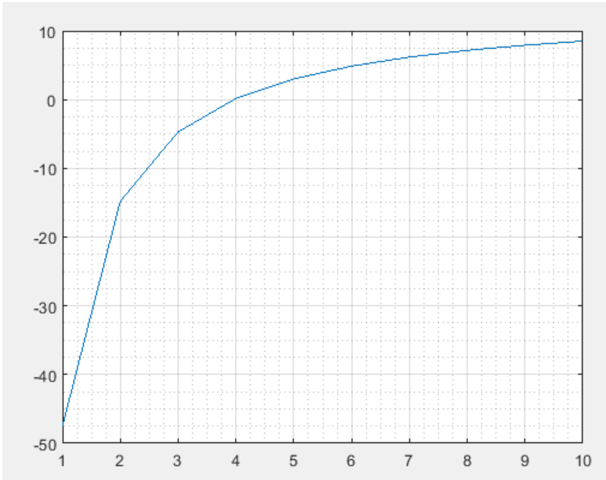


Figure 4.2: Plot of $f(c)$ against a range of c values

Confidence Interval of Estimates

A confidence interval is a derived range of values from sample statistics that is likely to contain the value of an unknown population parameter. It indicates that, if many repeated samples are taken from the population, a certain percentage of the resulting confidence interval is likely to contain the unknown population parameter[2].

If for an one-dimensional parameter θ, $l(\theta)$ is the corresponding log-likelihood function then let $l'(\theta)$ and $l''(\theta)$ be their first and second derivative respectively with respect to θ. Therefore the *observed information* of θ is defined as-

$$l(\hat{\theta}) =_{def} -l''(\hat{\theta}) \tag{4.26}$$

from statistical theory of maximum likelihood, variance and standard error can be estimated as-

$$\begin{gathered} \widehat{Var(\hat{\theta})} = \frac{1}{I(\hat{\theta}} = -\frac{1}{l''(\hat{\theta})} \\ SE = \widehat{SD(\hat{\theta})} = \sqrt{I(\hat{\theta})^{-1}} = \sqrt{\frac{-1}{l''(\hat{\theta})}} \end{gathered} \tag{4.27}$$

Similarly in case of two parameters ($\hat{c}$,$\hat{b}$) the *observed information matrix* (also known as *Hessian matrix*) can be obtained as-

$$I(\hat{c}, \hat{b}) =_{def} \begin{bmatrix} -\frac{\partial^2}{\partial c^2} l(c,b) & -\frac{\partial^2}{\partial c \partial b} l(c,b) \\ -\frac{\partial^2}{\partial b \partial c} l(c,b) & -\frac{\partial^2}{\partial b^2} l(c,b) \end{bmatrix}_{c=\hat{c}, b=\hat{b}} \tag{4.28}$$

[2]Definition is obtained form Minitab® 17 support

$$[I(\hat{c},\hat{b})]^{-1} = \begin{bmatrix} \widehat{Var(\hat{c})} & \widehat{Cov(\hat{c},\hat{b})} \\ \\ \widehat{Cov(\hat{c},\hat{b})} & \widehat{Var(\hat{b})} \end{bmatrix} \tag{4.29}$$

95% confidence interval of the estimated parameters can be calculated as-

$$\hat{c}e^{\pm 1.96\frac{\widehat{SD(\hat{c})}}{\hat{c}}}, \hat{b}e^{\pm 1.96\frac{\widehat{SD(\hat{b})}}{\hat{b}}} \tag{4.30}$$

From equation 4.20 observed information matrix for parameters (c, b) can be obtained as-

$$I(\hat{c},\hat{b}) =_{def} \begin{bmatrix} \sum_{j=1}^{m}\sum_{i=1}^{n}[t_i^u - t_{i-1}^u]^2\psi_1(c[t_i^u - t_{i-1}^u]) & -\frac{1}{b}\sum_{j=1}^{m}\sum_{i=1}^{n}[t_i^u - t_{i-1}^u] \\ \\ -\frac{1}{b}\sum_{j=1}^{m}\sum_{i=1}^{n}[t_i^u - t_{i-1}^u] & \frac{1}{b^2}\sum_{j=1}^{m}\sum_{i=1}^{n}[t_i^u - t_{i-1}^u] \end{bmatrix} \tag{4.31}$$

4.3 Gamma Process Simulation Methods

It is possible to approximate Gamma process with a limit of a compound Poisson process. However, Van Noortwijk (2009) argues that, since Gamma process has infinitely many jumps in each finite time interval, simulating Gamma process by simulating independent increment w.r.t. to tiny time increments is rather more efficient and mentions two simulation methods for sampling independent gamma process increments. These two methods are described briefly as follows-

4.3.1 Gamma Increment Sampling

Avramidis et al. (2003) terms this technique as Gamma Sequential Sampling (GSS) where independent samples $\delta i = x_i - x_{i-1}$ are drawn from Gamma density-

$$Ga(\delta|A(t_i) - A(t_{i-1}), b) = \frac{b^{A(t_i)-A(t_{i-1})}}{\Gamma(A(t_i) - A(t_{i-1}))}\delta^{[A(t_i)-A(t_{i-1})]-1}e^{-b\delta} \tag{4.32}$$

For $i = 1, 2, ..n$ and $x_0 = 0$.

4.3.2 Gamma Bridge Sampling

This method is well summarized from more than one literature by Van Noortwijk (2009). A sample is drawn that represents cumulative increment $X(t)$ in the interval $(0, t]$. Then another sample of cumulative increment $X(t/2)$ is drawn from conditional distribution of $X(t/2)$ given $X(t) = x$ such as:

$$f_{X(t/2)|X(t)}(y|x) = \frac{1}{x}Be(\frac{y}{x}|A(t/2), A(t) - A(t/2)) \tag{4.33}$$

For $0 \leq y \leq x$. It represents a transformed beta density on the interval $[0, x]$. Similarly, time interval $(0, t]$ is then divided into two more intervals $(0, t/2]$ and $(t/2, t]$. Then two more samples $X(t/4)$ and $X(3t/4)$ are drawn given the value of $X(t/2)$ and $X(t)$ respectively. The process continues in order to sample Gamma process path for 2^m time points such as: $t, t/2, t/4, 3t/4, .., 2^{-m}t, .., (1-2^{-m})$ for some positive integer m.

Gamma bridge sampling only allows equal length time interval and therefore Gamma increment sampling is utilized for the NHGP simulation in this thesis.

4.3.3 Detail Description of Simulation Process

During the meeting session with Statoil regarding the expectation from the project, it has been mentioned that, a simulator with a capability of calculating RUL under certain input values such as current condition, previous history and future usage profile could be of useful for decision makers. Although at present, condition monitoring data of the machines are not available but an attempt has been made to start developing a foundation of such simulator using randomly generated Gamma distributed data. The purpose of this simulator is of three folds as follows:

- Generate Gamma distributed data-set for a given set of parameter values
- Estimate parameter values if a data-set is provided
- Calculate RUL based on estimated parameters

The simulator is valid under following assumptions:

- Initial condition, $X(t = 0) = 0$
- At any inspection point at time t, degradation amount $X(t)$ is known accurately
- Amount of $X(t)$ represents the actual degradation amount meaning that data can be considered as direct CM data
- A failure threshold is known
- Exponential parameter u is known (this is not a critical assumption, it can also be estimated from the data-set along with other parameters and can be incorporated with current simulation process)

Following values are adopted for most of the simulation results in this document but these are readily adjustable depending on the requirements:

- Number of machines, $M = 100$
- Number of observations, $N = X(t = 0) + 100$
- Total time length, $T = 10$
- Time increment, $dt = 0.1$

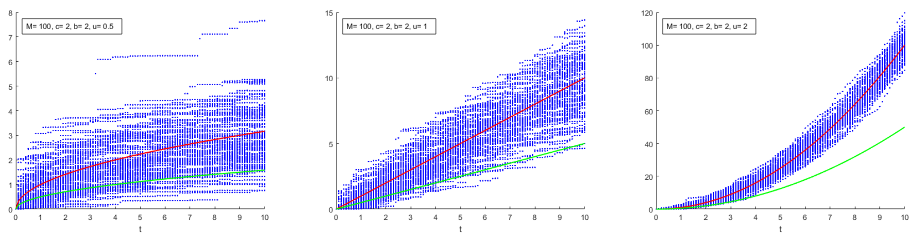
In terms of limitation, parameter estimation process has been done using Newton-Raphson method which requires to choose an initial c parameter value as an input of the function. Sometimes an adjustment of this initial input is required when estimated c is close to 0. The chosen initial c must be close enough to the estimated c in next iteration as it involves calculating equation 4.7 and it's derivative which are unable to handle negative values.

Contextual Definition of Threshold

The concept of threshold should be further clarified with respect to insulation degradation level and techincal failure of the machine. Above-mentioned degradation model only considers ageing of the insulation material and therefore it is not possible to set-up a threshold that indicates the technical failure of the EIS. This is because an aged insulation system is capable of operating under severe degraded condition and actual technical failure of a degraded insulation system requires an occurrence of transient or an operating error (Stone et al., 2004). Therefore in this context, exceeding a failure threshold denotes that the insulation system has reached that critical zone where a transient or an operating error will lead it to failure.

4.4 Realization of Degradation Paths

In order to observe the behavior of degradation paths, a set of Gamma distributed random paths have been generated with different combinations of parameter values. The behaviors are observed by changing one parameter value at a time keeping the other parameters constant. These are described below:



(a) u = 0.5 **(b)** u = 1 **(c)** u = 2

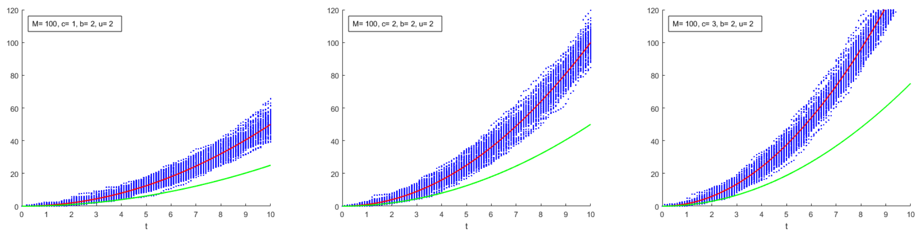
Figure 4.3: Behavior of shape function in terms of exponential parameter

4.4.1 Effect of Exponential Parameter u

Exponential parameter u regulates the concavity and convexity of trend shape (Gola and Nystad, 2011). For $u < 1$ the shape of degradation path is concave and for $u > 1$ it becomes convex. The path becomes linear when $u = 1$ and represents homogeneous Gamma process. In civil engineering application, some engineering knowledge exists in terms of parameter u regarding the deterioration of concrete. For example, for three types of degradation process such as diffusion-controlled ageing, reinforcement corrosion and sulfate attack the values 0.5, 1 and 2 are used respectively according to Mahmoodian and Alani (2013). Figure 4.3 illustrates the behaviors which are in agreement with figure 4.1.

4.4.2 Effect of Parameter c

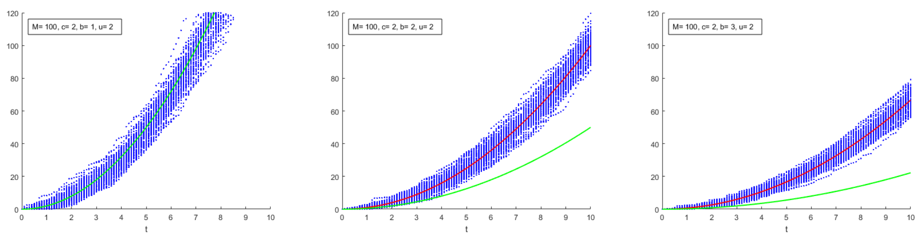
Increase in c parameter value influence the spread of the generated degradation paths which is not surprising as both parameter c and b determines the spread of Gamma probability distribution (Gola and Nystad, 2011). As the value of c increases from 1 to 3, the degradation paths are more spread out at the time increases. It can be clearly observed graphically by looking at the progression of variance as time increases (drawn as green) in figure 4.4 as oppose to the expected degradation over time (drawn as red).



(a) c = 1 **(b)** c = 2 **(c)** c = 3

Figure 4.4: Behavior of shape function in terms of parameter c

4.4.3 Effect of Scalar Parameter b



(a) b = 1 **(b)** b = 2 **(c)** b = 3

Figure 4.5: Behavior of shape function in terms of scalar parameter b

On the other hand, changing in scalar parameter shows an opposite behavior compare to shape parameter. Increasing b from 1 to 3 similarly, shows that the degradation paths become less spread out as time increases. Figure 4.5 clearly depicts this behavior. For

$b = 1$ the expected degradation and variance at time t is equal as the equations 4.13 indicates.

4.5 Parameter Estimation from Data

Parameter estimation is one of the important tasks in statistical data driven approaches for RUL prognostics. Quality of the estimation is critical as the parameters regulate the trend (degradation) shapes and incorrect estimation will certainly impact any results obtained by the estimated values. Therefore the validity of parameter estimation process described in section 4.2.2 need to be verified. To serve that purpose, a set of data generated using following input values-

Table 4.1: Input values for data generation

Item	M	N	T	dt	c	b	u
Values	100	100	10	0.1	2	2	2

Following the method described in section 4.2.2 for parameter estimation and confidence interval calculation, obtained results are quite satisfactory and consistent for other parameter values. An example of result is shown in table 4.2.

Table 4.2: Confidence interval calculation for c and b parameters

Parameter	Estimate	Confidence Level	Lower Bound	Upper Bound
c	2.0919	95%	2.0146	2.1722
b	2.0441		1.9584	2.1336

In real practice, the sample size and the interval of observations may depend on many practical aspects. For example, the compressor trains in Kollsnes processing plant are monitored in an opportunistic manner. Therefore, different combinations of number of units and observations have been analyzed to see the behavior of NHGP model in terms of parameter estimation quality and discussions are presented below.

4.5.1 Number of Sample Size

After selecting a training data set with known parameter values, the 95% confidence interval is calculated for the estimated parameters in terms of number of sample size. The estimated parameters are plotted with their associated confidence interval as orange and red lines. Figure 4.6 shows the values of estimated parameters starting from 100 samples reduced to only one sample.

For both of the estimates it can be seen that estimates are stable near the true value until 60% of the samples were considered. After disregarding 50% of the samples, the fluctuations in estimates are clearly evident. Level of confidence decreases along with the declining number of samples as expected.

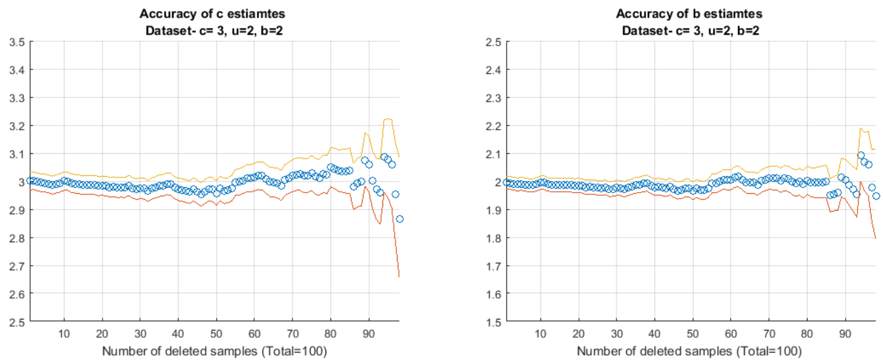


Figure 4.6: Accuracy of parameter estimates in terms of confidence interval w.r.t. sample size

4.5.2 Number of Observations

This section investigates the accuracy of estimated parameters of a sample size of 100 units with different combinations of observation strategies.

Ignoring Initial Observations

In case of partial discharge (PD) activity, it is known that, initial observation after installing a new equipment may not be very useful condition indicator because newly installed windings are subjected to higher PD magnitude and therefore initial PD measurement is best after about 6 months of operation (Stone, 2005).

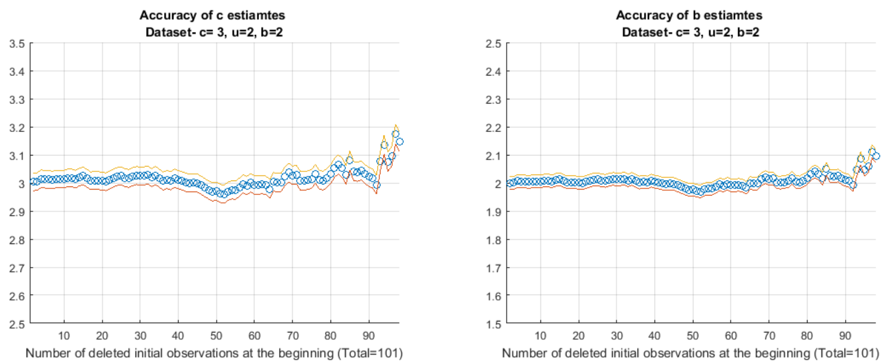


Figure 4.7: Accuracy of parameter estimates in terms of overlooking starting initial observations

Therefore, it could be interesting to see how the parameter estimation is affected when a number of initial observations are disregarded from the data-set. In this simulation, assuming that the condition is perfect at t=0, number of initial observations have been removed from a given data set.

Figure 4.7 shows the trend when deleting from 1 observation to 99 observations (only keeping the observation at t=0 and the final observation). Again it can be seen that, until removing about 50% of initial observations, the estimation is fairly stable before it starts to fluctuate.

Periodic Observation

Similarly a simulation has been conducted to see how periodic observations influence the parameter estimation quality. In this simulation, after the initial observation at t=0, a set of periodic observations are recorded with different intervals. For example, in case of a set of 100 data points and an interval of 3, 4th, 8th, 12th,.... data points are recorded until it reaches 100.

Following this process, table 4.3 shows the number of total observations when the interval ranges from 1 to 10.

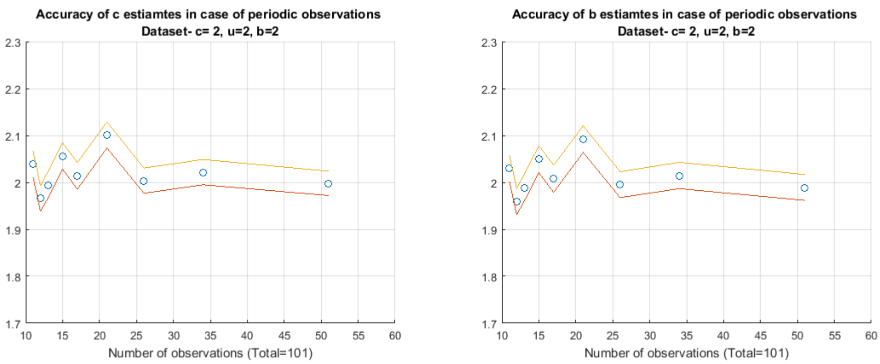


Figure 4.8: Accuracy of parameter estimates for periodic observations

Figure 4.8 shows the result of the simulation graphically and it is clearly evident that more the observation interval gets smaller, the estimation gets more accurate. Interestingly like previous simulation results, the estimation converges to a stable accuracy when the number of observations are about 50% or more.

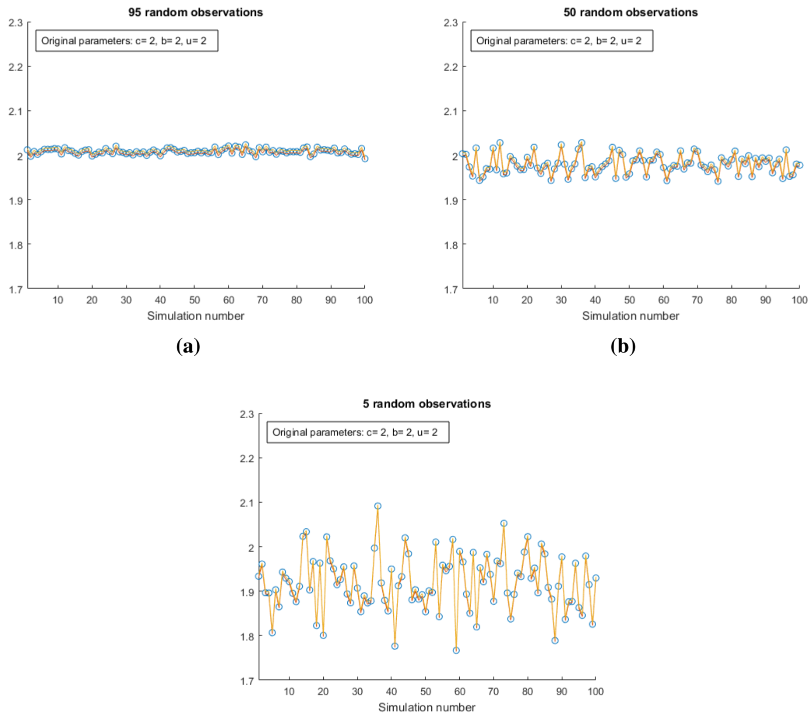
Table 4.3: Number of total observations corresponding to the observation interval

Interval of observations	1	2	3	4	5	6	7	8	9	10
Number of total observations	51	34	26	21	17	15	13	12	11	10

Opportunistic Observation

On-line PD monitoring is usually expensive and normally it's been done in an opportunistic basis for the rotary machines in Kollsnes processing plant. Therefore, it's of high interest to see how the accuracy of estimates is influenced for a non-periodic observation strategy.

The information about real practice of opportunistic inspections and frequency of inspections in Kollsnes are not available at this point. In general, the inspections are mainly



(c)

Figure 4.9: Accuracy of estimates in terms of opportunistic observations

conducted when the production demand is low and some machines are idle. Here it's been assumed that the inspection opportunities come in a random fashion. For a given data set of 100 data points, a specific number of data points are randomly chosen to estimate the parameters. For example, figure 4.9 shows how the accuracy of estimates are influenced when randomly selecting 5, 50 and 95 observations respectively and repeating the process for 100 times for each case.

When 95% observations are recorded then obviously the estimates are very stable and close to the true value. In case of randomly selected 50% of the observations are quite close to the true value ranging from 1.9 to 2.1 although the estimates are comparatively stable. When the number of selected observations gets as low as 5% of the original data set, the estimates become very unstable in a range between 1.7 to 2.2. Again it's noticeable that, even for the opportunistic observations, recording about 50% of the data randomly keeps the estimations fairly stable.

4.5.3 Optimal Inspection Strategy

It's been so far observed how quality of parameter estimation varies in terms of number of sample size (number of machines) and the number of observations (condition monitoring).

The 6 machines located at Kollsnes are assumed to be independent from each other meaning that, failure of one machine does not influence the failure(s) of any other machine(s). In such case, sample size for parameter estimation should always be limited to 1 and the inspection for condition monitoring may be optimized in terms of number of inspections and/or inspection strategy.

In order to understand the role of inspection strategy on how it's influencing the parameter estimation accuracy, an experiment has been set up where 1000 degradation paths are generated that represent degradation paths for 1000 identical machines. All paths are generated with same parameter value- $c = b = u = 2$. The total time length of the simulation is 10 unit and the current time is assumed to be 6 unit.

After that, 3 periodic and 7 opportunistic inspection strategies are considered where all the observations are taken in between the initial time and the current time at 6 unit. For each strategy b and c parameters are estimated for each machines and the average and variances are recorded. The definitions of inspection strategies are as follows-

- **Periodic 1** Every one other observations until current time (0.2.4...)
- **Periodic 2** Every second other observations until current time (0.3.6...)
- **Periodic 3** Every third other observations until current time (0.4.8...)
- **Opportunistic 1** More observations at the beginning stage than later stage approaching current time
- **Opportunistic 2** More observations at the later stage than the beginning
- **Opportunistic 3** More observations at the beginning stage than later stage approaching current time
- **Opportunistic 4** More observations at the later stage than the beginning
- **Opportunistic 5** All observations at the end stage than later stage approaching current time
- **Opportunistic 6** All observations at the beginning stage than the beginning
- **Opportunistic 7** All observations at the middle stage than later stage approaching current time

Table 4.4 summarizes the result of the experiment for parameter c. Result for parameter b shows exactly same trends as parameter c and thus not provided here. It is evident from the result that the total number of observations are the main criteria for an accurate estimation of the parameters. When the data is collected has little influence on the estimation. For instance, periodic 1, opportunistic 5, 6 and 7 collects same amount of data from different time points but both the estimations and the variances are very close to each other.

Table 4.4: Accuracy of estimated c parameter w.r.t. inspections strategies

Inspection strategy	Number of observations	c estimates	Variance
Full range of observations	101	2.05	N/A
Periodic 1	31	2.26	0.48
Periodic 2	22	2.34	0.53
Periodic 3	17	2.62	0.76
Opportunistic 1	14	2.71	0.85
Opportunistic 2	14	2.78	0.86
Opportunistic 3	9	3.26	2.17
Opportunistic 4	9	3.17	1.27
Opportunistic 5	22	2.34	0.54
Opportunistic 6	22	2.30	0.51
Opportunistic 7	22	2.38	0.57

4.6 Discussions

In this chapter, simulation technique is successfully implemented in order to generate Gamma distributed training data-set. Based on that, it's been shown how the shape of the degradation path changes with respect to changing of different parameter values. Most important insight of this chapter is that, dependency of the stability of parameter estimation on the total number of observations. According to this preliminary study, inspection interval and periodicity of the inspections play little importance compare to the total number of observations. For the same number of total observations for one machine, it does not matter much on the time points of the observed data.

This is an interesting insight in terms of decision maker's point of view under the assumption that Gamma process is suitable for the degradation modeling. If further study can optimize the minimum number of inspection data required for a reasonably accurate estimation, then management can flexibly plan inspection schedule.

Chapter 5

Prognostics of Remaining Useful Life

This chapter first develops the mathematical foundation of RUL estimation when the current condition is known. Then by utilizing the generated training data-set in previous chapter, the behavior of RUL under different scenario is discussed.

5.1 Relevant Definitions of RUL

In order to estimate a component RUL in advance with an acceptable level of uncertainty, either failure time probability based on failure time records can be obtained or the information of component deterioration trend during operation can be exploited and the latter approach maximizes the usage of component by allowing tailored maintenance planning (Nystad et al., 2012; Nystad, 2008). The general definition of RUL is already provided in chapter 2 but a closer look into age-based and state-based RUL definition is required for the clarification of this chapter.

5.1.1 Age-based RUL

Let an item is put into operation at time $t = 0$ and the time to failure for the item is denoted by T. If the item is still functioning at t then probability that the item will survive an additional length of h is,

$$R(h|t) = Pr(T > h + t|T > t) = \frac{Pr(T > h + t)}{Pr(T > t)} = \frac{R(h + t)}{R(t)} \tag{5.1}$$

$R(h|t)$ is called the conditional survivor function at age t and the Mean Residual Life (MRL) of the item at age t,

$$RUL(t) = \int_0^\infty R(h|t)dh = \frac{1}{R(t)} \int_t^\infty R(h)dh \tag{5.2}$$

This equation is only applicable when $R(h)$ is explicitly defined and numerical integration is not necessary to evaluate it (Govil and Aggarwal (1983)).

RUL distribution in terms of conditional lifetime distribution with the information that the component is functioning at time t is-

$$F_T(t) = Pr(T \leq t+h|T > t) = \frac{F_T(t+h) - F_T(t)}{1 - F_T(t)}, \forall t+h > t \tag{5.3}$$

In the time-based approach, the only available information at time t is whether the component is functioning or not. Thus the RUL depends on the age of the component only and not on the actual health condition of the component.

5.1.2 State-based RUL

Time-based approach may not be applicable when the component RUL depends on the actual health condition rather than the age of the component. In such situation, state-based approach is more applicable where remaining time to reach a pre-defined threshold level is defined by taking account of actual component state instead of it's survival time.

Under the assumption that degradation path follows a Gamma process, PDF of the degradation quantity $X(t)$ is defined as-

$$f_{X(t)}(x) = \frac{b^{A(t)}}{\Gamma(A(t))} x^{A(t)-1} e^{-bx} \tag{5.4}$$

Here $X(t)$ is the degradation indicator that are being measured and for the partial discharge, PD magnitude is the quantity of interest.

Now let a component's degradation level at time t is $X(t) = x_t$ and a pre-defined threshold is L. The state-based lifetime distribution of FPT is therefore can be defined as-

$$\begin{aligned} F_{T_L}(t) &= Pr(T_L \leq t) = Pr(X(t) \geq L) \\ &= \int_{x=L}^{\infty} f_{X(t)}(x)dx = \frac{\Gamma(A(t), Lb)}{\Gamma(A(t))} \end{aligned} \tag{5.5}$$

Here, $\Gamma(A(t), Lb)$ is the incomplete Gamma function[1].

If the shape function $A(t)$ is differentiable, the PDF of time to failure can be obtained by taking the derivative of equation 5.5 which is provided in (Van Noortwijk (2009)) as follows-

$$f_{T_L}(t) = \frac{A'(t)}{\Gamma(A(t))} \int_{Lb}^{\infty} \{log(x) - \psi(A(t))\} x^{A(t)-1} e^{-x} dx \tag{5.6}$$

Incorporating Current Health Condition

Now let assume that, CM of a component is possible to collect and the current health condition can be measured with reasonable precision. Knowing actual health condition at

[1] $\Gamma(A(t), y) = \int_{z=y}^{\infty} z^{A(t)-1} e^{-z} dz$ is the generic shape of gamma function. If $y = 0$ the gamma function is called complete and otherwise incomplete

current time can aid more accurate estimation of component's RUL as it varies with operating conditions and environment characteristics (Ghodrati et al. (2012)). If the degradation of a component at current time t is $X(t) = x_t$ then the conditional lifetime distribution can be written as-

$$\begin{aligned} F_{T_L}(t) &= Pr(T_L \leq t|X(t) = x_t) \\ &= Pr(X(t) > L|X(t) = x_t) \end{aligned} \tag{5.7}$$

Given the current component degradation level at time t, probability that a component survives an additional length of time h is of interest for the maintenance decision makers. The cumulative distribution function (CDF) of RUL under such assumption can be written as-

$$\begin{aligned} Pr(RUL \leq h) &= 1 - Pr(RUL > h) \\ &= 1 - Pr[X(t+h) < L|X(t) = x_t] \\ &= 1 - Pr[X(t+h) - x_t < L - x_t] \\ &= 1 - \int_0^{L-x_t} f_{A(h),b}(y)dy \\ &= 1 - \int_0^{L-x_t} \frac{b^{A(h)}}{\Gamma(A(h))} y^{A(h)-1} e^{-by} dy \end{aligned} \tag{5.8}$$

The PDF of RUL can be obtained by taking derivative of CDF with respect to h as follows-

$$\begin{aligned} f_{RUL}(h) &= \frac{d}{dh}[F_{T_L}] = \frac{d}{dh}[1 - Pr(RUL > h)] \\ &= -\frac{d}{dh}\left[\int_0^{L-x_t} \frac{b^{A(t+h)}}{\Gamma(A(t+h))} x^{A(t+h)-1} e^{-bx} dx\right] \end{aligned} \tag{5.9}$$

Utilizing Leibniz's rule for differentiation under integral sign, following expression is obtained which must be calculated numerically-

$$\begin{aligned} & f_{RUL}(h) \\ &= \int_0^{L-x_t} e^{-bx} \left[\frac{b^{c(t+h)^u} cux^{c(t+h)^u-1}(h+t)^{u-1}\left(\psi_0(c(t+h)^u) - ln(x) - ln(b)\right)}{\Gamma(c(t+h)^u)}\right] dx \end{aligned} \tag{5.10}$$

5.1.3 An Illustration of Condition-based RUL Estimation

The benefit of incorporating current health condition of the machines in RUL estimation is illustrated in figure 5.1. Here green lines represent Gamma distributed random degradation paths for 100 machines without any inspection before they reach a predefined threshold $L = 180$. The shape of all the degradation paths follow a power law model with parameters $c = 2,\ b = 2,\ u = 2$.

The thick blue line is the degradation path of one randomly selected unit. At current time $t = 7$, the degradation level of this unit is known to be 105. 100 random degradation

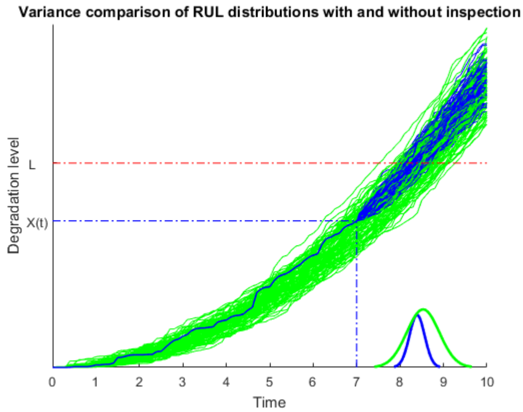


Figure 5.1: RUL distribution at current time t=7 for a threshold value L=150

paths from that point are simulated again with the same parameter values and the first hitting times of reaching L are recorded.

Advantage of calculating RUL by taking account of the current degradation level is immediately realized by Figure 5.2. If the current degradation level is known then the FHT is distributed more densely comparing to the situation when no information is available about the component degradation level. The uncertainty of RUL distribution is reduced when current degradation level is known which leads to a more precise estimation of RUL.

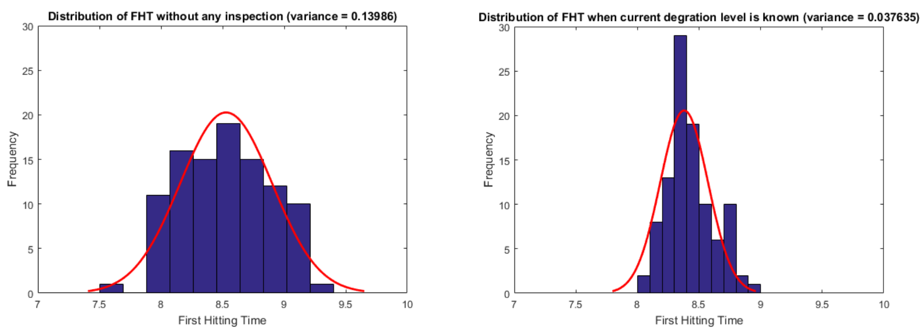
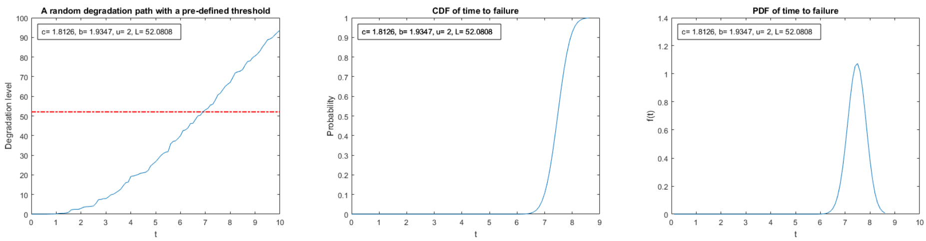


Figure 5.2: Comparison of RUL variances given the same threshold level

5.2 Results and Discussions

In this section, the data set generated in chapter 4 is utilized to investigate the behavior of RUL estimation in different circumstances. For example, figure 5.3 presents the result of CDF and PDF of a randomly selected a component's degradation path from the training data set. It's been assumed that the component is continuously monitored from initial perfect condition until it reaches a predefined threshold. Equations 5.5 and 5.6 are used to calculate CDF and PDF respectively.



(a) Selected degradation path **(b)** CDF of FHT **(c)** PDF of FHT

Figure 5.3: Lifetime distribution of FHT at time t

When a new machine is installed under the assumptions that, the degradation parameters are estimated with reasonable precision and the operating environment is stable over the period then lifetime modeling can answer some useful questions. It can give the probability of useful life exceeding a certain time length (from CDF) or useful life falling between a time interval (from area under the curve of PDF).

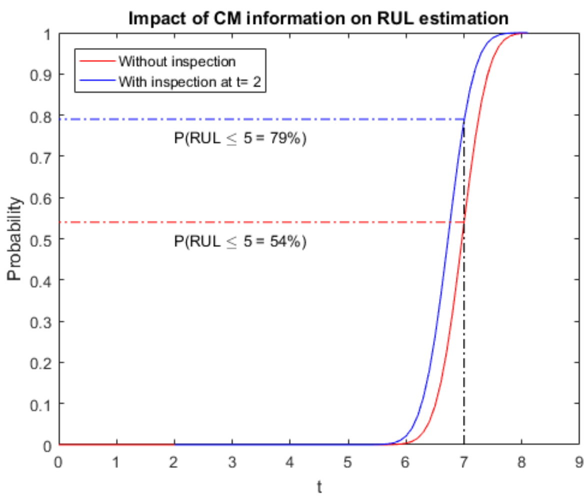
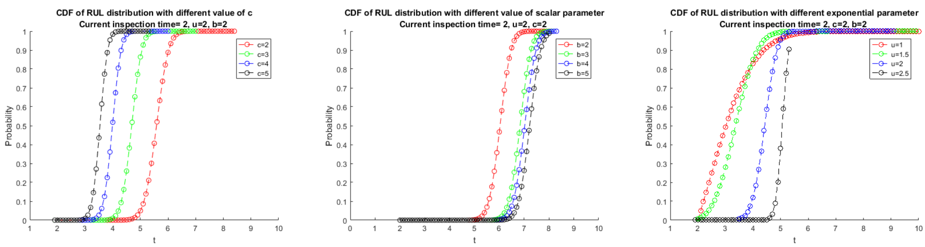


Figure 5.4: Comparison between lifetime modeling and RUL estimation

However, in general rotary machines are subjected to different operating conditions that has influence on component degradation as described in chapter 2. Therefore lifetime modeling may not be sufficient for predicting RUL accurately. In such situation, incorporating the knowledge of current health condition can significantly make a difference. Figure 5.4 shows a comparison between lifetime and RUL modeling for the same component when the component's actual condition is assumed to be known at time 2. It shows that the probability of that component surviving an additional 5 units of time is underestimated by the lifetime modeling which may lead the decision makers planning a preventive maintenance too late than it requires. The behavior of CDF of FHT under the assumption of a known current condition and a failure threshold are discussed below.

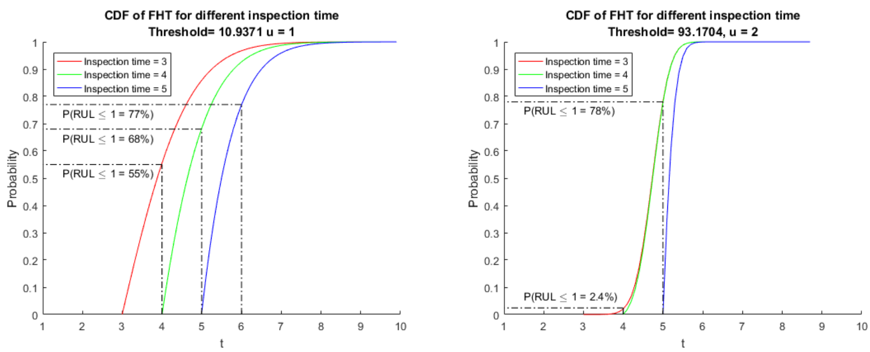
5.2.1 Influence of Parameters



(a) Selected degradation path **(b)** CDF of FHT **(c)** PDF of FHT

Figure 5.5: Behavior of CDF of FHT under different parameter values

How different parameters influence on the estimated CDF of FHT is illustrated in figure 5.5 under the assumption of a pre-defined threshold. (a) depicts the influence of the parameter c which is basically the constant term of the power law model used as shape function. The CDF becomes steeper and moves left along the x-axis as the value of c parameter increases. Which means both the probability of the unit failing before a time h and the degradation rate increases. CDF moves along the x-axis to the right as the value of b increases but the steepness seems to remain constant (figure 5.5 (b)). It should be noted that these results are consistent and in agreement with the behavior of NHGP degradation models w.r.t. different values of c and b parameters in figure 4.4 and 4.5 respectively.



(a) Homogeneous process **(b)** Non-homogeneous process

Figure 5.6: Comparison between homogeneous and non-homogeneous process

Finally (c) depicts the critical role of exponential parameter. When $u = 1$, it represents homogeneous Gamma process (red line). In such case degradation amount accumulates linearly over time unlike NHGP process where degradation rates become faster as the value of u increases. This phenomena is critical in terms of inspection intervals. Figure 5.6 shows the CDFs of FHT at 3 periodic inspection points at 3, 4 and 5 time unit for both homogeneous and NHGP cases. At each inspection point, the probability of the unit failing before one more time unit is presented.

It is clearly evident that, in case of NHGP, the probability of the unit surviving until one more inspection may drastically change after a certain point. This is consistent with the PD phenomena since PD does not cause a sudden breakdown rather they enhance chemical and physical ageing processes by breaking of polymer bonds of insulation material as PDs are repeated many times per voltage cycle (Martínez-Tarifa et al. (2010)).

5.2.2 Influence of Pre-defined Threshold

In threshold based RUL prognostics, determining a threshold accurately is of great importance. Nystad et al. (2012) point out that setting a high failure threshold value increases the risk of actual component failure while a conservative low threshold value may lead to unnecessary and counter productive maintenance interventions. Figure 5.7 clearly depicts the behavior of CDF with respect to setting up different threshold levels.

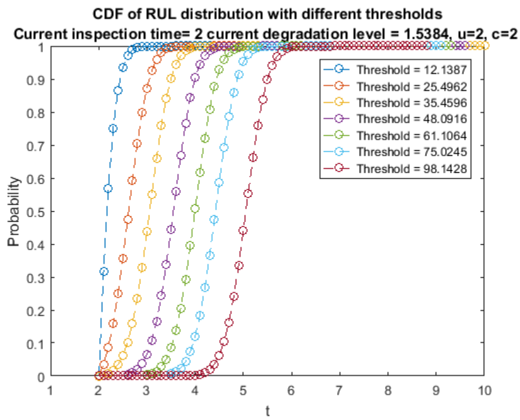
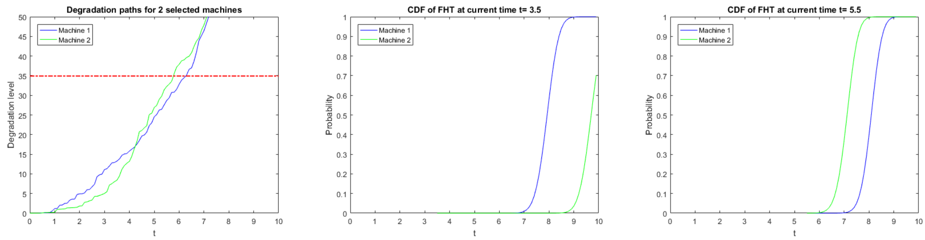


Figure 5.7: Behavior of CDF of FHT w.r.t. different threshold level

Decision process of setting-up a threshold level is often subjected to engineering experience, past data analysis and/or recommended standards (Si et al. (2011)). Although in this thesis a deterministic threshold is assumed to be known but random threshold consideration is possible which is briefly discussed in chapter 6.

5.2.3 Critical Role of Inspection

Referring to Zhu et al. (2001), it's been described in chapter 2 that, increase in PD magnitude is a better indicator than an absolute level of PD magnitude. Figure 5.8 illustrates that claim with a graphical representation.



(a) Degradation paths for 2 machines **(b)** CDF of FHT at 3.5 **(c)** CDF of FHT at 5.5

Figure 5.8: Potentiality of misguided decision making

From the degradation paths of machine 1 and 2 in figure 5.8 (a), although machine 2 degrades slowly at the beginning but after some point in time the degradation rapidly increases and outrun the degradation of machine 1 at around $t = 4$. Inspection before $t = 4$ indicates that machine 2 is in better condition than machine 1. However, another inspection at $t = 5.5$ reveals the actual situation. Figure 5.8 (b) and (c) illustrate the drastic differences in CDFs of FHT for both inspection times.

5.3 System Level Prognostics

The scope of this thesis was kept limited to only unit level RUL study but this section attempts to provide an insight on how the merit of an unit level RUL estimation can be transported into system level RUL estimation for a simplest case of 2 units. Gomes et al. (2013) estimated system level RUL by combining component's individual RUL information and Fault Tree Analysis (FTA) information of the system. A similar approach has been adopted here as well.

Let us consider a hypothetical scenario where 2 gas compressors are in operation. These machines are required to fulfill a specific level of production demand. They are under condition monitoring and a deterministic failure threshold is set. When a machine's health condition reaches the threshold point, it is assumed to be failed to fulfill the demand and it needs to be repaired in order to put in to operation again. Finally, failure of one machine is assumed to be independent from the failure of the other one. Two hypothetical seasonal scenarios are described below-

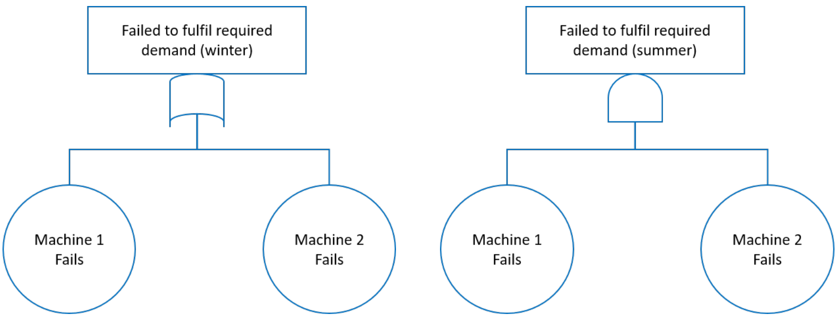
(a) During the winter season when the demand is high, both of the compressors are required to function properly. If one fails then other one can fulfill half of the demand but considering the economic aspect of highly expensive production loss the system is assumed to be failed and management would want to avoid such situations.

(b) During the summer time, the demand can be fulfilled by any of the machines but both of the machines are kept in operation and share the load of production demand. If one fails then other one takes the full load and it is assumed that it does not increase the probability of failure for the functioning machine.

Table 5.1: Degradation data for the machines

	Threshold	Current time	Degradation level	c	b
Machine 1	34.919	2.5	6.1919	2.55	2.57
Machine 2			5.5034	1.78	1.88

These two seasonal scenarios can easily be translated into fault tree diagram as in figure 5.9. Fault tree is basically a logic diagram in order to study the probable causes for a specific system failure, termed as *top event* (Rausand et al., 2004; Gomes et al., 2013). The top event (system failure) occurs when a sequence of events (failure) take place. For example, top event representing *failing to fulfill required demand in winter* occurs when any of the machine fails while top event representing *failing to fulfill required demand in summer* occurs when both of the machines fail. The method to calculate top event probability is briefly described in appendix A.2.2.



(a) Winter demand **(b)** Summer demand

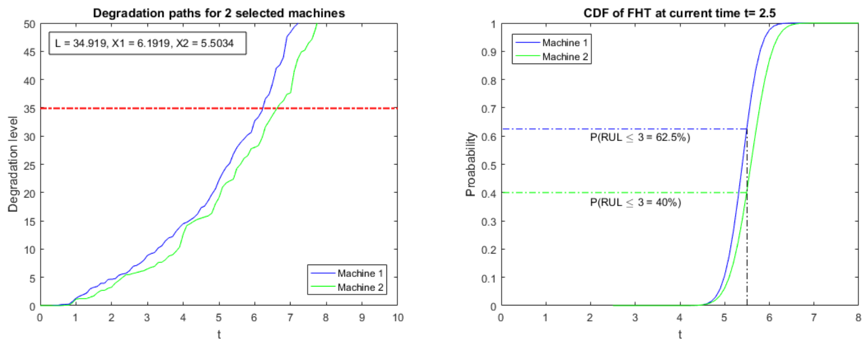
Figure 5.9: Seasonal demand requirement translated into FTA

From the generated data of 100 machines in previous chapter, 2 machines are randomly selected. Both of these machines are assumed to be regularly monitored and the parameters are estimated by collecting all the monitoring data until current time. At $t = 2.5$, the last inspection has been taken place. Table 5.1 summarizes the data-

The original degradation paths of the components and the CDF of FHT at current time are graphically presented in figure 5.10. From these two figures it is clearly visible that the component 2 is approaching faster towards failure. Now let assume a hypothetical situation where at the current time $t = 2.5$, management is interested to know the probability of the system surviving 3 more time unit which is until $t = 5.5$. For the component level the answer is quite straightforward as follows-

$$\begin{aligned} P(RUL_{Machine1} > 3) = 1 - P(RUL \leq 3) = 37.5\% \\ P(RUL_{Machine2} > 3) = 1 - P(RUL \leq 3) = 60\% \end{aligned} \tag{5.11}$$

However if the management is interested in the same question but in terms of system level then further adjustment is required.



(a) Degradation paths **(b)** CDF of FHT

Figure 5.10: Degradation paths and CDF of FHT for 2 machines

5.3.1 System Level RUL During Winter

In this scenario, both of the machines are required to function in order to maintain production demand (see figure 5.9 (a)). Any machine failure is considered as system failure and let the occurrence of basic events machine 1 and machine 2 failure before 3 additional time units are M1 and M2 respectively. Then the probability of the system failing before an additional 3 units of time can be obtained as follows-

$$\begin{aligned}
(RUL_{System} \leq 3) &= Pr(M1 \cup M2) \\
&= 1 - (1 - 0.625)(1 - 0.4) \\
&= 77.5\% \\
(RUL_{System} > 3) &= 22.5\%
\end{aligned} \tag{5.12}$$

Therefore, there is only 22.5% probability that the system will survive 3 more additional time unit. In this particular scenario, the system structure is in series combination. A series structure can not be more reliable than it's least reliable unit Rausand et al. (2004) and therefore by increasing the reliability of machine 1, the system reliability can be improved.

5.3.2 System Level RUL During Summer

For the summer period, only one machine is sufficient for fulfilling the production demand (see figure 5.9 (b)). Therefore, system RUL is obtained by-

$$\begin{aligned}
(RUL_{System} \leq 3) &= Pr(M1 \cap M2) \\
&= (0.625)(0.4) \\
&= 25\% \\
(RUL_{System} > 3) &= 75\%
\end{aligned} \tag{5.13}$$

Due to the simplicity of these particular system structures, the top event probabilities have been calculated directly. However, for a much complex systems, the fault trees can be transformed into its *cut sets* and *upper bound approximation* instead of using structure function. Refer to appendix A.1 for brief definitions of cut sets and upper bound approximation.

Chapter 6

Conclusion

This final chapter first discusses various important aspects and challenges in relation with the proposed degradation model and case-specific challenges for practical implication. Then the limitation of this study is briefly mentioned and based on that, possibilities of future works are discussed and some general recommendations are provided.

6.1 General Discussions

This thesis progressed based on the problem of high voltage rotary machine prognostics in system level. The primary approach to solve such complex problem was to develop a degradation model for a single machine and then further extend the model for incorporating into system level. For a single machine, Electrical insulation system has been identified as the main component of interest in terms of high voltage rotary machine failure based on literature review and expert knowledge involved in Kollsnes gas processing plant.

Thus far, degradation mechanism of an electrical insulation system in high voltage rotary machine has been discussed in order to find an appropriate condition indicator describing the actual health condition of the associated motor. Up until now, not enough evidences have been found for an universal prognostic condition indicator for high voltage rotary machine failures. The available physical tests are mainly used for the diagnostic purposes and none of them are validated as a good prognostic condition indicator yet.

Partial discharge activity shows some promising characteristics in order to become a potential candidate of a good condition indicator with lot of associated limitations needing attention and some of which are discussed in chapter 2. Due to it's well acceptability in industries as well as in Kollsnes gas processing plant, it has been assumed as the condition indicator in this thesis. Besides, although off-line partial discharge tests are more commonly used but unlike other diagnostic tests, on-line partial discharge monitoring is also possible (Stone et al., 2008; Renforth et al., 2015) which is an important factor to consider in condition based maintenance.

Realizing the rapid increase in PD activity over time, an unit level degradation model based on non-homogeneous Gamma process is proposed and behavior under different parameter values, inspection frequency, etc. are discussed. Associated parameters are calculated with 95% confidence interval and remaining useful life is estimated for different scenarios. Further, a transition from an unit level prognostics to system level prognostics is demonstrated. However, practicality of non-homogeneous Gamma process model in high voltage rotary machines is not validated yet mainly due to the lack of field data.

Data collection is one of the biggest challenge associated with validating the proposed model for high voltage rotary machine prognostics. It's been discussed in chapter 2 that, trending partial discharge activities of a single machine is a better degradation indicator than making comparison with other machines. In both situations however, some of the factors need to be in consideration while taking measurements such as, ensuring test instrument bandwidth, noise separation techniques, sensor types, operating voltages etc. Partial discharge activity data without considering these factors may not be useful or reliable for degradation modeling. Any relevant field data for the motors in Kollsnes gas processing plant are not available up to this point. Any systematically collected partial discharge database is also not found that could be useful to check the fitness of the proposed model. Nevertheless such database do exist as mentioned in chapter 2 but unfortunately the access is limited.

Another challenge associated with the threshold based degradation model arises from the difficulties in presetting a threshold itself. Throughout this thesis, a fixed failure threshold is assumed to be known beforehand which is an impractical assumption considering the fact that, partial discharge magnitudes are relative measures and thus an absolute measure is difficult to obtain from similar machines. Karsten Moholt AS, responsible company for the condition monitoring of the rotary machines in Kollsnes, provided a guideline regarding insulation quality with respect to partial discharge activity (refer to Table A.1 in appendix section A.1). This guideline is identical to the on-line partial discharge guideline[1] provided by High Voltage Partial Discharge Ltd (HVPD), an UK based service provider for on-line partial discharge testing and monitoring[2]. The guideline is specified for machines in the range of 3.3-15 kV. However these sort of guidelines are not universal and can be different based on many factors as discussed in chapter 2. Therefore credibility of such guidelines is still under question that needs to be addressed.

6.2 Limitations

One of the main limitation of this research is the unavailability of actual field data which has been discussed above in terms of associated challenges. Besides that, another limitation of the proposed degradation model is regarding operating environment and future usage profile of the machine. This is not a secret that both play an important role in ageing of machinery. However current stage of research is in preliminary stage and requires some validation before moving on to more complex degradation modeling and thus focus of this research has been kept limited accordingly. In addition, proposed model requires a predefined failure threshold in order to provide prognostics of remaining useful life. Finally,

[1]PD guideline by HVPD- `http://sites.ieee.org/houston/files/2016/01/2-HVPD-Night-2-On-line-Partial-Discharge-OLPD-Monitoring-of-Complete-HV-Networks-OG-Industry-Oct.14.pdf`

[2]HVPD- `http://www.hvpd.co.uk/`

as discussed in chapter 4, ageing process basically makes the machine vulnerable to failure under an occurrence of a transient or an operating error. This leaves a room for discussions regarding an appropriate approach for choosing an accurate threshold level, which has not been considered here due to practical limitations. Having mentioned the limitations, the model has potential for further improvement with future works.

6.3 Future Works and Recommendations

Although partial discharge activity has been assumed as a prognostic condition indicator through out this research; the research is not exhaustive regarding finding the most appropriate condition indicator. Further research is required in order to validate the claim that partial discharge is an acceptable indicator for the discussed purpose. The proposed non-homogeneous Gamma process model is valid for any condition indicator as long as the degradation trend is in the agreement with the properties of Gamma process. Therefore in this section, the discussions of potential future works focus only on the improvement of the model.

The most obvious improvement or extension of the proposed model is related to the determination of the failure threshold level. Unlike the assumption of a deterministic known threshold employed in this research, an assumption of a randomly distributed failure threshold is more practical. There are evidences of such implications as Abdel-Hameed (1975) defines the cumulative distribution function of first hitting time in terms of the probability density function of both deterioration trend and failure threshold under the assumption that, the threshold distribution is not dependent from deterioration distribution. Based on that, Nystad et al. (2012) conducts a case study of choke valve erosion used in offshore oil platform, to investigate associated problems with remaining useful life estimations when considering randomly distributed failure thresholds.

It was previously discussed in chapter 2 that, the measurement of the partial discharge activity is sensitive to many factors such as testing methods, noise separation techniques, operating voltage, etc. There are very minimal evidences thus far that the condition monitoring of motors in Kollsnes gas processing plant follow such systematic approach in terms of partial discharge activity measurement. Therefore further study regarding fitness of the proposed model to the actual case is questionable utilizing the available field data. However, there are evidences of systematic data collection as previously mentioned. For example, Stone and Warren (2004) investigate the effect of manufacturing winding age and insulation type on stator winding partial discharge levels based on data collected by *Iris Power Engineering*[3]. They also state that, there are collection of over 60,000 test results at the end of 2003 over the period of 10 years. By utilizing such databases, the proposed model can be studied more deeply in order to both validate and update the model. In addition, such database can also be utilized to understand the possible probability distribution of failure threshold.

Thus far, a single condition indicator has been assumed to describe the health condition of the motor condition. However the machines are operated in a complex operating environment under dynamic loads. Relying on a single condition indicator to describe

[3] Iris Power Engineering (`https://irispower.com/`

the actual health condition of such complex machinery is very unlikely to be enough. In addition, different factors such as operating conditions, usage profiles, etc. may have some associated factors. If there are such factors which have dependency with each others then further works are required to identify the factors and understanding the relationship between those factors. In such situations, multivariate Gamma process models can be investigated for specific case and Zhou et al. (2010) discuss such model for bivariate case.

The proposed model is capable of updating estimated parameters as the new condition monitoring information becomes available. Further comparative studies are possible in order to discuss appropriate approach. For example, Xu and Wang (2012) propose an adaptive Gamma process based model by considering the parameters as hidden state variables. Specially in case of insulation health, an adaptive model might be of great interest for more accurate prediction given the behavior described in subsection 5.2.3.

Gradual ageing process has been the main focus in the development of the degradation model thus far. However as described in section 4.3.3, transient (shock) is an important factor for the failure of the insulation system. Further researches are worth being conducted in order to combine the current degradation model with the probability of transient being taken place during a certain time period. Similar research works exist in literature. For example, Castro (2013) proposes condition based maintenance model by modelling the degradation with Gamma process and the initiation of degradation process with a non-homogeneous Poisson process.

Based on the conducted research with simulated data, it is difficult to conclude and recommend specific guidelines regarding the remaining useful life prognostics for the motors in Kollsnes gas processing plant. However, under the assumption that, degradation progression follows a Gamma process, some general concluding remarks can be presented. First of all, regardless of the approach taken for degradation modeling, useful data is a prerequisite for condition based maintenance. Therefore, much attention should be paid both regarding data collection and the quality of the collected data. In case of partial discharge monitoring, systematic approach should be taken so that the collected data is useful for comparison, trending and analysis. In addition, in case of installation of new rotary machines, an appropriate condition monitoring plan should be in consideration for quality data availability for the future assessment.

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Appendix A

Appendix

This appendix includes definitions and methods that are used but not explained in the main document. It also provides Matlab codes for main functions used in the simulation and estimation process.

A.1 Definitions

A.1.1 Compound Poisson Process

A jump stochastic process where jumps arrive according to a Poisson process. Unlike general Poisson process, the size of jumps is also random with a specified probability distribution.

For a Poisson process $\{N(t) : t \geq 0\}$ with a rate parameter λ, a compound Poisson process $\{X(t) : t \geq 0\}$ is defined as-

$$X(t) = \sum_{i=1}^{N(t)} D_i \tag{A.1}$$

$\{D_i : i \geq 1\}$ are i.i.d. random variables with a specified distribution function.

A.1.2 Condition Based Maintenance (CBM)

A maintenance support program that recommends maintenance actions based on condition monitoring information in order to avoid unnecessary maintenance tasks. Proper implementation of CBM helps reducing the number of unnecessary scheduled preventive maintenance operations which in turn significantly reduce maintenance cost (Jardine et al., 2006).

A.1.3 Cut Sets

According to Rausand et al. (2004), ”A cut set in a fault tree is a set of basic events whose occurrence (at the same time) ensures that the top event occurs. A cut set is said to be minimal if the set cannot be reduced without loosing its status as a cut set.”

A.1.4 Dewatering

Gas well dewatering is also known as gas well delequification which is a gemeral term to refer technologies used to remove water from the producing gas well that builds up due to the natural gas flow inside the well[1].

A.1.5 Maximum Likelihood Estimate (MLE)

Scholz (1985) defines MLE as follows-

Let $X = X_1, ..., X_n$ be a random vector of observations with a density $f_n(x|\Theta)$ over n-dimensional Euclidean space R^n. Θ is the unknown parameter vector contained in the parameter space $\Omega \subset R^s$. Then for any $\hat{\Theta} = \hat{\Theta}(x) \epsilon \Omega$ which maximizes $L(\Theta)$ over Ω is called *maximum likelihood estimate* of the unknown true parameter Θ.

Where $L(\Theta) = L_x(\Theta) = f_n(x|\Theta)$ considered as a function of $\Theta \epsilon \Omega$ for fixed x.

A.1.6 Upper Bound Approximation

Rausand et al. (2004) describes the top event probability in terms of upper bound approximation as follows:

Let a system is represented with k minimal cut sets $K_1, K_2, ...K_k$ then the system can be represented by a series structure of k minimal cut parallel structure.

Now let $\hat{Q}_j(t)$ is the probability of minimal cut parallel structure j fails at time t and $Q_0(t)$ is the top event failure probability. The top event probability is-

$$Q_0(t) \leq 1 - \prod_{j=1}^{k}(1 - \hat{Q}_j(t)) \tag{A.2}$$

The inequality sign represents that minimal cut sets may not be always independent meaning that the same basic events may represent several cut sets.

A.2 Methods

A.2.1 Newton-Raphson Method

Akram and ul Ann (2015) describes the Newton-Raphson method in detail which is method to find better approximations of the roots (zeroes) of a real valued function such as,

$$x : f(x) = 0 \tag{A.3}$$

[1]Source: http://j-jtech.com/services/gas-well-dewatering/

For a given real-valued function f and it's derivative f', the process of finding the root of f starts with an initial guess x_0. For a single variable and assuming that the function satisfies all the assumptions associated in the derivation of the formula, a better approximation x_1 is-

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} \tag{A.4}$$

Where $(x_1, 0)$ is the intersection of $x-$axis and the tangent of the graph of f at $(x_0, f(x_0))$.

The process is iterated until a sufficiently accurate value is found and therefore for a required n number of iterations $(n+1)$th approximation is-

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \tag{A.5}$$

An example of illustration of the approximation process is shown in figure A.1[2]

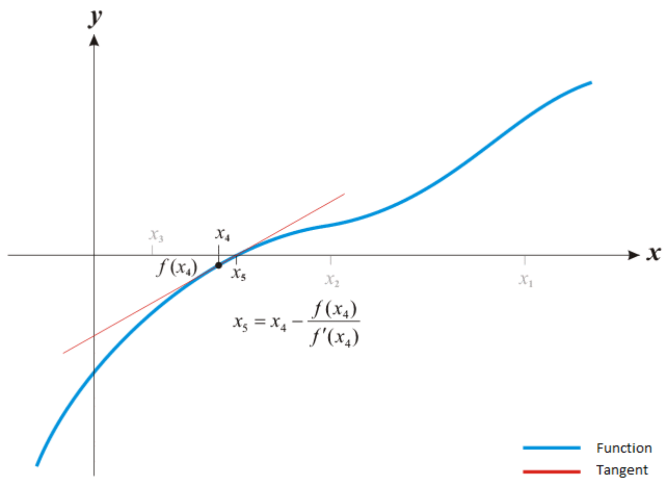

Figure A.1: Illustration of finding root of a given function

A.2.2 Fault Tree Analysis- Top Event Probability

Rausand et al. (2004) describes the method to calculate top event probability in terms of structure function as follows-

Let state vector for the structure at time t is $Y(t) = Y_1(t), Y_2(t), ..., Y_n(t)$. Then the structure function of the fault tree is-

$$\psi(Y(t)) = \psi(Y_1(t), Y_2(t), ..., Y_n(t)) \tag{A.6}$$

[2]Image source: `https://commons.wikimedia.org/wiki/File:NewtonIteration_Ani.gif`

For $\psi(Y(t)) = 1$, top event occurs at t.

If $Q(t)$ represents the probability of top event failure at time t then,

$$Q(t) = Pr(\psi(Y(t)) = 1) \tag{A.7}$$

Let $B_i(t)$ denote basic event B_i for $(i = 1, 2, .., n)$ at time t and $q_i(t)$ denote the unreliability of component i at t. Then for a single AND gate-

$$\begin{aligned} Q(t) &= Pr(B_1(t) \cap B_2(t) \cap ... \cap B_n(t)) \\ &= Pr(B_1(t)).Pr(B_2(t))...Pr(B_n(t)) \\ &= q_1(t).q_2(t)...q_3(t) \\ &= \prod_{i=1}^{n} q_i(t) \end{aligned} \tag{A.8}$$

Similarly for a single OR gate-

$$\begin{aligned} Q(t) &= Pr(B_1(t) \cup B_2(t) \cup ... \cup B_n(t)) \\ &= 1 - \prod_{i=1}^{n} (1 - q_i(t)) \end{aligned} \tag{A.9}$$

A.3 Resources

A.3.1 PD Guideline

Table A.1: PD guideline from Karsten Moholt AS

Insulation Quality	**PD (nC)**
Excellent	<2
Good	>2<4
Average	>4<10
Still acceptable	>10<15
Inspection recommended	>15<25
Unreliable	>25

A.4 Matlab Codes

A samples of main codes and functions are presented. Codes for graphs and decorative commands are not included due to space limitations.

A.4.1 NHGP Simulation, Parameter Estimation & Confidence Interval Calculation

```
% Definitions of variables and pre-allocation

c=2; u=2; b=2; bp=1/b;
T=10; inc=.1;
t1=transpose(0:inc:T);
t=repmat(t1,1,M);
M=100; N=length(t);
sp_inc=zeros(N,1);
sp=zeros(N,1);

g=zeros(N,M);
G=zeros(N,M);
xn=zeros(1,M);
lndeli=zeros(N,M);
cumsumlndeli=zeros(N,M);
sumlndeli=zeros(1,M);

p3=zeros(N,M);
cumsump3=zeros(N,M);
sump3=zeros(1,M);

% Calculating the shape function and function increment

for i=1:size(t,1)-1
    sp(i+1)=c*(power(t(i+1),u));
    sp_inc((i+1))=c*(power(t(i+1),u)-power((t(i)),u));
end

% Generate N number of gamma distributed observations
% for each of the M samples

for sample=1:1:M
    for i=1:size(t,1)-1
        g(i+1,sample)= (gamrnd(sp_inc(i+1),bp));
        G(i+1,sample)= g(i+1,sample)+G(i,sample);
        lndeli(i+1,sample)=log(g(i+1,sample));
        cumsumlndeli(i+1,sample)=lndeli(i+1,sample)+cumsumlndeli(i,sample);
        p3(i+1,sample) = (power(t(i+1,sample),u)-power(t(i,sample),u)).*lndeli(i+1,sample);
        cumsump3(i+1,sample)=p3(i+1,sample)+cumsump3(i,sample);
    end
    xn(sample) = G(N,sample);
    sumlndeli(sample)=cumsumlndeli(N,sample);
```

```
    sump3(sample)=cumsump3(N,sample);
end

totxn= sum(xn);
totsunlndeli = sum(sumlndeli);
totsump3=sum(sump3);

% Newton method to approximate the value of c

itermax =200;
eps = 1;
iter = 0;
CNM =0;
cest = 3.7;

while eps>=1e-4 && iter<=itermax
        CNM = cest - FirstFunM( N,M,t,cest,u,totxn,totsump3
            )/DiffFunC(cest,u,t,N);
        eps = abs(cest-CNM);
        cest = CNM;
        iter = iter+1;
end

% Estimate b with approximated c value

best=M*cest*power(t(N),u)/totxn;

% Standard error, variance & confidence interval
% computation of estimated parameters

varcest=var_cest( cest,u,t,N,M );
varbest=cest*M*(power(t(N),u))/power(best,2);

varcestinv=power(varcest,-1);
varbestinv=power(varbest,-1);
SEcest=sqrt(varcestinv);
SEbest=sqrt(varbestinv);
CIcestP=cest*exp((1.96*SEcest)/cest);
CIcestN=cest*exp(-1.96*SEcest/cest);
CIbestP=best*exp(1.96*SEbest/best);
CIbestN=best*exp(-1.96*SEbest/best);

CIcest=[CIcestN CIcestP];
CIbest=[CIbestN CIbestP];
```

Function: Calculation of Equation 4.24

```
function FFM = FirstFunM( N,M,t,cest,u,totxn,totsump3 )

psiti=zeros(N,1);
cumpsiti=zeros(N,1);

for i=1:N-1
    psiti(i+1) = (power(t(i+1),u)-power(t(i),u)).*(psi(cest
        .*(power(t(i+1),u)-power(t(i),u))));
    cumpsiti(i+1) = cumpsiti(i)+psiti((i+1));
end

part1=cumpsiti(N);
part2 = power(t(N),u)*log((M*cest*power(t(N),u))./totxn);
part3=totsump3/M;

FFM = part1 - part2 - part3;

end
```

Function: Calculation of Equation 4.25

```
function DFC = DiffFunC( cest,u,t,N )

DFC1=zeros(N,1);
cumsumDFC=zeros(N,1);

for i =1:N-1
    DFC1(i+1) = power((power(t(i+1),u)-power(t(i),u)),2)
        .*((psi(1,(cest.*(power(t(i+1),u)-power(t(i),u))))))
        ;
    cumsumDFC(i+1) = cumsumDFC(i)+DFC1((i+1));

end

DFC = cumsumDFC(N) - (power(t(N),u))/cest;
```

Function: Calculation of Variance

```
function VC = var_cest( cest,u,t,N,M )

DFC1=zeros(N,1);
cumsumDFC=zeros(N,1);

for i =1:N-1
```

```
    DFC1(i+1) = power((power(t(i+1),u)-power(t(i),u)),2)
        .*((psi(1,(cest.*(power(t(i+1),u)-power(t(i),u))))))
        ;
    cumsumDFC(i+1) = cumsumDFC(i)+DFC1((i+1));

end

VC = cumsumDFC(N)*M;
```

A.4.2 RUL Estimations From Training Data

```
load('b2c2u2.mat','t','g','G','cest','best','u');

choosesample=randi([1 100],1,1);
Threshold=70;
currenttime=20;

G=G(:,choosesample);
t=t(:,choosesample);

L=G(Threshold);
upperlimit=300;
dx=.1;

% Estimate parameters

[c,b]=calccbestfromdata(G,t,u,1);

% Calculate CDF of FHT from initial condition

numerator=zeros(length(t),1);
resultCDF=zeros(length(t),1);
denominator=1-CalcRULCDFNHGPnew(L,b,t(currenttime),
    upperlimit,dx,c,u);

for i=1:length(t)
    numerator(i)=1-CalcRULCDFNHGPnew((L-G(currenttime)),b,t
        (i),upperlimit,dx,c,u);
    resultCDF(i)=1-(numerator(i)/denominator);
end

% Calculate PDF of FHT from initial condition

resultpdf=zeros(length(t),1);

for i=1:length(t)
```

```
    resultpdf(i)=CalcRULPDFNHGP(t(i),dx,c,u,(L*b),upperlimit);
end

% Calculate CDF of FHT from current observation

currenttime=20;
currenttime=currenttime+1;
remainingtime=length(t)-currenttime;

numerator=zeros(length(t),1);
denominator=1-CalcRULCDFNHGPnew(L,b,t(currenttime),upperlimit,dx,c,u);
checkingextratime=(t(currenttime):.1:t(currenttime+remainingtime));
checkingextratime=transpose(checkingextratime);
resultcdfrul=zeros(length(checkingextratime),1);

for i=1:numel(checkingextratime)
    numerator(i)=1-CalcRULCDFNHGPnew((L-G(currenttime)),b,checkingextratime(i),upperlimit,dx,c,u);
    resultcdfrul(i)=1-(numerator(i)/denominator);
end
```

Function: Calculation of PDF from Initial Condition

```
function P = CalcRULPDFNHGP(t,dz,c,u,lowerlimit,upperlimit)

sp=c*power(t,u);
constantpart = c.*u.*power(t,(u-1))/gamma(sp);

spsub=sp-1;

z = lowerlimit:dz:upperlimit;

doint = (log(z)-psi(sp)).*(power(z,spsub)).*exp(-z);

fz=constantpart*doint;

P = sum(fz)*dz;

end
```

Function: Calculation of CDF from Initial Condition

```
function P = CalcRULCDFNHGPnew(L,b,t,upperlimit,dx,c,u)

```

```
sp=c*t^u;

firstpart = b^(sp)/gamma(sp);
x = L:dx:upperlimit;
fx = firstpart*x.^(sp-1).*exp(-b*x);
P = sum(fx)*dx;

end
```