A distributed parameter systems view of control problems in drilling

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Abstract: We give a detailed view of estimation and control problems raised by the drilling process where the distributed nature of the system cannot be ignored. In particular, we focus on the transport phenomena in Managed Pressure Drilling (MPD) and UnderBalanced Operations (UBO), as well as the time-delay mechanisms of the mechanical stick-slip vibrations. These industrial challenges raise increasingly difficult control questions for hyperbolic systems.

1. INTRODUCTION

The process of oil well drilling, schematically depicted on Figure 1, consists in boring a hole several kilometers deep into the ground, until a reservoir is reached. The drilling rig can be located on an onshore platform, an offshore platform (anchored on the sea bed) or on a drilling ship.

The process involves various physical phenomena of distributed nature, mainly propagation of mechanical waves, of pressure waves and one-dimensional multiphase flow. In this paper, we present several control problems raised by these phenomena where results from control and estimation of distributed parameter systems have potential to make an impact.

The main contribution of this paper is to formulate the estimation and control problems associated with drilling in an industry-relevant form. In particular, we put emphasis on the available sensors and actuators on actual rigs. Besides, we give a review of existing solutions and put them in perspective with the needs of the industry.

Drilling systems, as depicted on Figure 1, mainly consist of a mechanical part, composed of the rotary table, drill string and Bottom Hole Assembly (BHA), and a hydraulic part, consisting of the main pump, the inner part of the drill string, the annulus, and the outlet valve.

As a rule, steady operation of these systems translate into a better performance and safety. The process is subject, however, to various perturbations, uncertainties and instabilities that we detail throughout the paper. The paper is organized as follows. In Section 2 we describe the process of oil well drilling. In Section 3 we focus on the pressure control problem in Managed Pressure Drilling. In Section 4 we investigate the two-phase flow dynamics of UnderBalanced Operations. Finally, in Section 5, we present a novel control paradigm for the mechanical stick-slip problem.

2. DESCRIPTION OF THE PROCESS

To create the borehole, a long flexible series of connected pipes, referred to as the drillstring (or the drill pipes) is set into a rotating motion around its main axis by the rotary table, located at the surface facilities. At the other end of the drillstring, a cutting tool referred to as the drill bit chatters the rock. The operator's main mechanical inputs to the system are the speed of the rotary table and the weight exerted on the bit. These impact the performance of the cutting process, measured by the Rate-Of-Penetration (ROP).

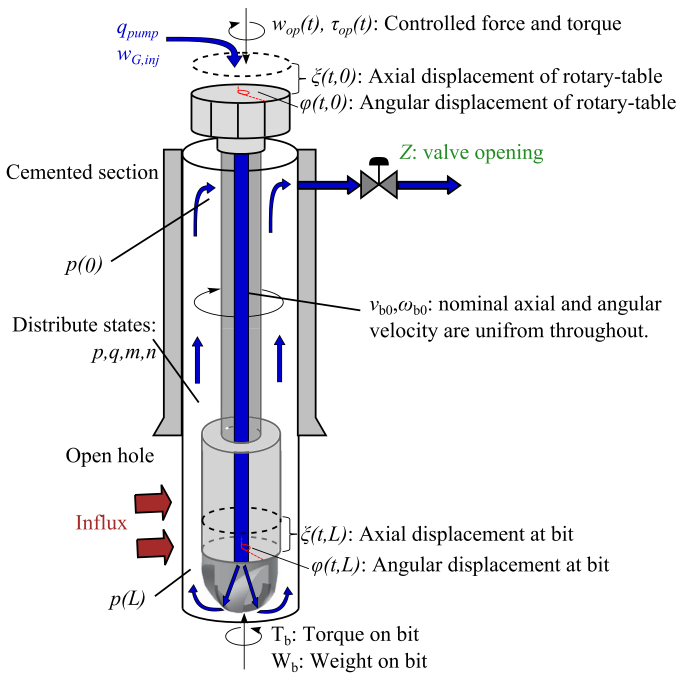

Fig. 1. Well schematic.

To evacuate the rock cuttings and pressurize the *open* part of the well [1], a drilling fluid is injected through the drill string from the surface, exits through the drill bit and flows back up the annular well to the surface, as depicted on Figure 1. To avoid collapse of the well or damage of the formation, the BottomHole Circulating Pressure (BHCP) must be kept between

[1] i.e. the part of the well that has not been cemented yet

pre-specified constraints. There are several ways to adjust the BHCP. In conventional drilling, the outflow of drilling fluid is free, and the only way to modify the pressure is to adjust the properties (density, viscosity,...) of the fluid. This modifies the pressure in a slow, quasi-static way. Dual Gradient Drilling techniques use two different drilling fluids at all times, which adds one degree of freedom to adjust the steady-state BHCP. Finally, in Managed Pressure Drilling (MPD) operations, the outflow of drilling fluid is regulated by a choke valve, which enables control of the fast pressure transients, and is considered as the only actuator in the rest of the paper.

In the next section, we detail the pressure control problem in Managed Pressure Drilling.

3. PRESSURE CONTROL IN MPD: SINGLE-PHASE FLOW DYNAMICS

The typical setup of a MPD system is schematically depicted on Figure 1. Mud is injected through the drillstring by the main pump. The outflow is regulated by a choke valve, the opening of which is the primary actuator. The equations describing the flow of mud in the annulus read as follows Aamo [2012]

$$\frac{\partial p}{\partial t} = \frac{\beta}{A}\frac{\partial q}{\partial x} \tag{1}$$

$$\frac{\partial q}{\partial t} = \frac{A}{\rho}\frac{\partial p}{\partial x} - \frac{F(q)}{\rho} + Ag\cos\theta(x) \tag{2}$$

where $x \in [0, L]$ and t are the space and time variables, respectively, with L being the length of the pipe, $p(x,t)$ is the pressure, $q(x,t)$ is the volumetric flow of mud, β is the bulk modulus of the drilling fluid, ρ is its density, A is the cross-sectional area of the annulus, $\theta(\cdot)$ its inclination and g is the gravity constant. The friction loss term F requires more attention and will be discussed in Section 3.1. The boundary conditions express that the inflow of mud is that imposed by the main pump, and the outflow is given by a valve equation

$$q(L,t) = q_{pump}, \quad q(0,t) = C_c Z\sqrt{\frac{1}{\rho}\max(p(L,t)-p_a, 0)} \tag{3}$$

where C_c is the choke constant, Z the choke opening, and p_a the atmospheric pressure.

3.1 Frequency-dependent friction

In the literature, when dealing with this problem, the viscous frication is typically modelled as a linear function of flow-rate, i.e. $F = kq$, where $k = 8\nu_0/r_0$ for laminar flow with ν_0 denoting fluid dynamic viscosity and r_0 the flow area radius. However at conditions of unsteady flow, this simple relation understates the actual viscous dissipation due to the 2-dimensional effects in the flow, usually referred to as the *Richardson annular effect*. Specifically, unsteady friction should be considered when the shear wave number $r_0(\omega/\nu_0)^{1/2}$ is greater than 5 [Stecki and Davis, 1986].

A popular way of incorporating unsteady friction is to use the relation described in Vítkovskŷ et al. [2006] (originally due to Brunone et al. [1995])

$$F(q) = kq + k_2\Big(\frac{\partial q}{\partial t} + \mathrm{sgn}\Big(q\frac{\partial q}{\partial x}\Big)\sqrt{\frac{\beta}{\rho}}\frac{\partial q}{\partial x}\Big),$$

where k_2 is determined empirically or from a high fidelity simulator, with typical values for laminar flow being around 0.1 Pezzinga [2000].

3.2 Control problem formulation

Using an appropriate coordinate transformation, the dynamics (1)–(3) rewrite as a 2–state linear hyperbolic system Aamo [2012] as follows

$$u_t(t,x) + \lambda(x)u_x(t,x) = \omega(x)v(t,x) \tag{4}$$

$$v_t(t,x) - \mu(x)v_x(t,x) = \sigma(x)u(t,x) \tag{5}$$

$$u(0,t) = U(t), \quad v(1,t) = qu(1,t) \tag{6}$$

where $U(t)$ is the new control variable. The primary objective of MPD is to maintain the BHCP $p(L)$ between pre-specified bounds. More precisely, it must satisfy

$$p_{\text{pore}} < p(L) < p_{\text{fracture}} \tag{7}$$

where p_{fracture} is the fracture pressure, above which one may damage the formation, and p_{pore} is the *pore*, or *reservoir* pressure: it is the pressure of the hydrocarbons trapped into the porous rock that constitutes the reservoir. Operating above the pore pressure ensures that no oil or gas is produced while drilling. Influxes from the reservoir are, in MPD, extremely undesirable, since the surface facilities are usually not able to handle them and they can lead to blowouts. The dynamics (4)–(6) are inherently stable and feature fast transients, but various barriers make tight control of the BHCP a difficult problem.

3.3 Lack of downhole sensors

Even though recent technologies such as wired drillpipes enable measurement of BHCP with a bandwidth compatible with real-time applications Craig et al. [2014], the overwhelming majority of drilling jobs is done without one. Control and estimation algorithm should consequently rely on topside measurements only.

3.4 Heave compensation

On offshore drilling operations, the facilities are subject to perturbations from the heave. When drilling on, a mechanical active heave compensation system prevents the drillstring from being affected by these. However, every 30 meters or so, this system must be de-activated to perform a so-called *connection*, i.e. to add some pipe length. The flow of mud is then stopped, but the drill string is then prone to *surge and swab* vertical movements which cause pressure variations in the mud, see Aarsnes et al. [2014b] for a thorough treatment. In previous efforts this have been modelled by considering that the inflow q_{pump} in (3) is zero but with a disturbance, see e.g. Aamo [2012], Landet et al. [2013] where this perturbation is modelled as a harmonic oscillator of known frequency ω. Equation (3) then becomes

$$q(L,t) = q_{\text{heave}}(t), \qquad \ddot{q}_{\text{heave}}(t) = \omega^2 q_{\text{heave}}(t) \tag{8}$$

An output feedback disturbance rejection control scheme for (1),(2),(8) is presented in Aamo [2012], relying on a backstepping observer-controller structure using topside measurements only. However, the assumptions that the heave perturbation spectrum is both known and single-frequency are not realistic. The stabilization of 2–state hyperbolic systems with an unmeasured harmonic perturbation, or a measured non-periodic one, remain open problems.

3.5 Gas kicks

A crucial control objective in MPD is to avoid gas influx from the reservoir, by keeping the BHCP above the pore pressure. Indeed, unlike in UnderBalanced Operations, the surface facilities are not prepared to handle arbitrary large amounts of gas. The pore pressure however, is usually only approximately known, and may suddenly increase while drilling, possibly above the current value of the BHCP. When this happens, gas and oil may enter the well: this unwanted influx is referred to as a *kick*. Gas kicks are particularly undesirable when uncontrolled, because of the following mechanism: the presence of gas in the well initially decreases the weight of the well column, which reduces the pressure, which in turn increases the gas influx. This positive feedback loop is inherently linked the interaction with the reservoir and the dynamics of multiphase flow. These can only be properly understood using a model for two-phase flow, which is the topic of the next Section.

4. GAS-KICKS AND UBO: TWO-PHASE FLOW DYNAMICS

In this Section, we focus on the dynamics of gas-liquid flow for drilling, which leads to the study of a particular class of 3–state hyperbolic systems. There are two occurrences of two-phase flow in drilling. The first one, as mentioned above, is when gas *undesirably* enters the well during a kick in MPD. The second one is when the BHCP is purposely maintained *below* the pore pressure. This drilling technique is referred to as UnderBalanced Drilling: there is a permanent influx of oil and gas from the reservoir. This not only increases the Rate of Penetration (ROP) but also prevents the drilling fluid from leaking into the formation, enhances the evacuation of rock cuttings and provides a "comfort margin" from the fracture pressure. The constant presence of gas in the system also slows the pressure dynamics down, as the speed of sound is much lower in gas than in a liquid. However, as will appear, it also severely complicates the overall dynamics of the system.

4.1 Physical modeling

Modeling of two-phase flow is a vast research area. Here, we focus on the particular class of *drift-flux* models Zuber [1965] which are best suited for low Gas-Oil Ratios Masella et al. [1998], as is typically the case in drilling. The models are based on two mass conservation laws, respectively for gas and liquid, and one combined momentum conservation law

$$\frac{\partial m}{\partial t} - \frac{\partial m v_L}{\partial s} = 0, \tag{9}$$

$$\frac{\partial n}{\partial t} - \frac{\partial n v_G}{\partial s} = 0, \tag{10}$$

$$\frac{\partial m v_L + n v_G}{\partial t} - \frac{\partial p + m v_L^2 + n v_G^2}{\partial s} = -(m+n) g \sin\phi(s) - F(m, n, v_G). \tag{11}$$

where $s \in [0, L]$ is the space variable, L being the length of the pipe, m and n are respectively the masses of liquid and gas per unit volume, v_L and v_G the velocities of liquid and gas and p is the pressure, which is assumed to be equal in the two phases. In the momentum equation (11), the term $(m+n)g \sin\theta$ represents the gravitational source term, while $F(m, n, v_G)$ accounts for frictional losses. Along with these conservation laws, algebraic relations ensure closure of the system: two equations relate the densities of gas and liquid to pressure, and the empirical slip law gives a relation between the velocities of gas and liquid of the following form Flåtten and Munkejord [2006]

$$v_G - v_L = \Phi(m, n, v_G) \tag{12}$$

Finally, boundary conditions express that the inflows of gas and liquid come both from the drillstring (mud and possibly

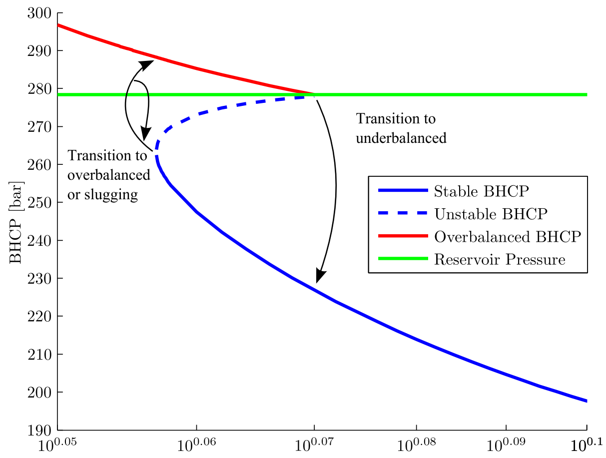

Fig. 2. Equilibrium values of the BHCP with respect to valve opening. For each value of the valve opening, there is at most 3 potential operating points [Aarsnes et al., 2014a].

gas injected by the operator), denoted by $W_{G,inj}$, $W_{L,inj}$, and be produced by reservoir with production coefficients k_L, k_G

$$Amv_l|_{s=0} = \rho_L q_{pump}(t) + k_L \max(P_{res} - p(0), 0), \tag{13}$$

$$Anv_g|_{s=0} = w_{G,inj}(t) + k_G \max(P_{res} - p(0), 0). \tag{14}$$

The total outflow satisfies a multiphase valve equation of the form

$$(mv_G + mv_L)|_{s=L} = C_v Z \sqrt{\bar{\rho}_m \max(p(L,t) - p_s, 0)} \tag{15}$$

where $\bar{\rho}_m$ the mixture density [2], C_v is the valve characteristic, p_s is the separator pressure, which is constant, and Z is the valve opening, which is the main actuator of the system. For more details on the physical modeling, the interested reader is referred to Aarsnes et al. [2014a].

4.2 Analysis of the steady-states

To find potential operating points in the presence of gas, one must solve Equations (9)–(12), along with the remaining closure and boundary relations mentioned above, at steady-state, which is a nonlinear two-point boundary value problem that must be numerically solved. For illustration purposes, we consider now the case of a dry gas well, i.e. where the reservoir contains only gas ($k_L = 0$). The following analysis is based on numerical results for various test cases and has proved to match the experience of field engineers. It is largely based on Aarsnes et al. [2014a]. For each value set of parameters, there are *at most* 3 physically meaningful solutions to the steady-state equations, one of them being overbalanced. This situation is depicted on Figure 2 where the equilibrium curves of the BHCP are plotted against the value of the valve opening, which is the main actuator of the system. One should notice in particular that the equilibrium points may appear and disappear when the opening of the valve is changed, causing hysteresis-like behaviors. Besides, the various equilibria have different stability properties. More precisely, it appears from simulations that the overbalanced equilibrium is stable (as mentioned in Section 3.2), and one of the two underbalanced steady-states is stable while the other is unstable. This motivates the refor-

[2] Actually, this is the simplest model for multiphase flow through a valve. The interested reader is referred to Schüller et al. [2003] for more involved models.

mulation of the dynamics in a form amenable to control and estimator design.

4.3 Quasilinear hyperbolic system formulation

Equations (9)–(12), along with the remaining closure and boundary relations rewrite as a 3–state quasilinear first-order hyperbolic system. When linearized around an equilibrium profile, it takes the following form

$$\begin{pmatrix} u \\ v_1 \\ v_2 \end{pmatrix}_t + \begin{pmatrix} \lambda_1(s) & 0 & 0 \\ 0 & -\mu_1(s) & 0 \\ 0 & 0 & -\mu_2(s) \end{pmatrix} \begin{pmatrix} u \\ v_1 \\ v_2 \end{pmatrix}_s = \Sigma(s) \begin{pmatrix} u \\ v_1 \\ v_2 \end{pmatrix} \tag{16}$$

with boundary conditions

$$u(t,0) = \begin{pmatrix} \rho_1 & \rho_2 \end{pmatrix} \begin{pmatrix} v_1(t,0) \\ v_2(t,0) \end{pmatrix} + U(t), \quad \begin{pmatrix} v_1(t,1) \\ v_2(t,1) \end{pmatrix} = \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} u(t,1) \tag{17}$$

where $\Sigma(\cdot)$ is the matrix of coupling coefficients and $U(t)$ the control input. The transport velocities are such that $-\mu(\cdot) < 0 < \lambda_1(\cdot) < \lambda_2(\cdot)$. Although some estimation and control results do exist for this class of systems, these are not yet applicable to the underbalanced drilling process. We now detail some of the questions relevant to UBO that remain open.

4.4 Stabilization of the unstable equilibrium

As pictured on Figure 2, there exists a potential operating point for which the BHCP is close to (yet below) the reservoir pressure. This point is particularly interesting to operate around because a small difference between BHCP and pore pressure yields a small influx of gas from the reservoir, which is desirable. Unfortunately, it is unstable.

This instability may lead to the system going overbalanced or, alternatively, experiencing *severe slugging*. This oscillatory behavior has been widely reported in the context of oil production (see, e.g. Dalsmo et al. [2002], Hu [2004], Storkaas [2005]) and is known to be detrimental to the facilities, among other reasons because of the large pressure fluctuations it induces. Both in the contexts of drilling and production, stabilizing an unstable equilibrium suppresses the occurence of severe slugging.

In Di Meglio et al. [2013], a stabilizing full-state feedback law is derived for (16),(17) along with an observer using *downhole* measurement. Although criteria for stability of quasilinear hyperbolic system do exist (see e.g. Coron et al. [2008], Vazquez et al. [2012]), no stabilizing control law has been designed for general 3–state quasilinear hyperbolic systems.

4.5 State Estimation

As mentioned above, the only observer result for systems of the form (16),(17) use uncollocated measurements, i.e. a measurement of $v(t,0)$. This means it needs downhole sensors to be implemented, which are usually unavailable in practice. It has been proved in Li [2009], however, that such systems with collocated measurements (i.e. measurements of $u_1(t,1)$ and $u_2(t,1)$) are observable. The method used there to prove this result is unfortunately not constructive and no design is, to our best knowledge, available. Such a result would be extremely valuable to the industry, if only for monitoring purposes: one could then estimate the amount and distribution of gas into the well, as well as the BHCP, from topside measurements.

4.6 Parameter identification

One of the difficulties of accurately monitoring the drilling process is the poor knowledge of downhole conditions. In particular, the characteristics of the reservoir, such as pore pressure or permeability of the rock, are only approximately known in advance by operators. Once again, few results enable estimation of uncertain parameters for hyperbolic systems. In Di Meglio et al. [2014], an adaptive observer using both downhole and topside sensors enables estimation of additive constant measurement biases. More precisely, modifying Equation (17) as

$$\begin{pmatrix} v_1(t,1) \\ v_2(t,1) \end{pmatrix} = \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} u(t,1) + \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} \tag{18}$$

the algorithm enables estimation of the uncertain parameters θ_1 and θ_2 from measurements of $u(t,0)$, $v_1(t,1)$ and $v_2(t,1)$. Significant improvements would be

- to rely on topside measurements only
- to simultaneously estimate q_1, q_2, θ_1 and θ_2
- to estimate the parameters of the nonlinear boundary conditions

As pointed out by Di Meglio et al. [2013], most of the results that will be developed for (16),(17) can straightforwardly be extended to systems with an arbitrary number of positive eigenvalues and one negative one. However, there are very few estimation and control results when there is more than one eigenvalue of each sign, or when the PDEs are coupled at their boundaries with Ordinary Differential Equations. Such problems do arise in drilling applications, as illustrated in the next section.

5. MECHANICAL INSTABILITIES

In this section, we focus on the mechanical part of the system, which consists of the rotary table, the drillstring and the bit, as depicted on Figure 1. As widely reported in the literature (see, e.g., Jansen [1993], Dunayevsky and Abbassian [1998]), it is subject to three main type of vibrations: axial, torsional and lateral ones. These deteriorate the performance of the process, cause fatigue on the equipment and lead to premature failing of the bit. The common approach to study these phenomena is to assume they are decoupled. Recently, the control community has particularly focused on the torsional vibrations that lead to the so-called *stick-slip* phenomenon, see e.g. Navarro-Lopez and Cortes [2007], Saldivar et al. [2011], Sagert et al. [2013], Bekiaris-Liberis and Krstic [2014]. Recent works (Richard et al. [2007]) suggest, however, that the mechanisms leading to stick-slip involve both the axial and torsional vibrations.

5.1 Stick-slip and bit bouncing

When drilling, the rotary table sets the drillstring into a rotation movement around its main axis. Under certain conditions, the system enters a limit cycle in which the velocity of the drilling bit oscillates between 0 (the stick phase) and around twice the velocity of the rotary table (the slip phase). To reproduce this behavior, many models assume that the friction due to the rock-on-bit interaction depends on the bit velocity (see, e.g., Navarro-Lopez and Cortes [2007], Canudas-de Wit et al. [2008], Ritto et al. [2009]). This assumption has led to the design of many control solutions to cope with stick-slip. In particular, Saldivar et al. [2011], Bresch-Pietri and Krstic

[2014], Bekiaris-Liberis and Krstic [2014] focus on the stability or stabilization of the torsional wave equation with a boundary condition corresponding to the velocity-dependent friction term.

A paradigm change Experiments reported in Richard et al. [2004] show however, that the friction does not intrinsically depend on the bit velocity. To explain the occurrence of the stick-slip oscillations, one must consider their coupling with the axial vibrations, referred to as bit bouncing. This is the topic of the next section.

5.2 Model Description

Axial and torsional vibrations in the drill string The dynamics of interest can be derived by assuming elastic deformations and using equations of continuity and state and the momentum balance for the axial and angular dynamics of the pipe. We denote the axial displacement by $\xi(t, x)$. The axial motion is described by (see Germay et al. [2009])

$$\frac{\partial^2 \xi}{\partial t^2}(t, x) - c_\xi^2 \frac{\partial^2 \xi}{\partial x^2}(t, x) = -k_\xi \frac{\partial \xi}{\partial t}(t, x) \tag{19}$$

where $c_\xi = \sqrt{\frac{E}{\rho}}$ is the wave propagation velocity, ρ is the pipe mass density, and E is Young's modulus and k_ξ is a damping constant. Equivalently for the angular motion, we denote the angular displacement in the string $\phi(t, x)$, the equations for the angular motion is

$$\frac{\partial^2 \phi}{\partial t^2}(t, x) - c_\phi^2 \frac{\partial^2 \phi}{\partial x^2}(t, x) = -k_\phi \frac{\partial \phi}{\partial t}(t, x) \tag{20}$$

where $c_\phi = \sqrt{\frac{G}{\rho}}$ is the wave velocity, G is the shear modulus and k_ϕ is a damping constant.

5.3 Downhole boundary condition: bit-rock interaction

The downhole conditions are found by expressing the weight-on-bit and torque-on-bit, which are the result of cutting forces and friction forces

$$w_b(t) = w_c(t) + w_f, \qquad \tau_b(t) = \tau_c(t) + \tau_f \tag{21}$$

In this paper, we focus on their expression in the no-stick phase, i.e. when the bit angular velocity is strictly positive [3]. As mentioned in 5.1, the friction weight w_f and torque τ_f are independent of the bit velocity, and are therefore constant. The cutting forces are given by

$$w_c(t) = a\zeta\epsilon d(t), \quad \tau_c(t) = \frac{1}{2}a^2\epsilon d(t), \tag{22}$$

where ϵ is the intrinsic specific energy of the rock, ζ is a number characterizing the inclination of the cutting force force and a is the bit radius. Finally, $d(t)$ is the combined depth of cut given by (see Fig 3)

$$d(t) = N(\xi(t, L) - \xi(t - t_N(t), L)). \tag{23}$$

where t_N is the delay time between two successive blades of the drill bit implicitly defined by

$$\phi(t, L) - \phi(t - t_N(t), L) = \frac{2\pi}{N}. \tag{24}$$

To derive the bottom boundary conditions, we must now give the relation between the weight and torque on bit and the states. Rigorously speaking, one should write separate wave equations

[3] For a more complete model, the interested reader is referred to Germay et al. [2009].

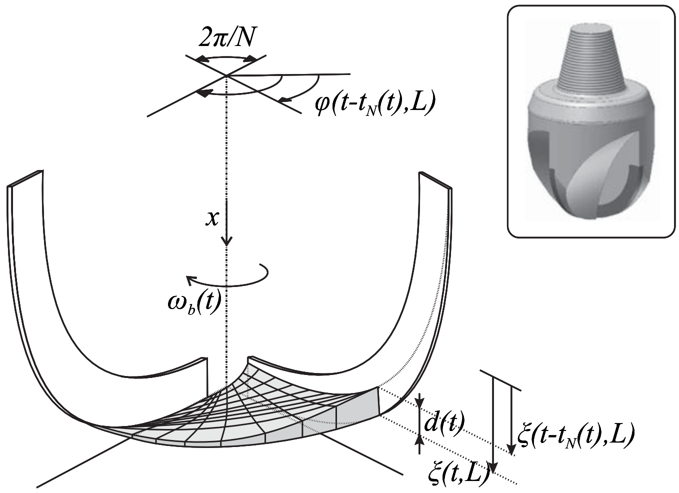

Fig. 3. Bit-rock interaction [Germay et al., 2009].

for the upper most part of the drill string and the BottomHole Assembly (BHA), which have different inertia, Young's modulus, etc. This procedure is adopted in Germay et al. [2009]. Another option is to neglect the difference in mechanical properties of the drill string as is done, e.g., in Richard et al. [2004]. Rather, we chose here an intermediate formulation, lumping the BHA into a single oscillator coupled with the drillstring. This is justified by the comparative length of the BHA ($\approx$ 200m) and the drillstring ($\approx$ 2000m). This yields the following relation

$$A_b \frac{\partial^2 \xi}{\partial t^2}(t, L) = -EA_b \frac{\partial \xi}{\partial x}(t, L) - w_b(t), \tag{25}$$

$$J_b \frac{\partial^2 \phi}{\partial t^2}(t, L) = -GJ_b \frac{\partial \phi}{\partial x}(t, L) - \tau_b(t), \tag{26}$$

where A_b, J_b are the area and polar inertia of the lowermost section of the drill string. Combining (22),(23),(26) yields the following boundary conditions

$$A_b \frac{\partial^2 \xi}{\partial t^2}(t, L) = -a\zeta\epsilon N\left[\xi(t, L) - \xi(t - t_N(t), L)\right] - w_f - EA_b \frac{\partial \xi}{\partial x}(t, L) \tag{27}$$

$$J_b \frac{\partial^2 \phi}{\partial t^2}(t, L) = -\frac{1}{2}a^2\epsilon N\left[\xi(t, L) - \xi(t - t_N(t), L)\right] - \tau_f - GJ_b \frac{\partial \phi}{\partial x}(t, L) \tag{28}$$

Notice that the time-delay $t_N(\cdot)$ is implicitly defined by (24) and therefore is state-dependent.

5.4 Topside boundary condition

At the topside boundary, we assume that the torque and weight on the drillstring are imposed by the operator, which yields

$$E_d A_d \frac{\partial \xi}{\partial x}(t, 0) = w_{op}(t), \qquad G_d J_d \frac{\partial \phi}{\partial x}(t, 0) = \tau_{op}(t), \tag{29}$$

Thus the full system is given by the distributed equations (19),(20) and boundary conditions (27)–(29).

5.5 Formulation as a coupled PDE-ODE system

The wave equations (19),(20) can be rewritten as a system of transport equations by considering the alternative set of variables

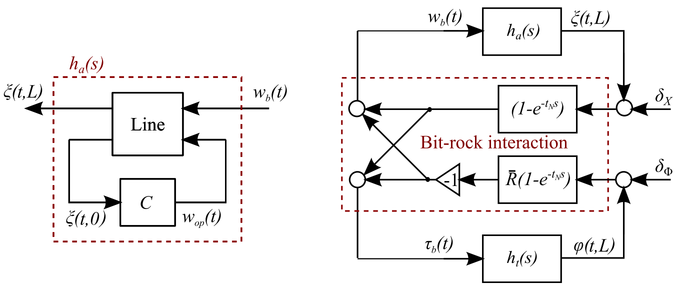

Fig. 4. Block diagram showing the regenerative effect of the Bit-rock interaction causing feedback to the drill string system [Aarsnes and Aamo, 2015].

$$u = \xi_t - c_\xi \xi_x, \qquad v = \xi_t + c_\xi \xi_x \tag{30}$$
$$\varphi = \phi_t - c_\phi \phi_x, \qquad \psi = \phi_t + c_\phi \phi_x \tag{31}$$

Denoting $X = \xi(t, L)$, $Z = \phi(t, L)$ the Equations (19), (20) along with boundary conditions (24)–(29) finally rewrites as the following system of transport PDEs coupled with a Delay Differential Equation

$$u_t + c_\xi u_x = -k_\xi(u + v)/2 \tag{32}$$
$$v_t - c_\xi v_x = -k_\xi(u + v)/2 \tag{33}$$
$$\varphi_t + c_\phi \varphi_x = -k_\phi(\varphi + \psi)/2 \tag{34}$$
$$\psi_t - c_\phi \psi_x = -k_\phi(\varphi + \psi)/2 \tag{35}$$
$$u(t, 0) = U(t) \tag{36}$$
$$\varphi(t, 0) = \Phi(t) \tag{37}$$
$$v(t, L) = -u(t, L) + 2\dot{X}(t) \tag{38}$$
$$\psi(t, L) = -\varphi(t, L) + 2\dot{Z}(t) \tag{39}$$

$$\ddot{X}(t) = -\alpha \left[X(t) - X(t - t_N(t))\right] - \beta \dot{X}(t) + \gamma u(t, L) \tag{40}$$
$$\ddot{Z}(t) = -\alpha' \left[X(t) - X(t - t_N(t))\right] - \beta' \dot{Z}(t) + \gamma' \psi(t, L) \tag{41}$$

where U, Φ are the control inputs, the delay $t_N(t)$ is implicitely defined by

$$Z(t) - Z(t - t_N(t)) = \frac{2\pi}{N} \tag{42}$$

and the parameters $\alpha, \alpha', \beta, \beta', \gamma, \gamma'$ are deduced from (27),(28). Additional boundary conditions should be derived for (32),(34) to be well-posed, similarly to Sagert et al. [2013].

5.6 Control problem

In Depouhon and Detournay [2014], a linear stability analysis of a lumped model shows the importance of the state-dependent delay $t_N(\cdot)$. Stabilization of state-dependent state delays systems has been investigated in Bekiaris-Liberis et al. [2012]. There, it is claimed that the result extends to systems with both state and input delays, which corresponds to (32)–(42). However, the result of Bekiaris-Liberis et al. [2012] uses measurements of the ODE states X and Z. In the case of drilling, it is more realistic to assume that only topside sensors are available. The problem of stabilizing a DDE with input delay remains, to our best knowledge, an open problem.

6. CONCLUSION

PDE control is a rapidly developing field, in particular thanks to Lyapunov and backstepping design methods, that complement pre-existing techniques (spectral, characteristics-based, etc.). This is particularly meaningful for the oil industry where, for a lot of problems discussed here, the distributed nature of the problem cannot be ignored: long delays, high frequency and underactuation / lack of sensors make PDE estimation and control particularly relevant. The models discussed in this paper can all be written under the same generic form

$$\frac{\partial}{\partial t}\begin{bmatrix} u \\ v \end{bmatrix}(t, x) + \begin{bmatrix} \Lambda & 0 \\ 0 & -M \end{bmatrix} \frac{\partial}{\partial x}\begin{bmatrix} u \\ v \end{bmatrix}(t, x) = E\begin{bmatrix} u \\ v \end{bmatrix}(t, x) \tag{43}$$
$$u(t, 0) = \rho v(t, 0) + U(t) \tag{44}$$
$$v(t, L) = q u(t, L) + C X(t) \tag{45}$$
$$\frac{\mathrm{d}X}{\mathrm{d}t} = AX(t) + A_1 X(t - t_N(X(t)) + Bu(t, 0) \tag{46}$$

With $\Lambda = \mathrm{diag}(\lambda_1, \ldots), M = \mathrm{diag}(\mu_1, \ldots) > 0$. illustrated on Figure 5.

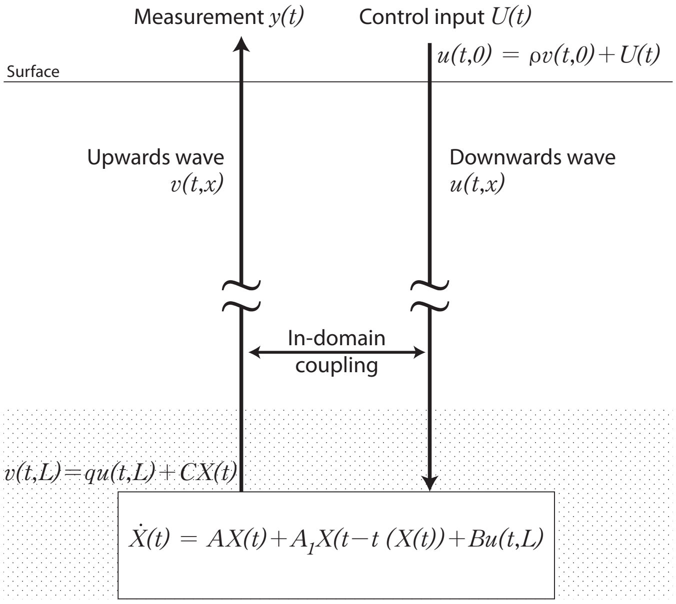

Fig. 5. Schematic view of the PDE control problems discussed in this paper.

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