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A Profit Maximisation Model for Operation of a Tidal Lagoon Power Plant

Determination of Optimal Operational Schedule and Potential Value Added by Integration in a High Intermittent Power System

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Submission date: June 2016
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Problem Definition

A stochastic profit maximisation model for operation of a tidal lagoon power plant including production head effects, power market prices and start-up costs will be developed. Optimal annual figures for production volumes and operational profit for specified technical characteristics will be determined and the effect of installed capacity will be discussed. Uncertainty in power market prices will be included. The value of short-term flexibility will be estimated based on different power market price characteristics. The Swansea Bay Tidal Lagoon project will be the case study for the model.

An extension to an already developed total system model for the European power system – the EMPIRE – to include tidal lagoon power generation will be developed. The extension will be based on the findings from the operational model presented above. Potential system cost reductions and environmental effects by integration of tidal lagoon technology in UK will be evaluated over 50 years' time horizon. Relevant policy cases and market design parameters will be discussed.

Preface

This thesis concludes our Master of Science at the Norwegian University of Science and Technology with specialisation in Applied Economics and Optimisation under the department of Industrial Economics and Technology Management. The research was conducted spring 2016.

The final scope and development of the research are formulated by the authors under guidance of Asgeir Tomasgard. The topic is based on an idea from David Curran in Statkraft. We propose extended research on the topic of new renewable power technologies to be accomplished in future. We hope our work can be a small step on the way towards sustainable energy supply.

We would like to thank our supervisor Professor Asgeir Tomasgard for interesting and stimulating discussions, and for guiding us in the right direction. We would also like to thank our co-supervisor Christian Skar for allowing us to use and expand the power system model EMPIRE and for enlightening help in the research. In addition, we would like to thank Gerardo Alfredo Perez-Valdes for providing the power system data series.

Further, we would like to thank Statkraft, and especially Knut Dyrstad, David Curran and Geir Fuglseth for a fruitful collaboration and for sharing their industry knowledge with us. They have inspired us to investigate new fields within renewable energy. Especially, we are grateful for Statkraft London receiving us in March and for an informative and interesting stay. We would also like to thank SINTEF for providing the necessary founds to finance our study trip to London.

Finally, thanks to family and friends for feedback on our thesis and support during the work.

Abstract

A stochastic profit maximisation model for operation of a tidal lagoon power plant including production head effects, power market prices and start-up costs is developed. The model is formulated as a two-stage stochastic mixed integer programming problem with bidding decisions in the first stage, and plant operation and real-time sales in the second stage. The Swansea Bay Tidal Lagoon project is taken as case study and the analysis is performed including sales in the United Kingdom (UK) day-ahead (spot) and intraday market for different power price characteristics. The largest annual energy production is estimated to 317 GWh and operational income is estimated to 15 million Great British Pounds (GBP) for current price characteristics. Two optimal power generation schedules based on different power price characteristics are identified and the value of short-term flexibility is estimated. A tidal lagoon power plant is shown to benefit from limited short-term operational flexibility and the value of flexibility is seen to increase with power price variance.

Further, the European power system model EMPIRE is extended to include tidal lagoon power resources. The extension comprise some flexibility in operational decisions by introducing a set of optional production schedules based on the findings in the above mentioned operational model. A tidal lagoon portfolio along the coast of Great Britain is used for a case study analysing the impact of developing and deploying tidal lagoon generation capacity in Great Britain. It is shown that the cost of energy from tidal lagoon power generators is high compared to other technologies. Only small emission reductions are accomplished by investing in tidal lagoon generators in Great Britain.

Sammendrag

En stokastisk profittoptimeringsmodell for planlegging av kraftproduksjon fra en tidevannslagune er utviklet. Variasjon i høydeforskjell mellom havnivå og vannnivå i reservoaret (fallhøyde), priser i kraftmarkedet og oppstartskostnader for turbiner er tatt hensyn til i formuleringen av problemet. Modellen er et to-stegs stokastisk, *Mixed Integer Programming* (MIP)-problem med bud i et day-ahead marked som førstestegsbeslutning og produksjonsplanlegging og salg i et sanntidsmarked som andrestegsbeslutninger. Et engelsk pilotprosjekt "The Swansea Bay Tidal Lagoon project" er brukt i et eksempelstudie. Analysen er utført basert på salg i de britiske day-ahead- og intraday-markedene for ulik kraftpriskarakteristikk. Høyeste årlig kraftproduksjon fra anlegget er estimert til 317 GWh og operasjonell inntekt er estimert til 15 millioner GBP for dagens priskarakteristikk. To optimale kraftproduksjonsmønstre er identifisert og verdien av korttidsfleksibilitet er estimert. Et kraftproduksjonsanlegg tilknyttet en tidevannslagune er vist å tjene på den begrensede korttidsfleksibiliteten anlegget besitter og det er vist at verdien av fleksibiliteten øker med økt varians i kraftpris.

Videre er den europeiske kraftsystemmodellen EMPIRE utvidet til å inkludere tidevannskraft fra tidevannslaguner. Modelltillegget inkluderer noe fleksibilitet i operasjonelle beslutninger ved å introdusere et sett av valgfrie produksjonsprofiler basert på funn i den operasjonelle modellen nevnt over. En portefølje av potensielle tidevannslaguner langs den engelske kysten er brukt i et studie for å analysere effekten av å bygge ut tidevannskraftpotensialet i Storbritannia. Resultatene viser at kostnaden per energienhet produsert i et kraftanlegg fra en tidevannslagune er høyere enn for andre elektrisitetsproduserende teknologier. Investeringer i tidevannslaguner i Storbritannia vil bare medføre marginale forbedringer i CO_2-utslippene i framtida.

Contents

List of Tables

List of Figures

Chapter 1

Introduction

In November 2015 the United Kingdom (UK) government presented its number one energy policy priority - security of electricity supply. Over time electricity demand and consumer electricity dependency are expected to increase with electrified equipments and changed consumer behaviour. On the supply side, a high share of intermittent energy is planned introduced to the UK power mix, providing uncertain and varying electricity generation depending on natural resource availability. If the share of flexible power generation capacity gets too small, power supply may be insufficient to cover total demand in scenarios of high system stress. Lack of security of energy supply is recognised as the future top priority energy problem in the UK.

The second top prioritised goal for UK energy policy is decarbonisation of power generation. UK is legally obliged to reduce carbon emissions by the national law, the Climate Change Act and bounded by the content of European Union (EU) climate targets and international climate agreements. In order to meet these obligations large changes in the energy sector are required and a shift towards renewable energy production and outfacing of coal are necessary. However, UK politicians have recently indicated that the decarbonisation obligations only will be met when security of supply is ensured. Hence, flexible power generation capacity is highly valued and predicted important in the future UK power mix.

Tidal range power is a renewable, predictable and intermittent power source limited by the tide cycle. UK has one of the world's largest tidal potentials and different technologies for tidal power generation are currently investigated. There are only a few developed and deployed tidal range power plants in the world, thus experience and knowledge about the technologies are limited. A private initiative develops tidal range projects for electricity generation from potential energy in a small sea water reservoir caused by the tide, called tidal lagoons. Funding for the tidal lagoon pilot project is in question and has received varying support from UK politicians. Short-term flexibility is said to be present, but within limits determined by the tidal cycle. This flexibility adds an opportunity cost to the power generated and allows for optimal scheduling of turbine dispatch and sales decisions. Thus, the technology can be a part of the solution for security of supply in a future low-carbon power market dominated by unstable and uncontrollable energy.

This work contributes to the research area of tidal range power planning by presenting a stochastic optimisation model with profit maximisation objective for operation of a tidal lagoon power plant. The main goal is determining optimal power generation schedule and bidding behaviour considering power price uncertainty in a day-ahead and intraday power market. The technical plant characteristics are modelled extensively, including production head effects, turbine start-up costs and location specific tide parameters, allowing for estimation of the available short-term operational flexibility. Further the findings are transferred to a European power system model in order to investigate the system impact by developing identified tidal lagoon power resources in the UK.

Chapter 2

Background

This chapter presents central background information within the scope of the work presented. Firstly, tidal energy and its limitations are presented. Then some insight is given on the political energy topic and finally, the current UK power mix and market design are presented.

2.1 Tidal Energy

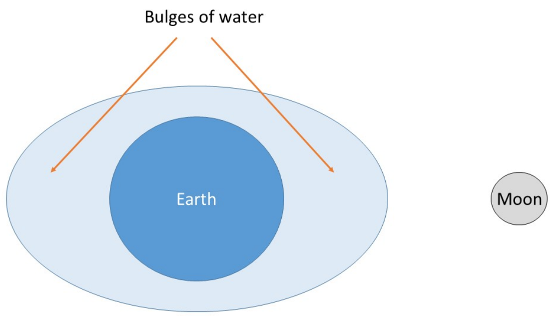
The differences in tide are caused both by the gravitational force acting on the earth from the moon and the sun, topography and the weather (44). The moon and sun pull the ocean towards itself causing high tide on the facing side of the earth and also on the opposite side of the earth due to the centripetal force when the system is in motion, see figure 2.1. Low tide occurs in between, i.e. 90 and 270 degrees from the facing side. Hence, high tide is experienced twice every 24 hours and the time from high to low tide is about six hours. Due to the certainty in earth and moon movements, the tide development caused by the gravitational force is known with little uncertainty at any point in time.

Meteorological factors affecting the tide – such as surface pressure and wind – varies over geographical areas and are associated with uncertainty. Topography, such as the shape of the seabed and coast line, affects the water flow when influenced by the forces mentioned.

2.1.1 Tidal Power Generation

Tidal power is a renewable and predictable energy source. Energy is produced by the surge of ocean waters during the rise and fall of the tide. Tidal energy produc-

Figure 2.1: Bulges of water causing high and low tide around the earth.



tion is considered to be in its infancy and there are only a few commercial sized tidal power plants operating in the world. Both kinetic and potential energy can be utilised in tidal energy technologies by *tidal stream* technologies and *tidal range* technologies, respectively. Tidal range technologies will be discussed further in this text.

Tidal range technology utilise the potential energy in the tidal range for electricity production. The tidal range in certain locations in the UK is about the largest seen in the world and the tidal resources in the UK is considered to hold a large potential for environmentally friendly electricity production. Traditionally tidal barrage has been used across tidal rivers, bays and estuaries. Recently tidal pool projects have been discussed as a more environmental friendly alternative with promising potential. Estimated total theoretical power potential per technology (12) is given in table 2.1:

Table 2.1: Theoretical tidal resources, UK (12)

Technology	**Estimated potential**	
Tidal stream	95 TWh/year	32 GW
Tidal range (barrage schemes)	96 TWh/year	45 GW
Tidal range (lagoon schemes)	25 TWh/year	14 GW

A tidal lagoon is a body of ocean water that is partly enclosed by a natural or constructed barrier. The wall surrounding the lagoon is equipped with turbines and adjustable gates allowing for a complete stop of water flow. When gates are open and the tide level is below the water level in the lagoon, water will flow

Figure 2.2: Tidal lagoon power generation during low tide

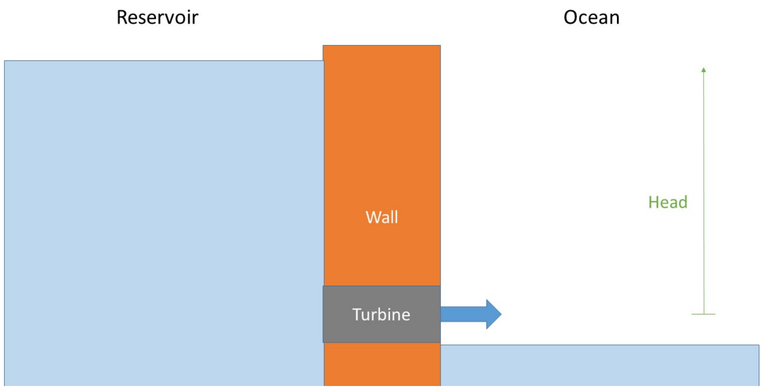
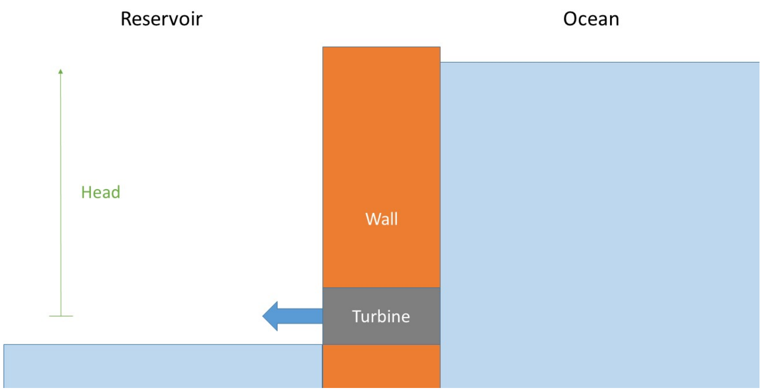


Figure 2.3: Tidal lagoon power generation during high tide



out of the lagoon through the turbines generating power, see figure 2.2. In case of the water level in the lagoon being below the tide level, water flows into the lagoon and power can again be produced, as illustrated in figure 2.3. Two-way power generation and four tidal extreme points every day allows for four power generation periods every day. Theoretically, a tidal lagoon can either provide a continuous energy flow or be controlled by an operator.

Currently, there are no operating tidal lagoons. A 300 MW tidal lagoon power plant at the Yalu River in China is planned for possible future construction [1]. In the UK a 340 MW tidal lagoon power plant in Swansea Bay, Wales is planned for

[1] http://apps1.eere.energy.gov/news/news_detail.cfm/news_id=8286 accessed 20-February-2016

possible realisation [2][3]. If realised this will be the largest tidal power plant in the world.

The tidal barrage technology functions similarly to a tidal lagoon, but the tidal barrage technology utilises the entire sea water content in a fjord or estuary enclosed from the sea, called a barrage. Turbines are installed in the wall allowing for power production when water flows through. The first commercial scale tidal power plant was the 240 MW Rance River estuary in Brittany in France from 1966. Today this is the second largest tidal plant in the world, only smaller than the 254 MW Sihwa Lake Tidal Power Station in South Korea, opened in 2011 [4].

For low-range hydro power generation, highest efficiencies are obtained by utilising a turbine with adjustable blades and gates, allowing for a wider range of turbine flows. That is, the choice of turbine technology affects both the feasible production head range and the potential power output. Commercial low-range turbines handle production heads down to 0.5 m [5].

Environmental Impact

The main environmental concerns for tidal range power production relates to potential local effects on ecology, tidal levels, water quality, hydrology, ground water and socioeconomic aspects(38). Impacts on fish and bird life are identified as the most pressing issue. The severity of environmental impacts depends on the sensitivity of the existing ecosystem, thus environmental evaluations are site specific considerations. Tidal lagoons are expected to have less environmental impact than tidal barrages. In contrast to the tidal barrage, the tidal lagoon can be located outside of the most ecology critical areas around estuaries(17), and two-way generation is shown to reduce ecological impacts significantly (2).

2.1.2 Modelling the Tide

The sum of gravitational forces acting on the sea level can be decomposed into a series of components called harmonic constituents. The harmonic constituents are tidal waves spreading across the surface of the earth, caused by the variation in speed, distance and angle between the moon, sun and the earth. The major constituents have periods around either 12 or 24 hours (20) and are described by (44):

[2] http://www.tidallagoonswanseabay.com/; accessed 23-February-2016

[3] Some references refer to the project as 240MW

[4] http://www.power-technology.com/features/featuretidal-giants---the-worlds-five-biggest-tidal-power-plants-4211218/; accessed 23-February-2016

[5] http://www.andritz.com/no-index/pf-detail?productid=9213; accessed 23-April-2016

$$h(t) = Acos(\sigma t - B) \tag{2.1}$$

where
$h(t)$ is the wave height at time t
A is the wave amplitude
σ is the periodic frequency
B is the phase

Then, the ocean response observed at a given location can be expressed as a corresponding sum of harmonic constituents. The complete expression for the tide height is given as (44):

$$H(t) = MW + \sum_i f_i H_i cos(\sigma_i t + (V_0 + u)_i - g_i) \tag{2.2}$$

where:
$H(t)$ is the ocean response at time t
MW is the middle water level [6]
H_i is the amplitude to the constituent i
g_i is the phase shift of constituent i

and known harmonic constants are:
f_i is the correction for variations in a 18.6 year cycle
σ_i is the periodic frequency in $rad/hour$ and
$(V_0 + u)$ is an astronomic argument describing the phase at $t = 0$.

The harmonic constants for each component at a given location can be found empirically by harmonic analysis of time series of surface variations or by numerical tidal models for the surrounding basin.

2.1.3 The Fluid Dynamics of Tidal Power

Tidal range power plants utilise the potential energy in water caused by the tide to produce electricity. Technical restrictions and equations for operational decision variables for a tidal lagoon is based on fluid mechanics. In this section relevant theory from fluid mechanics from (8) is presented.

When utilising potential energy for power generation in a tidal range power plant the production head is the difference between the ocean water level, given by the

[6] defined as the average water level over the last 19 years at a given location

tide, and the reservoir water level. For two-way generation the production head h is defined as the absolute value of the difference between the reservoir head h^{RES} and the current ocean or tide head H^T.

$$h = |h^{RES} - H^T| \tag{2.3}$$

The production head changes continuously during a tide cycle due to the variation in tide and possible changes in reservoir head. Reservoir head is a function of reservoir volume, thus a function of reservoir area and volume flow. For volume flow $q(t)$ at time t, defined in the direction out of the reservoir, reservoir area $A^R(t)$ at time t and initial reservoir head H_0^{RES} at time $t = 0$, reservoir head at time t, h_t^{RES} is determined by

$$h_t^{RES} = H_0^{RES} - \int_{t=1}^{t} \frac{q(t)}{A^R(t)} dt \tag{2.4}$$

The water velocity v in m/s caused by the production head h is calculated using the Bernoulli equation

$$v = \sqrt{2gh} \tag{2.5}$$

The corresponding turbine volume flow q, is calculated from the flow velocity v and the size of the turbine opening, given by the cross-sectional turbine area A_c.

$$q(t) = A_c v \tag{2.6}$$

The complete expression describing the volume flow through a turbine at time t is then

$$q(t) = A_c \sqrt{2gh} \tag{2.7}$$

For adjustable turbine opening, equation 2.7 describes the maximum obtainable turbine flow at a given production head.

For turbine efficiency function $\eta^T(q)$, generator efficiency η^G, density ρ in kg/m^3, gravitational acceleration g in m/s^2, production head h and turbine flow q, the power output P in a tidal range power plant is

$$P = \eta^T(q)\eta^G \rho g h q \tag{2.8}$$

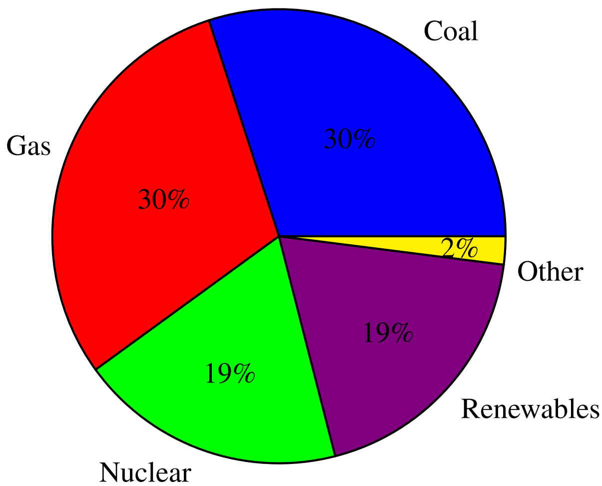
It follows that the power generated from a tidal range power plant is a nonlinear function δ of turbine flow q and production head h.

$$P = \delta(q, h) \tag{2.9}$$

2.2 UK Power Mix

Total electricity supplied to UK consumers in 2014 were 359 TWh, thereof 94 % generated internally and 6 % net imported (35). The most important energy sources for electricity generation were coal and gas. However, both renewables and nuclear sources cover a large share of total power mix, see figure 2.4. Marine energy sources, including all tidal energy sources only counted for 0.001% of total power generation in 2014, and 9 MW installed capacity. Total UK capacity mix is presented in figure 2.5[7].

Figure 2.4: Power mix UK by energy source as share of total generation (35)



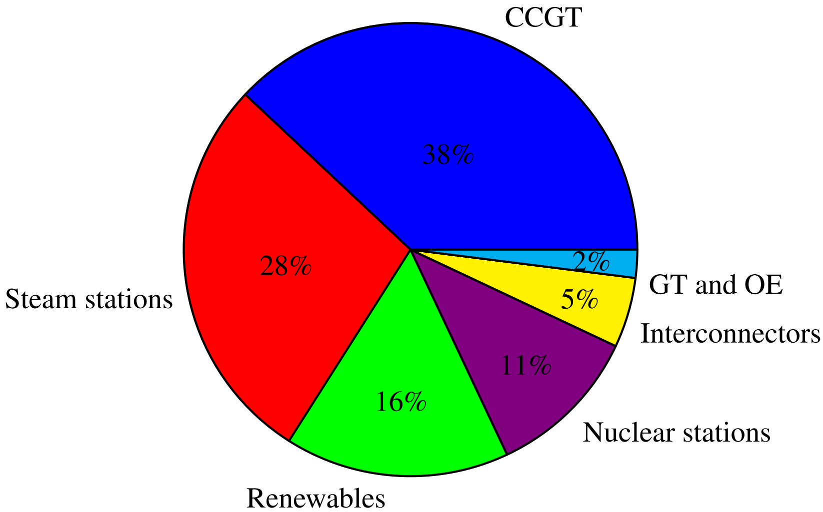
Energy sources can be evaluated at their degree of predictability and degree of flexibility, limiting their area of use. We define a perfectly predictable energy source as energy which availability is known, i.e it is possible to estimate potential power generation volume without uncertainty, but not necessarily possible to generate power when desired. On the other hand, a perfectly flexible energy source can be stored and generate any power – only limited by its installed capacity – whenever wanted. A short description of the main UK energy sources and their characteristics are presented below.

Coal

Coal is used for fuel in power plants and is classified as both perfectly predictable and flexible. Power generation by coal as fuel is recoginsed as the most CO_2

[7]Steam stations are conventional steam stations including 89 % coal fired stations, 6 % oil fired stations and 5 % gas fired stations. CCGT are combined cycle gas turbines. GT & OE are gas turbines and oil engines

Figure 2.5: Installed capacity UK 2014 by technology as share of total capacity (35)



emission intensive energy source [8]. The coal share of total electricity generation in the UK is decreasing and expected to decrease as a result of both today's national policy (51) and the European Union policy. From 2013 to 2014 the coal share of UK energy production decreased by 17 % (35).

Gas

Like coal, gas is used for fuel in power plants and classified both perfectly predictable and flexible. Gas fired power plants produce CO_2 when generating power, but the emissions only count for 50 % - 70 % of the amounts from burning coal[9]. Combined cycle gas turbine (CCGT) is a new gas fired power station technology utilising both a gas and steam turbine and claiming to generate 1.5 times the electricity obtained from the same fuel in conventional gas turbines[10]. The high flexibility and relatively low CO_2 emissions associated with new gas fired power plants make these technologies popular in combination with the planned higher share of intermittent renewable energy (51). From 2013 to 2014 the gas consumption for power production increased by 5.1 % (35).

[8] http://www.iea.org/topics/coal/ accessed 10-February-2016

[9] Comparison based on emissions as mass per energy content, given in https://www.eia.gov/tools/faqs/faq.cfm?id=73&t=11; accessed 10-February-2016

[10] https://powergen.gepower.com/resources/knowledge-base/combined-cycle-power-plant-how-it-works.html; accessed 10-February-2016

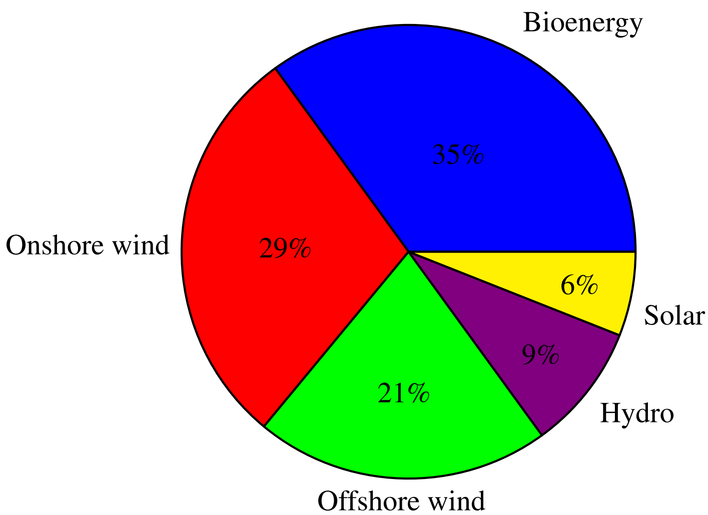
Nuclear

Nuclear power stations produce power from fission or fusion of atoms in a fuel [11]. Nuclear energy is classified as perfectly predictable but not flexible, thus suitable for base load only. Criticism regarding safety and handling of nuclear waste has made the technology unpopular. However, today nuclear power is an important part of the UK energy policy for decarbonisation and security of energy supply (51).

Renewables

Renewable power generation in the UK mainly comprises onshore and offshore wind energy, bioenergy, hydro energy and solar energy, see figures 2.6 and 2.7. Their characteristics varies between the technologies from the unpredictable and unflexible wind and solar energy to the somewhat predictable and highly flexible flexible hydro energy, and the perfectly predictable and perfectly flexible bioenergy. Renewables' share of total electricity generation increased by 21 % from 2013 to 2014 and total renewable installed capacity increased by 24 % (36).

Figure 2.6: Renewable power mix in UK 2014 as share of total renewable generation (35)

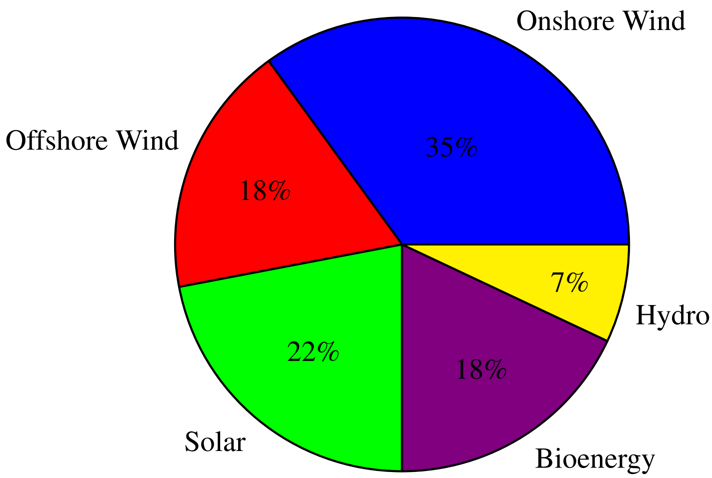


Interconnectors

The UK power grid is connected to surrounding power grids in France, the Netherlands and Ireland through five interconnectors (35). Total installed interconnector

[11] https://www.duke-energy.com/about-energy/generating-electricity/nuclear-how.asp; accessed 10-February-2016

Figure 2.7: Installed renewable capacity in UK 2014 as share of total renewable capacity, by technology (36)



capacity in 2014 were 4500 MW. The above mentioned net imports in 2014 of 6 % of total supply correspond to a 42 % increase in net imports from 2013. The largest share of net imports were transferred from France.

2.3 European and UK Climate and Energy Policy

This section describes the political energy context for the current UK power system. The first part is once summarised in (30).

The European Union (EU) has developed a common energy policy for its member countries. Its three main goals are[12]:

- Security of energy supply
- Competitive market for energy supply
- Sustainable energy consumption

In order to achieve these goals more energy will be produced and a higher share will be based on renewable energy sources. The EU will allow energy to cross borders in new pipelines and power lines across the union and an Internal Energy Market with common rules will be developed to integrate the independent energy markets. To ensure the right development in member countries the EU has

[12]https://ec.europa.eu/energy; accessed 08-May-2016

launched targets to be met by 2020, 2030 and 2050. By 2050, greenhouse gas emissions is to be reduced by 80-95% compared to the 1990 level.

The 2020 Energy Strategy is a ten-year strategy describing the priorities until 2020, including:

- Greenhouse gas emissions are reduced by at least 20 %
- Share of renewables in the consumed energy mix is at least 20 %
- Energy efficiency is to be improved by at least 20 %

The member countries in the European Union have committed to a binding agreement until 2030 to achieve:

- Share of renewables in the energy mix is at least 27 %
- Greenhouse gas emissions are reduced by at least 40 % compared to 1990 level

Other agreements on obtained targets by 2030 include electricity interconnection of 15 % inside the union and energy efficiency increase of at least 27 %.

Under the 2008 Climate Change Act the UK government committed to reduce CO2 emissions by 80% by 2050 relative to 1990 baseline (16). To reach these goals Great Britain is required to increase their share of renewable energy and phase out carbon intensive power production. One fifth of existing reliable capacity is expected to be phased out over the next years, while new investments are expected to mostly consist of wind power and nuclear power projects (51). Consequently, future production mix will consist of less reliable and flexible power generation capacity than today.

2.4 UK Power Markets

Due to the special characteristics of electricity, such as non-storability, the power system has to be carefully monitored and continuously balanced to avoid demand-supply unbalance. The transmission system operator (TSO), National Grid, is responsible for balancing the power system in the UK, ensuring that production corresponds to consumption at all time. On an overall level, three electricity markets exist to assist National Grid in balancing supply and demand: the day-ahead market, the intraday market and the ancillary service market. In addition, a capacity market has recently been implemented in the UK to ensure future security of supply.

In the day-ahead marked electricity is traded for delivery the following day, either on an hourly or half-hourly basis. In the intraday market participants can trade close to real time to balance their bids based on updated information such as weather forecasts. A trend in the UK power market is that the intraday market is less used, while the day-ahead market has a larger turnover. Finally, in the delivery hour the system is balanced by the TSO through activating balancing services provided in the ancillary service market.

Wholesale of electricity takes place on power exchanges. All generators operating in the UK have to submit final schedules to the TSO half an hour before real-time[13].

The APX UK Power exchange offers two day-ahead auctions where trading of electricity on an hourly and half-hourly basis takes place one day ahead of delivery. The auctions are double-sided blind auctions, meaning that the participants not can see other bids/offers in the auction. Participants in the two auctions must place orders (bids/offers) for the next day before 11:00 and 15:30, respectively. Supply and demand is then compared to find the price that clears the market. The clearing price is calculated for each hour of the following day.

The intraday market is used for balancing and trading of half-hour blocks of electricity. Trading takes place every hour and every day. All products in this market are automatically cleared.

2.4.1 The UK Electricity Market Reform

The electricity market reform can be seen as a reaction to the increasing threat to security of supply in the UK combined with the obligation to meet the EU 2020 goals. To ensure energy security the UK government implemented an electricity reform in 2014, The Energy Act 2013 (34). Two key mechanisms were included in the reform to ensure investments in necessary energy generation: Contracts For Difference (CFD) and the Capacity Market. The CFD scheme is discussed further in this work and an explanation follows below.

Contracts For Difference

The current system for subsidising renewable energy production in the UK is under change, transitioning from renewables obligation certificates [14] to CFD. Under the Contract For Difference system, low-carbon power production plants receive subsidies through receiving a predefined strike price [15] for sold electricity. The

[13] http://climatepolicyinitiative.org/wp-content/uploads/2011/12/Intraday-and-Wind-Integration-Paper.pdf; accessed 08-February-2016

[14] Renewables Obligation Certificates, see https://www.ofgem.gov.uk/environmental-programmes/renewables-obligation-ro; accessed 08-February-2016

[15] price reflecting the cost of investing in low carbon technology

government pays the difference between the strike price and the reference price[16]. In case of the reference electricity price exceeding the strike price, the difference is paid by the power produced to the government. The strike price increases expected revenues of low-carbon power plants, as well as reducing risk by ensuring predictable and stable revenues. As a result renewable power production technologies become price competitive to more mature power production technologies. CFD grants are secured through auctions. There are different auctions for predefined groups of technologies. The current commissioned budget for CFD support in the UK is in total 325 million GBP[17].

[16] average electricity price

[17] https://www.gov.uk/government/collections/electricity-market-reform-contracts-for-difference; accessed 08-March-2016

Chapter 3

Literature Review/Related Work

In this chapter relevant research on tidal range power planning is presented. A considerable amount of research is done on optimising hydro power investments and operation within the field of operational research, whereas limited research is done on tidal range power planning. Firstly, relevant work directly related to tidal range power plant planning is presented, before a selection of relevant research done on hydro power planning is described. Low range hydro power planning has similar characteristics to tidal range power planning and is relevant for the scope of this work. Finally, an overview of the most relevant published total power system models for Part II of this thesis is given.

3.1 Tidal Range Power Models and Studies

To the authors' knowledge there are only a few developed optimisation models optimising production from tidal range power plants. Some research has been performed using simulation models to determine the potential power output of tidal range plants, and to analyse design parameters. There are also a number of publications focusing on quantifying the tidal power potential in the UK.

3.1.1 Optimisation Models

Operation of the La Rance tidal power plant in France is analysed using an optimisation model developed in 1982. The model is presented in (29). Optimal operation is decided using a dynamic programming method. The model chooses between five different operational modes and optimises flow on the turbines when production takes place. Head variations are considered in the model, but production head is assumed constant in each period. Time is discretised into intervals of

10 minutes. The duration of the optimisation is one week followed by an interval of 12 hours. Variation in price and bidding strategies are not considered in the work.

Other models maximising power generation from tidal range power plants used to evaluate design parameters, are presented in (33), (31) and (13). In (33) the authors present a methodology to maximise the total energy generated by a tidal power plant with reservoir constraints. A Genetic Algorithm heuristic is used to decide optimal dispatch of turbines, considering a one-year horizon and one-way generation. Furthermore, a case study on estuary Bacanga, located in São Luís, Brazil is performed. Significant gain from optimal turbine dispatch in tidal power plants is achieved in the study. In (31) a dynamic programming algorithm is proposed to determine optimal operation of an one-way generation barrage tidal power plant, aiming to maximise the energy generation over a tidal cycle. The algorithm can be used to calculate the annual energy generation for different technical configurations, thus an optimal design of a tidal power plant can be determined. The algorithm is used to find optimal design of a tidal power plant at Hansthal creek of the Gulf of Kachchh in the Indian state of Gujarat. The results show that the minimum relative cost of energy per unit is achieved for an installed turbine capacity of 1200 MW. (13) present a power optimisation algorithm using Lagrange equations to solve the problem.

3.1.2 Simulation Models and Other Studies

More research is done using simulation models to analyse the power potential of tidal range power plants and associated design parameters.

(1) aims to determine the annual power outputs in two specified tidal barrage power plant locations in the UK. The study focuses on turbine characteristics, and improved characteristics from a new[1] low-range turbine in particular. Calculations of annual energy produced with varying input parameters are performed by iterative simulation, and estimated optimal values are decided. In (7) the potential electricity generation from operation of five major estuary barrages on the West Coast of the UK is estimated by using 0-D and 2-D numeric simulation. The results demonstrate that with 22 GW of installed capacity more than 33 TWh of electricity should be attainable per year. According to the authors this represents close to 10% of present UK demand. A similar analysis is performed in (2) where 0-D and 2-D modelling are used to determine optimal operation of tidal lagoon generators along the coast of North Wales. The research focuses on environmental impacts and changes in the natural ecosystem of developing tidal lagoon generation. Total annual energy production has also been calculated. (14) estimate the

[1] New in 2010 when the article was published

feasible power production from a barrage plant in the Bacanga estuary in Brazil, considering constrains such as urban considerations.

In (53) the obtainable energy gains from introducing pumping in a tidal range power plant are investigated. Through simulation net energy produced is determined for a range of turbine and pump efficiencies and two cases of tidal cycle amplitudes. Values for turbine and pump efficiencies are shown to be overestimated in similar research and highly affect the results of up to estimated 6 % annual energy increase.

Some work has also been published estimating the overall tidal power potential in the UK, analysing expected profitability and identifying key barriers for different tidal power technologies. Examples of such analyses can be found in (22, 41, 39, 12).

3.2 Hydro Power Models

3.2.1 Linearisation

For hydro power more research is completed. A decent amount of work is published optimising hydro power operation considering head variations. Varying head lead to non-linearity in the original problem.

An important question when formulating optimisation problems with varying head is how to handle the non-linearity. It is also important to evaluate which elements to include in the modelling, such as whether or not to include start-up costs and ramping constraints. In (9) a profit maximisation model for a power company that comprises several cascaded plants along a river basin is presented. The authors propose a 0/1 mixed integer linear programming model which can be used to account for the non-linear and non-concave three-dimensional relationship between the power produced, the water discharged and the production head. The model can be used for systems of several connected hydro power units. The main contribution of the model is how the relationship between head, discharge and power output is handled. In the proposed model, the authors discretise the head-flow-power relationship into a set of curves depending on the reservoir content. Then a precise piecewise linear approximation of each curve is performed. The non-concavity is handled through the use of binary variables. The model has been successfully tested on realistic case studies. For simplicity the authors chose to use three discretised curves for the flow-power relationship, corresponding to three levels of head. For improved accuracy the authors suggest to increase the number of curves used, however this will increase the size of the model and affects the solution time.

In (6) the authors present a model which considers head effects through an enhanced linearisation method based on the method presented in (9). The improvement of the model consists of the addition of some technical characteristics, such as ramping constraints, and the tightening of the approximation for the relationship between head, discharge and power. A set of volume intervals and a set of breakpoints are used to represent the three-dimensional relationship between head, discharge and power. Instead of approximating the power production by selecting a point on a single piecewise linear function, the enhanced method approximates it through a weighted combination of values computed for the two extremes of the head interval. This improvement of the linearisation method is the main contribution of the paper. The results show that the proposed model allows high solution accuracy and is solvable with modern mixed integer linear programming (MILP) software. However, the performance of the model is heavily dependent on problem size.

Similarly, the authors of (19) also aim to determine a feasible operation of the hydro units in a hydro chain, while trying to meet demand. The model includes variation in head by approximation through meshing and triangulation. A piecewise linear approximation is used to describe the relationship between head, power and discharge, so that the resulting triangular meshes require two binary variables for each grid point. This proves to be an accurate approximation, but the model performance depend strongly on problem size. The problem size could be reduced by beginning with a simple mesh.

A method for detailed unit scheduling of a hydro power plant comprising a series of interconnected reservoirs is presented in (4). Due to the interconnection of reservoirs, plant head affects the results and is included by linearisation of the production-discharge equation. The efficiency curve is also linarised. The problem is solved by an iterating loop of model building and optimisation for the entire planning period in each iteration. Based on the previous solution, including production and discharge variables, constraints are added to the problem or updated in the succeeding iteration. The test examples show that turbine start-up costs can be reduced by 50 % if included in the model.

(10) describes and compares three different methods for the piecewise linear approximation of functions of two variables. The presented methods are:

- The one-dimensional method, which uses a one-variable piecewise linearisation technique with special ordered sets for a discretised set of y values.
- The rectangle method was first introduced in (6). This is an improved version of the one-dimensional method, given by a better approximation on the y axis.

- The triangle method, which can be seen as an extension to the one-variable technique to a two-variable technique. The technique is based on the definition of three-dimensional triangles, see (50, 3).

The study considers solution error, problem size and solution time when evaluating the methods. The findings show advantages and drawbacks with all three methods. The triangle method provides the most accurate solutions, but at the cost of increased problem size and solution time. The quality of the solutions returned by the one-dimensional method is generally not good, and the percentage errors are significantly higher than for the other methods.

3.2.2 Bidding Strategies

To the authors knowledge there are no developed profit maximisation optimisation models for tidal lagoon power scheduling including realistic modelling of the power market. When optimising production based on profit maximisation, price considerations and bidding strategies are important factors. Considering hydro power scheduling several developed and well tested models include bidding strategies and price uncertainty. In (15) the authors have developed a model for optimal bidding in a day-ahead market taking market price uncertainty into consideration. Electricity price scenarios are generated by a statistical model based on the ARMA method. The model also includes optimal production and is demonstrated on a Norwegian hydro power producer acting in the Nordic power market. Finally, the value of including uncertainty in the model is discussed. In (27) the bidding problem in short-term markets for a hydro power producer taking into account plant operation and complex market specific power price uncertainty, is developed. Both day-ahead and short-time power price characteristics are investigated and requirements for representation of each market price uncertainty are presented.

3.3 Power Sector Models

In this section relevant literature for part II is presented.

The European Model for Power system Investment with (high shares) of Renewable Energy (EMPIRE), is a dynamic capacity expansion model for the European power system presented in (47). The model is used with an extension in part II of this thesis. EMPIRE optimises the development of the European power system, minimising investment and production costs with respect to generation and transmission capacity. Further description of the model can be found in chapter 10. In (47) EMPIRE is used in a decarbonisation study of the European power system for two different cases, one with transmission investments and one without.

Furthermore, an earlier version of EMPIRE is used in (45) to analyse the cost minimising investment plan required to achieve a predefined low-carbon power generation mix. Earlier versions of EMPIRE have also been used by Zero Emission Platform for several studies, (54, 55, 56), of CCS deployment in Europe.

The remaining content of this section is previously summarised in (47).

There are some other similar investment models to EMPIRE that should be mentioned here, that either include short-term uncertainty or long-term dynamics such as EMPIRE. TIMES is a model considering both long-term and short-term dynamics, such as EMPIRE, presented in (43). Similar to EMPIRE a multi-horizon approach is used to limit the problem size of the stochastic formulation. The authors use TIMES in a study focusing on how uncertainty in wind power production affects the Danish heat- and electricity sector and the importance of credible modelling of operation when wind power production constitutes an increasing share of the power production mix.

The power system models E2M2 and a further developed version of the EMPS model are presented in (24) and (49), respectively. Both models include an extensive modelling of the operational phase compared to EMPIRE. EMPS and E2M2 only consider investment periods sequentially, lacking some of the long term dynamics EMPIRE incorporates. Two power system models including long-term dynamics are LIMES-EU+ presented in (21) and DIMENSION presented in (40). Both are deterministic optimisation based investment models. Two different stochastic versions of DIMENSION are presented in (18) and (32), including one type of uncertainty each, respectively long-term (strategic) uncertainty and short-term (operational) uncertainty. The latter excludes long-term dynamics. In (25) DIMENSION is used for an extensive decarbonisation study of the European power sector.

To the authors knowledge no power system model similar to EMPIRE has been used to investigate the impact of tidal range power resources previously.

Chapter 4

Theory

In this chapter major concepts from optimisation theory relevant for the work performed are presented. In section 4.1, the area of mixed integer programming and associated solution algorithms are explained. Then, in section 4.2 special ordered sets and a specific area of application are presented. Finally, the field of stochastic programming is presented in section 4.3.

4.1 Mixed Integer Programming

Mixed integer programming (MIP) is modelling with both continuous variables and variables restricted to only take integer values (28). The topic is similar to integer programming (IP), where all variables must take integer values. Both fields of programming differentiate from linear programming (LP), allowing for continuous values for all variables. To a MIP and IP problem, large computational resources may be required both to find a feasible solution and proving its optimality.

Branch and Bound

Branch and Bound (B&B) is a method for finding a solution to an IP problem or a MIP problem (28) (48). Firstly, the integrality restrictions are relaxed and the optimal LP solution is obtained, providing an optimistic bound to the IP or MIP solution. If the LP solution is an infeasible solution to the problem – that is, if the integrality constraints are violated – two sub problems are created. The sub problems are similar to the LP problem but each considers an exclusive feasible region excluding the updated LP solution. This division of the feasible region is called *branching*. If the LP solutions to the two sub problems are infeasible solutions to the IP or MIP problem, the optimistic bound for the respective feasible region is updated and new sub problems are created, dividing the current feasible region into smaller regions excluding the current LP solution. The best feasible solution obtained at any point in time provides a pessimistic bound for the IP or MIP solution. This procedure is repeated until optimal solution is found or any stopping criteria is met, creating a system of parent and child nodes called the B&B tree.

During the B&B algorithm, a range of choices are available, such as node selection for further investigation, selection of sub problem to investigate first and variable selection for further branching. Another choice is whether and to what degree the current LP solution should be strengthened, leading to the next topic, the *branch and cut* algorithm.

Branch and Cut

Branch and Cut (B&C) is an extension to the B&B algorithm where valid inequalities or cuts are added to the LP problem in each node before branching on integer variables(28). The cuts strengthen the LP relaxation of the IP or MIP problem by restricting the feasible region and the region containing the LP solution specifically. No feasible solution is removed from the solution space, thus the cuts added in one node are valid in all other nodes. Increased number of cuts added reduces the number of nodes potentially required for investigation but also increases the problem size and the required solution time per node.

Heuristics

Use of heuristics during the search for an IP or MIP solution may reduce the required time for finding a feasible solution and/or improve the feasible solution found (48). The best feasible solution provides the pessimistic bound for the problem during the B&B search, thus finding better feasible solutions increases the number of pruned nodes and reduces total computational time. However, computational resources are sacrificed when heuristics are run and the per node run time may increase.

4.2 Special Ordered Sets

Special ordered sets are sets of variables defined in a specified order where each variable value is restricted by the values of the other variables in the set.

For a special ordered set of type 1 at most one variable in the set can be non-zero. In a special ordered set of type 2 (SOS2) at most two variables in the set can be non-zero and non-zero variables must be adjacent in the specified order.

A possible area of use for SOS2 is piecewise linearisation of a non-linear function $f(x)$. The linear pieces connect specified points $(x, f(x))$ on the function. Weighting variables of SOS2 holds the weight associated with each point. In this way each point $(x, g(x))$ on the linearised function is an interpolation between at most two consecutive values of $f(x)$.

4.2.1 Non-Linear Functions of Two Variables

In (52) a method for linearisation of non-linear functions of two or more variables using SOS2 is presented. For the two independent variables x, y and the non-linear function $z = f(x, y)$ a two-dimensional grid of M points in the x dimension and N points in the y dimension are specified together with the values of x and y in point nm, (X_m, Y_n). Then, the non-linear function $z = f(x, y)$ can be approximated by fixing the variables to the specified grid values (X_m, Y_n) for all grid points (m, n) and calculate a weighted sum of $f(X_m, Y_n)$ where the weights of each grid point (m, n) are given by the weighting variables λ_{nm}. The weighting variables must sum to one.

$$x = \sum_m \sum_n X_m \lambda_{nm} \tag{4.1}$$

$$y = \sum_m \sum_n Y_n \lambda_{nm} \tag{4.2}$$

$$z = \sum_m \sum_n f(X_m, Y_n) \lambda_{nm} \tag{4.3}$$

$$\sum_m \sum_n \lambda_{nm} = 1 \tag{4.4}$$

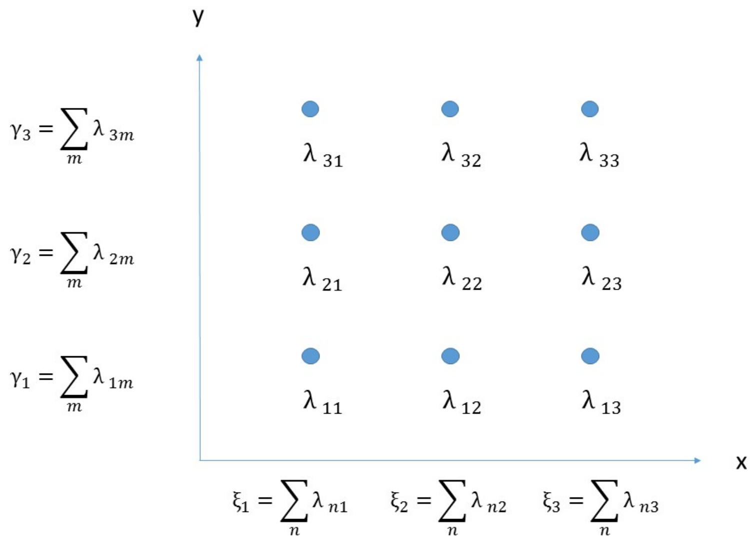
For interpolation in both the x and y dimension to work as intended, at most four weighting variables corresponding to four adjacent grid points can be non-zero. This condition is obtained by forcing at most two adjacent grid rows to contain non-zero weighting variables and at most two adjacent grid columns to contain non-zero weighting variables. Variables equal to the sum of weighting variables over a row or column in the x, y grid are created, see figure 4.1.

$$\xi_m = \sum_n \lambda_{nm} \quad \forall m \tag{4.5}$$

$$\gamma_n = \sum_m \lambda_{nm} \quad \forall n \tag{4.6}$$

The condition is imposed by taking each variable sets $\{\xi_1, ..., \xi_M\}$ and $\{\gamma_1, ..., \gamma_N\}$ as SOS2.

Figure 4.1: The two-dimensional grid illustrated for three grid points in each dimension. Weighting variables and both sets of SOS2 variables are shown.



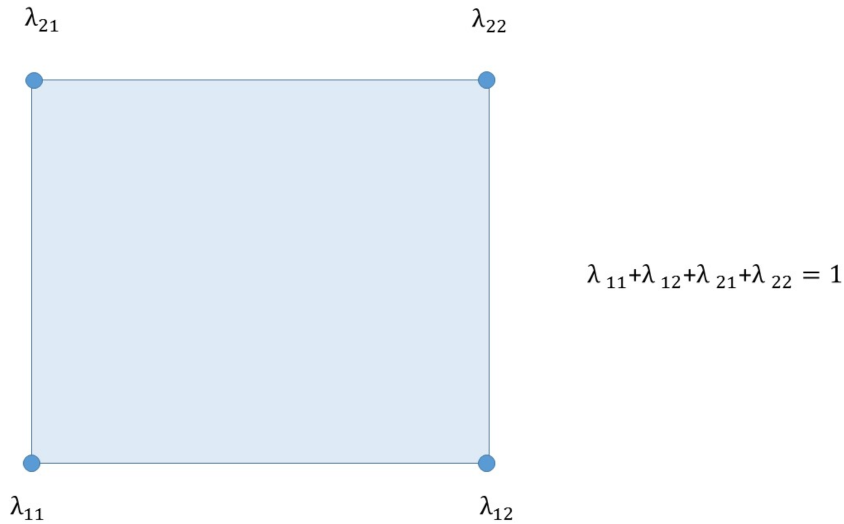
An interpolation in two dimensions described by four weighting variables corresponding to neighbouring grid points provides a correct but not an unique description of a combination of two variables (x, y). That is, the same combination of the two variables (x, y) can be described by many different combination of values for the same four weighting variables, see figure 4.2. A unique formulation is described by at most three non-zero weighting variables corresponding to neighbouring grid points, see figure 4.3. This condition can be imposed by ensuring at most two diagonals in the grid parallel with the line covering the points $(1, 1), (2, 2), (3, 3)...$ contains non-zero weighting variables and that those diagonals are adjacent.

To impose such an unique formulation another set of variables ζ_i is defined, each variable holding the sum of weighting variables corresponding to grid points sitting on a diagonal in the grid, as illustrated in figure 4.4. Each ζ_i corresponds to exactly one diagonal and the set of all ζ is taken as SOS2.

$$\zeta_i = \sum_m \lambda_{(m+i-M)m} \quad i = \{1, 2, ..., (N + M - 1)\} \tag{4.7}$$

If variables in one dimension need to be fixed to grid point values interpolation is done in one dimension only and the constraints 4.7 are redundant.

Figure 4.2: The area of solutions found by interpolation between the four adjacent weighting variables corresponding to the points $(1,1)$, $(2,1)$, $(1,2)$ and $(2,2)$. Note how each point in the shaded area can be explained by a number of different combinations of the four weighting variables.



4.3 Stochastic Programming

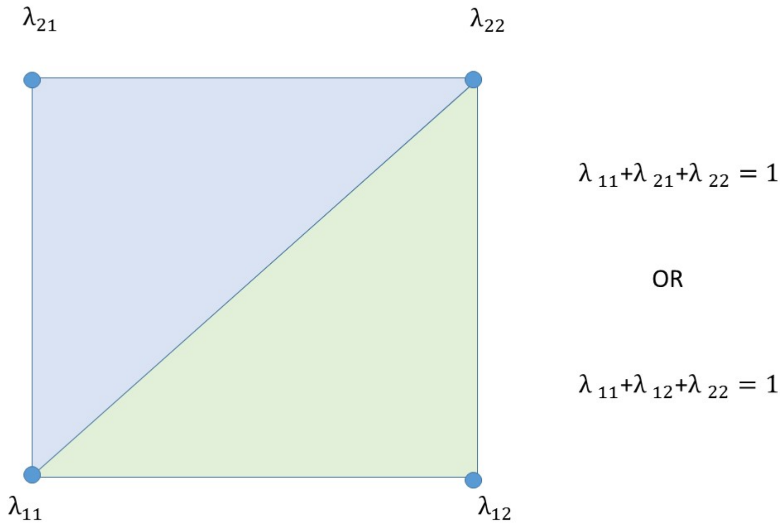
Stochastic programming is modelling accounting for a range of different realisations of data elements. The method is used when data inputs are uncertain and the solution to the problem varies with outcomes of the uncertain element. Because the possible outcomes are considered collectively, the solution will be less dependent on a specified outcome of the uncertain element and thus more robust to a range of possible outcomes.

In stochastic programming the model is divided into two or more *stages* where each stage is defined as a point in time when new information becomes available and a decision can be made. The future is more uncertain in the first stage than in the second stage and decreases with increasing stages. This modelling structure allows for delaying a decision until uncertainty is reduced, often called a *recourse* decision (23).

4.3.1 Scenario Generation

In stochastic programming, uncertainty in future realisation is described by a discretised set of possible outcomes. Each outcome is hereby called a *scenario* and

Figure 4.3: The two areas of solutions found by interpolation either between the three adjacent weighting variables corresponding to the points $(1, 1)$, $(1, 2)$ and $(2, 2)$ (blue area) or between the weighing variables corresponding to the points $(1, 1)$, $(2, 1)$ and $(2, 2)$ (green area). Note how each point in the two shaded areas can be explained by a unique combination of the three corresponding weighting variables, only.



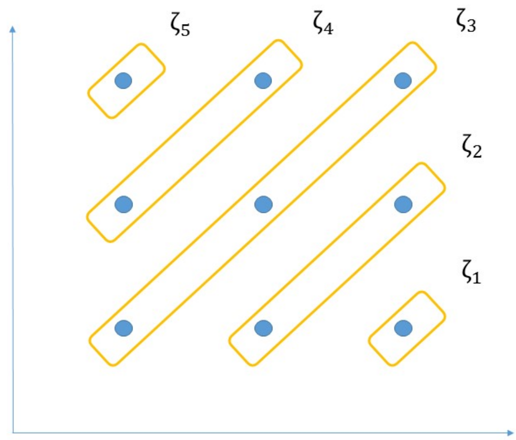
the collection of discretisesed outcomes is called a *scenario tree*, produced by a scenario generation procedure. Concepts for scenario generation methods relevant for the scope of this work are presented below. A complete description of more complex scenario generation methods can be found in (26).The following theory is based on (26) and once summarised in (30).

Scenario generation by a sampling procedure is either done by picking random values from the real parameter or by using some prediction method and sampling randomly from the error distribution. The samples are then used as scenarios to represent possible outcomes. In a path-based method, the scenarios contain possible realisations of the random variable for a range of consecutive points in time, called a *path*. Uncertainty is associated with which path to be realised but the future is assumed certain within the duration of each path.

4.3.2 Measuring the Quality of a Scenario Generation Procedure

When representing reality by a scenario tree, the solution obtained from solving the problem should provide an objective value as close as possible to the real op-

Figure 4.4: The variables holding the sum of weighting variables over each diagonal in the grid.



timal value when calculated for values from reality. That is, the priority is the performance of the obtained solution when put out in reality. This performance is measured by an *optimality gap*, defined as the gap between the real objective value from the real objective solution (a solution about impossible to obtain) and the real objective value from the approximated solution obtained from solving on a scenario tree. Thus, a scenario generation procedure providing a solution with a small optimality gap is the desired measure of goodness for the scenario generation procedure. However, when this property can not be tested, *stability* is an appropriate property replacement. A description of the stability requirement based on (26) follows below.

Stability

Out-of-sample stability for a scenario generation method providing a set of approximated solutions found when solving on its supplied scenario trees, is obtained if the solutions provide approximately the same true objective function values. That is, the set of solutions perform about equally when incorporated in reality. When the true objective function is unavailable a *weaker form of out-of-sample stability* can be tested. The scenario generation method is then said to be out-of-sample stable if it produce two scenario trees i and j with corresponding objective functions F_i and F_j and problem solutions x_i and x_j, and the three following requirements are satisfied:

$$F_i(x_i) \approx F_i(x_j) \tag{4.8}$$

$$F_j(x_j) \approx F_j(x_i) \tag{4.9}$$

$$F_i(x_i) \approx F_j(x_j) \tag{4.10}$$

That is, the approximated objective functions should produce about similar values for all approximated solutions obtained when solving on the scenario trees from the scenario generation method in question.

A scenario generation method is *in-sample stable* if the last requirement above, 4.10 is satisfied. That is, the set of objective values obtained from solving the set of approximated problems produced by the scenario generation procedure, are about equal.

Part I

The Operational Model

Chapter 5

Model Outline

In this chapter the profit optimisation model for operation of a tidal lagoon plant is presented. Core components of the model together with main assumptions, approximations and solution strategies are described.

5.1 Problem Description

A tidal lagoon power plant generates power when water flows through turbines in the lagoon wall surrounding the lagoon or constructed reservoir. The water flow is caused by the production head, corresponding to a difference in water level on the two sides of the lagoon wall due to the natural tide variation. By closing the sluices in the wall over some time, larger head difference, thus power generation potential is gained. The opportunity to delay power generation is the core of the following presented problem. When is the value of the reservoir water largest and what is this operational flexibility worth?

Power can be produced when water flows both into the reservoir on *flood* tide and out of the reservoir on *ebb* tide. This generation scheme allows for power generation every tidal *half cycle* resulting in approximately four production cycles in 24 hours. Within a six-hours tidal half cycle, hereby also referred to as a *production cycle*, flexibility in power generation is achieved by controlling the water flow through turbines. Unlike traditional hydro power operation, long-term energy storage is not possible.

The operational problem with profit maximisation objective for a tidal lagoon power plant takes into account power market prices and uncertainty in price realisation and power delivery obligation when planning both the production schedule and sales decisions. The tide cycle is assumed perfectly predictable for any point in time causing the power generation potential to be certain over the planning horizon. Thus, low risk is associated with delivery obligations. In addition, the flexibility allows for trade in close to real-time markets and exploitation of short-term price variations. Water flow and corresponding power sales decisions are chosen in order to maximise profit within allowable time and generation limits dictated by the tide cycle.

5.2 Model Design

The objective of the model is to maximise profits from sales of power generated by a tidal lagoon power plant. Power is sold either by in-advance bidding in the day-ahead power market and corresponding later delivery obligation, or by sales in a real-time power market. Operating costs are driven by turbine start-ups.

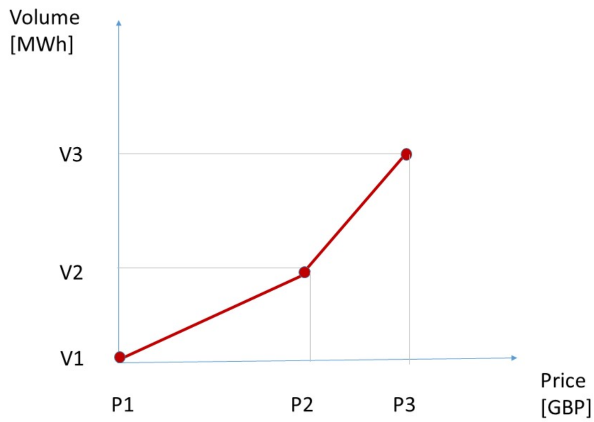
Power production is restricted by the natural tide cycle, installed plant capacity and turbine operating conditions. The tide cycle dictates the possible range of production head at any point in time, and the allowable power production period. Reservoir size and geometry affect reservoir water volumes, and reservoir head as a function of volume or flow. Turbine specifics determines allowable water flow range, turbine efficiency and upper bound on power generation.

5.2.1 Model Structure

Each production cycle problem is divided into two *stages*, where a stage is defined as a point in time when new information becomes available and a decision is made. In the first stage the producer decides on bidding volumes placed in the day-ahead power market for every hour in the production cycle in question. The decision is made with uncertainty in both the day-ahead and real-time power market prices. For the day-ahead market a linear bidding function is used by specifying a set of fixed bid point prices and allowing for optimal selection of corresponding bid point volumes, see illustration in figure 5.1. The procedure is similar to the method presented in (15). The day-ahead delivery obligation is revealed with the realised day-ahead power price and is the power volume found by interpolation between the bid volumes corresponding to the adjacent bid prices.

In the second stage both realisation of real-time market prices and realisation of the day-ahead delivery obligations based on the prior bidding decisions, for all six hours in the production cycle become available. The energy sold in the real-time market for all six hours in the production cycle and the corresponding energy generation are decided. Sales in the real-time market are modelled as a real-time recourse decision. All sales volumes are limited by power production.

Figure 5.1: The linear power producer bidding curve combining three bid points associated with chosen bidding volumes corresponding to each specified bid point price.



Price uncertainty in the model is described by a collection of possible *scenarios*. Each scenario contains pairs of realisations for the day-ahead and real-time power prices for all periods in a specified production cycle.

In reality, bidding decisions in the day-ahead market are decided for 24 consecutive hours (or 48 consecutive half-hours), while sales in the real-time market can be decided separately for each half-hour up to 30 min ahead of delivery. By considering all six hours in each stage, the above mentioned problem independence is achieved and a simplified model with only two stages can be obtained, see figure 5.2.

5.2.2 Time Discretisation

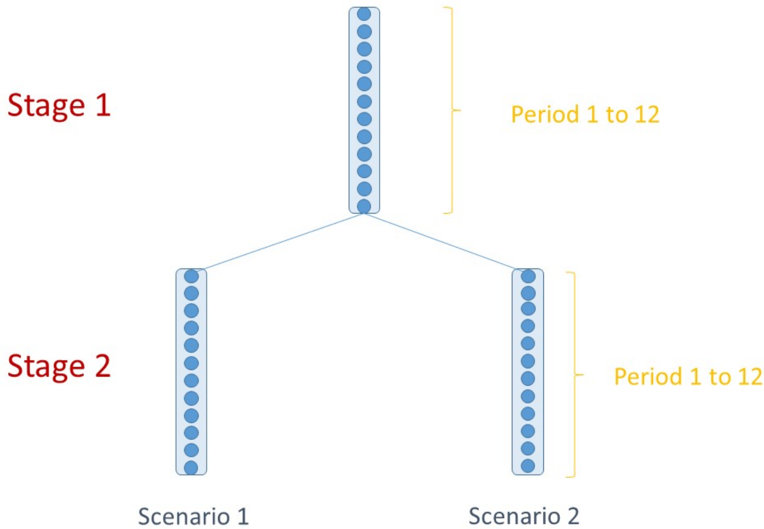
Time is discretisised into *periods* of 30 minutes, for which each decision is made. That is, turbine flow is assumed adjustable every 30 minutes and sales decisions are made for 30 minutes intervals. Further, turbine flow and power generation are assumed constant during a production period, i.e. energy production E over a period is

$$E = pT \tag{5.1}$$

where p is power produced in the period and T is the period duration in hours.

The time discretisation is chosen small enough to reflect the operational flexibility of the plant. On the other hand, the discretisation is chosen large enough to obtain different tide values in every period.

Figure 5.2: Model structure for each production cycle problem illustrated with two scenarios and six hours represented by 12 production periods with time discretisation of 30 minutes.



A production cycle is defined to start and end in two consecutive tidal extreme points, respectively. Thus, the duration of a production cycle varies. The duration of each production cycle is adjusted, either prolonged or shortened by a few minutes to achieve an integer number of periods in each production cycle. As a result the number of periods in a production cycle varies between 12 and 13 with the tide frequency.

5.2.3 Additional Assumptions

When modelling the operation of a tidal lagoon power plant certain assumptions and simplifications have been necessary.

An important assumption is that the reservoir water level is assumed to be at sea level at the end and beginning of every production cycle, hence the production head is zero at every tide peak (high and low). As a consequence, the reservoir is required to be completely filled up or emptied during each production cycle. Consequently, power generation is constrained within a production cycle only. The problem thus decomposes into independent optimisation problems for each production cycle and the model presented is a collection of all independent problems. The independence between consecutive production cycles allows for separate solution of each tidal half-cycle operation problem.

Production head is assumed constant during a period and equals the initial value in the beginning of the period. The reservoir head varies with turbine flow and reservoir area as described in equation 2.4. The reservoir is modelled as even surfaced and cylindrically shaped, hence reservoir area is constant over time.

To completely fill up or empty the reservoir sluicing is necessary. Flow through the turbines, and therefore power production, is only possible for volume flows larger than a minimum flow. Sluicing is assumed to level out the remaining difference between reservoir and sea level. Sluicing capacity is assumed to be sufficient and not handled explicitly in the model, but completed momentarily at the end of a production cycle.

5.2.4 Operational Costs

Fixed costs and energy proportional operational costs are assumed to only affect the total profitability of a tidal lagoon power plant and not the operational decisions. Thus, these costs are neglected in the problem. Turbine start-up costs are assumed important in determining optimal operation, as stated in (4), and chosen to be included in the model. Turbines are forced to shut down by the end of every production cycle.

5.3 Linearisation of the Problem

Most optimisation programs are still only able to solve linear problem. This is also the case for the software used here, Mosel Xpress. To linearise the problem, special ordered sets have been used.

The power-discharge equation 2.8 and maximum volume flow (limited by head), equation 2.7, are non-linear functions in the original problem and requires additional effort in solving compared to an LP problem. The equations are repeated for comprehensibility:

$$p = \eta^T(q)\eta^G \rho g h q$$

$$q(t) = A_c\sqrt{2gh}$$

When solving traditional hydro-scheduling problems a common simplification is assuming constant production head, i. e. constant reservoir head even when emptying the reservoir, and solving the resulting LP problem with linear solvers. As explained in appendix B, due to low production head and high relative head variation during power production in a tidal lagoon, assuming constant production head results in a power estimating error exceeding an acceptable value. Thus, head effects have been included in the modelling. In solving the non-linear problem, the following strategies were identified:

1. Using a non-linear problem solver
2. Linearisation of non-linear power function solved iteratively (CPLEX or other solver necessary)
3. MIP problem with pre-generation of production profiles
4. MIP problem with column generation of production profiles
5. MIP problem with linearisation of the non-linear power function

For solution purposes and integration in the EMPIRE model, the non-linear operational tidal lagoon problem is linearised. An explanation of the linearisation of the power-discharge equation follows below. The maximum volume flow restriction 2.7 has been relaxed in the linearised formulation of the problem and is instead handled in the preprocessing of the problem as described in section 8.3.1.

5.3.1 Linearisation of the Power-discharge Equation

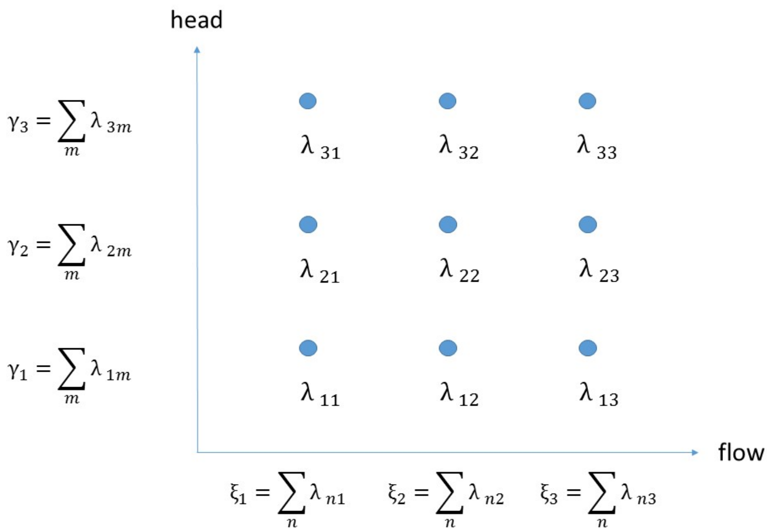
The linearisation method is chosen in order to capture both the production head and turbine flow effects on power production. Hence, a procedure allowing for continuous values of both variables are necessary. The complexity level captured is higher than the rectangle method presented in (10) and implemented in (9), and similar to both the triangle method described in (10), and the approach presented in (6).

The non-linear power-discharge equation is linearised by generating a two-dimensional grid of combinations of point values for reservoir head and turbine flow with corresponding values for power production. For each value of turbine flow and reservoir head, power is calculated according to the power-discharge equation for a tidal lagoon as given in equation 5.2.

$$p = \eta^T(q)\eta^G \rho g |h^{RES} - H^T| q \tag{5.2}$$

where $\eta^T(q)$ is a function for turbine efficiency, η^G is the generator efficiency, h^{RES} is reservoir head, H^T is tide head, q is turbine volume flow, ρ is density and g is gravity.

Figure 5.3: The two-dimensional flow-head grid illustrated for three grid points in each dimension. Weighting variables and both sets of SOS2 variables are shown.

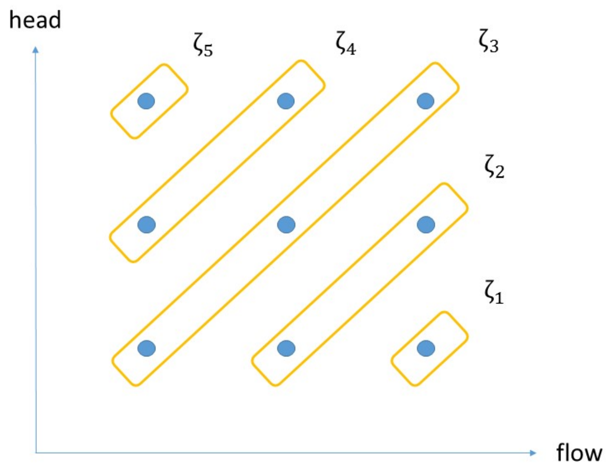


Continuous values for both turbine flow and reservoir head are allowed for by introducing weighting variables for interpolation in both the head and flow dimension. Reservoir head, turbine flow and power at any point in time are defined as a weighted sum of the grid point values. Hence, the power function is approximated as a piecewise linear function of two variables.

Interpolation in both the turbine discharge and reservoir head dimension is supported by introduction of two SOS2 according to the explanation in section 4.2 and illustrated in figure 5.3. Thus, any value for turbine flow and reservoir head are ensured to be a linear interpolation of at most four adjacent grid point values. That is, any of the two variables for turbine flow and reservoir head are an interpolated value of at most two adjacent point values in its dimension. The power variable, which is a function of both variables, can take an interpolated value of all four point values.

To ensure an unique description of a turbine flow-reservoir head combination, additional variables with imposed SOS2 restriction, equalling the sum of weighing variables over the diagonals in the grid are added to the problem, as described in section 4.2 and illustrated in figure 5.4. Hence, any solution for turbine discharge, reservoir head and power generated is an interpolated value of at most *three* adjacent point values. Note that the feasible region is unchanged with the introduction

Figure 5.4: The uniqueness SOS2 variables holding the sum of weighting variables over the grid diagonals.



of these restrictions, the addition affects the number of identical solutions only. The complete mathematical formulation is presented in 6.2.

Chapter 6

Mathematical Model

In this chapter the mathematical model for the operational profit optimisation problem for a tidal lagoon power plant is presented. Firstly, the non-linear model is presented followed by a linearisation of the problem.

6.1 The Non-linear Model

Sets and Indices

$\mathcal{C}$ Set of production cycles c. $\mathcal{C}^{EBB} \subseteq \mathcal{C}$ is the set of all production cycles c originating in high tide and $\mathcal{C}^{FLOOD} \subseteq \mathcal{C}$ is the set of all production cycles c originating in low tide.

$\mathcal{T}_c$ Set of time periods t in production cycle c, $= \{1..T_c\}$.

$\mathcal{N}$ Set of identical turbines n, $= \{1..N\}$.

$\mathcal{S}$ Set of scenarios for realisation of power market prices, indexed by s.

Ω Set of bid points in the day-ahead power market, indexed by ω

Parameters

P^{DA}_{cts} Power price for sales in the day-ahead market in production cycle c and period t for price scenario s, in $\frac{GBP}{MWh}$

$P^{BID}_{ct\omega}$ Specified point value price for bid ω in the day-ahead market for delivery in production cycle c and period t, in $\frac{GBP}{MWh}$

P^{RT}_{cts} Power price for sales in the real-time market for delivery in production cycle c and period t, given price scenario s, in $\frac{GBP}{MWh}$

T Duration of each time period t in hours

C^{OPR} Operational costs occurring at every turbine start-up, in GBP

η^G Generator efficiency

η^T Turbine flow dependent function for turbine efficiency

ρ Water density in $\frac{kg}{m^3}$

g Gravitational constant in $\frac{m}{s^2}$

V^0_c Volume in m^3 at beginning of each production cycle $c \in \mathcal{C}^{EBB}$ or volume at end of each production cycle $c \in \mathcal{C}^{FLOOD}$. Equals total volume flow out of or into reservoir during production cycle c.

Q^{MAX} Maximum allowed volume flow through each turbine in $\frac{m^3}{s}$

Q^{MIN} Minimum allowed volume flow through each turbine in $\frac{m^3}{s}$

A Reservoir area in m^2

H^T_{ct} Tide head in production cycle c and period t in m above/below middle water level

Pr_s Probability for realisation of price scenario s

A^C Turbine cross sectional area in m^2

F^{PROD} Production factor converting the production unit into the sales unit MWh

Variables

q_{ctns} Volume flow in production cycle c and time period t through turbine n given scenario s, in $\frac{m^3}{s}$

h^{RES}_{cts} Reservoir head in production cycle c and beginning of time period t given scenario s, in m above/below middle water level

h_{cts} Production head in production cycle c and beginning of time period t given scenario s, in m

w_{cts}	Power produced in production cycle c and time period t for scenario s, in W
$x^{DA}_{ct\omega}$	Bidding volume for bid point ω corresponding to the specified bid price $P^{BID}_{ct\omega}$ for delivery in the day-ahead market in production cycle c and period t, in MWh
y^{DA}_{cts}	Energy sold in the day-ahead market for delivery in production cycle c and period t given price scenario s, in MWh
y^{RT}_{cts}	Energy sold for delivery in the real-time market for delivery in production cycle c and period t for scenario s, in MWh
l_{cs}	Spilled water in production cycle c given scenario s, in m^3
α^{START}_{ctns}	Binary variable = 1 if turbine n is started up in production cycle c and time period t given scenario s, 0 otherwise
α^{RUN}_{ctns}	Binary variable = 1 if turbine n is running in production cycle c and time period t given scenario s, 0 otherwise

The Objective Function

The objective function maximises expected profit over all scenarios from sales in the day-ahead and real-time power market, including turbine start-ups costs.

$$\max \left\{ \sum_{s \in \mathcal{S}} Pr_s \left(\sum_{c \in \mathcal{C}} \sum_{t \in \mathcal{T}_c} \left(P^{DA}_{cts} y^{DA}_{cts} + P^{RT}_{cts} y^{RT}_{cts} \right) - \sum_{c \in \mathcal{C}} \sum_{t \in \mathcal{T}_c} \sum_{n \in \mathcal{N}} C^{OPR} \alpha^{START}_{ctns} \right) \right\} \tag{6.1}$$

Restrictions

Production head h_{cts} for production cycle c, time period t and scenario s is the difference in reservoir head h^{RES}_{cts} to tide level H^T_{ct}

$$h_{cts} = |h^{RES}_{cts} - H^T_{ct}|, \quad c \in \mathcal{C}, t \in \mathcal{T}_c, s \in \mathcal{S} \tag{6.2}$$

For production cycle c, time period t and scenario s, tide reservoir head h^{RES}_{cts} equals reservoir head in period $t-1$ *less* a reservoir geometry dependent function of any flow in period $t-1$ if c is *ebb* tide. If c is *flood* tide, h^{RES}_{cts} equals reservoir head in period $t-1$ *plus* a reservoir geometry dependent function of any flow in period $t-1$. For cylindrical reservoir this is

$$h_{cts}^{RES} = h_{c(t-1)s}^{RES} - \frac{T\sum_{n\in\mathcal{N}} q_{c(t-1)ns}}{A}, \quad c \in \mathcal{C}^{EBB}, t \in \mathcal{T}_c\backslash\{1\}, s \in \mathcal{S} \tag{6.3}$$

$$h_{cts}^{RES} = h_{c(t-1)s}^{RES} + \frac{T\sum_{n\in\mathcal{N}} q_{c(t-1)ns}}{A}, \quad c \in \mathcal{C}^{FLOOD}, t \in \mathcal{T}_c\backslash\{1\}, s \in \mathcal{S} \tag{6.4}$$

Reservoir head in the first period in a production cycle c and for scenario s, h_{c1s}^{RES},is assumed to equal the tide level at $t = 1$, H_{c1}^{T}.

$$h_{c1s}^{RES} = H_{c1}^{T}, \quad c \in \mathcal{C}, s \in \mathcal{S} \tag{6.5}$$

Volume flow through turbine n in production cycle c, time period t for scenario s, q_{ctns}, is limited by the reservoir head h_{cts}^{RES} as described by equation 2.7.

$$q_{ctns} \leq A^{C}\sqrt{2gh_{cts}}, \quad c \in \mathcal{C}, t \in \mathcal{T}_c, n \in \mathcal{N}, s \in \mathcal{S} \tag{6.6}$$

Volume flow through turbine n in production cycle c, time period t for scenario s, q_{ctns}, is limited by turbine n to be in operating mode, $\alpha_{ctns}^{RUN} = 1$, and maximum and minimum flow through each turbine, Q^{MAX} and Q^{MIN} respectively.

$$q_{ctns} - Q^{MAX}\alpha_{ctns}^{RUN} \leq 0, \quad c \in \mathcal{C}, t \in \mathcal{T}_c, n \in \mathcal{N}, s \in \mathcal{S} \tag{6.7}$$

$$q_{ctns} - Q^{MIN}\alpha_{ctns}^{RUN} \geq 0, \quad c \in \mathcal{C}, t \in \mathcal{T}_c, n \in \mathcal{N}, s \in \mathcal{S} \tag{6.8}$$

Total volume flow out of (or into) the reservoir in cycle c, time period t given scenario s is limited by maximum reservoir volume at beginning (or end) of cycle c, V_c^0. The gap between maximum reservoir volume and total volume flow in production cycle c and scenario s, l_{cs}, is spill and will flow through sluices at no power generation.

$$\sum_{t\in\mathcal{T}_c}\sum_{n\in\mathcal{N}} q_{ctns} + l_{cs} = V_c^0, \quad c \in \mathcal{C}, s \in \mathcal{S} \tag{6.9}$$

Total power produced w_{cts} in production cycle c and time period t, given scenario s is the sum of power produced in all n turbines, and is given by generator efficiency η^G, turbine efficiency function η^T, water density ρ, constant of gravity g, production head h_{cts} and volume flow through turbine n, q_{ctns}. The turbine efficiency η^T is a function of the volume flow through the turbine q_{ctns} and described in appendix D.

$$w_{cts} = \sum_{n\in N} \eta^G\eta^T(q)\rho g h_{cts} q_{ctns}, \quad c \in \mathcal{C}, t \in \mathcal{T}_c, s \in \mathcal{S} \tag{6.10}$$

The sum of power sold for delivery in production cycle c and period t for scenario s in both the day-ahead market y^{DA}_{cts} and the real-time market y^{RT}_{cts} must equal power produced in production cycle c and time period t for scenario s. The production factor F^{PROD} converts the production unit into sales unit.

$$y^{DA}_{cts} + y^{RT}_{cts} = F^{PROD} w_{cts}, \quad c \in \mathcal{C}, t \in \mathcal{T}_c, s \in \mathcal{S}, \tag{6.11}$$

The day-ahead sales obligation for delivery in production cycle c, period t and realisation of price scenario s is determined by linear interpolation between the adjacent bid quantities $x^{DA}_{ct\omega}$ corresponding to the specified bid prices $P^{BID}_{ct\omega}$ for bid ω

$$y^{DA}_{cts} = \frac{P^{DA}_{cts} - P^{BID}_{ct(\omega-1)}}{P^{BID}_{ct\omega} - P^{BID}_{ct(\omega-1)}} x^{DA}_{ct\omega} + \frac{P^{BID}_{ct\omega} - P^{DA}_{cts}}{P^{BID}_{ct\omega} - P^{BID}_{ct(\omega-1)}} x^{DA}_{ct(\omega-1)}$$

$$\text{if} \quad P^{BID}_{ct(\omega-1)} \leq P^{DA}_{cts} < P^{BID}_{ct\omega}, \quad c \in \mathcal{C}, t \in \mathcal{T}_c, s \in \mathcal{S}, \omega \in \Omega/\{1\} \tag{6.12}$$

Turbine n is started in cycle c and time period t for scenario s if it is running in production cycle c and time period t and not running in production cycle c and time period $t-1$

$$\alpha^{RUN}_{c(t-1)ns} + \alpha^{START}_{ctns} - \alpha^{RUN}_{ctns} \geq 0, \quad c \in \mathcal{C}, t \in \mathcal{T}_c \backslash T^0_c, n \in \mathcal{N}, s \in \mathcal{S} \tag{6.13}$$

Turbine n is assumed to require a start-up in the first running period t of every production cycle c, i.e. the turbine is standing prior to the start of each production cycle c

$$\alpha^{START}_{ctns} - \alpha^{RUN}_{ctns} = 0, \quad c \in \mathcal{C}, t \in \mathcal{T}^0, n \in \mathcal{N}, s \in \mathcal{S} \tag{6.14}$$

The number of symmetrical solutions for all turbines n are reduced by

$$q_{ctns} - q_{ct(n+1)s} \geq 0, \quad c \in \mathcal{C}, t \in \mathcal{T}_c, n \in \mathcal{N} \backslash N, s \in \mathcal{S} \tag{6.15}$$

Turbine discharge q_{ctns}, power production w_{cts}, sales quantities x^{DA}_{ct} and y^{RT}_{cts}, and spill l_{cs} can only take non-negative values.

$$q_{ctns} \geq 0, \quad c \in \mathcal{C}, t \in \mathcal{T}_c, n \in \mathcal{N}, s \in \mathcal{S} \tag{6.16}$$

$$x^{DA}_{ct\omega} \geq 0, \quad c \in \mathcal{C}, t \in \mathcal{T}_c, \omega \in \Omega \tag{6.17}$$

$$w_{cts}, y^{DA}_{cts}, y^{RT}_{cts} \geq 0, \quad c \in \mathcal{C}, t \in \mathcal{T}_c, s \in \mathcal{S} \tag{6.18}$$

$$l_{cs} \geq 0, \quad c \in \mathcal{C}, s \in \mathcal{S} \tag{6.19}$$

Reservoir head h_{cts}^{RES} and production head h_{cts} are free variables.

$$h_{cts}^{RES}, h_{cts} \quad isfree, \quad c \in \mathcal{C}, t \in \mathcal{T}_c, s \in \mathcal{S} \tag{6.20}$$

The variables controlling the turbine operating states and start-ups are binary variables.

$$\alpha_{ctns}^{RUN}, \alpha_{ctns}^{START} \in \{0,1\}, \quad c \in \mathcal{C}, t \in \mathcal{T}_c, n \in \mathcal{N}, s \in \mathcal{S} \tag{6.21}$$

6.2 Linearised Model

The model is linearised by linearisation of the power-discharge restriction 6.10 and relaxation of the head-flow restriction 6.6. SOS2 are introduced to describe two-dimensional interpolation between specified values of turbine flow and reservoir head.

Additional Sets and Indices

$\mathcal{R}$ — Set of grid break points in the flow dimension $= \{1, ..., R\}$, indexed by r

$\mathcal{K}$ — Set of grid break points in the head dimension $= \{1, ..., K\}$, indexed by k

$\mathcal{I}$ — Set of constraints reducing the degree of freedom in interpolation with weighting variables, $= \{1, 2, ..., (R + K - 1)\}$, indexed by i

Additional Parameters

Q_r — Predefined point value for volume flow through any turbine, for break point r, in $\frac{m^3}{s}$

H_{ctk}^{RES} — Predefined point value for reservoir head defined for break point k in production cycle c and period t, in m above/below middle water level.

W_{ctrk} — Predefined point values for power produced in production cycle c and period t for the combination of flow and reservoir head given by break-point (r, k), in W

Additional Variables

λ_{ctnsrk} — Weighting variable for break point (r, k) in production cycle c and period t for turbine n and scenario s.

h^{RES}_{ctns} A variable holding the value of reservoir head in production cycle c and period t seen by turbine n for scenario s and needed in the formulation with turbine dependent weighting variables λ_{ctnsrk}

ξ_{ctnsr} Variables allowing for interpolation of weighting variables in two dimensions. For a given production cycle c, time period t, turbine n, scenario s and flow point value r, this variable equals the sum of weighting variables over the head dimension or the rth *column* in the grid.

γ_{ctsk} Variables allowing for interpolation of weighting variables in two dimensions. For a given production cycle c, time period t, turbine n, scenario s and reservoir head point value k, this variable equals the sum of weighting variables over the flow dimension or the kth *row* in the grid.

ζ_{ctnsi} Variables used for reduction of degree of freedom in interpolation of weighting variables. For a given production cycle c, time period t, turbine n and scenario s this variable equals the sum of weighting variables over the ith diagonal in the grid.

Additional Restrictions

Turbine volume flow in production cycle c and period t given scenario s through turbine n is the weighted sum of the predefined turbine flows for all break points (r, k), Q_r, determined by the weighting variables λ_{ctnsrk}

$$q_{ctns} = \sum_{r\in\mathcal{R}} \sum_{k\in\mathcal{K}} Q_r \lambda_{ctnsrk}, \quad c \in \mathcal{C}, t \in \mathcal{T}_c, n \in \mathcal{N}, s \in \mathcal{S} \tag{6.22}$$

Reservoir head in production cycle c and period t seen by turbine n, given scenario s, is the weighted sum of the predefined reservoir head for all break points (r, k) H^{RES}_{ck} determined by the weighting variables λ_{ctnsrk}

$$h^{RES}_{ctns} = \sum_{r\in\mathcal{R}} \sum_{k\in\mathcal{K}} H^{RES}_{ck} \lambda_{ctnsrk}, \quad c \in \mathcal{C}, t \in \mathcal{T}_c, n \in \mathcal{N}, s \in \mathcal{S} \tag{6.23}$$

Further, reservoir head in production cycle c and period t given scenario s, is turbine independent and must be fixed to a common value for all turbines h^{RES}_{cts}

$$h^{RES}_{cts} = h^{RES}_{ctns}, \quad c \in \mathcal{C}, t \in \mathcal{T}_c, n \in \mathcal{N}, s \in \mathcal{S} \tag{6.24}$$

The power equation 6.10 is replaced by the following constraint. Power produced in production cycle c and period t given scenario s, is the weighted sum of the predefined power production values for breakpoints (r, k), W_{ctsrk}, determined by the weighting variables λ_{ctnsrk}

$$w_{cts} = \sum_{r \in \mathcal{R}} \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{N}} W_{ctrk} \lambda_{ctnsrk}, \quad c \in \mathcal{C}, t \in \mathcal{T}_c, s \in \mathcal{S} \tag{6.25}$$

The weighting variables for turbine n in production cycle c and period t must sum to 1 if the turbine is running and sum to 0 otherwise

$$\sum_{r \in \mathcal{R}} \sum_{k \in \mathcal{K}} \lambda_{ctnsrk} = \alpha_{ctns}^{RUN}, \quad c \in \mathcal{C}, t \in \mathcal{T}_c, n \in \mathcal{N}, s \in \mathcal{S} \tag{6.26}$$

At most four weighting variables corresponding to adjacent break points (r, k) can be non-zero. This is achieved by defining the variables ξ_{ctnsr} for production cycle c, period t, turbine n, scenario s and flow point value r, and similarly γ_{ctnsk} for reservoir head point value k, as the sum of all weighting variables λ_{ctnsrk} in the reservoir head and turbine flow dimension, respectively. Further, a SOS2 requirement is imposed on both the sum of ξ_{ctnsr} variables $\forall \mathcal{R}$, and the sum of γ_{ctnsk} variables $\forall \mathcal{K}$

$$\xi_{ctnsr} = \sum_{k \in \mathcal{K}} \lambda_{ctnsrk}, \quad c \in \mathcal{C}, t \in \mathcal{T}_c, n \in \mathcal{N}, s \in \mathcal{S}, r \in \mathcal{R} \tag{6.27}$$

$$\{\xi_{ctns1}, ..., \xi_{ctnsR}\} is SOS2 \quad c \in \mathcal{C}, t \in \mathcal{T}_c, n \in \mathcal{N}, s \in \mathcal{S} \tag{6.28}$$

$$\gamma_{ctnsk} = \sum_{r \in \mathcal{R}} \lambda_{ctnsrk}, \quad c \in \mathcal{C}, t \in \mathcal{T}_c, n \in \mathcal{N}, s \in \mathcal{S}, k \in \mathcal{K} \tag{6.29}$$

$$\{\gamma_{ctns1}, ..., \gamma_{ctnsK}\} is SOS2 \quad c \in \mathcal{C}, t \in \mathcal{T}_c, n \in \mathcal{N}, s \in \mathcal{S} \tag{6.30}$$

The degree of freedom in interpolation is reduced by ensuring at most two adjacent diagonals in the grid to take non-zero values. A third set of variables, ζ_{ctnsi} is defined as the sum of weighting variables over diagonal i for production cycle c, period t, turbine n and scenario s. The set of variables are then taken as SOS2.

$$\zeta_{ctnsi} = \sum_{r \in \mathcal{R}} \lambda_{ctnsr(i+r-R)}, \quad c \in \mathcal{C}, t \in \mathcal{T}_c, n \in \mathcal{N}, s \in \mathcal{S}, i \in \mathcal{I} \tag{6.31}$$

$$\{\zeta_{ctns1}, \zeta_{ctns2} ..., \zeta_{ctns(R+K-1)}\} isSOS2 \quad c \in \mathcal{C}, t \in \mathcal{T}_c, n \in \mathcal{N}, s \in \mathcal{S} \quad (6.32)$$

The weighting variables together with all sums of weighting variables are non-negative

$$\lambda_{ctnsrk} \geq 0, \quad c \in \mathcal{C}, t \in \mathcal{T}_c, n \in \mathcal{N}, s \in \mathcal{S}, r \in \mathcal{R}, k \in \mathcal{K} \quad (6.33)$$

$$\xi_{ctnsr} \geq 0, \quad c \in \mathcal{C}, t \in \mathcal{T}_c, n \in \mathcal{N}, s \in \mathcal{S}, r \in \mathcal{R} \quad (6.34)$$

$$\gamma_{ctnsk} \geq 0, \quad c \in \mathcal{C}, t \in \mathcal{T}_c, n \in \mathcal{N}, s \in \mathcal{S}, k \in \mathcal{K} \quad (6.35)$$

$$\zeta_{ctnsi} \geq 0, \quad c \in \mathcal{C}, t \in \mathcal{T}_c, n \in \mathcal{N}, s \in \mathcal{S}, i \in \mathcal{I} \quad (6.36)$$

The turbine dependent reservoir head variables are free

$$h^{RES}_{ctns} \quad isfree, \quad c \in \mathcal{C}, t \in \mathcal{T}_c, n \in \mathcal{N}, s \in \mathcal{S} \quad (6.37)$$

Constraints 6.10, 6.2 and 6.6 are removed and the complete linear model is then described by equations 6.1, 6.3-6.5, 6.7-6.9 and 6.11-6.37.

Chapter 7

Scenario Generation and Stability Testing

In this chapter the scenario generation methods utilised in the computational study conducted in 8 are presented, and a measure of their quality is briefly discussed.

7.1 Scenario Generation Methods

In order to test the model on power market prices with different characteristics than those experienced the last four years, two different scenario generation methods are developed and explained below. The first method produces a scenario tree based on current power market price characteristics, whereas the second method provide a scenario tree with increased price variance.

Hourly day-ahead and half-hourly intraday power prices over the last four years are downloaded from the APX Power Exchange [1]. The intraday market prices are intrepreted as real-time market prices. The hourly day-ahead prices are converted into half-hourly prices by replicating each value once.

Scenario Generation Method 1

The first scenario generation method is a sampling procedure. For each production cycle, S days are drawn from historical price data corresponding to S price

[1] see: http://www.apxgroup.com/market-results/apx-power-uk/ukpx-rpd-historical-data; accessed 05-March-2016. The day-ahead prices are found under the name *APX Power UK Auction & BritNed historical data* and the intraday prices under the name *APX Power UK RPD historical data*. Both prices correspond to the years 2012-2015

scenarios. Seasonality in power prices are accounted for by drawing from the set of days belonging to the same month as the production cycle in question. Each price scenario contains both a day-ahead price and a real-time price for all periods in the production cycle. Specifically, for P periods in a given production cycle c, each scenario contains P consecutive pairs of power prices starting in the initial half-hour for production cycle c. The consecutive prices are used to account for price correlation between consecutive periods in a production cycle and correlation between day-ahead and real-time prices are considered by drawing both prices for a given scenario from the same historical day. See appendix A for explanation of correlation and calculation of correlation between the two power prices. Four historical years and 30 drawable days may provide up to 120 possible scenarios for each production cycle.

Scenario Generation Method 2

The second scenario generation procedure assumes normally distributed power market prices of specified mean and standard deviation. Further, the current mean price levels experienced both over the year and during a day are assumed to exist in the future. These characteristics are captured by firstly calculating historical mean and standard deviation for day-ahead and intraday power prices for each half-hour during the day over a month from the historical time series. That is, $12 \times 48 = 576$ different pairs of mean and standard deviation for each market price are calculated. Then, price scenarios for production cycle c and time period t are randomly drawn from a normal distribution of mean equalling the calculated historical value for the corresponding month and half-hour, and standard deviation as two times the calculated value in the corresponding month and half-hour.

7.2 Scenario Generation Stability Testing

Stability is tested for both scenario generation procedures on a reduced version of the final model. The test version is run for five hours for each of 10 selected production cycles representing both the tidal characteristics and the power market price characteristics. Ideally, stability should have been tested on the final model. However, due to long solution times the results from the reduced test version are used to indicate stability in the solutions obtained from the scenario generation methods when utilised on the final models.

Stability is tested on the two scenario generation methods with two different number of generated scenarios. Due to the bidding function, less than 10 scenarios in the model is not desired as it would reduce the uncertainty in the model considerably. That is, when the number of scenarios approaches the number of bid point intervals, the uncertainty approaches zero. Stability were expected to be obtained

Table 7.1: In-sample Stability

	Current Price Var.		High Price Var.	
S	10	15	10	15
Mean Obj. [k GBP]	15798	15649	18350	17844
Std. Dev. [k GBP]	228	558	234	43
Std. Dev./Mean Obj.	1.4%	3.6%	1.3%	0.2%

for a number of scenarios exceeding 50, however, initial testing shows this level of scenario tree size would not be possible to solve within the scope of this work. The number of scenarios subject to stability testing are thus chosen to 10 and 15.

For both scenario generation methods, three scenario trees of size 10 are generated. For scenario generation method 1, three scenario trees of size 15 are generated whereas two scenario tress of size 15 are generated for scenario generation method 2[2]. The sample sizes are considered small and a result of limited computational resources.

In-sample Stability

In-sample stability is tested by solving all scenario tree specific problems and comparing the corresponding objective values. Mean and sample standard deviation are calculated for the set of obtained objective function values corresponding to each scenario generation method and scenario tree size. The results are shown in table 7.1. The sample size of two and three are clearly insufficient for evaluation of stability in solutions from the scenario generation methods due to the high variations in sample characteristics. For scenario generation method 1, standard deviation increases with scenario tree size, opposite behaviour than expected for larger sample sizes. The problematic sample size is even more visible for the high-variance price case with scenario tree size of 15. The large reduction in objective function standard deviation for scenario tree size 15 is unlikely to be a result of sufficient scenario tree size.

Out-of-sample Stability

The weaker form of out-of-sample stability described in section 4.3.2 is tested for both scenario generation procedures on the same two scenario tree sizes and sample sizes explained above. According to the test procedure, objective function values are calculated for all combinations of objective functions and solutions within same scenario tree size, and the resulting mean and standard deviation of

[2]The difference in sample size for scenario trees of size 15 for scenario generation method 2 is caused by technical problems

Table 7.2: Out-of-sample Stability

	Current Price Var.		High Price Var.	
S	10	15	10	15
Mean Obj. [k GBP]	16638	16552	16987	17385
Std. Dev. [k GBP]	398	581	2129	2189
Std. Dev./Mean Obj.	2.4%	3.5%	12.5%	12.6%

objective functions values are calculated and compared. The results are shown in table 7.2. Again, objective function standard deviation for scenario generation method 1 increases with increasing scenario tree size and the sample size is interpreted insufficient for stability evaluation. The results indicates highly unstable solutions from scenario generation method 2.

Chapter 8

Computational Study

In this chapter the implementation and solution specifics of the operational tidal lagoon profit maximisation model are presented. The study is performed with the Swansea Bay Tidal Lagoon project as case study. If accepted for financial support by the UK government the Swansea project will be the UK pilot tidal lagoon project.

In section 8.1, problem parameters such as plant facility specifics and market paramters, are presented. Then, in section 8.2 an explanation of applied problem reduction techniques follows. In section 8.3 additional measures taken when implementing the problem are described, comprising problem preproccesing, addition of valid inequalities and adjustments to solution algorithm control parameters. Section 8.4 presents the test cases and the problem results are presented in section 8.5. Finally, sources of errors are discussed in section 8.6

8.1 Model Input Parameters

All input parameters to the case study except the already discussed price data, are presented in this section. If available, data from the Swansea Bay Tidal Lagoon project are used.

8.1.1 Tidal Power Production Facility

Grid of combinations of point values for reservoir head and turbine flow with corresponding values for power production have been generated for every time period in every production cycle, according to the method presented in section 5.3. Resulting power output is location and facility specific, as it depends on the local tide cycle, turbine characteristics and reservoir design. For this study possible grid points are generated based on the Swansea tide cycle and the plant specifications described later in this section.

Modelling the Tide in Swansea

The Swansea tide cycle over a full year is modelled using the harmonic analysis described in section 2.1.2. For modelling purposes the middle water level could be set equal zero, $MW = 0$, since the energy potential in the surge of the tide only depends on the change in height over time. In addition, the correction for variations in a 18.6 year cycle, f_i, is neglected.

When considering superpositions of partial tides, the universal standard time and corresponding astronomic arguments, $(V_0 + u)_i$, should be included. Calculation of the astronomic arguments used is done following a method described by Schwiderski in (42). A slightly different notation is used by Schwiderski where the astronomic argument (V_0+u) is referred to as χ. This notation is adopted here [1].The astronomic argument is slightly time dependent [2]. The tide cycle used in the model is calculated using astronomic arguments calculated for 01.01.2016. Updating of the arguments over time has been neglected. The simplified expression used for the harmonic analysis is

$$H(t) = \sum_i H_i cos(\sigma_i t + \chi_i - g_i) \tag{8.1}$$

where:
$H(t)$ is the tide height at time t
H_i is the amplitude to the constituent i
σ_i is the periodic frequency of constituent i
χ_i is the astronomic argument of constituent i
g_i is the phase shift of constituent i

In this work, the tide cycle is calculated based on four constituents [3]. The geographical dependent constants for harmonic analysis are provided by different institutions or organisations subject to the county in question. The United Kingdom Hydrographic Office collects and publishes tidal data for the UK coastline. The data used for generating the Swansea tide cycle is published in (37). Some correc-

[1]A different notation has also been used for the other constants. However, this notation is not adapted here.
[2]Schwiderski suggest that these constants should be updated annually
[3]The constituents used in the analysis are: M_2, S_2, K_1 and O.

tions were made to the generated tide cycle after verifying with the published tide tables for Swansea. The corrections are described in appendix C.

Turbine and Reservoir Characteristics

The reservoir is modelled as a cylinder with 1.5 km radius[4]. Installed turbine capacity is taken to $340MW$, 34 turbines of 10 MW each. Total capacity equals the planned capacity in the Swansea Bay tidal lagoon project.

Turbine specific information about the efficiency of commercial turbines is hardly available. The turbine specifications used in this study are based on available data about the turbines planned used in the Swansea project [5] combined with available data for the La Rance tidal range project in France and turbine specifications used in a similar analysis presented in (33). Adjustments in turbine parameters are performed in order to meet the available data of 0.5 m minimum production head for low-range turbines [6].

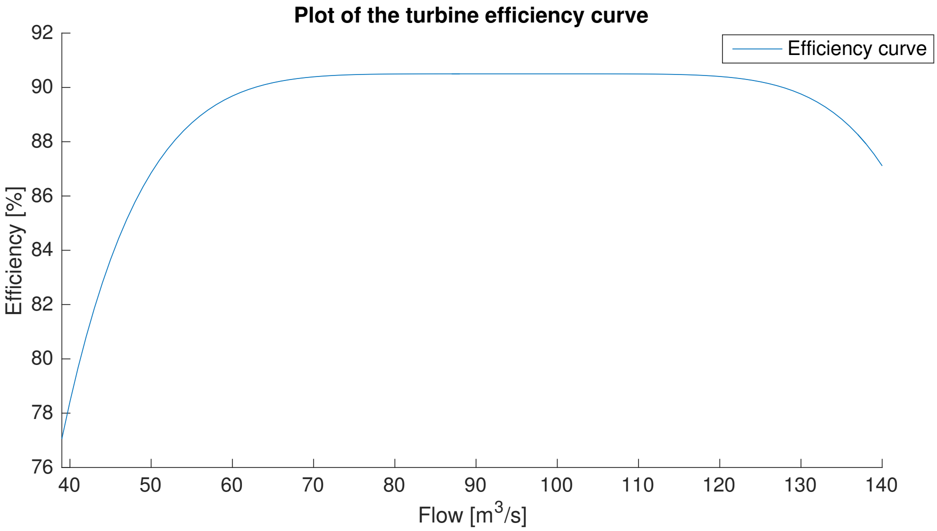
The turbine efficiency curve, see figure 8.1 is obtained from (33). Turbine efficiency is a function of the turbine specific characteristics and turbine flow, see appendix D for details.

[4]Reservoir geometry is decided based on photomontages presented of the lagoon on the project website http://www.tidallagoonswanseabay.com/the-project/see-the-lagoon/56/; accessed 03-March-2016

[5]http://www.power-technology.com/features/featuretidal-giants---the-worlds-five-biggest-tidal-power-plants-4211218/ accessed 03.05.16

[6]http://www.andritz.com/hy-bulb-turbines accessed 03.05.16

Figure 8.1: Plot of the turbine efficiency curve



Costs

As explained in chapter 5, only turbine start-up costs are included. The cost is set to 5 GBP. Turbine start-up costs are interpreted as variable operating costs and model profit is interpreted as plant profit before adding fixed operating costs.

8.1.2 Power Markets and The Bidding Function

For each production cycle and time period a fixed number of point values for day-ahead bid prices are chosen as input to the model. The number of price intervals chosen is the lowest number still reflecting different selling behaviour for a low, intermediate and high price realisation. Thus, three price intervals and four bid points are used. The bid point prices are selected in order to obtain equally many day-ahead price realisations in the price interval between any of them.

8.2 Problem Reduction

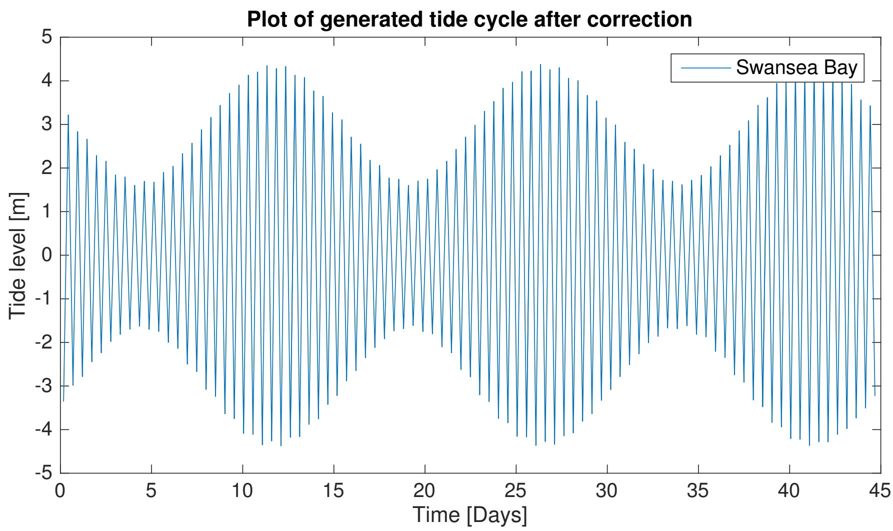
Due to binary turbine variables, extended use of SOS2 and price uncertainty, each production cycle problem becomes difficult to solve. Further, the entire set of production cycle problems is large and recognised too time consuming solving within the scope of this work. Problem size for three production cycles with different tidal characteristics are presented in table 8.1. If scaling this up for the entire problem (1409 cycles) solving the problem would take 7045 hours, or 281 days, and give a solution with duality gap in the range of (at least) 6%-58%. Clearly, techniques for problem size reduction are necessary. An explanation of two actions taken in order to reduce the problem size follows.

Table 8.1: Problem Size for a selection of production cycles, including 15 scenarios, 2 Turbines and 4×5 Grid Points. The duality gaps are obtained when solving for 5 hours.

Production Cycle	Number of Variables	Number of Constraints	Duality Gap
200	10413	8354	28%
218	11013	8579	6%
1108	11752	9554	58%

8.2.1 Representative Production Cycles

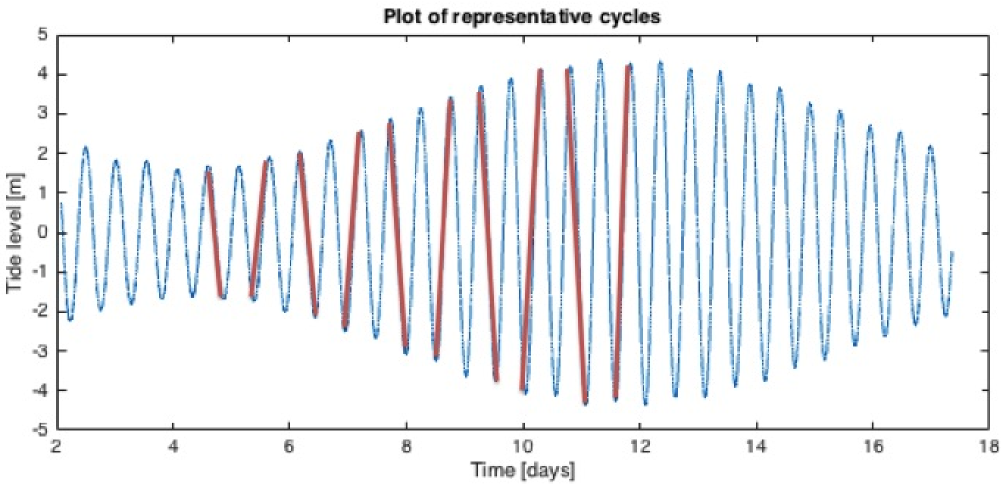
Figure 8.2: Plot of the corrected tide cycle over 45 days



Due to the periodical tide cycle with negligible seasonal variations, the tidal development over a year can be represented by a reduced number of cycles multiplied by a scaling factor. As seen in figure 8.2, the tide cycle period is 14 days and consists of two symmetrical seven-days parts in the generated tide cycle. Due to some seasonality in power prices, different solutions may be obtained for similar tide cycles at different times of the year. Four equally long price seasons, hereby called *seasons*, are defined, and each are equally represented in the reduced formulation of the problems.

Firstly, four 14-days tide periods are chosen, one from each defined season. Secondly, each 14-days period is represented by 10 six-hours production cycles, evenly spread over one of the symmetrical seven-days parts as illustrated in figure 8.3. Hence, both price seasonality and tidal variation are accounted for. By up-scaling the 40 carefully selected production cycles, the annual 1409 production cycles can be represented by the aggregated model.

Figure 8.3: Illustration of how representative cycles are selected from a seven day period



8.2.2 Reducing Turbine Running Flexibility

Each turbine is associated with binary variables and variables of SOS2, hence the number of turbines significantly increases the required computational resources.

The reservoir size is scaled down in the implementation in order to reduce the number of optional operating turbines, thus reducing binary variables in the operational model. By scaling up the income and costs correspondingly in the objective function, the power plant can be approximated by an aggregation of smaller, symmetrical plants. The idea is taken from (15). The approximated problem is smaller and much easier to solve. However, flexibility in choosing the number of running turbines during operation is lost with the approximation. Due to the scaling described, the number of modelled turbines is reduced from 34 to two. That is, considering the objective function scaling factor, the real option is to run either zero, 17 or 34 turbines.

8.3 Problem Implementation

In this subsection, the implementation of the problem is described. Additional actions are taken to enhance the solution of the problem. Firstly, adjustments to the model formulation are presented followed by a description of adjustments to the solution algorithm.

8.3.1 Preprocessing

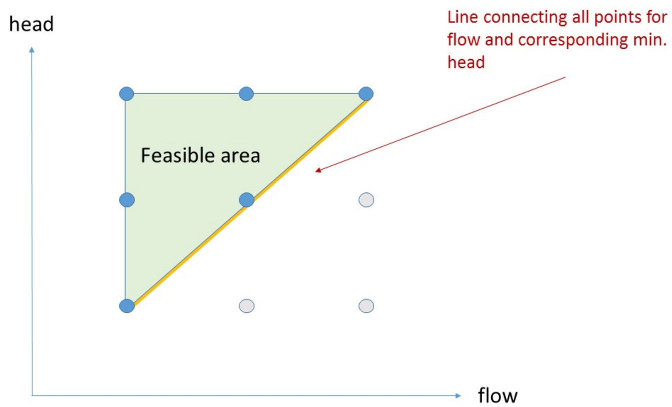
The problem size is further reduced by preprocessing of the weighting variables. For all break points in the reservoir head dimension $k \in \mathcal{K}$, any reservoir head parameter H_{ck}^{RES} exceeding the maximum or minimum limits for production cycle c, is identified. If removing the corresponding break point in the flow-head grid does not remove any feasible areas from the solution space, the weighting variable associated with that break point is removed from the problem.

Linearisation of Flow-head Restriction

The upper limit on volume flow through a turbine as a function of production head, as described by equation 2.7, is non-linear and relaxed in the linear formulation of the problem. To ensure feasible solutions of reservoir head and turbine flow, the reservoir head parameter in the flow-head grid is adjusted. For each production cycle the reservoir head parameter range from the minimum head dictated by the minimum turbine flow level (H_q^{MIN} for $q = Q^{MIN}$) to the maximum reservoir head obtainable in the production cycle in question. This formulation leads to increasing number of infeasible flow-head combinations with increasing flow in the grid. Again, weighting variables for grid points corresponding to infeasible flow-head combinations are removed from the problem if not removing any feasible areas from the solution space.

In the generated flow-head grid, all feasible combinations of reservoir head and turbine flow lie on or above a piecewise linear line connecting the grid points for a specific flow and its corresponding minimum reservoir head, see figure 8.4. An important consideration is to ensure no feasible areas are removed. When implementing the uniqueness SOS2 diagonals presented in 4.2.1 the diagonals must be defined parallel with the diagonal piecewise linear line connecting the (q, H_q^{MIN}) grid points. Note that infeasible operating points due to other constraints on flow and reservoir head are handled explicitly by linear constraints in the model.

Figure 8.4: The flow-head grid with the line restricting the feasible area due to the flow specific minimum head restriction 6.6, highlighted. Note that the grid diagonals for the SOS2 uniqueness variables are defined in the same direction.



8.3.2 Valid Inequalities

A problem specific valid inequality has been identified and added to the implementation of each production cycle problem. The valid inequality limits the total power generation during a production cycle to the production cycle specific upper limit. This upper limit is found by solving the production cycle power maximisation problem to optimality and take the resulting objective value as upper limit on production cycle power generation in the profit maximisation problem. The valid inequality reduces the feasible region for the LP relaxation of the problem, hence strengthening the LP relaxation and reducing the potential number of nodes for investigation in the branch and bound algorithm.

8.3.3 Solution Algorithm Control Parameters

The problems are implemented and solved using the FICO Xpress-IVE Optimizer software. The solver allows for specifying a range of solution algorithm settings. In general, each production cycle problem is different and thus requires different actions in order to optimise the solution algorithm. A range of solution algorithm settings have been tested for solution enhancement. For simplicity, only algorithm control parameters that are expected to improve the solution to a large share of the production cycle problems are changed. The implemented settings are presented below.

Cutting Strategy and Heuristic Strategy

The branch and bound algorithm integrated in the solver is found to be more effective when extending the use of both added cuts and heuristics during the search for a MIP solution. That is, a branch and cut algorithm explained in 4.1 together with heuristics are used. The cutting strategy determines the number of cuts to be generated and added to the problem during the branch and bound algorithm (48). This number is fixed to a high value by choosing *aggressive cut strategy* for the control parameter *cutstrategy*. The use of heuristics is extended beyond the default *automatic selection of heuristic strategy* by setting the control parameter *heurstrategy* to *extensive heuristic strategy*. This setting allows for use of heuristics of all solver predefined complexity levels during the branch and bound search.

Branch Point in SOS2 Variables

When branching on SOS2 variables in the branch and bound algorithm, the set is divided into two subsets where each part form one of the two subproblems described in section 4.1. The split point in the set is specified by input values for variable coefficients. This spit point should be chosen in order to avoid evaluating subproblems with unrealistic good optimistic bounds obtained from solutions suggesting interpolation between non-adjacent weighting variables.

For the set of variables holding the sum of weighting variables over each grid column, defined in equation 6.27 in the mathematical model, the split point results in branching into one part containing the two first variables in the ordering and the other part containing the two last variables in the ordering. The set of variables holding the sum of weighting variables over each grid row, defined in equation 6.29 in the mathematical model, is split into a part containing the two first variables in the ordering and a part containing the three last variables in the ordering.

8.4 Test Cases

The profit maximising operational problem for a tidal lagoon is run for two power market price cases. The corresponding power optimisation problem is developed and hereby referred to as problem 2, whereas the profit maximisation problem is referred to as problem 1. All instances are explained below.

Case 1: Current Power Market Price Case

The current power market price case represents the profit optimisation problem given current power market price characteristics. Within each independent production cycle problem, each scenario represents a historical path of power prices from the last four years. The obtained solutions can be interpreted as a good measure of annual plant income and operational pattern for a similar technology with similar characteristics running in near future.

Case 2: High variance power market prices

The high variance market price case represents a possible future state of the power market, influenced by a high share of intermittent power generation driving prices up and down with intermittent generation. Note that this case is not a prediction for future power markets but a representation of one out of many possible states with different market characteristics than observed today.

Problem 2: Maximisation of Power Generation

The corresponding power maximising model over each production cycle is developed and run. The model is based on the same technical assumptions but power market prices and bidding/selling decisions are neglected.

8.5 Results

Both problems were solved with the linear programming solver FICO Xpress-IVE Optimizer. Case 1 and 2 were solved on a HP BL686 G7 with 4 x AMD Opteron 6274 2.2 GHz processor and 128 Gb RAM. Each production cycle problem was first solved for five hours without the valid inequalities presented in section 8.3, then solved for four hours with the valid inequalities implemented. The final solution is obtained by choosing the best solution for each production cycle and case among the two runs. Problem 2 was solved to optimality for all production cycle problems on a HP Compac Elite 8300 with Intel(R) Core(TM) i7-3770 3.40 GHz processor and 16 Gb RAM.

This section presents the obtained results to the profit maximisation problem and the corresponding power maximisation problem. Firstly, the quality of the obtained solutions is discussed. Then power generation schedules for both problems and cases are evaluated, followed by a presentation and analysis of the sales decisions. Finally, the plant profitability is discussed.

8.5.1 Quality in Solution

When limiting solution time, the obtained solution is not necessarily optimal or optimality might not be proved. Thus, a gap between the best obtained solution and the upper bound may exist. For both case 1 and 2, when not implementing the valid inequalities presented in section 8.3, these gaps are up to 62% of the upper bound, see figure 8.5 for the case 1 solution characteristics. Case 2 solution characteristics are similar. When the valid inequalities are implemented, the upper bound decreases in most production cycle solutions, hence the duality gap decreases. The objective values are mostly not improved. See figure 8.6 for dulaity gaps in final solutions for case 1 and 2.

Figure 8.5: Top and middle: Objective value and upper bound for solution for each case 1 production cycle problem solved with and without the valid inequality presented in section 8.3. Bottom: Duality gaps for solution with and without the valid inequality.

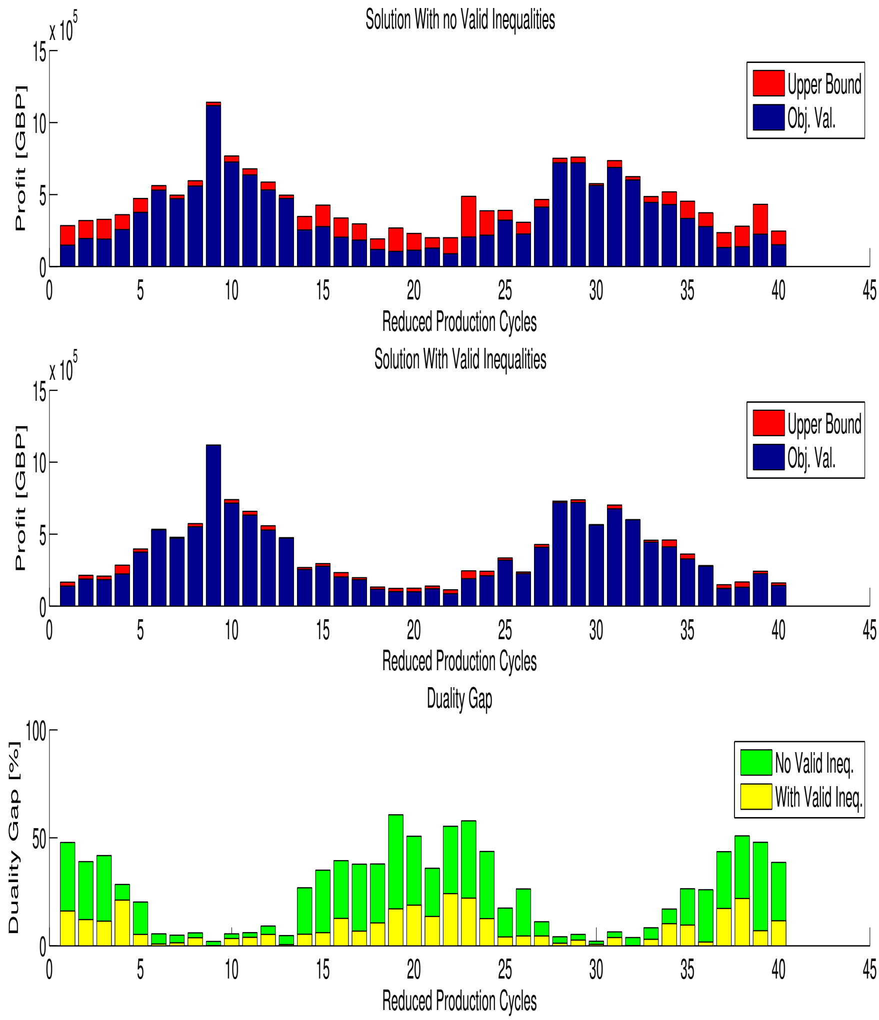
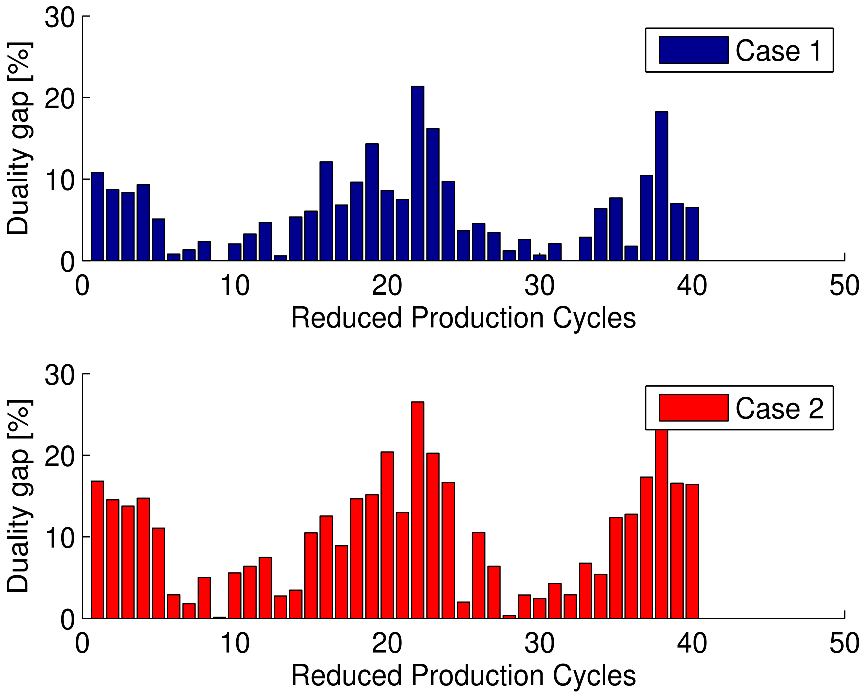


Figure 8.6: Top: Duality gap for the final solution to each case 1 production cycle problem. Bottom: Duality gap for the final solution to each case 2 production cycle problem.



8.5.2 Power Generation

Power generation specific results are presented for the two cases of the profit optimisation problem and the power optimisation problem. Firstly, these results are presented and compared by taking a scenario specific solution as base scenario for the cases of the profit maximisation problem. Secondly, each case is presented in further detail and scenario specific solutions are compared.

In figure 8.7 the case 1 power generation solution for the base scenario over a full model year is illustrated. Note that the reduced cycles shown here are not consecutive in reality. The non-zero bar shaped parts of the graph represent power generation for the modelled set of production cycles. Note how the power generation from the selected production cycles varies over the model year. Maximum power generation over a production cycle varies between $60MW$ and $240MW$.

Figure 8.7: Case 1 base scenario power generation during a model year

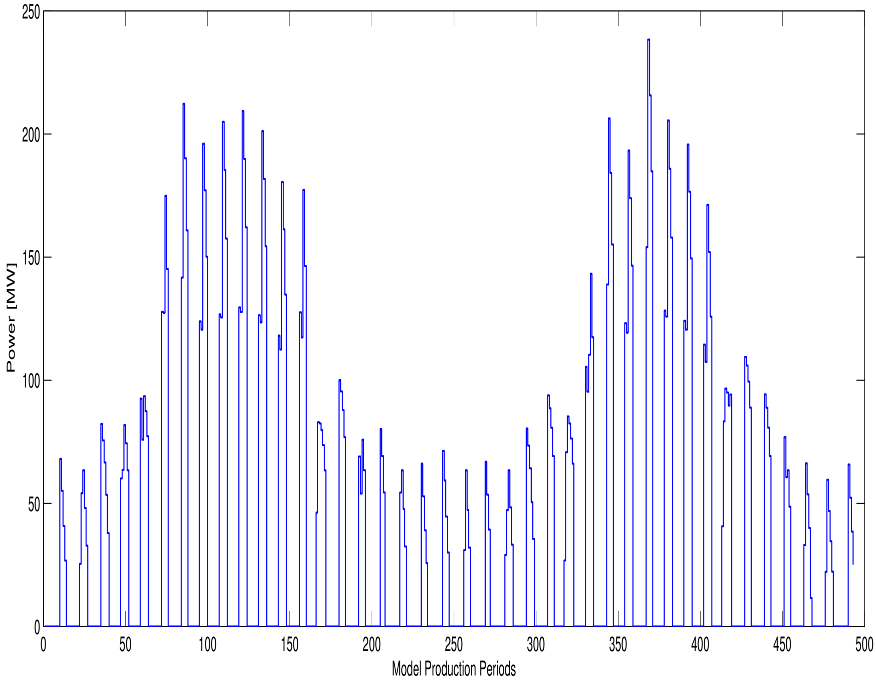
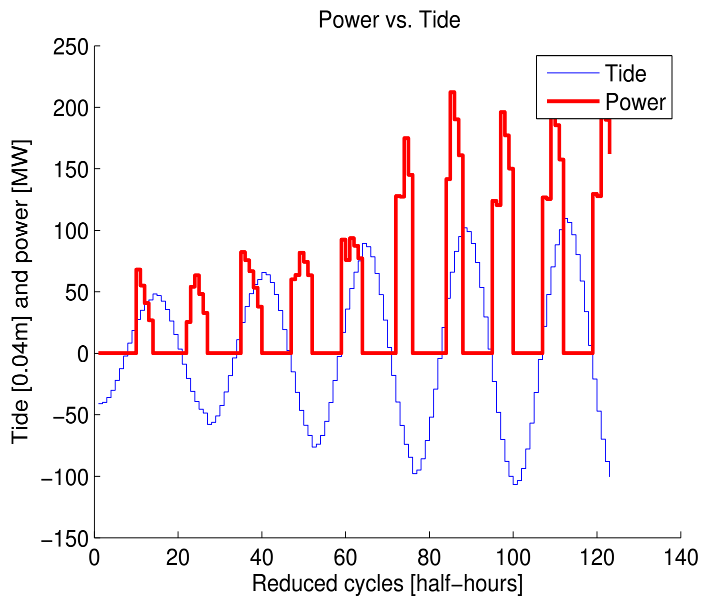


Figure 8.8: Case 1 base scenario power production for representative production cycles in model season 1 vs. the corresponding tide.



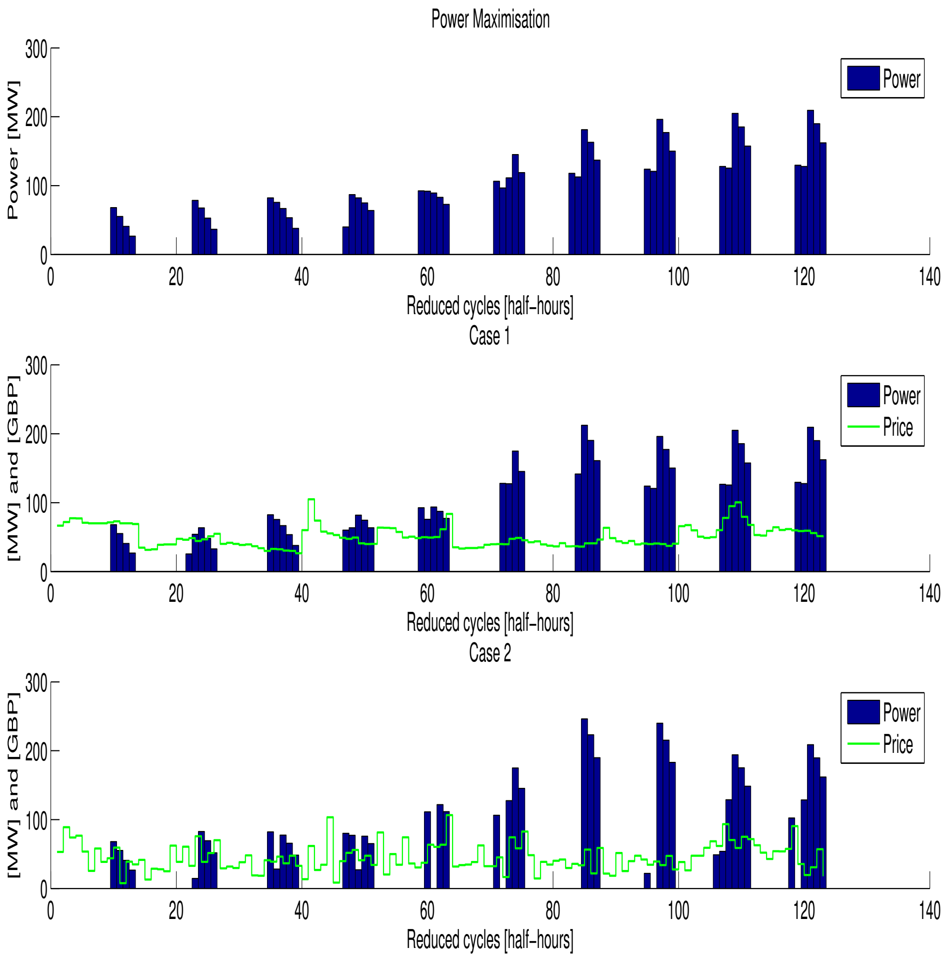
In figure 8.8 the case 1, base scenario power production versus the tide is illustrated. The tide is measured as height above middle water level. Note how power generation is limited by the tide cycle Generation is zero in the two to four first hours of every production cycle, hence in the succeeding period after any tidal extreme point.

Profit Optimisation Versus Power Optimisation

Annual energy generation is estimated to at most 317 GWh for the power maximisation problem, whereas the solution to the two profit maximisation problems gives annual energy generation of 313 GWh and 304 GWh for case 1 and case 2, respectively This corresponds to an annual energy reduction of 1.3% and 4.1% compared to the power maximisation value. Hence, annual plant capacity factor[7] is 0.11 for both cases.

[7]The capacity factor is calculated as $E/(C * H)$ where E is annual energy generation in MWh, C is installed capacity in MW and H is annual hours.

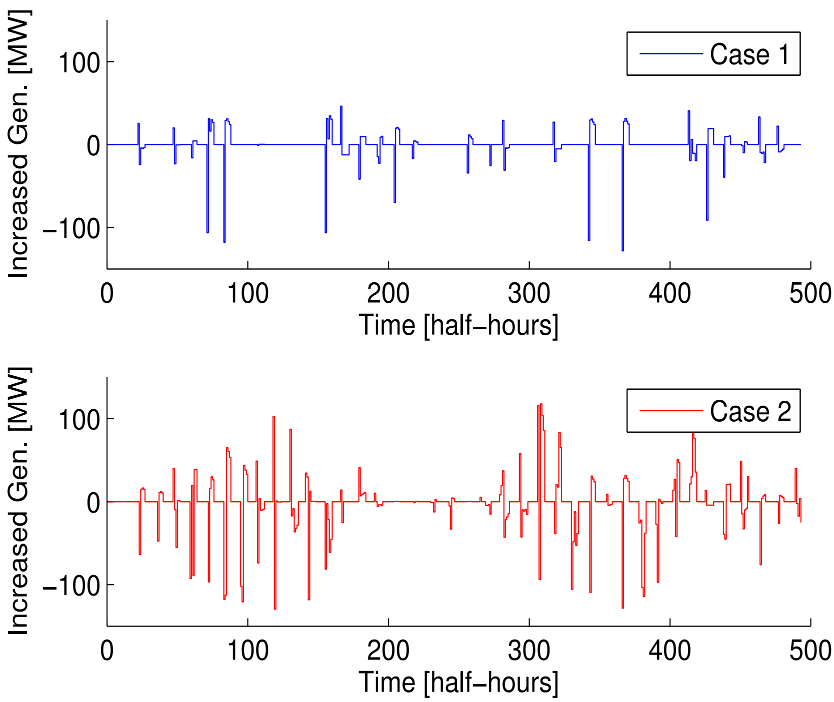
Figure 8.9: Top: Season one power maximisation generation pattern. Middle and bottom: Case 1 and case 2 base scenario power generation pattern vs the real-time market price.



In figure 8.9 the season one generation solutions for the power optimisation problem and the two cases of the profit optimisation problem are presented. The two profit optimisation base scenario solutions are plotted versus its corresponding real-time price. Comparing the three power generation plots for each plot it can be seen some differences from the top to the middle and bottom plot within a certain time interval of each production cycle. By looking at production cycle seven (for half-hour ≈ 85) and comparing the middle plot to the top plot, it can be seen that power production is delayed in return of increased production during the price increase the last periods of the production cycle. In the bottom plot for production cycle ten (for half-hour $\approx$ 120) the power generation is seen to follow the power price, by increasing production during high-price periods in the beginning and end of the production cycle, and reducing production when the price is low in between. The limitation in generation flexibility can be seen in both case one and two by studying the zero power production during the price increase between production cycle three and four, at half-hour ≈ 42.

During the model year, the power maximisation schedule consists of $2 - 2.5$ consecutive hours of power generation for all production cycles. The profit maximisation schedules consist of both shorter and longer duration period of generation. The shortest period of generation is 1.5 hours and the longest is 3.5 consecutive hours. In contrast to the power maximisation problem, the profit maximisation schedule also has more than one continuous period of production during a production cycle. The longest non-continuous period of generation within a production cycle is 4 hours, consisting of one period of non-zero production followed by three periods of zero production and finally four periods of non-zero production. Within a production cycle, the power maximisation solution consists of continuous production periods only.

Figure 8.10: Change in power generation in MW for case 1 and 2 base scenario above the power maximisation solution.



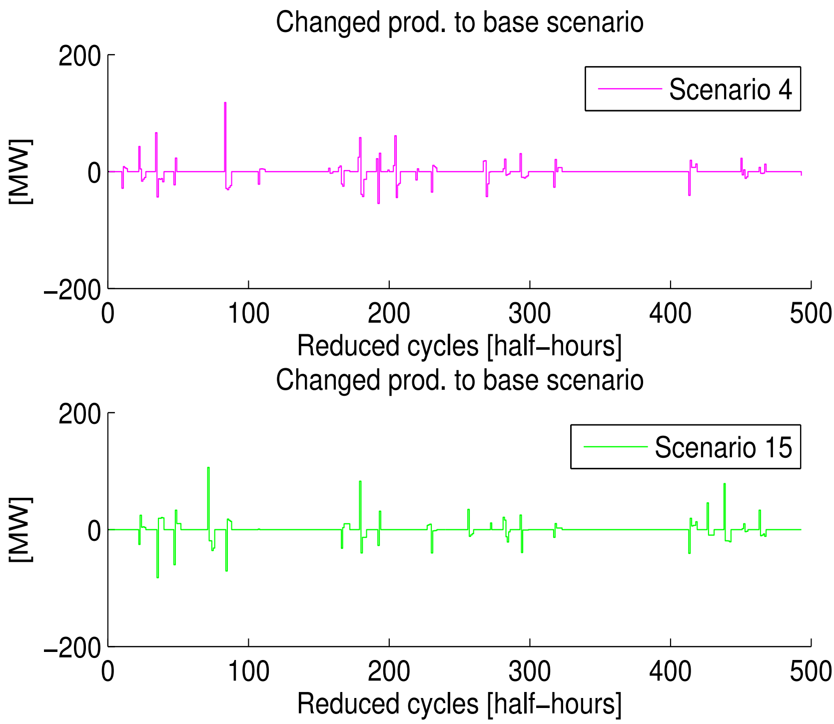
The power generation schedule varies some from the power maximising problem to the profit maximising problems and within the profit maximising problem for the two cases, see figure 8.10. When comparing all profit maximisation solutions to the power maximisation solution, the average changed production for all scenarios to the power maximisation problem (when only considering the periods with production), are $10.0MW$ and $23.3MW$ for case 1 and case 2, respectively. These figures correspond to 10.3% and 24.0% of average power optimisation production in non-zero production periods.

Case 1 Power Generation

Within case 1, the production schedule varies some from one price scenario to another, see figure 8.11. The change in generation for all scenarios to the base scenario, is on average 9.6 MW when only considering production periods with non-zero generation in the base scenario solution, or 9.8 % of average base scenario generation in non-zero periods.

For the base scenario during periods with non-zero power production all turbines are running 98% of the time. That is, during periods with power production all 34 turbines run almost all the time, whereas the the intermediate level of 17 turbines are used 2% of the time.

Figure 8.11: Case 1: Change in generation in MW for a set of selected scenarios to the base scenario

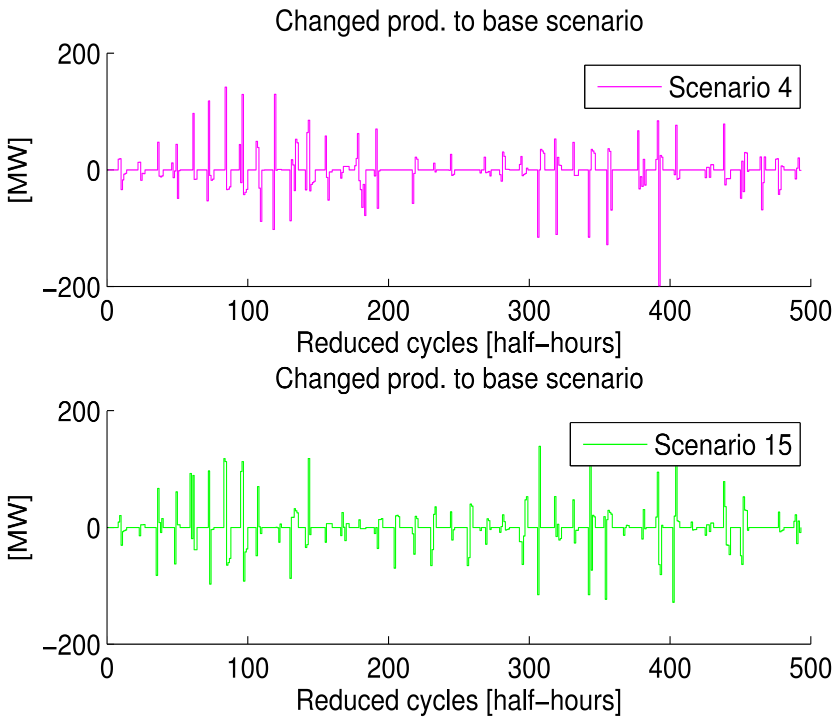


Case 2 Power Generation

Within case 2, the production schedule varies from one price scenario to another, see figure 8.12. The change in generation for all scenarios to the base scenario, is on average 29.9 MW when only considering production periods with non-zero generation in the base scenario solution, or 32.4 % of average base scenario generation in non-zero periods.

For the base scenario during periods with non-zero power production all turbines run 91% of the time, whereas the intermediate level of half turbine capacity is used 9% of the time.

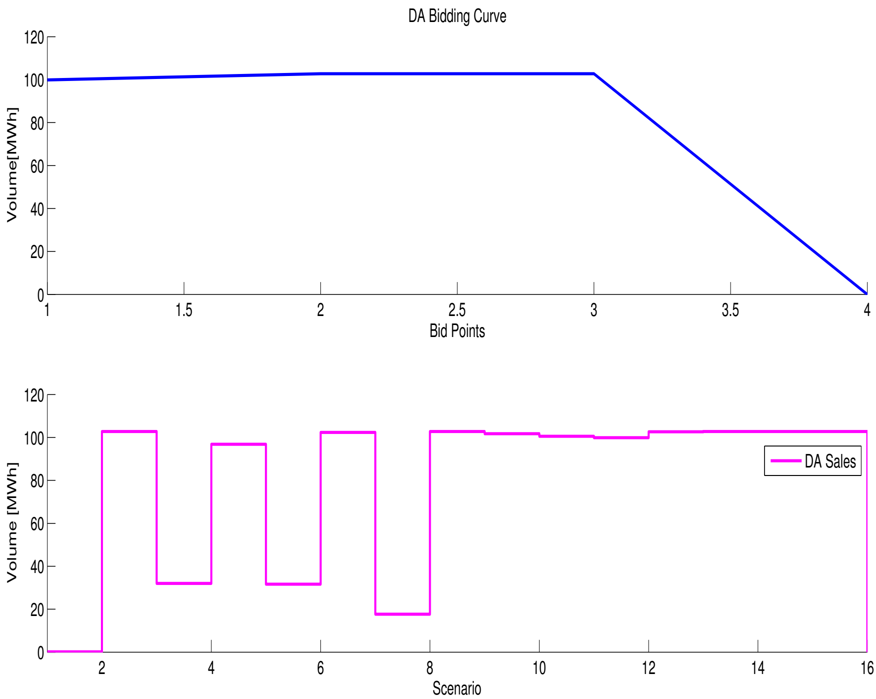
Figure 8.12: Case 2: Change in generation in MW for a set of selected scenarios to the base scenario



8.5.3 Sales Decisions

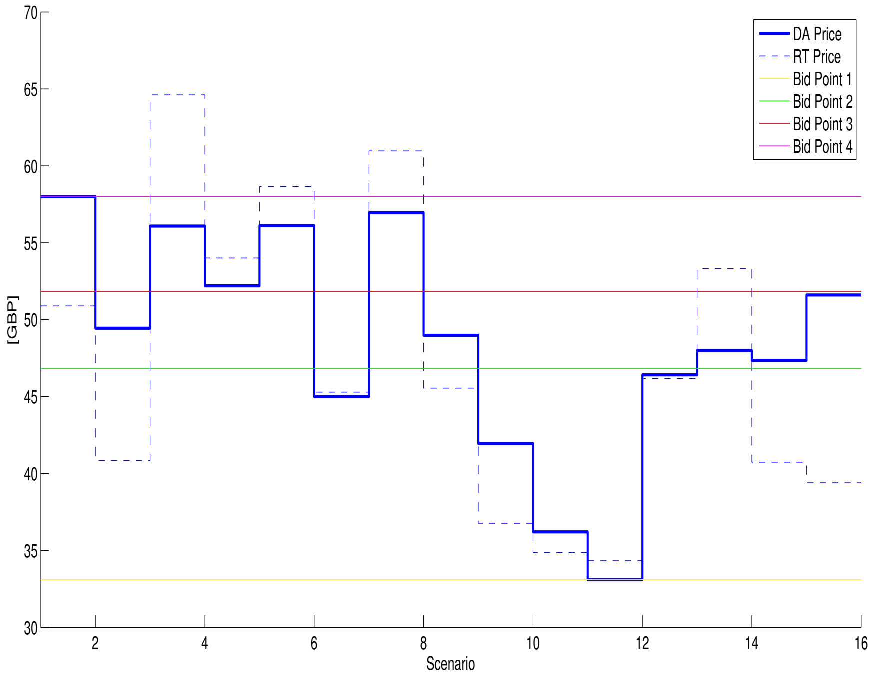
During the model year the average share of all sales for all scenarios, given case 1, are 45% for the day-ahead market and 55% for the real-time market. For case 2 during the model year, the average share of all sales for all scenarios are 41% and 59% for the day-ahead and real-time market, respectively.

Figure 8.13: Top: Case 1 day-ahead bidding curve for production cycle 1362 and period 10. Bottom: Case 1 day-ahead sales realisations for production cycle 1362 and period 10.



A typical bidding curve for a certain production cycle and period for case 1 is shown in the top plot in figure 8.13. Four optimal bid volumes are allocated to the four bid point prices and the piecewise linear line connecting the four point values make up the bidding curve. The three linear parts of the curve are hereby referred to as bid price intervals and are numbered from one to three, according to the bid point on their left side. For each day-ahead price realisation, the corresponding delivery obligation is given by the volume value on the curve. Non-zero and slightly increasing bidding volumes are seen for increasing bid prices in price interval one and two, whereas bidding volumes decrease towards zero with increasing day-ahead price realisation in bid price interval three. The bottom plot in figure 8.13 shows the realised day-ahead sales for all case 1 scenarios.

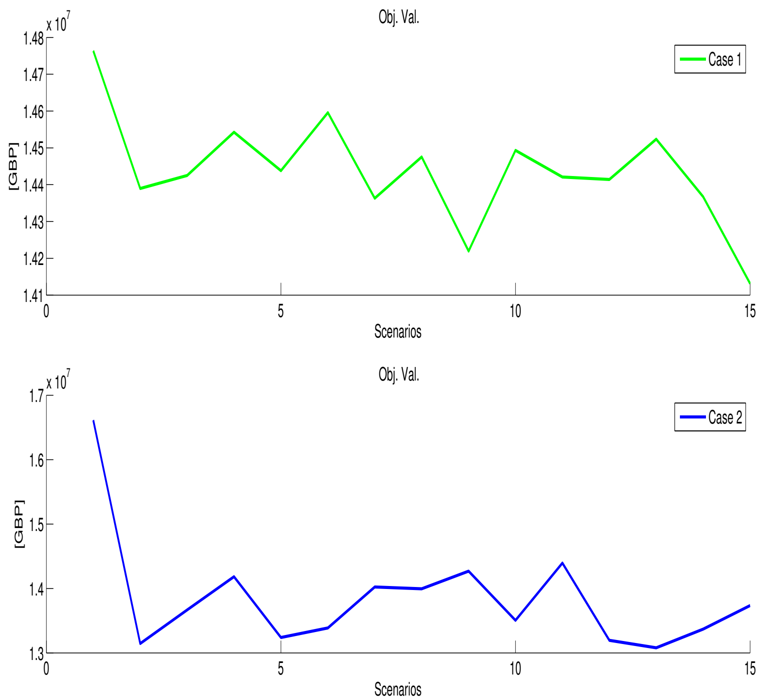
Figure 8.14: Case 1 day-ahead price realisation for production cycle 1362 and period 10 versus the bid point prices and real-time price realisation.



The optimal day-ahead bid volumes are chosen based on the possible power price realisations included in the model. In figure 8.14 the 15 day-ahead price scenarios are plotted versus the corresponding real-time price and the bid point prices. $P1$ refers to bid price one etc. Note how $1/3$ of the day-ahead price realisations lay in each of the three bid price intervals. The experienced decreasing bidding volume with increased day-ahead price realisation in bid price interval three can be understood by comparing all day-ahead price realisations in the third price interval. For four out of five scenarios the real-time price realisation is higher than the day-ahead price realisation. If realisation of any of the scenarios within bid price interval three, the expected income for selling the available power in the real-time market is higher than for selling the same amount in the day-ahead market. Hence, smaller bidding volumes for bid price interval three increases expected profit.

The value of short-term flexibility is estimated by calculating the annual income for all scenario specific solutions on the base scenario prices. That is, the sales decisions associated with a certain scenario are put into the the objective function for the base scenario, and the resulting objective function values are compared. For case 1, the average base scenario objective value for non-base scenario solutions is 14.4 million GBP compared to the optimal base scenario value of 14.8 million GBP, see figure 8.15. The average change in objective value for the non-optimal solutions to the optimal solution is 0.37 million GBP, leading to an average increase in objective value from the non-optimal solutions to the optimal solution of 2.4%. Performing the same analysis for case 2, the average base scenario objective value for the non-optimal solutions is 13.7 million GBP whereas the objective function value for the optimal solution is 16.6 million GBP. Then the average objective value change with non-optimal solution to the optimal objective value is 3.0 million GBP, and the average objective value improvement from the non-optimal solutions to the optimal solution is 21.7%.

Figure 8.15: Objective function value for the base scenario prices and all scenario specific solutions. The base scenario optimal objective value is plotted as scenario 1.



8.5.4 Profitability

The annual profit from operating the Swansea Bay Tidal Lagoon is estimated to 14.7 million GBP for case 1 and current power price characteristics. For power markets with increased variance, as described by case 2, the annual profit is estimated to 16.8 million GBP. Total start-up costs are estimated to 0.24 million GBP and 0.29 million GBP, for case 1 and 2 respectively.

Based on estimates for annual fixed operating costs for a UK tidal lagoon from (39), the annual Swansea Bay tidal lagoon profit would be 8.5 million GBP and 10.6 milion GBP for case 1 and 2, respectively. When considering the estimated investment cost from the same source and 120 years plant life time, the *return on investment* for the two price cases would be 0.33% and 0.74%, respectively[8].

8.6 Model Shortcomings

In this section, identified weaknesses in the model are presented. The shortcomings are assumed to impact the performance of the model and should be considered when analysing the previously presented results.

The reservoir size has a major impact on annual power output by limiting total power generation over a production cycle. Comparing utilised power capacity with total installed capacity of $340MW$, it becomes clear that a significant share of installed capacity is unused in all production cycles. Hence, increasing the reservoir size will increase total power generation. Similarly, given the reservoir size, installed capacity is more than sufficient, driving investment costs up. Due to unused capacity, the level of installed capacity in the model is high compared to realised production. Hence, the ratio of reservoir size to installed capacity used in the model is low. This results in low annual energy output and poor power plant capacity factor.

Further, the reservoir and turbine capacity scaling presented in section 8.2.2 greatly reduces the flexibility in turbine operation. Aggregating the turbine dispatch decisions removes some of the dynamics in optimal turbine adjustment to both operational and market conditions.

Moreover, representing the power price development seen over a full year, by four months of historical price data as described in section 8.2.1, might neglect important price characteristics and overestimate the impact of rare price incidents. Further, the definition of price seasons can be a source of error. The division into four three-months periods with similar average mean only, neglects the higher moments of the data. In addition, the scenario generation procedure combined with

[8]The rate of return is calculated by the Microsoft Excel integrated formula *rate*

the number of scenarios are insufficient to comprise the real uncertainty in the tidal lagoon operating decisions when placing day-head bids. The day-ahead bidding decisions are shown to be highly affected by the scenario specific prices.

The discretisations used for time representation and linearisation purposes are sources of error in the model. The time discretisation limits operational flexibility as turbine dispatch is constant within the defined period. Further, production head is assumed constant over a period. Regarding the linearisation, some error in the results is expected to arise from treating non-linear functions as piecewise linear functions. This error will decrease with increasing number of break points in the flow-head grid.

Considering the input parameters used in the computational study, the price data is taken from the years 2012 to 2015. High risk is associated with future power prices and some estimates predict a higher average power price than included in the input data to the model (39). Hence, the annual plant income presented above might be underestimated. In addition, the used turbine characteristics and performance at low head are uncertain and based on estimates. Future performance is even more uncertain, but low-range turbine performance is expected to improve with development of tidal range technologies (1). Hence, the power output presented might be an underestimate for future generation potential.

Finally, most of the problems are not proved to be optimal, hence better solutions than presented may exist.

Chapter 9

Conclusion and Further Work

9.1 Conclusion

The profit maximisation problem developed reveals an optimal power generation schedule on average varying by 10.3% from the average production cycle power generation found by solving the corresponding power optimisation problem. With increased power market price variance, the optimal generation schedule distinguishes even more from the power optimisation generation schedule. In case of double power market price variance the average change in optimal generation schedules is found to be 24.0%. Including power prices in the optimisation problem is shown to reveal a new optimal generation schedule. This indicates that the producer benefits from utilising available operational flexibility when considering power prices. The difference in optimal generation schedule, hence income gains, are shown to increase with increased power price variance.

The annual value of short-term flexibility in a tidal lagoon power plant operating in the current UK power market is estimated to 0.37 million GBP and a 2.4% increase in annual plant income. This value is shown to increase considerably with increased power market price variance. The annual value of short-term flexibility is estimated to 3.0 million GBP and a 21.7% gain in annual income for the high-variance power price case. Hence, the available operational flexibility in a tidal lagoon is shown to provide limited income gains in a power market with current price characteristics. However, with the high share of intermittent power capacity planned introduced to the European and UK power market in near future, the power price characteristics may change. If the power market realises increased price variance in future, active plant operation based on a profit optimisation is expected to provide significant income gains.

Optimal plant capacity has not been found and the model results are insufficient for estimating good levels of installed capacity. The optimal level must be decided by including investment costs in the model. Revenues will only increase with installed turbine capacity until a certain point limited by reservoir size. In that case, a large share of plant capacity will be unused in lower-range production cycles and investment costs per energy generated will be high. This is clearly not a profitable level of plant capacity. In the solutions presented a large share of installed capacity is unused in all production cycles. Low capacity utilisation and capacity proportional costs point towards optimal plant capacity to be at a lower level than the model capacity.

The 340 MW modelled tidal lagoon power plant based on the Swansea Bay project characteristics is shown to generate up to 317 GWh annually. For comparison, the Swansea Bay Tidal Lagoon project owner states an annual energy generation of 500 GWh and 58% increase from the amount presented in this work. Due to the high dependency between reservoir size and total energy available, the two values can not be directly compared without using the same input values for reservoir size. However, considering the possible differences in input parameters the authors still judge the 58% annual increase in energy output unrealistically high. The results presented indicate an overestimated figure for annual plant energy generation presented by the Swansea Bay Tidal Lagoon project owner.

The tidal lagoon plant income is calculated to about 15.0 million GBP per year based on the current market price characteristics. For a power market with twice the variance seen today, the annual income is estimated to 17.1 million GBP. When comparing with industry cost estimates a tidal lagoon operating in the current power market will provide a return on the plant investment of 0.33%. Investment returns above 3% require a 70 % decrease in investment costs. Higher returns are expected for increased power price variance. Hence, plant profitability will depend heavily on the technology cost level. Current realisation of the Swansea Bay Tidal Lagoon project is not shown to be profitable.

9.2 Further Work

The model presented has a number of shortcomings the authors propose for improvement in future work. First and foremost, techniques for enhanced solution must be done in order to facilitate increased complexity to the problem. Either further addition of problem specific valid inequalities combined with problem specific heuristics or a completely different formulation may be necessary. Focus on finding better solver settings is also expected to provide smaller improvements in solution time.

Increasing the number of turbines subject to unique operation and some sort of capacity optimisation will increase profits and should be done in future work. Focus should be put on drivers for variable costs and aim to integrate more realistic variable operating costs in the model. An extended model including an increased number of scenarios and either utilising a scenario reduction method or a more complex scenario generation method is also worth investigating. Specifically, a scenario generation method taking into account future prediction may provide more realistic results for future operation.

Part II

The Total System Model

Chapter 10

Introduction to EMPIRE

In this chapter the European Model for Power system Investment with (high shares) of Renewable Energy (EMPIRE) developed by Christian Skar, is presented. The content of this chapter is meant to provide the reader with necessary insight in the foundation of the work which the scope of this thesis is based on. A full presentation of the stochastic power system investment model is given in (47).

The EMPIRE is a dynamic capacity expansion model for the European power system. The objective is minimising total system cost for the entire European power system over a 40-years time horizon. Optimal figures for investments in production capacity, investments in transmission capacity and operation of installed generation capacity are determined. The model can take into consideration different policy scenarios and support schemes.

The model is formulated as a multi-horizon stochastic linear program considering uncertainty in load and intermittent generation. The solution contains optimal figures for country-wise investment decisions at 5-years intervals and energy production in each operational stage. The EMPIRE is based on the following assumptions, presented in (46):

- Perfect competition between power producers
- Generation capacity is aggregated over each defined technology
- Investment decisions can take continuous values
- Transmission lines can be built and operated independently

- Inelastic demand

- Perfect foresight about fuel prices, carbon price and load development

10.1 Conceptual Model

The objective of the model is to minimise total cost, comprising costs of investments for generation, transmission and storage capacity, operational costs for generation, and the cost of lost load. Subsidies, carbon price and other policy schemes are included in the generation cost parameters.

The optimal solution is constrained to balance load in all geographical areas, balance storage (charge and discharge) and by technical restrictions. Production is limited by installed capacity and investments are limited by technology and location limits. The EMPIRE includes a wide selection of technologies, covering renewable, fuel based, intermittent, flexible and reliable types of energy sources. Both new technologies and mature technologies are included. The operational modelling is simplified, only the most important technical constraints are included, such as limited thermal generation ramping and water availability for hydro power. Power exchange is limited by interconnector capacities.

10.2 Model Structure

A multi-horizon formulation has been applied in order to limit the problem size. This formulation considers two time-scales, a long-term (strategic) time scale of five years discretisation and a short-term (operational) time-scale of hourly time discretisation. This formulation is facilitated by assuming both future strategical and operational decisions to be independent of current operational realisations.

10.2.1 Geographical Representation

The European power system is divided into a set of system *nodes*, where each node is a near independent subsystem and represents one country. Demand and generation is node specific and power can be exchanged between nodes through interconnectors of specified capacities. Each technology is represented by an aggregated generator for each node, hence the aggregated generator represents the entire generation capacity for the given technology located in the node. This is not the case for location specific power generators, such as wind power generators and solar power generators. A set of location specific generators with unique production profiles based on available natural resources are used to represent these technologies.

10.2.2 Time Discretisation and Aggregation

The model time horizon covers the years 2010 to 2050. Investment decisions are made every five years and operational decisions are made every hour. Generation capacity is assumed available from the start of the five-years investment period when the investment decision was made. Investment costs are assumed to be payed upfront.

The model year is represented by a reduced set of operational hours. The reduced set contains selected intervals of consecutive hours, called seasons. Four seasons represent regular load seasons each containing 48 consecutive hours. In addition, six extreme load seasons are defined, each containing five consecutive hours. Thus, a model year consists of 222 operational hours.

10.2.3 Uncertainty

The EMPIRE is a stochastic model taking into account operational uncertainty in load profiles and renewable power production, comprising uncertainty in generation profiles for solar and wind energy and water inflow for hydro power production. Long-term uncertainty or strategic uncertainty is neglected.

10.3 Scenario Generation

Stochastic scenarios of realisation of load profiles, onshore and offshore wind production profiles and photovoltaics (PV) production profiles represent the mentioned operational uncertainty in the model. Each scenario contains a path of five model years containing certain hourly realisations of all the stochastic parameters. In order to preserve correlation between data series the scenario generation method constructs scenarios by sampling a range of consecutive hours from historical data and taking the historical realisations associated with the same hours as realisations for a given scenario.

Chapter 11

Model Outline of Extension

The EMPIRE has been extended to include tidal lagoon power technologies based on the findings in Part I. In order to keep the complexity in the extended model within desired solvable limits, the formulation of the operational tidal power model has been simplified. Two versions of the tidal extension has been developed, mainly differentiated by degree of operational flexibility.

11.1 Problem Description

The total European power system problem with tidal lagoon power extension aims to determine the cost optimal future energy mix for Europe when tidal lagoon power is included or available for inclusion in the technology portfolio. In order to meet demand in all defined power subsystems at any point in time during the time horizon, investment decisions are made for generation capacity, transmission capacity and storage capacity. Investments in power generation capacity are chosen for in order to meet power supply requirements. The operational decisions optimise deployment of all installed capacity subject to uncertain power demand and renewable generation.

The first version of the extension to EMPIRE adds the option to invest in a set of tidal lagoon power plants. If a plant is chosen, its power generation is fixed to a determined production schedule. The second version of the extension allows for both choosing whether to invest in a set of tidal lagoon power plant and what operational schedule to utilise.

11.2 Model Design

The model structure, the time discretisation, the uncertainty and the aggregation techniques in the original formulation of EMPIRE are unchanged in the extension. All technical tidal lagoon restrictions presented in the operational tidal lagoon problem in part I are excluded from the extension to EMPIRE in order to limit problem complexity. Instead, feasibility in operating points is ensured by careful selection of power production parameters.

Similar to the original EMPIRE formulation, the first version of the problem is a linear programming problem with continuous tidal lagoon investment and production variables. For any hour the production variable is set to a production parameter times generator installed capacity. The production parameters are defined in a production pattern, explained in 11.3. In other words, if decided to invest in tidal lagoon power, the power production in all succeeding periods are predetermined and no operational flexibility exists.

The second version of the extension includes flexibility in production but no flexibility in investment volume. The problem is modelled as a pattern formulation where multiple optional power production patterns are included in the model, representing a feasible power generation schedule. Each power generation pattern is associated with a binary variable defined for each tidal generator, season, investment period and scenario. The binary variable equals one if the generation pattern is used. When a pattern is in use, the following production during the season is determined for the tidal generator in question. That is, the scheduling decision is made for approximately $1/4$ of the model year and the operational flexibility is limited to changing operational schedule once each season and limited to the power pattern values. Also investment variables are binary variables equalling one if the generator is invested in. If chosen for investment a predefined tidal lagoon capacity will be installed. Hence, the second version of the tidal lagoon extension is formulated as a MIP problem and requires increased computational resources solving.

11.3 Power Generation Patterns

A power generation pattern is defined as a power generation schedule for every hour in a model year for a tidal lagoon power plant, hereby referred to as a tidal generator. The pattern is generator specific and consists of hourly power production amounts. Most important, the pattern is based on the findings in the operational tidal lagoon model presented in part I and ensures feasible power production. That is, all technical restrictions and tide cycle modelling are accounted for in the operational model and ensured in the system model by fixing production to a confirmed feasible schedule. Thus, all technical restrictions can be excluded from the system model.

Chapter 12

Mathematical Model: Extension to EMPIRE

The mathematical formulation of the extension to EMPIRE including tidal lagoon power production, will be presented here.

The objective function and all general constraints regarding installation decisions, generation decisions, balancing of the system and technical restrictions for other technologies are handled in EMPIRE. Only tidal lagoon power specific investments and operations are included in the extension. All technical restrictions limiting power generation is handled by ensuring feasibility of production patterns.

As described in chapter 11 two versions of the tidal lagoon power production extension are developed. The first version is based on predetermined production, whereas the second version includes some flexibility in production by allowing for choosing between production patterns for each season. Version one of the extension is presented in 12.1 followed by the description of the second version in 12.2.

12.1 Version 1: LP formulation

Sets

Sets defined in the extension only:

$\mathcal{G}^{Tide}$ Set of tidal power generators where $\mathcal{G}^{Tide} \subseteq \mathcal{G}$, indexed by g

Sets defined in EMPIRE, used in the extension:

$\mathcal{G}$ Set of generators, indexed by g

$\mathcal{I}$ Set of investment time periods, indexed by i

$\mathcal{S}$ Set of seasons, indexed by s

$\mathcal{H}$ Set of operational hours in a model year, indexed by h

$\mathcal{H}_s$ Set of operational hours in season s of a model year, indexed by h

Ω Set of stochastic scenarios ($|\Omega| = O$), indexed by ω

Parameters

Parameters defined in the extension only:

P_{gsh} Energy produced on tidal generator g in season s and hour h, given in MWh per MW installed capacity [MWh/MW]

Variables

Variables defined in EMPIRE, used in the extension:

$y^{gen}_{\omega ihg}$ Power produced on generator g in hour h and investment period i for scenario w, in W

v^{gen}_{ig} Installed generation capacity for generator g in investment period i, in W.

Restrictions

Energy produced on generator g in investment period i and hour h for scenario ω is constrained by the energy parameter P_{gsh} for generator g, season s and hour h, and installed capacity for generator g and investment period i.

$$y^{gen}_{\omega ihg} = P_{gsh} v^{gen}_{ig} \quad \omega \in \Omega, i \in \mathcal{I}, s \in \mathcal{S}, h \in \mathcal{H}_s, g \in \mathcal{G}^{Tide} \tag{12.1}$$

12.2 Version 2: MIP formulation

The sets and variables used in this version of the extension are mostly the same as in version one. Additional sets, parameters and variables necessary in the formulation are presented.

Sets

Sets defined in the extension only:

$\mathcal{P}_g$ Set of feasible production patterns for tidal generator g for a full model year, indexed by p

Parameters

Parameters defined in the extension only:

CAP_g Minimum investment capacity for generator g if chosen for investment

P_{gshp} Energy produced on tidal generator g in season s and hour h if production pattern p is used, given in MWh per MW installed capacity [MWh/MW]

Variables

Variables defined in the extension only

$\delta_{\omega isgp}$ Binary variable equal to 1 if production pattern p is used for generator g in season s and investment period i for scenario ω.

Restrictions

Energy generated in hour h on tidal power generator g in investment period i for scenario ω is given by the production parameter P_{gshp} for pattern p and season s, and the corresponding pattern controlling binary variable $\delta_{\omega isgp}$.

$$y^{gen}_{\omega ihg} = \sum_{p \in \mathcal{P}_g} P_{gshp} CAP_g \delta_{\omega isgp} \quad \omega \in \Omega, i \in \mathcal{I}, s \in \mathcal{S}, h \in \mathcal{H}_s, g \in \mathcal{G}^{Tide} \tag{12.2}$$

Installed capacity in generator g and investment period i must be at least the minimum capacity value for generator g CAP_g if the generator is producing energy in investment period i, season s and scenario ω, hence the corresponding pattern controlling binary variable $\delta_{\omega isgp}$ is 1. Similarly, production according to any pattern p can only happen on generator g in investment period i and season s for scenrio ω if generator g has been invested in.

$$v^{gen}_{gi} - \sum_{p \in \mathcal{P}_g} CAP_g \delta_{\omega isgp} \geq 0 \quad \omega \in \Omega, i \in \mathcal{I}, s \in \mathcal{S}, g \in \mathcal{G}^{Tide} \tag{12.3}$$

Only one production pattern p can be used for tidal generator g in investment period i and season s for scenario ω.

$$\sum_{p \in \mathcal{P}_g} \delta_{\omega isgp} = 1 \quad \omega \in \Omega, i \in \mathcal{I}, s \in \mathcal{S}, g \in \mathcal{G}^{Tide} \tag{12.4}$$

The pattern controlling variable for pattern p on generator g in investment period i and season s δ_{gspiw} is binary.

$$\delta_{\omega isgp} \in [0, 1] \quad \omega \in \Omega, i \in \mathcal{I}, s \in \mathcal{S}, g \in \mathcal{G}, p \in \mathcal{G}^{Tide} \tag{12.5}$$

Chapter 13

Computational Study

In this chapter, a computational study evaluating the impact of developing tidal lagoon generators along the coast of Great Britain (GB) is performed. The methodology of the study will be presented in 13.1. Here will parameters, included locations and generated tide cycles used in the modelling be given. In 13.2 will the performed tests be defined and the policy cases used in the study be described. Then will the results be reported in 13.3, before shortcomings of the modelling are discussed in 13.4.

13.1 Model Parameters

The presented computational study is performed based on the data set used in a decarbonisation study of the European power system presented in (47). The data set is based on the EU reference case 2013 published by the European Commission (11). The EU reference case determines long-term conditions for the dynamics of the European power system. This includes fuel prices, development of load profiles and carbon price development as a climate policy. In addition, cost data is collected for tidal lagoon power production.

Three stochastic scenarios are used, resulting in a total of 666 dispatch hours being considered for each investment period. For further information about the stochastic data the reader is referred to (47).

Investment costs and fixed and variable operation and maintenance (O&M) costs for tidal lagoon power production are obtained from (39). The utilised costs are based on cost reductions obtained by economics of scale and assume investments projects in the scale of 1800 MW.

13.1.1 Modelling the Tide

To allow for a number of location specific tidal lagoon power plants in the extension, the location specific tide cycles from selected locations have been analysed.

Included locations

Several potential locations for tidal lagoon power development along the coast of GB are included in the study [1]. Since the tide cycles are location dependent, the production potential and timing vary along the coast.

Figure 13.1: Map with included tidal lagoon power generation locations in GB

The geographical locations used in the study are given in figure 13.1. The locations are concentrated in two tidal areas of GB. The location specific tide cycles are plotted in figure 13.2 and 13.3 using the same method as in 8.1.1.

[1]The chosen locations are based on the project portfolio of Tidal Lagoon Power, http://www.tidallagoonpower.com/; accessed 26-January-2016

Figure 13.2: Plot of the tide cycles over 48 hours for the specified locations

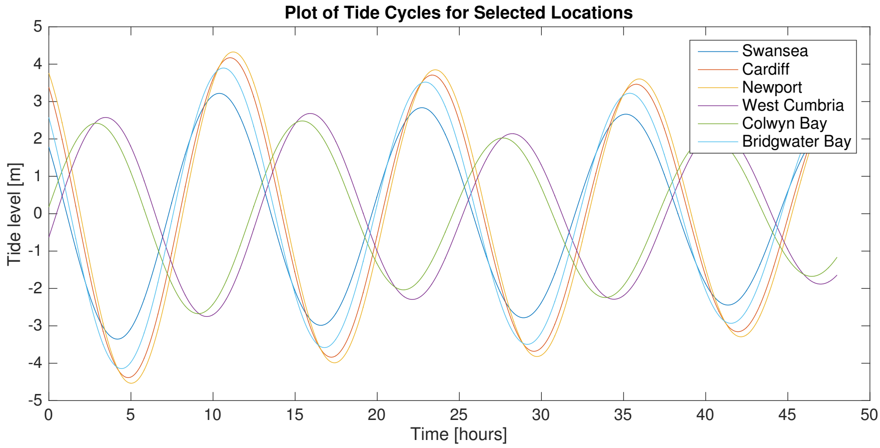
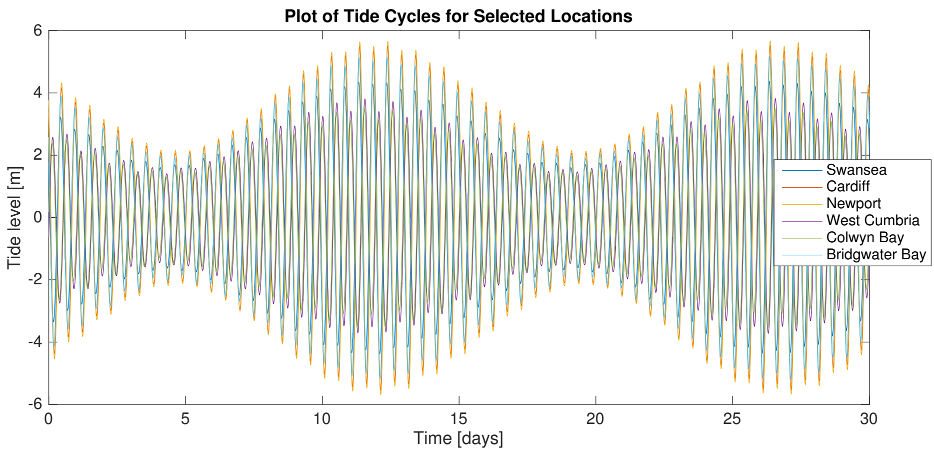


Figure 13.3: Plot of the tide cycles over 30 days for the specified locations



The tide cycle plots illustrate considerable difference between the two areas and only smaller differences between the locations within an area. Two important factors should be mentioned. Firstly, the West Cumbria and Colwyn Bay tide cycles have a phase delay of approximately five hours to the rest of the cycles. This means that maximum power production will take place at a different time of day. Secondly, there is a significant difference in amplitude meaning that the head difference, and therefore also power potential, varies from location to location.

Aggregated Cycles

To limit the problem size, the locations are aggregated into two areas each represented by a location specific generator in the model. The aggregated generators and associated potential capacity is given in table 13.1. The locations are divided between the two aggregated generators as follows:

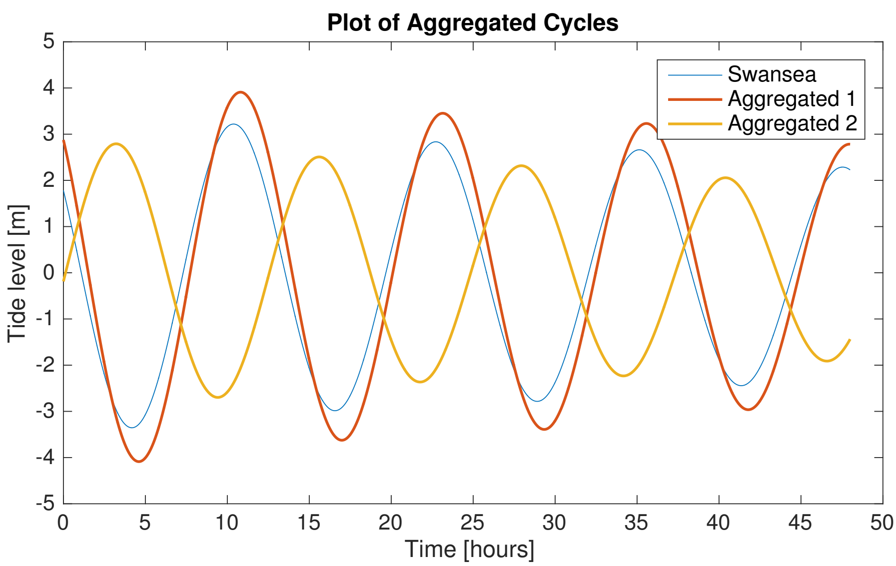
- Generator 1: Swansea, Cardiff, Newport, Bridgwater Bay
- Generator 2: West Cumbria, Colwyn Bay

Table 13.1: Capacity of aggregated tidal lagoon generators

Location	Assumed Capacity [MW]
Generator 1	5740
Generator 2	3600

An average tide cycle for each area is calculated and used when generating power generation patterns for the two generators.

Figure 13.4: Plot over 48 hours of the aggregated cycles and the Swansea tide cycle



Correction factors are calculated to adjust for the difference in amplitude and phase between the aggregated tide cycles and the Swansea cycle. These factors have then been used to adjust the generated production patterns from Swansea Bay as described in 13.1.2. Figure 13.4 illustrates the difference between the tide cycles used for generator 1, generator 2 and the Swansea tide cycle.

13.1.2 Pattern Generation

The tidal power production patterns used for the European study are based on the results from the computational study performed in chapter 8, which again is based on the tide cycle in Swansea Bay. To adjust to the location specific tide cycles corresponding to generator 1 and 2 used in this study, corrections have been necessary. The adjustments corrects the power potential and the timing of the production by using the correction factors calculated as described above.

By using selected optimal power production schedules from part I and correcting for locations, two unique power generation patterns for each aggregated tidal lagoon generator are developed:

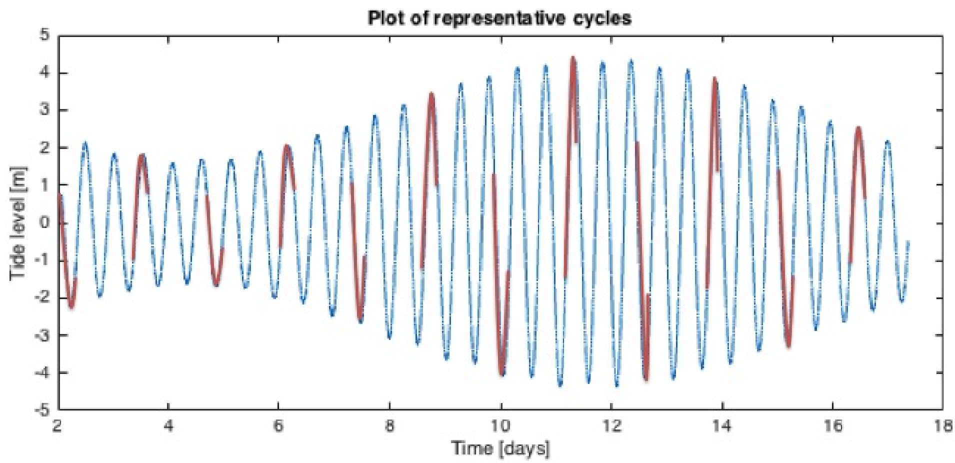
- Pattern 1: Power maximisation schedule
- Pattern 2: Profit maximisation schedule (current power price characteristics)

13.1.3 Operational Hours in the Model

A reduced number of hours are used to represent each season in a model year as described in 10.2.2. However, selection of model hours for tidal lagoon power production is done in order to obtain a realistic representation of annual power production values. The tidal power potential changes significantly during a month and a day. A small number of consecutive hours can therefore end up representing a full season by production cycles either with high power potential or with low power potential. To avoid this several blocks of six consecutive hours spread out over a longer period are used to represent each season.

To preserve the hourly time dependency obtained in production schedules from part I, a 24 hours time step is used between each block of chosen consecutive model hours. Thus, both daily and monthly characteristics of the tidal power generation schedules are preserved. In addition, the approach ensures that the tidal lagoon power generation data included correspond on a daily level with other data series included in EMPIRE.

Figure 13.5: Illustration of how representative cycles are distributed to preserve daily and monthly characteristics



13.2 Test Cases

Firstly, notation used to differentiate between the test instances is presented. Then case specific policy parameters are described and finally the defined policy cases are presented.

13.2.1 Notation

The following notation are used for the two versions of the extension:

- V1: LP-formulation with pre-determined production
- V2: MIP-formulation with some flexibility in production

The following notation are used for the two patterns included in the model:

- P1: Pattern 1, power production maximisation
- P2: Pattern 2, profit maximisation with normal price variance

In V1 predefined production is defined by including one pattern, whereas in V2 both patterns are included as optional production schedules.

The following notation are used for the two generators included in the model. The generators are used for all test instances:

- G1: Generator 1
- G2: Generator 2

Applied restrictions on investments

All policy cases are tested both without restrictions on investments, referred to as *free investments*, and with fixed tidal lagoon investments, referred to as *fixed investments*. When testing with *free investments* the power generation capacity mix is optimised without any restrictions on investments.

When *fixed investments* are applied, investments in tidal lagoon generators are restricted to a predefined level. For V1, total tidal lagoon investments are fixed to 9340MW, installed in year 2015. For V2 total tidal lagoon investments are fixed to 9340MW, installed within year 2025.

13.2.2 Policy Parameters

To comprise the effect of political context, five policy cases are used in the testing. The incorporated political parameters defining the cases are: total enforced emission reductions, line expansions, nuclear policies and implementation of the low carbon subsidy Contracts for Difference (CFD) in GB. An explanation of the parameters and cases follows.

Total Emission Reductions

When total enforced emission reductions are applied emissions must be linearly reduced with 80% percent by year 2050.

Line Expansions

When line expansions are allowed, investments in transmission capacity can be made to improve the power exchange limitations.

No Nuclear

If no nuclear is allowed, no new investments in nuclear generators are permitted in France and Great Britain.

Contracts for Difference

When CFD are used, CFD are modelled as implemented in Great Britain. Technologies supported by CFD in the modelling are wind generators, PV generators, nuclear generators and tidal lagoon generators. Total cost of operating generators of these technologies will be reduced because of the subsidies and the total system cost is reduced equally. In Great Britain CFD are financed by the government, thus the associated cost is not included in the resulting model objective. The CFD strike prices are given in 13.2. When used in the modelling an average power price of 65GBP/MWh is assumed.

Table 13.2: Assumed CFD strike prices given by technology

Technology	CFD strike price [GBP/MWh]
Tidal	130
Offshore	120
Onshore	80
Solar	80
Nuclear	120

13.2.3 Cases

Case 1: 80% emission reduction

Policy case 1 enforce 80% emission reductions by 2050 with no line expansion. Nuclear power can be used, but without extra support. This is used as the base case in this study.

Case 2: Base Case

Policy case 2 does not have an absolute emission limit. No line expansion is possible. Nuclear power can be used.

Case 3: With transmission expansion

Policy case 3 enforce 80% emission reductions by 2050. Line expansion is possible and nuclear power can be used.

Case 4: With CFDs in GB

Policy case 4 enforce 80% emission reductions by 2050 with no line expansion. CFD support is included in Great Britain. Nuclear can be used.

Case 5: No new nuclear investments in GB and France

Policy case 5 enforce 80% emission reductions by 2050 with no line expansion. New investments in nuclear power is not legal in Great Britain or France.

13.3 Results

In this section results from the described test instances are presented. Only the most relevant results for evaluating tidal lagoon investments are included. Results are presented for V1, Case 2 (Base Case) and only for other policy cases if the results deviate significantly from the base case. If not specified otherwise, P1 is taken as power generation input. Overall, the V2 results do not differentiate significantly from the presented V1 results. Only the operational results for V2 are therefore discussed.

Firstly optimal investments in tidal lagoon generators are presented and resulting capacity and generation mix analysed. Then impact on emissions is described and total system costs presented. Finally, operational results from V2 are presented.

All test instances are solved on the linear solver FICO Xpress-IVE Optimizer. All tests on V1 are solved on a HP dl165 G6 with 2 x AMD Opteron 2431 2.4 GHz processor and 24 Gb RAM, while all tests on V2 are solved on a HP BL686 G7 with 4 x AMD Opteron 6274 2.2 GHz processor and 128 Gb RAM. Problem size and solution time is given in table 13.3.

Table 13.3: Problem size, model complexity and solution time for the two versions of the model extension is given

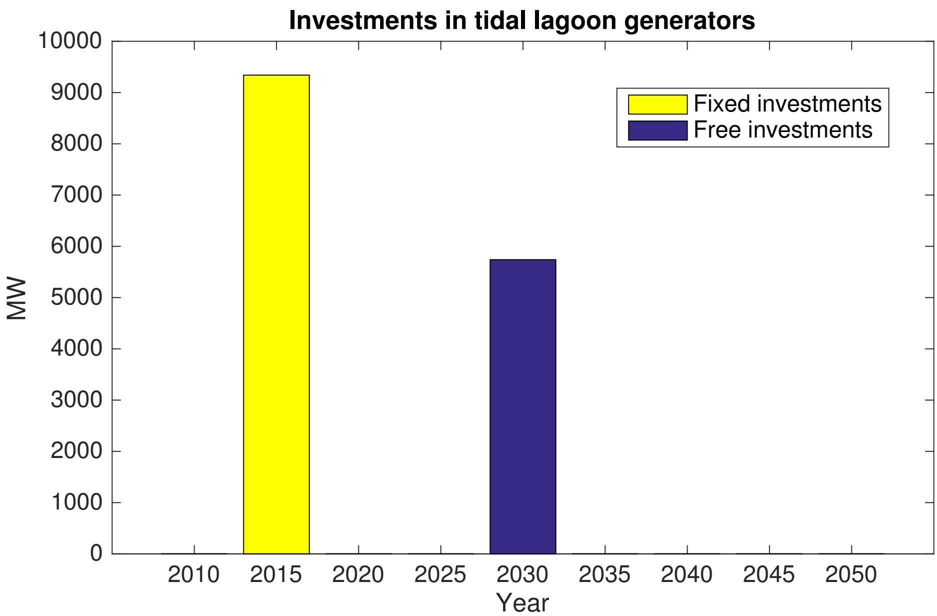
Test instances	# of variables	# of constraints	# of binary variables	Solution time [hours]
V1, case 2 free investments	5 728 674	8 258 981	0	0.79
V1, case 2 fixed investments	5 728 674	8 258 983	0	0.80
V2, case 2 fixed investments	5 729 634	8 259 943	960	65.73

13.3.1 Power Generation Capacity

Results from running with *free investments* gives no tidal lagoon generation capacity in the optimal capacity mix for all cases, except case 4. The optimal solution for case 4 contain investments in tidal lagoon generator 1 only. No investments are made in generator 2 in any of the free investment solutions.

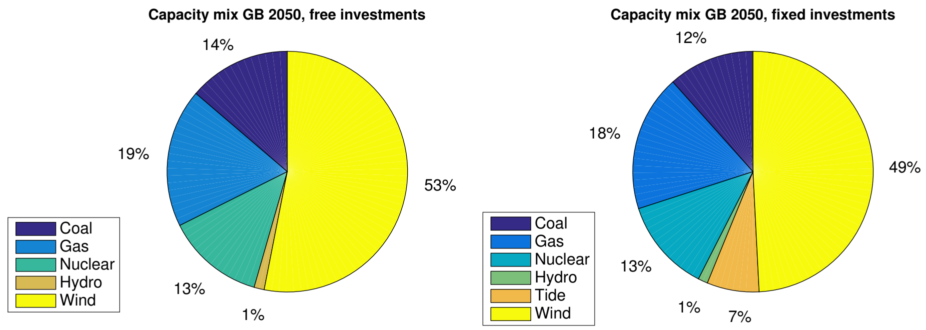
The resulting investments in tidal lagoon generation capacity during the time horizon for the *free investments* solution compared to the *fixed investments* solution for case 4 are illustrated in figure 13.6.

Figure 13.6: Illustration of investment time and size in tidal lagoon generators given case 4 with *free investments* and *fixed investments*



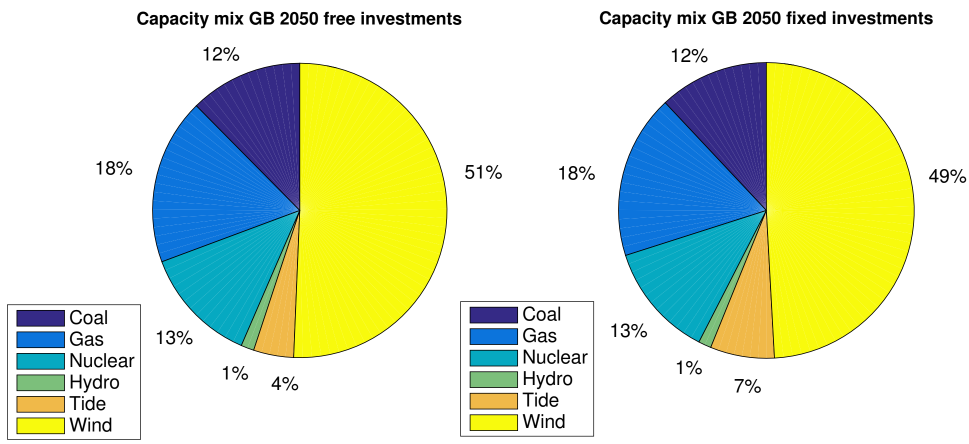
When restricting tidal lagoon investments to *fixed investments*, installed tidal lagoon power generation capacity corresponds to 7% of total GB capacity in 2050, see figure 13.7. Compared to the *free investments* solution, tidal lagoon generation capacity replaces wind, coal and gas generation capacities.

Figure 13.7: Plot of resulting generation capacity mix for Great Britain in 2050 for both *free investments* and *fixed investments* in tidal lagoon generators



For policy case 4, investments in tidal lagoon generators are part of the optimal *free investments* solution, equalling 4% of total installed capacity. Resulting capacity mix for the *free investments* and the *fixed investments* solutions are given in 13.8. Notice how optimal capacity mix for the *fixed investments* solution is the same as for case 2. Comparing the case 4 solutions, increased tidal lagoon generation capacity replaces wind generation capacity.

Figure 13.8: Plot of resulting generation capacity mix for Great Britain in 2050 for case 4 given both *free investments* and *fixed investments*in tidal lagoon generators



13.3.2 Generation Mix

All generation results used in this subsection are expected values of generation based on the results for the different scenarios in 2050.

Total tidal lagoon power generation when *fixed investments* are applied corresponds to 2% of total generation, as illustrated in figure 13.9. Resulting capacity mix for the *free investments* solution and *fixed investments* are given in figure 13.10. Despite the difference in installed capacity, the results show that tidal lagoon generation in both instances correspond to 2% of total generation.

Figure 13.9: Plot of resulting generation mix for Great Britain in 2050 for both *free investments* and *fixed investments* in tidal lagoon generators

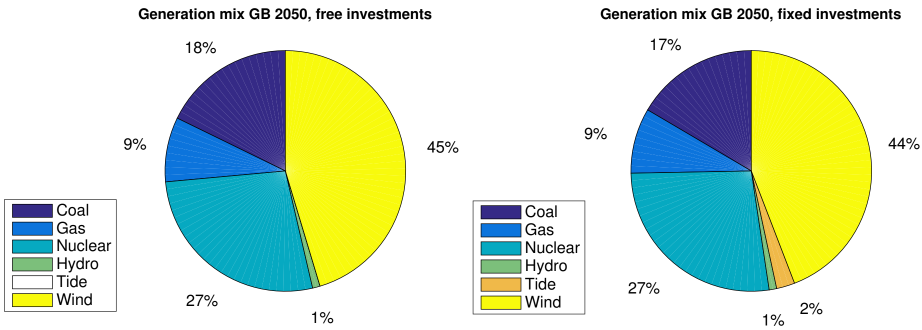
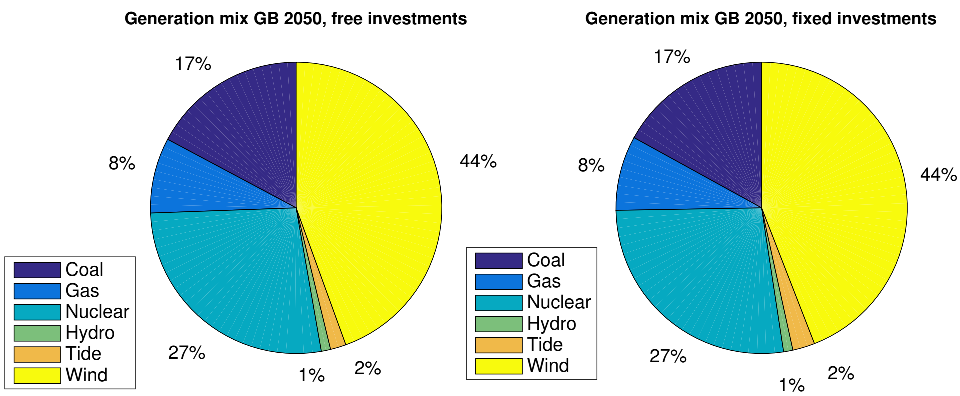


Figure 13.10: Plot of resulting generation mix for Great Britain in 2050 for case 4, both for *free investments* and *fixed investments*in tidal lagoon generators



Applying *fixed investments*, compared to the *free investments* solution, tidal lagoon generation replace generation from wind, coal and nuclear generators, as illustrated in figure 13.11. The change in generation mix for case 4, *free investments* and *fixed investments*, compared to optimal solution without CFD (Case 1) are given in figure 13.12. Here generation from wind, coal and gas generators are reduced the most.

Figure 13.11: Plot of resulting change in generation mix for Great Britain in 2050 for *fixed investments* in tidal lagoon generators

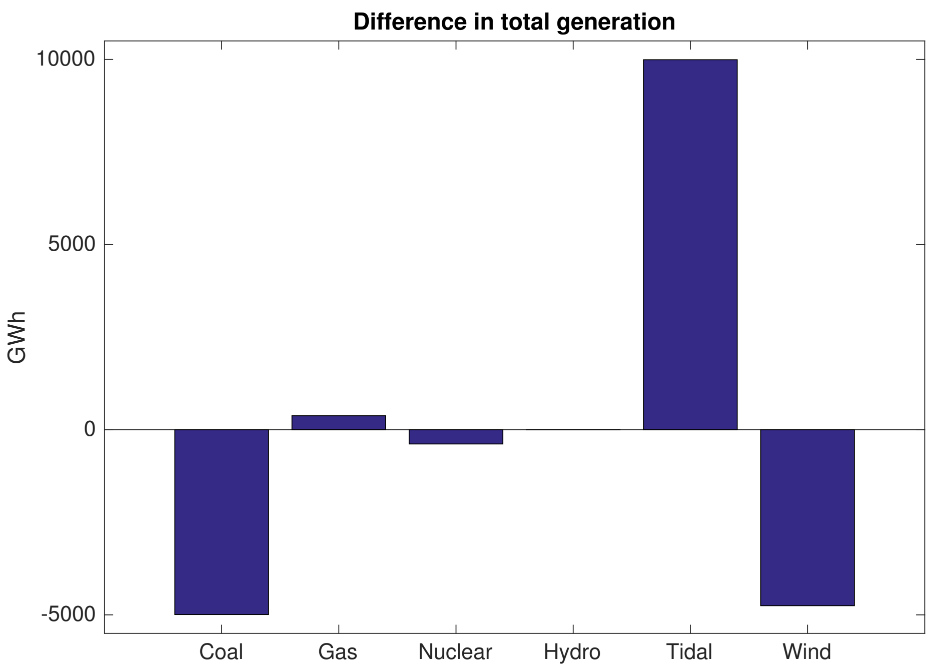


Figure 13.12: Plot of change in generation mix for Great Britain in 2050 for case 4 given *free investments* and *fixed investments* (when introducing CFD)

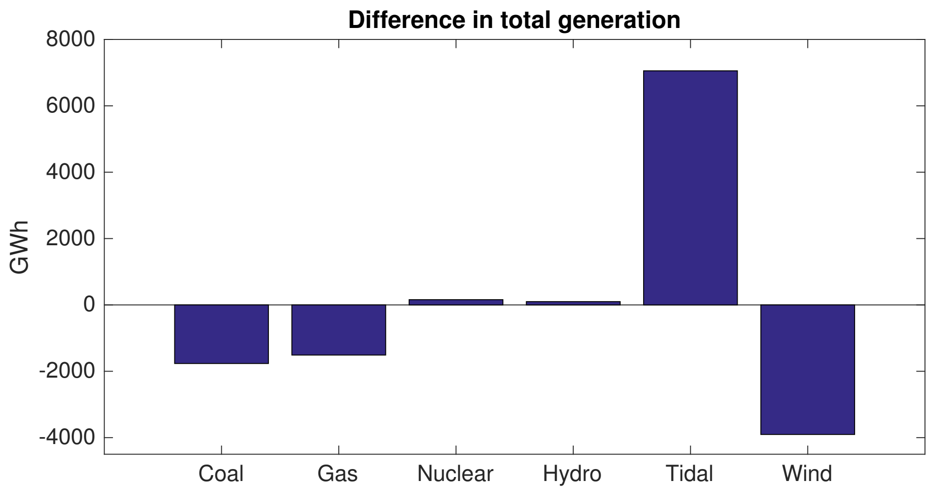
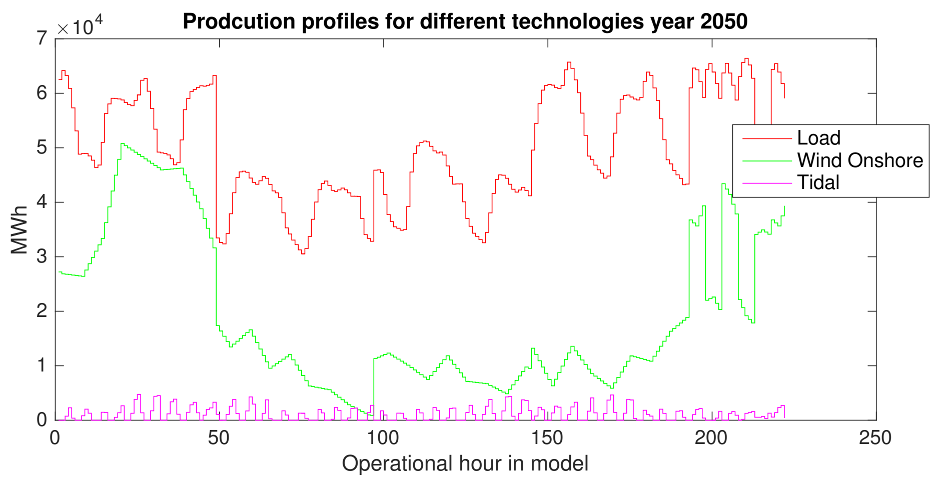


Figure 13.13: Plot of production in MWh over a model year representing year 2050 for different technologies



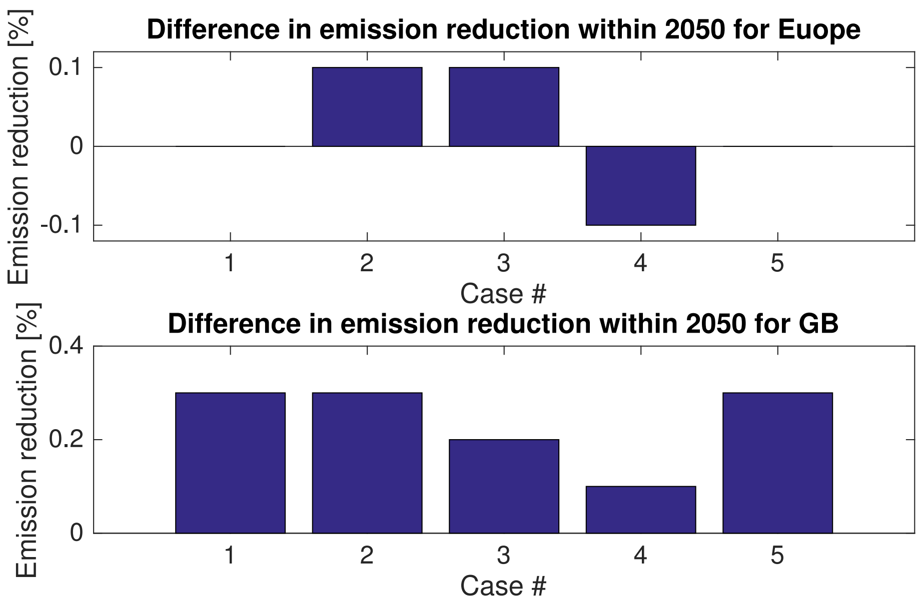
In figure 13.13, production from tidal lagoon generators are compared to wind generation and load in 2050 for Great Britain. The results are plotted over a model year.

Neglecting the obvious difference in production scale, other observations can be made comparing production schedules and the load profile. The plot illustrates significant changes in load and wind generation over time. Wind generation seems to follow a seasonal pattern, where generation either is relatively high or low over a consecutive number of hours. Tidal lagoon power generation follows a predictable cycle with high and low generation periods, but zero production occurs regularly every 2-5 hours, for 1-3 hours. Tidal power production varies with higher frequency than both wind production and load.

13.3.3 Emissions

The accomplished emission reduction by 2050 in both Europe and GB specifically is above 80 % for all test instances. The improvement in emission reduction in Great Britain and Europe when *fixed investments* are applied, compered to the *free investments* solutions, are shown in 13.14 for all cases. Emissions are improved in Great Britain for all policy cases and in Europe for case 2 and 3. In Europe, worse or identical emission reductions are obtained for case 1,4 and 5. Accomplished emission reductions are improved by up to 0.1% for Europe and 0.3% for Great Britain.

Figure 13.14: Plot of change in emission reduction achieved by 2050 in Europe and Great Britain for *fixed investments* in tidal lagoon generators for policy case 1-5



13.3.4 Total Costs

The objective value of the solutions correspond to total system costs of investing in generation and transmission capacity, operation of installed capacity and lost load. The resulting objective values are used to evaluate the additional cost of investing in tidal lagoon generators.

The cost of investing in tidal lagoon generators can be found by comparing total system costs of the *free investments* solutions with total system costs of the *fixed investments* solutions. The difference in cost represent the additional cost of implementing the predefined investment strategy for tidal lagoon generators. The differences in total system costs are shown in table 13.4 for case 2 and 4. For case 2 the cost of applying *fixed investments* is approximately 11.0 billion GBP, equalling 1.17 million GBP per MW installed generation capacity. This is a significant cost for developing and operating 9340 MW of tidal lagoon power, but a small fraction of the total system costs.

In case 4 investments in tidal lagoon generators are part of optimal solution both instances. Comparing total cost of the two solutions for case 4 gives the additional cost of forcing the model to invest in the predefined way. The difference is given in table 13.4 and shows a loss of a approximately 3.2 billion GBP when *fixed investments* are applied. This corresponds to 0.89 million GBP per MW additional installed capacity.

Total costs increase when power generation pattern 2 is applied.

Table 13.4: Additional costs of applying *fixed investments* given in million GBP for case 2 and 4. The cost figures is also given as share of total system cost and per MW installed tidal lagoon generation capacity

	Case 2	**Case 4**
Additional cost [mill. GBP]	11 175	3 246
% of total system cost	0.74	0.23
Cost per MW installed [mill. GBP/MW]	1.17	0.89

To evaluate the impact on system costs of implementating CFD in GB, total system costs for case 1 and case 4 are compared in table 13.5. The difference in costs gives a total system cost reduction of approximately 7.4% when implementing CFD.

Table 13.5: Difference in total system costs with CFD in million GBP for *free investments*

Total cost case 1 [mill. GBP]	1 516 104
Total cost case 4 [mill. GBP]	1 404 210
Difference [mill. GBP]	111 894
Cost reduction [%]	7.38

Total CFD support given to all technologies and the amount given to tidal lagoon generators are presented in the table 13.6. Support given to tidal lagoon generators constitute a share of less than 2% of total CFD support. Notice that the total CFD support given sums up to approximately 116 billion GBP, while the achieved cost reduction sums up to approximately 112 GBP. Hence, 96% of the support results in a direct system cost reduction.

Table 13.6: Total CFD support in the period 2010-2050 given in million GBP and share used on tidal lagoon generators

Total support [mill. GBP]	115 986
Tidal support [mill. GBP]	2 262
Share [%]	**1.95**

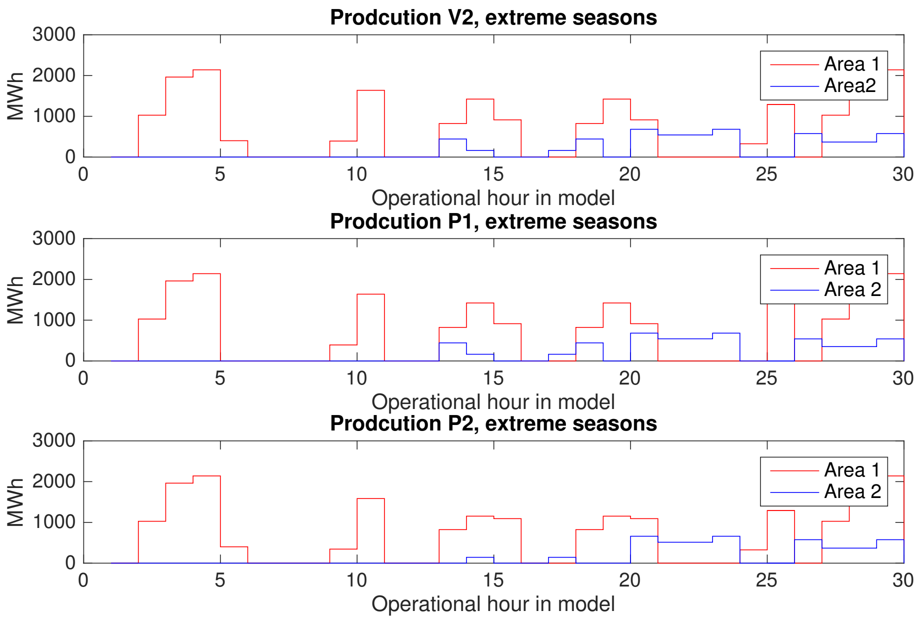
13.3.5 Operational Schedule

In this subsection operational results for V2, case 2 with *fixed investments* are presented. For *free investments* the solutions for V1 and V2 are identical. Optimal production schedules are given for the two tidal lagoon generators, G1 and G2.

Comparing the operational profiles for the different investment periods show that the production profiles are identical for all years except the years 2045 to 2049. The resulting operational profiles for the different scenarios are identical. The following operational results in this section are reported for scenario 1.

Comparing optimal generation schedules for year 2050 on G1 and G2 to the pre-defined production patterns, it is seen that the power maximisation pattern, P1, is used the most. For regular seasons P1 is utilised 100% of the hours on both generators. The model switches between using P1 and P2 in extreme seasons. On G1, both P1 and P2 are used in 50% of the extreme seasons whereas on G2, P1 is used in five of six extreme seasons. The optimal extreme season generation schedule on G1 and G2 is plotted in figure 13.15.

Figure 13.15: Plot of production in MWh over the extreme seasons. Subplot 1 illustrate the optimal generation schedule. Subplot 2 and 3 illustrate P1 and P2 respectively.



13.4 Model Shortcomings

The low complexity in operational decisions, removing or reducing the operational flexibility, is a major shortcoming in the model. Limiting production to predetermined production patterns constrains the short term dynamics. As a result, value of adapting production to extreme demand periods or low supply situations is lost. The real option to increase production during system stress is neglected.

The results indicate that the intentional operational flexibility included in V2 is insufficient in gaining additional value. Firstly, the production patterns are not generated based on the system conditions. Thus, tidal lagoon power generation does not adapt to increased power demand or reduced system supply. Based on the operational limitations in EMPIRE a perfectly fitted power generation pattern for each system realisation can be developed. That is, for three scenarios in the model three perfect power generation patterns are sufficient to obtain optimal tidal lagoon power generation schedules. These patterns would allow for power generation adjustments to system demand within the limitations of tidal lagoon technology. The current pattern generation method is obviously insufficient for this purpose.

Secondly, the discretisation of operational decisions for tidal lagoons restricts a power generation pattern to be used for an entire season. This greatly constrains the flexibility in the model.

These shortcomings are demonstrated in the results by the optimal generation schedules being identical for all scenarios and only one pattern being utilised in the regular season. The only utilisation of pattern flexibility is seen in the shorter seasons.

Generation of location specific tidal cycles and corresponding production patterns are assumed a source of inaccuracy in the computational study. Firstly, a simplified harmonic analysis is used to generate the cycles. Secondly, some of the location specific differences, especially difference in phase, are lost when aggregating the cycles. Finally, the correction method used to generate production patterns are just an adjustment of a previously generated pattern for a different location. The method does not consider optimal production decisions, hence the production patterns are not optimised subject to the location specific tide cycles. Additionally, the selection of operational hours for the tidal power generation pattern is not necessarily a good representation of real power generation potential over a year. The average power generation included in the model might be both an overestimate or an underestimate.

The costs and technical parameters for tidal lagoon technology used in the computational study are collected independently of the parameters used for the other technologies. Hence tidal lagoon costs utilised might be based on different assumptions than the original data set in EMPIRE. In addition, publicly available cost data for tidal lagoons are quite uncertain due to the immaturity of the technology. The power generation patterns are based on the computational study in part I and shortcomings in the generation of this data are previously discussed.

Chapter 14

Conclusion and Further Work

14.1 Conclusion

The achieved energy generation from tidal lagoon generators compared to installed capacity is identified as an important challenge for competitiveness of tidal lagoon generators to other technologies. Installed tidal lagoon capacity of 7% of total Great Britain power capacity is shown to generate only 2% of total energy. High investment costs compared to accomplished production result in high cost per produced energy unit. Despite the political context in the model favouring low-carbon technologies, tidal lagoon generators are not shown to be part of the optimal GB power mix in the future. To improve competitiveness of tidal lagoon generators measures to increase production per unit installed capacity or significantly reduce investments costs must be taken. Overall, tidal lagoon generators are evaluated to not be competitive to other renewable technologies and are not considered to be a natural part of the optimal generator mix for Great Britain in the future.

For the discussed portfolio of tidal lagoon projects in Great Britain to be optimal for investments prior to 2020, cost reductions of approximately 11 billion GBP or subsidies will be necessary. The cost figures presented are based on predefined tidal lagoon investments. It is clear that investing at a later time or only in parts of the available capacity would affect these cost figures. Investments in tidal lagoon generators are shown to be optimal with extra support through CFD subsidies, but only for generators with high tidal energy potential. In reality, CFD subsidies do not reduce the total cost, but reallocate it in order to improve the competitiveness of supported technologies. The implemented CFD support system would overall cost the UK government up to approximately 116 billion GBP, resulting in cost reductions of 112 billion GBP divided on selected power producers. This is a high subsidy cost considering the current commissioned budget for CFD support in the UK in total is 325 million GBP. The current UK CFD scheme is therefore not proved sufficient to ensure competitiveness of tidal lagoons.

If introducing tidal lagoon technology into the GB power market, the technology is expected to replace generation from wind and coal generators. Wind generation is expected to be reduced the most, thus only small improvement in CO_2 emissions will follow. Only tidal production replacing generation from carbon intensive technologies, such as coal, will contribute to emission reductions. All test instances reach the EU target of 80% emission reduction by 2050, independently of tidal lagoon investments. Hence, the tidal lagoon portfolio included in the study, is shown to only slightly affect achieved emission reductions.

Tidal lagoon generators have been claimed to contribute to security of supply in power systems with high share of intermittent, unreliable energy production. However, limited system gains are identified from combining production from tidal lagoon generators with wind production, given the used load profiles in this study. Tidal lagoon power production is predictable, but only to a certain degree controllable and periods with zero production will occur several times a day. This affects and reduces the security a tidal lagoon generator can provide in the power system. However, if tidal lagoons are developed over a wide spectre of coastal areas with different tide cycles, the duration of hours with zero power production will be reduced.

Optimal system power generation schedule for a tidal lagoon deviates from the profit maximisation schedule. The power maximisation pattern is favoured to the profit maximisation pattern in EMPIRE due to the system cost of reduced total energy generation exceeding the gained value of delaying production for high stress periods. It can be seen that the model extension is insufficient to evaluate the value added to the power system of introducing operational flexibility by tidal lagoon generation.

14.2 Further Work

The authors propose improvements on the pattern generation procedure to be the main focus for further research on the work presented. Two alternatives are identified. Firstly, column generation of production patterns in a subproblem for input to a main (system) problem, will provide better patterns. The main problem optimises the system problem based on current production pattern, and the reduced cost of the current solution is sent to the subproblem. The subproblem generates an improved pattern and ensures feasibility. As explained three high quality patterns would be sufficient.

Secondly, increasing the number of included production patterns and the frequency of operational decisions will improve the operational flexibility and performance in the model. However, this approach increases the problem size and would require more sophisticated solution methods. In addition, the method for selection of power generation patterns should be improved.

Chapter 15

Final Remarks

Utilisation of operational flexibility combined with plant management according to a profit maximisation solution will add some value for a tidal lagoon power plant operator. The value is shown to be minor in the current UK power market, but important income gains are identified for increased power market price variance. However, the operational flexibility is seen to add low value to the power system security of supply due to low energy potential compared to demand and insufficient operational flexibility to generate continuously over a day. Further, current tidal lagoon technology cost estimates are high and a large cost decrease is shown necessary for profitable plant realisation and optimal integration in the power system. Subsidies allow for a cost decrease delay, but the current political willingness to finance renewable power technologies is shown insufficient alone for tidal lagoon realisation.

Moreover, the development of tidal lagoon generators is shown not to be crucial for reaching the EU long-term emission reduction targets. Only a small impact on emission levels are proven when evaluating the effect of including tidal lagoon generators in the European power system.

Despite increased energy potential by technology development and increased tidal lagoon capacity, the scalability is limited by the number of suitable locations, regarding both the tidal range and environmental and social concerns. Currently, tidal lagoon power generation technology is concluded not valuable neither for an investor or the power system as a whole.

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Appendix A

Price Correlation

Correlation in wholesale prices for two objects, A and B, is a measure of the tendency of the two prices to move together and for historical prices it is defined (5):

$$\frac{1}{T-1}\frac{\sum_{t=1}^{T}(P_t^A - \overline{P^A})(P_t^B - \overline{P^B})}{SD^A SD^B} \tag{A.1}$$

Where T is the number of historical measurements or sample size, P_t^A and P_t^B are the historical prices for object A and B, $\overline{P^A}$ and $\overline{P^B}$ are the historical mean price over the time horizon and SD^A and SD^B are the historical standard deviation over the time horizon.

The correlation between the day-ahead and intraday prices over the years 2012-2015 is calculated to 0.490.

Appendix B

Error From Neglecting Head Effects

A useful simplification when solving hydro-scheduling problems is to assume constant head. To evaluate the consequences of such a simplification when scheduling tidal lagoon power production the following error estimates were conducted. The analyse was completed using a simple, constant head model for testing. The results indicated that such a simplification would reduce the quality of the solution considerably. The final optimisation model was therefore developed to include variations in head.

The model used for testing assumed linear relationship between volume flow through turbines and power production. This assumption included assumptions about constant efficiency with varying flow and constant height difference in reservoir water level with tide during power production.

The tidal cycle was divided into tidal half-cycles, each with a corresponding constant height difference used in the power equation. This constant was set equal to the average height difference in each tidal half cycle, depending on reservoir geometry. Time was discretised into periods of 15 min. To obtain realistic values for power production with constant reservoir height in all periods, only periods satisfying a criteria for corresponding tide level were used as possible power production periods. The criteria was that the difference in initial reservoir height at beginning of a tidal half cycle with the tide at a point in time t within the half cycle, had to be greater than a specified minimum height difference H^{MIN}. From this value a number of power production periods for each tidal half cycle were specified.

Table B.1: Error of power production with constant height to varying height for cylindrical reservoir

radius(t=0)	flow	error
400	Qmin	0.012
200	Qmin	0.052
500	Qmax	0.017
200	Qmax	0.052
500	(Qmin,Qmax)	0.093

The effect of the above mentioned assumptions was analysed by calculations of efficiency with varying flow and power productions with varying height difference. For volume flow through a turbine in the interval (Qmin, Qmax) the maximum error of constant efficiency was 2.0%. With constant volume flow the error is below 5.2% for cylindrical reservoir, below 3.2% for conical reservoir and below 9.3% for for varying volume flow. The error of power production with constant height to power production with varying height is shown for a cylindrical reservoir in B.1. Initial reservoir radius were chosen such that calculation of power production with both constant and continuous height ends with an empty reservoir. With varying volume flow and small reservoir volume the error of height difference approximation is evaluated to be significant.

Appendix C

Corrections to Generated Swansea Tide Cycle

Controlling values from the generated Swansea tide cycle against published data for a random selection of dates revealed the following:

- On an monthly and yearly scale spring and neap tide match the published tide data well.
- The total head difference (the amplitude) during spring tide was satisfying on average. The total head difference during leap tide was too high on average. Corrections were made.
- On a daily scale high and low tide took place at the opposite time, meaning that when there should be low tide the generated cycle was in high tide and the other way around. This was corrected by adjusting the phase. The correction resulted in high and low tide to take place approximately at the right time, with only small errors in timing.
- On a daily scale the head difference was a little too high or too low in some periods. No corrections have been made.

The corrections mentioned above has been implemented. The generated tide cycle with corrections match well on a yearly and monthly scale with the published tide table. There are some inaccuracies in head difference on a daily scale leading to overestimating of the energy potential in some periods and underestimating in other periods. This error is not assumed to have a great impact on the final results. There are also some inaccuracies in the timing of high and low tide daily. This affect the matching with prices, but is not assumed to have a significant impact on the overall analysis.

Figure C.1: Plot of the tide cycle over 48 hours before and after the corrections. This plot clearly illustrates the effect of the phase change.

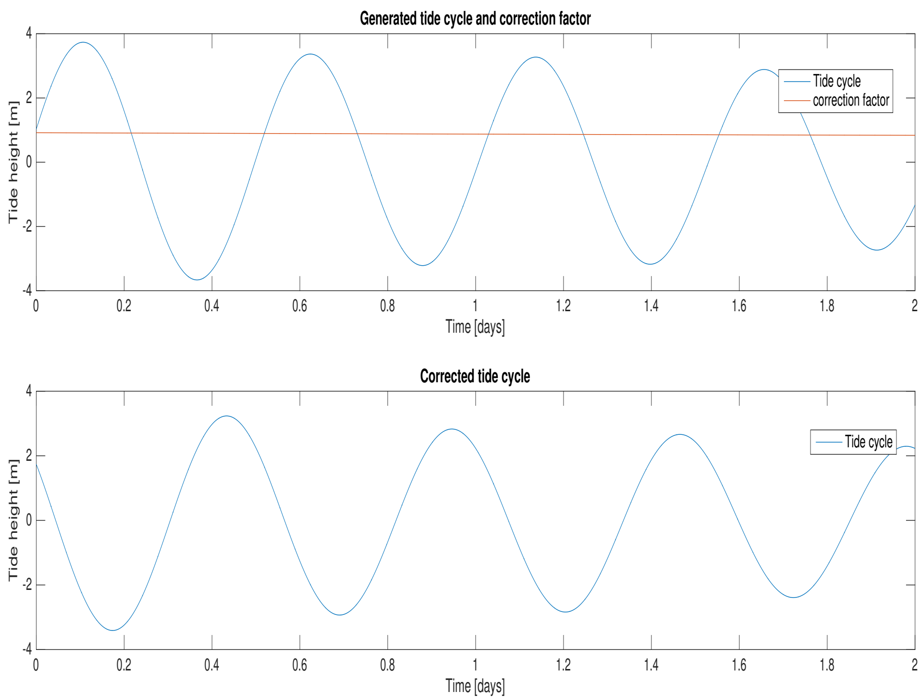
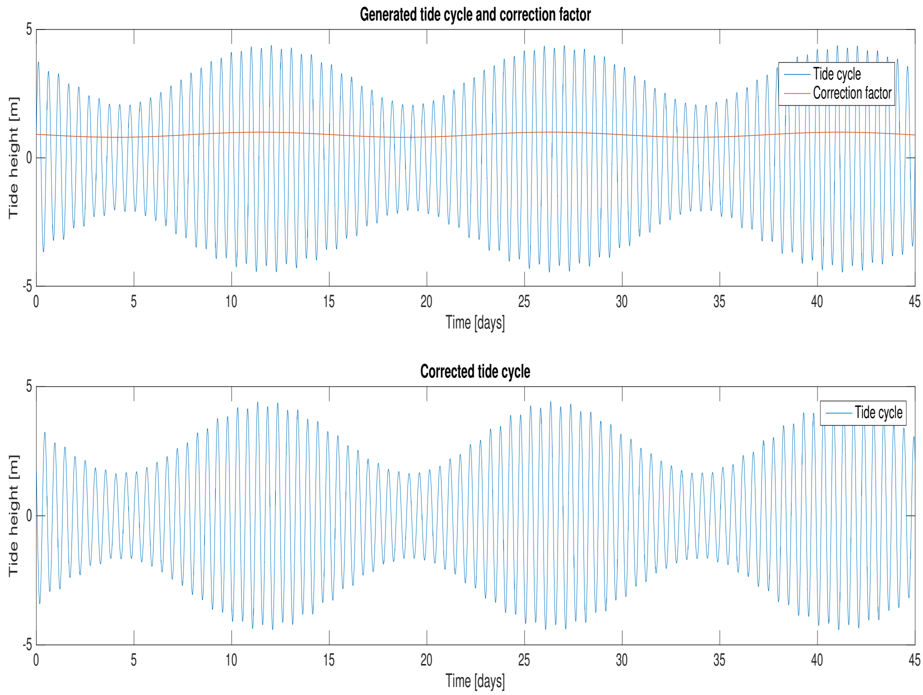


Figure C.2: Plot of the tide cycle over 45 days before and after the corrections. This plot clearly illustrates the effect on the tide height difference for neap tide from the corrections.



Appendix D

Turbine Characteristics

The turbine efficiency is a function of the dimensionless parameters α, β, χ and δ, as well as the nominal flow Q_n and the volume flow through the turbine q. This is given in equation D.1.

$$\eta^T(q) = \{1 - [\alpha|1 - \beta\frac{q}{Q_n}|^{\chi}]\} * \delta \tag{D.1}$$

Table D.1: Turbine specifications used in the modelling

Turbine characteristics			
α	3.5	Q_{min}	39 m^3/s
β	1.333	Q_{max}	140 m^3/s
χ	6	Minimum head	0.5 m
δ	0.905	A_c	12.56 m
Q_n	127 m^3/s		