

Path Planning for Formation Control of Autonomous Vehicles

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Abstract. In this paper a two-stage approach is introduced for optimum path planning of a team of autonomous vehicles in an environment cluttered with obstacles. The vehicles are requested to move in formation from an initial point to a final point. The Bump-Surface concept is used for the representation of the environment while the formation of the vehicles is presented by a deformable Delaunay triangulation. The proposed approach is presented in detail and test cases with multiple vehicles are simulated to demonstrate the efficiency of the method.

Keywords. Path Planning, Formation Control, Autonomous Vehicles

1 Introduction

Teams of autonomous vehicles are widely used in many applications, where the vehicles are requested to meet formations or other constraints to accomplish complex tasks, such as transportation of large objects [1], localization and mapping [2], search and rescue missions [3]. This interest is motivated by the necessity of having more vehicles performing tasks which are more difficult to perform with only one vehicle, for instance surveillance missions [4]. Furthermore, the motion in formation is particularly important when spatially distributed tasks have to be accomplished, like for instance, source seeking missions [5].

In this paper we present an approach for the path planning problem for a multi-vehicle system. In particular, we consider a multi-vehicle system which is consisting of autonomous vehicles. The objective is to find an optimal path for each vehicle which

connects an initial point with a final point while simultaneously the vehicles should be moving in a given formation.

The path planning problem for formations control of a team of autonomous vehicles has been investigated [6]-[8]. In [6], the authors presented a method based on rapidly exploring random trees (RRT) for path planning of formations with under-actuated vehicles. This method randomly samples the environment and chooses a free collision configuration for each vehicle. The authors revised the classical RRT to generate feasible paths for non-holonomic vehicles. Furthermore, they designed a priority strategy, which makes the vehicles to move in a given formation. The work [7] describes a method based on Voronoi Fast Marching (VFM) for formations of fully actuated mobile robots. This method can be classified as a potential field method but avoids the drawbacks related to local minima. In [8] an abstract manifold A was defined which is the product of two manifolds G and S . The manifold G is a Lie Group, which captures information about the orientation and position of every vehicle, while S is a manifold, which captures information about the shape of the group of vehicles. The states in the two manifolds G and S are controlled independently.

In this paper we extend the method proposed in [9] and [10] to multi-vehicle systems consisting of autonomous vehicles. The vehicles should be moving in a given formation. By using the proposed approach, it is possible to obtain simultaneously an optimum path, for each vehicle. Each path is constructed considering both the environment constraints and the formation constraints.

The main contribution of this paper is the introduction of a method for the path planning of a flexible formation of n autonomous vehicles in an environment cluttered with static obstacles. For the first time the formation relationship is represented by a deformable Delaunay triangulation, which has the ability to find a solution even when the vehicles are requested to move through narrow passages. Furthermore, the smoothness of the path is obtained by controlling the angles between the control-polygon segments, which define the system's path. A multiplicity of optimization criteria and constraints could be incorporated easily to the formulated optimization problem according to the mission requirements of the team of vehicles.

2 Basic assumptions and the two stage approach

It is assumed that a formation of autonomous vehicles should move in a 2D environment which is cluttered with known prohibited areas (obstacles-danger zones). A formal statement of the problem, the assumptions and the structure of the proposed method are given in this section.

2.1 The path planning problem for a formation of autonomous vehicles

Consider a team of $n \geq 3$ autonomous vehicles which should move from an initial to a goal location by keeping the desired formation in a 2D environment cluttered with known static obstacles. The basic assumptions are:

- Each vehicle is represented by a point which is moving only forward.
- Each vehicle is requested to move from an initial point S_i to a goal point G_i inside the desired formation, which should not split, while the length of the path should be minimum.
- The path of each vehicle should be smooth.
- The formation is modeled as a deformable polygon. The user defines both the minimum and maximum allowed formations of the polygon. The maximum formation is the desired one.

2.2 The geometry and the representation of the formation

We assume that a team of autonomous vehicles is enclosed in a deformable convex polygon, while no splits are allowed. The vertices of the convex polygon are the “external” vehicles. Fig.1 shows a visual representation of a team of 15 point-vehicles.

The convex polygon is used in order to take the advantage of the fact that the centroid \mathbf{R} always lies inside the polygon. Furthermore, by using convex polygon, we avoid to increase the problem’s complexity which is happen when we use non-convex polygons.

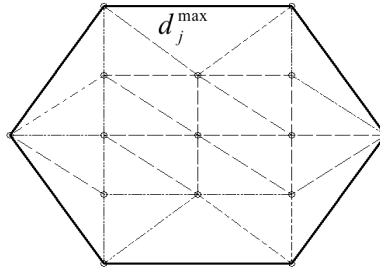


Fig. 1. A team of 15 vehicles (black circles). The corresponding: Delaunay triangles (dash lines) and the convex hull (black bold line).

The Delaunay triangulation [12] is used to facilitate the geometric relations between the vehicles within a geometry, where constraints could be defined easily. Generally, Delaunay triangulation is characterized by its simplicity and its “economy” in data storage. Furthermore, the Delaunay triangulation is independent of the order in which the points are processed. For a team of n vehicles we have, $2n - 2 - m$ triangles and $3n - 3 - m$ edges, where m is the number of vehicles on the convex hull. The length of each edge $d_j, j = 3, \dots, 3n - 3 - m$ is associated with the constraint $d_j^{\min} \leq d_j \leq d_j^{\max}$, where d_j^{\min} is the minimum safe distance and d_j^{\max} is the maximum, which is the desired distance between a pair of vehicles.

3 First stage: the sub-optimal path for the minimum size formation

In this stage, the minimum size formation is considered as a fixed shape and the shortest path is searched on the Bump-Surface. For the construction of the Bump-Surface representing a given 2D environment, a normalized workspace \mathbf{W} is constructed by linearly mapping the initial environment to $[0,1]^2$. The construction of the corresponding Bump-Surface S is obtained by a straightforward extension of the Z-value algorithm [9].

It is assumed that, the team has a reference point which lies at the centroid of the polygon representing the minimum size formation. The reference point traces a path $\mathbf{R}(s)$ in the normalized \mathbf{W} which starts from the given start point and terminates at the desired goal point. In order to define $\mathbf{R}(s)$ we use a B-Spline curve [13] to represent the path of the fixed minimum size formation:

$$\mathbf{R}(s) = \sum_{h=0}^{Q-1} N_h^2(s) \mathbf{p}_h, 0 \leq s \leq 1 \quad (1)$$

where, Q is the number of control points \mathbf{p}_h , $N_h^2(s)$ are the B-Spline basis functions and 2 is the curve degree. The goal of the proposed global path planning strategy is the determination of the position of $(Q-2)$ control points \mathbf{p}_h which define the requested path $\mathbf{R}(s)$.

3.1 Safe optimum motion of the minimum formation

A safe path $\mathbf{R}(s)$ is one that (i) does not collide with the obstacles and (ii) it is smooth. Following the results from [10], the arc length of $\mathbf{R}(s)$ approximates the length L of its image $\mathbf{S}(\mathbf{R}(s))$ on the Bump-Surface S as long as $\mathbf{R}(s)$ lies onto the flat areas of S . Furthermore, in order to take into account the shape of the formation, a set of feature points \mathbf{A}_i is selected on its boundary according to its shape and the requested accuracy [10].

Taking the above analysis into consideration, the path planning problem is formulated as an optimization problem which is described by,

$$\text{minimize } E_{comp} = \left\{ L, \sum_{i=1}^m \sum_{a=0}^{N_p} H_i^a, 1/\varphi_1, \dots, 1/\varphi_{Q-2} \right\}, \text{subject to } \varphi_h \leq 180, h=1, \dots, Q-2 \quad (2)$$

where N_p denotes the number of points taken on $\mathbf{R}(s)$ to discretize it, φ_h is the h -th angle between the control-polygon segments h and $h+1$ $\mathbf{R}(s)$ and H_i is the ‘‘flatness’’

of $\mathbf{A}_i(s)$ on S . The $\mathbf{R}(s)$ follows the shape of the defining control polygon which is derived by connecting the control points \mathbf{p}_h [13].

Then, a Genetic Algorithm is adopted in order to search for a solution to the formulated optimization problem (Eq.(2)). A floating point representation scheme is selected since the coordinates of the control points and the angles of the control polygon are real numbers. A fitness assignment strategy based on Pareto-optimal solutions called GPSIFF [14] is implemented. The following three genetic operators were selected. Reproduction: the proportional selection strategy is adopted, where chromosomes are selected to reproduce their structures in the next generation with a rate proportional to their fitness. Crossover: the one-point crossover was adopted. Mutation: a boundary mutation is used.

4 Stage 2: Determining the smooth path of each vehicle in the deformable formation.

With the reference path $\mathbf{R}(s)$ derived by the first stage, the path of every vehicle is determined considering the location of the vehicles in the desired formation. Since the formation has to pass through areas, where the minimum size formation is able to move with safety, then in the second phase a deformable formation is considered.

A deformation cost function is formulated with an optimum cost at the desired formation. In this stage each vehicle has its own “independent” smooth path but along its path it has to respect the desired formation. In order to ensure that the vehicles $n-m$ do not cross the border of the convex hull, the following condition is taken into account. At every point $\mathbf{R}(s_a), a=1, \dots, N_p$ of the path $\mathbf{R}(s)$ the location of the vehicle $\mathbf{R}^i, i=1, \dots, n-m$ is computed by the convex combination of the m vehicles which define the convex hull. Therefore,

$$\mathbf{R}^i = \sum_{f=1}^m w_f^i \mathbf{R}_f, i=1, \dots, n-m \quad (3)$$

where, the weight factors w_f^i satisfy,

$$w_f^i \geq 0 \text{ and } \sum_{f=1}^m w_f^i = 1, i=1, \dots, n-m \quad (4)$$

The goal of the proposed path planning strategy is the determination of the $Q-2$ control points \mathbf{q}_h^i , which define the requested path $\mathbf{R}^i(s)$ for the i -th vehicle given by the same equation as Eq.(1).

In order to take into account that the formation of the vehicles should adapt to the geometric characteristics of the environment while simultaneously trying to keep the desired shape, the following deformation function is proposed:

$$CD_j = e^{k_j}, j=3, \dots, 3n-3-m, \text{ and } k_j = \begin{cases} |d_j - d_j^{\max}|, & d_j \in [d_j^{\min}, d_j^{\max}] \\ \text{a very big value (defined by the user),} & \text{otherwise} \end{cases} \quad (5)$$

Eq. (5) gives a penalizing function, which takes the optimum value when $d_j = d_j^{\max}$ and the worst when $d_j \notin [d_j^{\min}, d_j^{\max}]$. The minimum size polygonal shape of the formation is obtained when $d_j = d_j^{\min}$.

According to the above requirements the derived objective function is a vector which is represented by,

$$\min E_{comp} = \{L^1, \dots, L^n, CD_1, \dots, CD_j\}, j = 3, \dots, 3n - 3 - m, \text{subject to} \begin{cases} w_j^i \geq 0 \\ \sum_{j=1}^m w_j^i = 1, i = 1, \dots, n - m \end{cases} \quad (6)$$

where L^i is the path's length for the i -th vehicle which is computed in a similar way as in Stage 1. In the optimization problem defined by Eq. (6), the optimization variables are the control points which define the path $\mathbf{R}^i(s)$ of each vehicle and the weight factors w_j^i .

A Micro-GA is used to search for a sub-optimum path of each vehicle. The main characteristics of the developed Micro-GA are the following: A floating point representation scheme is selected for the chromosome syntax. Each chromosome represents a possible $\mathbf{R}^i(s)$ as a sequence of the unknown control points \mathbf{q}_h^i . A fitness assignment strategy based on Pareto-optimal solutions is implemented.

It should be noticed that the quality of the individual solution generated in the initial phase plays a critical role in determining the quality of the final optimal solution. Thus, the solution which is derived from Stage 1 (the control points which define the path $\mathbf{R}(s)$), is used as an initial solution in the Micro-GA (seeding) for each vehicle's path $\mathbf{R}^i(s)$. This helps the Micro-GA to converge in short time to the sub-optimal path for each vehicle. The same genetic operators as in Stage 1 are used except the mutation operator which is ignored. In most cases, a maximum number of iterations (generations) is defined in advance for the termination. However, it is difficult to determine beforehand the number of generations needed to find near-optimum solutions. Thus, an assessment of the quality level of the Genetic Algorithm is made on-line. The proposed algorithm terminates either when the maximum number of generations is achieved or when the same best chromosome appears for a maximum number of generations.

5 Simulations

The performance of the proposed method is investigated through a number of simulation experiments for a variety of formations moving in 2D environments. All simulations are implemented in Matlab. In all test cases, the grid size is set to $N_g = 100$. For the first stage, the control parameters of the GA are the following: population size=250, maximum number of generations=500, crossover rate =0.75, boundary mutation rate=0.004. For the Micro-GA we set: population size=50, maximum number of generations=30, crossover rate =0.75. It is worth noting that the selection of

the appropriate control settings is the result of extensive experimental efforts with various control schemes adopted following the indications of the literature.

Test Case: We assume the environment of fig.2. Here, a team of 15 vehicles is requested to move in formation from the initial points S_i to goal points $G_i, i = 1, \dots, 15$. The convex hull is defined by six vehicles. A visual representation of both the initial and final formations, the computed path $\mathbf{R}(s)$ and the corresponding Delaunay triangles are shown in Fig.2. Each vehicle's path is defined by 8 control points. The computed solution takes about 10.24 minutes. Furthermore, Fig. 2 shows the formation of the convex hull while the team of vehicles is passing through narrow passages, where the formation is not just shrunk but it changed its shape autonomously to adapt to the environment.

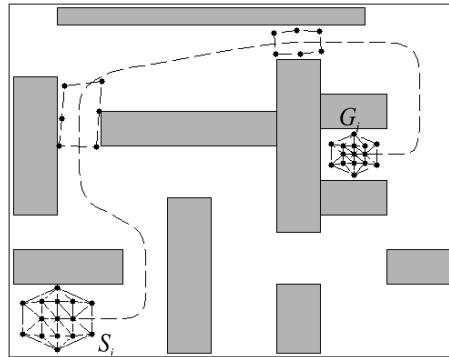


Fig. 2. The resulting solution path $\mathbf{R}(s)$, the initial and final convex hull with the corresponding Delaunay triangles and the trace of the convex hull in two different time instances.

Despite the fact that the problem under consideration is off-line, computational time results with respect to the number of the vehicles is of immense interest. The variation of CPU's time is indicative of the problem complexity. In these experiments, the environment is the one shown in Fig.2 and the number of control points is constant, while the number of vehicles is changed from 3 to 15. Fig. 3 shows that CPU time increases almost linearly with the increase in the number of vehicles.

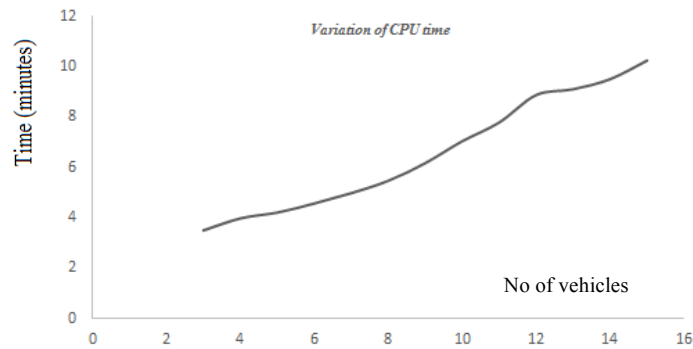


Fig. 3. A CPU time study

6 Conclusion

A new approach for the path planning of a deformable formation of autonomous vehicles is proposed. The Delaunay triangulation is proved to be very convenient for the modeling of a deformable formation since the extreme formations (minimum and maximum allowed formations) are defined by the user and thus we can easily determine the limits of the distances between the vehicles. Furthermore, the smoothness of the path is obtained by controlling the angles between the control-polygon segments, which define the system's path.

In future work the proposed approach should be extended to semi-known 2D and 3D environments, and in addition to the motion planning the guidance and control of the vehicles should be considered.

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