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Analysis of the Role of Energy Storage in Power Markets with Strategic Players

Vegard Skonseng Bjerketvedt

Master of Energy and Environmental Engineering
Submission date: February 2017
Supervisor: Magnus Korpås, IEL

Norwegian University of Science and Technology
Department of Electric Power Engineering

Preface

This thesis is written at the Department of Electrical Power Engineering at the Norwegian University of Science and Technology (NTNU) under the supervision of Professor Magnus Korpås and Ph.D. candidate Martin Kristiansen in cooperation with SINTEF Energy Research AS. I would like to extend my gratitude to my supervisor Magnus Korpås for the enthusiastic guidance and support during the entire thesis work. I would also like to thank Martin Kristiansen for feedback and interesting views on this thesis. Steven Gabriel for an intense and informative course in complementarity models and equilibrium. Anna Guan also deserves gratitude for linguistic improvements and help during the entire work.

Trondheim, February 3, 2017.

Vegard Skonseng Bjerketvedt

Abstract

Energy storage has gained increasing popularity in both industry and research during the last decade, due to its valuable flexibility service for power systems. Some claim that energy storage may have a central role in the European power system towards a cost-efficient de-carbonization. In order to gain useful insight regarding the incorporation of energy storage technologies in large-scale market simulators and investment models, we present a thorough evaluation of its material impact on market prices and welfare. A storage facility is modelled under perfect competition and imperfect competition, in order to study the effects of potential strategic behavior at the supply side of an energy only market.

The objective of this Master's thesis is to investigate the role of energy storage in power markets with strategic players. The power market and the players` strategies are modeled by applying complementarity theory. The models are formulated as Mixed Complementarity Problem (MCP) and Mathematical Program with Equilibrium Constraints (MPEC), which is developed to mimic the strategic behavior of both conventional power generators and energy storages. Several simulations have been conducted in order to analyze the influence of strategic game of the energy storage, where the storage has either been operated as price setter or price taker.

A case study consisting of one generator and one energy storage unit is carried out in order to evaluate the effect of strategic behavior. This study reveals that the intra-day price variations get smoother as more storage capacity is added to the system. If the operator behaves strategic, it will exercise market power in order to increase its profits, but it is shown that the magnitude of market power is limited by the level of production capacity. At 93 % of the optimal production capacity, the energy storage facility can have a significant impact on market prices. During morning and evening peak demand, the market price increases from 40 EUR/MWh to 69 EUR/MWh due to strategic behavior in terms of withholding production capacity. The results point out the effects of strategic behavior of an energy storage in an imperfect power market. The proposed study has led to the conclusion that the qualitative effect of the ownership of the storage unit is clearly present. At the same time, the quantitative results emerge as realistic, but these are still heavily dependent on the underlying assumptions and input parameters.

Sammendrag

Energilagring har opplevd en økende interesse blant industri og forskning de siste årene på grunn av sin verdifulle fleksibilitet i kraftsystemet. Flere påstår at energilager vil spille en sentral rolle i fremtidens kostnadseffektive de-karboniserte europeiske kraftsystem. For å oppnå innsikt om implementeringen av energilagerteknologier i storskala markeds simulatorer og investeringsmodeller, presenteres det en grundig evaluering av påvirkningene på markedspriser og velferd. Et energilager er modellert i et marked med perfekt konkurranse og imperfekt konkurranse, for å kunne studere effekter av potensiell strategisk adferd på tilbudssiden av markedet.

Formålet med denne masteroppgaven er å undersøke rollen til et energilager i et kraftmarked med strategiske aktører. Kraftmarkedet og aktørene er modellert ved bruk av komplementaritet teori. Modellene er formulert som blandede komplementaritet problemer og matematiske program med likevekts begrensninger, som er utviklet for å etterligne den strategiske adferden til både konvensjonelle kraftprodusenter og energilager. Flere simuleringer har blitt gjennomført for å analysere påvirkningen av energilagerets strategiske spill, hvor lageret opererer enten som en prissetter eller en pristaker.

Et case-studie bestående av en generator og et energilager ble gjennomført for å vurdere effektene av strategisk spill. Studiet viser at prisvariasjonene reduseres når et energilager blir introdusert i kraftsystemet. Om lagerets adferd er strategisk, vil markedsmakt bli utøvd for å øke egen profitt. Det viser seg at størrelsesordenen på markedsmakten er tett knyttet til kapasitetsbegrensningene for produksjon. Ved 93% av optimal produksjons-kapasitet, har energilageret stor påvirkning på markedsprisen. Ved høy etterspørsel på morgenen og ettermiddagen øker markedsprisen fra 40 EUR/MWh til 69 EUR/MWh grunnet lagerets strategiske valg. Resultatet tydeliggjør påvirkningen strategisk spill av et energilager ved imperfekt konkurranse har. Studiet fører til konklusjonen om at det eksisterer en kvalitativ effekt på grunn av lagerets strategiske spill. De kvantitative resultatene fremstår som realistiske, likevel er disse avhengig av modellens antagelser og input parameter.

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Abbreviations

CS – Consumer Surplus
KKT – Karush-Kunh-Tucker
LP – Linear Program
MCP – Mixed Complementarity Problem
MPEC – Mathematical Program with Equilibrium Constraint
MSNE – Mixed Strategy Nash Equilibrium
NLP – Non-Linear Program
PC – Perfect Competition
PM – Profit Maximizing
PS – Producer Surplus
PSNE – Pure Strategy Nash Equilibrium
QP – Quadratic Programs
RES – Renewable Energy Sources
SFEM – Supply Function Equilibrium Models
SS – Social Economic Surplus
TSO – Transmission System Operator

1 Introduction

1.1 Thesis Motivation and Description

Renewable energy sources have lately gained much attention in the energy sector due to their vast potential in reducing the dependence on fossil fuels. There has been an increased call for the technology development of renewable energy sources because of the issues regarding climate change concerns as well as consumer efforts. Sources such as wind and solar are considered as climate friendly, but the drawbacks of these sources are the variable and uncertain generation. The variability of these sources leads to the deployment of energy storage as an essential component of future energy systems.

To determine the potential role of energy storage in the energy system of the future, it is important to examine economic impacts in developing such systems. There have been conducted several studies on how energy storage can be utilized in an effective way in a power system. A common feature on these studies is the assumption of perfect competition, which suggests that all market players operate as price takers. Assuming perfect competition implies that the market participants' expect they have no influence on the market price, which is not always the case. As a result of this, the assumption may limit the reliability of the outcome of a power market to some extent. Hence, the role of strategic players on energy storage has to be further examined.

This Master's thesis is a continuation of the project thesis "*Analyzing the Investment Impact of Strategic Player with Market Power*." Reduced investment cost in energy storage makes storage technologies highly relevant for future power systems. The benefits associated with generating flexibility will therefore be valued. The debate and analysis of the utilization of the energy storage are often done under ideal conditions such as perfect competition, where no form of strategic behavior exists. This thesis will therefore focus on including the role of strategic players.

To the author's knowledge, the attempt to model a strategic behaving energy storage unit has not previously been studied before. Previous relevant work rather focus on analyzing how energy storages can reduce market power in a monopoly. (Yujian Ye, et al., 2016)

The role of energy storage in power markets with strategic players will be analyzed by applying complementarity theory. The models are formulated as Mixed Complementarity Problem (MCP) and Mathematical Problem with Equilibrium Constraints (MPEC), which are developed in order to mimic the strategic behavior of both conventional generating firms and energy storage firms.

The main objectives of the thesis are therefore to:

- Analyze the effects of imperfect competition in a power market
- Examine the effects of energy storages units in a power market
- Investigate the effects of different operating strategies of the energy storage unit in a power market

1.2 Modeling Software

General Algebraic Modelling System (GAMS) is the modeling tool applied in this thesis. GAMS is a high-level modeling system for optimization and mathematical programming. The system is tailored for complex, large scale modeling applications, and allows to build large models that can be reformulated for new model instances.

For the MCPS, the Path solver has been applied, and for the MPEC, KNITRO 10.0 are used.

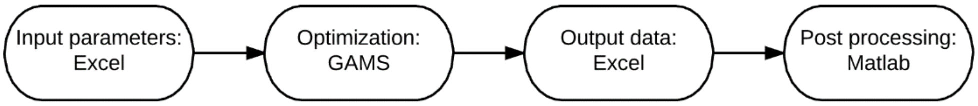

Figure 1-2 Data processing

Input parameters are exported from Excel to GAMS where the model gets solved. The results from the optimization are exported to a new Excel file, which is further exported to Matlab for post-processing of the data.

2 Theory

2.1 Power Markets

2.1.1 Energy Management and Markets

The power market is the arena where the supply and demand side of a market meet. Each representative has their objective, and together the representatives find a joint solution, also known as a market equilibrium. The overall purpose of arranging a market is to seek for an efficient allocation of resources.

Electricity systems comprise several physical challenges compared to other commodity markets. The electricity has to be consumed and generated at the same time, requiring a continuous flow of energy. Moreover, the consumption has significant seasonal and intra-day variations, whereas the production cost of conventional energy has an increasing marginal cost as well as capacity constraints. The capacity of storing a large amount of energy is also highly restricted and expensive. However, electricity is still considered as inevitable for most of the society. The lack of flexibility in both production and consumption will, therefore, be potentially abused by firms excreting market power in a deregulated energy market.

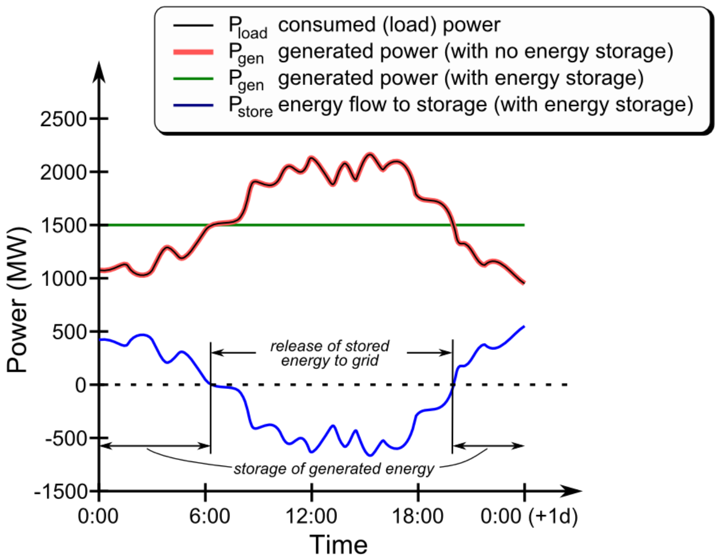

Figure 2-1 Balancing demand and supply *(Wikichesteredit, 2017)*

Figure 2-1, shows a simplistic and illustrative view of the balance of the consumption and production, with and without the possibility of storing energy. The increasing penetration of renewables will increase the variation in generation profile, as shown in the red graph. Moreover, the blue graph in Figure 2-1 represents the operation of energy storage; this illustrates the benefit of energy storage; the energy can be stored at low prices and excess capacity, oppositely the storage will generate power at high demand.

2.1.2 Market Power

By analyzing the liberalization process of the European power markets, the question of the existence of market power and its influence appears to be particularly relevant. The market shares of the largest generating firm are presented in Table 1. From 2000 to 2014, there has been a reduction in the largest firms' market shares as a result of the liberation process of the European power market.

Country	Market share of largest generating firm in the market in % (2000)	Market share of largest generating firm in the market in % (2014)	Number of main electricity generating companies with +5% market share (2003)	Number of main electricity generating companies with +5% market share (2014)
Belgium	91.1	59.8	2	2
Denmark	36.0	36.6	2	3
Germany	34.0	32.0	4	4
Spain	51.8	23.8	5	4
France	90.2	86.8	1	2
Italy	46.7	29.0	4	3
Sweden	49.5	42.9	3	3
UK	20.6	29.3'	6	7
Norway	30.6	30.5	6	3

Table 1 Market share and number of main electrical generating companies' in power markets I EU *(eurostat, u.d.)*

The number of main electricity generating companies with a market share above 5 % has fluctuated the last decade, without a clear up- or downward trend. The amount of the major generating firms in each market varies from each European power market. Despite a long liberation process, the statistics indicate that there may be a possibility for the different utility firms to exert market power.

In recent years, the effects of market power have received increasing attention. Imperfect competition and game theory have been introduced to the investment and marked clearing models (Ventosa, et al., 2005). Limited competition can arise for several reasons. The most common issues are due to limited transmission and capacity of production. Lise suggests the effect of market power in a fully liberalized European power market could have a price response up to 20% higher than marginal costs, caused by dry weather and transmission constraints (Lise, et al., 2008).

Moreover, the demand for electricity is relatively inelastic. The consumers are dependent on their power consumption and are therefore willing to pay a high price despite only a moderate decrease in consumption. Estimates of the elasticity of the Nordic power markets are roughly 0.025-0.035 (Arve Halseth, 1998). Halseth estimates are relatively conservative. Gribkovskaia suggests a demand elasticity on the interval 0.025-0.10 for the Norwegian power market (Gribkovskaia , 2015). The elasticity of demand is not a fixed economical parameter. The literature makes clear distinctions between short-run and long- run elasticity. Electricity is highly inelastic in the short-run, which justifies the Halseth and Gribkovskaias low estimates of the elasticity of demand (Anon., 2017). In the long-run electricity could be substituted with natural gas, and the demand become elastic (Ros, 2015). In the development of the future power system, i.e. through the evolving SmartGrids, demand responses are predicted to play a key role by several researchers, resulting in a more elastic short-run demand (Ros, 2015).

2.2 Economic Theory

This section is a brief introduction to the economic theory, which explains the necessary theory and foundation for modeling energy markets. This section covers theory about demand curves and elasticity, perfect competition as well as oligopoly and Cournot competition. The theory is presented superficially, and are meant to be demonstrative.

2.2.1 The Inverse Demand Curve and Demand Elasticity

Modeling the demand for one product can be done in several ways. The fixed-price scenario is when consumers are willing to pay regardless of the quantum, while fixed-quantum is referred to when customers have the infinite willingness to pay for a particular quantum of the good. Fixed-price and fixed-quantum are both individual extreme cases of the inverse demand curve representation of consumer behavior.

The downward sloping demand curve, Figure 2-2 Downward sloping inverse demand curve with and without price cap, represents the property of decreasing marginal utility of consumption, also known as the law of demand. The equation for the linear inverse demand curve is expressed as:

$$p(q) = a - b \cdot q \qquad (1)$$

where $p(q)$ denotes the price the consumers are willing to pay for the quantity demanded q. The parameter a represents the intersection on the y-axis, and parameter b is the slope of the demand curve.

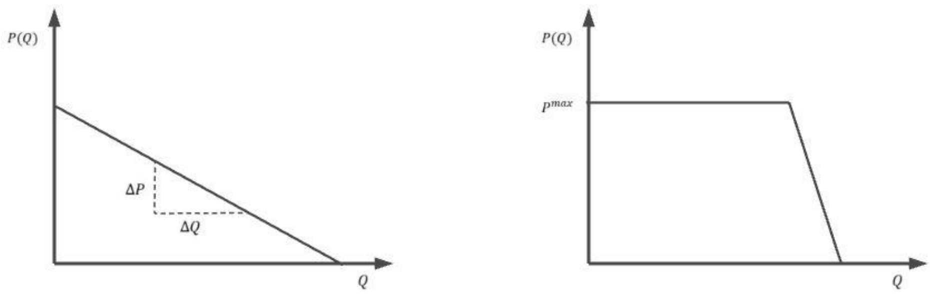

Figure 2-2 Downward sloping inverse demand curve with and without price cap

The elasticity of demand ε, represents the relative change in quantum to the change in price.

$$\varepsilon = \frac{\Delta Q/Q}{\Delta P/P} \quad (2)$$

Elasticity is the measurement of the responsiveness of the consumers due to changes in price on quantum. The constant b is the slope of the linear inverse demand curve. However, the elasticity ε varies along the curve,

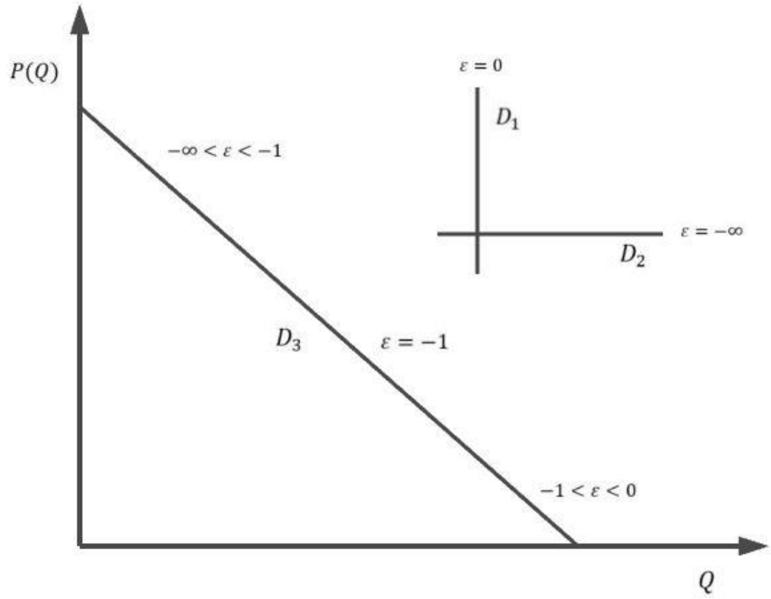

Figure 2-3 Elasticity of demand

Figure 2-3 shows different scenarios of demand and their corresponding elasticity. Demand is considered as perfectly inelastic when the demand curve is vertical as D_1, where there are no substitutes for the good and the consumers will buy regardless the market price. D_2 illustrates a perfectly elastic demand curve, where there are perfect substitutes for the product and the willingness to pay is equal regarding the quantity consumed. D_3 is an inverse downward sloping demand curve. In the upper left corner, the elasticity of demand is relatively elastic, this means that one percentage in change will cause an even larger percentage change in the quantum consumed. Equation (2) explains the relationship of a decreasing elasticity of demand as a result of increased quantum along the linear inverse demand curve. Thus, at one point the elasticity will become unit elastic until it turns relatively inelastic.

2.2.2 Perfect Competition

Perfect competition is a market structure where there exists a significant number of firms that supply a strictly homogeneous good to a large number of consumers. All market participants are rational and perfectly informed. Each firm faces a perfectly elastic demand curve, see D_2 in Figure 2-3 Elasticity of demand. The firms are profit maximizing agents without any influence on the market price. Hence, they set their price to marginal cost of production.

$$\Pi_i = p \cdot q_i - C_i(q_i) \tag{3}$$

$$\frac{\partial \Pi_i}{\partial q_i} = p - MC_i(q_i) = 0 \tag{4}$$

There are no long-run economic profits, but in the short-run, there may exist possibilities of both profits or losses. Several economists criticize perfect competition for modeling agents as passive, resulting in an unrealistic outcomes. It is naïve to assume that firms with a significant market share believes that their decisions have no impact on the market (Hogan, 2011).

2.2.3 Oligopoly and Cournot Competition

In an oligopolistic market structure, there are a small number of competing firms. Homogeneity of the good is not required, and there are possibilities of entry barriers into the market. Under this structure, the firms recognize their impact on the market and will exercise market power to maximize profits.

The most applied market structure for modeling imperfect competition in power markets is Cournot competition (Ventosa, et al., 2005). Each firm i faces a cost function $C_i(q_i)$, where q_i is the quantity produced by the firm. Equation (1) represent the inverse demand curve, where $q = \sum_i^n q_i$ is the consumed quantity and equation (5) is firm i`s profit.

$$\Pi_i = p(q) \cdot q_i - C_i(q_i) \tag{5}$$

Assuming perfect information, each firm knows the competing firms' response to every possible strategy. The Cournot firms supply a quantity which is the best response to every other firm's known strategy. By the deriving, the reaction functions of each firm, the Cournot game gets solved analytically.

The Cournot game can be represented as a duopoly game with two symmetric firms A and B, facing the inverse demand curve, equation (1), and a constant marginal cost of production c. Their profit functions are given by:

$$\Pi_A = p(q_A + q_B) \cdot q_A - C_A(q_A) \tag{6}$$
$$\Pi_B = p(q_A + q_B) \cdot q_B - C_B(q_B) \tag{7}$$

Maximizing each individual profit Π_i with respect to the supplied quantity q_i:

$$\frac{\partial \Pi_A}{\partial q_A} = a - 2b \cdot q_A - b \cdot q_B - c = 0 \tag{8}$$
$$\frac{\partial \Pi_B}{\partial q_B} = a - 2b \cdot q_B - b \cdot q_A - c = 0 \tag{9}$$

By inserting equations (8) into equation (9) and solve for q_A and q_B, the firms' reaction functions are then:

$$q_A^*(q_B) = \frac{a-c}{2b} - \frac{q_B}{2} \tag{10}$$
$$q_B^*(q_A) = \frac{a-c}{2b} - \frac{q_A}{2} \tag{11}$$

Note that the reaction functions are decreasing as the competitor increases its supply. For this symmetric Cournot game, the market equilibrium is:

$$(q_A^*, q_B^*, p^*) = \left(\frac{a-c}{3b}, \frac{a-c}{3b}, \frac{a+2c}{3}\right) \tag{12}$$

The equilibrium of the game is a Nash equilibrium, which is an intersection of the two reaction functions. In equilibrium, none of the players have any economic incentive to change their output. Nash equilibriums are therefore considered as strong in game theory, see section 2.3.2 The Solution Concept and Equilibrium.

2.2.4 Different types of market competition

Other models for oligopolistic competition are the Stackelberg structure. The difference between the Stackelberg game and the Cournot game is that under the Stackelberg game, one firm acts as leader, exploiting the first mover advantage, and play the optimal quantity knowing the reaction of the following firms`, who act as Cournot players. Under both market structures, the firms' decision variables are quantity.

Under Bertrand competition, the firms play a price, and the price determines the quantity of demanded. Bertrand competition is also applied for modeling imperfect competition in power markets, but often not preferred cause the equilibrium price will be the marginal cost of production when firms are symmetric (Ventosa, et al., 2005).

Cournot and Bertrand competition are special cases of Supply Function Equilibrium Models (SFEM). In SFEM the agent game in both price and quantity, offering block bids to the market. SFEM has large challenges caused by complexity and computational time. Despite the realistic representation of the behaviour of the producers, Cournot behaviour is often the preferred representation of the producer. (Ventosa, et al., 2005).

Cournot, Stackelberg and Bertrand competition have this common assumption; no forms of a corporation. By limiting quantity supplied or introducing a price floor, the cartel members could exercise market power. The economic incentives for the oligopoly players to achieve a monopoly solution through corporation are often not present, and the strategy "cheating" will often be dominant.

2.2.5 Monopoly

A firm is defined as a monopolist if the firm has the total control of the markets' total supplied quantum. The markets' inverse demand curve is the monopolists` residual demand. By merging firm A and B and maintaining all assumptions from Oligopoly and Cournot Competition, the Cournot game becomes a monopoly. The monopolist profit function and the first-order condition are given by:

$$\Pi = p(q) \cdot q - C(q) \tag{13}$$

$$\frac{\partial \Pi}{\partial q} = a - 2b \cdot q - c = 0 \qquad (14)$$

The monopolist increases its supplied quantity until the marginal revenue equals marginal cost of production, profit is maximized. The market equilibrium of the monopoly is:

$$(q^*, p^*) = \left(\frac{a-c}{2b}, \frac{a+c}{2}\right) \qquad (15)$$

The increased market power leads to a less efficient market equilibrium with price higher then marginal cost of production and a low supplied quantity compared to the other market structures.

Figure 2-4 presents the different market structures' explained in this section and the firms' respective decision variables.

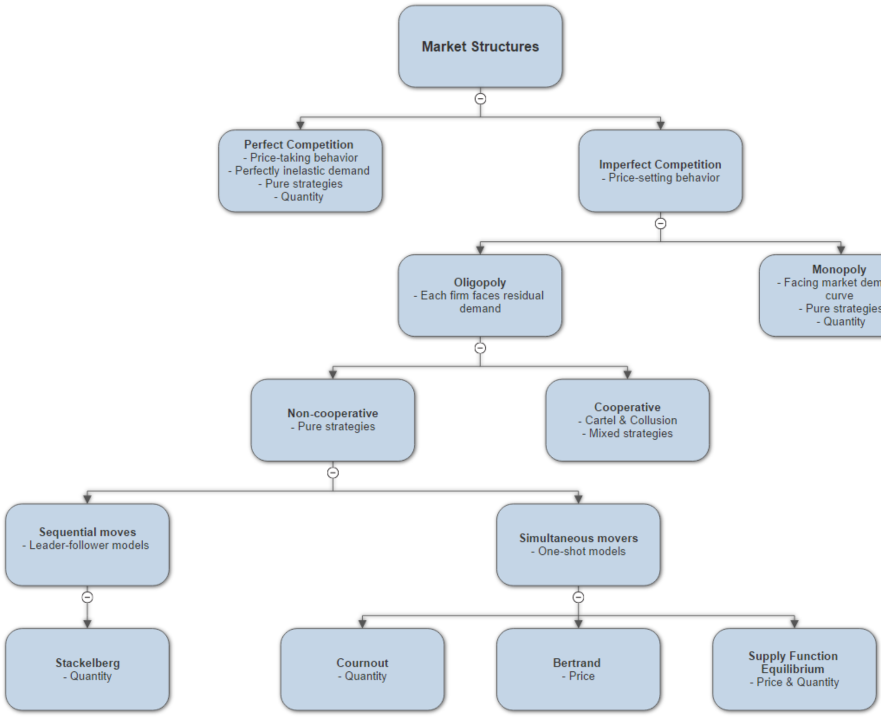

Figure 2-4 Market structures and decision variables

2.2.6 Social Economic Welfare and Dead Weight Loss

Social economic surplus (SS) is the micro economic measurement for the efficiency of resource allocation An increased SS is an unconditional improvement of the allocation of resources. Consumer surplus (CS) and producer surplus (PS) are the two components of SS, where SS is expressed as:

$$SS = CS + PS \tag{16}$$

Where the consumer surplus is given by:

$$CS = \int_0^{q^*} p(q) - p^* \, dq \tag{17}$$

This value represents the total benefit the consumer experiences for consuming quantity $q^*$to the market price p^*, the consumers' willingness to pay subtracting the actual payment.

The producer surplus is given by:

$$PS = \int_0^{q^*} p^* - MC(q) \ dq \tag{18}$$

The producer surplus is also known as profits, which is the total revenue subtracting the cost of production. Figure 2-5, illustrates the market equilibrium.

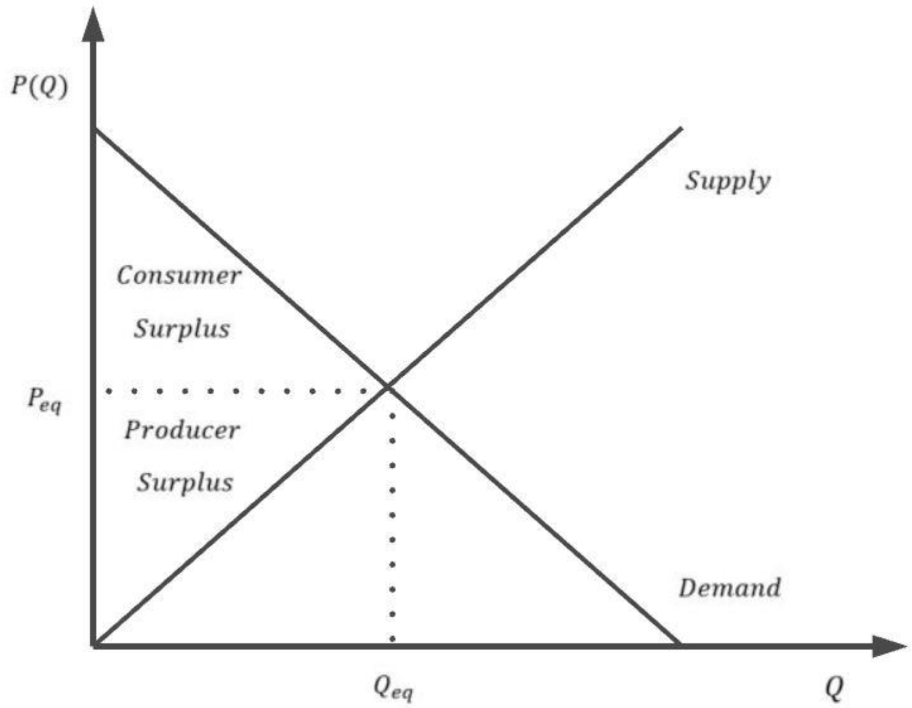

Figure 2-5 Market Equilibrium

Externalities are the economic costs which are the deviation from the perfect competition market equilibrium. Market power is an externality; the profits rise, the consumer surplus declines and the socio economic surplus is reduced. Deadweight loss is the reduction in SS, and the cost related to the externality.

The market equilibriums vary widely due to their associated market structure, as shown in Table 2 Market equilibriums under imperfect competition, present both the theoretical and the analytical equilibriums of a special case for the different market structures. The negative effect on SS increases with the externality market power. The interaction between or the lack of market participants prevents an effective resource allocation.

Competition	P	q_A	q_B	q_{tot}	π_A	π_B	π_{tot}	SS
Perfect	c	–	–	$\frac{a-c}{b}$	0	0	0	$\frac{(a-c)^2}{2b}$
Bertrand	c	–	–	$\frac{a-c}{b}$	0	0	0	$\frac{(a-c)^2}{2b}$
Cournot	$\frac{a+2c}{3}$	$\frac{a-c}{3b}$	$\frac{a-c}{3b}$	$\frac{2(a-c)}{3b}$	$\frac{(a-c)^2}{9b}$	$\frac{(a-c)^2}{9b}$	$\frac{2(a-c)^2}{9b}$	$\frac{4(a-c)^2}{9b}$
Stackelberg	$\frac{a+3c}{4}$	$\frac{a-c}{2b}$	$\frac{a-c}{4b}$	$\frac{3(a-c)}{4b}$	$\frac{(a-c)^2}{8b}$	$\frac{(a-c)^2}{16b}$	$\frac{3(a-c)^2}{16b}$	$\frac{15(a-c)^2}{32b}$
Monopoly	$\frac{a+c}{2}$	–	–	$\frac{a-c}{2b}$	–	–	$\frac{(a-c)^2}{4b}$	$\frac{3(a-c)^2}{8b}$
Perfect	4	–	–	48	0	0	0	4608
Bertrand	4	–	–	48	0	0	0	4608
Cournot	36	16	16	32	512	512	1024	2048
Stackelberg	28	24	12	36	576	288	864	2160
Monopoly	52	–	–	24	–	–	1152	1728

Table 2 Market equilibriums under imperfect competition, a = 100, b = 2 and c = 4

2.3 Game Theory

This sections covers an introduction to relevant the game theory. The basic idea behind game theory is to analyse the outcome of the behaviour of the participating agents. The first subsection presents a brief introduction to fundamental game theory, followed by a subsection about the concept of equilibrium.

2.3.1 Fundamental Game Theory

Game theory is "the study of mathematical models of conflict cooperation between intelligent rational decision-makers" (Meyerson, 1991). The field has been under tremendous development the last decades; game theory is a useful tool when analyzing the effects of the market participants' behavior.

A game is defined by a set of players, $N = \{1:n\}$. Each player $i \in I$ faces a number of strategies x_i, where these are player $i`s$ set of pure strategies. A pure strategy provides a complete definition of how a player behaves, it determines the decisions a player will make for all situations. Each player's strategy set is the set of available pure strategies. The collection of all possible pure strategies is given by:

$$X = X_1 \times X_2 \times \ldots \times X_n = \{ (x_1, x_2, \ldots, x_n) | x_i \in X_i \} \tag{19}$$

The utility function $u_i(x)$ is defined for every player i. The input in the utility functions are the strategy set x corresponding player i preferred strategy and the other players' response to $i`s$ played strategy.

$$x = x_1, x_2, \ldots, x_n \in X \tag{20}$$

A cornerstone in game theory is the rationality assumption, where an agent will always maximize its own utility. This agent will under no circumstances play a strategy leaving the agent itself worse off. The output of the utility function is always quantified as a real number $u_i(x) \in R$.

2.3.2 The Solution Concept and Equilibrium

A solution concept is defined as a set of rules for the players' actions. Thus, a solution concept is used to predict the outcome of a defined game. The most applied solution concept is the concept of equilibrium. There exist several forms of equilibrium, where these are classified according to the likelihood to hold. The strongest form is when there exists a dominant strategy among the players. Strategy x_i is dominant, if and only if, it yields the highest utility regardless of all other players' actions.

$x_i \in X$ is dominant if $u_i(x_{i,}, x_{-i}) \geq u_i(x_i^*, x_{-i})$ for all $x_i^* \in X_i$ and $x_{-i} \in X_{-i}$, where $\mathrm{x_{-i}}$ is the set of all vectors of the pure strategies with the i`th element removed.

If $u_i(x_i, x_{-i}) \geq u_i(x_i^*, x_{-i})$, the strategy will be strictly dominant. If $u_i(x_i, x_{-i}) \geq u_i(x_i^*, x_{-i})$ and $u_i(x_i, x_{-i}) > u_i(x_i^*, x_{-i})$ for a minimum of one $x_i \in X_i$ the strategy will be weakly dominant (Lamberson, 2009).

A dominant strategy will always be the outcome if it exists. This is due to the fact that utility maximization is independent on the other players' strategies. In many games, a dominant strategy does not exist. In the absence of a dominate strategy, a weaker form of equilibrium occurs. The best response to $x_{-i} \in X_{-i}$ is the pure strategy x_i if and only if $u_i(x_i, x_{-i}) \geq u_i(x_i^*, \mathrm{x_{-i}})$ for all x_i^* the equilibrium will be denoted as a Pure Strategy Nash Equilibrium (PSNE) if $\mathrm{x_i}$ is the best response to $\mathrm{x_{-i}}$ for all i. There exists a PSNE when none of the agents has an incentive to change their behavior, at in the Cournot game in section 2.2.3

Another form of Nash equilibrium is the Mixed Strategy Nash Equilibrium (MSNE). Assuming that the players are only interested in average return, the players can choose to play a mix of pure strategies with an optimal probability distribution instead of playing a pure strategy with the probability of one. The game rock-paper-scissor has a MSNE. In such a game there will be no dominant strategies or a PSNE. A randomized sequence of the three pure strategies with the probability distribution $p = {}^1\!/_3$ will gain the highest average return in this game, which classifies the equilibrium solution as a MSNE.

2.4 Complementarity Modeling

Due to the deregulation of energy markets complementarity modeling has gained an increasing popularity in search for a robust modeling approach of strategic behavior to aid decision-makers. The complementarity modeling generalizes linear programs (LP), convex nonlinear programs (NLP) and convex quadratic programs (QP). The optimality constraints of the problems are given by the Karush-Kuhn-Tucker (KKT) conditions for the different agents'.

2.4.1 Nonlinear Optimization and KKT Conditions

The KKT conditions are the optimality requirements of the first order conditions for a solution in nonlinear programming. By allowing inequality constraints, the KKT approach is a generalization of the Lagrange multipliers approach to nonlinear programming. In this section, the necessary and sufficient conditions will be presented.

The standard form formulation of a nonlinear optimization program is given below:

$$max\, f(\boldsymbol{x}) \quad (21)$$

$$Subjct\ to:$$

$$\forall i: g_i(\boldsymbol{x}) \leq b_i \quad (22)$$

$$\boldsymbol{x} \geq 0 \quad (23)$$

Vector $\boldsymbol{x} = (\boldsymbol{x}_1, x_2, \dots, x_{\boldsymbol{n}})$ is the solution vector to the nonlinear program, where $f(\boldsymbol{x})$ is the objective function and $g_i(\boldsymbol{x})$ constraints. Table 3 Necessary and sufficient conditions for optimality summarizes the necessary and sufficient conditions for optimality of the program.

Problem	***Necessary Conditions for Optimality***	***Also Sufficent if***
One − variable unconsstrained	$\frac{df}{dx} = 0$	$f(x)$ *concave*
Multivariable unconstrained	$\frac{df}{dx_j} = 0 \quad (j = 1,2,\dots,n)$	$f(\boldsymbol{x})$ *concave*
Constrained, nonnegativity constraints only	$\frac{df}{dx_j} = 0 \quad (j = 1,2,\dots,n)$ or ≤ 0 if $x_j = 0$	$f(\boldsymbol{x})$ *concave*
General constrained problem	*Karush − Kuhn − Tucker conditions*	$f(\boldsymbol{x})$ *concave and* $g_i(\boldsymbol{x})$ *convex for* $(i = 1,2,\dots,m)$

Table 3 Necessary and sufficient conditions for optimality *(Hiller & Liberman, 2010)*

Theorem. (Hiller & Liberman, 2010) Assume that *f(**x**)*, *g₁(**x**)*, *g₂(**x**)*, . . ., *g_m(**x**)* are differentiable functions satisfying certain regularity conditions. Then $\mathbf{x}^*$ = (x_1^*, x_2^*, . . . , x_n^*) can be an optimal solution for the nonlinear programming problem only if there exist *m* numbers u_1, u_2, ..., u_m such that all the following KKT conditions are satisfied:

$$\forall j: \frac{\partial f}{\partial x_j} - \sum_{i=1}^{m} u_i \frac{\partial g_i}{\partial x_j} \leq 0, at\ \boldsymbol{x} = \boldsymbol{x}^* \quad (24)$$

$$\forall j: x_j^* \left(\frac{\partial f}{\partial x_j} - \sum_{i=1}^{m} u_i \frac{\partial g_i}{\partial x_j} \right) = 0, at\ \boldsymbol{x} = \boldsymbol{x}^* \quad (25)$$

$$\forall i: g_i(x^*) - b_i \leq 0 \quad (26)$$

$$\forall i: u_i \cdot [g_i(x^*) - b_i] = 0 \quad (27)$$

$$\forall j: x_j^* \geq 0 \quad (28)$$

$$\forall i: u_i \geq 0 \quad (29)$$

Corollary. Assume that *f(**x**)* is a concave function and that $g_1(\mathbf{x})$, $g_2(\mathbf{x})$, . . ., $g_m(\mathbf{x})$ are convex functions (i.e., this problem is a convex programming problem), where all these functions satisfy the regularity conditions. Then $\mathbf{x}^*$ = (x_1^*, x_2^*, . . . , x_n^*) is an optimal solution if and only if all the conditions of the theorem are satisfied.

In special cases the problem can be solved analytically, if a closed-form solution can be derived. Generating a solution from the KKT conditions are usually done through an optimization algorithm and solved by numerically methods.

2.4.2 Linear Complementarity Problems

Linear Complementarity Problems (LCP) are exclusively linear, contains only decision variables and exogenous parameter. The LCP are denoted $LCP(M, q)$ for a given matrix $M \in R^{nxn}$ and vector $q \in R^n$, and seeks a vector $x \in R^n$ satisfying the following constraints:

$$x \geq 0 \quad (30)$$

$$Mx + q \geq 0 \quad (31)$$

$$x^T \cdot (Mx + q) = 0 \quad (32)$$

2.4.3 Non-linear Complementarity Problems

Unlike the LCPs, the restriction in a Nonlinear Complementarity Problem (NCP) is nonlinear. NCP with respect to a mapping $f: R^n \to R^n$, denoted as $NCP(f)$, in order to find the vector $x \in R^n$ such that:

$$x \geq 0 \tag{33}$$

$$f(x) \geq 0 \tag{34}$$

$$x^T \cdot f(x) = 0 \tag{35}$$

2.4.4 Variational Inequalities

"Variational inequality (VI) theory permits to formulate and analyze a variety of equilibrium problems. The theory provides qualitatively analyzing the problems in terms of existence and uniqueness of solution, stability and sensitivity analysis, and providing us with algorithms with accompanying convergence for computational purposes" (Nagurney, 2016)

Defining a Variational Inequality Problem, the finite dimensional variational inequality problem $VI(F, K)$ has to determine a vector $x^* \in K \to R^n$, such that:

$$\forall x \in K: F(x^*)^T \cdot (x - x^*) \geq 0 \tag{36}$$

or, equivalently

$$\forall x \in K: \langle F(x^*)^T, x - x^* \rangle \geq 0 \tag{37}$$

where F is given as a continuous function from $K \to R^n$, K is a given closed convex set, and $\langle ., . \rangle$ denotes the inner product in n dimensional Euclidean space.

2.4.5 Mixed Complementarity Problems

Mixed Complementarity Problems (MCP) consist of equality, inequality and complementarity constraints. MCPs does not have any form of an object function, only constraints. LCPs, NCPs and VIs can be expressed and solved as a MCP.

The MCP formulation is particularly dexterous when solving multiplayer games using mathematical modeling. It is different from standard optimization techniques as MCPs is solved by satisfying all optimality requirements given by the KKT conditions to the problem, see Figure 2-6 MCP, structure . The solution of a MCP formulation generates the optimal solution for all players in a multi-player game simultaneously by determining the value of each complementarity variable with respect to its complementarity constrain

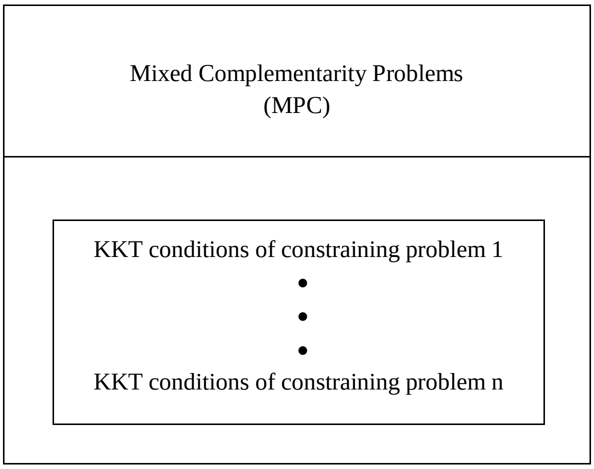

Figure 2-6 MCP, structure *(Gabriel, et al., 2013)*

By defining the Lagrangian function to each player, each player's individual optimization problem and deriving the optimality conditions, the MCP can be formulated. The aggregated KKT conditions of the multi-player game represents the optimization for all players in the equilibrium problem.

2.4.6 Mathematical Programs with Equilibrium Constraints

Mathematical Programs with Equilibrium Constraints (MPEC) are a model class where the object function is one single players' optimization problem subject to own constraints and the optimality constraints from other equilibrium problems, see Figure 2-7 MPEC structure .

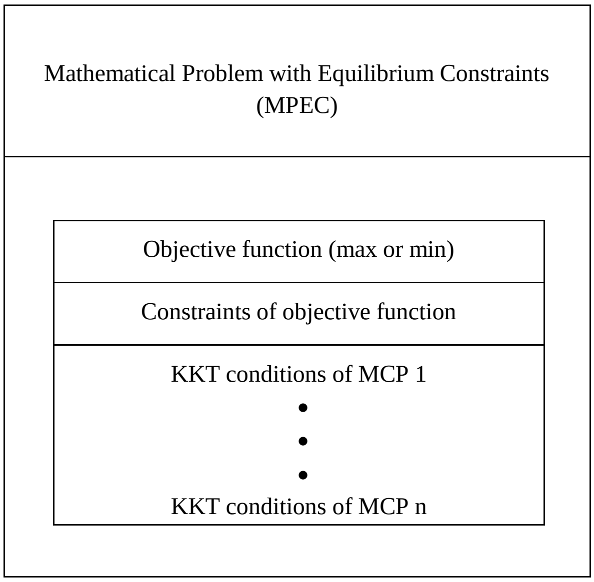

Figure 2-7 MPEC structure *(Gabriel, et al., 2013)*

The bi-level Stackelberg game from section 6.2.4 Different types of market competition can be formulated as an MPEC. The objective function of the Stackelberg leader will be the top-level, where the bottom -level will be the optimality constraints of the rest of the market participants.

MPECs are difficult and computationally challenging to find a unique optimal solution, as a result of the problem in general are non-convex and non-differentiable, and the FOCs are not sufficient for optimality. (Midthun, 2007)

3 Methodology

Modeling power markets is challenging, combining the physical laws of electricity and the interactions between the market participants creates complex scenarios. In this chapter, the models are explained in detail. Each agents' optimality constraints are derived by applying complementarity theory. Generating firms and energy storage companies change their behavior in the various scenarios, which is explained in the relevant subsections.

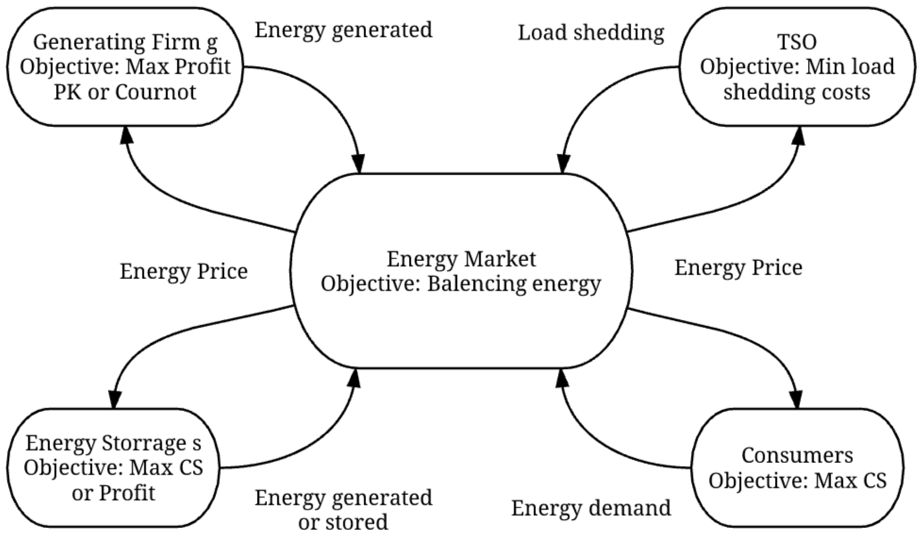

Figure 3-1 Model illustration, with agents' objective and decision variables

3.1 Declaration

Sets:

$g \in G$: $Set\ of\ generating\ firms$
$s \in S$: $Set\ of\ storage\ units$
$h \in H$: $Set\ of\ hours$

Parameters:

α $[-]$: $Cournot\ parameter$
CL_s $[-]$: $Converter\ efficiency$
W_s^{max} $[MW]$: $Power\ limits\ of\ storage\ unit\ s$
E_s^{max} $[MWh]$: $Energy\ capacity\ of\ storage\ unit\ s$
$V_g^{prod.cap}$ $[MW]$: $Production\ capacity\ for\ generating\ firm\ g$
$V_{g,h}^{ren}$ $[MW]$: $Renewable\ energy\ produced\ by\ generating\ firm\ g\ at\ hour\ h$
P^{cap} $[€/MWh]$: $Maximum\ price$
a_h^d $[€/MWh]$: $Constant\ benefit\ coefficient\ of\ demand\ at\ hour\ h$
b_h^d $[€/MWh^2]$: $Linear\ benefit\ coefficient\ of\ deman\ at\ hour\ h$
$b_{g,h}^c$ $[€/MWh^2]$: $Linear\ cost\ coefficient\ of\ generating\ firm\ g\ at\ hour\ h$
$c_{g,h}^c$ $[€/MWh^3]$: $Quadratic\ cost\ coefficient\ of\ generating\ firm\ g\ at\ hour\ h$

Primal Variables:

$\lambda_h[€/MWh]$: $Energy\ price\ at\ hour\ h$
$e_{s,h}^{stored}[MWh]$: $Amount\ of\ energy\ stored\ in\ unit\ s\ in\ hour\ h$
$w_{s,h}^{stored}$ $[MWh]$: $Energy\ stored\ by\ storage\ unit\ s\ in\ hour\ h$
$w_{s,h}^{gen}$ $[MWh]$: $Generation\ output\ of\ storage\ unit\ s\ in\ hour\ h$
$v_{g,h}^{conv}$ $[MWh]$: $Conventinal\ generation\ output\ by\ generating\ firm\ g\ in\ hour\ h$
ls_h $[MWh]$: $Amount\ of\ load\ shedding\ in\ hour\ h$
d_h $[MWh]$: $Demand\ in\ hour\ h$

Dual Variables:

$\xi_{s,h}[€hMWh]$: $Value\ of\ one\ unit\ of\ energy\ by\ storage\ s\ at\ hour\ h$
$\iota_{s,h}[€hMWh]$: $Scarcity\ rent\ of\ capacity\ of\ storage\ unit\ s\ at\ hour\ h$
$\mu_{g,h}[€hMWh]$: $Scarcity\ rent\ of\ capacity\ for\ generating\ firm\ g\ in\ hour\ h$
$\nu_{s,h}[€hMW]$: $Scarcity\ rent\ of\ converter\ capacity\ of\ storage\ s\ at\ hour\ h$

Functions:

$MC_{g,h}(v_{g,h}^{conv})$ $[€MWh]$: $Marginal\ cost\ of\ production\ for\ generating\ firm$

3.2 Demand Side

Electricity is a normal and strictly homogenous good. The consumers have a decreasing marginal benefit of consumption and the only preference is the price. Equation (38) presents the consumers benefit function and equation (39) the marginal benefit of consumption.

$$B_h(d_h) = \int MB_h(d_h) \tag{38}$$

$$MB_h(d_h) \ = a_h^d - b_h^d \cdot d_h \tag{39}$$

The consumers are represented by one rational optimization agent, maximizing consumer surplus over all hours h, thus the total benefit subtracting purchased costs is then:

$$max\ CS = \sum_{h \in H} B_d(d_h) - \lambda_h \cdot d_h \tag{40}$$

$$Subjct\ to:$$

$$\forall d: d_h \geq 0 \tag{41}$$

The quantity $d_h$is the decision variable, which is non-negative. The price λ_h are determined by the market clearing where the consumers act as price-takers, with no influence on the price. The consumers will always behave as price-takers in this model. However, consumer behavior will change through demand elasticity, the responsiveness to the price.

The Lagrangian function is given by:

$$\mathcal{L} = \sum_{h \in H} B_h(d_h) - \lambda_h \cdot d_h \tag{42}$$

The KKTs with respect to d_h:

$$\forall h: MB_h(d_h) - \lambda_h \leq 0 \tag{43}$$

$$d_h \geq 0 \tag{44}$$

$$\forall h: (MB_h(d_h) - \lambda_h) \cdot d_h = 0 \tag{45}$$

The consumer will increase its demand d_h as long marginal benefit $MB_h(d_h)$ do not exceeds the market price λ_h, equation (43).

3.3 Energy Market

The energy market is driven by the forces of demand and supply, balancing the energy consumed and produced. Equation (46) represents the energy balance of the system for each hour h.

$$\forall h: \sum_{g \in G} (V_{g,h}^{ren} + v_{g,h}^{conv}) + \sum_{s \in S} \left(w_{s,h}^{gen} - w_{s,h}^{stored} \right) + \ ls_h \geq d_h \tag{46}$$

The market price of energy is found by applying the complementarity slackness theorem on equation , where λ_h is the complementarity variable. (Mikuláˇs Luptáˇcik, u.d.) The market will be in equilibrium when the equation of the complementarity restriction, equation (47)-(49), are obeyed. This implies that the price of energy will rise until the demand and supply are balanced, or the price cap is reached and load shedding occurs.

$$\forall h: \left(\sum_{g \in G} (V_{g,h}^{ren} + v_{g,h}^{conv} + \sum_{s \in S} \left(w_{s,h}^{gen} - w_{s,h}^{stored} \right) - ls_h - d_h \right) \geq 0 \tag{47}$$

$$\lambda_h \geq 0 \tag{48}$$

$$\forall h: \left(\sum_{g \in G} (V_{g,h}^{ren} + v_{g,h}^{conv} + \sum_{s \in S} \left(w_{s,h}^{gen} - w_{s,h}^{stored} \right) - ls_h - d_h \right) \cdot \lambda_h = 0 \tag{49}$$

3.4 Transmission System Operator

The Transmission System Operators (TSO) objective is to maximize the consumer surplus and prevent the market price from exceeding the price cap given by the regulators. For market prices below the price cap the TSO will not shed any load. In scenarios where market prices reach the price cap the TSO will prevent the price from increasing further, then load shedding will occur. The amount of load shedding is not limited, and the TSO will only prevent the price of exceeding the given price cap, equation(50).

$$P^{cap} - \lambda_h \geq 0 \tag{50}$$

The complementarity slackness theorem applied on equation (50), where ls_h is the complementarity variable and equation (51)-(53) gives the complementarity restrictions.

$$\forall h: P^{cap} - \lambda_h \geq 0 \tag{51}$$

$$ls_h \geq 0 \tag{52}$$

$$\forall h: (P^{cap} - \lambda_h) * ls_h = 0 \tag{53}$$

3.5 Power Producers

The objective function of the power producers is given by equation (55). According to the rationality assumption, each generating firm g maximizes the firms' profit by supplying a quantity, $v_{g,h}^{conv} + V_{g,h}^{ren}$, to the market at price λ_h. Thus, the firm will earn a profit, found by subtracting the cost of production from revenue for all hours h. The power producers face a capacity constraint on production, as expressed in equation (55).

The firm's optimization problem is given by:

$$\forall g: max\, \Pi_g = \sum_{h \in H} \lambda_h * (V_{g,h}^{ren} + v_{g,h}^{conv}) - C_{g,h}(v_{g,h}^{conv}) \tag{54}$$

$Subject\ to$:

$$\forall g, \forall h: V_g^{prod.cap} - v_{g,h}^{conv} \geq 0 \tag{55}$$

$$\forall g, \forall h: v_{g,h}^{conventinal} \geq 0 \tag{56}$$

The production costs are represented by a continuous convex quadratic function, equation (57),

$$C_{g,h}\left(v_{g,h}^{conv}\right) = (b_g^c + c_g^c * v_{g,h}^{conv}) \cdot v_{g,h}^{conv} \tag{57}$$

b_g^c is the linear and c_g^c is the quadratic cost parameter. The marginal cost of production for renewable energy is assumed to be zero. Therefore, $V_{g,h}^{renewable}$ is declared as a parameter.

3.5.1 Perfect Competition

Under the assumption of perfect competition, firms act as a price-takers, believing that the quantity supplied does not affect the market equilibrium price. The market price is given by equation (58) .

$$\lambda_h = MB_h(d_h) = a_h^d - b_h^d \cdot d_h \tag{58}$$

The Lagrangian function of the power producers' optimization problem is defined in equation (59).

$$\forall g: \mathcal{L}_g = \sum_{h \in H} \lambda_h \cdot (V_{g,h}^{ren} + v_{g,h}^{conv}) - C_g(v_{g,h}^{conv}) \tag{59}$$

$$+\sum_{h\in H}\mu_{g,h}\cdot(V_g^{prod.cap}-v_{g,h}^{conv})$$

By derivation of the Lagrangian function, the optimality conditions may be derived.

With respect to $v_{g,h}^{conv}$:

$$\forall g,\forall h{:}\,\lambda_h - MC_{g,h}\big(v_{g,h}^{conv}\big)-\mu_{g,h}\leq 0 \tag{60}$$

$$\forall g,\forall h{:}\,v_{g,h}^{conv}\geq 0 \tag{61}$$

$$\forall g,\forall h{:}\,\big(\lambda_h - MC_{g,h}\big(v_{g,h}^{conv}\big)-\mu_{g,h}\big)\cdot v_{g,h}^{conv}=0 \tag{62}$$

With respect to $\mu_{g,h}$:

$$\forall g,\forall h{:}\,V_g^{prod.cap}-v_{g,h}^{conv}\geq 0 \tag{63}$$

$$\forall g,\forall h{:}\,\mu_{g,h}\geq 0 \tag{64}$$

$$\forall g,\forall h{:}\,\big(V_g^{prod.cap}-v_{g,h}^{conv}\big)\cdot\ \mu_{g,h}=0 \tag{65}$$

Equation (60) to (62) are the optimality and complementarity constraints for the power producer under perfect competition. Equation (60) requires that the market prices λ_h do not exceed the sum of marginal cost of production $MC_{g,h}\big(v_{g,h}^{conventinal}\big)$ and the scarcity rent of production capacity, $\mu_{g,h}$ each hour. The power generating firm will increase its supply until the restriction is not violated.

3.5.2 Cournot Competition

In the Cournot game, generating firms act as price setters. The quantity supplied by the firms affect the market price. Therefore, will the firms supply the quantity which gives them the highest profit due to expected supplied quantity by competitors and the demand elasticity.

The Lagrangian function of the generating firms is thus given by:

$$\begin{aligned}\forall g{:}\,\mathcal{L}_g = &\sum_{h\in H} MB_h(d_h)\cdot(V_{g,h}^{ren}+\ v_{g,h}^{conv})-C_g(v_{g,h}^{conv})\\ &+\sum_{h\in H}\mu_{g,h}\cdot(V_g^{prod.cap}-v_{g,h}^{conv})\end{aligned} \tag{66}$$

And the KKT with respect to $v_{g,h}^{conv}$:

$$\forall g: -\alpha * b_h^d \cdot (v_{g,h}^{conv} + V_{g,h}^{ren}) + MB_h(d_h) - MC_{g,h}(v_{g,h}) - \mu_{g,h} \leq 0 \tag{67}$$

$$\forall g: v_{g,h}^{conv} \geq 0 \tag{68}$$

$$\forall g: \left(-\alpha \cdot b_h^d \cdot \left(v_{g,h}^{conv} + V_{g,h}^{ren}\right) + MB_h(d_h) - MC_{g,h}(v_{g,h}) - \mu_{g,h}\right) \cdot v_{g,h}^{conv} = 0 \tag{69}$$

KKT with respect to $\mu_{g,h}$:

$$\forall g, \forall h: V_g^{prod.cap} - v_{g,h}^{conventinal} \geq 0 \tag{70}$$

$$\forall g, \forall h: \mu_{g,h} \geq 0 \tag{71}$$

$$\forall g, \forall h: \left(V_g^{prod.cap} - v_{g,h}^{conventinal}\right) \cdot \mu_{g,h} = 0 \tag{72}$$

The optimality constraints for a perfect competition and Cournot competition have several similarities, as shown in equation (67) - (69). The key difference is in equation (67). Under perfect competition, the firm supplied a quantity that required marginal cost $MC_{g,h}(v_{g,h})$ equal to the market price λ_h. The first term equation (67) presents the effect on market price by producing one unit extra, increased production results in decreasing market price. Consumers' marginal benefit $MB_h(d_h)$ is directly affected by the actual consumption d_h , and indirectly affected by the firms' production decisions $v_{g,h}^{conventinal}$. Since d_h is the sum of supplied quantity by both competing generating firms and storage units, the optimal production is not only a function of the consumer behavior and production cost, but also as a function of the competing firms' optimal response to own decisions.

3.6 Energy Storage Units

The energy storage units obtain their profits by intra-day arbitrage trade, which implies storing at low prices and generating at higher prices. Equation (73) is each unit's individual objective function, where $w_{g,h}^{stored}$, $w_{g,h}^{generated}$ and $e_{s,h}^{stored}$ are the decision variables. The storage units face restrictions on energy capacity, equation (74), and power capacity, equation (75). Equation (76) and (77) keep track of the energy level in the storage unit. The energy level in a storage unit after a period is the last period's energy level subtracting generated energy or adding the stored energy calibrating for converter losses, CL_s. The energy balanced is round coupled for the set of all hours, equation (77).

$$\forall s\text{: } max\ \Pi_s = \sum_{h \in H} \lambda_h \cdot (w_{s,h}^{gen} - w_{s,h}^{stored}) \tag{73}$$

$$Subject\ to:$$

$$\forall s, \forall h\text{: } E_s^{max} - e_{s,h}^{stored} \geq 0 \tag{74}$$

$$\forall s, \forall h\text{: } W_s^{max} - w_{s,h}^{stored} - w_{s,h}^{gen} \geq 0 \tag{75}$$

$$\forall s, \forall h = 1\text{: } e_{s,H}^{stored} + CL_s \cdot w_{s,1}^{stored} - w_{s,1}^{gen} - e_{s,1}^{stored} \geq 0 \tag{76}$$

$$\forall s, \forall h > 1\text{: } e_{s,h-1}^{stored} + CL_s \cdot w_{s,h}^{stored} - w_{s,h}^{gen} - e_{s,h}^{stored} \geq 0 \tag{77}$$

$$\forall s, \forall h\text{: } w_{s,h}^{gen} \geq 0 \tag{78}$$

$$\forall s, \forall h\text{: } w_{s,h}^{stored} \geq 0 \tag{79}$$

$$\forall s, \forall h\text{: } e_{s,h}^{stored} \geq 0 \tag{80}$$

The energy storage unit can act as both price-taker and price-setter. Therefore, the optimality constraint changes as their strategy changes.

3.6.1 Perfect Competition

The Lagrangian function for the energy storages operating as price taker is given by equation (81):

$$\begin{aligned} \forall s\text{: } \mathcal{L}_s = \sum_{h \in H} \lambda_h \cdot \left(w_{s,h}^{gen} - w_{s,h}^{stored}\right) \\ + \xi_{s,1} \cdot \left(e_{s,H}^{stored} + CL_s \cdot w_{s,1}^{stored} - w_{s,1}^{gen} - e_{s,1}^{stored}\right) \\ + \sum_{h \in H} \xi_{s,h} \cdot \left(e_{s,h-1}^{stored} + CL_s \cdot w_{s,h}^{stored} - w_{s,h}^{gen} - e_{s,h}^{stored}\right) \\ + \sum_{h \in H} \iota_{s,h} \cdot \left(E_s^{max} - e_{s,h}^{stored}\right) \end{aligned} \tag{81}$$

$$+\sum_{h\in H}\nu_{s,h}\cdot\left(W_s^{max}-w_{s,h}^{stored}-w_{s,h}^{gen}\right)$$

The storage units behave as the generating firms in section 2.2.2 Perfect Competition, they will supply energy as long the cost of storing are covered by the market price. Modeling the storages as firms under perfect competition will equivalent to maximizing the consumer surplus.

KKT conditions with respect to $w_{s,h}^{gen}$:

$$\forall s,\forall h\colon \lambda_h-\xi_{s,h}+\nu_{s,h}\le 0 \tag{82}$$
$$\forall s,\forall h\colon w_{s,h}^{gen}\ge 0 \tag{83}$$
$$\forall s,\forall h\colon (\lambda_h-\xi_{s,h}+\nu_{s,h})\cdot\ w_{s,h}^{gen}=0 \tag{84}$$

KKT conditions with respect to $w_{s,h}^{stored}$:

$$\forall s,\forall h\colon -\lambda_h+CL_s\cdot\xi_{s,h}-\nu_{s,h}\le 0 \tag{85}$$
$$\forall s,\forall h\colon w_{s,h}^{stored}\ge 0 \tag{86}$$
$$\forall s,\forall h\colon\left(-\lambda_h+CL_s*\xi_{s,h}-\nu_{s,h}\right)\cdot\ w_{s,h}^{stored}=0 \tag{87}$$

KKT conditions with respect to $e_{s,h}^{stored}$:

All hours, except last

$$\forall s,\forall h<H\colon \xi_{s,h+1}-\xi_{s,h}-\iota_{s,h}\le 0 \tag{88}$$
$$\forall s,\forall h<H\colon e_{s,h}^{stored}\ge 0 \tag{89}$$
$$\forall s,\forall h<H\colon (\xi_{s,h+1}-\xi_{s,h}-\iota_{s,h})\cdot e_{s,h}^{stored}=0 \tag{90}$$

Last hour

$$\forall s,\forall h=H\colon \xi_{s,1}-\xi_{s,H}-\iota_{s,H}\le 0 \tag{91}$$
$$\forall s,\forall h=H\colon e_{s,H}^{stored}\ge 0 \tag{92}$$
$$\forall s,\forall h<H\colon (\xi_{s,1}-\xi_{s,H}-\iota_{s,H})\cdot e_{s,H}^{stored}=0 \tag{93}$$

KKT conditions with respect to $\xi_{s,h}$:

First hour

$$s, \forall h = 1:\ e^{stored}_{s,H} + CL_s * w^{stored}_{s,1} - w^{generated}_{s,1} -\ e^{stored}_{s,1} \geq 0 \tag{94}$$

$$\forall s, h = 1: \xi_{s,1} \geq 0 \tag{95}$$

$$\forall s, \forall h = 1:\ (e^{stored}_{s,H} + CL_s \cdot w^{stored}_{s,1} - w^{generated}_{s,1} -\ e^{stored}_{s,1}) \cdot \xi_{s,1} = 0 \tag{96}$$

Rest of hours

$$\forall s, \forall h > 1:\ e^{stored}_{s,h-1} + CL_s \cdot w^{stored}_{s,h} - w^{generated}_{s,h} -\ e^{stored}_{s,h} \geq 0 \tag{97}$$

$$\forall s, h > 1: \xi_{s,h} \geq 0 \tag{98}$$

$$\forall s, \forall h > 1:\ (e^{stored}_{s,h} + CL_s \cdot w^{stored}_{s,h} - w^{generated}_{s,h} -\ e^{stored}_{s,h}) \cdot \xi_{s,h} = 0 \tag{99}$$

KKT conditions with respect to $\iota_{s,h}$:

$$\forall s, \forall h: E^{max}_s - e^{stored}_{s,h} \geq 0 \tag{100}$$

$$\forall s, \forall h: \iota_{s,h} \geq 0 \tag{101}$$

$$\forall s, \forall h: (E^{max}_s - e^{stored}_{s,h}) \cdot \iota_{s,h} = 0 \tag{102}$$

KKT conditions with respect to $\nu_{s,h}$

$$\forall s, \forall h: W^{max}_s - w^{stored}_{s,h} - w^{generated}_{s,h} \geq 0 \tag{103}$$

$$\forall s, \forall h: \nu_{s,h} \geq 0 \tag{104}$$

$$\forall s, \forall h: (W^{max}_s - w^{stored}_{s,h} - w^{generated}_{s,h}) \cdot \nu_{s,h} = 0 \tag{105}$$

$\xi_{s,h}$, is the value of one unit stored energy hour h for storage unit s. The storages will supply the market as long the value of stored energy exceeds the market price λ_h and the valuation of converting capacity $\nu_{s,h}$, equation (82). On the other hand, the unit will store energy if the relationship opposite and converter losses are covered, equation (85). Equation (88) – (93) determines the energy level in the storages based on the value of stored energy from time step to time step. The energy balance of the storage is pinned down by the equation from (94) to (99). The last time step is round coupled with the first, ensuring that storages do not generated more energy than it stores. The reaming equations (100) to (105) are the operating constraints on effect and energy capacity for the storage units.

3.6.2 Profit maximizing storage unit

When the energy storage unit operates as an arbitrage player with market power, are the model formulated as an MPEC, where the arbitrage player is the top-level problem. The objective function and restriction are the same as earlier, equation (73) - (80). Differently from the perfect competition strategy is that the storages operator observes a new price for every possible strategy played for storing and generating energy. The energy storages look down at the bottom-level and see all agent optimality constraints. The arbitrages players maximize the profit function subject to the other players expected behavior.

3.7 Model implementation and data input

3.7.1 **Input Parameters**

The model presented earlier in this chapter are relatively simplistic. Assumptions are made for reducing the complexity of the power market. The input parameters are selected for the purpose of giving a realistic representation of the power market.

The model is general, and have possibilities to have heterogeneous firms with intra-day variations in restrictions. All cases presented in this thesis have homogenous firms with no changes in operations restrictions intra-day.

$\mathrm{Cl_s}$	$\mathrm{V_{g,h}^{prod.cap}}$	$\mathrm{W_s^{max}}$	E_s^{max}	$\mathrm{V_{g,h}^{ren}}$	$\mathrm{b_h^d}$	$\mathrm{b_{g,h}^c}$	$\mathrm{c_{g,h}^c}$	$\mathrm{P^{cap}}$
0,9	10000	10000	10000	0	7	0,2	2	400

Table 4 Input parameters, base case

Table 4 and Figure 3-1present all input parameters for the base case. The firms’ production capacities are set high, so they will never be binding in the base case. The net efficiency of the energy storage is 90 %, which is reasonable (Ferreira , et al., 2013).

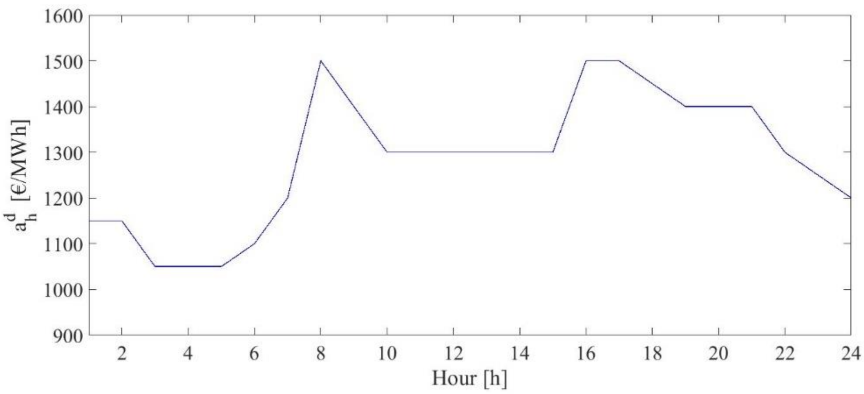

Figure 3-1 Constant benefit coefficient of demand at hour h

The constant benefit coefficient of demand at hour h are determined for the purpose of capture intra-day variations of demand, low utility by night and high utility at the morning and afternoon. The linear benefit coefficient is constant for all hours, this is both simplistic and realistic as the elasticity of demand increases with higher constant benefit coefficient.

The quadratic cost functions cost parameters are set to mimic the increasing marginal cost for production. Simulations as the cost of production should represent a realistic price.

3.7.2 Input Parameter Section 4.2

In section 4.2 Scarce Production Capacity constraints on both producers and storages are introduced.

$V_{g,h}^{prod.cap}$, W_s^{max} and E_s^{max} are the only parameter which changes. The new total production capacity is set to 170 MW and 200 MW for the different cases. The energy storages operation constraints are set to $W_s^{max} = 20$ MW and $E_s^{max} = 25$MWh.

In subsection 4.2.1 Shadow prices on production, storage effect and storage energy capacity, the different restrictions are presented table 5

W_s^{max} / E_s^{max}	$0/0$	$10/25$	$15/37.5$	$20/50$	$25/62.5$	$30/75$

Table 5 Energy storage constraints in subsection 4.2.1

3.7.3 Software

The models are implemented in General Algebraic Modeling System (GAMS). GAMS uses a high-level modeling language and allows the user to focus on the modeling approach by offering a simple setup and interface. GAMS allows the complementarity format; hence this program will be used in this Master thesis.

The MCP formulation is solved with PATH 4.7, a solver that's based on Newtons and Lemkes method. (Ferris & Pang, 1997). For the MPEC formulation the KINTRO 10.0 are used. The KINITRO package provides an efficient and robust solution for large-scale general problems, it is also efficient for solving minor complementarity problems. (Anon., u.d.)

3.7.4 Hardware

All simulations of the model are run on Apple MacBook Pro with OS Windows 10 and an Intel Core i5-5257U CPU 2.70GHz. The computational challenges of the model are considered small, and the equipment is more than adequate.

4 Results

This chapter presents the results from the base case and the specific scenarios for the market model presented in chapter 3. The primary focus of the thesis is to study the effects of energy storage and strategic behavior by the market participants. The following issues will be outlined and further discussed.

- How does imperfect competition affect a power market without energy storages?
- How does the energy storage affect the power market when operating as a consumer surplus maximizing agent?
- How does the ownership of the energy storage unit affect the power market?

4.1 Base Case

The base case is a purely qualitative study, where the goal of the simulations is to prove and target potential outcomes of strategic behavior in the energy market with and without energy storages units. The generating firms and energy storage units are assumed to be symmetric. Hence, the discussion concerning quantities will be on an aggregate level.

4.1.1 Imperfect Competition and Market Equilibriums

By analyzing the market equilibriums without the possibility of storing energy, a clear evidence of the effects of strategic behavior by the generating firms appears. The Cournot players have the possibility of exerting market power by reducing the supplied quantity resulting in both higher market price and increased total profit. The firms' reaction functions and the mechanism for price-setting are thoroughly explained in section 2.2.3.

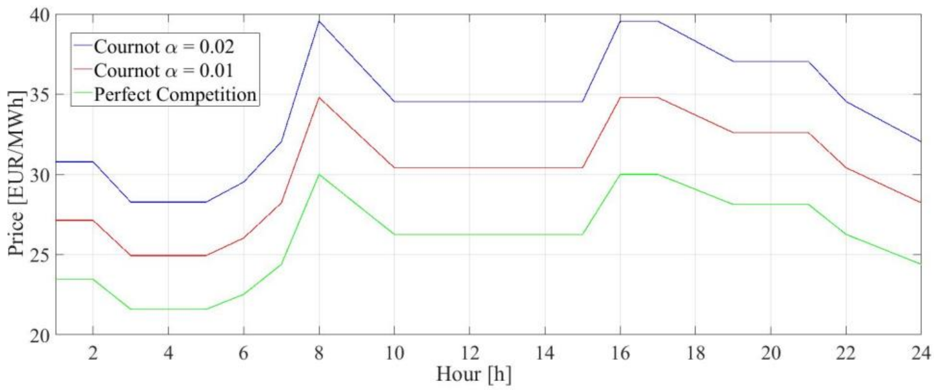

Figure 4-1 Market price, perfect competition vs. imperfect competition

Figure 4-1 and figure 4-2 shows the price profile and profits over a 24 hour time-series. The firms believe they can affect the market price with a factor of $\alpha * b_h^d$ per unit, when reducing the supplied quantity. An increased α is similar to escalating market power.

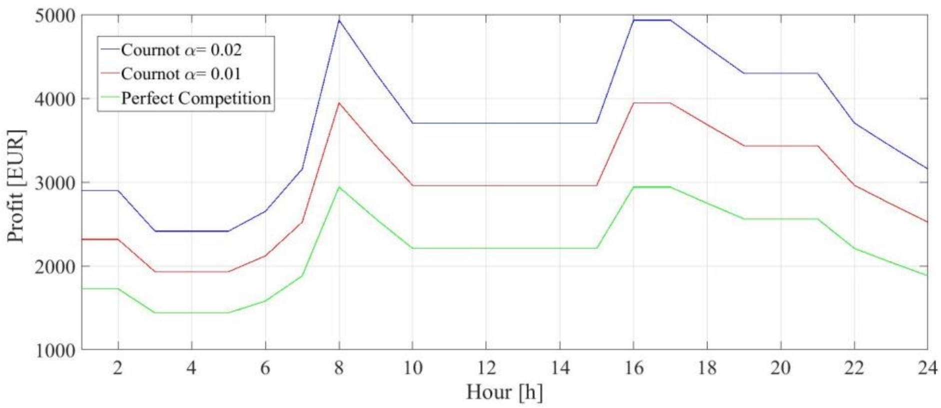

Figure 4-2 Total profit, perfect competition vs. imperfect competition

The increased market price result in higher profits for the generating firms. The demand curve represents an inverse relationship between the market price and the consumed quantity, the quantity under imperfect competition is as expected at a lower level than under perfect competition, as shown in figure 4-3.

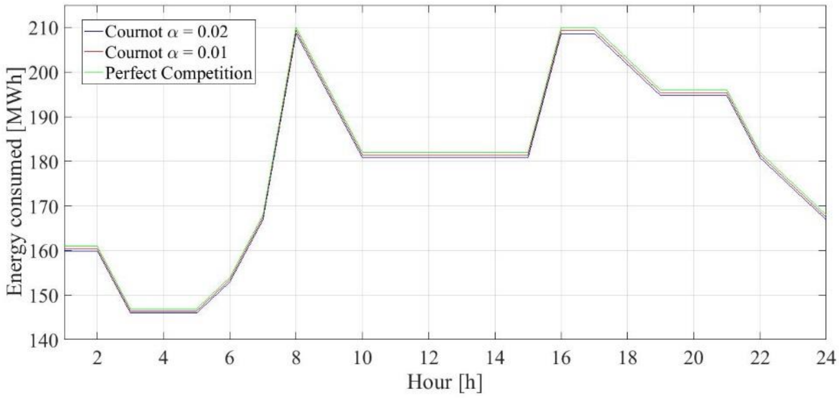

Figure 4-3 Energy consumed, perfect competition vs. imperfect competition

The elasticity of demand is central for the changes in the market equilibrium. Small changes in quantity will cause large effects on the market price. The Cournot players reduce their quantity by less than 1 %, resulting in a large response on the market price of a 50 % increase in peak-hour. This is consistent with the power market theory presented in section Market Power. The magnitude of the reaction on price and quantity are therefore reasonable. The elasticity of demand varies on the interval 2 - 3 %, which are considered low but still realistic for a Nordic power market (Gribkovskaia , 2015).

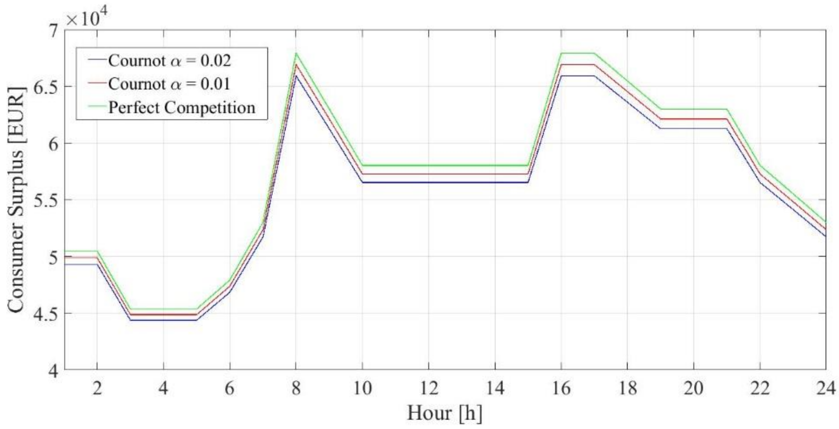

Figure 4-4 Consumer surplus, perfect competition vs. imperfect competition

During high demand periods, such as hour 8 and hour 16-18, the prices are relatively high compared to the other periods with lower demand. The consumers' utility of consumption is higher in these periods. Figure 4-4 presents the consumer surplus for each hour under different strategies by the firms. The preferences of the consumer lead to intra-day variations in price greater than the effects of market power.

The increased profits and the reduced consumer surplus has only a marginal impact on total social economical welfare. Despite the deviation from the perfect competition equilibrium, the new equilibriums are in the neighborhood of the optimal solution. Nevertheless, there have been large welfare transfers from the consumers to the producers due to the market power.

4.1.2 Energy Storage Units and Market Equilibrium

Introducing the possibility of storing energy in the model affected the market equilibrium. The effects are analyzed in a perfectly competitive market with energy storage units maximizing consumer surplus.

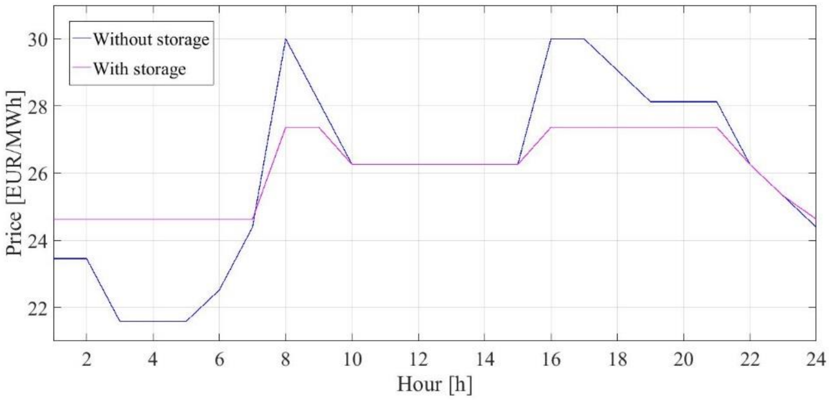

Figure 4-4 Market price, with and without energy storage

Figure 4-4 presents the price-profile for a 24-hour simulation with and without energy storages. The price-profile without energy storages shows significant price volatility compared the scenario with energy storages. When energy storage is utilized the intra-day price relationship, minimum price divided by maximum price increases from 0.72 to 0.9. The energy storages are not bounded by its respective power and energy constraints since the relation between min and max price equals the net efficiency of the storage unit.

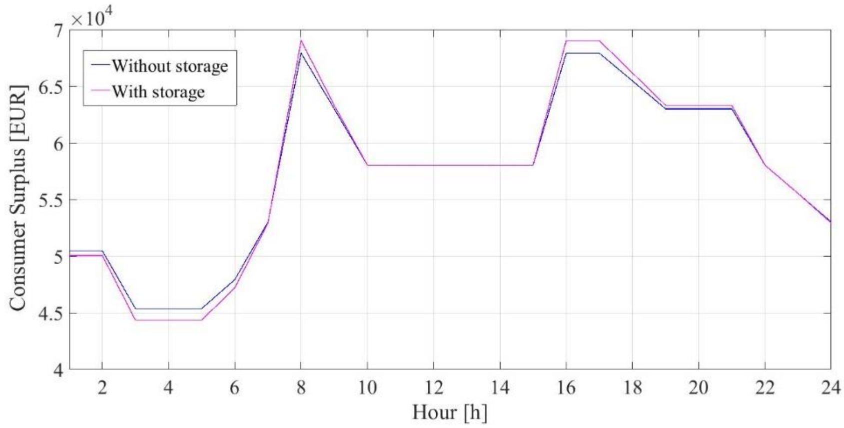

Figure 4-5 Consumer surplus, with and without energy storage

The energy storage shifts the consumption to the hours where the utility of consumption is at its peak. The consumers prefer higher consumption in peak-demand hours instead of high total quantity of consumption. Figure 4-5 Consumer surplus, with and without energy storage presents the hour-by-hour consumer surplus, the total consumer surplus is higher when energy storages are installed.

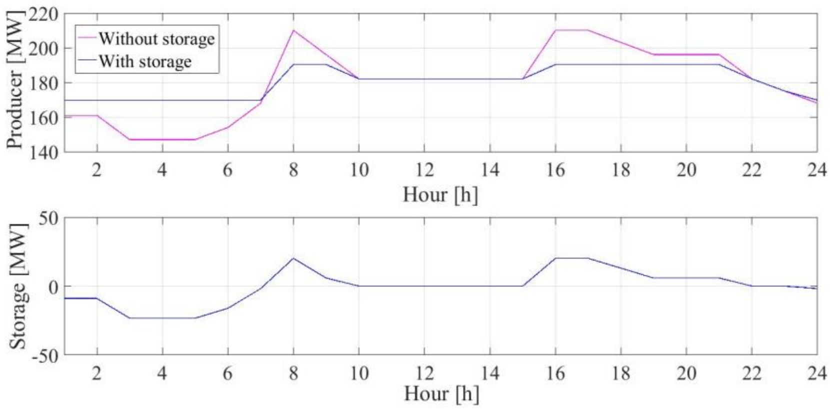

Figure 4-6 Producer and storage load profiles

The generating firms face a quadratic cost function, where the marginal cost of production is rising. The cost of storing is the market-price or margianal-cost of production adjusted for losses. In the base case, there are savings in shifting the production from high-demand to low-demand hours. The savings are relatively small, 0.4 %. Although the quantitative effect is little, the effect is still significant. The savings in production cost depend only on the slope of the production cost curve. In cases where the marginal cost curve is steeper, the utilization of the storage unit would increase.

The results in this section so far present only simulations under perfect competition. Under imperfect competition, known as Cournot behavior, the CS maximizing storage unit will influence the market in a similar way as earlier in this subsection. Figure 4-7, shows the same pattern as presented in Figure 4-6.

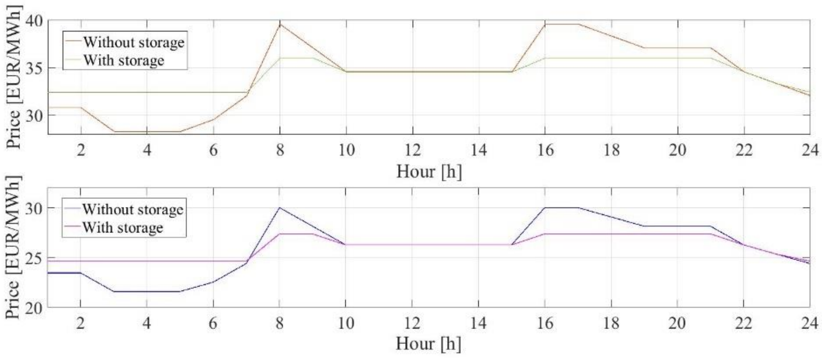

Figure 4-7 Market price perfect competition vs. imperfect competition with and without energy storage

The energy storage unit stores and generates close to the same amount of energy regardless of the producing firms' behavior. The utilization of the energy storage is expected to be higher under imperfect competition. However, the constant benefit coefficient a dominates marginal cost of production c in equation (12) due to the gradual increase in marginal cost of production. This results in approximately the same reduction of supply by the Cornout players each hour. The minimal price divided by the maximum price of the Cournot game intra-day without energy storage is the same as under perfect competition, 0.72. With more rapidly increasing marginal cost for production the operation of the storage will deviate from operations under perfect competition operating generators.

4.1.3 Effects of ownership of the storage units

The storage unit has the advantage of storing energy in periods with low price, and generate at high prices. This technical feature may be used for several purposes, depending on the ownership of the unit. Earlier the energy storage has maximized consumer surplus, this mimics the behaviour of smaller storages owned by the consumer or a perfect competitive market. Other relevant behaviour of the storage is the mimic of an arbitrage player. The arbitrage player is expected to drive the price up and the quantity down in order to maximize own profit at the expense of the consumers. Hence, two different ownerships are assessed; i) consumer oriented, and ii), producer oriented.

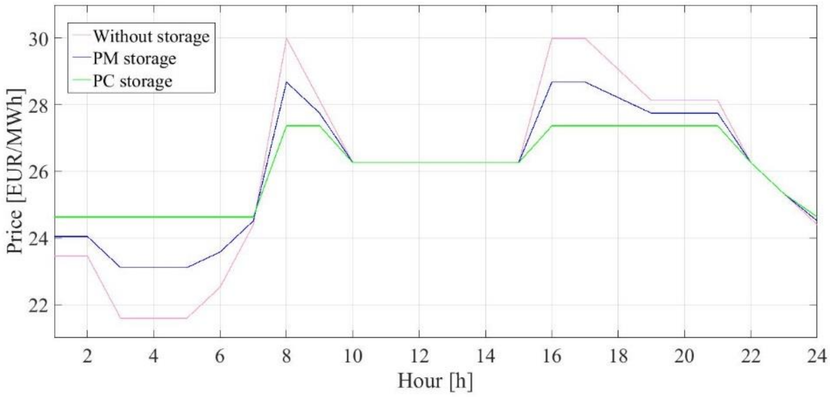

Figure 4-8 Market price, under different storage operation strategies, profit maximizing (PM) and perfect competitive (PC)

Figure 4-8 Market price, under different storage operation strategies clarifies the effect on price due to the ownership and behavior of the storage unit. The arbitrage player (PM) drives up the market price by restricting the supply relative the consumer surplus maximizing storage unit.

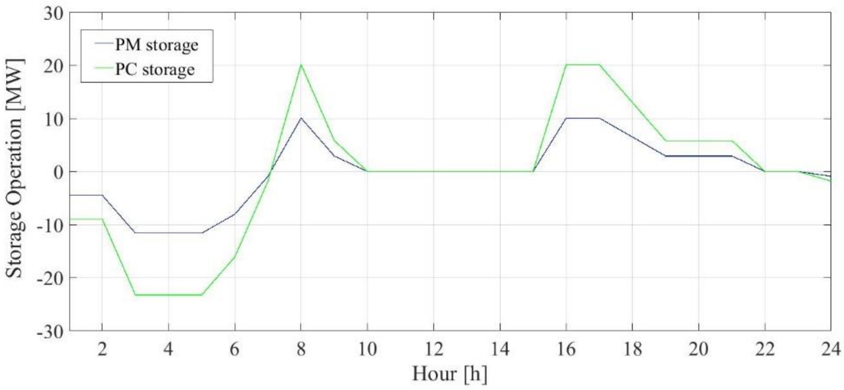

Figure 4-9 Energy generation by storages, under different storage operation strategies

The operation of the storage units is presented in figure 4-9, both strategies have the same pattern of storing at low price hours and generating at high price hours. The total profit for the consumer surplus maximizing agent is in total zero. The storage shifts the intra-day consumption by utilizing the flexibility of the storage.

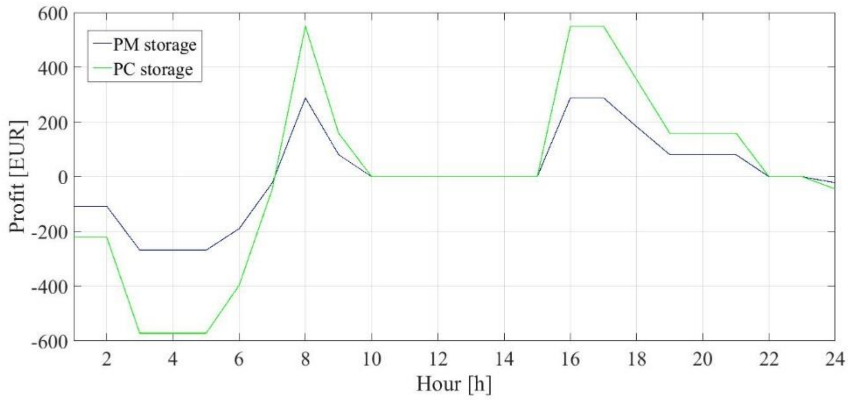

Figure 4-10 Storage units' profit, under different storage operation strategies

The arbitrage player obtains a positive total profit for the 24 hours. Both storages shift the consumption towards high demand periods. Nevertheless, the arbitrage player drives the price up in high demand periods relative to the consumer surplus maximizing storage. Despite the low level of changes in quantity the price level increases with 4.8% during peak-demand hours.

The effect of the strategic behavior by the arbitrage player may be considered marginal. In a well-functional power market with high degree of competition and high production capacity, the effects of the arbitrage player will not be considered as potential for distort competition. Nevertheless, the arbitrage player drives the market price upwards. In a micro grid, there is a potential lack of capacity or rapidly increasing marginal cost of production for serving the peak-demand hours. The effect of the arbitrage player behavior will therefore be reasonable to believe could be increased

4.2 Scarce Production Capacity

In this section, the role of the energy storage unit will be further analyzed under restricted generation capacity. Reducing the hourly generation capacities to 200 MW and 170 MW has great impact on the power market. In the perfect competition base case with no storage the peak-generation was 215 MW at hour 8, 16 and 17. Peak-production capacity is now reduced to 80 % and 93% of optimal. The energy storing unit has now an installed power capacity on 20 MW and an energy capacity of 25MWh.

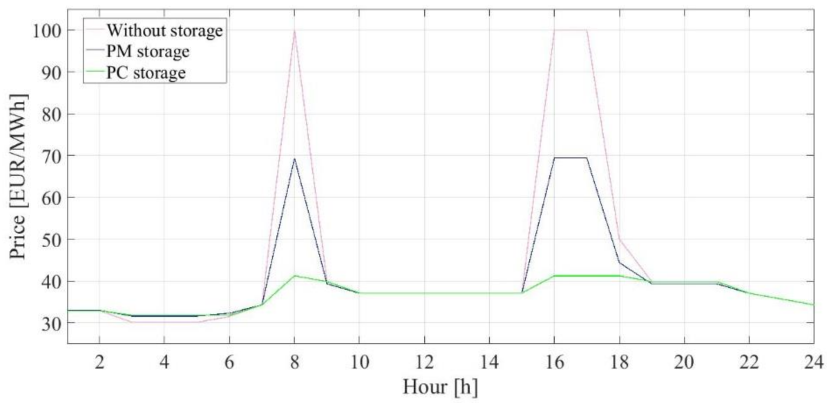

Figure 4-11 Market prices, under different storage operation strategies and production capacity of 200 MW

The effects caused by the reduction in production capacity are similar for both cases. The reduction of production capacity leads to increased energy prices, as illustrated in figure 4-11 and figure 4-13. The effect of ownership and strategy outlined in subsection 4.1.3 is present and enlarged as a result of the limited production capacity. Reducing the production capacity to 93 % of optimal effect, results in 70 % increase in energy prices at peak-hour demand caused by the strategic play. The effects of the reduced production capacity are not present when the energy storage maximize consumer surplus.

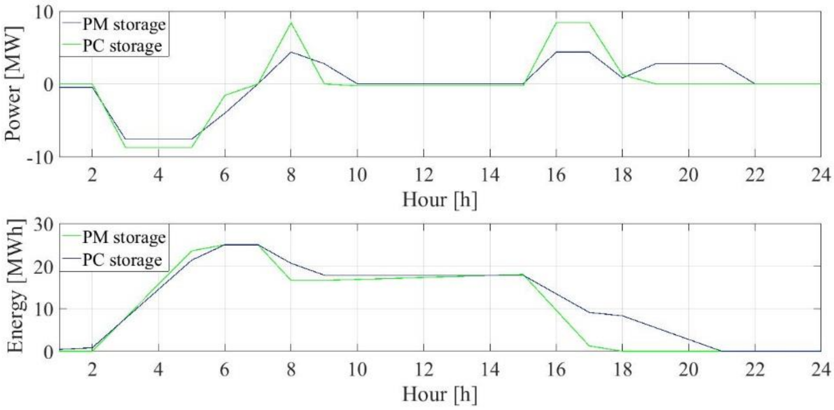

Figure 4-12 Generation profile and energy level for storage units under different storage operation strategies and production capacity of 200 MW

The generation profile for the two operation strategies are outlined in figure 4-12. The CS maximizing unit (PC) generates at higher levels at peak-hour visa vi the arbitrage player (PM).

PC storage generates 8.39 MW in hour 8, when the price peaks. To achieve profit the PM storage reduces the amount of generation in hour 8 to 4.37 MW, for then generating 2.77 MW in hour 9. The intra-day price variation makes it profitable generating at hour 9. The PM storage exploits the reduced production capacity in hour 8 to gain excess profit. In hour 9 the PC storage decides to produce since higher generation in hour 16 and 17 will have a negative opportunity cost. The operation of the energy storage will follow the same pattern for the peak-demand in hour 16 and 17. The PC storage generates when the utility of consumption peaks, at the same time the PM storage exploits the reduced production capacity and generate less in peak hours and more when the intra-day price difference exceeds the converter losses.

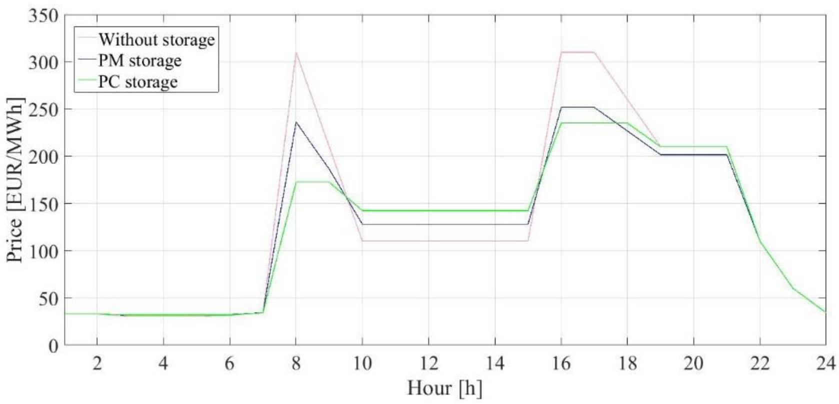

Figure 4-13 Market prices, Market price, under different storage operation strategies and production capacity of 170 MW

A small reduction in production capacity proved to have major effects on operation of the energy storages, and the storages effects on the energy market. With further reduction of capacity, to 170 MW, the effects follow the same pattern and the effects are also enlarged, figure 4-13.and figure 4-14

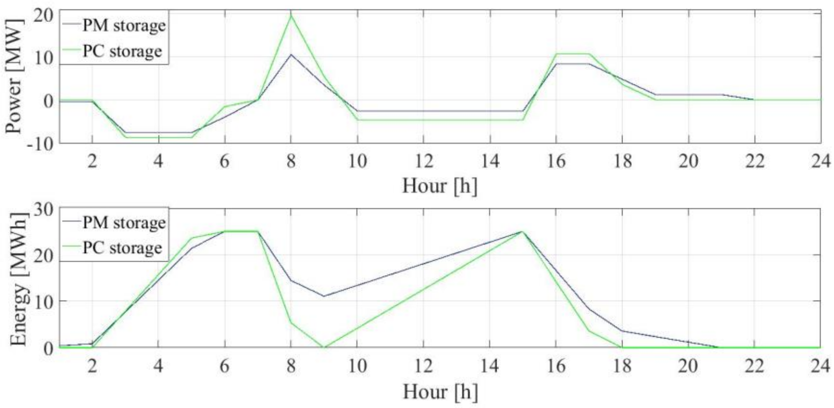

Figure 4-14 Generation profile and energy level for storage units under different storage operation strategies and production capacity of 170 MW

Between the morning at 8 peak demand and afternoon peak demand 16-17 both storage unit stores energy despite high midday price. The PC storage is constrained by generation (MW) and storing capability (MWh). At the same time the PM storage is only restricted by the capability of storing energy (MWh). In next subsection the valuation of the investment incentives will be discussed. So far, clear evidence is presented of how production flexibility can be advantageously used for gaining excess profit. The consumers are suffering great losses when the energy storage operates as a profit maximizing agent, as shown in Figure 4-15 Consumer surplus under different storage operations strategies and production capacity of 200 MW and figure 4-16

Consumer surplus under different storage operations strategies and production capacity of 170 MW

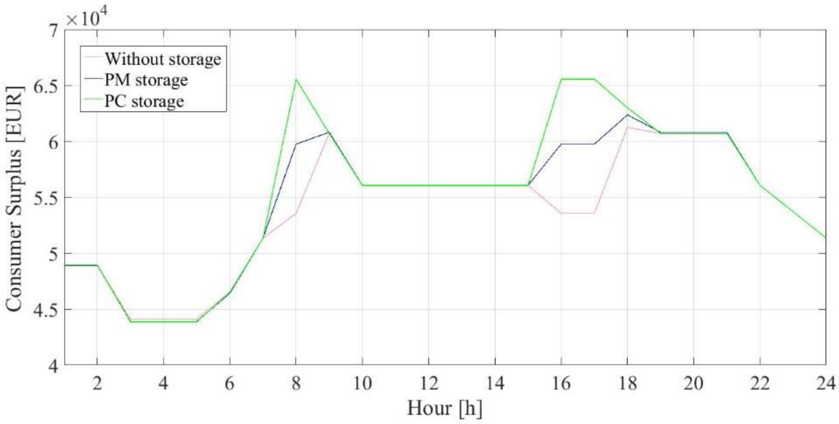

Figure 4-15 Consumer surplus under different storage operations strategies and production capacity of 200 MW

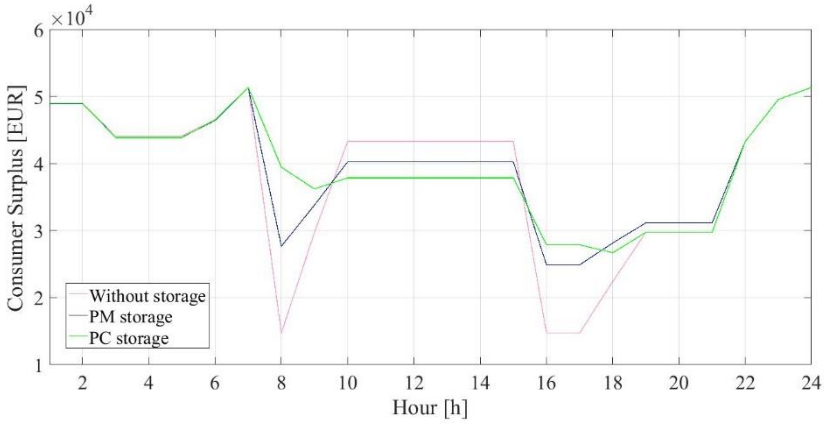

Figure 4-16 Consumer surplus under different storage operations strategies and production capacity of 170 MW

The consumer surplus in peak demand hours is clearly effected by the strategic behaviour of the energy storage unit.

4.2.1 Shadow prices on production, storage power and energy capacity

The storage energy and power capacity have a major influence on the valuation of future investment. With binding restrictions on generation capacity and storage operations, the shadow price of the capacity constraint represent the value of one unit extra. The shadow price is the measurement of the investment incentive, the highest expected investment cost an agent will except.

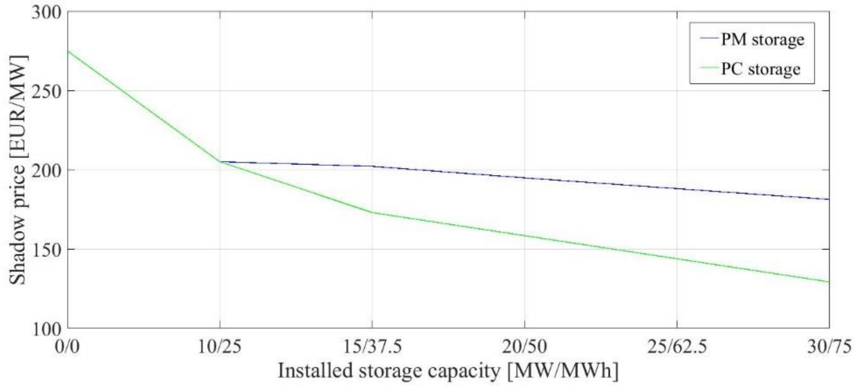

Figure 4-17 Peak shadow price on production capacity for generators, under different storage operation strategies and generator production capacity of 170 MW

As expected a high initial capacity of the storage will reduce valuation of generation capacity and the incentive to invest. The valuation of generation capacity by the perfectly competitive generators are higher when the energy storage operates as an arbitrage player, as a result of the higher peak-prices. From figure 4-17, the incentives for investments increases with reductions in competition and size of existing storages.

The energy storage unit has both restrictions on power generation and energy capacity. The valuation of increased power capacity on storage operation by the arbitrage player are low, although the optimal generation capacity of the producers is reduced to 80 %. The investment incentives on power capacity above 10 MW, 5% of hourly consume in hour 8, are more or less not existing. The investment incentives increase rapidly when the storage unit's capacity declines below 10 MW.

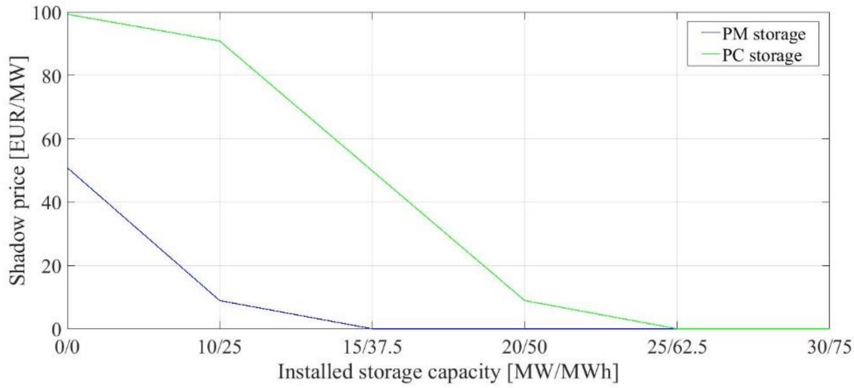

Figure 4-18 Peak shadow price on storage power capacity, under different storage operation strategies and generator production capacity of 170 MW

The storage with CS maximizing objective values increased power capacity definitely higher than the arbitrage player, figure 4-19. Nevertheless, real investment cost for energy storage units as lithium-ion batteries surpass the shadow values of the power capacity for both players. *(Anon., 2016)*

The same pattern of higher valuation of extra capacity is recognized for the energy capacity constraint. PC storage values increased energy capacity significantly higher than the PM storage, as illustrated in figure 4-19

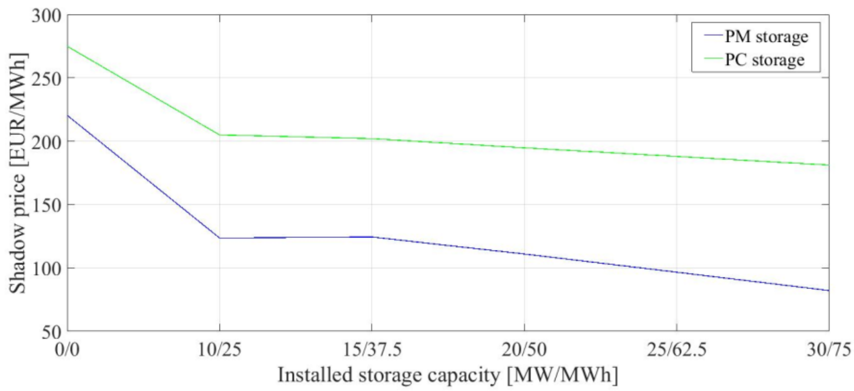

Figure 4-19 Peak shadow price on storage energy capacity under different storage operation strategies and generator production capacity of 170 MW

There is a clear tendency that under reduced generation capacity that consumers value increased capacity of both power and energy far more than the PM storage unit.

5 Discussion

5.1 Overall discussion

This section collects and summarizes the major findings in the previous chapter. These will be further discussed, where the role of energy storage in power markets with strategic players will be emphasized. Subsection 4.1.1 clarified the effect of market power for the generators. By reducing the supply quantity, the firms experienced an increase in both price and profit. A small change in supplied quantity caused a great change in price, which is regarded as realistic when considering an inelastic demand. The effect of market power become clearer as the Cournot-parameter α increases. This is in line with earlier studies on imperfect competition. (Willems, 2000)

The intraday price variations get reduced as the energy storage in a perfectly competitive market is introduced. The price variations are reduced to the efficiency of the energy storage. The results are as expected in a perfectly competitive power market without any capacity constraints. Moreover, the price taking storage is placed in a market with a Cournot player, which caused to a reduction in intraday price variations. The fact that the price variations are equal to the efficiency of the energy storage is caused by the lack of capacity constraints for the storage. Earlier studies (Oudalov, et al., 2008) that focus on the optimal size of an energy storage in a power market show similar behavior in trying to equalize the intraday variations, where the storage capacity is at the same time limited because of high investment costs.

The owner's objective of the energy storage determines how the operation develops. Price variations will occur in a market with increasing marginal costs of production, and also with a demand that vary from hour to hour. An energy storage that is owned by the consumers will reduce the intraday price variations until it equals the efficiency. The results are therefore reasonable when the arbitrage player reduce the supply relatively to the consumer-owned energy storage, and by that obtain an increase in profit. In a market without any constraints in production and slowly rising marginal costs during production, the effect of a strategic game is expected to be small, but the effect is still existing. Generally, few researchers have studied this area of expertise related to market power and energy storage. However, the few earlier studies have shown that the energy storage has the effect of reducing the market power of a monopolist (Yujian Ye, et al., 2016). The results in this study also support this conclusion, as the price of the energy market is reduced to during peak-demand hours.

The strategic playing energy storage has the ability to exercise market power and thereby increase own profit. What happens when capacity constraints are introduced to the market? Generally, the role of an energy storage in a power system depends strongly on what kind of power system the energy storage is located. In order to exercise market power, there must be either an increase in marginal costs or restrictions on production. In section 4.2, the generators production capacity is reduced compared to the base case. Thus, there is a clear relationship between reduced production capacity and increased market power. In case of tighter restrictions, the strategic energy storage has the possibility to increase the profit at the expense of the consumers. The shadow prices for the storage capacity are presented in subsections 4.2.1. As the strategic playing storage has the desire in reducing the supply, compared to the consumer owned storage, the shadow prices will naturally become lower. Thus, this provides the basis of the fact that the consumers will value a higher capacity in energy storages when the generation capacity is reduced.

The Lazared report (Anon., 2016) estimates todays levelized cost of storage (LCOS) to be at minimum 355 USD/MWh, and a potential reduction in capital cost by 38 % the next five years. Although the shadow prices of capacity were relative modest, in subsection 4.2.1. The investment incentives in storage capacity is within the range of an optimistic development in costs.

5.2 Limitations

This section provides an evaluation of the validity of the model and the following results in chapter 4. The weaknesses and limitations are therefore highlighted and further discussed.

First of all, the results of the model are highly dependent on the assumptions and the input parameters. The market structures of perfect and imperfect competition are solely assumptions of the state of a power market, which indeed decides the outcome of the model. In order for a strategic playing energy storage to obtain an increase in profit, comparing with a price taking storage, the competitors have to face limitations or increasing marginal costs. The constraints that lead to imperfect competition is central in considering the model itself, but also in evaluating the validity of the results.

The models are simplistic descriptions of a complex reality. Comprehensive extensions of the model can be conducted in order to obtain a more realistic representation of such complex problems. The market is presented with the features of continuous supply and demand curves. However, this representation does not account for the start and stop costs for the production units, which implies non-continuous curves.

The producers' marginal costs are rising, which results to the effect of intraday price variations as the demand will vary during the day. The marginal costs in the presented cases are assumed to be rising, but still increasing in a conservative way. In the base case, the increasing marginal costs provided a possibility for the energy storage to increase its own profit. A steeper curve of the marginal cost would leave the basis of an increase in market power, while a gentler curve would have reduced the possibility. In order to quantify the effect of the market power, a more precise representation of generator portfolio is necessary. It is crucial in considering whether the peak demand will be covered by the renewable energy with low marginal costs or peak power plants with high marginal costs.

The instabilities and variations of renewable energy can result in lack of production capacity during a day at different hours. It has therefore been conducted simulations with restricted production capacity, leading to that market power can be used. The realistic scenarios and the input parameters will then vary in response to the different markets with varying solar and wind conditions.

It is not only conditions on the supply side that determine the effect of market power. The consumers' price sensitivity on the demand play also a central role in the effect of market power. In the presented scenarios, the price elasticity is observed to be 2-3%. This is considered as a low price elasticity, which means that the consumer hardly reduces its power consumption at higher prices. Again, there are huge variations for the different power markets. The models are able to capture such effects, while it is also difficult to verify the results. However, an increase in price elasticity is still realistic in considering other European and American markets (Ros, 2015). Demand response is also expected to be significant in future power markets, both for private consumption and in industries. The process of installing Smart Meters is expected to take place in the entire EU, and this will probably lead to an increase in the consumers' price sensitivity. These representations of the consumers can limit the quality of the quantitative results.

However, there are possibilities for both import and export from other nodes in larger power systems. The transmission will offer the same flexibility as an energy storage would. When the prices are higher, the import will come from the neighboring node, while in case of lower prices the neighboring node will be exported to. In case of price variations between the nodes, the power market of the energy storage will be reduced if the possibility of transmission has been present. The lack of transmission provides the possibility for the existence of market power.

6 Conclusion

An investigation of the role of energy storage in a power market with different market structures has been conducted by modeling and applying complementarity theory. The generators and energy storages are modeled both as price takers and price setters. The effect from both the producers' and the storages' use of market power has been carefully analyzed. Moreover, the role of energy storage has also been modeled in different scenarios with increasing marginal costs in production, with and without constraints on production capacity. The overall objective is thus to study what kind of effects a strategic playing energy storage may provide.

By using complementarity theory, the models have been developed in order to recreate the different market structures. The problems are formulated as Mixed Complementarity Problems (MCP) and Mathematical Problems with Equilibrium Constraints (MPEC), which are solved in the modeling tool General Algebraic Modelling System (GAMS). The input parameters are selected in order to obtain a realistic representation of the market equilibriums

The study revealed that the Cournot producers can reduce the amount of supply, in order to increase own profit. The market equilibrium is designated as a Nash-Equilibrium, which means that none of the players sees any incentives to deviate from their own strategy. As energy storage is introduced to the power system, the intra-day price variations will be reduced.

The role of energy storage is highly affected by the assumptions of the market situation. The strategic playing energy storage exploited the benefit of market power in all the investigated scenarios. The energy storage has the possibility of exerting market power when constraints on production capacity is introduced. The investment incentives of the energy storage showed variations for the different strategies; the size of the incentives became less compared to real investments costs for energy storage technologies. Nevertheless, this is still within the range of realistic future investment costs.

The proposed study has led to the conclusion that the ownership of the energy storage can provide an idea about the effects of power market. The qualitative results show clearly the existence of a strategic behavior in energy storage, where these effects appear to be realistic. However, the quantitative results are still highly dependent on the models assumptions and input parameters, which should be further considered.

"*all perfect markets are perfect in the same way: all imperfect markets imperfect in their own different way*" – Paul Krugman

7 Further Work

The model in this thesis presents a simplistic description of a complex market. Several exciting expansions and scenarios should be further explored. The uncertainty related to the Renewable Energy Sources (RES) may have a major influence in how the energy storage will be operated under different strategies and ownership. The role of RES, in terms of solar and wind, will play an increasing role in the future power system. Hence, exploring this area of expertise is highly relevant.

As stated in the discussion part, one relevant expansion of the model is to include several nodes with transmission constraints. The storage unit will then face new scenarios as the other nodes may supply the same flexibility. The possibility of importing and exporting from other nodes offers the same flexibility as the storage. The potential benefits of future reduction in investment costs in energy storages are; savings as a result of reduced investments and also the scaling of transmission grid that is in favor of investing energy storing systems.

This work has primary focused on the analysis of the supply-side of the power market. However, the demand response becomes highly relevant when Smart-meters are installed. The continuity and availability of information makes it possible for consumers to respond quicker on price incentives, which will lead to increased flexibility of consumption. Thus, introducing representing agents for consumers group will be a realistic and relevant extension of the model.

The analysis is only concentrated on intra-day, 24 hours. Extending the time horizon as well as accounting for the investments in production capacity and energy storages, will take the modeling one step further into a wider understanding of the role of energy storage in power markets with strategic players. However, the extensions of the models are indeed an important way in the search of a realistic representation, but further work should also consider and validate the input parameters in order to obtain an applicable model of the complex problem.

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Appendix A Alternative Formulation

Several attempts and modeling approaches where conducted in the search for a realistic representation of a strategic behaving energy storage. In this appendix the Cournot approaches will be briefly presented and discussed.

The Cournot storage have the same optimization problem as the energy storage units presented in section 3.6, equation (73)-(80). From the optimization problem, the KKT conditions and the optimality constraints are derived.

$$\forall s:\ max\, \Pi_s = \sum_{h \in H} \lambda_h \cdot (w_{s,h}^{gen} - w_{s,h}^{stored}) \tag{73}$$

$Subject\ to$:

$$\forall s, \forall h: E_s^{max} - e_{s,h}^{stored} \geq 0 \tag{74}$$

$$\forall s, \forall h: W_s^{max} - w_{s,h}^{stored} - w_{s,h}^{gen} \geq 0 \tag{75}$$

$$\forall s, \forall h = 1: e_{s,H}^{stored} + CL_s \cdot w_{s,1}^{stored} - w_{s,1}^{gen} - e_{s,1}^{stored} \geq 0 \tag{76}$$

$$\forall s, \forall h > 1: e_{s,h-1}^{stored} + CL_s \cdot w_{s,h}^{stored} - w_{s,h}^{gen} - e_{s,h}^{stored} \geq 0 \tag{77}$$

$$\forall s, \forall h: w_{s,h}^{gen} \geq 0 \tag{78}$$

$$\forall s, \forall h: w_{s,h}^{stored} \geq 0 \tag{79}$$

$$\forall s, \forall h: e_{s,h}^{stored} \geq 0 \tag{80}$$

KKT with respect to $w_{s,h}^{gen}$:

$$\forall s, \forall h: -b_h^d * (w_{s,h}^{gen} - w_{s,h}^{stored}) + MB_h(d_h) - \xi_{s,h} + \nu_{s,h} \leq 0 \tag{106}$$

$$\forall s, \forall h: w_{s,h}^{gen} \geq 0 \tag{107}$$

$$\forall s, \forall h: (-b_h^d * (w_{s,h}^{gen} - w_{s,h}^{stored}) + MB_h(d_h) - \xi_{s,h} + \nu_{s,h} * w_{s,h}^{gen} = 0 \tag{108}$$

KKT with respect to $w_{s,h}^{stored}$:

$$\forall s, \forall h: b_h^d * \left(w_{s,h}^{gen} - w_{s,h}^{stored}\right) - MB_h(d_h) + CL_s * \xi_{s,h} - \nu_{s,h} \leq 0 \tag{109}$$

$$\forall s, \forall h: w_{s,h}^{stored} \geq 0 \tag{110}$$

$$\forall s, \forall h: \left(b_h^d * \left(w_{s,h}^{gen} - w_{s,h}^{stored}\right) - B_h(d_h) + CL_s * \xi_{s,h} - \nu_{s,h}\right) * w_{s,h}^{stored} = 0 \tag{111}$$

Equation (106) is the optimality constraint regarding generation, and has clear similarities to the Cournot power generating firms' optimality constraint on production, equation (112). The Cournot storage and producers both see how they affect the price by reducing the quantity supplied in the market, the first product in equation (106), $-b_h^d * \left(w_{s,h}^{gen} - w_{s,h}^{stored}\right)$.

$$\forall s, \forall h: -b_h^d * (w_{s,h}^{gen} - w_{s,h}^{stored}) + MB_h(d_h) - \xi_{s,h} + \nu_{s,h} \leq 0 \quad (106)$$
$$\forall g, \forall h: -b_h^d * (v_{g,h}^{conv} + V_{g,h}^{ren}) + MB_h(d_h) - MC_{g,h}(v_{g,h}) - \mu_{g,h} \leq 0 \quad (112)$$

The difference between the storage and the power producer is the valuation of production cost. The storages valuation is time-dependent $\xi_{s,h}$ and are determined by equation (113)

$$\forall s, \forall h < H: \xi_{s,h+1} - \xi_{s,h} - \iota_{s,h} \leq 0 \quad (113)$$

Which evaluates the value of energy for all hours h, in relationship to each other. The Cournot storage determines the valuation of stored energy considering the valuation for all other hours, this gives a realistic operation of the storage.

The KKT conditions with respect to $w_{s,h}^{stored}$ are presented in equation (109). This formulation is mathematically correct, but do not lead to realistic behavior or results. The energy storage believes that it can reduce the market price by storing more energy. The storage can affect the price by reduce supplied energy, but cannot reduce the consumers demand by increasing own demand. The result of this belief it that the storage does not store energy at the lowest price, which is suboptimal.

The Cournot storage will get a more realistic behavior if $-b_h^d * (w_{s,h}^{gen} - w_{s,h}^{stored})$ are moved from the optimality constraint for storing energy, this means it do not believe it can manipulate the price when storing energy. The restriction ensures that energy get stored if the storage values energy in hour h higher than the market price.

The representation of the energy storages as a Cournot player is realistic when the storage unit faces other Cournot players with the same market power. In an energy market where the producers act as price-takers the belief that the storage can influence the price is wrong. The Cournot player act as if it can control the price, although it is not necessarily correct, resulting in low profits. The Cournot formulation is static, and do not adapt the market situation as the MPEC formulation of the energy storage, as discussed earlier in this thesis, and are therefore just presented in the appendix. The result from the Cournot behaving storage are presented on page 68 and 69.

Appendix B Results

Base Case

Producer: Perfect Competition Storage: Not existing							
	λ_h	d_h	v_h^{conv}	w_h^{gen}	w_h^{stored}	e_h^{stored}	ls_h
1	23.4579439	160.934579	160.934579	0	0	0	0
2	23.4579439	160.934579	160.934579	0	0	0	0
3	21.588785	146.915888	146.915888	0	0	0	0
4	21.588785	146.915888	146.915888	0	0	0	0
5	21.588785	146.915888	146.915888	0	0	0	0
6	22.5233645	153.925234	153.925234	0	0	0	0
7	24.3925234	167.943925	167.943925	0	0	0	0
8	30	210	210	0	0	0	0
9	28.1308411	195.981308	195.981308	0	0	0	0
10	26.2616822	181.962617	181.962617	0	0	0	0
11	26.2616822	181.962617	181.962617	0	0	0	0
12	26.2616822	181.962617	181.962617	0	0	0	0
13	26.2616822	181.962617	181.962617	0	0	0	0
14	26.2616822	181.962617	181.962617	0	0	0	0
15	26.2616822	181.962617	181.962617	0	0	0	0
16	30	210	210	0	0	0	0
17	30	210	210	0	0	0	0
18	29.0654206	202.990654	202.990654	0	0	0	0
19	28.1308411	195.981308	195.981308	0	0	0	0
20	28.1308411	195.981308	195.981308	0	0	0	0
21	28.1308411	195.981308	195.981308	0	0	0	0
22	26.2616822	181.962617	181.962617	0	0	0	0
23	25.3271028	174.953271	174.953271	0	0	0	0
24	24.3925234	167.943925	167.943925	0	0	0	0

Producer: Cournot Competition $\alpha = 0.01$ Storage: Not existing							
	λ_h	d_h	v_h^{conv}	w_h^{gen}	w_h^{stored}	e_h^{stored}	ls_h
1	27.1308803	160.409874	160.409874	0	0	0	0
2	27.1308803	160.409874	160.409874	0	0	0	0
3	24.9417792	146.436889	146.436889	0	0	0	0
4	24.9417792	146.436889	146.436889	0	0	0	0
5	24.9417792	146.436889	146.436889	0	0	0	0
6	26.0363298	153.423381	153.423381	0	0	0	0
7	28.2254308	167.396367	167.396367	0	0	0	0
8	34.792734	209.315324	209.315324	0	0	0	0
9	32.603633	195.342338	195.342338	0	0	0	0
10	30.4145319	181.369353	181.369353	0	0	0	0
11	30.4145319	181.369353	181.369353	0	0	0	0
12	30.4145319	181.369353	181.369353	0	0	0	0
13	30.4145319	181.369353	181.369353	0	0	0	0
14	30.4145319	181.369353	181.369353	0	0	0	0
15	30.4145319	181.369353	181.369353	0	0	0	0
16	34.792734	209.315324	209.315324	0	0	0	0
17	34.792734	209.315324	209.315324	0	0	0	0
18	33.6981835	202.328831	202.328831	0	0	0	0
19	32.603633	195.342338	195.342338	0	0	0	0
20	32.603633	195.342338	195.342338	0	0	0	0
21	32.603633	195.342338	195.342338	0	0	0	0
22	30.4145319	181.369353	181.369353	0	0	0	0
23	29.3199814	174.38286	174.38286	0	0	0	0
24	28.2254308	167.396367	167.396367	0	0	0	0

Producer: Cournot Competition $\alpha = 0.02$ Storage: Not existing							
	λ_h	d_h	v_h^{conv}	w_h^{gen}	w_h^{stored}	e_h^{stored}	ls_h
1	30.7799443	159.888579	159.888579	0	0	0	0
2	30.7799443	159.888579	159.888579	0	0	0	0
3	28.2729805	145.961003	145.961003	0	0	0	0
4	28.2729805	145.961003	145.961003	0	0	0	0
5	28.2729805	145.961003	145.961003	0	0	0	0
6	29.5264624	152.924791	152.924791	0	0	0	0
7	32.0334262	166.852368	166.852368	0	0	0	0
8	39.5543175	208.635097	208.635097	0	0	0	0
9	37.0473538	194.707521	194.707521	0	0	0	0
10	34.54039	180.779944	180.779944	0	0	0	0
11	34.54039	180.779944	180.779944	0	0	0	0
12	34.54039	180.779944	180.779944	0	0	0	0
13	34.54039	180.779944	180.779944	0	0	0	0
14	34.54039	180.779944	180.779944	0	0	0	0
15	34.54039	180.779944	180.779944	0	0	0	0
16	39.5543175	208.635097	208.635097	0	0	0	0
17	39.5543175	208.635097	208.635097	0	0	0	0
18	38.3008357	201.671309	201.671309	0	0	0	0
19	37.0473538	194.707521	194.707521	0	0	0	0
20	37.0473538	194.707521	194.707521	0	0	0	0
21	37.0473538	194.707521	194.707521	0	0	0	0
22	34.54039	180.779944	180.779944	0	0	0	0
23	33.2869081	173.816156	173.816156	0	0	0	0
24	32.0334262	166.852368	166.852368	0	0	0	0

Producer: Perfekt Competition Storage: Perfect Competition							
	λ_h	d_h	v_h^{conv}	w_h^{gen}	w_h^{stored}	e_h^{stored}	ls_h
1	24.6306862	160.767045	169.730147	0	8.9631018	9.70518166	0
2	24.6306862	160.767045	169.730147	0	8.9631018	17.7719733	0
3	24.6306862	146.481331	169.730147	0	23.2488161	38.6959078	0
4	24.6306862	146.481331	169.730147	0	23.2488161	59.6198423	0
5	24.6306862	146.481331	169.730147	0	23.2488161	80.5437768	0
6	24.6306862	153.624188	169.730147	0	16.105959	95.0391398	0
7	24.6306862	167.909902	169.730147	0	1.82024467	96.67736	0
8	27.3674291	210.376082	190.255718	20.1203631	0	76.556997	0
9	27.3674291	196.090367	190.255718	5.83464878	0	70.7223482	0
10	26.2616822	181.962617	181.962617	0	0	70.7223482	0
11	26.2616822	181.962617	181.962617	0	0	70.7223482	0
12	26.2616822	181.962617	181.962617	0	0	70.7223482	0
13	26.2616822	181.962617	181.962617	0	0	70.7223482	0
14	26.2616822	181.962617	181.962617	0	0	70.7223482	0
15	26.2616822	181.962617	181.962617	0	0	70.7223482	0
16	27.3674291	210.376082	190.255718	20.1203631	0	50.6019851	0
17	27.3674291	210.376082	190.255718	20.1203631	0	30.4816221	0
18	27.3674291	210.376082	190.255718	12.9775059	0	17.5041162	0
19	27.3674291	203.233224	190.255718	5.83464878	0	11.6694674	0
20	27.3674291	196.090367	190.255718	5.83464878	0	5.8348186	0
21	27.3674291	196.090367	190.255718	5.83464878	0	0	0
22	26.2616822	181.962617	181.962617	0	0	0	0
23	25.3271028	174.953271	174.953271	0	0	0	0
24	24.6306862	167.909902	169.730147	0	1.82024467	1.63839002	0

Producer: Cournot α = 0.01 Storage: Perfect Competition							
	λ_h	d_h	v_h^{conv}	w_h^{gen}	w_h^{stored}	e_h^{stored}	ls_h
1	28.5232735	160.210961	169.297491	0	9.08652962	9.92756135	0
2	28.5232735	160.210961	169.297491	0	9.08652952	18.1054379	0
3	28.5232735	145.925247	169.297491	0	23.3722439	39.1404574	0
4	28.5232735	145.925247	169.297491	0	23.3722439	60.1754769	0
5	28.5232735	145.925247	169.297491	0	23.372244	81.2104966	0
6	28.5232735	153.068104	169.297491	0	16.2293871	95.8169449	0
7	28.5232735	167.353818	169.297491	0	1.94367243	97.5662501	0
8	31.6925261	209.758211	189.526763	20.2314484	0	77.3348017	0
9	31.6925261	195.472496	189.526763	5.94573393	0	71.3890678	0
10	30.4145319	181.369353	181.369353	0	0	71.3890678	0
11	30.4145319	181.369353	181.369353	0	0	71.3890678	0
12	30.4145319	181.369353	181.369353	0	0	71.3890678	0
13	30.4145319	181.369353	181.369353	0	0	71.3890678	0
14	30.4145319	181.369353	181.369353	0	0	71.3890678	0
15	30.4145319	181.369353	181.369353	0	0	71.3890678	0
16	31.6925261	209.758211	189.526763	20.2314484	0	51.1576197	0
17	31.6925261	209.758211	189.526763	20.2314484	0	30.9261716	0
18	31.6925261	202.615353	189.526763	13.0885909	0	17.8375807	0
19	31.6925261	195.472496	189.526763	5.94573393	0	11.8918468	0
20	31.6925261	195.472496	189.526763	5.94573393	0	5.94611316	0
21	31.6925261	195.472496	189.526763	0	0	0.00037947	0
22	30.4145319	181.369353	181.369353	0	0	0.00037947	0
23	29.3199814	174.38286	174.38286	0	0	0.00037947	0
24	28.5232735	167.353818	169.297491	0	1.94367247	1.74968469	0

Producer: Cournot $\alpha = 0.02$ Storage: Perfect Competition							
	λ_h	d_h	v_h^{conv}	w_h^{gen}	w_h^{stored}	e_h^{stored}	ls_h
1	32.3905608	159.658491	168.836449	0	9.17795756	10.0917553	0
2	32.3905608	159.658491	168.836449	0	9.17795756	18.3519171	0
3	32.3905608	145.372777	168.836449	0	23.4636718	39.4692218	0
4	32.3905608	145.372777	168.836449	0	23.4636718	60.5865264	0
5	32.3905608	145.372777	168.836449	0	23.4636718	81.7038311	0
6	32.3905608	152.515634	168.836449	0	16.3208147	96.3925643	0
7	32.3905608	166.801348	168.836449	0	2.03510041	98.2241547	0
8	35.989512	209.144355	188.830622	20.3137332	0	77.9104215	0
9	35.989512	194.858641	188.830622	6.02801894	0	71.8824025	0
10	34.54039	180.779944	180.779944	0	0	71.8824025	0
11	34.54039	180.779944	180.779944	0	0	71.8824025	0
12	34.54039	180.779944	180.779944	0	0	71.8824025	0
13	34.54039	180.779944	180.779944	0	0	71.8824025	0
14	34.54039	180.779944	180.779944	0	0	71.8824025	0
15	34.54039	180.779944	180.779944	0	0	71.8824025	0
16	35.989512	209.144355	188.830622	20.3137332	0	51.5686693	0
17	35.989512	209.144355	188.830622	20.3137332	0	31.2549361	0
18	35.989512	202.001498	188.830622	13.1708761	0	18.08406	0
19	35.989512	194.858641	188.830622	6.02801894	0	12.056041	0
20	35.989512	194.858641	188.830622	6.02801894	0	6.02802208	0
21	35.989512	194.858641	188.830622	6.02801894	0	3.1385E-06	0
22	34.54039	180.779944	180.779944	0	0	3.1385E-06	0
23	33.2869081	173.816156	173.816156	0	0	3.1385E-06	0
24	32.3905608	166.801348	168.836449	0	2.03510041	1.83159351	0

Producer: Cournot $\alpha = 0.01$ Storage: Cournot $\alpha = 0.01$							
	λ_h	d_h	v_h^{conv}	w_h^{gen}	w_h^{stored}	e_h^{stored}	ls_h
1	28.3392723	160.237247	168.123014	0	7.88576766	8.61547642	0
2	28.3392723	160.237247	168.123014	0	7.88576766	15.7126673	0
3	28.0499881	145.992859	166.27652	0	20.2836609	33.9679621	0
4	28.0499881	145.992859	166.27652	0	20.2836609	52.2232569	0
5	28.0499881	145.992859	166.27652	0	20.2836609	70.4785517	0
6	28.1946302	153.115053	167.199767	0	14.0847143	83.1547946	0
7	28.4839144	167.359441	169.046262	0	1.68682104	84.6729335	0
8	32.1022108	209.699684	192.141771	17.5579133	0	67.1150202	0
9	31.8129266	195.455296	190.295276	5.16002007	0	61.9550001	0
10	30.4145319	181.369353	181.369353	0	0	61.9550001	0
11	30.4145319	181.369353	181.369353	0	0	61.9550001	0
12	30.4145319	181.369353	181.369353	0	0	61.9550001	0
13	30.4145319	181.369353	181.369353	0	0	61.9550001	0
14	30.4145319	181.369353	181.369353	0	0	61.9550001	0
15	30.4145319	181.369353	181.369353	0	0	61.9550001	0
16	32.1022108	209.699684	192.141771	17.5579133	0	44.3970868	0
17	32.1022108	209.699684	192.141771	17.5579133	0	26.8391735	0
18	31.9575687	202.57749	191.218523	11.3589667	0	15.4802068	0
19	31.8129266	195.455296	190.295276	5.16002007	0	10.3201867	0
20	31.8129266	195.455296	190.295276	5.16002007	0	5.16016667	0
21	31.8129266	195.455296	190.295276	5.16002007	0	0.00014659	0
22	30.4145319	181.369353	181.369353	0	0	0.00014659	0
23	29.3199814	174.38286	174.38286	0	0	0.00014659	0
24	28.4839144	167.359441	169.046262	0	1.68682104	1.51828553	0

Producer: Cournot $\alpha = 0.02$ Storage: Cournot $\alpha = 0.02$							
	λ_h	d_h	v_h^{conv}	w_h^{gen}	w_h^{stored}	e_h^{stored}	ls_h
1	32.0522277	159.706825	166.95682	0	7.24999573	8.61547642	0
2	32.0522277	159.706825	166.95682	0	7.24999573	15.7126673	0
3	31.5256039	145.496342	164.031133	0	18.5347905	33.9679621	0
4	31.5256039	145.496342	164.031133	0	18.5347905	52.2232569	0
5	31.5256039	145.496342	164.031133	0	18.5347905	70.4785517	0
6	31.7889158	152.601583	165.493977	0	12.8923931	83.1547946	0
7	32.3155395	166.812066	168.419664	0	1.60759833	84.6729335	0
8	36.7383506	209.037378	192.990837	16.0465418	0	67.1150202	0
9	36.2117269	194.826896	190.065149	4.76174697	0	61.9550001	0
10	34.54039	180.779944	180.779944	0	0	61.9550001	0
11	34.54039	180.779944	180.779944	0	0	61.9550001	0
12	34.54039	180.779944	180.779944	0	0	61.9550001	0
13	34.54039	180.779944	180.779944	0	0	61.9550001	0
14	34.54039	180.779944	180.779944	0	0	61.9550001	0
15	34.54039	180.779944	180.779944	0	0	61.9550001	0
16	36.7383506	209.037378	192.990837	16.0465418	0	44.3970868	0
17	36.7383506	209.037378	192.990837	16.0465418	0	26.8391735	0
18	36.4750387	201.932137	191.527993	10.4041444	0	15.4802068	0
19	36.2117269	194.826896	190.065149	4.76174697	0	10.3201867	0
20	36.2117269	194.826896	190.065149	4.76174697	0	5.16016667	0
21	36.2117269	194.826896	190.065149	4.76174697	0	0.00014659	0
22	34.54039	180.779944	180.779944	0	0	0.00014659	0
23	33.2869081	173.816156	173.816156	0	0	0.00014659	0
24	32.3155395	166.812066	168.419664	0	1.60759834	1.51828553	0

Producer: Perfekt Competition Storage: MPEC							
	λ_h	d_h	v_h^{conv}	w_h^{gen}	w_h^{stored}	e_h^{stored}	ls_h
1	24.0443151	160.850812	165.332363	0	4.48155086	464.007128	0
2	24.0443151	160.850812	165.332363	0	4.48155086	468.040523	0
3	23.1097356	146.698609	158.323017	0	11.624408	478.502491	0
4	23.1097356	146.698609	158.323017	0	11.624408	488.964458	0
5	23.1097356	146.698609	158.323017	0	11.624408	499.426425	0
6	23.5770253	153.774711	161.82769	0	8.05297943	506.674106	0
7	24.5116048	167.926914	168.837036	0	0.91012276	507.493217	0
8	28.6837146	210.188041	200.127859	10.0601816	0	497.433035	0
9	27.7491351	196.035838	193.118513	2.91732445	0	494.515711	0
10	26.2616822	181.962617	181.962617	0	0	494.515711	0
11	26.2616822	181.962617	181.962617	0	0	494.515711	0
12	26.2616822	181.962617	181.962617	0	0	494.515711	0
13	26.2616822	181.962617	181.962617	0	0	494.515711	0
14	26.2616822	181.962617	181.962617	0	0	494.515711	0
15	26.2616822	181.962617	181.962617	0	0	494.515711	0
16	28.6837146	210.188041	200.127859	10.0601816	0	484.455529	0
17	28.6837146	210.188041	200.127859	10.0601816	0	474.395348	0
18	28.2164248	203.111939	196.623186	6.48875302	0	467.906595	0
19	27.7491351	196.035838	193.118513	2.91732445	0	464.98927	0
20	27.7491351	196.035838	193.118513	2.91732445	0	462.071946	0
21	27.7491351	196.035838	193.118513	2.91732445	0	459.154621	0
22	26.2616822	181.962617	181.962617	0	0	459.154621	0
23	25.3271028	174.953271	174.953271	0	0	459.154621	0
24	24.5116048	167.926914	168.837036	0	0.91012273	459.973732	0

Producer: Cournot $\alpha = 0.01$ Competition Storage: MPEC							
	λ_h	d_h	v_h^{conv}	w_h^{gen}	w_h^{stored}	e_h^{stored}	ls_h
1	27.8270769	160.310418	164.853682	0	4.54326478	578.70977	0
2	27.8270769	160.310418	164.853682	0	4.54326478	582.798708	0
3	26.7325264	146.181068	157.86719	0	11.6861219	593.316218	0
4	26.7325264	146.181068	157.86719	0	11.6861219	603.833728	0
5	26.7325264	146.181068	157.86719	0	11.6861219	614.351237	0
6	27.2798016	153.245743	161.360436	0	8.11469335	621.654461	0
7	28.3743522	167.375093	168.346929	0	0.97183629	622.529114	0
8	33.2426301	209.536767	199.421043	10.115724	0	612.41339	0
9	32.1480795	195.407417	192.43455	2.9728669	0	609.440523	0
10	30.4145319	181.369353	181.369353	0	0	609.440523	0
11	30.4145319	181.369353	181.369353	0	0	609.440523	0
12	30.4145319	181.369353	181.369353	0	0	609.440523	0
13	30.4145319	181.369353	181.369353	0	0	609.440523	0
14	30.4145319	181.369353	181.369353	0	0	609.440523	0
15	30.4145319	181.369353	181.369353	0	0	609.440523	0
16	33.2426301	209.536767	199.421043	10.115724	0	599.324799	0
17	33.2426301	209.536767	199.421043	10.115724	0	589.209075	0
18	32.6953548	202.472092	195.927797	6.54429547	0	582.66478	0
19	32.1480795	195.407417	192.43455	2.9728669	0	579.691913	0
20	32.1480795	195.407417	192.43455	2.9728669	0	576.719046	0
21	32.1480795	195.407417	192.43455	2.9728669	0	573.746179	0
22	30.4145319	181.369353	181.369353	0	0	573.746179	0
23	29.3199814	174.38286	174.38286	0	0	573.746179	0
24	28.3743522	167.375093	168.346929	0	0.97183633	574.620832	0

Producer: Cournot $\alpha = 0.02$ Competition Storage: MPEC							
	λ_h	d_h	v_h^{conv}	w_h^{gen}	w_h^{stored}	e_h^{stored}	ls_h
1	31.5852525	159.773535	164.362514	0	4.58897876	610.686555	0
2	31.5852525	159.773535	164.362514	0	4.58897876	614.816636	0
3	30.3317706	145.66689	157.398726	0	11.7318359	625.375288	0
4	30.3317706	145.66689	157.398726	0	11.7318359	635.93394	0
5	30.3317706	145.66689	157.398726	0	11.7318359	646.492592	0
6	30.9585116	152.720213	160.88062	0	8.16040732	653.836959	0
7	32.2119935	166.826858	167.844409	0	1.01755054	654.752755	0
8	37.7719148	208.889726	198.73286	10.1568666	0	644.595888	0
9	36.5184329	194.783081	191.769072	3.01400952	0	641.581878	0
10	34.54039	180.779944	180.779944	1.4133E-08	0	641.581878	0
11	34.54039	180.779944	180.779944	1.5678E-08	0	641.581878	0
12	34.54039	180.779944	180.779944	1.6987E-08	0	641.581878	0
13	34.54039	180.779944	180.779944	1.8082E-08	0	641.581878	0
14	34.54039	180.779944	180.779944	1.8991E-08	0	641.581878	0
15	34.54039	180.779944	180.779944	1.8204E-08	0	641.581878	0
16	37.7719148	208.889726	198.73286	10.1568666	0	631.425012	0
17	37.7719148	208.889726	198.73286	10.1568666	0	621.268145	0
18	37.1451738	201.836404	195.250966	6.58543808	0	614.682707	0
19	36.5184329	194.783081	191.769071	3.01400952	0	611.668697	0
20	36.5184329	194.783081	191.769071	3.01400952	0	608.654688	0
21	36.5184329	194.783081	191.769071	3.01400952	0	605.640678	0
22	34.54039	180.779944	180.779944	1.9176E-08	0	605.640678	0
23	33.2869081	173.816156	173.816156	0	0	605.640678	0
24	32.2119935	166.826858	167.844409	0	1.01755053	606.556474	0

Scarce Production Capacity and Flexibility

Producer: Perfect Competition Storage: Not Existing, $V^{prod.cap}$ = 200 MW							
	λ_h	d_h	v_h^{conv}	w_h^{gen}	w_h^{stored}	e_h^{stored}	ls_h
1	32.9166667	159.583333	159.583333	0	0	0	0
2	32.9166667	159.583333	159.583333	0	0	0	0
3	30.1388889	145.694444	145.694444	0	0	0	0
4	30.1388889	145.694444	145.694444	0	0	0	0
5	30.1388889	145.694444	145.694444	0	0	0	0
6	31.5277778	152.638889	152.638889	0	0	0	0
7	34.3055556	166.527778	166.527778	0	0	0	0
8	100	200	200	0	0	0	0
9	39.8611111	194.305556	194.305556	0	0	0	0
10	37.0833333	180.416667	180.416667	0	0	0	0
11	37.0833333	180.416667	180.416667	0	0	0	0
12	37.0833333	180.416667	180.416667	0	0	0	0
13	37.0833333	180.416667	180.416667	0	0	0	0
14	37.0833333	180.416667	180.416667	0	0	0	0
15	37.0833333	180.416667	180.416667	0	0	0	0
16	100	200	200	0	0	0	0
17	100	200	200	0	0	0	0
18	50	200	200	0	0	0	0
19	39.8611111	194.305556	194.305556	0	0	0	0
20	39.8611111	194.305556	194.305556	0	0	0	0
21	39.8611111	194.305556	194.305556	0	0	0	0
22	37.0833333	180.416667	180.416667	0	0	0	0
23	35.6944444	173.472222	173.472222	0	0	0	0
24	34.3055556	166.527778	166.527778	0	0	0	0

Producer: Perfect Competition Storage: Perfect Competition, $V^{prod.cap}$ = 200 MW							
	λ_h	d_h	v_h^{conv}	w_h^{gen}	w_h^{stored}	e_h^{stored}	ls_h
1	32.9166667	159.583333	159.583333	0	0	0	0
2	32.9166667	159.583333	159.583333	0	0	0	0
3	31.8364198	145.45194	154.182099	0	8.73015872	7.85714285	0
4	31.8364198	145.45194	154.182099	0	8.73015873	15.7142857	0
5	31.8364198	145.45194	154.182099	0	8.73015873	23.5714286	0
6	31.8364198	152.594797	154.182099	0	1.58730159	25	0
7	34.3055556	166.527778	166.527778	0	0	25	0
8	41.2606169	208.39134	200	8.39134044	0	16.6086596	0
9	39.8611111	194.305556	194.305556	0	0	16.6086596	0
10	37.1345552	180.409349	180.672776	0	0.26342677	16.8457436	0
11	37.1345552	180.409349	180.672776	0	0.26342677	17.0828277	0
12	37.1345552	180.409349	180.672776	0	0.26342678	17.3199118	0
13	37.1345552	180.409349	180.672776	0	0.26342679	17.556996	0
14	37.1345552	180.409349	180.672776	0	0.26342679	17.7940801	0
15	37.1345552	180.409349	180.672776	0	0.2634268	18.0311642	0
16	41.2606169	208.39134	200	8.39134044	0	9.63982374	0
17	41.2606169	208.39134	200	8.39134044	0	1.2484833	0
18	41.2606169	201.248483	200	1.2484833	0	0	0
19	39.8611111	194.305556	194.305556	0	0	0	0
20	39.8611111	194.305556	194.305556	0	0	0	0
21	39.8611111	194.305556	194.305556	0	0	0	0
22	37.0833333	180.416667	180.416667	0	0	0	0
23	35.6944444	173.472222	173.472222	0	0	0	0
24	34.3055556	166.527778	166.527778	0	0	0	0

Producer: Perfect Competition Storage: MPEC, $V^{prod.cap}$ = 200 MW							
	λ_h	d_h	v_h^{conv}	w_h^{gen}	w_h^{stored}	e_h^{stored}	ls_h
1	33.0066872	159.570473	160.033436	0	0.46296298	0.41666668	0
2	33.0066872	159.570473	160.033436	0	0.46296296	0.83333335	0
3	31.6177984	145.483172	153.088992	0	7.6058201	7.67857144	0
4	31.6177984	145.483172	153.088992	0	7.6058201	14.5238095	0
5	31.6177984	145.483172	153.088992	0	7.6058201	21.3690476	0
6	32.3122428	152.526822	156.561214	0	4.03439153	25	0
7	34.3055556	166.527778	166.527778	0	0	25	0
8	69.3918919	204.372587	200	4.37258687	0	20.6274131	0
9	39.3224474	194.382508	191.612237	2.77027027	0	17.8571429	0
10	37.0833333	180.416667	180.416667	0	0	17.8571429	0
11	37.0833333	180.416667	180.416667	0	0	17.8571429	0
12	37.0833333	180.416667	180.416667	0	0	17.8571429	0
13	37.0833333	180.416667	180.416667	0	0	17.8571429	0
14	37.0833333	180.416667	180.416667	0	0	17.8571429	0
15	37.0833333	180.416667	180.416667	0	0	17.8571429	0
16	69.3918919	204.372587	200	4.37258687	0	13.484556	0
17	69.3918919	204.372587	200	4.37258687	0	9.11196911	0
18	44.3918919	200.801158	200	0.8011583	0	8.31081081	0
19	39.3224474	194.382508	191.612237	2.77027027	0	5.54054054	0
20	39.3224474	194.382508	191.612237	2.77027027	0	2.77027027	0
21	39.3224474	194.382508	191.612237	2.77027027	0	0	0
22	37.0833333	180.416667	180.416667	0	0	0	0
23	35.6944444	173.472222	173.472222	0	0	0	0
24	34.3055556	166.527778	166.527778	0	0	0	0

Appendix C GAMS Codes

Perfect Competitive Generators and Storages

C:\Users\vegardsb\Desktop\FirstTry\Model1.gms 24. januar 2017 13:38:33 **Page 1**

```
SETS
g firms /g1, g2, g3/
s storrage firms /s1, s2, s3/
h hours /h1*h24/
hsub1(h) /h1*h23/
hsub2(h) /h2*h24/

PARAMETERS
Cl(s)
enInst(s)
capInst(s)
ad(h)
bd(h)
bc(h,g)
cc(h,g)
capProd(h,g)
Vren(h,g)
Pmax
;

$CALL GDXXRW input.xls trace=3 Squeeze=N Index=Index_1!a1
$GDXIN input.gdx
$LOAD Cl enInst capInst ad bd bc cc capProd Vren Pmax
$GDXIN

display Cl, enInst, capInst, ad, bd, bc, cc, capProd, Vren, Pmax;

POSITIVE VARIABLES
vconv(h,g) energy generated by firm i in period h
wgen(h,s) energy generated by storage s in period h
wstor(h,s) energy stored by storage s in period h
en(h,s) energy stored unit s time h
d(h) energy demand in hour h
ls(h) load shedding in hour h

lamda(h) price in periode h
my(h,g) dual to production capacity constraint
xi(h,s) dual to energy balanse of storrage
v(h,s) dual to capacity constraint of storage unit s
l(h,s) dual to energy capacity of storage unit s

EQUATIONS

MarkClear_lamda(h)
TSO_ls(h)
PowerProducer_vconv(h,g)
PowerProducer_my(h,g)
EnergyStorage_wgen(h,s)
EnergyStorage_wstor(h,s)
EnergyStorage_l(h,s)
EnergyStorage_v(h,s)
EnergyStorage_en1(h,s)
EnergyStorage_en2(h,s)
EnergyStorage_xi1(h,s)
EnergyStorage_xi2(h,s)
Demand_d(h)

;
*MarketClear/Energy Balance
MarkClear_lamda(h).. sum(g,vconv(h,g)+Vren(h,g)) + sum(s,(wgen(h,s) - wstor(h»
,s))) - d(h) + ls(h) =g= 0 ;
```

```
TSO_ls(h).. - lamda(h) + Pmax =g= 0 ;
*PowerProducer
PowerProducer_vconv(h,g).. -lamda(h) + bc(h,g) + 2*cc(h,g)*vconv(h,g) + my(h,»
g) =g= 0 ;
PowerProducer_my(h,g).. -vconv(h,g) + capProd(h,g) =g= 0 ;
*EnergyStorage
EnergyStorage_wgen(h,s).. -lamda(h) + xi(h,s) + v(h,s) =g= 0;
EnergyStorage_wstor(h,s).. lamda(h) - xi(h,s)*CL(s) + v(h,s) =g= 0 ;
EnergyStorage_l(h,s).. enInst(s) - en(h,s) =g= 0 ;
EnergyStorage_v(h,s).. capInst(s) - wgen(h,s) - wstor(h,s) =g= 0 ;
EnergyStorage_en1(h,s)$hsub1(h).. -xi(h+1,s) + xi(h,s) + l(h,s) =g= 0 ;
EnergyStorage_en2('h24',s).. -xi('h1',s) + xi('h24',s) + l('h24',s) =g= 0 ;
EnergyStorage_xi1('h1',s).. en('h24',s) + wstor('h1',s)*CL(s) - wgen('h1',s) »
- en('h1',s) =g= 0 ;
EnergyStorage_xi2(h,s)$hsub2(h).. en(h-1,s) + wstor(h,s)*CL(s) - wgen(h,s) - »
en(h,s) =g= 0 ;
*Demand
Demand_d(h).. lamda(h) - (ad(h)-bd(h)*d(h)) =g= 0;

MODEL

ESS1/
MarkClear_lamda.lamda,
TSO_ls.ls

PowerProducer_vconv.vconv,
PowerProducer_my.my,

EnergyStorage_wgen.wgen,
EnergyStorage_wstor.wstor,
EnergyStorage_l.l,
EnergyStorage_v.v,
EnergyStorage_en1.en,
EnergyStorage_en2.en,
EnergyStorage_xi1.xi,
EnergyStorage_xi2.xi,

Demand_d.d/
;

Solve
ESS1 using mcp
;

Parameters TotEnergyProd, TotStored, TotGenByStor, Balance(h), Virk;

TotEnergyProd = sum(h,sum(g,vconv.l(h,g))) ;
TotStored = sum(h,sum(s,wstor.l(h,s))) ;
TotGenByStor = sum(h,sum(s,wgen.l(h,s))) ;
Balance(h) = sum(g,(vconv.l(h,g)+Vren(h,g)))+sum(s,(wgen.l(h,s)-wstor.l(h,s))»
) - d.l(h);
Virk = TotGenByStor/TotStored;

Display vconv.l, TotEnergyProd ;
Display wstor.l, TotStored ;
Display wgen.l, TotGenByStor ;
Display Balance;
Display Virk

execute_unload 'results.gdx';
*Solution variables
```

```
execute 'gdxxrw results.gdx Squeeze=N var=vconv rng=vconv!a1';
execute 'gdxxrw results.gdx Squeeze=N var=wgen rng=wgen!a1';
execute 'gdxxrw results.gdx Squeeze=N var=wstor rng=stor!a1';
execute 'gdxxrw results.gdx Squeeze=N var=en rng=en!a1';
execute 'gdxxrw results.gdx Squeeze=N var=d rng=d!a1';
execute 'gdxxrw results.gdx Squeeze=N var=ls rng=lsa1';
execute 'gdxxrw results.gdx Squeeze=N var=lamda rng=lamda!a1';
execute 'gdxxrw results.gdx Squeeze=N var=my rng=my!a1';
execute 'gdxxrw results.gdx Squeeze=N var=xi rng=xi!a1';
execute 'gdxxrw results.gdx Squeeze=N var=v rng=v!a1';
execute 'gdxxrw results.gdx Squeeze=N var=l rng=l!a1';
*Input Parameters
execute 'gdxxrw results.gdx Squeeze=N par=Cl rng=Cl!a1';
execute 'gdxxrw results.gdx Squeeze=N par=enInst rng=enInst!a1';
execute 'gdxxrw results.gdx Squeeze=N par=capInst rng=capInst!a1';
execute 'gdxxrw results.gdx Squeeze=N par=ad rng=ad!a1';
execute 'gdxxrw results.gdx Squeeze=N par=bd rng=bd!a1';
execute 'gdxxrw results.gdx Squeeze=N par=bc rng=bc!a1';
execute 'gdxxrw results.gdx Squeeze=N par=cc rng=cc!a1';
execute 'gdxxrw results.gdx Squeeze=N par=capProd rng=capProd!a1';
execute 'gdxxrw results.gdx Squeeze=N par=Vren rng=Vren!a1';
execute 'gdxxrw results.gdx Squeeze=N par=Pmax rng=Pmax!a1';

*execute 'gdxxrw results.gdx Squeeze=N txt=C(i,b) rng=sim!B8';

```

Perfect Competitive Storages and Cournot Generators

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```
SETS
g firms /g1, g2, g3/
s storrage firms /s1, s2, s3/
h hours /h1*h24/
hsub1(h) /h1*h23/
hsub2(h) /h2*h24/

PARAMETERS
C1(s)
enInst(s)
capInst(s)
ad(h)
bd(h)
bc(h,g)
cc(h,g)
capProd(h,g)
Vren(h,g)
Pmax
;

*Loading parameters
$CALL GDXXRW input.xls trace=3 Squeeze=N Index=Index_1!a1
$GDXIN input.gdx
$LOAD C1 enInst capInst ad bd bc cc capProd Vren Pmax
$GDXIN

display C1, enInst, capInst, ad, bd, bc, cc, capProd, Vren, Pmax;


POSITIVE VARIABLES
vconv(h,g) energy generated by firm i in period h
wgen(h,s) energy generated by storage s in period h
wstor(h,s) energy stored by storage s in period h
en(h,s) energy stored unit s time h
d(h) energy demand in hour h
ls(h) load shedding in hour h

lamda(h) price in periode h
my(h,g) dual to production capacity constraint
xi(h,s) dual to energy balanse of storrage
v(h,s) dual to capacity constraint of storage unit s
l(h,s) dual to energy capacity of storage unit s

EQUATIONS

MarkClear_lamda(h)
TSO_ls(h)
PowerProducer_vconv(h,g)
PowerProducer_my(h,g)
EnergyStorage_wgen(h,s)
EnergyStorage_wstor(h,s)
EnergyStorage_l(h,s)
EnergyStorage_v(h,s)
EnergyStorage_en1(h,s)
EnergyStorage_en2(h,s)
EnergyStorage_xi1(h,s)
EnergyStorage_xi2(h,s)
Demand_d(h)

;
*MarketClear/Energy Balance
```

```
MarkClear_lamda(h).. sum(g,vconv(h,g)+Vren(h,g)) + sum(s,(wgen(h,s) - wstor(h»
,s))) - d(h) + ls(h) =e= 0 ;
TSO_ls(h).. - lamda(h) + Pmax =g= 0 ;
*PowerProducer
PowerProducer_vconv(h,g)..  - ad(h) + bd(h)*d(h) + 0.02*bd(h)*vconv(h,g) + (b»
c(h,g) + 2*cc(h,g)*vconv(h,g)) + my(h,g) =g= 0 ;
PowerProducer_my(h,g).. -vconv(h,g) + capProd(h,g) =g= 0 ;
*EnergyStorage
EnergyStorage_wgen(h,s).. -lamda(h) + xi(h,s) + v(h,s) =g= 0;
EnergyStorage_wstor(h,s).. lamda(h) - xi(h,s)*CL(s) + v(h,s) =g= 0 ;
EnergyStorage_l(h,s).. enInst(s) - en(h,s) =g= 0 ;
EnergyStorage_v(h,s).. capInst(s) - wgen(h,s) - wstor(h,s) =g= 0 ;
EnergyStorage_en1(h,s)$hsub1(h).. -xi(h+1,s) + xi(h,s) + l(h,s) =g= 0 ;
EnergyStorage_en2('h24',s).. -xi('h1',s) + xi('h24',s) + l('h24',s) =g= 0 ;
EnergyStorage_xi1('h1',s).. en('h24',s) + wstor('h1',s)*CL(s) - wgen('h1',s) »
- en('h1',s) =g= 0 ;
EnergyStorage_xi2(h,s)$hsub2(h).. en(h-1,s) + wstor(h,s)*CL(s) - wgen(h,s) - »
en(h,s) =g= 0 ;
*Demand
Demand_d(h).. lamda(h) - (ad(h)-bd(h)*d(h)) =g= 0;

MODEL

ESS1/
MarkClear_lamda.lamda,
TSO_ls.ls,
PowerProducer_vconv.vconv,
PowerProducer_my.my,
EnergyStorage_wgen.wgen,
EnergyStorage_wstor.wstor,
EnergyStorage_l.l,
EnergyStorage_v.v,
EnergyStorage_en1.en,
EnergyStorage_en2.en,
EnergyStorage_xi1.xi,
EnergyStorage_xi2.xi,
Demand_d.d/
;

Solve
ESS1 using mcp
;

Parameters TotEnergyProd, TotStored, TotGenByStor, Balance(h), Virk;

TotEnergyProd = sum(h,sum(g,vconv.l(h,g))) ;
TotStored = sum(h,sum(s,wstor.l(h,s))) ;
TotGenByStor = sum(h,sum(s,wgen.l(h,s))) ;
Balance(h) = sum(g,(vconv.l(h,g)+Vren(h,g)))+sum(s,(wgen.l(h,s)-wstor.l(h,s))»
) - d.l(h);
Virk = TotGenByStor/TotStored;

Display vconv.l, TotEnergyProd ;
Display wstor.l, TotStored ;
Display wgen.l, TotGenByStor ;
Display Balance;
Display Virk

execute_unload 'results.gdx';
*Solution variables
execute 'gdxxrw results.gdx Squeeze=N var=vconv rng=vconv!a1';
```

```
execute 'gdxxrw results.gdx Squeeze=N var=wgen rng=wgen!a1';
execute 'gdxxrw results.gdx Squeeze=N var=wstor rng=stor!a1';
execute 'gdxxrw results.gdx Squeeze=N var=en rng=en!a1';
execute 'gdxxrw results.gdx Squeeze=N var=d rng=d!a1';
execute 'gdxxrw results.gdx Squeeze=N var=ls rng=ls!a1';
execute 'gdxxrw results.gdx Squeeze=N var=lamda rng=lamda!a1';
execute 'gdxxrw results.gdx Squeeze=N var=my rng=my!a1';
execute 'gdxxrw results.gdx Squeeze=N var=xi rng=xi!a1';
execute 'gdxxrw results.gdx Squeeze=N var=v rng=v!a1';
execute 'gdxxrw results.gdx Squeeze=N var=l rng=l!a1';
*Input Parameters
execute 'gdxxrw results.gdx Squeeze=N par=Cl rng=Cl!a1';
execute 'gdxxrw results.gdx Squeeze=N par=enInst rng=enInst!a1';
execute 'gdxxrw results.gdx Squeeze=N par=capInst rng=capInst!a1';
execute 'gdxxrw results.gdx Squeeze=N par=ad rng=ad!a1';
execute 'gdxxrw results.gdx Squeeze=N par=bd rng=bd!a1';
execute 'gdxxrw results.gdx Squeeze=N par=bc rng=bc!a1';
execute 'gdxxrw results.gdx Squeeze=N var=cc rng=cc!a1';
execute 'gdxxrw results.gdx Squeeze=N var=capProd rng=capProd!a1';
execute 'gdxxrw results.gdx Squeeze=N var=Vren rng=Vren!a1';
execute 'gdxxrw results.gdx Squeeze=N var=Pmax rng=Pmax!a1';

*execute 'gdxxrw results.gdx Squeeze=N txt=C(i,b) rng=sim!B8';

```

Cournot Storages and Cournot Generators

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```
SETS
g firms /g1, g2, g3/
s storrage firms /s1, s2, s3/
h hours /h1*h24/
hsub1(h) /h1*h23/
hsub2(h) /h2*h24/

PARAMETERS
Cl(s)
enInst(s)
capInst(s)
ad(h)
bd(h)
bc(h,g)
cc(h,g)
capProd(h,g)
Vren(h,g)
Pmax
;

*Loading parameters
$CALL GDXXRW input.xls trace=3 Squeeze=N Index=Index_1!a1
$GDXIN input.gdx
$LOAD Cl enInst capInst ad bd bc cc capProd Vren Pmax
$GDXIN

display Cl, enInst, capInst, ad, bd, bc, cc, capProd, Vren, Pmax;

POSITIVE VARIABLES
vconv(h,g) energy generated by firm i in period h
wgen(h,s) energy generated by storage s in period h
wstor(h,s) energy stored by storage s in period h
en(h,s) energy stored unit s time h
d(h) energy demand in hour h
ls(h) load shedding in hour h

lamda(h) price in periode h
my(h,g) dual to production capacity constraint
xi(h,s) dual to energy balanse of storrage
v(h,s) dual to capacity constraint of storage unit s
l(h,s) dual to energy capacity of storage unit s

EQUATIONS

MarkClear_lamda(h)
TSO_ls(h)
PowerProducer_vconv(h,g)
PowerProducer_my(h,g)
EnergyStorage_wgen(h,s)
EnergyStorage_wstor(h,s)
EnergyStorage_l(h,s)
EnergyStorage_v(h,s)
EnergyStorage_en1(h,s)
EnergyStorage_en2(h,s)
EnergyStorage_xi1(h,s)
EnergyStorage_xi2(h,s)
Demand_d(h)
;

*MarketClear/Energy Balance
MarkClear_lamda(h).. sum(g,vconv(h,g)+Vren(h,g)) + sum(s,(wgen(h,s) - wstor(h»
```

```
,s))) - d(h) + ls(h) =e= 0 ;
TSO_ls(h).. -lamda(h) + Pmax =g= 0 ;
*PowerProducer
PowerProducer_vconv(h,g)..  - ad(h) + bd(h)*d(h) + 0.02*bd(h)*vconv(h,g) + (b»
c(h,g) + 2*cc(h,g)*vconv(h,g)) + my(h,g) =g= 0 ;
PowerProducer_my(h,g).. -vconv(h,g) + capProd(h,g) =g= 0 ;
*EnergyStorage
EnergyStorage_wgen(h,s).. - ad(h) + bd(h)*d(h) + 0.02*bd(h)*(wgen(h,s)-wstor(»
h,s)) + xi(h,s) + v(h,s) =g= 0 ;
EnergyStorage_wstor(h,s).. ad(h) - bd(h)*d(h) - 0.02*bd(h)*(wgen(h,s)-wstor(h»
,s)) - xi(h,s)*CL(s) + v(h,s) =g= 0 ;

EnergyStorage_l(h,s).. enInst(s) - en(h,s) =g= 0 ;
EnergyStorage_v(h,s).. capInst(s) - wgen(h,s) - wstor(h,s) =g= 0 ;
EnergyStorage_en1(h,s)$hsub1(h).. -xi(h+1,s) + xi(h,s) + l(h,s) =g= 0 ;
EnergyStorage_en2('h24',s).. -xi('h1',s) + xi('h24',s) + l('h24',s) =g= 0 ;
EnergyStorage_xi1('h1',s).. en('h24',s) + wstor('h1',s)*CL(s) - wgen('h1',s) »
- en('h1',s) =g= 0 ;
EnergyStorage_xi2(h,s)$hsub2(h).. en(h-1,s) + wstor(h,s)*CL(s) - wgen(h,s) - »
en(h,s) =g= 0 ;
*Demand
Demand_d(h).. lamda(h) - (ad(h)-bd(h)*d(h)) =g= 0;

MODEL

ESS1/
MarkClear_lamda.lamda,
TSO_ls.ls
PowerProducer_vconv.vconv,
PowerProducer_my.my,
EnergyStorage_wgen.wgen,
EnergyStorage_wstor.wstor,
EnergyStorage_l.l,
EnergyStorage_v.v,
EnergyStorage_en1.en,
EnergyStorage_en2.en,
EnergyStorage_xi1.xi,
EnergyStorage_xi2.xi,
Demand_d.d/
;

Solve
ESS1 using mcp
;

Parameters TotEnergyProd, TotStored, TotGenByStor, Balance(h), Virk,ProfitLag»
erTot, ProfitGenTot;

TotEnergyProd = sum(h,sum(g,vconv.l(h,g))) ;
TotStored = sum(h,sum(s,wstor.l(h,s))) ;
TotGenByStor = sum(h,sum(s,wgen.l(h,s))) ;
Balance(h) = sum(g,(vconv.l(h,g)+Vren(h,g)))+sum(s,(wgen.l(h,s)-wstor.l(h,s))»
) - d.l(h);
Virk = TotGenByStor/TotStored;
ProfitLagerTot = sum(h,sum(s,lamda.l(h)*(wgen.l(h,s)-wstor.l(h,s))));
ProfitGenTot = sum(h,sum(g,lamda.l(h)*vconv.l(h,g)-((bc(h,g)+cc(h,g)*vconv.l(»
h,g))*vconv.l(h,g))));
Display vconv.l, TotEnergyProd ;
Display wstor.l, TotStored ;
Display wgen.l, TotGenByStor ;
Display Balance;
```

```
Display Virk ;
Display ProfitLagerTot
Display ProfitGenTot
*tap per tidsperiode for energilageret

execute_unload 'results.gdx';
*Solution variables
execute 'gdxxrw results.gdx Squeeze=N var=vconv rng=vconv!a1';
execute 'gdxxrw results.gdx Squeeze=N var=wgen rng=wgen!a1';
execute 'gdxxrw results.gdx Squeeze=N var=wstor rng=stor!a1';
execute 'gdxxrw results.gdx Squeeze=N var=en rng=en!a1';
execute 'gdxxrw results.gdx Squeeze=N var=d rng=d!a1';
execute 'gdxxrw results.gdx Squeeze=N var=ls rng=ls!a1';
execute 'gdxxrw results.gdx Squeeze=N var=lamda rng=lamda!a1';
execute 'gdxxrw results.gdx Squeeze=N var=my rng=my!a1';
execute 'gdxxrw results.gdx Squeeze=N var=xi rng=xi!a1';
execute 'gdxxrw results.gdx Squeeze=N var=v rng=v!a1';
execute 'gdxxrw results.gdx Squeeze=N var=l rng=l!a1';
*Input Parameters
execute 'gdxxrw results.gdx Squeeze=N par=Cl rng=Cl!a1';
execute 'gdxxrw results.gdx Squeeze=N par=enInst rng=enInst!a1';
execute 'gdxxrw results.gdx Squeeze=N par=capInst rng=capInst!a1';
execute 'gdxxrw results.gdx Squeeze=N par=ad rng=ad!a1';
execute 'gdxxrw results.gdx Squeeze=N par=bd rng=bd!a1';
execute 'gdxxrw results.gdx Squeeze=N par=bc rng=bc!a1';
execute 'gdxxrw results.gdx Squeeze=N var=cc rng=cc!a1';
execute 'gdxxrw results.gdx Squeeze=N var=capProd rng=capProd!a1';
execute 'gdxxrw results.gdx Squeeze=N var=Vren rng=Vren!a1';
execute 'gdxxrw results.gdx Squeeze=N var=Pmax rng=Pmax!a1';
```

Top-level MPEC Storage and Perfect Competitive Generators

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```
SETS

g firms /g1, g2,g3/
s storrage firms /s1, s2, s3/
h hours /h1*h24/
hsub1(h) /h1*h23/
hsub2(h) /h2*h24/

PARAMETERS
Cl(s)
enInst(s)
capInst(s)
ad(h)
bd(h)
bc(h,g)
cc(h,g)
capProd(h,g)
Vren(h,g)
Pmax
;

*Loading parameters
$CALL GDXXRW input.xls trace=3 Squeeze=N Index=Index_1!a1
$GDXIN input.gdx
$LOAD Cl enInst capInst ad bd bc cc capProd Vren Pmax  lamdaInn Tcost Tcap
$GDXIN

display Cl, enInst, capInst, ad, bd, bc, cc, capProd, Vren, Pmax,  lamdaInn, »
Tcost, Tcap; ;

POSITIVE VARIABLES
vconv(h,g) energy generated by firm i in period h
wgen(h,s) energy generated by storage s in period h
wstor(h,s) energy stored by storage s in period h
en(h,s) energy stored unit s time h
d(h) energy demand in hour h
ls(h) load shedding in hour h

lamda(h) price in periode h
my(h,g) dual to production capacity constraint
xi(h,s) dual to energy balanse of storrage
v(h,s) dual to capacity constraint of storage unit s
l(h,s) dual to energy capacity of storage unit s

VARIABLES
NegStackObj negative of Stack objtive
;

EQUATIONS

StackObjDef Stackelberg objective defination

EnergyStorage_wgen(h,s)
EnergyStorage_wstor(h,s)
EnergyStorage_l(h,s)
EnergyStorage_v(h,s)
EnergyStorage_en1(h,s)
EnergyStorage_en2(h,s)
EnergyStorage_xi1(h,s)
EnergyStorage_xi2(h,s)
MarkClear_lamda(h)
```

```
TSO_ls(h)
TSO_uimp(h)
TSO_uexp(h)
TSO_dirac(h)
PowerProducer_vconv(h,g)
PowerProducer_my(h,g)
Demand_d(h)
;

*EnergyStorage
StackObjDef..   sum(h,sum(s,(lamda(h)*(wgen(h,s)-wstor(h,s)))))- NegStackObj »
=e= 0 ;

EnergyStorage_l(h,s).. - enInst(s) + en(h,s) =l= 0 ;
EnergyStorage_v(h,s).. - capInst(s) + wgen(h,s) + wstor(h,s) =l= 0 ;
EnergyStorage_xi1('h1',s).. -en('h24',s) - wstor('h1',s)*CL(s) + wgen('h1',s)»
 + en('h1',s) =l= 0 ;
EnergyStorage_xi2(h,s)$hsub2(h).. -en(h-1,s) - wstor(h,s)*CL(s) + wgen(h,s) +»
 en(h,s) =l= 0 ;

*MarketClear/Energy Balance
MarkClear_lamda(h).. sum(g,vconv(h,g)+Vren(h,g)) + sum(s,(wgen(h,s) - wstor(h»
,s))) - d(h) + ls(h) =g= 0 ;
TSO_ls(h).. - lamda(h) + Pmax =g= 0 ;
*PowerProducer
PowerProducer_vconv(h,g).. -lamda(h) + 2*cc(h,g)*vconv(h,g) + my(h,g) =g= 0 ;
PowerProducer_my(h,g).. -vconv(h,g) + capProd(h,g) =g= 0 ;
*Demand
Demand_d(h).. lamda(h) - (ad(h)-bd(h)*d(h)) =g= 0;

MODEL

ESS1/

StackObjDef,
EnergyStorage_l,
EnergyStorage_v,
EnergyStorage_xi1,
EnergyStorage_xi2,
MarkClear_lamda.lamda,
TSO_ls.ls
PowerProducer_vconv.vconv,
PowerProducer_my.my,
Demand_d.d/
;
Option MPEC=KNITRO;
ESS1.optfile = 1 ;

Solve
ESS1 maximizing NegStackObj using mpec ;
;

Parameters TotEnergyProd, TotStored, TotGenByStor, Balance(h), Virk;

TotEnergyProd = sum(h,sum(g,vconv.l(h,g))) ;
TotStored = sum(h,sum(s,wstor.l(h,s))) ;
TotGenByStor = sum(h,sum(s,wgen.l(h,s))) ;
Balance(h) = sum(g,(vconv.l(h,g)+Vren(h,g)))+sum(s,(wgen.l(h,s)-wstor.l(h,s))»
) - d.l(h);
Virk = TotGenByStor/TotStored;

```

```
Display vconv.l, TotEnergyProd ;
Display wstor.l, TotStored ;
Display wgen.l, TotGenByStor ;
Display Balance;
Display Virk

execute_unload 'results.gdx';
*Solution variables
execute 'gdxxrw results.gdx Squeeze=N var=vconv rng=vconv!a1';
execute 'gdxxrw results.gdx Squeeze=N var=wgen rng=wgen!a1';
execute 'gdxxrw results.gdx Squeeze=N var=wstor rng=stor!a1';
execute 'gdxxrw results.gdx Squeeze=N var=en rng=en!a1';
execute 'gdxxrw results.gdx Squeeze=N var=d rng=d!a1';
execute 'gdxxrw results.gdx Squeeze=N var=ls rng=lsa1';
execute 'gdxxrw results.gdx Squeeze=N var=lamda rng=lamda!a1';
execute 'gdxxrw results.gdx Squeeze=N var=my rng=my!a1';
execute 'gdxxrw results.gdx Squeeze=N var=xi rng=xi!a1';
execute 'gdxxrw results.gdx Squeeze=N var=v rng=v!a1';
execute 'gdxxrw results.gdx Squeeze=N var=l rng=l!a1';
*Input Parameters
execute 'gdxxrw results.gdx Squeeze=N par=Cl rng=Cl!a1';
execute 'gdxxrw results.gdx Squeeze=N par=enInst rng=enInst!a1';
execute 'gdxxrw results.gdx Squeeze=N par=capInst rng=capInst!a1';
execute 'gdxxrw results.gdx Squeeze=N par=ad rng=ad!a1';
execute 'gdxxrw results.gdx Squeeze=N par=bd rng=bd!a1';
execute 'gdxxrw results.gdx Squeeze=N par=bc rng=bc!a1';
execute 'gdxxrw results.gdx Squeeze=N par=cc rng=cc!a1';
execute 'gdxxrw results.gdx Squeeze=N par=capProd rng=capProd!a1';
execute 'gdxxrw results.gdx Squeeze=N par=Vren rng=Vren!a1';
execute 'gdxxrw results.gdx Squeeze=N par=Pmax rng=Pmax!a1';

*execute 'gdxxrw results.gdx Squeeze=N txt=C(i,b) rng=sim!B8';

```

Top-level MPEC Storage and Cournot Generators

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```
SETS

g firms /g1, g2,g3/
s storrage firms /s1, s2, s3/
h hours /h1*h24/
hsub1(h) /h1*h23/
hsub2(h) /h2*h24/

PARAMETERS
Cl(s)
enInst(s)
capInst(s)
ad(h)
bd(h)
bc(h,g)
cc(h,g)
capProd(h,g)
Vren(h,g)
Pmax
;

*Loading parameters
$CALL GDXXRW input.xls trace=3 Squeeze=N Index=Index_1!a1
$GDXIN input.gdx
$LOAD Cl enInst capInst ad bd bc cc capProd Vren Pmax  lamdaInn Tcost Tcap
$GDXIN

display Cl, enInst, capInst, ad, bd, bc, cc, capProd, Vren, Pmax,  lamdaInn, »
Tcost, Tcap; ;

POSITIVE VARIABLES
vconv(h,g) energy generated by firm i in period h
wgen(h,s) energy generated by storage s in period h
wstor(h,s) energy stored by storage s in period h
en(h,s) energy stored unit s time h
d(h) energy demand in hour h
ls(h) load shedding in hour h

lamda(h) price in periode h
my(h,g) dual to production capacity constraint
xi(h,s) dual to energy balanse of storrage
v(h,s) dual to capacity constraint of storage unit s
l(h,s) dual to energy capacity of storage unit s

VARIABLES
NegStackObj negative of Stack objtive
;

EQUATIONS

StackObjDef Stackelberg objective defination

EnergyStorage_wgen(h,s)
EnergyStorage_wstor(h,s)
EnergyStorage_l(h,s)
EnergyStorage_v(h,s)
EnergyStorage_en1(h,s)
EnergyStorage_en2(h,s)
EnergyStorage_xi1(h,s)
EnergyStorage_xi2(h,s)
MarkClear_lamda(h)
```

```
TSO_ls(h)
TSO_uimp(h)
TSO_uexp(h)
TSO_dirac(h)
PowerProducer_vconv(h,g)
PowerProducer_my(h,g)
Demand_d(h)
;

*EnergyStorage
StackObjDef..    sum(h,sum(s,(lamda(h)*(wgen(h,s)-wstor(h,s)))))- NegStackObj »
=e= 0 ;

EnergyStorage_l(h,s).. - enInst(s) + en(h,s) =l= 0 ;
EnergyStorage_v(h,s).. - capInst(s) + wgen(h,s) + wstor(h,s) =l= 0 ;
EnergyStorage_xi1('h1',s).. -en('h24',s) - wstor('h1',s)*CL(s) + wgen('h1',s)»
 + en('h1',s) =l= 0 ;
EnergyStorage_xi2(h,s)$hsub2(h).. -en(h-1,s) - wstor(h,s)*CL(s) + wgen(h,s) +»
 en(h,s) =l= 0 ;

*MarketClear/Energy Balance
MarkClear_lamda(h).. sum(g,vconv(h,g)+Vren(h,g)) + sum(s,(wgen(h,s) - wstor(h»
,s))) - d(h) + ls(h) =g= 0 ;
TSO_ls(h).. - lamda(h) + Pmax =g= 0 ;
*PowerProducer
PowerProducer_vconv(h,g).. -lamda(h)  + bc(h,g) + 2*cc(h,g)*vconv(h,g) + my(h»
,g) =g= 0 ;
PowerProducer_my(h,g).. -vconv(h,g) + capProd(h,g) =g= 0 ;
*Demand
Demand_d(h).. lamda(h) - (ad(h)-bd(h)*d(h)) =g= 0;

MODEL

ESS1/

StackObjDef,
EnergyStorage_l,
EnergyStorage_v,
EnergyStorage_xi1,
EnergyStorage_xi2,
MarkClear_lamda.lamda,
TSO_ls.ls
PowerProducer_vconv.vconv,
PowerProducer_my.my,
Demand_d.d/
;
Option MPEC=KNITRO;
ESS1.optfile = 1 ;

Solve
ESS1 maximizing NegStackObj using mpec ;
;

Parameters TotEnergyProd, TotStored, TotGenByStor, Balance(h), Virk;

TotEnergyProd = sum(h,sum(g,vconv.l(h,g))) ;
TotStored = sum(h,sum(s,wstor.l(h,s))) ;
TotGenByStor = sum(h,sum(s,wgen.l(h,s))) ;
Balance(h) = sum(g,(vconv.l(h,g)+Vren(h,g)))+sum(s,(wgen.l(h,s)-wstor.l(h,s))»
) - d.l(h);
Virk = TotGenByStor/TotStored;
```

```

Display vconv.l, TotEnergyProd ;
Display wstor.l, TotStored ;
Display wgen.l, TotGenByStor ;
Display Balance;
Display Virk

execute_unload 'results.gdx';
*Solution variables
execute 'gdxxrw results.gdx Squeeze=N var=vconv rng=vconv!a1';
execute 'gdxxrw results.gdx Squeeze=N var=wgen rng=wgen!a1';
execute 'gdxxrw results.gdx Squeeze=N var=wstor rng=stor!a1';
execute 'gdxxrw results.gdx Squeeze=N var=en rng=en!a1';
execute 'gdxxrw results.gdx Squeeze=N var=d rng=d!a1';
execute 'gdxxrw results.gdx Squeeze=N var=ls rng=lsa1';
execute 'gdxxrw results.gdx Squeeze=N var=lamda rng=lamda!a1';
execute 'gdxxrw results.gdx Squeeze=N var=my rng=my!a1';
execute 'gdxxrw results.gdx Squeeze=N var=xi rng=xi!a1';
execute 'gdxxrw results.gdx Squeeze=N var=v rng=v!a1';
execute 'gdxxrw results.gdx Squeeze=N var=l rng=l!a1';
*Input Parameters
execute 'gdxxrw results.gdx Squeeze=N par=Cl rng=Cl!a1';
execute 'gdxxrw results.gdx Squeeze=N par=enInst rng=enInst!a1';
execute 'gdxxrw results.gdx Squeeze=N par=capInst rng=capInst!a1';
execute 'gdxxrw results.gdx Squeeze=N par=ad rng=ad!a1';
execute 'gdxxrw results.gdx Squeeze=N par=bd rng=bd!a1';
execute 'gdxxrw results.gdx Squeeze=N par=bc rng=bc!a1';
execute 'gdxxrw results.gdx Squeeze=N par=cc rng=cc!a1';
execute 'gdxxrw results.gdx Squeeze=N par=capProd rng=capProd!a1';
execute 'gdxxrw results.gdx Squeeze=N par=Vren rng=Vren!a1';
execute 'gdxxrw results.gdx Squeeze=N par=Pmax rng=Pmax!a1';

*execute 'gdxxrw results.gdx Squeeze=N txt=C(i,b) rng=sim!B8';

```