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Optimal design of heat exchanger networks with pressure changes.

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MASTERKONTRAKT

- uttak av masteroppgave

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Institutt **Institutt for industriell økonomi og teknologiledelse**	
Studieprogram **Industriell økonomi og teknologiledelse**	Hovedprofil **Anvendt økonomi og optimering**

3. Masteroppgave

Oppstartsdato **15. jan 2016**	Innleveringsfrist **10. jun 2016**
Oppgavens (foreløpige) tittel **Optimal design of heat exchanger networks with pressure changes.**	

Oppgavetekst/Problembeskrivelse
Purpose
The use of optimization for the design of heat exchanger networks is an established field in process synthesis. When considering heat exchange with compression and expansion, however, the process streams are no longer fixed, and this represents a new and major challenge in these problems. The corresponding design task has been referred to as simultaneous work and heat exchange networks (WHENs). The main purpose of this thesis is to develop new superstructures and models that are needed to formulate and solve the design and optimization problem for WHENs. Department of Energy and Process Engineering (EPT) – NTNU will provide some of the needed technical information about the fundamental insight about work and heat integration as well as data for relevant case studies.

Main contents:
1. Describe the optimization problem related to design of work and heat exchange networks.

2. Formulate one or more models for the design problem.

3. Implement the one or more sub model(s) for the problem by use of appropriate software.

4. Test the model(s) with real data.

5. Discuss the applicability and adequacy of the model(s) as decision support tools for the given problem.

Hovedveileder ved institutt **Professor Bjørn Nygreen**	Medveileder(e) ved institutt **Professor Truls Gundersen, EPT**
Ekstern bedrift/institusjon **SINTEF Energy Research**	Ekstern veileder ved bedrift/instutisjon **Chao Fu**
Merknader **1 uke ekstra p.g.a påske.**	

4. Underskrift

Student: Jeg erklærer herved at jeg har satt meg inn i gjeldende bestemmelser for mastergradsstudiet og at jeg oppfyller kravene for adgang til å påbegynne oppgaven, herunder eventuelle praksiskrav.

Partene er gjort kjent med avtalens vilkår, samt kapitlene i studiehåndboken om generelle regler og aktuell studieplan for masterstudiet.

NTNU 15/1-2016
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Sted og dato

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Originalen lagres i NTNUs elektroniske arkiv. Kopi av avtalen sendes til instituttet og studenten.

Preface

This work is my master thesis in the master programme of industrial Economics and Technology Management at the Norwegian University of Science and Technology (NTNU). My technological specialization is in process engineering, and my economical specialization is in optimization, so I was very lucky to get a project where I can use my knowledge from both fields. It has been interesting to dive deeper into heat and work integration, handle nonlinear optimization and to learn new software. I would like to thank my supervisor, Professor Bjørn Nygreen at the Department of Industrial Economics and Technology Management, for facilitating this project project and for sharing his knowledge about optimization and LaTeX. I am also thankful to Professor Truls Gundersen at the Department of Energy and Process Engineering and Chao Fu from SINTEF Energy RESEARCH for coming up with the project description and for useful lectures, test cases, comments and insights about heat and work integration.

Trondheim, 10th of June 2016

Preben Maurstad Uv

Abstract

Due to climate change is it desirable to improve the energy efficiency of production systems. The environmental impact will decrease with increasing energy efficiency and it will even reduce the operational cost. Heat integration is a much used method for improving the energy efficiency and the annualized cost. The usual heat exchanger network synthesis problem is to design a heat exchanger network that minimizes the annualized cost. A set of hot streams which needs to be cooled and a set of cold streams which need to be heated is given. When the possibility of expanding or compressing some of the streams is added, the problem gets more complex. Without pressure manipulation the streams are continuously heated/cooled from the supply temperature to the target temperature. Pressure manipulation changes the temperature of the fluid going through the turbine/compressor, so the stream is split into two parts. This can cause the streams to change identity and makes the modeling and optimization more difficult. The objective is to minimize the hot and cold utilities, minimize compression work and maximize the work produced by turbines. Because of the difference in energy quality is exergy going to be used instead of energy. Fu and Gundersen (2015b) have made some insights and a graphical procedure that finds a good solution to the problem. The procedure and theorems have not been received as well as the authors hoped, so an optimization model not based on their insights was of interest. A MINLP model, which did not utilize the insights from Fu and Gundersen (2015b), was developed in the project thesis. The model handled expansion of hot and cold streams above ambient. In this master thesis have the model been extended to also include compression and below ambient temperatures. The MINLP model did not always find the optimal solution, so a new model with the use of insights was developed. The model became a much simpler LP model and have for all test cases found an equally good or better solution than the first model and the graphical procedure. Both models have been implemented in GAMS and have been tested on several test cases. The concept of simultaneous work and heat integration is quite novel and the models developed in this thesis is just a step in the right direction. The end will probably be an optimization model which designs the work and heat exchange network with respect to the annualized costs.

Sammendrag

Grunnet klimaforandringer er det ønskelig og forbedre energieffektiviteten til produksjonsanlegg. Miljøpåvirkningen minker ved økende energieffektivitet og operasjonskostnadene reduseres også. Varmeintegrasjon is en mye brukt metode for å øke energieffektiviteten og operasjonskostnadene. Den vanlige målsetningen for av varmeintegrasjon er å designe et varmevekslernettverk som minimaliserer både drifts-og investeringskostnadene. Det er gitt et sett med varme strømmer som trenger å kjøles ned og et sett med kalde strømmer som skal varmes opp. Når strømmene i tillegg får muligheten til å ekspanderes eller komprimeres blir problemet mer komplekst. Uten trykkforandringer vil strømmene bli kontinuerlig varmet opp/kjølt ned fra start til slutt temperaturen. Trykkforandringer endrer også temperaturen på strømmene, så den kontinuerlig strømmen blir nå delt inn i en del før og ett trykkforandringsenheten. Dette kan føre til at strømmen midlertidig endre identitet og optimering og modelleringen vanskeligere. Målsetningen er å minimere den eksternt tilførte varmen og kjølingen, minimere arbeider i kompressorer og maksimere arbeid produsert av turbinene. På grunn av forskjellen i energikvalitet vil eksergi bli brukt istedenfor energi. Fu and Gundersen (2015b) har kommet med noen teoremer og en grafisk metode for dette problemet. Innsekten deres har ikke blitt så godt mottatt som de har ønsket, så de ønsket en optimeringsmodell som ikke er basert på deres teoremer for å uavhengig test teoremene og metoden sin. En slik MINLP-modell ble utviklet i prosjektarbeidet for ekspansjon av varme og kalde strømmer over omgivelsestemperatur. In denne masteroppgaven har modellen blitt utvidet slik at det strømmene også kan komprimeres og ha temperaturer omgivelsestemperaturen. MINLP-modellen fant ikke alltid optimal løsning av seg selv og det ble derfor utviklet en ny modell basert på teoremene til Fu and Gundersen (2015b). Denne modellen ble en mye enklere LP-modell og fant for alle test-casene en like god eller bedre løsning enn MINLP-modellen og den grafiske metoden. Begge modellene har blitt implementert i GAMS og har blitt testet på flere test-cases. Konseptet med integrasjon av både varme og arbeid er relativt nytt, så modellene som er utviklet in denne oppgaven er kun et steg i riktig retning. Det endelige målet vil være å få en modell som designer hele nettverket og minimerer investerings-og driftskostnadene.

Innhold

Kapittel 1

Introduction

1.1 Background

Due to the climate change it is desirable to improve the energy efficiencies of production plants. The environmental impact will decrease with increasing energy efficiency and it will even reduce the operational cost. Heat integration is a much used method for improving the energy efficiency and the annual cost. If a system consists of a hot stream that needs to be cooled and a cold stream that need to be heated, this can be achieved in several different ways. One way would to be using cooling water (cold utility) to cool the hot stream and hot steam (hot utility) to heat the cold stream. A method using heat integration would let the hot and cold stream heat exchange with each other and then use hot and cold utility for the rest of the heating and cooling. The extra heat exchanger will increase the investment cost, but will at the same time reduce the operational cost. So there is a trade-off between increasing the energy efficiency and the investment cost. The heat integration problem is complex and optimization is an appropriate tool for this task. Gundersen and Næss (1988) provides a good review of the subject. A simultaneous MINLP model Yee and Grossmann (1990) exists, but have trouble with industrial sized problems. There is also a sequential framework by Anantharaman and Gundersen (2007) for optimizing the heat exchanger network.

The usual heat exchanger network synthesis problem is to design a heat exchanger network that minimizes the annualized cost. A set of hot streams which needs to be cooled and a set of cold streams which need to be heated is given. All the streams have a specified heat capacity flow rate and supply and target temperatures. Heat exchangers, hot utilities (heaters) and cold utilities (coolers) are placed such that there always is a minimum temperature difference of ΔT between streams that are matched. ΔT is not fixed for the overall problem and should be optimized. This can be done directly in the model or iteratively. When the possibility of expanding or compressing some of the streams is added, the problem gets more complex. Without pressure manipulation the streams are continuously heated/cooled from the supply temperature to the target temperature. Pressure manipulation changes the temperature of the fluid going through the turbine/compressor, so now the stream might be split into two parts: One before the fluid enters the turbine/compressor and one after. Another complicating factor is that the inlet and outlet temperature of the turbine/compressor do not have to be between the supply and target temperature. This means that a cold stream might act as a hot stream for certain temperature ranges. Let a cold stream have a supply temperature of 300°C and a target temperature of 380°C. The stream is split into two parts by an expander and the inlet and outlet temperature of the expander is 200°C and 100°C, respectively. Then the two parts will have different identities. The first part operates between 300°C and 200°C and therefore acts as an hot stream. The second part operates between 100°C and 380°C and therefore acts as an cold stream. In some cases it is optimal to split the streams that needs pressure manipulation into two or more parts with a turbine/compressor each. This introduces nonlinearities into the problem and it complicates the problem even further. Fu and Gundersen (2015b) have come up with some insights and a graphical procedure that helps with to fix inlet and outlet temperatures for turbines/compressors in a way that minimizes the use of useful energy (exergy). The goal of this thesis is to improve the optimization model developed in the project these and then use insights to improve the model. Since the goal is to minimize the exergy instead of the annual cost it is actually not necessary to design the whole heat exchanger network. This simplifies to problem substantially compared to the case where the whole heat exchanger network has to be designed.

Kapittel 2

Heat and work integration

2.1 Exergy

The objective for the problem is to minimize the hot and cold utility consumption, minimize the compression work and maximize the output from the turbines. The quality of the different energy forms is quite different. The work input/output from the compressor/turbine is of very high quality (usually electricity) and can be almost fully converted to any other energy form. The cold utility is on the other hand usually of very low quality (usually water at low temperature) and can not effectively be converted to other energy forms. So using 1 MW of work to save 1 MW of cold utility is a very bad deal. It is even a bad deal to use 1 MW of work to save 1 MW of hot utility. The work could have been used to drive a heat pump which produced 4-6 MW of hot utility instead. Exergy is the useful part of the energy. So exergy can be seen as the potential to convert energy into work. The three main types of exergy is mechanical, thermomechanical and chemical exergy. Thermomechanical exergy can be split into temperature and pressure based exergy. The work linked to the compressor/turbine is assumed to be pure exergy, while the exergy linked to the hot and cold utility is purely temperature based. The exergy is calculated based on a reference state, usually the surrounding environment, so the exergy from the cold utility will be zero if reference temperature is the same as the the cold utility temperature. If Q is the heat energy content, T is the temperature of the heat and T_0 is the reference temperature, then the exergy can be calculated by (2.2).

$$E_x = Q\left(1 - \frac{T_0}{T}\right) \quad , T \geq T_0 \tag{2.1}$$

If the heat has a temperature below the ambient temperature will equation (2.2) give a negative result. This is clearly wrong, so a new equation is used when T is below ambient temperature. Both equations are only valid when the temperatures are given in Kelvin. This avoids cases with division by zero.

$$E_x = Q\left(\frac{T_0}{T} - 1\right) \quad , T \leq T_0 \tag{2.2}$$

It is useful to use exergy instead of energy when energy forms of different qualities are used.

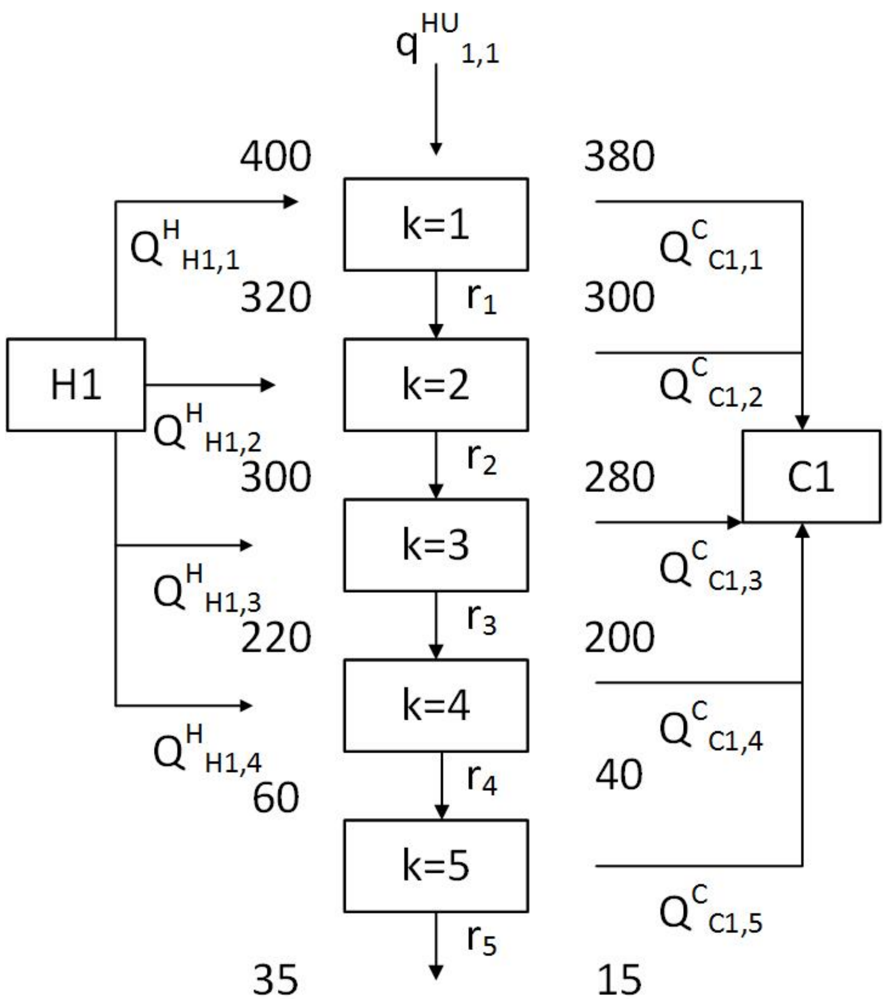
2.2 Heat Cascade

The heat cascade is usually used for calculating the minimum hot and cold utility consumption, but now it will be used to minimize the exergy consumption. The heat cascade consists of temperature intervals based on the supply temperature of the given hot and cold streams. Since there has to be a temperature difference (ΔT) between streams that are exchanging heat, there will be a hot and a cold side in the heat cascade. See Figure 2.1 for an illustration of a heat cascade with one hot and one cold stream. For each temperature interval it will be calculated a heat balance and heat will be allowed to cascade to lower temperature intervals. The heat going from one interval to next is called a heat residual. The heat can be transferred upwards since it is impossible to directly transfer heat from a cold source to a hotter source. Heat can be provided from hot utility sources (hot steam, flue gas, condenser, etc.) at different temperatures and from hot streams that are passing through the interval. Heat is consumed by the cold streams that are passing through and by the cold utilities (cooling water, reboiled, etc.). Since the heat only can be transferred downwards in the heat cascade, is it always be possible to design a heat exchanger network with the same energy and exergy consumption as the minimum found from the heat cascade. The result obtained from the heat cascade is therefore a lower bound for the energy and exergy consumption. A heat exchanger network designed on a basis of this does often have a lot of heat exchangers and stream splits, which makes the investment costs too high.

The heat capacity flow rate (MCP) is assumed to be constant. This means that the heat supplied/demanded by a hot/cold stream to a temperature interval can be calculated as in equation (2.3) and (2.4). The min/max functions are necessary because the target temperature might be inside a temperature interval. This can be avoided by adding the target temperatures to the heat cascade, but this increase the number of equations and variables. Q_k^H and Q_k^C is set to zero if the temperature interval is not between the supply and target temperature.

$$Q_k^H = MCP\left[T_k^H - \max\left(T_{k+1}^H, T^T\right)\right] \tag{2.3}$$

$$Q_k^C = MCP\left[\min\left(T_k^C, T^T\right) - T_{k+1}^C\right] \tag{2.4}$$



Figur 2.1: Example of a heat cascade with one hot and cold stream

2.3 Pinch analysis

If a temperature interval in the heat cascade does not transfer heat to the next interval, means it that a pinch point have been found. A pinch point is therefore a point where no heat should be transferred across. The pinch point will be associated with the hot and cold side temperature located between the two intervals that does not exchange heat. So the pinch temperatures are 220 and 200°C if $r_3 = 0$ in Figure 2.1. The pinch point decomposes the cascade into two parts. Above the pinch temperature there will be a heat deficit, which means that hot utility must be added. Below the pinch temperature there is a heat surplus, which needs to be taken care of by cold utility. This means that any heat transferred from above the pinch point to below the pinch point will increase both the hot and the cold utility consumption. The pinch decomposition have lead to the concept of correct integration, which addresses how different utilities (compressor, turbine, heat exchanger, distillation column and heat pump) should be integrated. A turbine cools down the passing fluid and should therefore be placed in the heat surplus region below the pinch temperature. This contradicts usual rule that a turbine should be placed at high temperatures since this maximizes the work produced. A compressor adds heat to the fluid that is compressed and should therefore be placed in the heat deficit area above the pinch temperature. This does also contradict the usual rule that a compressor should be placed at a low temperature since this minimizes the compression work. So one must consider the trade-off between heat integration and work production/generation when placing turbines/compressors. The exergy consumption is a good way to solve the trade-off, since it takes energy quality into account. The two rules regarding the placement of compressors/turbines was stated in Aspelund et al. (2007) as heuristics. By using these heuristics, it is possible to find the best compression/expansion scheme for a hot/cold stream. Wechsung et al. (2011) presents a superstructure for hot and cold streams if the streams goes through three pressure manipulations. A hot stream should be cooled down to pinch and then be compressed since this adds heat above the pinch. Then it should be cooled to pinch and then be expanded so that it adds cooling below the pinch. When the stream is heated up to pinch again it should be compressed so that it again adds heat above the pinch. The stream is then cooled until it reaches the target temperature. One could add several extra steps with pressure manipulations, but it will come to a point where the annualized extra investment cost is larger than the savings from the increased heat integration.

Kapittel 3

Optimiziation of heat and work integration

The goal for the optimization is to minimize the exergy consumption for a simultaneous heat and exchange network. The network will consist a set of hot streams and a set of cold streams with fixed MCP and supply and target temperature. There will also be hot and cold utilities with fixed temperatures which covers the external heating/cooling demand. Some of the streams will also have a supply and target pressure. This means that the stream must go through a pressure changing unit for reaching its target pressure. The stream can at supply temperature be splitter into several stream parts that are merged together again at the target temperature. Each stream split part can only change pressure once and the pressure difference must be the same as between the target and supply pressures. The temperature difference ΔT is assumed to be fixed. The pressure change is assumed to be a polytropic process with a fixe κ. The actual heat and work exchange network will not be designed, only the inlet and outlet temperatures of the pressure changing units, the hot/cold utilities and the exergy consumption will be calculated. It is also assumed that all streams can exchange heat with each other. The work produced by turbines and the work added by compressors is assumed to be electricity with a 100% exergy content. Only the temperature based exergy of the hot and cold utilities will be looked at. The exergy content can then be calculated by using the Carnot factor.

3.1 Related Work

Aspelund et al. (2007) proposed two heuristics regarding the correct placement of expanders/compressors in a heat exchanger network. Wechsung et al. (2011) used these rules to make an MINLP optimization formulation for the synthesis of sub-ambient heat exchanger network including compression and expansion. The model combines pinch analysis, exergy analysis and mathematical programming to obtain an optimal network. The article also presents a superstructure for the optimal sequence of expansion/compression and heating/cooling for hot and cold streams. Fu and Gundersen (2015b) showed that the heuristics do not always hold and present several theorems and a graphical procedure for integration of expanders above the ambient temperature. Onishi et al. (2014) presents a MINLP optimization model for simultaneous synthesis of work exchange networks with heat integration. The model includes a superstructure where expanders/compressors might be located on several different shafts or standalone shafts. The model utilizes the superstructure from Wechsung et al. (2011) and minimizes the total annualized cost. Onishi et al. (2015) presents a MINLP model for retrofitting heat exchanger networks, wherein the pressure recovery of process streams is performed to enhance energy integration. Dowling (2015) made a mathematical model which minimizes a weighted combination of work and hot utility for synthesis of a heat exchanger network with above ambient compression. There is one large limitation with the model. It does not take into consideration that the outlet temperatures of the compressors is potential pinch candidates.

3.2 Symbols and notation for the models

Indices		Description
s, y	-	stream
k, i, j	-	temperature interval
m, n	-	stream split part
u	-	hot or cold utility source
b	-	branch, $b \in B$
z	-	segment, $z \in Z$
Sets		Description
S	-	Set of streams
S^P	-	Set of streams with pressure manipulation
S^H	-	Set of hot streams
S^C	-	Set of cold streams
S^{CC}	-	Set of cold streams with compression
S^{CH}	-	Set of hot streams with compression
S^{EC}	-	Set of cold streams with expansion
S^{EH}	-	Set of hot streams with expansion
S^{CWP}	-	Set of cold streams without pressure manipulation
S^{HWP}	-	Set of hot streams without pressure manipulation
M_s	-	Set of stream split parts for stream $s \in S^P$
K_s^{CBTS}	-	Temperature intervals below T_s^S on the cold side of the heat cascade for stream $s \in S^{EC} \cup S^{CC}$
K_s^{CBTST}	-	Temperature intervals between T_s^T and T_s^Son the cold side of the heat cascade for stream $s \in S^{EC} \cup S^{CC}$
K_s^{CATT}	-	Temperature intervals above T_s^T on the cold side of the heat cascade for stream $s \in S^{EC} \cup S^{CC}$
K_s^{CATS}	-	Temperature intervals above T_s^S on the cold side of the heat cascade for stream $s \in S^{EC} \cup S^{CC}$
K_s^{CBTT}	-	Temperature intervals below T_s^T on the cold side of the heat cascade for stream

$s \in S^{EC} \cup S^{CC}$

K_s^{HBTST} - Temperature intervals between T_s^T and T_s^S on the hot side of the heat cascade for stream $s \in S^{EH} \cup S^{CH}$

K_s^{HBTS} - Temperature intervals below T_s^S on the hot side of the heat cascade for stream $s \in S^{EH} \cup S^{CH}$

K_s^{HATT} - Temperature intervals above T_s^T on the hot side of the heat cascade for stream $s \in S^{EH} \cup S^{CH}$

K_s^{HATS} - Temperature intervals above T_s^S on the hot side of the heat cascade for stream $s \in S^{EH} \cup S^{CH}$

K_s^{HBTT} - Temperature intervals below T_s^T on the hot side of the heat cascade for stream $s \in S^{EH} \cup S^{CH}$

K_{sk}^{HBTSK} - Temperature intervals which are below T_s^S and above k on the hot side of the heat cascade for stream $s \in S^{EH} \cup S^{CH}$

K_{sk}^{CATSK} - Temperature intervals which are above T_s^S and above k on the cold side of the heat cascade for stream $s \in S^{EC} \cup S^{CC}$

K_{sk}^{HATTK} - Temperature intervals which are above T_s^T and above k on the hot side of the heat cascade for stream $s \in S^{EH} \cup S^{CH}$

K_{sk}^{CBTTK} - Temperature intervals which are below T_s^T and above k on the cold side of the heat cascade for stream $s \in S^{EC} \cup S^{CC}$

K_s^{CTS} - A set with the temperature interval located directly below the supply temperature for a cold stream $s \in S^{EC} \cup S^{CC}$

K_s^{CTT} - A set with the temperature interval located directly below the target temperature for a cold stream $s \in S^{EC} \cup S^{CC}$

K_s^{HTS} - A set with the temperature interval located directly above the supply temperature for a hot stream $s \in S^{EH} \cup S^{CH}$

K_s^{HTS} - A set with the temperature interval located directly above the supply temperature for a hot stream $s \in S^{EH} \cup S^{CH}$

U^H - Set of hot utilities.

U^C - Set of Cold utilities.

B_s - Branches for stream $s \in S^P$

Z_{bs}	-	Stream segments for stream $s \in S^P$, branch $b \in B_s$
Parameters		Description
T_s^S	-	Supply temperature for stream $s \in S^P$
T_s^T	-	Target temperature for stream $s \in S^P$
T_k^H	-	Temperature on the hot side of the heat cascade interval $k = 1..\lvert K\rvert + 1$
T_k^C	-	Temperature on the cold side of the heat cascade interval $k = 1..\lvert K\rvert + 1$
MCP_s	-	Product of mass flow rate and specific heat capacity for stream s
T_0	-	Ambient temperature
P_s^S	-	Supply pressure for stream $s \in S^P$
P_s^T	-	Target pressure for stream $s \in S^P$
Q_{sk}^H	-	Heat supplied from hot stream $s \in S^{HWP}$ to interval k
Q_{sk}^C	-	Heat demand for cold stream $s \in S^{CWP}$ in interval k
Q_{sk}	-	Heat supply/demand for stream $s \in S \setminus S^P$ for interval k
Q_{skbz}^P	-	Heat supply/demand for stream $s \in S^P$, branch $b \in B_s$, segment $z \in Z_{bs}$ for interval k
T_{sb}^{Δ}	-	Temperature difference for the pressure changing unit for stream $s \in S^P$, branch $b \in B_s$
T_u^{HU}	-	Temperature of hot utility u
T_u^{CU}	-	Temperature of cold utility utility u
ΔT	-	Temperature difference between hot and cold side
κ_s	-	Heat capacity ratio for stream $s \in S^P$
η_s	-	Polytropic efficiency for pressure changing unit for stream $s \in S^P$
Δ	-	A small value
Variables		Description
r_k	-	Heat residual from interval k.
q_{skm}^C	-	Heat demand for cold stream $s \in S^{CC} \cup S^{EC}$ from interval k
q_{skm}^H	-	Heat supplied by the hot stream $s \in S^{CH} \cup S^{EH}$ to interval k
q_{ku}^{HU}	-	Hot utility consumption of source u to interval k
q_{ku}^{CU}	-	Cold utility consumption for source u to interval k
t_{sm}^{in}	-	Inlet temperature for pressure changing unit for stream $s \in S^P$, part m
t_{sm}^{out}	-	Outlet temperature for pressure changing unit for stream $s \in S^P$, part m
$t_{skm}^{in,D}$	-	Dummy inlet temperature for pressure changing unit for stream $s \in S^P$, part m

in interval k

$t^{out,D}_{skm}$ - Dummy outlet temperature for pressure changing unit for stream $s \in S^P$, part m in interval k

α^{in}_{skm} - Interpolation fraction for t^{in}_{sm} in interval $k = 1..|K|+1$

α^{out}_{skm} - Interpolation fraction for t^{out}_{sm} in interval $k = 1..|K|+1$

a^{in}_{skm} - Variable used to calculate t^{in}_{skm}

a^{out}_{skm} - Variable used to calculate $t^{out,in}_{skm}$

f_{sm} - Fraction for stream split part m of stream $s \in S^P$

f^{B}_{sb} - The fraction of stream $s \in S^P$, that goes through branch $b \in B_s$

q^{SHWP}_{skm} - Heat supplied to above a potential pinch point in interval k

q^{SCWP}_{skm} - Heat supplied demanded above a potential pinch point in interval k

q^{HP}_{skm} - Heat supplied to above a potential pinch point in interval k from a stream with pressure manipulation

q^{CP}_{skm} - Heat required by a cold stream above a potential pinch point in interval k for a stream with pressure manipulation

3.3 Model without the use of insight

A model which could handle expansion of hot and cold streams above ambient temperature was developed in the project thesis. The model did not utilize the insight from Fu and Gundersen (2015b) and did thus end up being a MINLP model. This model have been expanded to include compression of hot and cold streams and to handle temperatures below ambient temperature. Sections 3.3.1-3.3.7 is based on the project thesis. The model is based on a minimum utility model, Gundersen (1991), and have been modified to include the effects of pressure manipulation, splitting streams and is minimizing the use of exergy instead of utility cost. The heat cascade is made from the supply temperature of the streams and some other key temperatures that is needed for all the equations to work correctly. To make the model user friendly was it developed an algorithm that sets up the heat cascade by first listing all the necessary temperatures and then sort the list while deleting duplicating temperatures. A heat balance must be calculated at each temperature interval. Then the heat supplied/demanded by the streams without pressure manipulation can be preprocessed, while it is calculated during the optimization for the streams with pressure change. The equations for heat supplied/demanded by hot and cold streams with expansion was derived in the project thesis, while the equations for compression was derived in this master thesis. The equations turn out to be non smooth and a reformulation of these equations was done in the project thesis. The derivation of the equations are quite tedious and they have not been changed in the master thesis. The equations have therefore not been added to this thesis. Some errors in the model was also identified and removed by deriving new equations for four different special temperature intervals. The possibility of having below ambient conditions was included by altering the objective function and by including variables for cold utilities.

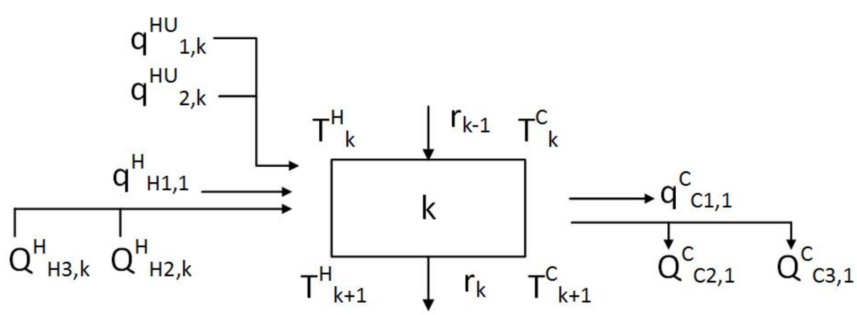
3.3.1 Objective function

The usual objective for HEN design is to minimize the annualized cost. Using the heat cascade to calculate the minimum hot and cold utility consumption will not give a complete heat exchanger network. It is therefore not possible to calculate the investment cost for the heat exchangers. Grossmann et al. (1998) made an optimization model which designs the whole heat exchanger network. The number of heat exchangers and the heat exchanger area is then possible to calculate, which enables the model to minimize the total annualized cost instead of just the energy efficiency. The model includes both nonlinearities and binary variables. One of the objectives for the model developed here is to test the theorems and graphical procedure from Fu and Gundersen (2015b). Their method minimizes the consumed exergy and does not consider costs at all. It is therefore unnecessary to include the equations from Grossmann et al. (1998) to check this. This also keeps be new model simpler and easier to solve by not adding unnecessary binary variables and nonlinearities. The objective for this model will therefore be to minimize the exergy consumption. Negative exergy consumption means that exergy is produced instead. So for cases with negative exergy consumption will the model actually maximize produced exergy. The first part of the objective function is the exergy content of the hot utility consumption. q_{ku}^{HU} is the hot utility consumption from source u to heat cascade interval k and is multiplied with the Carnot factor. The expression have been made general by allowing several hot utility sources at different temperatures, T_u^{HU}. The second part is the work produced by the turbines. The work is calculated by multiplying the turbine efficiency, η_s, with the $MCP_s f_{sm}$ flowing through the turbine and the temperature difference in the turbine.

$$\min e_x = \sum_{k \in K} \sum_{u \in U_k^H} \left(1 - \frac{T_0}{T_u^{HU}}\right) q_{ku}^{HU} - \sum_{s \in S^{EH} \cup S^{EC}} \sum_{m \in M_s} \eta_s MCP_s f_{sm} \left(t_{sm}^{in} - t_{sm}^{out}\right) \tag{3.1}$$

3.3.2 Heat balance for a temperature interval

The reason for using a heat cascade is to ensure thermodynamical feasibility. The heat cascade divides the continuous temperature scale into intervals with inlet and outlet temperatures. Thermodynamical feasibility is ensured if heat is only transferred downwards the cascade. It is therefore necessary to perform a heat balance for each interval. See Figure 3.1. The heat residual r_{k-1} is coming from the previous interval and r_k is transferred to the next one. Hot utility q_{ku}^{HU} may be added from different sources. q_{ku}^{HU} is associated with a temperature T_v^{HU} and can only add heat to the temperature intervals with $T_k^H \leq T_v^{HU}$. Heat from streams without pressure manipulation, Q_{sk}^H, and heat from streams with pressure manipulation, q_{sk}^H, is added, while heat to streams without pressure manipulation, Q_{sk}^C, and heat from streams with pressure manipulation, q_{sk}^C is subtracted.



Figur 3.1: Illustration of the heat balance for a temperature interval

$$r_1 + \sum_{s \in S^{EC}} \sum_{m \in M_s} q_{s,1,m}^C - \sum_{s \in S^{EH}} \sum_{m \in M_s} q_{s,1,m}^H - \sum_{u \in U_1^H} q_{1,u}^{HU} = \sum_{s \in S^{HWP}} Q_{s,1}^H - \sum_{s \in S^{CWP}} Q_{s,1}^C \tag{3.2}$$

$$r_k - r_{k-1} + \sum_{s \in S^{EC}} \sum_{m \in M_s} q^C_{skm} - \sum_{s \in S^{EH}} \sum_{m \in M_s} q^H_{skm} - \sum_{u \in U^H_k} q^{HU}_{ku} = \sum_{s \in S^H} Q^H_{sk} - \sum_{s \in S^C} Q^C_{sk} \quad , k \in K \setminus \{1\} \tag{3.3}$$

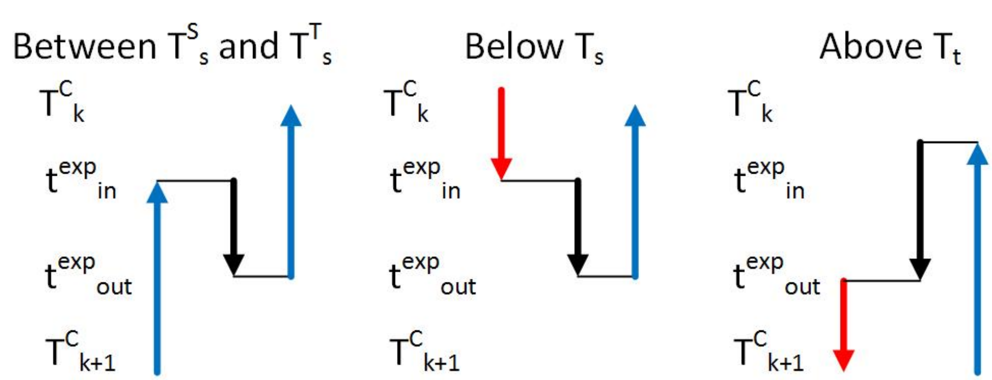
$$r_k \geq 0 \quad , k \in K \tag{3.4}$$

r_k is the heat residual going from one temperature interval to the next and it has to be non-negative for the heat to be transferred in the right direction in the heat cascade. The heat balance also calculates q^{HU}_{kv}, which is the hot utility demand from source v. Q^C_{sk} and Q^H_{sk} is preprocessed and will be 0 if the stream does not go through heat interval k. Q^H_{sk} and Q^C_{sk} is calculated with equation 3.5 and 3.6, respectively. q^C_{skm} and $q^H_{skm}$is the heating requirement for the cold streams with pressure manipulations.

$$Q^H_{sk} = MCP_s \left(T^H_k - T^H_{k+1} \right) \quad , s \in S^{HWP}, k \in K \tag{3.5}$$

$$Q^C_{sk} = MCP_s \left(T^C_k - T^C_{k+1} \right) \quad , s \in S^{CWP}, k \in K \tag{3.6}$$

3.3.3 Heat demand for a cold stream with expansion



Figur 3.2: Behavior of a cold stream with expansion inside a temperature interval

Figure 3.2 shows how a stream may behave inside a temperature interval, depending on the location in the heat cascade. The blue arrows shows where the stream has a heat demand and the red arrows shows where the stream supplies heat. The black arrow shows where the stream goes through the turbine, here the stream does not transfer any heat. A cold stream without pressure manipulation will start at the supply temperature and will continuously increase the temperature up to the target temperature. When a stream passes through a turbine it will make a jump downwards in temperature. This is illustrated to the left in Figure 3.2. In some cases it will be beneficial for the stream to be cooled below the supply temperature or heated above the target temperature. This is illustrated in the middle and to the right in the figure.

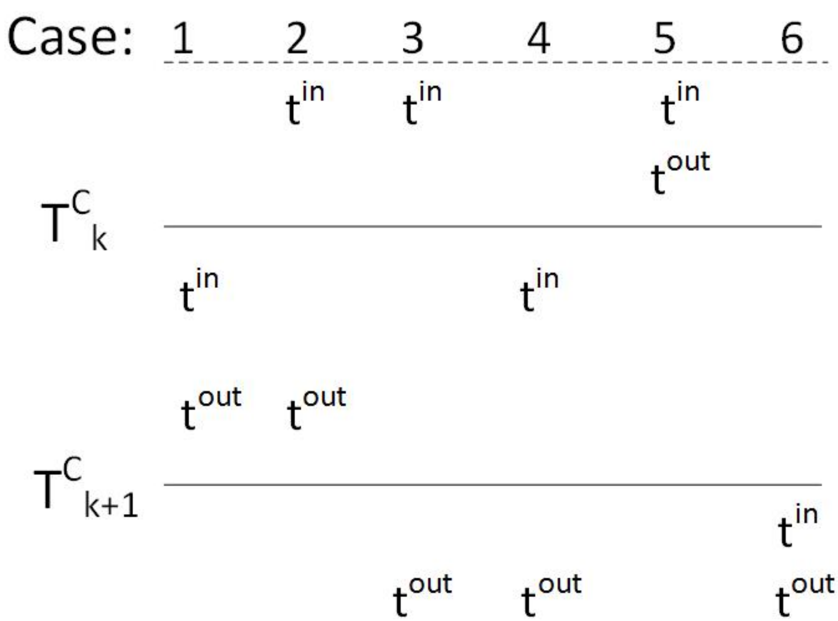
The left illustration shows the regular case where the inlet and outlet temperature is between the target and supply temperature. Here the stream will enter the interval at a temperature of T^C_{k+1} and be heated up (blue arrow) until it reaches the inlet temperature of the turbine. Here the stream is expanded (black arrow), which makes the temperature drop. The stream will then continue to be heated up (blue arrow) until it leaves the interval at the temperature of T^C_{k+1}.

The middle illustration shows the case where the stream enters an interval below the supply temperature. To reach an interval below the supply temperature does the stream have to supply heat instead of demanding it. The stream will enter the interval with a temperature of T^C_k and then go downwards until it reaches the inlet temperature of the turbine. Since the stream now supplies heat, it is illustrated with a red arrow. The turbine makes the stream temperature drop further (black arrow). Since the stream have to end up at the target temperature, which is higher than T^C_k, it will have to be heated up (blue arrow) again to T^C_k, where it enters the next interval.

The third illustration shows the case where both the inlet and outlet temperature is above the target temperature. The stream will enter the interval at a temperature of T^C_{k+1} and then be heated up (blue arrow) until it reaches the inlet temperature. Here the temperature will drop when the stream goes through the turbine. The stream have to end up at the target temperature, which is below T^C_{k+1}. The stream does therefore have to supply heat until the stream temperature until it leaves the interval with a temperature of T^C_{k+1}. The last part is illustrated with a red arrow since the stream supplies heat between the outlet of the turbine and the end of the interval.

Figure 3.2 does only show the case where both the inlet and outlet temperature of the expander are inside the temperature interval, but the inlet and outlet temperature may also be above or below the given interval. This makes 6 cases in total as shown in Figure 3.3. The diagrams in

Figure 3.2 can be altered to illustrate the cases where one or both of the expander temperatures are outside the interval by altering the line connected to the inlet and outlet temperature. For example if the inlet temperature is above T_k^C should the left blue arrow in the diagram for above T_t (right illustration) would be extended up to T_k^C. If the inlet temperature is below T_{k+1}^C would the left blue arrow would actually not be drawn at all. By using the different diagrams in Figure 3.2 it is possible to find the heating requirement for each of the cases in Figure 3.3, which is done in Table 3.2, 3.3 and 3.4. For case 1 (both inlet and outlet temperature inside the interval) in an interval between the supply and target temperature does one get the left illustration in Figure 3.2. Here the stream got a heat demand from T_{k+1}^C to t^{in} and then from t^{out} to T_k^C. The total heat demand in that interval then becomes: $q_{sk}^C = MCP_s\left(t^{in} - T_{k+1}^C\right) f_s + MCP_s\left(T_k^C - t^{out}\right) f_s$. The same derivation was done to the 5 other cases in Figure 3.3 and then again for the intervals below the supply temperature and above the target temperature. The goal was to find general equations which were valid for as many cases as possible.



Figur 3.3: The different possible cases for expander inlet and outlet temperatures

Table 3.2 shows the heat demand for all the 6 cases for an interval between the supply and target temperature. The inlet and outlet temperatures of the turbine is variables, so the model needs to decide which of the six equations to use depending on the inlet and outlet tempera-

Case 1:	$q^C_{skm} = MCP_s f_{sm}($	t^{in}_{sm}	-	T^C_{k+1}	+	T^C_k	-	t^{out}_{sm}	$)$
Case 2:	$q^C_{skm} = MCP_s f_{sm}($	T^C_k	-	T^C_{k+1}	+	T^C_k	-	t^{out}_{sm}	$)$
Case 3:	$q^C_{skm} = MCP_s f_{sm}($	T^C_k	-	T^C_{k+1}	+	T^C_k	-	T^C_{k+1}	$)$
Case 4:	$q^C_{skm} = MCP_s f_{sm}($	t^{in}_{sm}	-	T^C_{k+1}	+	T^C_k	-	T^C_{k+1}	$)$
Case 5:	$q^C_{skm} = MCP_s f_{sm}($	T^C_k	-	T^C_{k+1}	+	T^C_k	-	T^C_k	$)$
Case 6:	$q^C_{skm} = MCP_s f_{sm}($	T^C_{k+1}	-	T^C_{k+1}	+	T^C_k	-	T^C_{k+1}	$)$

Tabell 3.2: q^C_{skm}-equations for intervals between T^T_s and T^S_s

tures. This must be done by making one general equation with some variables that detect the correct case depending on the inlet and outlet temperatures. Table 3.2 shows that the second and third term inside the parenthesis remains the same for all of the six cases. The first and fourth term does on the other hand change. It turns out that the first term is t^{in}_{sm} when the inlet temperature is inside the interval, T^C_k when the inlet temperature is above the interval and T^C_{k+1} when the inlet temperature is below the interval. The same pattern is found for the fourth term, but now it is dependent of the outlet temperature. One way to make a general equation would be to introduce a temperature variable for the first and fourth term, which choses the correct temperature among the three cases. This can be done by introducing $t^{in,D}_{skm}$ and $t^{out,D}_{skm}$ and define them as equation 3.7.

$$t^{in/out,D}_{skm} = \begin{cases} T^C_k & \text{if } t^{in/out}_{sm} \geq T^C_k \\ t^{in/out}_{sm} & \text{if } T^C_k \geq t^{in/out}_{sm} \geq T^C_{k+1} \\ T^C_{k+1} & \text{if } T^C_k \geq t^{in/out}_{sm} \end{cases} \quad , s \in S^{EC}, k \in K, m \in M_s \tag{3.7}$$

3.7 can be modeled by using min/max functions. Equations A.21 and 3.9 is introduced to do this. Another way to calculate $t^{in,D}_{skm}$ and $t^{out,D}_{skm}$ would be to introduce a binary variable for each T^C_k which keeps track of whether the inlet/outlet temperature is above or below a given T^C_k. This would introduce a lot of binary variables and relations which used the big M-method. This would make the LP realization less thigh and therefore increase the solution time. The author did therefore decide to not pursue this alternative. By utilizing equation (A.21) and (3.9), it possible to derive the general equation (3.10), which handles all the six cases.

$$t^{in,D}_{skm} = \max\left(\min\left(t^{in}_{skm}, T^C_k\right), T^C_{k+1}\right) \quad , s \in S^{EC}, k \in K, m \in M_s \tag{3.8}$$

$$t_{skm}^{out,D} = \max\left(\min\left(t_{skm}^{out}, T_k^C\right), T_{k+1}^C\right) \quad , s \in S^{EC}, k \in K, m \in M_s \tag{3.9}$$

$$q_{skm}^C = MCP_s f_{sm}\left(t_{skm}^{in,D} - T_{k+1}^C + T_k^C - t_{skm}^{out,D}\right) \quad , s \in S^{EC}, k \in K_s^{CBTST}, m \in M_s \tag{3.10}$$

Case 1:	$q_{skm}^C = MCP_s f_{sm}($	T_k^C	-	t_{sm}^{out}	-	$[T_k^C$	-	t_{sm}^{in}	$])$
Case 2:	$q_{skm}^C = MCP_s f_{sm}($	T_k^C	-	t_{sm}^{out}	-	$[T_k^C$	-	T_k^C	$])$
Case 3:	$q_{skm}^C = MCP_s f_{sm}($	T_k^C	-	T_{k+1}^C	-	$[T_k^C$	-	T_k^C	$])$
Case 4:	$q_{skm}^C = MCP_s f_{sm}($	T_k^C	-	T_{k+1}^C	-	$[T_k^C$	-	t_{sm}^{in}	$])$
Case 5:	$q_{skm}^C = MCP_s f_{sm}($	T_k^C	-	T_k^C	-	$[T_k^C$	-	T_k^C	$])=0$
Case 6:	$q_{skm}^C = MCP_s f_{sm}($	T_k^C	-	T_{k+1}^C	-	$[T_k^C$	-	T_{k+1}^C	$])=0$

Tabell 3.3: q_{skm}^C-equations for intervals below T_s^S

Table 3.3 shows the heat demand for all the 6 cases for an interval below the supply temperature. The table shows that there are only the second and fourth term inside the parenthesis that changes for the different cases. The second term is dependent on the outlet temperature and the fourth term is dependent on the inlet temperature. So the second term can be changed out with $t_{skm}^{out,D}$ and the fourth term with $t_{skm}^{in,D}$. The general equation then becomes:

$$q_{skm}^C = MCP_s f_{sm}\left(T_k^C - t_{skm}^{out,D} - \left[T_k^C - t_{skm}^{in,D}\right]\right) \quad , s \in S^{EC}, k \in K_s^{CBTS}, m \in M_s \tag{3.11}$$

Case 1:	$q_{skm}^C = MCP_s f_{sm}($	t_{sm}^{in}	-	T_{k+1}^C	-	$[t_{sm}^{out}$	-	T_{k+1}^C	$])$
Case 2:	$q_{skm}^C = MCP_s f_{sm}($	T_k^C	-	T_{k+1}^C	-	$[t_{sm}^{out}$	-	T_{k+1}^C	$])$
Case 3:	$q_{skm}^C = MCP_s f_{sm}($	T_k^C	-	T_{k+1}^C	-	$[T_{k+1}^C$	-	T_{k+1}^C	$])$
Case 4:	$q_{skm}^C = MCP_s f_{sm}($	t_{sm}^{in}	-	T_{k+1}^C	-	$[T_{k+1}^C$	-	T_{k+1}^C	$])$
Case 5:	$q_{skm}^C = MCP_s f_{sm}($	T_k^C	-	T_{k+1}^C	-	$[T_k^C$	-	T_{k+1}^C	$])=0$
Case 6:	$q_{skm}^C = MCP_s f_{sm}($	T_{k+1}^C	-	T_{k+1}^C	-	$[T_{k+1}^C$	-	T_{k+1}^C	$])=0$

Tabell 3.4: q_{skm}^C-equations for intervals above T_s^T

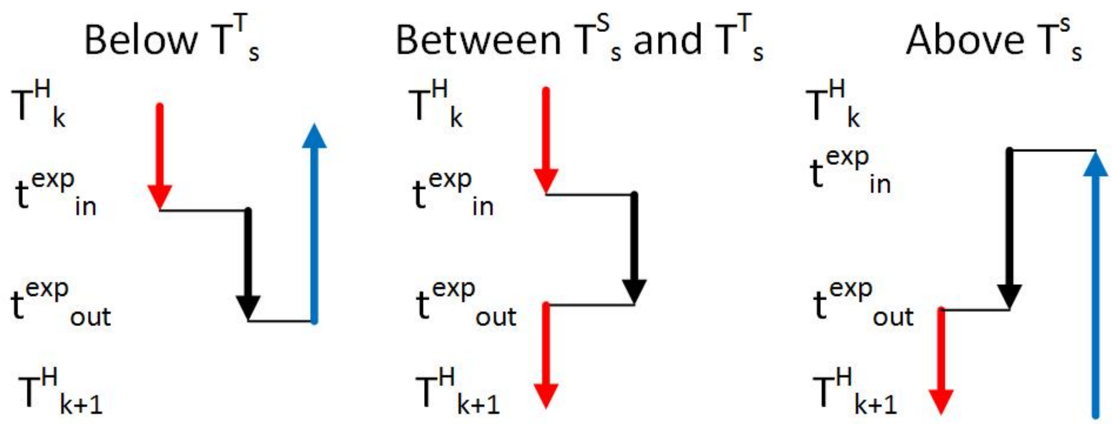
Table 3.4 shows the heat demand for all the 6 cases for an interval above the target temperature. The table shows that there are only the first and third term inside the parenthesis that changes for the different cases. The first term is dependent on the inlet temperature and the third term is dependent on the outlet temperature. So the first term can be changed out with $t_{skm}^{in,D}$ and the third term with $t_{skm}^{out,D}$. The general equation then becomes:

$$q_{skm}^{C} = MCP_s f_{sm}\left(t_{skm}^{in,D} - T_{k+1}^{C} - \left[t_{skm}^{out,D} - T_{k+1}^{C}\right]\right) \quad , s \in S^{EC}, k \in K_s^{CATT}, m \in M_s \tag{3.12}$$

Equation (3.11) does contain T_k^C two times with the opposite sign. The two equation can therefore be simplified by removing both of the T_k^C terms. The same is true for (3.12), but now it is the T_{k+1}^C terms that can be removed. The two equations does now become identical and can be written as equation (3.13) instead.

$$q_{skm}^{C} = MCP_s f_{sm}\left(t_{skm}^{in,D} - t_{skm}^{out,D}\right) \quad , s \in S^{EC}, k \in K_s^{CBTS} \cup K_s^{CATT}, m \in M_s \tag{3.13}$$

3.3.4 Heat demand for a hot stream with expansion



Figur 3.4: Behavior of a hot stream with expansion inside a temperature interval

Figure 3.4 shows how a hot stream will behave in an interval the heat cascade. The left illustration shows the case where both the inlet and outlet temperature is below the target temperature. Since the interval is below the target temperature will the stream enter the interval with a temperature T_k^H and then supply heat (red arrow) until it reaches the inlet temperature of the turbine. The stream expands (black arrow) through the turbine, which decreases the temperature further. The stream have to end up at the target temperature and thus have to be heated up to T_k^H where it leaves the interval. The last part is illustrated with a blue arrow since the stream now is heated up instead of being cooled down.

The middle illustration shows the case where both the inlet and outlet temperature is between the supply and target temperature. The stream will enter the interval with a temperature of T_k^H and then supply heat (red arrow) until it reaches the inlet of the turbine. Here the temperature drops (black arrow) and the stream continues to supply heat (red arrow) until it leaves the interval with a temperature of T_{k+1}^H.

The right illustration shows the case where both the inlet and outlet temperature is above the supply temperature. Since the interval is above the supply temperature will the stream enter the interval from below with a temperature of T_{k+1}^H. It stream will be heated up until it reaches the inlet of the turbine. This is illustrated with a blue arrow. The temperature drops (black arrow) when the stream is expanded through the turbine. Then the stream will supply heat (red arrow) until it is cooled down to T_{k+1}^H where it leaves the interval.

Figure 3.4 does only show the cases where both the inlet and outlet temperature of the turbine is inside the given temperature interval, but the inlet and outlet temperature may also be above or below the given interval. This makes 6 cases in total as shown in figure 3.3. By using the different diagrams in Figure 3.4 it is possible to find the heating requirement for each of the cases in Figure 3.3, which is done in Table 3.5, 3.6 and 3.7. For case 1 (both inlet and outlet temperature inside the interval) in an interval between the supply and target temperature does one get the middle illustration in Figure 3.4. Here the stream supplies heat from T_k^H to t^{in} and then from t^{out} to T_{k+1}^H. The total heat supplied in that interval then becomes: $q_{sk}^H = MCP_s\left(T_k^H - t^{in}\right) f_s + MCP_s\left(t^{out} - T_{k+1}^H\right) f_s$. The same derivation was done to the 5 other cases in Figure 3.3 and then again for the intervals below the target temperature and above the supply temperature. The goal was again to find general equations which were valid for as many cases as possible.

Table 3.5 shows the heat supply for all the 6 cases for an interval between the supply and target temperature. The table shows that there are only the second and third term inside the parenthesis that changes for the different cases. The second term is dependent on the inlet temperature and the third term is dependent on the outlet temperature. So the second term can be

Case 1:	$q^H_{skm} = MCP_s f_{sm}($	T^H_k	-	t^{in}_{sm}	+	t^{out}_{sm}	-	T^H_{k+1}	)
Case 2:	$q^H_{skm} = MCP_s f_{sm}($	T^H_k	-	T^H_k	+	t^{out}_{sm}	-	T^H_{k+1}	)
Case 3:	$q^H_{skm} = MCP_s f_{sm}($	T^H_k	-	T^H_k	+	T^H_{k+1}	-	T^H_{k+1}	)
Case 4:	$q^H_{skm} = MCP_s f_{sm}($	T^H_k	-	t^{in}_{sm}	+	T^H_{k+1}	-	T^H_{k+1}	)
Case 5:	$q^H_{skm} = MCP_s f_{sm}($	T^H_k	-	T^H_k	+	T^H_k	-	T^H_{k+1}	)
Case 6:	$q^H_{skm} = MCP_s f_{sm}($	T^H_k	-	T^H_{k+1}	+	T^H_{k+1}	-	T^H_{k+1}	)

Tabell 3.5: q^H_{skm}-equations for intervals between T^S_s and T^T_s

changed out with $t^{in,D}_{skm}$ and the third term with $t^{out,D}_{skm}$. The general equation then becomes:

$$q^H_{skm} = MCP_s f_{sm}\left(T^H_k - t^{in,D}_{skm} + t^{out,D}_{skm} - T^H_{k+1}\right), s \in S^{EH}, k \in K^{HBTST}_s, m \in M_s \tag{3.14}$$

Case 1:	$q^H_{skm} = MCP_s f_{sm}($	T^H_k	-	t^{in}_{sm}	-	$[T^H_k$	-	t^{out}_{sm}	])
Case 2:	$q^H_{skm} = MCP_s f_{sm}($	T^H_k	-	T^H_k	-	$[T^H_k$	-	t^{out}_{sm}	])
Case 3:	$q^H_{skm} = MCP_s f_{sm}($	T^H_k	-	T^H_k	-	$[T^H_k$	-	T^H_{k+1}	])
Case 4:	$q^H_{skm} = MCP_s f_{sm}($	T^H_k	-	t^{in}_{sm}	-	$[T^H_k$	-	T^H_{k+1}	])
Case 5:	$q^H_{skm} = MCP_s f_{sm}($	T^H_k	-	T^H_k	-	$[T^H_k$	-	T^H_k	])
Case 6:	$q^H_{skm} = MCP_s f_{sm}($	T^H_k	-	T^H_{k+1}	-	$[T^H_k$	-	T^H_{k+1}	])

Tabell 3.6: q^H_{skm}-equations for intervals below T^T_s

Table 3.6 shows the heat supply for all the 6 cases for an interval below the target temperature. The table shows that there are only the second and fourth term inside the parenthesis that changes for the different cases. The second term is dependent on the inlet temperature and the fourth term is dependent on the outlet temperature. So the second term can be changed out with $t^{in,D}_{skm}$ and the fourth term with $t^{out,D}_{skm}$. The general equation then becomes:

$$q^H_{skm} = MCP_s f_{sm}\left(T^H_k - t^{in,D}_{skm} - \left[T^H_k - t^{out,D}_{skm}\right]\right), s \in S^{EH}, k \in K^{HBTT}_s, m \in M_s \tag{3.15}$$

Table 3.7 shows the heat supply for all the 6 cases for an interval above the supply temperature. The table shows that there are only the first and third term inside the parenthesis that changes for the different cases. The first term is dependent on the outlet temperature and the third term is dependent on the inlet temperature. So the first term can be changed out with $t^{out,D}_{skm}$ and the third term with $t^{in,D}_{skm}$. The general equation then becomes:

Case 1:	$q^H_{skm} = MCP_s f_{sm}($	t^{out}_{sm}	-	T^H_{k+1}	-	$[t^{in}_{sm}$	-	T^H_{k+1}])
Case 2:	$q^H_{skm} = MCP_s f_{sm}($	t^{out}_{sm}	-	T^H_{k+1}	-	$[T^H_k$	-	T^H_{k+1}])
Case 3:	$q^H_{skm} = MCP_s f_{sm}($	T^H_{k+1}	-	T^H_{k+1}	-	$[T^H_k$	-	T^H_{k+1}])
Case 4:	$q^H_{skm} = MCP_s f_{sm}($	T^H_{k+1}	-	T^H_{k+1}	-	$[t^{in}_{sm}$	-	T^H_{k+1}])
Case 5:	$q^H_{skm} = MCP_s f_{sm}($	T^H_k	-	T^H_{k+1}	-	$[T^H_k$	-	T^H_{k+1}])=0
Case 6:	$q^H_{skm} = MCP_s f_{sm}($	T^H_{k+1}	-	T^H_{k+1}	-	$[T^H_{k+1}$	-	T^H_{k+1}])=0

Tabell 3.7: q^H_{skm}-equations for intervals above T^S_s

$$q^H_{skm} = MCP_s f_{sm}\left(t^{out,D}_{skm} - T^H_{k+1} - \left[t^{in,D}_{skm} - T^H_{k+1}\right]\right), s \in S^{EH}, k \in K^{HATS}_s, m \in M_s \tag{3.16}$$

The streams are hot, so the hot side of the heat cascade must be used. Therefore does $t^{in,D}_{skm}$ and $t^{out,D}_{skm}$ need new equations for the hot streams:

$$t^{in,D}_{skm} = \max\left(\min\left(t^{in}_{skm}, T^H_k\right), T^H_{k+1}\right), s \in S^{EH}, k \in K, m \in M_s \tag{3.17}$$

$$t^{out,D}_{skm} = \max\left(\max\left(t^{out}_{skm}, T^H_k\right), T^H_{k+1}\right), s \in S^{EH}, k \in K, m \in M_s \tag{3.18}$$

Equation (3.15) does contain T^H_k two times with the opposite sign. The two equation can therefore be simplified by removing both of the T^H_k terms. The same is true for equation (3.16), but now it is the T^H_{k+1} terms that can be removed. The two equations does now become identical and can be written as equation(3.19) instead.

$$q^H_{skm} = MCP_s f_{sm}\left(t^{out,D}_{skm} - t^{in,D}_{skm}\right), s \in S^{EH}, k \in K^{HBTT}_s \cup K^{HATS}_s, m \in M_s \tag{3.19}$$

3.3.5 Turbine

The turbine is modeled as an polytropic process with polytropic efficiency η_T.

$$t^{out}_{sm} - \left(\frac{P^T_s}{P^S_s}\right)^{\eta_s \frac{\kappa_s - 1}{\kappa_s}} t^{in}_{sm} = 0, \quad s \in S^{EH} \cup S^{EC}, m \in M_s \tag{3.20}$$

Equation (3.20) shows that the variable t_{sm}^{out} is actually not necessary, but it is kept in the model since it makes the model more readable. The equation is only valid if the temperature is used in the unit Kelvin. This is done by adding 273.15 to t_{sm}^{in} and t_{sm}^{out} in the implementation.

3.3.6 Stream split

Fu and Gundersen (2015b) discovered that it is not always optimal to let the stream only go through one turbine. When a stream goes through a turbine it supplies cooling to the system and if this cooling is larger than the cooling demand of the system, it will be better to split the stream into several branches. Each branch will have its own turbine which may operate at different temperatures. This feature have been modeled by including a fraction variable, f_{sm}, which shows how much of the MCP_s that goes into stream split part m. It is difficult to find out how many branches a stream will be split into. Tests show that two branches is usually enough, but there has been a case where three branches were needed.

When a stream is split into several branches it possible to get several solutions that are mathematically different, but that in practice are identical. If a stream is split into two branches, then there will be two optimal solutions. The first solution may have fractions 0.7 and 0.3, with temperatures of 200°C and 400°C. The other optimal solution would be fractions of 0.3 and 0.7 and temperatures of 400°C and 200°C. If this symmetry is broken, it will reduce the search space and make the problem faster to solve. The symmetry is broken by restriction (3.22), which forces fractions with lower indices to be greater than or equal to the fractions with higher indices. So only the first of the aforementioned hypothetical solutions would be valid.

$$\sum_{m \in M_s} f_{sm} = 1 \quad , s \in S^{EH} \cup S^{EC} \tag{3.21}$$

$$f_{sm} \geq f_{sm+1} \quad , s \in S^{EH} \cup S^{EC}, m \in M_s \setminus \{|M_s|\} \tag{3.22}$$

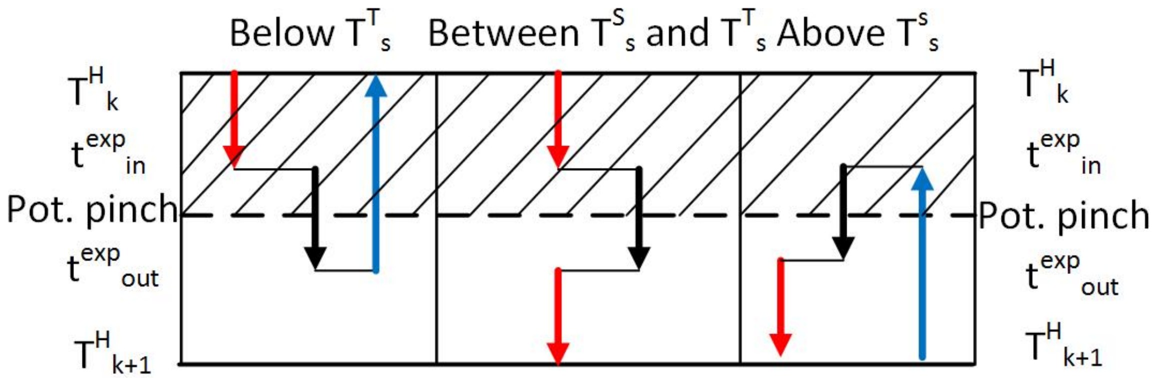
$$f_{sm} \geq 0 \quad , s \in S^{EH} \cup S^{EC}, m \in M_s \tag{3.23}$$

3.3.7 Potential Pinch point at outlet of pressure changing unit

A heat cascade will only give thermodynamically feasible solutions if the temperature intervals are chosen correctly. The model as it is now will not handle the case where the pinch point is located inside a temperature interval. The heat residual from the temperature interval will be zero, but the residual from the actual pinch point may be negative, which is thermodynamically infeasible. To prevent this from happening, it is important that the heat residual from all the potential pinch points is calculated. The pinch point will always be located at the beginning of a stream and this is why the heat cascade is based on the supply temperatures for the streams. A pressure changing unit does in practice divide a stream into two separate streams. The supply temperature of the first stream is the same as for the original stream. The supply temperature of the second stream however is actually the outlet temperature of the pressure changing unit. The outlet temperature is an variable that is found during the optimization, so it is not possible to put this temperature into the heat cascade beforehand. t_{sm}^{out} is the outlet temperature for stream s, part m. It will most likely be located inside a temperature a temperature interval and the heat residual r_{skm}^{out} from this temperature must be calculated and forced to be non-negative. It is not given which temperature interval the t_{sm}^{out}-variables will be located inside. One way to solve this would be to use binary variables to keep track of which interval each if the outlet temperatures is associated with and then calculate the heat residual from t_{sm}^{out} in these intervals. Another way would be to find general equations for all intervals based on $t_{skm}^{out,D}$ instead. Both methods requires a restriction for each outlet temperature inside each temperature interval. The first method also needs the same number of new binary variables with restrictions, while the second method can reuse variables. Because of this did the second method seem more appealing and was chosen to be derived and implemented.

The heat residual, r_{skm}^{out}, is calculated by doing a heat balance inside a temperature interval from $T_k^{H/C}$ to $t_{skm}^{out,D}$. The temperature interval specific $t_{skm}^{out,D}$ is used instead of the real t_{sm}^{out} because the equations need to valid even if t_{sm}^{out} is not located inside the interval. If t_{sm}^{out} is located above the interval will $t_{skm}^{out,D}$ be equal to $T_k^{H/C}$ and the heat residual of the previous interval will be calculated. If t_{sm}^{out} is located below the interval will $t_{skm}^{out,D}$ be equal to $T_{k+1}^{H/C}$ and the interval's actual heat residual will be calculated. Equation (3.24) calculates heat residual r_{skm}^{out} and is based on equation (3.56). r_{skm}^{out} is equal to the residual from the previous interval, minus the heating requirement from cold streams with and without pressure manipulation and plus the heat supplied from hot streams with and without pressure manipulation. The heating requirements have been changed so that they only include the heat transferred from the beginning of the interval to the outlet temperature $t_{skm}^{out,D}$.

$$
\begin{aligned}
&r_{skm}^{out} - r_{k-1} + q_{skm}^{CP} - q_{skm}^{HP} - \sum_{u \in U_k^H} q_{ku}^{HU} - \sum_{s \in S^{HWP}} MCP_s \left(T_k^H - t_{skm}^{out,D} \right) \\
&+ \sum_{s \in S^{CWP}} MCP_s \left(T_k^C - t_{skm}^{out,D} \right) = 0, s \in S^{EH} \cup S^{EC}, k \in K \setminus \{1, |K|\}, m \in M_s
\end{aligned}
\tag{3.24}
$$



Figur 3.5: Behavior of a hot stream with expansion and a potential pinch point inside a temperature interval

Figure 3.5 shows how a hot stream with expansion may behave inside a temperature interval. The left illustration shows the case where both the inlet and outlet temperature of the turbine is located inside an interval below the target temperature. The stream enters the interval with a temperature of T_k^H and supplies heat (red arrow) until it reaches the inlet of the turbine. Heat is supplied from T_k^H to the maximum of the potential pinch point (black dashed line), $t_{skm}^{out,D}$, and the inlet of the turbine, $t_{ykn}^{in,D}$. The heat supplied above the potential pinch point does then become $MCP_y \left(T_k^H - \max\left(t_{ykn}^{in,D}, t_{skm}^{out,D} \right) \right)$. $t_{ykn}^{in,D}$ is the inlet temperature of a turbine from the same or another stream y, split n. Then the temperature drops during the expansion (black arrow), before the stream is heated up (blue arrow) to T_k^H. The heating requirement above the potential pinch point becomes T_k^H minus the maximum of the potential pinch point (black dashed line), $t_{skm}^{out,D}$, and the outlet of the turbine, $t_{ykn}^{out,D}$: $MCP_y \left(T_k^H - \max\left(t_{ykn}^{out,D}, t_{skm}^{out,D} \right) \right)$. The general equation then becomes:

$$q_{skm}^{HP} = \sum_{y \in S^{EH}} \sum_{n \in M_y} MCP_y f_{yn} \left(\max\left(t_{ykn}^{out,D}, t_{skm}^{out,D}\right) - \max\left(t_{ykn}^{in,D}, t_{skm}^{out,D}\right)\right)$$
$$s \in S^{EH} \cup S^{EC}, k \in K_s^{HBTT}, m \in M_s \tag{3.25}$$

The middle illustration in Figure 3.5 shows a hot stream with both inlet and outlet temperature located inside a temperature interval between the supply and target temperature may behave. The stream enter the interval from above and supplies heat (red arrow) until it reaches the inlet of the turbine. The heat supplied above the potential pinch point then becomes $MCP_y \left(T_k^H - \max\left(t_{ykn}^{in,D}, t_{skm}^{out,D}\right)\right)$. The temperature drops through the turbine (black arrow) and the stream continues to supply heat (red arrow) until it leaves the interval with a temperature of T_{k+1}^H. Only the heat supplied above the potential pinch point is of interest, so the stream supplies heat from the maximum of the outlet temperature and the potential pinch point to the potential pinch point. The heat supplied then becomes: $MCP_y \left(\max\left(t_{ykn}^{out,D}, t_{skm}^{out,D}\right) - t_{skm}^{out,D}\right)$. The general equation becomes:

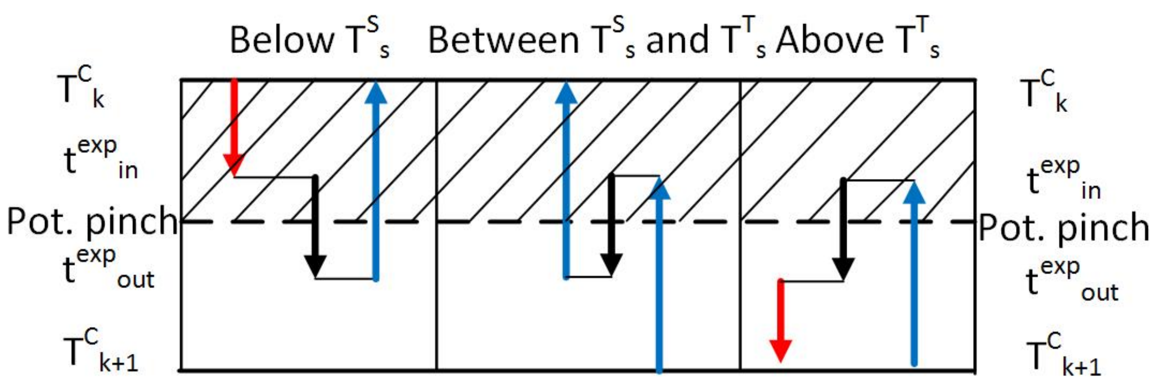
$$q_{skm}^{HP} = \sum_{y \in S^{EH}} \sum_{n \in M_y} MCP_y f_{yn} \left(T_K^H - \max\left(t_{ykn}^{in,D}, t_{skm}^{out,D}\right) + \max\left(t_{ykn}^{out,D}, t_{skm}^{out,D}\right) - t_{skm}^{out,D}\right)$$
$$s \in S^{EH} \cup S^{EC}, k \in K_s^{HBTST}, m \in M_s \tag{3.26}$$

The right illustration in Figure 3.5 shows a hot stream with both inlet and outlet temperature located inside a temperature interval above the supply temperature may behave. The stream enter the interval from below and is heated up (blue arrow) until it reaches the inlet of the turbine. The heating requirement above the potential pinch point (black dashed line) then becomes $MCP_y \left(\max\left(t_{ykn}^{in,D}, t_{skm}^{out,D}\right) - t_{skm}^{out,D}\right)$. The temperature drops through the turbine (black arrow) and the stream supplies heat (red arrow) until it leaves the interval with a temperature of T_{k+1}^H. Only the heat supplied above the potential pinch point is of interest, so the stream supplies heat from the maximum of the outlet temperature and the potential pinch point to the potential pinch point. The heat supplied then becomes: $MCP_y \left(\max\left(t_{ykn}^{out,D}, t_{skm}^{out,D}\right) - t_{skm}^{out,D}\right)$. The general equation becomes:

$$q_{skm}^{HP} = \sum_{y \in S^{EH}} \sum_{n \in M_y} MCP_y f_{yn} \left(\max\left(t_{ykn}^{out,D}, t_{skm}^{out,D} \right) - \max\left(t_{ykn}^{in,D}, t_{skm}^{out,D} \right) \right)$$
$$s \in S^{EH} \cup S^{EC}, k \in K_s^{HATS}, m \in M_s \tag{3.27}$$

Equation (3.25) and (3.27) are in fact identical and can be merged together to one equation:

$$q_{skm}^{HP} = \sum_{y \in S^{EH}} \sum_{n \in M_y} MCP_y f_{yn} \left(\max\left(t_{ykn}^{out,D}, t_{skm}^{out,D} \right) - \max\left(t_{ykn}^{in,D}, t_{skm}^{out,D} \right) \right)$$
$$s \in S^{EH} \cup S^{EC} k \in K_s^{HATS} \cup K_s^{HBTT}, m \in M_s \tag{3.28}$$



Figur 3.6: Behavior of a cold stream with expansion and a potential pinch point inside a temperature interval

Figure 3.6 shows how a cold stream with expansion may behave inside a temperature interval. The left illustration shows the case where both the inlet and outlet temperature of the turbine is located inside an interval below the supply temperature. The stream enters the interval with a temperature of T_k^C and supplies heat (red arrow) until it reaches the inlet of the turbine. Heat is supplied from T_k^C to the maximum of the potential pinch point (black dashed line), $t_{skm}^{out,D}$, and the inlet of the turbine, $t_{ykn}^{in,D}$. The heat supplied above the potential pinch point does then become $MCP_y \left(T_k^C - \max\left(t_{ykn}^{in,D}, t_{skm}^{out,D} \right) \right)$. $t_{ykn}^{in,D}$ is the inlet temperature of a turbine from the same or another stream y, split n. Then the temperature drops during the expansion (black arrow), before the stream is heated up (blue arrow) to T_k^C. The heating requirement above the potential pinch point for the heating becomes T_k^C minus the maximum of the pinch point, $t_{skm}^{out,D}$, and the outlet of the turbine, $t_{ykn}^{out,D}$: $MCP_y\left(T_k^C - \max\left(t_{ykn}^{out,D}, t_{skm}^{out,D}\right)\right)$. The general equation then becomes:

$$q_{skm}^{CP} = \sum_{y \in S^{EC}} \sum_{n \in M_y} MCP_y f_{yn} \left(\max\left(t_{ykn}^{in,D}, t_{skm}^{out,D}\right) - \max\left(t_{ykn}^{out,D}, t_{skm}^{out,D}\right)\right)$$
$$s \in S^{EH} \cup S^{EC}, k \in K_s^{HBTT}, m \in M_s \tag{3.29}$$

The middle illustration in Figure 3.6 shows how a cold stream with both the inlet and outlet temperature is located inside a temperature interval between the supply and target temperature may behave. The stream enter the interval from below and is heat up (blue arrow) until it reaches the inlet of the turbine. The heating requirement above the potential pinch point then becomes $MCP_y\left(T_K^C - \max\left(t_{ykn}^{in,D}, t_{skm}^{out,D}\right)\right)$. The temperature drops through the turbine (black arrow) and the stream continues to be heated up (blue arrow) until it leaves the interval with a temperature of T_k^C. Only the heating requirement above the potential pinch point is of interest, so the stream is heated from the maximum of the outlet temperature and the potential pinch point to T_k^C. The heat requirement for this part then becomes: $MCP_y\left(\max\left(t_{ykn}^{out,D}, t_{skm}^{out,D}\right) - t_{skm}^{out,D}\right)$. The general equation becomes:

$$q_{skm}^{CP} = \sum_{y \in S^{EC}} \sum_{n \in M_y} MCP_s f_{yn} \left(T_K^C - \max\left(t_{ykn}^{out,D}, t_{skm}^{out,D}\right) + \max\left(t_{ykn}^{in,D}, t_{skm}^{out,D}\right) - t_{skm}^{out,D}\right)$$
$$s \in S^{EH} \cup S^{EC}, k \in K_s^{HBTST}, m \in M_s \tag{3.30}$$

The right illustration in Figure 3.6 shows how a cold stream with both inlet and outlet temperature is located inside a temperature interval above the target temperature may behave. The stream enter the interval from below and is heated up (blue arrow) until it reaches the inlet of the turbine. The heating requirement above the potential pinch point (black dashed line) then becomes $MCP_y\left(\max\left(t_{ykn}^{in,D}, t_{skm}^{out,D}\right) - t_{skm}^{out,D}\right)$. The temperature drops through the turbine (black arrow) and the stream supplies heat (red arrow) until it leaves the interval with a temperature of

T^C_{k+1}. Only the heat supplied above the potential pinch point is of interest, so the stream supplies heat from the maximum of the outlet temperature and the potential pinch point to the potential pinch point. The heat supplied then becomes: $MCP_y \left(\max \left(t^{out,D}_{ykn}, t^{out,D}_{skm} \right) - t^{out,D}_{skm} \right)$. The general equation becomes:

$$q^{CP}_{skm} = \sum_{y \in S^{EC}} \sum_{n \in M_y} MCP_s f_{yn} \left(\max \left(t^{in,D}_{ykn}, t^{out,D}_{skm} \right) - \max \left(t^{out,D}_{ykn}, t^{out,D}_{skm} \right) \right)$$
$$s \in S^{EC} \cup s^{EH}, k \in K^{HBTT}_s, m \in M_s \tag{3.31}$$

Equation (3.29) and (3.31) are in fact identical and can be merged together to one equation:

$$q^{CP}_{skm} = \sum_{y \in S^{EC}} \sum_{n \in M_y} MCP_s f_{yn} \left(\max \left(t^{in,D}_{ykn}, t^{out,D}_{skm} \right) - \max \left(t^{out,D}_{ykn}, t^{out,D}_{skm} \right) \right)$$
$$s \in S^{EC} \cup s^{EH}, k \in K^{HBTT}_s \cup K^{HATS}_s, m \in M_s \tag{3.32}$$

The max functions causes the equations to be non smooth, they were therefore reformulated by using a non linear approximation instead. The max function was modeled like equation (3.33), where Δ is a small number.

$$\max(x, y) = 0.5 \left(x + y + \sqrt{(x-y)^2 + (\Delta)^2} - \Delta \right) \tag{3.33}$$

3.3.8 Compression

3.3.8.1 Objective function

Extending the model so it handles both expansion and compression requires some changes to old equations and some new equations. Work is added to an compressor to increase the pressure of the stream passing through and this must be included in the objective function. The only difference between calculating the exergy content for compression and expansion is the exponential for the efficiency, η_s. For compression the efficiency will have an exponent of -1. If the efficiency is 100%, then the objective function would not have to change. A third term is added to the new objective function (3.34) to include the effect of an efficiency lower than 100%.

$$\begin{aligned}\min \sum_{k\in K}\sum_{u\in U_k^H}\left(1-\frac{T_0}{T_u^{HU}}\right)q_{ku}^{HU} - \sum_{s\in S^{EH}\cup S^{EC}}\sum_{m\in M_s}\eta_s MCP_s f_{sm}\left(t_{sm}^{in}-t_{sm}^{out}\right)\\ -\sum_{s\in S^{CH}\cup S^{CC}}\sum_{m\in M_s}\eta_s^{-1} MCP_s f_{sm}\left(t_{sm}^{in}-t_{sm}^{out}\right)\end{aligned} \tag{3.34}$$

3.3.8.2 Compressor

The pressure change in a compressor is handled almost identical as to change through a turbine. Again the only difference is the exponent of the efficiency.

$$t_{sm}^{out}-\left(\frac{P_s^T}{P_s^S}\right)^{\eta_s^{-1}\frac{\kappa-1}{\kappa}} t_{sm}^{in}=0, \quad s\in S^{CH}\cup S^{CC}, m\in M_s \tag{3.35}$$

3.3.8.3 Heat balance

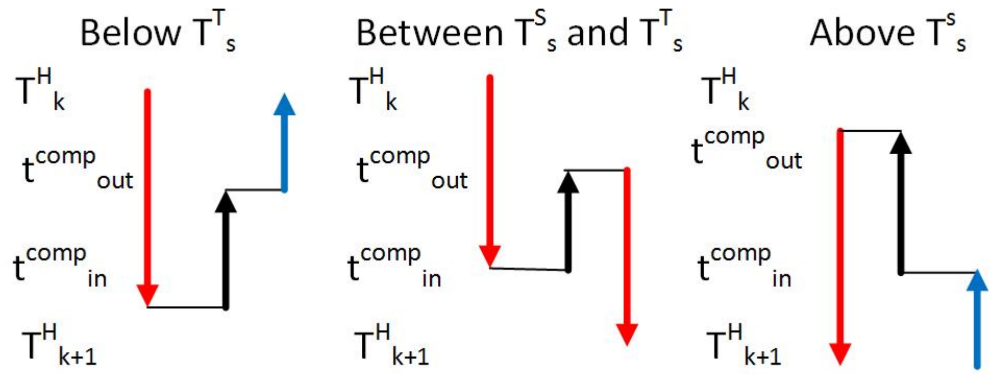
The heat balance does also change and needs to include the heating requirements for the streams with compression. It turns out that the heating requirement equations for streams with compression is identical to equivalent equations for the streams with expansion. The only change that needs to be made to the heat balance equations is to extend the sets so that the streams with compression is added. Equation (3.2) and (3.3) is changed into (3.36) and (3.37).

$$r_1+\sum_{s\in S^{EC}\cup S^{CC}}\sum_{m\in M_s} q_{s,1,m}^C-\sum_{s\in S^{EH}\cup S^{CH}}\sum_{m\in M_s} q_{s,1,m}^H-\sum_{v\in V_1} q_{1,v}^{HU}=\sum_{s\in S^{HWP}} Q_{s,1}^H-\sum_{s\in S^{CWP}} Q_{s,1}^C \tag{3.36}$$

$$r_k-r_{k-1}+\sum_{s\in S^{EC}\cup S^{CC}}\sum_{m\in M_s} q_{skm}^C-\sum_{s\in S^{EH}\cup S^{CH}}\sum_{m\in M_s} q_{skm}^H-\sum_{u\in U_k^H} q_{ku}^{HU}=\sum_{s\in S^H} Q_{sk}^H-\sum_{s\in S^C} Q_{sk}^C \quad, k\in K\setminus\{1\} \tag{3.37}$$

3.3.8.4 Heat supply for a hot stream with compression

Figure 3.7 shows how a hot stream with compression will behave if both the inlet and outlet of the compressor is inside a heat cascade temperature interval. The left illustration shows the case where the interval is below the target temperature of the stream. The stream will enter the interval with a temperature of T_k^H and then supply heat (red arrow) until it reaches the inlet tempe-



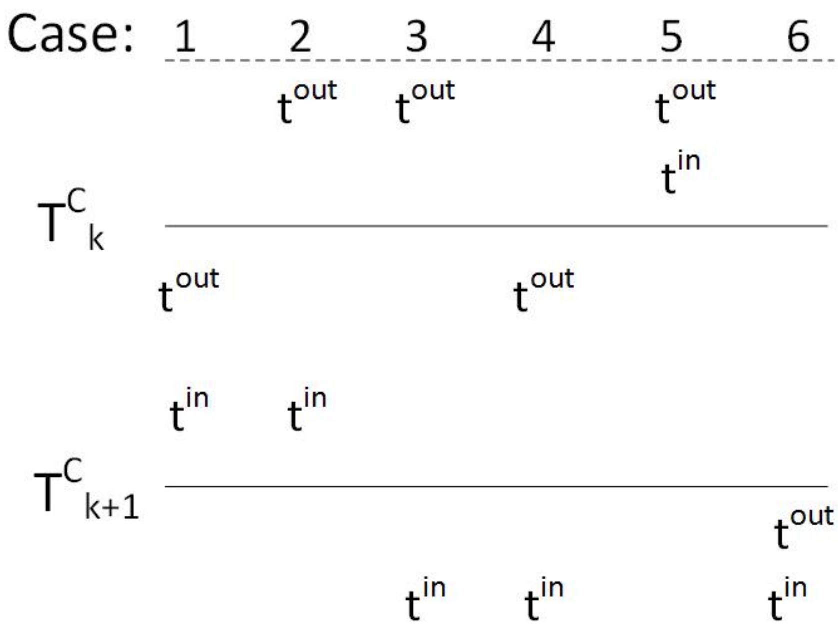
Figur 3.7: Hot stream behavior for streams with compression

rature of the compressor. The stream will then increase its temperature through the compressor (black arrow). The stream will now have to be heated up (blue arrow) to the target temperature. The target temperature is higher than T_k^H, which means that the stream leaves the interval with a temperature of T_k^H.

The middle illustration shows the case where both the inlet and outlet temperature is inside a temperature interval between the supply and target temperature. The stream enters the interval with a temperature of T_k^H and then supplies heat (red arrow) until it reaches the inlet of the compressor. The stream will then increase its temperature through the compressor (black arrow). The target temperature is below the interval, so the stream will continue to supply heat (red arrow) until it leaves the interval with a temperature of T_{k+1}^H.

The right illustration shows the case where both the inlet and outlet temperature is inside an interval above the supply temperature. Since the interval lies above the supply temperature will the stream enter the interval with a temperature of T_{k+1}^H. The stream will be heated up (blue arrow) the inlet of the compressor. The stream will then increase its temperature through the compressor (black arrow). The target temperature is below the temperature interval, so the stream will supply heat (red arrow) until it leaves the interval with a temperature of T_{k+1}^H.

Figure 3.7 does only show the cases where both the inlet and outlet temperature of the compressor is located inside the given temperature interval, but the inlet and outlet temperature may also be above or below the given interval. This makes 6 cases in total as shown in figure 3.8. By using the different diagrams in Figure 3.7 it is possible to find the heating requirement for each of the cases in Figure 3.8, which is done in Table 3.8, 3.9 and 3.10. For case 1 (both inlet and outlet temperature inside the interval) in an interval between the supply and target temperature, one gets the middle illustration in Figure 3.7. Here the stream supplies heat from T_k^H to t^{in} and then from t^{out} to T_{k+1}^H. The total heat supplied in that interval then becomes: $q_{sk}^H = MCP_s\left(T_k^H - t^{in}\right) f_s + MCP_s\left(t^{out} - T_{k+1}^H\right) f_s$. The same derivation was done to the 5 other cases in Figure 3.8 and then again for the intervals below the target temperature and above the supply temperature. The goal was again to find general equations which were valid for as many cases as possible. Table 3.8 shows the heating requirement for all the 6 cases for an interval



Figur 3.8: The different possible cases for compressor inlet and outlet temperatures

Case 1:	$q_{skm}^H = MCP_s f_{sm}($	T_k^H	-	t_{sm}^{in}	+	t_{sm}^{out}	-	T_{k+1}^H	)
Case 2:	$q_{skm}^H = MCP_s f_{sm}($	T_k^H	-	t_{sm}^{in}	+	T_k^H	-	T_{k+1}^H	)
Case 3:	$q_{skm}^H = MCP_s f_{sm}($	T_k^H	-	T_{k+1}^H	+	T_k^H	-	T_{k+1}^H	)
Case 4:	$q_{skm}^H = MCP_s f_{sm}($	T_k^H	-	T_{k+1}^H	+	t_{sm}^{out}	-	T_{k+1}^H	)
Case 5:	$q_{skm}^H = MCP_s f_{sm}($	T_k^H	-	T_k^H	+	T_k^H	-	T_{k+1}^H	)
Case 6:	$q_{skm}^H = MCP_s f_{sm}($	T_k^H	-	T_{k+1}^H	+	T_{k+1}^H	-	T_{k+1}^H	)

Tabell 3.8: q_{skm}^H-equations for intervals between T_s^S and T_s^T

between the supply and target temperature. The table shows that there is only the second and third term inside the parenthesis that changes for the different cases. The second term is dependent on the inlet temperature and the third term is dependent on the outlet temperature. So the second term can be changed out with $t_{skm}^{in,D}$ and the third term with $t_{skm}^{out,D}$. The general equation then becomes:

$$q_{skm}^{H} = MCP_s f_{sm} \left(T_k^H - t_{skm}^{in,D} + t_{skm}^{out,D} - T_{k+1}^H \right), s \in S^{CH}, k \in K_s^{HBTST}, m \in M_s \tag{3.38}$$

Case 1:	$q_{skm}^H = MCP_s f_{sm}($	T_k^H	-	t_{sm}^{in}	-	$[T_k^H$	-	t_{sm}^{out}	])
Case 2:	$q_{skm}^H = MCP_s f_{sm}($	T_k^H	-	t_{sm}^{in}	-	$[T_k^H$	-	T_k^H	])
Case 3:	$q_{skm}^H = MCP_s f_{sm}($	T_k^H	-	T_{k+1}^H	-	$[T_k^H$	-	T_k^H	])
Case 4:	$q_{skm}^H = MCP_s f_{sm}($	T_k^H	-	T_{k+1}^H	-	$[T_k^H$	-	t_{sm}^{out}	])
Case 5:	$q_{skm}^H = MCP_s f_{sm}($	T_k^H	-	T_k^H	-	$[T_k^H$	-	T_k^H	])
Case 6:	$q_{skm}^H = MCP_s f_{sm}($	T_k^H	-	T_{k+1}^H	-	$[T_k^H$	-	T_{k+1}^H	])

Tabell 3.9: q_{skm}^H-equations for intervals below T_s^T

Table 3.9 shows the heating requirement for all the 6 cases for an interval below the target temperature. The table shows that there is only the second and fourth term inside the parenthesis that changes for the different cases. The second term is dependent on the inlet temperature and the fourth term is dependent on the outlet temperature. So the second term can be changed out with $t_{skm}^{in,D}$ and the fourth term with $t_{skm}^{out,D}$. The general equation then becomes:

$$q_{skm}^{H} = MCP_s f_{sm} \left(T_k^H - t_{skm}^{in,D} - \left[T_k^H - t_{skm}^{out,D} \right] \right), s \in S^{CH}, k \in K_s^{HBTT}, m \in M_s \tag{3.39}$$

Case 1:	$q_{skm}^H = MCP_s f_{sm}($	t_{sm}^{out}	-	T_{k+1}^H	-	$[t_{sm}^{in}$	-	T_{k+1}^H	])
Case 2:	$q_{skm}^H = MCP_s f_{sm}($	T_k^H	-	T_{k+1}^H	-	$[t_{sm}^{in}$	-	T_{k+1}^H	])
Case 3:	$q_{skm}^H = MCP_s f_{sm}($	T_k^H	-	T_{k+1}^H	-	$[T_{k+1}^H$	-	T_{k+1}^H	])
Case 4:	$q_{skm}^H = MCP_s f_{sm}($	t_{sm}^{out}	-	T_{k+1}^H	-	$[T_{k+1}^H$	-	T_{k+1}^H	])
Case 5:	$q_{skm}^H = MCP_s f_{sm}($	T_k^H	-	T_{k+1}^H	-	$[T_k^H$	-	T_{k+1}^H	])=0
Case 6:	$q_{skm}^H = MCP_s f_{sm}($	T_{k+1}^H	-	T_{k+1}^H	-	$[T_{k+1}^H$	-	T_{k+1}^H	])=0

Tabell 3.10: q_{skm}^H-equations for intervals above T_s^S

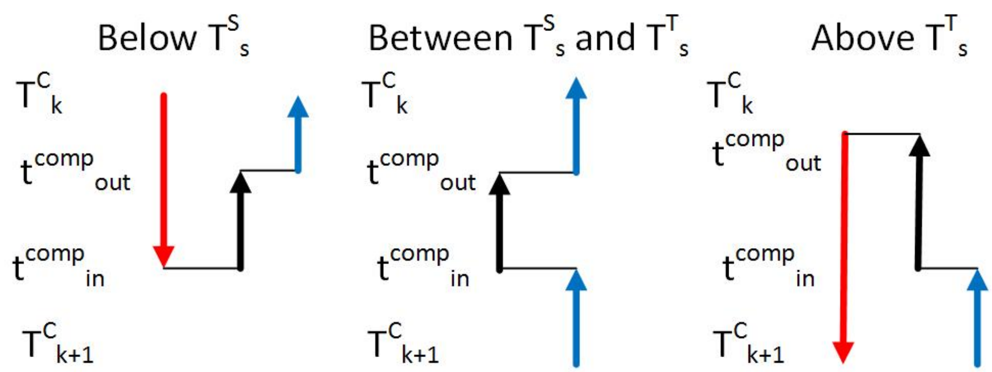
Table 3.10 shows the equation for the heat supply for all the 6 cases for an interval above the supply temperature. The table shows that there is only the first and third term inside the parenthesis that changes for the different cases. The first term is dependent on the outlet temperature and the third term is dependent on the inlet temperature. So the first term can be changed out with $t_{skm}^{out,D}$ and the third term with $t_{skm}^{in,D}$. The general equation then becomes:

$$q_{skm}^{H} = MCP_s f_{sm} \left(t_{skm}^{out,D} - T_{k+1}^{H} - \left[t_{skm}^{in,D} - T_{k+1}^{H} \right] \right), s \in S^{CH}, k \in K_s^{HATS}, m \in M_s \tag{3.40}$$

Equation (3.39) does contain T_k^H two times with the opposite sign. The two equation can therefore be simplified by removing both of the T_k^H terms. The same is true for equation (3.40), but now it is the T_{k+1}^H terms that can be removed. The two equations does now become identical and can be written as equation (3.41) instead.

$$q_{skm}^{H} = MCP_s f_{sm} \left(t_{skm}^{out,D} - t_{skm}^{in,D} \right), s \in S^{CH}, k \in K_s^{HBTT} \cup K_s^{HATS}, m \in M_s \tag{3.41}$$

3.3.8.5 Heat requirement for a cold stream with compression



Figur 3.9: Cold stream behavior for streams with compression

Figure 3.9 shows how a cold stream with compression will behave if both the inlet and outlet of the compressor is inside a heat cascade temperature interval. The left illustration shows the case where both the inlet and outlet temperature is inside a temperature interval below the supply temperature. Since the interval is located below the supply temperature will the stream enter the interval with a temperature of T_k^C. The stream will supply heat (red arrow) until it reaches the inlet of the compressor. The stream will then increase its temperature through the compressor (black arrow). The target temperature is above the interval, so the stream will be heated up (blue arrow) until it leaves the interval with a temperature of T_k^C.

The middle illustration shows the case where the inlet and outlet of the compressor is located inside an interval between the supply and target temperature. The stream enters the interval with a temperature of T_{k+1}^C and is then heated up (blue arrow) until it reaches the inlet of the compressor. The stream will then increase its temperature through the compressor (black arrow). The stream continues to be heated up (blue arrow) after the compressor until it leaves the interval with a temperature of T_k^C.

The right illustration shows the case where the inlet and outlet of the compressor is inside an interval above the target temperature. Since the interval is located above the target temperature will the stream enter the interval with a temperature of T_{k+1}^C. The stream will be heated up (blue arrow) until it reaches the inlet of the compressor. The stream will then increase its temperature through the compressor (black arrow). The supply temperature is located below the interval so the stream will now supply heat (red arrow) until it leaves the interval with a temperature of T_{k+1}^C.

Figure 3.9 does only show the case where both the inlet and outlet temperature of the compressor is inside the temperature interval, but the inlet and outlet temperature may be above or below the given interval. This makes 6 cases in total as shown in figure 3.8. By using the different diagrams in Figure 3.9 it is possible to find the heating requirement for each of the cases in Figure 3.8, which is done in Table 3.11, 3.12 and 3.13. For case 1 (both inlet and outlet temperature inside the interval) in an interval between the supply and target temperature one get the left illustration in Figure 3.9. Here the stream got a heat demand from T_{k+1}^C to t^{in} and then from t^{out} to T_k^C. The total heat demand in that interval then becomes: $q_{sk}^C = MCP_s\left(t^{in} - T_{k+1}^C\right) f_s + MCP_s\left(T_k^C - t^{out}\right) f_s$. The same derivation was done to the 5 other cases in Figure 3.8 and then again for the intervals below the supply temperature and above the target temperature. The goal was to find general equations which were valid for as many cases as possible.

Table 3.11 shows the heat demand for all the 6 cases for an interval below the supply temperature. The table shows that there is only the first and fourth term inside the parenthesis that changes for the different cases. The first term is dependent on the inlet temperature and the fourth term is dependent on the outlet temperature. So the first term can be changed out with

Case 1:	$q^C_{skm} = MCP_s f_{sm}($	t^{in}_{sm}	-	T^C_{k+1}	+	T^C_k	-	t^{out}_{sm}	)
Case 2:	$q^C_{skm} = MCP_s f_{sm}($	t^{in}_{sm}	-	T^C_{k+1}	+	T^C_k	-	T^C_k	)
Case 3:	$q^C_{skm} = MCP_s f_{sm}($	T^C_{k+1}	-	T^C_{k+1}	+	T^C_k	-	T^C_k	)
Case 4:	$q^C_{skm} = MCP_s f_{sm}($	T^C_{k+1}	-	T^C_{k+1}	+	T^C_k	-	t^{out}_{sm}	)
Case 5:	$q^C_{skm} = MCP_s f_{sm}($	T^C_k	-	T^C_{k+1}	+	T^C_k	-	T^C_k	)
Case 6:	$q^C_{skm} = MCP_s f_{sm}($	T^C_{k+1}	-	T^C_{k+1}	+	T^C_k	-	T^C_{k+1}	)

Tabell 3.11: q^C_{skm}-equations for intervals between T^T_s and T^S_s

$t^{in,D}_{skm}$ and the fourth term with $t^{out,D}_{skm}$. The general equation then becomes:

$$q^C_{skm} = MCP_s f_{sm}\left(t^{in,D}_{skm} - T^C_{k+1} + T^C_k - t^{out,D}_{skm}\right) \quad, s \in S^{CC}, k \in K^{CBTST}_s, m \in M_s \tag{3.42}$$

Case 1:	$q^C_{skm} = MCP_s f_{sm}($	T^C_k	-	t^{out}_{sm}	-	$[T^C_k$	-	t^{in}_{sm}	])
Case 2:	$q^C_{skm} = MCP_s f_{sm}($	T^C_k	-	T^C_k	-	$[T^C_k$	-	t^{in}_{sm}	])
Case 3:	$q^C_{skm} = MCP_s f_{sm}($	T^C_k	-	T^C_k	-	$[T^C_k$	-	T^C_{k+1}	])
Case 4:	$q^C_{skm} = MCP_s f_{sm}($	T^C_k	-	t^{out}_{sm}	-	$[T^C_k$	-	T^C_{k+1}	])
Case 5:	$q^C_{skm} = MCP_s f_{sm}($	T^C_k	-	T^C_k	-	$[T^C_k$	-	T^C_k	])=0
Case 6:	$q^C_{skm} = MCP_s f_{sm}($	T^C_k	-	T^C_{k+1}	-	$[T^C_k$	-	T^C_{k+1}	])=0

Tabell 3.12: q^C_{skm}-equations for intervals below T^S_s

Table 3.12 shows the heat demand for all the 6 cases for an interval below the supply temperature. The table shows that there is only the second and fourth term inside the parenthesis that changes for the different cases. The second term is dependent on the outlet temperature and the fourth term is dependent on the inlet temperature. So the second term can be changed out with $t^{out,D}_{skm}$ and the fourth term with $t^{in,D}_{skm}$. The general equation then becomes:

$$q^C_{skm} = MCP_s f_{sm}\left(T^C_k - t^{out,D}_{skm} - \left[T^C_k - t^{in,D}_{skm}\right]\right) \quad, s \in S^{CC}, k \in K^{CBTS}_s, m \in M_s \tag{3.43}$$

Table 3.13 shows the heat demand for all the 6 cases for an interval above the target temperature. The table shows that there is only the first and third term inside the parenthesis that changes for the different cases. The first term is dependent on the inlet temperature and the

Case 1:	$q^C_{skm} = MCP_s f_{sm}($	t^{in}_{sm}	-	T^C_{k+1}	-	$[t^{out}_{sm}$	-	T^C_{k+1}	])
Case 2:	$q^C_{skm} = MCP_s f_{sm}($	t^{in}_{sm}	-	T^C_{k+1}	-	$[T^C_{k}$	-	T^C_{k+1}	])
Case 3:	$q^C_{skm} = MCP_s f_{sm}($	T^C_{k+1}	-	T^C_{k+1}	-	$[T^C_{k}$	-	T^C_{k+1}	])
Case 4:	$q^C_{skm} = MCP_s f_{sm}($	T^C_{k+1}	-	T^C_{k+1}	-	$[t^{out}_{sm}$	-	T^C_{k+1}	])
Case 5:	$q^C_{skm} = MCP_s f_{sm}($	T^C_{k}	-	T^C_{k+1}	-	$[T^C_{k}$	-	T^C_{k+1}	])=0
Case 6:	$q^C_{skm} = MCP_s f_{sm}($	T^C_{k+1}	-	T^C_{k+1}	-	$[T^C_{k+1}$	-	T^C_{k+1}	])=0

Tabell 3.13: q^C_{skm}-equations for intervals above T^T_s

third term is dependent on the outlet temperature. So the second first can be changed out with $t^{in,D}_{skm}$ and the third term with $t^{out,D}_{skm}$. The general equation then becomes:

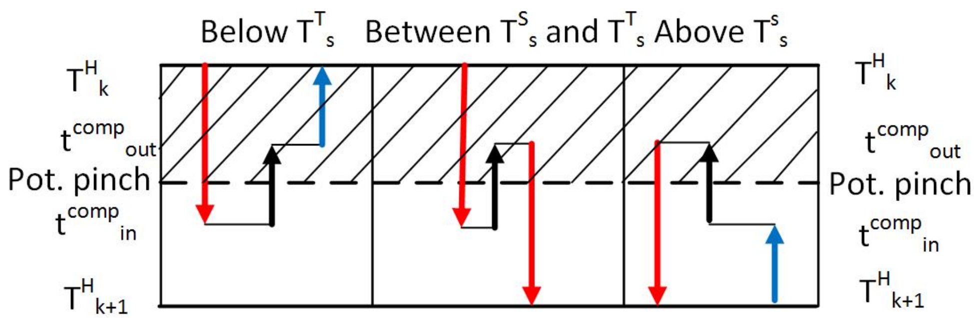
$$q^C_{skm} = MCP_s f_{sm}\left(t^{in,D}_{skm} - T^C_{k+1} - \left[t^{out,D}_{skm} - T^C_{k+1}\right]\right) \quad , s \in S^{CC}, k \in K^{CATT}_s, m \in M_s \tag{3.44}$$

Equation (3.43) does contain T^C_k two times with the opposite sign. The two equation can therefore be simplified by removing both of the T^C_k terms. The same is true for (3.44), but now it is the T^C_{k+1} terms that can be removed. The two equations does now become identical and can be written as equation (3.45) instead.

$$q^C_{skm} = MCP_s f_{sm}\left(t^{in,D}_{skm} - t^{out,D}_{skm}\right) \quad , s \in S^{CC}, k \in K^{CBTS}_s \cup K^{CATT}_s, m \in M_s \tag{3.45}$$

3.3.8.6 Heat residual for the outlet of a compressor

The outlet of the compressor is the beginning of a stream segment and is therefore a potential pinch point. Section 3.3.7 showed the derivation of the equations needed to find heat residual from the outlet of an turbine and similar equations must be derived for the outlet of a compressor. Figure 3.10 shows how a hot stream with compression may behave inside a temperature interval. The left illustration shows the case where both the inlet and outlet temperature of the compression is located inside an interval below the target temperature. The stream enters the interval with a temperature of T^H_k and supplies heat (red arrow) until it reaches the inlet of the compressor. Heat is supplied from T^H_k to the maximum of the potential pinch point (black dashed line), $t^{out,D}_{skm}$, and the inlet of the compressor, $t^{in,D}_{ykn}$. The heat supplied above the potential pinch point does then become $MCP_y\left(T^H_k - \max\left(t^{in,D}_{ykn}, t^{out,D}_{skm}\right)\right)$. $t^{in,D}_{ykn}$ is the inlet tem-



Figur 3.10: Behavior of a hot stream with compression and a potential pinch point inside a temperature interval

perature of a compressor from the same or another stream y, split n. Then the temperature increases during the compression (black arrow), before the stream is heated up (blue arrow) to T_k^H. The heating requirement above the potential pinch point becomes T_k^H minus the maximum of the potential pinch point (black dashed line), $t_{skm}^{out,D}$, and the outlet of the compressor, $t_{ykn}^{out,D}$: $MCP_y\left(T_k^H - \max\left(t_{ykn}^{out,D}, t_{skm}^{out,D}\right)\right)$. The general equation then becomes:

$$q_{skm}^{HP} = \sum_{y \in S^{CH}} \sum_{n \in M_y} MCP_y f_{yn}\left(\max\left(t_{ykn}^{in,D}, t_{skm}^{out,D}\right) - \max\left(t_{ykn}^{out,D}, t_{skm}^{out,D}\right)\right) \qquad s \in S^{CH} \cup s^{CC}, k \in K_s^{HBTT}, m \in M_s \tag{3.46}$$

The middle illustration in Figure 3.10 shows a hot stream with both inlet and outlet temperature is located inside a temperature interval between the supply and target temperature may behave. The stream enter the interval from above and supplies heat (red arrow) until it reaches the inlet of the compressor. The heat supplied above the potential pinch point (black dashed line) then becomes $MCP_y\left(T_k^H - \max\left(t_{ykn}^{in,D}, t_{skm}^{out,D}\right)\right)$. The temperature increases through the compressor (black arrow) and the stream continues to supply heat (red arrow) until it leaves the interval with a temperature of T_{k+1}^H. Only the heat supplied above the potential pinch point is of interest, so the stream supplies heat from the maximum of the outlet temperature and the potential pinch point to the potential pinch point. The heat supplied then becomes: $MCP_y\left(\max\left(t_{ykn}^{out,D}, t_{skm}^{out,D}\right) - t_{skm}^{out,D}\right)$. The general equation becomes:

$$q_{skm}^{HP} = \sum_{y\in S^{CH}} \sum_{n\in M_y} MCP_y f_{yn} \left(T_K^H - \max\left(t_{ykn}^{in,D}, t_{skm}^{out,D}\right) + \max\left(t_{ykn}^{out,D}, t_{skm}^{out,D}\right) - t_{skm}^{out,D}\right)$$
$$s \in S^{CH} \cup s^{CC}, k \in K_s^{HBTST}, m \in M_s \tag{3.47}$$

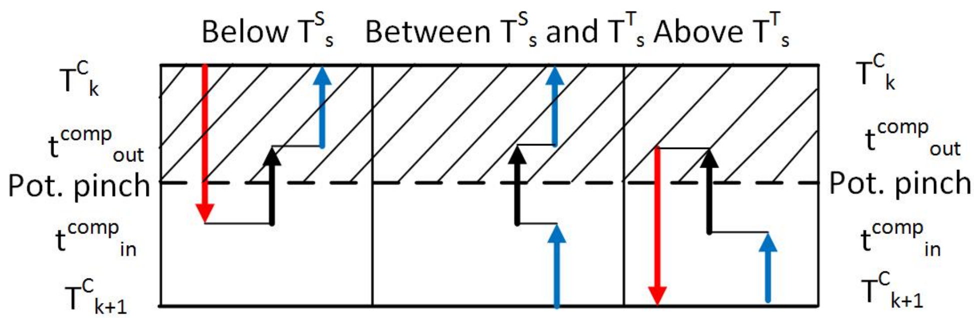
The right illustration in Figure 3.10 shows a hot stream with both inlet and outlet temperature is located inside a temperature interval above the supply temperature may behave. The stream enter the interval from below and is heated up (blue arrow) until it reaches the inlet of the compressor. The heating requirement above the potential pinch point (black dashed line) then becomes $MCP_y\left(\max\left(t_{ykn}^{in,D}, t_{skm}^{out,D}\right) - t_{skm}^{in,D}\right)$. The temperature increases through the compressor (black arrow) and the stream supplies heat (red arrow) until it leaves the interval with a temperature of T_{k+1}^H. Only the heat supplied above the potential pinch point is of interest, so the stream supplies heat from the maximum of the outlet temperature and the potential pinch point to the potential pinch point. The heat supplied then becomes: $MCP_y\left(\max\left(t_{ykn}^{out,D}, t_{skm}^{out,D}\right) - t_{skm}^{out,D}\right)$. The general equation becomes:

$$q_{skm}^{HP} = \sum_{y\in S^{CH}} \sum_{n\in M_y} MCP_y f_{yn} \left(\max\left(t_{ykn}^{out,D}, t_{skm}^{out,D}\right) - \max\left(t_{ykn}^{in,D}, t_{skm}^{out,D}\right)\right)$$
$$s \in S^{CH} \cup s^{CC}, k \in K_s^{HATS}, m \in M_s \tag{3.48}$$

Equation (**??**) and (3.48) are in fact identical and can be merged together to one equation:

$$q_{skm}^{HP} = \sum_{y\in S^{CH}} \sum_{n\in M_y} MCP_y f_{yn} \left(\max\left(t_{ykn}^{out,D}, t_{skm}^{out,D}\right) - \max\left(t_{ykn}^{in,D}, t_{skm}^{out,D}\right)\right)$$
$$s \in S^{CH} \cup s^{CC}, k \in K_s^{HATS} \cup K_s^{HBTT}, m \in M_s \tag{3.49}$$

Figure 3.11 shows how a cold stream with expansion may behave inside a temperature interval. The left illustration shows the case where both the inlet and outlet temperature of the compressor is located inside an interval below the supply temperature. The stream enters the interval with a temperature of T_k^C and supplies heat (red arrow) until it reaches the inlet of the compressor. Heat is supplied from T_k^C to the maximum of the potential pinch point (black dashed line), $t_{skm}^{out,D}$, and the inlet of the compressor, $t_{ykn}^{in,D}$. The heat supplied above the potential pinch



Figur 3.11: Behavior of a cold stream with compression and a potential pinch point inside a temperature interval

point does then become $MCP_y\left(T_k^C - \max\left(t_{ykn}^{in,D}, t_{skm}^{out,D}\right)\right)$. $t_{ykn}^{in,D}$ is the inlet temperature of a compressor from the same or another stream y, split n. Then the temperature drops during the compression (black arrow), before the stream is heated up (blue arrow) to T_k^C. The heating requirement above the potential pinch point for the heating becomes T_k^C minus the maximum of the pinch point, $t_{skm}^{out,D}$, and the outlet of the compressor, $t_{ykn}^{out,D}$: $MCP_y\left(T_k^C - \max\left(t_{ykn}^{out,D}, t_{skm}^{out,D}\right)\right)$. The general equation then becomes:

$$q_{skm}^{CP} = \sum_{y \in S^{CC}} \sum_{n \in M_y} MCP_y f_{yn}\left(\max\left(t_{ykn}^{out,D}, t_{skm}^{out,D}\right) - \max\left(t_{ykn}^{in,D}, t_{skm}^{out,D}\right)\right)$$
$$s \in S^{CC} \cup s^{CH}, k \in K_s^{HBTT}, m \in M_s \tag{3.50}$$

The middle illustration in Figure 3.11 shows how a cold stream with both the inlet and outlet temperature located inside a temperature interval between the supply and target temperature may behave. The stream enter the interval from below and is heat up (blue arrow) until it reaches the inlet of the compressor. The heating requirement above the potential pinch point then becomes $MCP_y\left(\max\left(t_{ykn}^{in,D}, t_{skm}^{out,D}\right) - t_{skm}^{out,D}\right)$. The temperature increases through the compressor (black arrow) and the stream continues to be heated up (blue arrow) until it leaves the interval with a temperature of T_k^C. Only the heating requirement above the potential pinch point is of interest, so the stream is heated from the maximum of the outlet temperature and the potential pinch point to T_k^C. The heat requirement for this part then becomes: $MCP_y\left(\max\left(T_k^C - t_{ykn}^{out,D}, t_{skm}^{out,D}\right)\right)$. The general equation becomes:

$$q^{CP}_{skm} = \sum_{y \in S^{CC}} \sum_{n \in M_y} MCP_s f_{yn} \left(T^C_K - \max\left(t^{out,D}_{ykn}, t^{out,D}_{skm}\right) + \max\left(t^{in,D}_{ykn}, t^{out,D}_{skm}\right) - t^{out,D}_{skm} \right)$$
$$s \in S^{CC} \cup s^{CH}, k \in K^{HBTST}_s, m \in M_s \tag{3.51}$$

The right illustration in Figure 3.11 shows a cold stream with both inlet and outlet temperature is located inside a temperature interval above the target temperature may behave. The stream enter the interval from below and is heated up (blue arrow until it reaches the inlet of the compressor. The heating requirement above the potential pinch point (black dashed line) then becomes $MCP_y \left(\max\left(t^{in,D}_{ykn}, t^{out,D}_{skm}\right) - t^{in,D}_{skm}\right)$. The temperature increases through the compressor (black arrow) and then the stream supplies heat (red arrow) until it leaves the interval with a temperature of T^C_{k+1}. Only the heat supplied above the potential pinch point is of interest, so the stream supplies heat from the maximum of the outlet temperature and the potential pinch point to the potential pinch point. The heat supplied then becomes: $MCP_y \left(\max\left(t^{out,D}_{ykn}, t^{out,D}_{skm}\right) - t^{out,D}_{skm}\right)$. The general equation becomes:

$$q^{CP}_{skm} = \sum_{y \in S^{CC}} \sum_{n \in M_y} MCP_s f_{yn} \left(\max\left(t^{out,D}_{ykn}, t^{out,D}_{skm}\right) - \max\left(t^{in,D}_{ykn}, t^{out,D}_{skm}\right) \right)$$
$$s \in S^{CC} \cup s^{CH}, k \in K^{HBTT}_s, m \in M_s \tag{3.52}$$

Equation (3.50) and (3.52) are in fact and can be merged together to one equation:

$$q^{CP}_{skm} = \sum_{y \in S^{CC}} \sum_{n \in M_y} MCP_s f_{yn} \left(\max\left(t^{out,D}_{ykn}, t^{out,D}_{skm}\right) - \max\left(t^{in,D}_{ykn}, t^{out,D}_{skm}\right) \right)$$
$$s \in S^{CC} \cup s^{CH}, k \in K^{HBTT}_s \cup K^{HATS}_s, m \in M_s \tag{3.53}$$

Comparing all the new equations for streams with compression with the equations for expansion shows that they are in fact identical. At first this might seem a bit strange since compression adds heat and expansion adds cooling. Compression does also increases the temperature of the stream while expansion does the opposite. Since compression increases the temperature will the temperature difference through the compressor have an opposite sign than the difference through a turbine. It can therefore make sense that the same equations can be used for both compression and expansion. In the complete version of the model in Appendix A.1 have the all the equations for expansion and compression, with the exception of equation (3.20) and (3.35), been combined. Another discovery is that the introduction of r_{skm}^{out} and its accompanying equations actually makes r_k and its accompanying equations obsolete. r_{skm}^{out} vil for all temperature intervals except for one for each stream split part m, be equal to either r_k or r_{k+1}. The reason why r_k and its accompanying equations will not be removed is that r_{skm}^{out} is heavy to compute. It contains a lot non linearities and will do the same calculations several times. It is also only on very rare occasions that r_{skm}^{out} will be more binding that r_k. The whole reason for including r_{skm}^{out} was handle the cases where a new pinch point is located at the outlet of a pressure changing unit. This does not happen often, so it is a better solution to let r_{skm}^{out} be a free variable and only force it to be non-negative if it actually is negative for a case. All the test cases have first been solved with r_{skm}^{out} being a free variable and then again with r_{skm}^{out} being non-negative if one of the r_{skm}^{out} variables were negative.

3.3.9 Below ambient temperature

The difference between above and below ambient situations is that the formula for exergy changes, which in turn makes the cold utility more valuable than the hot utility. From an energy perspective there is not any difference between above and below ambient conditions. Both cases usually have both hot and cold utility consumption. In the case with above ambient conditions is usually the cold utility temperature around the ambient temperature, which makes the cold utility exergy content negligible. With below ambient conditions however are the tables turned. Now the hot utility temperature will be close to the ambient temperature and have negligible exergy content. So for cases with below ambient temperatures it is actually the cold utility consumption that is of interest. The objective function needs another term for the exergy content of the cold utility. The cold utility consumption in temperature interval k from source u, q_{ku}^{CU}, is multiplied with the below ambient exergy factor for the source.

$$\begin{aligned}
&\min \sum_{k\in K}\sum_{u\in U_k^H}\left(1-\frac{T_0}{T_u^{HU}}\right)q_{ku}^{HU}+\sum_{k\in K}\sum_{u\in U_k^C}\left(\frac{T_0}{T^{CU}}-1\right)q_{ku}^{CU}\\
&-\sum_{s\in S^{EC}\cup S^{EH}}\sum_{m\in M_s}\eta_s MCP_s f_{sm}\left(t_{sm}^{in}-t_{sm}^{out}\right)-\sum_{s\in S^{CC}\cup S^{CH}}\sum_{m\in M_s}\eta_s^{-1} MCP_s f_{sm}\left(t_{sm}^{in}-t_{sm}^{out}\right)
\end{aligned} \tag{3.54}$$

To find and calculate the cold utility consumption there also have to be made some changes to the heat balance equations. Equation (3.56) is the general equation and is valid for all the temperature intervals except the first and last one. The change made here was to add the sum of all q_{ku}^{CU} on the left side of the equal sign. The first interval (equation 3.55) needs its own equation since heat residual r_0 does not exist. Earlier it had been assumed that there was only one source of cold utility, so heat residual $r_{|K|}$ was used find the cold utility consumption. This assumption was made since the cold utility exergy content was negligible for above ambient conditions. Now that the cold utility consumption have to be calculated for different sources, it is necessary to make a unique equation (3.57) for the last interval without the $r_{|K|}$-variable. All the cold utility consumption would have ended up in that variable instead of the q_{ku}^{CU}-variables because it is not present in the objective function.

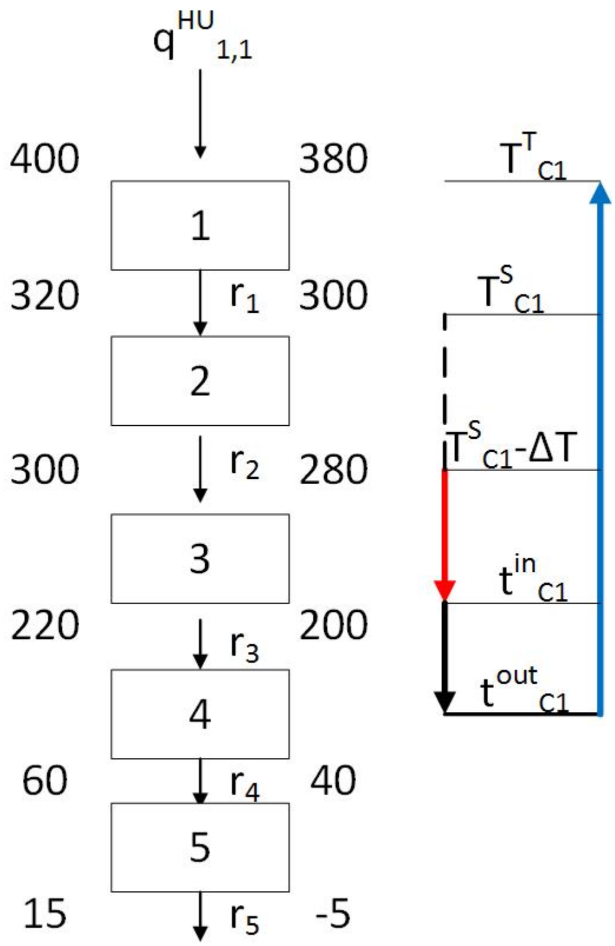
$$r_1+\sum_{s\in S^{EC}\cup S^{CC}}\sum_{m\in M_s}q_{s,1,m}^{C}-\sum_{s\in S^{EH}\cup S^{CH}}\sum_{m\in M_s}q_{s,1,m}^{H}-\sum_{u\in U_1^H}q_{1,u}^{HU}=\sum_{s\in S^{HWP}}Q_{s,1}^{H}-\sum_{s\in S^{CWP}}Q_{s,1}^{C} \tag{3.55}$$

$$\begin{gathered}
r_k-r_{k-1}+\sum_{s\in S^{EC}\cup S^{CC}}\sum_{m\in M_s}q_{skm}^{C}-\sum_{s\in S^{EH}\cup S^{CH}}\sum_{m\in M_s}q_{skm}^{H}-\sum_{u\in U_k^H}q_{ku}^{HU}+\sum_{u\in U_k^C}q_{ku}^{CU}=\sum_{s\in S^H}Q_{sk}^{H}-\sum_{s\in S^C}Q_{sk}^{C}\\
k\in K\setminus\{1,|K|\}
\end{gathered} \tag{3.56}$$

$$-r_{|K|-1}+\sum_{s\in S^{EC}\cup S^{CC}}\sum_{m\in M_s}q_{s,|K|,m}^{C}-\sum_{s\in S^{EH}\cup S^{CH}}\sum_{m\in M_s}q_{s,|K|,m}^{H}+\sum_{u\in U_{|K|}^C}q_{|K|,u}^{CU}=\sum_{s\in S^H}Q_{s,|K|}^{H}-\sum_{s\in S^C}Q_{s,|K|}^{C} \tag{3.57}$$

3.3.10 Special temperature intervals

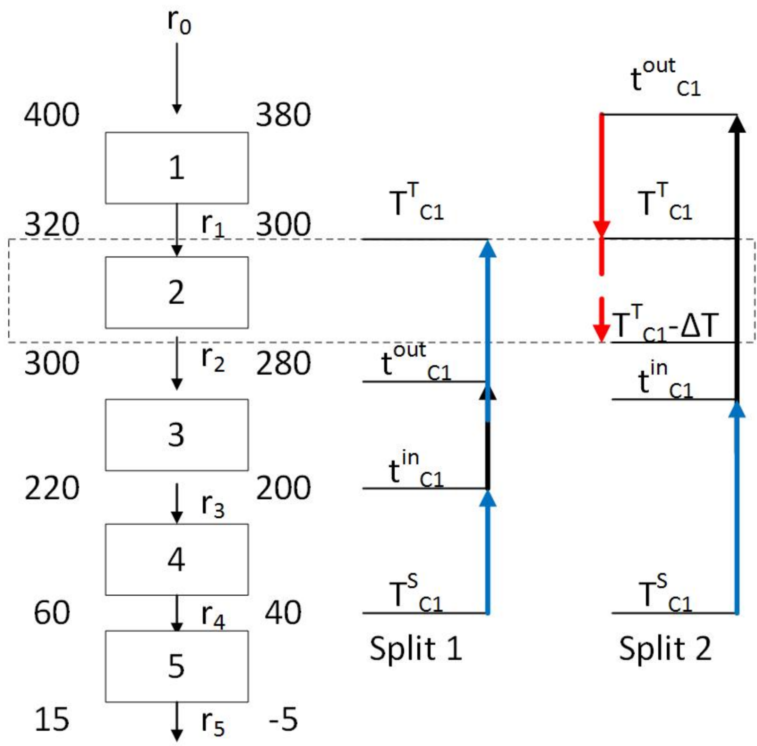
The fact that there has to be a minimum temperature difference when heat is transferred between two streams led to the heat cascade having two sides. One for hot streams and one for cold streams. This is not problematic for streams without pressure change, since these streams will not change identity throughout the heat cascade. When streams change identity is it important to be aware at which temperature the stream can start to supply/receive heat from. Figure 3.12 illustrates one of the challenges that occurs when a stream changes identity. The stream starts at 300°C and supplies heat (red arrow) until it reaches the inlet of the turbine with a temperature of 220°. The temperature drops through the turbine (black arrow) and it receives heat (blue arrow) until it reaches the target temperature of 380°. Although the supply temperature is 300°C, can the stream not supply heat to another stream in the interval 300-280°C (cold side of the heat cascade) because of the required temperature difference. If both inlet and outlet temperatures are located above the supply temperature, then will temperature interval 2 be like a regular interval below the supply temperature and equation (3.45) can be used. If one or both are located below, like in this case, will the interval be like an interval between the supply and target temperature instead. Then equation (3.42) should be used instead. In the project thesis this was wrongly solved by using equation (3.42) for the interval below the supply temperature. This gave the correct result for the one test case which had expansion of a cold stream, but would have given a wrong result if there had been a test case where the stream did not temporarily changed identity. A similar problem arose for a test case with compression as seen in Figure 3.13. Here a cold stream was split into two parts with one compressor each. For the first split was the compressor located between the supply and target temperature, so there is no identity change. For the second split was the compressor outlet located above the target temperature, which means that the stream will temporarily act as an hot stream between the outlet and the target temperature. An error arose in the second temperature interval for the second split. The interval is between the supply and target temperature and equation (3.42) should be used. This is correct for split 1, but is actually wrong for the second split. The second split is temporarily hot when it reaches the target temperature. This means that the hot side of the heat cascade must be used and that the stream should end at the target temperature on the hot side. The red dashed line have been added to illustrate this. So temperature interval 2 is actually above the target temperature for the second case, which means that equation (3.45) should be used instead.



Figur 3.12: Behavior of a cold stream with expansion and identity change

Both the temperature interval directly below the supply and below the target temperature needs a new general equation which handles the different scenarios correctly. The different cases which needs a new general equation are:

1. Cold stream with expansion/compression: The temperature interval below T^S
2. Cold stream with expansion/compression: The temperature interval below T^T
3. Hot stream with expansion/compression: The temperature interval above below T^S
4. Hot stream with expansion/compression: The temperature interval above below T^T



Figur 3.13: Illustration of the special case

3.3.10.1 Cold stream with expansion/compression: The temperature interval below T^S

Here equation (3.42) should be used if the inlet of the pressure changing unit is above T^S and equation (3.45) for the opposite case. The difference between the equations is that equation (3.42) got an extra (T_k^C-T_{k+1}^C) term. Since the variable a_{skm}^{in} can tell whether the inlet of the turbine/compressor is located above, inside or below a given temperature interval, it can be used together with (T_k^C-T_{k+1}^C) to develop a general equation. The term (T_k^C-T_{k+1}^C) should only be present when the turbine/compressor is located below the supply temperature. a_{skm}^{in} is equal to zero when this is true. The general equation then becomes:

$$q_{skm}^C = MCP_s f_{sm} \left(t_{skm}^{in,D} - t_{skm}^{out,D} + \left(1 - a_{skm}^{in}\right)\left(T_k^C - T_{k+1}^C\right)\right) \quad , s \in S^{CC} \cup S^{EC}, k \in K_s^{CTS}, m \in \ M_s \tag{3.58}$$

3.3.10.2 Cold stream with expansion/compression: The temperature interval below T^T

Here equation (3.42) should be used if the outlet of the pressure changing unit is below T^T and equation (3.45) for the opposite case. The difference between the equations is that equation (3.42) got an extra (T_k^C-T_{k+1}^C) term. Since the variable a_{skm}^{out} can tell whether the outlet of the turbine/compressor is located above, inside or below a given temperature interval, it can be used together with (T_k^C-T_{k+1}^C) to develop a general equation. The term (T_k^C-T_{k+1}^C) should only be present when the turbine/compressor is located below the target temperature. a_{skm}^{out} is equal to zero when this is true. The general equation then becomes:

$$q_{skm}^C = MCP_s f_{sm}\left(t_{skm}^{in,D} - t_{skm}^{out,D} + \left(1 - a_{skm}^{out}\right)\left(T_k^C - T_{k+1}^C\right)\right) \quad , s \in S^{CC} \cup S^{EC}, k \in K_s^{CTT}, m \in M_s \tag{3.59}$$

3.3.10.3 Hot stream with expansion/compression: The temperature interval above T^S

Here equation (3.38) should be used if the inlet of the pressure changing unit is above T^S and equation (3.40) for the opposite case. The difference between the equations is that equation (3.38) got an extra (T_k^H-T_{k+1}^H) term. Since the variable a_{skm}^{in} can tell whether the inlet of the turbine/compressor is located above, inside or below a given temperature interval, it can be used together with (T_k^H-T_{k+1}^H) to develop a general equation. The term (T_k^H-T_{k+1}^H) should only be present when the turbine/compressor is located above the supply temperature. a_{skm}^{in} is equal to zero when this is true. The general equation then becomes:

$$q_{skm}^H = MCP_s f_{sm}\left(t_{skm}^{in,D} - t_{skm}^{out,D} + a_{skm}^{in}\left(T_k^C - T_{k+1}^C\right)\right) \quad , s \in S^{CH} \cup S^{EH}, k \in K_s^{HTS}, m \in M_s \tag{3.60}$$

3.3.10.4 Hot stream with expansion/compression: The temperature interval above T^T

Here equation (3.38) should be used if the outlet of the pressure changing unit is above T^T and equation (3.40) for the opposite case. The difference between the equations is that equation (3.38) got an extra (T_k^H-T_{k+1}^H) term. Since the variable a_{skm}^{out} can tell whether the outlet of the turbine/compressor is located above, inside or below a given temperature interval, it can be used together with (T_k^H-T_{k+1}^H) to develop a general equation. The term (T_k^H-T_{k+1}^H) should only be present when the turbine/compressor is located above the target temperature. a_{skm}^{out} is equal to zero when this is true. The general equation then becomes:

$$q^{H}_{skm} = MCP_s f_{sm} \left(t^{in,D}_{skm} - t^{out,D}_{skm} + a^{out}_{skm} \left(T^C_k - T^C_{k+1} \right) \right) \quad , s \in S^{CH} \cup S^{EH}, k \in K^{HTT}_s, m \in M_s \quad (3.61)$$

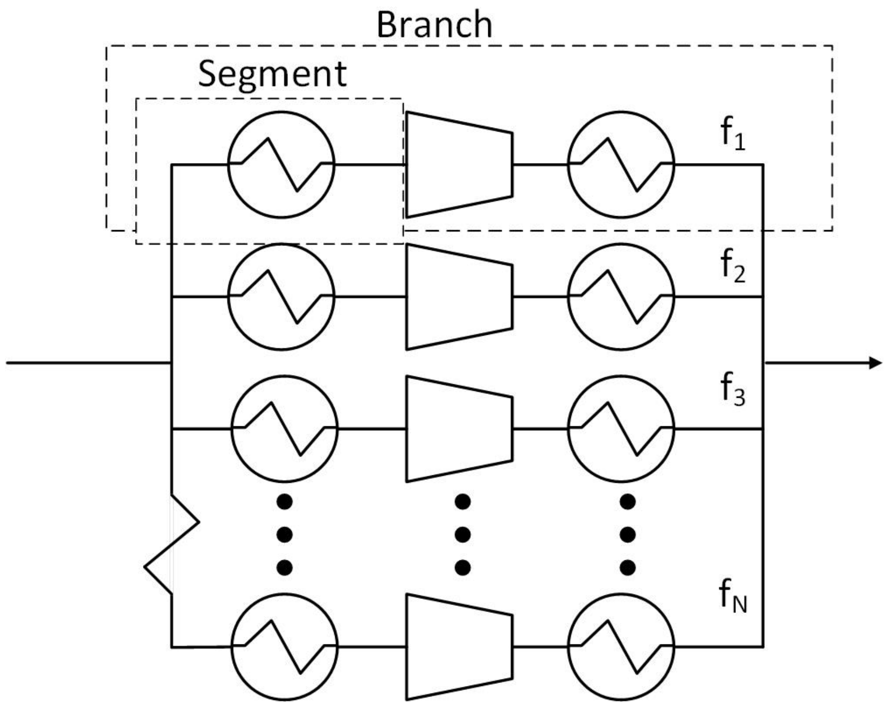
3.4 Model with the use of insight

3.4.1 Intro

Fu and Gundersen (2015a) contains theorems regarding the optimal placement of pressure changing units in a heat and work exchange network. The theorems state that the optimal placement is either at the the pinch point, at hot utility temperature or ambient temperature. This means that there is no need for the inlet/outlet temperatures of the pressure changing units to be continuous variables. The feasible region is drastically reduced by turning the continuous temperatures into only a few discrete choices. If one fixes the inlet/outlet temperature of a stream, it can be split into two segments with fixed inlet and outlet temperatures. The original stream with pressure manipulation might be split into several branches with one pressure manipulating unit for each branch, so the MCP going through each branch will have to be decided. The MCP is the only variable, so the equations used to calculate the heat heat demand/supply for each stream segment in each temperature interval become linear. The integral properties of the old model does also disappear, which turns the problem into a LP.

3.4.2 Splitting the streams

Figure 3.14 shows a new superstructure for the splitting of streams into branches and segments. Each stream with pressure manipulation will be split into branches with different inlet and outlet temperatures for the pressure manipulating unit. The unit will divide each branch into two segments. The first segment will have the same supply temperature as the original stream and the target temperature will be the inlet temperature of the unit. The supply temperature of the second segment is the outlet temperature of the unit and the target temperature will be the target temperature of the original stream. The identity of each stream segments is decided by the relative position of the supply and target temperature. This method will only work if the streams are split into the correct branches; Each stream with pressure manipulation needs branches where the inlet to the pressure changing unit is at hot or cold utility temperature, pinch temperature or ambient temperature. The introduction of pressure changing units into a heat exchanger network may introduce new pinch points or change the pinch point. In some cases



Figur 3.14: Superstructure for the splitting of the streams with pressure manipulation

it will be optimal to place pressure changing units starting at these new pinch points, so extra branches are needed. Fortunately it is possible to locate these new pinch points without too much trouble. New pinch points will always be located at the supply temperature of a stream. This means that each of the original supply temperatures is a potential pinch point. The pressure changing units that are already placed at hot/cold utility, pinch or ambient temperatures will have stream segments starting at the outlet temperature of the unit. The supply temperature of these stream segments may also become new pinch points, so new branches needs to be included to take care of these cases as well. Due to the cooling effect of expansion and the deficit of heat below the pinch, an expander should be placed below the pinch and vice versa for a compressor. This means that there is no need to include the branches where expansion occurs above the original pinch and branches where compression occurs below the pinch. The number of branches is almost halved by using this insight.

3.4.3 Set, indices, parameters and variables

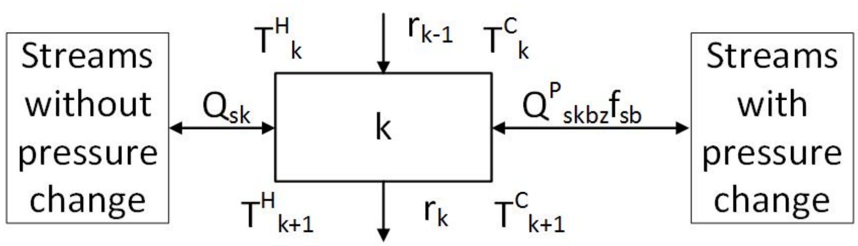
3.4.4 Objective function

The objective is to minimize the exergy usage of the heat and work exchange network. It can be expressed as

$$\min \sum_{k\in K}\sum_{u\in U_k^H}\left(1-\frac{T_0}{T_u^{HU}}\right)q_{ku}^{HU}+\sum_{k\in K}\sum_{u\in U_k^C}\left(\frac{T_0}{T^{CU}}-1\right)q_{ku}^{CU} + \sum_{s\in S^P}\sum_{b\in B_s}T_{sb}^{\Delta}MCP_{sb}f_{sb}^{B} \tag{3.62}$$

Her the first term is the hot utility exergy, which is the hot utility added to the network multiplied with the Carnot factor. The second term is work needed or produced by the pressure changing units. Here each stream with pressure manipulation have been split into branches with one pressure manipulation unit at each branch. The work added to/produced by each unit is calculated as the temperature difference between the inlet and outlet for each unit multiplied with the MCP and the fraction of the stream going through the branch.

3.4.5 Heat balance



Figur 3.15: Illustration of the heat balance for a temperature interval

Heat must always be transferred from a higher temperature to a lower temperature. It is also required to be a minimum temperature difference between the hot and the cold temperature. To make sure of this, a heat cascade is created with several temperature intervals. By using a heat balance at each interval the heat residuals r_k can be calculated. So if r_k is forced to be non-negative, we know for sure that the heat transfer is thermodynamically feasible and that it is possible to design a corresponding heat and work exchange network. The heat balance for a temperature interval can be derived from Figure 3.15. The heat residual going out of an interval is equal to the heat residual coming in plus the heat added and subtracted by all the streams passing through the interval. Q^P_{skbz} and Q_{sk} are the heat supplied to/from streams with and without pressure manipulation, respectively. Q^P_{skbz} and Q_{sk} will be positive if the stream is hot, negative if the stream is cold and 0 for both if the temperature interval is not between the supply and target temperature. Q^P_{skbz} and Q_{sk} are calculated in the preprocessing step by multiplying the MCP with the temperature difference in the interval for all the intervals between the supply and target temperature. Equation (3.63)-(3.65) is the heat balance for a temperature interval and equation (3.66) forces the residuals to be non-negative and thus thermodynamical feasible.

$$r_1 - \sum_{u \in U^H_k} q^{HU}_{1,u} - \sum_{s \in S^P} \sum_{b \in B_s} \sum_{z \in Z_{sb}} Q^P_{s,1,bz} f^B_{sb} = \sum_{s \in S \setminus S^P} Q_{s,1} \tag{3.63}$$

$$r_k - r_{k-1} - \sum_{u \in U^H_k} q^{HU}_{ku} + \sum_{u \in U^C_k} q^{CU}_{ku} - \sum_{s \in S^P} \sum_{b \in B_s} \sum_{z \in Z_{sb}} Q^P_{skbz} f^B_{sb} = \sum_{s \in S \setminus S^P} Q_{sk} \quad , k \in K \setminus \{1, |K|\} \tag{3.64}$$

$$-r_{|K|-1} + \sum_{u \in U^C_k} q^{CU}_{|K|,u} - \sum_{s \in S^P} \sum_{b \in B_s} \sum_{z \in Z_{sb}} Q^P_{s,|K|,bz} f^B_{sb} = \sum_{s \in S \setminus S^P} Q_{s,|K|} \tag{3.65}$$

$$r_k \geq 0 \quad , k \in K \tag{3.66}$$

3.4.6 Stream split and mass balance

To make sure that the mass balance is maintained, it is necessary to force the sum of fractions to be 1. This is done by equation (3.67). Equation(3.68) is added to make sure that the stream flows in the correct direction. A negative f_{sb} would mean that a turbine would work as a compressor and increase the pressure instead of dropping it. This must be avoided since the outlet temperatures and thus the supply temperatures of the second segment for each branch would be wrong.

$$\sum_{b \in B_s} f^B_{sb} = 1 \quad , s \in S^P \tag{3.67}$$

$$f^B_{sb} \geq 0 \quad , s \in S^P, b \in B_s \tag{3.68}$$

3.4.7 Locating the pinch point with minimum utility model

The minimum utility model developed underneath is used to locate the pinch point and to double check results from the model. It is similar, but simpler than the two other models. This model assumes that there are no pressure changes. This means that all streams goes continuously from supply to target temperature. The identity and MCP flow does not change. This makes it easy to calculate the heat supply/demand in each interval for each stream. Equation (3.4) and (**??**) are used to do the calculations. The objective for this model is to minimize the hot utility consumption. The only restriction is that the heat residual from a temperature interval must be non-negative, so a heat balance is performed for each interval to calculate the heat residuals.

$$\min \sum_{U \in U^H_1} q^{HU}_{ku} \tag{3.69}$$

$$r_k - r_{k-1} = \sum_{s \in S^H} Q^H_{sk} - \sum_{s \in S^C} Q^C_{sk} \quad , k \in K \tag{3.70}$$

$$r_k \geqslant 0, k \in K \tag{3.71}$$

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Implementation

4.1 Global optimization

The problem is a MINLP, which makes it hard to optimize. The nonlinearities causes non convexities which causes most solvers to get stuck in local optima. It is therefore hard to prove optimality. The discrete variables does also make the problem more challenging to solve. It is therefore necessary to use global optimization software to solve this problem. GAMS (General Algebraic Modeling System) Rosenthal (2015) is a suitable modeling language because it can utilize several different solvers.

4.1.1 DICOPT

The default solver for MINLP is DICOPT Grossmann (2002), which stands for DIscrete and Continuous OPTimizer. The solver is based on three key ideas: outer approximation algorithm, equality relaxation and augmented penalty. The problem is decomposed into a master problem (MIP) and a sub problem (NLP). The algorithm solves a series of NLP and MIP problems. The default stop criteria is to stop when the NLP sub problem starts worsening.

4.1.2 BARON

Since DICOPT could not find the global optimal solution for some of the cases was another solver was also used; BARON Rosenthal (2015). BARON stands for the Branch And Reduce Optimization Navigator and is a global solver. BARON is combining constraint propagation, interval analysis, and duality in its reduce arsenal with advanced branch-and-bound optimization concepts. BARON could not handle variables declared as SOS2, so the reformulation with binary had to be used when the problem was solved with BARON.

DICOPT did not need any bounds or good starting point to find a solution. BARON did on the other hand require quite good bounds for it to find a solution quickly. One way to find a lower bound was to solve the relaxed problem (RMINLP) first and then set the lower bound to the relaxed solution. A way to find a good upper bound is to run the model without letting the streams with pressure manipulations to split.

4.2 GAMS

The model is implemented in GAMS 23.7.3. DICOPT uses CPLEX for the LP subproblems and CONOPT3 for the NLP subproblems. BARON uses ILOG CPLEX for the LP subproblems and MINOS for the NLP subproblems.

4.3 Implementation of model without the use of insights

Most of the implementation of the model without the use of insights was carried out during the project work. The implementation has been extended to include the possibility of compression and sub ambient temperatures. The equations and restrictions were the same for compression and expansion, so it was only the logic for creating sets which had to be altered. Since the plan always was to reformulate the model using insights, have the some parts of the model been simplified in the model. None of the test cases had multiple hot or cold utility sources, so a single variable, hu, was used for the hot utility consumption and $r_{|K|}$ for cold utility consumption. The polytropic efficiency, η_s, was assumed to be 100%, so it was not included in the implementation. An extra term was added to the objective function to introduce the exergy content of the cold utility consumption, which was necessary for sub ambient cases.

The implementation consists of data input, creating the heat cascade, create the necessary sets and then the model. The data input is further explained in 4.4.1. To enable the heat cascade to ensure thermodynamic feasibility is it important to use the correct temperatures. The pinch point will be located at the beginning of a stream, so all supply temperatures must be included. The identity of the stream is also of importance, since the supply temperature of a hot stream belongs to the hot (left) side of the heat cascade, while it belongs on the cold side (right) for a cold stream. The hot utility and ambient temperature must also be included since pressure change may happen at these temperatures. Some of the equations assumes that none of the streams have a target temperature inside an interval, so all the target temperatures are added as well. The $\alpha_{skm}^{in/out}$-variables requires an interval above or below the supply/target temperatures, so these are also added. The heat cascade requires the temperature intervals to be ordered in a descending order, so a simple algorithm was developed and implemented to sort the temperatures and delete duplicate values. Then could the heat demand (Q_k^C) and demand (Q_k^H) for each temperature interval be calculated for the streams without pressure change. The rest of the sets which were dependent on the heat cascade could also be found. BARON could not handle SOS2 variables, so an alternative implementation (equation 4.1) with the use of binary variables ($\delta_{skm}^{in/out}$) were implemented in the file BaronsS.gms. This file have to be included in Model.gms if BARON is going to be used instead of CONOPT.

$$\begin{aligned}
\alpha_{s,1,m}^{in/out} - \delta_{s,1,m}^{in/out} \leq 0 & \qquad s \in S^P, m \in M_s \\
\alpha_{skm}^{in/out} - \delta_{s,k-1,m}^{in/out} - \delta_{skm}^{in/out} \leq 0 & \qquad k \in K \setminus \{1, |K|\}\, s \in S^P, m \in M_s \\
\alpha_{s,|K|,m}^{in/out} - \delta_{a,|K|-1,m}^{in/out} \leq 0 & \qquad s \in S^P, m \in M_s \\
\sum_{k=1..|K|-1} \delta_{skm}^{in/out} = 1 & \qquad , s \in S^P, m \in M_s
\end{aligned} \tag{4.1}$$

Some equations and relations have been added or altered in the implementation to reduce the number of nonlinearities. f_{sm} is multiplied with $t_{sm}^{in} - t_{sm}^{out}$ or $t_{skm}^{in,D} - t_{skm}^{out,D}$ several places. Δt_{sm} and Δt_{sm} is introduced as the two temperature differences and are used in conjunction with f_{sm} instead.

4.4 Implementation of model with the use of insights

The implementation in its entirety have been separated into four parts:

1. Data input
2. Locating the original pinch point
3. Create branches and segments
4. Optimization

4.4.1 Data input

In this file the user must enter the necessary data for describing and solving the problem. The number of hot and cold streams must be entered together with their corresponding supply and target temperature and the MCP. The naming convention is H1, H2, ... for hot streams and C1, C2, ... for cold streams. The streams with pressure change must also be specified together with supply and target pressure and κ. It is also necessary to specify the hot and cold utility temperature, the ambient temperature and minimum temperature difference (ΔT).

4.4.2 Locating the original pinch point with a heat integration model

It is necessary to locate the pinch point of the original streams to create all the necessary branches and segments. The model from section 3.4.7 will find the pinch point and is implemented in the Pinch.gms- file. The temperature intervals in the heat cascade is taken from the supply temperatures of the original streams. These a sorted and duplicate temperatures are removed. The supply and target temperatures are used together with the MCP values to calculate the heat requirement (Q^H_{sk} or Q^C_{sk}) for each stream in each temperature interval. The only variables are the heat residuals and the hot utility consumption. Only two constraints are the heat balance for each temperature and the non-negativity constraint for the heat residuals, r_k. This is a very small LP model and with the CPLEX solver it is solved in less than a tenth of a second. The model can also be used to verify the solutions from the heat and work optimization models. After the pressure changing units have been placed will the system be a regular heat exchanger network where all the streams have fixed supply and target temperature and MCP values. This model can check calculate the hot and cold utility consumption and compare it with the results from the heat and work optimization models. If the results deviate it means that the proposed solution is thermodynamically infeasible and that there is something wrong with the heat and work integration model and/or implementation.

4.4.3 Create branches and segments

The use of insight led to the choice of inlet and outlet temperature for the pressure changing units from being continuous to being a discrete choice. The insight can predict a set of inlet and outlet temperatures where one or several will be part of the optimal solution. This part of the implementation identifies the set of inlet and outlet temperatures and creates the branches with the related segments corresponding to the given inlet and outlet temperatures. The insights have led to this list of inlet and outlet temperatures that are of interest:

1. Outlet temperature at the target temperature
2. Inlet temperature at the pinch temperature
3. Inlet temperature at pinch temperatures created by pinch expansion/compression
4. Inlet at hot utility temperature for above ambient expansion
5. Inlet at ambient temperature for above ambient compression

4.4.3.0.1 Outlet temperature at the target temperature In some cases will the problem be degenerate and will have multiple inlet and outlet temperature arrangements that give the same minimum exergy consumption. One or several of the many potential solutions will be covered by the other branches in the list, but there is one solution that in many way seems better and should therefore be included. There are two benefits for letting the compression/expansion end at the target temperature. It is beneficiary to only have one pressure changing unit instead of two and the stream is not separated into two different segments. If the stream had been separated into two segments instead, would this increase the chance of needing small and expensive heat exchangers when the actual network is to be designed.
The outlet temperature of the stream is known beforehand and set to the target temperature.

This means that there is not a second segment. The first segment Goes from the supply temperature to the inlet of the pressure changing unit. Equation (3.20) or (3.35) is used to calculate the inlet temperature based on the outlet temperature. The calculated inlet temperature decides the identity of the stream segment. The identity is hot if the inlet temperature is below the supply temperature and cold for the opposite case. So a new stream with the supply temperature of the original stream and a outlet temperature equal to the inlet temperature of the pressure changing unit is created. The segment does also get the same MCP as the original stream and the temperature difference in the pressure changing unit is calculated. Sets (B_s and Z_{bs}) linking the newly created stream with the original stream is also updated.

4.4.3.0.2 Inlet temperature at pinch temperature and pinch temperatures created by pinch expansion/compression The pinch point for the original problem have been calculated, so it is fairly straight forward to include a branch for all the streams with pressure manipulation. The branch will consist of two segments. The first segment will have the same supply temperature as the original stream. The target temperature will be the calculated pinch point. The pinch point is actually two different temperatures, with one for hot streams and one for cold streams. A key finding from the project thesis was that the pinch temperature should be decided by the identity of the stream at the inlet of the pressure changing unit and not by the identity of the original stream. So it necessary to check the identity of the first branch segment before the target temperature is decided. The supply temperature of the second stream segment is set to the outlet temperature of the pressure changing unit, which is calculated by equation (3.20) or (3.35). The target temperature is set to the original target temperature. The identity of the second stream segment is decided by the relative position of the supply and target temperature. B_s and Z_{bs} is updated with the new branch and segments, the MCP is set and the temperature difference through the pressure changing unit is also calculated.

The expansion/compression beginning at the pinch temperature may lead to the creation of other pinch points. It is therefore necessary to create branches with pressure changing units at all the potential new pinch points. The potential new pinch points is the supply temperatures of the original streams. A loop goes through the supply temperatures and evaluates whether it is necessary to add a branch or not. The branch is not created if the outlet temperature is too high (about 50°C higher than the hot utility temperature) or too low (about 50°C lower than the ambient temperature). The brach will have one stream segment before and after the pressure changing unit. The first stream will go from the supply temperature of the original stream to the inlet of the pressure changing unit. The identity of the stream decides whether the hot or cold potential pinch temperature should be used. The outlet temperature is then calculated and this becomes the supply temperature of the second segment. Then the temperature difference in the pressure changing unit is calculated. Then the identity of the second is decided and the MCP, B_s and Z_{bs} are also updated. When an additional pinch point is created it will be a region between the two pinch points. This region will be in full heat balance and it not necessary to add any additional heating or cooling.This means that half of the potential pinch points can be ignored since they lead introduce heating/cooling into region in heat balance. Compression adds heat and should therefore on happen on potential pinch point above the original pinch point., while the opposite is true for expansion.

For cases with small pressure changes may a new pinch point be created by the outlet of a pinch compressor/turbine. The outlet of a pressure changing unit is in fact the beginning of a stream and therefore a potential pinch point. It is possible to calculate the inlet temperature of a pressure changing unit starting at this potential pinch point since the original pinch point is known. The tricky part is to decide whether the hot or cold pinch temperature should be used in each step. First is the inlet temperature set to the original pinch candidate corresponding to the identity of the stream at the pinch. The outlet of the pinch compressor/turbine is then calculated using equation (3.20) or (3.35). This temperature is the new potential pinch point and is the basis for the inlet temperature for this branch. The identity of the stream segment going from the supply temperature to the new potential pinch point decides whether the hot or cold potential pinch temperature should be used. The question is then whether the potential pinch temperature is originally hot or cold. This is actually decided by the identity of the stream segment starting at the outlet of the pinch expansion/compression. When the inlet temperature have been set can the outlet temperature for this branch be calculated by equation (3.20) or (3.35). The procedure for getting the correct inlet and outlet temperature for this case is therefore as follows:

1. Check the identity of the stream at the original pinch. Hot identity = hot pinch and cold identity = cold pinch.

2. Calculate the outlet temperature of a compressor/turbine starting at the correct original pinch.

3. Find the identity of the stream segment beginning at the outlet of the pinch compressor/turbine. This shows whether the calculated outlet temperature is the hot or cold potential pinch temperature.

4. Find the identity of the stream segment entering the potential pinch point (outlet temperature for the original pinch compressor/turbine) and use this to choose the correct potential pinch temperature. The inlet temperature for the pressure changing unit for this branch is set to this temperature.

5. Calculate the outlet temperature of the pressure changing unit starting at the correct potential pinch temperature. Then the identity of the second segment can be decided and MCP, B_s and Z_{bs} can be updated.

A potential pinch point originating from pinch compression/expansion will only be a potential pinch for streams with the same type of pressure change (compression/expansion). If the second pressure changing type is different than the pinch compression/expansion it will add heat/cooling to the region between the two pinch points. This region is in heat balance, so expansion/compression that disturbs this balance should be avoided. So for the second generation potential pinch point it is necessary to loop over streams with expansion and compression separately.

4.4.3.0.3 Inlet at hot utility temperature for above ambient expansion and at ambient temperature for above ambient compression In some cases it may be optimal to do all or parts of the expansion at the hot utility temperature or at ambient temperature for compression. The procedure for creating the branches and stream segments for these two cases is identical to the one for the original pinch temperature and potential pinch candidates (excluding second generation). So to make the implementation easier were the ambient and hot utility temperature were together with the original pinch added to the loop which created the branches for the potential pinch points.

4.4.4 Optimization

A new heat cascade must be created, before the the model i ready for optimization. The supply temperature of the new stream segments are potential new pinch points and must there be included in the heat cascade together with the original supply temperatures. The temperatures are collected and then sorted. Duplicates are also removed. Then the heat requirement for all streams to all the heat cascade intervals can be calculated. The only variables are the split fraction (f_{sb}), heat residual (r_k) and the hot utility consumption (hu). Since none of the test cases have been with multiple hot utility sources, was the hot utility consumption modeled with a single variable, hu. The only restrictions are the heat balances for each temperature interval, mass balances for the stream branches and the non-negativity constraint for the heat residuals. The results from the optimization are the minimum exergy consumption, the inlet and outlet temperature of the pressure changing units and the MCP fractions going through the units.

Kapittel 5

Computational study

8 test cases have been solved by both optimization models and have been compared with the results from the graphical procedure. The test cases and the results from the graphical procedures have been provided by co-supervisors Truls Gundersen and Fu Chao. Table 5.1 compares the minimum exergy consumptions for the different test cases achieved by the graphical procedure and the two models. The model with the use of insights does does either achieve the same or better results than the graphical procedure. The model without the use of insights is more variable and gets both better and worse results than the graphical procedure. The results from each test case is presented more in detail

Cases	Graphical procedure	Model without insight	Model with insight
1	168.2 kW	157.5 kW (1.3 s)	157.5 kW (0.50 s)
2	143.3 kW	152.2 kW (0.8 s)	134.6 kW (0.54 s)
3	-205.8 kW	-205.8 kW (0.50 s)	-205.8 kW (0.50 s)
4	-203.3 kW	-203.3 kW (0.50 s)	-203.3 kW (0.45 s)
5	-206.4 kW	-203.77 kW (0.65 s)	-206.3 kW (0.48 s)
6	-470.2 KW	-436.23 kW (0.51 s)	-470.4 kW (0.53 s)
7	175.6 kW	178.1 kW (0.6 s)	175.6 kW (0.53 s)
8	54.6 kW	63.0 kW (1.2 s)	52.28 kW (0.58 s)

Tabell 5.1: Results for test case 1-8

5.1 Test case 1

Here both models found a better solution than the graphical procedure. The reason for this is the new insight from the project thesis regarding the the voice of pinch temperature for pinch expansion/compression. If this new insight is incorporated into the graphical procedure it will find the same result. This case does consist of three streams and only one of streams changes pressure.

	Graphical procedure	**Model without insight**	**Model with insight**
Exergy consumption [kW]	168.2	157.5	157.5
Hot utility consumption [kW]	740	740	740
Work added [kW]	-255.0	-265.7	-265.7
Inlet/outlet temperature [°C]	200/72.5	220/87.1	220/87.1
MCP [kW/°C]	2	2	2

Tabell 5.2: Results for test case 1

5.2 Test case 2

Here the model with the use of insight found a better solution, while the other model found a worse solution than the one from the graphical procedure. The reason is once again that the graphical procedure did not take into account the identity of a stream at the inlet and outlet of the turbine. By using this insight together with the procedure yields the same result as for the model with the use of insight. The model without the use of insight was not able to the optimal solution by using CONOPT as the solver. When inlet/outlet temperatures and split fractions was set to the optimal solution did the model give the correct hot and cold utility consumption. This mean that the optimal solutions is intact a feasible solution in the model, but that it is not capable of finding it. BARON did not find the optimal solution within 15 minutes. It requires thigh bounds for several variables to find the optimal solution within reasonable time. The insights from the graphical can be used to find such bounds, but this goes against the purpose of the insight free model. The model with the use of insight found the optimal solution and it did in fact require three turbines to do so.

	Graphical procedure	Model without insight	Model with insight
Exergy consumption [kW]	143.3	152,3	134.6
Hot utility consumption [kW]	923.1	853.4	720
Work added [kW]	-384.7	-336.3	-353.5
Inlet/outlet temperature [°C]	220/106.4 400/245	220/106.4 210.4/99.0	220/106.4 400/225 126.4/34.4
MCP [kW/°C]	1.94 1.06	1 2	2 0.54 0.46

Tabell 5.3: Results for test case 2

5.3 Test case 3

Here both models and the graphical procedure found the same optimal exergy consumption. The model with the use of insight and the graphical procedure have the same inlet and outlet temperatures for both the turbines, while the model without the use of insight found a solution with only one turbine. This last result is very favorable compared the the first result when it comes to investment costs. This case tests what happens when the cooling from expansion at the pinch point exceeds the cooling demand for the system. The solutions proposed by the model without the use of insights found an inlet and outlet temperature for a turbine which matched the cooling provided below the pinch with the cooling demand. The inlet and outlet temperatures can be calculated and a branch with corresponding stream segment can be created and included in the model with the use of insight. The pinch point is found during the pre processing and so is the cooling demand for the system. A heat balance below the pinch point gives: MCP$(T^{Pinch} - T^{Out})$=Cooling demand. Solving for T^{Out} gives: $T^{Out} = T^{Pinch} - \frac{\text{Cooling demand}}{MCP}$. For this case the cooling demand is 450 kW and the pinch point is 220°C. This gives an outlet temperature of 70°C, which is what the model without the use of insight found.

5.4 Test case 4

Both models and the graphical procedure found the same result. This is a very simple case because of the huge pressure ratio. The temperature difference in the turbine is so big that there are very few possible inlet and outlet temperatures.

	Graphical procedure	Model without insight	Model with insight
Exergy consumption [kW]	-205.8	-205.8	-205.8
Hot utility consumption [kW]	691.0	691.0	691.0
Work added [kW]	-601.0	-601.0	-601.0
Inlet/outlet temperature [°C]	400/151.9 220/38.2	270.3/70	400/151.9 220/38.2
MCP [kW/°C]	0.84 2.16	3	0.84 2.16

Tabell 5.4: Results for test case 3

	Graphical procedure	Model without insight	Model with insight
Exergy consumption [kW]	-203.3	-203.3	-203.3
Hot utility consumption [kW]	1060	1060	1060
Work added [kW]	-809.6	-809.6	-809.6
Inlet/outlet temperature [°C]	400/-4.8	400/-4.8	400/-4.8
MCP [kW/°C]	2	2	2

Tabell 5.5: Results for test case 4

5.5 Test case 5

The graphical procedure and the model with the use of insight found the exact same solution. There is a small difference in exergy consumption due to some round off errors. The model without the use of insight did not find the optimal solution. The BARON solver did not find the optimal solution within reasonable time either. The model did however find a result which arguable is better than the optimal solution. The solution does only use one turbine and does only have a small exergy penalty of 2.6 kW (1.25%). This is a difficult case since the cooling from the expansion creates a new pinch point, which leads to more binding constraints (non-negativity of the heat residuals). This is also the first case with four streams. It seems like the model without the use of insights have difficulties with finding the optimal solutions when there are several pinch points (like for test case 2).

5.6 Test case 6

The graphical procedure and the model with the use of insight found the exact same solution. There is a small difference in exergy consumption due to some round off errors. The model without the use of insight did not find the optimal solution. The BARON solver did not find

	Graphical procedure	Model without insight	Model with insight
Exergy consumption [kW]	-206.4	-203.8	-206.3
Hot utility consumption [kW]	350	350	350
Work added [kW]	-406.6	-404.0	-406.4
Inlet/outlet temperature [°C]	330/167.5 160/43.3	226.7/92.016	330/167.5 160/43.3
MCP [kW/°C]	1.23 1.77	3	1.23 1.77

Tabell 5.6: Results for test case 5

the optimal solution within reasonable time either. Although the solution only consists of one turbine, is the exergy penalty so big that the solution from the graphical procedure and the other model is still considerably better.

	Graphical procedure	Model without insight	Model with insight
Exergy consumption [kW]	-470.2	-436.2	-470.4
Hot utility consumption [kW]	819	738.8	818.5
Work added [kW]	-938.6	-858.8	-938.5
Inlet/outlet temperature [°C]	400/218.7 160/43.3	258.1/115	400/218.7 160/43.3
MCP [kW/°C]	3.69	6	3.69 2.31

Tabell 5.7: Results for test case 6

5.7 Test case 7

This is the first case with both compression and expansion. The graphical procedure and the model with the use of insight gets the same result, while the other model did not find the optimal solution. It seems like the other have get problems when stream splitting is required. The solution it found does only have one turbine and one compressor, while the optimal solution requires two of both. The exergy penalty is only 2.5 kW (1.4%) and it saves two pressure changing units. So it is arguable a better solution than the optimal one when it comes to costs.

	Graphical procedure	Model without insight	Model with insight
Exergy consumption [kW]	175.6	178.1	175.6
Hot utility consumption [kW]	37.6	27.688	37.5
Work added [kW]	-158.3 312.4	-189.7 352.0	-158.4 312.5
Inlet/outlet temperature [°C]	210/123.2 110/41.2 190/291.4 300/425.5	254.9/160 262.7/380	400/218.7 160/43.3 160/43.3 160/43.3
MCP [kW/°C]	1.15 0.85 2.66 0.34	2 3	1.15 0.85 2.66 0.34

Tabell 5.8: Results for test case 7

5.8 Test case 8

This is a case with both compression and expansion and is the only case with below ambient temperatures. The model without the use of insight did again find a solution with only one turbine and one compressor. The model with the use of insight found a slightly better solution than the graphical procedure. Instead of letting the second compressor start at the newly created pinch point, does this solution let the compressor start at the supply temperature instead. This reduces the compression work and enables the first pinch turbine to take a larger portion of the MCP, which increases the work produced from expansion. The cold utility consumption increases with 10.2 kW because of this, but the decrease in net added work makes up for this and more.

	Graphical procedure	**Model without insight**	**Model with insight**
Exergy consumption [kW]	54.6	63.0	52.7
Cold utility consumption [kW]	13.0	6.9	23.2
Work added [kW]	-99.1 135.5	-97.1 139.2	-103.5 123.8
Inlet/outlet temperature [°C]	-71/-125.4 -131/-169.3 -75/-31.6 -25/27.2	-93/-141.5 -61.3/-14.9	-71/-125.4 131/-169.3 -75/-31.6 -135/-104.7
MCP [kW/°C]	1.40 0.60 2.51 0.49	2 3	1.67 0.33 2.51 0.49

Tabell 5.9: Results for test case 8

Kapittel 6

Concluding remarks

The goal for this thesis have been to develop a complete model for minimizing exergy consumption for a work and heat exchange network and then improve the model using insights. The model without the use of insights became a MINLP, which are computational difficult to solve. CONOPT found good results quickly and sometimes even the optimal solution. It generally had trouble with finding the optimal solution when the streams with pressure change required splitting. CONOPT did instead find solutions without stream splits. Some of these solutions are highly interesting, since the exergy penalty was small compared to the cost reduction from removing a pressure changing unit. By using insights it was possible to fix the inlet and outlet temperatures for the pressure changing units at some interesting temperatures, which led to the model becoming a LP. The model then became very easy to solve to optimality and it did for all cases find a solution which was equally good or better than the solutions from the other model and the graphical procedure. The models does not design the actual work and heat exchange network, but can guarantee that it is possible to design a network with the given utility consumption and work production from the solutions. The models are therefore suitable for targeting. When the pressure changing units and MCP values have been fixed does the problem actually become a heat exchanger network problem. It exists both manual methods and optimization models for designing both exergy optimal solution or more cost effective solutions. The graphical procedure is quite time consuming for larger cases, so the model with the use of insight can be helpful for quickly testing new theories and cases regarding heat and work exchange integration. There are some scenarios which the models have not been tested on. This has mainly to do the with the lack test cases and time consuming graphical procedure for finding the optimal solution. None of the test cases have had scenarios with two pressure changing units of the same type. It has either been a single pressure changing unit or one of each kind. With two streams with the same type of pressure change may cause some trouble for the graphical procedure. If the cooling/heating provided by the units exceeds the cooling/heating demand for the system will there be a question of how much of each unit should be allowed to be compressed/expanded at the pinch. The model should presumably handle this case, but the graphical procedure may have to add some new theorems for solving this problem. None of the test cases included a hot stream with compression. Equations for this scenario has been derived, but not thoroughly tested. It makes more sense to expand a hot stream instead of compressing it since a hot stream is cooled down. This is probably why most of the test cases have had expansion of hot streams instead of compression. All the test cases have been strictly either above the ambient or below the ambient temperature. The ambient temperature will become a pinch point because of the hot and cold utility at this temperature will have zero exergy content. This will effectively decompose the problem into two separate problems, one above and one below the ambient temperature.

6.1 Further work

The research about work and heat exchange is quite novel and there are several possible modifications which can be done to the models. The reason for including streams with pressure manipulation into the heat exchanger network is to often to reduce the hot and/or the cold utility consumption. The pressure ratio for the streams with pressure change not likely to be fixed and should be a variable instead. It would also be natural to let one stream with the same supply and target temperature go through a series of pressure changes with the pressure ratios as variables. If the choice of pressure ratios is set to be discrete choice with binary variables, would the model with the use of insight be turned into a IP. If the pressure ratio on the other hand is going to continuous will the model get nonlinearities. The end game will be to also consider costs and designing the whole heat and work exchange network. This will introduce more nonlinearities and binary variables. It has also been assumed that the hot and cold utilities are isothermal, but is not always the case. So a further extension to the models would be to include utilities with supply and target temperatures. The model without the use of insights did in some cases find a solution without stream splits, which were nearly as good as the optimal solution. An extension to model with insights would be to let it find this solution as well and then force the model to only use one branch per stream with pressure manipulation. This can be done by letting f_{sb}^{B} be a SOS1 set. The solutions without streams splits had the same pinch points as for the optimal solution, so it should be possible to find the inlet and outlet temperature of a single pressure changing achieving this after the model have been solved.

Bibliografi

Anantharaman, R. and Gundersen, T. (2007). The sequential framework for heat exchanger network synthesis - network generation and optimization. *Chemical Engineering transactions*, 12.

Aspelund, A., Berstad, D., and Gundersen, T. (2007). An extended pinch analysis and design procedure utilizing pressure based exergy for subambient cooling. *Applied Thermal Engineering*, 27(16):2633–2649.

Dowling, A. (2015). Mathematical programming approaches to balance heat-work trade-offs in above ambient systems.

Fu, C. and Gundersen, T. (2015a). Appropriate placement of compressors and expanders in above ambient processes. *Chemical Engineering Transactions*, Journal 45(pages 643-648).

Fu, C. and Gundersen, T. (2015b). Integrating expanders into heat exchanger networks above ambient temperature. *AIChE*, 61(10):3404–3422.

Grossmann, I., Yeomans, H., and Kravanja, Z. (1998). A rigorous disjunctive optimization model for simultaneous flowsheet optimization and heat integration. *Computers & chemical engineering*, Voume 22:Pages 157–164.

Grossmann, I. E. (2002). *DICOPT: A Discrete Continuous Optimization Package*. GAMS Development Corporation, Washington D.C.

Gundersen, T. (1991). *The use of mathematical programming in process synthesis*. Norsk Hydro a.s, Research Centre.

Gundersen, T. and Næss, L. (1988). The synthesis of cost optimal heat exchanger networks. *Comput. chem. Engng*, 12(6):503–530.

Onishi, V., Ravagnani, M., and Caballero, J. (2014). Simultaneous synthesis of work exchange networks with heat integration. *Chemical Engineering Science*, 112:87–107.

Onishi, V., Ravagnani, M., and Caballero, J. (2015). Retrofit of heat exchanger networks with pressure recovery of process streams at sub-ambient conditions. *Energy Conversion and Management*, 94:377–393.

Rosenthal, R. E. (2015). *Gams, A user's Guide*. GAMS Development Corporation, Washington Dc, USA.

Wechsung, A., Aspelund, A., Gundersen, T., and Bartion, P. I. (2011). Synthesis of heat exchanger networks at subambient conditions with compression and expansion of process streams. *AIChE*, 57(8):2090–2108.

Yee, T. F. and Grossmann, I. E. (1990). Simultaneous optimization models for heat integration. Technical report, Carnegie Institute of Technology.

Figurer

Tabeller

Appendices

Tillegg A

Complete model without the use of insight

$$\min \sum_{k \in K} \sum_{u \in U_k^H} \left(1 - \frac{T_0}{T_u^{HU}}\right) q_{ku}^{HU} + \sum_{k \in K} \sum_{u \in U_k^C} \left(\frac{T_0}{T^{CU}} - 1\right) q_{ku}^{CU}$$
$$- \sum_{s \in S^{EC} \cup S^{EH}} \sum_{m \in M_s} \eta_s MCP_s f_{sm} \left(t_{sm}^{in} - t_{sm}^{out}\right) - \sum_{s \in S^{CC} \cup S^{CH}} \sum_{m \in M_s} \eta_s^{-1} MCP_s f_{sm} \left(t_{sm}^{in} - t_{sm}^{out}\right) \tag{A.1}$$

$$r_1 + \sum_{s \in S^{EC} \cup S^{CC}} \sum_{m \in M_s} q_{s,1,m}^C - \sum_{s \in S^{EH} \cup S^{CH}} \sum_{m \in M_s} q_{s,1,m}^H - \sum_{u \in U_1^H} q_{1,u}^{HU} = \sum_{s \in S^{HWP}} Q_{s,1}^H - \sum_{s \in S^{CWP}} Q_{s,1}^C \tag{A.2}$$

$$r_k - r_{k-1} + \sum_{s \in S^{EC} \cup S^{CC}} \sum_{m \in M_s} q_{skm}^C - \sum_{s \in S^{EH} \cup S^{CH}} \sum_{m \in M_s} q_{skm}^H - \sum_{u \in U_k^H} q_{ku}^{HU} + \sum_{u \in U_k^C} q_{ku}^{CU} = \sum_{s \in S^H} Q_{sk}^H - \sum_{s \in S^C} Q_{sk}^C$$
$$k \in K \setminus \{1, |K|\} \tag{A.3}$$

$$-r_{|K|-1} + \sum_{s \in S^{EC} \cup S^{CC}} \sum_{m \in M_s} q_{s,|K|,m}^C - \sum_{s \in S^{EH} \cup S^{CH}} \sum_{m \in M_s} q_{s,|K|,m}^H + \sum_{u \in U_{|K|}^C} q_{|K|,u}^{CU} = \sum_{s \in S^H} Q_{s,|K|}^H - \sum_{s \in S^C} Q_{s,|K|}^C \tag{A.4}$$

$$q_{skm}^H = MCP_s f_{sm} \left(T_k^H - t_{skm}^{in,D} + t_{skm}^{out,D} - T_{k+1}^H\right), s \in S^{EH} \cup S^{CH}, k \in K_s^{HBTST} \setminus K_s^{HTT}, m \in M_s \tag{A.5}$$

$$q^{H}_{skm} = MCP_s f_{sm}\left(t^{out,D}_{skm} - t^{in,D}_{skm}\right), s \in S^{EH} \cup S^{CH}, k \in K^{HBTT}_s \cup K^{HATS}_s \setminus K^{HTS}_s, m \in M_s \tag{A.6}$$

$$q^{H}_{skm} = MCP_s f_{sm}\left(t^{in,D}_{skm} - t^{out,D}_{skm} + a^{in}_{skm}\left(T^{C}_{k} - T^{C}_{k+1}\right)\right) \quad , s \in S^{CH} \cup S^{EH}, k \in K^{HTS}_s, m \in M_s \tag{A.7}$$

$$q^{H}_{skm} = MCP_s f_{sm}\left(t^{in,D}_{skm} - t^{out,D}_{skm} + a^{out}_{skm}\left(T^{C}_{k} - T^{C}_{k+1}\right)\right) \quad , s \in S^{CH} \cup S^{EH}, k \in K^{HTT}_s, m \in M_s \tag{A.8}$$

$$q^{C}_{skm} = MCP_s f_{sm}\left(t^{in,D}_{skm} - T^{C}_{k+1} + T^{C}_{k} - t^{out,D}_{skm}\right) \quad , s \in S^{EC} \cup S^{CC}, k \in K^{CBTST}_s \setminus K^{CTS}_s, m \in M_s \tag{A.9}$$

$$q^{C}_{skm} = MCP_s f_{sm}\left(t^{in,D}_{skm} - t^{out,D}_{skm}\right) \quad , s \in S^{EC} \cup S^{CC}, k \in K^{CBTS}_s \cup K^{CATT}_s \setminus K^{CTT}_s, m \in M_s \tag{A.10}$$

$$q^{C}_{skm} = MCP_s f_{sm}\left(t^{in,D}_{skm} - t^{out,D}_{skm} + \left(1 - a^{in}_{skm}\right)\left(T^{C}_{k} - T^{C}_{k+1}\right)\right) \quad , s \in S^{CC} \cup S^{EC}, k \in K^{CTS}_s, m \in M_s \tag{A.11}$$

$$q^{C}_{skm} = MCP_s f_{sm}\left(t^{in,D}_{skm} - t^{out,D}_{skm} + \left(1 - a^{out}_{skm}\right)\left(T^{C}_{k} - T^{C}_{k+1}\right)\right) \quad , s \in S^{CC} \cup S^{EC}, k \in K^{CTT}_s, m \in M_s \tag{A.12}$$

$$\begin{aligned} r^{out}_{skm} - r_{k-1} + q^{CP}_{skm} - q^{HP}_{skm} - \sum_{u \in U^{H}_{k}} q^{HU}_{ku} - \sum_{s \in S^{HWP}} MCP_s\left(T^{H}_{k} - t^{out,D}_{skm}\right) \\ + \sum_{s \in S^{CWP}} MCP_s\left(T^{C}_{k} - t^{out,D}_{skm}\right) = 0, s \in S^{P}, k \in K \setminus \{1, |K|\}, m \in M_s \end{aligned} \tag{A.13}$$

$$\begin{aligned} q^{HP}_{skm} = \sum_{y \in S^{EH} \cup S^{CH}} \sum_{n \in M_y} MCP_y f_{yn}\left(T^{H}_{K} - \max\left(t^{in,D}_{ykn}, t^{out,D}_{skm}\right) + \max\left(t^{out,D}_{ykn}, t^{out,D}_{skm}\right) - t^{out,D}_{skm}\right) \\ s \in S^{P}, k \in K^{HBTST}_s, m \in M_s \end{aligned} \tag{A.14}$$

$$q^{HP}_{skm} = \sum_{y \in S^{EH} \cup S^{CH}} \sum_{n \in M_y} MCP_y f_{yn} \left(\max\left(t^{out,D}_{ykn}, t^{out,D}_{skm}\right) - \max\left(t^{in,D}_{ykn}, t^{out,D}_{skm}\right) \right)$$
$$s \in S^P k \in K^{HATS}_s \cup K^{HBTT}_s, m \in M_s \tag{A.15}$$

$$q^{CP}_{skm} = \sum_{y \in S^{EC} \cup S^{CC}} \sum_{n \in M_y} MCP_s f_{yn} \left(T^C_K - \max\left(t^{out,D}_{ykn}, t^{out,D}_{skm}\right) + \max\left(t^{in,D}_{ykn}, t^{out,D}_{skm}\right) - t^{out,D}_{skm} \right)$$
$$s \in S^P, k \in K^{HBTST}_s, m \in M_s \tag{A.16}$$

$$q^{CP}_{skm} = \sum_{y \in S^{EC} \cup S^{CC}} \sum_{n \in M_y} MCP_s f_{yn} \left(\max\left(t^{in,D}_{ykn}, t^{out,D}_{skm}\right) - \max\left(t^{out,D}_{ykn}, t^{out,D}_{skm}\right) \right)$$
$$s \in S^P, k \in K^{HBTT}_s \cup K^{HATS}_s, m \in M_s \tag{A.17}$$

$$t^{out}_{sm} - \left(\frac{P^T_s}{P^S_s}\right)^{\eta_s^{-1} \frac{\kappa-1}{\kappa}} t^{in}_{sm} = 0, \quad s \in S^{CH} \cup S^{CC}, m \in M_s \tag{A.18}$$

$$t^{out}_{sm} - \left(\frac{P^T_s}{P^S_s}\right)^{\eta_s \frac{\kappa-1}{\kappa}} t^{in}_{sm} = 0, \quad s \in S^{EH} \cup S^{EC}, m \in M_s \tag{A.19}$$

$$t^{in,D}_{skm} = \max\left(\min\left(t^{in}_{skm}, T^C_k\right), T^C_{k+1}\right) \quad , s \in S^{EC} \cup S^{CC}, k \in K, m \in M_s \tag{A.20}$$

$$t^{in,D}_{skm} = \max\left(\min\left(t^{in}_{skm}, T^H_k\right), T^H_{k+1}\right) \quad , s \in S^{EH} \cup S^{CH}, k \in K, m \in M_s \tag{A.21}$$

$$\textstyle\sum_{m \in M_s} f_{sm} = 1 \quad , s \in S^{EH} \cup S^{EC} \tag{A.22}$$

$$f_{sm} \geq f_{sm+1} \quad , s \in S^{EH} \cup S^{EC}, m \in M_s \setminus \{|M_s|\} \tag{A.23}$$

$$f_{sm} \geq 0 \quad , s \in S^{EH} \cup S^{EC}, m \in M_s \tag{A.24}$$

$$r_k \geq 0 \quad , k \in K\{1, |K|\} \tag{A.25}$$

$$r_{skm}^{out} \geq 0 \quad , s \in S^P, k \in K\{1, |K|\}, m \in M_s \tag{A.26}$$

Tillegg B

The complete model with the use of insight

$$\min \sum_{k\in K} \sum_{u\in U_k^H} \left(1 - \frac{T_0}{T_u^{HU}}\right) q_{ku}^{HU} + \sum_{k\in K} \sum_{u\in U_k^C} \left(\frac{T_0}{T^{CU}} - 1\right) q_{ku}^{CU} + \sum_{s\in S^P} \sum_{b\in B_s} T_{sb}^{\Delta} MCP_{sb} f_{sb} \tag{B.1}$$

$$r_1 - \sum_{u\in U_k^H} q_{1,u}^{HU} - \sum_{s\in S^P} \sum_{b\in B_s} \sum_{z\in Z_{sb}} Q_{s,1,bz}^P f_{sb} = \sum_{s\in S\setminus S^P} Q_{s,1} \tag{B.2}$$

$$r_k - r_{k-1} - \sum_{u\in U_k^H} q_{ku}^{HU} + \sum_{u\in U_k^C} q_{ku}^{CU} - \sum_{s\in S^P} \sum_{b\in B_s} \sum_{z\in Z_{sb}} Q_{skbz}^P f_{sb} = \sum_{s\in S\setminus S^P} Q_{sk} \quad , k\in K\setminus\{1,|K|\} \tag{B.3}$$

$$-r_{|K|-1} + \sum_{u\in U_k^C} q_{|K|,u}^{CU} - \sum_{s\in S^P} \sum_{b\in B_s} \sum_{z\in Z_{sb}} Q_{s,|K|,bz}^P f_{sb} = \sum_{s\in S\setminus S^P} Q_{s,|K|} \tag{B.4}$$

$$\sum_{b\in B_s} f_{sb} = 1 \quad , s\in S^P \tag{B.5}$$

$$f_{sb} \geq 0 \quad , s\in S^P, b\in B_s \tag{B.6}$$

$$r_k \geq 0 \quad , k\in K \tag{B.7}$$

Tillegg C

Test cases

C.1 Test case 1

Stream	T_S [°C]	T_T [°C]	MCP [kW/°C]	ΔH [kW]	p_T [bar]	p_T [bar]
H1	400	60	3	1020	-	-
C1	300	380	2	160	3	1
C2	200	380	6	1080	-	-

Tabell C.1: Stream data for test case 1

C.2 Test case 2

Stream	T_S [°C]	T_T [°C]	MCP [kW/°C]	ΔH [kW]	p_S [bar]	p_T [bar]
H1	400	110	2	580	-	-
H2	400	280	3	360	2.5	1
C1	200	380	8	1440	-	-

Tabell C.2: Stream data for test case 2

Stream	T_S [°C]	T_T [°C]	MCP [kW/°C]	ΔH [kW]	p_S [bar]	p_T [bar]
H1	400	130	2	540	-	-
H2	400	130	3	810	5	1
C1	200	380	8	1440	-	-

Tabell C.3: Stream data for test case 3

Stream	T_S [°C]	T_T [°]	MCP [kW/°C]	ΔH [kW]	p_S [bar]	p_T [bar]
H1	400	60	3	1020	-	-
H2	400	280	2	240	25	1
C1	200	380	8	1440	-	-

Tabell C.4: Stream data for test case 4

C.3 Test case 3

C.4 Test case 4

C.5 Test case 5

Stream	T_S [°C]	T_T [°C]	MCP [kW/°C]	ΔH [kW]	p_S [bar]	p_T [bar]
H1	400	60	3	1020	3	1
H2	330	80	9	2250	-	-
C1	15	220	6	1230	-	-
C2	140	380	8	1920	-	-

Tabell C.5: Stream data for example 5

C.6 Test case 6

Stream	T_S [°C]	T_T [°C]	MCP [kW/°C]	ΔH [kW]	p_S [bar]	p_T [bar]
H1	400	60	6	2040	3	1
H2	330	80	9	2250	-	-
C1	15	220	6	1230	-	-
C2	140	380	8	1920	-	-
C3	40	380	3	1020	-	-

Tabell C.6: Stream data for example 6

Tillegg D

Gams code

D.1 Model without insight

D.1.1 Main code

```
$Include Cases/Testcase8.gms
Sets
SCWP(SALL)     Cold streams without pressure manipulation
SHWP(SALL)     Hot streams without pressure manipulation
SP(SALL)       All streams with pressure manipulation
*SALL2(SALL)    Copy of SALL /H1*H5,C1*C5/
;

SCWP(SALL)$SC(SALL)  = yes;
SCWP(SALL)$SCP(SALL)  = no;
SHWP(SALL)$SH(SALL)  = yes;
SHWP(SALL)$SHP(SALL)  = no;
SP(SALL)$SCP(SALL)  = yes;
SP(SALL)$SHP(SALL)  = yes;




Sets
KALL              all possible intervals                     /1*20/
KTEMP(KALL)       temperature intervals
KALL2(KALL)       copy of all possible intervals             /1*20/
KALL3(KALL)       copy of all possible intervals             /1*20/
KALL4(KALL)       copy of all possible intervals             /1*20/
KHATTK(KALL,KALL2,SALL)  map between KALL and KALL2. Used to calculate aout
KHCTS(KALL,KALL2)        map between KALL and KALL2. Used to calculate ain
KCBTTK(KALL,KALL2,SALL)  map between KALL and KALL2. Used to calculate aout
KT2(KALL2)        copy of temperature intervals
K(KALL)           cascade intervals
K2(KALL)          cascade intervals without first interval
First(KALL)       first cascade interval
MSYM(M)           set for symmetry equations
;

MSYM(M)  = yes$( ord(M) lt card(M) );

* Hot and cold side temperatures for the heat cascade and parameters
* used for creating and sorting the temperature intervals
Parameter
Tc(KALL)          cold side heat cascade temperature
Th(KALL)          hot side heat cascade temperature
Nk                number of temperature intervals
Index     /1/
Stop      /0/
Temp      /0/
Iter      /1/
Iterlim
Tmax
;
***Creating the heat cascade***
*Collecting all necessary temperature intervals
If(Thu > T0,
Tc(KALL)$( ord(KALL) eq Index ) = Thu + 100;
Index = Index + 1;
);
LOOP (SALL$SH(SALL),
Tc(KALL)$( ord(KALL) eq Index ) = Ts(SALL) - Dt;
Index = Index + 1;
Tc(KALL)$( ord(KALL) eq Index ) = Tt(SALL) - Dt;
Index = Index + 1;
);

LOOP (SALL$SC(SALL),
Tc(KALL)$( ord(KALL) eq Index ) = Ts(SALL);
```

```
Index = Index + 1;
Tc(KALL)$( ord(KALL) eq Index ) = Tt(SALL);
Index = Index + 1;
);

Tmax=SMAX(KALL,TC(KALL));
LOOP (SALL$SCP(SALL),
Tc(KALL)$( ord(KALL) eq Index ) = Ts(SALL) - Dt;
Index = Index + 1;
Tc(KALL)$( ord(KALL) eq Index ) = Tt(SALL) - Dt;
Index = Index + 1;
If ( Tt(SALL) < ( Tmax ),
         Tc(KALL)$( ord(KALL) eq Index ) = Tt(SALL) - Dt;
         Index = Index + 1;
);
);

Tc(KALL)$( ord(KALL) eq index ) = T0 - Dt;
Index = Index + 1;

LOOP (SALL$SHP(SALL),
Tc(KALL)$( ord(KALL) eq Index ) = Ts(SALL) ;
Index = Index + 1;

Tc(KALL)$( ord(KALL) eq Index ) = Tt(SALL) ;
Index = Index + 1;

);



Nk = Index-1;
*Sorting and deleting duplicating temperature intervals
While (Stop = 0,
         Stop=1;
         Loop ( KALL$( ord(KALL) < Nk),
                  If( Tc(KALL) eq Tc(KALL + 1),
                           Iterlim = Nk - 1 - ord(KALL) ;
                           For ( iter = 0 to Iterlim,
                                    Tc(KALL + iter) = Tc( KALL + (Iter + 1) )
                           );
                  Tc( KALL + ( Nk - ord(KALL) ) ) = 0;
                  Nk = Nk - 1;

                  );
                  if( Tc(KALL) < Tc(KALL + 1),
                           Temp = Tc(KALL);
                           Tc(KALL)= Tc(KALL + 1);
                           Tc(KALL + 1) = Temp;
                           Stop = 0;
                  );
         );
);

KTEMP(KALL)                = yes$(ord(KALL) lt NK + 1);
KT2(KALL2)                 = yes$(ord(KALL2) lt NK);
K(KALL)                    = yes$(ord(KALL) lt NK);
K2(KALL)                   = yes$(ord(KALL) lt NK);
K2(KALL)$(1 eq ord(KALL) ) = no;
First(KALL)                = yes$(ord(KALL) eq 1);
TH(KTEMP)                  = TC(KTEMP) + Dt;

Sets
HCPH(KALL,SALL)  Temp. int. hot side for hot streams between Tt and Ts
```

```
HCPC(KALL,SALL)  Temp. int. cold side for cold streams between Tt and Ts
KHATT(KALL,SALL) Temp. int. hot side above Tt
KCBTT(KALL,SALL) Temp. int. cold side below Tt
KCATS(KALL,SALL) Temp. int. cold side above Ts
KHBTS(KALL,SALL) Temp. int. hot side below Ts
SHB(SALL,KALL)   Hot streams pressent in temp. int. k  between Ts and Tt
SCB(SALL,KALL)   Cold streams pressent in temp. int. k between Ts and Tt
SHO(SALL,KALL)   Hot streams pressent in temp. int. k  outside Ts and Tt
SCO(SALL,KALL)   Cold streams pressent in temp. int. k  outside Ts and Tt
;

KHATT(KALL,SALL)$(SP(SALL) and Th(KALL) >= Tt(SALL) and Tt(SALL) < Tmax + 50
      and ord(KALL) <= NK ) = yes;
KCBTT(KALL,SALL)$(SP(SALL) and Tc(KALL)<= Tt(SALL) and ord(KALL)<= Nk) = yes;
KCATS(KALL,SALL)$(SP(SALL) and Tc(KALL)>= Ts(SALL) ) = yes;
KHBTS(KALL,SALL)$(SP(SALL) and Th(KALL)<= Ts(SALL) and ord(KALL)<= Nk) = yes;
SHB(SALL,KALL)$(Th(KALL)<=Ts(SALL)and Th(KALL+1)>=Tt(SALL)and SHP(SALL))=yes;
SCB(SALL,KALL)$(Tc(KALL)> Ts(SALL)and Tc(KALL+1)< Tt(SALL)and SCP(SALL))=yes;
SHO(SALL,KALL)$(( Th(KALL) > Ts(SALL) or Th(KALL+1) < Tt(SALL)) and SHP(SALL)
and ord(KALL) < Nk)=yes;
SCO(SALL,KALL)$(( Tc(KALL)<= Ts(SALL) or Tc(KALL+1)>= Tt(SALL)) and SCP(SALL)
and ord(KALL) < Nk)=yes;

HCPH(KALL,SALL)$( Th(KALL)<=Ts(SALL) and Th(KALL)>Tt(SALL) and SH(SALL))=yes;

HCPC(KALL,SALL)$( Tc(KALL)>Ts(SALL) and Tc(KALL)<=Tt(SALL) and SC(SALL))=yes;

Parameters
Qh(SALL,KALL)              Heat from hot stream s to interval k
Qc(SALL,KALL)              Heat requirement for cold stream s to interval k
Pressureratio(SALL)        The temperature ratio calculated from the pressures
Exfachot                   Hot exergy factor
Exfaccold                  Hot exergy factor
Epshin(KALL,SALL)          Small number
Epscin(KALL,SALL)          Small number
Epshout(KALL,SALL)         Small number
Epscout(KALL,SALL)         Small number
Chatt(SALL)                Cardinality of KHATT
Ccbtt(SALL)                Cardinality of KCBTT
Delta                      Small value /0.001/;
;

*Calculating the cardinality of KHATT and KBCTT
Chatt(SP) = 0;
Ccbtt(SP) = 0;
Loop( KALL$(KTEMP(KALL)),
         LOOP( SP,
                 if ( Tt(SP) <= Th(KALL) and Tt(SP) < Tmax + 50,
                         Chatt(SP) = Chatt(SP) + 1;
                 );
                 if ( Tc(KALL) <= Tt(SP),
                         Ccbtt(SP) = Ccbtt(SP) + 1;
                 );
         );
);


KHATTK(KALL,KALL2,SALL)$( SP(SALL) and ( ord(KALL2) >= ord(KALL) )
and ( ord(KALL2) < NK ) and ( ord(KALL) <= Chatt(SALL) ) ) = yes;
KHCTS(KALL,KALL2)$( ord(KALL2) >= ord(KALL) and  ord(KALL2 ) < NK ) = yes;
KCBTTK(KALL,KALL2,SALL)$( SHP(SALL) and ord(KALL2) > ( NK - Ccbtt(SALL) )
and ord(KALL2) > ( ord(KALL) - a - (Nk- Ccbtt(SALL)) )
and ord(KALL) > Chatt(SALL) and ord(KALL2) < NK ) = yes;
KCBTTK(KALL,KALL2,SALL)$( SCP(SALL) and ord(KALL2) > ( NK - Ccbtt(SALL) )
```

```
and ord(KALL2) > ( ord(KALL) - b - (Nk- Ccbtt(SALL)) )
and ord(KALL) > Chatt(SALL) and ord(KALL2) < NK ) = yes;
Display Nk,Ccbtt,a;
* Calculating Qh and Qc
Qh(SALL,KALL)$(HCPH(KALL,SALL) )   = MCP(SALL) *
                                     ( Th(KALL) - max( Th(KALL+1), Tt(SALL) ));
Qc(SALL,KALL)$HCPC(KALL,SALL)      = MCP(SALL) *
                                     ( min( TC(KALL), Tt(SALL) ) - Tc(KALL+1));
Epshin(KALL,SHP)$( Tc(KALL) eq Ts(SHP) ) = 0.001;
Epscin(KALL,SCP)$( TH(KALL) eq Ts(SCP) ) = 0.001;
Epshout(KALL,SCP)$( Th(KALL) eq Tt(SCP) ) = 0.001;
Epscout(KALL,SHP)$( Tc(KALL) eq Tt(SHP) ) = 0.001;
Th(KTEMP)= Tc(KTEMP) + Dt;
Exfachot  = (1 - ( T0 + 273.15 ) / ( Thu + 273.15 ) );
Exfaccold = (( T0 + 273.15 ) / ( Tcu + 273.15 ) - 1 );
Pressureratio(SP) = ( Ps(SP) / Pt(SP) )**( ( Kappa(SP) - 1 ) / Kappa(SP) );
Variables
ex                       Objective variable: Exergy
;
Positive Variables
r(KALL)                  acc. heat in int. k
ain(SALL,KALL,M)         Sum of alphas up to k
aout(SALL,KALL,M)        Sum of alphas up to k
f(SALL,M)                flow variable
hu                       hot utility
;
Variables
tin(SALL,M)             Inlet temperatur of expander
tout(SALL,M)            Outlet temperatur of expander
toutD(SALL,KALL,M)      Dummy outlet temperature for expander
tinD(SALL,KALL,M)       Dummy innlet temperature for expander
qhvar(SALL,KALL,M)      Heat from hot stream in int. k
qcvar(SALL,KALL,M)      Heat to cold stream in int. k
dted(SALL,KALL,M)       Dummy temperature difference for turb.&comp.
dte(SALL,M)             Temperature difference between tin and tout
rex(SALL,KALL,M)        Heat accumulation for extra potential pinch
;
Equations
Objective                Exergy produced
Qh1(SALL,KALL,M)         Heat from hot stream s above Ts or Below Tt
Qh2(SALL,KALL,M)         Heat from hot stream s between Ts and Tt
Qh3(SALL,KALL,M)         Heat from hot stream s for a special case
Qc1(SALL,KALL,M)         Heat to cold stream s between Ts-Dt and Tt
Qc2(SALL,KALL,M)         Heat to cold stream s below Ts or above tt
Qc3(SALL,KALL,M)
Qc4(SALL,KALL,M)
FirstR                   Heat accumulation first stage
Rk(KALL)                 Heat accumulation
Dteeq(SALL,KALL,M)       Dummy temperature difference
Dteq(SALL,M)             Temperature difference
Texp(SALL,M)             Expander temperatures
Teinalpha(SALL,M)        Calculating alpha_in
```

```
Teoutalpha(SALL,M)          Calculating alpha_out
Aineq(SALL,M,KALL2)         Calculating a_in
Aouteq(SALL,M,KALL2)        Calculating a_out
Tindeqc(SALL,M,KALL)        Calculating teinD cold side
Toutdeqc(SALL,M,KALL)       Calculating teoutD cold side
Tindeqh(SALL,M,KALL)        Calculating teinD hot side
Toutdeqh(SALL,M,KALL)       Calculating teoutD hot side
Alphaineq(SALL,M)           Sum of alpha_in
AlphaOuteq(SALL,M)          Sum of alpha_out
Floweq(SALL)                Sum of f
Symfloweq(SALL,M)           Symmetry breaking
Rextra(SALL,KALL,M)         Heat accumulation for potential pinch (teout)
TinUpper(SALL,M)            Upper bound for inlet temperature
TinLower(SALL,M)            Lower bound for inlet temperature
;

*****************  SOLVE WITH SOS2 SET and DICOPT  ********************
*$ontext
SOS2 variables
alphain(SALL,M,KALL)      Interpolation fraction for teinD
alphaout(SALL,M,KALL)     Interpolation fraction for teoutD
;
*$offtext

*****************  SOLVE WITH Binary variables and BARON ***************
*$include BaronS



****************************   The equations   *************************

*Objective function. Exergy
Objective..
ex              =E=   Exfachot * hu
                    + Exfaccold * sum(KALL$( ord(KALL) = (NK - 1)), r(KALL) )
                    - sum((SP,M), Mcp(SP) * f(SP,M) * dte(SP,M));


*Calculating qh and qc

Qh1(SHP,KALL,M)$( ( Th(KALL) > Ts(SHP) or Tc(KALL) < Tt(SHP) )
and ord(KALL) < Nk)..
qhvar(SHP,KALL,M)     =E= Mcp(SHP) * f(SHP,M) * (0 - dted(SHP,KALL,M));

Qh2(SHP,KALL,M)$( Tc(KALL ) > Tt(SHP) and Ts(SHP) >= Th(KALL) )..
qhvar(SHP,KALL,M)      =E= Mcp(SHP) * f(SHP,M) *
( Th(KALL) - max(Th(KALL+1),Tt(SHP)) - dted(SHP,KALL,M) );

Qh3(SHP,KALL2,M)$( Tc(KALL2)= Tt(SHP) )..
qhvar(SHP,KALL2,M)    =E= Mcp(SHP) *  f(SHP,M) *
( sum(KALL$KHATT(KALL,SHP),alphaout(SHP,M,KALL)) * ( Th(KALL2) - Th(KALL2+1))
 - dted(SHP,KALL2,M) );

Qc1(SCP,KALL,M)$( Tt(SCP) > Tc(KALL) and Ts(SCP ) < Tc(KALL)
and ord(KALL)< Nk)..
qcvar(SCP,KALL,M)     =E= Mcp(SCP) * f(SCP,M) *
( dted(SCP,KALL,M) + min( Tc(KALL),Tt(SCP) ) - Tc(KALL+1) );

Qc2(SCP,KALL,M)$((Tc(KALL) < Ts(SCP) or Tc(KALL) > Tt(SCP))
and ord(KALL) < Nk)..
qcvar(SCP,KALL,M)      =E= Mcp(SCP) * f(SCP,M) * dted(SCP,KALL,M) ;

Qc3(SCP,KALL,M)$( Tc(KALL) = Tt(SCP) )..
qcvar(SCP,KALL,M)    =E= Mcp(SCP) *  f(SCP,M) *
```

```
( (1 - aout(SCP,KALL,M))* ( Th(KALL) - Th(KALL+1) )
 + dted(SCP,KALL,M) );
Qc4(SCP,KALL,M)$( Tc(KALL) = Ts(SCP) )..
qcvar(SCP,KALL,M)    =E= Mcp(SCP) *  f(SCP,M) *
((1 - ain(SCP,KALL,M))* ( Tc(KALL) - Tc(KALL+1) )
 + dted(SCP,KALL,M) );
*Calculating heat accumulation
FirstR(KALL)$FIRST(KALL)..
r(KALL)                =E= hu
                        + sum( SHWP,  Qh(SHWP,KALL) )
                        + sum( (SHP,M), qhvar(SHP,KALL,M) )
                        - sum( SCWP, Qc(SCWP,KALL) )
                        - sum( (SCP,M), qcvar(SCP,KALL,M) );
Rk (KALL)$K2(KALL)..
r(KALL)                =E= r(KALL - 1)
                         + sum( SHWP,  Qh(SHWP,KALL) )
                         + sum( (SHP,M), qhvar(SHP,KALL,M) )
                         - sum( SCWP,  Qc(SCWP,KALL) )
                         - sum( (SCP,M), qcvar(SCP,KALL,M) );
*Temperature difference between tein and teout and between teinD and teoutD
Dteeq(SALL,KALL,M)$(SP(SALL) and ord(KALL) < Nk) ..
dted(SALL,KALL,M)     =E= tinD(SALL,KALL,M) - toutD(SALL,KALL,M);
Dteq(SALL,M)$SP(SALL)..
dte(SALL,M)           =E= tin(SALL,M) - tout(SALL,M);
*Turbine/compressor
Texp(SALL,M)$SP(SALL)..
tin(SALL,M)     =E= ( tout(SALL,M) + 273.15 ) * Pressureratio(SALL) - 273.15;
*Calculating alpha values
Teinalpha(SALL,M)$SP(SALL)..
tin(SALL,M)    =E= sum( KALL$KCATS(KALL,SALL),
                      ( Tc(KALL) + Epscin(KALL,SALL)) * alphain(SALL,M,KALL))
                 + sum( KALL$KHBTS(KALL,SALL),
                      ( Th(KALL) - Epshin(KALL,SALL)) * alphain(SALL,M,KALL))»
;
Teoutalpha(SALL,M)$SP(SALL)..
tout(SALL,M) =E= sum( KALL$KHATT(KALL,SALL),
                     ( Th(KALL) + Epshout(KALL,SALL)) * alphaout(SALL,M,KALL)»
)
                + sum( KALL$KCBTT(KALL,SALL),
                     ( Tc(KALL) - Epscout(KALL,SALL) )  *
                     alphaout(SALL,M,KALL +(Chatt(SALL) - Nk + Ccbtt(SALL))))»
;
*Calculating a values
Aineq(SALL,M,KALL2)$(KT2(KALL2) and SP(SALL) )..
ain(SALL,KALL2,M)   =E= sum( KALL$KHCTS(KALL,KALL2), alphain(SALL,M,KALL) );
```

```
Aouteq(SALL,M,KALL2)$(KT2(KALL2) and SP(SALL) )..
aout(SALL,KALL2,M)=E= sum(KALL$KHATTK(KALL,KALL2,SALL),alphaout(SALL,M,KALL))
                    + sum(KALL$KCBTTK(KALL,KALL2,SALL),alphaout(SALL,M,KALL))»
;


*Interpolating to calculate teinD/teoutD

Tindeqc(SCP,M,KALL)$( ord(KALL) < Nk )..
tinD(SCP,KALL,M)  =E= Tc(KALL) * ain(SCP,KALL,M)
                       + Tc(KALL+1) * ( 1 - ain(SCP,KALL,M) );

Toutdeqc(SCP,M,KALL)$( ord(KALL) < Nk )..
toutD(SCP,KALL,M)  =E= Tc(KALL) * aout(SCP,KALL,M)
                      + Tc(KALL+1) * ( 1 - aout(SCP,KALL,M) );

Tindeqh(SHP,M,KALL)$( ord(KALL) < Nk )..
tinD(SHP,KALL,M)   =E= Th(KALL) * ain(SHP,KALL,M)
                      + Th(KALL+1) * ( 1 - ain(SHP,KALL,M) );

Toutdeqh(SHP,M,KALL)$( ord(KALL) < Nk )..
toutD(SHP,KALL,M)  =E= Th(KALL) * aout(SHP,KALL,M)
                     + Th(KALL+1) * ( 1 - aout(SHP,KALL,M) );


*Sum of alhpas

Alphaineq(SALL,M)$( SCP(SALL) or SHP(SALL) )..
1       =E= sum( KALL$KCATS(KALL,SALL), alphain(SALL,M,KALL) )
          + sum( KALL$KHBTS(KALL,SALL), alphain(SALL,M,KALL) );

AlphaOuteq(SALL,M)$( SCP(SALL) or SHP(SALL) )..
1       =E= sum( KALL$KHATT(KALL,SALL), alphaout(SALL,M,KALL) )
          + sum( KALL$KCBTT(KALL,SALL),
            alphaout(SALL,M,KALL + (Chatt(SALL) - Nk + Ccbtt(SALL) )) );



*Sum of fractions

Floweq(SP)..
1       =E= sum( M,f(SP,M) );



*Breaking symmetery

Symfloweq(SP,M)$MSYM(M)..
f(SP,M)             =G= f(SP,M+1);

*Upper and øpwer bounds for the inlet temperature
TinUpper(SALL,M)$SP(SALL)..
tin(SALL,M) =L= Thu;

TinLower(SALL,M)$SP(SALL)..
tin(SALL,M) =G= Tcu;


*Extra heat accumulation for extra potential pinch point

Rextra(SP,KALL,M)$(Tmax+Dt>Th(KALL) * Pressureratio(SP) and ord(KALL) < Nk)..
rex(SP,KALL,M)   =E= r(KALL-1) + sum(SALL$SHWP(SALL),Qh(SALL,KALL) *
                                  ( 1 - aout(SP,KALL,M) ) )
```

```
                               - sum( SALL$SCWP(SALL),Qc(SALL,KALL) *
                                ( 1 - aout(SP,KALL,M) ) )

                               + sum( (M2,SALL)$SHB(SALL,KALL),
Mcp(SALL) * f(SALL,M2) * (Th(KALL) - ( tinD(SALL,KALL,M2) + toutD(SP,KALL,M)
+ Sqrt(Sqr(tinD(SALL,KALL,M2) - toutD(SP,KALL,M)) + Sqr(delta)) - delta )/2
+ ( toutD(SALL,KALL,M2) + toutD(SP,KALL,M)
+ Sqrt( Sqr(toutD(SALL,KALL,M2) - toutD(SP,KALL,M)) + Sqr(delta))- delta)/2
- toutD(SP,KALL,M)) )

                              - sum((M2,SALL)$SCB(SALL,KALL),
Mcp(SALL) * f(SALL,M2) * (Tc(KALL) - (toutD(SALL,KALL,M2) + toutD(SP,KALL,M)
+ Sqrt( Sqr(toutD(SALL,KALL,M2) - toutD(SP,KALL,M)) + Sqr(delta)) -delta)/2
+ ( tinD(SALL,KALL,M2) + toutD(SP,KALL,M)
+ Sqrt( Sqr(tinD(SALL,KALL,M2) - toutD(SP,KALL,M)) + Sqr(delta) ) -delta)/2
- toutD(SP,KALL,M)) )

                              + sum( (M2,SALL)$(SHO(SALL,KALL)  ),
Mcp(SALL) * f(SALL,M2) *( (toutD(SALL,KALL,M2) + toutD(SP,KALL,M)
+ Sqrt(Sqr(toutD(SALL,KALL,M2) - toutD(SP,KALL,M)) + Sqr(delta) ) -delta)/2
- ( tinD(SALL,KALL,M2) + toutD(SP,KALL,M)
+ Sqrt(Sqr(tinD(SALL,KALL,M2) - toutD(SP,KALL,M)) +Sqr(delta) ) - delta)/2))

                             - sum( (M2,SALL)$SCO(SALL,KALL),
Mcp(SALL) * f(SALL,M2) * ( toutD(SALL,KALL,M2) + toutD(SP,KALL,M)
+ Sqrt( Sqr(toutD(SALL,KALL,M2) - toutD(SP,KALL,M)) + Sqr(delta)) - delta)/2
-( tinD(SALL,KALL,M2) + toutD(SP,KALL,M)
+ Sqrt( Sqr(tinD(SALL,KALL,M2) - toutD(SP,KALL,M)) + Sqr(delta) ) -delta)/2)
;

Model HeatCascade/all/ ;



Solve HeatCascade using minlp minimizing ex ;


Parameter Work(SALL);
Work(SP) = MCP(SP) * sum(M,f.l(SP,M) * ( tin.l(SP,M) - tout.l(SP,M) ) );

Display Work,ex.l,hu.l;
```

D.1.2 Baron implementation

```
*********************   SOLVE WITH BARON   ***********************

Positive variables
alphain(SALL,M,KALL)      Interpolation fraction for teind
alphaout(SALL,M,KALL)     Interpolation fraction for teoutd
;
Binary variables
din(SALL,M,KALL)          SOS2 substitute binary variable
dout(SALL,M,KALL)         SOS2 substitute binary variable
;

Equations
Firstsos2in(SALL,M)
Alphasos2in(SALL,M,KALL)
Lastsos2in(SALL,M,KALL)
Firstsos2out(SALL,M)
Alphasos2out(SALL,M,KALL)
Lastsos2out(SALL,M,KALL)
Sumdin(SALL,M)
Sumdout(SALL,M);
;
Firstsos2in(SALL,M)$SP(SALL)..
0 =L= din(SALL,M,"1")-alphain(SALL,M,"1");
Alphasos2in(SALL,M,KALL)$(SP(SALL)and ord(KALL) < Nk and ord(KALL) >1)..
0 =L= din(SALL,M,KALL)+din(SALL,M,KALL-1)-alphain(SALL,M,KALL);
Lastsos2in(SALL,M,KALL)$(SP(SALL) and ord(KALL) = NK)..
0 =L= din(SALL,M,KALL-1)-alphain(SALL,M,KALL);

Firstsos2out(SALL,M)$SP(SALL)..
0 =L= dout(SALL,M,"1")-alphaout(SALL,M,"1");
Alphasos2out(SALL,M,KALL)$(SP(SALL)and ord(KALL) < (Nk+2) and ord(KALL) >1)..
0 =L= dout(SALL,M,KALL)+dout(SALL,M,KALL-1)-alphaout(SALL,M,KALL);
Lastsos2out(SALL,M,KALL)$(SP(SALL) and ord(KALL) = (Nk+2))..
0 =L= dout(SALL,M,KALL-1)-alphaout(SALL,M,KALL);
Sumdin(SALL,M)$(SP(SALL))..
sum(KALL$(ord(KALL)<=Nk),din(SALL,M,KALL)) =E= 1;
Sumdout(SALL,M)$(SP(SALL))..
sum(KALL$(ord(KALL)<=(Nk+2)),dout(SALL,M,KALL)) =E= 1;
option MINLP = BARON;
```

D.2 Gams code model with insight

D.2.1 Main file with optimization

```
$Include Cases/Testcase8.gms
$include Pinch
$include Branches
$include Cascade

**************************  Optimization  **************************

Variables
ex_                 Exergy;

Positive Variables
r_(KALL)            Heat residual
hu_                 Hot utility consumption
f(SALL,SALL2)       MCP fraction;

Equations
ObjFunc             Objective function
Firsteq
HB(KALL)            Heat Balance
MB(SALL)            Mass Balance;


ObjFunc..
ex_        =E=    (1 - (T0 + 273.15)/(Thu + 273.15)) * hu_
+ ((T0 + 273.15)/(Tcu + 273.15)-1) * sum(KALL$(ord(KALL)=(Nk-1)),r_(KALL))
+ sum(SALL$SP(SALL),
sum(SALL2$B1(SALL,SALL2),Td(SALL,SALL2) * MCP(SALL2) * f(SALL,SALL2) ));

Firsteq..
r_("1")    =E= hu_
+ sum(SALL$SP(SALL), sum(SALL2$B1(SALL,SALL2),f(SALL,SALL2)*
                         sum(SALL3$Z(SALL,SALL2,SALL3),QP(SALL3,"1"))))
+ sum(SALL$SW(SALL), Q(SALL,"1"));

HB(KALL)$(ord(KALL) > 1 and ord(KALL) < Nk )..
r_(KALL)   =E= r_(KALL-1)
       + sum(SALL$SP(SALL), sum(SALL2$B1(SALL,SALL2),f(SALL,SALL2)*
                         sum(SALL3$Z(SALL,SALL2,SALL3),QP(SALL3,KALL))))
       + sum(SALL$SW(SALL), Q(SALL,KALL));

MB(SALL)$SP(SALL)..
1         =E= sum(SALL2$B1(SALL,SALL2),f(SALL,SALL2));

option limrow = 20;
Model NewModel/ObjFunc, Firsteq, HB, MB/;
Solve NewModel using lp minimizing ex_ ;


*Calculating work and printing results

Parameter Work(SALL)
tin_
tout_
MCP_;
Work(SALL)$SP(SALL)=
sum(SALL2$B1(SALL,SALL2), Td(SALL,SALL2) * MCP(SALL2) * f.l(SALL,SALL2) );

Display ex_.l,Work,hu_.l,f.l;

Loop((SALL,SALL2)$B1(SALL,SALL2),
  if(f.l(SALL,SALL2)>0,
    tin_ = Tt(SALL2);
    tout_ = sum(SALL3$(Z(SALL,SALL2,SALL3) ),
            Ts(SALL3));
```

```
    tout_ =tout_ - Ts(SALL2);
    Display "Inlet temperature: ",tin_;
    Display "Outlet temperature: ",tout_;
    MCP_ = MCP(SALL) * f.l(SALL,SALL2);
    Display "MCP: ",MCP_;
);
);

```

D.2.2 Hot utility model

```
**************** Hot utility model *************************

*$Include Test7b.gms


Set
S(SALL);
S(SALL)= SH(SALL)+SC(SALL);


Sets
KALL              all possible intervals                  /1*100/
KTEMP(KALL)       temperature intervals
K(KALL)           cascade intervals
K2(KALL)
First(KALL)       first cascade interval
;


Parameter Tc(KALL) cold side heat cascade temperature
NK
Index     /1/
Stop      /0/
Count     /0/
Temp      /0/
Iter      /1/
Iterlim;
;

LOOP (SALL$SH(SALL),
Tc(KALL)$(ord(KALL) = Index) = Ts(SALL) - Dt;
Index = Index + 1;
);
LOOP (SALL$SC(SALL),
Tc(KALL)$(ord(KALL) = Index) = Ts(SALL);
Index = Index + 1;
);
*Tc(KALL)$(ord(KALL) = Index) = Thu - Dt;
*Index = Index + 1;
Tc(KALL)$(ord(KALL) = Index) = T0;
Index = Index + 1;
Tc(KALL)$(ord(KALL) = Index) = Tcu;

Scalar NK number of heat cascade intervals;
Nk=Index;


While (Stop = 0,
         Stop=1;
         Loop ( KALL$( ord(KALL) < Nk),
                 If( Tc(KALL) eq Tc(KALL + 1),
                         Iterlim = Nk - 1 - ord(KALL) ;
                         For ( iter = 0 to Iterlim,
                                 Tc(KALL + iter) = Tc( KALL + (Iter + 1) )
                         );
                 Tc( KALL + ( Nk - ord(KALL) ) ) = 0;
                 Nk = Nk - 1;

                 );
                 if( Tc(KALL) < Tc(KALL + 1),
                         Temp = Tc(KALL);
                         Tc(KALL)= Tc(KALL + 1);
                         Tc(KALL + 1) = Temp;
                         Stop = 0;
```

```
                    );
          );
);

KTEMP(KALL)                  = yes$(ord(KALL) lt NK + 1);
K(KALL)                      = yes$(ord(KALL) lt NK);
K2(KALL)                     = yes$(ord(KALL) lt NK);
K2(KALL)$(1 eq ord(KALL))    = no;
First(KALL)                  = yes$(ord(KALL) eq 1);
Display Nk,Tc,KTEMP,K;

Sets
HCPH(KALL,SALL) Over Ts
HCPC(KALL,SALL) Over Ts
;

Parameters
Th(KALL)                  hot side heat cascade temperature
Qh(SALL,KALL)             Heat from hot stream s to interval k
Qc(SALL,KALL)             Heat requirement for cold stream s to interval k
;
Th(KTEMP)=Tc(KTEMP) + Dt;


HCPH(KALL,SALL)$(KTEMP(KALL) and SH(SALL)) = yes$(
(Th(KALL+1) >= Tt(SALL) or (Tt(SALL) <= Th(KALL) and Tt(SALL) >= Th(KALL+1)))
                                                  and Th(KALL)<= Ts(SALL) );
HCPC(KALL,SALL)$(KTEMP(KALL) and SC(SALL)) = yes$( Tc(KALL+1) >= Ts(SALL) and
(Tc(KALL)<= Tt(SALL) or (Tt(SALL) <= Tc(KALL) and Tt(SALL)>= Tc(KALL+1) )));

Qh(SALL,KALL)$HCPH(KALL,SALL) = Mcp(SALL) *
                                  ( TH(KALL)- max( TH(KALL+1), Tt(SALL) ) );
Qc(SALL,KALL)$HCPC(KALL,SALL) = Mcp(SALL) *
                                  ( min( TC(KALL), Tt(SALL)) - Tc(KALL+1) );

Variables
hu                       hot utility
;

Positive Variables
r(KALL)                  acc. heat in int. k
;

Equations
FirstR                    heat accumulation
Rk(KALL)                  heat accumulation
;

FirstR(KALL)$FIRST(KALL)..        r(KALL)        =E= hu
                                                   + sum( SH, Qh(SH,KALL) )
                                                   - sum( SC, Qc(SC,KALL) );
Rk  (KALL)$K2(KALL)..             r(KALL)        =E= r(KALL-1)
                                                   + sum( SH, Qh(SH,KAll) )
                                                   - sum( SC, Qc(SC,KALL) );

Model HeatCascade/all/;

Solve HeatCascade using lp minimizing hu ;

Scalar
OrdSh
Ordsc;

Ordsh=card(SH);
```

```
Ordsc=card(SC);

Display hu.l,Tc,Qh,Qc;
```

D.2.3 Creating branches

```
****************** Creating branches and segments *****************
Scalar Tpinch/0/
Hotpinch /0/
Tcold
Temp;
Tcold=min(Tcu,T0)


loop( KALL$(ord(KALL) < Nk),
   if( r.l(KALL) = 0,
     Tpinch = Tc(KALL+1)
   );
);

Hotpinch = Tpinch;
Parameter Tout1           Outlet temperature of the turbine
Pressureratio(SALL)       Pressureratio for expansion or compression
;

Pressureratio(SP) = ( Ps(SP) / Pt(SP) )**( ( Kappa(SP) - 1 ) / Kappa(SP) );


Sets B1(SALL,SALL)
Z(SALL,SALL,SALL);
Parameters
Tin
Tin1
Tou
Td(SALL,SALL)
Hot
Thuout
Temp;

* Loop for creating branches and segments for interesting temperatures
LOOP( SALL$(SCP(SALL) or SHP(SALL)),
    if(Ts(SALL)<Tt(SALL),
*         Cold stream
          Tin1 = (Tt(SALL) + 273.15 ) * Pressureratio(SALL) - 273.15;
    else
*         Hot stream
          Tin1 = (Tt(SALL) + 273.15 ) * Pressureratio(SALL) - 273.15;
    );

    if (Tin1 > Ts(SALL) and Tin1 < Thu + 100 and Tin1 > Tcold,
      ordsc = ordsc + 1;
      SCNEW(SALLC)$(ord(SALLC) =  ordsc) = yes;
      Ts(SALLC)$(ord(SALLC) =  ordsc) = Ts(SALL);
      Tt(SALLC)$(ord(SALLC) =  ordsc) = Tin1;
      MCP(SALLC)$(ord(SALLC) =  ordsc) = MCP(SALL);
      B1(SALL,SALLC)$(ord(SALLC) =  ordsc)  = yes;
      Z(SALL,SALLC,SALLC2)$(ord(SALLC) = ordsc and ord(SALLC2) =ordsc)= yes;
      Td(SALL,SALLC)$(ord(SALLC) =  ordsc) = Tt(SALL) - Tin1;
    elseif Tin1 <= Ts(SALL) and Tin1 < Thu + 100 and Tin1 > Tcold,
      ordsh = ordsh + 1;
      SHNEW(SALLH)$(ord(SALLH) =  ordsh) = yes;
      Ts(SALLH)$(ord(SALLH) =  ordsH) = Ts(SALL);
      Tt(SALLH)$(ord(SALLH) =  ordsH) = Tin1;
      MCP(SALLH)$(ord(SALLH) =  ordsH) = MCP(SALL);
      B1(SALL,SALLH)$(ord(SALLH) =  ordsH)  = yes;
      Z(SALL,SALLH,SALLH2)$(ord(SALLH) = ordsh and ord(SALLH2)= ordsh) =yes;
      Td(SALL,SALLH)$(ord(SALLH) =  ordsH) = Tt(SALL) - Tin1  ;
      );

*         Create branches and segments for potential pinch points
```

```
Thuout = (Thu + 273.15 ) / Pressureratio(SALL) - 273.15;
LOOP ( KALL$(Th(KALL)<= Thu and Th(KALL)> Tcold
       and ((Th(KALL)<> Ts(SALL) or Tc(KALL)<> Ts(SALL))
       or Ts(SALL) = Thu)and ord(KALL)<= Nk),

     Temp = Th(KALL);

     if(Ts(SALL)<Th(KALL),
*        Cold stream
         Tout1 = (Tc(KALL) + 273.15 ) / Pressureratio(SALL) - 273.15;
     else
*        Hot stream
         Tout1 = (Th(KALL) + 273.15 ) / Pressureratio(SALL) - 273.15;
     );

   if (Th(KALL) > Ts(SALL)and Tout1 < Thu + 50
       and (Tout1 > Tcold - 50 or Thuout < Tcold),
      ordsc = ordsc + 1;
      SCNEW(SALLC)$(ord(SALLC) =  ordsc) = yes;
      Ts(SALLC)$(ord(SALLC) =  ordsc) = Ts(SALL);
      Tt(SALLC)$(ord(SALLC) =  ordsc) = Tc(KALL);
      MCP(SALLC)$(ord(SALLC) =  ordsc) = MCP(SALL);
      B1(SALL,SALLC)$(ord(SALLC) =  ordsc)  = yes;
      Z(SALL,SALLC,SALLC2)$(ord(SALLC) = ordsc and ord(SALLC2 =ordsc)=yes;
      Td(SALL,SALLC)$(ord(SALLC) =  ordsc) = Tout1 - Tc(KALL);
      hot = 0;
   elseif Tout1 < Thu + 50 and (Tout1 > Tcold - 50 or Thuout < Tcold),
      ordsh = ordsh + 1;
      SHNEW(SALLH)$(ord(SALLH) =  ordsh) = yes;
      Ts(SALLH)$(ord(SALLH) =  ordsH) = Ts(SALL);
      Tt(SALLH)$(ord(SALLH) =  ordsH) = Th(KALL);
      MCP(SALLH)$(ord(SALLH) =  ordsH) = MCP(SALL);
      B1(SALL,SALLH)$(ord(SALLH) =  ordsH)  = yes;
      Z(SALL,SALLH,SALLH2)$(ord(SALLH) = ordsh and ord(SALLH2)=ordsh)=yes;
      Td(SALL,SALLH)$(ord(SALLH) =  ordsH) = Tout1 - Th(KALL) ;
      Hot = 1;
   );
   if (Tout1 > Tt(SALL) and Tout1 < Thu + 50
       and (Tout1 > Tcold - 50 or Thuout < Tcold),
     ordsh = ordsh + 1;
     SHNEW(SALLH)$(ord(SALLH) =  ordsh) = yes;
     if (Hot = 1,
      Z(SALL,SALLH,SALLH2)$(ord(SALLH)=(ordsh-1)and ord(SALLH2)=ordsh)=yes;
     else
      Z(SALL,SALLC,SALLH2)$(ord(SALLC) =ordsc and ord(SALLH2)=ordsh)= yes;
      );

      Ts(SALLH)$(ord(SALLH) =  ordsh) = Tout1;
      Tt(SALLH)$(ord(SALLH) =  ordsh) = Tt(SALL);
      MCP(SALLH)$(ord(SALLH) =  ordsh) = MCP(SALL);
   elseif Tout1 < Thu + 50 and (Tout1 > Tcold - 50 or Thuout < Tcold) ,
     ordsc = ordsc + 1;
     SCNEW(SALLC)$(ord(SALLC) =  ordsc) = yes;
     if (Hot = 1,
      Z(SALL,SALLH,SALLC2)$(ord(SALLH) =ordsh and ord(SALLC2)= ordsc)= yes;
     else
      Z(SALL,SALLC,SALLC2)$(ord(SALLC)=(ordsc-1)and ord(SALLC2)=ordsc)=yes;
      );
     Ts(SALLC)$(ord(SALLC) =  ordsc) = Tout1;
     Tt(SALLC)$(ord(SALLC) =  ordsc) = Tt(SALL);
     MCP(SALLC)$(ord(SALLC) =  ordsc) = MCP(SALL);
      );
   );
*  Second pinch generation
```

```
  Loop(SALL2$(ord(SALL2) = ord(SALL)),

   if(Ts(SALL) >= Tpinch and Ts(SALL) > Tt(SALL),
       Hotpinch = Tpinch + Dt;
   elseif   Ts(SALL)> Tpinch,
       Hotpinch = Tpinch + Dt;
   );
   Tin1 = (Hotpinch + 273.15 ) / Pressureratio(SALL) - 273.15;

   if(Tt(SALL)>= Tin1,
*           Cold pinch
       if(Ts(SALL)> Tin1,
*           Hot stream
         Tin1 = Tin1 + Dt;
       );
   else
*           Hot pinch
       if(Ts(SALL)<Tin1,
*           Cold stream
         Tin1 = Tin1 - Dt;
       );
   );
   Tout1 = (Tin1 + 273.15 ) / Pressureratio(SALL2) - 273.15;
   if (Tin1 > Ts(SALL2) and Tin1 < (Thu + 50) and Tin1 > (Tcold - 50)
       and Tout1 < (Thu + 50) and Tout1 > (Tcold - 50),
          ordsc = ordsc + 1;
          SCNEW(SALLC)$(ord(SALLC) =  ordsc) = yes;
          Ts(SALLC)$(ord(SALLC) =  ordsc) = Ts(SALL2);
          Tt(SALLC)$(ord(SALLC) =  ordsc) = Tin1;
          MCP(SALLC)$(ord(SALLC) =  ordsc) = MCP(SALL2);
          B1(SALL2,SALLC)$(ord(SALLC) =  ordsc)  = yes;
          Z(SALL2,SALLC,SALLC2)$(ord(SALLC) =  ordsc
          and ord(SALLC2) = ordsc)= yes;
          Td(SALL2,SALLC)$(ord(SALLC) =  ordsc) = Tout1-Tin1;
          Hot = 0;
   elseif Tin1 <= Ts(SALL2) and Tin1 < (Thu + 50) and Tin1 > (Tcold - 50)
       and Tout1 < (Thu + 50) and Tout1 > (Tcold - 50),
          ordsh = ordsh + 1;
          SHNEW(SALLH)$(ord(SALLH) =  ordsh) = yes;
          Ts(SALLH)$(ord(SALLH) =  ordsH) = Ts(SALL2);
          Tt(SALLH)$(ord(SALLH) =  ordsH) = Tin1;
          MCP(SALLH)$(ord(SALLH) =  ordsH) = MCP(SALL2);
          B1(SALL2,SALLH)$(ord(SALLH) =  ordsH)  = yes;
          Z(SALL2,SALLH,SALLH2)$(ord(SALLH) =  ordsh
          and ord(SALLH2)= ordsh) = yes;
          Td(SALL2,SALLH)$(ord(SALLH) =  ordsH) = Tout1-Tin1 ;);
          Hot = 1;
   if (Tout1 > Tt(SALL) and Tout1 < (Thu + 50) and Tout1 > (Tcold - 50)
       and Tout1 < (Thu + 50) and Tout1 > (Tcold - 50),
      ordsh = ordsh + 1;
      SHNEW(SALLH)$(ord(SALLH) =  ordsh) = yes;
     if (Hot = 1,
      Z(SALL,SALLH,SALLH2)$(ord(SALLH)=(ordsh-1)and ord(SALLH2)=ordsh)=yes;
     else
      Z(SALL,SALLC,SALLH2)$(ord(SALLC) = ordsc and ord(SALLH2)=ordsh)=yes;
      );

      Ts(SALLH)$(ord(SALLH) =  ordsh) = Tout1;
      Tt(SALLH)$(ord(SALLH) =  ordsh) = Tt(SALL);
      MCP(SALLH)$(ord(SALLH) =  ordsh) = MCP(SALL);
   elseif Tout1 < (Thu + 50) and Tout1 > (Tcold -50)
    and Tout1 < (Thu + 50) and Tout1 > (Tcold - 50),
    ordsc = ordsc + 1;
    SCNEW(SALLC)$(ord(SALLC) =  ordsc) = yes;
```

```
        if (Hot = 1,
         Z(SALL,SALLH,SALLC2)$(ord(SALLH) = ordsh and ord(SALLC2)=ordsc)=yes;
        else
         Z(SALL,SALLC,SALLC2)$(ord(SALLC)=(ordsc-1)and ord(SALLC2)=ordsc)=yes;
           );
        Ts(SALLC)$(ord(SALLC) =  ordsc) = Tout1;
        Tt(SALLC)$(ord(SALLC) =  ordsc) = Tt(SALL);
        MCP(SALLC)$(ord(SALLC) =  ordsc) = MCP(SALL);
          );

       );
);
```

D.2.4 Creating the heat cascade

```
**************** Creating the heat cascade ********************


*Creating heat temperature intervals
Index = 1;

LOOP (SALL$(SH(SALL) or SHNEW(SALL)),
Tc(KALL)$(ord(KALL) = Index) = Ts(SALL) - Dt;
Index = Index + 1;
);
LOOP (SALL$(SC(SALL) or SCNEW(SALL)),
Tc(KALL)$(ord(KALL) = Index) = Ts(SALL);
Index = Index + 1;
);
Tc(KALL)$(ord(KALL) = Index) = T0;

Scalar NK number of heat cascade intervals;
Nk=Index;
Stop=0;
While (Stop = 0,
         Stop=1;
         Loop ( KALL$( ord(KALL) < Nk),
                 If( Tc(KALL) eq Tc(KALL + 1),
                         Iterlim = Nk - 1 - ord(KALL) ;
                         For ( iter = 0 to Iterlim,
                                 Tc(KALL + iter) = Tc( KALL + (Iter + 1) )
                         );
                 Tc( KALL + ( Nk - ord(KALL) ) ) = 0;
                 Nk = Nk - 1;

                 );
                 if( Tc(KALL) < Tc(KALL + 1),
                         Temp = Tc(KALL);
                         Tc(KALL)= Tc(KALL + 1);
                         Tc(KALL + 1) = Temp;
                         Stop = 0;
                 );
         );
);


* Calculating the heat requirement for each stream, branch and segment
Parameter Qp(SALL,KALL)
Q(SALL,KALL);

Th(KALL)$(ord(KALL)<=Nk) = Tc(KALL) + Dt;

HCPH(KALL,SALL)$(ord(KALL) < Nk and (SH(SALL) or SHNEW(SALL) )) = yes$(
(Th(KALL+1) >= Tt(SALL) or (Tt(SALL) <= Th(KALL) and Tt(SALL) >= Th(KALL+1)))
and Th(KALL)<= Ts(SALL) );

HCPC(KALL,SALL)$(ord(KALL) < Nk and SC(SALL) or SCNEW(SALL))= yes$(Tc(KALL+1)
>= Ts(SALL) and ( Tc(KALL)<= Tt(SALL) or (Tt(SALL) <= Tc(KALL)
and Tt(SALL)>= Tc(KALL+1) ) ) );

Qp(SALL,KALL)$(HCPH(KALL,SALL) and SHNEW(SALL)) = Mcp(SALL) *
                                ( TH(KALL)- max( TH(KALL+1), Tt(SALL) ) );
Qp(SALL,KALL)$(HCPC(KALL,SALL) and SCNEW(SALL)) = - Mcp(SALL) *
                                ( min( TC(KALL), Tt(SALL)) - Tc(KALL+1) );
Q(SALL,KALL)$(HCPH(KALL,SALL) and SH(SALL)) = Mcp(SALL) *
                                ( TH(KALL)- max( TH(KALL+1), Tt(SALL) ) );
Q(SALL,KALL)$(HCPC(KALL,SALL) and SC(SALL)) = - Mcp(SALL) *
                                ( min( TC(KALL), Tt(SALL)) - Tc(KALL+1) );

```

D.2.5 Example of data input

```
*Test case 1
Sets
SALL              All possible streams             /H1*H50, C1*C50/
SH(SALL)          Hot streams                      /H1/
SC(SALL)          Cold streams                     /C1*C2/
SCP(SALL)         Cold streams w. press. manip.    /C1/

Parameters
Ts(SALL)          supply temperature
/ H1 400
C1 300
C2 200/
Tt(SALL)          target temperature
/ H1 60
C1 380
C2 380/
MCP(SALL)         mass flow multiplied with cp for stream s
/ H1 3
C1 2
C2 6 /
Ps(SALL)           Supply pressure
/ C1 300/
Pt(SALL)           Target pressure
/ C1 100/
Dt                Minimum temperature difference
/20/
T0                Ambient temperature
/15/
Thu               Hot utility temperature
/400/
Tcu               Hot utility temperature
/15/
Kappa(SALL)       Kappa
/C1 1.4/
;

Sets
SALL2(SALL)                                       /H1*H50, C1*C50/
SALL3(SALL)                                       /H1*H50, C1*C50/
SALLH(SALL)                                       /H1*H50/
SALLC(SALL)                                       /C1*C50/
SALLH2(SALL)                                      /H1*H50/
SALLC2(SALL)                                      /C1*C50/
SHNEW(SALL)
SCNEW(SALL)
SHP(SALL)         Hot streams w. press. manip.
SP(SALL)          All streams with pressure manipulation
SW(SALL)          Streams without pressure manipulation
M                 Parts for SCP and SHP            /1*2/
M2(M)             Copy of parts                    /1*2/
;
*SALL2(SALL)$(SC(SALL) or SH(SALL)) =yes;
SHP(SALL)           = no;
SP(SALL)$SCP(SALL)  = yes;
SP(SALL)$SHP(SALL)  = yes;
SW(SALL)            = yes;
SW(SALL)$SP(SALL)  = no;

Scalar a /0/;
Scalar b /2/;
```