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Noise and dissipation in magnetoelectronic nanostructures

Doctoral thesis
for the degree of philosophiae doctor

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Abstract

This thesis adresses electric and magnetic noise and dissipation in magnetoelectronic nanostructures. Charge and spin current fluctuations are studied in various nanosized metallic structures consisting of both ferromagnetic and non-magnetic elements. The interplay between current and magnetization fluctuations, and the relation of these fluctuations to the electric and magnetic dissipation of energy, are considered. Special focus is on the enhancement of magnetization damping due to so-called spin pumping, which is shown to be directly connected to thermal spin current fluctuations.

Two fundamental sources of current noise are considered: Thermal noise and shot noise. Since a spin current polarized transverse to the magnetization is absorbed in a ferromagnet, transverse spin current fluctuations exert a fluctuating torque on the magnetization. This spin-transfer torque causes significant magnetization fluctuations in nanoscale ferromagnets. In single-domain ferromagnets in contact with normal electric conductors, the spin-current induced magnetization noise is directly connected to the magnetization damping caused by spin pumping. In non-uniformly magnetized ferromagnets, the spin-current induced magnetization noise is related to a nonlocal tensor damping that reflects the spatial variation of the magnetization. At low temperatures, spin current shot noise in the presence of an applied bias is the dominant contribution to the magnetization noise.

In spin valves, two ferromagnets are separated by a thin normal metal spacer. The interaction of the ferromagnets affects their magnetization noise and damping, which are shown to vary with the relative magnetic orientation of the ferromagnets. Due to giant magnetoresistance, the magnetization fluctuations cause resistance noise. The resistance noise is identified as a prominent source of electric noise at relatively high current densities. The noise level can vary substantially with the relative magnetic orientation.

Preface

The work presented in this thesis was done at the Department of Physics, NTNU, Trondheim during the years 2003-2008. The experimental work presented in paper I was done at the Surface Physics Lab at Simon Fraser University, Vancouver, Canada during the spring of 2003.

The thesis presents studies of noise and dissipation in various nanosized magnetoelectronic structures. Five papers form the basis of the thesis. The work was financed by the Research Council of Norway through the FUNMAT/NANOMAT program.

I would like to thank my supervisor Arne Brataas for his patience and support as I struggled with this thesis. Without his help I certainly would not have succeeded. I would also like to thank my highly skilled collaborators on four of the papers, Gerrit E. W. Bauer and Yaroslav Tserkovnyak. Their contributions made a big difference. I thank Bret Heinrich and Georg Woltersdorf for the collaboration on paper I. My colleagues at the Department of Physics, especially my office roommates Roman Shchelushkin, Hans Joakim Skadsem and Håvard Haugen, deserve thanks for their friendliness and helpfulness. Hans Joakim and Mariann Heldahl is thanked for proofreading parts of this thesis.

Trondheim, March 2008

Jørn Foros

List of Papers

Paper [1]
Scattering of spin current injected in Pd(001)
Jørn Foros, Georg Woltersdorf, Bret Heinrich, and Arne Brataas
Journal of Applied Physics **97**, 10A714 (2005)

Paper [2]
Magnetization noise in magnetoelectronic nanostructures
Jørn Foros, Arne Brataas, Yaroslav Tserkovnyak, and Gerrit E. W. Bauer
Physical Review Letters **95**, 016601 (2005)

Paper [3]
Resistance noise in spin valves
Jørn Foros, Arne Brataas, Gerrit E. W. Bauer, and Yaroslav Tserkovnyak
Physical Review B **75**, 092405 (2007)

Paper [4]
Electric and magnetic noise and dissipation in ferromagnetic single- and double-layers
Jørn Foros, Arne Brataas, Gerrit E. W. Bauer, and Yaroslav Tserkovnyak
To be submitted to Physical Review B

Paper [5]
Current-induced noise and damping in non-uniform ferromagnets
Jørn Foros, Arne Brataas, Yaroslav Tserkovnyak, and Gerrit E. W. Bauer
Submitted to Physical Review B [arXiv:0803.2175]

Contents

Chapter 1

Introduction

The sound of heavy traffic downtown, bad radio reception, snow on the TV-screen - there are many examples of noise in our everyday lives. By noise then, we typically understand unwanted sound or disturbance. In physics and electronics, on the other hand, noise refers specifically to the random fluctuations or variations of observable quantities. For example, in a conductor, the current fluctuates due to thermally induced random movement of the conducting electrons. Similarly, the magnetization in ferromagnets fluctuates due to thermal variations of the direction of the microscopic atomic magnetic moments.

Dissipation of energy is another fundamental property of physical systems. When a system is driven from its lowest energy equilibrium state by some external perturbation, and then left alone, it will typically return to equilibrium by dissipating energy - the system is damped. To sustain motion in a damped system, a continuous input of energy is needed. For example, a simple pendulum will inevitably stop in its equilibrium state - hanging vertically - unless you give it a push once in a while.

In many cases, noise is unwanted. Transmitting information through a wire, you don't want electronic noise to obscure the signal. Reading information from your magnetic hard disk, you want the small magnetic regions representing the bits to be in stable states with only small fluctuations, to ensure that the read-out is correct. However, in some cases, noise can also be desirable. Writing to your magnetic hard disk, you need to be able to switch the magnetic bits. Thermal fluctuations can aid in this process, yielding faster switching. Similarly, damping may some times be desirable, some times not. Strong damping of the magnetization aids in stabilizing the state of the magnetic bits, making stored information less likely to degrade. But strong damping also makes it harder to switch the bits in the writing process.

Though not obvious, noise and damping are closely related. In fact, the equilibrium noise (i.e., the fluctuations observed in the equilibrium state) and the dissipation of energy in a physical system are just two sides of the same coin. To understand this, let us as an example consider the Brownian motion of small particles suspended in some liquid medium. Floating around in the medium, the particles experience random impacts with the liquid molecules due to the latters thermal motion. As a result, the particles move in a random, irregular pattern. Suppose next that some external driving force capable of moving the particles is applied. Due to impacts with the liquid particles, such motion exhibits friction and hence is damped. Thus, it is the same microscopic mechanism that is responsible for the damping of the forced motion, as is responsible for the random motion in equilibrium. This common origin suggests that the out-of equilibrium damping and the equilibrium fluctuations are related. The relation is very general, and is known as the fluctuation-dissipation theorem (FDT) [6, 7]. The FDT has been successfully applied to a variety of systems.

As we have seen, both noise and damping are fundamental properties of physical systems, and hence important for our understanding of nature. The FDT tells us that the equilibrium noise of a system determines the damping, and vice versa. Investigation of noise thus provides an alternative route to gaining insight on the dissipative processes out of equilibrium. Electric current noise in mesoscopic conductors [8] has drawn significant interest in recent years, and serves as another example illustrating the FDT. In equilibrium, the current noise, commonly known as Johnson-Nyquist noise [9, 10], is proportional to the conductivity. The conductivity (or its inverse, the resistivity) is a measure of the frictional force on the conducting electrons when a voltage is applied. Hence, equilibrium noise and out-of-equilibrium dissipation of energy are related.

The examples above show that the study of noise and damping is important both from a fundamental and technological point of view. This thesis concerns the study of noise and damping in various magnetoelectronic nanostructures. These are nano-sized structures with both electrical and magnetic properties, typically consisting of various numbers of alternating magnetic and non-magnetic metallic layers. The combination of normal metals with good conductivity, such as copper or silver, and ferromagnetic metals with large magnetic moments, such as cobalt or iron, into a single structure, has proved to lead to novel physical phenomena and technological devices. Perhaps the best example is the hard disk read head presently used in computers, which is based on an effect known as giant magnetoresistance (GMR), to be discussed in the next chapter. In fact, in the last two decades, research on magnetoelectronic structures and the phenomena they show has grown into a field of its own known as magnetoelectronics or spintronics. The researchers that discovered GMR were awarded the Nobel prize in physics 2007.

Five papers form the basis of the thesis. Paper I reports on an experimental study, while the other four papers are theoretical in nature. The unifying theme is the interplay between electric current noise and magnetization noise, and the damping or dissipation of energy related to the noise. In magnetoelectronic nanostructures, spin-polarized current noise causes the magnetization in ferromagnets to fluctuate due to the so-called spin-transfer torque. This effect is investigated in paper II for a single uniform (monodomain) ferromagnet sandwiched by two normal metals. In paper V, the same effect is studied in non-uniformly magnetized ferromagnets. The damping of the magnetization, which by the FDT is related to the equilibrium magnetization noise, is also considered. Paper IV addresses spin-polarized current noise in various kinds of ferromagnetic double-layers with a normal metal spacer, i.e., so-called spin valves. The resulting magnetization noise and related damping are considered. Paper III, and also paper IV, addresses resistance noise induced by magnetization noise in spin valves. Paper I presents experimental work on magnetization damping, related to the theoretical results of paper II. The paper discusses enhanced magnetization damping in thin iron (Fe) layers connected to the non-magnetic metal palladium (Pd). The enhancement of the damping is due to "pumping" of spins from the Fe layer to the Pd layer.

Before proceeding, a short survey of the scientific progress in the relevant areas is in order. There has been [11, 12, 13, 14, 15] and is still [16, 17, 18] a substantial amount of research on the nature of magnetization damping. The so-called Gilbert damping constant, which parametrizes the magnetization damping, was phenomenologically introduced in 1955 [11, 12]. Recently Kohno *et al.* [16] and Skadsem *et al.* [17] investigated its microscopic origin, while Gilmore *et al.* [18] used first principles to estimate its value in the most common ferromagnetic metals Fe, Co and Ni. The Gilbert damping in ferromagnets connected to a conducting environment has been shown to be considerably enhanced as compared to the intrinsic damping in isolated ferromagnets [19, 20, 21, 22]. With regards to the noise properties of magnetoelectronic nanostructures, a number of experimental and theoretical studies have been carried out. Shot noise in hybrid ferromagnetic-normal metal structures has been studied first by Bulka *et al.* [23], and subsequently by many others [24, 25, 26, 27, 28]. The fluctuations of the order parameter in ferromagnets, such as Barkhausen noise due to irregular movement of domain walls [29, 30], have been studied by the magnetism community for a long time. Brown [31] analyzed the thermal fluctuations of the magnetization vector in small single-domain ferromagnets in 1963. The presence of magnetization noise has been shown to be important for the process of magnetization reversal [32, 33, 34, 35]. Thermally assisted depinning of a narrow domain wall under a spin-polarized current has been observed [36], and thermally-assisted current-driven domain wall motion has been studied theoretically [37, 38]. Resistance or voltage noise due to magnetization fluctuations in spin valves has received attention lately [39, 40, 41, 42]. Despite this large body of work, the topic of electric and magnetic noise and dissipation is far from exhausted.

The thesis is organized as follows. In the next chapter, some preliminary concepts are reviewed. In Ch. 3, I explain the Landauer-Büttiker theory of spin-dependent electron transport in mesoscopic magnetic and non-magnetic conductors, with focus on spin current noise. In Ch. 4 I sketch how the transport theory can be used to investigate current-induced magnetization noise and damping in various magnetoelectronic nanostructures, the details of which are the subject of the papers. Finally, at the very back, the papers are included. I emphasize that the following chapters introduce and explain basic theory and concepts needed for the studies reported in the papers. Only to a small extent do they rederive or restate the calculations and results of the papers. It is my hope that the chapters are an understandable and readable introduction to the papers.

Chapter 2

Preliminary concepts

In this chapter I will briefly review some concepts that are central to the thesis.

2.1 Basic spintronic effects

In addition to its charge, the electron carries intrinsic angular momentum, or *spin*. Discovered by the two Dutch physicists Goudsmit and Uhlenbeck in 1925, the existence of the electron spin explained both the fine structure in atomic spectra and the Zeemann effect, which at the time were not understood. The spin is a vector, but being a quantum mechanical property, it is also quantized, and if measured it can only yield the value $-1/2$ or $1/2$, or "up" or "down", in any direction.

A voltage across a conductor generates a current. Electrons propagate through the conductor, resulting in a net transport of charge that may be used to light your apartment or power your TV. The spin of the electrons tag along for the ride, of course, but that's usually it. While the charge of the electron provides desired functionality, the spin typically does not. Spintronics (spin-based electronics), or magnetoelectronics, is the name of

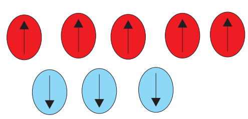

Figure 2.1: A spin current is a net flow of spin angular momentum, i.e., there are for instance more spin-up electrons (red circles) than spin-down electrons (blue circles) that propagate through the conductor.

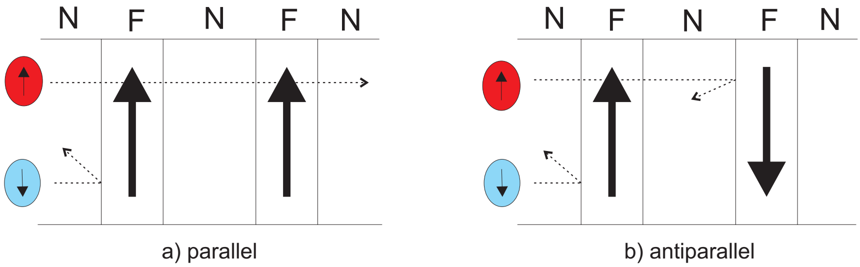

Figure 2.2: A simple explanation of GMR. Two thin ferromagnetic (F) films are separated by a normal metal (N), forming a spin valve. The thick arrows indicate the direction of magnetization in the ferromagnets. The ability of the electrons to transmit through the system depends on their spin and the relative magnetic orientation of the films. Red circles represent spin-up electrons, and blue circles represent spin-down electrons (with spin quantization axis along the direction of magnetization in the ferromagnets). The dotted lines indicate the ease at which the electrons can propagate through the structure in the a) parallel and b) antiparallel configurations.

a young technology that seeks to change just that [43, 44, 45, 46]. As the name suggests, in spintronics one tries to use the electron spin in addition to (or instead of) the charge in electronic appliances. In this context, a spin-polarized current, or simply spin current, is an important quantity. A spin current is a net flow of spin angular momentum, as sketched in Fig 2.1. In ferromagnets, spin-up and spin-down electrons have different conductivities due to the exchange splitting of the electron bands, resulting in a spin-polarization of the current. As the spin is a property of the electron, a spin current may be accompanied by a flow of charge, but not necessarily.

The field of spintronics started with the discovery of giant magnetoresistance (GMR) in 1988 [47, 48, 49, 50]. GMR is seen in structures of alternating magnetic and non-magnetic normal metal layers, and is the dependence of electric transport on the relative magnetization direction of the magnetic layers. Today, GMR is the leading technology for read heads in computer hard disks. To explain GMR, I refer to Fig. 2.2 showing a so-called spin valve, i.e., two ferromagnetic layers separated by a normal metal layer. An electric current is flowing left to right, perpendicular to the interfaces. In a), the ferromagnets are magnetized in parallel, while in b) they are antiparallel. The ease at which the electrons can propagate through the structure is not the same in the two frames, due to the spin-dependent band structure of the ferromagnets. In a), spin-up electrons (red circles)

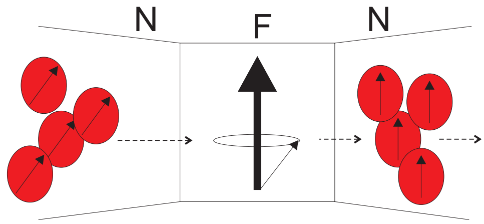

Figure 2.3: A simple explanation of spin-transfer torque. A few electrons that are spin-polarized in a direction differing from the direction of magnetization are incident from a normal metal (N) on a ferromagnet (F). When the electrons enter the magnetic field in the ferromagnet, their spins start precessing. The net average result is a loss of the spin component polarized transverse to the magnetization, which by conservation of angular momentum is equivalent to a torque on the ferromagnet.

can more easily propagate through than the spin-down ones (blue circles), as illustrated by the dotted lines. In b), neither spin-up nor spin-down electrons can easily propagate through. The overall effect is that the electrical resistance is lower in the parallel configuration of a) than in the antiparallel one of b). Albert Fert and Peter Grünberg were awarded the nobel prize in physics in 2007 for this discovery [50]. Resistance *noise* caused by GMR and fluctuating magnetizations in spin valves is considered in papers III and IV.

GMR is a result of the effect that the ferromagnets have on the electric current. Reversely, a spin-polarized current flowing through a ferromagnet can affect the magnetization via the so-called spin transfer torque [51, 52, 53, 54]. The torque is caused by the absorption of the vector component of the spin current polarized transverse to the magnetization. The absorption happens on the length scale of the ferromagnetic coherence length [55, 56, 57, 58], which in transition metals is only a couple of monolayers. For an explanation of this effect, see Fig. 2.3 showing a few electrons incident on a ferromagnet from a normal metal. These electrons are spin-polarized in a direction differing from the direction of magnetization, and their spins will therefore undergo precession when they enter the magnetic field in the ferromagnet. The individual spins will typically precess on different length scales [59]. Averaging over all electrons, the net effect is a loss of the spin component polarized transverse to the magnetization, as illustrated to the right in the figure. By conservation of angular momentum, this implies a torque on the magnetization, that can cause the magnetization to precess or even reverse direction. Spin-transfer torque and current-induced magnetization dynamics have generated a lot of interest over the last years [60, 61, 62, 63, 64], and are central ingredients in this thesis. In several of the papers, a fluctuating spin-transfer torque due to spin current fluctuations is considered.

The spin valve in Fig. 2.2 is a so called current-perpendicular-to-the-plane (CPP) spin valve. While it is easier to illustrate the GMR-effect in such a spin valve, it is its close relative the current-in-the-plane (CIP) spin valve which currently is used in hard disk read heads. CPP read heads are believed to be a serious alternative to the current CIP ones in the future, promising better performance. In this respect, understanding and controlling magnetization noise and damping in CPP-heads is important, and a motivation for the work presented in this thesis. The noise properties of CPP spin valves and CPP nanopillar multilayers are subject to much ongoing research [41, 65, 42].

Magnetic random access memory (MRAM) is another spintronic device that in the future might find its way into most personal computers. This memory has the advantage over todays RAM that it is non-volatile [66].

2.2 Ferromagnetism

As we have seen, ferromagnets are essential elements in spintronics, and in the work reported in this thesis. A brief introduction to ferromagnetism and magnetization dynamics is therefore in order.

Due to the Pauli exclusion principle and the strong exchange interaction between electron spins, some metals, such as iron, cobalt and nickel, exhibit ferromagnetism [67]. Their atomic magnetic moments lign up in a common direction, giving them a net magnetization even in the absence of applied external fields. This spontaneous and permanent magnetization is used in a variety of applications, from simple compass needles to advanced spintronic devices such as GMR read heads and MRAM.

However, as a result of the magnetostatic dipolar interaction, *all* the atomic magnetic moments in a ferromagnetic sample typically do not align in the same direction. Instead, the ferromagnetic order is broken up into domains, in which all atomic moments are aligned, but where different domains are magnetized in different directions. The area between domains over which the magnetization direction changes, is called a domain wall [64]. Other, more exotic, kinds of magnetic ordering are also possible. For example, in the γ-phase of iron, the magnetization has been observed to form a so-called spin spiral. Paper V considers magnetization noise and damping in such non-uniformly magnetized ferromagnets. All the other papers focus

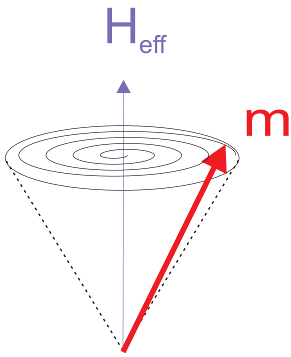

Figure 2.4: A simple illustration of the basic features of the LLG equation. If displaced from the equilibrium direction along $\mathbf{H}_{\text{eff}}$, the magnetization vector will undergo damped precessional motion.

on ferromagnetic films that are sufficiently thin that the magnetization is uniform, or single-domain. In such ferromagnets, the total magnetization is just the algebraic sum of all the atomic moments, and the ferromagnets act as giant magnetic molecules or *macrospins* [68].

The magnetization dynamics of single-domain ferromagnets is well described by the Landau-Lifshitz-Gilbert (LLG) equation of motion [69, 11, 12]

$$\frac{d\mathbf{m}}{dt} = -\gamma \mathbf{m} \times \mathbf{H}_{\text{eff}} + \alpha_0 \mathbf{m} \times \frac{d\mathbf{m}}{dt}, \tag{2.1}$$

where $\mathbf{m}$ is the unit magnetization vector of the ferromagnet, γ is the gyromagnetic ratio, and $\mathbf{H}_{\text{eff}}$ the effective magnetic field. Although magnetization is quantum mechanical in origin, this classical equation of motion has proved successful in describing the behaviour of ferromagnets [70]. The equation describes only transverse magnetization dynamics, since both terms on the right hand side involves cross products in $\mathbf{m}$. Longitudinal changes in the magnetization, i.e., changes in the magnetization magnitude, are typically costly in energy, and are not considered in this thesis. The effective field $\mathbf{H}_{\text{eff}}$ can be obtained from an energy functional, and usually includes contributions both from internal anisotropy and demagnetizing fields, externally applied fields, as well as dipolar and exchange coupling to possible neighbouring ferromagnets. The second term on the right hand side of the LLG equation describes damping of the magnetization motion. α_0, called the Gilbert damping constant [11, 12], parametrizes this dissipation of energy. As noted in the previous chapter, the origin and nature of α_0 is an active area of research. Paper I presents experimental work on enhancement of the Gilbert damping due to so-called "spin pumping"[21, 22]. Fig. 2.4 illustrates the basic features of the LLG equation for the case where an external perturbation has displaced the magnetization from its equilibrium direction along $\mathbf{H}_{\text{eff}}$. The first term on the right hand side of the LLG equation is a torque, making the magnetization vector precess around the direction of the effective magnetic field. The damping term causes the precession to diminish and the magnetization vector to gradually approach $\mathbf{H}_{\text{eff}}$ - the lowest energy direction. Note that this is a somewhat simplified picture, in which $\mathbf{H}_{\text{eff}}$ has been simply taken as a constant. The LLG equation is used also to describe dynamics in non-uniform ferromagnets.

The LLG equation can be modified to take into account the spin-transfer torque discussed in the previous section. By conservation of angular momentum, this gives an extra term $\gamma \mathbf{I}_s^{\text{abs}}/M$ on the right hand side, where $\mathbf{I}_s^{\text{abs}}$ is the absorbed (vector) spin current, and M is the total magnetic moment of the ferromagnet. This modification of the LLG equation is used in several of the papers.

The magnetization noise, or more precisely, the fluctuations of the magnetization vector, can be examined using the LLG equation and the fluctuation-dissipation theorem. This is illustrated in the following section.

2.3 The fluctuation-dissipation theorem

The fluctuation-dissipation theorem (FDT) was first formulated by Callen and Welton in 1951 [6]. However, the relation between fluctuations and dissipation for e.g. Brownian motion and electric currents have been known longer. Presentations of the FDT have been given by Landau and Lifshitz [7] and by Morandi, Napoli and Ercolessi [71], and by White for magnetic systems [72]. The usefulness of the FDT in investigating magnetization noise and damping has been discussed lately [73, 74].

The FDT relates the equilibrium noise of physical systems to their response to external perturbations. More precisely, the equilibrium fluctuations of the physical quantity that characterizes the system are related to the out-of-equilibrium dissipation of energy. Examples of this general connection has already been given in terms of Brownian motion and electric current noise. In this thesis, focus is on magnetic systems and magnetization noise. In the following, I briefly present the FDT applied to such systems, showing that the equilibrium magnetization fluctuations are related to the dissipative part of the magnetic susceptibility, i.e., the Gilbert damping constant.

A single-domain ferromagnet may be characterized by its uniform unit magnetization vector $\mathbf{m}$. The spontaneous equilibrium fluctuations at time t are $\delta\mathbf{m}(t) = \mathbf{m}(t) - \langle\mathbf{m}(t)\rangle$, where the brackets denote statistical averaging at equilibrium. The fluctuations are most conveniently analyzed in terms of the correlator $S_{ij}(t-t') = \langle\delta m_i(t)\delta m_j(t')\rangle$, where i and j denote Cartesian components. Since we consider the equilibrium state of the system, S_{ij} only depends on the time difference $t-t'$. The classical FDT states that S_{ij} is related to the magnetic susceptibility:

$$S_{ij}(t-t') = \frac{k_B T}{2\pi M}\int d\omega e^{-i\omega(t-t')}\frac{\chi_{ij}(\omega) - \chi^*_{ji}(\omega)}{i\omega}, \tag{2.2}$$

where T is the temperature, M is the total magnetic moment of the ferromagnet, and $\chi_{ij}(\omega)$ is the ij-component of the (Fourier-transformed) magnetic susceptibility. Notice that the fluctuations vanish as $T \to 0$. The susceptibility is defined by the linear magnetic response to an external magnetic field $\mathbf{h}^{(\mathrm{ext})}(t)$:

$$\Delta m_i(t) = \sum_j \int dt' \chi_{ij}(t-t') h_j^{(\mathrm{ext})}(t'), \tag{2.3}$$

where $\mathbf{\Delta m}(t)$ is the change in magnetization caused by the field. The susceptibility can be evaluated with the aid of the LLG equation. Including $\mathbf{h}^{(\mathrm{ext})}(t)$ in Eq. (2.1), and then linearizing in the small quantities $\mathbf{h}^{(\mathrm{ext})}(t)$ and $\mathbf{\Delta m}(t)$, yields

$$\chi^{-1} = \frac{1}{\gamma}\begin{bmatrix} \gamma|\mathbf{H}_{\mathrm{eff}}| - i\omega\alpha_0 & i\omega \\ -i\omega & \gamma|\mathbf{H}_{\mathrm{eff}}| - i\omega\alpha_0 \end{bmatrix}. \tag{2.4}$$

The inverse susceptibility is here written in matrix (tensor) form in the space (plane) orthogonal to the equilibrium magnetization direction. Note that the effective field has been assumed magnetization independent, as was done also in the explanation related to Fig. 2.4 in the previous section. This simplification has no effect on the calculation of noise, since in this case only the dissipative part of the susceptibility plays a role. Inserting the inverse of Eq. (2.4) into Eq. (2.2) determines the equilibrium noise.

The spontaneous equilibrium fluctuations $\delta\mathbf{m}(t)$ may be regarded as caused by a fictitious random magnetic field $\mathbf{h}(t)$ with zero mean. An alternative form of the FDT can be derived in terms of the correlator $\langle h_i(t)h_j(t')\rangle$ of this field. To this end, simply note that Eq. (2.3) implies that $\delta m_i(\omega) = \sum_j \chi_{ij}(\omega)h_j(\omega)$, in Fourier space. Inverting this relation, it follows from Eq. (2.2) that

$$\langle h_i(t)h_j(t')\rangle = \frac{k_B T}{2\pi M}\int d\omega e^{-i\omega(t-t')}\frac{[\chi^{-1}_{ji}(\omega)]^* - \chi^{-1}_{ij}(\omega)}{i\omega}. \tag{2.5}$$

Inserting Eq. (2.4) into Eq. (2.5), then yields the well-known result [31]

$$\langle h_i(t)h_j(t')\rangle = \frac{2k_BT\alpha_0}{\gamma M}\delta_{ij}\delta(t-t'), \tag{2.6}$$

where i and j denote components orthogonal to the equilibrium magnetization direction. This expression relates the equilibrium magnetization fluctuations, given in terms of **h**, to the damping or dissipation of energy in the ferromagnet. **h** may be explicitly included in Eq. (2.1) to give a stochastic form of the LLG equation.

Papers II and V consider enhancement of the equilibrium magnetization fluctuations in respectively uniform and non-uniform ferromagnets due to thermal spin current fluctuations. The corresponding enhancement of the damping is also considered, by using the theory described above. Guided by the FDT, paper IV discusses in detail magnetization fluctuations and damping as a function of the magnetic configuration in spin valves.

Chapter 3

Spin-dependent mesoscopic electron transport

This thesis is concerned with nanosized or mesoscopic physical systems. These systems are much smaller than macroscopic objects, but considerably larger than individual atoms. With the demand for increasingly smaller yet functional components in electronic appliances, such systems have become very important. Recent advances in fabrication technology allow them to be manufactured in a controlled way. One or more dimensions of a mesoscopic system is in the nano-micron range, and as a result, quantum mechanics may be important for understanding the system properties. It is thus appropriate to take a quantum mechanical approach to electron transport in these systems. This is commonly done by using the so-called Landauer-Büttiker (LB) formalism [75, 8, 76].

In the following, the basics of the LB approach will be explained. Since this thesis is concerned with magnetoelectronic nanostructures, the generalization of the formalism to spin-resolved transport is considered. The formalism is the basis for the investigation of spin current fluctuations in the papers. This chapter is divided into three sections. In the first, the LB theory is introduced, following the presentation in Refs. [75] and [8]. Charge and spin current fluctuations are then considered within this framework. Magnetoelectronic circuit theory, a semiclassical formulation of the LB formalism convenient for treating layered ferromagnet-normal metal structures, is briefly explained in the third section.

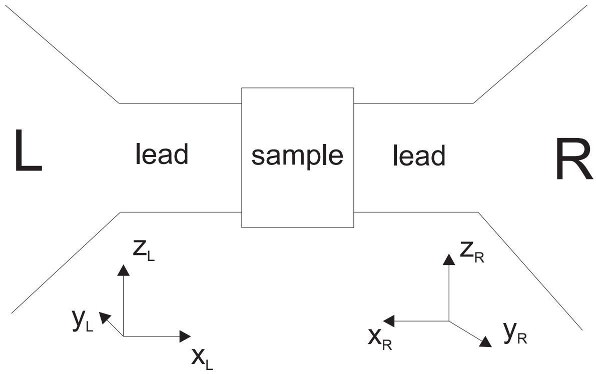

Figure 3.1: A mesoscopic sample (conductor) connected to two large reservoirs. The narrow contacts, or leads, serve as electron waveguides. Local Cartesian coordinate systems are defined in the leads such that positive x-direction is towards the sample on both sides.

3.1 Transport as a scattering problem

The LB formalism constitutes a scattering approach to electric transport in mesoscopic conductors. It is a quantum mechanical theory in which currents are calculated in terms of probabilities for the electron states to transmit through the conductor. The transmission probabilities are assumed to be known from a quantum mechanical calculation. The theory, as it will be presented here, does not include interactions between the conducting electrons (Coulomb interaction), or between electrons and phonons. It is restricted to conductors small enough that scattering can be assumed elastic, implying that electron transport is considered perfectly phase-coherent. The theory is applicable to systems in the stationary regime. The current is calculated using second quantized field operators, forming a many-particle theory in which electrons can be created and destroyed. These operators incorporate the Pauli principle so that the correct Fermi statistics are imposed on the electrons.

I consider a mesoscopic sample or conductor, for example a thin metallic film, as shown in Fig. 3.1. The sample is contacted to two terminals or electrodes, labelled L (left) and R (right), by which a bias voltage across the sample can be applied. The terminals are assumed to be large reservoirs in (thermal) equilibrium, so that they are characterized by a common constant temperature T and respective constant chemical potentials μ_L and μ_R. For equilibrium to be achieved, scattering in the reservoirs must be inelastic, as opposed to the sample. The electron states at energy E in the reservoirs are then occupied according to the Fermi-Dirac distribution function

$$f_A(E) = \frac{1}{e^{(E-\mu_A)/k_BT} + 1}, \tag{3.1}$$

where $A = L, R$ denotes reservoir. The reservoirs act as sources and sinks of electrons. The energies of the emitted electrons are determined by the Fermi-Dirac distribution in the reservoirs. As for absorption of electrons, the reservoirs are assumed to be perfect sinks; any electron propagating from the sample towards either reservoir is absorbed by the reservoir. As pictured in Fig. 3.1, the contacts between the reservoirs and the sample are narrow, so that the sample constitutes only a small perturbation of the reservoirs. This justifies the assumption of equilibrium in the reservoirs. In the following, the narrow ballistic contacts guiding electrons to and from the sample will be called *leads*. The leads are in some sense just a convenient theoretical invention in which the propagating electron states may be defined.

The sample is in this picture viewed as a scatterer limiting the transport of electrons between the reservoirs. Electrons emitted from the reservoirs are guided by the ballistic leads to the sample, where they may be reflected or transmitted to the other reservoir. The electron states in the leads are correspondingly divided into "incoming"and "outgoing"states, respectively moving towards or away from the sample. These states do not mix, since there is no scattering in the leads, and since the reservoirs are perfect sinks. The conductance is completely determined by the reflection and transmission probabilites of the sample, which are conveniently expressed in terms of a scattering matrix. The sample will in the following be taken to be a metallic ferromagnet, and the reservoirs to be non-magnetic normal metals. I will consider spin resolved electron transport through the ferromagnet.

The Hamiltonian in the leads are assumed to be separable, so that the electron wave function can be decomposed into a longitudinal and a transverse part. Longitudinal motion along the leads is free and hence described by a plane wave, while motion transverse to the leads is governed by a confining potential and hence quantized. The electron states are accordingly $e^{ik_{Am}x_A}\phi_{Am\alpha}(y_A, z_A)$, where k_{Am} is the continuous electron wave number for longitudinal motion, and $\phi_{Am\alpha}(y_A, z_A)$ is the quantized transverse wave function. The subscripts A and α denote respectively lead (L, R) and spin $(\uparrow, \downarrow)$, m is a discrete index associated with the quantized transverse motion, and x_A, y_A and z_A are local Cartesian coordinates in lead A, defined in Fig. 3.1. The energy of the electron states is $E_{Am}(k_{Am}) = E_{Am}(0) + \hbar^2 k_{Am}^2/(2m_e)$, where the first term is the quantized transverse energy independent of k_{Am}, and the second term is the longitudinal free-electron energy. m_e is the electron mass. The states $\phi_{Am\alpha}(y_A, z_A)$ will in the following be referred to as transverse channels. The total number of channels participating in the propagation of electrons is finite, limited by the energy the electrons possess as they enter the leads (the Fermi energy in the reservoir).

As electrons in incoming states in the leads reach the sample, they can either be reflected or transmitted to the other reservoir. This process is described by scattering states. For an electron incident on the sample in transverse channel m in the left lead, the scattering state is

$$\psi_{Lm\alpha}(\mathbf{r}_L) = e^{ik_{Lm}x_L}\phi_{Lm\alpha}(y_L, z_L) + \sum_{n\beta}\sqrt{\frac{v_{Lm}}{v_{Ln}}}s_{LLnm\beta\alpha}e^{-ik_{Ln}x_L}\phi_{Ln\beta}(y_L, z_L) \tag{3.2}$$

in the left lead, and

$$\psi_{Lm\alpha}(\mathbf{r}_R) = \sum_{n\beta}\sqrt{\frac{v_{Lm}}{v_{Rn}}}s_{RLnm\beta\alpha}e^{-ik_{Rn}x_R}\phi_{Rn\beta}(y_R, z_R), \tag{3.3}$$

in the right lead. Eq. (3.2) consists of the incoming wave plus reflected waves generated in all transverse channels in the left lead, while Eq. (3.3) are the generated transmitted waves in all transverse channels in the right lead. $v_{Lm} = \hbar k_{Lm}/m_e$ is the electron velocity in transverse channel m in lead L, and $s_{LLnm\beta\alpha}$ is an element of the scattering matrix of the sample, to be further discussed later. This particular element describes the reflection of an electron incident on the sample in state (m, α) in lead L to state (n, β) in the same lead, propagating in the opposite direction. It is useful to note here that the scattering matrix relates "current amplitudes"(square root of velocity times wave amplitude) in the two leads, and not wave amplitudes. This is convenient, since due to current conservation, the scattering matrix is then unitary. This is also the reason why the carrier velocity appears in the above expression for the scattering states.

The most general electron wave can be constructed as a superposition of the above stationary scattering states:

$$\Psi(\mathbf{r}_L, \mathbf{r}_R, t) = \frac{1}{\sqrt{2\pi}}\sum_{Am\alpha}\int dk_{Am}\psi_{Am\alpha}a_{Am\alpha}e^{-iE_{Am}t/\hbar} \tag{3.4}$$

where $a_{Am\alpha}$ are the amplitudes of the basis states, and t is the time. The next step is to let this general wave state become a field operator in the language of second quantization. The procedure for this is as follows [75]:

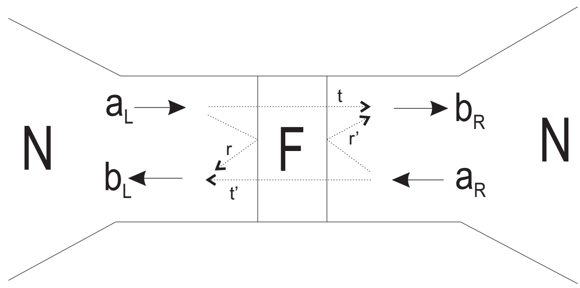

Figure 3.2: A thin ferromagnetic (F) film is sandwiched by normal metals (N). Electric transport in the system is evaluated in terms of transmission probabilities for the electron states, with the aid of second quantized annihilation and creation operators. The operators shown in the figure are annihilation operators, with the $\hat{a}$-operators annihilating electrons moving towards the ferromagnet, and the $\hat{b}$-operators annihilating electrons moving away from the ferromagnet. Also shown are the reflection and transmission matrices $r = s_{LL}, r' = s_{RR}, t = s_{RL}, t' = s_{LR}$ (see Eq. (3.5)).

Replace the amplitudes $a_{Am\alpha}$ by operators $\hat{a}_{Am\alpha}$ that annihilate electrons in incoming state (m, α) in lead A, with wave number k_{Am} and energy E_{Am}. Introduce also adjoint operators $\hat{a}^{\dagger}_{Am\alpha}$, which create electrons in incoming state (m, α) in lead A. For convenience, change from a k-space integral in Eq. (3.4) to an integral over energy. In this energy representation, the annihilation and creation operators must be properly renormalized [75]: $\hat{a}_{Am\alpha} \to \hat{a}_{Am\alpha}\sqrt{\hbar v_{Am}}$ and $\hat{a}^{\dagger}_{Am\alpha} \to \hat{a}^{\dagger}_{Am\alpha}\sqrt{\hbar v_{Am}}$. Finally, introduce also operators $\hat{b}_{Am\alpha}$ and $\hat{b}^{\dagger}_{Am\alpha}$, which respectively annihilate and create electrons in *outgoing* states (m, α) in lead A. These operators are defined by their relation to the $\hat{a}$-operators given by the scattering matrix. For the annihilation operators, the relation reads (see Fig. 3.2)

$$\begin{pmatrix} \hat{b}_L \\ \hat{b}_R \end{pmatrix} = \begin{pmatrix} s_{LL} & s_{LR} \\ s_{RL} & s_{RR} \end{pmatrix} \begin{pmatrix} \hat{a}_L \\ \hat{a}_R \end{pmatrix}, \tag{3.5}$$

where spin indices have been omitted for simplicity. The operators are here written as vectors in the space spanned by the transverse channels, while s_{RL}, for instance, is the scattering *sub*matrix in the same space for electron transmission from the left side of the ferromagnet to the right. A similar matrix relation holds for the creation operators. Including spin indices, Eq. (3.5) reads $\hat{b}_{Am\alpha} = \sum_{Bn\beta} s_{ABmn\alpha\beta}\hat{a}_{An\beta}$, where $A, B = L, R$. In the rest of the thesis, the following convention will be used: Whenever an annihilation or creation operator or a scattering matrix element is written *without* transverse channel index, it is to be understood as a vector/matrix in transverse channel space. On the other hand, if one of these ojects is specified *with* such an index, it is to be understood as an element in this space. For convenience, I define $r \equiv s_{LL}, r' \equiv s_{RR}, t \equiv s_{RL}, t' \equiv s_{LR}$ for use in the following (see Fig. 3.2).

With the aid of the annihilation and creation operators, spin resolved field operators for each of the leads can be constructed from Eq. (3.4):

$$\begin{aligned}\hat{\Psi}_{A\alpha}(\mathbf{r},t) &= \sum_m \int dE e^{-iEt/\hbar} \frac{\phi_{Am\alpha}(y,z)}{\sqrt{hv_{Am}(E)}} \\ &\times \left[\hat{a}_{Am\alpha}(E)e^{ik_{Am}x} + \hat{b}_{Am\alpha}(E)e^{-ik_{Am}x}\right] \end{aligned} \tag{3.6}$$

and

$$\begin{aligned}\hat{\Psi}^{\dagger}_{A\alpha}(\mathbf{r},t) &= \sum_m \int dE e^{iEt/\hbar} \frac{\phi^{*}_{Am\alpha}(y,z)}{\sqrt{hv_{Am}(E)}} \\ &\times \left[\hat{a}^{\dagger}_{Am\alpha}(E)e^{-ik_{Am}x} + \hat{b}^{\dagger}_{Am\alpha}(E)e^{ik_{Am}x}\right], \end{aligned} \tag{3.7}$$

where I have suppressed the indices on the coordinates. The object of interest for the study of charge and spin transport, the 2×2 current operator in spin space, is readily found from these. In lead A, the operator reads (tilde denotes a matrix in spin space)

$$\tilde{\hat{I}}_A = \begin{pmatrix} \hat{I}_A^{\uparrow\uparrow} & \hat{I}_A^{\uparrow\downarrow} \\ \hat{I}_A^{\downarrow\uparrow} & \hat{I}_A^{\downarrow\downarrow} \end{pmatrix}, \tag{3.8}$$

with components

$$\hat{I}_A^{\alpha\beta}(x,t) = \frac{\hbar e}{2im_e}\int dydz[\hat{\Psi}^{\dagger}_{A\beta}(\mathbf{r},t)\frac{\partial}{\partial x}\hat{\Psi}_{A\alpha}(\mathbf{r},t) - \left(\frac{\partial}{\partial x}\hat{\Psi}^{\dagger}_{A\beta}(\mathbf{r},t)\right)\hat{\Psi}_{A\alpha}(\mathbf{r},t)]. \tag{3.9}$$

Notice that this is the total current flowing in the longitudinal direction in lead A. Inserting Eqs. (3.6) and (3.7) into Eq. (3.9) yields

$$\hat{I}_A^{\alpha\beta}(t) = \frac{e}{h}\int dEdE' e^{i(E-E')t/\hbar}[\hat{a}^{\dagger}_{A\beta}(E)\hat{a}_{A\alpha}(E') - \hat{b}^{\dagger}_{A\beta}(E)\hat{b}_{A\alpha}(E')]. \tag{3.10}$$

In arriving at this simple form for the current, it has been assumed that the energies E and E' are close to each other, which is the case for observable quantities such as average current and current noise [8]. In the following, expressions are simplified by disregarding spin flip processes in the ferromagnet, and assuming that the ferromagnet is thicker than the magnetic coherence length.

The charge and spin currents $\hat{I}_{c,A}(t)$ and $\hat{\mathbf{I}}_{s,A}(t)$ are given by

$$\hat{I}_{c,A}(t) = \sum_{\alpha} \hat{I}_A^{\alpha\alpha}(t) \tag{3.11}$$

and

$$\hat{\mathbf{I}}_{s,A}(t) = \hbar/(2e) \sum_{\alpha\beta} \boldsymbol{\sigma}^{\alpha\beta} \hat{I}_A^{\beta\alpha}(t), \tag{3.12}$$

where $\tilde{\boldsymbol{\sigma}} = (\tilde{\sigma}_x, \tilde{\sigma}_y, \tilde{\sigma}_z)$ is the vector of Pauli matrices. These expressions give the instant currents at time t, and hence provide information on both the time-averaged currents and the current fluctuations. Current fluctuations, which is the focus of this thesis, are considered in the next section. It is instructive to first consider the average currents. To do so, it is necessary to evaluate quantum statistical averages of products of creation and annihilation operators, i.e.,

$$\langle a^\dagger_{Am\alpha}(E) a_{Bn\beta}(E') \rangle = \delta_{AB}\delta_{mn}\delta_{\alpha\beta}\delta(E-E')f_A(E). \tag{3.13}$$

The brackets should here be understood as a statistical average of the quantum mechanical expectation value of the operator-product in a many-particle state. Eq. (3.13) yields the result $\langle \hat{I}_{c,L} \rangle = -\langle \hat{I}_{c,R} \rangle = \langle \hat{I}_c \rangle$, i.e., charge current is conserved, as it should be. The value of the charge current is

$$\langle \hat{I}_c(t) \rangle = \langle \hat{I}^{\uparrow\uparrow}(t) \rangle + \langle \hat{I}^{\downarrow\downarrow}(t) \rangle = \frac{1}{e} \int dE (G^\uparrow + G^\downarrow)(f_L - f_R). \tag{3.14}$$

Here

$$G^\alpha = \frac{e^2}{h} \mathrm{Tr}(1 - r_\alpha r_\alpha^\dagger) \tag{3.15}$$

is the spin-dependent conductance of the ferromagnet, with the definition $r_\alpha = s_{LL\alpha}$, and the trace is over transverse channel space. Since transport involves only electrons around the Fermi level, and the scattering matrix varies slowly with energy, the energy dependence of G^α can to a good approximation be neglected. Hence, G^α can be evaluated at the Fermi level and taken outside the integral in Eq. (3.14). Noting that $\int dE f_A \approx \mu_A$ as long as the temperature is not too high in the reservoirs, Eq. (3.14) then takes the form of Ohm's law: $\langle \hat{I}_c \rangle = GV$, where $G = G^\uparrow + G^\downarrow$ is the total conductance, and $V = (\mu_L - \mu_R)/e$ is the applied voltage. Hence, the quantum mechanical LB formalism, appropriate for mesoscopic transport, reproduces the classical Ohm's law. But it also gives an explicit expression for the conductance in terms of microscopic reflection and transmission probabilities. These should be found from a quantum mechanical calculation.

For the spin current it similarly follows that $\langle\hat{\mathbf{I}}_{s,L}(t)\rangle = -\langle\hat{\mathbf{I}}_{s,R}(t)\rangle = \langle\hat{\mathbf{I}}_s(t)\rangle$, where

$$\langle\hat{\mathbf{I}}_s(t)\rangle = \frac{\mathbf{m}\hbar}{2e}(\langle\hat{I}^{\uparrow\uparrow}(t)\rangle - \langle\hat{I}^{\downarrow\downarrow}(t)\rangle) = \frac{\mathbf{m}\hbar}{2e^2}\int dE(G^{\uparrow} - G^{\downarrow})(f_L - f_R). \quad (3.16)$$

$\mathbf{m}$ is the unit magnetization vector of the ferromagnet, that defines the spin quantization axis. Hence, the spin current is collinear with the magnetization, as expected. As discussed in Ch. 2, this does not give rise to a spin-transfer torque on the ferromagnet. To get a non-collinear spin current, and thus a spin-transfer torque, a layered structure with more than one ferromagnet (such as a spin valve) is needed. It is also necessary that the magnetizations of the ferromagnets are non-collinear. Magnetoelectronic circuit theory, which is built on the above theory, is useful for treating such problems, and is the topic of Sec. 3.3.

Unlike the average spin current, the fluctuations of the spin current can in general be non-collinear to the magnetization, even for a system with only one ferromagnet. This is investigated below.

3.2 Current fluctuations

Of special interest in this thesis are the fluctuations of the current from its average value. Both charge and spin current fluctuations can be evaluated within the framework presented in the previous section. Two fundamental types of current noise will be considered in the following: Thermal noise and shot noise. The first manifests itself as fluctuations in the occupation numbers of the electron channels incident on the sample. It is caused by thermal equilibrium fluctuations in the reservoirs, and scales with temperature. Shot noise, as opposed to thermal noise, is an out-of-equilibrium phenomenon. It is present when a voltage is applied across the sample, and even at zero temperature. Shot noise is due to the discreteness of the electron charge, and the probabilistic scattering of electrons as they are incident on the sample. It scales with the applied voltage. Other sources of noise include $1/f$ noise, diffusion noise, generation-recombination noise and quantum noise [77]. Neither of these sources are considered here. In the following, focus will be on low-frequency noise. In this limit, charge and spin is conserved, not only on average, but instantaneously.

The charge current fluctuations in lead A are $\delta\hat{I}_{c,A}(t) = \hat{I}_{c,A}(t) - \langle\hat{I}_{c,A}(t)\rangle$, where $\hat{I}_{c,A}(t)$ is the charge current at time t as defined in the previous section. The spin current fluctuations are similarly $\delta\hat{I}_{s_i,A}(t) = \hat{I}_{s_i,A}(t) - \langle\hat{I}_{s_i,A}(t)\rangle$, where i ($i = x, y$ or z) denotes the vector component of the spin current. The z-axis is chosen along the magnetization direction, and is the spin quantization axis. Both the charge and spin current fluctuations are most conveniently analyzed in terms of their respective correlation functions

$$\begin{aligned} S_{c,AB}(t-t') &= \langle \delta\hat{I}_{c,A}(t)\delta\hat{I}_{c,B}(t')\rangle \\ &= \langle \hat{I}_{c,A}(t)\hat{I}_{c,B}(t')\rangle - \langle \hat{I}_{c,A}(t)\rangle\langle \hat{I}_{c,B}(t')\rangle \end{aligned} \tag{3.17}$$

and

$$\begin{aligned} S_{ij,AB}(t-t') &= \langle \delta\hat{I}_{s_i,A}(t)\delta\hat{I}_{s_j,B}(t')\rangle \\ &= \langle \hat{I}_{s_i,A}(t)\hat{I}_{s_j,B}(t')\rangle - \langle \hat{I}_{s_i,A}(t)\rangle\langle \hat{I}_{s_j,B}(t')\rangle. \end{aligned} \tag{3.18}$$

These correlators can be evaluated with the aid of the current operator in the previous section. It is then necessary to evaluate the expectation value of four annihilation and creation operators. This is done by noting that the creation and annihilation operators obey the anticommutation relation

$$\{\hat{a}^\dagger_{Am\alpha}(E), \hat{a}_{Bn\beta}(E')\} = \delta_{AB}\delta_{mn}\delta_{\alpha\beta}\delta(E-E'). \tag{3.19}$$

The anticommutator of two creation or two annihilation operators vanishes. Similar relations hold also for the $\hat{b}$-operators. From these relations and Eq. (3.13) it follows that

$$\begin{aligned} &\langle \hat{a}^\dagger_{Ak\alpha}(E_1)\hat{a}_{Bl\beta}(E_2)\hat{a}^\dagger_{Cm\gamma}(E_3)\hat{a}_{Dn\delta}(E_4)\rangle \\ &\qquad - \langle \hat{a}^\dagger_{Ak\alpha}(E_1)\hat{a}_{Bl\beta}(E_2)\rangle\langle \hat{a}^\dagger_{Cm\gamma}(E_3)\hat{a}_{Dn\delta}(E_4)\rangle \\ &= \delta_{AD}\delta_{BC}\delta_{kn}\delta_{lm}\delta_{\alpha\delta}\delta_{\beta\gamma}\delta(E_1-E_4)\delta(E_2-E_3)f_A(E_1)[1-f_B(E_2)], \end{aligned} \tag{3.20}$$

where the subscripts A, B, C, D denote leads, k, l, m, n denote transverse channels, and $\alpha, \beta, \gamma, \delta$ denote spin. The identity

$$\sum_{CD} \mathrm{Tr}(s^\dagger_{AC\alpha}s_{AD\beta}s^\dagger_{BD\beta}s_{BC\alpha}) = \delta_{AB}M_A, \tag{3.21}$$

which follows from the unitarity of the scattering matrix when spin flip processes are disregarded, is also needed in the calculation of the current fluctuations. Here the trace is over the space of the transverse channels, and M_A is the total number of channels in lead A.

In general, the total noise is not a simple superposition of pure thermal noise and pure shot noise, but a complicated function of both temperature and voltage. Still, it is convenient to treat the two noise sources independently, as is done in the following.

Thermal noise

Holding the reservoirs at the same chemical potential, i.e., $f_L = f_R = f$, while allowing the temperature to be finite, there will be no shot noise in the system, only thermal equilibrium noise. Using Eqs. (3.20) and (3.21), and noting that $f(1-f) = k_B T(-\partial f/\partial E)$, the zero-frequency charge current noise is

$$\begin{aligned} S^{(\mathrm{th})}_{c,AA}(\omega = 0) &= \int d(t-t') S^{(\mathrm{th})}_{c,AA}(t-t') \\ &= 2k_B T(G^{\uparrow} + G^{\downarrow}), \end{aligned} \tag{3.22}$$

where the superscript (th) emphasizes that these are thermally induced fluctuations. The result for $S^{(\mathrm{th})}_{c,AB}(\omega = 0)$, where $B \neq A$, differs from this expression only by a minus sign, since positive current direction is defined towards the ferromagnet on both sides, and charge current is conserved. Eq. (3.22) is the well-known Johnson-Nyquist noise [8], related to the dissipation of energy of the conducting electrons, i.e., the resistance, in accordance with the FDT. Similarly, the zero-frequency thermal spin current noise is

$$\begin{aligned} S^{(\mathrm{th})}_{ij,AB}(\omega = 0) &= \int d(t-t') S^{(\mathrm{th})}_{ij,AB}(t-t') \\ &= \frac{\hbar k_B T}{8\pi} \sum_{\alpha\beta} \sigma_i^{\alpha\beta} \sigma_j^{\beta\alpha} \mathrm{Tr}[2\delta_{AB} - s^{\dagger}_{BA\alpha} s_{BA\beta} - s^{\dagger}_{AB\beta} s_{AB\alpha}], \end{aligned} \tag{3.23}$$

where the scattering matrices should be evaluated at the Fermi energy. This spin current correlator is studied in papers II and IV, and I refer to these for further details. In particular, it is found that this correlator is non-zero both for components collinear and non-collinear to the magnetization. Hence, these spin current fluctuations can cause a fluctuating spin-transfer torque on the magnetization, to be discussed in Ch. 4.

Shot noise

Allowing a bias $\mu_L - \mu_R = eV$ between the reservoirs, shot noise will be present. It is now convenient to take the temperature to be zero, so that there is no thermally induced noise. The noise correlators are calculated using Eq. (3.20), and the relations $f_A(1-f_A) = 0$ and $\int dE (f_L - f_R)^2 = e|V|$ that hold at zero temperature. The zero-frequency charge current shot noise becomes [8]

$$\begin{aligned} S^{(\mathrm{sh})}_{c,AA}(\omega=0) &= \int d(t-t')S^{(\mathrm{sh})}_{c,AA}(t-t') \\ &= \frac{e^3}{h}|V|[\mathrm{Tr}(r^\dagger_\uparrow r_\uparrow t^\dagger_\uparrow t_\uparrow)+\mathrm{Tr}(r^\dagger_\downarrow r_\downarrow t^\dagger_\downarrow t_\downarrow)]. \end{aligned} \tag{3.24}$$

The scattering matrices should be evaluated at the Fermi energy, and $r_\alpha = s_{LL\alpha}$ and $t_\alpha = s_{RL\alpha}$. The superscript (sh) emphasizes that this is shot noise. The result for $S^{(\mathrm{sh})}_{c,AB}(\omega=0)$, where $B \neq A$, differs from Eq. (3.24) by a minus sign. Similarly, the zero-frequency spin shot noise is

$$\begin{aligned} S^{(\mathrm{sh})}_{ij,AB}(\omega=0) &= \int d(t-t')S^{(\mathrm{sh})}_{ij,AB}(t-t') \\ &= \frac{\hbar}{8\pi}\sum_{\alpha\beta}\sigma_i^{\alpha\beta}\sigma_j^{\beta\alpha}\int dE\sum_{C\neq D} f_C(1-f_D) \\ &\quad \times\mathrm{Tr}[s^\dagger_{AC\alpha}s_{AD\beta}s^\dagger_{BD\beta}s_{BC\alpha}]. \end{aligned} \tag{3.25}$$

The spin shot noise was studied in papers II and IV, and I refer to these for further details. As for the thermal noise, this correlator is non-zero for components non-collinear to the magnetization, implying a fluctuating spin-transfer torque on the magnetization.

3.3 Magnetoelectronic circuit theory

So far I have been studying a single ferromagnetic film sandwiched by reservoirs (voltage sources). Experimentally and for device applications, ferromagnetic multilayers are however more interesting. Magnetoelectronic circuit theory was developed by Brataas, Bauer and Nazarov [56, 57, 59] as a tool to determine transport properties of magnetoelectronic multilayers. This theory is a semiclassical formulation of the spin-resolved LB scattering formalism presented in Sec. 3.1.

The idea of the magnetoelectronic circuit theory is to divide the system into resistive elements (scatterers), nodes (low resistance interconnectors), and reservoirs (voltage sources). The nodes and reservoirs are described by distribution functions, and the current between them is calculated quantum mechanically using the LB scattering theory. The system of interest here is shown in Fig. 3.3. It is a spin valve, i.e., two ferromagnetic films F_1 and F_2 separated by a normal metal N. The ferromagnets are viewed as scatterers and the sandwiched normal metal as a node. The outer normal metals L (left) and R (right) are taken to be large reservoirs in thermal equilibrium,

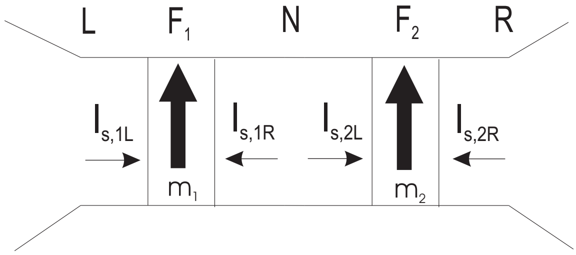

Figure 3.3: A spin valve with two ferromagnets F_1 and F_2 with unit magnetization vectors $\mathbf{m}_1$ and $\mathbf{m}_2$, here shown in the parallel configuration $\mathbf{m}_1 = \mathbf{m}_2$. The currents in the system are evaluated close to the interfaces, with positive directions defined in the figure, using magnetoelectronic circuit theory.

characterized by Fermi-Dirac distribution functions $f_L = f(E - \mu_L)$ and $f_R = f(E - \mu_R)$. Here μ_L and μ_R are the respective chemical potentials, so that $\mu_L - \mu_R = eV$ is the applied voltage. The ferromagnets limit the current, and are characterized by scattering matrices, as introduced Sec 3.1. The node is a new circuit element, not present in the "traditional" LB theory. For the magnetoelectronic circuit theory to be valid, the momentum distribution of the electrons in the node must be approximately isotropic and constant in space. This can be achieved by irregularities in the node's shape or impurity or disorder scattering, and is satisfied in most metallic systems [59].

Due to the spin-dependent conductances of the ferromagnets, there can be a non-zero spin potential (i.e., a potential difference for electrons with opposite spin directions) in the node when a voltage is applied [59]. If the magnetization vectors $\mathbf{m}_1$ and $\mathbf{m}_2$ of the respective ferromagnets are collinear, i.e., parallel or antiparallel, the direction of this non-equilibrium spin potential, or *spin accumulation*, is either along $\mathbf{m}_1$ or $\mathbf{m}_2$. The same goes for the direction of polarization of a possible spin current flowing in the system. If $\mathbf{m}_1$ and $\mathbf{m}_2$ are non-collinear, the situation is more complex and interesting. In general, neither the spin current nor the spin accumulation is then collinear with any of the magnetizations. It is in this case necessary to adopt a 2×2 matrix representation in spin space for the potentials, currents, and conductances. The node is hence in general characterized by both a non-zero scalar (charge) distribution function f_{cN}, and a non-zero vector spin distribution function $\mathbf{f}_{sN}$, that may be combined into a semiclassical distribution matrix

$$\tilde{f}_N = \tilde{1} f_{cN} + \tilde{\boldsymbol{\sigma}} \cdot \mathbf{f}_{sN} = \begin{pmatrix} f_N^{\uparrow\uparrow} & f_N^{\uparrow\downarrow} \\ f_N^{\downarrow\uparrow} & f_N^{\downarrow\downarrow} \end{pmatrix}, \tag{3.26}$$

in 2×2 spin space. Here $\tilde{1}$ is the unit matrix and $\tilde{\boldsymbol{\sigma}}$ is the vector of the Pauli matrices.

As in conventional circuit theory, Kirchhoff's current law has a central place in magnetoelectronic circuit theory. Disregarding spin flip processes in the normal metal node, Kirchhoff's "extended" law states that both charge and spin currents are conserved in the node. The charge current is furthermore conserved throughout the entire system. The spin current, however, is in general not, since in passing through a ferromagnet, the component polarized transverse to the magnetization is lost. This effect, described already in Ch. 2, is the cause of the spin-transfer torque, and is due to the short coherence length for transverse spin currents in ferromagnets. Because of this, it is convenient to define local spin currents in four different places in the spin valve shown in Fig. (3.3). The local currents are evaluated close to the F|N-interfaces. $\mathbf{I}_{s,1L}$, for example, denotes the spin current evaluated on the left side of ferromagnet F_1, with positive current direction defined in the figure.

The basic formalism needed to evaluate the charge and spin currents has already been given in Sec. 3.1. The necessary object is Eq. (3.10), which provides complete information on both charge and spin transport. The calculation of average charge and spin currents involve the quantum statistical average (3.13) of products of creation and annihilation operators belonging to the outer normal metal reservoirs. In addition, the quantum statistical average of products of creation and annihilation operators defined in the middle normal metal node (N) is needed. The important ansatz in magnetoelectronic circuit theory is that the latter average reads

$$\langle a^\dagger_{Nm\alpha}(E) a_{Nn\beta}(E') \rangle = \delta_{mn} \delta(E - E') f_N^{\beta\alpha}(E), \tag{3.27}$$

where $f_N^{\beta\alpha}$ is defined in Eq. (3.26). This semiclassical ansatz states that the expectation value of the number operator in the node can be equated to an isotropic distribution function in spin space. The distribution function is assumed independent of the transverse electron channels, analogous to the Fermi-Dirac distribution functions in the reservoirs. The ansatz is satisfied when the scattering of electron wave functions in the node is sufficiently chaotic, as due to irregularities in the node's shape or disorder scattering.

It follows that the average charge current on e.g. the right side of ferromagnet F_1 is [59]

$$\langle I_{c,1R}\rangle = \frac{1}{e}\int dE\{G_1^{\uparrow}(f_{cN}+\mathbf{f}_{sN}\cdot\mathbf{m}_1-f_L)+G_1^{\downarrow}(f_{cN}-\mathbf{f}_{sN}\cdot\mathbf{m}_1-f_L)\} \tag{3.28}$$

and the average spin current is [59]

$$\begin{aligned}\langle \mathbf{I}_{s,1R}\rangle \;=\; & \frac{\hbar}{2e^2}\int dE\{\mathbf{m}_1[G_1^{\uparrow}(f_{cN}+\mathbf{f}_{sN}\cdot\mathbf{m}_1-f_L) \\ & -G_1^{\downarrow}(f_{cN}-\mathbf{f}_{sN}\cdot\mathbf{m}_1-f_L)] \\ & +2\mathrm{Re}G_{1R}^{\uparrow\downarrow}\mathbf{m}_1\times(\mathbf{f}_{sN}\times\mathbf{m}_1)+2\mathrm{Im}G_{1R}^{\uparrow\downarrow}\mathbf{f}_{sN}\times\mathbf{m}_1\}. \end{aligned} \tag{3.29}$$

Here G_1^{α} is the spin-dependent conductance of F_1 defined in the previous section, which governs the charge current and the component of the spin current collinear to the magnetization. The transverse, or "mixing", conductance [56]

$$G_{1R}^{\uparrow\downarrow} = \frac{e^2}{h}\mathrm{Tr}(1-r'_{\uparrow}r'^{\dagger}_{\downarrow}) \tag{3.30}$$

can be understood as the conductance for the component of the spin current transverse to the magnetization. Here r'_{α}, defined in Sec. 3.1, is the spin-dependent scattering matrix for electron reflection at the right interface of F_1. As transverse spin currents are quickly absorbed in ferromagnets, the mixing conductance can be viewed as an interface conductance (resistance). It is for this reason that it is necessary in Eq. (3.30) to denote on which side (L or R) of the ferromagnet it is defined. In general, $G_{1L}^{\uparrow\downarrow}$ may differ from $G_{1R}^{\uparrow\downarrow}$. For convenience, a 2×2 conductance matrix

$$\tilde{G} = \begin{pmatrix} G_{\uparrow} & G_{\uparrow\downarrow} \\ G_{\downarrow\uparrow} & G_{\downarrow} \end{pmatrix}, \tag{3.31}$$

in spin space can be defined in terms of all the spin-resolved conductances. It follows from Eq. (3.30) that $G_{\downarrow\uparrow} = G_{\uparrow\downarrow}^*$.

To completely determine the currents, it remains to find the unknown potential $\tilde{f}_N$. This is done by invoking Kirchhoff's current law for the normal metal node:

$$\begin{aligned}\langle I_{c,1R}\rangle + \langle I_{c,2L}\rangle &= 0 \qquad (3.32)\\ \langle \mathbf{I}_{s,1R}\rangle + \langle \mathbf{I}_{s,2L}\rangle &= 0, \qquad (3.33)\end{aligned}$$

where on both sides of the node, positive current direction is defined in the direction away from it, see Fig. 3.3. From the first of these equations, and

Eq. (3.28), the average charge current is [59]

$$\langle I_c \rangle = \frac{GV}{2}\left(1 - P^2 \frac{1-\cos\theta}{1-\cos\theta+\eta+\eta\cos\theta}\right). \tag{3.34}$$

Note that there is no reference as to where in the spin valve this is evaluated, since charge current is conserved throughout the system. The charge current is a function of the relative orientation of the magnetizations $\mathbf{m}_1$ and $\mathbf{m}_2$, given by the angle $\cos\theta = \mathbf{m}_1 \cdot \mathbf{m}_2$. It is for simplicity assumed here that the ferromagnets have identical conductance parameters; $G = G^{\uparrow} + G^{\downarrow}$ is defined as the total conductance of each of the ferromagnets, $P = (G^{\uparrow} - G^{\downarrow})/G$ as the polarization, and $\eta = 2G^{\uparrow\downarrow}/G$ as the relative mixing conductance. (It is assumed that the left and right interfaces of the ferromagnets are identical, i.e., $G_L^{\uparrow\downarrow} = G_R^{\uparrow\downarrow} = G^{\uparrow\downarrow}$).

From Eqs. (3.29) and (3.33), the spin-transfer torque on each of the ferromagnets can be found. The torque is due to the absorption of the spin current component transverse to the magnetization. The component collinear to the magnetization is not absorbed. Hence, the spin-transfer torque on e.g F_1 is simply given by $\boldsymbol{\tau}_1 = \langle \mathbf{I}_{s,1L} \rangle + \langle \mathbf{I}_{s,1R} \rangle$, i.e., the difference between the spin currents evaluated on the left and right sides. This yields [59]

$$\boldsymbol{\tau}_1 = -\mathbf{m}_1 \times (\mathbf{m}_1 \times \mathbf{m}_2) \frac{P\eta}{\sin^2(\theta/2) + \eta\cos^2(\theta/2)} \frac{\langle I_c \rangle}{2}, \tag{3.35}$$

where $\langle I_c \rangle$ is given above. Notice that the torque vanishes when the ferromagnets are collinear.

Magnetoelectronic circuit theory is used in paper IV, where electric and magnetic noise in spin valves are considered. Current fluctuations rather than average currents are the focus of this paper.

Chapter 4

Current-induced magnetization noise and damping

In the previous chapter, a suitable description of charge and spin currents in mesoscopic structures containing ferromagnetic elements was introduced. This formalism will now be used to look at the interplay between spin currents and magnetization in such structures, with focus on spin-current induced magnetization noise and damping . Two physical phenomena are central: Spin-transfer torque and spin pumping. Through the spin-transfer torque, spin current fluctuations in magnetoelectronic nanostructures cause random fluctuations of the magnetization vector. This magnetization noise can in turn lead to resistance noise, due to the GMR-effect. Spin pumping can be viewed as the inverse effect of the spin-transfer torque. It is the emission of spin angular momentum from a ferromagnet whose magnetization vector changes in time, and can be shown to cause an appreciable enhancement of the magnetization damping [52, 21, 22]. The LLG equation, discussed in Ch. 2, is useful for studying these effects. As explained in Ch. 2, equilibrium magnetization noise and damping are closely connected. This connection will be explored in the following.

This chapter is divided into three parts, corresponding to the different systems that are considered in this thesis: i) Single monodomain ferromagnet, ii) spin valve, and iii) single non-uniform ferromagnet. In a mostly qualitative manner, it describes the main ideas behind the research reported in the papers, and states some of the main results.

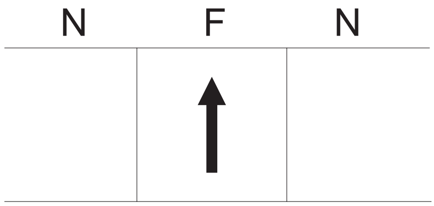

Figure 4.1: A single monodomain ferromagnet (F) sandwiched by normal metals (N). The arrow indicates the direction of magnetization.

4.1 Single monodomain ferromagnet

It is natural to start with perhaps the simplest kind of structure that may be called magnetoelectronic: A single nanoscale ferromagnet sandwiched by normal metals, as shown in Fig. 4.1. This structure was already considered in the context of transport in the previous chapter. The ferromagnet is assumed to be sufficiently thin so that the internal exchange coupling renders all atomic magnetic moments parallel [68]. The ferromagnet is then monodomain and behaves like a giant magnetic moment, or macrospin, with unit magnetization vector $\mathbf{m}$.

Spin pumping and enhanced Gilbert damping

If the direction of the magnetization vector is changing in time (precesses) due to, e.g., the action of an external driving field, a spin current can be emitted from the ferromagnet into the surrounding normal metals [21]. This out-of-equilibrium phenomenon has been termed spin pumping. The effect can be analyzed within the LB mesoscopic transport formalism, although in the form presented in Ch. 3, this formalism applies to systems with magnetization in static equilibrium. When the magnetization direction precesses, the scattering matrix of the ferromagnet, which is an object in spin space, becomes time-dependent. The response of the conducting electrons to this variation can be calculated in the adiabatic approximation, since the precession period of the magnetization is typically several orders of magnitude larger than the relevant time scale for electron transport [22]. This enables the use of Eq. (3.10) to find corrections to Eqs. (3.28) and (3.29) linear in the time-dependence of the scattering matrix. Tserkovnyak *et al.* [21] found that for ferromagnets thicker than the magnetic coherence length, the correction to the charge current is zero, while the correction to the spin current is

$$\mathbf{I}_s^{\text{pump}} = \frac{\hbar}{4\pi}\left(\text{Re}g^{\uparrow\downarrow}\mathbf{m}\times\frac{d\mathbf{m}}{dt}\right), \tag{4.1}$$

where $g^{\uparrow\downarrow} = hG^{\uparrow\downarrow}e^2$ is the dimensionless mixing conductance, of which the typically small imaginary part has been neglected. A precessing magnetization hence causes a non-equilibrium flow of spins into the neighbouring normal metals, but no flow of charge. Notice that this “pumped”spin current is present also in the absence of applied voltage, as long as there is some external perturbation causing a non-zero $d\mathbf{m}/dt$.

The spin pumping is necessarily associated also with pumping of energy out of the ferromagnet, and hence ferromagnetic relaxation [22]. By conservation of angular momentum, emission of spins leads to a modification of the LLG equation; an extra term $\gamma\mathbf{I}_s^{\text{pump}}/M$, where M is the total magnetic moment of the ferromagnet, must be included on the right hand side. This is equivalent to an enhancement

$$\alpha_0 \to \alpha = \alpha_0 + \frac{\gamma_0\hbar\text{Re}g^{\uparrow\downarrow}}{4\pi M} \tag{4.2}$$

of the Gilbert damping constant. This expression is valid when the surrounding environment is a perfect spin sink, such that the pumped spin current never returns to the ferromagnet. The spin-pumping induced enhancement of the Gilbert damping has been verified by a number of experiments [19, 20, 22, 1]. Paper I is one such experiment. It adresses spin pumping from Fe to Pd, and the resulting enhancement of the Gilbert damping. Emphasis is on the attenuation of spin angular momentum in the Pd layer [78], and the dependence of this process on the thickness of the layer. The experimental technique used is ferromagnetic resonance (FMR), which is the phenomenon of resonant absorption of the energy of a high-frequency magnetic field in ferromagnetic substances [79, 80, 81]. The Gilbert damping is determined from the linewidth of the FMR absorption curve [19].

Spin-current induced magnetization noise

From Eqs. (3.23) and (3.25) in Ch. 3 it can be shown (see papers II,IV) that spin current fluctuations polarized transverse to the magnetization are absorbed in the ferromagnet, whereas those polarized collinear with the magnetization are not. By angular momentum conservation, absorption of fluctuating spin currents implies a random torque acting on the magnetization. In paper II it is shown how this fluctuating spin-transfer torque results in an increased level of magnetization noise in the ferromagnet. With the aid of the LLG equation, the noise is conveniently described by translating the fluctuating torque into a fictitious random magnetic field $\mathbf{h}'(t)$, similar to the field $\mathbf{h}(t)$ introduced in Ch. 2. The explicit fluctuations of the magnetization vector are then expressed by the response of the magnetization to this field. It is shown that $\mathbf{h}'(t) = \mathbf{h}^{(\mathrm{th})}(t) + \mathbf{h}^{(\mathrm{sh})}(t)$, where $\mathbf{h}^{(\mathrm{th})}(t)$ is due to thermal spin current fluctuations, and $\mathbf{h}^{(\mathrm{sh})}(t)$ is due to spin shot noise. These two random fields are independent, and their respective correlation functions are

$$\langle h_i^{(\mathrm{th})}(t) h_j^{(\mathrm{th})}(t') \rangle = 2k_B T \frac{\alpha'}{\gamma M} \delta_{ij} \delta(t - t'), \tag{4.3}$$

and

$$\langle h_i^{(\mathrm{sh})}(t) h_j^{(\mathrm{sh})}(t') \rangle = \frac{\hbar}{4\pi} \frac{e|V|}{M^2} \delta_{ij} \delta(t - t') [\mathrm{Tr}(r_\uparrow r_\uparrow^\dagger t'_\downarrow t'^\dagger_\downarrow) + \mathrm{Tr}(r'_\downarrow r'^\dagger_\downarrow t_\uparrow t_\uparrow^\dagger)]. \tag{4.4}$$

Here i and j denote Cartesian components, and

$$\alpha' = \frac{\gamma \hbar \mathrm{Re} g^{\uparrow\downarrow}}{4\pi M} \tag{4.5}$$

coincides with the enhancement of the Gilbert damping constant due to spin pumping. The relation between the equilibrium part of the noise $\mathbf{h}^{(\mathrm{th})}(t)$ and α' is seen to be in perfect accordance with the FDT (see Ch. 2 or papers II,IV). Hence, the thermal spin current noise is the noise process related to the enhanced dissipation of energy by spin pumping, and calculating the noise is an alternative route to finding the damping.

4.2 Spin valve

As noted earlier, spin valves, such as the one shown in Fig. 4.2, is of great technological importance. Two single-domain ferromagnets are separated by a thin normal metal layer, through which they can interact. A static non-local exchange coupling is mediated by electrons through the normal metal spacer [82], and a static dipolar coupling is caused by stray magnetic fields. In addition, non-equilibrium spin currents between the ferromagnets couple them dynamically [83, 22]. This is due to spin pumping from each of the ferromagnets, that subsequently is absorbed by the other ferromagnet as a spin-transfer torque, coupling the dynamics of the magnetizations. Depending on the interlayer couplings, the ferromagnets may in the ground state be oriented with their magnetizations either in parallel or antiparallel. This is largely determined by the thickness of the middle normal metal. By

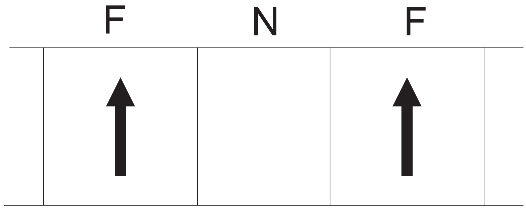

Figure 4.2: A spin valve consisting of two ferromagnets (F) separated by a normal metal (N). The arrows indicate that the spin valve is shown in the parallel magnetic configuration.

pinning one of the layers, other magnetic configurations are also possible. The configuration can be manipulated by external fields or electric currents via the spin-transfer torque. Applying a sufficiently strong external field, both magnetizations will be forced to point in the direction of the field, irrespectively of the original configuration. In Fig. 4.2, the spin valve is in the parallel configuration.

The magnetoelectronic circuit theory described in the previous chapter is very useful for calculating spin currents and spin-transfer torque in spin valves. Paper IV concerns spin current fluctuations and the resulting fluctuating spin-transfer torque in spin valves. The induced magnetization noise is calculated for various magnetic configurations, both for spin valves where one ferromagnet is pinned and for spin valves where neither ferromagnet is pinned. The FDT is used to find the Gilbert damping that is related to the noise. It is shown that the noise and damping vary with the magnetic configuration.

Paper III, and also paper IV, focuses on resistance noise induced by magnetization fluctuations. Due to GMR, the resistance R is a function of the angle θ between the magnetizations. When the direction of the magnetization vectors fluctuate, due to spin current fluctuations or thermal noise processes intrinsic to the ferromagnets, θ and hence R fluctuate. The resistance noise is characterized by the correlation function

$$S_R(t-t') = \langle \Delta R(t) \Delta R(t') \rangle \tag{4.6}$$

where $\Delta R(t) = R(t) - \langle R(t) \rangle$. Since the magnetization fluctuations typically are small, it is reasonable to calculate the resistance noise to their lowest non-vanishing order. In paper III, the importance of the interactions between the ferromagnets for the noise is emphasized. It turns out that both the static dipolar and exchange couplings, as well as the dynamic spin-exchange coupling, play major roles.

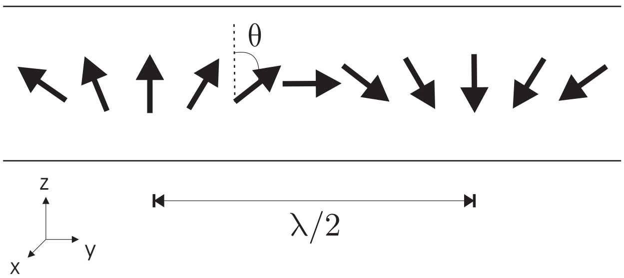

Figure 4.3: An example of a non-uniform ferromagnet. The magnetization $\mathbf{m}(y)$ rotates with wavelength λ in the yz-plane, forming a so-called spin spiral.

Resistance noise has received attention from experimenters lately [39, 84, 41, 42]. Smith and Arnett [39] demonstrated that resistance noise resulting from thermal magnetization fluctuations in magnetoresistive read heads can contribute to a significant portion of the heads total noise power. Covington *et al.* [42] measured resistance noise in magnetoelectronic nanostructures with up to 15 magnetic layers, finding a large difference in noise power between the parallel and antiparallel magnetic configurations. In paper III I connect my theoretical results to this latter experiment.

4.3 Single non-uniform ferromagnet

The single domain picture of ferromagnets is sound for sufficiently thin ferromagnets. Larger ferromagnets typically consist of a number of regions or domains, separated by domain walls. An exotic example of a non-uniform ferromagnet is shown in Fig. 4.3. It is a so-called spin spiral with constant pitch, which may be found in the γ-phase of iron [85] and in some rare earth metals [86]. Half a wavelength of this structure may serve as a simple profile of a domain wall.

In paper V, spin-current induced magnetization noise and damping in non-uniformly magnetized ferromagnets are investigated. Central in this work is the adiabatic spin-transfer torque, which is a transfer of spin angular momentum from propagating electrons to the magnetization of the ferromagnet. The transfer happens as the spin of the electrons adapt to the changing magnetization direction. The magnetization is assumed to be slowly varying on the scale of the magnetic coherence length. The torque reads [64]

$$\boldsymbol{\tau}(y) = \mathbf{m}(y) \times [\mathbf{m}(y) \times d\mathbf{I}_s(y)/dy] \tag{4.7}$$

where $\mathbf{m}(y)$ is the magnetization, for simplicity assumed to be varying only in the y-direction, and $\mathbf{I}_s(y) = I_s\mathbf{m}(y)$ is the spin current, assumed to be anywhere perfectly collinear with the magnetization. In paper V, it is shown how spin current fluctuations lead to enhanced magnetization noise via this torque. The current-induced magnetization noise is well described by introducing a random magnetic field, similar to the field introduced for single-domain ferromagnets, but in the present case inhomogenous and anisotropic. The FDT is used to relate the noise to the magnetization damping, which in general is shown to be a nonlocal tensor. For illustration, the results are applied to the spin spiral shown in Fig. 4.3.

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Paper I

Scattering of spin current injected in Pd(001)

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Scattering of spin current injected in Pd(001)

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We have studied spin pumping in Pd/Fe(001) ultrathin crystalline films prepared on GaAs(001) by ferromagnetic resonance (FMR). FMR measurements show that the Pd(001) overlayers lead to an appreciable attenuation of the spin current, which was generated by the precessing magnetization of Fe. Pd overlayers thicker than about 10 nm act as perfect spin sinks. It is argued that the loss of spin coherence in Pd is caused by scattering with spin fluctuations. © *2005 American Institute of Physics*. [DOI: 10.1063/1.1853131]

I. INTRODUCTION

Tserkovnyak *et al.*[1] showed that a precessing magnetization can generate a spin current into an adjacent normal-metal (NM) layer. The pumped spin current at the interface between the ferromagnetic (FM) layer and NM is given by

$$\mathbf{j}_{\text{spin}} = \frac{\hbar}{4\pi} g^{\uparrow\downarrow} \mathbf{n} \times \frac{\partial \mathbf{n}}{\partial t}, \tag{1}$$

where $\mathbf{n}$ is the unit vector along the magnetic moment $\mathbf{M}$, and $g^{\uparrow\downarrow}$ is the interface mixing conductance per unit area in units of e^2/h.[1] For interfaces with some degree of diffuse scattering $g^{\uparrow\downarrow}$ is close to the number of transverse channels in NM, $\Sigma_{m,n}\delta_{m,n}$, see Refs. 2–4. In simple metals with a spherical Fermi surface this sum is given by

$$g^{\uparrow\downarrow} = \frac{k_F^2}{4\pi} \approx 0.85\left(\frac{N}{2}\right)^{2/3}, \tag{2}$$

where k_F is the Fermi wave vector and N is the density of electrons in NM. Equation (2) is valid in the limit that the mean free path in the NM film is larger than its thickness. In magnetic double layers FM1/NM/FM2 the spin current injected by FM1 into NM can be absorbed by the ferromagnetic layer FM2. The transverse component of the spin current in NM is entirely absorbed at the NM/FM2 interface.[5,6] Consequently, the spin current results in an interface Gilbert-like damping for the ferromagnetic layer FM1. For small precessional angles the spin current $\mathbf{j}_{\text{spin}}$ is almost entirely transverse. For good spin sinks, the Gilbert damping is given by the conservation of the total spin momentum and is equal to

$$\alpha = \gamma\hbar \frac{g^{\uparrow\downarrow}}{4\pi M_s} \frac{1}{d_1}, \tag{3}$$

where γ is the gyromagnetic ratio, M_s is the saturation magnetization, and d_1 is the thickness of the ferromagnetic layer FM1. The inverse dependence of the Gilbert damping on the film thickness clearly testifies to its interfacial origin. In this case the layer FM1 acts as a spin pump and the layer FM2 acts as a spin sink. The spin pump and spin sink effects have been thoroughly quantitatively studied in Au/Fe/Au/Fe/GaAs(001) structures, see, e.g., Refs. 7 and 8. The quantitative comparison with spin pumping theory is very good.[8] The strength of spin pumping at RT was found to be only 14% lower than that predicted by theory, and it was in excellent agreement at He temperatures. This is an important result. In magnetic double layer structures spin dynamics studies can be carried out with a perfect spin sink, allowing one to determine the full strength of spin pumping.

The spin pump effect can also be observed in single FM films surrounded by NM layers, provided that the pumped spin current is transported away from the FM/NM interface. Interface damping was studied in NM/Py/NM sandwiches by Mizukami *et al.*,[9] where NM=Pt, Pd, Ta, and Cu. The NM layers were 5 nm thick. No interface damping was observed with the Ta and Cu layers. Tserkovnyak *et al.* explained the lack of interface damping in (Ta,Cu)/Py/(Cu,Ta) structures by long spin-diffusion lengths in Cu and Ta. The 5-nm-thick Cu and Ta do not provide effective spin sinks. However, a substantial interface damping was observed in both the Pt and Pd layers. The results by Mizukami *et al.* were obtained on samples prepared by sputtering. Since Pd and Pt have a strong tendency to intermix with 3*d* transition elements it is interesting to compare the results obtained from samples prepared by sputtering with samples prepared by molecular-beam epitaxy (MBE) techniques. The purpose of this paper is to study the spin pump effect in Pd overlayers using crystalline epitaxial Pd/Fe(001) structures which were prepared by MBE, where the intermixing between the Fe and Pd is known to be minimal.[10]

II. SAMPLE GROWTH AND FMR MEASUREMENTS

Metallic *n*Pd/16Fe(001) films were grown on GaAs(001) by MBE using epi-ready GaAs(001) semi-

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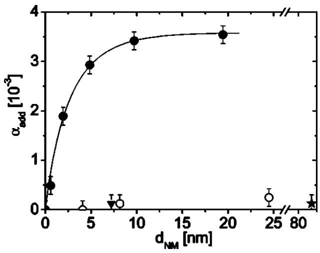

FIG. 1. The additional Gilbert damping α_{add} arising from spin pumping as a function of the Pd film thickness. The Pd data are shown in (●). For comparison several points are shown for Au (○), Ag (▼), and Cu(★). The solid line was obtained by fitting the Pd data using Eq. (8). λ_{dec} was found to be 9 nm. 1 ML of Pd corresponds to 0.2 nm.

insulating templates, see details in Ref. 11. n was between 3 and 200. The integers represent the number of monolayers (ML). All films were covered with Au for protection in ambient conditions. Pd has a lattice mismatch of 4.4% with respect to Fe and 4.9% with respect to Au, and therefore samples with a sufficient thickness of Pd are affected by the relaxation of lattice strain. The presence of a self-assembled network of misfit dislocation half loops was observed by plan view transmission electron microscopy (TEM).[12] Above a Pd thickness of 100 ML the network of self-assembled misfit dislocations leads to strong two magnon scattering. Therefore, the study of intrinsic damping had to be carried out for the Pd films thinner than 100 ML (20 nm).[12] The damping was investigated by ferromagnetic resonance (FMR) at 24 and 36 GHz.

III. RESULTS AND DISCUSSION

The role of Pd in the propagation of a spin current was investigated by monitoring the FMR linewidth as a function of the Pd overlayer thickness in nPd/16Fe/GaAs(001) structures. The FMR linewidth has two contributions: (a) the intrinsic Gilbert contribution corresponding to the Fe film bulk damping and (b) the contribution from the pumped spin current, which is dissipated in the Pd layer and thus contributes to the Fe interface damping. The additional Gilbert damping arising from spin pumping is shown in Fig. 1.

For comparison some results with Au, Ag, and Cu overlayers are shown. Clearly, Pd is different than the noble metals. For the Pd layers thicker than 10 nm (50 ML) the additional Gilbert damping saturates, i.e., Pd acts as a perfect spin sink. For the Au, Ag, and Cu overlayers the contribution from spin pumping remains so small that it is within the accuracy of our FMR measurements. Au, Ag, and Cu in this thickness range behave as spin accumulators, not spin sinks. Spin pumping from the Fe layer increases the spin momentum in Cu, Ag, and Au, and the resulting backflow of spin current nearly compensates the spin pumping, resulting in a zero interface current and a negligible additional damping.

In order to discuss the spin pumping contribution in Pd it is informative to first estimate the momentum electron mean free path. The sheet resistance of the two samples 20Au/50Pd/16Fe/GaAs(001) and 20Au/16Fe/GaAs(001) was measured by means of the van der Pauw technique. The sheet resistances were found to be 9.6Ω/□ and 19.9Ω/□. Since the Pd layer contributes in parallel to the overall sheet resistance one can conclude that the sheet resistance for the 50-ML-thick Pd film is approximately 18.7Ω/□. This sheet resistance leads to the resistivity $\varrho = 18.2\ \mu\Omega$ cm. This value is about two times bigger than that of bulk Pd, $\rho_{Pd}^{bulk} = 10.8\ \mu\Omega$ cm.[13] The measured resistivity allows one to estimate the mean free path λ_m using a simple formula (valid only for a spherical Fermi surface)

$$\frac{1}{\rho} = \frac{e^2 N \lambda_m}{m^* v_F}, \tag{4}$$

where e is the elementary charge, m^* is the electron effective mass, N is the density of electrons, and v_F is the Fermi velocity. The number of conduction electrons (Γ-centered electron sheet) per Pd atom was found to be 0.37.[14] This results in the carrier density $N = 2\times 10^{15}$ cm^{-3}. The effective mass of the conduction electrons is $m^* \sim 2m$, where m is the free-electron mass and the Fermi velocity is $v_F = 5.6\times 10^7$ cm/s.[14] This results in $\lambda_m \simeq 9$ nm. One should realize that the sheet resistance in the Pd thin film is mostly determined by diffuse scattering at the interfaces. Therefore, $\lambda_m = 9$ nm significantly underestimates the mean free path inside the Pd film. A similar behavior was found for the Au thin films grown on Fe/GaAs(001).[15] Since the momentum mean free path in our Pd overlayers is larger than the film thicknesses the spin-diffusion theory[16] is not applicable in the interpretation of our results.

We will demonstrate below that the mean free spin decoherence length is less than the momentum mean free path in our Pd samples. In this limit, the spin flow pumped by the Fe layer is gradually attenuated in Pd before the momentum of the electron is changed. The spin flow pumped by Fe into Pd decays, preventing the net pumped spin momentum from returning back to the Fe film after reflection at the outer Pd interface. When the thickness of the sample is less than the mean free path, the backflow of the spin current can thus be described by

$$\mathbf{I}_s^{back} = \mathbf{I}_s^{pump} e^{-2d_{Pd}^{eff}/\lambda_{dec}}, \tag{5}$$

where d_{Pd}^{eff} is the effective thickness of the Pd film and λ_{dec} is the mean decoherence length. $\mathbf{I}_s^{pump}$ is given by Eq. (1). The factor 2 in the exponent appears because the effective thickness of the Pd film for the spin current making it back to Fe is twice the film thickness. The effective Pd film thickness d_{Pd}^{eff} is larger than d_{Pd}. The ratio $d_{Pd}^{eff}/\lambda_{dec}$ can be estimated by including the length of the electron path propagating under an angle θ with respect to the film normal. This calculation includes only electrons at the Fermi surface participating in spin pumping. For a spherical Fermi surface one can write

$$e^{-2d_{Pd}^{eff}/\lambda_{dec}} = \frac{1}{\pi k_F^2}\int_0^{k_F} 2\pi k_\parallel dk_\parallel e^{-2d_{Pd}/\cos\theta\lambda_{dec}}, \tag{6}$$

where $k_\parallel$ is the component of the k vector parallel to the interface and $\cos\theta = [1-(k_\parallel/k_F)^2]^{0.5}$. The net spin current across the interface is then given by

$$\mathbf{I}_s = \mathbf{I}_s^{\text{pump}}(1 - e^{-2d_{\text{Pd}}^{\text{eff}}/\lambda_{\text{dec}}}), \quad (7)$$

which leads to the enhanced Gilbert damping due to spin pumping

$$\alpha_{\text{Pd}} = \frac{\gamma\hbar g^{\uparrow\downarrow}}{4\pi M_s d_{\text{Fe}}}(1 - e^{-2d_{\text{Pd}}^{\text{eff}}/\lambda_{\text{dec}}}). \quad (8)$$

The thickness dependence in Fig. 1 can be fit by two independent parameters, $g^{\uparrow\downarrow}$ and λ_{dec}. The resulting parameters are $g^{\uparrow\downarrow}=0.9\times10^{15}$ cm^{-2} and $\lambda_{\text{dec}}=9$ nm. When the Pd layer is thicker than the momentum mean free path, there might still be a backflow of electron $I_s^{\text{back}}/I_s^{\text{pump}} \sim \exp(-2\lambda_m/\lambda_{\text{dec}})$. For $\lambda_m=\lambda_{\text{dec}}$ gives $I_s^{\text{back}}/I_s^{\text{pump}} \sim 0.1$. The backflow would lead to a smaller value of the measured spin mixing conductance than that expected from the electron band calculations. Realizing that the mean free path inside the Pd layer is larger than the Pd layer thickness this correction is small in our samples.

From Eq. (2) one can estimate the spin mixing conductance, $g^{\uparrow\downarrow}=0.5$ and 0.7×10^{15} cm^{-2} assuming 0.37 Γ electrons/atom[14] and 0.55 *s-p* electrons/atom,[17] respectively. This is in reasonable agreement with the experimentally required value of 0.9×10^{15} cm^{-2} considering that the band structure of Pd is complex and Eq. (2) can be considered only a crude approximation. First-principles band calculations are required to account for complexity of the Pd band structure which can affect both the spin mixing conductance and Shavrin resistance. λ_{dec} is comparable to the momentum mean free path, $\lambda_m=9$ nm obtained from the crude interpretation of the sheet resistance.

The mean free path of electrons in the Pd layer is larger, see above; this implies that the spin decoherence happens on the shorter length scale than the bulk momentum scattering. A good exponential fit in Fig. 1 suggests that the interface diffuse momentum scattering at the Pd/air interface does not affect the spin decoherence in Pd. The spin current is randomized mostly inside the Pd layer. Bulk Pd is known to have strong spin electron-electron correlation effects having a large Stoner enhancement factor resulting in enhanced paramagnetic susceptibility compared to Ag, Au, and Cu.[18] Associated local fluctuating magnetic moments (paramagnons) are believed to make Pd suitable, under the right conditions, for establishing a long-range ferromagnetic state.[19] One can envision that paramagnons in Pd can lead to an effective long-range decoherence of spin current. This means that the direction of the pumped spin momentum gets randomized by large spin fluctuations inside Pd; and, consequently, the spin momentum backflow loses its net spin momentum and is unable to compensate the spin current generated by spin pumping. The spin mixing conductance in our samples is lower than that required to interpret the data by Mizukami *et al.* Their measurements require $g^{\uparrow\downarrow}=1.4\times10^{15}$ cm^{-2} for 5-nm-thick Pd. This is by a factor of 1.6 bigger than that observed in our studies. The difference between these two experiments can be due to the difference in sample preparation. The results by Mizukami *et al.* suggest that sputtering leads to an enhanced value of $g^{\uparrow\downarrow}$ and thus the intermixing of FM and Pd increases the strength of spin pumping.

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Paper II

Magnetization noise in magnetoelectronic nanostructures

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Magnetization Noise in Magnetoelectronic Nanostructures

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By scattering theory we show that spin current noise in normal electric conductors in contact with nanoscale ferromagnets increases the magnetization noise by means of a fluctuating spin-transfer torque. Johnson-Nyquist noise in the spin current is related to the increased Gilbert damping due to spin pumping, in accordance with the fluctuation-dissipation theorem. Spin current shot noise in the presence of an applied bias is the dominant contribution to the magnetization noise at low temperatures.

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Time-dependent fluctuations of observables ("noise") are a nuisance for the engineer, but also a fascinating subject of study for the physicist. The thermal current fluctuations in electric circuits, as well as the Poissonian current fluctuations due to the discrete electron charge emitted by hot cathodes, are classical textbook subjects. The fluctuations of the order parameter in ferromagnets, such as Barkhausen noise due to moving domain walls, have been studied by the magnetism community for almost a century. Recently, it has been discovered that electronic noise is dramatically modified in nanostructures. Theoretical predictions on the suppression of charge shot noise in quantum devices have been confirmed experimentally [1]. Spin current fluctuations, i.e., spin shot noise, is as yet a purely theoretical concept [2]. In nanoscale magnetism, thermal noise plays an important role by activating magnetization reversal of ferromagnetic clusters [3]. Charge shot noise in ferromagnetic spin valve devices has been discussed as well [4,5]. Interesting new questions have been raised by recent experimental studies on the dynamics of nanoscale spin valves [6–8] in which electric transport is affected by the magnetization direction of the ferromagnetic elements. Central to these studies is the spin-transfer torque exerted by a spin-polarized current on the magnetization causing it to precess or even reverse direction [9–11]. Covington *et al.* [8] interpreted the observed dependence of noise spectra in nanopillar spin valves on bias current direction in terms of this spin torque, but a full consensus has not yet been reached [12].

In a normal metal the average current of net spin angular momentum (spin current) vanishes, but its fluctuations are finite. In this Letter we demonstrate that equilibrium and nonequilibrium spin current noise in normal metals is directly observable in hybrid ferromagnet-normal metal structures: The noise exerts a fluctuating spin-transfer torque on the magnetization vector causing an observable magnetization noise. The theory of noise in magnetoelectronic devices requires a consistent treatment of fluctuations in the currents as well as the magnetization. We demonstrate that thermal spin current fluctuations are instrumental for the spin-pumping-enhanced Gilbert damping in magnetic multilayers [13], and that spin shot noise should be observable at low temperatures. The better understanding of noise in ferromagnetic spin valves should aid the development of next-generation magnetoelectronic and magnetic memory devices.

The magnetization noise in isolated single-domain ferromagnets is well described by the Landau-Lifshitz-Gilbert (LLG) equation of motion

$$\frac{d\mathbf{m}}{dt} = -\gamma\mathbf{m} \times [\mathbf{H}_{\mathrm{eff}} + \mathbf{h}^{(0)}(t)] + \alpha_0\mathbf{m} \times \frac{d\mathbf{m}}{dt}, \quad (1)$$

where $\mathbf{m}$ is the unit magnetization vector, γ the gyromagnetic ratio, $\mathbf{H}_{\mathrm{eff}}$ the effective magnetic field, and α_0 the Gilbert damping constant. The stochastic torque $\mathbf{m} \times \mathbf{h}^{(0)}(t)$ describes thermal agitation in terms of a random field $\mathbf{h}^{(0)}(t)$ with zero average and a white noise correlation function [14]

$$\langle h_i^{(0)}(t)h_j^{(0)}(t')\rangle = 2k_BT\frac{\alpha_0}{\gamma M_s \mathcal{V}}\delta_{ij}\delta(t-t'). \quad (2)$$

Here i and j are Cartesian components, k_BT the thermal energy, M_s the saturation magnetization, and $\mathcal{V}$ the volume of the ferromagnet. The magnetization noise depends on the Gilbert damping α_0 that parametrizes the dissipation of magnetic energy in the ferromagnet. The relation between noise and damping is a corollary of the fluctuation-dissipation theorem (FDT) [14].

In ferromagnets in contact with normal conductors, fluctuating spin currents contribute to the magnetization noise through the spin-transfer torque. The torque is caused by the absorption of only that component of the spin current that is polarized transverse to the magnetization. This happens on the length scale of the magnetic coherence length λ_c [15–17]. In transition metals, λ_c amounts to only a couple of monolayers. A second ingredient needed to understand the noise properties is the inverse effect of the spin torque, often referred to as "spin pumping" [9,13]: a ferromagnet with a changing magnetization direction in contact with conductors emits a spin current. The loss of angular momentum is equivalent to an enhancement of the

Gilbert damping constant such that $\alpha_0 \rightarrow \alpha_0 + \alpha'$ [13]. There is ample evidence that the enhancement α', to be explicitly defined later, can become much larger than α_0 [18].

We consider hybrid structures of a ferromagnet (F) in good electric contact with normal metals (N), such as an N|F|N structure (Fig. 1), with an applied current or voltage bias (a lateral structure in which the ferromagnet is on top of the current carrying normal metal would also serve to illustrate our ideas). At nonzero temperatures the (spin) current through the interface(s), and thus the spin torque, fluctuates. When a bias is applied, the spin current fluctuates even at zero temperature giving spin shot noise. We show in the following that the fluctuations of the magnetization vector due to thermal and shot noise can be described by an effective random field $\mathbf{h}(t)$. The thermal magnetization noise is governed by the FDT, i.e., the relation between the noise amplitude and the Gilbert damping is preserved, with the damping constant $\alpha_0 \rightarrow \alpha_0 + \alpha'$. In other words, the thermal spin current noise is identified as the microscopic process that ensures validity of the FDT in the presence of spin pumping.

We use the Landauer-Büttiker (LB) scattering approach [1] generalized to describe spin transport [4] for a thin ferromagnetic film sandwiched by normal metals (Fig. 1). The LB-approach evaluates current in terms of transmission probabilities for propagating electron states. Assuming that the longitudinal (perpendicular to the F/N interfaces) and transverse electronic motion in the normal metal leads are separable, the $\alpha\beta$ component of the 2×2 current operator in spin space at time t on the left side of the ferromagnetic film reads [4]

$$\hat{I}_L^{\alpha\beta}(t) = \frac{e}{h} \int dE dE' e^{i(E-E')t/\hbar} [a_{L\beta}^{\dagger}(E) a_{L\alpha}(E') - b_{L\beta}^{\dagger}(E) b_{L\alpha}(E')]. \tag{3}$$

Here $a_{L\alpha}^{(\dagger)}(E)$ and $b_{L\alpha}^{(\dagger)}(E)$ are vectors in the space of transverse modes (transverse motion is quantized) that annihilate (create) electrons with spin α and energy E in the left lead moving towards or leaving the ferromagnet, respectively. The scattering properties of the ferromagnet relates the b operators to the a operators;

$$\begin{pmatrix} b_L \\ b_R \end{pmatrix} = \begin{pmatrix} s_{LL} & s_{LR} \\ s_{RL} & s_{RR} \end{pmatrix} \begin{pmatrix} a_L \\ a_R \end{pmatrix} \tag{4}$$

where spin indices have been omitted for simplicity and s_{RL}, for instance, is the scattering matrix (in transverse mode space) for electron transmission from the left side of the ferromagnet to the right. The charge and spin currents are $I_{c,L}(t) = \Sigma_\alpha \hat{I}_L^{\alpha\alpha}(t)$ and $\mathbf{I}_{s,L}(t) = -\hbar/(2e)\Sigma_{\alpha\beta}\hat{\boldsymbol{\sigma}}^{\alpha\beta}\hat{I}_L^{\beta\alpha}(t)$, where $\hat{\boldsymbol{\sigma}}$ is a vector of the Pauli matrices. With the quantum mechanical expectation value $\langle a_{Ln\alpha}^{\dagger}(E) a_{Lm\beta}(E')\rangle = \delta_{mn}\delta_{\alpha\beta}\delta(E-E')f(E-\mu_L)$, where f is the Fermi-Dirac distribution and μ_L is the chemical potential in the left normal metal, the average charge and spin currents can be obtained [16]. The charge current fluctuations on the left side of the ferromagnet are given by the correlation function $S_{c,LL}(t-t') = \langle \Delta I_{c,L}(t) \Delta I_{c,L}(t')\rangle$, where $\Delta I_{c,L}(t) = I_{c,L}(t) - \langle I_{c,L}(t)\rangle$ is the fluctuation of the charge current from its average value. Expressions are simplified in the following by assuming that the normal metals are either very large or support strong spin-flip scattering, such that a spin current emitted by the ferromagnet never returns. We also assume that the ferromagnet is thicker than the magnetic coherence length. Furthermore, we disregard spin-flip processes in the ferromagnet, which is allowed when the spin-flip length is longer than the coherence length. We assume that the (noise) frequencies are much smaller than all relevant energy scales; the temperature, the applied voltage, and the exchange splitting in the ferromagnet. This assumption is implicit in Eq. (2) and in adiabatic spin-pumping theory [13], and is well justified up to ferromagnetic resonance frequencies in the GHz regime. The average magnetization direction is taken to be along the z axis.

Let us consider first the unbiased trilayer with zero average current. At a temperature $T \neq 0$ the instant current at time t does not vanish due to thermal fluctuations. The zero frequency thermal charge current noise is found by Fourier transforming the current correlation function. The result is $S_{c,LL}^{(\mathrm{th})}(\omega = 0) = 2k_B T(e^2/h)(g^{\uparrow} + g^{\downarrow})$, where $g^{\alpha} = \mathrm{Tr}(1 - r_\alpha r_\alpha^{\dagger})$ is the dimensionless spin-dependent conductance. $r_\alpha \equiv s_{LL\alpha}$ should be evaluated at the Fermi energy, and the trace is over the space of the transverse modes. This is the well-known Johnson-Nyquist noise that relates the dissipative element, i.e., the electric resistance, to the noise, as required by the FDT.

More interesting is the correlation

$$S_{ij,KK'}(t-t') = \langle \Delta I_{s_i,K}(t) \Delta I_{s_j,K'}(t')\rangle \tag{5}$$

between the i (vector) component ($i = x, y$, or z) of the

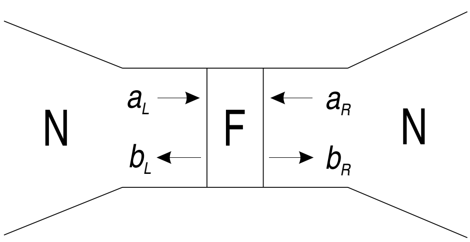

FIG. 1. The transport properties of a thin ferromagnet sandwiched between two large normal metals are evaluated using annihilation and creation operators for the propagating electron states (only annihilation operators are shown here). $a_{L(R)}$ and $b_{L(R)}$ annihilate an incoming and outgoing electron in the left (right) lead, respectively, and are related by the scattering properties of the ferromagnet [see Eq. (4)].

spin current on side K ($=L$ or R) and the j component ($j = x, y$, or z) on side K' ($=L$ or R). The zero-frequency thermal spin current noise becomes

$$S^{(\mathrm{th})}_{ij,KK'} = \frac{\hbar k_B T}{8\pi} \sum_{\alpha\beta} \hat{\sigma}_i^{\alpha\beta} \hat{\sigma}_j^{\beta\alpha} \mathrm{Tr}(2\delta_{KK'} - Q^{\alpha\beta}_{KK'} - Q^{\beta\alpha}_{K'K}), \quad (6)$$

where σ_i ($i = x, y, z$) denotes one of the Pauli matrices and $Q^{\alpha\beta}_{KK'} = s_{KK'\alpha} s^{\dagger}_{KK'\beta}$ should be evaluated at the Fermi energy. The xx and yy components of the thermal spin current noise, $S^{(\mathrm{th})}_{xx,LL}$ and $S^{(\mathrm{th})}_{yy,LL}$, are governed by the real part of the dimensionless mixing conductance [16] $g_L^{\uparrow\downarrow} = \mathrm{Tr}(1 - r_\uparrow r_\downarrow^\dagger)$. Furthermore, $S^{(\mathrm{th})}_{xx,LL} \neq S^{(\mathrm{th})}_{xx,LR}$ (and similar for the yy component) since the transverse spin current is not conserved at the interface. By angular momentum conservation, absorption of the fluctuating spin current implies random torques acting on the magnetization. On the other hand (in the absence of spin-flip scattering) $S^{(\mathrm{th})}_{zz,LL} = S^{(\mathrm{th})}_{zz,LR}$ since a spin current polarized parallel to the magnetization is allowed to traverse the ferromagnet.

We now turn to the effect of the fluctuating torques on the magnetization vector. To this end the LLG Eq. (1) must be generalized by substituting $d\mathbf{m}/dt \to d\mathbf{m}/dt + \gamma \mathbf{I}_{s,\mathrm{abs}}/(M_s \mathcal{V})$, where $M_s\mathcal{V}$ is the total magnetization of the ferromagnet and $\mathbf{I}_{s,\mathrm{abs}} = \mathbf{I}_{s,L} + \mathbf{I}_{s,R}$ is the spin current absorbed by the ferromagnet. (Note that on both sides of the ferromagnet positive current direction is towards to F/N interface; see Fig. 1.) The mean $\langle \mathbf{I}_{s,\mathrm{abs}} \rangle$ vanishes for the single ferromagnet considered here, but the fluctuations $\langle (\mathbf{I}_{s,\mathrm{abs}})^2 \rangle$ do not. The thermal magnetization noise of the isolated magnet is given by Eq. (2). Proceeding from Eq. (6), we find the thermal fluctuations of the torque to be of exactly the same form and therefore represented by a new, statistically independent random field $\mathbf{h}^{(\mathrm{th})}(t)$ with correlation function

$$\langle h_i^{(\mathrm{th})}(t) h_j^{(\mathrm{th})}(t') \rangle = 2 k_B T \frac{\alpha'}{\gamma M_s \mathcal{V}} \delta_{ij} \delta(t - t'), \quad (7)$$

where α' is defined by

$$\alpha' = \frac{\gamma \hbar \,\mathrm{Re}(g_L^{\uparrow\downarrow} + g_R^{\uparrow\downarrow})}{4\pi M_s \mathcal{V}} \quad (8)$$

and where i and j label axes perpendicular to the magnetization direction. The condition that the ferromagnet is thicker than the coherence length allowed us to disregard terms like $\mathrm{Tr}(t_\uparrow t_\downarrow^\dagger)$, where $t_\alpha \equiv s_{RL\alpha}$. The expression for α' is identical to the enhancement of the Gilbert damping in adiabatic spin-pumping theory [13]. We conclude that the enhanced magnetization noise in N|F|N sandwiches can be described by an effective random field $\mathbf{h}(t) = \mathbf{h}^{(0)}(t) + \mathbf{h}^{(\mathrm{th})}(t)$, associated with the enhanced Gilbert constant $\alpha = \alpha_0 + \alpha'$. Basically, we extended the LLG with a (Langevin) thermal agitation term given by $\mathbf{h}^{(\mathrm{th})}(t)$ to capture the increased noise that, according to the FDT, must exist in the presence of spin pumping. We proved that the thermal spin current noise is the underlying microscopic mechanism. Large magnetization noise is expected in thin magnetic layers in which α' dominates α_0 [18]. The small imaginary part of the mixing conductance does not appear explicitly in Eq. (8). Via a renormalized gyromagnetic ratio γ [13], it affects $\mathbf{h}^{(0)}(t)$ and $\mathbf{h}^{(\mathrm{th})}(t)$ identically, keeping the FDT intact.

The shot noise is most easily evaluated at zero temperature. Evaluating the zero-frequency charge shot noise we find $S^{(\mathrm{sh})}_{c,LL} = S^{(\mathrm{sh})}_{c,LR}$, reflecting charge conservation. Using Eq. (5) the zero-frequency spin shot noise at $T = 0$ is

$$S^{(\mathrm{sh})}_{ij,KK'} = \frac{\hbar}{8\pi} \sum_{\alpha\beta} \hat{\sigma}_i^{\alpha\beta} \hat{\sigma}_j^{\beta\alpha} \int dE \sum_{K'' \neq K'''} \mathrm{Tr}(W^{\alpha\beta}_{KK'K''K'''}) \times f_{K'''}(1 - f_{K''}), \quad (9)$$

where $i, j = x$ or y, $K'', K''' = L$ or R, and $W^{\alpha\beta}_{KK'K''K'''} = s_{K'K'''\alpha} s^\dagger_{KK'''\alpha} s_{KK''\beta} s^\dagger_{K'K''\beta}$. Nonconservation of the transverse spin shot noise implies a fluctuating torque as above. Using Eq. (9) we obtain the magnetization noise induced by the spin shot noise,

$$\langle h_i^{(\mathrm{sh})}(t) h_j^{(\mathrm{sh})}(t') \rangle = \frac{\hbar}{4\pi} \frac{e|V|}{M_s^2 \mathcal{V}^2} \delta_{ij} \delta(t - t') [\mathrm{Tr}(r_\uparrow r_\uparrow^\dagger t'_\downarrow t'^\dagger_\downarrow) + \mathrm{Tr}(r'_\downarrow r'^\dagger_\downarrow t_\uparrow t_\uparrow^\dagger)], \quad (10)$$

where $\mu_L - \mu_R = eV$ is the applied voltage and $r_\alpha \equiv s_{LL\alpha}$, $r'_\alpha \equiv s_{RR\alpha}$, $t_\alpha \equiv s_{RL\alpha}$, and $t'_\alpha \equiv s_{LR\alpha}$. A number of terms in the second sum in Eq. (9) have been disregarded using the condition that the ferromagnet is thicker than the coherence length. Equation (10) vanishes with the exchange splitting only if these terms are included.

In order to compare the shot noise, Eq. (10), with the thermal noise, Eq. (7), we consider a symmetric N|F|N structure (Fig. 1) with clean interfaces that conserve the transverse momentum of scattering electrons. We adopt a simple semiclassical approximation in which an incoming electron is totally reflected when its kinetic energy is lower than the potential barrier of the ferromagnet, and transmitted with unit probability otherwise. In terms of the exchange splitting $\Delta U = U_\uparrow - U_\downarrow$, where $U_{\uparrow(\downarrow)}$ is the potential barrier for spin-up (down) electrons, the combination of scattering coefficients is simplified to

$$\mathrm{Tr}(r_\uparrow r_\uparrow^\dagger t'_\downarrow t'^\dagger_\downarrow) + \mathrm{Tr}(r'_\downarrow r'^\dagger_\downarrow t_\uparrow t_\uparrow^\dagger) = M \frac{\Delta U}{E_F}, \quad (11)$$

where M is the number of transverse modes and E_F the Fermi energy in the normal metal. With $\mathrm{Tr}(r_\uparrow r_\downarrow^\dagger) \approx 0$, which usually holds for intermetallic interfaces, the mixing conductance reduces to $g_L^{\uparrow\downarrow} = g_R^{\uparrow\downarrow} = M$. The condition for a significant contribution of shot noise to the magnetization noise can thus be written $eV > k_B T E_F/\Delta U$. For

$\Delta U \sim E_F/5$ and typical experimental voltage drops in nanoscale spin valves this condition is $T \lesssim 10$ K. At low temperatures we therefore predict an observable crossover from thermal to shot noise dominated magnetization noise as a function of the applied bias.

The effective random field $\mathbf{h}(t)$ is not directly observable, but its correlation function is readily translated into that of the magnetization vector $\mathbf{m}(t)$. Linearizing the LLG equation (including spin pumping) in terms of small deviations $\Delta\mathbf{m}$ from the equilibrium direction $\hat{z}$, we obtain the power spectrum of the x component of the magnetization vector $S_x(\omega) = \int d(t-t')e^{i\omega(t-t')}\langle\Delta m_x(t)\Delta m_x(t')\rangle$,

$$S_x(\omega) = 2\gamma\frac{\alpha k_B T}{M_s \mathcal{V}} \times \frac{\omega^2 + \omega_y^2 + \alpha^2\omega^2}{(\omega^2 - \omega_0^2 + \alpha^2\omega^2)^2 + \alpha^2\omega^2(\omega_x + \omega_y)^2}, \tag{12}$$

and similarly for the y component. Here shot noise has been disregarded, α is the spin-pumping-enhanced Gilbert constant, $\omega_0 = \sqrt{\omega_x\omega_y}$ is the ferromagnetic resonance frequency, and ω_x and ω_y are determined by the leading terms in the magnetic free energy expansion near equilibrium, where x and y are taken along the principal axes transverse to z. Note that Eq. (12) is proportional to the imaginary (dissipative) part of the transverse spin susceptibility in accordance with the FDT. It therefore reflects both the enhanced broadening of the ferromagnetic resonance as well as the enhanced low-frequency magnetization noise. Including shot noise increases the prefactor of Eq. (12) with a bias dependent term.

Rebei and Simionato recently investigated magnetization noise in ferromagnetic thin films using an *sd* model [12], and found results similar to our Eq. (12). We believe that our approach based on the scattering theory of transport is more general and, not being based on a specific model for the electronic structure, accessible to first-principles calculations [19], and better suited to treat more complicated devices. Also, Rebei and Simionato did not attempt to evaluate the shot noise contribution to the magnetization noise.

In conclusion, we demonstrate that the magnetization noise in nanoscale ferromagnets is increased by contacting with a conducting environment. The effect is explained by the transfer of transverse spin current fluctuations in the normal conductors to the ferromagnetic order parameter. Both thermal and shot noise generate effective random magnetic fields felt by the magnetization. The thermal magnetization noise increases in the same way as the Gilbert damping of the mean-field magnetization dynamics, in accordance with the fluctuation-dissipation theorem. Just like the spin-pumping induced broadening of the ferromagnetic resonance, the low-frequency magnetization noise is strongly enhanced in thin ferromagnetic films covered by a few monolayers of a strong spin-flip scattering metal such as Pt. At easily accessible lower temperatures the effect of shot noise dominates that of thermal noise.

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Paper III

Resistance noise in spin valves

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Resistance noise in spin valves

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Fluctuations of the magnetization in spin valves are shown to cause resistance noise that strongly depends on the magnetic configuration. Assisted by the dynamic exchange interaction through the normal-metal spacer, the electrical noise level of the antiparallel configuration can exceed that of the parallel one by an order of magnitude, in agreement with recent experimental results.

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The dynamics of nanoscale spin valve pillars, in which electric currents flow perpendicular to the interface planes (CPP), attracts much interest.[1–3] The giant magnetoresistance (GMR) of such pillars of ferromagnetic films separated by normal metal makes them attractive as future read heads in magnetic hard-disk drives. However, Covington *et al.*[3] found that the performance of CPP-GMR heads might be degraded by enhanced low-frequency resistance noise. They ascribed this effect to the spin-transfer torque, i.e., the torque exerted by a spin polarized current on the magnetizations of the ferromagnetic layers.[4–6] Rebei and Simionato,[7] on the other hand, favored micromagnetic disorder as an explanation. More recently, electrical noise measurements have been carried out on CPP nanopillar multilayers with up to 15 magnetic layers.[8] Interestingly, the noise power was found to be suppressed by more than an order of magnitude by aligning the magnetizations from antiparallel to parallel in an external magnetic field.

The noise properties of small metallic structures pose challenges for theoretical physics[9] to which ferromagnetism adds a novel dimension.[10–13] The thermal fluctuations of single domain magnetic clusters have been described already 50 years ago by Brown.[10] Recently, it has been shown that by contacting a ferromagnet with a conducting environment, the magnetization fluctuations are enhanced compared to the bulk value.[13] CPP spin valves offer an opportunity to detect the enhanced magnetization noise electrically by the GMR effect, but the new degree of freedom of a fluctuating detector magnetization complicates the picture in a nontrivial way. The better understanding of the noise properties of CPP nanopillar spin valves reported in the present Brief Report should therefore be of interest for basic physics as well as for applications.

In spin valves, two sources of thermal noise must be taken into account: Direct agitation of the magnetizations due to intrinsic processes[10] and thermal spin-current fluctuations outside the ferromagnets that affect the magnetizations by means of the spin-transfer torque.[13] Here, we disregard spin-current shot noise, assuming a sufficiently small external current bias. We calculate the magnetization noise for the parallel (P) and antiparallel (AP) magnetic configurations using the stochastic equations of motion for the magnetization vectors in the macrospin model. When the relative orientation between the magnetizations fluctuates, so does, via the GMR, the electrical resistance. We show that due to static (exchange and dipolar) and dynamic (nonequilibrium spin-exchange) couplings between the ferromagnets, the resistance noise strongly depends on the magnetic configuration and applied magnetic field.

The thermal agitation of the magnetizations is conveniently described by introducing stochastic magnetic fields acting on the ferromagnets.[10,13] The fluctuations of the magnetizations, and hence the resistance noise, can then be expressed by the transverse magnetic response (susceptibility) of the magnetizations to these stochastic fields. The magnetic response of spin coherent hybrid structures depends on static and dynamic interactions between the magnetic elements, and therefore differs strongly from that of bulk systems. In spin valves, a static nonlocal exchange coupling is mediated by electrons through the normal-metal spacer, and a static dipolar coupling is caused by stray magnetic fields. Additionally, each ferromagnet couples to an external magnetic field. All these couplings affect the stability and response of the magnetic ground state, and therefore the resistance noise, by favoring either the P or AP configurations. For typical spacer thicknesses considered here and in experiments,[8] the nonlocal exchange and dipolar couplings both favor the AP configuration. Naturally, an external magnetic field favors and stabilizes the P configuration. From these simple considerations, we may expect already a dependence of the resistance noise on the magnetic configuration and applied field. The message of this Brief Report is that much more is going on, however.

The dynamic interaction in spin valves is due to *nonequilibrium* spin currents between the ferromagnets.[14,15] A ferromagnet emits spins when its magnetization changes in time ("spin pumping"),[16] which subsequently may be absorbed by the other ferromagnet as a spin-transfer torque.[14] This "dynamic exchange" couples the small-angle dynamics of the magnetizations. The coupled dynamics may be analyzed in terms of collective spin-wave-like modes[15] that govern the magnetic response, and hence the resistance noise. As we will see, the mode that governs the resistance noise in the P configuration is damped more than the respective AP mode. We show that this leads to a substantial lowering of the resistance noise level in the P configuration as compared to the AP. As discussed below, and somewhat surprisingly, this conclusion holds even though the stochastic noise fields are

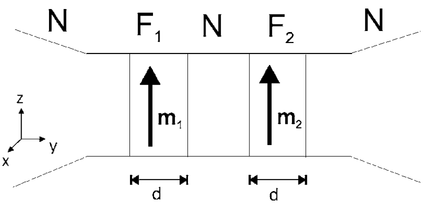

FIG. 1. A spin valve consists of two ferromagnetic thin films F_1 and F_2 separated by a normal-metal spacer N and connected to normal-metal reservoirs. The ferromagnets have magnetizations $\mathbf{m}_1$ and $\mathbf{m}_2$ (here in the parallel configuration), the same thickness d, and equal contact conductances.

stronger for the P mode than for the AP mode.

The resistance noise induced by magnetization fluctuations in spin valves is thus determined by the combined effects of the dynamic exchange coupling, static nonlocal exchange and dipolar couplings, and external magnetic field, and as a result, varies substantially with the magnetic configuration. In particular, we find that when the ferromagnets are ordered antiparallel, the noise level can be much higher than when they are parallel. Our results thus offer an explanation of the experimental findings by Covington *et al.*[8]

We consider a spin valve as pictured in Fig. 1. Two ferromagnetic films with magnetizations $\mathbf{m}_1(t)$ and $\mathbf{m}_2(t)$ (where t is the time) are separated by a thin normal-metal spacer and connected to normal-metal reservoirs. Due to thermal intrinsic and spin-current noise, the magnetizations are subject to fluctuations $\delta\mathbf{m}_1(t)=\mathbf{m}_1(t)-\langle\mathbf{m}_1\rangle$ and $\delta\mathbf{m}_2(t)=\mathbf{m}_2(t)-\langle\mathbf{m}_2\rangle$ from their time-averaged values. The ferromagnets are thicker than the magnetic coherence length so that they perfectly absorb any incoming spin current polarized transverse to the magnetization direction.[17–19] Furthermore, spin-flip processes in the middle normal metal are disregarded, which is usually allowed for CPP spin valves. The ferromagnets can then effectively communicate by means of the dynamic exchange coupling.[14,15] The static nonlocal exchange and dipolar couplings can both be described by a Heisenberg coupling $-J\mathbf{m}_1\cdot\mathbf{m}_2$, where J is the coupling strength, favoring parallel (antiparallel) alignment when $J>0$ ($J<0$). We focus on the situation in which the externally applied currents or voltages are sufficiently small to not affect the device dynamics. For simplicity, we take the spin valve to be symmetric (i.e., the two ferromagnets are identical) and consider only collinear magnetic configurations. Assuming that the static coupling J is negative, the antiparallel state is the ground state without applied external fields, while the parallel state is achieved by applying a sufficiently strong external magnetic field forcing the magnetizations to align.

The resistance noise is characterized by the correlation function

$$S(t-t')=\langle\Delta R(t)\Delta R(t')\rangle, \tag{1}$$

where $\Delta R(t)=R(t)-\langle R\rangle$. The noise is caused by fluctuations in the magnetizations via the dependence of the resistance $R(t)$ on the angle θ between the magnetizations. Close to collinear configurations, $R(t)$ can be expanded in the small fluctuations $\delta\mathbf{m}_1(t)$ and $\delta\mathbf{m}_2(t)$ as

$$R[\mathbf{m}_1(t)\cdot\mathbf{m}_2(t)]\approx R(\pm1)\mp\frac{1}{2}[\delta\mathbf{m}^{\mp}(t)]^2\left(\frac{\partial R}{\partial\cos\theta}\right)_{\mathrm{P/AP}}, \tag{2}$$

where the upper (lower) signs hold for the P (AP) orientation, $\delta\mathbf{m}^{\mp}(t)=\delta\mathbf{m}_1(t)\mp\delta\mathbf{m}_2(t)$, and the differential on the right-hand side should be evaluated for $\mathbf{m}_1\cdot\mathbf{m}_2=\cos\theta=1$ (P) or $\cos\theta=-1$ (AP). Equation (2) inserted into Eq. (1) expresses the resistance noise in terms of the magnetization fluctuations $\delta\mathbf{m}^{\mp}(t)$. Assuming that the fluctuations are Gaussian distributed,[10] we can employ Wick's theorem[20] and obtain

$$S_{\mathrm{P/AP}}(t-t')=\frac{1}{2}\left(\frac{\partial R}{\partial\cos\theta}\right)^2_{\mathrm{P/AP}}\sum_{i,j}S^2_{m_i^{\mp}m_j^{\mp}}(t-t'), \tag{3}$$

where $S_{m_i^-m_j^-}(t-t')=\langle\delta m_i^-(t)\delta m_j^-(t')\rangle$, $S_{m_i^+m_j^+}(t-t')=\langle\delta m_i^+(t)\delta m_j^+(t')\rangle$, and the summation is over all Cartesian components ($i,j=x$, y, or z). Only the difference between the magnetization vectors $\delta\mathbf{m}^-(t)$ (the antisymmetric mode) induces noise when the magnetizations are parallel, whereas only the sum $\delta\mathbf{m}^+(t)$ contributes when they are antiparallel.

The fluctuations $\delta\mathbf{m}^{\mp}(t)$ are the solutions of the stochastic Landau-Lifshitz-Gilbert (LLG) equation of motion for the magnetizations, which, when augmented to include thermal spin-current noise, dynamic exchange coupling, and static exchange and dipolar couplings, reads[15]

$$\frac{d\mathbf{m}_k}{dt}=-\mathbf{m}_k\times[\omega_0\hat{\mathbf{z}}+\omega_c(\mathbf{m}_k\cdot\hat{\mathbf{x}})\hat{\mathbf{x}}+\omega_x\mathbf{m}_l+\gamma\mathbf{h}_k(t)]$$
$$+(\alpha_0+\alpha')\mathbf{m}_k\times\frac{d\mathbf{m}_k}{dt}-\alpha'\mathbf{m}_l\times\frac{d\mathbf{m}_l}{dt}. \tag{4}$$

Here, $k,l=1,2$ denotes ferromagnets 1 or 2, $\omega_0\hat{\mathbf{z}}=\gamma\mathbf{H}_0$, where γ is the gyromagnetic ratio and $\mathbf{H}_0$ an external field applied along the z axis, $\omega_x=\gamma J/M_sd$ parametrizes the static couplings (d is the thickness of the ferromagnets and M_s the saturation magnetization), and α_0 is the intrinsic Gilbert damping constant. We have also included an in-plane anisotropy field $\omega_c(\mathbf{m}_k\cdot\hat{\mathbf{x}})\hat{\mathbf{x}}=\gamma\mathbf{H}_c$ along the x axis. $\alpha'\mathbf{m}_{1(2)}\times d\mathbf{m}_{1(2)}/dt$ is the (dimensionless) spin current emitted by ferromagnet 1 (2) (Ref. 16) that is subsequently absorbed by ferromagnet 2 (1), giving rise to the dynamic exchange coupling. The parameter $\alpha'=(\gamma\hbar\,\mathrm{Re}\,g^{\uparrow\downarrow})/(8\pi M_s\mathcal{V})$ (Ref. 15) governs the strength of the dynamic exchange coupling, where $g^{\uparrow\downarrow}$ is the dimensionless interface spin-mixing conductance (of which we have disregarded a small imaginary part),[17] and $\mathcal{V}$ is the volume of a ferromagnet. If desired, spin currents emitted to the outer normal-metal reservoirs can also be included, simply by making the substitution $\alpha_0\to\alpha_0+\alpha'$. Finally, $\mathbf{h}_k(t)$ is the effective time-dependent stochastic field representing the thermal agitation of ferromagnet k. We write $\mathbf{h}_k(t)=\mathbf{h}_k^{(0)}(t)+\mathbf{h}'_k(t)$, where $\mathbf{h}_k^{(0)}(t)$ describes the intrinsic thermal noise and $\mathbf{h}'_k(t)$ describes the (statistically independent) noise induced by spin current fluctuations via the spin-transfer torque.[13] $\mathbf{h}_k^{(0)}(t)$ has zero average and a white noise correlation function that satisfies the fluctuation-dissipation theorem[10] (FDT):

$$\langle h_{k,i}^{(0)}(t)h_{k,j}^{(0)}(t')\rangle = 2k_BT\frac{\alpha_0}{\gamma M_s\mathcal{V}}\delta_{ij}\delta(t-t'). \tag{5}$$

Here, i and j are Cartesian components and k_BT is the thermal energy.

The spin-current-induced field $\mathbf{h}_k'(t)$ can be determined using magnetoelectronic circuit theory[17] and the results of Ref. 13. Requiring conservation of charge and spin in the normal-metal spacer[11] and taking into account thermal fluctuations of the distribution function in the same spacer,[11] we arrive at the following results:[21] For collinear configurations, the spin-current-induced noise field $\mathbf{h}_k'(t)$ is given by [compare with Eq. (5)]

$$\langle h_{k,i}'(t)h_{k,j}'(t')\rangle = 2k_BT\frac{\alpha'}{\gamma M_s\mathcal{V}}\delta_{ij}\delta(t-t'). \tag{6}$$

Here, $k=1,2$, and i and j label components perpendicular to the magnetization direction. Furthermore, $\mathbf{h}_1'(t)$ and $\mathbf{h}_2'(t)$ are not statistically independent,

$$\langle h_{1,i}'(t)h_{2,i}'(t')\rangle = -\langle h_{1,i}'(t)h_{1,i}'(t')\rangle, \tag{7}$$

due to current conservation. In accordance with the FDT, the total noise field $\mathbf{h}_k(t)=\mathbf{h}_k^{(0)}(t)+\mathbf{h}_k'(t)$ is thus proportional to the total damping $\alpha=\alpha_0+\alpha'$, where α' is the enhancement of the Gilbert damping due to emission of spin currents, as defined above.

The anisotropy field and the negative exchange and/or dipolar coupling ($\omega_x<0$) align the ferromagnets antiparallel along the x axis when the external field is turned off. Then, $\mathbf{m}_k(t)\approx\pm\hat{\mathbf{x}}+\delta\mathbf{m}_k(t)$ for $k=1,2$, where $\delta\mathbf{m}_k\approx\delta m_{k,y}\hat{\mathbf{y}}+\delta m_{k,z}\hat{\mathbf{z}}$ are the transverse fluctuations induced by the stochastic noise fields. Linearizing the LLG equation in $\delta\mathbf{m}_k$, we can evaluate the magnetization noise $S_{m_i^+m_j^+}(t-t')$ using Eqs. (5)–(7), and find the resistance noise from Eq. (3). A strong external field enforces a parallel magnetic configuration. Disregarding a sufficiently weak anisotropy field in this case, $\mathbf{m}_k(t)\approx\hat{\mathbf{z}}+\delta\mathbf{m}_k(t)$, where $\delta\mathbf{m}_k\approx\delta m_{k,x}\hat{\mathbf{x}}+\delta m_{k,y}\hat{\mathbf{y}}$. This may be used to find $S_{m_i^-m_j^-}(t-t')$ and subsequently $S_P(t-t')$.

The zero-frequency resistance noise $S_{\mathrm{P/AP}}(\omega'=0)=\int d(t-t')\langle\Delta R(t)\Delta R(t')\rangle_{\mathrm{P/AP}}$ thus becomes

$$S_{\mathrm{P/AP}}(0) = \frac{2}{\pi}\left(\frac{2\gamma k_BT}{M_s\mathcal{V}}\right)^2\left(\frac{\partial R}{\partial\cos\theta}\right)^2_{\mathrm{P/AP}}\int d\omega\, X_{\mathrm{P/AP}}, \tag{8}$$

where

$$X_{\mathrm{P}} = \frac{[\omega^2+(\omega_t-\omega_c)^2]^2+(\omega^2+\omega_t^2)^2+2\omega^2(2\omega_t-\omega_c)^2}{2\alpha_t^{-2}\{[\omega^2-\omega_t(\omega_t-\omega_c)]^2+\omega^2\alpha_t^2(2\omega_t-\omega_c)^2\}^2} \tag{9}$$

for the parallel configuration and

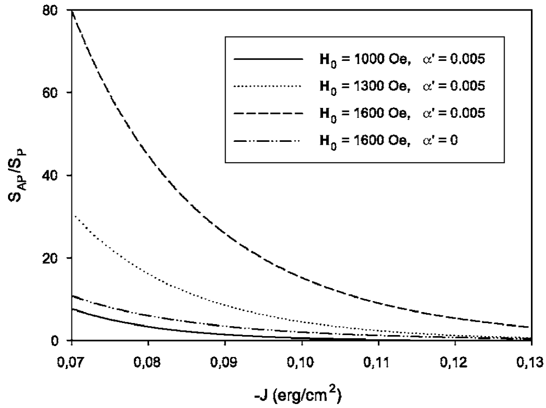

FIG. 2. The ratio $S_{\mathrm{AP}}/S_{\mathrm{P}}$ of the noise powers as a function of the coupling strength $-J$, for some values of the applied external field in the parallel configuration (in the antiparallel configuration, the external field is zero). The damping has been set to $\alpha_0=0.01$ and the anisotropy field to $\omega_c/\gamma=10$ Oe, with the experiments by Covington *et al.* (Ref. 8) in mind.

$$X_{\mathrm{AP}} = \left(\frac{\omega^2\alpha_t+\omega_c^2\alpha_0}{[\omega^2+\omega_c(2\omega_x-\omega_c)]^2+4\omega^2(\omega_x\alpha_0-\omega_c\alpha)^2}\right)^2 \tag{10}$$

for the antiparallel configuration. Here, we set the external field to zero for the antiparallel configuration and assume small damping, $\alpha\ll1$. The integration over frequency in Eq. (8) reflects the quadratic dependence of the resistance noise on the magnetization noise in the time domain [see Eq. (3)]. $\omega_t=\omega_0+2\omega_x$ and $\alpha_t=\alpha_0+2\alpha'$ (note the difference with $\alpha=\alpha_0+\alpha'$) are the frequency and damping of the antisymmetric mode $\delta\mathbf{m}^-(t)$ in the P configuration.[15] The differential $\partial R/\partial\cos\theta$, as calculated by magnetoelectronic circuit theory,[17] depends only weakly on the magnetic configuration[21] and is taken in the following to be a constant. The ratio $S_{\mathrm{AP}}/S_{\mathrm{P}}$ of the noise powers as a function of the static coupling strength $-J$ is shown in Fig. 2 for some values of the applied external field in the parallel configuration. As expected, the noise ratio increases with increasing external field, since this field stabilizes the P configuration. It is also easily understood that the noise ratio decreases with increasing coupling strength, because the coupling stabilizes the AP configuration while destabilizing the P configuration.

Figure 2 emphasizes the importance of including the dynamic exchange coupling. If disregarded, i.e., $\alpha'=0$, the ratio $S_{\mathrm{AP}}/S_{\mathrm{P}}$ is substantially smaller. To understand this surprising result, consider the derivation of the expressions for S_{P} and S_{AP}: The noise S_{P} is caused by the antisymmetric mode $\delta\mathbf{m}^-(t)=\delta\mathbf{m}_1(t)-\delta\mathbf{m}_2(t)$, which, as can be seen from Eq. (9), is strongly damped by $\alpha_t=\alpha_0+2\alpha'$.[15] The noise S_{AP} in the AP configuration, on the other hand, is caused by the mode $\delta\mathbf{m}^+(t)$, which is relatively weakly damped. Since, according to the FDT, a larger damping is associated with stronger stochastic fields, the mode $\delta\mathbf{m}^-(t)$ in P should be agitated stronger than the $\delta\mathbf{m}^{+}(t)$ AP mode. At first sight, our results for the effect of α' on the ratio S_{AP}/S_{P} thus seem to violate the FDT. However, as emphasized above, the damping affects not only the stochastic fields but also the magnetic response of the magnetization to these fields. Since the resistance noise depends *quadratically* on the magnetization noise and *quartically* on the linear-response function, a relatively suppressed response of the antisymmetric P mode turns out to be more important than the increased stochastic fields. As a result, S_{P} is significantly reduced as compared to S_{AP} when the dynamic exchange is included.

We conclude from Fig. 2 that, depending on parameters such as the exchange coupling and the applied magnetic field, the noise power can be much higher in the antiparallel than in the parallel configuration, in agreement with the experimental results by Covington *et al.*[8] on multilayer pillars. In these experiments, the magnetizations reached the parallel alignment for external magnetic field of $\gtrsim$1500 Oe. Whereas we treated spin valves with two ferromagnetic films, Covington *et al.* dealt with multilayers of 4–15 magnetic films. However, the difference between the noise properties of bilayers and multilayers should be quite small, since the only local structural difference is the number of neighboring ferromagnets. This assertion is supported by the experiments by Covington *et al.* that did not reveal strong differences for nanopillars with 4–15 layers.

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Erratum: Resistance noise in spin valves [Phys. Rev. B 75, 092405 (2007)]

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(Dated: March 29, 2008)

PACS numbers: 75.75.+a, 72.70.+m, 72.25.Mk

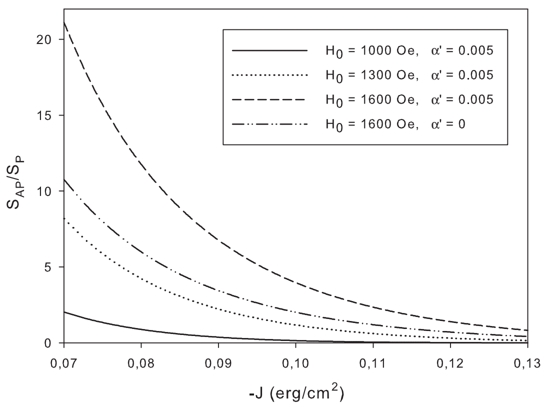
The stochastic field correlator depends on the configuration. Eq. (7) is correct for the parallel configuration of the spin valve. However, for the antiparallel configuration it should read

$$\langle h'_{1,i}(t)h'_{2,i}(t')\rangle = \langle h'_{1,i}(t)h'_{1,i}(t')\rangle.$$

Eq. (10) then becomes

$$X_{\mathrm{AP}} = \left(\frac{\omega^2\alpha_0 + \omega_c^2\alpha_t}{[\omega^2 + \omega_c(2\omega_x - \omega_c)]^2 + 4\omega^2(\omega_x\alpha_0 - \omega_c\alpha)^2}\right)^2,$$

which leads to quantitative changes in Fig. 2 as shown below. The qualitative conclusions of the paper remain unaffected.



Paper IV

Electric and magnetic noise and dissipation in ferromagnetic single- and double-layers

To be submitted to Physical Review B

Electric and magnetic noise and dissipation in ferromagnetic single- and double-layers

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The interplay between current and magnetization fluctuations in nanostructures with alternating magnetic and non-magnetic layers is investigated. We use scattering theory and magnetoelectronic circuit theory to calculate charge and spin current fluctuations. Via the spin-transfer torque, spin current noise causes a significant enhancement of magnetization fluctuations. The equilibrium current and magnetization noise are related to respectively the resistance and magnetization damping. We demonstrate that calculation of the magnetization noise is a fruitful way of obtaining the magnetization damping. Special focus is on spin valves in which one of the ferromagnets is pinned, which are systems frequently studied in experiments. We find that there is a difference in magnetization noise and damping between the parallel, antiparallel and perpendicular magnetic configurations. For the perpendicular configuration, the noise and damping are anisotropic. Due to giant magnetoresistance (GMR), the magnetization fluctuations in spin valves induce resistance noise, which is identified as a prominent source of electric noise at relatively high current densities. The resistance noise is shown to vary considerably with the magnetic configuration, partly due to the dependence of the GMR sensitivity on the configuration. The contribution from spin current fluctuations to the resistance noise is shown to be significant. Resistance noise can be converted into voltage noise, which is an experimentally accesible quantity that may be employed to verify our results.

PACS numbers: 72.70.+m, 72.25.Mk, 75.75.+a

I. INTRODUCTION

In magnetoelectronic nanostructures, integrated ferromagnetic elements produce new functionality. The interplay between magnetism and electricity in these structures shows great promise for device applications. Giant magnetoresistance (GMR) is already the leading technology used for read heads in magnetic hard disk drives. Considerable progress is made in research on possible future applications, such as magnetic random access memory and spin based transistors. The performance of such devices is amongst other factors determined by their electric and magnetic noise and dissipation of energy. A thorough understanding of noise and dissipation is hence essential.

The equilibrium voltage noise across conductors was investigated by Johnson in 1928[1] and explained by Nyquist[2]. The noise power was found to be directly proportional to the electric dissipation, i.e., the resistance. As electronic elements are made increasingly smaller, current noise in mesoscopic conductors has gained a lot of interest[3]. Following the birth of magnetoelectronics, also the noise properties of hybrid non magnetic-ferromagnetic structures, to which magnetization adds a novel dimension, has received attention. In ferromagnets, a charge current is accompanied by a spin current, i.e., a non-equilibrium flow of spin angular momentum. Both charge and spin-polarized current noise in layered structures of alternating ferromagnets and normal metals have been investigated theoretically[4–9].

The thermal fluctuations of the magnetization vector in small single-domain ferromagnets were analyzed by Brown in 1963[10]. He introduced a stochastic field acting on the magnetization to account for thermal agitation. Lately, it has been shown that the magnetization noise is increased in ferromagnetic films contacted to normal metals[11,12]. This is due to spin current fluctuations originating in the external circuit, that are subsequently absorbed by the magnetization in the ferromagnet, hence giving a fluctuating spin-transfer torque[13–16] on it. The presence of magnetization noise has been shown to be important for the process of magnetization reversal[17–20].

In ferromagnets, the magnetization vector may be excited from its equilibrium direction by an external magnetic field or an applied spin current. The process in which it dissipates energy and returns to equilibrium is usually parametrized by the Gilbert damping constant, phenomenologically introduced in 1955[21,22]. There is still a substantial amount of research on the nature of this damping. Recently Kohno *et. al*[23] and Skadsem *et. al*[24] investigated its microscopic origin, while Gilmore *et. al*[25] used first principles to estimate the value of the Gilbert constant in the most common ferromagnetic metals Fe, Co and Ni. The Gilbert damping in ferromagnets connected to a conducting environment has been shown to be considerably enhanced as compared to the intrinsic damping in isolated ferromagnets[26,27].

Noise and dissipation are closely related. The fluctuation-dissipation theorem (FDT)[28,29] states that there is a one-to-one correspondence between the equilibrium noise of a physical system, and the systems response to external perturbations, i.e., its dissipation of energy. Specifically, the equilibrium current noise in a conductor is related to the resistance, as found and explained by Johnson and Nyquist. Likewise, the equilibrium fluctuations of the magnetization in ferromagnets is connected to the Gilbert damping. The usefulness of the FDT in investigating magnetization noise and damping has been discussed lately[30,31].

In the present paper, we investigate the interplay between current and magnetization noise in layered nanostructures consisting of alternating magnetic and nonmagnetic films. The FDT is used to relate the equilibrium electric and magnetic noise to the corresponding dissipation of energy, and we show that this yields correct results for the resistance and Gilbert damping. Focus is on magnetization noise caused by spin current fluctuations via the spin-transfer torque. We start by reviewing the noise in the simple system of a single mono-domain ferromagnet sandwiched by normal metals[11]. We provide an in-depth guide to the key results of Ref. 11, presenting technical details previously omitted. Both thermal current noise, which is always present, and shot noise, which requires an applied bias, are taken into account, and the resulting magnetization noise and related Gilbert damping are derived. We go on to consider spin valves, i.e., two mono-domain ferromagnetic films separated by a thin normal metal. We consider the experimentally relevant case where one of the ferromagnets is pinned, as opposed to our work on spin valves in Ref. 32, in which both ferromagnets were taken to be susceptible to fluctuations. Magnetoelectronic circuit theory[33–35] is used to calculate the charge and spin current fluctuations, and the resulting magnetization noise and related Gilbert damping of the free ferromagnet are found. The discussion is limited to the parallel (P), antiparallel (AP) and perpendicular (90°) magnetic configurations, and it is shown that the magnetization noise and Gilbert damping differ in these configurations. For the 90° configuration, the magnetization noise and damping are anisotropic.

Direct measurement of magnetization fluctuations in ferromagnets is a difficult task. Spin valves provide an opportunity to indirectly measure magnetization noise via resistance noise (or voltage noise), hence offering an experimental check[36,37] of our results. The magnetically induced resistance noise is a result of GMR. We obtain analytical expressions for the noise in both the P, AP and 90° configurations. The role of the GMR sensitivity, $\partial R/\partial\theta$, where R is the resistance and θ is the relative angle between the magnetizations, is explored. The dependence of the sensitivity on θ leads to a sizeable difference in resistance noise in the P, AP and 90° configurations. The resistance noise can be converted into current noise, and adds to the Johnson-Nyquist noise also present in spin valves. It is a prominent contribution at relatively high current densities[36,37], and should be of importance for the performance of spin valve read heads[30]. We show that the contribution from spin current fluctuations to the resistance noise is significant.

We end this paper by considering resistance noise in the P and AP configurations for spin valves where neither of the ferromagnets are pinned, but rather are identical. As both ferromagnets now will fluctuate, interactions between them become very important. We point out that depending on these interactions, there may be a large difference in noise level between the P and AP configurations. Our results for these spin valves include previously presented results[32] as a limiting case. In Ref. 32, the demagnetizing field inside the ferromagnets, which is due to the intralayer magnetic dipolar interaction, was not included, and the dependence of the GMR sensitivity on θ was neglected.

The paper is organized as follows. We begin by reviewing the fluctuation-dissipation theorem, applied to magnetic systems. In section III, the noise properties of a single ferromagnetic thin film sandwiched by normal metals is worked out in detail, emphasizing the relation of the noise to the damping. Then, in section IV, we consider current noise, magnetization noise, and magnetization damping in spin valves, and use the results to calculate the resistance noise induced by GMR. In section V we draw our conclusions.

II. FLUCTUATION-DISSIPATION THEOREM

The fluctuation-dissipation theorem (FDT) relates the noise properties of physical systems in equilibrium to their response when they are subject to external perturbations. More precisely, the equilibrium fluctuations of the physical quantity that characterizes the system is related to the out-of-equilibrium dissipation of energy. For example, in an electric conductor, the fluctuations in the electric current is related to the conductivity, as shown and explained by Johnson and Nyquist[1,2]. Similarly, in magnetic systems, the equilibrium fluctuations in the magnetization are related to the dissipative part of the magnetic susceptibility. In the following we briefly describe the FDT applied to magnetic systems.

A single-domain ferromagnet may be characterized by its uniform unit magnetization vector $\mathbf{m}$. Its time-dependent equilibrium fluctuations are described by the correlator $\langle \delta m_i(t)\delta m_j(t')\rangle$, where $\delta m_i(t) = m_i(t) - \langle m_i(t)\rangle$. Here the brackets denote statistical averaging at equilibrium, and i and j denote Cartesian components. The classical FDT states that these fluctuations are related to the magnetic susceptibility:

$$\langle \delta m_i(t)\delta m_j(t')\rangle = \frac{k_B T}{2\pi M_s V}\int d\omega e^{-i\omega(t-t')} \times \frac{\chi_{ij}(\omega) - \chi^*_{ji}(\omega)}{i\omega}, \quad (1)$$

where M_s is the saturation magnetization, T is the temperature, V is the volume of the ferromagnet, and $\chi_{ij}(\omega)$ is the ij-component of the magnetic susceptibility (in Fourier space). The susceptibility is defined by the linear causal magnetic response to an external driving field $\mathbf{H}^{(\mathrm{dr})}(t)$:

$$\Delta m_i(t) = \sum_j \int dt' \chi_{ij}(t-t') H_j^{(\mathrm{dr})}(t'), \tag{2}$$

where $\Delta m_i(t)$ is the change in magnetization caused by the driving field.

An alternative form of the FDT may be derived by introducing a fictitious stochastic magnetic field $\mathbf{h}^{(0)}(t)$ with zero mean, and viewing the fluctuations as caused by the action of this field. The properties and strength of $\mathbf{h}^{(0)}(t)$ is determined by demanding that it produces the correct fluctuations $\delta\mathbf{m}(t)$. From Eq. (2) it follows that $\delta m_i(\omega) = \sum_j \chi_{ij}(\omega) h_j^{(0)}(\omega)$ in Fourier space. Inverting this relation, it follows that the correlator of the stochastic field is

$$\begin{aligned}\langle h_i^{(0)}(t) h_j^{(0)}(t')\rangle &= \frac{k_B T}{2\pi M_s V}\int d\omega e^{-i\omega(t-t')} \\ &\times \frac{[\chi_{ji}^{-1}(\omega)]^* - \chi_{ij}^{-1}(\omega)}{i\omega}, \end{aligned} \tag{3}$$

where $\chi_{ij}^{-1}(\omega)$ is the ij-component of the Fourier transformed inverse susceptibility. We will later make use of this second version of the FDT.

III. SINGLE FERROMAGNET

The magnetization dynamics of an isolated single-domain ferromagnet is well described by the Landau-Lifshitz-Gilbert (LLG) equation[21,38]

$$\frac{d\mathbf{m}}{dt} = -\gamma_0 \mathbf{m}\times\mathbf{H}_{\mathrm{eff}} + \alpha_0 \mathbf{m}\times\frac{d\mathbf{m}}{dt}, \tag{4}$$

where γ_0 is the gyromagnetic ratio, $\mathbf{H}_{\mathrm{eff}}$ the effective magnetic field, and α_0 the Gilbert damping constant. The effective field includes internal anisotropy and demagnetizing fields, and any externally applied fields. The equilibrium magnetization noise is given by Eq. (1), where Eq. (4) may be used to evaluate the magnetic susceptibility. To find the susceptibility, first note that the equilibrium direction of the magnetization is in the direction of the effective static field $\mathbf{H}_{\mathrm{eff}}$. Next, let the weak external driving field $\mathbf{H}^{(\mathrm{dr})}(t)$ be applied transverse to the equilibrium direction, modifying the LLG equation by the substitution $\mathbf{H}_{\mathrm{eff}} \to \mathbf{H}_{\mathrm{eff}} + \mathbf{H}^{(\mathrm{dr})}(t)$. Then the magnetization is excited from the equilibrium direction $\mathbf{m}_0 = \mathbf{H}_{\mathrm{eff}}/|\mathbf{H}_{\mathrm{eff}}|$ such that $\mathbf{m}(t) \approx \mathbf{m}_0 + \Delta\mathbf{m}(t)$, where the small change in magnetization $\Delta\mathbf{m}(t)$ is perpendicular to $\mathbf{m}_0$. Linearizing the LLG equation in $\Delta\mathbf{m}(t)$ we find the transverse inverse susceptibility tensor

$$\chi^{-1} = \frac{1}{\gamma_0}\begin{bmatrix} \gamma_0\mathbf{H}_{\mathrm{eff}} - i\omega\alpha_0 & i\omega \\ -i\omega & \gamma_0\mathbf{H}_{\mathrm{eff}} - i\omega\alpha_0 \end{bmatrix} \tag{5}$$

written in matrix/tensor form in the plane orthogonal to $\mathbf{m}_0$. Note that we for simplicity have assumed that the effective field $\mathbf{H}_{\mathrm{eff}}$ is a constant, although it in general depends on the magnetization direction $\mathbf{m}$. This simplification has no effect on the result of the calculation of noise.

The LLG equation can explicitly describe both magnetization dynamics and noise, by including the stochastic field $\mathbf{h}^{(0)}(t)$ given by Eq. (FDT2) on the right hand side so that $\mathbf{H}_{\mathrm{eff}} \to \mathbf{H}_{\mathrm{eff}} + \mathbf{h}^{(0)}(t)$. From Eqs. (3) and (5)[10],

$$\langle h_i^{(0)}(t) h_j^{(0)}(t')\rangle = 2k_B T \frac{\alpha_0}{\gamma_0 M_s V}\delta_{ij}\delta(t-t'). \tag{6}$$

This expression relates the equilibrium fluctuations of the magnetization to its dissipative properties, the Gilbert damping.

Eq. (4) with the stochastic field $\mathbf{h}^{(0)}(t)$ included describes well the magnetization dynamics and noise of a single-domain ferromagnet isolated from the outside world. When the ferromagnet is contacted to a conducting environment, interesting new effects that modify both the dynamics and noise come in to play. These effects are considered in the following.

If the direction of the magnetization vector is changing in time due to, e.g., the action of an external driving field, spins can be emitted from the ferromagnet to the surroundings[26]. For ferromagnets thicker than the magnetic coherence length, this "pumped"spin current is given by

$$\mathbf{I}_s^{\mathrm{pump}} = \frac{\hbar}{4\pi}\left(\mathrm{Re}g^{\uparrow\downarrow}\mathbf{m}\times\frac{d\mathbf{m}}{dt} - \mathrm{Im}g^{\uparrow\downarrow}\frac{d\mathbf{m}}{dt}\right), \tag{7}$$

where the material parameter $g^{\uparrow\downarrow}$ is the dimensionless conductance for electrons with spin oriented perpendicular to the magnetization direction (the "mixing conductance")[33–35]. By conservation of angular momentum this leads to the appearance of an extra term $\gamma\mathbf{I}_s^{\mathrm{pump}}/(M_s V)$ on the right hand side of the LLG equation, which can be shown[26] to be equivalent to a renormalization of the Gilbert damping and the gyromagnetic ratio:

$$\frac{1}{\gamma_0} \to \frac{1}{\gamma} = \frac{1}{\gamma_0}\left(1 - \frac{\gamma_0\hbar\mathrm{Im}g^{\uparrow\downarrow}}{4\pi M_s V}\right), \tag{8}$$

$$\alpha_0 \to \alpha = \frac{\gamma}{\gamma_0}\left(\alpha_0 + \frac{\gamma_0\hbar\mathrm{Re}g^{\uparrow\downarrow}}{4\pi M_s V}\right). \tag{9}$$

These expressions are valid when the surrounding environment is a perfect spin sink, such that the pumped spin current never returns to the ferromagnet. Since the imaginary part of the mixing conductance typically is small, the renormalization of γ_0 will hereafter be neglected.

The inverse effect of spin pumping is also important: A spin-polarized current may exert a torque on the ferromagnet, leading to precessional motion of the magnetization, or even magnetization reversal. This spin-transfer torque[13–16] is due to the absorption of the component of the spin current polarized transverse to the magnetization[33,39]. By conservation of angular momentum, the LLG equation must be modified by including a term $-\gamma_0 \mathbf{I}_{s,\text{abs}}/(M_s V)$ on the right hand side, where $\mathbf{I}_{s,\text{abs}}$ is the absorbed spin current, to take this into account. The absorption of spin angular momentum happens on the length scale of the magnetic coherence length[33,39,40], which for transition metals is of the order of a nanometer. In the following we shall only consider ferromagnets thicker than this length, so that spin absorption is complete.

Recently we have shown[11] that due to the spin-transfer torque, the magnetization noise in magnetoelectronic nanostructures can be considerably increased as compared to an isolated ferromagnet. At elevated temperatures, thermal fluctuations in the spin current exert a fluctuating torque on the magnetization, increasing the noise. For a ferromagnet sandwiched by normal metals, the enhancement of the noise is described by a stochastic field $\mathbf{h}^{(\text{th})}(t)$ similar to the intrinsic field $\mathbf{h}^{(0)}(t)$. $\mathbf{h}^{(\text{th})}(t)$ has correlation function[11]

$$\langle h_i^{(\text{th})}(t) h_j^{(\text{th})}(t') \rangle = 2 k_B T \frac{\alpha'}{\gamma M_s V} \delta_{ij} \delta(t - t'), \tag{10}$$

where

$$\alpha' = \frac{\gamma \hbar \text{Re} g^{\uparrow\downarrow}}{4\pi M_s V} \tag{11}$$

is the enhancement of the Gilbert damping due to spin pumping (see Eq. (9)). Assuming $\mathbf{h}^{(0)}(t)$ and $\mathbf{h}^{(\text{th})}(t)$ are statistically independent, the total noise is thus given by $\mathbf{h}(t) = \mathbf{h}^{(0)}(t) + \mathbf{h}^{(\text{th})}(t)$. We know that the total damping is $\alpha = \alpha_0 + \alpha'$, and from Eqs. (6) and (10) we see that the total noise is related to the total damping, in agreement with the FDT. Hence, the thermal spin current noise is the noise process related to the enhanced dissipation of energy by spin pumping, and calculating the noise is an alternative route to finding the damping. In thin ferromagnetic films, α' can be of the same order, or larger than, α_0[27]. In the following subsections we will give a detailed derivation of the result Eq. (10). We will also evaluate the shot noise contribution to the magnetization noise, which is important at low temperatures[11]. We may note here that Eq. (10) may be found also by direct application of Eq. (3) to the LLG equation with spin pumping included.

A. Scattering theory

The system at study is a thin ferromagnetic film connected to two large normal metals through two leads, as shown in Fig. 1. We assume that the normal metals are perfect spin sinks and that the ferromagnet is thicker than the magnetic coherence length, which in a simple two-band ferromagnet reads $\lambda_c = \pi/(k_\uparrow - k_\downarrow)$, where $k_{\uparrow(\downarrow)}$ is the spin-dependent Fermi wave vector. The normal metals are characterized by Fermi-Dirac distribution functions f_L and f_R and corresponding chemical potentials μ_L and μ_R, where L and R refer to the left and right sides. They are held at a common temperature T. We use the Landauer-Büttiker (LB) scattering theory[3] to evaluate the spin current fluctuations, and then the LLG equation to calculate the resulting magnetization noise.

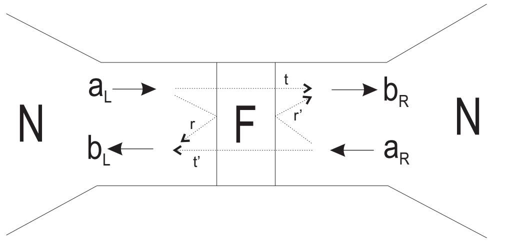

FIG. 1: A thin ferromagnetic (F) film is sandwiched by normal metals (N). The current fluctuations in the system are evaluated in terms of transmission probabilities for the electron states, with the aid of second quantized annihilation and creation operators. The operators shown in the figure are annihilation operators, with the a-operators annihilating electrons moving towards the ferromagnet, and the b-operators annihilating electrons moving away from the ferromagnet. Also shown are the reflection and transmission matrices r, r', t, t' (see Eq. (14)), for simplicity without spin indeces.

The idea of the LB approach is to evaluate the current from a microscopic viewpoint, in terms of transmission probabilities for the electron states. The ferromagnetic film is viewed as a scatterer, limiting the propagation of electrons between the normal metals. The scattering properties of the ferromagnet together with the occupation numbers in the normal metals determine the transport properties of the system. Assuming that the transverse and longitudinal motion in the leads are separable, the electron states are described by a continuos wave vector for the longitudinal motion, and a discrete mode index for the quantized transverse motion. Introducing creation and annihilation operators, the LB formalism[3,41] generalized to describe spin transport gives

$$\begin{aligned} \hat{I}_A^{\alpha\beta}(t) &= \frac{e}{h} \int dE dE' e^{i(E-E')t/\hbar} [a_{A\beta}^\dagger(E) a_{A\alpha}(E') \\ &\quad - b_{A\beta}^\dagger(E) b_{A\alpha}(E')]. \end{aligned} \tag{12}$$

for the $\alpha\beta$-component of the 2×2 current operator in spin space at time t on side A (= L(left) or R(right)) of the ferromagnetic film. Here $a_{A\alpha}^{(\dagger)}(E)$ and $b_{A\alpha}^{(\dagger)}(E)$ are vectors in the space of the transverse modes that annihilate (create) electrons with spin α and energy E in lead A moving towards or away from the ferromagnet, respectively (see Fig. 1). The a-operators are related to the b-operators by the scattering properties of the ferromagnet:

$$b_{A\alpha}(E) = \sum_{B\beta} s_{AB\alpha\beta}(E) a_{B\beta}(E), \tag{13}$$

where $s_{AB\alpha\beta}$ is the scattering submatrix in transverse mode space for incoming electrons with spin β in lead B $(= L$ or $R)$ scattered to outgoing states in lead A with spin α. The summation is over $B = L, R$ and over spin $\beta =\uparrow,\downarrow$. A similar relation holds for the creation operators. Suppressing for simplicity spin indeces, Eq. (13) can be written in the more transparent way (see Fig. 1)

$$\begin{pmatrix} b_L \\ b_R \end{pmatrix} = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix} \begin{pmatrix} a_L \\ a_R \end{pmatrix}, \qquad (14)$$

where $r = s_{LL}$, $r' = s_{RR}$, $t = s_{RL}$ and $t' = s_{LR}$. In the following we shall for simplicity disregard spin-flip processes in the ferromagnet, so that $s_{AB\alpha\beta} = s_{AB\alpha}\delta_{\alpha\beta}$. Due to current conservation, the scattering matrix defined by Eq. (14) is unitary.

The charge and spin currents $I_{c,A}(t)$ and $\mathbf{I}_{s,A}(t)$ are given by the current operator: $I_{c,A}(t) = \sum_\alpha \hat{I}_A^{\alpha\alpha}(t)$ and $\mathbf{I}_{s,A}(t) = -\hbar/(2e)\sum_{\alpha\beta}\hat{\boldsymbol{\sigma}}^{\alpha\beta}\hat{I}_A^{\beta\alpha}(t)$, where $\hat{\boldsymbol{\sigma}} = (\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z)$ is the vector of Pauli matrices. The average charge and spin currents are evaluated using the quantum statistical average $\langle a^\dagger_{Am\alpha}(E)a_{Bn\beta}(E')\rangle = \delta_{AB}\delta_{mn}\delta_{\alpha\beta}\delta(E-E')f_A(E)$ of the product of one creation and one annihilation operator. Here m and n denote transverse modes and $f_A = f(E-\mu_A)$ is the Fermi-Dirac distribution function in normal metal A, with μ_A the chemical potential. The creation and annihilation operators furthermore obey the anticommutation relation

$$\{a^\dagger_{Am\alpha}(E), a_{Bn\beta}(E')\} = \delta_{AB}\delta_{mn}\delta_{\alpha\beta}\delta(E-E'). \qquad (15)$$

The anticommutator of two creation or two annihilation operators vanish. Similar relations hold for the b- operators. From these relations it follows that

$$\begin{aligned} &\langle a^\dagger_{Ak\alpha}(E_1)a_{Bl\beta}(E_2)a^\dagger_{Cm\gamma}(E_3)a_{Dn\delta}(E_4)\rangle \\ &-\langle a^\dagger_{Ak\alpha}(E_1)a_{Bl\beta}(E_2)\rangle\langle a^\dagger_{Cm\gamma}(E_3)a_{Dn\delta}(E_4)\rangle \\ &\qquad = \delta_{AD}\delta_{BC}\delta_{kn}\delta_{lm}\delta_{\alpha\delta}\delta_{\beta\gamma} \\ &\times\delta(E_1-E_4)\delta(E_2-E_3)f_A(E_1)[1-f_B(E_2)], \end{aligned} \qquad (16)$$

where the subscripts A, B, C, D denote leads, k, l, m, n denote transverse channels, and $\alpha, \beta, \gamma, \delta$ denote spin. This expression will be needed in the calcuation of the current fluctuations. We will also use the identity

$$\sum_{CD} \mathrm{Tr}(s^\dagger_{AC\alpha}s_{AD\beta}s^\dagger_{BD\beta}s_{BC\alpha}) = \delta_{AB}M_A \qquad (17)$$

which follows from the unitarity of the scattering matrix. Here the trace is over the space of the transverse modes, and M_A is the number of transverse modes in lead A.

The charge and spin current fluctuations are given by the correlators

$$S_{c,AB}(t-t') = \langle \delta I_{c,A}(t)\delta I_{c,B}(t')\rangle \qquad (18)$$

and

$$S_{ij,AB}(t-t') = \langle \delta I_{s_i,A}(t)\delta I_{s_j,B}(t')\rangle, \qquad (19)$$

respectively. Here $\delta I_{c,A}(t) = I_{c,A}(t) - \langle I_{c,A}(t)\rangle$ is the deviation of the charge current from its average value in lead A at time t, and $\delta I_{s_i,A}(t)$ is the deviation of the vector component i ($i = x, y$ or z) of the spin current. We use a coordinate system for the spin current in which the z-axis is along the magnetization direction, which defines the spin quantization axis. Two fundamental types of current noise will be considered in the following: Thermal noise and shot noise. In general, the total noise is not a simple superposition of pure thermal noise and pure shot noise. Still, it is convenient to treat the two noise sources independently, as is done in the following. We will consider low-frequency noise.

B. Thermal current noise

At equilibrium $f_L = f_R = f$, and the average current is zero. The fluctuations are however finite, due to the non-zero temperature. Thermal current noise manifests itself as fluctuations in the occupation numbers of the electron channels incident on the sample. Using Eqs. (12), (13), (16) and (17) we evaluate the zero frequency noise $S^{(\mathrm{th})}_{c,AA}(\omega=0) = \int d(t-t')S^{(\mathrm{th})}_{c,AA}(t-t')$ and find

$$S^{(\mathrm{th})}_{c,AA}(\omega=0) = \frac{2e^2}{h}k_BT(g^\uparrow + g^\downarrow), \qquad (20)$$

where $g^\alpha = \mathrm{Tr}(1 - r^\dagger_\alpha r_\alpha)$ is the spin-dependent dimensionless conductance of the ferromagnet, to be evaluated at the Fermi energy. The superscript (th) emphasizes that the fluctuations are due to thermal agitation, and the trace is over the space of the transverse modes. To arrive at this result we made use of the relation $f(1-f) = k_BT(-\partial f/\partial E)$. The result for $S^{(\mathrm{th})}_{c,AB}(\omega=0)$, where $B \neq A$, differs from the above expression only by a minus sign, since positive current direction is defined towards the ferromagnet on both sides, and charge current is conserved. Eq. (20) is the well-known expression for Johnson-Nyquist noise, relating the equilibrium current noise to the out-of-equilibrium dissipation of energy, in accordance with the FDT.

The thermal spin current noise follows in a similar way. The zero frequency noise becomes

$$\begin{aligned} S^{(\mathrm{th})}_{ij,AB}(0) \;=\; & \frac{\hbar k_BT}{8\pi}\sum_{\alpha\beta}\sigma_i^{\alpha\beta}\sigma_j^{\beta\alpha}\mathrm{Tr}[2\delta_{AB} \\ & -s^\dagger_{BA\alpha}s_{BA\beta} - s^\dagger_{AB\beta}s_{AB\alpha}], \end{aligned} \qquad (21)$$

where the scattering matrices should be evaluated at the Fermi energy. From this we get e.g.

$$S^{(\mathrm{th})}_{zz,AA} = \frac{\hbar}{4\pi}k_BT(g^\uparrow + g^\downarrow) \qquad (22)$$

for the correlator of the longitudinal (polarized parallel with the magnetization) spin current components. Similarly,

$$S^{(\mathrm{th})}_{xx,AA} = S^{(\mathrm{th})}_{yy,AA} = \frac{\hbar}{4\pi}k_BT(g_A^{\uparrow\downarrow} + g_A^{\downarrow\uparrow}) \qquad (23)$$

for the correlators of the transverse (polarized perpendicular to the magnetization) spin current components. The transverse (or "mixing") conductance $g_A^{\uparrow\downarrow}$ describes electrons with spin polarization perpendicular to the magnetization propagating towards the interface on side A of the ferromagnet. We have $g_L^{\uparrow\downarrow} = \mathrm{Tr}[1 - r_\uparrow (r_\downarrow)^\dagger]$ and $g_L^{\downarrow\uparrow} = (g_L^{\uparrow\downarrow})^*$, and $g_R^{\uparrow\downarrow} = \mathrm{Tr}[1 - r'_\uparrow (r'_\downarrow)^\dagger]$ and $g_R^{\downarrow\uparrow} = (g_R^{\uparrow\downarrow})^*$. We see that just like the charge current noise, Eq. (20), the spin current noise obeys the FDT. The spin current correlators are proportional to the conductances for the respective spin current components. Note that $S^{(\mathrm{th})}_{zz,AA}$ only differs from $S^{(\mathrm{th})}_{c,AA}$ by the factor $\hbar^2/(2e)^2$, which is the square of the conversion factor from charge to spin currents.

For the cross correlations, we find e.g. $S^{(\mathrm{th})}_{zz,LR} = -S^{(\mathrm{th})}_{zz,LL}$, reflecting that the component of the spin current polarized parallel to the magnetization is conserved in the ferromagnet. On the other hand, $S^{(\mathrm{th})}_{xx,LR} = S^{(\mathrm{th})}_{yy,LR} = 0$, since the spin current components perpendicular to the magnetization are absorbed over the length scale of the ferromagnetic coherence length.

C. Shot noise

Shot noise, as opposed to thermal noise, is an out-of-equilibrium phenomenon. It is present when a voltage is applied across the sample (i.e., $\mu_L \neq \mu_R$), and even at zero temperature. Shot noise is due to the discreteness of the electron charge, and the probabilistic scattering of electrons as they are incident on the sample. To evaluate the shot noise, let $\mu_L - \mu_R = eV$ with V the applied voltage, and let the temperature be zero, so that there is no thermal noise. We are here only concerned with the current fluctuations, although in this case also the average charge current is non-zero. The average spin current accompanying the average charge current does not exert a torque on a single ferromagnet, since the spin current is polarized along the direction of magnetization. Using again Eqs. (12), (13), and (16) we find the well-known result[3]

$$S^{(\mathrm{sh})}_{c,AA}(\omega = 0) = \frac{e^3}{h}|V|[\mathrm{Tr}(r_\uparrow^\dagger r_\uparrow t_\uparrow^\dagger t_\uparrow) + \mathrm{Tr}(r_\downarrow^\dagger r_\downarrow t_\downarrow^\dagger t_\downarrow)] \quad (24)$$

for the correlator of the charge shot noise. Here the scattering matrices should be evaluated at the Fermi energy, and the superscript (sh) emphasizes that we are considering shot noise. We have used the relations $f_A(1-f_A) = 0$ and $\int dE (f_L - f_R)^2 = e|V|$. The result for $S^{(\mathrm{sh})}_{c,AB}(\omega = 0)$, where $B \neq A$, differs from Eq. (24) by a minus sign. Similarly, we find

$$\begin{aligned} S^{(\mathrm{sh})}_{ij,AB}(\omega = 0) &= \frac{\hbar}{8\pi} \sum_{\alpha\beta} \hat{\sigma}_i^{\alpha\beta} \hat{\sigma}_j^{\beta\alpha} \int dE \sum_{CD} f_C(1 - f_D) \\ &\times \mathrm{Tr}[s^\dagger_{AC\alpha} s_{AD\beta} s^\dagger_{BD\beta} s_{BC\alpha}] \end{aligned} \quad (25)$$

for the spin current shot noise. As for the thermal noise we find from this expression $S^{(\mathrm{sh})}_{zz,LR} = -S^{(\mathrm{sh})}_{zz,LL}$ and $S^{(\mathrm{th})}_{xx,LR} = S^{(\mathrm{th})}_{yy,LR} = 0$, reflecting that the component of the spin current polarized parallel to the magnetization is conserved in the ferromagnet, while the spin current components perpendicular to the magnetization are absorbed.

D. Magnetization noise and damping

The absorption of transverse thermal spin current noise and spin shot noise in the ferromagnet implies a fluctuating spin-transfer torque on the magnetization. The resulting increment of the magnetization noise is calculated using Eq. (4), which by conservation of angular momentum is modified by the spin torque $-\gamma_0 \mathbf{I}_{s,\mathrm{abs}}/(M_s V)$ in the presence of spin currents. Here $\mathbf{I}_{s,\mathrm{abs}} = \mathbf{I}_{s,L} + \mathbf{I}_{s,R}$ is the (instantaneously) absorbed spin current. (Recall that on both sides of the ferromagnet, positive current direction is defined towards the magnet.) Since $\mathbf{I}_{s,\mathrm{abs}}$ is perpendicular to $\mathbf{m}$, we may write $\mathbf{I}_{s,\mathrm{abs}} = \mathbf{m} \times [\mathbf{m} \times \mathbf{I}_{s,\mathrm{abs}}]$, such that the modified stochastic LLG equation reads

$$\begin{aligned} \frac{d\mathbf{m}}{dt} &= -\gamma_0 \mathbf{m} \times [\mathbf{H}_{\mathrm{eff}} + \mathbf{h}^{(0)}(t)] + \alpha_0 \mathbf{m} \times \frac{d\mathbf{m}}{dt} \\ &+ \frac{\gamma_0}{M_s V} \mathbf{m} \times [\mathbf{m} \times \mathbf{I}_{s,\mathrm{abs}}]. \end{aligned} \quad (26)$$

In the present case, the average absorbed spin current $\langle \mathbf{I}_{s,\mathrm{abs}} \rangle$ is zero, but the fluctuations $\delta \mathbf{I}_{s,\mathrm{abs}}(t)$ are finite. We can thus define $\mathbf{h}(t) = -1/(M_s V) \mathbf{m} \times \delta \mathbf{I}_{s,\mathrm{abs}}(t)$ as a stochastic "magnetic" field describing the spin current (thermal or shot) noise, analogous to the intrinsic noise field $\mathbf{h}^{(0)}(t)$. It follows that

$$\langle h_i(t) h_i(t') \rangle = \frac{1}{M_s^2 V^2} \sum_{AB} S_{jj,AB}(t - t') \quad (27)$$

and

$$\langle h_i(t) h_j(t') \rangle = -\frac{1}{M_s^2 V^2} \sum_{AB} S_{ji,AB}(t - t') \quad (28)$$

for $i, j = x, y$; $i \neq j$. The component of the field parallel to the magnetization is of no interest, since it has no effect on the magnetization. Assuming that the frequency of the current noise is low compared to all relevant energy scales (the temperature, applied voltage, and exchange splitting), we can approximate $S_{ij,AB}(t - t') \approx S_{ij,AB}(\omega = 0)\delta(t - t')$. Using Eq. (21) we then find the already advertised result

$$\langle h_i^{(\mathrm{th})}(t) h_j^{(\mathrm{th})}(t') \rangle = 2k_B T \frac{\alpha'}{\gamma_0 M_s V} \delta_{ij} \delta(t - t') \quad (29)$$

for the thermally induced spin current stochastic field. Here i may equal j, and $\alpha' = \gamma_0 \hbar \mathrm{Re}(g_L^{\uparrow\downarrow} + g_R^{\uparrow\downarrow})/(4\pi M_s V)$

is the spin-pumping enhancement of the Gilbert damping. From Eq. (3) we see that this result is in agreement with the FDT, with the Gilbert damping given by $\alpha = \alpha_0 + \alpha'$.

For the stochastic field describing the shot noise, we similarly find

$$\langle h_i^{(\mathrm{sh})}(t) h_j^{(\mathrm{sh})}(t')\rangle = \frac{\hbar}{4\pi}\frac{e|V|}{M_s^2 V^2}\delta_{ij}\delta(t-t')[\mathrm{Tr}(r_\uparrow r_\uparrow^\dagger t'_\downarrow t'^\dagger_\downarrow) \mathrm{Tr}(r'_\downarrow r'^\dagger_\downarrow t_\uparrow t_\uparrow^\dagger)] \quad (30)$$

using Eq. (25) and the unitarity of the scattering matrix. For a simple model for the scattering coefficients, it has been shown that the shot noise-induced stochastic field may dominate the thermally induced one for typical experimental voltage drops in nanoscale spin valves at low temperatures[11]. In the following we shall consider room temperature, allowing us to neglect shot noise.

IV. SPIN VALVE

We now proceed to consider the noise properties of spin valve nanopillars, i.e., two ferromagnets F_1 and F_2 with respective unit magnetization vectors $\mathbf{m}_1$ and $\mathbf{m}_2$ separated by a thin normal metal spacer N, as shown in Fig. 2. We first assume that F_2 is pinned, and hence disregard any fluctuations of this ferromagnet. Later, we will consider fluctuations also of this ferromagnet. In experiments on spin valves, it is common to have one of the ferromagnets pinned. The pinning can be achieved by e.g. antiferromagnetically coupling the ferromagnet to a third magnet, or by making it much bigger than the other.

The magnetization noise in F_1 is given by intrinsic noise plus noise due to thermally fluctuating spin currents. The intrinsic noise is the same as for solitary ferromagnets, as considered in the previous section, and as given by Eq. (6). The spin current induced noise on the other hand, is affected by the presence of the second ferromagnet, and so differs from that calculated in the preceeding section. To evaluate the spin current noise, we use magnetoelectronic circuit theory[33–35]. Requiring charge and spin conservation in the middle normal metal, the circuit theory enables us to consistently determine the current fluctuations in the system.

Due to giant magnetoresistance (GMR), the fluctuations of $\mathbf{m}_1$ cause resistance noise. While magnetization noise is difficult to measure directly, it may be indirectly measured through resistance noise. Resistance noise in spin valves is hence an interesting quantity to consider, that offers an experimental check of our results. Resistance noise is also interesting from a fundamental viewpoint, and may be important for the sensitivity of GMR read heads for magnetic storage.

This section is organized as follows. First the magnetoelectronic circuit theory is introduced and used to calculate the current noise in spin valves. Then the spin current noise is translated into a stochastic field that acts on F_1, following the recipe in the Sec. III D. The related Gilbert damping is found with the aid of the FDT for both the parallel(P), antiparallel(AP) and perpendicular(90°) magnetic configurations. Using the LLG equation, we next calculate the explicit fluctuations of the magnetization vector induced by the stochastic field, and the resulting resistance noise. We end this section by considering spin valves in which the ferromagnets are identical and hence equally susceptible to fluctuations.

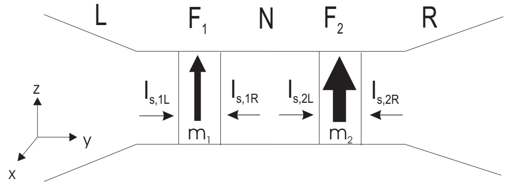

FIG. 2: A spinvalve with two ferromagnets F_1 and F_2 with unit magnetization vectors $\mathbf{m}_1$ and $\mathbf{m}_2$, here shown in the parallel (P) configuration $\mathbf{m}_1 = \mathbf{m}_2 = \hat{\mathbf{z}}$. The magnetization of F_2 is pinned. The currents in the system are evaluated close to the interfaces, with positive directions defined in the figure, using magnetoelectronic circuit theory.

A. Circuit theory

Magnetoelectronic circuit theory was developed by Brataas, Bauer and Nazarov[33–35] as a tool to determine transport properties of magnetoelectronic multilayers, such as the spin valve shown in Fig. 2. The theory is a semiclassical formulation of the spin-resolved LB scattering formalism explained in Sec III A. The idea of the theory is to divide the system at study into resistive elements (scatterers), nodes (low resistance interconnectors), and reservoirs (voltage sources). The nodes and reservoirs are characterized by distribution functions, and the current between them is calculated quantum mechanically using LB scattering theory. We view the ferromagnets as scatterers, the sandwiched normal metal layer as a node, and the outer normal metals L (left) and R (right) as large reservoirs. The reservoirs are in thermal equilibrium, and hence characterized by Fermi-Dirac distribution functions $f_L = f(E-\mu_L)$ and $f_R = f(E-\mu_R)$, where μ_L and μ_R are the respective chemical potentials. Depending on the relative orientation of the magnetization vectors $\mathbf{m}_1$ and $\mathbf{m}_2$, there can be a non-equilibrium accumulation of spins on the normal metal node. The node is hence in general characterized by both a non-zero scalar (charge) distribution function f_{cN}, and a non-zero vector spin distribution function $\mathbf{f}_{sN}$. For convenience, f_{cN} and $\mathbf{f}_{sN}$ may be combined into a distribution matrix $\hat{f}_N = \hat{1} f_{cN} + \boldsymbol{\sigma}\cdot\mathbf{f}_{sN}$ in 2×2 spin space, where $\hat{1}$ is the unit matrix and $\boldsymbol{\sigma}$ is the vector of the Pauli matrices.

As in the previous sections, the ferromagnets are assumed thicker than the magnetic coherence length, and spin-flip processes are disregarded in both the ferromagnets and the middle normal metal node. The node is furthermore assumed to be sufficiently chaotic, so that $\hat{f}_N$ is isotropic and constant in space. As shown in Fig. 2, currents are evaluated close to the F|N-interfaces.

The average charge and spin currents follow from Eq. (12). The important ansatz of magnetoelectronic circuit theory is the quantum statistical average $\langle a^\dagger_{Am\alpha}(E)a_{Bn\beta}(E')\rangle = \delta_{AB}\delta_{mn}\delta(E-E')f_A^{\beta\alpha}(E)$. Here $a_{Bn\beta}$ is the annihilation operator for electrons propagating from normal metal A ($A = L, R$ or N) towards one of the ferromagnets, and $f_A^{\beta\alpha}$ is the $\beta\alpha$-component of the 2×2 semiclassical distribution matrix $\hat{f}_A$ in spin space on normal metal A. For the reservoirs ($A = L$ or R), we simply have $f_A^{\beta\alpha} = \delta_{\beta\alpha}f(E-\mu_A)$. On e.g. the right side of ferromagnet F_1, the average charge current is[34,35]

$$\begin{aligned}\langle I_{c,1R}\rangle &= \frac{e}{h}\int dE\{g_1^\uparrow(f_{cN}+\mathbf{f}_{sN}\cdot\mathbf{m}_1-f_L)\\ &\quad +g_1^\downarrow(f_{cN}-\mathbf{f}_{sN}\cdot\mathbf{m}_1-f_L)\}\end{aligned}\tag{31}$$

and the average spin current

$$\begin{aligned}\langle \mathbf{I}_{s,1R}\rangle &= \frac{1}{4\pi}\int dE\{\mathbf{m}_1[g_1^\uparrow(f_{cN}+\mathbf{f}_{sN}\cdot\mathbf{m}_1-f_L)\\ &\quad -g_1^\downarrow(f_{cN}-\mathbf{f}_{sN}\cdot\mathbf{m}_1-f_L)]\\ &\quad +2\mathrm{Re}g_{1R}^{\uparrow\downarrow}\mathbf{m}_1\times(\mathbf{f}_{sN}\times\mathbf{m}_1)\\ &\quad +2\mathrm{Im}g_{1R}^{\uparrow\downarrow}\mathbf{f}_{sN}\times\mathbf{m}_1\}.\end{aligned}\tag{32}$$

Here g_1^α is the spin-dependent dimensionless conductance of F_1, and $g_{1R}^{\uparrow\downarrow}$ is the mixing conductance of the interface between F_1 and the middle normal metal. The average charge and spin currents on the other side of F_1 is easily deduced by recalling that the charge current and the component of the spin current polarized along the magnetization is conserved through the ferromagnet, while the transverse spin current is absorbed. Hence $\langle I_{c,1L}\rangle = -\langle I_{c,1R}\rangle$ and

$$\begin{aligned}\langle \mathbf{I}_{s,1L}\rangle &= \frac{1}{4\pi}\int dE\{\mathbf{m}_1[g_1^\uparrow(f_L-f_{cN}-\mathbf{f}_{sN}\cdot\mathbf{m}_1)\\ &\quad -g_1^\downarrow(f_L-f_{cN}+\mathbf{f}_{sN}\cdot\mathbf{m}_1)].\end{aligned}\tag{33}$$

Similar expressions hold for the currents evaluated on the left and right sides of F_2. For simplicity we shall in the following take the ferromagnets to have identical conductance parameters.

Since spin-flip processes are disregarded, both charge and spin are conserved on the middle normal metal node:

$$\langle I_{c,1R}\rangle + \langle I_{c,2L}\rangle = 0 \tag{34}$$

$$\langle \mathbf{I}_{s,1R}\rangle + \langle \mathbf{I}_{s,2L}\rangle = 0 \tag{35}$$

Eqs. (31)-(35) give four equations for the four unknown components of the distribution matrix $\hat{f}_N$ on the middle normal metal node. Solving this set of equations and then using again Eq. (31), yields $\langle I_{c,1L}\rangle = -\langle I_{c,1R}\rangle = \langle I_{c,2L}\rangle = -\langle I_{c,2R}\rangle \equiv I_c = G_vV$, where $V = (\mu_L-\mu_R)/e$ is the applied voltage, and[35]

$$G_v = \frac{e^2 g}{2h}\left(1-P^2\frac{1-\cos\theta}{1-\cos\theta+\eta+\eta\cos\theta}\right) \tag{36}$$

is the conductance of the spin valve, that depends on the angle $\theta = \arccos(\mathbf{m}_1\cdot\mathbf{m}_2)$. We have defined $g = g^\uparrow + g^\downarrow$ as the total conductance of each of the ferromagnets, $P = (g^\uparrow - g^\downarrow)/g$ as the polarization, and $\eta = 2g^{\uparrow\downarrow}/g$ as the relative mixing conductance.

B. Current noise

We consider low-frequency current noise. In this limit, charge and spin are instantaneously conserved on the normal metal node. This implies that also the charge and spin current fluctuations are conserved:

$$\Delta\hat{I}_{1R}(t) + \Delta\hat{I}_{2L}(t) = 0, \tag{37}$$

where we have written the current fluctuations as a 2×2 matrix in spin space. $\Delta\hat{I}_{1R}(t) = \hat{1}\Delta I_{c,1R}(t) - 2e/\hbar\boldsymbol{\sigma}\cdot\Delta\mathbf{I}_{s,1R}(t)$ denotes the fluctuations in the lead to the right of F_1, with $\Delta I_{c,1R}(t)$ the charge current fluctuations and $\Delta\mathbf{I}_{s,1R}(t)$ the spin current fluctuations. As a result of this conservation law, the distribution matrix $\hat{f}_N$ on the node must be fluctuating. The current fluctuations can hence be written

$$\Delta\hat{I}_{1R(2L)}(t) = \delta\hat{I}_{1R(2L)}(t) + \frac{\partial\langle\hat{I}_{1R(2L)}\rangle}{\partial\hat{f}_N}\delta\hat{f}_N(t), \tag{38}$$

where $\delta\hat{f}_N(t)$ are the fluctuations of the distribution matrix. $\delta\hat{I}_{1R(2L)}(t)$ are the intrinsic fluctuations when $\delta\hat{f}_N(t) = 0$, coinciding with the fluctuations calculated for single ferromagnets in the previous section. Expression (38) applies also to the current fluctuations evaluated on the left side of ferromagnet F_1 and the right side of ferromagnet F_2. In the following, we consider thermal current noise, recalling from Sec. III D that for typical voltage drops in spin valves, shot noise is only important at low temperatures. We shall for simplicity limit our discussion to the cases where the magnetizations are oriented either parallel, antiparallel or perpendicular. Experimentally, these are the most relevant configurations.

Using Eqs. (31), (32), (37) and (38) together with the results in Sec. III, we can evaluate the charge and spin current fluctuations in the spin valve. The correlator of the charge current fluctuations is given by the conductance (36):

$$\int d(t-t')\langle\Delta I_c(t)\Delta I_c(t')\rangle = 2k_BTG_v(\theta). \tag{39}$$

In the low-frequency regime considered here, charge current fluctuations are conserved in the ferromagnets as well as the normal metal node. There is hence no need to specify where in the spin valve the fluctuations in the above correlator are evaluated. The result holds for the parallel, antiparallel, and perpendicular configurations, as well as any other configuration, as dictated by the FDT. G_v can easily be a factor of two larger in the parallel than in the antiparallel configuration, hence causing a substantial difference in the noise levels. Later we will see that via resistance noise, magnetization fluctuations are an additional source of electric noise in spin valves.

The spin current correlator $\langle \Delta I_{s_i,A}(t)\Delta I_{s_j,B}(t')\rangle$, where i and j denote Cartesian components and $A(B) = 1L, 1R, 2L$ or $2R$, is found in a similar manner. Although the spin current fluctuations are conserved in the middle normal metal node, they are not in the ferromagnets, since the ferromagnets absorb transverse spin angular momentum. Consequently, the spin current correlator depends on where in the spin valve it is evaluated. Rather than stating the explicit spin current correlators, we shall proceed to evaluate the magnetization fluctuations caused by the spin current noise.

C. Magnetization noise and damping

The current-induced stochastic field acting on F_1 is found with the recipe given in Sec. III D. In the following we give results valid in the parallel, antiparallel or perpendicular configurations. By using the FDT, we also find the related Gilbert damping in all three cases.

1. Parallel configuration

Let us start with the parallell magnetic configuration, $\mathbf{m}_1 \cdot \mathbf{m}_2 = 1$. We find

$$\langle h_i^{(\mathrm{th})}(t)h_j^{(\mathrm{th})}(t')\rangle_P = 2k_BT\frac{\alpha_{sv}}{\gamma_0 M_s V}\delta_{ij}\delta(t-t') \qquad (40)$$

for the thermal spin current-induced stochastic magnetic field in ferromagnet F_1 in this case. Here i, j label vector components perpendicular to the magnetization,

$$\alpha_{sv} = \frac{3\gamma_0\hbar \mathrm{Re} g^{\uparrow\downarrow}}{8\pi M_s V}, \qquad (41)$$

and the mixing conductance $g^{\uparrow\downarrow}$ has been taken to be the same for all four F|N-interfaces. In arriving at this result, we have made explicit use of the assumption that the middle normal metal node is chaotic, which explicitly means that $\langle \delta I_{s_i,1R}(t)\delta I_{s_j,2L}(t')\rangle = 0$, i.e. there are no correlations across the node. α_{sv} can be shown by the FDT to be the spin-pumping enhancement of the Gilbert damping corresponding to the increased noise. This is done by following the steps outlined for a single ferromagnet, i.e., Eqs. (3)-(5). Note that for a ferromagnetic film in a spin valve, the effective field $\mathbf{H}_{\mathrm{eff}}$ in the LLG equation includes contributions due to exchange coupling to the second film. This has however no effect on the stochastic field and Gilbert damping.

The above result coincides with the result (29) for the single ferromagnet sandwiched by normal metals, with the exception of a factor 3/4. The spin current-induced enhancement of the magnetization noise and damping is thus three quarters of what it is for a single ferromagnet. We note that the enhancement of the damping may be found also in a more direct way: Using Eqs. (7) and (32) we can compute the net spin angular momentum leaving each of the ferromagnets when the magnetizations are slightly out of equilibrium, and by conservation of angular momentum infer the corresponding enhancement of the Gilbert damping constant. This yields the result (41) as it should. The 3/4-suppression as compared to single ferromagnets is due to the inability of the middle normal metal node to absorb spin angular momentum. Since the node is chaotic, it divides any incoming flow of spin angular momentum in two, transmitting one half and reflecting the other.

2. Antiparallel configuration

For the antiparallel configuration ($\mathbf{m}_1 \cdot \mathbf{m}_2 = -1$), we find in the same way

$$\langle h_i^{(\mathrm{th})}(t)h_j^{(\mathrm{th})}(t')\rangle_{AP} = \langle h_i^{(\mathrm{th})}(t)h_j^{(\mathrm{th})}(t')\rangle_P, \qquad (42)$$

i.e., the current-induced noise and damping is the same in the AP and P configurations. In arriving at this result we have neglected the small imaginary part of the mixing conductance. Including it would give a small correction to the damping parameter, as well as a renormalization of the gyromagnetic ratio. The noise and damping would then no longer be identical in the P and AP configurations.

3. Perpendicular configuration

We assume for this configuration that F_2 by some means has been pinned along the x-direction, and that the equilibrium direction of $\mathbf{m}_1$ is the z-axis. We then find

$$\langle h_x^{(\mathrm{th})}(t)h_x^{(\mathrm{th})}(t')\rangle_{\mathrm{Perp}} = 2k_BT\frac{\alpha'_{xx}}{\gamma_0 M_s V}\delta(t-t'), \qquad (43)$$

$$\langle h_y^{(\mathrm{th})}(t)h_y^{(\mathrm{th})}(t')\rangle_{\mathrm{Perp}} = 2k_BT\frac{\alpha'_{yy}}{\gamma_0 M_s V}\delta(t-t'), \qquad (44)$$

where

$$\alpha'_{xx} = \frac{3\gamma_0\hbar \mathrm{Re} g^{\uparrow\downarrow}}{8\pi M_s V}$$

$$\alpha'_{yy} = \frac{\gamma_0\hbar \mathrm{Re} g^{\uparrow\downarrow}}{4\pi M_s V}\left[2 - \frac{\eta(2-P^2+2\eta)}{2(1+\eta)(1-P^2+\eta)}\right] \qquad (45)$$

by the FDT are the current-induced enhancement of the Gilbert damping. The cross correlators

$\langle h_x^{(\mathrm{th})}(t) h_y^{(\mathrm{th})}(t')\rangle_{\mathrm{Perp}} = \langle h_y^{(\mathrm{th})}(t) h_x^{(\mathrm{th})}(t')\rangle_{\mathrm{Perp}} = 0$. The enhancement of noise and damping in perpendicular spin valves is hence anisotropic. To accomodate this, the damping term in the LLG equation for F_1 must be modified to $\mathbf{m}_1 \times \overleftrightarrow{\alpha} d\mathbf{m}_1/dt$, where the Gilbert damping is a tensor (in the plane perpendicular to the magnetization):

$$\overleftrightarrow{\alpha} = \begin{pmatrix} \alpha_0 + \alpha'_{xx} & 0 \\ 0 & \alpha_0 + \alpha'_{yy} \end{pmatrix}. \tag{46}$$

Note that the damping tensor must be written inside the cross product in the damping term to ensure that the LLG equation preserves the length of the unit magnetization vector.

D. Resistance noise

The intrinsic stochastic field $\mathbf{h}^{(0)}$ plus the above calculated current-induced stochastic field $\mathbf{h}^{(\mathrm{th})}$ describe the total magnetization noise in F_1. The explicit fluctuations of the magnetization vector can be calculated using the LLG equation, with these stochastic fields as input quantities. Magnetization fluctuations in turn cause resistance noise, due to the GMR-effect. GMR is the dependence of the resistance on the relative angle of the magnetizations of the ferromagnetic films, i.e., the dot product $\mathbf{m}_1 \cdot \mathbf{m}_2$. Resistance noise may be an important factor for the sensitivity of GMR read heads[37]. Covington *et al.*[36] identified GMR-induced resistance noise as an important source of electric noise in current-perpendicular-to-the-plane (CPP) spin valves. CPP spin valves are considered an alternative for the current-in-the-plane spin valves currently used in GMR read heads. Resistance noise is interesting also from a fundamental viewpoint. We focus on the zero-frequency resistance noise

$$S_R(\omega = 0) = \int d(t-t')\langle \Delta R(t)\Delta R(t')\rangle, \tag{47}$$

where $\Delta R(t)$ are the random fluctuations of the resistance at time t.

Resistance noise can be measured indirectly, via voltage or current noise. It is an out-of-equilibrium noise source, that requires a non-zero voltage or current bias. Assuming constant voltage bias, the resistance noise translates into current noise. The current noise can then be compared with the Johnson-Nyquist noise discussed in Sec IV B. It will be shown that at high current densities, as e.g. those employed in experiments on current-induced magnetization dynamics in spin valves, the magnetization-induced noise can be the dominant contribution to the electric noise. The high current densities considered are not so high that shot noise dominates over Johnson-Nyquist noise, consistent with our assumption that shot noise may be neglected.

In the following, the resistance noise in the parallel, antiparallel and perpendicular configurations are derived. Recall that the magnetization in ferromagnet F_2 is assumed pinned. The analysis of resistance noise in the case of two fluctuating magnetizations is left for the next section.

1. *Parallel configuration*

The total stochastic field in F_1 cause fluctuations $\delta\mathbf{m}_1(t) = \mathbf{m}_1(t) - \langle\mathbf{m}_1\rangle$ of the magnetization from its time-averaged equilibrium value. For the parallel configuration $\langle\mathbf{m}_1\rangle = \mathbf{m}_2$, such that the dot product of the magnetizations is $\mathbf{m}_1\cdot\mathbf{m}_2 = 1 - \frac{1}{2}\delta\mathbf{m}_1^2$. For small fluctuations we can expand the resistance to first order in $\delta\mathbf{m}_1^2$, giving

$$R(\mathbf{m}_1\cdot\mathbf{m}_2) \approx R(1) - \frac{1}{2}\delta\mathbf{m}_1^2\frac{\partial R(1)}{\partial\cos\theta}, \tag{48}$$

where $\cos\theta = \mathbf{m}_1\cdot\mathbf{m}_2$, with θ the angle between the magnetization directions. The resistance noise correlator then becomes

$$\begin{aligned}\langle\Delta R(t)\Delta R(t')\rangle_P &= \langle R(t)R(t')\rangle_P - \langle R(t)\rangle_P\langle R(t')\rangle_P \\ &= \frac{1}{4}\left(\frac{\partial R(1)}{\partial\cos\theta}\right)^2 [\langle\delta\mathbf{m}_1^2(t)\delta\mathbf{m}_1^2(t')\rangle_P \\ &\quad -\langle\delta\mathbf{m}_1^2(t)\rangle_P\langle\delta\mathbf{m}_1^2(t')\rangle_P], \end{aligned}\tag{49}$$

where the brackets denote statistical averaging at the equilibrium P configuration. Assuming that the fluctuations of the magnetization vectors are gaussian distributed, we can employ Wick's theorem[42], which states that fourth order moments of the fluctuations can be evaluated in terms of the sum of products of second order moments. We then arrive at

$$\begin{aligned}\langle\Delta R(t)\Delta R(t')\rangle_P &= \frac{1}{2}\left(\frac{\partial R(1)}{\partial\cos\theta}\right)^2 \\ &\quad\times\sum_{ij}\langle\delta m_{1,i}(t)\delta m_{1,j}(t')\rangle_P^2, \end{aligned}\tag{50}$$

where i and j denote Cartesian components. Since the magnetization fluctuations are small, we may disregard their longitudinal component. The summation can hence be limited to the Cartesian components transverse to the average magnetization. It remains to determine the correlator of these fluctuations.

The derivative appearing in the above expression, i.e., the GMR sensitivity, is determined by magnetoelectronic circuit theory. From Eq. (36) we find

$$\frac{\partial R(1)}{\partial\cos\theta} = -\frac{hP^2}{e^2 g\eta}, \tag{51}$$

which is to be inserted in Eq. (50).

The correlator of the magnetization fluctuations in Eq. (50) is found with the LLG equation. To this end, we define a coordinate system as shown in Fig. 2 such that the ferromagnetic films are in the xz-plane. The LLG equation reads

$$\frac{d\mathbf{m}_1}{dt} = -\gamma_0 \mathbf{m}_1 \times [\mathbf{H}_{\text{eff}} + \mathbf{h}(t)] + (\alpha_0 + \alpha_{sv})\mathbf{m}_1 \times \frac{d\mathbf{m}_1}{dt}, \tag{52}$$

where the effective field $\mathbf{H}_{\text{eff}}$ includes internal anisotropy and demagnetizing fields, any externally applied fields, and in addition dipolar and exchange coupling to F_2. The total stochastic field $\mathbf{h}(t) = \mathbf{h}^{(0)}(t) + \mathbf{h}^{(\text{th})}(t)$ includes both the intrinsic field $\mathbf{h}^{(0)}(t)$ and the spin-current induced field $\mathbf{h}^{(\text{th})}(t)$. $\mathbf{h}^{(0)}(t)$ is the same as that for a single ferromagnet (see section III) and independent of the magnetic configuration of the spin valve, while $\mathbf{h}^{(\text{th})}(t)$ was calculated in the previous section. α_0 is the intrinsic Gilbert damping constant related to $\mathbf{h}^{(0)}(t)$ by the FDT, and α_{sv} is the spin-pumping enhancement of the damping related to $\mathbf{h}^{(\text{th})}(t)$. The effective field can in general be written $\mathbf{H}_{\text{eff}} = \mathbf{H}_0 + \mathbf{H}_a + \mathbf{H}_d + \mathbf{H}_e$, where $\mathbf{H}_0$ is the external field, $\mathbf{H}_a$ is the in-plane anisotropy field, $\mathbf{H}_d$ is the out-of-plane demagnetizing field, and $\mathbf{H}_e$ represents both the dipolar and exchange fields. The external field and anisotropy field are both taken along the z-axis. We parametrize these fields by writing $\gamma\mathbf{H}_0 = \omega_0\hat{\mathbf{z}}$ and $\gamma\mathbf{H}_a = \omega_a(\mathbf{m}_1 \cdot \hat{\mathbf{z}})\hat{\mathbf{z}}$, defining the quantities ω_0 and ω_a as measures of the strengths of these fields. The demagnetizing field is pointing out-of-plane, i.e. along the y-axis, supressing out-of-plane components of the magnetization. We may therefore set $\gamma\mathbf{H}_d = -\omega_d(\mathbf{m}_1 \cdot \hat{\mathbf{y}})\hat{\mathbf{y}}$, where ω_d gives the strength of this field. The dipolar and exchange couplings may both be described by the Heisenberg coupling energy $-J\mathbf{m}_1 \cdot \mathbf{m}_2$, which favorizes a parallel magnetic configuration if the coupling strength $J > 0$ and an antiparallell if $J < 0$. This translates into the field $\gamma\mathbf{H}_e = \omega_e\mathbf{m}_2$, where $\omega_e = \gamma J/M_s d$.

The P configuration is reached by applying a sufficiently strong external field, forcing $\langle\mathbf{m}_1\rangle$ to align along the direction of the pinned $\mathbf{m}_2$, which we define as the $+z$-direction. Linearizing the LLG equation in the transverse fluctuations $\delta\mathbf{m}(t) \approx \delta m_x(t)\hat{\mathbf{x}} + \delta m_y(t)\hat{\mathbf{y}}$, we find the magnetization noise correlator

$$\langle\delta m_i(t)\delta m_j(t')\rangle_P = \frac{\gamma_0 k_B T\alpha}{\pi M_s V}\int d\omega e^{-i\omega(t-t')}U_{ij}, \tag{53}$$

by using the correlators for the stochastic fields. Here

$$U_{xx} = \frac{[\omega^2 + (\omega_t + \omega_d)^2]}{[\omega^2 - \omega_t(\omega_t + \omega_d)]^2 + \omega^2\alpha^2(2\omega_t + \omega_d)^2}, \tag{54}$$

$$U_{xy} = \frac{-i\omega(2\omega_t + \omega_d)}{[\omega^2 - \omega_t(\omega_t + \omega_d)]^2 + \omega^2\alpha^2(2\omega_t + \omega_d)^2}, \tag{55}$$

$$U_{yy} = \frac{(\omega^2 + \omega_t^2)}{[\omega^2 - \omega_t(\omega_t + \omega_d)]^2 + \omega^2\alpha^2(2\omega_t + \omega_d)^2}, \tag{56}$$

$$U_{yx} = -Z_{xy}, \tag{57}$$

and we have defined $\alpha = \alpha_0 + \alpha_{sv}$ and $\omega_t = \omega_0 + \omega_a + \omega_e$. To arrive at the above expressions we have assumed small damping, i.e., $\alpha_0^2, \alpha_{sv}^2 \ll 1$. For the zero-frequency resistance noise $S_P(0) = \int d(t-t')\langle\Delta R(t)\Delta R(t')\rangle_P$, Eq. (53) inserted into Eq. (50) yields

$$S_P(0) = \frac{1}{\pi}\left(\frac{hP^2}{e^2 g\eta}\right)^2\left(\frac{\gamma_0 k_B T\alpha}{M_s V}\right)^2 \times \int d\omega(U_{xx}^2 + U_{yy}^2 - 2U_{xy}^2). \tag{58}$$

To gain insight into this rather complicated expression, it is convenient to make some simplifications. Although the demagnetizing field, which serves to stabilize the magnetization in the plane of the film, is important to get the right magnitude of the noise, we can gain physical understanding by disregarding it. Setting $\omega_d = 0$, we find

$$S_P(0) = \left(\frac{\gamma_0 k_B T}{M_s V}\right)^2\left(\frac{hP^2}{e^2 g\eta}\right)^2\frac{1}{\omega_t^3\alpha}. \tag{59}$$

Understandably, the resistance noise is greatly influenced by ω_t, the total strength of the external, anisotropy, dipolar and exchange fields. Both the external and anisotropy field stabilizes the magnetization, hence lowering the noise. The dipolar and exchange field either stabilizes or destabilizes the magnetization, depending on the sign of the material specific coupling constant J. We note also that the Gilbert damping plays a significant role for the resistance noise. Even though the FDT tells us that large damping gives large magnetization fluctuations, we see from Eq. (59) that the resistance noise decreases with increasing damping. This is because the damping not only determines the noise, but also the magnetic response to the internal and external fields. The latter effect turns out to limit the resistance noise more than the first effect enhances it. From this observation, we see also the importance of including spin current noise and spin pumping (i.e., the parameter α_{sv}) in the calculation, since α_{sv} can be of the same order as α_0[27].

Resistance noise is measured indirectly, either through current or voltage noise. In our case, the external voltage applied by the reservoirs is assumed to be a constant nonfluctuating quantity. Hence, the resistance noise causes measurable current noise. Converting resistance noise to current noise follows easily from Ohms law. Comparing with the Johnson-Nyquist noise calculated in Sec. IV B, we find that the magnetization-induced noise can dominate at typical current densities applied in experiments on current-induced magnetization dynamics in spin valves. In making this comparison, which depends on many material parameters, it is important to use Eq. (58) and not Eq. (59), since the demagnetizing field has a large effect on the magnitude of the magnetization-induced noise. The magnetization-induced noise is most prominent when the ferromagnets are small, since the ratio of Johnson-Nyquist noise to magnetization-induced noise scales with the volume of the ferromagnets.

2. Antiparallel configuration

Assuming $J < 0$, the anisotropy field and the dipolar and exchange couplings make the AP configuration ($\langle \mathbf{m}_1 \rangle = -\mathbf{m}_2$) the ground state when the external magnetic field is turned off. Following the recipe in the previous subsection, we find that the resistance noise in this case is

$$\langle \Delta R(t) \Delta R(t') \rangle_{AP} = \frac{1}{2} \left(\frac{\partial R(-1)}{\partial \cos\theta} \right)^2 \times \sum_{ij} \langle \delta m_{1,i}(t) \delta m_{1,j}(t') \rangle^2_{AP}, \tag{60}$$

where the GMR sensitivity

$$\frac{\partial R(-1)}{\partial \cos \theta} = -\frac{hP^2\eta}{e^2 g(1-P^2)^2}. \tag{61}$$

Evaluating the magnetization noise correlators from Eq. (52) linearized, the zero frequency resistance noise becomes

$$\begin{aligned} S_{AP}(0) &= \int d(t-t') \langle \Delta R(t) \Delta R(t') \rangle_{AP} \\ &= \frac{1}{\pi} \left(\frac{hP^2\eta}{e^2 g(1-P^2)^2} \right)^2 \left(\frac{\gamma_0 k_B T \alpha}{M_s V} \right)^2 \\ &\times \int d\omega (V_{xx}^2 + V_{yy}^2 - 2V_{xy}^2), \end{aligned} \tag{62}$$

where $V_{ij} = U_{ij}(\omega_t \to \omega_s)$ with $\omega_s = \omega_a - \omega_e$ (recall that $\omega_e < 0$). To gain physical understanding of this expression, we again disregard the demagnetizing field and get

$$S_{AP}(0) = \left(\frac{\gamma_0 k_B T}{M_s V} \right)^2 \left(\frac{hP^2\eta}{e^2 g(1-P^2)^2} \right)^2 \frac{1}{\omega_s^3 \alpha}. \tag{63}$$

As expected, the resistance noise decreases with increasing ω_s. The anisotropy, dipolar and exchange fields stabilizes the magnetization, with the dipolar and exchange fields playing a role similar to that of the external field in the P configuration. The Gilbert damping enters in the same way as for the P configuration.

Comparing $S_P(0)$ with $S_{AP}(0)$, we see that the expression for the resistance noise in the P and AP configurations are very similar, except for the prefactor due to the GMR sensitivity. The value of the noise may be very different though, depending on the strength of the external field in the P configuration, and the strength of the dipolar and exchange fields. If we consider the special case $\omega_t = \omega_s$,

$$\frac{S_P}{S_{AP}} = \frac{(1-P^2)^4}{\eta^4}. \tag{64}$$

For e.g. $P = 0.5$ and $\eta = 1$, this becomes ≈ 0.3, showing that GMR sensitivity can induce a substantial difference in noise level between the P and AP configurations. S_{AP} may be translated into current noise in the same manner as S_P.

3. Perpendicular configuration

Assuming that F_2 is pinned in the x-direction, instead of the z-direction, the perpendicular state $\langle \mathbf{m}_1 \rangle \cdot \mathbf{m}_2 = 0$ is realized. The average direction of F_1 is dictated by the anisotropy field to be along the z-axis, as explained in the previous subsections. In the following we assume that the geometry of the ferromagnets and the distance between them are such that the interlayer exchange and dipolar coupling are so weak that they may be neglected. This simplification is convenient when analyzing the perpendicular configuration.

Expanding the resistance to first order in the fluctuations $\delta \mathbf{m}_1$, we find in this case

$$\langle \Delta R(t) \Delta R(t') \rangle_{\text{Perp}} = \left(\frac{\partial R(0)}{\partial \cos\theta} \right)^2_{\text{Perp}} \langle \delta m_{1x}(t) \delta m_{1x}(t') \rangle. \tag{65}$$

The resistance noise in the perpendicular configuration is hence second order in the magnetization fluctuations, unlike the P and AP configurations, where the noise is fourth order in the magnetization fluctuations. The GMR sensitivity

$$\frac{\partial R(0)}{\partial \cos\theta} = -\frac{4hP^2\eta}{e^2 g(1+\eta-P^2)^2}, \tag{66}$$

with the aid of Eq. (36). Linearizing Eq. (52) and using the correlators Eqs. (43) and (44) for the stochastic field we find

$$\langle \delta m_{1x}(t) \delta m_{1x}(t') \rangle = \frac{\gamma_0 k_B T}{\pi M_s V} \int d\omega e^{-i\omega(t-t')} \frac{\omega^2(\alpha_0 + \alpha'_{yy}) + (\omega_p + \omega_d)^2(\alpha_0 + \alpha'_{xx})}{[\omega^2 - \omega_p(\omega_p + \omega_d)]^2 + \omega^2[\omega_p(2\alpha_0 + \alpha'_{xx} + \alpha'_{yy}) + \omega_d(\alpha_0 + \alpha'_{xx})]^2}, \tag{67}$$

where $\omega_p = \omega_0 + \omega_c$. We then arrive at the zero-frequency resistance noise

$$S_{\text{Perp}}(0) = \frac{2\gamma_0 k_B T}{M_s V} \left(\frac{4hP^2\eta}{e^2 g(1+\eta-P^2)^2} \right)^2 \frac{\alpha_0 + \alpha'_{xx}}{\omega_p^2}. \tag{68}$$

We see that this is quite different from the expressions for the noise in the P and AP configurations. In particular, the damping appears here in the numerator. Note also that the noise is independent of the demagnetizing field. Depending on parameters, $S_{\text{Perp}}(0)$ can differ substantially from $S_P(0)$ and $S_{AP}(0)$. However $S_{\text{Perp}}(0)$ is not necessarily much larger than $S_P(0)$ and $S_{AP}(0)$, as one could expect from the fact that $S_{\text{Perp}}(0)$ is second order in the magnetization flucations, while $S_P(0)$ and $S_{AP}(0)$ are fourth order. This is due to the fact that $\langle \delta m_{1x}(t)\delta m_{1x}(t')\rangle$ is a rapidly varying function, so that when integrated over, it yields a small result. This means that in calculating the resistance noise in the perpendicular configuration, we should include also the next order term, proportional to $\langle \delta m_{1x}(t)\delta m_{1x}(t')\rangle^2$. However, the prefactor of this term is typically much smaller than the prefactor of the first order term, hence justifying neglecting it.

E. Two identical ferromagnets

So far we have considered one of the ferromagnets to be pinned. As a final remark on noise in spin valves, we now take a look at the case where the ferromagnets are identical and hence equally susceptible to fluctuations. In the same manner as in the preceeding section, we calculate the magnetization and resistance noise, focusing now only on the P and AP configurations.

The fluctuations of F_1 are $\delta\mathbf{m}_1(t) = \mathbf{m}_1(t) - \langle\mathbf{m}_1\rangle$ and those of F_2 are $\delta\mathbf{m}_2(t) = \mathbf{m}_2(t) - \langle\mathbf{m}_2\rangle$. As before, we choose the z-axis so that the time-averaged equilibrium values are $\langle\mathbf{m}_1\rangle = \langle\mathbf{m}_2\rangle = \hat{\mathbf{z}}$ for the parallel configuration, and $\langle\mathbf{m}_1\rangle = -\langle\mathbf{m}_2\rangle = \hat{\mathbf{z}}$ for the antiparallel. The dot product of the magnetizations is $\mathbf{m}_1\cdot\mathbf{m}_2 = \pm 1 \mp \frac{1}{2}(\delta\mathbf{m}^{\mp})^2$, where the upper (lower) sign holds for the P (AP) orientation and $\delta\mathbf{m}^{\mp} = \delta\mathbf{m}_1 \mp \delta\mathbf{m}_2$. For small fluctuations, we can expand the resistance to first order in $(\delta\mathbf{m}^{\mp})^2$, finding

$$R(\mathbf{m}_1\cdot\mathbf{m}_2) \approx R(\pm) \mp \frac{1}{2}(\delta\mathbf{m}^{\mp})^2\frac{\partial R(\pm 1)}{\partial(\cos\theta)}. \qquad (69)$$

The resistance noise is then

$$\begin{aligned}\langle \Delta R(t)\Delta R(t')\rangle_{P/AP} &= \langle R(t)R(t')\rangle_{P/AP} - \langle R(t)\rangle_{P/AP}\langle R(t')\rangle_{P/AP} \\ &= \frac{1}{4}\left(\frac{\partial R(\pm1)}{\partial\cos\theta}\right)^2[\langle(\delta\mathbf{m}^{\mp})^2(\delta\mathbf{m}^{\mp})^2\rangle_{P/AP} - \langle(\delta\mathbf{m}^{\mp})^2\rangle_{P/AP}\langle(\delta\mathbf{m}^{\mp})^2\rangle_{P/AP}], \qquad (70)\end{aligned}$$

which by employing Wick's theorem becomes

$$\begin{aligned}\langle \Delta R(t)\Delta R(t')\rangle_{P/AP} &= \frac{1}{2}\left(\frac{\partial R(\pm1)}{\partial\cos\theta}\right)^2 \\ &\quad\times\sum_{ij}\langle \delta m_i^{\mp}(t)\delta m_j^{\mp}(t')\rangle^2_{P/AP}. \end{aligned} \qquad (71)$$

The GMR sensitivity was calculated in the preceeding section, while the correlator of the magnetization fluctuations remains to be determined.

Letting the subscripts k and l refer to ferromagnet 1 or 2, the LLG equation in this case reads

$$\begin{aligned}\frac{d\mathbf{m}_k}{dt} &= -\gamma_0\mathbf{m}_k\times[\mathbf{H}_{\text{eff}} + \mathbf{h}_k(t)] \\ &\quad + (\alpha_0+\alpha_{sv})\mathbf{m}_k\times\frac{d\mathbf{m}_k}{dt} + \frac{\alpha_{sv}}{3}\mathbf{m}_l\times\frac{d\mathbf{m}_l}{dt},\end{aligned} \qquad (72)$$

where the effective field $\mathbf{H}_{\text{eff}}$ was described in the previous section, and is assumed equal for both ferromagnets. The stochastic field is also the same as that given in the previous section, but an additional effect must be taken into account: Due to current conservation, the spin-current induced part of the stochastic fields of the two ferromagnets are not independent of each other. With the spin current noise calculated in Sec. IV B, and following the recipe in section III D, we find

$$\langle h_{1,i}^{(\text{th})}(t)h_{2,j}^{(\text{th})}(t')\rangle_P = -2k_BT\frac{\alpha_{sv}/3}{\gamma_0 M_s V}\delta_{ij}\delta(t-t') \qquad (73)$$

for the P configuration, and

$$\langle h_{1,i}^{(\text{th})}(t)h_{2,j}^{(\text{th})}(t')\rangle_{AP} = 2k_BT\frac{\alpha_{sv}/3}{\gamma_0 M_s V}\delta_{ij}\delta(t-t'). \qquad (74)$$

for the AP configuration (as before i,j label components perpendicular to the magnetization direction). α_{sv} is defined in Eq. (41). Naturally, the intrinsic fields $\mathbf{h}_1^{(0)}$ and $\mathbf{h}_2^{(0)}$ are independent of each other. The last term in the LLG equation is the portion of the (dimensionless) spin current pumped from ferromagnet l (see section III) that is transmitted to, and subsequently absorbed by ferromagnet k. Since the normal metal node is chaotic, this amounts to one third of the net total spin current pumped out of ferromagnet l. Both ferromagnets completely absorb any incoming spin currents polarized transverse to their magnetizations, due to the assumption that they are thicker than the magnetic coherence length. The mutual spin pumping and following spin absorption between the ferromagnets couple them dynamically in what is called the dynamic spin-exchange coupling[27,43]. Naturally, this coupling was not present in spin valves with one ferromagnet pinned.

Linearizing the Eq. (72) in $\delta\mathbf{m}_k(t)$ we can now evaluate the desired magnetization noise correlators, to be inserted in Eq. (71). For the zero-frequency resistance noise $S_{P/AP}(0) = \int d(t-t')\langle\Delta R(t)\Delta R(t')\rangle_{P/AP}$ this yields

$$S_P(0) = \frac{1}{\pi}\left(\frac{hP^2}{e^2 g\eta}\right)^2\left(\frac{2\gamma_0 k_B T}{M_s V}\right)^2 \times \int d\omega (Z_{xx}^2 + Z_{yy}^2 - 2Z_{xy}^2). \tag{75}$$

for the P configuration, and

$$S_{AP}(0) = \frac{1}{\pi}\left(\frac{hP^2\eta}{e^2 g(1-P^2)^2}\right)^2\left(\frac{2\gamma_0 k_B T}{M_s V}\right)^2 \times \int d\omega (X_x^2 + X_y^2) \tag{76}$$

for the AP configuration. Here

$$Z_{xx} = \frac{\alpha_t[\omega^2 + (\omega_i+\omega_d)^2]}{[\omega^2 - \omega_i(\omega_i+\omega_d)]^2 + \omega^2\alpha_t^2(2\omega_i+\omega_d)^2}, \tag{77}$$

$$Z_{xy} = \frac{-i\omega\alpha_t(2\omega_i+\omega_d)}{[\omega^2 - \omega_i(\omega_i+\omega_d)]^2 + \omega^2\alpha_t^2(2\omega_i+\omega_d)^2}, \tag{78}$$

$$Z_{yy} = \frac{\alpha_t(\omega^2 + \omega_i^2)}{[\omega^2 - \omega_i(\omega_i+\omega_d)]^2 + \omega^2\alpha_t^2(2\omega_i+\omega_d)^2}, \tag{79}$$

and

$$X_x = \frac{\omega^2\alpha_s + (\omega_c+\omega_d)^2\alpha_t}{[\omega^2 + (\omega_c+\omega_d)(2\omega_e - \omega_c)]^2 + \omega^2(2\omega_x\alpha_s - 2\omega_c\alpha - \omega_d\alpha_t)^2}, \tag{80}$$

$$X_y = \frac{\omega^2\alpha_s + \omega_c^2\alpha_t}{[\omega^2 + \omega_c(2\omega_e - \omega_c - \omega_d)]^2 + \omega^2(2\omega_x\alpha_s - 2\omega_c\alpha - \omega_d\alpha_s)^2}. \tag{81}$$

For convenience, we have defined $\alpha_s = \alpha_0 + 2\alpha_{sv}/3$, $\alpha_t = \alpha_0 + 4\alpha_{sv}/3$, $\alpha = \alpha_0 + \alpha_{sv}$ (note the difference between α, α_s and α_t), and $\omega_i = \omega_0 + \omega_a + 2\omega_x$. To arrive at the above expressions we have assumed small damping, i.e., $\alpha_0^2, \alpha_{sv}^2 \ll 1$.

Comparing with the results in the previous section, we see that Eq. (75) is similar to Eq. (58), while Eq. (76) differs considerably from Eq. (62). This is due to the static dipolar and exchange couplings, and the dynamic spin-exchange coupling, whose effects on the noise are changed by the presence of the second fluctuating ferromagnet. In particular, the latter coupling causes the Gilbert damping constant to enter Eqs. (75) and (76) differently. It can be shown that Eq. (75) decreases with the external field, and Eq. (76) decreases with the dipolar and exchange coupling, as expected. It should be noted that the noise level in general is higher when both ferromagnets fluctuate, than when only one fluctuates. Hence, comparing with Johnson-Nyquist noise, magnetically induced resistance noise is even more important in the present case.

There are, as we see from the above expressions, a number of material parameters that determine the resistance noise. Depending on these parameters, the noise level in the P configuration can differ substantially from that in the AP configuration. Note that Eq. (76) reproduces the corresponding result of Ref. 32 when the demagnetizing field is disregarded, i.e., when $\omega_d \to 0$. Eq. (75) reproduces the corresponding result of Ref. 32 when $\omega_a \to 0$ and $\omega_d \to -\omega_a$, since in this reference the external field was taken perpendicular to the anisotropy field. It was shown in Ref. 32 that the value of $S_P(0)$ can differ considerably from $S_{AP}(0)$ in typical experimental spin-valve setups. Furthermore, the importance of the dynamic spin-exchange coupling on this difference was pointed out. However, the role of the GMR sensitivity on the noise was not explored. Here we point out that the GMR sensitivity can cause a significant difference in noise between the P and AP configurations, as shown in the previous section. We note also that it is important to include the demagnetizing field to get the right magnitude of the noise. Suppressing out-of-plane deviations, the demagnetizing field serves to stabilize the magnetizations, and hence lowering the resistance noise. It acts however in the same way for both the P and AP configurations, and is hence of less importance for the comparison of $S_P(0)$ and $S_{AP}(0)$.

V. CONCLUSIONS

Using scattering theory and magnetoelectronic circuit theory, we have demonstrated the effect of spin current fluctuations on the magnetization in ferromagnetic single- and double-layers. Via a fluctuating spin-transfer torque, the current noise causes significantly enhanced magnetization noise, that in spin valves vary with the magnetic configuration. The noise is related to the magnetization damping by the FDT, and can be experimentally detected as resistance noise. The contribution from spin current noise to resistance noise is considerable, and may be an issue for the next generation magnetoresistive spin valve read heads.

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Paper V

Current-induced noise and damping in non-uniform ferromagnets

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Current-induced noise and damping in non-uniform ferromagnets

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In the presence of spatial variation of the magnetization direction, electric current noise causes a fluctuating spin-transfer torque that increases the fluctuations of the ferromagnetic order parameter. By the fluctuation-dissipation theorem, the equilibrium fluctuations are related to the magnetization damping, which in inhomogeneous ferromagnets acquires a nonlocal tensor structure. In biased ferromagnets, shot noise can become the dominant contribution to the magnetization noise at low temperatures. Considering spin spirals as a simple example, we show that the current-induced noise and damping is significant.

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Electric currents induce magnetization dynamics in ferromagnets. Three decades ago, Berger [1, 2] showed that an electric current passing through a ferromagnetic domain wall exerts a torque on the wall. The cause of this spin-transfer torque is the reorientation of spin angular momentum experienced by the electrons as they adapt to the continually changing magnetization. Subsequently, it was realized that the same effect may also be present in magnetic multilayers [3]. In the latter case, the torque may cause reversal of one of the layers, while in the former, it may cause domain wall motion. The early ideas have been confirmed both theoretically and experimentally [4].

Recently, the importance of noise in current-induced magnetization dynamics has drawn attention. Although often noise is undesired, it may in some cases be quite useful. Wetzels *et al.* [5] showed that current-induced magnetization reversal of spin valves is substantially sped up by an increased level of current noise. The noisy current exerts a fluctuating torque on the magnetization [6]. Ravelosona *et al.* [7] reported observation of thermally assisted depinning of a narrow domain wall under a current. Thermally-assisted current-driven domain wall motion has also been studied theoretically [8, 9]. Motivated by recent experiments, Duine *et al.* [9] derived finite-temperature current-driven domain wall velocities.

The present paper addresses current-induced magnetization noise in non-uniformly magnetized ferromagnets. The spatial variation of the magnetization direction gives rise to increased magnetization noise; by a fluctuating spin-transfer torque, electric current noise causes fluctuations of the magnetic order parameter. We take into account both thermal current noise and shot noise, and show that the resulting magnetization noise is well represented by introducing fictitious stochastic magnetic fields. By the fluctuation-dissipation theorem (FDT), the thermal stochastic field is related to the dissipation of energy, or damping, of the magnetization. The FDT hence constitutes a simple and efficient way to evaluate the damping, providing also a physical explanation in terms of current noise and spin-transfer torque. Since the correlator of the stochastic field in general is inhomogenous and anisotropic, the damping is a nonlocal tensor. As a simple and illuminating example we consider ferromagnetic spin spirals, for which the field correlator and damping become spatially independent. It is shown that for spirals with relatively short wavelength ($\sim$ 20nm), the current-induced noise and damping is substantial. Since half a wavelength of a spin spiral can be considered as a simple model for a domain wall, this suggests that current-induced magnetization noise and damping should be an issue for narrow domain walls.

It is instructive to start with an introduction to the FDT for *uniform* (single-domain) ferromagnetic systems, characterized by a time-dependent unit magnetization vector $\mathbf{m}(t)$ and magnetization magnitude M_s (the saturation magnetization). The spontaneous equilibrium noise of such *macrospins* is conveniently described by the correlator $S_{ij}(t-t') = \langle \delta m_i(t) \delta m_j(t') \rangle$, where $\delta m_i(t) = m_i(t) - \langle m_i(t) \rangle$ is the random deviation of the magnetization from the mean value at time t. The brackets denote statistical averaging at equilibrium, while i and j denote Cartesian components transverse to the equilibrium (average) direction of magnetization. Applying a weak external magnetic field $\mathbf{h}^{(\mathrm{ext})}(t)$, the magnetization can be excited from the equilibrium state. Assuming linear response, the resulting change in magnetization is

$$\Delta m_i(t) = \sum_j \int dt' \chi_{ij}(t-t') h_j^{(\mathrm{ext})}(t'), \qquad (1)$$

defining the magnetic susceptibility $\chi_{ij}(t-t')$ as the causal response function. The FDT relates this susceptibility to the equilibrium noise correlator [10]:

$$S_{ij}(t-t') = \frac{k_B T}{M_s V} \int d\omega e^{-i\omega(t-t')} \frac{\chi_{ij}(\omega) - \chi_{ji}^*(\omega)}{i2\pi\omega}, \qquad (2)$$

where T is the temperature and V is the volume of the ferromagnet.

The spontaneous equilibrium fluctuations $\delta\mathbf{m}(t)$ may be regarded to be caused by a fictitious random magnetic field $\mathbf{h}(t)$ with zero mean. We can derive an alternative form of the FDT in terms of the correlator $\langle h_i(t)h_j(t')\rangle$. To do so, simply note that Eq. (1) implies that $\delta m_i(\omega) = \sum_j \chi_{ij}(\omega)h_j(\omega)$ in Fourier space. Inverting this relation, it follows from Eq. (2) that

$$\langle h_i(t)h_j(t')\rangle = \frac{k_B T}{M_s V}\int d\omega e^{-i\omega(t-t')}\frac{[\chi_{ji}^{-1}(\omega)]^* - \chi_{ij}^{-1}(\omega)}{i2\pi\omega}, \tag{3}$$

where $\chi_{ji}^{-1}(\omega)$ is the ji-component of the Fourier transformed inverse susceptibility tensor.

The magnetic susceptibility can be found from the Landau-Lifshitz-Gilbert (LLG) equation of motion. The stochastic LLG equation describes magnetization dynamics and noise in both uniform as well as non-uniform ferromagnets, and reads

$$\frac{d\mathbf{m}}{dt} = -\gamma\mathbf{m}\times[\mathbf{H}_{\text{eff}} + \mathbf{h} + \mathbf{h}^{(\text{ext})}] + \alpha_0\mathbf{m}\times\frac{d\mathbf{m}}{dt}. \tag{4}$$

Here γ is the gyromagnetic ratio, $\mathbf{H}_{\text{eff}}$ is an effective static magnetic field determining the equilibrium state, $\mathbf{h}(t)$ is the above random noise-field, $\mathbf{h}^{(\text{ext})}(t)$ is the weak excitation introduced in Eq. (1), and α_0 is the Gilbert damping constant. Linearizing this equation in the magnetic response to $\mathbf{h}^{(\text{ext})}(t)$, we find the inverse susceptibility

$$\chi^{-1} = \frac{1}{\gamma}\begin{bmatrix} \gamma|\mathbf{H}_{\text{eff}}| - i\omega\alpha_0 & i\omega \\ -i\omega & \gamma|\mathbf{H}_{\text{eff}}| - i\omega\alpha_0 \end{bmatrix} \tag{5}$$

written in matrix (tensor) form in the plane normal to the equilibrium magnetization direction. Note that the static field has here been assumed local and magnetization independent. While not valid in most realistic situations, this simple form for the effective field captures the key physics of interest here, since only the dissipative part of the susceptibility (the Gilbert damping term) affects the noise. Inserting Eq. (5) into Eq. (3), we get the well-known result [11]

$$\langle h_i(t)h_j(t')\rangle = \frac{2k_B T\alpha_0}{\gamma M_s V}\delta_{ij}\delta(t-t'), \tag{6}$$

where i and j denote components orthogonal to the equilibrium magnetization direction. This expression relates the equilibrium noise, in terms of $\mathbf{h}$, to the damping or dissipation of energy in the ferromagnet. It may be noted that in thin ferromagnetic films in good electrical contact with a metal, the equilibrium noise and corresponding Gilbert damping has been shown to be substantially enhanced. This is due to the transfer of transverse spin current fluctuations in the neighbouring metal to the magnetization [6, 12].

We now turn our attention to a more complex system, i.e., a metallic ferromagnet whose direction of magnetization $\mathbf{m}$ is varying along some direction in space, say, the y-axis. It is assumed that the spatial variation is adiabatic, i.e., slow on the scale of the ferromagnetic coherence length. The ferromagnet is furthermore assumed to be translationally invariant in the x- and z-directions, and its magnetization magnitude is taken to be constant and equal to the saturation magnetization M_s. In general, the dynamics and fluctuations of such a magnetization *texture* depend on position. Due to the spatial variation of the magnetization, *longitudinal* (i.e., polarized parallel with the magnetization) spin current fluctuations transfer spin angular momentum to the ferromagnet. The resulting enhancement of the magnetization noise is described by introducing a random magnetic field, whose correlator is inhomogenous and anisotropic, unlike Eq. (6). By the FDT, the correlator is related to the magnetization damping, that acquires a nonlocal tensor structure. In the following we make use of the fact that the time scale of the fluctuations is much longer than the time set by relevant energy scales, such as the temperature, the applied voltage, and the exchange splitting, as implicitly done already in Eq. (6). We shall disregard spin-flip processes and the associated noise.

It is convenient to transform the magnetization texture to a rotated reference frame, defined in terms of the equilibrium (average) magnetization direction $\mathbf{m}_0(y) = \langle\mathbf{m}(y,t)\rangle$ of the texture. The three orthonormal unit vectors spanning this position-dependent frame is $\hat{\mathbf{v}}_1 = \hat{\mathbf{v}}_2\times\hat{\mathbf{v}}_3$, $\hat{\mathbf{v}}_2 = (d\mathbf{m}_0/dy)/|d\mathbf{m}_0/dy|$ and $\hat{\mathbf{v}}_3 = \mathbf{m}_0$. The local gauge

$$U(y) = \left[\ \hat{\mathbf{v}}_1(y)\ \ \hat{\mathbf{v}}_2(y)\ \ \hat{\mathbf{v}}_3(y)\ \right]^T, \tag{7}$$

transforms the magnetization, and hence the relevant equations involving the magnetization, to this frame. That is, $U\mathbf{m}_0(y) \equiv \tilde{\mathbf{m}}_0 = \hat{\mathbf{z}}$, where the tilde indicates a vector in the transformed frame. We note also that $U\hat{\mathbf{v}}_1 = \hat{\mathbf{x}}$ and $U\hat{\mathbf{v}}_2 = \hat{\mathbf{y}}$, and that U is orthogonal, i.e., $U^{-1} = U^T = [\hat{\mathbf{v}}_1\ \hat{\mathbf{v}}_2\ \hat{\mathbf{v}}_3]$.

We consider a charge current I flowing through the ferromagnet along the y-axis. Assuming that the equilibrium magnetization direction $\mathbf{m}_0(y)$ changes adiabatically, the electrons spins align with the changing magnetization direction when propagating through the texture. The spin current is then anywhere longitudinal, and hence given by $\mathbf{I}_s(y) = I_s\mathbf{m}_0(y)$. The alignment of the electrons spins causes a torque $\boldsymbol{\tau}(y) = -d\mathbf{I}_s(y)/dy$ on the ferromagnet. Since $d\mathbf{I}_s(y)/dy$ is perpendicular to $\mathbf{m}_0(y)$, the torque can be written $\boldsymbol{\tau}(y) = \mathbf{m}_0(y)\times[\mathbf{m}_0(y)\times d\mathbf{I}_s(y)/dy]$, or $\tilde{\boldsymbol{\tau}}(y) = U\boldsymbol{\tau}(y) = \tilde{\mathbf{m}}_0\times[\tilde{\mathbf{m}}_0\times U d\mathbf{I}_s(y)/dy]$ in the transformed representation. When $I = 0$, which we will take in the following, $\tilde{\boldsymbol{\tau}} = 0$, on average. However, at $T \neq 0$ thermal fluctuations of the spin current result in a fluctuating spin-transfer torque

$$\Delta\tilde{\boldsymbol{\tau}}(y,t) = \Delta I_s(t)\tilde{\mathbf{m}}_0\times[\tilde{\mathbf{m}}_0\times U\frac{d\mathbf{m}_0(y)}{dy}], \tag{8}$$

where $\Delta I_s(t)$ are the time-dependent spin current fluctuations with zero mean, propagating along the y-direction.

The action of the fluctuating torque on the magnetization is described by the LLG equation if we, by conservation of angular momentum, add the term $-\gamma\Delta\boldsymbol{\tau}/(M_s A)$ on the right hand side. Here A is the cross section (in the xz-plane) of the ferromagnet. Linearizing and transforming the LLG equation to the rotated reference frame, it is seen that the fluctuating torque (8) can be represented by a random magnetic field $\tilde{\mathbf{h}}'(y,t) = \Delta I_s(t)/M_s A)[\tilde{\mathbf{m}}_0 \times U d\mathbf{m}_0(y)/dy]$, analogous to $\mathbf{h}(t)$ discussed above. Using Eq. (7)

$$\tilde{\mathbf{h}}'(y,t) = -\frac{\Delta I_s(t)}{M_s A}\left|\frac{d\mathbf{m}_0(y)}{dy}\right|\hat{x}, \tag{9}$$

i.e., the (transformed) current-induced random field points in the x-direction.

The longitudinal spin current fluctuations $\Delta I_s(t)$ can be found by Landauer-Büttiker scattering theory [6, 13]. Disregarding spin-flip processes, the spin-up and spin-down electrons flow in different and independent channels. In the low-frequency regime, in which charge is instantly conserved, longitudinal spin current fluctuations are perfectly correlated throughout the entire ferromagnet. This applies also to ferromagnets in which the magnetization changes adiabatically in space and time [4]. Hence, the thermal spin current fluctuations are given by [6, 13]

$$\langle \Delta I_s(t)\Delta I_s(t')\rangle = \frac{\hbar^2}{(2e)^2}2k_B T(G_\uparrow + G_\downarrow)\delta(t-t'), \tag{10}$$

where $G_{\uparrow(\downarrow)}$ is the conductance for electrons with the spin aligned (anti)parallel with the magnetization. This expression is simply the Johnson-Nyquist noise generalized to spin currents [6]. We find from Eqs. (9) and (10)

$$\langle \tilde{h}'_x(y,t)\tilde{h}'_x(y',t')\rangle = \frac{2k_B T\beta_{xx}(y,y')}{\gamma M_s V}\delta(t-t') \tag{11}$$

for the correlator of the current induced random field. Here we have defined

$$\beta_{xx}(y,y') = \frac{\gamma\hbar^2\sigma}{4e^2 M_s}\left|\frac{d\mathbf{m}_0(y)}{dy}\right|\left|\frac{d\mathbf{m}_0(y')}{dy}\right| \tag{12}$$

with $\sigma = (G_\uparrow + G_\downarrow)L/A$ the total conductivity. Recall that $\tilde{h}'_y(t) = \tilde{h}'_z(t) = 0$. Eqs. (11) and (12) describes the *nonlocal* and *anisotropic* magnetization noise due to thermal current fluctuations in adiabatic non-uniform ferromagnets. The noise vanishes with the spatial variation of the magnetization vanishes. The random field is spatially correlated throughout the ferromagnet, as a consequence of Eq. (10).

According to the FDT, the thermal noise is related to the magnetization damping. Since the noise correlator (11) is inhomogeneous and anisotropic, the corresponding damping must in general be a nonlocal tensor. To evaluate the damping, we hence need the spatially resolved version of the FDT, which reads

$$\begin{aligned}\langle \delta\tilde{m}_i(y,t)\delta\tilde{m}_j(y',t')\rangle &= \frac{k_B T}{2\pi M_s A}\int d\omega e^{-i\omega(t-t')} \\ &\quad\times\frac{\chi_{ij}(y,y',\omega) - \chi^*_{ji}(y',y,\omega)}{i\omega},\end{aligned} \tag{13}$$

in the transformed representation. Here $\delta\tilde{\mathbf{m}}(y,t) = U\delta\mathbf{m}(y,t) = \delta m_x(y,t)\hat{\mathbf{x}} + \delta m_y(y,t)\hat{\mathbf{y}}$, and the susceptibility is defined by

$$\Delta\tilde{m}_i(y,t) = \sum_j \int\int dy'dt'\chi_{ij}(y,y',t-t')\tilde{h}_j^{(\mathrm{ext})}(y',t'). \tag{14}$$

This is analogous to Eq. (1), but with the external field and magnetic excitations transformed: $\tilde{h}_j^{(\mathrm{ext})}(y,t) = Uh_j^{(\mathrm{ext})}(y,t)$ and $\Delta\tilde{\mathbf{m}}(y,t) = U\Delta\mathbf{m}(y,t)$. The susceptibility in the rotated reference frame differs from Eq. (5) and has to be determined. It may be noted that it is straightforward to generalize Eqs. (13) and (14) to the case of general three-dimensional dynamics.

We may substitute $\tilde{h}_j^{(\mathrm{ext})}(y',t') \to \tilde{h}'_j(y',t')$ in Eq. (14) to find the explicit fluctuations of the magnetization vector caused by the spin-transfer torque. Combining this expression with Eqs. (13) and (11), we arrive at an implicit integral equation for the unknown susceptibility. The nonlocal tensor damping can then be inferred from the susceptibility. Instead of finding a numerical solution for an arbitrary texture, we consider here a ferromagnetic spin spiral as shown in Fig. 1, for which the description of magnetization noise can be mapped onto the macrospin problem. A simple analytical result can then be found, allowing for a comparison with Eq. (6), and hence an estimate of the relative strength and importance of the current-induced noise and damping.

Spin spirals can be found in some rare earth metals [14] and in the γ-phase of iron [15], and are described by $\mathbf{m}_0(y) = [0, \sin\theta(y), \cos\theta(y)]$, where $\theta(y) = 2\pi y/\lambda = qy$, with λ the wavelength of the spiral. Then $d\mathbf{m}_0(y)/dy =$

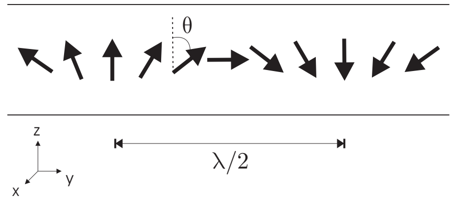

FIG. 1: An example of a non-uniform ferromagnet. The magnetization rotates with wavelength λ in the yz-plane, forming a spin spiral.

$[0, \cos\theta(y), -\sin\theta(y)]$ so that $|d\mathbf{m}_0(y)/dy| = q$. As emphasized earlier, our theory is applicable when the wavelength is much larger than the magnetic coherence length. For transition metal ferromagnets, the coherence length is of the order of a few ångström. From Eq. (12) we find $\beta_{xx} = \gamma\hbar^2\sigma q^2/(4e^2 M_s)$. The current-induced noise correlator (11) for spin spirals is hence homogeneous,

$$\langle \tilde{h}'_x(t)\tilde{h}'_x(t')\rangle = \frac{2k_BT\beta_{xx}}{\gamma M_s V}\delta(t-t'), \qquad (15)$$

similar to Eq. (6), but anisotropic. The problem of relating noise to damping in terms of the FDT can therefore be mapped exactly onto the macrospin problem: The transformation (7) can be used to show that equations analogous to Eqs. (1)-(6) are valid for the spin spiral, when analyzed in the rotated reference frame. It is then seen that the damping term corresponding to Eq. (15) is

$$\tilde{\mathbf{m}} \times \overleftrightarrow{\beta}\frac{d\tilde{\mathbf{m}}}{dt} \qquad (16)$$

in the transformed representation. Here $\overleftrightarrow{\beta} = \mathrm{diag}\{\beta_{xx}, 0\}$ is the 2×2 tensor Gilbert damping in the xy-plane. Hence, β_{xx} is the enhancement of the Gilbert damping caused by the spatial variation of the magnetization and the spin-transfer torque. Due to its anisotropic nature, $\overleftrightarrow{\beta}$ is inside the cross product in Eq. (16), in this way ensuring that the LLG equation preserves the length of the unit magnetization vector $\tilde{\mathbf{m}}$.

To get a feeling for the significance of the current-induced noise and damping, we evaluate $\overleftrightarrow{\beta}$ numerically for a spin spiral with wavelength 20 nm, and compare with α_0. Taking parameter values for α_0, M_s, and σ from Refs. [16–19], we find $\beta_{xx} \approx 5\alpha_0$ for Fe (with $\alpha_0 = 0.002$), and $\beta_{xx} \approx 4\alpha_0$ for Co (with $\alpha_0 = 0.005$). Hence, current-induced noise and damping in spin spirals can be substantial. Considering half a wavelength of the spin spiral as a simple domain wall profile, these results furthermore suggest that a significant current-induced magnetization noise and damping should be expected in narrow domain walls in typical transition metal ferromagnets. This should have consequences for both field- and current-induced domain wall depinning and motion, in which magnetization noise and damping play central roles [7, 9, 20]. An increased level of magnetization noise should aid wall depinning. Recent theoretical and experimental advances suggest that the velocity of a current-driven wall is inversely proportional to the Gilbert damping [4]. The increased noise and the tensor nature of the Gilbert damping should be taken into account in micromagnetic simulations.

So far we have only considered thermal current noise; let us finally turn to shot noise. With the voltage V across the ferromagnet turned on, a nonzero current I flows in the y-direction. Disregarding spin-flip processes, the resulting spin current shot noise is [6, 13]

$$\langle \Delta I_s^{(\mathrm{sh})}(t)\Delta I_s^{(\mathrm{sh})}(t')\rangle = \frac{\hbar^2}{(2e)^2}eVFG\delta(t-t') \qquad (17)$$

at zero temperature. Here the superscript (sh) emphasizes that we are now looking at shot noise. The Fano factor F is between 0 and 1 for non-interacting electrons [21]. If the length of the metal exceeds the electron-phonon scattering length λ_{ep}, shot noise vanishes [13, 21]. λ_{ep} is strongly temperature dependent, and can at low temperatures exceed one micron in metals. To find the contribution from shot noise to the magnetization noise, simply replace Eq. (10) with Eq. (17) in the above calculation of the random-field correlator. In experiments on current-induced domain wall motion [4], typical applied current densities are about $j = 10^8$ A/cm^2. At low temperatures, the ratio of shot noise to thermal current noise, $eVF/2k_BT$, can then exceed unity for long (but not longer than λ_{ep}) ferromagnetic (e.g. Fe) wires. Shot noise can hence be expected to be the dominant contribution to the magnetization noise at low temperatures.

In summary, we have calculated current-induced magnetization noise and damping in non-uniform ferromagnets. Taking into account both thermal and shot noise, we evaluated the fluctuating spin-transfer torque on the magnetization. The resulting magnetization noise was calculated in terms of a random magnetic field. Employing the FDT, the corresponding enhanced Gilbert damping was identified for spin spirals.

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