

Trygve Olsen

## **A Comparison of Four Different Diversification Strategies in the Norwegian Market with Portfolios Consisting of Stocks and Bonds**

A comparison of risk and return in portfolios of stocks and bonds in the Norwegian market based on equal weighting, 60/40, mean-variance, and risk parity

Trondheim, mai 2014



Høgskolen i Sør-Trøndelag  
Handelshøgskolen i Trondheim

Trygve Olsen

## **A Comparison of Four Different Diversification Strategies in the Norwegian Market with Portfolios Consisting of Stocks and Bonds**

**A comparison of risk and return in portfolios of stocks and bonds in the Norwegian market based on equal weighting, 60/40, mean-variance, and risk parity**

## **En sammenligning av fire forskjellige diversifiseringsstrategier i det norske markedet med porteføljer bestående av aksjer og obligasjoner**

**En sammenligning av risiko og avkastning i porteføljer av aksjer og obligasjoner i det norske markedet basert på likevektning, 60/40, mean-variance, og risikoparitet**

**HIST, Handelshøyskolen  
i Trondheim, Biblioteket,  
Postboks 2320  
N-7004 Trondheim**

Spesialiseringsretning:	Finansiering og Investering
Veileder:	Khine Kyaw

**Høgskolen i Sør-Trøndelag  
Handelshøyskolen i Trondheim**

Høgskolen har intet ansvar for synspunkter eller innhold i oppgaven.  
Framstillingen står utelukkende for studentens regning og ansvar.

Trygve Olsen

## **A Comparison of Four Different Diversification Strategies in the Norwegian Market with Portfolios Consisting of Stocks and Bonds**

A comparison of risk and return in portfolios of stocks and bonds in the Norwegian market based on equal weighting, 60/40, mean-variance, and risk parity

Trondheim, mai 2014

Høgskolen i Sør-Trøndelag  
Handelshøyskolen i Trondheim

Høgskolen har intet ansvar for synspunkter eller innhold i oppgaven.  
Framstillingen står utelukkende for studentens regning og ansvar.

## **Preface and Acknowledgements**

This thesis was written in my final semester of my master's degree in financing and investing at Trondheim Business School.

In this thesis I take a closer look at the Norwegian market, and compare risk and return in portfolios consisting of stocks and bonds based on equal weighting, 60/40 allocation, mean-variance optimization and risk parity. The objective is to illuminate different investment strategies based on the portfolios risk and return, and examine how investors can make strategic investment decisions depending on their risk profile and target return.

The reason I have chosen this topic is the simple explanation that I find investments and portfolio management very interesting. During the years in school I have learned about this topic in theory. As my master thesis it came natural to take a closer look at the Norwegian market and see how theory holds up in practice. In addition to this, I have work in this field between my bachelor and master degree.

**Supervisor:** Khine Kyaw

I would like to thank my supervisor Khine Kyaw for good advice and academic discussions during the process. I would also like to thank Stein Frydenberg at Trondheim Business School for giving me inspiration to select this topic for the thesis, and the instruction of how to use Datastream for collecting data.

The contents and opinions given in this thesis are the sole responsibility of the author.

---

Trygve Olsen

Trondheim, Thursday, May 22. 2014

## Abstract

This thesis takes a closer look on the Norwegian bond and stock market in the period 1998-2012 and compares different portfolio allocations. The problem statement is as follows: “A comparison of risk and return in portfolios of stocks and bonds in the Norwegian market based on equal weighting, 60/40, mean-variance, and risk parity”

Applied relevant theory is the framework of modern portfolio theory and the mean-variance optimization as well as the Sharpe ratio and the risk parity approach.

The portfolio construction and calculation of portfolio weights are obtained through the use of Excel and the Problem Solver.

The results show that the naïve equal weighting portfolio and the 60/40 stock/bond portfolio outperforms both the mean-variance and the risk parity portfolio. The mean-variance portfolio realizes the highest Sharpe ratio in line with the objective of the methodology, followed by the risk parity portfolio. Both portfolios realizes Sharpe ratios superior to the equal weighting and 60/40 portfolios.

The risk parity portfolio has some interesting characteristics such as equal return correlation to stocks and bonds making the portfolio truly diversified and not solely dependent on stock or bond returns, and it has equal risk contribution to the portfolios total risk from both stocks and bonds.

In the absence of leverage, however, the expected return of the risk parity portfolio and the mean-variance portfolio is too low to be compelling for most investors. By applying leverage to the risk parity portfolio or the mean-variance portfolio up to the naïve equal weighting or traditional 60/40 portfolios risk-levels, an investor can theoretically achieve higher returns. This depends, however, on the cost of leverage.

## Sammendrag

Denne avhandlingen ser nærmere på det norske obligasjons - og aksjemarkedet i perioden 1998-2012 og sammenligner ulike porteføljeallokeringer. Problemstillingen er som følger: "En sammenligning av risiko og avkastning i porteføljer av aksjer og obligasjoner i det norske markedet basert på likevektning, 60/40, mean-variance, og risiko paritet"

Anvendt relevant teori er moderne porteføljeteori og mean-variance optimalisering, samt Sharpe ratio og risiko paritet. Porteføljekonstruksjon og beregning av porteføljevektene er løst gjennom bruk av Excel og problemløseren.

Resultatene viser at den naive likevekts-porteføljen og 60/40 porteføljen bestående av aksjer/obligasjoner prester bedre enn både mean-variance og risiko paritet porteføljen. Mean-variance porteføljen realiserer den høyeste høyest Sharpe ratio i tråd med målet til metoden, fulgt av risiko paritet porteføljen. Begge porteføljer realiserer overlegne Sharpe ratio i forhold til likevektings - og 60/40 porteføljen.

Risiko paritet porteføljen har noen interessante egenskaper som lik avkastningskorrelasjon til aksjer og obligasjoner noe som gjør denne porteføljen virkelig diversifisert og ikke bare avhengig av aksje - eller obligasjonsavkastning, og den har likt risikobidrag til porteføljens totale risiko fra både aksjer og obligasjoner.

Uten belåning er imidlertid den forventede avkastningen til risiko paritet porteføljen og mean-variance porteføljen for lav til å være et fristende alternativ for de fleste investorer. Ved å belåne risiko paritet porteføljen eller mean-variance porteføljen opp til den naive likevektede eller tradisjonelle 60/40 porteføljenes risikonivåer, kan en investor i teorien oppnå høyere avkastning. Dette avhenger imidlertid av kostnaden ved belåning.

# TABLE OF CONTENTS

<b>1. INTRODUCTION .....</b>	<b>9</b>
1.2 THE STRUCTURE OF THE THESIS .....	11
<b>2. LITERATURE REVIEW AND THEORY .....</b>	<b>11</b>
2.1 THE HEURISTIC REALITY.....	11
2.2 THE RATIONAL THEORY .....	12
2.3 MODERN PORTFOLIO THEORY .....	12
2.4 LIMITATIONS.....	14
2.5 RISK PARITY .....	15
2.6 SHARPE RATIO .....	16
<b>3. DATA.....</b>	<b>17</b>
3.1 EQUITY (STOCKS).....	17
3.2 FIXED INCOME (BONDS) .....	17
3.3 RISK-FREE ASSET.....	17
<b>4. METHODOLOGY.....</b>	<b>18</b>
4.1 DEFINITIONS OF THE DIFFERENT PORTFOLIOS .....	18
4.1.1 EQUAL WEIGHTING PORTFOLIO .....	18
4.1.2 60/40 PORTFOLIO.....	18
4.1.3 MEAN-VARIANCE PORTFOLIO .....	18
4.1.4 RISK PARITY PORTFOLIO .....	18
4.2 PORTFOLIO CONSTRUCTION .....	19
4.2.1 CONSTRAINTS ON WEIGHTS.....	20
4.3 RISK PARITY PORTFOLIO CONSTRUCTION .....	20
4.3.1 CONSTRAINTS.....	21
4.4 SUMMARY OF PORTFOLIO CONSTRUCTION.....	21
<b>5. RESULTS.....</b>	<b>22</b>
5.1 PERFORMANCE.....	22
5.2 TRADITIONAL RISK-RETURN FRONTIER .....	23
5.3 RISK-ADJUSTED RETURNS .....	25
5.4 RETURN CORRELATIONS .....	26
<b>6. DISCUSSION .....</b>	<b>28</b>
6.1 RETURNS, VOLATILITY AND SHARPE RATIOS .....	28

6.2	INVESTOR'S CONSTRAINTS .....	31
6.3	LIMITATIONS .....	32
<b>7.</b>	<b>CONCLUSION.....</b>	<b>33</b>
<b>8.</b>	<b>REFERENCES .....</b>	<b>34</b>
<b>9.</b>	<b>APPENDIX .....</b>	<b>36</b>
9.1	NORWEGIAN BOND AND STOCK INDEX: CUMULATIVE GROWTH... ..	36
9.2	TIME SERIES OF PORTFOLIO WEIGHTS.....	36
9.3	THE EFFICIENT FRONTIER, 1998-2012.....	39



## TABLE SUMMARY

**TABLE 1:** Return correlations of the different portfolios with stocks and bonds 1998-2012, based on monthly returns.....27

**TABLE 2:** Excess return, volatility and Sharpe ratios for the different portfolios 1998-2012, based on monthly data.....29

## FIGURE OVERVIEW

<b>FIGURE 1:</b> Equal Weighting, 60/40, Mean-Variance, and Risk Parity Portfolios: Cumulative Growth 1998-2012. All portfolios are rebalanced monthly to maintain constant weights.....	22
<b>FIGURE 2:</b> Efficient Frontier of portfolios of stocks and bonds in the Norwegian market, 1998-2012.....	24
<b>FIGURE 3:</b> Sharpe ratios of different stock/bond portfolios, 1998-2012. All figures are calculated on a monthly basis.....	25

# 1. Introduction

**Problem Statement:** “A comparison of risk and return in portfolios of stocks and bonds in the Norwegian market based on equal weighting, 60/40, mean-variance, and risk parity”

Diversification is an important word in the world of finance and investing. The concept can most easily be described as not putting all of your eggs in one basket when making investment decisions. The following simple example has perfect negative correlation, but should clarify the concept of diversification. Suppose an investor has two investment opportunities at hand, investment A and investment B. Investment A drops down by 20 % the first year and recovers by 25 % the second year. Investment B goes up by 25 % the first year and then drops by 20 % the second year. Both investments have had zero cumulative growth individually after two years. If the investor had diversified with equal weighting (50% in A and 50 % in B) the investment would have yielded 2.5% each year with rebalancing, leaving the cumulative return after two years at 5.0625 %.

By diversifying investments across different investment opportunities or different asset classes it is possible to lower the risk of the invested portfolio substantially. In terms of total return, however, the diversified portfolio will probably lose to a less diversified portfolio. The upside is the improved risk-adjusted return that comes with diversification.

In practice, institutional investors and large pension fund portfolios usually have a 60/40 allocation between stocks and bonds, with 60 % invested in stocks and 40 % invested in bonds. The portfolios also include other asset classes, but generally with very small weights invested in them. In this way the alternative asset classes only contribute to the return and risk at the margin. This is also the case regarding the asset allocation for the Norwegian Pension Fund (Folketrygdfondet 2014).

Modern portfolio theory has its roots in the framework developed by Markowitz (1952). In his paper he was the first to describe the dual goal of an investor, to maximize return and minimize risk. He describes how a portfolios total variance is reduced when the portfolio consists of assets with low covariance, and how the portfolios optimal composition changes

and yields higher returns as the investor accepts higher risk (variance). This theory laid the groundwork for portfolio management as we know it today.

In recent years a new approach to asset allocation called risk parity has received more attention. The concept can most easily be described as diversifying by risk instead of money. This means that each asset class in a portfolio has equal risk contribution to the portfolio's total risk.

This thesis compares the four different portfolio allocations (equal weighting, 60/40, mean-variance and risk parity) and illuminates the different investment strategies based on the portfolio's risk and return.

As data and modelling tool, Microsoft Excel 2012 is used as this program is offered in a standard form. The advantage by using this program is that it is highly available at approximately zero cost. The disadvantage is that the data analysis and graph opportunities related to the efficient frontier is not optimal.

The thesis uses historical return data for a Norwegian stock and bond index, which is used in the calculations. One limitation of the thesis is this two asset case, and not the inclusion of multiple assets in addition to the time period restricted to 15 years. This is a result of the bond index return history that did not go further back than 1998.

Earlier studies show a variety of results regarding the performance of different portfolios. In this thesis the results show that historical performance of the risk parity portfolio is more similar to the mean-variance (tangency) portfolio than the equal weighting or the traditional 60/40 portfolios. The well-established mean-variance (tangency) portfolio has the highest realized Sharpe ratio. The newer risk parity approach also has a high risk-adjusted return, but both portfolios do not outperform the naïve equal weighting portfolio and the traditional 60/40 portfolio. Both the mean-variance and risk parity portfolios have substantially lower risk than the equal weighting and the 60/40 portfolios, due to a higher allocation to bonds. This makes the mean-variance and risk parity portfolios less aggressive, but also lowers the expected return. An investor can cope with this by applying leverage to the risk parity or mean-variance portfolios up to the naïve equal weighting or traditional 60/40 portfolio's risk-levels and accomplish higher returns. This depends, however, on the cost of leverage.

## **1.2 The Structure of the Thesis**

The thesis starts off with the chapter “Literature Review and Theory” to give the reader an introduction to the field of study and a summary of relevant theory applied to give answer to the problem statement, followed by the data chapter. This chapter explains which data is used in this thesis, and where the data is collected. Further follows the methodology chapter with a short explanation of the four different portfolios, and an explanation of the portfolio construction. The result chapter presents the results of this thesis before the discussion chapter discusses different investment decisions and portfolio selection depending on investors risk profile and other constraints. The thesis is concluded with the conclusion chapter.

## **2. Literature Review and Theory**

### **2.1 The Heuristic Reality**

Cognitive psychology explains how we collect, process and apply information. It focuses on how individuals actually make decisions, as opposed to how they should make rational decisions. The starting point is that individuals have a limited capacity to process information, and that we usually use the rules of thumb when the decisions are too complicated. In many cases this yields good decisions and help to simplify the decisions for us. However, this may cause systematic biases in our assessments that can lead to suboptimal choices. Such insights from cognitive psychology are increasingly being used to explain the observed financial behavior of investors. Economist’s uses expected utility theory, probability theory and the theory of rational expectations when making investment decisions. In other words it says what we should do. Behavioral Finance, however, explain the choices made by investors and their assessment of the decision. When making these judgments we often use mental shortcuts known as heuristics. We use heuristics to help us reach decisions quickly and efficiently.

The prospect theory developed by Kahneman and Tversky (1979) is a heuristic bias that explains that we often prefer simple heuristic decision strategies rather than normative and rational strategies. The fundamental ideas of prospect theory are that reference points exist, and that losses loom larger than corresponding gains. By reference point means the earlier

state relative to which gains and losses are evaluated. The theory shows that when facing a gamble people prefer a sure gain compared to a situation with an uncertain gain. In other words that we act very risk-averse in situations regarding gains. On the other hand the theory shows that we act risk-seeking when facing situations that have a sure loss compared with a situation of a possible bigger loss, but with a small chance of no loss at all. In other words in choosing between a sure gain and an uncertain gain with a higher expected return, many choose the sure gain. If choosing between a sure loss and an uncertain loss with higher expected negative value, more are willing to take the risk and choose the uncertain alternative.

## **2.2 The Rational Theory**

The basic assumption in financial theory is that investors are rational. They seek to maximize their expected utility, which is tied to the amount left at the end of the investment period. In other words; investors are greedy. The choices of investments are made under uncertainty, and if an investor could choose among assets with 100 percent safe returns, the investor would choose the asset that offers the highest return. In this situation no portfolio is necessary. In reality however, different asset classes has different returns, and assets with higher expected returns usually involves higher risk. Risk can be explained as the variation in returns. By combining assets with low covariance investors can reduce the variance in the portfolio and thus lower the risk of the portfolio. This thesis assumes that investors are rational and thus approve the use of financial theory.

## **2.3 Modern Portfolio Theory**

Modern portfolio theory (MPT) has its roots in the framework developed by Markowitz (1952). In Harry Markowitz's original paper "Portfolio Selection" published in the Journal of Finance in 1952, he is the first to describe the dual goal of an investor. The first goal is to maximize the return, and the second goal is to minimize risk (variance). He describes how a portfolios total variance is reduced when the portfolio consists of assets with low covariance, and how the portfolios optimal composition changes and yields higher returns as the investor accepts higher risk (variance). In other words, MPT considers how an investor should choose

a portfolio with a good trade-off between risk and return. This trade-off is often illustrated in the traditional mean-variance diagram, where every possible combination of assets can be plotted on a graph with the standard deviation (risk) on the horizontal axis and return on the vertical axis. The line that connects these combinations of assets with the lowest amount of risk for a given amount of return, or the highest level of return for a given level of risk, is called the efficient frontier.

The minimum variance opportunity set of combinations of risky assets can be defined as follows:

The minimum variance opportunity set is the locus of risk and return combinations offered by portfolios of risky assets that yields the minimum variance for a given rate of return (Copeland, Weston and Shastri 2005, 121).

The efficient set can be defined as follows:

The efficient set is the set of mean-variance choices from the investment opportunity set where for a given variance (or standard deviation) no other investment opportunity offers a higher mean return (Copeland, Weston and Shastri 2005, 123).

The MPT model was later extended by James Tobin (1958) by the introduction of a risk-free asset. This made it possible to leverage portfolios on the efficient frontier, and led to the two-fund separation and the capital market line. The two fund separation can be defined as follows:

Each investor will have a utility-maximizing portfolio that is a combination of the risk-free asset and a portfolio (or fund) of risky assets that is determined by the line drawn from the risk-free rate of return tangent to the investors efficient set of risky assets (Copeland, Weston and Shastri 2005, 135).

The line connecting the risk-free rate and the efficient set is known as the capital market line. By applying leverage, portfolios on the capital market line are able to outperform portfolios on the efficient set.

Sharpe (1964), Linter (1965) and Mossin (1966) extended Markowitz' model to the capital asset pricing model (CAPM). For his work, Markowitz won the Nobel Prize in Economic Sciences for his theory along with Merton Miller and William F. Sharpe in 1990. Given strong assumptions, the CAPM says that the market portfolio sits on the efficient frontier, and all investors should hold the market portfolio, leveraged or deleveraged with positions in the risk-free asset.

## **2.4 Limitations**

It is worth mentioning that portfolio theory is a model developed in a hypothetical world. This means that financial models are based on many assumptions.

The Efficient Market Hypothesis, who is the basis of all financial models, was first introduced by Fama (1965), where he defined market efficiency as a market where a large numbers of rational and risk averse investors trade actively to maximize profits and minimize risks on the basis of the same information which is freely available to all the investors at the same time. Further, all stock prices should fully reflect all relevant information and stock prices adjust quickly to new information.

Fama reviewed the theory, and in 1970 he published the paper "Efficient Capital Markets: A Review of Theory and Empirical Work" (Fama 1970). He defines three types of efficiency based on the type of information to be understood in the phrase "all prices fully reflect all relevant information". The three types of efficiency are weak, semi-strong and strong-from efficiency.

Portfolio theory is based on many assumptions about investors and the opportunity set. In addition to assumptions already mentioned, investors are price takers and have homogeneous expectations about asset returns that have a normal distribution. Investors do not need to pay



taxes or transaction costs. There exists a risk-free asset, so that investors can lend or borrow unlimited at the risk-free rate.

Many of the assumptions for portfolio theory are quite strong, and have been criticized. As mentioned, behavioral finance has proven that the assumption that all investors are rational is wrong. Insider trading and other anomalies also affect the symmetry of information to investors. Transaction costs and taxes are also affecting investors return in reality, and investors cannot lend or borrow at the risk-free rate in unlimited amounts. However, portfolio theory is the fundament for portfolio management used today.

## **2.5 Risk Parity**

Some of its theoretical components were developed in the 1950s and 1960s after Markowitz presented his portfolio theory. The term risk parity, however, was first used by Edward Qian, of PanAgora Asset Management, when he authored a white paper in 2005; “Risk Parity Portfolios: Efficient Portfolios through True Diversification” (Qian 2005).

Edward Qian (2011) also authored a paper on risk parity and diversification in 2011. In this paper he gives an overview of the risk parity concept and presents theoretical arguments for risk parity portfolios. In this paper Qian shows the possibility of risk targeting at the portfolio level to achieve higher total returns, by using leverage if necessary. By creating the risk parity line, which is the line that connects the risk-free asset and the risk parity portfolio (with an unchanged Sharp ratio) he shows that risk targeting is possible.

The risk parity approach has received more attention in recent years. This approach is an alternative approach to investment diversification. In traditional 60/40 equity/bond portfolios the diversification looks well balanced when viewed from the perspective of cash invested in each asset class. Looked at from the risk contribution from each asset class to the portfolio, the 60/40 portfolio is not well balanced. The stock market’s volatility is substantially greater than the bond market’s volatility. As a result the portion invested in the stock market dominates the risk in the portfolio. The return of the 60/40 portfolio earns much of its return from exposure to equity risk and little from exposure to bond risk. From the risk exposure perspective this portfolio is under-diversified. The risk parity approach has a simple fix for this problem. Instead of diversifying by cash invested, diversify by risk. In the case of a

portfolio consisting of stocks and bonds this means that an investor must diversify in a manner that gives each asset class equal risk contribution to the portfolio. Generally this implicates a larger portion invested in low-risk assets than in high-risk assets. This results in a less aggressive portfolio with expected returns lower than the traditional 60/40 portfolio. The risk parity portfolio is more fixed income heavy, which lower the portfolios volatility and return. Investors can deal with this problem by leveraging up the portfolio to increase expected return and risk to desired levels.

The risk parity portfolio performance relative to other asset allocation alternatives can be highly dependent on the time period and the asset classes included (Chaves et.al 2011). The authors of this paper show that the risk parity portfolio is highly sensitive to inclusion of assets. This is one of the key messages in this paper.

## 2.6 Sharpe Ratio

The Sharpe ratio was developed by Nobel Prize winner William F. Sharpe (1966). The Sharpe ratio measures risk-adjusted return and can show whether a portfolios return are due to smart investment decisions or a result of excess risk. It is calculated by dividing a portfolios excess return by the portfolios standard deviation of the returns. Excess return is found by subtracting the risk-free rate from the return of the portfolio. The Sharpe ratio formula is:

$$SR = \frac{r_p - r_f}{\sigma_p}$$

If a portfolio has a high Sharpe ratio, this indicates that the portfolio has had good risk-adjusted development. If the Sharpe ratio is negative, this indicates that a risk-free asset would perform better than the portfolio in question.

### **3. Data**

The time period investigated in this paper is from 01.01.1998-31.12.2012, which gives a time period of 15 years. To get more observations (data points), monthly return data is used. This gives 180 observations of monthly returns. Because of a higher exposure to short term volatility, weekly observations are not used.

#### **3.1 Equity (stocks)**

For the equity asset class, OSEBX, which is the main index for stocks in the Norwegian market, is used. The index consists of a representative sample of all listed shares on the Oslo Stock Exchange (Oslo Børs 2014). The monthly returns are calculated from the set of daily stock prices over the time period. The data is collected using the financial database Datastream.

#### **3.2 Fixed Income (bonds)**

For the fixed income asset class, the Norwegian Bond Index is used. This is a reference bond index for the Norwegian Pension Fund (SPN) consisting of investment grade bonds. The data is collected from “folketrygdfondets” internet page, who has published the historical returns in monthly time series to promote transparency (Folketrygdfondet 2014).

#### **3.3 Risk-free asset**

As the risk-free rate, the 1 Month NIBOR is used. The data is collected from the homepage of Professor Bernt Arne Ødegaard at the University of Stavanger, Norway ([finance.bi.no/~bernt/](http://finance.bi.no/~bernt/) 2014).

NIBOR - Norwegian Interbank Offered Rate- is a collective term for the Norwegian money market rates with different maturities (Finans Norge 2014). NIBOR is often considered as the markets risk-free rate, and is thus relevant in financial calculations.

## **4. Methodology**

### **4.1 Definitions of the different portfolios**

In this paper four different portfolios are included. These portfolios are the naïve equal weighting portfolio, the traditional 60/40 portfolio, the mean-variance (tangency) portfolio and the risk parity portfolio.

Using historical monthly return data for stocks and bonds, monthly weighted return is calculated for each of the portfolios.

#### **4.1.1 Equal Weighting Portfolio**

The reason why the naïve equal weighting portfolio is included is that it diversifies in the simplest manner by allocating the capital in all assets equal. Investors do not need to assume any knowledge regarding the distribution of the asset class returns. The asset allocation is based on the number of assets in the portfolio; each asset as a weight equal to  $1/N$  where  $N$  is the number of assets. In the two asset case of stocks and bonds, 50 percent is allocated to stocks and 50 percent is allocated to bonds.

#### **4.1.2 60/40 Portfolio**

The traditional 60/40 portfolio who allocates 60 percent to stocks and 40 percent to bonds is included because of its well-known use in practice. Large institutional investors and pension funds usually have a 60/40 allocation of its invested capital.

#### **4.1.3 Mean-Variance Portfolio**

The mean-variance portfolio is included because of its well-known theoretic framework, and the impact that the portfolio theory has had on portfolio management to this day.

#### **4.1.4 Risk Parity Portfolio**

Risk parity is an approach that has received more attention in recent years. This approach does not have the same well established framework as modern portfolio theory, and is included as an alternative approach to diversifying investments. The risk parity concept is simple and intuitive: diversify by risk, not by money. In other words each asset class

contributes with the same amount of risk to the portfolio.

## 4.2 Portfolio Construction

For the two asset case the portfolio return and standard deviation is calculated as:

Portfolio return: 
$$E(r_p) = x_i E(r_i) + x_j E(r_j)$$

Where  $x_i$  and  $x_j$  denotes weights allocated to each asset,  $E(r_i)$  and  $E(r_j)$  denotes expected return for each asset.

Portfolio standard deviation: 
$$\sigma_p = \sqrt{x_i^2 \sigma_{x_i}^2 + x_j^2 \sigma_{x_j}^2 + 2x_i x_j \text{Cov}(x_i, x_j)}$$

The portfolio variance is  $\sigma_p^2$ , and the covariance for the two asset case is calculated as:

$$\text{Cov}(x_i, x_j) = E[(x_i - E(x_i))(x_j - E(x_j))]$$

The covariance is a measure of the way two variables move in relation to each other. If the covariance is positive, the variables move in the same direction. If the covariance is negative, the variables move in the opposite direction. The covariance is an extremely important concept because it is the appropriate measure of the contribution of a single asset to portfolio risk.

The variance-covariance matrix in the two asset case is defined as:

$$\begin{vmatrix} \text{var}(x_i) & \text{cov}(x_i, x_j) \\ \text{cov}(x_j, x_i) & \text{var}(x_j) \end{vmatrix}$$

Where  $\text{cov}(x_i, x_j)$  is the covariance between the returns of asset  $x_i$  and asset  $x_j$ .

Excel Solver is used to find the two programming problems.

Programming problem 1 (finding the minimum-variance opportunity set):

$$\text{MIN } \sigma^2(r_p) \quad \text{subject to} \quad E(r_p) = K$$

The solver's objective is to minimize portfolio variance by changing portfolio weights, given the constraints on weights. The minimum-variance opportunity set is found by finding all combinations that give the lowest risk for a given return.

Programming problem 2 (finding the efficient set):

$$\text{MAX } E(r_p) \quad \text{subject to} \quad \sigma^2(r_p) = K$$

The solver's objective is to maximize portfolio return by changing portfolio weights, given the constraints on weights. The efficient set is the locus of highest returns for a given risk, and shows all efficient combinations of stock/bond allocations.

The mean-variance (tangency) portfolio is found as the portfolio with the highest achieved Sharpe ratio.

#### 4.2.1 Constraints on Weights

The weights allocated to each asset cannot be negative, in other words this constraint does not allow short selling. In reality the majority of portfolios are indeed long portfolios. This constraint is added to approach the preferences of the majority of investors. Further, the sum of all weights is equal to 1, which means that the portfolio cannot be levered.

#### 4.3 Risk Parity Portfolio Construction

The risk parity concept is simple and intuitive: diversify by risk, not by money. In the two asset case this generally implies a higher weight allocated to lower risk assets (bonds) and a lower weight allocated to high risk assets (stocks). The concept is that each asset class contributes with the same amount of risk to the portfolio.

For the two asset case the risk contribution is given by:

$$RC_1 = \frac{w_1^2 \sigma_1^2 + \rho w_1 w_2 \sigma_1 \sigma_2}{w_1^2 \sigma_1^2 + 2\rho w_1 w_2 \sigma_1 \sigma_2 + w_2^2 \sigma_2^2} \quad RC_2 = 1 - RC_1$$

The total variance (the denominator) is composed of variances and twice the covariance. The risk contribution from an asset is equal to the ratio of the sum of its variance and covariance (the numerator) to the total variance.

### **4.3.1 Constraints**

In addition to the constraints for the mean-variance portfolio construction (no negative weights, and the sum of all weights is equal to 1), the risk parity condition represented by the formulas above is added as a constraint in Excel Solver when portfolio weights are calculated.

## **4.4 Summary of Portfolio Construction**

All figures are calculated on a monthly basis, and the portfolios weighted returns are calculated with monthly rebalancing of the portfolios. By using the calculated weights allocated to stocks and bonds for the mean-variance and risk parity portfolio the historical performance of these portfolios are calculated with monthly rebalancing. These weights are fixed percentages and held constant over the time period, and needs monthly rebalancing as the returns vary. This results in a static model of the efficient frontier. Most discussions of the mean-variance analysis are restricted to static models. Hence, investors can only make decisions at the beginning of the investment period and must wait for the results without adjusting the portfolio weights until the end of the period. In dynamic investment analysis the portfolio weights are continuously changed according to the observed market asset prices. The static methodology is a simple analysis compared to dynamic models, but it illustrates the concept of the mean-variance optimization. One drawback of the static model is that it is less realistic than the dynamic model.

## 5. Results

### 5.1 Performance

Figure 1 show the historical cumulative growth of 100 NOK invested at the beginning of 1998 and the portfolios performance to the end of 2012. The figure includes the simple equal weighting portfolio, the 60/40 portfolio, the mean-variance portfolio and the risk parity portfolio.

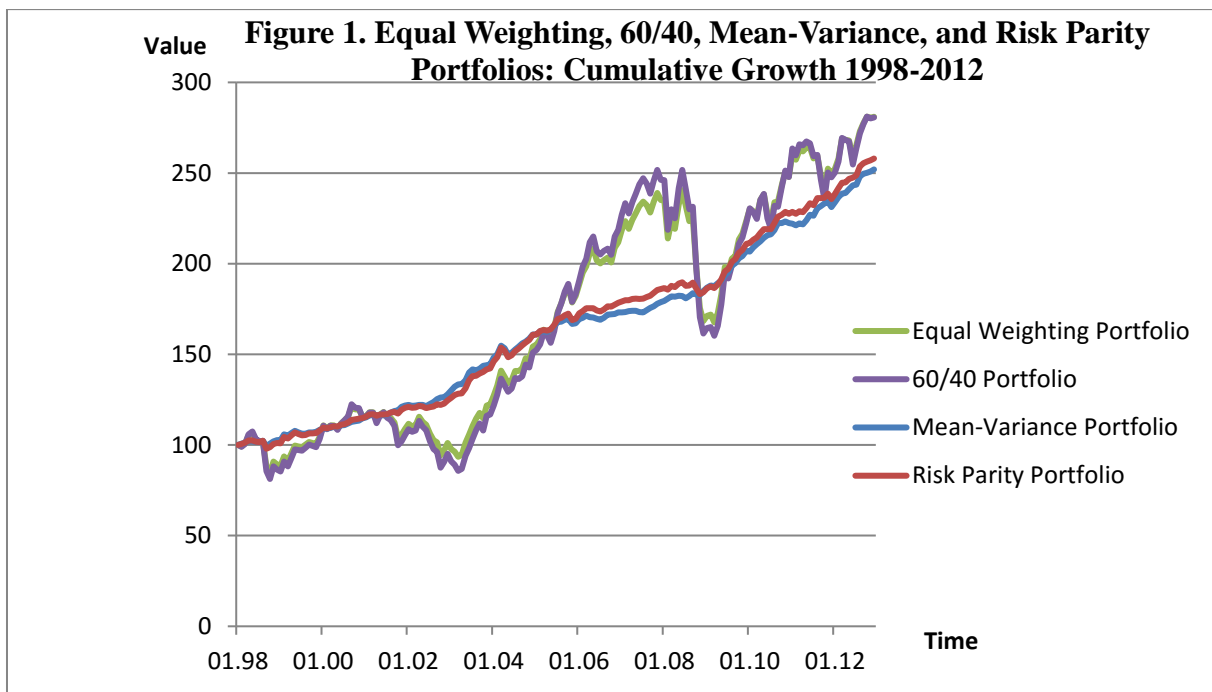


Figure 1. This figure shows total cumulative growth of 100 NOK invested in portfolios of stocks and bonds in the Norwegian market, 1998-2012. The Equal weighting portfolio allocates 50 percent to stocks and 50 percent to bonds and is rebalanced monthly to maintain constant weights. The 60/40 portfolio allocates 60 percent to stocks and 40 percent to bonds and is rebalanced monthly to maintain constant weights. The mean-variance (tangency) portfolio targets the highest Sharpe ratio and is rebalanced monthly to maintain constant weights. The risk parity portfolio targets an equal risk allocation to stocks and bonds and is rebalanced monthly to maintain constant weights.

The figure shows that the best of the alternatives above from a return point of view is the simple equal weighting portfolio and the 60/40 portfolio. Under this alternatives 100 NOK will have grown to 282 NOK over the 15 year period. This is probably why the traditional



60/40 portfolio is so frequently used by large institutional investors, the returns. The mean-variance (tangency) portfolio has grown to 253 NOK and the risk parity portfolio to 259 NOK over the period.

Since both the equal weighting and the 60/40 portfolios include a larger portion allocated to stocks this might not come as a surprise. As mentioned the majority of risk and return in portfolios consisting of stocks and bonds generally come from stocks. However, the bond index has performed very well over the time period, resulting in high realized returns combined with low risk. This makes the mean-variance and the risk parity portfolio perform not much weaker than the equal weighting and 60/40 portfolio despite their relative high weights allocated to bonds.

## 5.2 Traditional Risk-Return Frontier

The trade-off between risk and return is traditionally illustrated by using a mean-volatility diagram. Figure 2 shows the efficient frontier of stock/bond allocation, along with the tangency line. The bond index had a monthly return of 0.51 % with a volatility of 0.81 %. The stock index had a monthly return of 0.77 % with a volatility of 6.92 %. The efficient set connecting these two points represents all possible portfolios of stock/bond allocation. The tangency line is the line that connects the risk-free rate with the efficient set. For an investor this means the combination of investments in the risk-free asset and the mean-variance (tangency) portfolio of risky assets. As the figure shows, the monthly risk-free rate (1 Month NIBOR) averaged to 0.36 % over the period. This is represented by the point on the y-axis. The mean-variance (tangency) portfolio allocates 4.87 % to stocks and 95.13 % to bonds, with a monthly return of 0.52 % and a volatility of 0.80 %. This portfolio is the portfolio with the highest realized Sharpe ratio. The mean-variance portfolio has an allocation very close the minimum-variance portfolio which allocates a larger portion to bonds (97.19 %) and a smaller portion to stocks (2.81 %). The risk parity portfolio allocates 10.44 % to stocks and 89.56 % to bonds, with a monthly return of 0.53 % and a volatility of 0.95 %. This risk parity portfolio allocation between stocks and bonds is the one that gives each asset (stocks and bonds) equal risk contribution to the portfolios total risk. As figure 2 shows, the risk parity portfolio is quite close to the mean-variance portfolio. The naïve equal weighting portfolio allocates 50 % to stocks and 50 % to bonds, while the traditional 60/40 portfolio allocates 60 % to stocks and 40 % to bonds. The equal weighting portfolio has a monthly

return of 0.64 % and a volatility of 3.43 %, while the 60/40 portfolio has a monthly return of 0.66 % and a volatility of 4.12 %.

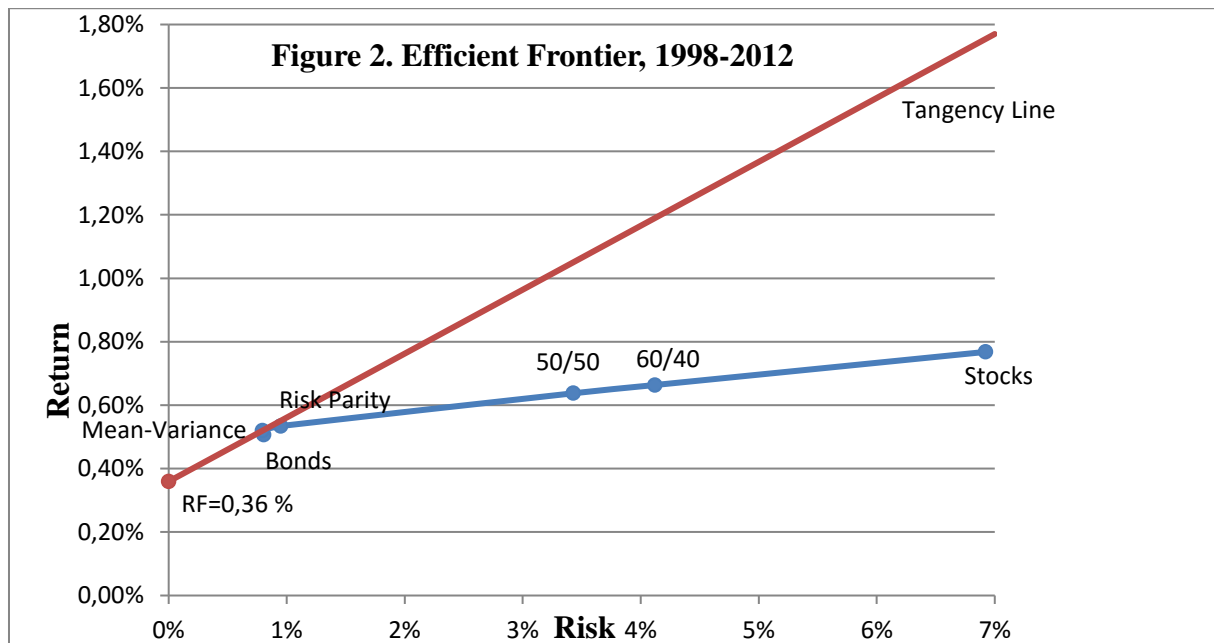


Figure 2. This figure shows the efficient frontier of portfolios of stocks and bonds in the Norwegian market from 1998-2012. The tangency line is the line that connects the risk-free asset with the efficient set.

Modern portfolio theory says that an optimal portfolio is somewhere on the line connecting the risk-free rate and the efficient set of risky assets. Risk-averse investors should allocate their portfolio in a manner that invests some money in cash (the risk-free asset), and the rest in the mean-variance (tangency) portfolio. This portfolio is represented by the tangency line between the risk-free asset and the efficient set. Risk-tolerant investors should invest in a manner that involves the use of leverage. In this case they should borrow at the risk-free rate, rather than invest in the risk-free rate, and invest more than 100 percent in the mean-variance (tangency) portfolio. This portfolio is represented by the tangency line that that extends beyond the point where the tangency line and the efficient set intersects.

### 5.3 Risk-Adjusted Returns

It is important for investors to take a closer look on the risk-adjusted return.

The Sharpe ratio which measures risk-adjusted return can show whether a portfolios return are due to smart investment decisions or a result of excess risk. Figure 3 show the Sharpe ratio of the different stock and bond portfolios.

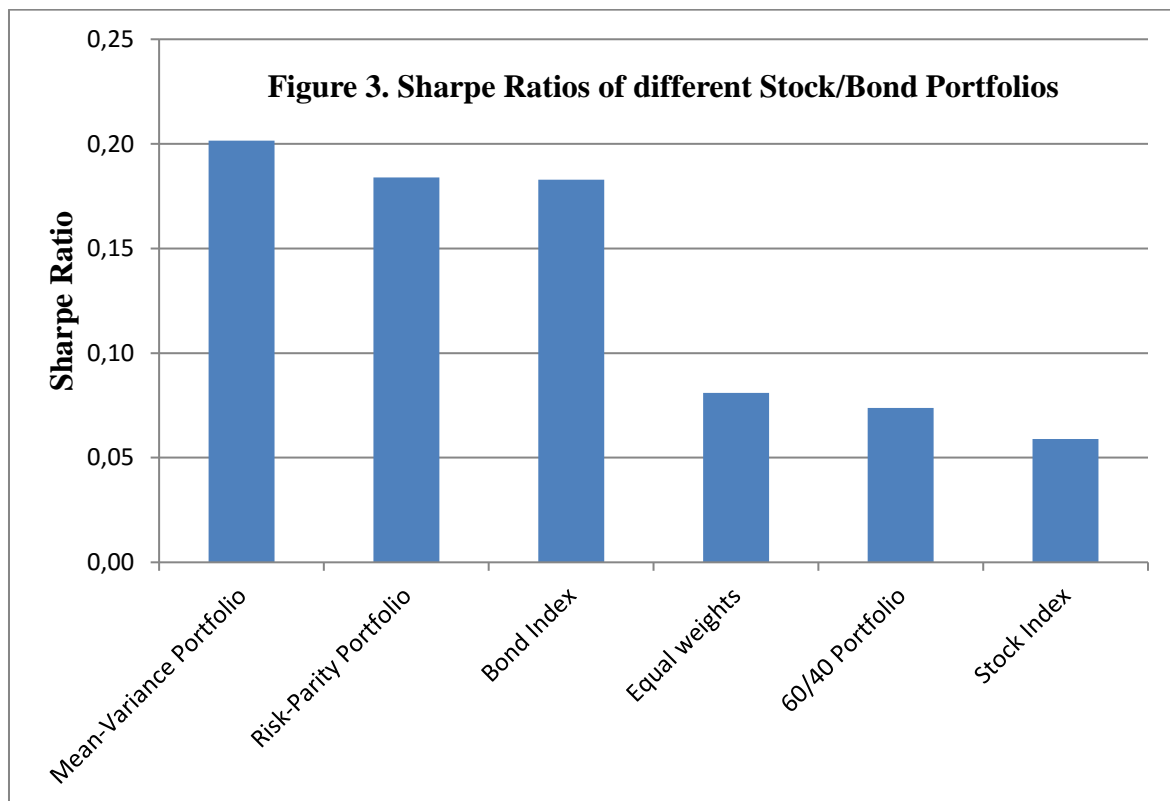


Figure 3. This figure shows Sharpe ratios of mean-variance, risk parity, bonds, equal weighting, 60/40 and stocks. The Sharpe Ratio is calculated by subtracting the risk-free rate from the portfolios return and divides the result (excess return) by the standard deviation of the portfolios return. All figures are calculated on a monthly basis.

From this perspective the mean-variance (tangency) portfolio has the highest Sharpe ratio of 0.201, which supports the objective of the methodology, followed by the risk parity portfolio with a Sharpe ratio of 0.184. At the extreme case of 100 percent invested in bonds, the Sharpe ratio due to non-diversification is 0.183, which is a relative high Sharpe ratio for this asset class. This can be explained by high realized returns at little risk over the time period. The realized Sharpe ratio for bonds is marginally worse than the Sharp ratio of the risk parity

portfolio. The mean-variance (tangency) portfolio and risk parity portfolio appears to be superior over the traditional 60/40 portfolio with a Sharpe ratio of 0.074 and the simple naïve equal weighting portfolio, with Sharpe ratio of 0.081. At the other extreme with 100 percent invested in stocks, the Sharpe ratio due to non-diversification is 0.059, which is the lowest in the sample. This is a result of the high risk for this asset class.

By using these results it is obvious that the mean-variance portfolio is the portfolio with the best trade-off between risk and return, followed by the risk parity portfolio. The traditional 60/40 portfolio, which outperforms both the mean-variance portfolio and the risk parity portfolio measured in return, clearly has a higher risk exposure.

Risk-tolerant investors can by applying leverage target their desired risk level and accomplish higher returns. This is equivalent to moving along the tangency line beyond the point where the tangency line and the efficient set intersects in figure 2. By leveraging up the mean-variance portfolio in such a manner, it is possible to target the same risk level as the 60/40 portfolio and realize higher returns than the 60/40 portfolio.

## **5.4 Return Correlations**

From an investors standpoint it is useful to analyze the different portfolio characteristics. One measure is the return correlations of the different portfolios with stocks and bonds as illustrated in table 1. Correlation can be described as a measure of how two variables move in relation to each other. When calculating correlation, we get the correlation coefficient, which ranges between -1 and 1. A correlation coefficient of 1 is called perfect positive correlation, and it implies that as one variable moves up or down, the other variable will move in the exact same direction. On the other hand a correlation coefficient of -1 is called perfect negative correlation, and it implies as one variable moves up or down, the other variable will move in the exact opposite direction. A correlation coefficient of 0 is called no correlation, and the two variables move completely random in relation to each other.

**Table 1. Return correlations of the different portfolios with stocks and bonds**

<b>Correlations</b>	<b>Stocks</b>	<b>Bonds</b>
Equal Weights	0.993	-0.019
60/40	0.997	-0.058
Mean-Variance	0.293	0.908
Risk Parity	0.657	0.657

Table 1. This table shows the return correlations of the different portfolios with stocks and bonds. The time period is 1998-2012, and the correlations are calculated by using monthly returns.

The equal weighting portfolio and the 60/40 portfolio have extremely high positive correlations with stocks (0.993 and 0.997) and extremely low negative correlation (almost zero) with bonds (-0.019 and -0.058). In general, these types of portfolios consisting of a large allocation in stocks, the majority of the portfolios risk and return come from stocks. This is because stocks have higher volatility and higher expected returns than bonds. The results of the portfolios correlation with stocks and bonds in table 1 back up this concept regarding the returns. The return of the equal weighting portfolio and the 60/40 portfolio is highly dependent on stock returns and not dependent on bond returns. The mean-variance (tangency) portfolio has a lower positive correlation with stocks (0.293) and a very high positive correlation with bonds (0.908). In other words the mean-variance portfolios return is substantially more dependent on bond returns rather than stock returns. This can be explained by the high weight allocation to bonds (95.13 %) in this portfolio. The risk parity portfolio has equal positive correlation with both stocks and bonds (0.657), which makes this portfolio truly diversified and not solely dependent on stock or bond returns.

## 6. Discussion

### 6.1 Returns, Volatility and Sharpe Ratios

From a return point of view the naïve equal weighting portfolio and the traditional 60/40 portfolio outperforms the mean-variance and the risk parity portfolios as shown in figure 1. The equal weighting portfolio is diversified in a cash manner, but not in a risk manner. The same can be said for the traditional 60/40 portfolio. It looks well balanced from a cash point of view, but not from the risk perspective: the portfolio allocate a higher percentage to higher risk assets (stocks) and a lower percentage to lower risk assets (bonds). Over the time period stocks had a monthly volatility of 6.92 %, while bonds had a monthly volatility of only 0.81 %. The reason many large pension funds use the 60/40 stock/bond allocation is the returns, and over the time period stocks had a monthly return of 0.77 %, while bonds had a monthly return of 0.51 %.

The outperforming returns of the equal weighting and the 60/40 portfolios is illustrated in table 2. Both portfolios have a substantially higher monthly excess return over the risk-free rate than the mean-variance and risk parity portfolios. The traditional 60/40 portfolio has a monthly excess return of 0.30 % which is the highest in the sample. Closest to the 60/40 portfolio is the equal weighting portfolio with excess return of 0.28 %. The risk parity portfolio and the mean-variance portfolio have excess returns of 0.17 % and 0.16 % respectively.

From a risk point of view the 60/40 portfolio is the portfolio with the highest monthly volatility (4.12 %), followed by the equal weighting portfolio (3.43 %). This is a result of the relative high weights in stocks. The risk parity portfolio has a volatility of 0.95 %, while the mean-variance portfolio has a volatility of 0.80 %. This is a result of the high weights in bonds.

The column on the far right side in table 2 summarizes the portfolios risk-adjusted returns, measured by the Sharpe ratio. By this measure the mean-variance and the risk parity portfolios is superior to the 60/40 and the equal weighting portfolio.

**Table 2. Excess Return, Volatility and Sharpe Ratio**

	<b>Excess Return over 1M NIBOR</b>	<b>Volatility</b>	<b>Sharpe Ratio</b>
Equal Weighting	0.28 %	3.43 %	0.08
60/40	0.30 %	4.12 %	0.07
Mean-Variance	0.16 %	0.80 %	0.20
Risk Parity	0.17 %	0.95 %	0.18

Table 2. This table shows excess return, volatility and Sharpe ratios for the different portfolios. All numbers are calculated based on monthly data. Excess return is calculated by subtracting the risk-free rate (1 month NIBOR) from the portfolios return.

So which alternative should an investor choose?

There are many factors that affect the choice of an investment decision for an investor. The most important one is the investors risk profile. A risk-averse investor clearly makes investment decisions not solely on return targeting, but also compares the portfolios risk (volatility). From this standpoint it is natural to compare risk-adjusted returns, measured by the portfolios Sharpe ratio. From the alternative portfolios in this thesis, the mean-variance and the risk parity portfolio would be preferred over the naïve equal weighting and the traditional 60/40 portfolios. Figure 1 clearly shows that the mean-variance and the risk parity portfolios is less volatile and has more stable returns than the equal weighting and 60/40 portfolios.

Risk-tolerant investors and investors focusing on return targeting would probably not be so interested in the portfolios risk (volatility), rather achieving a highest possible return. From this standpoint an unlevered portfolio would allocate a larger portion invested in stocks which has higher expected returns than bonds. The portfolio alternatives in this thesis that achieves the highest cumulative returns are the naïve equal weighting and the 60/40 portfolios (see figure 1).

If the unlevered assumption is relaxed, investors can apply leverage to the portfolio. By applying leverage investors can increase expected portfolio return. From a risk-adjusted point of view an investor can apply leverage to the risk parity or mean-variance portfolios up to the naïve equal weighting or traditional 60/40 portfolios risk-levels and accomplish higher returns. This depends, however, on the cost of leverage.

Modern portfolio theory states that an optimal portfolio is somewhere on the line connecting the risk-free rate and the efficient set of risky assets (see figure 2). Risk-averse investors should allocate their portfolio with some money in cash (the risk-free asset), and the rest in the mean-variance (tangency) portfolio, while risk-tolerant investors should use leverage and borrow at the risk-free rate, rather than invest in the risk-free rate, and invest more than 100 percent in the mean-variance (tangency) portfolio. The results in this thesis show that the portfolio with the best trade-off between risk and return is the mean-variance portfolio. By leveraging up the mean-variance portfolio it is possible for investors to target the same risk level as the 60/40 portfolio and realize higher returns than the 60/40 portfolio.

The risk parity portfolio has some interesting characteristics such as equal return correlation to stocks and bonds making the portfolio truly diversified and not solely dependent on stock or bond returns, and it has equal risk contribution to the portfolios total risk from both stocks and bonds. In the absence of leverage, however, the expected return of the risk parity portfolio and the mean-variance portfolio is too low to be compelling for most investors. By applying leverage to the risk parity portfolio or the mean-variance portfolio, an investor can theoretically achieve return in line with the traditional 60/40 portfolio with a lower level of risk than the 60/40 portfolio. An important point, however, is that the advantage of a levered portfolio declines as the cost of leverage increases, and a higher level of leverage increases the possibility of financial distress. This might impose the investor with extra costs related to financial distress. Due to its high allocation in fixed income assets, the levered risk parity and mean-variance portfolios will be more sensitive to interest rate movements. High interest rates will over the long run boost the portfolios because of the relative high weights in fixed income assets, while stocks will be less favored in such an environment and stock prices will have a negative impact. For the equity-heavy 60/40 and equal weighting portfolios stocks will over the long run get a positive boost from lower interest rates.

During the time period of 1998-2012 the Norwegian market has been influenced by several international incidents. The burst of the IT-Bubble after the millennium led to a decline in the international stock markets. This resulted in a declining trend for the Norwegian stock market during the years of 2001-2003. As figure 1 shows, the equity-heavy equal weighting and the 60/40 portfolios suffered the most from a declining stock market during this period.

In the period prior to the financial crisis in 2008, the stock market had a tremendous positive development. Equity-heavy portfolios such as the 60/40 and the equal weighting portfolios



outperformed the risk parity and the mean-variance portfolios substantially (see figure 1). As a result of the financial crisis, the whole world's money markets were hit, and the price of risk rose sharply. Many banks and other financial institutions became very reluctant to lend money and interest rates rose while the world's stock markets plunged. This resulted in bigger losses for the equity-heavy portfolios than the more fixed income-heavy portfolios. The mean-variance and the risk parity portfolios fared reasonably well during this period. These findings indicate that the income-heavy portfolios (mean-variance and risk parity) hold up better in times of declining stock markets and recessions as a result of less exposure to stocks and a higher exposure to bonds. In times of strong economic growth and rising prices in the stock markets, equity-heavy portfolios are obviously preferable. A higher allocation to stocks boosts the returns of these portfolios substantially. Whether this holds up in less extreme situations could be an interesting topic for further research.

## **6.2 Investor's Constraints**

Risk-return is the classic trade-off in the universe of investments. In practice investors also have other constraints which limit their investment opportunities.

For institutional investors, like large pension funds it might be difficult to implement levered portfolios in practice. Many institutional investors are highly regulated and restrict the use of leverage in their portfolios. This is probably one of the reasons for the often used 60/40 stock/bond allocation for institutional investors.

For private investors the use of leverage is easier to implement in practice. It is less restricted and usually depends on each investor's credit limit and risk profile. This thesis shows that the mean-variance is the portfolio with the best trade-off between risk and return, and this portfolio would be a good alternative to apply with leverage. A drawback of this alternative is that the mean-variance optimization methodology can be difficult to implement for private investors due to challenges associated with the estimation of expected returns and covariance's with accuracy.

When making investment decisions investors need to take into account their liquidity needs during the investment period. It is needless to say that it would be a stupid idea to invest all of the financial wealth in an investment opportunity, if the near future involves liquidity needs.

If such liquidity needs are known in advance the alternatives are either to not invest the part of capital that is needed and to keep this amount in the risk-free asset (or the bank), or to invest this amount in money market or fixed income assets with lower risk and involves no binding of the money invested. To invest the needed amount of capital in high risk assets such as the more volatile stock market should be avoided.

Investors who are highly liquid or have high income are better candidates to apply leverage to their portfolio. They are less prone to financial distress and are less likely to miss margin calls.

Another factor that influences investment decisions is the investor's time frame for the investment. Clearly a longer investment period would be less exposed to short term volatility than an investment with a shorter time frame. This will affect investor's investment decisions depending on their risk profile.

### **6.3 Limitations**

In this thesis the calculated weights allocated to stocks and bonds for the mean-variance and risk parity portfolios are held constant over the time period. This static model of the efficient frontier is a simplified model. In this case investors can only make decisions at the beginning of the investment period and must wait for the results without adjusting the portfolio weights until the end of the period. This methodology is a simple analysis compared to dynamic models, but it illustrates the concept of the mean-variance optimization. In dynamic models the portfolio weights are continuously changed according to the observed market asset prices. One drawback of the static model is that it is less realistic than the dynamic model, and this makes the results less reliable compared to dynamic models.

In general, theoretic models give a strong simplified picture of the reality. The concept is to clarify the points one want to explain. This makes theoretical models incomplete representations of the reality, but the models have to be simplified in order to work.

## 7. Conclusion

This thesis shows that the well-established mean-variance portfolio has the highest realized Sharpe ratio in line with the objective of the methodology. The newer risk parity approach also has a high risk-adjusted return, but both portfolios do not outperform the naïve equal weighting portfolio and the traditional 60/40 portfolio.

The risk parity portfolio has some interesting characteristics such as equal return correlation to stocks and bonds making the portfolio truly diversified and not solely dependent on stock or bond returns, and it has equal risk contribution to the portfolios total risk from both stocks and bonds. In the absence of leverage, however, the expected return of the risk parity portfolio and the mean-variance portfolio is too low to be compelling for most investors. By applying leverage to the risk parity portfolio or the mean-variance portfolio up to the naïve equal weighting or traditional 60/40 portfolios risk-levels, an investor can theoretically achieve higher returns. This depends, however, on the cost of leverage.

There are some limitations in this paper. For starters the efficient frontier is a static model and the calculated weights for the mean-variance and the risk parity portfolios are held constant over the time period. They are only rebalanced each month to maintain constant weights, and are not recalculated each month according to observed asset market prices as a dynamic model will. The bond index did not go further back than 1998. A longer time period and more observations would be preferable. In addition to this, the bond index has realized high monthly returns and a high Sharpe ratio over the time period, resulting in extremely high weights in bonds for the mean-variance portfolio.

The inclusion of more asset classes could be an interesting topic for further research.

## 8. References

- Chaves, Denis, Jason Hsu, Feifei Li and Omid Shakernia. 2011. "Risk Parity Portfolio vs. Other Asset Allocation Heuristic Portfolios". *The Journal of Investing*. Vol. 20. No. 1. 108-118.....16
- Copeland, Thomas E., J. Fred Weston and Kuldeep Shastri. 2005. *Financial Theory and Corporate Policy*. Fourth Edition. Pearson Addition Wesley.....13
- Fama, E. F. 1965. "The behavior of stock-market prices". *Journal of Business*. 38 (1). 34–105.....14
- Fama, E. F. 1970. "Efficient capital markets: A review of theory and empirical work". *The Journal of Finance*. 25 (2). 383–417.....14
- finance.bi.no/~bernt/. 2014. My Finance Resources. Norwegian Financial Data. Asset Pricing. Risk Free Monthly Rates. n.d.  
[http://finance.bi.no/~bernt/financial\\_data/ose\\_asset\\_pricing\\_data/NIBOR\\_monthly.txt](http://finance.bi.no/~bernt/financial_data/ose_asset_pricing_data/NIBOR_monthly.txt)....17
- Finans Norge. 2014. NIBOR – Norwegian Inter Bank Offered Rate. n.d.  
<http://www.fno.no/Hoved/Fakta/Verdipapirer-og-kapitalforvaltning/Faktaark-verdipapirer-og-kapitalforvaltning-A---A/Pengemarkedsrenter/NIBOR/>.....17
- Folketrygdfondet. 2014. Investerer i Norge og Norden. n.d.  
<http://www.folketrygdfondet.no/vaar-forvaltning/category235.html>.....9
- Folketrygdfondet. 2014. Månedlige historiske avkastningstall. n.d.  
<http://www.folketrygdfondet.no/avkastning-over-tid/category370.html>.....17
- Kahneman, Daniel and Amos Tversky. 1979. "Prospect Theory: An Analysis of Decision Under Risk". *Econometrica*. 263-291.....11
- Lintner, John. 1965. "The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets". *The Review of Economics and Statistics*. Vol. 47. 13-37.....14
- Markowitz, Harry M. 1952. "Portfolio selection". *Journal of Finance*. 7 (1). 77-91.....9,12
- Mossin, Jan. 1966. "Equilibrium in a capital asset market". *Econometrica*. 34. 768-783.....14

Oslo Børs. 2014. Oslo Børs Benchmark Index. n.d.  
[http://www.oslobors.no/markedsaktivitet/stockIndexOverview?newt\\_\\_ticker=OSEBX....17](http://www.oslobors.no/markedsaktivitet/stockIndexOverview?newt__ticker=OSEBX....17)

Qian, Edward. 2005. "Risk Parity Portfolios: Efficient Portfolios through True Diversification." PanAgora Asset Management.  
<http://www.panagora.com/assets/PanAgora-Risk-Parity-Portfolios-Efficient-Portfolios-Through-True-Diversification.pdf.....15>

Qian, Edward. 2011. "Risk Parity and Diversification". The Journal of Investing. Vol. 20. No. 1. 119-127.....15

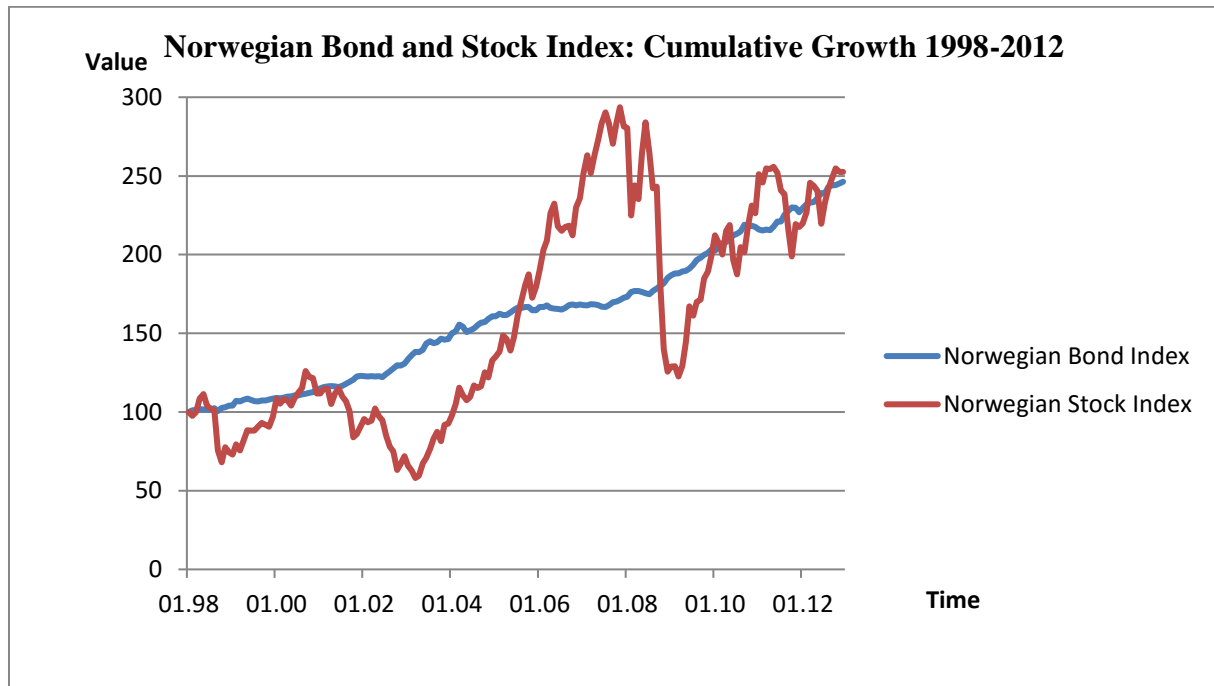
Sharpe, William F. 1964. "Capital asset prices: A theory of market equilibrium under conditions of risk". Journal of Finance. Vol. 19. 425-442.....14

Sharpe, William F. 1966. "Mutual Fund Performance". The Journal of Business. Vol. 39. No. 1. 119-138.....16

Tobin, James. 1958. "Liquidity Preference as Behavior Towards Risk". The Review of Economic Studies. 25. 1. 65-86.....13

## 9. Appendix

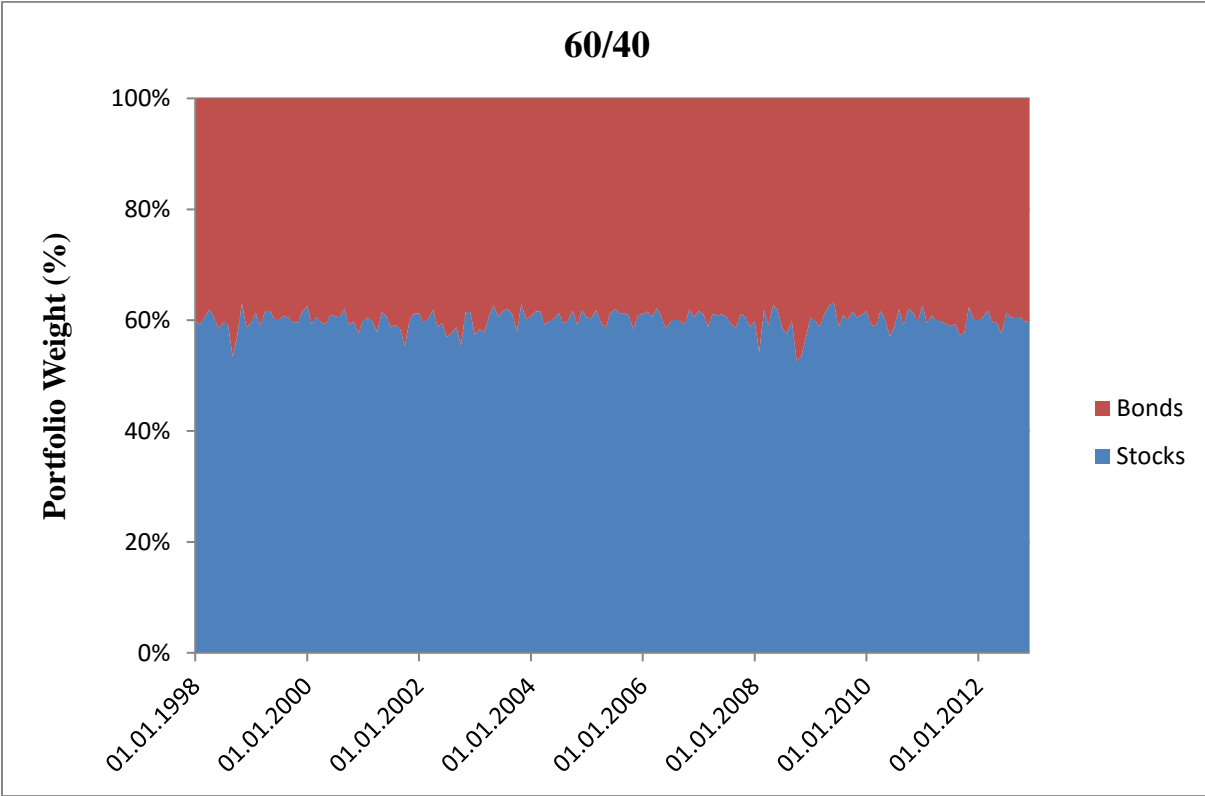
### 9.1 Norwegian Bond and Stock Index: Cumulative Growth 1998-2012

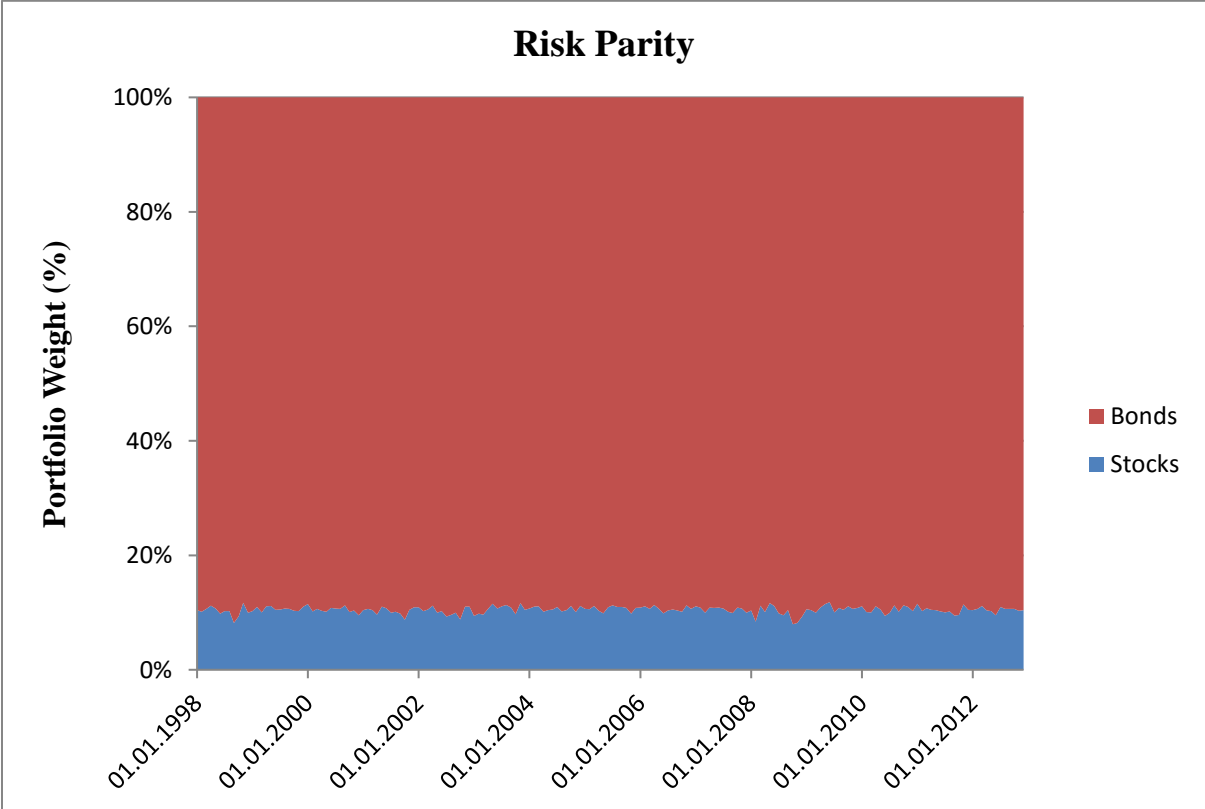
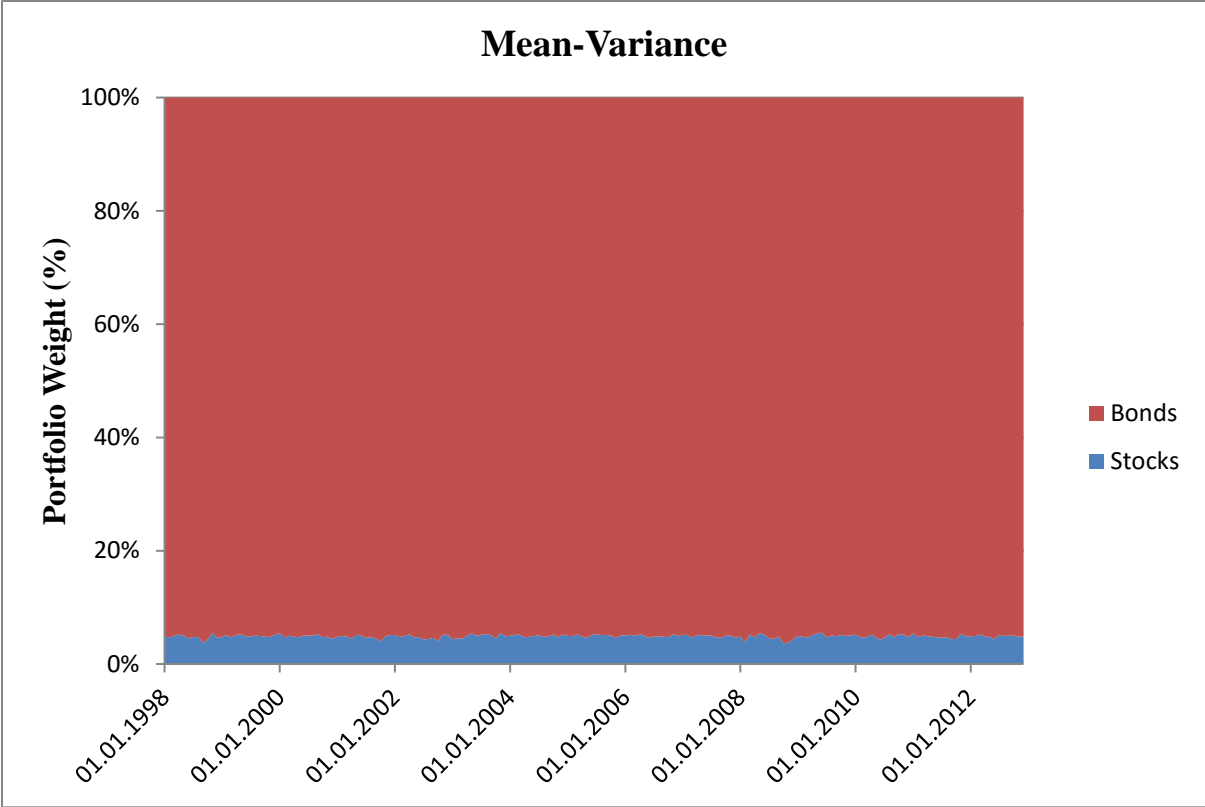


This figure shows total cumulative growth of 100 NOK invested in the stock and bond index in the Norwegian market, 1998-2012. The bond index grew to NOK 247 at the end of 2012, while the stock index grew to NOK 254 respectively.

### 9.2 Time Series of Portfolio Weights

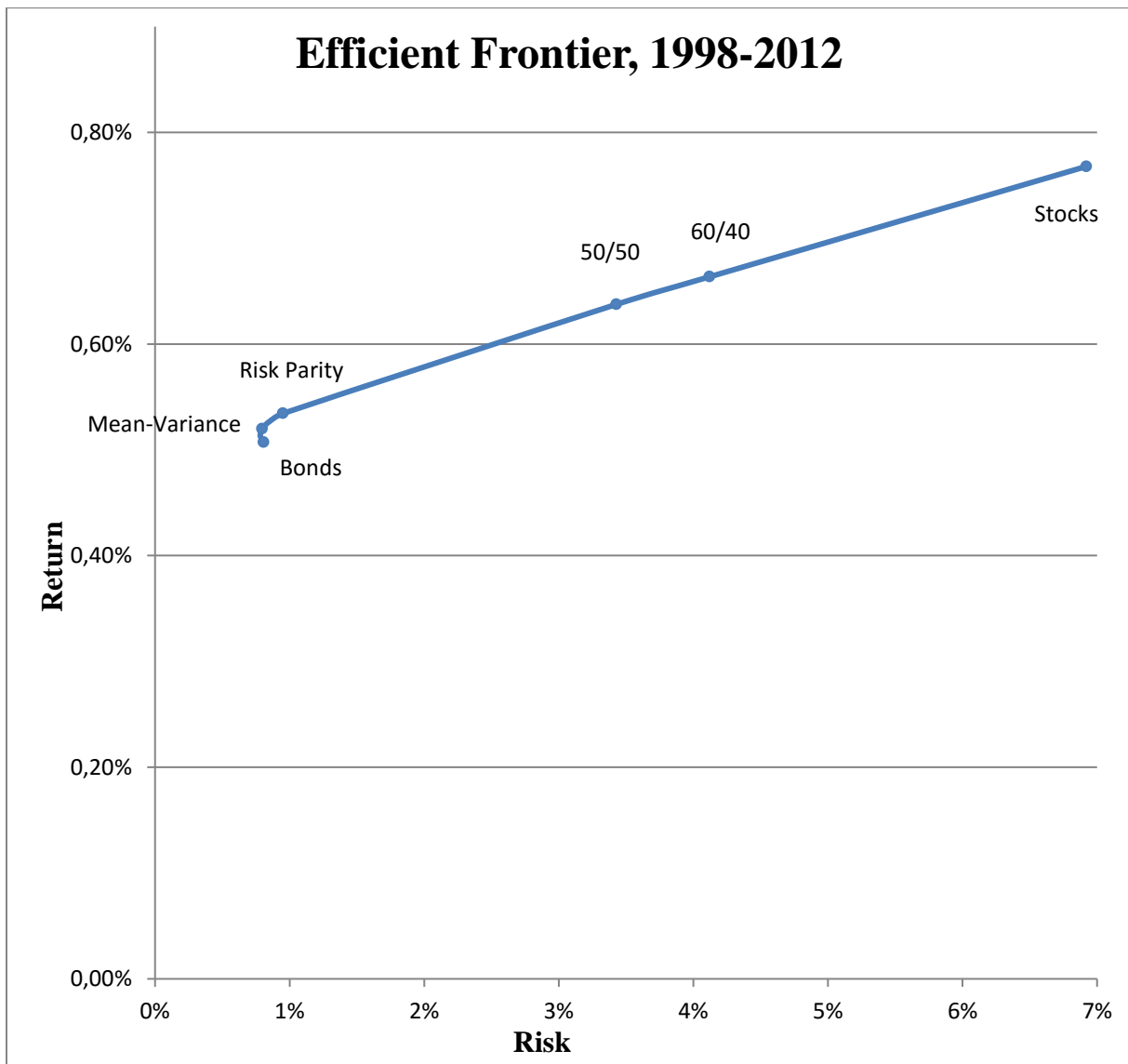
The charts below compare the time series of portfolio weights for the four different portfolio allocation strategies with monthly rebalancing. The 60/40 portfolio and the equal weighting portfolio have the highest turnover. This is a result of the high weights allocated to stocks which have a larger variation in returns. The mean-variance portfolio clearly has the lowest turnover. The risk parity portfolio also has a relative low turnover. These two portfolios have a much larger allocation to bonds, which reduces the turnover due to more stable returns.







### 9.3 The Efficient Frontier, 1998-2012



This figure shows the enlarged efficient frontier of portfolios of stocks and bonds in the Norwegian market from 1998-2012.