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Strategic Location of Heterogeneous Resources for the Fire and Rescue Service

Emilie Krutnes Engen
Jonas Gamme Nouri

Industrial Economics and Technology Management
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Supervisor: Henrik Andersson, IØT

Norwegian University of Science and Technology
Department of Industrial Economics and Technology Management

Preface

This master thesis has been prepared during the spring 2016 at the Norwegian University of Science and Technology, Department of Industrial Economics and Technology Management. The thesis studies emergency response planning (ERP) and unlike most of the existing literature in operational research, it focuses on the fire and rescue service (FRS). Optimization methods are used to quantitatively evaluate FRS performance and demonstrate the applicability of a mathematical approach as a strategic and tactical decision support tool. The model developed in this master thesis has been applied to a real life fire and rescue situation in Oslo, through the assistance of the local FRS capacity, Oslo Brann- og Redningstjeneste (BRE).

We thank our supervisor, Professor Henrik Andersson, for excellent guidance in all aspects of the project. Your assistance has been invaluable for the final result. We also extend our sincerest thanks to our co-supervisor, Associate Professor Tobias Andersson Granberg at Linköping University (LiU), for sharing his extensive expertise and experience in the field.

Furthermore, we would like to thank the representatives from BRE, Fredrik Frøland, Viggo Stavnes, and Anne Kløw, for providing insight into the FRS. Their perspective have been crucial in giving applicability to the mathematical approach of the project.

Abstract

The fire and rescue service (FRS) in Norway is responsible for responding to a number of different emergency calls. Keeping an adequate level of preparedness is essential in order to respond quickly and efficiently to emergency fire and rescue calls. This requires extensive planning at the strategic, tactical, and operational levels. Operational research and optimization models are important support tools for decision makers in emergency response planning (ERP). However the previous research on FRS is limited, compared to emergency medical service (EMS), which has been at the interest of operational researchers since the 1960's.

When considering the FRS, the preservation of life and property is the main objective. The problem studied in this report is the Maximum Preservation Location Problem for Heterogeneous Accidents and Response Units (MPLP-HAR). It aims at finding the optimal location for the FRS resources in an area that gives the highest level of preparedness, in order to respond to a demand for heterogeneous accidents. To solve the problem, an integer programming model has been developed. The objective function uses a preservation function that captures the distinct ability to preserve life and property in the event of an emergency. Hence, the preservation function reflects the level of preparedness.

The model distinguishes between first and total response time. In this case, these response times refer to the time it takes for the FRS to initiate and fully provide emergency relief at the accident site, respectively. Response units respond to different accident types with different resource requirements. Demand for a given accident type is determined based on historical call data. The results show that the model can be used as a support tool for various types of location decisions at the strategic and tactical levels.

The computational study includes both a technical and an economical part. The technical study determines the appropriate formulation and solution approach for the problem. The economical studies is performed on the region of Oslo, in corporation with Oslo Brann- og Redningstjeneste (BRE). After validation, the model is able to provide quantifiable measures of preparedness in the region and evaluate the current operational state. Furthermore, the

MPLP-HAR is used to investigate a number of alternative resource locations and operational structures that could improve the level of preparedness. The MPLP-HAR is a strategic and tactical location decision support tool and its applicability increases with data accuracy.

Sammendrag

I Norge er Brann- og redningstjenesten den institusjonen med ansvar for beredskap ved brann og akutte ulykker. Det er kommunen selv som sørger for etablering og drift av et lokalt brannvesen. Brannvesenet har ansvar for å utføre forebyggende og beredskapsmessige oppgaver på en effektiv og sikker måte. Dette krever grundig planlegging på et strategisk, taktisk, og operasjonelt nivå. Operasjonsanalyse og optimeringsmodeller er viktige verktøy for beslutningstakere innenfor beredskapsplanlegging i nødetatene. Litteraturen innenfor operasjonsanalyse tyder riktignok på at brann- og redningstjenesten er blitt viet mindre oppmerksomhet enn akuttmedisinske tjenester, som har vært av interesse for fagfolk siden midten av 1960-tallet.

Brann og redningstjenesten har som formål å verne liv, helse, miljø, og materielle verdier. Det studerte problemet i denne rapporten er "the Maximum Preservation Location Problem for Heterogeneous Accidents and Response Units" (MPLP-HAR). Modellen tar sikte på å finne den optimale ressurslokasjonen for brann- og redningstjenesten som gir den bevaringsevnen og beredskapen som skal til for å håndtere en etterspørsel for ulike ulykkestyper. En lineær blandet heltallsmodell er utviklet for å løse problemet. Objektivfunksjonen inneholder en bevaringsfunksjon som gjenspeiler evnen til å verne om liv, helse, miljø, og materielle verdier, når en nødsituasjon inntreffer. Bevaringsevnen reflekterer dermed beredskapen.

Modellen skiller mellom første og total responstid. Første responstid indikerer hvor lang tid brannvesenet bruker på å initiere en redningsinnsats på skadestedet. Total respons er oppnådd når alle påkrevde ressurser er tilgjengelige på skadestedet. Hver enkelt ulykkestype krever ulike redningskjøretøy med ulike kapasiteter. Ulykkesraten for hver ulykkestype, som utgjør etterspørselen, bestemmes av historiske data. Resultatene viser at modellen kan anvendes som et støttende verktøy for strategiske og taktiske beslutninger.

De matematiske studiene er delt inn i en teknisk og en økonomisk del. Modellens ønskede formulering og løsningsmetode fastsettes i den tekniske delen, og deretter er et økonomisk studie gjennomført i samarbeid med Oslo Brann- og Redningstjeneste (BRE). Etter en valid-

eringsundersøkelse av det kvantitative beredskapsmålet, anvendes modellen til å evaluere den nåværende operasjonelle situasjonen i Oslo. Deretter benyttes MPLP-HAR til å undersøke en rekke alternative beredskapsplaner og ressursallokeringer, foreslått av BRE, og dermed gi innsikt i deres påvirkning på beredskapen i området. Modellen er et verktøy for å støtte opp under strategiske og taktiske beslutninger og anvendbarheten styrkes i takt med kvaliteten på datagrunnlaget.

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1 Introduction

The first evidence of organized firefighting originates from ancient Rome. A number of serious fires in the city lead them to see the potential benefits of combating the flames to avoid the loss of life and property. However, it was not until the urbanization of Europe, during the industrial revolution in the 17^{th}, 18^{th}, and 19^{th} centuries, that an organized fire brigade became common in the major cities. Initially, the development of firefighting equipment, in particular the water pump, was important for the effectiveness of the brigades at the time. It quickly became apparent that a fire develops rapidly in built-up areas. Therefore, initializing immediate firefighting is crucial to succeed in saving life and property. The introduction and development of the fire engine has been a crucial part of the history of firefighting. From being horse-drawn steam engines that squirted limited amounts of water at close range, fire engines have become the iconic red trucks that are recognized all over the world today.

From the initial focus on technological advances and developing equipment, the modern development of the fire and rescue service (FRS) is particularly concerned with effectively responding to emergency calls and issues regarding the location of resources to areas where they can be used most efficiently. For a society, providing a FRS becomes a socioeconomic trade-off between costs of establishment and operation of the service and citizen safety and security.

The modern FRS is the organization responsible for providing emergency firefighting and rescue services, whenever life and property is in danger. Rescue services include a variety of tasks such as traffic accidents, seaborne- and height rescue, and incidents involving pollution, biological- and chemical hazards. The FRS is also part of keeping a societal disaster preparedness level in the event of for example natural disasters or acts of terrorism. Another important aspect of FRS is fire prevention, in order to reduce the risk and consequence of fires. The FRS represents one of the three main emergency services, along with the emergency medical service (EMS) and the police service. Although they have their distinct areas of responsibility, cooperation and intertwining responses are common.

In Norway, the municipality is responsible for establishing and operating an adequate FRS (Lovdata, 2015), but often several smaller municipalities cooperate in providing a common service. The organization consists of one or multiple fire stations, containing a fleet of vehicles and staffed by full- or part-time employees. Today there are 335 FRS institutions and totally 12 500 fire service employees in operational and administrative positions. However, only 3000 are full-time employees (DSB, 2016), meaning that FRS in Norway is heavily reliant on part-time employees. The size, diversity, and duties of the FRS vary across the country, depending on the distinctive nature of the jurisdiction.

Static planning and resource management have been traditional characteristics of emergency response planning (ERP) in FRS (Ulander, 2015). Emergencies are reported to a dispatching central that allocates emergency resources according to pre-determined criteria. Fixed teams of firefighters are assigned to fire engines, located at the fire station, waiting to be dispatched. In Sweden, however, dividing firefighters into smaller operational groups has gained ground as a new way of planning (Granberg et al., 2015). The smaller units can be used for preventive work or be strategically located in high risk areas for given time periods to maintain a high level of preparedness. In addition, having smaller teams spread across a larger geographical area reduces the average travel time to an accident site for the first team to be dispatched.

Preparedness is a way of organizing the FRS to ensure that correctly equipped personnel is predisposed to respond effectively to emergencies. The level of preparedness is a relevant performance measure for all ERP institutions. In FRS, the preparedness in a given region depends on the time it takes for the right resources to perform emergency relief to a set of stochastic accidents that might occur. Good preparedness is thus characterized by an effective response where expected demand is high (Granberg et al., 2015).

In FRS, the objective is to minimize the damage done to life and property in the event of an emergency and take sufficient measures to prevent them from happening in the first place. The basis for their mandate and responsibility comes from the Fire and Explosion Prevention Act (Brann- og Eksplosjonsvernloven) issued by the Ministry of Justice (Justis- og Beredskapsdepartementet) and adhered by the Directorate for Civil Protection and Emergency

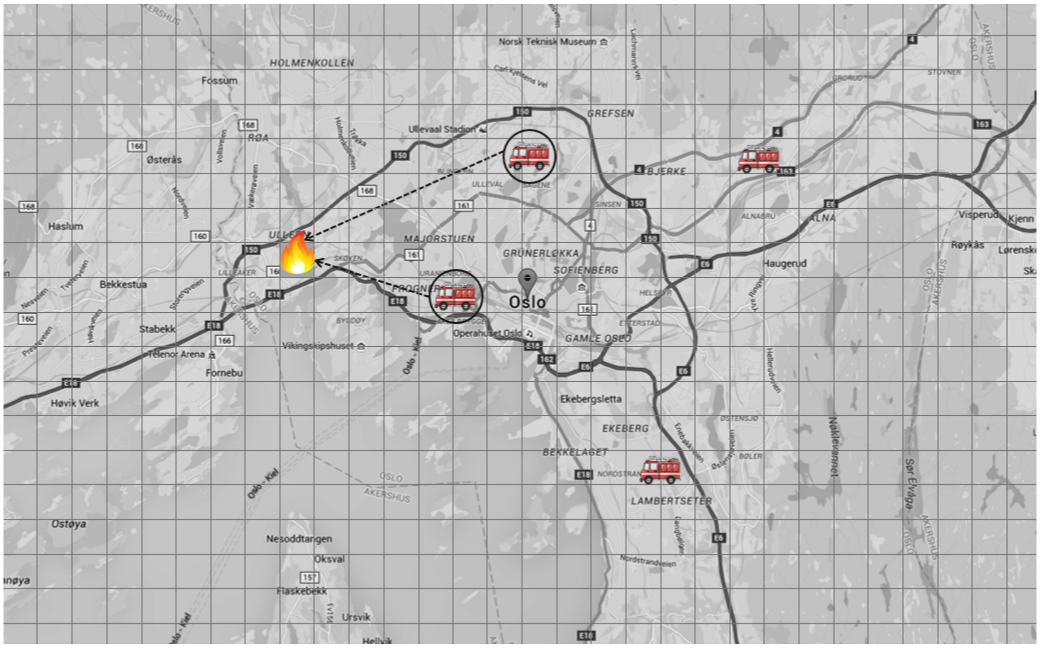

Figure 1: Allocation of first and second response unit to an accident site

(Direktoratet for Samfunnssikkerhet og Beredskap). In addition to the statuary responsibilities, each municipality devise their own risk analysis to identify the distinctive characteristics of the jurisdiction. Considerations here include geographic and demographic characteristics as well as infrastructure and organizational structure that could reveal distinctive objectives. The statuary responsibilities of the respective FRS institutions are seen as a minimum standard of service, and often their individual ambition exceeds the centrally determined specifications.

When relating effective response to the FRS objectives, it is clear that the ability to preserve life and property in the event of an emergency characterizes preparedness. Having defined preparedness as being the general performance measure of ERP, this report defines a preservation function as part of the performance measure for FRS. The preservation is a function of the response time and type of accident. Response time is defined as the time it takes from the emergency is reported to the FRS response units are ready to perform emergency relief at the site of the accident. The first response refers to the time it takes for the first unit to respond and the total response is the time it takes until all required units are ready to operate at the accident site. The decreasing ability to preserve life and property with time is distinctively different for the different types of accidents. Generally, a fire develops exponentially in severity and the ability to preserve life and property upon arrival therefore show an exponential decrease with increased response time.

Keeping an adequate level of preparedness requires extensive planning on different levels, characterized by their perspectives and objectives. Firstly, there is strategic planning, concerning long term perspectives. Typically, strategic decisions involve the development of infrastructure and siting of fire stations and resources. Next, there is tactical planning, involving decisions on how to efficiently utilise the existing network over a mid- to long-term planning horizon in order to achieve strategic objectives. At the short term level, there is operational planning, where the day-to-day operational decisions are made.

Traditionally, strategic and to some extent tactical planning, have been devoted a lot of attention from operational researchers (Ulander, 2015), while operational decisions have mainly been left to decision makers in FRS. The distinction between the levels are related to planning horizon, organizational involvement, and input parameters. The strategic and tactical levels typically use the simplification of static planning. This means that all inputs are considered constant over the course of the planning process. Problems and decisions at this level are therefore suitable for scientific modelling and analysis. Operational decisions are often viewed as dynamic planning problems that require real-time information and are therefore left to FRS professionals that use experience and knowledge to make qualified decisions. This report is concerned with static planning at the strategic and tactical levels, where solutions with different characteristics are easily compared, given a static objective.

The focus is to demonstrate how an optimization approach can be used to develop a decision support tool for decision makers by solving the Maximal Preservation Location Problem for Heterogeneous Accidents and Response Units (MPLP-HAR). Therefore, much emphasis is devoted to capture the characteristics of the real-life problem in the modelling phase and to develop a realistic, yet applicable decision support tool.

Outline This report first presents a review of relevant literature on ERP in general, with a specific focus on FRS. This is to give the reader insight into the field and gain experience from previous studies. After thoroughly reviewing existing literature on the field, the report turns to modelling and solving the problem at hand. Firstly, preparedness is defined and a performance measure is established. The key to providing a useful planning tool for decision makers in the FRS, is to quantify preparedness. A methodology for doing so is presented.

Next, the existing literature on related location problems is examined mathematically. Although they may not have been initially intended to model problems in ERP, location problems have been essential in resource allocation planning in emergency services. These models have also been the inspiration for the MPLP-HAR. The mathematical model is motivated and formulated in the subsequent section. The main objective is to determine the optimal location for FRS resources that maximizes the level of preparedness in the area.

A technical study is then conducted to determine the desirable formulation and solution approach for the MPLP-HAR. Next, a case study of an actual FRS organization is introduced. The current operational state of Brann- og Redningsetaten in Oslo (BRE) and the requirements and opportunities it is facing, are examined. The distinct characteristics of BRE form the basis for the modelling choices and input data in the problem.

The qualitative data and the method for obtaining them are presented next, before they are applied in a computational study. Model parameters are determined through trials. The MPLP-HAR is then used to evaluate the current operational state of BRE and investigate a number of scenarios. Finally, the findings of the computational study are discussed and the applicability of the model is evaluated. Lastly, the report suggests aspects and extensions for future studies.

2 Literature

This section presents relevant literature on the topic of emergency response planning (ERP) in general and fire and rescue service (FRS) in particular. The purpose is to give the reader key background information on existing operational research on the field and insights to previous written material used by the authors of this report. The section starts by defining ERP and looks into its importance and application in different institutions.

2.1 Emergency Response Planning

Emergency Response Planning (ERP) is the field of operational research concerned with finding the optimal allocation of emergency response resources. The overall objective is to prevent the loss of lives and minimize the damage done to property, society, and surroundings in the event of an emergency. It is performed by municipal and corporate institutions alike and is imposed by regulations at a number of governmental levels and also influenced by non-regulatory considerations. At the municipality level, the emergency medical service (EMS), the fire and rescue service (FRS), and the police service are institutions that are set to have a level of preparedness to respond to emergencies in their jurisdiction. For these institutions, ERP has become an increasingly complex task (Erickson, 1999). Continuously compounding social, technical, and political developments contribute to the operational complexity of maintaining an adequate level of preparedness.

In the context of ERP, resource allocation is a plan for assigning available resources to achieve one or several objectives. Dispatching and reallocation are issues that regard what resources to dispatch to a given request and how to reallocate previously allocated resources to satisfy the new request. Furthermore, ERP requires modeling of demand, travel times, and defining performance measures and objectives. The problems in ERP are addressed for both long-term and short-term perspectives. The planning is traditionally divided into strategic, tactical, and operational planning. In addition, the problems are separated into static and dynamic planning, where static modeling assumes deterministic information and dynamic models account for parameter changes over the course of the planning period. These aspects are discussed in greater detail in this section.

According to a survey of relevant studies in Ulander (2015), the majority of the articles published are concerned with strategic and tactical planning. This also implies a focus on static planning in the operational research community on the field. Facility and resource location is the focus of most of the researched problems. In optimization, the mathematical problems are categorized as either facility location, assignment, or covering problems. These are usually formulated as integer programming problems and often they use binary formulations for the decision variables. However, many of these formulations are adaptable to operational planning through dynamic programming with continuously updated input variables.

2.2 Emergency Services

The previous literature has focused on ERP in the three main institutions that are making up the traditional emergency services; emergency medical service, the police service, and fire and rescue service. Of the three, EMS has particularly been devoted attention from operational researchers and ambulance allocation and dispatching is a widely studied problem (Ulander, 2015).

EMS is the provision of out-of-hospital immediate medical care and transportation of patients to hospitals (Ingolfsson, 2013). EMS has come a long way since the first ambulance was developed in the early 19^{th} century. Today, EMS planning is a challenging process due to the variability in frequency, location, and severity of incidents. Research in the field has received more attention over the last decades, with an increasing number of publications. Modern technology and availability of data have made it possible to model and forecast expected demand and predict response times more accurately. This means one is able to perform EMS planning and management in a more efficient manner (Ingolfsson, 2013).

The police is the emergency service responsible for responding to any incident of law violation or disturbance of the peace in public. This includes a vast variety of calls, but most incidents

Table 1: Literature survey concerning strategic planning (Ulander, 2015)

Strategic planning level		
Static planning	Dynamic planning	Problem
Hakimi (1965)		Switching station distribution
Hogg (1968)		Station location
Toregas et al. (1971)		Emergency facilities location
Schreuder (1981)		Fire station location
Church and ReVelle (1974)		Facility location
Schreuder (1981)		Fire station location
Badri et al. (1998)		Fire station location
Batta and Mannur (1990)		Fire station location
Marianov and ReVelle (1992)		Response team location
Schilling et al. (1979)		Resource location
ReVelle and Snyder (1995)		Facility location
Kolesar and Blum (1973)		Correlation problem
Erkut et al. (2008)		Facility location
Erkut et al. (2009)		Facility location
Beraldi and Bruni (2009)		Location and allocation
Leyva (2011)		Facility location
Coskun and Erol (2010)		Location and allocation
Chevalier et al. (2012)		Location and allocation
Halpern (1979)		Correlation problem

trigger the same response from one response unit. In the police service, the location of the majority of police patrols at any given time is usually determined by the patrol sectors, rather than by stationary facilities (Larson, 1974). The vehicles are assigned a sector of the jurisdiction to patrol and the statistical location depends on the relative amount of time they spend in the various parts of the sector. Ensuring adequate preparedness then becomes a problem of finding an efficient distribution of police patrol sectors (Curtin et al., 2010).

Table 2: Literature survey concerning tactical planning (Ulander, 2015)

Tactical planning level		
Static planning	Dynamic planning	Problem
Curtin et al. (2010)		Patrol area distribution
Beraldi and Bruni (2009)		Location and allocation
Coskun and Erol (2010)		Location and allocation
Chevalier et al. (2012)		Location and allocation
Schilling et al. (1979)		Resource location
	Zaki et al. (1997)	Location and allocation
	Andersson and Särdqvist (2007)	Location and allocation
	Dahlgren et al. (2009)	Location and allocation
	Schmid and Doerner (2010)	Location and allocation
Geroliminis et al. (2009)		Resource location
Yang et al. (2004)		Districting problem
Carter et al. (1972)		Districting problem
Church et al. (2001)		Staffing problem

Compared with the other emergency services, FRS predisposes relatively few response units that on average face a low, but varied demand (Ulander, 2015). However, an emergency usually results in a long engagement involving a considerable amount of resources. In addition to emergency calls, the FRS also engages in fire preventive work and other preplanned engagements, such as inspections, maintenance, and education. Response units and staff are traditionally fixed to the fire stations, thus facility location is an important aspect of keeping a high level of preparedness.

2.3 Operational Research in the FRS

When looking into operational research and mathematical modelling for the FRS, one finds that resource management and allocation decisions are made on several different levels. These

Table 3: Literature survey concerning operational planning (Ulander, 2015)

Operational planning level		
Static planning	Dynamic planning	Problem
	Ulander (2015)	Resource reallocation
	Andersson and Värbrand (2007)	Resource dispatching
	Swersey (1982)	Resource dispatching
	Ignall et al. (1982)	Resource dispatching
	Haghani and Yang (2007)	Resource dispatching
	Kolesar and Walker (1974)	Resource reallocation
	Dimopoulou and Giannikos (2004)	Structuring problem
	Ng and Raff (2008)	Structuring problem
Larson (1974)		Vehicle location
Yang et al. (2004)		Districting problem
Carter et al. (1972)		Districting problem

are presented in greater detail. The way parameters are perceived in planning is also examined towards the end of this section.

2.3.1 Strategic Planning

Strategic planning is concerned with long-term decisions that normally engage large parts of the organization (Ulander, 2015). The financial impacts are usually significant as these planning decisions involve large investments in infrastructure and network. Decisions include facility construction, resource investment, service area determination, and standardizations. The long-term planning horizon and financial impacts of these decisions have made strategic planning a very popular field for operational researchers and a number of different aspects have been the subject of optimization studies.

The siting of fire stations is an example of a strategic decision that has been thoroughly studied. Different approaches have been used to solve this problem, such as the p-median approach. The p-median was originally formulated as a problem were the objective is to find the optimal distribution of a set of switching centers that all flows in the network must pass through. The total length of flow to all nodes is minimized (Hakimi, 1965). In other words, resources from a number of switching centers must reach all nodes in the network at the lowest possible total cost. Hakimi (1965) examines the case were the police service is set to distribute a minimum number of policemen in a highway network, such that no one is farther away from a policeman than a distance d. Applied to the context of FRS, the switching centers represent fire stations and the objective is to minimize the travel distance or time it takes to reach all demand zones in the jurisdiction. In the real world, decision makers have a set of feasible sitings of fire stations. Finding the optimal siting allocation of a given number of fire stations, using a formulation of the p-median problem, was presented by Hogg (1968).

The use of the distance d as maximum distance to a switching center for any node in the network means that the p-median problem can easily be extended to a formulation in which it finds the distribution for the minimum number of switching centers needed to provide a given coverage level for the network. The Location Set Covering Model (LSCM) was formulated by Toregas et al. (1971), primarily intended for the citing of emergency services. The upper limit, d, can represent the distance or response time from the emergency service to a node in the network. The formulation is particularly suitable for problems where the emergency service has a statutory responsibility for a certain response time. Such is usually the case with FRS. The p-median problem turns towards having a long service time in areas where the demand is low. When incorporating a cost for the different citing candidates, the problem indicates the minimum-cost location of fire stations. A relevant extension of the LSCM is to introduce the possibility of dual coverage (Schreuder, 1981). This means that certain high risk areas can be covered by more than one facility, thus reducing response time when resources from more than one facility is needed or in instances where some given resources are unavailable.

The number of available facilities and resources are however not always sufficient to cover the demand in all nodes in the network. The Maximal Covering Location Problem (MCLP) takes this into account and seeks to maximize the population covered by a given service standard, i.e. within a threshold time or travel distance (Church and ReVelle, 1974). An extension of this is presented in Schilling et al. (1980), where both population and property are taken into account. The article presents a formulation that incorporate different types of vehicles, crews, accident calls, and multiple objectives. Although reaching the demand as quickly as possible is the most common strategic objective in FRS, multi-objective models have also been developed (Badri et al., 1998). In addition to travel times, facility location can be evaluated on a number of tangible and intangible criteria, for example with technical, political, and economic objectives. Because covering location models seek to cover the maximum population possible, no further considerations are made for the population that is not covered. Additional constraints can however be implemented to ensure that even though parts of the population are not covered by a threshold service standard, it is still favourable for them to live as close to a service station as possible.

Recognizing that responding to a fire accident usually requires the efforts from multiple response units, with a variety of resources and competencies, requires optimization models accordingly. Batta and Mannur (1990) developed a covering model, based on the LSCM and MCLP, were multiple response units are needed at an emergency site. Another extension, presented by Marianov and ReVelle (1992), defines response teams, consisting of different types of response units. These can then be strategically located to achieve an improved coverage standard. As a more applicable location decision model, Schilling et al. (1979) extend the location decision to include also the distribution of specialized equipment and/or manpower among the service sites. Introducing different types of resources, including a heterogeneous fleet of response units, allows decision makers to specify a service standard of required units. ReVelle and Snyder (1995) introduced the possibility of combining response units from different emergency services, by studying an optimization problem in which FRS units and ambulances are located within the same facilities.

The average response distance in an area is inversely proportional to the square root of the number of locations where FRS response units are available to respond to incidents (Kolesar and Blum, 1973). Given resource constraints, the square-root function can be used to find an optimal resource allocation or describe response time consequences of a given allocation.

Compared to EMS, FRS is considered to have a lower resource utilization rate overall (Ulander, 2015). That is, emergency calls are relatively scarce. A valid assumption for earlier covering location models have therefore been that all resources are available at all times. However, engagements for an emergency call have a considerably longer duration. Consequently, modern FRS institutions are taking the possibility of multiple simultaneous accidents into account (BRE, 2015), meaning that certain response units might be busy when a new emergency is reported. The busy probability for EMS response units were modeled in the Maximum Expected Survival Location Problem (MEXSLP) by Erkut et al. (2008).

Another probabilistic model for determining the optimal location of facilities in congested emergency systems was formulated by Beraldi and Bruni (2009). The authors argue that the busy probability is closely related to the number of emergency calls and therefore choose to model uncertainty in demand directly. Another source of uncertainty is the travel time uncertainty. This can arise from either traffic and congestion or driving conditions, meaning that different times of the day, weather conditions, or seasons might effect the travel time. In turn, this means uncertainty in response times for the emergency services. In an extension of the MEXSLP, the travel time uncertainty is modeled in an ambulance location context (Erkut et al., 2008). However, the improvements to the solution quality of incorporating travel time uncertainty in the model are less significant than including a busy probability (Erkut et al., 2009).

The significant financial impact of strategic planning makes cost an important planning aspect, which has been studied more extensively in recent years. Factors contributing to the total cost of the system are expenses related to research and forecasting, infrastructure build-up, and resource acquirement (Leyva, 2011). Coskun and Erol (2010) developed an optimal location model for EMS where the objective is to minimize the cost of the system. The efficient utilization of FRS resources were examined by Chevalier et al. (2012). They assumed a given service standard and attempted to minimize the set-up costs of the required facilities. The problem could be solved at both the strategic and tactical levels as staff and equipment were included in the allocation plan along with facilities. In a broader strategic planning perspective, one also consider cost effectiveness in relation to preventive work. Halpern (1979) argues that it is occasionally more cost efficient to install detection alarm systems in order to improve the fire protection in an area, rather than investing in additional fire stations. In addition to detecting a fire at an early stage, preventive systems can automatically provide accurate information of the exact position of the fire site.

2.3.2 Tactical Planning

The tactical planning level is characterized by its mid- to long planning horizon and involves decisions that seek to efficiently utilise the existing network in order to achieve strategic objectives. Planning periods ranges typically from weeks to months and years. The planning is usually done periodically and includes personnel planning, equipment management, resource utilization and allocation, and at-the-site performance. Financial impacts of tactical planning amounts from acquiring new resources and equipment, fleet size changes, route diversions, and other changes in day-to-day operations (Ulander, 2015). Tactical planning is related to strategic planning in that the located facilities need to be adequately equipped in order to serve their purpose effectively. Combining the two planning levels can be done by modelling both the strategic facility location and determining what resources to locate at these facilities (Beraldi and Bruni, 2009, Coskun and Erol, 2010, Chevalier et al., 2012, Schilling et al., 1979).

The distribution of resources between facilities, such as response units between fire stations, is a good example of a tactical planning decision, as opposed to a strategic planning decision of locating the facilities themselves. Resource dispatching decisions and response priorities (Zaki et al., 1997), as well as relocation of resources to temporal changes in demand (Andersson and Särdqvist, 2007) are examples of studied problems in tactical planning. By specifying required resources to any type of accidents, such as traffic accidents or residential fires, Andersson and Särdqvist (2007) proposed a model that specify the location of each unit, rather than having each station responding to the accidents. The approach can be evaluated on various problems and objective functions, such as the p-median or the MCLP.

From the traditional stationary resource allocation, i.e. that response units and crews are confined to fire stations at all times, the use of smaller, non-stationary units have become a popular problem to study. These non-stationary units are not confined to any specific location, but can move freely in order to facilitate and support the tactical planning process (Dahlgren et al., 2009). In addition to reducing the first response time by having smaller units positioned near high risk areas, the use of non-stationary units enables a more effective use of service personnel. Smaller units can perform for example preventive work inspections, instead of having everyone in the traditional 4-5 member crew perform the task.

Uncertainty in ERP is also considered at the tactical planning level. Schmid and Doerner (2010) considered the location and allocation of EMS units that are subject to changes in travel time. As discussed above in Section 2.3.1, considering the busy probability of response units is of importance for tactical planning as well (Geroliminis et al., 2009). A consequence of having certain response units engaged in ongoing emergencies is that the service level is reduced for certain types of accidents. Differentiating accident characteristics with respect to resource requirements enables the model to have an accurate description of the preparedness level for each accident type.

The dispatching decision is one of the most important aspects of tactical planning. Deciding what resources to send to a given incident is often done by assigning response areas (Ulander, 2015). Specific fire stations and resources are assigned areas for which they have a given responsibility in case of an emergency. Response areas may vary with geography, demography, or resource availability at any given time. The assignment of an area to one fire station is coordinated with the other stations in the organization. Minimizing response time and balancing the workloads by assigning response areas for fire stations is an important part of tactical planning (Yang et al., 2004). Response areas could also be assigned to units rather that stations, which would be of particular interest in combination with the use of non-stationary units (Carter et al., 1972).

Cost of manpower is the single largest cost factor in providing emergency services (Church et al., 2001), and is therefore also one of the most central elements in efficient service provision. Church et al. (2001) discusses how to determine the optimal combination of geographical staffing and equipment to efficiently meet the demand. A number of complicating issues such as work hour restrictions, geographical considerations, and types of shifts, makes personnel planning a complex problem to optimally determine.

2.3.3 Operational Planning

Finally, day-to-day operational, real-time decisions are the subject of operational planning. For the FRS, dispatching of response units and personnel are typical operational decisions. Planning at the operational level is required when there are very short-term variations in demand, typically with a time range of days, hours, or minutes. Costs associated with this type of planning are usually insignificant compared to the other planning levels and smaller parts of the organization are involved in the decision process. However, because the planning horizon is short, detailed, real-time information and frequent updates are required to make good decisions.

In FRS, operational planning consists of allocation, dispatching, and re-allocation decisions, based on the current operational state. By accessing real-time information, dispatchers are able to use the tactical planning models for response area assignments in operational planning as well (Yang et al., 2004, Carter et al., 1972). This form of dynamic resource reallocation to changing situations is of particular interest in instances when response units from the fire brigade are dispatched to an emergency call and parts of the jurisdiction suffers from a degradation in preparedness. An early model was developed by Kolesar and Walker (1974), that evaluates and suggests reallocation alternatives. The expected duration of the emergency, the expected response time to new incoming alarms, the risk situation, the coverage criterion, expected duration of the relocation, the urgency of relocation, and the travel distance between relocation sites are factors that determine the relocation procedure.

Other operational planning issues concern dispatching decisions with respect to the number of units (Swersey, 1982) and the different types of vehicles (Ignall et al., 1982) to send to a certain incident. Today, this is often done at the strategic or tactical level, and FRS institutions devise courses of action specifying the responses to a set of classified accidents (BRE, 2015).

In operational planning, dispatching decisions are based on urgency of the emergency and the expected demand of current and future calls. Andersson and Värbrand (2007) developed a model for ambulance management where the objective was to keep an adequate level of preparedness in an area by locating and relocating ambulances. The algorithm determines relocation sites for the EMS when the preparedness level drops below a certain threshold value. Another real-time decision support tool for emergency unit dispatchers was developed by Haghani and Yang (2007), whom presented a model in which dispatched units can switch routes according to updated information after already being dispatched to an accident site.

Previous models have considered the seriousness of the incident, time of the day and season, and expected losses at both the incident and expected future emergencies that are directly affected. In recent years, decisions regarding the rescue effort on the accident site have been studied as part of operational planning (Dimopoulou and Giannikos, 2004). An example is the physical positioning of firefighters to prevent the fire from spreading further (Ng and Raff, 2008).

2.3.4 Static Planning

For most of the papers discussed above, the assumption of static planning is made. Static planning is characterized by having known input data that does not change over the course of the planning process. This is almost universally a simplification of the ever-changing real world being modeled. However, in instances where the dynamic aspects are not dominant, the simplification could still yield useful results (Ulander, 2015). The assumption is most valid in strategic and tactical planning where problem parameters are considered to be constant within the bounded planning period. When for example decision makers address the problem of locating a FRS facility, the relevant information for all candidate locations are known in advance. Then the characteristics of the facility do not change over the course of the pre-defined planning period.

Static planning problems are usually easy to define and model. The problem is solved efficiently, with a reasonable computational capacity. Solutions with different characteristics are often easily comparable, as the objectives of the models are static. This means that decision makers can compare facility location solutions, given different available resources. It is useful to evaluate for example the effects of additional response units and crews when deciding on the number and location of fire stations in FRS. It is worth mentioning however, that although the assumption of static planning is most widespread at the strategic and tactical level, typical operational planning problems also have static elements. In resource dispatching, decision makers often have pre-defined rules regarding what stations to alert or which resources or vehicles to send, depending on the situation at hand.

2.3.5 Dynamic Planning

In dynamic planning, the input data changes over the course of the planning period. Parameters such as demand, travel times, and available resources are generally not known in advance, but rather revealed with time. Information is updated continuously also after the initiation of a solution process, leading to possible re-allocations of already dispatched resources. Mechanisms for updating information is therefore required in the solution method. As a result of the time-dependent nature in dynamic models there are some limitations regarding choice of solution method. The models must be solved efficiently, and this often implies the use of heuristic solution methods (Kolesar and Walker, 1974).

The most relevant problem in dynamic FRS planning is deciding in real-time the number and types of resources that is dispatched to an accident site (Ignall et al., 1982, Swersey, 1982). The dynamic reallocation of entire fire companies have also been studied (Kolesar and Walker, 1974). Dynamic planning has also been used in the firefighting problem (Ng and Raff, 2008), were real-time information of the fire condition and manpower status is used to effectively combat the flames.

Due to the complexity, dynamic planning has traditionally taken a back-seat to static planning in operational research, but is becoming increasingly popular, primarily at the operational planning level. Having up-to-date information on resource location and incident status is crucial in dispatching and reallocating resources. Such models have primarily been developed in EMS (Andersson and Särdqvist, 2007).

2.4 Problem Data

All FRS planning problems at any one of the planning levels involve input data. As discussed above, there is a distinction in how the input data is perceived as either static (2.3.4) or dynamic (2.3.5). When solving location problems, a representation of the geographical area at hand is required. To facilitate the solution process, this is done by dividing the area into a set of zones represented by nodes in a network. Both resources and demand are concentrated in these nodes (ReVelle and Snyder, 1995). The size of each zone varies. A very fine division of the area means that one is able to capture more of the identity and characteristic of each node. However, this also rapidly increases the size of the problem and thus also the required computational capacity.

The nodes in the network are given a demand for fire protection for a given expected accident (Batta and Mannur, 1990). The demand can be based on a number of parameters such as population, historical data, infrastructure, or other special identities or considerations. The required service standard for the different nodes may vary (BRE, 2015), but in cover formulations the objective is to cover as much demand as possible within a certain threshold time or distance (Church and ReVelle, 1974). The desired parameter for deciding demand can then also decide the exact location of the node in relation to the others in the network. Typically, this is decided by accident or population density.

A subset of the demand nodes then becomes facility nodes where potential resources can be located. This can either be facilities, response units, or personnel. Usually there are restrictions to potential facility nodes and therefore it may be beneficial to have a subset to reduce the number of potential solutions (Ulander, 2015). Resource location problems require complete data overview and a defined resource requirement, in relation to accident demand. Depending on the objective function, travel times or distances between the nodes are essential information i FRS problems. This parameter is usually pre-calculated and taken in as a matrix. The distances are known from shortest path calculations of feasible arcs in the network, and the travel times can depend on the response unit in question, the times of the day, weather conditions, traffic, or other factors.

Staffing problems considers scheduling and aims to find the optimal staffing plan to meet a given expected demand. This means looking into both scheduling and location. An integrated approach is presented in Church et al. (2001). Staffing problems require information on personnel capabilities and demand for competencies at the accident site, as well as HR and legal working environment considerations.

2.5 Preparedness

Preparedness is a term used to express the ability the FRS has to respond to a certain accident. In emergency response, making correct and immediate decisions is crucial in order to achieve high performance. With a good overview of the preparedness in the area, the FRS is able to locate resources in a manner that improves the total preparedness. As a result, a critical part of ERP is being able to measure the level of preparedness. Through the scope of optimization, the preparedness measure is used as the objective. However, there are a great deal of challenges related to formulating a quantitative preparedness measure, due to the variation in scenarios and how the level of preparedness is perceived.

2.5.1 Preparedness Definition

The term preparedness is difficult to define as there exists no general definition of preparedness that is applicable for a specific situation. According to Stevenson (2015) preparedness is *"a state of readiness, especially for war"*. In Ulander (2015) preparedness for FRS is defined as their *"ability to respond to accidents"*. Another definition related to emergency and disaster response state preparedness as *"the knowledge and capacities to effectively anticipate, respond to, and recover from, the impacts of likely, imminent or current hazard events or conditions"* (Granberg, 2013).

2.5.2 Methodology

Because there is no general definition of preparedness, the term needs to be evaluated for a specific situation. In order to quantify preparedness, Granberg (2013) introduced a methodology based on four steps.

Select Event and Perspective To construct a preparedness measure it is important to identify the event and perspective to which the measure applies. The event describes the situation one is preparing for and the perspective describes the point of view to which the measure is used.

Select Indicators After selecting an appropriate event and perspective, a selection of quantifiable indicators are identified. As the preparedness measure for FRS assess several events, the number of indicators increases. For a certain accident type, different factors are included and the relevance of these factors vary according to the specific incident. A study conducted for the FRS in Sweden suggested a set of relevant indicators for measuring preparedness. This included the response time for required resources and the expected number of accidents (Granberg et al., 2015). By dividing the area into a set of geographical zones, each zone has a given response time and an accident specific demand.

Combine Indicators After a set of relevant indicators are selected, the indicators are combined into an index for calculating the level of preparedness. The indicators selected often vary in units and in order to combine these into an index, it may be useful to scale the indicators by including a weighting factor (Granberg, 2013).

In previous research, some general preparedness measures have been developed. In ERP, preparedness is calculated by dividing the area into different zones. Granberg (2013) suggests a calculation of the preparedness measure, p_j, in each zone j, for EMS.

$$p_j = \frac{1}{c_j} \sum_{l=1}^{L_j} \frac{\gamma^l}{{t_j}^l} \tag{1}$$

$$\gamma^1 > \gamma^2 > ... > \gamma^{L_j} \tag{2}$$

The demand in each zone j is given by c_j and L_j represents the number of ambulances that contribute to the preparedness in the given zone j. γ^l is the contribution factor for ambulance l and t_j^l is the travel time to zone j for ambulance l. The ambulances are arranged such that the index $l = 1$ represents the closest ambulance, $l = 2$ the second closest, and so on.

In the case of EMS, the accidents may be considered homogeneous in that all resources that respond to a set of accidents are equal. The FRS responds to variety of emergencies with different resource requirements and it is therefore beneficial to consider the accidents heterogeneous. If the preparedness measure is to be calculated for a set of different events, one general measure cannot apply to all (Granberg, 2013). In the case of FRS, multiple rescue operations are performed for different accident types. As a result, a specific accident type requires a certain preparedness measure. Granberg et al. (2015) suggests the following expression, which is a general preparedness measure for a certain accident type $a \in A$ in zone $j \in J$.

d_{aj}	the expected number of accident of type a in zone j
V_{paj}	the response time for requested personnel resource p that contributes to preparedness for handling accident of type a in zone j
T_{uaj}	the response time for requested vehicle u that contributes to preparedness for handling accident of type a in zone j
B_{ra}	the number of personnel resources with a certain competence r for accident of type a
D_{ka}	the number of vehicles of type k needed for accident of type a in zone j
γ	the weighting factor for contribution of expected demand d_{aj}

$$p_{aj} = (d_{aj})^{\gamma} \quad \frac{\displaystyle\sum_{p \in P} V_{paj} + \sum_{u \in U} T_{uaj}}{\displaystyle\sum_{r \in R} B_{ra} + \sum_{k \in K} D_{ka}} \qquad \forall a \in A, j \in J \tag{3}$$

Validate the Measure After creating the preparedness measure, a validation process should be conducted. This is done to ensure that the preparedness measure is in line with the intended purpose. Granberg (2013) suggests two validation procedures, both a sensitivity analysis and a face validity. The former involves evaluating the output data by changing the input data. The latter is a validation method were the results are evaluated by system experts and validated by established measures.

2.5.3 Preparedness in FRS

The methodology has been applied in an EMS context (Andersson and Värbrand, 2007) and more recently also in FRS (Granberg et al., 2015). Although both preparedness measures are important aspects of a decision support tool in ERP, it is a more complex process to find a preparedness measure in FRS compared to EMS (Granberg et al., 2015). The FRS responds to a set of diversified accidents that require different resources. Therefore, both demand and response units must be treated as heterogeneous sets. Furthermore, in EMS the emergency calls usually only require the response of a single ambulance, whereas multiple response units are often required for emergency calls in FRS.

To reduce complexity and develop an applicable qualitative preparedness measure, certain simplifications need to be made. The number of events that are included in the measure can be limited. The FRS responds to a large number of different emergency calls, but the preparedness is mostly interesting to investigate for incidents requiring an urgent response. Granberg et al. (2015) suggests selecting two specific events, fire in buildings and traffic accidents, to be part of the preparedness measure. These events occur frequently and their relative severity dictates a need for immediate response. To address the issue of multiple response units having different response times, the average response time of the required units is used in the preparedness measure in this case.

This report presents an alternative preparedness measure for FRS in Section 8. The same methodology has been applied to devise an appropriate measure for the problem at hand.

3 Location Problems

In this section, a selected few location problems and their mathematical formulations are examined. These have all previously been extensively utilised to solve problems in ERP and they have served as inspiration for the mathematical formulation for the MPLP-HAR in this report. More importantly however, existing location problems have been an important part of the validation process of the developed model presented in this report.

Initially in ERP, location problems were applied for EMS to locate ambulance bays and ambulances. However, the problems can be generalized to apply to the location of any given resource. Here, the resources are assumed to be a response unit responding to an incident. All of the location problems presented in this section consider incidents that require the response from a single unit only. In order to locate the units, the modeled area is divided into a set of zones, where each zone is a geographic location with an expected demand. A subset of the demand zones are possible locations for the response units, referred to as candidate locations. The location problems aim to locate a set of units to a geographical location, based on a given objective.

3.1 Covering Problems

The covering problems locate response units aiming to cover all demand zones in an area by a given service standard and are based on the traditional set cover problem. Toregas et al. (1971) introduced the Location Set Covering Model (LSCM) as a model to minimize the number of fire stations to be located to meet the pre-defined service requirements. LSCM is a binary programming model that minimize the number of units needed in order to cover all demand zones within a given time or distance limit. Due to limitations in LSCM, the Maximal Coverage Location Problem (MCLP) was developed by Church and ReVelle (1976). The MCLP fixes the number of units and maximize coverage within a given a time or distance limit. A formulation of the MCLP is given below.

Sets

I the set of demand nodes i

J the set of candidate locations j, $J \subset I$

Parameters

D_i the demand in node i

T^C the coverage time standard

T_{ij} the travel time from node i to j

T_d the pre-travel delay

C the maximum number of response units

A_{ij} $\begin{cases} 1, & \text{if demand node } i \text{ is covered by candidate location } j\text{, i.e. } T_{ji} + T_d \leq T^C \\ 0, & \text{otherwise} \end{cases}$

Variables

y_i $\begin{cases} 1, & \text{if demand node } i \text{ is covered} \\ 0, & \text{otherwise} \end{cases}$

x_j $\begin{cases} 1, & \text{if candidate location } j \text{ is selected} \\ 0, & \text{otherwise} \end{cases}$

Mathematical Formulation

$$max \sum_{i \in I} D_i y_i \tag{4}$$

$$\sum_{j \in J} A_{ij} x_j \geq y_i \quad i \in I \tag{5}$$

$$\sum_{j \in J} x_j \leq C \tag{6}$$

$$y_i \in \{0, 1\} \quad i \in I \tag{7}$$

$$x_j \in \{0,1\} \quad j \in J \tag{8}$$

The objective function (4) maximizes total demand covered. Constraints (5) state that demand node i is only covered if at least one candidate location that covers i is selected and Constraints (6) limit the number of selected candidate locations to the maximum number of vehicles C. Constraints (7) and (8) are binary constraints for the decision variables y_i and x_j.

3.2 p-median

As opposed to the covering problems, that focus on covering as much demand as possible, the p-median problem attempts to minimize the total or average travel time from demand zones to the closest of C response units. The p-median problem was first introduced by Hakimi (1965) and further developed by ReVelle and Swain (1970). A p-median formulation is described below.

Sets

I the set of demand nodes i

J the set of candidate locations j, $J \subset I$

Parameters

D_i the demand in node i

T_{ij} the travel time from node i to j

T_d the pre-travel delay

C the maximum number of response units

Variables

$$y_{ij} \begin{cases} 1, & \text{if demand node } i \text{ is covered by candidate location } j \\ 0, & \text{otherwise} \end{cases}$$

$$x_j \begin{cases} 1, & \text{if candidate location } j \text{ is selected} \\ 0, & \text{otherwise} \end{cases}$$

Mathematical Formulation

$$min \sum_{i \in I} D_i \sum_{j \in J} T_{ij} y_{ij} \tag{9}$$

$$\sum_{i \in I} y_{ij} \leq |I| x_j \quad j \in J \tag{10}$$

$$\sum_{j \in J} y_{ij} \geq 1 \quad j \in J \tag{11}$$

$$\sum_{j \in J} x_j \leq C \tag{12}$$

$$y_{ij} \in \{0, 1\} \quad i \in I, j \in J \tag{13}$$

$$x_j \in \{0, 1\} \quad j \in J \tag{14}$$

The objective function (9) minimizes the total travel time in a given area. Constraints (10) ensure that only selected candidate locations can cover demand zones. Constraints (11) require that all demand zones have to be covered at least once. Constraints (12) require that the number of selected candidate locations corresponds to the given number of available response units, C. Constraints (13) and (14) are binary constraints for the decision variables y_{ij} and x_j.

3.3 Maximum Survival Location Problem

For EMS there have been studies questioning the traditional covering approach to the location problem (Erkut et al., 2009). The Maximum Survival Location Problem (MSLP), separates itself from the MCLP and the p-median problem in its attempt to maximize the expected number of survivors (Erkut et al., 2009). The element of introducing a maximum survival approach emphasises the importance of a quick response in ERP, based on survival statistics with respect to time. A formulation for the MSLP is given below.

Sets

I the set of demand nodes i

J the set of candidate locations j, $J \subset I$

Parameters

D_i	the demand in node i
T_{ij}	the travel time from node i to j
T_d	the pre-travel delay
$S(T_{ij} + T_d)$	the survival probability in i if it is served by j, as a function of time
C	the maximum number of response units

Variables

$$y_{ij} \begin{cases} 1, & \text{if demand node } i \text{ is covered by candidate location } j \\ 0, & \text{otherwise} \end{cases}$$

$$x_j \begin{cases} 1, & \text{if candidate location } j \text{ is selected} \\ 0, & \text{otherwise} \end{cases}$$

Mathematical Formulation

$$max \sum_{i \in I} D_i \sum_{j \in J} S(T_{ij} + T_d) y_{ij} \tag{15}$$

$$\sum_{i \in I} y_{ij} \leq M x_j, \quad j \in J \tag{16}$$

$$\sum_{j \in J} y_{ij} = 1, \quad i \in I \tag{17}$$

$$\sum_{j \in J} x_j \leq C \tag{18}$$

$$y_{ij} \in \{0, 1\} \quad i \in I, j \in J \tag{19}$$

$$x_j \in \{0,1\} \quad j \in J \tag{20}$$

The objective function (15) maximize the expected number of survivors. Other than the objective function, the constraints in MSLP are equivalent to the constraints in the p-median problem.

3.4 Comparing Location Problems

The cover formulations use a fixed service standard and consider all demand zones to be either covered or not covered, depending on whether the response time is below or above the defined threshold value. The threshold time value for a given emergency is indicated at T^C. The objective value is constant for all response times $t \leq T^C$ and thus the MCLP makes no distinction between response times as long as they are within a given service standard.

In order to graphically represent the objective as a function of response time, the negative value of the p-median objective function is maximized. The response time is proportional to the objective value and thus a reduced response time linearly increases the objective value. As opposed to the MCLP discussed above, the p-median formulation is able to capture the benefits of reducing the response time as much as possible.

The formulation of the MSLP differs from the p-median only in the objective function. In the MSLP the negative relation between increases in response time and the objective value is not linear, but rather exponential. The idea is to underline the importance of quick response time and showcase the decreasing trend of the ability to provide emergency relief with time. The initial minutes are weighted as the most critical. A graphical representation of the objective values with time for the MCLP, p-median, and MSLP is presented in Figure 2. As opposed to the MCLP, the objective value of the p-median and MSLP does not necessarily reach zero as the response time increases beyond a thresholds value. This indicates that any coverage of demand is beneficial, even if response time is significant.

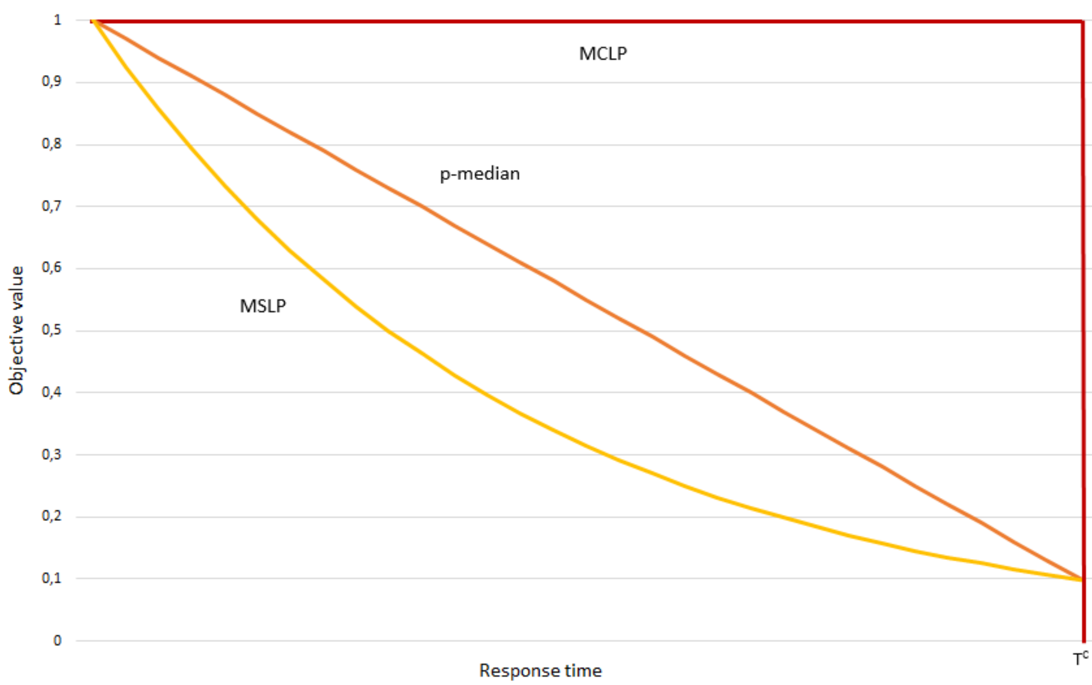

Figure 2: Comparison of the objective values in MCLP, p-median, and MSLP

3.5 Evaluation of Location Problems

Erkut et al. (2009) suggests a comparison of the MCLP, the p-median, and the MSLP, as all the problems are solved in a deterministic fashion, assuming the availability of all vehicles. The main difference between the three models is the performance measure in the objective function. The MSLP replaces a zero-one coverage and average response time with a probability of survival.

An example, presented by Erkut et al. (2009), illustrates how the limitation of the covering models can result in large measurement errors.

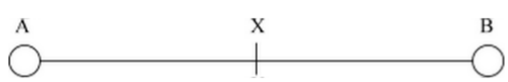

Figure 3: Example demonstrating the limitation of covering problems (Erkut et al., 2009)

Assuming two demand locations A and B and one candidate location X, illustrated in Figure 3. The distances between X and the two locations A and B are equivalent, for instance 9 minutes. A covering model with a time threshold of 9 minutes would consider both locations covered and give the optimal location at point X, disregarding the demand in the two locations. Assuming the demand in node A is 10 and node B is 1, and the probability of survival for response time t is given by e^{-t}, the expected number of survivors s is given in equation (21).

$$E[s] = D(A) \times P(s_A) + D(B) \times P(s_B) = 10 \times \mathrm{e}^{-9} + 1 \times \mathrm{e}^{-9} = 0.001358 \tag{21}$$

If the candidate location instead is placed at point A, the expected number of survivors s increase, as presented in equation (22).

$$E[s] = D(A) \times P(s_A) + D(B) \times P(s_B) = 10 \times \mathrm{e}^{0} + 1 \times \mathrm{e}^{-18} \approx 10 \tag{22}$$

The latter expression gives an expected survival over 7000 better than the optimal covering solution. Even though the probability of surviving in location B is close to none existing, this exaggerated example illustrates the covering models limited ability to discriminate between different response times and how this results in poor location decisions.

Looking now closer at the performance measure in MCLP, p-median, and MSLP. The problems can be compared further, by computing the objective function given in equation (23).

$$max \sum_{i \in N} D_i \sum_{j \in M} p_{ij} y_{ij} \tag{23}$$

where the performance measure is referred to as p_{ij}. The difference in performance measure between the three models is describes in Table 4.

Table 4: Comparison of the performance measure in MCLP, p-median, and MSLP

Model	p_{ij}
MCLP	$\begin{cases} 1, & 0 \leq (T_{ij} + T_d) \leq T_c \\ 0, & (T_{ij} + T_d) > T_c \end{cases}$
p-median	$-(T_{ij} + T_d)$
MSLP	$S(T_{ij} + T_d)$

An empirical study, conducted by Erkut et al. (2009), compared the three problems using data from a city in Canada. The problems were solved to optimality using 180 demand zones and 16 candidate locations. Figure 4 compares the objective value of MSLP and MCLP and p-median, respectively for C station locations, ranging from 1 to 16.

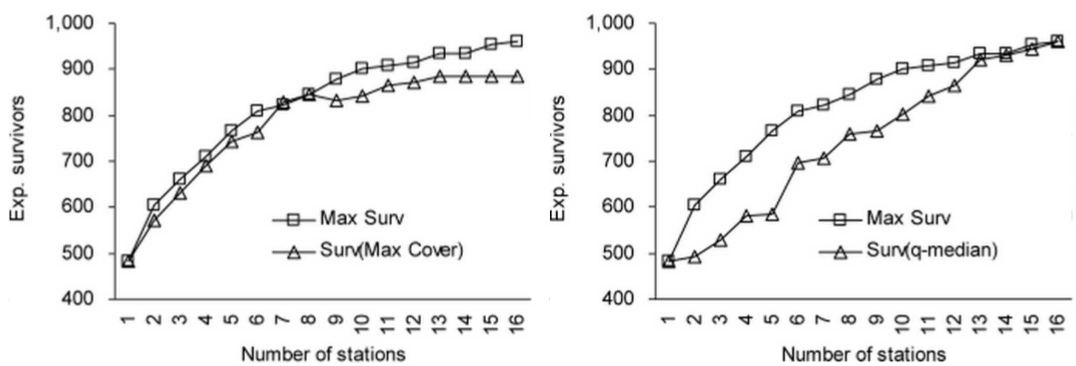

Figure 4: Comparison of the optimal solution in MSLP with MCLP and p-median

The empirical study illustrates that using MCLP or p-median can result in poor decision making when choosing station locations, compared to using MSLP. Erkut et al. (2009) argues that for the MCLP, the expected number of survivors is 7.7% lower than for the MSLP. For the p-median, the expected number of survivors is up to 23.5% lower. All models are evaluated using the MSLP formulation. In addition, the p-median is weak when C is between 8 and 9. This is because the p-median model chooses to reduce the longest response time instead of the shortest response time. In this case, shortening the longest response time would not increase the survivability. As shown in the figure, with $C > 13$ the MCLP gives the same solution as for $q = 13$. By contrast, the MSLP is able to increase the expected number of survivors for each additional station. The objective value in MSLP is more relevant in deciding how many stations to build, because it measures the survivability explicitly. For a decision maker, a performance measure stating the survivability of the system is more meaningful than the total coverage.

As argued in the study discussed above, the coverage problems are integer programs that are easily solved and simple to communicate (Erkut et al., 2009). However in MCLP, optimality is given by full coverage and thus the problem requires an extensive amount of response units, disregarding the system costs. Another issue is the assumption of deterministic and known response times, when in reality there are numerous factors affecting response times, such as traffic, pre-travel delay, weather, local events, and time of day (Erkut et al., 2008). These effects are disregarded in the problems described above. The MCLP, p-median, and MSLP takes the pre-travel delay into account, but also assumes this value to be deterministic and known.

In MSLP, the slope of the survival graph and the factors that can reverse the development of the slope with time, need to be identified. In addition, a proper definition of what it means to have survived is needed. In Erkut et al. (2009) this is done in the case of cardiac arrests for EMS. Using the MSLP in FRS planning decisions would require the same procedure for all types of accidents. As FRS includes both the potential loss of lives and material assets, a performance measure including both aspect would be valued.

All the models described above are deterministic and assume that the nearest response units are available at all times. Erkut et al. (2008) argues that in the case of EMS, systems are designed for low utilization, but most EMS systems have a utilization rate of at least 25%. That is, the response unit are incapacitated a quarter of the time. Therefore a near zero utilization may not be a realistic assumption. Furthermore, response times are assumed to be deterministic. Yet, actual response times are highly variable both with respect to travel and pre-travel times. Extensions to the MSLP have therefore been made to deal with these two types of uncertainty (Erkut et al., 2008). Firstly, there is the Maximal Expected Survival Location Problem (MEXSLP). It is based on the MSLP, but introduces a stochastic variable indicating whether or not a response unit is busy. Secondly, Erkut et al. (2008) also introduces the Maximal Survival Location Problem with Probabilistic Response time (MSLP + PR) where the objective function is extended to include an expression for the expected probability of survival as a function of the pre-computed, known response time, for a given demand location. Finally, in the Maximal Expected Survival Location Problem with Probabilistic Response time (MEXSLP + PR) both sources of uncertainty are incorporated in a formulation structurally similar to the MSLP.

The effects of incorporating uncertainty into the problems have been showcased in a computational comparison of the location covering equivalents of these survival problems (Erkut et al., 2009). This study found that taking both uncertainties into account would yield a problem that far outperformed the simple deterministic covering problem in deviation from best-known solution. Furthermore, only accounting for either availability or travel time uncertainty individually, as is done in the MEXSLP and MSLP + PR respectively, would yield significant improvements. Interestingly, incorporation of availability improves the solution quality more than including travel-time uncertainty. Therefore, there is reason to believe that deterministic planning is more relevant i FRS, as the utilization rate is lower than in EMS.

The location problems introduced in this section are all developed in the context of EMS, where incidents are considered homogeneous. In the case of FRS a similar approach would not be applicable, as different accidents require a different measure of preparedness. This report therefore looks more closely into the construction of a preparedness measure that apply to different accident types in Section 8.

4 Problem Description

The problem analyzed in this report is a station location and resource allocation problem in fire and rescue service (FRS). It is a strategic and tactical decision support tool for FRS organizations that are establishing or rearranging infrastructure. The objective is to maximize the level of preparedness in a pre-defined area, by giving the FRS an optimal ability to preserve life and property in the case of an emergency. The area is divided into a set of demand zones and a subset of the demand zones are considered potential facility or resource locations. The problem considers a heterogeneous set of accidents with different resource requirements and perceptions of preparedness. The demand for each accident type varies across the demand zones according to their likelihood of occurrence in any given zones. As the different accident types have different resource requirements, the location problem considers individual response units from a set of vehicle types with specific capabilities. The units can differ in equipment or staffing level within the same vehicle type.

Preparedness in FRS is the ability to respond to emergencies, and good preparedness is being able to respond efficiently and effectively to emergencies where the demand is high. The objective function is a function of the expected demand for the different accident types in each demand zone and the preservation value as a function of the response times of the responding units. The problem distinguishes between first and total response and the preservation value decreases as a function of both time measures. The first ready-to-operate unit at the accident site is considered to provide first response, while full response is assumed to be achieved when the last required unit can provide support to the operating units. The response time thus considers both the travel time to the emergency location and any required preparation time before being dispatched and upon arrival.

Because the accident types require different sets of resources, the importance of the first response may vary according to which type of resource it is provided by. Different response units are staffed and equipped differently, and the ability to provide immediate emergency relief upon arrival, thus slowing the decreasing preservation trend, varies accordingly. The role of each response unit is decided by the type of unit in question, its current location in relation to the location of the emergency, response unit characteristics, and the nature of the current emergency. For certain accident types, however, only one response unit is required and hence total response is achieved by the first responding unit.

The problem is thus to locate fire stations and allocate resources in order to maximize the preparedness level in an area. It is intended as a decision support tool for determining the location of permanent facilities and an appropriate fleet to efficiently utilize existing infrastructure. The problem is denoted the Maximum Preservation Location Problem for Heterogeneous Accidents and Response Units (MPLP - HAR).

5 Mathematical Model

In this section, the developed mathematical model is presented. Firstly, the assumptions behind the formulation are outlined, to give an understanding of the problem at hand. Next, the complete mathematical formulation is presented.

5.1 Model Description

The developed model solves the Maximum Preservation Location Problem for Heterogeneous Accidents and Response Units (MPLP-HAR). The problem attempts to locate vehicles in order to maximize the overall preservation in the area. The area is divided into several zones, each representing a smaller geographic area. The zones have unique characteristics and differ with respect to area, population, infrastructure, and risk profile. This means that all zones have a given demand for each accident type, based on their respective expected rate of occurrence. All the zones are considered both a demand zone and a possible candidate location for the vehicles.

The problem treats the response units within each vehicle type to be treated as heterogeneous. Response units differ with respect to equipment and staffing levels and can therefore possess different sets of competencies. The number of available units are given as an input, and hence the busy probability of a given unit is not considered. In other words, all units are assumed to be available at all times. This simplification is made due to the low general utilization level in FRS and the high regulatory dimensional standards that are imposed. Adding a busy probability would complicate the model unnecessary, compared to the actual operational situation, were this rarely is an issue.

As FRS responds to several different emergency calls, heterogeneous accidents with unique preservation functions are considered. However, the situation is simplified by grouping the accidents into a number of categories, depending on accident severity and the required resources. Emergency calls that fall under the same accident type must trigger the same response from the FRS as the requirements for each accident type is a predefined parameter.

The response time is defined as the time it takes from the emergency is reported to the dispatched units are ready to operate at the site of the accident. This includes the preparation time required for each unit prior to departure from the original location, the travel time to the accident site, and an intervention time upon arrival before emergency relief efforts can commence. Preparation and intervention time are predetermined depending on the vehicle type. The formulation can also regard travel times as vehicle dependant, as maneuverability in traffic and top speed can vary with the type of vehicle in question. As previously discussed, travel time uncertainty has a minor impact on the solution and is therefore not considered.

The response time is divided into first and total response time. First response is achieved when the first applicable unit is ready to operate at the accident site. Total response is assumed to be provided when the final required unit is operational and providing emergency relief. In FRS, most emergency calls trigger the response from multiple units of different vehicle types. However, simple rescue calls require no more than one base unit at the accident site. In these cases total response is achieved by the single dispatched unit.

The preservation function is used in the objective function. It is calculated for each accident type and depends on the response time of the first and total response units. For all accident types, the preservation value is considered to be 100% when the vehicles are dispatched and sent to the accident site. Hence, the model disregards the damage done in the time period before the emergency is reported to a dispatcher. For all accident types, the preservation value initially decreases exponentially with respect to time. When the first unit arrives, the function becomes a linearly decreasing function, and when the last unit arrives, the decreasing trend of the preservation value is stopped. The function is illustrated in Figure 8, where the first and total response arrive after 10 and 20 minutes, respectively. In accidents that require first response only, the preservation value is considered constant after the arrival of the first response unit. The exact identities of the preservation function are examined closely in Sections 9 and 10.

5.2 Mathematical Formulation MPLP-HAR

Sets

I	the set of demand zones, i
J	the set of candidate locations, j, k
A	the set of accident types, a
A^S	the set of accident types that require more than one unit, $A^S \subset A$
V	the set of vehicle types, v
U	the set of units, u, o

Parameters

λ_a	the severity factor for accident type a
D_{ia}	the demand in zone i for accident type a
P_{iaju}	the preservation value for accident type a in zone i, with first and total response provided by unit u from zone j
P_{iajuko}	the preservation value for accident type a in zone i, with first response provided by unit u from zone j and total response from unit o in zone k
C_v	the maximum number of available units of type v
E_{av}	the required number of vehicles of type v to an accident a
T^T_{jiu}	the travel time from zone j to i for unit u
T^P_u	the preparation time for unit u
T^I_u	the intervention time for unit u
R_{uv}	$\begin{cases} 1, & \text{if unit } u \text{ is of vehicle type } v \\ 0, & \text{otherwise} \end{cases}$
Q_{au}	$\begin{cases} 1, & \text{if unit } u \text{ can provide first response for accident type } a \\ 0, & \text{otherwise} \end{cases}$

Variables

$$x_{ju} \quad \begin{cases} 1, & \text{if unit } u \text{ is located in zone } j \\ 0, & \text{otherwise} \end{cases}$$

$$s_{iaju} \quad \begin{cases} 1, & \text{if unit } u \text{ located in zone } j \text{ contributes to accident } a \text{ in zone } i \\ 0, & \text{otherwise} \end{cases}$$

$$w_{iaju} \quad \begin{cases} 1, & \text{if unit } u \text{ located in zone } j \text{ provides first and total response to accident } a \\ & \text{in zone } i \\ 0, & \text{otherwise} \end{cases}$$

$$w_{iajuko} \quad \begin{cases} 1, & \text{if unit } u \text{ located in zone } j \text{ provides first response to accident } a \text{ in zone } i \\ & \text{and total response is provided by the last response unit } o \text{ located in zone } k \\ 0, & \text{otherwise} \end{cases}$$

Mathematical Formulation

$$max \sum_{i \in I} \sum_{a \in A} \lambda_a D_{ia} (\sum_{u \in U} \sum_{j \in J} \sum_{o \in U} \sum_{k \in J} P_{iajuko} w_{iajuko} + \sum_{u \in U} \sum_{j \in J} P_{iaju} w_{iaju}) \tag{24}$$

$$\sum_{j \in J} x_{ju} = 1 \quad u \in U \tag{25}$$

$$\sum_{j \in J} \sum_{u \in U} R_{uv} s_{iaju} \geq E_{av} \quad i \in I, a \in A, v \in V \tag{26}$$

$$\sum_{j \in J} \sum_{u \in U} Q_{au} w_{iaju} = 1 \quad i \in I, a \in A/A^S \tag{27}$$

$$\sum_{j \in J} \sum_{k \in J} \sum_{u \in U} \sum_{o \in U} Q_{au} w_{iajuko} = 1 \quad i \in I, a \in A^S \tag{28}$$

$$w_{iaju} \leq x_{ju} \quad i \in I, a \in A/A^S, j \in J, u \in U \tag{29}$$

$$w_{iajuko} \leq x_{ju} \quad i \in I, a \in A^s, j, k \in J, u, o \in U \tag{30}$$

$$w_{iajuko} \leq x_{ko} \quad i \in I, a \in A^s, j, k \in J, u, o \in U \tag{31}$$

$$s_{iaju} \leq x_{ju} \quad i \in I, a \in A, j \in J, u \in U \tag{32}$$

$$\sum_{j \in J} \sum_{u \in U} \sum_{k \in J} \sum_{o \in U} (T^T_{kio} + T^P_o + T^I_o) w_{iajuko} \geq \sum_{j \in J} (T^T_{jip} + T^P_p + T^I_p) s_{iajp} \quad i \in I, a \in A^S, p \in U \tag{33}$$

$$x_{ju} \in \{0, 1\} \quad j \in J, u \in U \tag{34}$$

$$w_{iajuko} \in \{0, 1\} \quad i \in I, a \in A^S, j \in J, k \in K, u, o \in U \tag{35}$$

$$w_{iaju}, s_{iaju} \in \{0, 1\} \quad i \in I, a \in A, j \in J, u \in U \tag{36}$$

The objective function (24) maximizes the preservation value based on the first and total response, weighted by the expected demand in the area and a severity factor for each accident type. Constraints (25) make sure that there is exactly one of each unit. The units responding to the emergencies must meet the resource requirements for any given accident type, as dictated by (26). Constraints (27) ensure that a qualified first response unit is provided. In instances where more than one unit is needed, (28) require that both first and total response is provided, by making sure that both the first and last response units are dispatched. Constraints (29)-(31) ensure that the dispatched units are available at their initial candidate location. The contributing units must come from candidate locations were the units are available, as assured by Constraints (32). Constraints (33) ensure that the last response unit is actually last, meaning that all required units have arrived when the last response unit arrives. Finally, the binary identities of the variables are given in Constraints (34)-(36).

5.3 Model Improvements

The size and complexity of the problem grow rapidly with the number of accident types, response units, and zones that are considered. For a real world application, the magnitude of these sets are often stretching upwards from a dozen to the hundreds. This means that the formulation described above can contain a vast number of variables, w_{iajuko}. In order to reduce computational time and increase the applicability of the MPLP-HAR, it is necessary to investigate potential improvements to the model.

5.3.1 Reduce the Number of Constraints

The nature of the model formulation dictates the construction of a vast number of variables in the problem. The same can be said for the number of restrictions. However, reducing the number of restrictions by compiling duplicates, reduces the computational time without altering the solution space. Equation (37) is a suggested replacement for Constraints (30) - (31) in the original formulation.

$$2w_{iajuko} \leq x_{ju} + x_{ko} \quad i \in I, a \in A^s, j, k \in J, u, o \in U \tag{37}$$

Furthermore, the restrictions can be aggregated by summation. For every total response unit, o, responding from candidate location k, there can be no more than one first response unit, u, responding from candidate location j. This means that the sum of all variables w_{iajuko}, over j and u, can not exceed x_{ko}. Equation (38) summarizes this fact and is meant as a replacement the Constraints for (30) in the original formulation. Similarly, the same argument is used to replace Constraints (31) in the original formulation, with the expression in equation (39).

$$\sum_{j \in J} \sum_{u \in U} w_{iajuko} \leq x_{ko} \quad i \in I, a \in A^S, k \in J, o \in U \tag{38}$$

$$\sum_{k \in J} \sum_{o \in U} w_{iajuko} \leq x_{ju} \quad i \in I, a \in A^S, j \in J, u \in U \tag{39}$$

Another approach to reducing the number of restrictions is through a Big-M formulation. The strategy is to find an upper limit to the number of variables that can be generated. The tighter this upper limit can be determined, the more effective the formulation becomes. In this case, for each accident type a, in demand zone i, there will always be only one first response unit, u, responding from candidate location j. This identity can utilised to limit the number of constraints by a Big-M formulation. Through a summation over accident types a and demand zones i, the total number of non-zero variables variables, w_{iaju}, can be limited to a multiple of the accident types and the demand zones, as shown in equation (40). The same summation can be made over the number of responding units, s_{iaju}, expressed in equation (41). In fact, the upper limit can be accurately determined through either the multiple of accident types and demand zones or units and vehicle zones. The first option is chosen in this instant, as this gives the tightest upper limit.

$$\sum_{i \in I} \sum_{a \in A/A^S} w_{iaju} \leq |I| \cdot |A| x_{ju} \quad j \in J, u \in U \tag{40}$$

$$\sum_{i \in I} \sum_{a \in A} s_{iaju} \leq |I| \cdot |A| x_{ju} \quad j \in J, u \in U \tag{41}$$

For emergencies requiring multiple response units, the restrictions can be limited by a summation of the binary variable, w_{iajuko}. Equations (42) and (43) exhibit a combination of a Big-M formulation and the aggregated restrictions from equations (38) and (39).

$$\sum_{i \in I} \sum_{a \in A^S} \sum_{j \in J} \sum_{u \in U} w_{iajuko} \leq |I| \cdot |A| x_{ko} \quad k \in J, o \in U \tag{42}$$

$$\sum_{i \in I} \sum_{a \in A^S} \sum_{k \in J} \sum_{o \in U} w_{iajuko} \leq |I| \cdot |A| x_{ju} \quad j \in J, u \in U \tag{43}$$

5.3.2 Symmetry Breaking Constraints

Symmetric solutions occur when there exist multiple equivalent solutions in a problem. This creates problems for solution algorithms, as equivalent search regions are explored unnecessarily more than just once (Fahle et al., 2001). Breaking symmetry then involves removing those solutions that are technically different, but practically similar. An example of such symmetry comes when a homogeneous set of vehicles are treated like individual units in an assignment or routing problem. If the vehicles, i, comes from a homogeneous set, I, it has no practical implication whether if it is vehicle $i = 1$ or $i = 2$ that is assigned to a route. These are however mathematically different, and thus there are symmetric solutions in the problem.

Removing symmetry from the problem can be done by adding symmetry breaking constraints. These are constraints that are only satisfied by one assignment in each equivalence class. In the MPLP-HAR, the response units are treated individually as units u, as well as being from a set of vehicle types, v. Often times however, the units within each vehicle type are considered equal. This means that there are potential symmetric solutions in the problem. Equation (44) is a suggested symmetry breaking constraint that can be added whenever the units are considered homogeneous within each vehicle type. It prevents the problem from including solutions where all u's are considered to be assigned to the candidate locations j, unless multiple response units are located in the same location.

$$\sum_{j \in J} j x_{ju} \leq \sum_{j \in J} j x_{ju+1} \quad v \in V, u \in U \quad | \quad R_{uv} = 1 \quad \& \quad R_{u+1,v} \tag{44}$$

5.4 Heuristic Solutions

As discussed, for most real-life FRS situations the problem involves dozens of demand zones, candidate locations, vehicle types, and response units. This implies that the formulation of the MPLP-HAR results in an extensive number of binary variables, w_{iajuko}. In turn this means that solving the problem to optimality requires extensive computational capacity.

Now, because so many binary variables are generated, relatively few of them are assigned a non-zero value. This in fact implies that most of the variables, w_{iajuko}, contain very little information about the optimal solution to the problem.

The trade-off between solution quality and computational time is always a consideration that is present in optimization studies. In order to develop an applicable decision support tool, heuristic solution methods are investigated to reduce the computational time. The general uncertainty of the input data in most ERP problems means that a near optimal solution can still give useful results. A heuristic is a method that generates a solution within reasonable computational time, but without any guarantee with respect to the solution quality or ability to determine how close to optimality the solution is (Lundgren et al., 2010). Heuristics vary from simple rules of thumb to sophisticated algorithms, designed to solve a specific class of problems.

5.4.1 LP-Relaxation

A linear programming (LP) relaxation of an integer programming problem removes the integrality constraint and so allows non-integer rational solution. The LP-relaxation facilitates the branch-and-bound solution algorithm by obtaining bounds for the integer programming. In general, the relaxation is an approximation of the original problem and a solution of the relaxed problem provides useful information about the original problem.

This heuristic strategy is used to relax the binary constraints for the variables w_{iajuko}, w_{iaju}, and s_{iaju}. The binary unit location variables, x_{ju}, are determined first and then the other variables are assigned a rational value. Fractional values are given whenever this is beneficial to the objective value of the problem.

5.4.2 Stochastic Variable Generation

In a resource allocation problem a considerable amount of the location alternatives can be ruled out by intuition. Locating for example a sea vessel far away from any coastal lines or lakes would intuitively be sub-optimal. The best way to reduce the number of variables is to eliminate these seemingly unnecessary variables from the formulation. However, this requires much knowledge of the data set in question. A simpler approach is to randomly generate only a subset of all variables and search for a near-optimal solution in that data set. In this particular instance, were most variables w_{iajuko} and w_{iaju} are equal to zero, there is reason to believe that this method can provide useful results. Especially since there are potentially large numbers of symmetric solutions if one makes the assumptions of homogeneous response units within each vehicle type.

A simple heuristic is therefore devised where the stochastic variable *random* determines whether or not the binary variables w_{iajuko} or w_{iaju} are generated. The parameter $\delta \in [0, 1]$ dictates the proportion of all variables that are generated by the optimization software. A pseudo code summarizing the heuristic approach is presented in Algorithm 1.

Algorithm 1 Stochastic Variable Generation

```
function CreateSubsetOfVariables(variable_matrix,δ)
    for all w in variable_matrix do
        if random ≤ δ then
            function CreateVariable(w)
            end function
        end if
    end for
end function
```

5.4.3 Model with Homogeneous Units

When assuming that all units of a single vehicle type are equal, the model can be reformulated by replacing the unit's set U with a set for vehicle types, V. All parameters and variables with indices for first and last response units, u and v, will instead have indices for vehicle types m and n. In order to capture the number of different vehicle types located in each zone, the binary location variable, x_{ju}, in the initial model formulation is replaced by an integer location variable, x_{jm}. In addition, an integer variable, y_{iajm}, is added, defining the number of vehicles of type m located in zone j that respond to accident a in demand zone i. The redefined model is explained in detail in Appendix A.

The simplification of assuming homogeneous units within the same vehicle type is considered a valid assumption in many instances. As discussed in Section 5.3.2, having homogeneous units creates a great number of symmetric solutions in the original MPLP-HAR formulation. The alternative formulation is intended to reduce the sets and number of restrictions, in order to solve large problems to optimality within reasonable computational time. Although the formulation looses the aspect of distinguishing between units, these differences are often insignificant and the applicability of the MPLP-HAR increases when it is able to handle larger location problems.

6 Technical Study

The purpose of the technical study is to determine the best formulation of the the MPLP-HAR as well as the most efficient computational approach to find optimal solutions. First the aspect of computational time is presented. Then the effects of the model improvements suggested in Section 5.3 are investigated. The mathematical model is implemented and tested for different input arguments. The implementation is done in Mosel and solved with Xpress IVE 64 bit Optimization Suite, Version 7.8.0. The solution is run on an Intel CORE i7 vPro processor.

The technical study is conducted using a realistic test case where the size of the problem resembles a real-life FRS situation. The area is divided into 15 demand zones, i, all considered potential candidate locations, j and k. There are five different accident types, a, where four of them require multiple response units. Seven different vehicles types, v, are considered and there are 18 response units, u, in total. The parameter specifications are summarized in Table 5.

Table 5: Input parameter specification

Input parameters	Sets	Indices	Size
Demand zones	I	i	15
Candidate locations	J	j, k	15
Accident types	A	a	5
Accident types - multiple units	A^S	a	4
Vehicle types	V	v	7
Number of units	U	u,o	18

The percentage gap between the best bound and the best integer solution found is a good indication of the solution quality. As the search proceeds through the branch-and-bound tree, the best integer solution improves. The gap is calculated using the expression from equation (45). Notice that IVE Xpress calculates the gap using the best bound as the denominator, rather that the best solution found.

$$Gap = 100 \times \frac{\textit{Best bound - Best sol}}{\textit{Best sol}} \tag{45}$$

6.1 Computational Time

The acceptable solution time and accuracy to an optimization problem varies with the case at hand. This is generally a trade-off where a lower computing time is favoured in operational planning and an accurate solution is more important in strategic and tactical planning. In addition, near optimal solutions can be acceptable in instances where there is considerable uncertainty to the input parameters (Lundgren et al., 2010).

The MPLP-HAR is a strategic and tactical decision support tool, which emphasis solution quality and accepts longer computational time. The problem has therefore initially given a run-time of 10 hours. It is generally difficult to determine the optimal run-time for a tactical and strategic planning problem, but it is always desirable to reduce it if possible. The problem has also been solved with a run-time of 5 hours and 30 minutes. However, with the 30 minute run-time the model is unable to produce any feasible solutions.

Another concern when solving large-scale problems, is the required computer memory. The use of vast computer memory limits the computational time to a maximum of about 10 hours for this problem, as the computer runs out of memory before the optimal solution is found. It struggles to find any solutions, let alone feasible solutions, whenever the problem becomes large enough. This implies that IVE Xpress is unable to solve the LP-relaxation of the problem to optimality in the root node and therefore also unable to commence the branch-and-bound solution strategy. This is the case when the original MPLP-HAR formulation is computed, denoted Test 0 (T0). For T0, the optimization software is unable to find a solution at any of the tested run-times. There are also examples where the LP-relaxation is solved in the root node and an upper bound is established, but no feasible solutions are found in the nodes of the branch-and-bound tree.

From the general findings in Table 7 there are only minor differences in the optimal solutions for the 5 hours and 10 hours tests. The benefits of considerably increasing the run-time past 5 hours are minimal. For the computational study of FRS problems in Section 10, a standard run-time of 5 hours is utilised. The computational time is considered acceptable as the MPLP-HAR is strategic and tactical decision support tool. The 5 hours test is also sufficient to generate solutions of a desired quality, when given model improvements are implemented.

6.2 Model Improvements

In order to reduce the size of the problem, several model improvements are suggested. First, alternative constraints are suggested to reduce the number of columns generated. Reducing the number of constraints contribute to a reduced computing time in each node generated in the branch-and-bound tree. Next, symmetry breaking constraints are included in the formulation. This removes a number of mathematically unique solutions that, for practical purposes, are considered equal. Finally, suggested heuristic solution methods are investigated.

Table 6: Description of the different tests performed

		Test description
Test	Constraints	Comment
T0	-	The initial model
T1	(37)	One constraint for w_{iajuko}
T2	(38),(39)	w_{iajuko} aggregated over J and U
T3	(42),(43)	w_{iajuko} aggregated over I, A, J and U
T4	(40)	w_{iaju} aggregated over I and A
T5	(41)	s_{iaju} aggregated over I and A
T6	(37),(40)	T1 and T4 combined
T7	(37),(41)	T1 and T5 combined
T8	(38)-(40)	T2 and T4 combined
T9	(38),(39),(41)	T2 and T5 combined
T10	(40),(42),(43)	T3 and T4 combined
T11	(41)-(43)	T3 and T5 combined
T12	(37),(40),(41)	T1, T4 and T5 combined
T13	(38)-(41)	T2, T4 and T5 combined
T14	(40)-(43)	T3, T4 and T5 combined

6.2.1 Reduce the Number of Constraints

In the original problem formulation, presented in Section 5, the number of restrictions increases rapidly with growing sets. Particularly, Restrictions (30) and (31), that are defined for all sets, give cause for concern. By replacing some sets of constraints with combined or aggregated sets, the number of constraints are reduced. An alternative way of reducing the number of restrictions is by introducing a Big-M formulation to a number of the restrictions. The idea is to limit the number of restrictions by finding an upper bound for the number of binary variables that can take a non-zero value. That is, the sum of all binary variables in the problem cannot exceed a given value M. The effectiveness of this method depends on how tight the upper bound is, i.e. how small one is able to set the value M.

Different combinations of the model improvements (37)-(41), described in Section 5.3, are tested with at a run-time of 5 hours and 10 hours, respectively. The test T0 is the original model without any of the improvements. T1 - T14 are different combinations of the model improvements that attempt to reduce the number of restrictions. From Table 7 it is apparent that Constraints (38) and (39) are needed to provide acceptable solutions. In this case, a gap of 5% is considered acceptable, and thus T8, T9, and T13 are acceptable with a 5 hours run-time. This implies that combining the aggregated Constraints (38) and (39) with one or both Constraints (40) and (41) yields the best model formulation. T8 is the formulation that yields the best result in the 10 hours test. Because T2, T8, T9, and T13 yield the best results for both the 5 hours and 10 hours test, these solutions form the basis for further technical tests.

The main reason for the improvements by introducing Constraints (37)- (41), is the dramatic reduction of constraints in the problem. Solving the problem with the original formulation, as is done in T0, generates more than 8,7 million rows, while the most effective tests contains less than 1% of that.

Table 7: Testing the model with additional constraints

		5 hours					10 hours				
Test	Rows	Nodes	LP	Best bound	Best sol.	Gap	Nodes	LP	Best bound	Best sol.	Gap
T0	8773998	0	-	-	-	-	0	-	-	-	-
T1	4400058	0	-	-	-	-	0	-	-	-	-
T2	58398	323	0.16724	0.16546	0.10936	51.13%	883	0.16724	0.16546	0.15873	4.24%
T3	26538	0	-	-	-	-	-*	-	-	-	-
T4	8770218	0	-	-	-	-	0	-	-	-	-
T5	8754288	0	-	-	-	-	0	-	-	-	-
T6	4396218	0	-	-	-	-	1	0.16956	0.16634	-	-
T7	4380288	0	-	-	-	-	0	-	-	-	-
T8	**54618**	**712**	**0.16824**	**0.16550**	**0.15925**	**3.92%**	**1587**	**0.16824**	**0.16549**	**0.16069**	**2.99%**
T9	38688	1347	0.16846	0.16601	0.15749	5.41%	4370	0.16846	0.16601	0.16010	3.69%
T10	22758	0	-	-	-	-	-*	-	-	-	-
T11	6828	17	0.16959	0.16865	-	-	-*	-	-	-	-
T12	4376508	0	-	-	-	-	0	-	-	-	-
T13	34908	581	0.16933	0.16621	0.16016	3.78%	2551	0.16941	0.16620	0.16033	3.66%
T14	3048	17	0.17053	0.16991	0.08433	101.49%	-*	-	-	-	-
Best solution			0.16824	0.16550	0.15925	3.92%		0.16824	0.16549	0.16069	2.99%

* No results due to out of memory error

6.2.2 Symmetry Breaking Constraints

Another method for reducing the problem size, is the inclusion of symmetry breaking constraints. By assuming that all units of the same vehicle type are equal, symmetry breaking constraints for vehicle types with multiple units are included in the model. The results are presented in Table 8.

Table 8: Testing the model including symmetry breaking constraints

Test	Rows	Cols	Nodes	LP	Best bound	Best sol.	Gap
T2	58409	1041505	444	0.16724	0.16483	0.15738	4.73%
T8	54629	1041505	832	0.16824	0.16536	0.15987	3.44%
T9	38699	1041505	2331	0.16846	0.16599	0.15746	5.42%
T13	34919	1041505	1115	0.16941	0.16601	0.15719	5.61%

Compared to the initial model, T2 shows a considerable improvement in the objective value by including symmetry breaking constraints, and T8 yields a slightly better objective value. On the contrary, T9 and T13 both experience a reduction in the objective value. This might be due to the large number of nodes generated. In a large branch-and-bound tree the algorithm can get stuck in a sub-tree that do not contribute to improve the solution.

6.3 Heuristic Solutions

As discussed in Section 5.4, heuristics are relevant for the MPLP-HAR and a number of heuristic solution approaches are therefore investigated. First, a simple LP-relaxation is conducted. Then a stochastic variable generation is applied and finally, the problem is modeled assuming homogeneous units.

6.3.1 LP Relaxations

The LP-relaxation is performed by relaxing the binary restrictions on all variables except for the solution variables, x_{ju}. The solution is then inserted into the original problem, fixing $x_{ju} = \bar{x}_{ju}$. When resolving the problem, the optimal objective value is found. This is tested using the model formulation T8, as this yields the best results in the previous tests performed. The solutions obtained from the LP-relaxation are described in Table 9.

Table 9: Testing T8 with LP-relaxations

Test	Rows	Cols	Nodes	LP	Best bound	Best sol.	Gap
MIP	54618	1041505	712	0.16824	0.16550	0.15925	3.92%
LP	54618	1041505	0	0.16824	0.16738	0.16671	0.40%
$\bar{x}_{ju}$	54636	1041505	1	0.16671	0.16090	0.16090	0.00%

The best solution's objective value from the LP-relaxation is an upper bound to the original MIP problem. When fixing the solutions $\bar{x}_{ju}$, the objective value is lower than the solution obtained in the LP-relaxation. Hence, the solution is not optimal. When solving the LP-relaxation, some of the w_{iajuko}, w_{iaju}, and s_{iaju} variables are not binary. The solution is therefore considered a heuristic, as the variables are not kept binary in the LP-relaxed solution. However the LP-relaxation reduces the computational time to approximately one hour.

6.3.2 Stochastic Variable Generation

The stochastic variable generation algorithm, explained in Section 5.4.2, is used to generate a stochastic subset of the w_{iajuko} and w_{iaju} variables. The number of variables generated is decided by the δ parameter. The initial number of the w_{iajuko} variables are equal to the product of the size of the sets used, $|I| \cdot |A| \cdot |J|^2 \cdot |U|^2$. The number calculates to a total of 5467500 variables. Of these the number of non-zero variables range between 18 and 75. This implies that the percentage of non-zero variable range between 0.00033% and 0.001372%.

Knowing how many variables that need to be generated to obtain a sufficiently good solution is difficult. The heuristic is tested for three δ values, 0.01, 0.05 and 0.1 using the model formulation from T8. In Table 13 the results from the tests are given.

Table 10: Testing T8 with stochastic variable generation

δ	Rows	Cols	Nodes	LP	Best bound	Best sol.	Gap
1	54618	1041505	712	0.16824	0.16550	0.15925	3.92%
0.1	54618	123733	2149	0.16742	0.16261	0.15807	2.87%
0.05	54618	73365	4202	0.16582	0.16089	0.15622	2.99%
0.01	54618	32362	10707	0.15891	0.15341	-	100.00%

The results indicate that a stochastic variable generation approach can provide useful solutions if an appropriate δ value is chosen. The best result is found when 10% of the w_{iajuko} and w_{iaju} variables are generated, i.e. δ is set to 0.1. When δ is equal to 0.01 the MPLP-HAR is unable to find a feasible solution. The best bound is even determined to be lower than the best solutions from the other tests, implying that the subset of variables generated are to small to provide a useful solution.

The initial algorithm generates the variables randomly. As previously discussed, however, a better approach to find the optimal solution is to establish a criteria upon which the random variables can be generated. Next, the algorithm is therefore altered with a random percentage of the variables for all j's, u's, k's, and o's only. This implies that once the location and unit is determined for first and total response, they are eligible for all possible accidents, a, in any demand zone, i. In other words, the best objective value is determined with given unit locations. The new algorithm is tested for δ equal to 0.1, which provides the best results in the previous test. The results are summarized in Table 11. The new algorithm shows improvement in the objective value, which illustrates the effect of applying selective conditions to the stochastic variable generation algorithm.

Table 11: Testing new version of stochastic generation

#	δ	Rows	Cols	Nodes	LP	Best bound	Best sol.	Gap
1	0.1	54618	123733	2149	0.16742	0.16261	0.15807	2.87%
2	0.1	54618	124875	2240	0.16765	0.16300	0.15969	2.08%

6.3.3 Model with Homogeneous Units

By assuming that all units within a certain vehicle type are equal, the model can be redefined, as described in Section 5.4.3 and Appendix A. The results from this model is presented in Table 12. Notice that the number of rows in T0 is relatively high, compared to the other tests. The number of rows after the pre-solve operations in IVE Xpress is markedly lower than the initial number of rows. This implies that this model improvement is inefficient. The formulations of T9 and T13 are not regarded in the redefined model, due to the redefinition of constraints that result in these improvements being inappropriate.

Table 12: Testing the redefined model where only vehicle types are considered

Test	Rows	Cols	Nodes	LP	Best bound	Best sol.	Gap
T0	1341352	83860	5396	0.16661	0.15929	0.15423	3.28%
T2	30952	83860	18000	0.16390	0.15900	0.15689	1.35%
T8	29482	83860	18000	0.16552	0.15912	0.15551	2.32%

The results from Table 12 indicate that T2, and secondly T8, perform the best under this assumption. The reduction in problem size from the assumption of homogeneous response units, enables all of the formulations to examine an increased number of nodes. The solution quality is then also improved accordingly.

6.4 Comparing Heuristic Solutions

A comparison of the objective values in the heuristic solutions is presented in Table 13. The respective solutions are evaluated in the original MPLP-HAR with the T8 formulation. All tests are solved to optimality, as the x_{ju} variables are fixed. The objective values indicate that the original problem, with the T8 formulation, provides solutions of comparable quality to both the LP-relaxation and variable stochastic generation approaches.

Table 13: Comparing the objective value from the different heuristics inserted in the original model

Model	LP	Best bound	Best sol.	Gap
Original	0.16539	0.16022	0.16022	0.00%
LP	0.16671	0.16090	0.16090	0.00%
Stochastic	0.16429	0.16083	0.16083	0.00%
Redefined	0.16358	0.15742	0.15742	0.00%

In addition to the objective values, it is important to consider the actual response unit locations, given by the x_{ju} variables, when evaluating the solution quality of heuristic solution methods. All three heuristic solutions are structurally similar to the one obtained by the original model. First response is given considerable emphasis through the preservation function, resulting in a characteristic spread of the units that enable them to quickly reach all demand zones in the network. Any redundant response units are concentrated in high demand areas. However, in the LP-relaxation, a number of the w_{iajuko} and w_{iaju} variables take a fractional value. Now, this does not reflect an operational state as it would suggest that a unit u or o, in candidate location j or k, would only partially be providing first or total response to an accident a in demand zone i. This occurs because the convex-nature of the preservation function yields a higher objective value from compiling fractional w_{iajuko} and w_{iaju} values, as opposed to having strictly binary variables.

7 Case Study

In this section, the fire and rescue service in Oslo (Brann- og Redningsetaten, BRE) is presented. BRE is examined to understand how FRS is performed in Oslo. Initially, their organizational structure and resource management are described. Further, the accident classification and routines for emergency response are outlined and their perception of response time and preparedness is discussed. Finally, this section looks into some of the upcoming challenges imposed by BRE.

The information presented in this section is collected from The Annual Report 2014 (BRE, 2014b), The Statistics Report 2014 (BRE, 2014a), and The Preparedness Analysis 2015 (BRE, 2015), supplemented by meetings and interviews with representatives from BRE.

7.1 The FRS in Oslo

BRE is responsible for performing FRS in the municipality of Oslo. Together with other local capacities of emergency response in Norway they are part of the National Fire and Rescue Service (Direktorat for samfunnssikkerhet og beredskap, DSB). DSB is under the control of the Norwegian Government and all the local FRS distributors receive common statutory requirements from the Goverment. In addition, the each of the local FRS capacities develop a Risk Analysis to uncover individual risks in their specific municipality.

7.2 Organizational Structure

FRS in Oslo has an Emergency Service with a total of 315 employees spread across seven full-time operational fire stations. Four of these are inner city stations and are located at Briskeby, Sagene, Bryn, and the main station located inside of Ring 1. Each of these stations are staffed with at least six firefighters at all times. Outside the city centre, the FRS has three stations, located at Smestad, Grorud, and Kastellet, staffed with teams of four firefighters. In total, the emergency staff consists of at least 38 firefighters at all times. The available emergency staff is summarized in Table 14. In addition, FRS in Oslo is staffed with an Operations Chief, a Rescue Department with four employees, two aspirants, and a supporting staff of eight employees. The Rescue Department is used for incidents related to dangerous chemicals and are hence not a part of the emergency preparedness staff. The aspirants are students training to become firefighters. As they are not certified firefighters they are not included in the staff, according to Norwegian Laws and Regulations (Lovdata, 2002). The supporting staff consists of three ladder units staffed with a crew of two firefighters and a fire vessel, also staffed with two firefighters. Altogether, at any given day, the FRS in Oslo has the capacity of keeping a staff of 54 employees.

Table 14: FRS operational teams in Oslo (BRE, 2015)

Emergency staff		
Station	No. of teams	Total staff
Briskeby	2	6
Sagene	2	7
Bryn	2	6
Main station	2	7
Smestad	1	4
Grorud	1	4
Kastellet	1	4
Total staff	11	38

7.3 Resources

An FRS organization requires a number of resources to perform its tasks. This includes different vehicles with the necessary equipment and competent personnel.

In the case of an emergency, every base unit is staffed with three to five firefighters: a driver, a team leader, and one to three rescue divers. Rescue divers are firefighters with self-contained breathing apparatus (SCBA) capacity. The rescue diver vehicle is staffed with three to five rescue divers, including a driver. The aerial apparatus is staffed by a driver and a ladder-operator, while the tanker truck requires a driver. The skippers and machine suits operate the seaborne vessel. Each firefighter has its own set of competencies and can deploy a number of different roles in the line-up. In addition, fire officers follow the emergency response in command vehicles, whenever there is need for further coordination between units at the accident site. At any given time, the FRS in Oslo has the capacity of keeping a total staff of 54 firefighters on call, where 38 of these are in the emergency staff. Table 15 illustrates Oslo's resource allocation status, as of March 2015. The primary resource location is the station to which the resources are assigned. Often however, response units and personnel are positioned at different locations in conjunction with training, coursing, or preventive work.

Table 15: Resource allocation at the different fire stations in Oslo (BRE, 2015)

FRS resources in Oslo							
Resources	Briskeby	Sagene	Bryn	Main	Smestad	Grorud	Kastellet
Staff							
Operations Chief	0	0	0	1	0	0	0
Lieutenant	1	1	1	1	1	1	1
Interim Lieutenant	3	3	2	4	1	1	1
Firefighter	4	5	3	8	2	2	2
Aspirant	0	0	0	0	1	0	1
Vehicles							
Base unit	1	1	1	1	1	1	1
Rescue diver unit	1	1	1	1	0	0	0
Ladder unit	1	1	0	1	0	0	0
Rescue boat	0	0	0	1	0	0	0
Pollution unit	0	0	0	1	0	0	0
Rescue unit	0	0	0	1	0	0	0
Tow truck	0	0	0	1	0	0	0

In addition, BRE has a port in the city harbour with a rescue boat, a work boat, and an emergency pollution vehicle, staffed with a interim lieutenant and a firefighter. These units are located in the same district as the Main station, and are therefore assumed to have the same location as the resources currently at the Main station location. All stations are also equipped with one or several backup vehicles of different types. These are not staffed on a regular basis and hence not part of their emergency preparedness. They are used whenever there are mechanical issues or scheduled maintenance on vehicles in the operational fleet.

7.4 Accidents

In 2014 BRE received 13.400 emergency calls, where almost half of these were classified as false or unnecessary calls. Fires and service to other emergency services received the largest amount of calls. Other categories are traffic and other accidents, accident with dangerous goods, drowning, and water accidents. These categories can be further divided into several sub-groups. A graphical schedule of all the different calls are summarized in Table 16 with data from the last three years (BRE, 2014a).

Table 16: A complete graphical schedule of all calls from the last three years

Number of accidents in Oslo			
Accident types	2014	2013	2012
Fire accidents	2731	2529	2352
Traffic and other accidents	450	492	453
Accidents with dangerous goods	33	30	23
Drowning accidents	73	56	45
Water accidents	341	348	398
Service to other emergency services	2750	2387	2003
False or unnecessary alarms	6554	5947	5547
Total	13400	12260	11311

When BRE receives a call at the 110-central, the dispatcher categorizes the call according to a set of criteria, based on the type of accident and the severity of the incident. The dispatcher is supported by a dispatching system that displays the location and status of all response units in the area. Based on the given emergency location and calculated response times for the required units, the system suggests which units to dispatch. However, the system does not take traffic and other variable factors into account and therefore the dispatcher may choose a different dispatching strategy, based on his or her own experience. When the dispatcher allocates units to an accident, the fire stations in question are alarmed. The teams dispatched to the accident prepare and depart from their initial locations. When they arrive at the accident site, some additional preparation is needed before the rescue operation begins. The number and type of response units that are dispatched varies depending on the accident classification and severity.

7.5 Emergency Response Routines

Emergencies are reported to the 110-central in Oslo and from here the emergency response is organized. As soon as the emergency call is received, it is classified into one of the categories based on the information received from the caller. When the emergency is reported through an automatic alarm, the classification is done automatically. Each category has a pre-defined standard response with respect to the number of each response unit that is dispatched. Based on the the nature of the ongoing accident, a computer program suggests what units to dispatch depending on their current status and position in relation to the accident site. The suggestion can however be overruled by the 110-professionals if there are special considerations, with respect to either staff, equipment, or logistics, that would suggest an alternative optimal dispatching plan. The average response time from the call is received to the first unit arrives at the accident site is six minutes and six seconds, but this time is often less if the emergency requires so (BRE, 2016).

BRE operates with units organized into rescue teams. A rescue team consists of a base unit, a rescue diver vehicle, and an aerial apparatus. For emergency calls where a lifesaving effort is required, the routine involves dispatching one rescue team. Examples of such calls include minor fires or unclassified alarm calls. For larger fires or accidents, more than two rescue teams are dispatched. Such emergencies are characterized by a long duration of involvement, additional operational management, and specialized internal and external competencies are required. This includes fires in building complexes or institutions, forest or tunnel fires, or natural disasters.

There are also emergency calls where only a single response unit is required. These include smaller fires outside of built up areas, water leakages and ambulance calls to the rescue vessel. BRE is building capacity to be able to respond to simultaneous emergency calls from all three severity classifications (BRE, 2015). Handling three simultaneous emergency calls is an adjustment from the objective of dimensioning for two simultaneous calls, stated in 2010 (ROS 2010). By having this ambition, BRE is going beyond the statuary responsibility of emergency response for the FRS, that only state a maximum response time for the first emergency to occur, stated in the jurisdiction (Lovdata, 2002).

A number of calls do however require specialized equipment and resources. Examples include drownings and accidents at sea, which require the use of a boat, or emergency pollutions, biological-, chemical-, and environmental hazards, which require the emergency pollution vehicle. In addition, a number of the FRS engagements are so-called non-emergency calls. Salvage rescue is an example of such calls, as well as pre-planned inspections and preventive work. These will not be considered further, as the focus of this report is on planning for emergency calls that require immediate response.

The vehicle capacity and its ability to perform at the site is also dependant on the staffing and equipment level. A base unit can be manned by up to five firefighters at a time, meaning that there are two teams of rescue divers in addition to a team leader. In that case, one of these teams are ready to start the rescue dive immediately upon arrival and the responding unit has a larger capacity to handle other tasks in this critical initial phase. However, the current staffing level do not allow for all base units to be manned by five firefighters, and often times they are manned by only three or four. In both these cases, the driver has to change into his or her diver equipment before two teams are operational, which is estimated to take an additional two minutes. The rescue procedure is standardized for all these staffing levels, and the optimal allocation of manpower then becomes a question of efficiently utilising available resources. However, having at least four firefighters on each base unit means that the first team of rescue divers are ready to operate upon arrival. This is a considerable advantage, as the first minutes are the most critical in order to save lives (BRE, 2015).

Emergency response routines are largely effected by at-the-site operational decisions. To get access to the accident site, certain units might occasionally utilise alternative routes, other than the calculated shortest path. The shortest path might be too narrow for certain vehicles to operated properly or obstructed by other response units. To further facilitate logistics at the accident site, a number of units might be dispatched with an intended delay, giving the operations chief an opportunity to establish an overview of the accident before the resources arrive. This is particularly relevant when considering that not all units are required at the same time. A tanker truck for example, will not be needed until the fire engine runs dry from its own on-board water supply. This will however not be the focus of this report, as it considers planning at a strategic and tactical level, and these considerations are addressed solely on an operational level.

7.6 Response Time

The lapse of a rescue process is divided into several time periods, as illustrated in Figure 5. The time from a call is received until the rescue team is alarmed is referred to as the dispatching time. The preparation time is the time from the alarm is received to the rescue team departs from the original location and the travel time is the time from departure to arrival at the accident site. When arriving at the site, the rescue team preforms some last preparations before the rescue operation is initialized. This is referred to as the intervention time. The operation time refers to the duration of the rescue efforts. This often constitutes a large part of the total rescue process. Note that the distances in Figure 5 do not reflect the actual relations between the time slots. After the rescue operation is completed, the rescue team travels back to the station and performs after service work, such as water refill. It is not until this process is completed, that the response unit is available to respond to a new call.

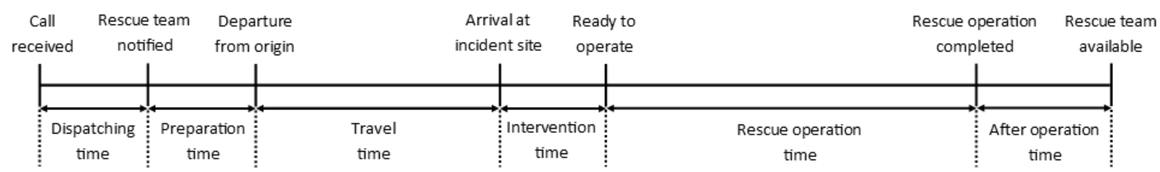

Figure 5: The time lapse of the rescue process

When performing strategic and tactical planning, the performance is evaluated primarily with respect to the response time. The response time is defined as the time it takes from a call is received until the rescue team begins operating at the accident site (DSB, 2002). This includes the dispatching time, preparation time, travel time, and intervention time. The effort time is defined by BRE as the time it takes from a station receives an alarm until the rescue team is ready to operate at the accident site. Thus, it disregards the dispatching time. The effort time of a vehicle in FRS is illustrated in Figure 6. The preparation time and intervention times show less variations than the travel time, and may be considered fixed. The travel time is calculated based on the travel distance and speed limits along the route. The speed limits are set based on what is considered prudent at a certain distance, from a risk consideration, and the FRS should therefore not plan to drive at a higher speed. However, according to BRE the travel time show variations between the different vehicles. The rescue diver vehicle has a better maneuverability than the base- and ladder unit. In addition, the travel time also varies according to other factors such as the route of travel, traffic, and special events. As the travel time may be both reduced and increased according to a set of uncertain factors, calculating the travel time based on the speed limits and the shortest path from origin to destination, is assumed to be a reasonable simplification.

The statuary maximal total response time, required by DSB, is determined according to a set of criteria. The response time in congested areas, where the risk of extensive spread of fire is high, should not exceed 10 minutes. In urban areas, the response time should not exceed

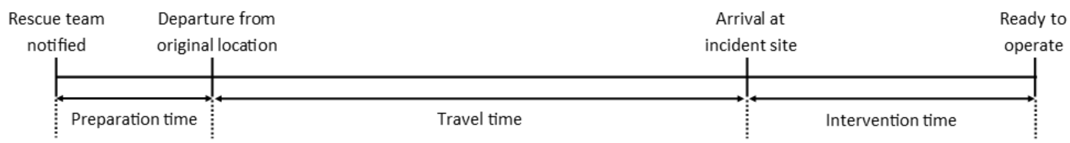

Figure 6: The effort time of a vehicle in FRS

20 minutes. In rural areas, the maximal required response time is 30 minutes (DSB, 2002). In addition, there are a number of special considerations regarding response times for high risk buildings, such as schools, hospital, hotels, and other public institutions.

It is important to keep in mind that the response time in this case is defined and calculated from the point in time when the emergency is reported to the FRS. In that sense, it does not consider the time it takes from the accident commences to the dispatched units reach the site. The response time is in some cases heavily reliant on witnesses with means of communication to report the emergency. The use of smoke detectors, alarm systems, and surveillance equipment is therefore an important tool in detecting and reporting emergencies at an early stage with accurate details of location and development.

7.7 Preparedness Analysis

BRE is a particularly interesting organization to study in this context. Firstly, is it the largest FRS institution in Norway with the most infrastructural facilities, the largest fleet of vehicles, and the most manpower. BRE also keeps detailed information and statistics regarding emergency calls in the area, which is prerequisite to perform accurate optimization studies in ERP.

7.7.1 Resource and Facility Expansion

The preparedness analysis in BRE (2015) reveals a few discrepancies in the current operational state of the organization and a few challenges that will have to be met in the years to come. These issues are addressed through a number of improvement measures. According to the preparedness analysis, Oslo is the fastest growing capital in Europe and the population will reach 660 000 within 1-2 years (BRE, 2015). Today Oslo have 634 463 inhabitants (SSB, 2015). At this population level, BRE has a regulatory imposed responsibility to include another team of firefighters to their operational state (Lovdata, 2002), consisting of at least four firefighters and a fire engine. At the current rate of emergency calls in the area, BRE has expressed the need to be able to handle three simultaneous emergency calls of varying severity.

The geographic expansion of the municipality also implies that BRE has to make strategic changes to the infrastructure. In the preparedness analysis, the main station is suggested relocated from the city center to Bryn. This implies establishing a new facility in the city center that can provide preparedness for the area inside Ring 2. Furthermore, the analysis concludes that the current fire station at Kastellet should be relocated and suggests establishing a new facility and in the southeastern part of the jurisdiction. All the fire stations along the perimeter should have a staff of at least three firefighters with SCBA-capacity, to ensure the ability to efficiently handle building fires.

The new main station at Bryn is intended to get a new ladder unit, in addition to the existing three units located at the inner stations. Today, the aerial apparatuses are favourably located, but the demand dictates the acquirement of a fourth. In time, the organization is also intended to acquire a new fire engine with foam capacity and the ability for a radio controlled water canon.

7.7.2 Special Considerations

In the Norwegian Fire and Explosion Prevention Act (Lovdata, 2015) the minimum service standard for response times are stated, as previously discussed. In built-up areas with a high risk of proliferation or particularly severe damage to life or property, the first response time cannot exceed 10 minutes. In addition to the city center, this also includes hospitals, accommodations, assembly facilities, and other institutions, categorized as special fire objects. In otherwise densely populated areas, the maximum first response time is 20 minutes and otherwise it is set to 30 minutes. These regulations are imposed on all municipalities in the country, and when it comes to BRE, most of the areas within the city center would be categorized as densely populated areas and have maximum response time accordingly.

The special fire objects include tunnels longer than 500 meters, and Oslo have more of these than any other municipality. These require special attention and are a particular challenge for rescue divers because technical limitations makes it difficult to perform over long time periods and distances. Being the capital and most populous city also brings about special considerations for FRS. Firstly, many important societal functions are run from Oslo and they are vulnerable to fire accidents and at risk for sabotage and terrorism. Secondly, an emergency in the city's underground metro can have particularly severe consequences. Finally, the jurisdiction is a national hub for infrastructure and includes highways, tunnels, train stations, and harbours with busy traffic and transportation of dangerous goods. The high concentration of special fire objects means that BRE also must utilise considerable resources to conduct required preventive work and inspections at these sites.

8 Preparedness in FRS

Preparedness measures are used to evaluate how prepared an emergency response institution is to handle an event that has yet to occur, e.g. accidents or emergencies (Granberg et al., 2015). In previous literature there exist no general definition of preparedness (Andersson and Värbrand, 2007) and therefore no universal quantifiable measure. However, it is important to understand and measure the level of preparedness for any institution that is responsible for emergency and disaster response, in order to evaluate the effects of improvement measures(Granberg, 2013). In this section, a quantified preparedness measure is developed from the methodology presented in Granberg (2013). This measure is developed for an FRS application and forms the basis for the objective function in the mathematical model.

8.1 Select Event and Perspective

In this report, the preparedness is measured and evaluated for FRS, and hence an organizational perspective is used. In the context of FRS, the measure is conducted for multiple events. This includes a large variety of different accident types. In cooperation with BRE, the accident types are aggregated into different categories, based on the resource requirements and the accident severity. The categories identified are: fire accidents, fire alarms, traffic accidents, sea rescue, and alarms. Constructing a measure encompassing multiple accident types increases the number of factors that needs to be involved and thus also the complexity of the preparedness measure. The purpose of preparedness for the BRE is to support ERP and resource management, including decisions related to location, allocation, and relocation of resources. However, this report focuses on location of resources on a strategic and tactical planning level.

8.2 Select Indicators

In order to measure the preparedness, given the event and perspective, quantifiable indicators are needed. Granberg (2013) have identified three main indicators that affect the preparedness in FRS, namely demand and response time. These indicators are somewhat extended to incorporate the number of events that are relevant in FRS in this section. Firstly, there is the likelihood that an emergency occurs in a particular area. The area in question is divided into a number of smaller demand zones that each have an expected rate at which the different accidents occur. In this case there are a number of different accident types that each require a unique set of resources. Therefore each zone i is given a demand factor for each accident type, a, D_{ia}. The total demand for the area sums up to one. That is, each D_{ia} is given a fractional value proportional to their its rate of occurrence.

In addition to the relative rate of occurrence, the different accident types also differ in severity. Although the seriousness of an accident varies according to the specific incident, each accident type has a general degree of severity. For instance are fire accidents and sea rescue considered more serious than traffic accidents, alarm calls, and other rescue services. In the case of fires and sea rescue the response time is considered substantial, relative to the other accident types. As a result, each accident type is given a severity factor, λ_a, that reflect the general need for immediate response for the given accident type a.

The expected number of FRS response units that can be dispatched to a given emergency is defined. The nature of the accident, a, dictates not only what type of emergency response units that are required, but also how many of each vehicle type. Because some accidents require more than a single resource or response unit, the response time is divided into first and total response. The first response is the time it takes for the first required unit to respond, and the total response represents the time it takes for all required resources to be ready to operate at the accident site. Both the response time and expected demand are quantifiable indicators. The response time requires input locations, travel distance, and speed. To the FRS it is important to focus on both on first and total response, when considering preparedness. In this case, the first and total response units are determined based on their initial location in relation to the accident site. The locations j and k of the available first and total response units u and o, required to an accident a occurring in demand zone i, are summarized in the binary variables w_{iajuko}. They indicate whether unit u located in zone j provides first response and unit o in zone k provides total response to accident type a in zone i.

Finally, timing is a crucial indicator of good preparedness. It is important to measure the time it takes for a response unit to respond to an accident in any given demand zone. The response time includes both travel time and necessary preparations. The time it takes to reach the emergency site also affects the ability to perform effective emergency relief, as the accident severity increases with time. The different dispatched units also have capabilities that are required for different accident types. All these aspects are summarized into a preservation parameter, P_{iajuko}. Coupled with the location variable w_{iajuko}, the preservation parameter incorporates first and total response times and gives a value for different accident types depending on the vehicle type dispatched.

Next, the determined indicators must be combined to a preparedness measure. In this case the indicators consists of a demand parameter, an accident severity factor, and the first and total response times, represented by a preservation function and a location variable.

8.3 Combine the Indicators

Based on the indicators discussed above, equation (46) proposes a combination that makes up the preparedness measure.

$$P(\lambda_a, D_{ia}, t^F, t^T) = \sum_{i \in I} \sum_{a \in A} \lambda_a D_{ia} \sum_{u \in U} \sum_{j \in J} \sum_{o \in U} \sum_{k \in J} P_{iajuko} w_{iajuko} \tag{46}$$

The indicators must be combined in such a way that the measure possess the desired qualities. In the case of ERP in FRS it is considered good preparedness to have available and qualified units respond to the areas where the demand is high. The area and accident specific demand parameter, D_{ia}, is therefore multiplied linearly with the severity factor for each accident type, λ_a, and the vehicle location variable, w_{iajuko}. The value is summed over all accident and vehicle types, and all vehicle- and demand zones.

The desired property for the response times is to be negatively correlated with the value of the preparedness measure. That is, the preparedness improves as the response times are reduced. The preservation parameter must therefore have a negative development with respect to the response time. The structural identity of the preservation function is presented in equation 47. The preservation function includes a exponential and linear part that both decrease as a weighted multiple of the first and total response times, t^F and t^T.

$$P_{iajuko}(t^F, t^T) = e^{-\alpha_a \cdot t^F} - \beta_{a,u} \cdot (t^T - t^F) \tag{47}$$

The preparedness value is relative is relative and therefore it is meaningless to evaluate a given operational state on its own, using this preparedness measure. The value for one given state is however able to tell us how well an organising of the institution performs in relation to another. The preparedness measure can be used in an optimization problem to evaluate a location solutions of w_{iajuko} in an objective function of a preparedness maximization problem.

8.4 Validate the Measure

In Granberg (2013), three methods are used to evaluate the preparedness measure. This includes validation by FRS professionals, validation by simulation, and comparison to existing measures. In the computational study in Section 10, the validation methods are displayed in greater detail. FRS professionals were consulted throughout the process of developing the preparedness measure in this study. Firstly, the structure of the measure was evaluated to ensure that the right indicators were included and weighted according to significance. Moreover, the measure must reflect good levels of preparedness in the eyes of professionals. The generated solutions are therefore evaluated by representatives from BRE in order to ensure that the measure reflects their understanding of preparedness.

Through simulation and application of the preparedness measure the identities, and characteristics can be validated. The measure is in this case validated through a sensitivity analysis of the indicators. By applying the measure to a known problem or case study, it can be observed that the changes in indicator identity yield the desired changes to the preparedness value. This would typically involve a sensitivity analysis on response time or demand to see that the preparedness level changes accordingly.

The final validation method involves the use of existing problems and methods. In Section 10, the developed preparedness measure is used in the objective function to solve the MPLP-HAR. The results are compared to the results from a p-median problem and a Maximum Coverage Location Problem. Even though these problems have a slightly different view of preparedness, they are expected to follow the same general trends when it comes to the perception of good preparedness. Seeing that the developed measure behaves structurally similarly to established preparedness measures of ERP resource locations is an important part of the validation process.

9 Data

The basis for the computational study is the geographical area and municipality of Oslo. This section describes the characteristics of the provided input data. Data accuracy is crucial for the MPLP-HAR to be a useful decision support tool for FRS professionals and in this case the data reflects the current operational state of BRE in Oslo.

9.1 Methodology

The data has been collected using a number of different approaches. The accident data is collected from BRE and consists of accident statistics from the first quarter of 2016. BRE implemented a new semi-automatic system for reporting emergency responses in 2015, and has started obtaining statistics since January of 2016. The new system is named BRIS and integrates reporting from dispatchers and firefighters, as well as a post-accident evaluation. The dispatcher enters in any information obtained from the reporter of the emergency. This can later be altered once the circumstances of the emergency has been clarified. Firefighters can also provide information about the response times and rescue operation time from the response units. However, this specific information has not been used in the computational study of this report, as the FRS professionals are still adapting to the routines of applying the new system and the data from this particular aspect is uncertain. The data set used includes address and postal code for the emergency, accident type, and the number and type of resources dispatched.

The data set had some discrepancies and therefore some corrective changes were made. First, due to lacking information about the district of the accidents, a data set of postal codes in all city districts was collected from the Agency for Planning and Building Services in Oslo (Oslo Plan- og Bygningsetat, PBE). Some of the addresses were also lacking a postal code or displayed erroneous postal codes. Using a Python script and a Google Maps API the correct postal codes for each address was obtained. The final input demand was calculated using Microsoft Excel. Further, the travel times were calculated using a Python script and a

Google Maps API. The travel time matrix that was obtained, was used in the calculation of the preservation values.

Based on the number and type of resources, the different accident types were aggregated into five accident types. The process of finding appropriate accident types and general resource requirements was consulted by FRS professionals at BRE. Emergency calls were then classified in the determined categories based on the nature of the emergency, required resources, and accident severity. A preservation function was then formulated in corporation with operational representatives from BRE and later generated using MatLab R2015a.

9.2 Demand Zones

The region of Oslo is divided into 15 different zones, where each city district represents a demand zone in the problem formulation. The region with its respective city districts is illustrated in Figure 7. Notice that *Sentrum* is not a district, but is operated by St. Hanshaugen. Hence, these two areas are considered one district. Each demand zone is represented by a node, and the nodes are positioned in the centre of each district. In Section 10.2.1 the nodes are positioned at the site of the current station location in districts with a fire station. The other nodes are positioned in the geographic centre of the districts. This is done in order to more accurately evaluate the current station locations. Notice that the demand in these districts are assumed to be located at the given node. However, the travel time within a district is estimated to two minutes. In demand zones where the node is positioned at the location of the fire station, rather than at the geographical centre of the zone, the travel time from the station to the centre of the zone is included in the travel time within the zone. This is to more accurately estimate the travel time in the districts where the fire station is located close to the district border. Independent of how the nodes are positioned, deviations from the actual travel times cannot be avoided.

Generally, however, the number of demand zones considered in ERP location problems is significantly more than 15. Real-life problems usually contain anywhere between 100 to 3000 demand zones to accurately determine the location of a resource or a facility. In addition to

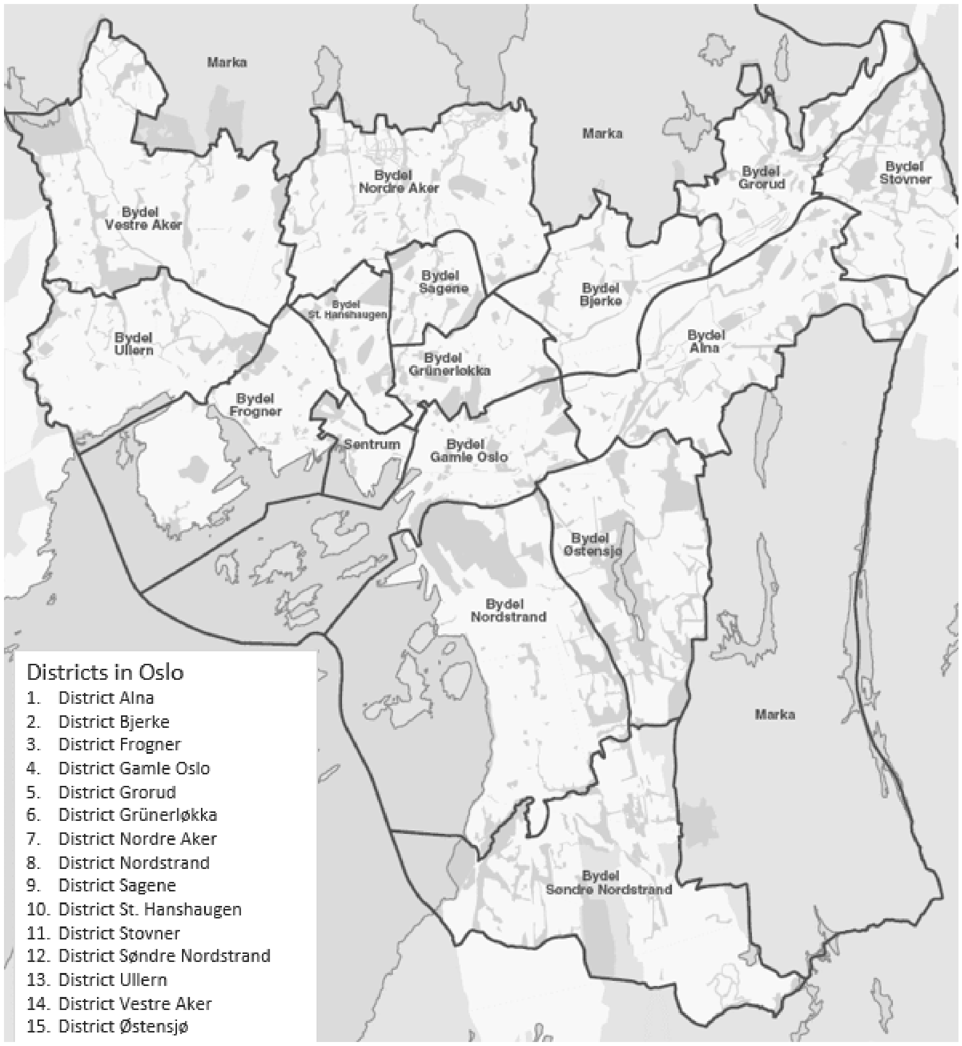

Figure 7: Map of the city districts in Oslo

city districts, the 666 postal codes in Oslo, and their geographical coordinates, were available as possible demand zones. In some cases, it was beneficial to utilise these postal codes, rather than city districts, to achieve a finer demand zones division. The zones were then combined into the desired number of demand zones by pairing two and two postal codes, starting with the geographically closest ones. The coordinates of the lowest number postal code then becomes the center of the new zone. The process is repeated until the postal codes are reduced to an appropriate number of demand zones. Eventually, the initially paired zones are merged with postal codes from all sides. Keeping the center of the demand zone in the initially paired zones is therefore seen as a reasonable assumption for determining the location of the node in the new zone. This approach of generating additional demand zones and potential candidate locations increases the size of the problem dramatically. Merging postal codes is therefore suitable for evaluating a subset of the area were a division of city districts is required.

9.3 Candidate Locations

The mathematical model incorporates the candidate locations in a subset J of the demand zones I. Initially, all demand zones are assumed to be potential candidate locations, implying that a response unit may be located in any one of the zones. Some zones however, are not appropriate locations for FRS resources, as a result of different factors, such as economics, insufficient infrastructure, or regulatory and property considerations. It has therefore been important to discuss and evaluate the solutions with FRS professionals to understand their real-life applicability.

A more realistic approach to determining a resource location in FRS is through a thorough evaluation of the area prior to a mathematical study. The evaluation aims to reveal a number of candidate locations that can be evaluated by the optimization model. In a few particular instances, BRE has indeed identified areas or candidate locations for the siting of FRS resources, that can be used as input data. These instances are examined closer in the computational study, in Section 10. It is in such instances that the MPLP-HAR can be the most useful decision support tool.

9.4 Accident Types

When the 110-central receives a call, the call is classified according to a set of criterion that determines the accident type, described in Table 16 in Section 7. For the purpose of this study, the accidents are categorised in five different types according to the resources required and the accident severity: fire accident, alarm calls, traffic accident, sea rescue, and rescue calls. This simplification captures the majority of the emergency calls that are made and more importantly includes the most urgent emergencies that require immediate response. That is, the high volume categories of unnecessary calls or false alarms are not considered any further, for example. Table 17 summarizes the number of calls received in the first quarter of 2016.

Table 17: Aggregated call data by accident type from Q1, 2016

Accidents in Oslo		
#	Accident type	Total
1	Fire accidents	251
2	Alarm calls	1522
3	Traffic accidents	36
4	Sea rescue	4
5	Rescue calls	622
	Total	2435

In cooperation with BRE the severity of the different accident types is evaluated. Each accident type is given a severity factor, related to the average urgency of the given accident type. This is described in Table 18. It is worth noticing that the urgency of a specific call can deviate from this factor.

Table 18: Severity factor describing the urgency for each accident type

Severity factor, λ_a		
#	Accident type	Severity factor
1	Fire accidents	0.3
2	Alarm calls	0.2
3	Traffic accidents	0.1
4	Sea rescue	0.3
5	Rescue calls	0.1

Fire accidents are given most attention in this study, as this is FRS's primary task. In addition, the consequence of these accidents are generally the most severe. Traffic accidents are also relatively common, but the response time is not as crucial as for fire accidents. Water accidents do not occur that frequently, but the response time is crucial for survival. Other accidents includes a variety of different accidents and hence this constitutes a large proportion of total calls. The severity and need for immediate rescue service varies, but generally immediate response is not crucial for these emergencies.

9.5 Response Units

Today, there are 18 operational response units available at all times in Oslo. In addition, BRE has other units that may be used for certain accidents. There are seven different vehicle types: base unit, rescue diver unit, ladder unit, rescue boat, emergency pollution unit, rescue unit, and tow truck. The current number of available units is shown in Table 19.

Table 19: Available units of the different types in Oslo (BRE, 2015)

#	Unit type	No. of units
1	Base unit	7
2	Rescue diver unit	4
3	Ladder unit	3
4	Rescue boat	1
5	Pollution unit	1
6	Rescue unit	1
7	Tow truck	1
	Total	18

The different unit types respond to different accident types. The types of units that are required for a given accident, vary according to the specific accident type. In general, the required resources for providing total response for a given accident type is described in Table 20. In the case of accidents requiring more than a single response unit, all units are considered appropriate for conducting first response. However, the preservation ability varies with the vehicle type that provides first response. This is discussed thoroughly in Section 10.1. In Oslo, the different unit types are often located at the same initial position as the inner stations are organized into rescue teams of multiple response units. The order of arrival at an accident site then depends on the travel time differences or the dispatchers strategic decision.

Table 20: Required number of units for the different accident types

	Required units				
Unit	Fire	Alarm	Traffic accident	Sea rescue	Other rescue
Base unit	2	1	1	1	1
Rescue diver unit	2	1	0	1	0
Ladder unit	2	0	0	0	0
Rescue boat	0	0	0	1	0
Pollution unit	0	0	0	1	0
Rescue unit	0	0	1	0	0
Tow truck	0	0	1	0	0

It is worth noting that all response units are treated as vehicles that are able to respond using the public road network. This assumption is made as the response times are calculated using Google Maps. The most significant consequence of the assumption is that the rescue boat is modeled as a vehicle that uses the road network. This means that whenever the rescue boat travels from one sea-side zone to another, its response time resembles the time it takes to drive between the zones, and not the time it takes to sail between them. This is however a reasonable assumption for the case study of Oslo. The coastal demand zones, where sea rescue occurs, are aligned with proximity to a main road such that the travel time at sea can somewhat resemble that of a land based vehicle. A map of Oslo and it's city districts can be seen in Figure 7.

9.6 Demand

The real-life demand shows variation both in terms of location and time for the different accident types. In this report, the demand is classified by accident types and zones. Based on historical accident data from the first quarter of 2016, the different accident types are given a fraction of the total demand. Furthermore, the demand is calculated for each zone. For sea rescue, a subset of the demand zones with a coastal line are selected. The demand is divided equally between the given zones. This is due to the low frequency of sea rescue in the accident data set. The data set from 2016 show some discrepancies due to the low frequency in accident occurrence. The distribution of accidents between the different zones does not represent the actual distribution with accuracy. This is particularly the case in traffic accidents and sea rescue.

9.7 Response Time

The response times for the dispatched units are calculated based on their geographical position at the time of the emergency call, in relation to the emergency location. The travel times between all zones are given in integer minute values and the travel times are considered asymmetric. They are calculated as the shortest path from the centre of the origin zone to the centre of the destination zone. The software assumes the traveling speed is equivalent to the speed limits along the chosen route. Hence, the travel times are considered constant over time. That is, there are no variations due to traffic, different times of the day, seasonal or weather conditions. According to the FRS professionals at BRE, the different vehicle types are to a varying degree able to achieve the speed limits when traveling to an emergency location. The calculated travel times are therefore also multiplied by a vehicle specific factor that compensates for issues regarding maneuverability and driving with emergency lights. These are summarized in Table 21. As discussed in Section 9.5, the rescue boat's response time is calculated as a driving vehicle and has not been given a travel time factor as it is would be meaningless to discuss whether or not it is able to keep the speed limits.

The travel time within the same zone is assumed to be two minutes, and all feasible routes have a minimum traveling time of two minutes. This is due to the relative coarse zone division of the Oslo area into city districts. In addition to the travel times, the response times considered here consist of a preparation time, T_u^P, before departure from the vehicle location and an intervention time, T_u^I, upon arrival at the accident site. Both are vehicle specific, constant parameters. The minute values for these are presented in Table 21 and are based on requirements from BRE in this instance. BRE is however, also able to collect empirical data on preparation and travel times. As a response unit is dispatched to an emergency, the firefighters press a button that automatically registers the exact time at which the unit departs from its original location. Similarly, they can register the time of arrival by the touch of a button, meaning that as long these routines are followed, BRE can obtain accurate preparation and travel times. This system is recently implemented and firefighters are just getting used to these new reporting routines. Preparation and intervention times are therefore assumed to be equal for all vehicle types in this case.

The first response time is denoted t^F and the total response time is t^T, provided by the first and last response units, u and o. Intuitively, the first response unit has to arrive before the unit providing total response, and hence $t^F < t^T$ is required.

Table 21: The response time for different vehicle types

#	Vehicle type	Travel time factor	T_u^P [min]	T_u^I [min]
1	Base unit	1.05	1	2
2	Rescue diver unit	1.0	1	2
3	Ladder unit	1.1	1	2
4	Rescue boat	1.0	1	2
5	Emergency pollution unit	1.1	1	2
6	Tow truck	1.05	1	2
7	Rescue unit	1.05	1	2

9.8 Preservation Function

Based on the aggregated accident types, individual preservation functions are formulated for each type. Accurately determining the characteristics of the preservation function requires extensive knowledge of the accidents in question. Interviews and discussions with FRS professionals have therefore been critical in this process. The preservation function in the objective function represents the level of preservation that the dispatched units achieve with their efforts at the accident site. It is dependent on the first and total response time. If all required units are ready to operate at the accident site at precisely the same moment as the emergency commences, they are able to preserve 100% of life and property. That is, if both first and total response time is equal to zero, the preservation value is equal to one. In most cases however, both the first and total response time will have non-zero minute values. This implies a lower preservation value and thus also a lower objective value.

The ability to preserve life and property decreases in accordance with the proceeding of accident severity. In the case of fire accidents the preservation function decreases exponentially with time, until the first response unit is operational at the accident site. It is however assumed that once the first response is achieved, the firefighters are able to perform some of the most essential efforts at the scene. The different vehicle types have varying capabilities to different accident types, but generally the decreasing trend of the preservation value then turns from exponentially to linearly decreasing. The ability to fully stop the decreasing trend is achieved when all dispatched response units are ready to operate at the accident site. At this time, the preservation value is final.

In the case of fire, alarms, traffic accidents, and sea rescue the preservation function has two time dependent input parameters, t^F and t^T, where t^F is the first response time and t^T is the total response time. These are the accident types that require multiple response units. The preservation values depend on the time it takes for the first and total response units, u and o, to arrive at the accident a at site i from their initial location at j and k, respectively. P_{iajuko} is then a pre-calculated six dimensional matrix of preservation values between 0 and 1.

The function is a combination of an exponential and a linear function and hence includes an exponential and a linear decreasing factor. The exponential decreasing factor for accident type a is denoted α_a. Different accident types have a varying degree of severity in the initial phase, reflected in α_a. Depending on the accident types at hand, the different vehicles have different required capacities and the ability to initially preserve life and property depends on what unit arrives first at the site. This is reflected in the linear decreasing factor $\beta_{a,v}$, for accident type a when first response has been provided by a vehicle type v. The linearly decreasing trend is considered constant from the arrival of the first to the final response unit, regardless of the sequence and time of arrival of the respective dispatched units.

Emergency calls regarding rescue require only one response unit. In these cases the emergency a occurs in demand zone i, and the required unit u is located in vehicle zone j. The preservation function, P_{iaju}, is then dependent only on the first response time t^F and the exponential decrease factor α_a. Total response is considered to be achieved by the arrival of the first responding unit with the required capabilities. The number of required units of each vehicle type is summarized in Table 16 in Section 9.5. The preservation functions for the different accident types are summarized in Table 22.

Table 22: The preservation function for each accident type

#	Accident	Preservation function
1	Fire accidents	$P_1(t^F, t^T) = e^{-\alpha_1 \cdot t^F} - \beta_{1,u} \cdot (t^T - t^F)$
2	Alarm calls	$P_2(t^F, t^T) = e^{-\alpha_2 \cdot t^F} - \beta_{2,u} \cdot (t^T - t^F)$
3	Traffic accidents	$P_3(t^F, t^T) = e^{-\alpha_3 \cdot t^F} - \beta_{3,u} \cdot (t^T - t^F)$
4	Water accidents	$P_4(t^F, t^T) = e^{-\alpha_4 \cdot t^F} - \beta_{4,u} \cdot (t^T - t^F)$
5	Rescue calls	$P_5(t^F) = e^{-\alpha_5 \cdot t^F}$

10 Computational Study

This section presents the computational study. First, the characteristics of the preservation function are determined, through a sensitivity analysis of different decreasing slopes. To ensure its applicability, a verification study is conducted. Once the identities are found, the preservation function is used in the mathematical formulation to evaluate scenarios and investigate a number of relevant problems for BRE. In the computational study, only the results themselves are presented, without further discussions. In the following Section 11, the results are the subject of thorough discussion and conclusions related to the problem description are drawn.

For the purpose of this computational study, the units in each vehicle type are considered homogeneous. From the technical study in Section 6, the formulation denoted T8 is the most efficient formulation for the MPLP-HAR. Emphasising the trade-off between solution accuracy and computational time, the T8 formulation with a five hour computational time is considered desirable for the purpose of this report. This involves an accepted gap between the best bound and the optimal solution when no units are confined to any station location and all zones are considered candidate locations.

10.1 Preservation Determination

The preservation function aims to capture the nature of the accident at hand and verification from FRS professionals is therefore essential in giving a realistic representation of the ability to preserve life and property. Determining the values of the exponential and linear decreasing parameter, α_a and β_{av}, is done by studying a set of possible parameters in cooperation with BRE. The final results are analysed and verified by BRE professionals. The general formulation of the preservation function is presented in equation (48).

$$P_{iajuko}(t^F, t^T) = e^{-\alpha_a \cdot t^F} - \beta_{a,u} \cdot (t^T - t^F) \tag{48}$$

An important aspect of the preservation function is to capture the effects of the initial effort at the accident site. The arrival of the first response unit results in a reduced decreasing trend of the preservation function. Upon the arrival of the total response unit, the preservation function takes its final value. An example of the preservation value as a function of time is illustrated in Figure 8. In this case, the first response unit is ready to operate after 10 minutes and the total response is obtained after 20 minutes.

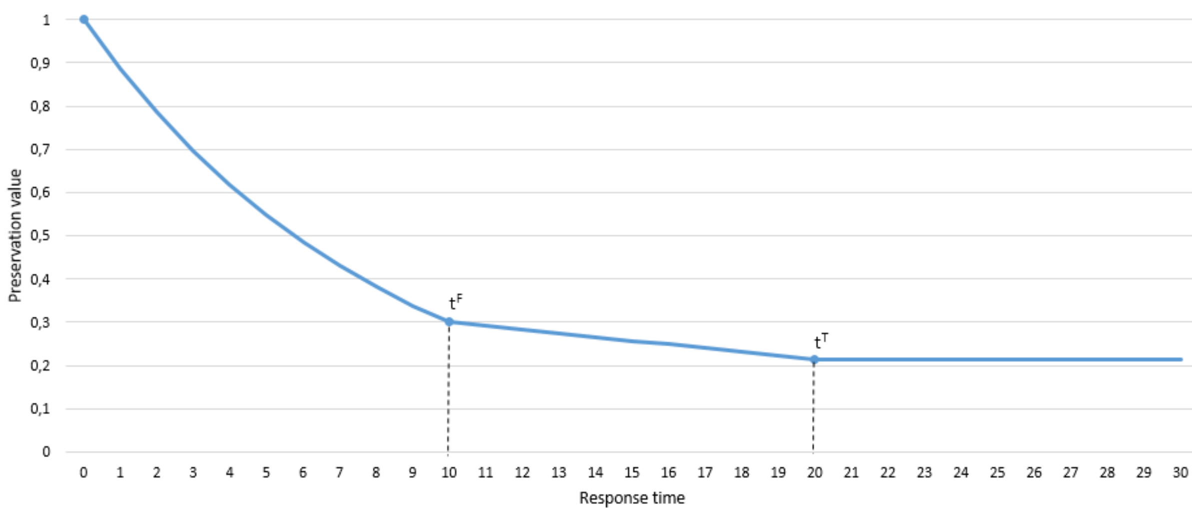

Figure 8: An example of the preservation function

10.1.1 Exponential Decrease Factor

The exponential decrease factor, α_a, determines the initial decrease in preservation for accident a before the arrival of the first response unit. Table 23 summarizes the preservation level obtained for the different decreasing factors tested. The preservation level is calculated according to the equations presented in Table 22 for α_a^1, α_a^2, and α_a^3, respectively. The relative need for immediate response is reflected in the different preservation values and the five accident types are fire accidents, traffic accidents, alarms, sea rescue and, rescue calls.

In Table 23, the preservation level is examined after 10 minutes. This response time is chosen based on the regulatory standards discussed in Section 7.6 and the values are chosen to highlight the changes in the optimal solution with differences in the preservation characteristics.

Table 23: Exponential decrease factor

Accident type	P-function	α_a^1	α_a^2	α_a^3
Fire	$P_{i1juko}(10, t^T)$	60%	40%	80%
Alarm	$P_{i2juko}(10, t^T)$	70%	50%	90%
Traffic	$P_{i3juko}(10, t^T)$	80%	60%	95%
Water	$P_{i4juko}(10, t^T)$	50%	30%	70%
Rescue	$P_{i5ju}(10)$	80%	60%	95%

However, an investigation of the slope of the exponentially decreasing function is necessary. The combination of an exponential and a linear decreasing factor, α_a and β_{av} requires that the decreasing trend is reduced with the arrival of the first response unit. Table 24 therefore summarizes the first derivative of the example values for each accident type after 15 minutes. 15 minutes is chosen to make sure that even a late arrival of the first response unit would have a relieving effect on the decreasing trend of the preservation value. Now, only the exponential component of the preservation function is included. Table 24 is therefore evaluated in relation to Table 25, containing the linear decreasing factors. As a result, the exponential decrease is combined with a linear decrease, where the linear decrease factor has a lower magnitude than the derivative of the exponential decrease function, with respect to the first response time. The required relationship between the parameters is given in equation (49). This is of course only a concern for accident types that require multiple response units.

$$-\beta_{av} > \frac{d}{dt^F} e^{-\alpha_a \cdot t^F} \tag{49}$$

10.1.2 Linear Decrease Factor

The linear decrease factor, β_{av}, depends on both the characteristics of the accident and the vehicle type of the first response unit. The ability to preserve life and property is enhanced with the capabilities to perform required efforts. Different β_{av} values are investigated in the tests, from a gentle to a relatively steep slope. In Table 25 the values for the first test,

Table 24: The derivative exponential decrease factor

#	α_a^1	α_a^2	α_a^3
$\frac{d}{dt^F}e^{-\alpha_1 \cdot t^F}$	-0.0237	-0.0232	-0.0160
$\frac{d}{dt^F}e^{-\alpha_2 \cdot t^F}$	-0.0209	-0.0245	-0.0090
$\frac{d}{dt^F}e^{-\alpha_3 \cdot t^F}$	-0.0160	-0.0237	-0.0047
$\frac{d}{dt^F}e^{-\alpha_4 \cdot t^F}$	-0.0245	-0.0198	-0.0209

with a moderate linear decrease factor β_{av}^1, are presented. Note that the absolute values are presented and that they constitute a decrease with time when applied in the preservation function. In the second test, the β_{av}^2's are the double of what they are in Table 25, giving a steeper decrease. In the third test, the linear decrease factor β_{av}^3 has half the magnitude of β_{av}^1. These relationships are given in equations (50) and (51). The different values are related to the α_a values, presented in Table 23.

Table 25: Linear decrease factor, β_{av}^1

$a \in \mathrm{A}^S / \beta_{av}$	β_{a1}	β_{a2}	β_{a3}	β_{a4}	β_{a5}	β_{a6}	β_{a7}
1	0.0075	0.0075	0.010	-	-	-	-
2	0.005	0.0075	-	-	-	-	-
3	0.005	-	-	-	-	0.005	0.005
4	0.0125	0.010	-	0.0075	0.015	-	-

$$\beta_{av}^2 = 2 \cdot \beta_{av}^1 \tag{50}$$

$$\beta_{av}^3 = \frac{1}{2} \cdot \beta_{av}^1 \tag{51}$$

10.1.3 Sensitivity Analysis

A combination of the different exponential and linear decreasing factors are examined to determine the characteristics of the preservation function. To determine α_a and β_{av}, the behaviour of the different preservation functions are evaluated by FRS professionals from BRE, to ensure the applicability of the model. The combination of parameters for the tests are summarized in Table 26.

Table 26: Preservation tests

Test	P1.1	P1.2	P1.3	P2.1	P2.2	P2.3	P3.1	P3.2	P3.3
α_a	α_a^1	α_a^1	α_a^1	α_a^2	α_a^2	α_a^2	α_a^3	α_a^3	α_a^3
β_{av}	β_{av}^1	β_{av}^2	β_{av}^3	β_{av}^1	β_{av}^2	β_{av}^3	β_{av}^1	β_{av}^2	β_{av}^3

Test P1 In the first tests, P1.1 - P1.3, the exponential decrease factor, α_a, has a moderate value. According the FRS professionals, these are the most realistic values and the relative severity of the accident types are captured quite accurately. The linear decrease values, β_{av}, varies as shown in Table 26. The results are displayed graphically in Figure 9. From the illustration it is clear that Test P1.3 gives the widest spread of resources by placing vehicles in 13 different zones. This results in the shortest average first response time for the three tests, of 3.9 minutes. These findings are unsurprising, as Test P1.3 gives the most emphasis to the first response time, relative to the total response. P1.2 gives the tightest collection of resources and the longest average first response time of 4.6 minutes. In this test, the linear decrease factor is the steepest and total response time is therefore given a higher priority, at the cost of the first response. Test P1.1 gives a moderate average first response time of 4.1 minutes. This illustrates that by having a steeper linear decrease, the arrival of the last responding unit is given more emphasis. As a result, the resources gather in high-demand zones and downgrades the importance of first response in low-demand zones.

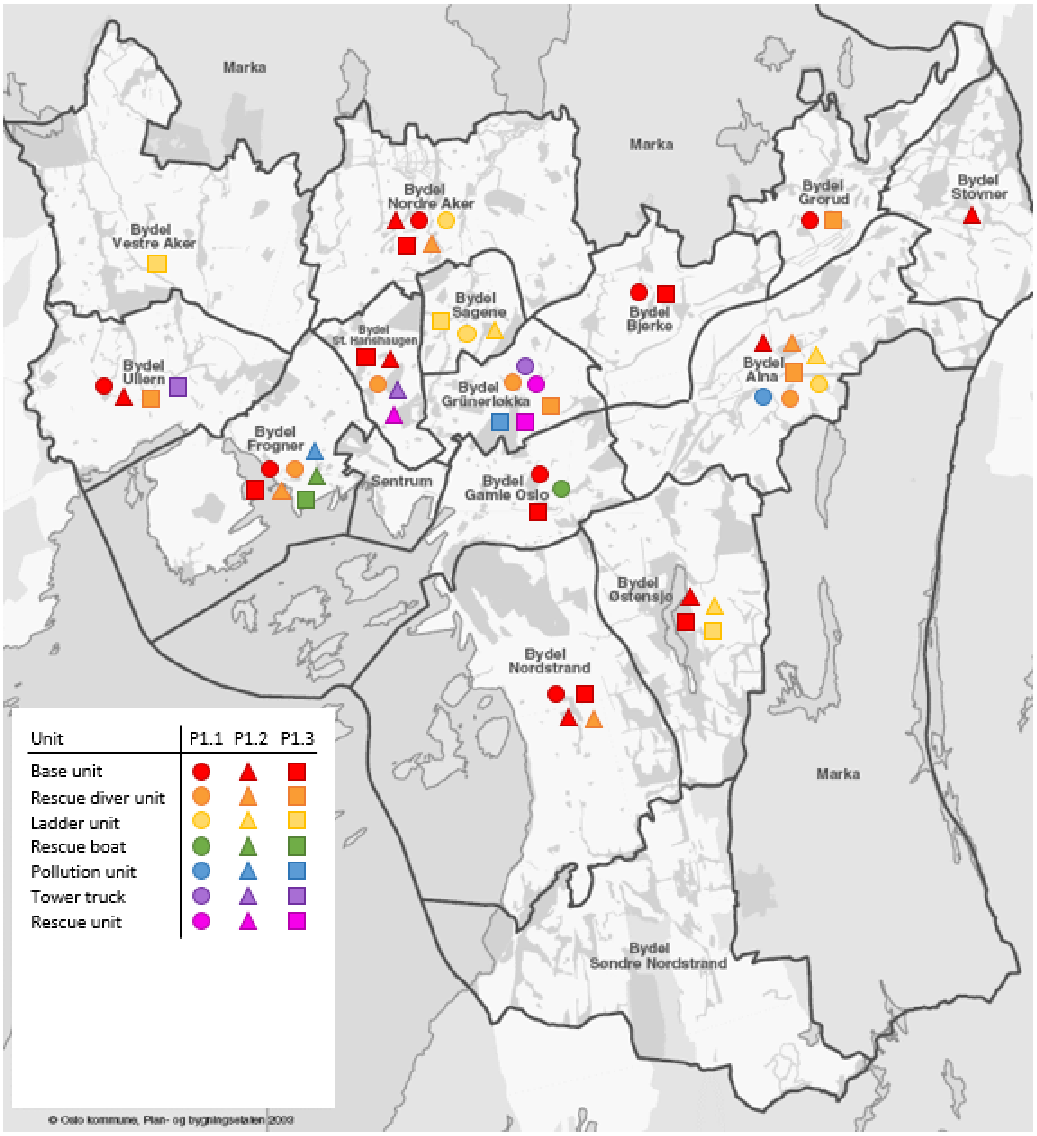

Figure 9: Locations for different preservation functions, P1

Test P2 In Test P2, the preservation function has a steep initial exponential decrease resulting in an increased emphasis on the first response. The results of Test P2 are illustrated in Figure 10. The relative importance of an efficient first response is evident in the average first response time. For Tests P2.1-P2.3, the average first response time to all zones is 4.1 minutes, compared to 4.2 minutes in Tests P1.1-P1.3. As the linear decrease factor is varied within Test P2, the solutions resemble the findings from Test P1.

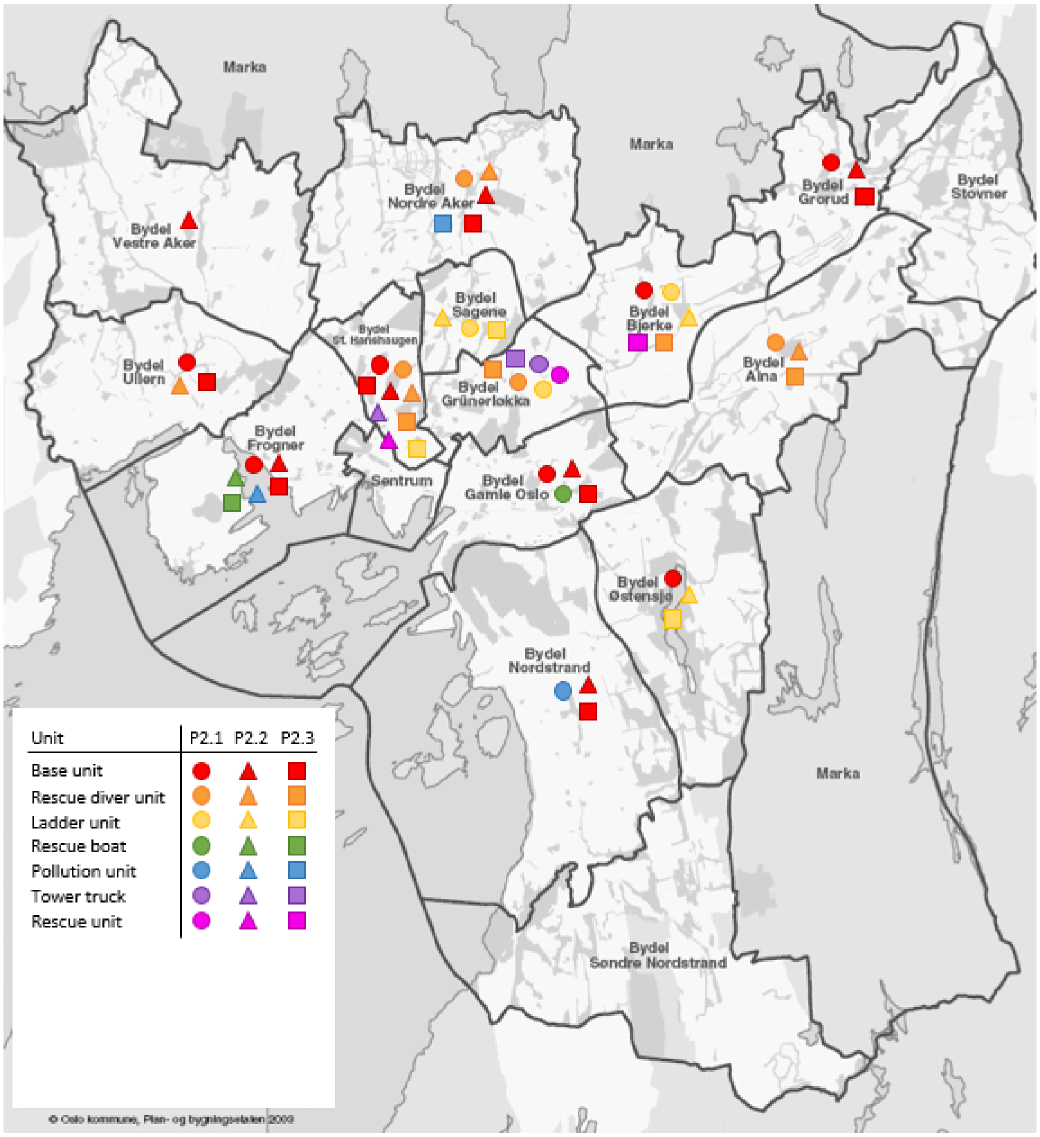

Figure 10: Locations for different preservation functions, P2

Test P3 Finally, in Test P3 the preservation function is using a gentle exponential decrease. This implies that the first response is given a low priority compared to the total response. Figure 11 displays a greater concentration of units in high-demand zones, compared to Tests P1 and P2. The average first response time to all zones is 4.6 minutes in Tests P3.1-P3.3, which constitutes a considerably increase compared to Tests P1 and P2. This is particularly evident for Test P3.2. Here, the slant exponential decrease is combined with a steep linear decrease, thus emphasising the relative importance the total response time and giving an average first response time to all zones of 4.8 minutes.

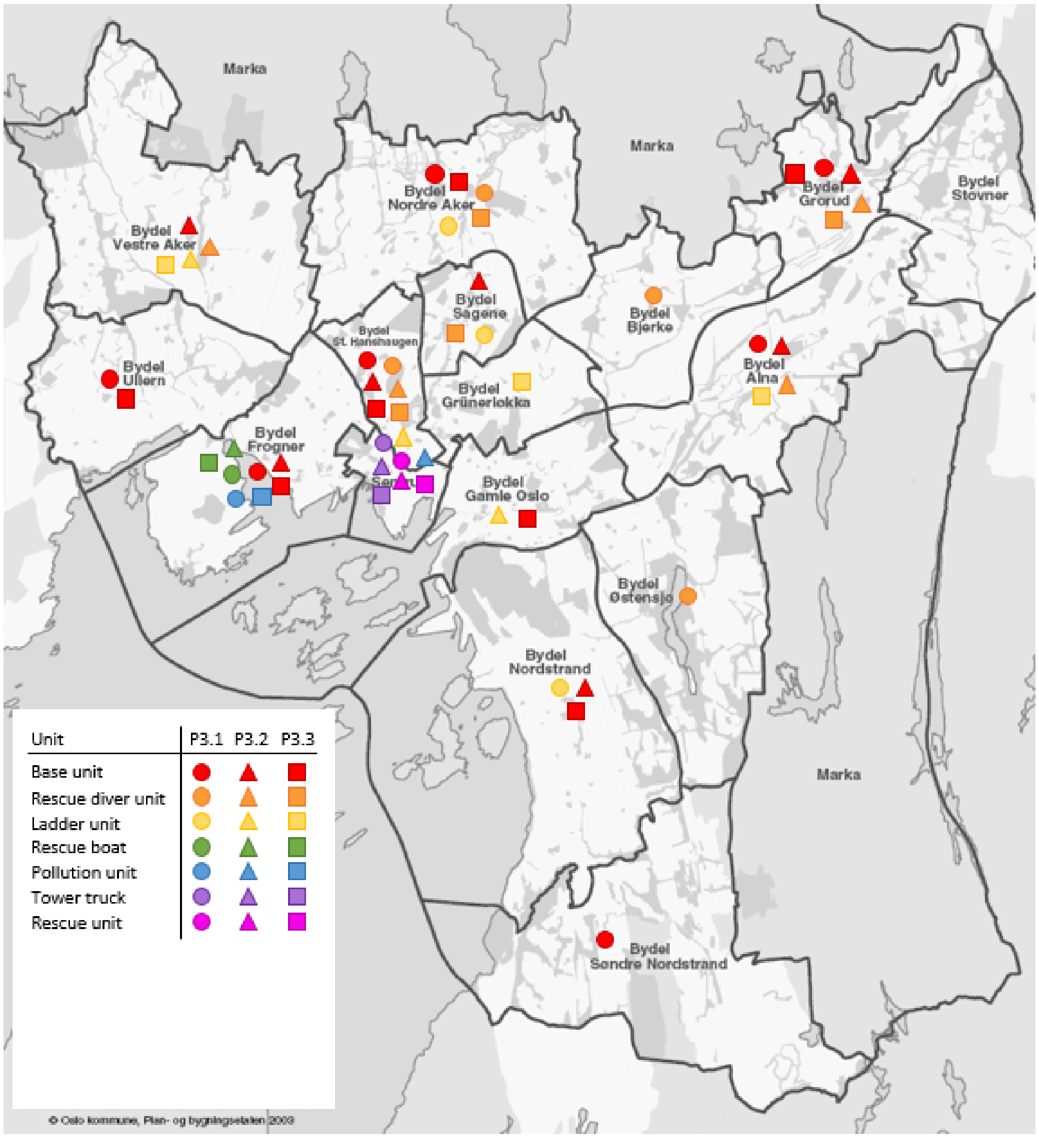

Figure 11: Locations for different preservation functions, P3

The three tests are all part of validating the preparedness measure and determining the identity of the preservation function. The different solutions in this sensitivity analysis indicate that the preparedness measure follows the desired weighting between first and total response. The purpose of the tests is to determine a preservation function that is in line with the FRS professionals' understanding of preparedness. They consider first response critical and a preservation function that captures this yields a desired solution. In addition, the choice of

α_a and β_{av} must account for the findings in Table 24. After close consultation from professionals at BRE, the P1.1 formulation for the MPLP-HAR is chosen for the computational study. The identity of this formulation is able to reflect good preparedness in its solution. Particularly, the formulation chooses to locate mostly single base units in the outer stations, while specialized response units are gathered in the city centre, resembling the division of inner and outer stations preferred by BRE. In addition, the generated solutions from this test result in short average first response times for the area.

10.1.4 Comparison to Existing Location Problems

An important part of determining the preservation function, and thus also the FRS preparedness measure, is model verification. Here, the performance of the developed model is compared to known location problems from existing literature. Along with the MPLP-HAR, the mathematical models discussed in Section 3, the p-median and MCLP, are used to evaluate the location problem for FRS resources in Oslo.

A few adjustments have been made in order to get comparable results from the verification study. None of the existing location problems account for a heterogeneous fleet of vehicles. For the purpose of preparedness measure comparison, these location problems are extended to account for a heterogeneous fleet. The comparison of the three location problems is performed by simply changing the objective function and keeping the constraints from the formulation of the MPLP-HAR. More precisely, the models are identical apart from the preservation functions, which are made to replicate the identities of the respective problems. For the MPLP-HAR, the preservation function with the selected parameters, determined in Section 10.1, is chosen. In the MCLP, the preservation value is binary, depending on whether or not the total response time exceeds a threshold value. For this comparison, the threshold value equals 20 minutes. Finally, the p-median has a linearly decreasing preservation function with a preservation value of 10% after 20 minutes. Again, the preservation value depends on the total response time only. The respective objective functions are illustrated in Figure 12.

A close assessment of the MCLP and the p-median problem would be needed at this point

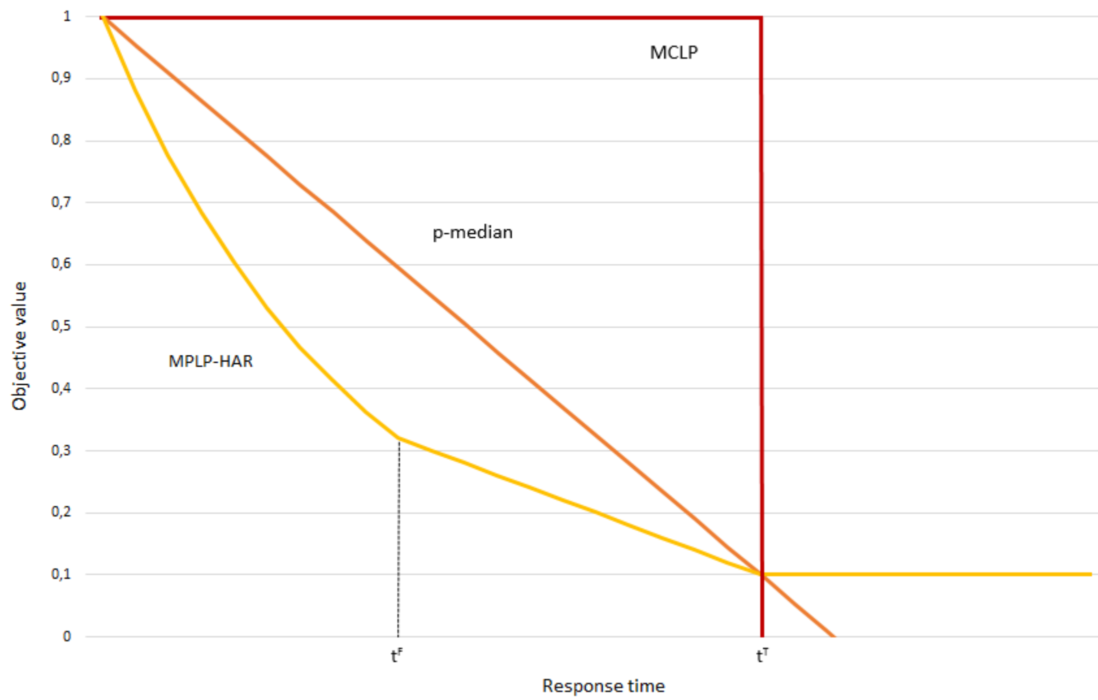

Figure 12: Comparison of the objective values in MCLP, p-median, and MPLP-HAR

to accurately determine the threshold time and the slope of the p-median's linear decrease, similar to the analysis done for the MPLP-HAR. For the purpose of this verification study, the threshold value for the cover problem is set according to the statuary requirement for response times in urban areas. Similarly, the preservation function in the p-median problem is formulated to reflect this threshold value. Given the number of resources within the relatively small geographical area of Oslo, the threshold time of 20 minutes is not difficult to accommodate. Total response is normally achieved in a significantly shorter time with the current operational state. Therefore a number of different solutions can perform well in a cover problem and this indicates a discrepancy in the cover formulation. The MCLP attempts to find a satisfactory solution instead of continuously searching for a optimal solution with respect to response time.

The results from the comparison of the three problems are summarized in Table 27. Each solution is evaluated using the objective function of all three problems. The MCLP, p-median, and MPLP-HAR give structurally the same optimal resource locations. The response units show a slightly greater geographical spread in the p-median problem and the MPLP-HAR, compared to the MCLP. A graphical illustration of the solutions is presented in Figure 13. L1 represent the MCLP, L2 the p-median problem, and L3 the MPLP-HAR.

Table 27: Comparing different locations models

Obj. val./Sol. val	MCLP	p-median	MPLP-HAR
MCLP	0.18348	0.18288	0.18348
p-median	0.13732	0.16056	0.15686
MPLP-HAR	0.14876	0.15931	0.15987

From Table 27 it is evident that the solutions from all three problems perform well when evaluated using the objective function for the cover formulation. A reason for the high and consistent objective values is the favorable threshold time value in the cover formulation. However, the solution from the developed MPLP-HAR achieves the same objective value as the MCLP and outperforms the p-median solution. Similarly, the MPLP-HAR performs better when evaluated as a p-median solution than the MCLP formulation. Intuitively, the p-median solution performs better than the MCLP solution when evaluated in MPLP-HAR, as the formulation developed in this report is based on the p-median problem.

10.2 Scenarios

This section is dedicated to investigating a number of current and future scenarios faced by BRE. The MPLP-HAR is applied, first to evaluate the current operational state of the FRS organization in Oslo. Thereafter a number of suggested improvements in the station structure and resource allocation are investigated to evaluate the effects on the preparedness level. The investigated future scenarios are formulated based on BRE's suggested improvements, stated in their preparedness analysis from 2015 (BRE, 2015).

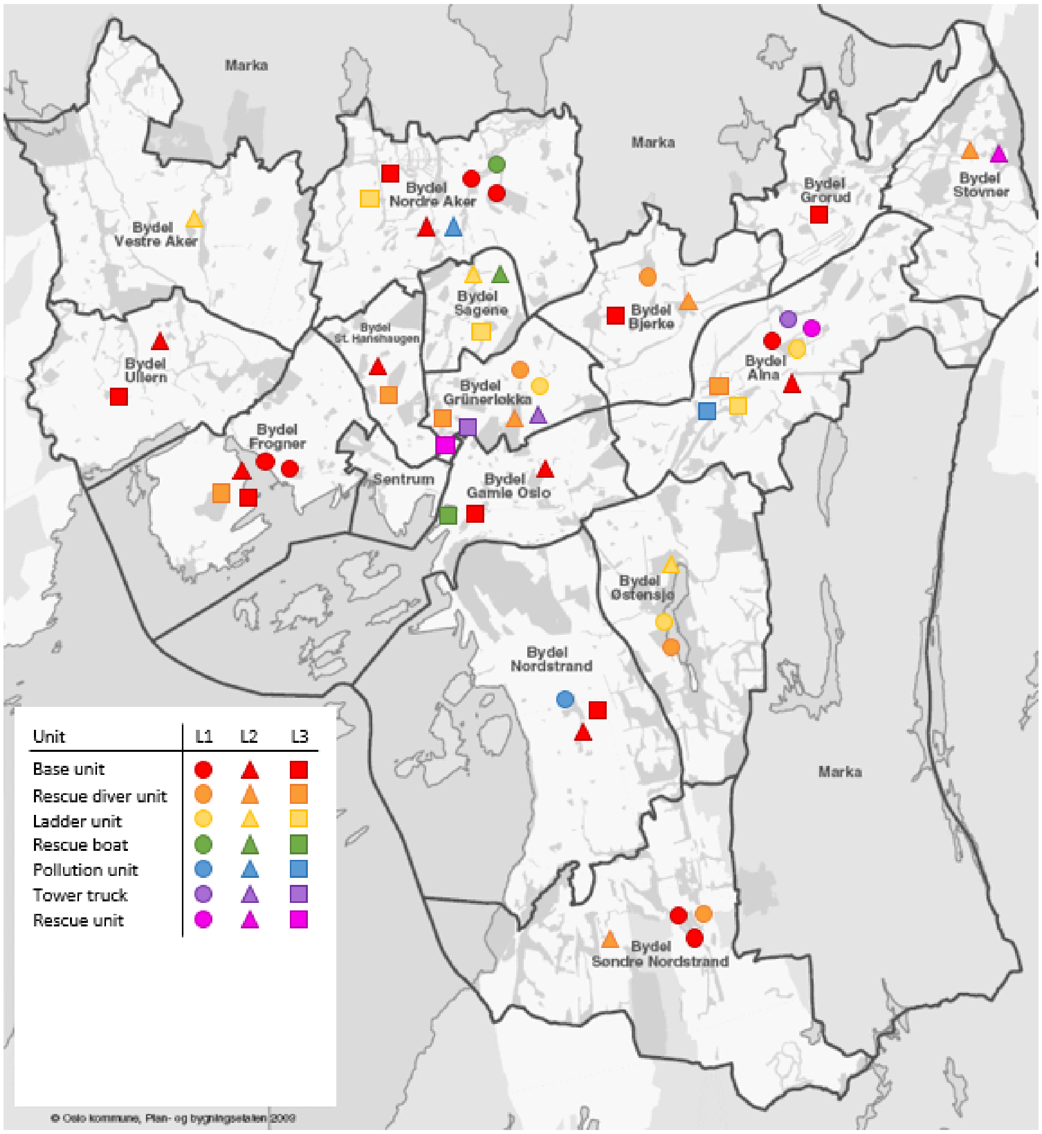

Figure 13: Comparing locations for the different location problems

10.2.1 Current Station Structure

The fire stations in Oslo are currently located as illustrated in Figure 14. The squares represent station location and the demand node in these districts and the circles represent the demand nodes in the other districts. By fixing all the variables $x_{ju} = \bar{x}_{ju}$ according to the current resource allocation, described in Table 15 in Section 7.3, the current preparedness level is evaluated. The rescue team operating the pollution unit and the rescue boat are currently located at Briskeby or Sagene, but according to BRE this staff will be relocated to Sentrum, when the Main station is moved to Bryn. In the tests performed, the pollution unit and rescue boat are located at the Main station.

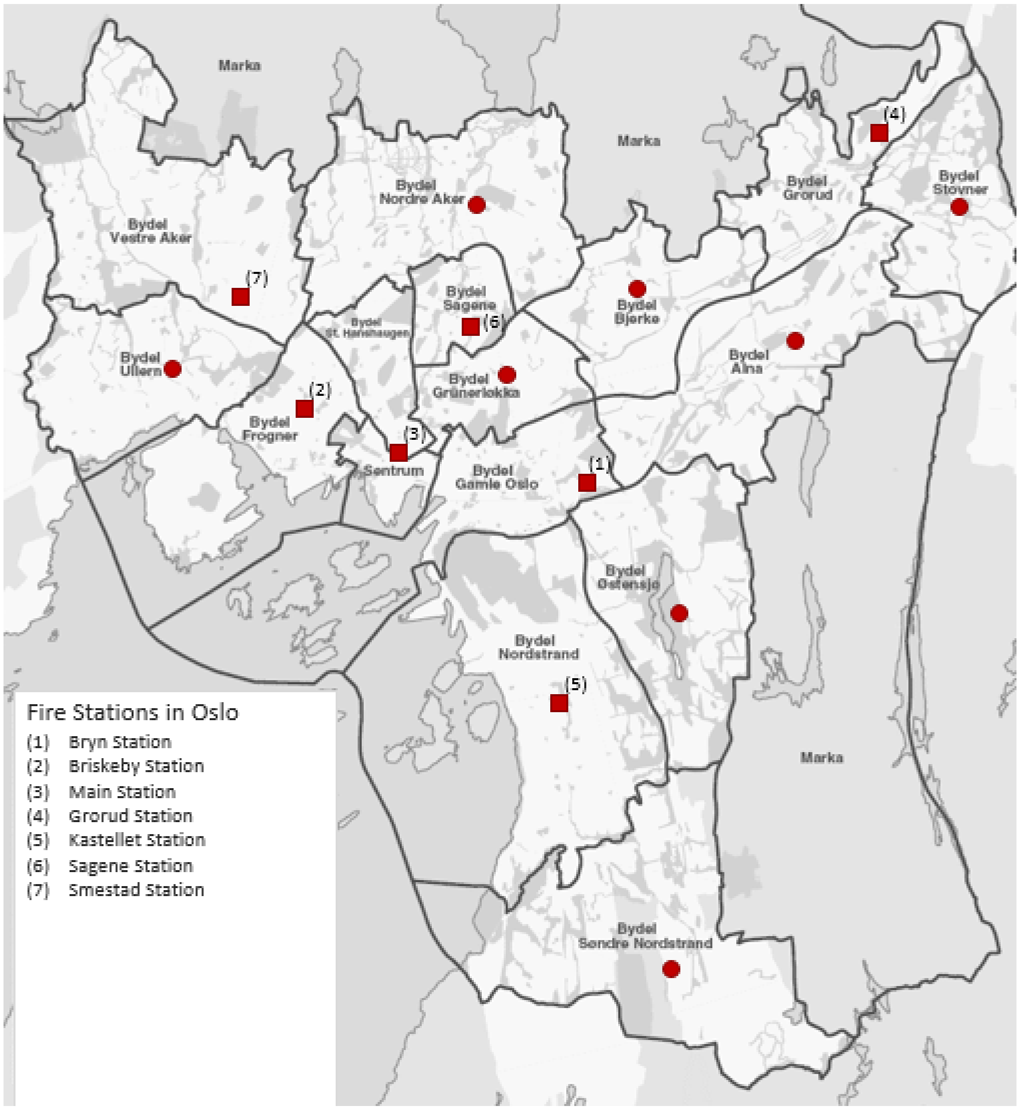

Figure 14: Current station location and demand nodes

Table 28: Objective value for current and optimal resource location

#	Best bound	Objective value	Gap
S1.1	0.16419	0.15607	5.20%
S1.2	0.15392	0.15392	0.00%

In Table 28 the optimal resource location, S1.1, given when x_{ju} are unconstrained, is compared to the current station location, S1.2. The solutions are illustrated in Figure 15. From the graphical representation it is clear that both the current and optimal location have focused the majority of the resources in the inner city districts, were the demand is high and the mean response time to districts on all sides is relatively low. However, the optimal location shows a tendency to position a single unit in a number of candidate locations, as opposed to the current structure were multiple units are located at the same station. The scattered location of units is favoured because the preservation function puts much emphasis on an efficient first response.

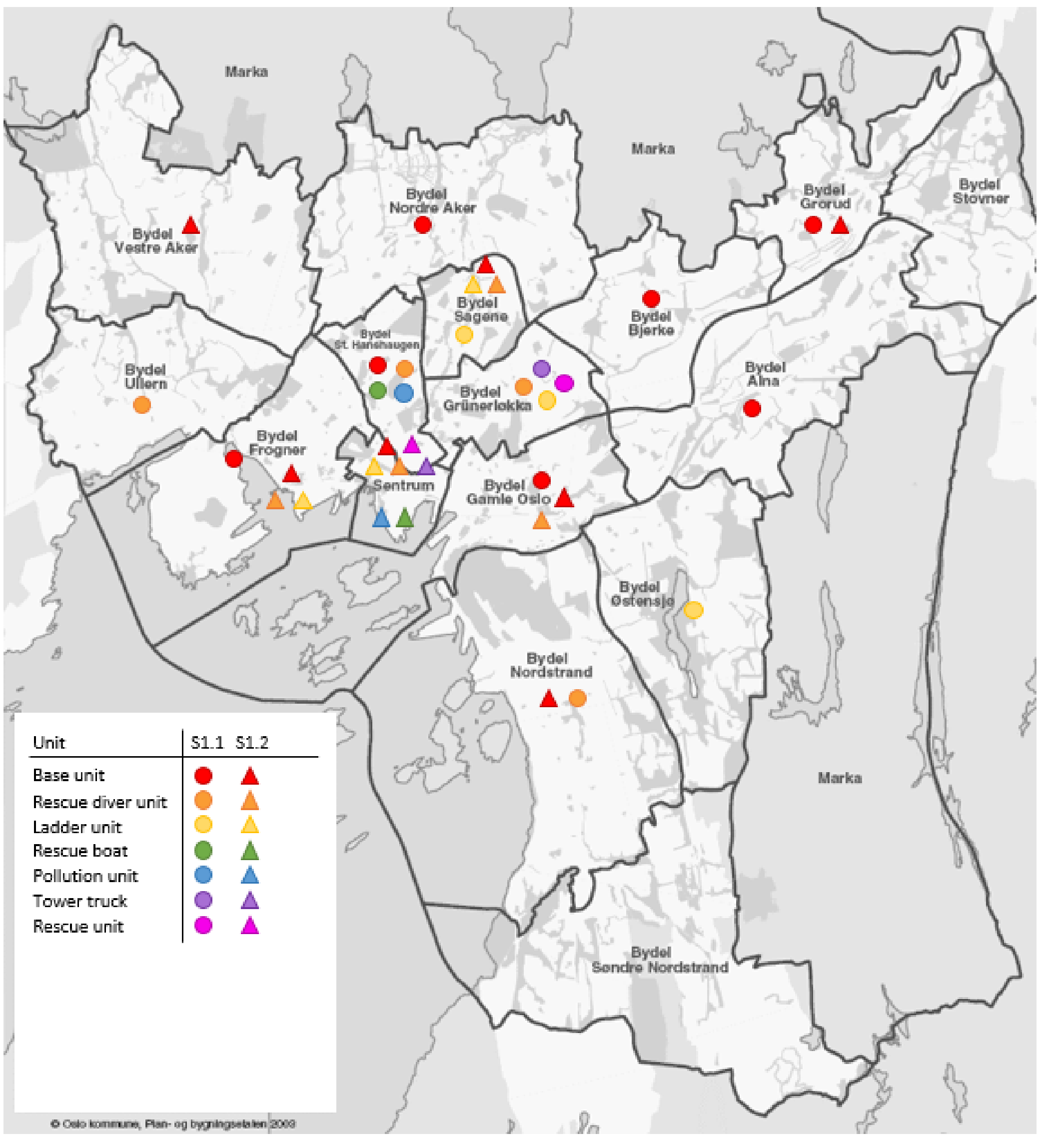

Figure 15: Current and optimal resource location

10.2.2 Relocation of the Main Station

BRE is planning to relocate the Main station from the city centre to Bryn and build a new City station. They have studied several possible locations for the new City station, as described in Section 7.7.1, and have narrowed the search down to three candidate locations. These have been selected based on geographical position, infrastructure access, and applicability of the property itself. The three candidate locations for the new City station, (3A'), (3B'), and (3C'), are illustrated in Figure 16. (1') represents the new Main station at Bryn and (3') the current location of the City station. By relocating the resources from the current City station to location (1') at Bryn, the candidate locations in the inner city are analyzed individually. The resources originally located at Bryn station are moved to (3'), (3A'), (3B'), and (3C'), respectively.

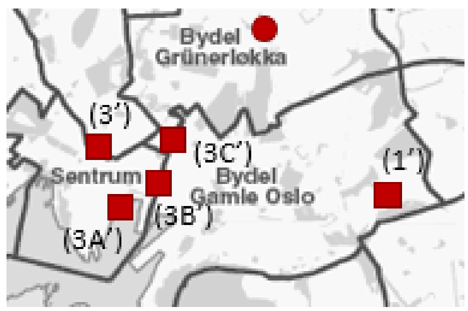

Figure 16: Possible new station locations in the city centre

The results of the study are presented in Table 29. The objective values represent the preparedness for the whole jurisdiction of Oslo. S2.1 represents the current station location, and in S2.2 the resources at the Main station are relocated to Bryn and the resources at Bryn are moved to the City station (3'). In S2.3, S2.4 and S2.5, the Main station is relocated to Bryn and the resources at Bryn are moved to location (3A'), (3B'), and (3C'), respectively. Relocating resources between the current Main station and Bryn shows improvement in the object value. Relocating the City station to (3A') has a negative effect on the overall preservation in the area, compared to keeping the original City station in the city centre. Location (3B') and (3C'), however, have a positive effect, and (3C') gives the largest increase in the objective value.

Table 29: Objective value for different locations of the station in the city centre

#	Location	Objective value
S2.1	(1),(3)	0.15392
S2.2	(1'),(3')	0.15442
S2.3	(1'),(3A')	0.15411
S2.4	(1'),(3B')	0.15505
S2.5	(1'),(3C')	0.15510

As illustrated in Figure 16, candidate location (3A') is closer to the sea than (3B') and (3C'). (3A') is therefore disadvantageous when it comes to responding to the high-demand zones further inland. Because there are no demand zones off-shore, the coastal location performs worse than (3B') and (3C'). It is however worth mentioning that locating a fire station at candidate location (3A') would facilitate the coordination between the harbour resources and the other operational and administrative units. Locating the City station in either (3'), (3B'), or (3C') will result in BRE keeping a separate harbour facility for the rescue boat and the emergency pollution unit, that they are required to have.

10.2.3 Relocation of Kastellet Station

In the years to come BRE is expecting to see a rapid development in settlements and changes in the demographic situation in Southeastern Oslo. In that regard, they are looking to relocate the fire station currently located at Kastellet. Potential candidate locations have not been investigated in detail, but proximity to the main roads E18 or E6 have been suggested. Two alternative locations, (5D) and (5E), are illustrated in Figure 17. The location of the City station may affect the relocation of Kastellet station. Therefore different combinations of the City station and new Kastellet station are investigated. The results are presented in Table 30.

The results indicate that location (5E) is preferred to (5D), independent of the location of the City station. The best objective values are achieved when the City station is located at (3C'),

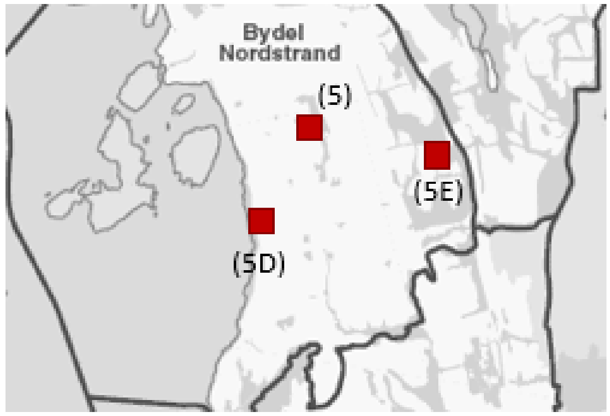

Figure 17: Current station location and demand nodes

Table 30: Objective value for different locations of Kastellet station

#	Fixed locations	Objective value
S3.1	(1),(3),(5D)	0.15386
S3.3	(1),(3),(5E)	0.15469
S3.2	(1'),(3'),(5D)	0.15436
S3.4	(1'),(3'),(5E)	0.15521
S3.5	(1'),(3A'),(5D)	0.15413
S3.6	(1'),(3A'),(5E)	0.15490
S3.7	(1'),(3B'),(5D)	0.15502
S3.8	(1'),(3B'),(5E)	0.15585
S3.9	(1'),(3C'),(5D)	0.15506
S3.10	(1'),(3C'),(5E)	0.15590

for both the original Kastellet location (5) and candidate locations (5D), and (5E). Solution S3.10 yields the best overall solution. However, the demand zone in district Nordstrand is also moved to (5E) in this case. Demand southwest of (5E) is therefore not considered, because there are no demand zones southwest of this node. When analysing the relocation of Kastellet station, one might want to divide Oslo Southeast into smaller demand zones in order to capture the demand distribution more accurately.

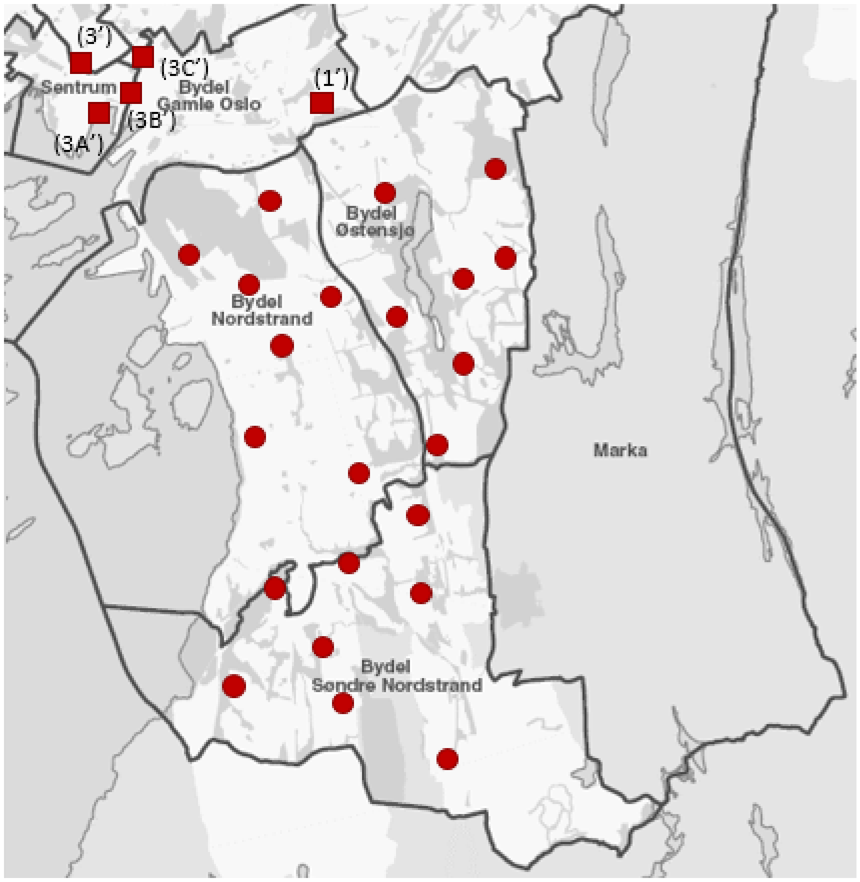

Figure 18: Demand zones in Southeastern Oslo

Oslo Southeast is therefore isolated and analysed by dividing the districts Nordstrand, Østensjø, and Søndre Nordstrand into smaller demand zones. The demand nodes, or the center of each demand zone, are illustrated in Figure 18. In addition, the new Main station at Bryn and the three possible locations for the City station are included. To make sure that the required response for all accident types are obtained, one ladder unit is included at the City station. This is done so that total response can be achieved for accidents that require two ladder units. The station at Kastellet is relaxed and the model first locates a base unit representing the optimal location for Kastellet station. The results are presented in Table 31. In all cases, postal code 1286 is suggested for the relocation of Kastellet station. The location of 1286 is presented in Figure 19. S3.13 yields the best result, with the City station located at (3A'). As opposed to the previous study of the whole of Oslo, location (3A') is considered better than (3C'), when looking at Oslo Southeast in isolation. This is because

(3A’) has the best response time to the southeastern region of Oslo, regardless of whether demand locations are located with proximity to E18 or E6. On the other hand, (3C’) is the favourable location for responding to emergencies in the northern and western parts of Oslo.

Table 31: Objective value and suggested location for fire station in the southeastern region

#	Fixed locations	Objective value	Station location
S3.11	(1),(3),(5)	0.12925	-
S3.12	(1’),(3’)	0.12930	1286
S3.13	(1’),(3A’)	0.13135	1286
S3.14	(1’),(3B’)	0.13014	1286
S3.15	(1’),(3C’)	0.12901	1286

10.2.4 Location of Additional Fire Station

In addition to relocating the current fire station at Kastellet, BRE is exploring the possibility of establishing an additional fire station in Oslo Southeast. First, Oslo Southeast is isolated and investigated to find an appropriate location to include an additional fire station with one base unit. Because the optimal solution may change with the location of the City station and the relocated station discussed in Section 10.2.3, different combinations of these stations are regarded in the study of alternative locations for the additional station in Oslo Southeast. The results from Oslo Southeast are then applied to all of Oslo to investigate how this effects the preparedness level in the whole jurisdiction. By relaxing the location of the additional base unit, other possible locations for the new fire station are studied.

In Table 32 the results from Oslo Southeast are presented. The different station structures are tested to investigate which combination yields the best result. Independent of the initial station structure, location 1286 is preferred. The results indicate that the new station structure gives better results than the current structure. The highest objective values are obtained in S4.3, S4.4, and S4.5, with the City station located at (3’), (3A’), and (3B’), respectively. According to this test, S4.4 yields the highest preservation value, with station locations in

Oslo Southeast at postal codes 1187 and 1286. The suggested locations are presented in Figure 19. Again, the solutions and objective values are found from a study of the isolated region and represent a possible sub-optimal solution in terms of preparedness for the whole jurisdiction.

Table 32: Objective value and suggested locations for two new fire stations in Oslo Southeast

#	Fixed locations	Objective value	Station location
S4.1	(1),(3),(5)	0.13264	1286
S4.2	(1),(3)	0.13458	0693,1286
S4.3	(1'),(3')	0.13508	0693,1286
S4.4	(1'),(3A')	0.13585	1187,1286
S4.5	(1'),(3B')	0.13522	0693,1286
S4.6	(1'),(3C')	0.13478	0693,1286

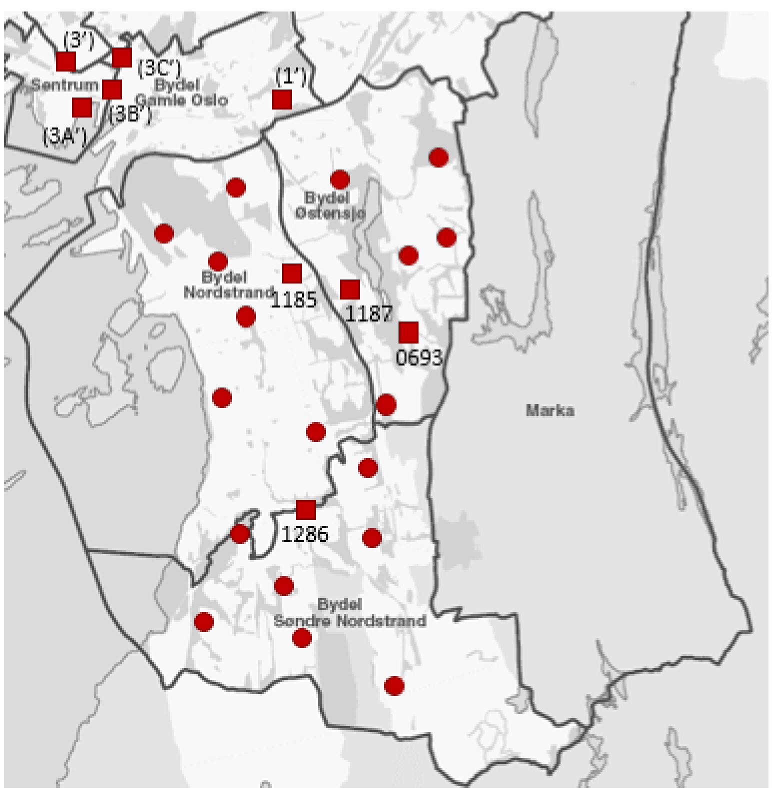

Figure 19: Possible station locations and demand nodes in Oslo Southeast

Originally, an additional ladder is fixed to the City station, as discussed in Section 10.2.3. This restriction is now relaxed such that the ladder unit can be located in any one of the southeastern candidate location. The results are presented in Table 33. Intuitively, relaxing the ladder improves the objective value. S4.4 still yields the highest objective value, now referred to as S4.10, with the ladder located at 1185.

Table 33: Objective value and suggested location for 2 new fire stations in the southeastern region including a ladder unit

#	Fixed locations	Objective value	Station location	Ladder location
S4.7	(1),(3),(5)	0.13331	1286	0693
S4.8	(1),(3)	0.13452	0693,1286	1185
S4.9	(1'),(3')	0.13606	0693,1286	1185
S4.10	(1'),(3A')	0.13762	1187,1286	1185
S4.11	(1'),(3B')	0.13622	0693,1286	1185
S4.12	(1'),(3C')	0.13580	0693,1286	1185

The best solution from the isolated study of the southeastern region, S4.10, is further analysed in relation to the whole jurisdiction of Oslo. In order to investigate the effects of an additional station, 1286 is included as both a demand zone and possible candidate location. The results from these tests are presented in Table 34. In S4.13 all resources are fixed according to current station allocation, including one base unit located at 1286. In S4.14 the resources are fixed according to the possible future station allocation from S3.13. In S4.15 the resources are fixed according to S3.10, including 1286. Finally, tests S4.16-S4.18 look into alternative locations for the additional base unit that is initially fixed to southeastern region. In these tests the location of one of the base units in Oslo Southeast is relaxed and the model suggests alternative locations for this unit, as indicated in the column *Free Unit Location*. The results clearly indicate that Nordre Aker is the preferred location for a potential new base unit, rather than any of the suggested southeastern locations.

Table 34: Objective value for fixed station locations

#	Station location	Free unit location	Objective value
S4.13	(1),(3),(5),1286	-	0.15492
S4.14	(1'),(3A'),1286,1187	-	0.15510
S4.15	(1'),(3C'),(5E),1286	-	0.155883
S4.16	(1),(3),(5)	Nordre Aker	0.15615
S4.17	(1'),(3A'),1286	Nordre Aker	0.15676
S4.18	(1'),(3C'),(5E)	Nordre Aker	0.15813

10.2.5 Reallocation of Resources for Fixed Station Locations

In order to test the optimal resource allocation, given a set of fire station locations, the different station locations are considered fixed. However, the resources are not fixed to a certain station, but are allocated by the model. This scenario is tested with different station locations, found in the previous studies. The results are presented in Table 35. In addition to the current situation in S5.1, the locations that are investigated in this case are chosen based on their performance in the previous tests. S5.2 performs well when the southeastern region of Oslo is studied in isolation, while S5.3 gives a better objective value when the whole jurisdiction of Oslo is included in the preparedness evaluation. A graphical representation of the respective resource locations is presented in Figure 20. The illustration reveals that both the solutions from S5.1 and S5.3 locate three ladder units in the same candidate location. This contradicts the intuition that no more than two ladder units should be located in a certain zone, as maximum two ladder units are required for any given accident type. Again, the problems are not solved to optimality in this case and by visual inspection one can determine that moving one of the ladders to another candidate location gives a slightly improved objective value. Knowledge of the problem and a critical inspection of the solutions are therefore important when mathematical approaches are used to solve real-life problems.

Table 35: Objective value for 7 possible station locations

#	Station location	Objective value
S5.1	(1),(3),(5)	0.15448
S5.2	(1'),(3A'),1286	0.15465
S5.3	(1'),(3C'),(5E)	0.15601

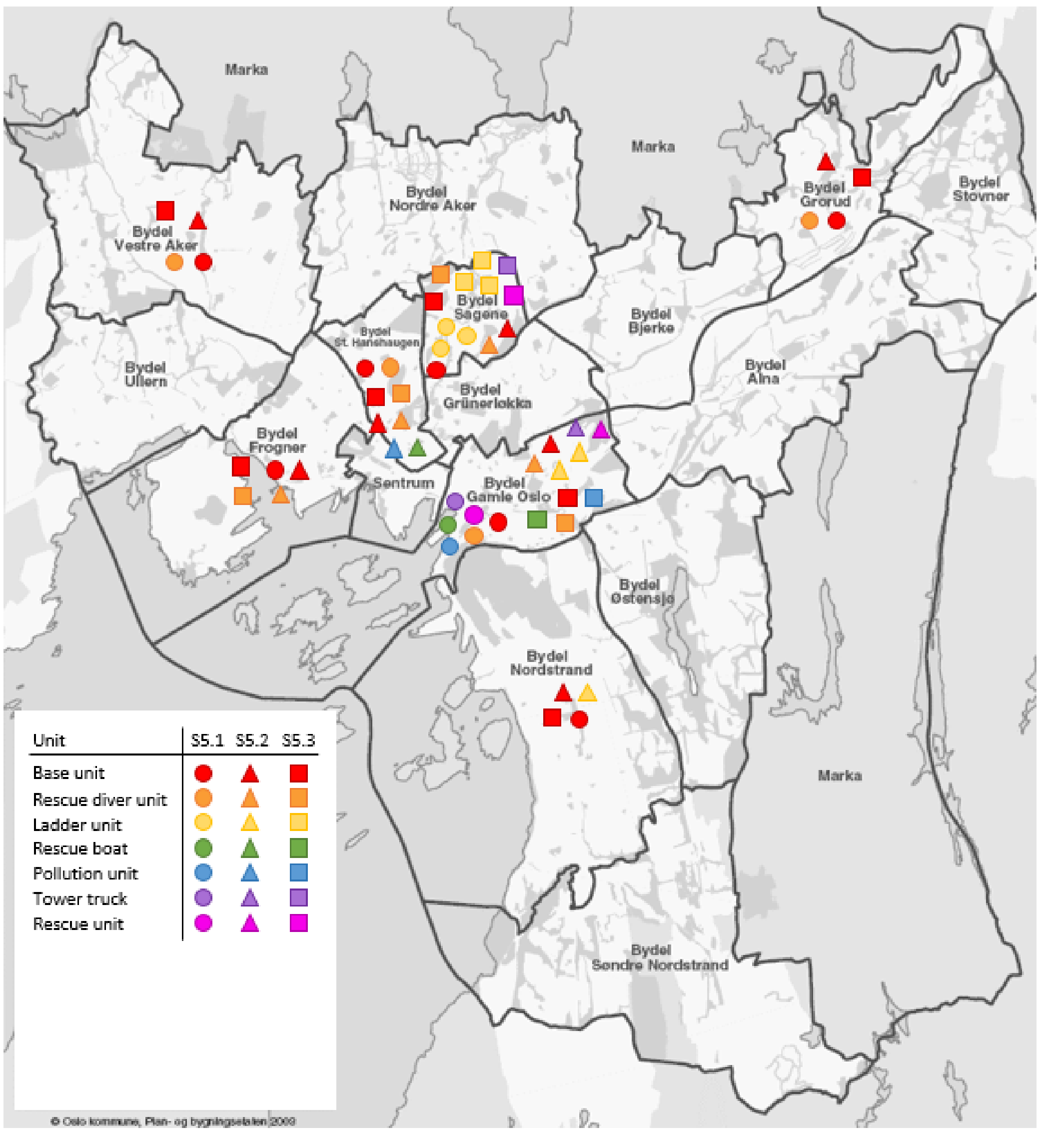

Figure 20: Reallocation of resources for fixed station location

11 Discussion and Conclusion

In this section, the findings from the computational study are discussed. First, the results of the technical study are assessed. Second, the modeling assumptions are addressed, to give reason for the choices made. Next, this section analyze the findings of the economic study for the real-life FRS situation in Oslo. Conclusions are drawn on the basis of the results, to make them useful for decision makers in FRS. The applicability of the model is also assessed in the light of modelling and data discrepancies, to identify potential sources of inaccuracy. Finally, this section looks into future studies and suggests potential extensions of the model.

11.1 Findings from the Technical Studies

The purpose of the technical study is to determine the most desirable formulation of the MPLP-HAR for the case at hand. Performing the technical study on a test case of similar proportions to the real-life problem investigated in the computational study, increase the ability to determine improvements effective for the FRS institution analyzed. There is no guarantee that the chosen formulation is preferred in an other case, as different model improvements can come more into their own if the particulars in the problem are changed.

From the technical study in Section 6, it is found that reducing the number of constraints, and thus also the number of rows in the problem, yields the best results for the test case. Specifically, introducing equations (38) and (39) from Section 5.3, in the test denoted T8, give a desirable formulation for the computational study. T8 performed well at both a 5 and 10 hours run-time and it is a simple aggregation of constraints that does not change the solution space. Heuristic approaches are also considered. The LP relaxation, the stochastic variable generation, or the redefined model all find useful solutions within 5 hours, and the LP-relaxation is even able to provide its solution in less than 5 hours. Heuristic solution methods are therefore considered as a viable option for solving the MPLP-HAR. For the computational study, formulation T8 is chosen instead of a heuristic solution approach. This formulation provides solutions of comparable quality to the heuristics whenever the problem becomes to large to solve to optimality and it is able to give the optimal solution when the problem size is limited.

The trade-off between solution quality and computational time is discussed extensively in Section 6.1. The section concludes that a gap of less than 5% yields acceptable results for this problem. The problem is therefore not always solved to optimality. Particularly when the input data is uncertain and contain inaccuracies, near-optimal solutions are acceptable. This means that the 5 hours test is sufficient to generate solutions of a desired quality and that is a reasonable time span for a decision support tool for strategic and tactical problems. The improvements that were found in having a 10 hours run-time for the problem were marginal and therefore the 5 hours run-time is chosen for the computational study. This is in fact a typical phenomenon in location problems. A number of near-optimal solutions are often very similar and the objective function is therefore relatively flat, meaning that the gain from extending the computational time is reduced rapidly.

11.2 Preparedness Validation

After determining the desired formulation of the MPLP-HAR in the technical study, the preparedness measure of the objective function was validated. As discussed in Section 8.4, there are three methods for validating a preparedness measure. Firstly, there is validation by dispatcher evaluation. FRS professionals from BRE were consulted for the initial development of the preservation function and their experience is the basis for the value of the exponential and linear decreasing factors, α_a and β_{av}. Representatives from BRE were also central in evaluating the solutions from tests P1 - P3, from Section 10.1. For the MPLP-HAR to be an applicable decision support tool for FRS professionals, the results must be in line with their perception of preparedness. Even though professionals possess a great deal of experience in the field, it is difficult to accurately determine the identity of the preservation ability with time, especially when they are asked to specify percentage values. Solution valuation is therefore also of great importance.

Next, a sensitivity analysis was performed, based on the information provided by BRE, with respect to the preservation function. The preparedness measure is closely related to the preservation function and the sensitivity analysis investigates the solution behaviour as the identity of the preservation function changes. Tests P1 - P3 display the expected changes in the solution structure as the preservation factors, α_a and β_{av}, are changed. Having a steep initial decrease emphasises the relative importance of a rapid first response. In these cases the solution structure tends to have resources spread over a wider area to ensure that a first responding unit quickly can reach any demand zone. This is particularly evident as all required units are assumed to be able to perform first response to a given accident in these tests. Similarly, the resources tend to concentrate on high demand zones whenever total response is given a high, relative importance in the preservation function.

11.2.1 Comparison to Existing Location Problems

As an important part of the preparedness and model verification process, the MPLP-HAR was compared to established location problems from existing literature. In this case, the objective function reflects the characteristics of three different problems, while the constraints from the formulation of the MPLP-HAR are kept unchanged. From the solutions, presented graphically in Figure 13, it appears as though the MPLP-HAR provides a solution structurally similar to the ones given by the MCLP and p-median location problems. In the case of BRE in Oslo, the MPLP-HAR is able to capture and emphasise the same operational characteristics as the existing problems in the comparison study. This is a good indication that the preparedness measure is applicable for determining appropriate locations for resources in FRS.

Solutions from the three location problems were also evaluated using the three different objective functions. The purpose is to examine how well the solutions perform in related objective functions. Intuitively, each problem achieves the best objective value when evaluated by their own objective function. The study reveals that the MPLP-HAR is closely related to the p-median problem, as the objective values are quite similar, indicating that both problems emphasises the same aspects. Interestingly, the MPLP-HAR solution outperforms both the p-median evaluated in the MCLP and the MCLP evaluated in the p-median objective function. All solutions in this case performs well when evaluated as cover models due to the relatively high threshold time value.

11.3 Scenarios

In this section the findings from the investigated scenarios are discussed. The MPLP-HAR is developed to be a decision support tool for strategic and tactical problems. The scenarios are chosen based on real-life challenges faced by BRE in Oslo. The scenarios also showcase the model applicability by solving a set of different types of problems that the MPLP-HAR is designed to evaluate.

11.3.1 Current Station Structure

Initially, the current station and resource location is analyzed to give a preparedness value for current operational state of the organization. Figure 14 displays the location of the nodes and the current station locations. Now, the MPLP-HAR provides an optimal objective value given the location of the response units, but this value does not provide any information on the level of preparedness in itself. The objective value must be seen in relation to the objective value of different operational structures.

The current resource location is therefore analyzed in relation to the optimal resource location. The two objective values are presented in Table 28 and a graphical representation of the response unit locations is given in Figure 15. For the current station location the objective value is solved to optimality, as the x_{ju} variables are pre-determined. The optimal location is not solved to optimality, but rather has a gap of 5.20%, implying that the true optimal objective value lies somewhere between the best solution and the best bound, indicated in Table 28.

The graphical representation reveals similarities between the two solutions in that both choose to focus on the inner city districts by positioning several units in central zones with high demand. However, the improvements in the objective value in the optimal resource location arise from having a single response unit positioned in 10 individual candidate locations. This scattered structure of the fleet indicates that efficient first response can be provided to large parts of the jurisdiction, as all required units are assumed to be able to provide first response. According to FRS professionals, the first response is crucial when it comes to preserving life and property in the event of an emergency, and the emphasis on first response is reflected in the objective function of the MPLP-HAR.

An apparent drawback of the optimal resource location structure is the need to establish FRS facilities in most of the city districts, as opposed to having few, large fire stations like today. This is financially costly, as economies of scale dictate that it is more efficient to keep multiple response units the same facility. It is not considered realistic for BRE to expand to 12 fire stations in the near future, but the findings from the optimal resource location should still be kept in mind. In Sweden, dividing traditional rescue teams into smaller operational groups, of for example individual response units, has gained ground as a new way of planning. The smaller groups can be strategically located in high risk areas for given time periods, to maintain a high level of preparedness without establishing permanent facilities at these locations. BRE in Oslo is recommended to look into the possibility of such organizing, based on the findings from the optimal resource location.

11.3.2 Strategic Station Relocations

BRE have decided to move the Main Station to Bryn. In doing so, they also plan to move the resources currently located at Bryn to a location in the city centre. Three possible candidate locations have been identified as appropriate new sites for a fire station and they were individually evaluated in relation to the new Main station at Bryn. The results are presented in Table 29. The best objective value and overall preparedness state for the jurisdiction is achieved by selecting to have the new City station located at (3C'). This is therefore the recommended station location from the candidates presented, based on the evaluation criteria of keeping a high level of overall preparedness.

The MPLP-HAR is designed to be a strategic and tactical decision support tool for FRS professionals and its full potential is utilised in problems like these. Professionals perform an initial analysis were resource requirements, economics, applicability, and regulatory issues are considered and end up with a number of suitable candidate locations. These candidate locations are then evaluated by the MPLP-HAR to find the optimal solution. Whenever the problem contains a limited number of response units and candidate locations, it is easily solved to optimality. It is also easier to accept the solution as feasible and optimal whenever a pre-computational assessment of the candidate locations has been made.

The study of relocating Kastellet station sets initially out to investigate whether proximity to the main roads E6 or E18 should be emphasised. Two suggested locations are tested, in addition to the current station location at Kastellet. The results indicate that the overall preparedness improves the most if the new station is located at (5E), in relation to E6. This location is favourable possibly because the response time to other high demand areas are reduced, compared to location (5) or (5D). The results are however not a sufficiently good measure of the preparedness in the southeastern region of Oslo. By investigating candidate locations (5), (5D), and (5E) all demand in the region is assumed to come from the candidate location itself and demand in parts of the region is therefore ignored.

To get a more accurate representation of the demand distribution in the region a finer division of demand zones is necessary. Increasing the number demand zones in Southeast Oslo means that the region needs to be isolated from the rest of jurisdiction in order for the MPLP-HAR to computationally evaluate the preparedness. The interesting finding from Table 32 is that City station location (3A') gives a better objective value in this case than (3C'), which is found to be the optimal location when considering the whole area of Oslo. The reason is that responding to Southeastern Oslo is better from (3A'), while (3C') has the advantage when it comes to other parts of the city.

This finding showcases the importance of considering preparedness in the jurisdiction as a whole. Isolating certain regions can give solutions that are optimal for that particular region, but not for the whole area. In a broader perspective, evaluating preparedness in the isolated region of Oslo might also yield sub-optimal solutions. BRE have three neighbouring FRS jurisdictions with Asker and Bærum to the west, Romerike to the north, and Follo to the east. Cooperation between these, and indeed all FRS institutions in the country, can contribute to an overall improved preparedness level in Norway.

The isolated studies of the southeastern region of Oslo brings forth an interesting observation regarding problem scalability. In these studies, the set of response units is reduced to the specific number units that are concerned in the problem. This in turn means that the MPLP-HAR is able to solve problems were the number of demand zones and candidate locations are increased, to optimality. The combined size of the sets makes up the complexity of the problem, and even though the relationship is not necessarily one-to-one, when some sets are reduced, others can be increased. It may therefore be necessary to divide the area and resources in complex ERP problems to be able to obtain a useful solution. In these situations, it is however important to evaluate the solutions in the broader perspective, as well.

11.3.3 Additional Facilities and Resource Management

The initial analysis, made in Section 10.2.4, to find the location of a potential additional fire station in the southeastern part of Oslo is again performed on an isolated region of the jurisdiction. The best way to interpret the results is therefore to see them as a basis for finding suitable candidate locations, as is done with (3A'), (3B'), and (3C'), and then do a study of these candidate locations. The approach requires an iterative process, where these results are the first step to finding the optimal location for an additional station.

A key observation one makes from these tests is that the optimal location for the additional fire station varies with choice of City station. The optimal solution consistently contains candidate location 1286, but depending on whether (3A'), or (3B')/(3C') is chosen for the City station, the optimal location for the additional station is either 1187 or 0963, respectively. This shows the importance of understanding long-term planning when it comes to establishing permanent FRS facilities. The choices that BRE makes for the location of the City station today effect the plans for new fire stations in the future. To achieve an optimal level of preparedness in the long-term, any decisions must be evaluated in the context of the entire planning period.

Another interesting finding from Table 34 is the suggested locations for the unconstrained response unit when the whole area is evaluated and one single station is located in Oslo Southeast. For all the investigated station locations in the city centre and southeast region, the additional unit is being located in Nordre Aker. This indicates that the overall preparedness level is improved more if the additional station with one base unit is located in this city district, rather than in Oslo Southeast, as BRE initially suggests. The result is however based on the current demand distribution in Oslo. The final decision must be based on projected demand for the planning period as well, rather than just historical data, that is used in these analysis.

A study of the optimal resource allocation with fixed station locations is performed in Section 10.2.5. Three different station locations are considered, based on the discussion of establishing a new station in the city centre and the relocation of the station at Kastellet. The solutions clearly distinguish between inner and outer stations with respect to resource location, as is done by BRE today. The current allocation of resources is summarized in Table 15 in Section 7.3. The MPLP-HAR does however suggest a strengthening of the outer stations by having two response units, rather than one base unit, for several station location scenarios. The interesting thing about this finding is that these changes are relatively easily implemented compared to establishing new facilities. It is therefore recommended for the BRE to strategically evaluate the allocation of its resources. The test could potentially be even more interesting to do if the problem is evaluated with time dependent demand. At day time, many people go to the city centre to work before returning to their suburban homes in the afternoon. If emergency response demand follows the same pattern, this would underline the idea of temporal strengthening of the outer zones.

11.4 Model Discrepancies

It is very difficult to model real-world processes with complete accuracy in operational research. Simplifications have to be made in order to get an applicable and understandable model. The following two sections identify some of the discrepancies and evaluate their significance. A distinction is made between modelling and data discrepancies. A model discrepancy involves a simplification to the mathematical formulation to the extent that it is unable to capture aspects of the real-life operational situation it is suppose to model. A perfect mathematical representation is always difficult to achieve in ERP, but some simplifying assumptions are more valid than others. Even though the MPLP-HAR includes a number of aspects that are useful in FRS location problems, such as a heterogeneous fleet of unique response units, heterogeneous accident types, a distinction between first and total response, and a preservation function, some aspects have been left out. Their importance and implication are discussed in this section.

11.4.1 Uncertainty

Uncertainty related to travel time and response unit availability is not taken into account. As previously discussed in Section 2.3.1, the impacts of uncertainty in travel time are minor (Erkut et al., 2008), and can be caused by for example conjunction and rush hour traffic. The data discrepancy in the travel times can however be more significant when evaluated for a FRS problem, as a fire engine experiences a lower degree of navigability than an ambulance. Particularly in built up areas, it is unrealistic for a fire engine to steadily keep the speed limits at all times. The travel time compensation factor is included to address this issue for each vehicle type, but it does not change over time or with the nature of the dispatching route. The uncertainty related to resource availability is by Erkut et al. (2008) considered more significant. However, most studies on the subject examine instances where the resource utility rate is far greater than in FRS, as for example in EMS. This factor have therefore been excluded from the modeling, as the resource availability is generally considered high in FRS. BRE in Oslo is dimensioned to handle three simultaneously occurring accidents, which in this particular instance contributes to the valid assumption of no uncertainty regarding resource availability.

11.4.2 Static Planning

For most of the papers discussed in Section 2, the assumption of static planning is made. Static planning is characterized by having known input data that does not change over the course of the planning process. This is almost universally a simplification of the ever-changing real world being modeled. However, in instances where the dynamic aspects are not dominant, such as in strategic or tactical level planning, the simplification can still yield useful results. The MPLP-HAR is intended to be a decision support tool at precisely these levels. Throughout the bounded planning period of the problem, which can stretch from weeks and months to years and decades, the problem parameter are considered constant. The relevant information for determining the location and resources needed for a new facility, such as demand, travel times, and accident characteristics, are generally perceived to not change with time. However, demographics can change and new parts of the jurisdiction are continuously developing. It is therefore of great importance for the FRS institution to consider settlement forecasts in the region when establishing infrastructure that is meant to last for years to come.

11.5 Data Discrepancies

The other main source of discrepancy comes from the input data. Data discrepancy means that the input data themselves are an inaccurate representation of the parameters in the real-world problem being modelled. The discrepancy can come from insufficient or inaccurate data. However, data discrepancies can be reduced by devoting a closer emphasis to securing accurate input data.

11.5.1 Demand Zones

The demand zones are a very important input parameter as they make up the nodes in the location problem. Essentially, each node represents a demand zone and all demand is accumulated to a single point. This is of course a simplification as emergencies can occur anywhere in zone. The location of the nodes do not only dictate the travel times between the zones, but also the travel time within each demand zone, from the center to the perimeter. The nodes are located in the geographical center of each demand zone, except for the candidate locations that contain response units, in which case the facility location also becomes the node. Inaccuracies can therefore occur whenever the facility is not located in the center of the demand zone.

The potential inaccuracies that are related to the demand zones becomes more severe as the size of the zones in the case study increases. In this report, the city districts are initially the basis for 15 demand zones. This is a very coarse division of the area and one of the major data discrepancies in the study. Dividing the area up into smaller demand zones reduces all of these discrepancies, but on the other hand makes the problem larger and harder to solve. In this case study, the MPLP-HAR was unable to produce a feasible solution when the number of demand zones were increased in the case when all response units are free to be located in any candidate location. The case studies where the whole region is considered, in which the city districts form the demand zones and all demand zones are potential candidate locations, are therefore assumed to contain considerable inaccuracies due to demand zone discrepancy.

However, for the location problems that focus on a sub-section of the total area or only a select few candidate locations, it is favourable to divide the the demand zones into a finer distribution. In Section 10.2.4, the three southeastern city districts in Oslo are divided into 22 demand zones. The increased number of demand zones yields a higher level of accuracy with respect to travel times and demand distribution. The geographical coordinates of postal codes in Oslo are publicly available and by iteratively combining the two closest postal codes into a single zone with one set of coordinates, the city districts were divided into demand zones. The coordinates from the lowest postal number in the first combination of two postal codes becomes the centre of the new demand zone. A closer investigation of the true center of the new demand zone can be conducted by examining the coordinates of all combined postal codes. However, the selected center is assumed to be combined with postal codes from all sides, and the simplification of keeping its coordinates is considered valid.

11.5.2 Candidate Locations

A candidate location is a node in the network were the emergency response resources can be positioned. In the computational study, the candidate locations are selected on different basis depending on the purpose of the conducted test. At times, all demand zones are considered possible candidate locations, even though not all of them may be suitable for the location of a FRS facility or response unit. This means that the results cannot be utilised directly to locate resources. In Section 10.2.2, three appropriate candidate locations for a city centre facility have been identified by BRE, and the MPLP-HAR can evaluate them directly with respect to the preparedness level in the area. In a study were the resource location problem is applied without a careful assessment of the candidate locations, it can however indicate regions of the area that suffer from low levels of preparedness. This analysis can therefore motivate a closer assessment of the region in question. Once a number of suitable candidate locations have been identified, a new evaluation can be performed to determine the optimal location.

11.5.3 Accident Types

The FRS responds to a great variety of emergency calls. BRE categorizes the calls into one of more than 100 categories. In cooperation with representatives of BRE, these categories are aggregated into an appropriate number accident types that are used in the model. This simplification is considered valid as many of the emergencies trigger the same response from the FRS and the categories are aggregated into accident types based on resource requirements and severity development. The possible inaccuracies in the aggregated accident types comes from variations in intervention and rescue operation times between different emergencies in the same accident type. The emergency response units are dispatched based on the initial classification of the accidents, but the perception of accident severity can change once professionals arrive at the site and get to inspect the emergency first hand.

11.5.4 Demand

The magnitude of demand in each demand zone is based on historical data and is proportional to the relative number of each accident type that have occurred in the zone. New routines for reporting and classifying emergencies were recently implemented and the available historical data used in this case study is therefore only based on emergency calls from the first quarter of 2016. Basing demand in a given zone solely on historical data, could also be problematic. Because the historical demand is based on data from recent months, and FRS emergencies occur relatively rarely, the demand might central around a few demand zones, while others are neglected. This is not because the demand necessarily is so low in the neglected areas, but rather because there has not been any accidents in these areas recently. Basing the demand solely on historical data would hence require extensive data from a longer historical time span. For regions of the jurisdiction that experience a rapid demographic development it is also recommended to consider the forecasts for future demand in the planning period. This is particularly important when the demand is the basis for establishing permanent emergency response facilities, such as a fire station.

11.5.5 Response Units

The response units in the case study are divided into vehicle types, as summarized in Table 19 in Section 9.5. For this case study, all units in a vehicle type are considered homogeneous. This means that for example the base units are assumed to have the exact same capabilities, when in fact they are set up differently with respect to staffing and equipment levels. The number of rescue divers on a base unit effects the intervention time before the response unit is considered fully operational upon arrival at the accident site. This discrepancy is considered minor, as the intervention time only makes up a small proportion of the total response time and the differences in emergency relief efforts are neglectable, according to FRS professionals.

As previously mentioned in Section 9.5, all response units are treated as vehicles that are able to respond to emergencies using the public road network. This includes the rescue boat. Obviously, this is an assumption that can cause discrepancies related to response times on the fjord. The reason for this simplification is the lack of public available data for offshore travel times.

11.5.6 Response Time

The response time consists of three elements; the preparation time prior dispatching, the travel time from the candidate location to the accident site, and the intervention time before emergency relief efforts commence upon arrival. The preparation and intervention time are largely independent of the initial resource location, the location of the accident, and the accident type. This means that these aspects of the response time mostly effect the relative objective value and not the solution to the problem. In this report, both the preparation and intervention times are constant. Further assessment of the effect of adding a constant delay to the travel time is however needed, before one can write off its significance completely.

The travel times are what dictate the solution. There are however potential data discrepancies within the travel times themselves. The data is obtained using Google Maps to calculate the travel times between postal codes, and should be critically assessed. Specifically, the data does not include variations in travel times at different times of the day and the routes include public road networks only. Response units are therefore assumed to keep the speed limit along their dispatching routes. The experience of FRS professionals suggest that not all vehicles are able to keep the up with the speed limits as they travel to an accident site, and a travel time compensation factor has therefore been added for every vehicle type. The travel time factors are summarized in Table 21 in Section 9, along with the preparation and intervention time values. The travel time factor is still assumed to be constant for all times of the day and any type of road, even though there are some variations along these aspects.

11.5.7 Preservation Function

The preservation function is an essential part of the performance measure in this report. Its accuracy and realism is therefore important. The structure of the preservation functions for the different accident types are ascertained with the help of FRS professionals. However, the numerical value of the parameters are estimated from a set of example values. Evaluating the realism of the preservation function would involve an assessment of multiple historical accidents within each accident type, to analyze whether the function is able to capture the distinct, real life preservation ability. A higher level of accuracy could be achieved by a closer study of the modeling parameters.

11.6 Heuristics

Heuristic solution methods are not applied in the computational study of this report, but as shown in the technical study in Section 6, they can provide viable results. The formulation of the MPLP-HAR means that it becomes a very large problem once it is applied to a real-world case study. Obtaining a solution to the problem within reasonable computational time might in some cases require the use of heuristics, and this report has suggested a few methods.

A heuristic solution method gives no guarantees for solution quality and one can never be sure to find the true optimal solution to the problem. However, heuristics are appropriate options in optimization problems were one is happy to find a sufficiently good solution. In ERP location problems it is usually very difficult to reach an optimal solution. Location problems tend to have a very flat objective function, as many different solutions provide a close to optimal objective value. The relative degree of uncertainty in the input data also makes it more acceptable to achieve a near-optimal solution, and the findings from heuristic solution methods can be considered useful in may cases.

11.7 Future Studies

Based on the findings of this report, recommendations for further studies are outlined in this section. It is important that aspects that make the model more applicable for decision makers are studied further. Introducing an economic perspective and costs with respect to the establishment of infrastructure and resource procurement, is a natural extension. Estimating a price on emergency response and particularly human life, is notoriously difficult. However, the cost of preparedness should be considered in a broader socioeconomic perspective. The objective would be to determine the optimal level of preparedness that is efficient for the society as a whole.

The MPLP-HAR can be used as an operational planning tool if dynamic information is implemented into the model. By updating the information according to incidents that occur in the planning period, response units can be reallocated to new locations, as the availability of resources change. This gives dispatchers an operational decision support tool. At this planning level the computational time is essential as input parameter change in real-time. The current formulation of the problem would therefore only be applicable to operational problems were there are a limited set of candidate locations or response units that were to be relocated. Heuristic solution methods can however be a way of solving the MPLP-HAR more efficiently. As previously discussed, a number of heuristic approaches were able to provide viable solutions, and a closer examination of appropriate heuristics is recommended for future studies.

Including scheduling in the ERP process can provide decision makers with a useful tool in their everyday work. Today, the scheduling is done manually by the operations chief, and it takes up much of his or her time. At any given time, the fire brigade requires a sufficient staffing level. Having both the right number and combination of firefighters on call is therefore an extensive planning process. Extending the model to including variables for staff members with different competencies can provide personnel planners with a useful tool for staff scheduling. In this instance it would also be natural to consider different staffing levels on each response unit as well, by further differentiating the capabilities of each unit.

As previously discussed, extending the model beyond the simplification that all response units are available at all times, is recommended. It can therefore also be useful to look into the planning of the engagements that the FRS have in addition to the emergency calls. Many of these engagements are pre-planned, such as training exercises, preventive work, and care and maintenance. It is however essential that these tasks can be performed without compromising the emergency preparedness level in the region. Coordination and scheduling of these activities is therefore important and can be incorporated into the ERP problem.

A local FRS might have certain infrastructural elements in their jurisdiction that require an elevated level of preparedness, as previously mentioned. For BRE in Oslo, this includes the underground metro, several long tunnels with heavy traffic, and a number of institutions at risk for sabotage or terrorism, discussed in the case study in Section 7. In addition, the FRS institutions in Norway are required by law to have a maximum response time of 10 minutes for special fire objects, such as hospitals, assembly rooms, and accommodations (DSB, 2002). To consider these aspects, the model can be extended to include upper limit constraints for the response time to a subset of the demand zones. This would resemble a cover formulation and can give an indication of the resources required to maintain a certain preparedness level, especially for a more extensive case study than Oslo.

For future studies of the MPLP-HAR the authors recommend a closer study of the input data, specifically the travel times and demand data. Increased travel time accuracy can be achieved by introducing temporal and spatial differentiation of the travel time factor, with respect to the time of day and type of road being used. It would also be beneficial to look into travel times at sea for the rescue boat. The basis for the demand needs to be revisited before new computational studies in the region can be conducted. In Oslo, BRE continuously updates its accident statistics, and a more complete picture of the demand distribution can be drawn as the number of reported emergencies goes up.

Although the three main emergency services have their distinct areas of responsibility, co-operation and intertwining responses are common. Therefore, it would be interesting to investigate the relation to the other emergency services closer. For example, firefighters could possess a set of competencies that would be useful for an EMS response, such as knowledge of first aid and CPR. Taking these considerations into account could form the basis for studying a problem that maximizes the overall preparedness, for all emergency services. Recently, Norway introduced a new Public Safety Network (Nødnett), allowing for direct communication between the three main emergency services. In light of this new found interest in emergency service coordination, dispatchers could also benefit from the ability to dispatch vehicles from any emergency service, solely depending on their location.

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A Mathematical Model - Heterogeneous Units

This Appendix presents an alternative model to the MPLP-HAR, were the units of the same vehicle types are considered equal. In many cases it is reasonable to assume heterogeneous units within the same vehicle type, and the following formulation eliminates a great number of symmetric solutions assosited to the assumption.

A.1 Mathematical Formulation

Sets

I	the set of demand zones, i
J	the set of candidate locations for vehicles, j, k
A	the set of accident types, a
A^S	the set of accident types that require more than one unit, $A^S \subset A$
V	the set of vehicle types, m,n

Parameters

λ_a	the severity factor for accident type a
D_{ia}	the demand in zone i for accident type a
P_{iajm}	the preservation value for accident type a in zone i, with first and full response provided by vehicle type m from zone j
P_{iajmkn}	the preservation value for accident type a in zone i, with first response provided by vehicle type m from zone j and full response from unit n in zone k
C_m	the number of available units of type m
E_{am}	the required number of vehicles of type m to an accident a
T^T_{jim}	the travel time from zone j to i for vehicle type m
T^P_m	the preparation time for vehicle type m
T^I_m	the intervention time for vehicle type m
Q_{am}	$\begin{cases} 1, & \text{if vehicle type } m \text{ can provide first response for accident type } a \\ 0, & \text{otherwise} \end{cases}$

Variables

x_{jm} number of vehicles of type m located in zone j

y_{iajm} number of vehicles of type m located in zone j that contributes to accident a in zone i

$$s_{iajm} \quad \begin{cases} 1, & \text{if a vehicle of type } m \text{ is located in zone } j \text{ contributes to accident } a \text{ in zone } i \\ 0, & \text{otherwise} \end{cases}$$

$$w_{iajm} \quad \begin{cases} 1, & \text{if a vehicle of type } m \text{ is located in zone } j \text{ provides first and full response to} \\ & \text{accident } a \text{ in zone } i \\ 0, & \text{otherwise} \end{cases}$$

$$w_{iajmkn} \quad \begin{cases} 1, & \text{if a vehicle of type } m \text{ located in zone } j \text{ provides first response to accident } a \text{ in} \\ & \text{zone } i \text{ and full response is provided by the a last response unit of vehicle} \\ & \text{type } n \text{ located in zone } k \\ 0, & \text{otherwise} \end{cases}$$

Mathematical Formulation

$$max \sum_{i \in I} \sum_{a \in A} \lambda_a D_{ia} (\sum_{j \in J} \sum_{m \in V} \sum_{k \in J} \sum_{n \in V} P_{iajmkn} w_{iajmkn} + \sum_{j \in J} \sum_{m \in V} P_{iajm} w_{iajm}) \tag{52}$$

$$\sum_{j \in J} x_{jm} = C_m \quad m \in V \tag{53}$$

$$\sum_{j \in J} y_{iajm} \geq E_{am} \quad i \in I, a \in A, m \in V \tag{54}$$

$$\sum_{j \in J} \sum_{m \in V} Q_{am} w_{iajm} = 1 \quad i \in I, a \in A/A^S \tag{55}$$

$$\sum_{j \in J} \sum_{m \in V} \sum_{k \in J} \sum_{n \in V} Q_{am} w_{iajmkn} = 1 \quad i \in I, a \in A^S \tag{56}$$

$$w_{iajm} \leq x_{jm} \quad i \in I, a \in A/A^S, j \in J, m \in V \tag{57}$$

$$w_{iajmkn} \leq x_{jm} \quad i \in I, a \in A^s, j, k \in J, m, n \in V \tag{58}$$

$$w_{iajmkn} \leq x_{kn} \quad i \in I, a \in A^s, j, k \in J, m, n \in V \tag{59}$$

$$y_{iajm} \leq x_{jm} \quad i \in I, a \in A, j \in J, m \in V \tag{60}$$

$$Ms_{iajm} \geq y_{iajm} \quad i \in I, a \in A, j \in J, m \in V \tag{61}$$

$$\sum_{j\in J}\sum_{m\in V}\sum_{k\in J}\sum_{n\in V}(T^T_{kin}+T^P_n+T^I_n)w_{iajmkn} \geq \sum_{j\in J}(T^T_{jip}+T^P_p+T^I_p)s_{iajp} \quad i \in I, a \in A^S, p \in V \tag{62}$$

$$x_{jm} \in \{1, ..., C_m\} \quad j \in J, m \in V \tag{63}$$

$$y_{iajm} \in \{1, ..., M\} \quad j \in J, m \in V \tag{64}$$

$$w_{iajmkn} \in \{0, 1\} \quad i \in I, a \in A^S, j \in J, k \in K, m, n \in V \tag{65}$$

$$w_{iajm}, s_{iajm} \in \{0, 1\} \quad i \in I, a \in A, j \in J, m \in V \tag{66}$$

The objective function (52) maximizes the preservation value based on the first and full response, weighted by the expected demand in the area and the severity factor. Constraints (53) make sure that the all available vehicles are allocated. The required number of the correct vehicle type are dispatched to the emergencies, as dictated by (54). Hence, the number of dispatched vehicles may be larger than required. Constraints (55) ensure that a qualified first response unit is provided. In instances where more than one unit is needed, (56) require that both first and full response is provided, by making sure that both the first and last response units are dispatched. Constraints (57)-(59) ensure that the vehicles dispatched are available in the given zones. The contributing units must come from candidate locations were the units are available, as ensured by Constraints (60). Using a big-M formulation, Constraints (61) make sure the vehicles contribute to the emergency when the number of contributing vehicles are larger than zero. In this case, M equals the maximum number of required units from any vehicle type m in any accident a, $max(E_{am})$. Hence, the big-M formulation is only necessary for whenever multiple units of the same vehicle type are needed to an accident time. In all other cases the M is equal to one. Constraints (62) ensure that the last response unit is actually last, meaning that all required vehicles have arrived when the last response vehicle arrives. Finally, the integer and binary identities of the variables are given in Constraints (63)-(66).