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AND POLLUTION
EMBODIED IN TRADE:**

Theoretical Development

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Production Factors and Pollution Embodied in Trade: Theoretical Development

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Abstract

Many studies have calculated the pollution resulting from consumption activities. A common assumption in many of these studies is that imports are produced using domestic technology. Studies of economic factors embodied in trade have shown that regional technology differences are important for modeling of international trade patterns. Therefore, it is likely that calculations of pollution embodied in international trade will require the consideration of regional technology differences. This article develops a framework based on input-output analysis (IOA) for calculating production factors and pollution embodied in trade. In its most general form, the model has significant data requirements. Consequently, many simplifications are discussed that lead to reductions in data requirements, without the introduction of large errors. The framework is applied to the case of Norway in a companion article (Peters et al., 2004).

Keywords

Input-output analysis; Embodied pollution; Embodied factors; Factors of production; CO₂; Trade;

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1 Introduction

Consumer demand is the central cause of all factor use and environmental pollutants; both directly and indirectly. When production for consumer demand occurs in the same country as consumption, then government policy can be used to regulate factor use and the release of pollutants. Increasing competition in economies open to trade has led to increasing production in foreign countries for domestic consumption. In particular, regulating the resulting pollution embodied in trade is becoming a critical issue to stem global pollution levels. This article develops a model to accurately quantify the factor use and pollution embodied in trade.

Wyckoff and Roop (1994) estimated that about 13% of the total carbon emissions of six of the largest OECD countries was embodied in manufactured imports in the mid-1980's. The total carbon emissions embodied in import was as high as 40% for France. Lenzen (1998) found that 31% of Australian emissions are exported, while 21% of emissions occur in foreign regions (relative to total Australian emissions). Consequently, Australia is a net exporter of carbon¹. Hertwich et al. (2002) found that over 70% of the emissions in Norway are associated with export. Compared to consumer demand, around one third of Norwegian emissions are embodied in imports. Further, Norway exports five times more carbon than it imports. Many other studies perform similar calculations and obtain similar results (e.g. Bullard and Herendeen, 1975; Gay and Proops, 1993; Battjes et al., 1998; Machado et al., 2001; Muradian et al., 2002; Ahmad and Wyckoff, 2003; Lenzen et al., 2004).

The economic literature on international trade is particularly interested in the factors of production embodied in trade (Samuelson, 1953; Vanek, 1968). The typical factors of production considered by economists are various disaggregation of labor, capital, and sometimes land. Mathematically, the determination of factors embodied in trade has many parallels with pollution embodied in trade. The Heckscher–Ohlin–Vanek (HOV) model commonly employed to describe international trade is found to fail dismally empirically (Trefler, 1993, 1995; Hakura, 2001). However, a relaxation of some of the model assumptions shows that the modified HOV model can relatively accurately model factor embodiments in trade (Hakura, 2001). Of particular importance is the need to relax the assumption of identical technology in each region.

A common weakness of the studies determining carbon embodied in trade is that they assume that the foreign countries use the same technology as the domestic country. Partly this is due to lack of data and partly due to a lack of a consistent theoretical framework. The error obtained by assuming identical technologies is likely to be small for countries with similar technologies and energy sources, but quite different when the countries have large technological differences. Perhaps of most concern is the factors and pollutants embodied in trade when the trading partners are developed and developing countries. Haukland (2004) shows that the total carbon dioxide emissions per unit output for Norway and China show a difference of up to three orders of magnitude for some sectors and pollutants. Only recently have studies emerged that consider directly different

¹Compared to the HOV model, discussed below, this would suggest Australia has a comparative advantage in carbon intensive goods. This more likely reflects a cost advantage in “energy” intensive goods compared with a pollution advantage in carbon intensive goods.

technology in each region (Ahmad and Wyckoff, 2003; Lenzen et al., 2004); however, both these studies assume that the developing countries have the same technology as developed countries.

The model developed in this article has many similarities with interregional and multi-regional input-output models (see for instance, Miller and Blair (1985)). Interregional studies have shown that feedback effects between regions are relatively small and the extra data requirements may not be justified (Round, 2001). However, in the present study, factor usage intensities vary widely between regions and consequently, small feedback effects can be amplified to significant amounts. A recent model with particular focus on CO₂ embodied in trade shows that uni-directional trade makes a significant difference, but multi-regional trade only offers improvements of up to a few percent (Lenzen et al., 2004).

Lenzen et al. (2004) developed a multi-region model using the make-use framework of input-output analysis (although still implicitly using the industry-technology assumption). The focus of that model is to determine the balance of trade for CO₂ emissions. The model presented here describes several simplification that can be used when calculating a trade balance for a given region. Ahmad and Wyckoff (2003) applied similar simplifications to determine the balance of trade for CO₂ emissions in OECD countries. They consider the emissions from total imports and exports, but their framework cannot be applied to arbitrary demands. The model developed in this article has a focus on arbitrary demands.

This paper develops a model to determine factors embodied in trade. The central framework of the model is input-output analysis (IOA). Several key issues are revealed in the development of the model, for consistency between future studies, these issues are discussed. The first sections address some important issues. The middle sections present the model. Companion studies applies the framework to Norway at a variety of scales (Peters et al., 2004; Briceno et al., 2005).

2 Input-output framework

The model developed in this article is based on input-output analysis (IOA) (Leontief, 1941; Miller and Blair, 1985; United Nations, 1999). In summary, the total output of the economy is given by intermediate consumption and final consumption,

$$x = Ax + y \tag{1}$$

where A is the interindustry requirements matrix and y represents various demands on the economy. Solving for the total output, x , gives

$$x = (I - A)^{-1}y \tag{2}$$

This gives the interpretation of the Leontief inverse; the change in output for unit change in demand. If entry j in y changes by unity, then column j of $(I - A)^{-1}$ gives the required output from all industry sectors.

It is possible to perform IOA calculations at both a commodity level and an industry level. For many studies it is desirable to construct the data at the commodity level

as this is what consumers purchase. Many countries collect IO data in the make-use framework and allow for secondary production (Miller and Blair, 1985; United Nations, 1999). Through the make-use framework it is usually possible to construct symmetric tables at the commodity level. Unfortunately, in some countries the symmetric tables are available only at the industry level. In addition, the factor use intensities are usually produced on an industry basis only. So at some stage the commodity detail has to be transformed to the industry detail. Consequently, most IO studies are performed at the industry level.

In this article it is assumed that the IO data are at the industry level; the number of industry sectors is given by n . Further, it is assumed that the entries of A are in monetary units. Mixed units can be used as described by Chapman (1974), Bullard and Herendeen (1975), and Duchin (2004).

2.1 Factor usage and pollutant emissions

Given the output of an economy, x , the factor input required to produce the output is given by

$$F(I - A)^{-1}y \quad (3)$$

where F is a $k^* \times n$ matrix representing the factor use intensities and k^* is the number of factors considered. The $F_{\alpha i}$ element is the input of factor α in the sector i per unit output of that sector.

Similarly, the pollution generated by an output of x is given by

$$G(I - A)^{-1}y \quad (4)$$

where G is an $l \times n$ matrix representing the pollution intensities and l is the number of pollutants considered. The $G_{\alpha i}$ element is the amount of pollutant α emitted in the sector i per unit output of that sector.

For the purpose of this paper, pollutants and factors are considered equivalent (compare with Lenzen (2001b)). The matrix F will be a $k \times n$ matrix, where $k = k^* + l$ is the total number of factors and pollutants considered. To this end, “factors embodied in trade” refers to both the traditional economic factors (labor, capital) and pollutants (carbon dioxide, toxic chemicals, etc) embodied in trade. The term “factor usage” refers to both the use of traditional factors in production and the pollution released from production processes.

2.2 Elements of demand

The ultimate objective of the model is to determine the factor usage resulting from a given demand of products in a domestic economy. Depending on the nature of the study, the model needs to be adaptable to a variety of demands. On a national level, one may be interested in the factor usage resulting from a total demand of the entire economy. On a smaller focus, one may be interested in the demand resulting from household consumption. Depending on the scale of interest, several issues arise.

The final demand is often broken down into numerous components, for example,

$$y = \underbrace{y^d + y^i}_{\text{private}} + \underbrace{y^g + y^{gi}}_{\text{government}} + y^c + y^m + y^e - M \quad (5)$$

where y^d is the private domestic demand on domestic production, y^i is the private domestic demand on imported products, $y^f = y^d + y^i$ is the final private domestic demand, y^g is the government domestic demand on domestic production, y^{gi} is the government domestic demand on imported products, y^c is gross fixed capital formation, y^m is the trade and transport margins, y^e exports, and M imports to both final demand and to interindustry demand.

This breakdown of final demand is by no means exhaustive, and ultimately depends on data availability and the nature of the application. Various demand terms are discussed throughout the text where relevant. Further details can be found in many references on input-output analysis, for example United Nations (1999). Some relevant demands are discussed now.

2.2.1 Household demands

The household demands are relatively straightforward. Demands on domestically produced products and imported products are usually distinguished. The imports are usually valued in basic prices. It is important to capture the impacts of both competitive and non-competitive imports; the issue of non-competitive imports is discussed further below. Of particular interest is studying the expenditure various households as opposed to total household expenditure (see Peters et al., 2004, for more details). When studying structural changes in an economy it is often useful to ensure that household income can cover household expenditure; one method to achieve this is through Social Accounting Matrices (United Nations, 1999).

2.2.2 Gross fixed capital formation

Gross fixed capital formation is measured by the total value of a producer's acquisitions, less disposals, of fixed assets (United Nations, 1993). It can be argued that interindustry gross capital formation should be included into intermediate inputs as it ultimately occurs to facilitate production (Lenzen, 2001b). Ultimately, the treatment of gross capital formation depends on the application being studied and the detail of the data available.

Gross capital formation can be "internalized" into the standard input-output framework (Lenzen, 1998, 2001b). Let the total non-capital demands on the economy be given by, y^t , and the gross capital formation by, y^c . The total output of the economy is given by

$$x = Ax + y^t + y^c \quad (6)$$

Following Lenzen (2001b), let

$$y^c = Kx \quad (7)$$

where K represents the capital requirements for a unit of output. Thus, the total output of the economy can also be given by

$$x = (A + K)x + y^t \quad (8)$$

Consequently, the total output of the economy can be expressed equivalently as,

$$x = (I - (A + K))^{-1}y^t = (I - A)^{-1}(y^t + y^e) \quad (9)$$

This equivalence shows that if the total demands on the economy are considered, then it does not matter if the capital requirements are internalized or not. In this case, there is little justification for constructing K . For arbitrary demands, however, the resulting output will vary depending on the treatment of capital. In this case, if it is assumed that capital is purchased to facilitate production, then capital should be internalized into the interindustry requirements matrix, $A \mapsto A + K$.

Whether capital should be internalized into the interindustry requirements matrix depends on the application being studied. For an economy heavily dependent on exports, the majority of capital expenditures might be in the expansion of exporting industries. Thus in a study of household demands the capital requirements might lead to unrealistically high outputs in some sectors as a result of capital expenditure in the exporting sectors. This would suggest that the capital demands should be broken down into separate categories for interindustry, government, private, export, and so on.

Another issue of importance is the type of capital expenditure. Some capital expenditure may be used for replacement, other capital might be used for expansion (e.g. Miller and Blair, 1985). Also, the capital expenditures may vary by large amounts from year to year. These points suggest that to correctly model capital requires the consideration of temporal effects. In this case, consideration should be given to dynamic IO models (Duchin and Szyld, 1985; Fleissner, 1990).

The data to construct the capital requirements matrix is not always readily available. To illustrate the difficulties in determining K it is worth noting that Lenzen (2001b) estimated K from “several disparate unpublished working estimates” and estimated the error in the elements of K to be 50-70%. Also, distinguishing between what is capital expenditure and what is an interindustry requirements for production can be difficult; the reporting industry might have a different view as the statistical office constructing the IO data.

2.2.3 Government demand

The relationship between government demand and private demand is not straightforward. It could be argued that if there was no private demand, then there would be no government demand. From this viewpoint, all government demand is a consequence of private demand and so the two should be treated together. In contrast, in a study focussing on the consequences of private consumption patterns government demand may be considered as separate from household demand. For instance, how is government expenditure on defence related to household consumption of food? On the contrary, some government expenditure, such as welfare, is directly used by households (United Nations, 1999).

The treatment of government demand in relation to private demand is likely to be dependent on the study being performed and on the detail of data available. For many studies, it may be worthwhile to internalize government expenditure into the interindustry requirements matrix as for capital in the previous section. Further, for many studies it may be important to capture changes to government income or expenditure. For instance,

a large shift in household expenditure away from personal transportation services to public transportation may have many indirect effects on government income and expenditure.

2.2.4 Trade and transport margins

If the IO data is valued in basic or producer prices then there may be a final demand category for trade and transport margins. As for capital and government demand, the treatment of trade and transport margins requires some consideration. In some cases, it may be possible to internalize trade and transport margins into the interindustry requirements matrix. Margins vary depending on the destination of the purchase—household, export, import—and may also vary with quantity. The treatment of trade and transport margins is particularly important for studies of household consumption. The trade and transport margins may vary for different household purchasers. Further, different types of the same product may have different margins; for instance, cheap and luxury cars. An important area that is often inadequately handled in the IOA is the emissions from international transportation; particularly of imports. The final treatment of trade and transport margins will depend on assumptions made for a particular study.

2.2.5 Exports

Exports are usually straightforward and it is not often distinguished where the exports go. The treatment of re-exports needs to be considered; and this may vary from country to country. For the framework here it is useful to distinguish between exports that go to final demand and exports that go to industry.

2.2.6 Imports

Imports can be considered as either competitive or non-competitive. The competitive imports are produced by the domestic economy, the non-competitive imports are not produced in the domestic economy. It is possible to deal with both types of imports through the make-use framework; although this requires assuming that the imports are produced with domestic technology. Since non-competitive imports are not produced in the domestic economy they are difficult to incorporate into IOA analysis.

Consider a non-competitive import. An extra row can be added to the make and use tables for this import. The region does not make this import (it is non-competitive) so in the make table this row consists of zeros. For the same reason, the domestic use matrix has zeros for this import. However, in the imported use matrix, the new row for the non-competitive import is non-zero and represents the use of the import by domestic industries.

From the make and use framework, it is then possible to construct the interindustry requirements matrices for domestically produced and imported products. At the commodity-commodity level, the interindustry requirements matrix for domestically produced products will have zeros in the row and column for the non-competitive import, but the interindustry requirements matrix for non-competitive imports has non-zero elements for this import. At the industry-industry level, all detail of the non-competitive imports are lost since the domestic economy does not produce the non-competitive imports. Thus

the standard make-use framework needs to be modified to account for non-competitive imports at the industry level. Appendix A gives an illustrative example of the problems in dealing with non-competitive imports.

3 Domestic outputs for arbitrary demands

In this section, the input-output structure to deal with imports and exports is discussed in detail. This work is a precursor to a detailed assessment of factors embodied in trade. For simplicity, it is assumed that all government demand and capital demand is included in private domestic demand, y^d . The notation used in this section is shown in Table 1.

Variable	Description
x	Output of the given economy
y^d	Final domestic demand on domestic production
y^{im}	Final domestic demand on imported production
y^{ex}	Final export demand on domestic production
$y^f = y^d + y^{im}$	Final domestic demand of consumers
$y^t = y^d + y^{ex}$	Total demand on domestic production
$m = A^{im}x + y^{im}$	Total imports to both domestic intermediate demand and domestic final demand
A^d	Interindustry requirements on domestic production
A^{im}	Interindustry requirements of imports
$A = A^d + A^{im}$	Total interindustry requirements
F^d	Industry requirements of domestic production factors

Table 1: The notation used in section 3.

In a given region, some of the imports go directly to the final consumer and the remainder go directly to fulfill intermediate demand in production processes. Exports are treated as final demand. This can be written more formally as (United Nations, 1999)

$$x = (A^d + A^{im})x + y^d + y^{ex} + y^{im} - m \quad (10)$$

A balance must hold for the imports,

$$m = A^{im}x + y^{im} \quad (11)$$

That is, the total imported is equal to total imported for intermediate demand plus the total imported for final demand. From this balance, it is easy to show that

$$x = A^d x + y^d + y^{ex} = A^d x + y^t \quad (12)$$

This result is intuitive, it states that imports do not change the total output of an economy unless there is a drop in demand for domestically produced goods, y^t , or domestically produced intermediate demand, A^d . For a more detailed discussion of these points see Appendix B.

It is important to note that these equations use A^d and not A . A represents the technology used in the economy, the A^d represents the amount of domestic inputs used by the technology. If there are no imports into intermediate demand, $A^{im} = 0$, then it follows that $A = A^d$. This rarely happens in reality.

From (12), the output for given demands can be determined. The total output for all demands on domestic production is

$$x = (I - A^d)^{-1}y^t \quad (13)$$

Recall that the Leontief inverse, $(I - A^d)^{-1}$, represents the output for a unit demand, as discussed earlier. So the Leontief inverse allows decomposition of separate demands. The output required for a domestic final demand of domestic production, y^d , is simply

$$x^d = (I - A^d)^{-1}y^d \quad (14)$$

The output required for a demand, y^{ex} , of exports is

$$x^{ex} = (I - A^d)^{-1}y^{ex} \quad (15)$$

Extending this further, the output for an arbitrary demand is

$$x^* = (I - A^d)^{-1}y^* \quad (16)$$

where y^* could represent government demand, capital demand, household demand, a unit demand on a particular sector, and so on. These outputs can also be written in terms of A instead of A^d ; this is discussed in Appendix C. Note, that if $A^{im} \neq 0$ then a proportion of production, given by $A^{im}x^*$ occurs in foreign regions.

By using the direct multiplier for factor usage, F^d , the total factors embodied in production can be determined,

$$E^* = F^d(I - A^d)^{-1}y^* \quad (17)$$

The term $F^d(I - A^d)^{-1}$ is often called a multiplier. The direct factors used are given by F^d , while the total factor use is given by $F^d(I - A^d)^{-1}$; the Leontief inverse captures all the indirect effects. Lenzen (2001b) considers different multipliers in detail. He compares the size of the multipliers when various components are incorporated into the multiplier expression; for instance, capital, imports, and so on.

Now, consider the factor usage embodied in a country's imports. To produce the arbitrary demand y^* in the domestic economy usually requires imports into production; the required imports are given by $m = A^{im}x^*$. The output in the foreign region i is then given by

$$x_i = (I - A_i^d)^{-1}m_i \quad (18)$$

where m_i is the import from country i and A_i^d is the relevant interindustry coefficients matrix in that country. The embodied factors are given by

$$E_i = F_i^d x_i = F_i^d (I - A_i^d)^{-1} m_i \quad (19)$$

where F_i^d is the factor use intensity in region i . In turn, this foreign region may require imports to produce the demand, m_i . In general, these indirect imports and embodied production factors may continue far upstream in the same way that interindustry demands continue far upstream on the domestic level. As a result, an import from one country indirectly includes imports from many other countries. The mathematical formulation to analyze this problem becomes more complex and is now presented.

4 Output and factors embodied in trade

This section now introduces the added complexities to calculate factors that are embodied in trade. In the previous section it was shown how to determine domestic outputs, x^* , for a given demand, y^* ; see (16). To produce y^* requires imports of $A^{im}x^*$. In general, the imports may come from a number of different regions with different technologies; each of these regions also places import demands on foreign economies. This leads to an infinite number of upstream imports. To simplify the presentation only two regions are considered initially, the more general case is formulated later.

For the case of multiple trading regions the notation needs to be modified to accommodate an index for each region. Table 2 shows the notation used in the multiregion formulations.

Name	Description
x_i	Output of region i .
y_i	Final domestic demand on domestic production (c.f., y^d).
y_{ij}	Import to final demand from region i to region j (c.f., y^{im}).
$m_{ij} = A_{ij}x_j + y_{ij}$	Import from region i to region j .
A_i^d	Interindustry requirements on domestic production in region i .
A_{ij}	Interindustry requirements of imports from region i to j .
$A_i = A_i^d + \sum_{j \neq i} A_{ij}$	Total interindustry requirements in region i .
F_i	Direct factor requirements in region i .

Table 2: The notation used for the multi-region formulations.

4.1 Two trading regions

Consider two trading regions,

$$x_1 = A_1x_1 + y_1 + m_{12} - m_{21} \quad (20)$$

and

$$x_2 = A_2x_2 + y_2 + m_{21} - m_{12} \quad (21)$$

Let region 1 be identified as the domestic economy. Region 2 is identified as the foreign economy. Note that the total imports from region 1 to region 2 are the same as the exports from region 1 to region 2. We want to determine the total output for an arbitrary demand in the domestic economy, region 1. To this end, put $y_2 = 0$.

There are several different scenarios that may arise, depending on the complexity considered. Ultimately, the choice of model depends on data availability. These cases are considered in turn.

4.1.1 Case 1: Region 1 imports to interindustry only and does not export

In this case the variables simplify as $m_{12} = 0$ and $m_{21} = A_{21}x_1$. The resulting system of equations after simplification are

$$x_1 = A_1^d x_1 + y_1 \quad (22)$$

where the imports have dropped out as discussed earlier, and

$$x_2 = A_2 x_2 + A_{21} x_1 \quad (23)$$

That is, the second region only produces for interindustry exports to region 1, further, $A_2 = A_2^d$ since region 2 does not have interindustry imports.

These equations can be solved as follows,

$$x_1 = (I - A_1^d)^{-1} y_1 \quad (24)$$

$$x_2 = (I - A_2)^{-1} [A_{21} x_1] = (I - A_2^d)^{-1} A_{21} (I - A_1^d)^{-1} y_1 \quad (25)$$

The total output resulting from the production of y_1 is given by $x_1 + x_2$. By using the factor use intensities in each region then the total factor use resulting from the production of y_1 is given by $F_1 x_1 + F_2 x_2$. The $F_1 x_1$ is the domestic factor use and $F_2 x_2$ is the factor use in the foreign region and consequently the factors embodied in trade.

If it is assumed that both regions have the same technology, $A = A_2 = A_1 = A_1^d + A_{21}$, then addition of (22) and (23) gives the total output

$$x = x_1 + x_2 = (I - A)^{-1} y_1 \quad (26)$$

and the total factor use

$$F(I - A)^{-1} y_1 \quad (27)$$

These last two equations offer a good check to our derivation. By assuming that each region has the same technology, is equivalent to assuming that all of the demand was produced in one region. That is, it is irrelevant where the imports are produced, so assume the imports are produced in region 1. Thus, the output is expected to be $(I - A)^{-1} y_1$. This check becomes particularly useful in the more complex scenarios.

4.1.2 Case 2: Region 1 imports to interindustry only and exports to region 2

In this case the variables simplify as $m_{12} = A_{12} x_2$ and $m_{21} = A_{21} x_1$. The system is considerably more complex as Case 1. For a given domestic demand, a proportion of the interindustry demand is imported, but in the production of the intermediate demand the foreign region may import from the domestic region, which in turn imports some intermediate demand from the foreign region, and so on.

The resulting system of equations after simplification is

$$x_1 = A_1^d x_1 + y_1 + A_{12} x_2 \quad (28)$$

$$x_2 = A_2^d x_2 + A_{21} x_1 \quad (29)$$

That is, region 1 and 2 trade between intermediate industries.

By adding these two equations and simplifying the total output for the demand y_1 is given by

$$x = x_1 + x_2 = A_1 x_1 + A_2 x_2 + y_1 \quad (30)$$

If it is assumed that the both regions use the same technology, $A = A_1 = A_2$ then it follows that

$$x = (I - A)^{-1} y_1 \quad (31)$$

which following the previous section is the expected result.

At this stage, there is no benefit in explicitly deriving expressions for x_1 and x_2 , however, for later generalization there is benefit of including the expressions in a matrix form,

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} A_1^d & A_{12} \\ A_{21} & A_2^d \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} y_1 \\ 0 \end{pmatrix} \quad (32)$$

This is essentially the interregional IO model presented by Miller and Blair (1985). Also compare this with Strømman and Gautepllass (2003).

The total factor use resulting from the demand is then given by

$$x = \begin{pmatrix} F_1 & F_2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \quad (33)$$

4.1.3 Case 3: General case for 2 regions

In this case the variables simplify as $m_{12} = A_{12} x_2 + y_{12}$ and $m_{21} = A_{21} x_1 + y_{21}$. This case is an increase in complexity of Case 2 to allow for imports directly to domestic consumers. In matrix form, the resulting equations are

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} A_1^d & A_{12} \\ A_{21} & A_2^d \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} y_1 + y_{12} \\ y_2 + y_{21} \end{pmatrix} \quad (34)$$

Four cases can be considered:

1. $y_1 \neq 0, y_{21} = 0, y_2 = 0, y_{12} = 0$: This is Case 2 above
2. $y_1 = 0, y_{21} = 0, y_2 \neq 0, y_{12} = 0$: This is Case 2 above, but region 2 is considered as the domestic economy, and region 1 is considered the foreign economy.
3. $y_1 = 0, y_{21} \neq 0, y_2 = 0, y_{12} = 0$: This allows a determination of the total output resulting from an import to domestic final demand in region 1. In this case, y_{21} is given exogenously.
4. $y_1 = 0, y_{21} \neq 0, y_2 = 0, y_{12} = 0$: Same as the previous point, but region 2 is considered as the domestic economy, and region 1 is considered the foreign economy.

These four cases, show that Case 2 can be used to described all possible scenarios of imports and exports both to interindustry demand and to final demand. By putting the relevant demands to zero, as in the four points above, it is possible to determine the output from each of the regions for the given demand. It is a case of simply summing all the respective outputs to get the desired result.

4.2 General m -region model

In this section the 2-region model is generalized to an m region model. For simplicity, only the domestic final demands in region 1 are considered; $y_i = 0$ for $i > 1$. Due to the symmetry of the problem, any region can be considered as the domestic economy by labeling it as region 1. The resulting system of equations solves for the output in the domestic economy,

$$x_1 = A_1^d x_1 + y_1 + \underbrace{\sum_{j \neq 1} (A_{1j} x_j + y_{1j})}_{\text{exports}} \quad \text{for } i = 1 \quad (35)$$

where the export terms are all exports from region 1 to interindustry and final demand in all other regions. The outputs in the foreign regions are,

$$x_i = A_i^d x_i + \underbrace{\sum_{j \neq i} (A_{ij} x_j + y_{ij})}_{\text{exports}} \quad \text{for all } i \neq 1 \quad (36)$$

where the exports terms are to interindustry and final demand in all other regions. As in Case 2 above, it is straightforward to show by summation that the desired result is obtained for identical technologies; this is left as an exercise for the reader. Writing these equations in matrix form gives

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_m \end{pmatrix} = \begin{pmatrix} A_1^d & A_{12} & A_{13} & \dots & A_{1m} \\ A_{21} & A_2^d & A_{23} & \dots & A_{2m} \\ A_{31} & A_{32} & A_3^d & \dots & A_{3m} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ A_{m1} & A_{m2} & A_{m3} & \dots & A_m^d \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_m \end{pmatrix} + \begin{pmatrix} y_1^d + \sum_{j \neq 1} y_{1j} \\ y_{21} \\ y_{31} \\ \vdots \\ y_{m1} \end{pmatrix} \quad (37)$$

The matrix is a generalization of the interindustry requirements, A , in standard IOA. The columns in the matrix represent the inputs into production; the off diagonal matrices are the imports into production from foreign regions. The rows represent the exports to foreign regions. The first element in the demand vector represents final demand on domestic production and the other elements are imports into final demand.

The factors embodied in trade from each region is given by $F_i x_i$ and the total factor use for a given demand is,

$$E = \sum_i F_i x_i \quad (38)$$

where F_i is the direct factor use intensity in each region, i .

The matrix equation, (37), is the same as the multi-regional models developed by Miller and Blair (1985). Lenzen et al. (2004) developed a similar model using the make-use framework. On collapsing their make-use blocks into symmetric matrices the same model results; they implicitly assume an Industry Technology Assumption. Ahmad and Wyckoff (2003) also used a similar model to calculate emissions embodied in trade.

4.3 General case: Simplifications

To solve the above system requires knowledge of the interindustry import shares in each region, A_{ij} . Consequently, this type of calculation requires significant data collection at a detail that is not currently available. This problem can be alleviated through several simplifications and approximations.

4.3.1 Direct trade dominates

Consider the factor usage for an arbitrary demand placed on the domestic economy (region 1). As a first approximation a similar approach to Case 1 for 2 regions can be taken. That is, the only interindustry trade that occurs is imports to industry in the domestic economy. In reality it is likely that the “second order” effects of interindustry trade amongst regions would be insignificant. For instance, Lenzen et al. (2004) find these effects to be around 1% (see their Table 7). Further, these terms are often assumed to be negligible in the similar interregional trade models (Round, 2001).

Mathematically, this simplification reduces (37) to,

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_m \end{pmatrix} = \begin{pmatrix} A_1^d & 0 & 0 & \dots & 0 \\ A_{21} & A_2^d & 0 & \dots & 0 \\ A_{31} & 0 & A_3^d & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ A_{m1} & 0 & 0 & \dots & A_m^d \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_m \end{pmatrix} + \begin{pmatrix} y_1^d + y_1^{ex} \\ y_{21} \\ y_{31} \\ \vdots \\ y_{m1} \end{pmatrix} \quad (39)$$

where

$$y_1^{ex} = \sum_{j \neq 1} (A_{1j}^{im} x_j + y_{1j}) \quad (40)$$

is the total exports from Norway to all regions to both industry and final demand.

4.3.2 Using trade shares to estimate A_{ij}

Data on A_{ij} is rarely available; however, many countries construct $A_i^{im} = \sum_{j \neq i} A_{ij}$. Also, data on the total trade flows between regions is often tabulated. Using this information, it is possible to estimate the share of trade flows from each region,

$$\{s_{ij}\}_k = \frac{\{m_{ij}\}_k}{\{m_{total}\}_k} \quad (41)$$

where $\{m_{ij}\}_k$ is the total imports of good k from region i to j and $\{m_{total}\}_k$ is the total imports of good k into region j . Calculations show that it is important to consider the trade shares in individual sectors and not the average of all sectors (Peters et al., 2004).

Once the vector s_{ij} has been constructed A_{ij} and y_{ij} can be estimated as

$$A_{ij} = s_{ij}A_i^{im} \quad (42)$$

and

$$y_{ij} = s_{ij}y_i^{im} \quad (43)$$

4.3.3 Grouping of like regions

Another way of reducing the data requirements is to assume that certain groups of countries have similar technology. This is particularly relevant for developing countries where sufficient data might not be readily available. For instance, if it is assumed that all Asian countries (except Japan) have similar technology to China, then Asia can be treated as one region with the technology of China. Similar approximations can be made for other grouped regions; North America, Europe, Africa, Latin America, and so on.

5 Simplifications for specific problems

As is discussed in Munksgaard et al. (2005) there are three scales of interest in consumption related issues; national, regional, and local. In the context of the work presented here, there are two scales; total demand (national) and arbitrary demand (regional and local). In the case of total demands, the above model has many simplifications. In particular, it is possible to perform the calculations without the import shares, A^{im} .

5.1 Trade balances

The study by Lenzen et al. (2004) focuses on a method of obtaining a CO₂ trade balance (Munksgaard and Pedersen, 2001). Given the desirability for determining the trade balance on a policy level, it is important that a reasonable estimate of the trade balance can be obtained by most national statistical offices. Fortunately, many simplifications result when considering the total demand of an economy.

For the following, consider a determination of the CO₂ trade balance for region 1. The total domestic emissions for the domestic demand is given by

$$E_1^d = F_1(I - A_1^d)^{-1}y_1^d \quad (44)$$

The total domestic emissions due to all exports from region 1 is given by

$$E_1^{ex} = F_1(I - A_1^d)^{-1} \sum_{j \neq 1} m_{1j} \quad (45)$$

The emissions in all other regions due to imports to region 1 is given by

$$E_i = F_i(I - A_i^d)^{-1} \sum_{j \neq i} m_{j1} \quad (46)$$

where m_{1j} is the total imports from region j to region 1 (including direct consumption of imports). The sum of all the foreign emissions,

$$E_1^{im} = \sum_{i \neq 1} E_i \quad (47)$$

is the emissions occurring in foreign regions due to production in region 1.

From this information different approaches can be used to allocate who is responsible for factor usage; or more relevantly, pollution (Munksgaard and Pedersen, 2001). If the producer is made responsible for emissions then the total emissions are

$$P_1 = E_1^d + E_1^{ex} \quad (48)$$

If the consumer is made responsible then,

$$C_1 = E_1^d + E_1^{im} \quad (49)$$

A trade balance can also be considered

$$B_1 = P_1 - C_1 = E_1^{ex} - E_1^{im} \quad (50)$$

The study by Ahmad and Wyckoff (2003) focused on trade balances and issues of allocating responsibility applied the above simplification to most OECD countries. In the limit of considering all countries, their method implicitly includes all multidirectional flows, without requiring the interindustry requirements of imports, A_{ij} . Although, if the study requires emissions for arbitrary demands, then the method used by Ahmad and Wyckoff (2003) cannot be applied.

Perhaps of more interest however, is the emissions occurring in non-OECD countries. The OECD countries may record the exports and imports to developing countries, but still the A^d matrix is required for the developing countries. This data may not be available in the detail required. To alleviate this problem somewhat, like regions can be grouped together as discussed earlier. Some total interindustry coefficient matrices, A , exist for developing countries, together with emissions intensities (China and India for example). Lenzen et al. (2004) use the interindustry requirements of Australia for the rest of the world and Ahmad and Wyckoff (2003) assumed that the rest of the world had the same technology as the USA². The results in Peters et al. (2004) show that it is important to include the technology differences of developing countries.

5.2 Error in using A and not A^d

The error in using A and not A^d can be viewed in two different ways. First, recall that if the output

$$x = (I - A)^{-1}y \quad (51)$$

is used then it is assumed that imports have been produced with domestic technology (see Case 2 for the 2-region model). So if A is used instead of A^d then multidirectional trade

²Note that Ahmad and Wyckoff (2003) looked at the likely impacts of including Chinese technology, but their final results did not include these impacts.

is included, but with domestic technology. This assumption may be adequate for many calculations, and in fact, it may be superior to use A instead of A^d if A^{im} is unavailable. With this in mind, it is still worth investigating the implications of using A and not A^d .

The error in using A and not A^d is given by

$$\epsilon = \frac{\|x - \bar{x}\|}{\|x\|} \quad (52)$$

where $x = (I - A^d)^{-1}y$ is the correct answer and $\bar{x} = (I - A)^{-1}y$ is the approximation. Further manipulations shows that

$$\epsilon = \frac{\|(I - A)^{-1}A^{im}x\|}{\|x\|} \quad (53)$$

which shows that the error decreases with smaller values of A^{im} .

In some cases, particularly for a region that trades primarily with countries with similar technologies, the errors of using A^d and ignoring interregional trade are likely to be greater than the error in capturing interregional trade in A . Although a verification of this would require a variety of case studies.

6 Other relevant issues

There are many other relevant technical issues that must be accounted for when performing IOA in a multiregional study. This section briefly discusses the main areas of concern.

6.1 Valuation

The choice of valuation needs to be considered; basic, producer, or purchaser prices. The different valuations differ primarily in the trade and transport margins, and taxes and subsidies. Typically margins and taxes are applied at different rates in different sectors and on different products. Even across the same goods, margins and taxes can differ for a variety of reasons (United Nations, 1999):

- Margins may vary depending on if the product is purchased from the producer, wholesaler or retailer.
- Different establishments offer different levels of service.
- Discounts may be given for bulk purchases.
- Brand names and luxury items may attract a larger margin.
- Mode of transport might vary for different customers.
- Taxes may vary depending on the end use of the product.

For these reasons it is more homogenous to work in basic prices than purchaser prices. Basic prices are more representative of the production value of a product rather than the market value. Although, within the aggregations used for products, it may be difficult to distinguish the margins for individual products. This makes calculations of individual consumption profiles difficult (Peters et al., 2004).

6.2 Inflation

The data covering a variety of regions is likely to come from a variety of sources and different time periods. Adjustments for inflation are required to make the data consistent for a given time period. Inflation is usually measured using the Consumer Price Index (CPI). In some regions it is possible to get the CPI for certain classes of goods; for instance, energy products.

6.3 Exchange rates

Since foreign regions are being compared with the domestic economy then exchange rates between regions must be considered. If the interindustry coefficients matrices, A , are in monetary units, then a currency conversion is not required as the entries are unitless coefficients. However, conversions may be required for trade flows and the factor use intensities.

A problem arises since one unit of a given currency may buy different quantities of a particular good in different countries. As an example of the likely errors it is worth considering the variation in cost of a commonly traded good by region. The export price of unwrought aluminium in 2000 varied considerably across regions³ (in kg/US\$); Norway 1.73, Sweden 1.70, United Kingdom 1.61, United States 1.44, Germany 1.67, Denmark 1.42, Japan 2.55, and China 1.45. These price differences are considerable and the choice of exchange rate will greatly alter the results.

There are two primary ways to convert the currencies between regions; Market Exchange Rate (MER) and Purchasing Price Parity (PPP). The MER is the market price of one currency in terms of another. It fluctuates on a daily basis; however, the data used in IOA is typically the total of annual transactions, and thus fluctuations in exchange rates can be averaged over a 12 month period. Further, the MER does not necessarily reflect relative prices of goods and services between countries. Due to these reasons the PPP was developed. It compares the prices of a basket of common household goods between regions. It is generally stable over time and gives a better reflection of the purchasing power of consumers in different regions. However, there are also some fundamental issues in the use of PPP such as the desired basket of goods varies in different regions, the basket of goods varies with wealth, the presence of bartered items, and various transaction costs are misrepresented. The differences are expected to be largest when comparing developed and developing countries. For further details refer to Samuelson (1964), Vachris and Thomas (1999), and Lafrance and Schembri (2002).

The example of unwrought aluminum above implies that PPP would be a better measure. It is also likely that the PPP will vary across sectors. Thus the accurate use

³Source: <http://www.unctad.org/>

of PPP will require a considerable amount of data collection. We are not aware of any studies that look at the variations of PPP across sectors, and whether the average PPP is a better measure of the average across industry sectors than MER.

6.4 Industry classification and aggregation

Often the data from different regions is collected at different levels of aggregation and industry classifications. To perform the analysis will require mapping the data, at some stage, to a consistent industry classification. For some classifications it is possible to obtain correspondence tables, otherwise, the correspondence tables need to be constructed by referring to the different classification descriptions. A further problem is that the classifications are often specific to a given economy. This means that it is often not possible to get a consistent match between different classifications.

Another issue with classifications is that some data is in an industry classification and other data is in a product classification. For example, survey's of consumer expenditure and trade flow data is usually in a product classification, while IO data is usually at the industry level. Usually there is not a one-to-one correspondence between product and industry classifications. This requires several assumptions and approximations to be made. For instance, Peters et al. (2004) ensure that when product data is mapped to the industry level, it balances with the industry outputs in the IO data.

6.5 Data sources

Data for a multiregional study unfortunately comes from a variety of sources. Each source often has its own aggregation, different methodologies, and different country coverage. These problems even occur within the same country; for instance, IO data and energy data may be at different aggregations. It would be ideal if data was available from one reliable source to ensure consistency and completeness. To a degree this is starting to happen; for example, the EU's EuroStat database⁴.

6.6 International transportation

Through most IO studies the impacts of international transportation is missed. One of the reasons for this problem is difficulties in where to allocated certain impacts due to the very international nature of the shipping industry. To estimate impacts due to international transportation would usually require an explicit analysis.

6.7 Competitive and non-competitive imports

It is important to capture the impacts of non-competitive imports. At the industry level, it is easy to loose information of non-competitive imports. The issues associated with non-competitive imports are discussed in Appendix A with a detailed example.

⁴<http://europa.eu.int/comm/eurostat>

6.8 Errors

Errors can enter into the calculations in many ways. The IO data always has an error associated with it (Lenzen, 2001a,b), in some cases the size of the errors are documented. Errors also arise in the adjustments for currency conversions, inflation, concordances, aggregation, and so on. In a multi-regional case specific data may not be available and needs to be estimated. The magnitude of these errors is often difficult to estimate, but the errors still need to be accounted for. Ideally, some sort of error analysis needs to be performed; such as Monte-Carlo analysis.

7 Conclusion

An important consideration in the determination of environmental impacts and factors embodied in trade is to account for the technology differences across regions. This article has developed a framework using IOA to explicitly determine the factors embodied in trade. In its general form, the model requires significant data collection. Several simplifications were suggested to reduce data requirements without introducing a large error. A companion article applies this framework to the case of Norway (Peters et al., 2004). Also, several technical issues in performing multiregional studies were discussed.

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A Constructing A^d , A^{cm} , and A^{ncm}

In this appendix we work through a hypothetical example for dealing with competitive and non-competitive imports through the make-use framework.

A.1 Given Data

Consider a hypothetical economy with three industries A, B, and C and five commodities α , β , γ , δ , and ϵ . ϵ is a non-competitive import. The economy does not import γ into industry. The make matrix is,

$$M = \begin{array}{c|ccc|c} & A & B & C & q \\ \hline \alpha & 156 & 24 & 0 & 180 \\ \beta & 9 & 80 & 0 & 89 \\ \gamma & 0 & 0 & 62 & 62 \\ \delta & 93 & 74 & 20 & 187 \\ \epsilon & 0 & 0 & 0 & 0 \\ \hline g & 258 & 178 & 82 & 518 \end{array} \quad (54)$$

The use of products produced domestically,

$$U^d = \begin{array}{c|ccc|c} & A & B & C & q \\ \hline \alpha & 10 & 20 & 5 & 25 \\ \beta & 20 & 18 & 6 & 44 \\ \gamma & 5 & 4 & 3 & 12 \\ \delta & 22 & 34 & 6 & 62 \\ \epsilon & 0 & 0 & 0 & 0 \end{array} \quad (55)$$

The use of competitive imports,

$$U^{cm} = \begin{array}{c|ccc|c} & A & B & C & q \\ \hline \alpha & 9 & 8 & 5 & 22 \\ \beta & 9 & 0 & 2 & 11 \\ \gamma & 0 & 0 & 0 & 0 \\ \delta & 7 & 12 & 2 & 21 \\ \epsilon & 0 & 0 & 0 & 0 \end{array} \quad (56)$$

The use of non-competitive imports,

$$U^{ncm} = \begin{array}{c|ccc|c} & A & B & C & q \\ \hline \alpha & 0 & 0 & 0 & 0 \\ \beta & 0 & 0 & 0 & 0 \\ \gamma & 0 & 0 & 0 & 0 \\ \delta & 0 & 0 & 0 & 0 \\ \epsilon & 4 & 3 & 2 & 9 \end{array} \quad (57)$$

The sum of the rows in the make matrix M is the total output of each product,

$$q^d = Mi = \begin{pmatrix} 180 \\ 89 \\ 65 \\ 187 \\ 0 \end{pmatrix} \quad (58)$$

The sum of the columns is the total output of each industry

$$g^d = M'i = \begin{pmatrix} 258 \\ 178 \\ 82 \end{pmatrix} \quad (59)$$

The total output of the domestic economy less the commodities used in domestic production gives the final demand by consumers on domestic production (see equation 4.5 in United Nations, 1999),

$$y^d = q^d - U^d i = \begin{pmatrix} 145 \\ 45 \\ 50 \\ 125 \\ 0 \end{pmatrix} \quad (60)$$

For this example, the import to final consumers is somewhat arbitrary

$$y^{cm} = \begin{pmatrix} 20 \\ 68 \\ 29 \\ 8 \\ 0 \end{pmatrix} \quad (61)$$

and

$$y^{ncm} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 67 \end{pmatrix} \quad (62)$$

The total imports is given by the final demand on imports, plus the imports used by industry,

$$q^{cm} = y^{cm} + U^{cm} i = \begin{pmatrix} 42 \\ 79 \\ 29 \\ 29 \\ 0 \end{pmatrix} \quad (63)$$

and

$$q^{ncm} = y^{ncm} + U^{ncm}_i = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 76 \end{pmatrix} \quad (64)$$

A.2 Constructing the share matrices

The market share matrix, $D = M'\hat{q}^{-1}$, describes which industries produce which products (it is an industry by product matrix). Each column must add to one. A complication arises in constructing the share matrix since some elements of the vector q^d may contain zeros due to non-competitive imports. If an element of q^d is zero then it means that no industry produces that particular commodity. Consequently, the corresponding column in the share matrix must be zero (note the transpose of rows and columns). That is, when constructing the share matrix, if an element of q^d is zero then the corresponding row of M is zeros and column of D is zeros. Thus

$$D = M'\hat{q}^{-1} = \begin{array}{c|ccccc} & \alpha & \beta & \gamma & \delta & \epsilon \\ \hline A & 0.8667 & 0.1011 & 0 & 0.4973 & 0 \\ B & 0.1333 & 0.8989 & 0 & 0.3957 & 0 \\ C & 0 & 0 & 1.0 & 0.1070 & 0 \end{array} \quad (65)$$

The use coefficient matrix, $B = U\hat{g}^{-1}$, represents the proportion of each commodity used by each industry. There is a matrix for both domestic use and the use of imports,

$$B^d = U^d\hat{g}^{-1} = \begin{array}{c|ccc} & A & B & C \\ \hline \alpha & 0.0388 & 0.1124 & 0.0610 \\ \beta & 0.0775 & 0.1011 & 0.0732 \\ \gamma & 0.0194 & 0.0225 & 0.0366 \\ \delta & 0.0853 & 0.1910 & 0.0732 \\ \epsilon & 0 & 0 & 0 \end{array} \quad (66)$$

$$B^{cm} = U^{cm}\hat{g}^{-1} = \begin{array}{c|ccc} & A & B & C \\ \hline \alpha & 0.0349 & 0.0449 & 0.0610 \\ \beta & 0.0349 & 0 & 0.0244 \\ \gamma & 0 & 0 & 0 \\ \delta & 0.0271 & 0.0674 & 0.0244 \\ \epsilon & 0 & 0 & 0 \end{array} \quad (67)$$

and

$$B^{ncm} = U^{ncm}\hat{g}^{-1} = \begin{array}{c|ccc} & A & B & C \\ \hline \alpha & 0 & 0 & 0 \\ \beta & 0 & 0 & 0 \\ \gamma & 0 & 0 & 0 \\ \delta & 0 & 0 & 0 \\ \epsilon & 0.0155 & 0.0169 & 0.0244 \end{array} \quad (68)$$

A.3 Industry Technology Assumption

The Industry Technology Assumption is used here. The Commodity Technology Assumption is not really an option since a method would be needed to convert the make and use matrices into square matrices. The advantages and disadvantages of each technology assumption are not discussed here, but it is noted that this is an active area of research (see Miller and Blair, 1985; United Nations, 1999).

A.3.1 Commodity-commodity level

The various interindustry requirements matrices can now be constructed. The commodity-commodity matrices are given by

$$A^d = B^d D = \begin{pmatrix} 0.0486 & 0.1049 & 0.0610 & 0.0703 & 0 \\ 0.0807 & 0.0987 & 0.0732 & 0.0864 & 0 \\ 0.0198 & 0.0222 & 0.0366 & 0.0224 & 0 \\ 0.0994 & 0.1803 & 0.0732 & 0.1258 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad (69)$$

$$A^{cm} = B^{cm} D = \begin{pmatrix} 0.0362 & 0.0439 & 0.0610 & 0.0417 & 0 \\ 0.0302 & 0.0035 & 0.0244 & 0.0200 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0.0325 & 0.0633 & 0.0244 & 0.0428 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad (70)$$

$$A^{ncm} = B^{ncm} D = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0.0157 & 0.0167 & 0.0244 & 0.0170 & 0 \end{pmatrix} \quad (71)$$

and the technology is given by

$$A = A^d + A^{cm} + A^{ncm} = \begin{pmatrix} 0.0848 & 0.1488 & 0.1220 & 0.1119 & 0 \\ 0.1109 & 0.1023 & 0.0976 & 0.1064 & 0 \\ 0.0198 & 0.0222 & 0.0366 & 0.0224 & 0 \\ 0.1319 & 0.2437 & 0.0976 & 0.1686 & 0 \\ 0.0157 & 0.0167 & 0.0244 & 0.0170 & 0 \end{pmatrix} \quad (72)$$

From these matrices the total output can be verified

$$x^d = (I - A^d)^{-1} y^d = \begin{pmatrix} 180 \\ 89 \\ 62 \\ 187 \\ 0 \end{pmatrix} \quad (73)$$

Alternatively,

$$x^d = (I - A)^{-1}(y^d + y^{cm} + y^{ncm} - q^{cm} - q^{ncm}) = \begin{pmatrix} 180 \\ 89 \\ 62 \\ 187 \\ 0 \end{pmatrix} \quad (74)$$

Both of these equations equate with the total output of the economy, q^d .

The total imports are given by

$$q^{cm} = y^{cm} + A^{cm}x^d \quad (75)$$

and

$$q^{ncm} = y^{ncm} + A^{ncm}x^d \quad (76)$$

Interestingly, it is possible to determine the amount of non-competitive imports at the commodity level. That is, $A^{ncm}x^d = q^{ncm} = (0, 0, 0, 0, 9)'$, also $A^{cm}x^d = q^{cm} = (22, 11, 0, 21, 0)'$.

A.3.2 Industry-industry level

The same matrices can be constructed at the industry-industry level,

$$A^d = DB^d = \begin{pmatrix} 0.0838 & 0.2026 & 0.0966 \\ 0.1086 & 0.1815 & 0.1029 \\ 0.0285 & 0.0429 & 0.0444 \end{pmatrix} \quad (77)$$

$$A^{cm} = DB^{cm} = \begin{pmatrix} 0.0473 & 0.0725 & 0.0674 \\ 0.0467 & 0.0327 & 0.0397 \\ 0.0029 & 0.0072 & 0.0026 \end{pmatrix} \quad (78)$$

$$A^{ncm} = DB^{ncm} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (79)$$

Note that this will always be zero at the industry level. The technology is given by

$$A = A^d + A^{cm} + A^{ncm} = \begin{pmatrix} 0.1311 & 0.2751 & 0.1641 \\ 0.1553 & 0.2141 & 0.1426 \\ 0.0314 & 0.0501 & 0.0470 \end{pmatrix} \quad (80)$$

From these matrices the total output can be verified

$$x^d = (I - A^d)^{-1}(Dy^d) = \begin{pmatrix} 258 \\ 178 \\ 88 \end{pmatrix} \quad (81)$$

Alternatively,

$$x^d = (I - A)^{-1}(Dy^d + Dy^{cm} + Dy^{ncm} - Dq^{cm} - Dq^{ncm}) = \begin{pmatrix} 180 \\ 89 \\ 62 \\ 187 \\ 0 \end{pmatrix} \quad (82)$$

Both of these equations equate with the total output of the economy at the industry level, g^d .

A.4 Discussion

The reason for the example calculations is to see what happens to the non-competitive imports in the make-use framework through the manipulations to construct the interindustry matrices. At the industry level, A^{ncm} is always zero, implying that industries do not use the competitive import, when in fact they do. The information is lost through converting the data from the commodity to industry level. This is since it is implicitly assumed through the Make matrix that the imports are made with the same technology as the domestic economy. Since the domestic economy does not make non-competitive imports, then this is always zero.

In contrast, the information on non-competitive imports is not lost at the commodity level. This is easy to explain. The balance of commodities states (e.g. United Nations, 1999, eqn 4.5),

$$q = y + Bg \quad (83)$$

where $q = q^d + q^{cm} + q^{ncm}$, $B = B^d + B^{cm} + B^{ncm}$, and $y = y^d + y^{cm} + y^{ncm}$. Substituting the make relationship, $g = Dq$, into this gives,

$$q = y + B(Dq) \quad (84)$$

This gives the data at the commodity-commodity level. Surprisingly, this last step does not assume that the imported goods are made with domestic technologies. The use coefficient matrices, B , tell us what proportion of the vector on its right is produced domestically, competitive import, or non-competitive import.

However, when converting to the industry level, (83) is pre-multiplied by D ,

$$g = Dq = Dy + DBg \quad (85)$$

It is in this step that it is assumed that the imports are made with domestic technology since all the vectors are multiplied by the make share matrix. Thus, at the commodity-commodity level details of competitive imports are not lost, it is the process of going to the industry level that the information is lost.

B Further analysis on imports

To further highlight how imports work in the IOA framework it is worth considering the interpretation of A , A^d , and A^{im} in more detail. Consider two cases: imports to final demand and imports to intermediate production.

First, consider the case of imports to final demand only, that is, the matrix $A^{im} = 0$. Thus (10) becomes

$$x = A^d x + y^d + y^{ex} + \underbrace{y^{im} - M}_{=0} = A^d x + y^d + y^{ex} \quad (86)$$

As the imports increase, y^d must decrease to satisfy the given domestic demand, y^f . That is, imports cause a drop in total output because the product is no longer produced domestically.

Second, consider the case of all imports going to intermediate production processes, that is, $y^{im} = 0$. Rewriting (10) gives

$$x = Ax + y^d + y^{ex} - M \quad (87)$$

Since $A = A^d + A^{im}$ then this equation becomes

$$x = A^d x + y^d + y^{ex} + \underbrace{A^{im} x - M}_{=0} = A^d x + y^d + y^{ex} \quad (88)$$

It should be clear that the total interindustry production coefficients, A , must remain constant for the given time period as A represents the technology of the economy. Thus, for an increase in A^{im} there is an equal decrease in A^d . Thus, as imports increase, A^d must decrease since A is constant.

Thus for the case of all imports going to intermediate demand, imports still has the net effect of reducing total output. It should not be hard to see that by considering the case where the imports go to both intermediate demand and to final demand the same results hold. Imports do not effect the output of an economy for the static framework considered in input-output analysis.

By combining the above arguments the net effect of imports is to reduce total output, since A and y^f are constant, that is,

$$x = Ax + y^f + y^{ex} - M \quad (89)$$

where M is the total of the imports going to both intermediate production and final demand.

C Outputs in terms of A

Equations (14), (15), and (16) can be rewritten using A instead of A^d . This has advantages if one wants to assume that a foreign economy has the same technology as the domestic one. The technologies of the different economies are related through the A not A^d .

The balance equation given by (10) can be rewritten as

$$x = Ax + y^d + y^{im} + y^{ex} - m \quad (90)$$

So the total output of the economy is also given by

$$x = (I - A)^{-1}(y^d + y^{im} + y^{ex} - m) \quad (91)$$

The output is the same as in (13). Note that the import, m , must remain in the equation.

To determine the domestic output required for an arbitrary demand, y^* , is more complex. Rewrite (16) as

$$x^* - A^d x^* = y^* \quad (92)$$

Then add the proportion of imports to both sides

$$x^* - A^d x^* - A^{im} x^* = y^* - A^{im} x^* \quad (93)$$

Simplifying,

$$(I - A)x^* = y^* - A^{im} x^* \quad (94)$$

Thus,

$$x^* = (I - A)^{-1}(y^* - A^{im} x^*) \quad (95)$$

It is noted, that the decomposition is much more complex than before. In fact, it is not possible to determine the value of x^* without rewriting it in the form of (16) since it occurs on both sides of the above equation.

These equations show that the use of A to determine the outputs of a given economy is not practical, unless determining total output. $A = A^d + A^{im}$ contains information about imports, and this information needs to be subtracted away from the final demands so that the output does not include the contributions from imports. To determine outputs in the domestic economy it is most practical to use A^d . A^{im} can be used to determine the share of imports used in domestic production. A is used if it is the only data available or if working with total outputs.

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