A two-layer structure for stabilization and optimization of an oil gathering network ⋆

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Abstract: In this work, we present the control and optimization of a network consisting of two gas-lifted oil wells, a common pipeline-riser system and a separator. The gas-lifted oil wells may be open-loop unstable. The regulatory layer stabilizes the system by cascade control of wellhead pressure measurements without needing bottom hole sensing devices. An economic Nonlinear Model Predictive Control (NMPC) based on the Multiple Shooting (MS) formulation is applied for optimization of the network operations. The optimization layer thus provides optimal settings for the regulatory controllers. The control structure has been validated by using the realistic OLGA simulator as the process, and using simplified models for Kalman filtering and the NMPC design. The simplified models are implemented in Modelica and fit to the Olga model to represent the main dynamics of the system. The proposed two-layer controller was able to stabilize the system and increase the economical outcome.

Keywords: Nonlinear Model Predictive Control, Oil Production Optimization, Extended Kalman filters, Control Structure design.

1. INTRODUCTION

In an offshore platform, the flow control of the oil wells is a key to attain good overall operational performance. The control of the producers assisted by gas-lift may be challenging due to oscillatory flow patterns known as *casing heading* and *density wave* (Bin and Golan, 2003). Moreover, even under stable well operations, oscillations known as *riser slugging* may originate in the pipeline-riser system that transport the production from the wellhead to the platform (Taitel, 1986).

The oscillatory flow behavior can be reduced or eliminated by increasing the pipeline back-pressure, *i.e.*, reducing the opening of the choke vale (Schmidt et al., 1980), or by increasing the lift-gas injection rate (Golan and Whitson, 1991). However, these solutions are not necessarily optimal from an economical point of view, and automatic feedback control has emerged as a viable alternative (Havre and Dalsmo, 2002).

Dynamic multiphase flow models are required to develop, analyze and tune well flow controllers. These models are typically built based on physical assumptions and vary in complexity. Detailed models are implemented in commercial multiphase flow simulators such as OLGA (Schlumberger, 2014). However, simplified low-order models are typically preferred for model based controllers (Eikrem et al., 2008; Jahanshahi and Skogestad, 2014). Moreover, when appropriately tuned, such models are sufficiently accurate for use in such controllers.

Feedback control solutions for wells assisted by gas-lift and pipeline-riser systems have been studied thoroughly during the last 30 years. Most of these works consider decoupled or independent wells and risers, for instance stabilization of slug flow in wells (Eikrem et al., 2008) or in pipelines/riser systems (Jahanshahi and Skogestad, 2014). Wells sharing the same riser may affect a common manifold pressure, hence, it is then required to analyze the dynamics of the sub-systems performing as a whole. Willersrud et al. (2013) addresses control and optimization of an oil gathering network with several wells, risers, a compressors and a separator with nonlinear model predictive control. However, the regulation capability in closed-loop was not studied. Nonlinear predictive control applied for regulatory control to such systems may be prohibitively computationally expensive. Therefore, in this work we assess the applicability of the simplified gas-lifted well and riser models described by Jahanshahi (2013) for dynamic optimization of a coupled system of wells feeding a riser. To this end, Nonlinear Model Predictive Control (NMPC) is applied to steer set-points of a regulatory layer implemented with PI controllers.

It is preferable to use a structured software platform for development and analysis of NMPC. Modelica is a convenient non-proprietary modeling language that assists to generate balanced-complexity models (Elgsæter et al., 2012). The models by Jahanshahi (2013) are translated to independent Modelica sub-models. The boundary conditions of these sub-models, which are given by pressures and flows, can be coupled to other sub-models or set to a constant. Moreover, Modelica compilers, such as OpenModelica [1], are able to check the consistency of the sub-models, and their interconnections. Therefore, Modelica assists in a bottom-up model development. Modelica compilers generate a *functional mock-up unit* (FMU), which is a stan-

[1] www.openmodelica.org

dard model component that can be shared with other applications. Subsequently, the resulting model may be imported to CasADi (Andersson, 2013) via the integration to the JModelica.org compiler. Casadi implements efficient automatic differentiation techniques and is interfaced to other numerical packages. This enables fast development of NMPC solutions, without needing deep knowledge on the implementation of Nonlinear Programming solvers or Automatic Differentiation tools.

In this work, the control structure is divided into two layers. A regulatory layer is designed after controllability analysis of the unstable system. This consists of cascade controllers for wells and SISO controllers for the pipeline-riser system. Then, the second layer implements production optimization by providing set-points to the lower layer. To this end, the simplified sub-models are parametrized and adjusted to a detailed model in OLGA. An Extended Kalman Filter (EKF) is developed using the simplified models and tuned to track the detailed model. Then, the NMPC is implemented using state feedback. In order to assess the performance of the controller, the network system is steered from an initial predetermined fixed set-point to an optimal point by the NMPC.

The paper is organized as follows. In Section 2, the network system is described, and then the simplified models and the modeling fitting are presented in Section 3. The control structure and its building blocks are described in Section 4. The numerical results are presented in Section 5, and finally, the main conclusions and remarks are summarized in Section 6.

2. SYSTEM DESCRIPTION

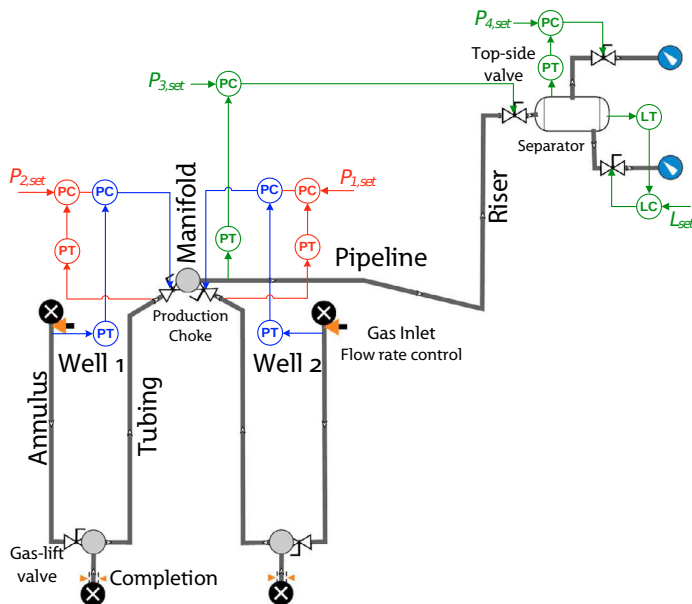The oil gathering system to be studied is modelled in the OLGA simulator and is represented in Figure 1. The

Fig. 1. OLGA-model: Oil gathering system with low level control structure.

network consists of two wells operated by gas-lift which feed a common pipeline-riser to a separator. The network contains 7 control inputs:

- Gas injection controlled by mass flow rate at the annulus top of each well.
- Production choke valve opening of each well.
- Top-side valve opening.
- Two valves downstream to the separator.

The wells are considered to be geometrically identical. These are vertical with tubing and annulus length of 2048 m. The tubing diameters are equal to 0.124 m., the annuli are represented by a cylindrical not-annular pipeline of 0.2 m. diameter, and the roughness coefficients are equal to 4.5E-5 m. The reservoir temperature is equal to 108 °C while the well inflow relation is considered linear with a coefficients of 2.47E-6 kg/s/Pa. The produced gas-oil-ratio (GOR) and water fraction (WCUT) are considered negligible. However, the reservoir pressures are different, being 160 bar for well 1 and 170 bar for well 2.

The pipeline length is 4300 m, where the last 2300 m has a negative inclination of 1° to mimic an undulated seabed. The riser has a height of 300 m. The pipeline and riser have a diameter of 0.2 m and a roughness of 2.8E-5 m. The separator is controlled to operate at a constant pressure of 5 bar.

In the OLGA simulator, the fluid properties can be specified by a black-oil model or can be supplied as PVT Tables. We use the PVT option in this work. The PVT tables are generated by PVTSim©. These tables store the fluid properties such as gas density (ROG), oil density (API), and gas mass fraction (RS) as functions of the pressure and temperature. The viscosity of the fluid model range from 0.2 to 1 cP, which is not sufficient to classify the fluid as heavy oil. The produced fluid is saturated and does not have free gas in a wide range of pressures. Due to this fluid conditions and the low reservoir pressure, the wells considered in this work are not *naturally flowing*. Therefore, gas-lift is required to assist the production.

3. SIMPLIFIED MODELS AND FITTING

The OLGA model described in Section 2 acts as the real system and is treated as a black-box model. However, we assume that some commonly available parameters of the system are given, such as the geometry and fluid properties at a given pressure and temperature.

Simplified models are built using representative parameters and first principles. We choose the models developed by Jahanshahi (2013) since these were successfully fit to the OLGA model. However, parameters of the simplified model had to be modified due to changes in the boundary conditions and the fluid model. Hence, in this section we present and discuss the simplified model parametrization and suitability to control the OLGA model.

3.1 Generalized submodel

We treat the gas-lift well and pipeline-riser as independent building components of the gathering network system. From a general perspective, any of these subsystems can be represented by the following ODE structure:

$$\dot{x}_s = f_s(x_s, u_s) \tag{1a}$$
$$y_s = h_s(x_s, u_s) \tag{1b}$$

where the subscript s refers to any subsystem in $\mathcal{S} = \{w_1, w_2, r_1\}$ which contains a reference to the wells and pipeline-riser. The separator is assumed to be operating at a constant pressure, which is the usual and reasonable assumption. The differential states x_s represent the mass of the phases liquid and gas contained in the subsystem s which evolve according to f_s. The function h_s defines the variables y_s which gathers the input pressures, and output mass flow rate variables for each phase.

The physical assumptions on each submodel are similar. The liquid phase is considered incompressible and the gas phase is modeled assuming the ideal gas law, with constant temperature and gas molecular weight.

3.2 Gas-lift well submodel

The annulus is modeled as a vertical cylindrical tank filled with gas at a constant temperature. The state of the annulus is fully defined by the contained mass of gas,

$$(\dot{m}_G)_a = (w_{G,in})_a - (w_G)_{inj}, \tag{2}$$

where $(w_{G,in})_a$ is the inlet gas flow rate to the annulus which is used as a control input and $(w_G)_{inj}$ is the injection rate from the annulus to the bottom of the tubing. The pressure at the annulus top, where the measurement is taken, is calculated based on the ideal gas law while the pressure at the injection point is considered to be the pressure at the top plus the pressure due to gas gravity.

The well tubing is modeled by two states, the mass of the gas and liquid in the well,

$$(\dot{m}_G)_w = \left(\frac{\eta}{\eta+1}\right) w_{res} + (w_G)_{inj} - (w_G)_{wh} \tag{3a}$$

$$(\dot{m}_L)_w = \left(\frac{1}{\eta+1}\right) w_{res} - (w_L)_{wh}, \tag{3b}$$

where η is the average mass ratio of gas and liquid produced from the reservoir which is assumed to be a known constant parameter of the well. $(w_G)_{wh}$ and $(w_L)_{wh}$ are the mass flow rates of gas and liquid at the well-head. The production mass rate w_{res} [kg/s] from the reservoir to the well is assumed to be described by a linear Inflow Performance Relationship (IPR). Similar to the annulus the pressure at the top of the well is calculated assuming the ideal gas law. Then, the gravity of the two-phase mixture and the friction in the tuning are taken into account to get the bottom-hole pressure. See (Jahanshahi, 2013) for the complete formulation.

3.3 Pipeline-riser submodel

The pipeline-riser is modeled by four states which are the masses of the gas and liquid phases inside the pipeline and the riser sections. The four state equations of this submodel are:

$$(\dot{m}_G)_p = (w_{G,in})_p - (w_G)_{rb} \tag{4a}$$

$$(\dot{m}_L)_p = (w_{L,in})_p - (w_L)_{rb} \tag{4b}$$

$$(\dot{m}_G)_r = (w_G)_{rb} - (w_{G,out})_r \tag{4c}$$

$$(\dot{m}_L)_r = (w_L)_{rb} - (w_{L,out})_r \tag{4d}$$

Here, the subscripts 'in', 'rb' and 'out' stand for 'inlet', 'riser base' and 'outlet' respectively. The mass flow rates at the riser base are calculated by valve equations, and

there are four tuning parameters in the pipeline-riser model which are used to fit the model to a real system or a detailed OLGA model. The model equations and the model-fitting procedure are given by Jahanshahi and Skogestad (2014).

3.4 Coupling submodels

Submodel equations represented by eq. (1) are coupled with mass and pressure balances. Moreover, every submodel has at the output boundary a valve equation:

$$|w^o|_1 = k\sqrt{\rho^o \max\left(p^o - p^i\right)} \tag{5}$$

where $w^o = (w_G^o, w_L^o)$ are the mass flow of gas phase and liquid phase, respectively and ρ^o is the estimated mixture density at the output. The pressures upstream and downstream the valves are p^o and p^i, respectively. The parameter k should be tuned following a procedure described in (Jahanshahi, 2013; Jahanshahi and Skogestad, 2014).

3.5 Model fitting

The simplified models include tuning parameters which are fit to the process. The tuning parameters must be chosen to match both the steady-state and dynamic behavior of the system. A good matching of the steady-state behavior of pressures and flow rates are necessary to find correct optimal settings. Moreover, the dynamic behavior (e.g. stability regions) is required to design the regulatory layer. We followed the model fitting procedure described by Jahanshahi and Skogestad (2014).

4. CLOSED-LOOP CONTROL

This work is focused on the control and automation layer of a multi-level offshore control hierarchy (Foss, 2012) and on the production optimization layer. Our suggested control structure is represented in Figure 2.
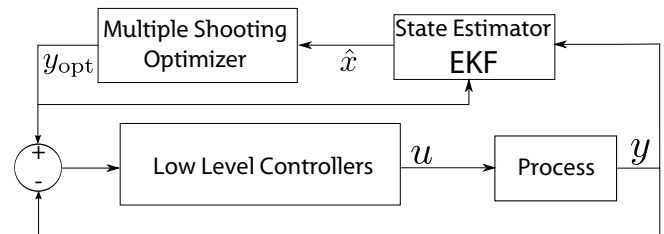
Fig. 2. Control structure

The controller can be separated in three main building components:

- Low level controller: The wells are controlled by cascade controllers which inner-loop measures the pressure at the top of the annulus and the outer-loop the pressure at the wellhead. The pressure at the inlet of the pipeline is controlled by a PI control loop which manipulates the valve at the top of the riser. Finally, for the separator, the liquid level is measured and controlled by a PI controller manipulating a liquid output valve; in the same way, the pressure is measured and controlled by a PI controller manipulating a

gas output valve. The pressure and level set-points are 5 bar and 20% of the separator height, respectively.

- State-estimator: The process measurements y are used to correct estimated dynamical states $\hat{x}$ of the system. This operation is performed on-line with an Extended Kalman Filter (EKF). The EKF uses the the simplified models of the wells and risers coupled to the low level controllers. Thus, the states being estimated correspond to the states of the simplified models and the state of the controllers

- Multiple Shooting (MS) optimizer: The MS optimizer takes as input the estimated states $\hat{x}$ of the EKF and computes an optimal trajectory y_{opt} for the pressure set points and an input flow rate for gas. The objective function considers the oil being produced and the gas being injected over a certain period, and penalizes the control effort being applied.

4.1 Low level control

Optimal gas-lift operating points under high lift-gas injection price are located in an unstable region where the casing-heading instability occurs. Therefore, low level controllers are required for stabilization. The gas-lift well has two degrees of freedom for control, the gas injection rate and the production choke valve. In this work we use the production choke for stabilization, see e.g. (Jahanshahi, 2013).

Downhole pressure measurements can be used for stabilizing flow. A simpler alternative are instruments placed on the wellhead and topside. In this work we combine wellhead pressure measurements in a cascade structure. In an earlier controllability analysis (Jahanshahi, 2013), it has been shown that the pressure measurement at the top of the tubing is not a suitable controlled variable in a SISO structure. The reason is the RHP (Right Half-Plane) zero dynamics associated with the pressure at the top of the tubing. With a SISO controller, this measurement reacts with an inverse response to input changes (Skogestad and Postlethwaite, 2005), that imposes unavoidable large peaks in the sensitivity transfer functions. However, when the tubing pressure is combined with other measurements, such as the annulus pressure, it is possible to design a controller with a low peak in its sensitivity transfer function (Jahanshahi, 2013). In the cascade control structure used in this work, the annulus pressure measurement is controlled by the valve and its set-point comes from the master control loop controlling the tubing pressure at a given set-point.

4.2 State estimation

The EKF is implemented in discrete time as in (Simon, 2006, p. 409). The model used within the filter consists of coupling the sub-models described in Section 3 and models for the low level controllers in Section 4.1.

The low level controllers are implemented within the OLGA-model, and their state variables are not available. Therefore, similar low level controllers are coupled with the simplified models and their states are estimated in the EKF.

All models are written in continuous time and discretized using the CVODES (Hindmarsh et al., 2005) integrators and CasADi (Andersson, 2013) for Automatic differentiation of the system equations. The EKF receives measurements every 10 sec., hence CVODES integrates the system and find the required sensitivities for this period of time.

The measurements used for state estimation are the wellhead pressures and the pipeline inlet pressure. Although more measurements are available, only measurements which are control variables in the regulatory layer are considered. The reason is that the regulatory layer forces these variables to track the same set-points in the model and in the plant. Thus, these measurements are unbiased in steady-state and therefore suitable for the Kalman filter algorithm. Additional measurements which contain steady-state bias deteriorate the estimation. Here, the estimation relies on a good model rather than on aggressive corrections due to measurements.

4.3 Multiple Shooting optimizer

The MS optimizer solves the following problem:

$$\min_{\Theta} \sum_{k \in \mathcal{K}} \left(-q_{\mathrm{o}}(\mathbf{x}_k, \mathbf{u}_k) + \alpha_{\mathrm{g}} q_{\mathrm{inj}}(\mathbf{x}_k, \mathbf{u}_k) \right) + \tag{6a}$$

$$\sum_{k \in \mathcal{K}} \left(\mathbf{u}_{k-1} - \mathbf{u}_k \right)^{\top} R_{\mathrm{u}} \left(\mathbf{u}_{k-1} - \mathbf{u}_k \right) + \tag{6b}$$

$$\left(\mathbf{u}_K - \mathbf{u}_{\mathrm{opt}} \right)^{\top} R_{\mathrm{u}}^{\mathrm{f}} \left(\mathbf{u}_K - \mathbf{u}_{\mathrm{opt}} \right) + \tag{6c}$$

$$\text{s.t.}: \quad \mathbf{x}_{k+1} = F(\mathbf{x}_k, \mathbf{u}_k), \quad k \in \mathcal{K}, \tag{6d}$$

$$\mathbf{y}_k = Y(\mathbf{x}_{k+1}, \mathbf{u}_k), \quad k \in \mathcal{K} \tag{6e}$$

$$\mathbf{b}_l^{\mathrm{x}} \le \mathbf{x} \le \mathbf{b}_u^{\mathrm{x}} \tag{6f}$$

$$\mathbf{b}_l^{\mathrm{y}} \le \mathbf{y} \le \mathbf{b}_u^{\mathrm{y}} \tag{6g}$$

$$\mathbf{b}_l^{\mathrm{u}} \le \mathbf{u} \le \mathbf{b}_u^{\mathrm{u}}, \tag{6h}$$

where the set of variables to be optimized Θ is composed of the state variables at the end of the shooting periods $(\mathbf{x}_2, \ldots, \mathbf{x}_{K+1})$ and the control variables $\mathbf{u}_k, k \in \mathcal{K}$. Hence, the problem is divided in K shooting periods ($\mathcal{K} = \{1, \ldots, K\}$), which are coupled by the MS state constraints (6d). The function F represents a simulation of the simplified models over a discretization period, which is chosen equal to 1 hour. The states $\mathbf{x}_{k+1}, k \in \mathcal{K}$ contains the state of the simplified models at the end of the corresponding shooting period. The initial state $\mathbf{x}_1$ is not a decision variable and it is estimated by the EKF. The objective function aim to maximize an economical value, given by the oil production and the gas injection at a given price α_{g}. Moreover, a penalty term that penalizes control changes is included in (6b) which can be tuned with the positive semi-definite matrix R_{u}. To this end, $\mathbf{u}_0$ is equal to the current set-points being applied to the process. Finally, the objective implements a final stage cost (6c), which penalizes the mismatch between the final inputs $\mathbf{u}_K$ and the steady-state optimal input $\mathbf{u}_{\mathrm{opt}}$. With this aim, optimal input $\mathbf{u}_{\mathrm{opt}}$ is computed off-line and the positive semi-definite matrices $R_{\mathrm{u}}^{\mathrm{f}}$ is tuned. Output constraints are implemented as bounds on the states (6f) and with bounds on the output variables $\mathbf{y}$ in the equations (6e) and (6g). Finally, input bounds are set in (6h).

Output constraints are required to keep the optimizer inside the physical limits of the system and away of unstable regions. Therefore, many constraints are implemented

to provide robustness to the optimization method. These include bounds on wellhead pressures, flow rates and mass fractions within the pipelines. The requirement of a large-set of output constraints makes the MS formulation the preferable choice as opposed to the most compact Single Shooting formulation. The cost of an iteration of the MS formulation is dictated by the number of state variables, which is low in this example. However, the cost of an iteration of the Single Shooting formulation depends on the number of output constraints being considered.

Problem (6) is solved with IPOPT (Wächter and Biegler, 2005). Observe that fulfilling tight tolerances of the optimality conditions for problem (6) can be computationally very expensive. Therefore the solver was limit to make 60 major iterations or to process during 30 minutes.

5. CONTROLLER PERFORMANCE

We apply the controller suggested in section 4 to control the OLGA model. We start the NMPC after 1 hour when the regulatory layer has settled and the Kalman filter has converged. The prediction horizon is set to forecast 16 hours and the discretization period of each shooting interval is 1 hour. We optimize 16 shooting intervals ($K = 16$), each containing 20 variables, corresponding to 5 controls and 15 states. However, the MS algorithm returns only the optimal control inputs for the next 1 hour. The optimal inputs consist of the gas injection rates of the two wells and the optimal set-points for the regulatory controllers.

The performance of the pressure controller for well #1 is shown in Fig. 3. The performance of the controller related to well #2 is similar. These are cascade controllers where the set-points for the master loops (tubing pressure) are given by the optimization layer and the slave loops manipulate the production choke valve openings. The production valves are opening gradually to increase the oil production rates. Nevertheless, they respect the constraints imposed for the controllability purpose. Since the wellhead pressures are used for the state estimation and they follow the optimal set-points, the measurements and estimates are very close. However, the modeling mismatch causes estimation errors for the annulus pressures and the openings of the valves.

Fig. 4 and Fig. 5 show the gas injection rates and oil production rates for the two wells. The optimal gas injection rates are calculated by the NMPC. As shown in the figures, the optimizer injects more gas to the wells to reach the optimal operation point which is dependent on the oil and gas prices. The estimation error is caused by process/model mismatch. Here, the normalized price of oil is 1 and the normalized price of gas equals 2.5, for each kg/s.

Fig. 6 shows the control of the pressure at the pipeline inlet. This controller manipulates the top-side valve and the optimal set-points are given by the NMPC. The estimation error of the inlet pressure is negligible because its measurement is used for state estimation and it is directly controlled by the regulatory layer. Nevertheless, the valve opening estimation suffers due to modeling error. Moreover, we observe a constraint violation in the transient
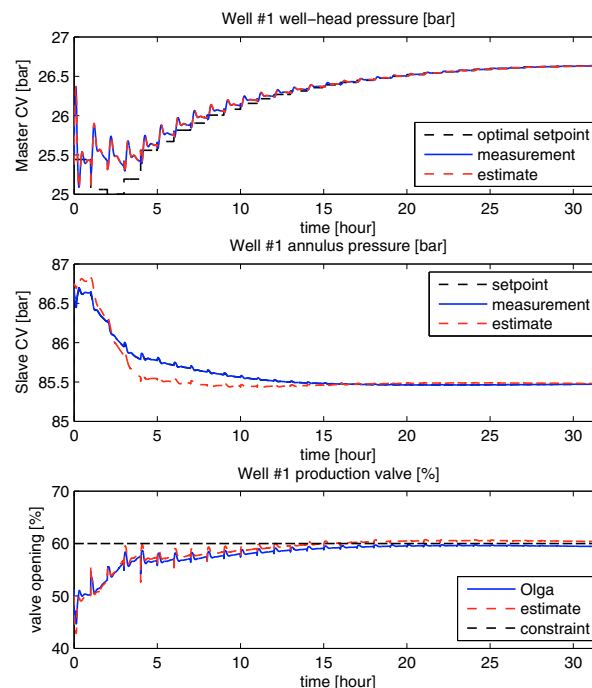

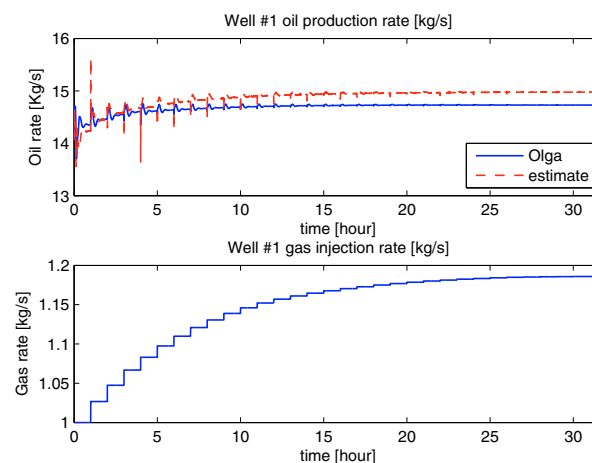Fig. 3. Well-head pressure of well #1

Fig. 4. Oil production rate of well #1

response because the optimization algorithm was halted (CPU-time limit) before and optimal solution is reached. However, this constraint is satisfied in steady-state. The proposed low level control structure and the EKF computational times are negligible compared to the sampling time of the plant. Hence, these are suitable for on-line applications. However, the NMPC solution is not solved to the default tolerances in IPOPT and it is halted after 30 minutes of execution, therefore a sub-optimal solution is used. Moreover, in order to keep the controller performance assessment independent of this computational time, the process simulator is paused during this computation. Nevertheless, observe that IPOPT is a general purpose solver and does not exploit structure of the MS formulation. Therefore, appropriate solvers may solve this problem in a feasible time for a closed-loop application (Diehl et al., 2009).
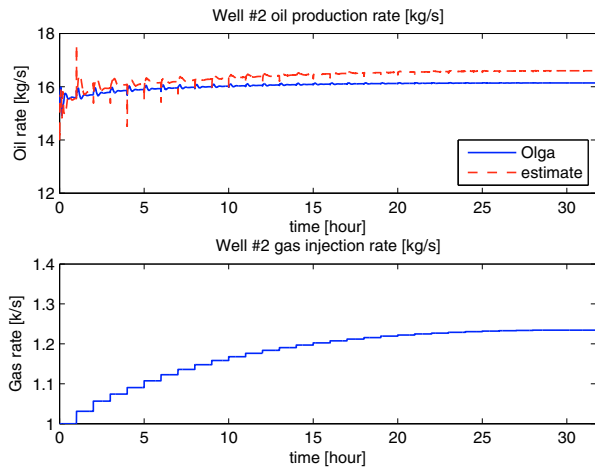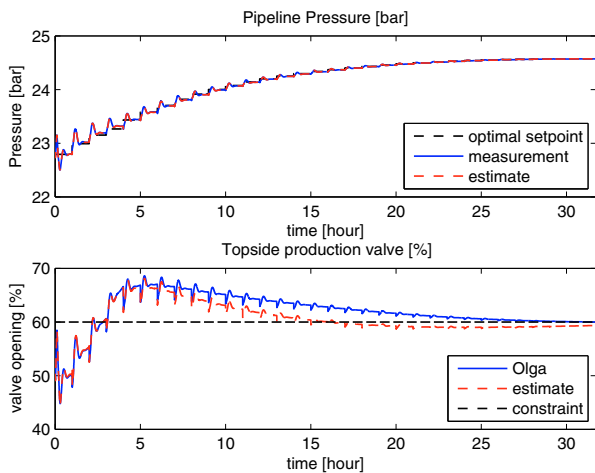
Fig. 5. Oil production rate of well #2

Fig. 6. Pressure at pipeline inlet

6. CONCLUSION

To our knowledge, this paper is the first publication considering regulatory control of a multiple well system and riser steered by an NMPC optimization layer. The structure work well by jointly calculating dynamic setpoint trajectories and ensuring stable flow conditions on a realistic simulator. Thus, it is a promising approach.

However, the optimization algorithm for NMPC is not fast enough to be used in closed-loop, therefore, further research developments should be carried out to exploit the structure of the Multiple Shooting formulation with specialized Nonlinear Programming solvers.

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