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A study of two-phase gas-liquid flow with viscous oil in a vertical pipe and annuli channel

Olusegun Damilare Bolarinwa

NTNU
Norges teknisk-
naturvitenskapelige universitet

Institutt for energi- og prosessteknikk

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MASTEROPPGAVE

for

Olusegun Bolarinwa

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A study of two-phase gas-liquid flow with viscous oil in a vertical pipe and annuli channel

En studie av to-fase gass-væskestrømmen med viskøs olje i et vertikalt rør og annular-kanalen

Bakgrunn og målsetting

The heavy oil production requires good understanding of multiphase flow in well (called as well hydraulics). The existing models used in the commercial software were developed on the basis of experimental observations of light oils in a pipe flow. Their applicability to the viscous oil well is questionable, in particular in the area that the gas-lift is used, for this situation, the flow will be in annular geometry

The objective of this work is to conduct the experimental study on the multiphase flow in annular geometry. In which the pressure gradient will be measured, and the gas-liquid flow regime will be identified. The influence of the pipe geometry will be investigated.

Oppgaven bearbeides ut fra følgende punkter

1 Conduct single phase liquid flow in an annular geometry
2 Conduct two-phase gas-liquid flow in an annular geometry with variation of flowrate and pipe inclination angles
3 Analysis of the experimental data
4 Assessment of the existing models for flow regime and pressure gradient.

” - ”

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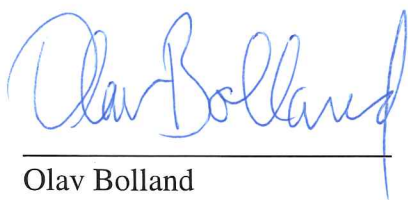
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Olav Bolland
Instituttleder

Zhilin Yang
Faglig ansvarlig/veileder

Medveileder(e): Ole Jørgen Nydal

ABSTRACT

The knowledge of gas-liquid flows in annulus geometries can be found in several applications such as the geothermal, nuclear, space, chemical and petroleum. Over time, the design of ducts with annulus geometries for transport of two phase gas-liquid flows has been dependent on the concept of the hydraulic diameter. This has, in many ways, led to oversizing of such ducts due to inadequate estimation of the friction factor of such annulus geometries. Flow pattern predictions for two phase flow in annulus geometries have also been explored but more work is still needed for better understanding of the subject.

The aim of this research is to examine single phase flow and two phase gas-liquid flow in annulus geometries. This research work was carried out at the new annulus geometry test rig at the Multiphase Flow Laboratory in the Department of Energy and Process, Norwegian University of Science and Technology. The inner diameter of the outer pipe is 90mm and the outer diameter for the inner pipe is 30mm. For single phase flow, experimental pressure drop measurements are made in a 6m horizontal annulus geometry set up to investigate the validity of existing models for estimating the friction factor and hence the pressure drop of annulus geometries based on several equivalent diameter concepts. Water and Oil are both used for the single phase flow experimental study. The Hydraulic diameter was inadequate to predict pressure drop in the annulus test section. However, the Knudsen diameter performed best and is suitable for prediction of the measured single phase pressure drop and friction factor in the annulus test section. For two phase gas-liquid flow, experimental investigation is carried out to observe flow patterns in a 6m inclinable annulus rig set up for angles of 15, 30, 45 and 60 degrees. This is done for both air-water and air-oil mixtures. Flow pattern maps are generated. It was observed that the angle of inclination, size of duct and viscosity of the working liquid all play have a considerable effect on the flow regimes observed, flow regimes transitions points and size of each flow regime region on flow pattern maps.

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NOMENCLATURE

SYMBOL	QUANTITY	UNIT
A	Cross sectional area	m^2
e	Eccentricity	$-$
ε	Roughness	m
d_1	Outer diameter of inner rod	m
d_2	Inner diameter of outer pipe	m
D_C	Diameter of casing	m
D_T	Diameter of tubing	m
U	Velocity	m/s
U_{sl}	Superficial velocity of liquid	m/s
U_{sg}	Superficial velocity of gas	m/s
U_l	Velocity of liquid	m/s
U_g	Velocity of gas	m/s
U_m	Mixture velocity	m/s
Q_g	Volumetric gas flow rate	m^3/s
Q_l	Volumetric liquid flow rate	m^3/s
Q	Volumetric flow rate	m^3/s

r_1	Outer radius of inner rod	m
r_2	Inner radius of outer pipe	m
r_{max}	Locus of maximum velocity	m
S	Perimeter of circular cross section	m
S_1	Perimeter of cross section of inner rod	m
S_2	Perimeter of cross section of outer pipe	m
K	Annulus pipe diameter ratio	$-$
Re	Reynolds number	$-$
D	Diameter	m
ρ	Density	kg/m^3
μ	Dynamic viscosity	kg/ms
τ	Shear stress	N/m^2
Δp_f	Pressure loss due to friction	N/m^2
ΔP	Pressure drop	N/m^2
L	Length	m
G	Mass flux	$kg/m^2 s$
λ	Volumetric flux	$-$
X	Lockhart Martinelli parameter	$-$

1. INTRODUCTION

As in many applications other than the one which is the subject of this thesis, multiphase flow and its associated technology has developed due to several attempts and researches to understand its complexities and underlying physics. Basically, multiphase flow refers to the physical phenomenon in which two or more phases flow concurrently in a conduit or channel. The behavior of the flow is more complex, particularly the properties of the flow such as the pressure drop and phase velocities when compared to single phase flow. The difference in densities and viscosities lead to different shear stresses at the wall which are different for each phase. Two phase flows are found in several practical applications in many industries such as the geothermal, nuclear, space, chemical and petroleum. In addition, specific applications can be found in equipment such as chemical and nuclear reactors, automobile engines as well as boilers and condensers (Bolarinwa, 2015).

We find and observe two phase flow during the production and transportation of oil and gas in the petroleum industry. In the past decades, efforts have been to predict the multiphase flow behavior in various systems. Multiphase flow is observed in a range of inclination from horizontal to vertical channels in wells as well as in flowlines. For oil or gas wells, reduction of pressure and temperature of the produced hydrocarbons as they flow from the reservoir into and up the wellbore could lead to the evolution of gas for oil wells or condensation of gas for gas wells, thereby creating a complex multiphase system (Bolarinwa, 2015). In some cases, in oil wells, we inject gas via the annulus to help the oil rise to the surface (Bratland, 2013) resulting in a multiphase flow which poses problems with prediction of the flow phenomenon and become more difficult to deal with as compared to single phase flow.

The complexity with multiphase flows is a direct consequence of the basic difference from single phase flows which is the physical distribution of the phases in the channel flow. These variations are called flow regimes or patterns. Various patterns are observed and their formation depends largely on the forces acting on the fluids. In turn, these forces which include turbulence and buoyancy are dependent on the properties of the flow such as the fluid properties

of the phases like density and viscosity, pipe geometry parameters like pipe diameter and inclination angle as well as the properties of the flow such as the flow rates (Bolarinwa, 2015).

In the case of the pipe geometry, studies have shown that changing the pipe geometry from circular to annular conduits have an influence on the behavior of the multiphase flow. We find these annular geometries with multiphase flows in many applications. Double pipe heat exchangers in terms of evaporation and condensation, which consist of two concentric pipes, are a typical example (Gschnaidtner, 2014). Annulus geometries also find application in the petroleum industry. It is customary to have flow in wells to occur through a tubing string. However, it has also been observed that many oil wells with high production rates produce through the casing/tubing annulus (Brill J. P., 1999). Multiphase flow in annular channels are observed in directional wells with sand screen, vertical wells with electrical submersible pumps (ESPs) and gas liquid inline separators (Colmanetti, H, Souza De Castro, Kjedlby, & Derks, 2015)

Another example, for which several researches in multiphase flow in annuli have been a direct consequence of, is in the drilling of oil wells. Typically, when a well is drilled to tap into the hydrocarbon reservoir, the equipment used consists of drill string and some sort of circulation system. As the drill bit at the end of the drill string cuts through the formation, debris or cuttings are transported up the annulus between the drill string and casing by the fluid which has been circulated down the well through the drill pipe. This is done to clean the drilled well of cuttings and debris. At the surface, the fluid is stripped of debris and cuttings and re-circulated down the drill pipe.

The hydrostatic pressure of the drilling fluid in the wellbore exceeds the pore pressure of the sedimentary formation for conventional drilling techniques such as overbalanced drilling (OBD). However, in various cases, one faces the possibility of fluid loss under extreme overbalanced conditions, where underbalanced drilling (UBD) techniques offer several advantages (D.B & F.B, 1994). The pressure of the drilling fluids in the borehole of UBD operations is less than the pore pressure of the sedimentary rock in comparison to OBD operations. Thus, the rate of penetration can be increased, the risk of lost circulation can be

minimized and the lifetime of the drilling bit can be prolonged (Guo & Ghalambor, 2002). Due to the lower pressure of the drilling fluid, there is reduced damage to the formation which is as a result of the fact that there is no inflow of drilling fluid into the permeable zone. Given the substantial advantages, the demand for UBD techniques steadily increases among gas and oil producers. In recent years, the exploitation of unconventional petroleum reservoirs including shale and tight-sandstone gas has become an alternative to natural gas. In this context, UBD of horizontal wellbores (UBHD) is the key for the successful exploitation of such unconventional reservoirs (Zou, 2013).

The primary advantages of horizontal wells over vertical wells include a larger drainage area, an increased production per well and hence a reduction of number of wells needed, higher producing rates and the ability to turn non accessible reservoirs into productive petroleum sources by avoiding barriers and protected landscape (Zou, 2013). Basically, there are three primary fluids used for Underbalanced Drilling: liquid, gas and two phase fluid. The underbalanced condition during UBD operations is generally achieved by the injection of gas into the drilling fluid thus leading to reduction in density and giving a light weight mixture or multiphase situation. Yet, by using a gas liquid mixture as the drilling fluid in UBD operations, the estimation of design parameters such as the pressure loss of the gas-liquid flow, the choice of the appropriate flow and the performance of the hole cleaning becomes more complex than for single-phase drilling fluids due to the interaction of the two phases (Gschnaidtner, 2014). However, compressibility values of liquid and gas vary from each other as pressure and temperature changes, the liquid fraction also changes. We must also remember that frictional pressure gradients are highly dependent on flow rates, flow regimes, fluid properties and pipe geometry. Therefore, phase behavior is a very important component in underbalanced drilling models (Air, 2015).

As a result of this phase behavior during UBD operations, two phase flow prediction techniques are used to predict several parameters such as pressure drops flow patterns, velocities, liquid holdup, and other parameters. These models used are usually the mechanistic two phase flow models. These models have been shown to perform better based on previous works and literature than the empirical models as they are based on the physical phenomena of the

complex fluid system as compared to the latter which is based purely on experimental data (Aladwani, 2003).

1.1 AIMS AND OBJECTIVES

The heavy oil production requires good understanding of multiphase flow in well (also called well hydraulics). The existing models used in the commercial software were developed on the basis of experimental observations of light oils in a pipe flow. Their applicability to the viscous oil well is questionable, in particular in the area that the gas-lift is used, for this situation, the flow will be in annular geometry

The objective of this work is to conduct the experimental study on the single and two phase gas-liquid flow in annular geometry inn which the single phase pressure gradient will be measured and the gas-liquid flow regime will be identified. The influence of the pipe geometry will be investigated.

The tasks in this research work include:
1. Conduct single phase liquid flow in an annular geometry
2. Conduct two-phase gas-liquid flow in an annular geometry with variation of flowrate and pipe inclination angles
3. Analysis of the experimental data
4. Assessment of the existing models for flow regime and pressure gradient.

1.2   PREVIOUS RESEARCH AND LITERATURE

Over the years, some research has been done to investigate and understand the physics of single phase and multiphase flow in annuli. The performance of the hydraulic diameter as well as other equivalent diameter concepts have been evaluated in their suitability in predicting pressure drop in single phase flow in annuli geometries. For multiphase flows through annuli, several correlations have been developed in making valid and useful predictions regarding liquid hold-up and pressure gradient. Many of these empirical correlations, which were developed without taking into cognizance the existing flow regimes, were developed to fit into experimental data empirically. These correlations were developed by:

1. Applying correlations that have been developed for predictions in pipe flow and extrapolating it to the annulus situation by using the idea of hydraulic diameter. In truth, several formulae have been used for the hydraulic diameter apart from the usual formula known as the area divided by the wetted perimeter.
2. Applying correlations which are developed from the experimental data from two phase flows in annuli, most of which are independent of the flow pattern.

Various models can be developed based on the data from experiments and their accuracy can be examined (Chen Y. , 2001). Indeed, the improved understanding of multiphase flow in pipes needs both an experimental and theoretical approach (Brill & Arlrachakaran, State of the Art of Multiphase flow, 1992). The theoretical approach over the past decades have been in the form of development of mechanistic models.

Four mechanistic models were developed earlier (Shoham, 2006). They include:

1. Homogenous No-Slip Model, where the multiphase mixture is assumed to be in a pseudo single phase with averaged fluid properties. Most importantly, it assumes there is no slippage between the phases
2. The Two Fluid or Separated Model in which the phases are treated separately by using the knowledge of single phase flow, particularly with respect to the friction factor and hydraulic diameter
3. Generalized Solutions developed by doing Similarity Analysis

4. Drift Flux Model, which treats the multiphase flow as a homogenous mixture in addition to taking into account the slippage between the phases. The drift flux model was first proposed by (Zuber & Findlay, 1965). Usually, this model is applied to vertical dispersed systems (Chen Y. , 2001)

As clearly seen, both empirical and mechanistic model are very much predicated on the knowledge of single phase flow. Single-phase and two-phase flows are the most common flow conditions of drilling fluids occurring during a drilling process. Single-phase fluids are widely used to transport the cuttings from the drilling to the surface in OBD operations while UBD operations generally rely on two-phase fluids.

As the focus of this study is in evaluating single phase friction factor in annulus geometries and in flow pattern observations for multiphase flow in inclined annulus geometries, the concepts and the research work on single phase and multiphase flow in annuli will be discussed in this chapter.

1.2.1 SINGLE PHASE FLOW IN ANNULI

The geometrical concept of an annulus is first established. The annulus is defined or described as the volume between two circular concentric or eccentric pipes, where the flow occurs through the area bounded by the outer pipe inner wall and the inner pipe outer wall. As noted by (Caetano, Shoham, & Brill, 1992), two geometrical parameters identify these configurations: the annulus pipe diameter ratio, and the degree of eccentricity. The annulus pipe diameter ratio accounts, to some extent, for the flow area and is expressed by

$$K = \frac{D_T}{D_C} \tag{1.1}$$

where D_T is the outer diameter of the inner pipe (tubing) and D_C is the inner diameter of the outer pipe (casing). Subsequently in this work, the outer diameter of the inner pipe (tubing) and inner diameter of the outer pipe (casing) will be treated with the notations d_1 and d_2. The degree of eccentricity accounts for the displacement of the inner pipe center from the outer pipe center and is expressed by

$$e = \frac{2DBC}{(D_C - D_T)} \tag{1.2}$$

where DBC is the distance between the pipe centers. Annuli can have eccentricity values varying from zero to one. Subsequently in this work, the outer diameter of the inner pipe (tubing) and inner diameter of the outer pipe (casing) will be treated with the notations d_1 and d_2. *Figure 1.1* shows cross sections of annuli with the same pipe diameter ratio value, K, and for eccentricities of 0.0, 0.5, and 1.0. (Caetano, Shoham, & Brill, 1992)
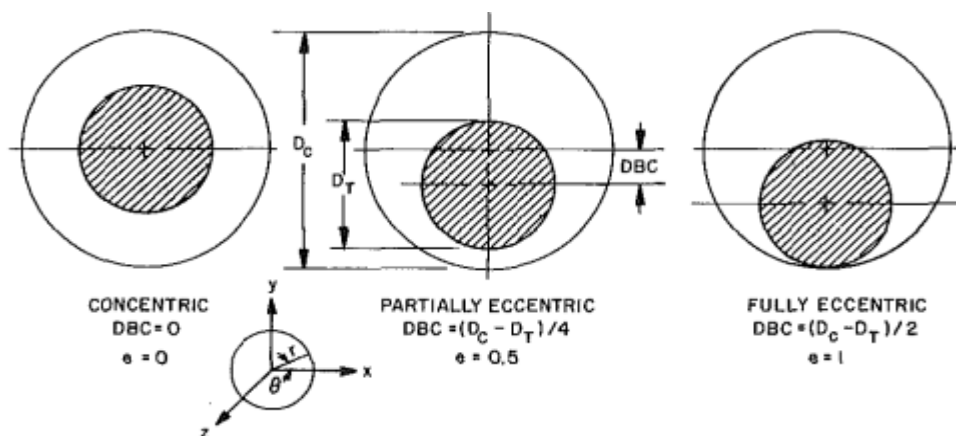

Figure 1.1. Annulus configurations (Brill J. P., 1999)

Several researches in the past have treated the flow through annuli based on the concept of the hydraulic diameter, which is four times the area for flow divided by the wetted perimeter. Based on this, when the arithmetic is done and simplified, the hydraulic diameter for annulus configurations is given by

$$D_H = D_C - D_T \qquad\qquad (1.3)$$

However, the hydraulic diameter is not always the most representative characteristic dimension for flow in an annulus. Some of these reasons, both experimental and analytical, are examined later in this chapter. The work by (Caetano, Shoham, & Brill, 1992) highlights the important need to have a clear understanding of flow in an annulus must be achieved in order to determine appropriate characteristic dimensions.

In previous work done on the subject of single phase flow in annuli, the concepts of single phase flow are still very much applicable as the flow regimes observed are the same. However, the velocity profile in annulus configurations is different and the analysis of the flow poses other considerations. **Nonetheless, the concepts in single phase pipe flow used for the understanding of single phase flow in annuli configurations must be mentioned**. This is mirrored in the work of (Fang, Xu, & Zhou, 2010) where it is highlighted that the single-phase friction factor of pipe flow is used for determining single-phase friction pressure drop in addition to being used as the foundation for pressure drop calculations of supercritical flow and two-phase flow.

In normal circular channels, single phase flow exists in different flow regimes primarily dependent on parameters that adequately describe the physics of the flow. Fluids are normally represented by a multitude of single particles flowing side by side in the fluid. They can also be seen as viscous mass. The particles in viscous fluids are considered as sensitive to the internal forces of neighboring particles, also known as viscous stress (Gschnaidtner, 2014). According to the no-slip condition the velocity of a moving fluid at the surface of a wall is assumed to be zero. In the flow of a viscous fluid the layer from the surface (velocity is zero)

to the location, where the fluid reaches its maximum velocity is known as the boundary layer (Gschnaidtner, 2014). In single phase pipe flow, because of the no-slip condition, the fluid velocity in a pipe changes from zero at the pipe wall to a maximum at the pipe center. However, it is observed that it is much easier to work with an average velocity which remains constant in incompressible flow when the cross-sectional area of the pipe is constant (Cengel & Cimbala, 2006). When the flow is fully developed, the average velocity is half of the maximum velocity. This is based on the assumption that the flow is laminar.

Single phase flow exists in three different flow regimes depending on the velocity of the flow. There are *Laminar flow regimes* in which the flow is characterized by smooth streamlines and highly ordered motion, as well as *Turbulent flow regimes* where the flow is characterized by velocity fluctuations and highly disordered motion. Observing from practical situations, the transition from laminar to turbulent flow is gradual and it occurs over some region, *Transition region*, in which the flow fluctuates between laminar and turbulent flows before it becomes fully turbulent. Most flows encountered in practice are turbulent. Laminar flow is encountered when highly viscous fluids such as oils flow in small pipes or narrow passages (Cengel & Cimbala, 2006).

In predicting a particular regime for a certain flow situation, the dimensionless number, Reynolds' number is used. It is the ratio of inertia forces to viscous forces in a fluid flow. The Reynolds' number is represented by

$$Re = \frac{\rho.U.D}{\mu} \qquad (1.4)$$

where ρ is the density, U is the average velocity, μ is the dynamic viscosity and D is the diameter of the pipe. According to (Cengel & Cimbala, 2006), transition from laminar to turbulent flow also depends on the degree of disturbance of the flow by surface roughness, pipe vibrations, and fluctuations in the flow. The flow in a circular pipe is laminar for Re ≤ 2300, turbulent for Re ≥ 4000, and transitional in between for most practical conditions. We obtain

all these single phase flow regimes in annulus geometries as well, albeit depending on a different value for the 'effective' or 'equivalent' diameter as the flow area is different as compared to circular pipes.

The pressure loss or pressure drop due to friction for all types of fully developed internal flows (laminar or turbulent flows, circular or noncircular pipes, smooth or rough surfaces, horizontal or inclined pipes) for most practical situations on friction factor (Cengel & Cimbala, 2006) as

$$\Delta p_f = f \frac{L}{D} \frac{\rho U^2}{2} \tag{1.5}$$

where f is the Darcy friction factor, sometimes called the Moody friction factor, which is a function of wall shear stress, τ and is denoted by

$$f = \frac{8\tau}{\rho U^2} \tag{1.6}$$

As such, it is usually required to determine the value of the friction factor to determine the frictional pressure gradient. However, we must first determine what flow regime the fluid is whether it is laminar or turbulent by examining the range in which the dimensionless number, Reynolds' number falls in.

For laminar flow, work done by Hagen and Poiseuille for pressure drop in a pipe with no elevation (assuming no acceleration and fully developed flow) gives an equation

$$\Delta P = \frac{32\mu LU}{D^2} \tag{1.7}$$

where U is the average velocity.

With no-slip condition at the wall and a velocity gradient of zero at the centre of the pipe gives the velocity profile of the fluid as the shape of a parabola with the maximum velocity at the centre of the pipe (Gschnaidtner, 2014). Combining the equations (1.5) and (1.7) and equating them to each other, we get the friction factor for laminar flow as

$$f = \frac{64}{Re} \tag{1.8}$$

As mentioned earlier, Transition flow occurs in the range of Reynolds numbers between 2300 and 4000. However, there have been observations suggesting that there are uncertainties with the Darcy friction factor value in this flow regime.

For turbulent flow, extensive research done in this regard shows that the velocity profile and pressure gradient are largely dependent and sensitive to the pipe wall properties such as the roughness. However, it has been observed from dimensional analysis that the effect of roughness on the velocity profile and pressure gradient depends on the relative roughness (ratio of roughness to inside diameter) and the dimensionless Reynolds' number. The dependence is not derived by theory but empirical relations through data from several experiments.

Nikuradse in 1933 and Colebrook in 1939 developed empirical relations for the friction factor in turbulent and transition (region between laminar and turbulent) flows respectively. However, a modification is made to couple the data from both transition and turbulent experiments performed, to form the modified Colebrook equation which is an implicit relation of the friction factor and it is given as

$$\frac{1}{\sqrt{f}} = -2.0\, log\left(\frac{\varepsilon/D}{3.7} + \frac{2.57}{Re\sqrt{f}}\right) \tag{1.9}$$

where ε is the roughness and D is the diameter.

A graphical plot of f as a function of Re was created by the American engineer Rouse from the Colebrook's equation. Two years later, Lewis F. Moody redrew Rouse's diagram into the form used in most practical situations today. Indeed, the implicit relations can only solved numerically or by a trial and error process.

However, several explicit approximations to the Colebrook equation have been developed. One of these which is one of the most accurate according to (Brill J. P., 1999) and easy to use is given by Zigrang and Sylvester given by

$$\frac{1}{\sqrt{f}} = -2.0 log(\frac{2\,\varepsilon/D}{3.7} - \frac{5.02}{Re} log\left(\frac{2\,\varepsilon/D}{3.7} + \frac{13}{Re}\right) \tag{1.10}$$

In 1983, the Haaland equation was proposed by S.E. Haaland. It is used to solve directly for the Darcy friction factor f for a full-flowing circular pipe. It is an approximation of the implicit Colebrook–White equation, but the discrepancy from experimental data is well within the accuracy of the data. In the Haaland equation there is no need to iterate the Darcy friction factor. The accuracy of the Darcy friction factor solved from this equation is claimed to be within about ±2 %, if the Reynolds number is greater than 3000 (Fox, Pritchard, & McDonald, 2010). The Haaland Equation is given as:

$$\frac{1}{\sqrt{f}} = -1.8\,log\left(\left(\frac{\varepsilon/D}{3.7}\right)^{1.11} + \frac{6.9}{Re}\right) \tag{1.11}$$

The Blasius equation is the most simple equation for solving the Darcy friction factor. Because the Blasius equation has no term for pipe roughness, it is valid only to smooth pipes. However, the Blasius equation is sometimes used in rough pipes because of its simplicity. The Blasius

equation is valid up to the Reynolds number 10^5 (Fox, Pritchard, & McDonald, 2010). However, (Kiijarvi, 2011) observed through experimental work that the Blasius equation cannot replace the Colebrook and Haaland equation, when the Darcy friction factor must be solved in a rough pipe. The Blasius equation is appropriate only in a smooth pipe. There are a number of other correlations for the

Single phase friction factor of pipe flow, whose ranges of validity were described by various authors. (Romeo, Royo, & Monzon, 2002) did a comprehensive analysis on the available correlations of the single-phase friction factor and ranked them (Fang, Xu, & Zhou, 2010). Yıldırım (2009) conducted the most comprehensive analysis of existing correlations for single-phase friction factor. He provided the maximum and minimum errors each correlation has in the ranges of $4000 \leq Re \leq 10^8$ and $10^{-6} \leq Re \leq 0.05$ where *Re* is the Reynolds number and *Rr* is the relative roughness. The Nikarudse equation, which was mentioned before, is the base for the turbulent smooth portion of the Moody diagram (Moody, 1944). However, it is implicit for friction factor *f*, thus needs iteration that is not convenient. The Filonenko equation alongside the Blasius equation are widely used for calculating turbulent flow in smooth pipes. Incropera and DeWitt (2001) gave the Filonenko equation applicable Re range of $3000 \leq Re \leq 10^6$. Other friction factor correlations proposed for turbulent flows include those by (Manadili, 1997), (Chen N. H., 1979) and (Romeo, Royo, & Monzon, 2002). In this work, their performance in comparison with the Haaland correlation is within the range of $\pm 0.1\%$. *In this work, the Haaland equation is used for all turbulent flow friction factor calculations.*

With all these studies, work has been done to show how these concepts in circular pipes can be used in annulus geometries. Similar to the characterization of single-phase flow through circular conduits, the flow through annular conduits can be subdivided into laminar, transitional and turbulent flow characteristics (Gschnaidtner, 2014).

1.2.1.1 LAMINAR FLOW IN ANNULI

Analytical solutions for the both the velocity profile and friction factor concentric annulus are given by Bird et al. (1976) (Caetano, Shoham, & Brill, 1992). As far as laminar flow is concerned, the normal assumption is that the flow is symmetrical around the central line of the inner and outer pipe for the single phase flow through a concentric annulus. We see this in a representation of the velocity profile as shown in *Figure 1.2*. Considering no-slip conditions at the surface of the outer and inner pipe and a velocity gradient of zero at the locus of maximum velocity, the theoretical position of the locus of the maximum velocity r_{max} can, in accordance with (Knudsen & Katz, Fluid Dynamics and Heat Transfer, 1958) be expressed as

$$r_{max} = \sqrt{\frac{r_2{}^2 - r_1{}^2}{2\,ln\left(\frac{r_2}{r_1}\right)}} \qquad (1.12)$$

where r_2 is the inner radius of the outer and r_1 the outer radius of the inner pipe. Using the radius of maximum velocity (Knudsen & Katz, Fluid Dynamics and Heat Transfer, 1958) proposed an analytical solution to calculate the point velocity:

$$u = 2.U.\frac{r_2{}^2 - r_1{}^2 - r_{max}{}^2\,ln\frac{r_2}{r}}{r_2{}^2 + r_1{}^2 - 2.r_{max}{}^2} \qquad (1.13)$$

The above equations were observed to be valid for Reynolds numbers of up to 700 (Prengle & Rofthus, 1955). They further observed that for Reynolds numbers in the range from 1,500 to 2,000 the maximum radius shifts more toward the outer radius of the core pipe than predicted by *Equation (1.12)*. As a result, (Prengle & Rofthus, 1955) suggest using another equation

instead of *Equation (1.12)* for a laminar flow and for Reynolds numbers just above 700 (Gschnaidtner, 2014).
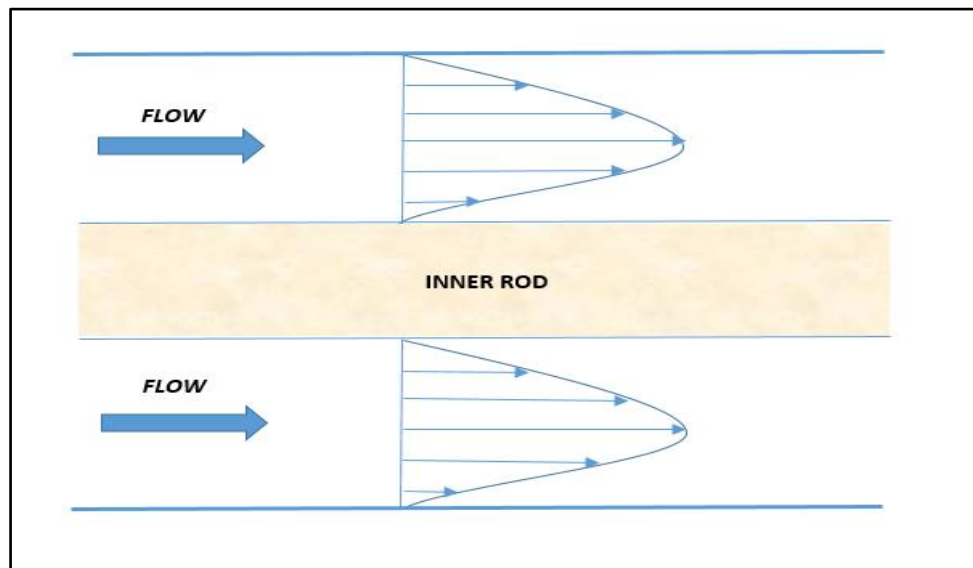
Figure 1.2. Velocity profile of Incompressible Viscous fluid in an Annulus

It is also observed by (Gschnaidtner, 2014) that the first approach for the friction factor estimation of laminar flow in annulus geometries would be to apply the hydraulic diameter concept as suggested by (Moody, 1944) and Fromm (1923). Normally, the shear stress on the wall of the pipe is given by

$$\tau = \frac{1}{8}\rho f U^2 \qquad\qquad (1.14)$$

and the pressure drop per unit length based on the shear stress is given by

$$\frac{dP}{dL} = \tau \frac{S}{A} \qquad (1.15)$$

Combining both equations, for circular pipes, the pressure drop per unit length of pipe is given as

$$\frac{dP}{dL} = \frac{1}{8}\rho f U^2 \frac{S}{A} \qquad (1.16)$$

where, ρ, f and U are the density of the fluid, the friction factor and the average velocity of the fluid in the pipe. S refers to the wetted perimeter and it is the circumference of inner wall of the circular pipe and A is the cross-sectional area of the pipe.

For an annulus geometry, the wetted perimeter will be the sum of S_1 and S_2 as shown in the figure below.

Figure 1.3. Annulus cross section with outer pipe and inner rod

Hence, the pressure drop per unit length is given by

$$\frac{dP}{dL} = \frac{1}{8}\rho f U^2 \frac{S_1 + S_2}{A} \tag{1.17}$$

where A is the cross sectional area and is given by

$$A = \frac{d_2{}^2 - d_1{}^2}{4} \tag{1.18}$$

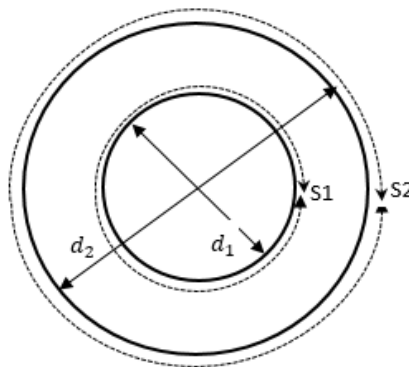Combining both equations and doing a bit of arithmetic, this results in

$$\frac{dP}{dL} = \frac{1}{2}\rho f U^2 \frac{1}{(d_2 - d_1)} \tag{1.19}$$

Consequently, this is simply means the pressure drop in the annulus can be predicted by the use of the pressure drop equations by using the hydraulic diameter in place of the diameter in the pressure drop equation. The hydraulic diameter, however, is considered as insufficient for estimating the frictional pressure drop for laminar flow in an annular geometry (Jones & Leung, 1981); (White, 2006); (Mironer, 1979). This is a consequence of the work by (Gnielinski, 2007) which suggests that the shear stress on the surface of the inner pipe pipe is higher than that on the surface of the outer pipe since the locus of the maximum velocity is shifted towards the inner pipe. Therefore, amongst others, (Knudsen & Katz, Fluid Dynamics and Heat Transfer, 1958), (Jones & Leung, 1981), as well as (Reed & Pilehvari, 1993) suggest, to modify the hydraulic diameter according to a function first derived by Wien (1900) and (Lamb, 1907). According to (Gschnaidtner, 2014), a function was proposed by Wien (1900) and (Lamb, 1907) that depends on the ratio $K = r_1/r_2$ which is evidently the same as $K = d_1/d_2$. The function is given as

$$\varphi(K) = \frac{(1-K)^2 . \ln K}{(1+K^2). \ln K + (1-K^2)} \tag{1.20}$$

and the Moody friction factor is then given by

$$f = \frac{64}{Re} . \varphi(K) \tag{1.21}$$

with Re as a function of the hydraulic diameter $d_2 - d_1$

(Gnielinski 2007) compared the modified friction factor above with experimental studies by (Koch and Feind 1958) and found the mean square error to be within the range of 2% (Gschnaidtner, 2014). This correlation is the same as the correlation that was presented in the work of (Caetano, Shoham, & Brill, 1992) which was developed by Bird (1976). However, in that research work, the Fanning friction factor, which is one-fourth of the Moody friction factor was used for analysis. According to Bird (1976), the laminar Fanning friction factor is given by

$$f_{CA} = \frac{16}{Re} \frac{(1-K)^2}{\left[\frac{1-K^4}{1-K^2} - \frac{1-K^2}{ln(1/K)}\right]} \tag{1.22}$$

Considering that this is Fanning friction factor, multiplying by a factor of 4 to get the Moody friction factor and doing a bit arithmetic rearranging, we observe that this function modifying the friction factor is the same as the one proposed by Wien (1900) and (Lamb, 1907).

Essentially, this means that this factor is adjusting the value of the hydraulic diameter. In essence, when doing calculations for the friction factor and pressure drop, the value used in place for the hydraulic diameter is a 'modified hydraulic diameter'. In the work by (Caetano, Shoham, & Brill, 1992) therefore, by using $K = d_1/d_2$, the modified hydraulic diameter used is given

$$D_H = \frac{(d_2 - d_1)\left[d_2{}^2 + d_1{}^2 - \frac{(d_2{}^2 - d_1{}^2)}{ln(d_2/d_1)}\right]}{(d_2 - d_1)^2} \qquad (1.23)$$

This relation is the same as the ones used in the works of (Reed & Pilehvari, 1993) and (Knudsen & Katz, Fluid Dynamics and Heat Transfer, 1958). *For the purpose of this work, this relation is termed as the Knudsen diameter*. Based on their research work, (Caetano, Shoham, & Brill, 1992) found this relation to be in good agreement with experimental measurements of the single phase friction factor in annulus geometries. Several other diameter correlations, often termed equivalent diameters, have been proposed as well. (Crittendon, 1959) proposed an equivalent diameter relation for annulus geometries. According to (Sorgun & Ozbayoglu, 2010), the Crittendon diameter is not accurate enough to determine the pressure drop through annular geometries (Gschnaidtner, 2014). The performance of several of these other equivalent diameters are examined in this research work in the laminar flow region.

1.2.1.2   TURBULENT FLOW IN ANNULI

Based on the concept of the hydraulic diameter, turbulent flow in annulus geometries occurs at Reynolds numbers above 4000. (Gschnaidtner, 2014) observes that (Prengle & Rofthus, 1955) found turbulent flow to be existent at Reynolds numbers above 2200 with the equivalent diameter as in *Equation 1.25*

Transition from laminar to turbulent annular flow, according to (Gschnaidtner, 2014), can be described corresponding to single-phase flow through a circular tube. The experimental investigations by (Knudsen & Katz, 1950) (Quarmby, 1967), (Nouri, Umur, & Whitelaw, 1993) and (Brighton & Jones, 1964)suggest that the turbulent velocity profile in an annulus geometries appears be flatter at the point of maximum velocity, which is similar to the observation with circular pipe geometries.

The point of maximum of velocity is not quite fully established and understood. Rothfus et al.(Rothfus *et al.*, 1950; 1955) and Knudsen and Katz (1950; 1958) observed that the locus of maximum velocity for fully turbulent flow is the about the same as for laminar flow. However, the research by (Quarmby, 1967) showed that the results are not accurate and found the location of the point of maximum velocity closer to the inner pipe at high Reynolds number in comparison with laminar flow. Following the experimental results of (Brighton & Jones, 1964), Rothfus et al. (1966) introduced a new formula for calculating the maximum radius (Gschnaidtner, 2014):

$$\frac{2.(r_{max} - r_1)}{(r_2 - r_1)} = \left(\frac{r_1}{r_2}\right)^{0.2} \tag{1.24}$$

Several works have been carried out to examine the pressure drop for turbulent flow in annulus geometries such as (Knudsen & Katz, 1958), (Rothfus *et al.*, 1950), (Quarmby, 1967), (Brighton & Jones, 1964), (Nouri, Umur, & Whitelaw, 1993), (Rehme, 1973), (Gnielinski, 2007), (Koch and Feind, 1958); (Sorgun & Ozbayoglu, 2010) and (Jones & Leung, 1981). All these works proposed correlations for equivalent diameters to estimate the friction factor and pressure drop for turbulent flow in annulus geometries. According to (Mironer, 1979), "the hydraulic diameter significantly underestimates the pressure drop of turbulent flows through annular passages" (Gschnaidtner, 2014). (Jones & Leung, 1981) showed that that the modified hydraulic diameter mentioned in the section on Laminar flow and used by (Caetano, Shoham, & Brill, 1992) is also applicable for the use in turbulent flow in annuli. Indeed, turbulent flow

in annuli was duly investigated by (Colmanetti, H, Souza De Castro, Kjedlby, & Derks, 2015), (Caetano, Shoham, & Brill, 1992) and (Gschnaidtner, 2014). For both (Colmanetti, H, Souza De Castro, Kjedlby, & Derks, 2015) and (Caetano, Shoham, & Brill, 1992), a vertical set up was used while (Gschnaidtner, 2014) used a horizontal set up. The work of (Colmanetti, H, Souza De Castro, Kjedlby, & Derks, 2015) made use of the equivalent of by (Sas-Jaworsky & Reed, 1998), which is observed to be the same as the one used for the investigation of laminar and turbulent flow in annuli by (Caetano, Shoham, & Brill, 1992).

The work by (Colmanetti, H, Souza De Castro, Kjedlby, & Derks, 2015) is for turbulent flow and suggests that the equivalent diameter in *Equation 1.23* performs very well for the pressure drop measurements when compared with the predicted pressure drop (Average Relative Error of 5%) in contrast to the performance of the hydraulic diameter (Average Relative Error, ARE greater than 40%). However they employed the Blasius friction factor correlation for smooth pipes. (Caetano, Shoham, & Brill, 1992) used a much more complex friction factor for its calculations for its friction factor prediction. Their work uses the implicit correlation based on the work of Gunn and Darling which is a Nikuradse type of expression. Because of its complexity, it is not employed in this work. (Gschnaidtner, 2014) also examined single phase turbulent flow in annuli. In his work, he observed that the performance of the hydraulic diameter is less than an ARE of -10% while the equivalent diameter which was confirmed to perform well by the works of Jones and Leung (1981), (Colmanetti, H, Souza De Castro, Kjedlby, & Derks, 2015) and (Caetano, Shoham, & Brill, 1992) to perform with an ARE of about 40%. He suggests that the best performing correlation is the one suggested by Rofthus et al (1966). The hydraulic diameters for all these works vary and hence could possibly have an effect on these results. This is something that is also examined in this work. *In this research work, the friction factor correlation used is the Haaland correlation as it is an explicit relation and a good approximation of the Colebrook equation.*

As for the hydrodynamic entrance length, (Quarmby, 1967) found the entrance length to be in the same order of magnitude as for circular tubes which was estimated to be about 30-40 times of the equivalent diameter

1.2.1.3 TRANSITIONAL FLOW IN ANNULI

It was established earlier that the transition region between the laminar and turbulent region exists in between a Reynolds number range from 2000 to 4000. However, some researchers have (Rofthus & Prengle, 1952) observed that the transition region for single phase flow in annulus has an extended range in comparison to pipe flow situation. In this context, turbulence at a Reynolds number of about 900 was observed by (Rofthus & Prengle, 1952). Additionally, the extend of the transition region was found by (Rofthus, Monrad, & Senecal, 1950) to be a function of the radius ratio $\frac{r_1}{r_2}$, with a longer range for smaller values of the radius ratio (Gschnaidtner, 2014).

Regarding the velocity profile, (Prengle & Rofthus, 1955) introduced a Reynolds number with an equivalent diameter based on the radius of maximum velocity as below

$$d_H = 2 \frac{(r_2{}^2 - r_1{}^2)}{r_2} \qquad (1.25)$$

(Gschnaidtner, 2014) observes there is no particular equation given for calculating the friction factor of transitional flow in annular cross sections, very much like the situation with single-phase flow through circular pipes and that neither the approaches for laminar flow nor for turbulent flow are accurate enough to estimate the friction factor for the transition zone.

1.2.1.4 EFFECT OF ECCENTRICITY

In this work, the effect of eccentricity of an annulus geometry is not examined experimentally. However, it is worth mentioning that some research has been done in regard to this. The case of an eccentric annulus and its effect has been examined by some researchers for Laminar flow.

The laminar velocity distribution in an eccentric annulus was studied by (Snyder & Goldstein, 1965). They also studied the friction factors in eccentric annular pipes and found the friction factor to change considerably by a factor of 2 with eccentricity. (Ozgen & Tosun, 1987) investigated how to determine the maximum velocity locus in an eccentric annulus. (Tosun, 1984) provides an equation to calculate the flow rate through an eccentric annulus as a function of eccentricity ratio and radius ratio, which is able to reproduce the exact values. Also, the research work by (Jonsson & Sparrow, 1965) observes that the friction factor is decreased by eccentricity. This is also observed in the experimental work by (Caetano, Shoham, & Brill, 1992) where the Fanning friction factor model performance clearly shows that the eccentricity leads to a decrease in the friction factor.

The effect of eccentricity on the velocity profile and the friction factor in the turbulent flow region in annuli geometries has also been investigated. The locus of maximum velocity was studied by (Wollfe & Clump, 1963), reaching the conclusion that the point of maximum velocity can be sufficiently described by the solution of (Heyda, 1959) of the Navier-Stokes equation for laminar flow (Gschnaidtner, 2014). Like their experimental work on laminar flow in annuli, (Jonsson & Sparrow, 1965) observed frictional factors to decrease with eccentricity. This also was observed for turbulent flow region by (Caetano, Shoham, & Brill, 1992) that the friction factor decreases with eccentricity, just as they observed for the Laminar flow region. (Jonsson & Sparrow, 1965) also made use of contour diagrams to describe the velocity field. (Kacker, 1973) examined the turbulent flow through eccentric annular geometries and proposed a correlation using the hydraulic diameter to estimate the friction factor. It was found to be less than for a concentric annular section. (Nouri, Umur, & Whitelaw, 1993) derived from their experiments also the phenomenon of a decreasing friction factor with an increasing eccentricity by 22.5 % (Gschnaidtner, 2014). Furthermore, a correlation based on the geometry factor for laminar flow was suggested by (Rehme, 1973) which performed accurately in comparison to experimental data for low diameter ratios (Gschnaidtner, 2014).

1.2.2 TWO PHASE FLOW CONCEPTS

Emphasis for this work is on the two phase flow pattern maps for in annuli geometries, particularly attempting to observe the effect of inclinations and pipe geometry. However, we must first proceed to establish the two phase flow concepts in pipes before proceeding to two phase flow in annuli channels as the same flow regimes observed in two phase pipe flow have been observed with two phase flow in annuli albeit with some 'modifications'. The presence of an extra phase or phases in a conduit or duct adds to the complexity of the analysis of the flow situation. In comparison to single phase flow where we observe laminar, transition and turbulent regimes, more flow regimes can be developed depending on the pipe geometry (diameter and pipe inclination) as well as the density and viscosity of the phases.

The complexity of multiphase flows has resulted in a high degree of empirical relations for describing or predicting flow behavior. Several empirical relations have been developed to predict flow pattern, slippage between the phases, friction factors and other parameters associated with multiphase flows (Brill J. P., 1999). Most books describe the common method for presenting the particular flow regimes using two-dimensional maps, often called flow pattern maps. An example of such a map is found below, many of which are generally plotted from experimental data obtained from extensive research (Gschnaidtner, 2014).
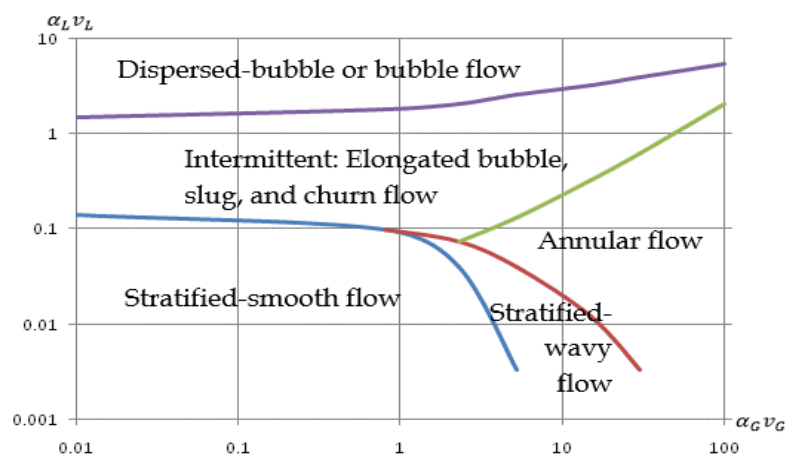

Figure 1.4. Typical two phase flow map (Bratland, 2013)

Flow pattern maps falls into two categories. One is the experimental flow pattern map generated directly from experimental data. It is completely empirical and limited to the data on which it is based. To account for the effects of fluid properties and pipe diameter, additional correlations must be introduced. On the other hand, Mechanistic flow pattern maps are developed from the analysis of physical transition mechanisms which are modeled by fundamental equations (Chen Y. , 2001). Empirical correlations are still required in the mechanistic model for the for closure relationships.

It is tacitly observed by (Bratland, 2013) that even though the flow regime maps are useful tools for getting estimating which flow regimes we can be expected from a set of input data, each map is not, general enough to be valid for other data sets. Interestingly, the maps are based on physical quantities as coordinates or dimensional groups. The physical quantities on such maps include superficial and mixture velocities, mass flow rates as well as liquid holdup and void fraction, while dimensionless coordinates including the Reynolds number. It is therefore important to describe these parameters quantitatively as below:

The superficial velocity of each phase is the ratio of the volumetric flow rate to the cross sectional area of the conduit and we have for both gas and liquid phases expressed as:

$$U_{sl/sg} = \frac{Q_{g/l}}{A} \qquad (1.26)$$

The mixture velocity which is assumed to be the velocity of the multiphase mixture is the sum of the superficial velocities of both phases. This is given as

$$U_m = \frac{Q_g + Q_l}{A} \qquad (1.27)$$

Also, the mass flux G is defined as the sum of the product of the superficial velocity and the density of each phase:

$$G = U_{sg}\rho_g + U_{sl}\rho_l \qquad (1.28)$$

Another quantity, the volumetric flux λ is defined as the ratio of the volumetric flow rate of the a phase to the total volumetric flow rate and it is given by

$$\lambda = \frac{Q_{g/l}}{Q_g + Q_l} \qquad (1.29)$$

In addition, we can consider the Reynolds' number for the two phase mixture by using the mixture velocity which is the sum of the superficial velocities and it is given by

$$Re = \frac{\rho \, U_m D}{\mu} \qquad (1.30)$$

By considering the cross sectional area occupied by the phases, we can get expressions for the phase velocities such as

$$U_{g/l} = \frac{Q_{g/l}}{A_{g/l}} \qquad (1.31)$$

The effects of fluid properties (density, viscosity and interfacial tension) for each phase as well as inclination angle and the pipe diameter are important and must be put into consideration in

the bid to understand and fully describe the phenomena observed in two phase flow systems. As observed by (Shoham, 2006), flow regimes or patterns in multiphase systems depends on the following variables:

- Geometrical Variables such as inclination angle and pipe diameter
- Operational Parameters such as flow rates of gas and liquid
- The physical properties of the phases such as density, viscosity etc.

Indeed, flow pattern determination remains one of the major challenges in understanding and analysis of two phase phenomena. Three basic flow patterns were suggested by Hubbard & Dukler (1966). They are:

1. Separated flow patterns: In this patterns, both phases exist as continuous phases. Examples are stratified flows (stratified smooth flow and stratified wavy flow) and annular flows:

2. Intermittent flow patterns: In these patterns, one of the phases exist as discontinuous. These include: flow, plug flow, churn flow

3. Dispersed flow patterns: In these patterns, one of the phases is well dispersed in the other phase. Examples include: bubble flow, dispersed bubble flow (finely dispersed bubbles exist in a continuous flowing liquid phase).

These are general pattern forms which can be evaluated in a more specific way by observing these patterns in vertical and horizontal flows.

1.2.2.1   HORIZONTAL FLOW REGIMES

The horizontal flow regimes are defined as follows. They are also shown in *Figure 2.7*.

1. **Stratified (Smooth and Wavy) Flow**: Stratified flow consists of two superposed layers of gas and liquid, formed by segregation under the influence of gravity.

2. **Intermittent (Slug and Elongated Bubble) Flow:** The intermittent flow regime is usually divided into two sub regimes: plug or elongated bubble flow and slug flow. The elongated bubble flow regime can be considered as a limiting case of slug flow, where

the liquid slug is free of entrained gas bubbles. Gas–liquid intermittent flow exists in the whole range of pipe inclinations and over a wide range of gas and liquid flow rates.

3. **Annular-Mist Flow**: During annular flow, the liquid phase flows largely as an annular film on the wall with gas flowing as a central core

4. **Dispersed Bubble Flow**: At high liquid rates and low gas rates, the gas is dispersed as bubbles in a continuous liquid phase. The bubble density is higher toward the top of the pipeline, but there are bubbles throughout the cross section.
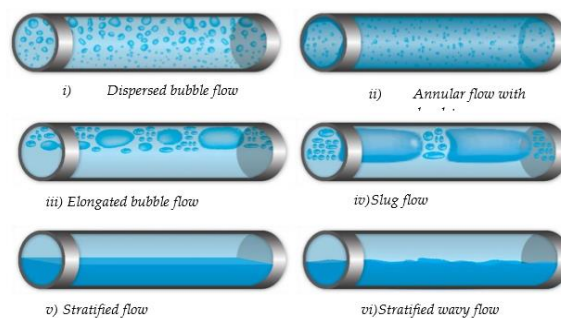
i) Dispersed bubble flow   ii) Annular flow with

iii) Elongated bubble flow   iv)Slug flow

v) Stratified flow   vi)Stratified wavy flow

Figure 1.5. Horizontal flow regimes (Bratland, 2013)

1.2.2.2   VERTICAL FLOW REGIMES

The vertical flow regimes are defined as follows.

1. **Bubble Flow:** The gas phase is distributed in the liquid phase as variable-size, deformable bubbles moving upward with zigzag motion. The wall of the pipe is always contacted by the liquid phase.

2. **Slug Flow**: Most of the gas is in the form of large bullet-shaped bubbles that have a diameter almost reaching the pipe diameter. These bubbles are referred to as "Taylor bubbles" move uniformly upward, and are separated by slugs of continuous liquid that bridge the pipe and contain small gas bubbles. The gas bubble velocity is greater than that of the liquid.

3. **Churn Flow**: If a change from a continuous liquid phase to a continuous gas phase occurs, the continuity of the liquid in the slug between successive Taylor bubbles is destroyed repeatedly by a high local gas concentration in the slug. This oscillatory flow of the liquid is typical of churn flow. It may not occur in small-diameter pipes. The gas bubbles may join and liquid may be entrained in the bubbles.

4. **Annular-Mist Flow**: Annular flow is characterized by the continuity of the gas phase in the pipe core. The liquid phase moves upward partly as a wavy film and partly in the form of drops entrained in the gas core.

1.2.2.3   PIPE INCLINATION EFFECTS (INCLINED FLOWS)

We observe that even though stratified regimes occur in horizontal flows, they are non-existent in vertical flows as the means of stratification will be impossible in this case. Intuitively, it would be expected that the stratified flows would gradually disappear as we make a transition from horizontal to vertical conditions. As such, we see the effect of pipe inclination which had been mentioned earlier as an important geometric variable in the formation of the flow regimes. It has been proven experimentally by (Shoham, 2006) that even a small change in the angle has a major effect in the transition from stratified flow to non-stratified flow. It has been observed that stratified flow is not seen in the experimental range of flow rates for upward inclinations higher than about 20°. However, stratified flow region is commonly observed up to -70° for downward flow (Chen Y. , 2001).

Generally, the flow regime in a near-horizontal pipe remains segregated for downward inclinations and changes to an intermittent flow regime for upward inclinations. An intermittent flow regime remains intermittent when tilted upward and tends to segregated flow pattern when inclined downward. The inclination should not however significantly affect the distributed flow regime.

1.2.2.4   FLOW PATTERN PREDICTION MODELS

Regardless of the fact that prediction models are not particularly explored in this work, previous work done ought to be mentioned as it does affect the understanding of flow patterns – a subject which qualitatively examined experimentally in this work. Over the years, models have been developed to understand better the flow pattern development and flow pattern transitions. These have been developed to enhance the proper prediction of multiphase phenomena at various conditions with respect to flow rates, pipe inclinations etc. (Kabir & Hasan, 1989) observes that prediction models for two phase flow phenomena fall into three basic categories:

1. **The Homogeneous model**: With this model, early researchers such as Wallis (1969) treated multiphase flow as a homogenous mixture of gas and liquid with averaged property values from the constituent phases. In other words, the model disregarded the fact that gas normally flows faster than liquid. Simply put, the model assumes no difference between the phase velocities. Models developed by (Hagedorn & Brown, 1965) and (Poettmann & Carpenter, 1952) are among examples of such models. The failure to account for slip between the phases limits the accuracy of these models in predicting the phenomena. In fact, (Brill J. P., 1999) observes that these models tend to under predict pressure because the prediction of the liquid volume is small. These have not found much use or application in the industry

2. **Separated model**: These models account for the slip between phases by the use of empirical liquid hold up correlations as well as empirical relations for the frictional interaction between the phases. These models are independent of the flow pattern or regime. Notable among these models are the (Beggs & Brill, 1973) correlation and (Mukherjee & Brill, 1983) correlation for horizontal flows. These models are of course based on extensive laboratory data.

   Regarding vertical flows, the (Lockhart & Martinelli, 1949) model was developed for vertical flows. In order to calculate the two-phase multiplier Lockhart and Martinelli defined the well-known Lockhart-Martinelli parameter X as the ratio of the liquid pressure drop to the gas pressure drop as below

$$X^2 = \frac{\left(\frac{dp}{dl}\right)_l}{\left(\frac{dp}{dl}\right)_g} \tag{1.32}$$

As such, it is then possible to make an estimation of the pressure drop using single-phase friction factors for each phase (Gschnaidtner, 2014)

3. **Flow pattern approach**: This method involves describing the flow pattern at sections of the pipe and applying the appropriate the right void fraction and pressure drop correlations for each flow pattern (Kabir & Hasan, 1989). For estimating the pressure drop and void fraction based on flow patterns, the flow pattern dependent model is used. The flow pattern model, also called mechanistic modelling approach, emerged in the early 80's. An example of such a model mechanistic model which is able to predict horizontal and near horizontal two phase flow was developed by (Taitel, Barnea, & Dukler, 1980). Flow pattern transitions are predicted analytically and it was shown every transition boundary can be represented by two dimensionless groups for each inclination angle (Barnea, Shoham, Taitel, & Dukler, 1979).
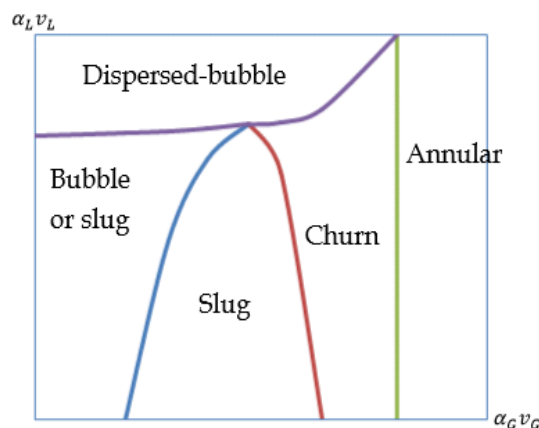

Figure 1.6. Flow pattern map for two phase vertical flow (Bratland, 2013)

1.2.2.5   TWO PHASE FLOW IN ANNULI

In the previous section effort has been made to describe the flow regimes and flow pattern maps for circular pipes. This is because the same flow regimes and similar flow pattern maps are observed with annulus geometries.

Several studies have been carried out to investigate two phase flow in annuli. Pressure drops and void fraction in horizontal air-water flow were studied by (Salcudean, Groeneveld, & Leung, 1983b) in several obstruction geometry were used, particularly including an annular geometry, and discovered that these geometries have a considerable effect on the phase distribution of slug and bubbly flow as well as the measured pressure drops.

(Osamusali & Chang, 1988) and (Osamusali, 1988) conducted theoretical and experimental studies on two-phase flow pattern and flow pattern transition in horizontal pipes and annulus geometries. (Osamusali, 1988) observed similar flow patterns for circular and annular pipe geometries by direct visual observation and by ring type capacitance transducers and presented the observed flow regimes in a flow pattern map. It was also observed that that the diameter ratio of the inner to the outer diameter has a considerable effect on the flow patterns observed as there was significant shift of the flow pattern transitions of two-phase flow through annuli when compared with circular pipe flow. The works by (Mendes, Rodriguez, Estevam, & Divonsir, 2013), (Osamusali, 1988), (Ekberg, Ghiaasiaan, Abdel-Khalik, Yoda, & Jester, 1999) and (Sunthankar, Kuru, Miska, & A, 2003) presented flow pattern maps for several annulus pipe geometries and observed marked differences in comparison with those observed with circular pipe geometries. (Mendes, Rodriguez, Estevam, & Divonsir, 2013) presented flow pattern maps for inclination angles of 45 and 90 degrees. They employed the use of a pressure signature technique to identify the flow pattern transitions.

In addition, (Caetano, Shoham, & Brill, 1992) studied single phase and two phase flow in an upward-vertical two-phase annular-duct flow. They examined the effect of eccentricity on the the flow patterns for air-water and air-kerosene mixtures. (Kelessidis & Dukler, 1989) worked on upward-vertical two-phase flow in an annular duct. Flow-patterns maps were shown as well as the transitions of flow patterns. . A model for transitions in upward vertical and inclined

annular duct flow was suggested by (Hasan & Kabir, 1992). They also studied the influence of inclination on flow patterns, however the model was validated only with data of upward-vertical flow. (Ekberg, Ghiaasiaan, Abdel-Khalik, Yoda, & Jester, 1999) used annulus geometries with narrow gaps for air-water flow pattern classifications. (Wongwises & Pipathattakul, 2006) also examined the effect of inclinations on flow regimes developed in annulus sections with annulus geometries with very small areas. .

(Ozbayoglu & Omurlu, 2007) carried out experimental and theoretical studies about two-phase flow through concentric annuli. A new mathematical model which is based on a new equivalent diameter concept to predict flow pattern and frictional pressure loss was used in their work. They compared their model with experimental studies and found it to be in good agreement. In addition, they also observed that flow pattern transition and the frictional pressure losses were considerably affected by the size of the duct and the viscosity of the liquid.

(Osgouei, Ozbayoglu, & Ozbayoglu, 2012) and (Osgouei R. E., 2010) proposed another classification of the two-phase flow pattern. Flow pattern maps were presented, in addition to using a special technique to describe the flow pattern transitions. By applying the flow pattern models by (Beggs & Brill, 1973) and (Taitel, Barnea, & Dukler, 1980) they concluded that they could not adequately predict flow pattern transitions as they employed the hydraulic diameter concept (Gschnaidtner, 2014).

Sorgun et al. (2011; 2013) made comparisons between experimental data and a computational fluid dynamics model for two phase flow patterns in annulus geometries. They observed that their computational fluid dynamics model can predict flow patterns and pressure losses with reasonable accuracy (Gschnaidtner, 2014).They also observed slightly higher pressure loss for concentric annular geometries compared to fully eccentric annulus geometries.

This work examines these flow patterns in annulus geometries and how the effect of pipe inclinations and viscosity of the fluid could affect the flow pattern regions and transitions.

Table 1-1. Equivalent diameter relations used for evaluation in this research work

NAME	AUTHOR(S)	EQUATION
Hydraulic		$$D_H = d_2 - d_1$$
Lamb	(Lamb, 1907)	$$d_H = \sqrt{d_2{}^2 + d_1{}^2 - \frac{\left(d_2{}^2 - d_1{}^2\right)}{\ln\left(\frac{d_2}{d_1}\right)}}$$
Knudsen	(Knudsen & Katz, Fluid Dynamics and Heat Transfer, 1958),	$$d_H = \frac{(d_2 - d_1)\left[d_2{}^2 + d_1{}^2 - \frac{\left(d_2{}^2 - d_1{}^2\right)}{ln(d_2/d_1)}\right]}{(d_2 - d_1)^2}$$
Prengle	(Prengle & Rofthus, 1955)	$$d_H = 2\frac{(r_2{}^2 - r_{max}{}^2)}{r_2}; \quad r_{max} = \frac{r_2{}^2 - r_1{}^2}{2\ln\frac{r_2}{r_1}}$$
Ozbayoglu	(Ozbayoglu & Omurlu, 2007)	$$d_H = (d_2 - d_1)\left[0.9983 - 1.4472\left(\frac{d_1}{d_2}\right)\right.$$ $$+ 7.2649\left(\frac{d_1}{d_2}\right)^2$$ $$- 16.833\left(\frac{d_1}{d_2}\right)^3$$ $$\left. + 13.564\left(\frac{d_1}{d_2}\right)^4\right]$$
Crittendon	(Crittendon, 1959)	$$d_H = \frac{1}{2}\left[\sqrt[4]{d_2{}^2 - d_1{}^2 - \frac{\left(d_2{}^2 - d_1{}^2\right)}{\ln\left(\frac{d_2}{d_1}\right)}}\right.$$ $$\left. + \sqrt{d_2{}^2 - d_1{}^2}\right]$$
Omurlu	(Omurlu, 2006)	$$d_H = \sqrt{d_2{}^2 - d_1{}^2}$$

2.   EXPERIMENTAL FACILITY AND SET UP

The experiment was carried out at the Multiphase Laboratory at the Department of Energy and Process at NTNU in a newly set up annulus test section. The experimental set up stands behind the existing horizontal loop in the Multiphase laboratory

The inclinable vertical loop which is fed with multiphase mixture of gas and liquid, consists primarily of

1. A long supporting beam to which the test section is to be supported on
2. A test section of solid pipe in an outer pipe to form the desired annulus geometry
3. A mixing section, which helps to couple the separate phases of gas and liquid together before entry into the test section
4. A separator, which separates the different phases after passing through the section in order to have the phases recycled again through the loop
5. A pressure transducer attached to the test section
6. A long flexible hose which joins the mixing section to the test section to allow for the development of the flow before entry into the test section

The test rig built consists of inner and outer pipe. This set up is mounted on the supporting beam assembly whose inclination can be changed as desired by the push of a button on a relay device. The inclinations can be changed from 0 – 75 degrees to the horizontal. Air, water and oil can be fed from the existing test section by the flows from the existing facilities in the laboratory into the mixing section. There is a flexible hose that runs in a loop on the ground delivering single or two phase flow from the mixing section to the test section.

2.1.   TEST SECTION

As mentioned earlier, the length of the test section is 6m with the annulus section having an outer pipe with inner diameter of 90mm and inner rod/pipe having an outer diameter of 36mm.

In building the test section, it was necessary to come up with a smart way of having a small pipe in the larger pipe to form the annulus.

The inner rod has 3 sections of rods of Lexan coupled together to from the required 6m inner rod. Coupling the sections together was done by drilling threaded holes in one 2m section to be fitted with a threaded butt from a successive 2m section. A model of how this is done is shown in *Figure 2.1*. The proposed model is to have 'spikes' in the form of screws at every 30 cm of the 2m section of the entire length of the inner rod. Three long screws are used at each axial positon with the screw heads resting on the inner wall of the Acrylic outside pipe providing stability. For the outer pipe, the lengths of Acrylic pipe were available in 1m sections and had to be joined in a proper way to achieve the required 6m length of test section and to also prevent leakages during the experiments. The 1 meter sections are joined with special connections as shown in the picture in *Figure 2.3*. The Acrylic outer pipe is then mounted on the Lattix beam by having them sit between half circle holders that are bolted to the body of the Lattix beam. This is also seen in *Figure 2.3*. These holders or mounts are at every 50 cm of the test section to ensure proper support of the latter.
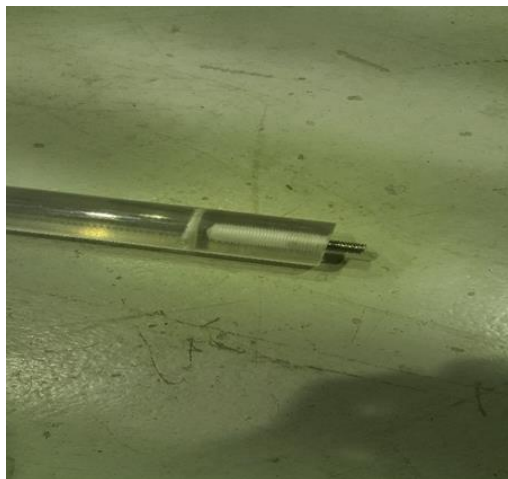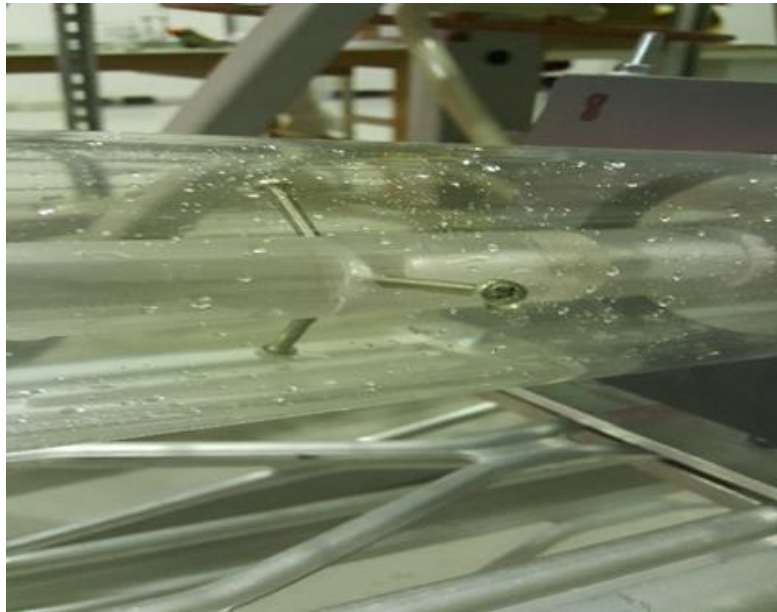

Figure 2.1. Lexan rod for the inner rod
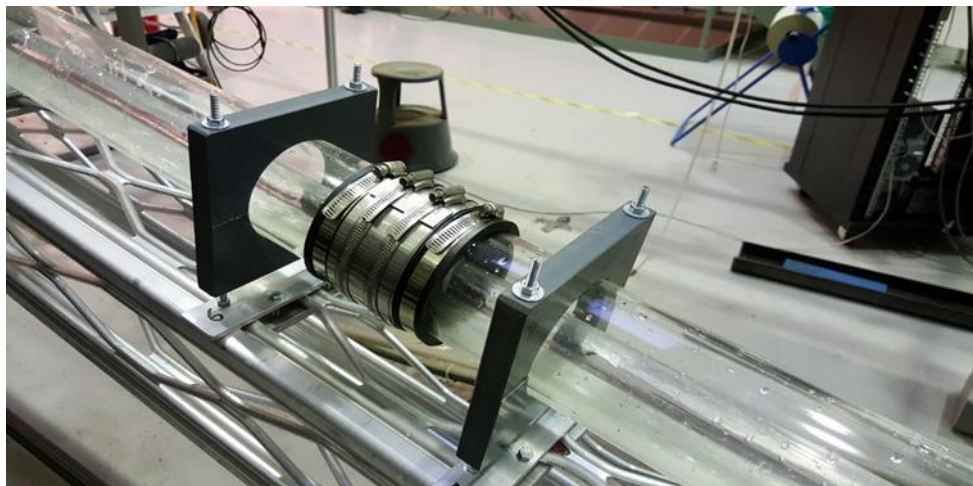
Figure 2.2. Internal of annulus test section

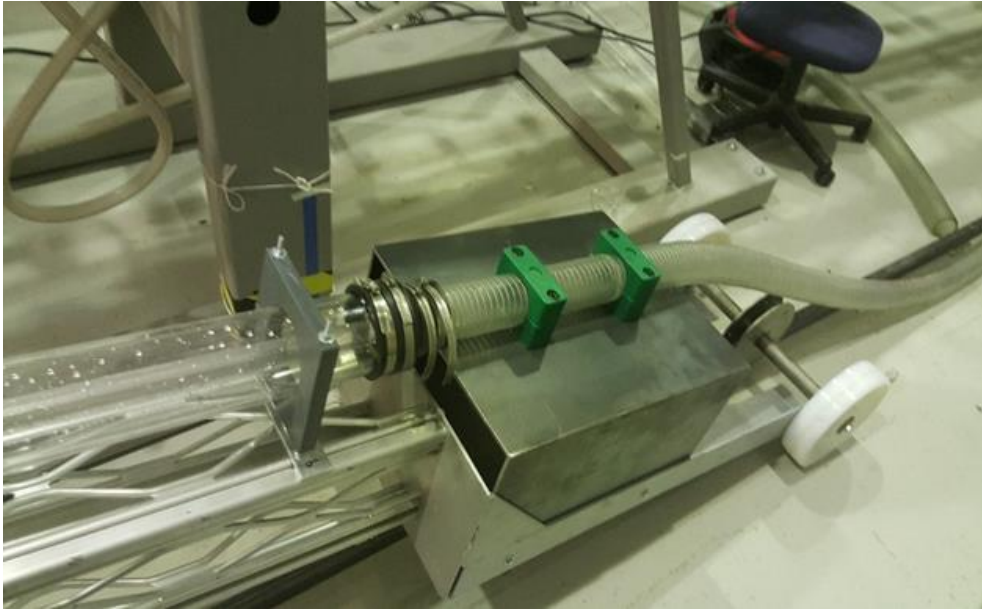*Figure 2.3. Test section with connections and supports*

Figure 2.4. Inlet to test section and trolley

Figure 2.5. Control device for changing inclinations; Support beam for test section

Attached to the system are several pressure transducers, control valves, pneumatic valves and pumps which are connected to a DAQ system and controlled through a LABVIEW program.

AIR FLOW LINE

It is supplied from the central line of the laboratory at 7 bara and is reduced to 4bara in the fluid feed line and then is reduced again by a control valve before flowing into the buffer tank. The flow rates of air are measured by means of a Coriolis flow meter before the control valve.

WATER FLOW LINE

Water is stored in an oil-water separator and supplied to the water line by centrifugal pumps. The liquid flow is then controlled through a control valve and the flow rates are measured by means of electromagnetic flow meters before the control valve.

OIL LINE

Very similar to the water line, oil is fed from the same oil-water separator and is fed to the oil line by centrifugal pumps. Included in the arrangement is also a control valve for regulating the amount of flow and the Coriolis meters for the flow rate measurements.

Figure 2.6. Mixing section

The measurement and observation area of the test section is located downstream from the inlet section. The measurement section includes a Fuji Electric France S.A.S. ® differential pressure transducer with an uncertainty of 0.16% of the range installed in order to measure frictional pressure losses and the average pressure inside the loop. No temperature control devices were installed as we make the relatively valid assumption that there is not much expectation with regards to ambient temperature change or change in temperature in the test section. The first line from the pressure transducer is placed at an acceptable distance from the inlet section to ensure the necessary entry length to ensure fully developed flow in the test section. The distance between the points on the test section connected to the pressure transducer is 1.73 m.

Figure 2.7. Differential pressure transducer

Figure 2.8. Rheometer for measuring   viscosity of liquid

Figure 2.9. Analytical Mass Balance

The properties of the fluids used for this experimental study are measured with apparatus in the lab. Viscosity and density of oil and water are measured using the Rheometer and Analytical mass balance. The fluid properties are given in the table below.

Table 2-1. Fluid Properties

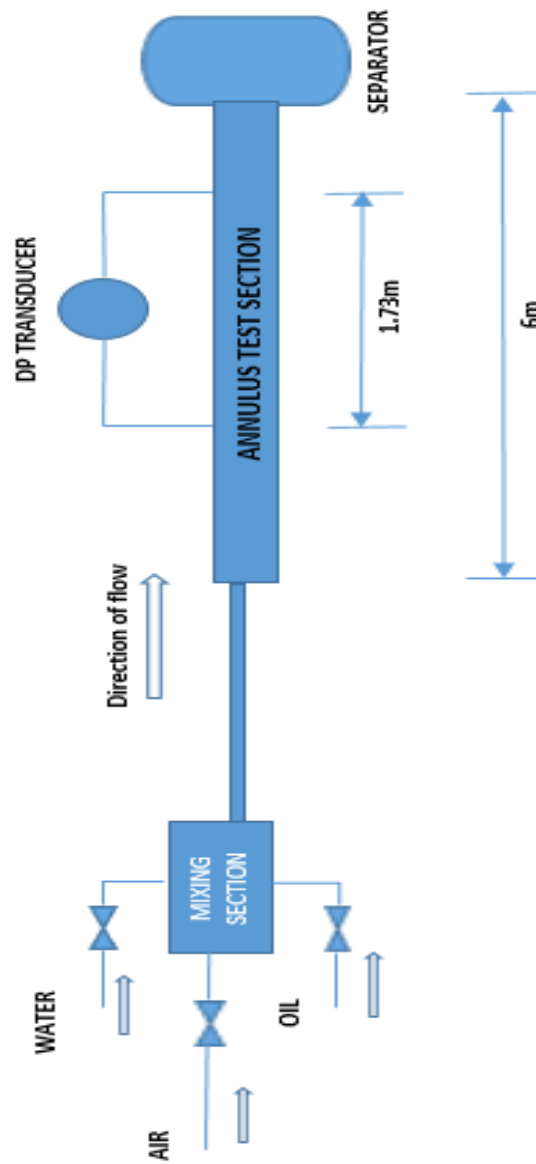| Fluid	Density (kg/m3)	Viscosity (cP)
Air	1.22	0.018
Water	1000	1.0
Oil	822	21

Figure 2.10. Schematic of Test section

Figure 2.11. The inclinable test rig with the annulus test section in operation

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3. EXPERIMENTAL STUDY OF SINGLE PHASE AND TWO PHASE FLOW IN ANNULI

As it has been mentioned earlier, this research involves experimental work which has been carried out to investigate the understanding of single phase and two phase flow in annuli. The experiments were carried out with the intention to make validating comparisons with existing models. For the single phase experiments, oil and water were the fluids used for investigation within the laminar and turbulent flow regions respectively. For two phase flow, experiments were carried out with air-oil and air-water and flow patterns were duly observed.

FLUID PROPERTIES

All experiments are performed at oil and water temperatures of about 15℃.

WATER

The water used in these experiments has a density of about 998kg/m^3 and viscosity of 1cP at 15℃. A green pigment is added to water for better visualization, particularly in two phase flow.

OIL

The oil used in these experiments (NEXBASE 8050) has a density of 822kg/m^3 and viscosity of 21cP at 15℃. The viscosity is measured by the use of TA Instruments® rheometer.

3.1. SINGLE PHASE FLOW EXPERIMENTS

In this study, single phase experiments were performed in the annulus test section with oil and water whose properties are as below. Flow rates and the corresponding pressure drop measurements are recorded by logging data through the National Instruments LabView program interface. The pressure drop measurements are the differential pressure readings from the tappings on the test section which are 1.73 m. The roughness of the pipe, ε, considered for

use in this work for turbulent flow calculations is $1.5 \times 10^{-6} m$ based on the roughness for drawn tubes.

In performing the single phase experiments, the inclinable test section is setting in place to a horizontal position by propping up the test section by the use of a fork lift. The position of the first tapping is placed such that it meets the requirement for the entry length to ensure fully developed flow. The entry length for turbulent flow is meant to be approximately a length of 50D and its value is about 3m from the inlet to the test section. The experiments are done for the Laminar flow region with oil and the Turbulent flow region with water.

The limitation on the capacity of the oil and water pumps made it inadequate to perform experiments to examine the Transition region between Laminar and Turbulent flow. The experimental data gathered are subsequently analyzed in the following chapter. When performing the single phase flow experiments, care was taken to ensure that gas bubbles were not trapped in the connecting hoses between the differential pressure transducer and test section.

The range of flow rates for single phase oil and single phase water experiments are given in the table below

Table 3-1. Range of Flow rates for Oil and Water

FLUID	RANGE OF FLOW RATES
OIL	4000 – 10040 kg/h
WATER	4.00– 10.00 l/s

3.2. TWO PHASE FLOW EXPERIMENTS

Air-water and air-oil mixtures are used for the two phase experimental investigation. Flow regimes are observed for both set of mixtures for various angles of inclinations and flow pattern maps are plotted. As the single phase flow, flow rates for the two phase flow are controlled through the opening interface of the National Instruments LabView program for controlling valves, pumps and compressors.

The experimental procedure for the two phase experiments is as follows:

a) The test section is set to the required angle by means of a Bosch angle meter for the set of experimental runs

b) The centrifugal pumps are controlled at the LabView interface to pump liquid (oil or water) into the test section with the valves also adjusted to the desired flow rate

c) Subsequently, the air is let in from the air flow line by the control valve through the PID controller and the air flow rate is adjusted to the desired value

d) After the two phase flow has reached some state of stability, the flow regime is observed and noted against the gas and liquid flow rates. Usually, the waiting period to achieve stabilization is between 2 – 3 minutes.

e) Pictures of the flow regimes are taken as well as videos are recorded for visual observations

f) Following all the experimental runs for a particular angle, the angle for the test section is reset by means of the Bosch meter and experimental runs are performed for the new set angle by repeating steps (b) – (e).

g) After this has been done for all the angles (15, 30, 45 and 60 degrees) for air-water mixture, all of the above is then repeated for the two phase air-oil.

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The initial intention of the experimental investigation was to observe flow patterns and plot flow pattern maps for various angles ranging from 0 – 90 degrees to the horizontal as well as pressure drop measurements for various flow regimes. However, the limitations regarding the range of the pressure transducer ruled out of the possibility carrying out two phase pressure drop measurements for this work, particularly for inclined situations. The gravitational portion of the pressure drop causes the pressure drop to exceed the range of the existing differential pressure transducers. Consequently, the pressure drop measurements can be investigated in further research work if pressure transducers with a wider range is installed. The vibration of the experimental set up when running high flow rates of gas and liquid also set a limitation for the flow rates that could be used for this study.

Also, in the investigation of flow regimes, the observation for 0 degrees (fully horizontal) is considered inaccurate. With the present existing inclinable test section, the only way to obtain the test section fully horizontal is to lift its inlet end above ground by the use of a forklift. The forklift is kept in position for this horizontal situation with the test section over 1.5m above ground. This creates a riser slugging situation most of the time as the two phase flow enters the test section through the hose rising from the ground. Hence, even with stratified flow upstream of the 'riser' at certain gas and liquid superficial velocities, it is impossible to ever obtain stratified flow in the test section with those superficial velocities. As such, the validity of flow pattern maps or pressure drop measurements for two phase flow for 0 degrees in this set-up is indeed questionable. Hence, the flow pattern maps for two phase air-water and air-oil for 0 degrees are not considered or presented in this work.

Table 3-2. Test Matrix for Air-Water Two Phase Flow

Water flow rate (l/s)	Air flow rate (kg/h)
2.66	2.58 - 29.8
3.5	2.58 - 24.59
4.89	2.58 - 24.60
6	2.58 - 24.61
7.12	2.58 - 24.62
7.67	2.58 – 24.62
7.81	2.58-12.4
7.92	2.58- 12.4

Table 3-3. Test Matrix for Air-Oil Two Phase Flow

Oil flow rate (kg/s)	Air flow rate (kg/h)
2.09	2.58 - 29.8
2.70	2.58 - 22.39
3.91	2.58 - 22.39
4.46	2.58 - 22.36
5.11	2.58 - 22.37
5.81	2.58 - 12.41
6.42	2.58-10.93
6.51	2.58-10.94

4. RESULTS AND DATA ANALYSIS

In this section, the results from the experiments performed in the annulus test section are presented. The results are presented for single phase and two phase experiments. For single phase flow, several correlations for equivalent diameters are evaluated and measured single phase friction factor is also presented. The results obtained are also compared with previous works. For two phase flow, flow regime maps for air-water and air-oil mixtures are presented.

4.1. SINGLE PHASE EXPERIMENTAL RESULTS

4.1.1. LAMINAR FLOW

For the single phase experiments in the annulus test section with oil, the measured experimental drop with corresponding mass flow rates are shown in the graph below. The predicted pressure drop values based on the friction factor for Laminar flow and the various diameter concepts are also shown in *Figure 4.1*. We observe that, as expected from the pressured drop equation, measured pressure drop increases with the mass flow rate of oil. Looking further, it is observed that the best performing diameter concept for the single phase experiments with oil (Laminar flow) is the Knudsen diameter. Even though the pressure drop calculated with Knudsen is under predicted, the under-prediction appears small when compared with the performance of other diameter concepts. The Hydraulic diameter does not seem to perform very well as expected based on the predicted pressure drop calculated with the use of the hydraulic diameter because of considerable under prediction of the pressure drop. The Omurlu diameter performs the least resulting in very high under prediction of the pressure drop when compared with experimental data.
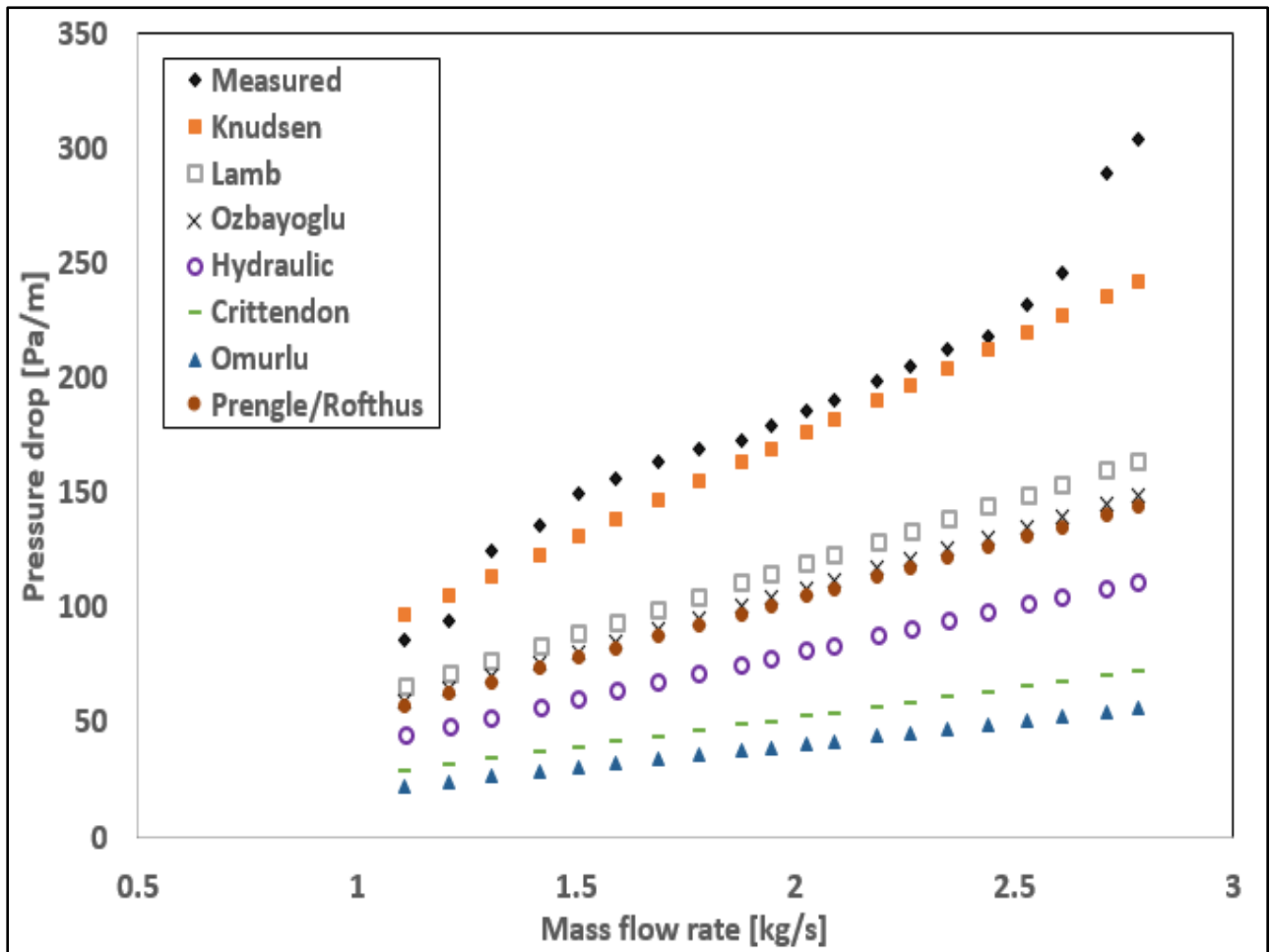
Figure 4.1. Pressure drop of single phase with the mass flow rate of oil in the annulus test section

The measured pressure drop per unit length is also plotted against the predicted pressure drops based on all diameter concepts which are considered for evaluation in this work. The graph is shown in *Figure 4.2*. From calculations and plots, we clearly see the good performance of the Knudsen diameter. The predicted pressure drop based on the Knudsen diameter is found to have an Average error of about -6% when compared with experiment. The calculation for ARE is given as

$$ARE = \left(\frac{\left(\frac{dP}{dL} \right)_{model} - \left(\frac{dP}{dL} \right)_{exp}}{\left(\frac{dP}{dL} \right)_{exp}} \right) \times 100 \qquad (4.1)$$

As predicted by theory which suggests that the Hydraulic diameter is unsuitable for accurate pressure drop predictions in annulus geometries, the predicted pressure drop based on the Hydraulic diameter has an ARE of almost -60%. The performance of other diameter concepts are also shown in the graph.

Table 4-1 below shows the values of each diameter concept based on the values of the outer diameter of the inner rod, d_1 which is 30 mm and the inner diameter of the outer pipe diameter, d_2 which is 90 mm. The Average Relative Errors of the predicted pressure drops based on each diameter are also presented. We observe clearly that ARE% increases with increasing value of the diameters.

Table 4-1. ARE% of Diameter Concepts for Laminar Flow.

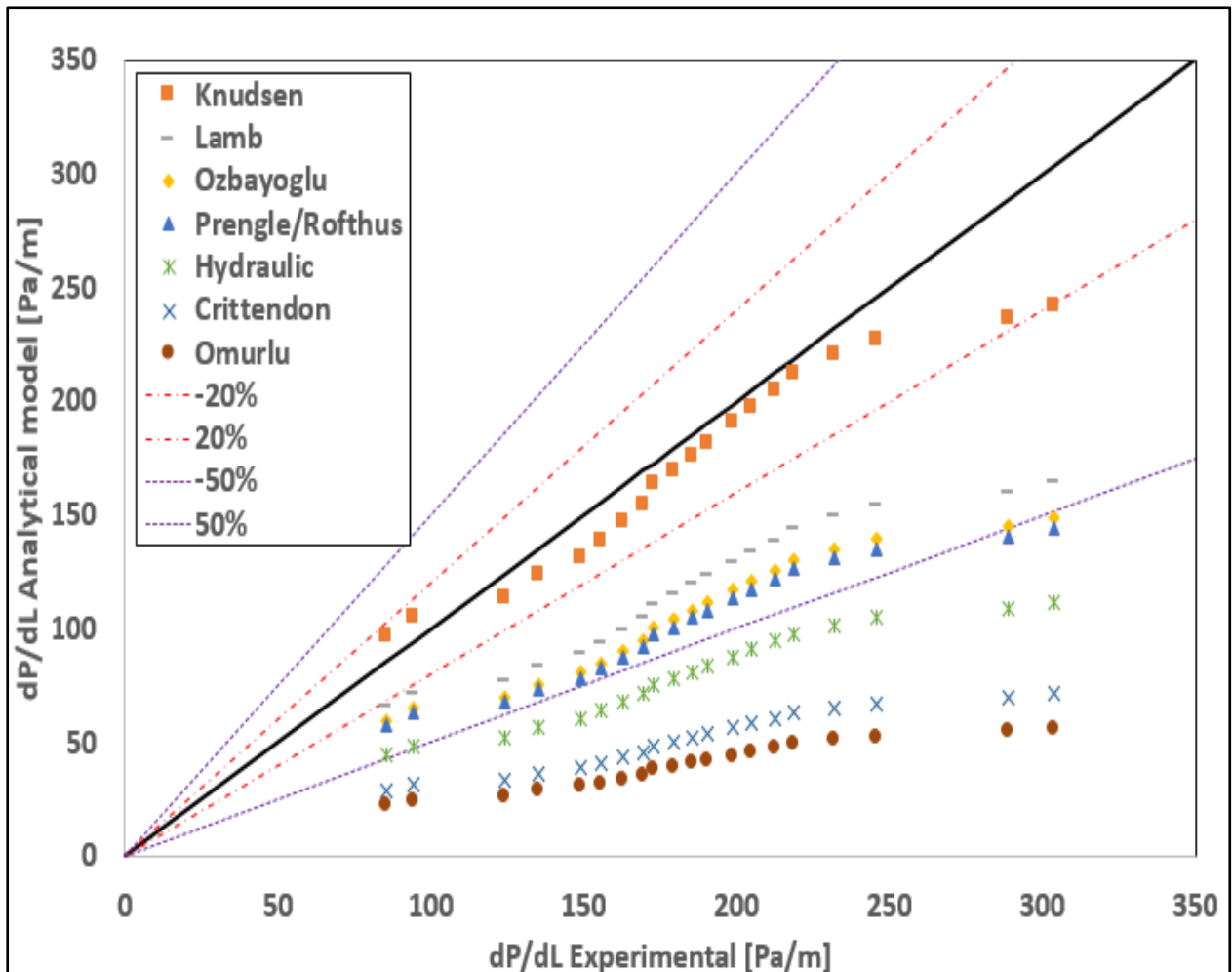| Diameters	Values (m)	Error%
Knudsen	0.04077	-6.15
Lamb	0.04946	-36.23
Ozbayoglu	0.05203	-42.37
Prengle/Rofthus	0.05359	-44.05
Hydraulic	0.06000	-56.66
Crittendon	0.07482	-72.13
Omurlu	0.08485	-78.33

Figure 4.2. Pressure drop of single phase oil in the annulus test section

4.1.2. TURBULENT FLOW

For the single phase experiments in the annulus test section with water, the measured experimental drop with corresponding volumetric flow rates are shown in the graph below. The predicted pressure drop values based on the Haaland friction factor correlation and the various diameter concepts are also shown in *Figure 4.3*. We see that, as expected from the pressured drop equation, the measured pressure drop increases with the volumetric flow rate of water. The best performing diameter concept for the single phase experiments with water, which is in the turbulent flow region, is the Knudsen diameter. The pressured drop based on the Knudsen

diameter is however slightly over predicted. The Hydraulic diameter does not seem to perform very well as expected based on the predicted pressure drop calculated with the use of the hydraulic diameter because of considerable under prediction of the pressure drop. The Omurlu diameter performs the least resulting in very high under prediction of the pressure drop when compared with experimental data.

The measured pressure drop per unit length is also plotted against the predicted pressure drops based on all diameter concepts which are considered for evaluation in this work. The graph is shown in *Figure 4.4*. From calculations and plots, we clearly see the good performance of the Knudsen diameter. The predicted pressure drop based on the Knudsen diameter is found to have an over prediction with Average error of about 5% when compared with experiment. As predicted by theory which suggests that the Hydraulic diameter is unsuitable for accurate pressure drop predictions in annulus geometries, the predicted pressure drop based on the Hydraulic diameter has an ARE of almost -35%. The performance of other diameter concepts are also shown in the graph.

A table is also presented below to show the values of each diameter concept based on the values of the outer diameter of the inner rod, d_1 which is 30 mm and the inner diameter of the outer pipe diameter, d_2 which is 90 mm. The Average Relative Errors of the predicted pressure drops based on each diameter are also presented. We observe clearly that ARE% increases with increasing value of the diameters just as was observed for laminar flow.
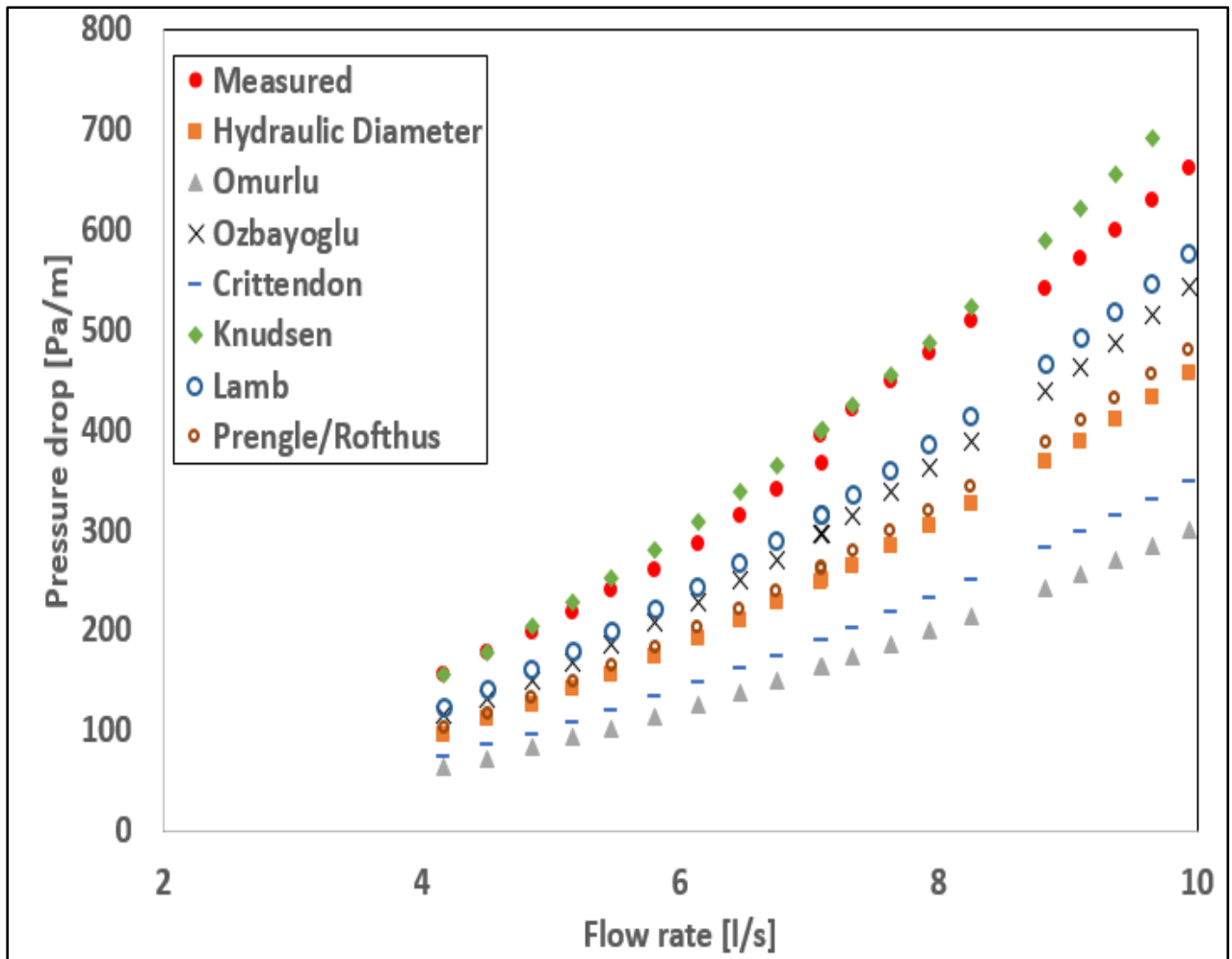
Figure 4.3. Pressure drop of single phase flow with the mass flow rate of water in the annulus test section
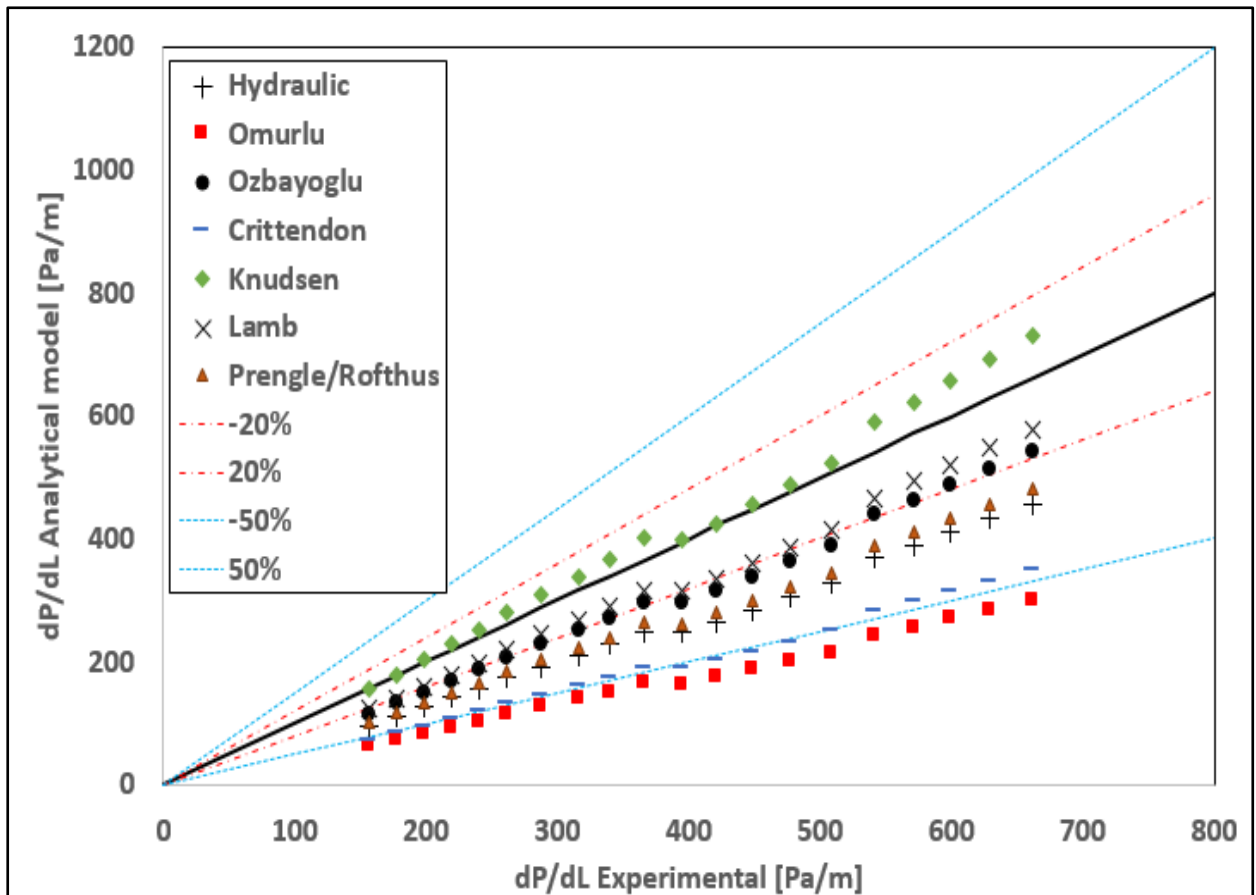
Figure 4.4. Pressure drop of single phase water in an annulus test section

Table 4-2. ARE% of Diameter Concepts for Turbulent Flow.

Diameters	Values (m)	Error %
Knudsen	0.04077	5.1
Lamb	0.04946	-16.8
Ozbayoglu	0.05203	-21.83
Prengle/Rofthus	0.05750	-30.8
Hydraulic	0.06000	-34.3
Crittendon	0.07482	-49.74
Omurlu	0.08485	-56.84

Following all these, friction factor is plotted against the Reynolds number to generate a chart similar to a Moody Chart. The measured friction factor and predicted friction factor are both plotted against Reynolds number. The measured friction factor is obtained from the pressure drop equation by using the pressure drop and flow rate measurements with the Knudsen diameter as depicted in *Equation 1.23* while the predicted friction factor is obtained by using the Haaland friction factor correlation with a Reynolds number based on the Knudsen diameter. We see from the graph that the values for the predicted friction factor and measured friction factor are in good agreement. It is observed however that for Laminar flow that measured friction factor is a slightly higher than that predicted by the model while it is slightly lower than that predicted for Turbulent flow. A possible explanation for this disparity could be the effect of viscosity of the different fluids used for the investigation of the laminar and turbulent flow regions.
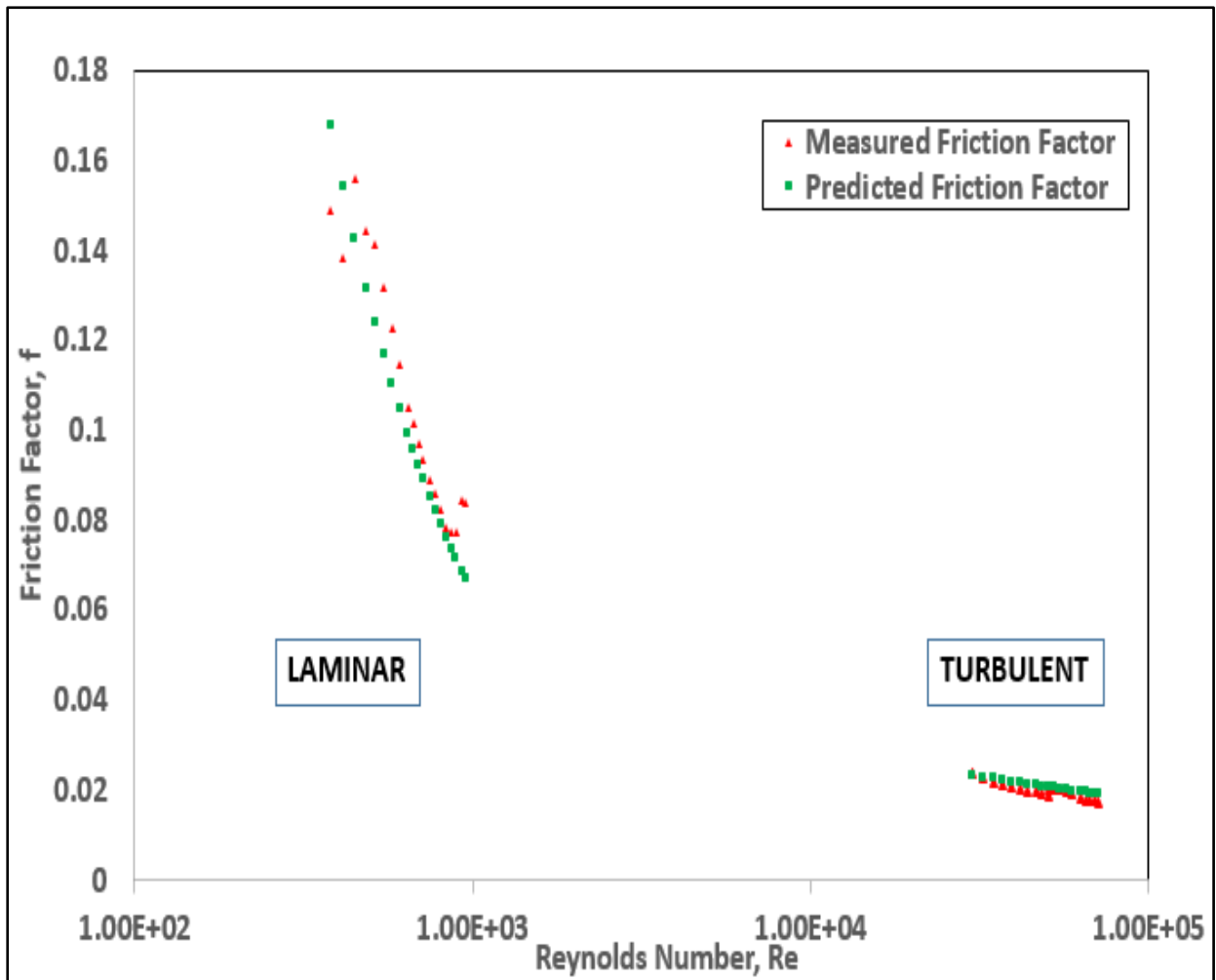
Figure 4.5. Friction factor model performance for annulus test section

4.1.3. COMPARISON WITH OTHER WORKS

The results from this work affirm that the concept of the hydraulic diameter is inadequate to model pressure drop, in the annulus test set-up and hence unable to capture properly essential fluid phenomena in annulus geometries with an average relative error of pressure drop prediction of -56% and -34% for laminar and turbulent flow respectively.

(Colmanetti, H, Souza De Castro, Kjedlby, & Derks, 2015) also confirm in their work, which was investigation for only turbulent flow, that the hydraulic diameter is inadequate based on the experimental investigation with an average relative error of pressure drop prediction above -40%. It must however be mentioned that size of gap in their annulus set-up is larger compared to this work with a hydraulic diameter of 95 mm. Their experiments were also performed in a vertical annulus set-up. The work by (Caetano, Shoham, & Brill, 1992), which investigated laminar and turbulent flow with a vertical concentric annulus test section of hydraulic diameter of 34 mm, also established that the hydraulic diameter was not sufficient for flow in annulus geometries, particularly for laminar flow.

However, (Gschnaidtner, 2014) suggests otherwise. Based on his experimental work for turbulent flow, he concludes that the hydraulic diameter is sufficient for annulus geometries with an average relative error of pressure prediction of less than -10%. This appears to be in contrast with most theoretical postulations that suggests the inadequacy of the hydraulic diameter for annulus geometries. However, the experimental set up for his work has a much smaller hydraulic diameter of 18.2mm which is half of that of the work by (Caetano, Shoham, & Brill, 1992) and much less than that used in this work and the work by (Colmanetti, H, Souza De Castro, Kjedlby, & Derks, 2015). The flow area is also very small which leads to very high pressure drops based on the high flow velocities in the annulus test section. Whether this affects the results obtained his work, however, remains to be seen or understood. Indeed, we see that from all the works that there seems to be a better ARE % with reducing hydraulic diameter. In other words, the smaller the size of the duct and hydraulic diameter, the better the performance of the hydraulic diameter concept. This is yet to be proven and can be investigated for other annulus geometries with different hydraulic diameter.

Interestingly, it is established in this work that the Knudsen diameter performs well for prediction of pressure drop and friction factor for laminar and turbulent flow in the annulus test section used for this research. It is in good agreement with the work of (Caetano, Shoham, & Brill, 1992), as we had established earlier in this work that the geometry factor used in their work as shown in *Equation 1.20* is the same as using the Knudsen diameter in place of the hydraulic diameter.

Likewise, this results from this research work is in good agreement with the work of (Colmanetti, H, Souza De Castro, Kjedlby, & Derks, 2015) which establishes that the use of the an equivalent diameter by Sas-Jaworsky, a direct equivalent of the Knudsen diameter, is adequate for prediction of pressure drop in annulus geometries

4.2. TWO PHASE FLOW EXPERIMENTAL RESULTS

For the two phase flow experimental investigation in this research work, flow pattern data were gathered for air-water and air-oil mixtures in an annulus test section using the existing concentric annulus test section in the Multiphase Laboratory. Based on established definitions for two phase flow regimes, flow patterns were observed and visually determined. The results from these observations are presented as flow pattern maps with the superficial gas and liquid velocities as the coordinates for the maps. Samples of the flow regimes observed for both air-water and air-oil are shown in the Appendix.

4.2.1. AIR-WATER EXPERIMENTS

The flow pattern maps for air-water two phase flows for test section angles of 15, 30, 45 and 60 degrees to the horizontal are presented in *Figure 4.6* to *Figure 4.9*. In all the cases of the angles, dispersed bubble flow is observed above a water superficial velocity of 1m/s with the least superficial air velocity of about 0.08m/s. It is however observed that the transitions from slug flow to churn flow starts earlier with increasing pipe inclination.

For an inclination of 15 degrees, the transition from slug to churn occurs at about a gas superficial velocity of 0.8 m/s while it occurs at about 0.5 m/s for an inclination of 60 degrees. This is a direct consequence of the situation that the slug region on the flow pattern map also appears to reduce with increasing inclinations. The transition from bubble to slug are also different for all the inclinations. Evidently, the angle inclinations have a considerable effect on the flow patterns developed and observed in the test section as well as the transition from one flow regime to another.

Dispersed bubbles are observed in all cases of inclinations at a superficial velocity of water about 1.4 m/s. This is plausible based on the requirement that the high flow rates of liquid will be required to disperse the gas flow. For the works by (Kelessidis & Dukler, 1989) and (Caetano, Shoham, & Brill, 1992), they observed that the dispersed bubble flow occurred at superficial velocity of 0.7 m/s and 1m/s respectively for fully vertical angles. (Kelessidis & Dukler, 1989) worked with a hydraulic diameter of 25.2 mm while (Caetano, Shoham, & Brill, 1992) worked with a hydraulic diameter of 34 mm. Based on this, (Caetano, Shoham, & Brill, 1992) concluded that the effect of hydraulic diameter, which refers to the size of the duct is an important parameter regarding the formation of the dispersed flow regime and generally flow regimes (Mendes, Rodriguez, Estevam, & Divonsir, 2013). Considering that the hydraulic diameter used in this work is 60 mm and the dispersed bubble flow is observed at superficial velocities of water as high as 1.5 m/s, it can be safely assumed that the size of the duct is an important parameter in flow pattern observation and transitions as duly observed and concluded by (Caetano, Shoham, & Brill, 1992).
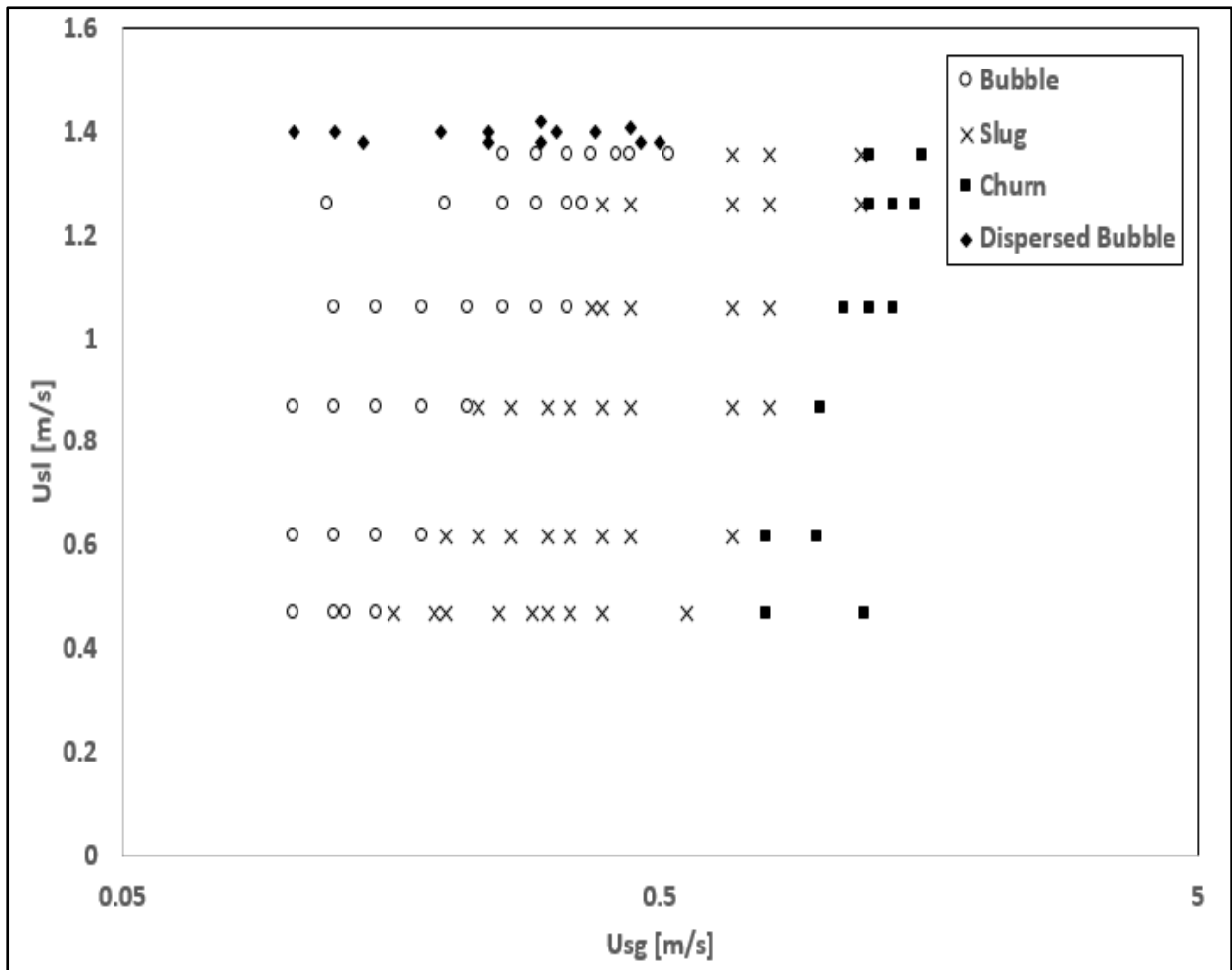
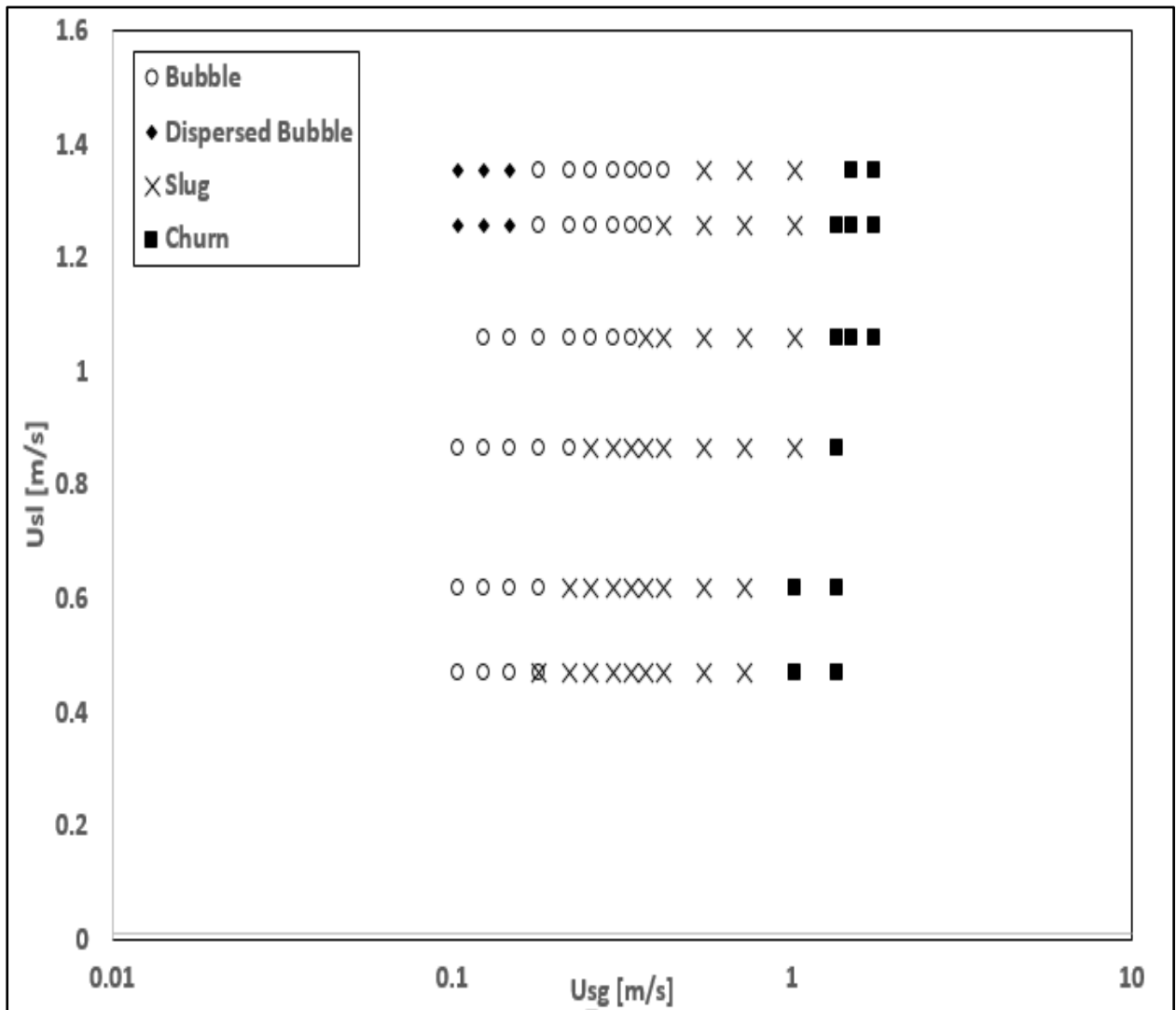*Figure 4.6. Air-water flow pattern map for inclination of 15 degrees*

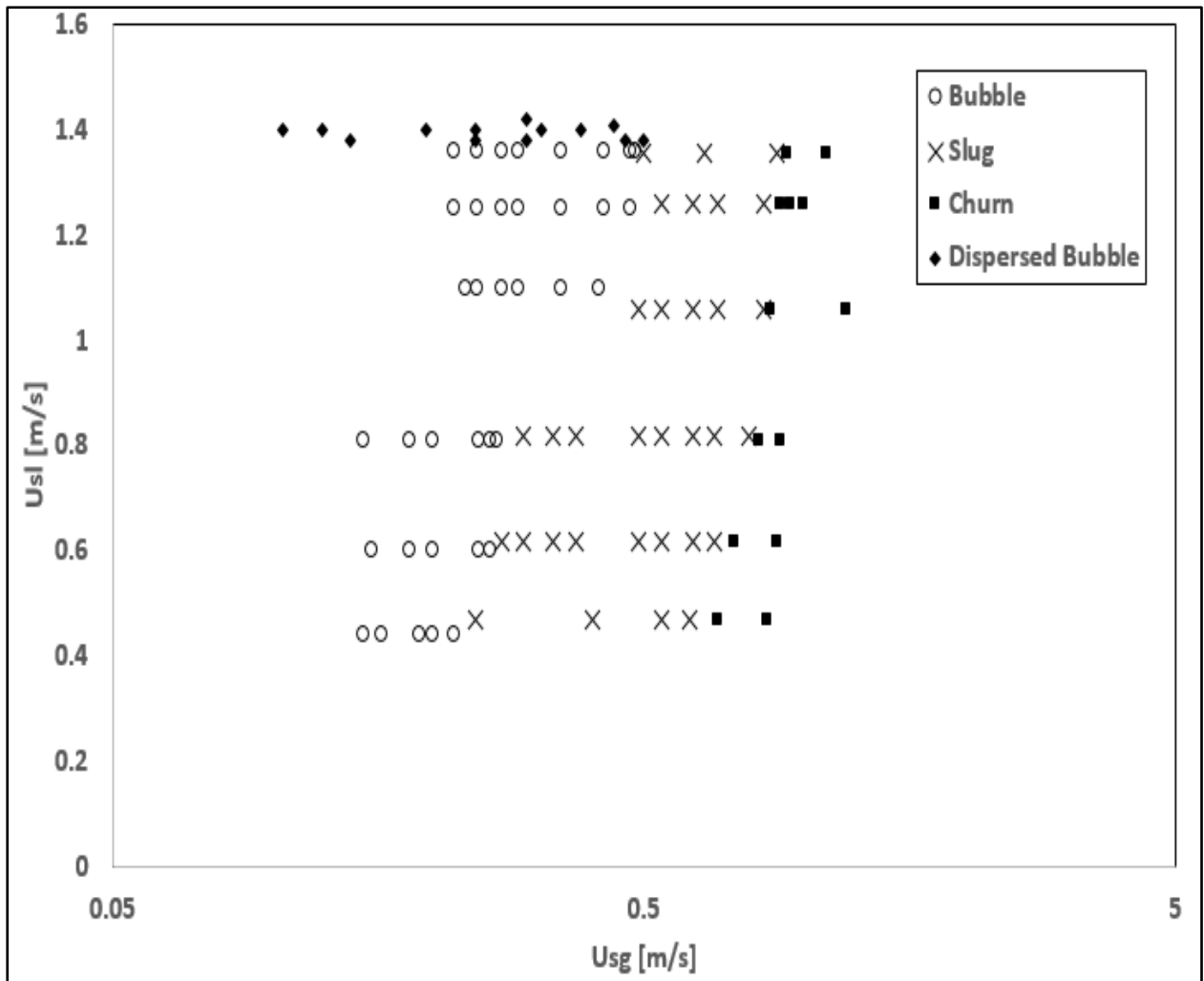*Figure 4.7. Air-water flow pattern map for inclination of 30 degrees*

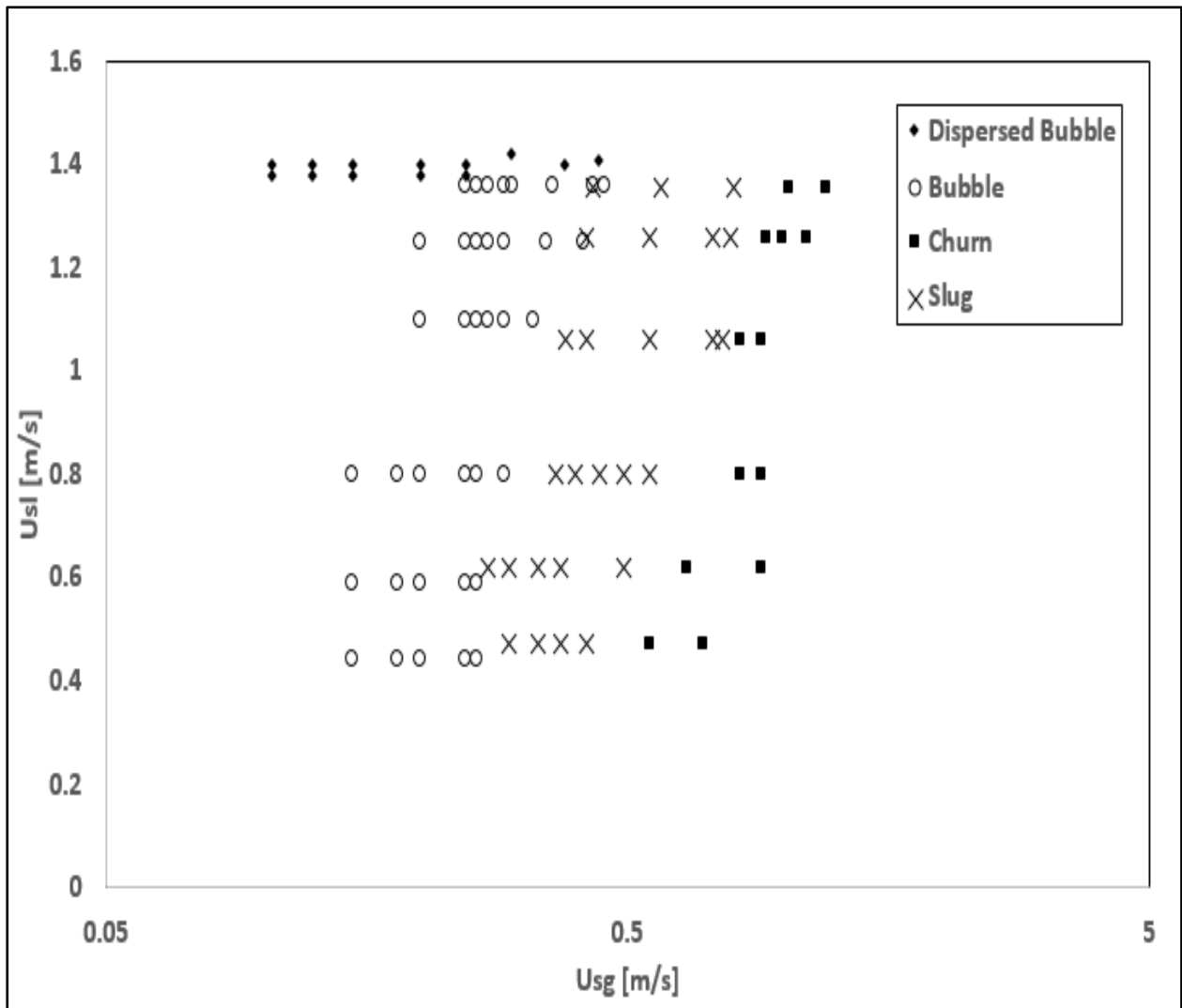*Figure 4.8. Air-water flow pattern map for inclination of 45 degrees*

Figure 4.9. Air-water flow pattern map for inclination of 60 degrees

4.2.2.AIR-OIL EXPERIMENTS

The flow pattern maps for air-oil two phase flows for test section angles of 15, 30, 45 and 60 degrees to the horizontal are shown in *Figure 4.10* to *Figure 4.13*. In all the cases of the angles, dispersed bubble flow is observed above a water superficial velocity of 1m/s just as with air-water two phase flow.

When the flow pattern maps per inclination angle for the air-oil mixture are compared with air-oil mixtures, it is observed that the transitions from bubble to slug flow, bubble to dispersed bubble flow and churn to annular flow transition appear to occur at earlier superficial velocities of air. This disparity when compared with air-water two phase flows is evidently caused by the properties of the oil, just as is suggested in the work of (Caetano, Shoham, & Brill, 1992), particularly the viscosity.

As with air-water two phase flow, the transition from slug to churn occurs at lower gas superficial velocities of air with increasing angles of inclinations. This is a direct consequence of the situation that the slug region on the flow pattern map also appears to reduce with increasing inclinations. Hence, these inclinations have a considerable effect on the flow patterns developed and observed in the test section as well as the transition from one flow regime to another for air-oil mixtures. When the flow patterns of air-oil two phase flow are compared per angle of inclination with its corresponding air-water flow pattern map (e.g. 15 degrees air-oil vs 15 degrees air-water), it is also observed that the slug region is comparatively smaller. This could be due to the effect of the viscosity of the oil as higher viscosity would naturally suppress interfacial waves associated with slug flows.
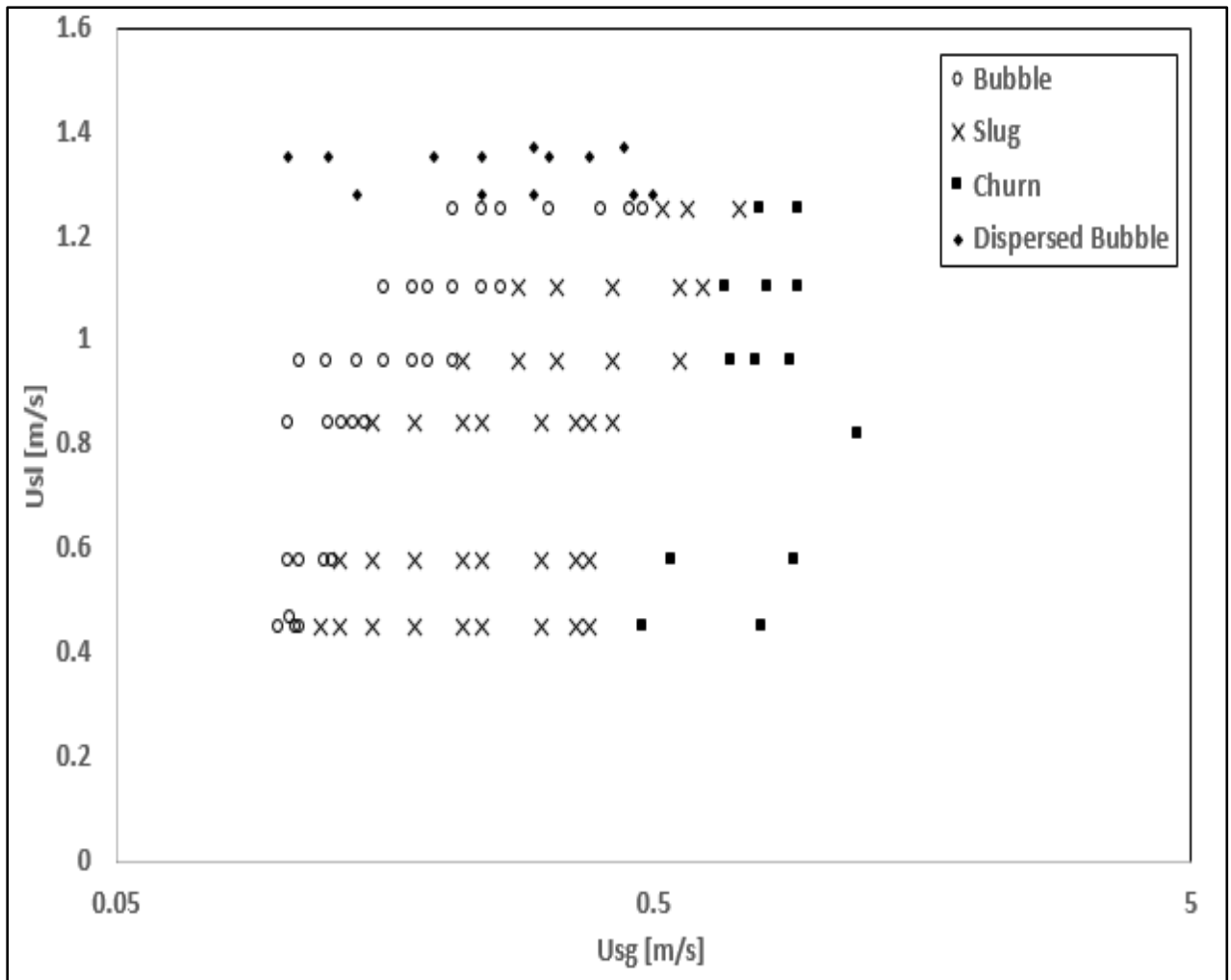
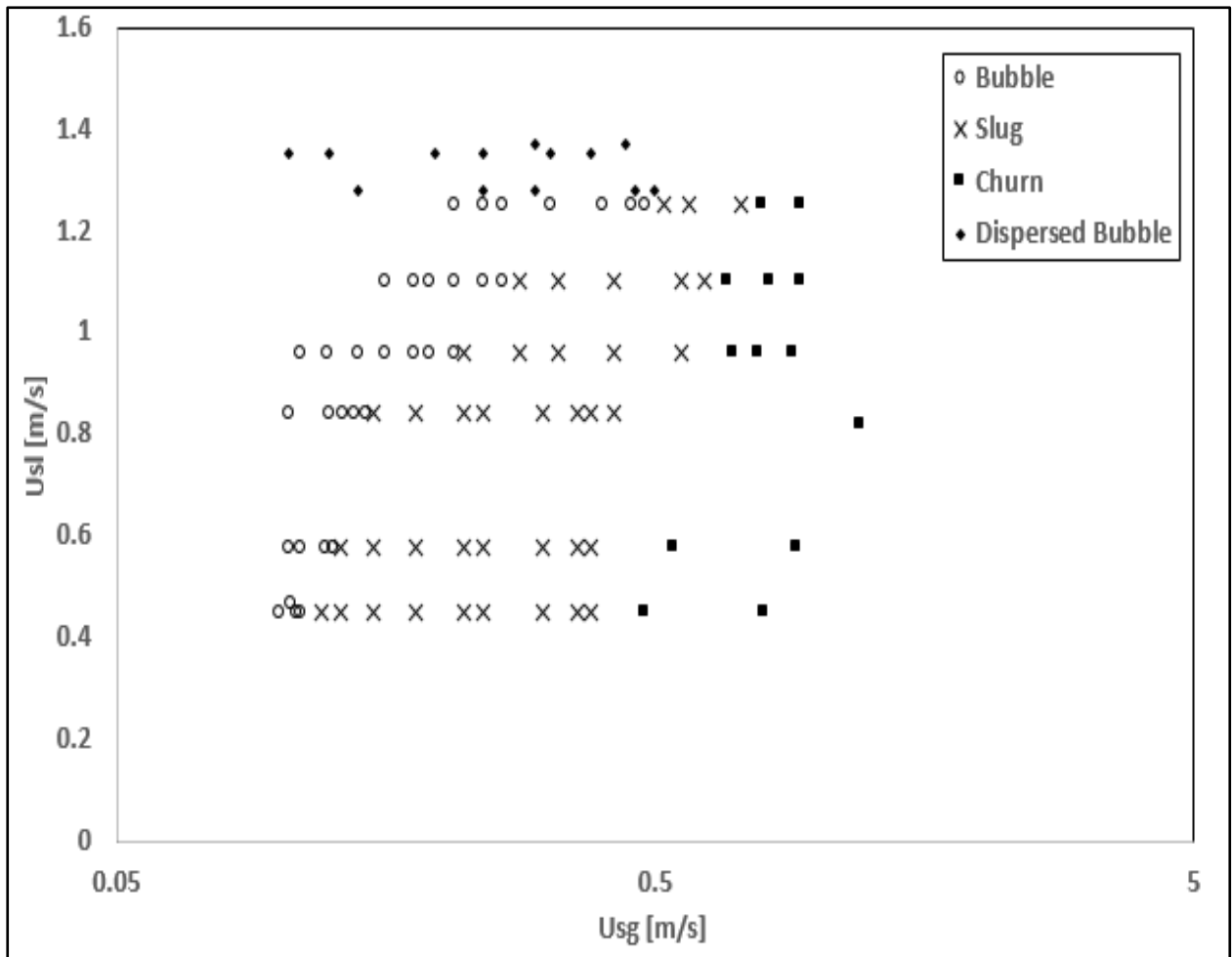*Figure 4.10. Air-Oil Flow Pattern map for inclination of 15 degrees*

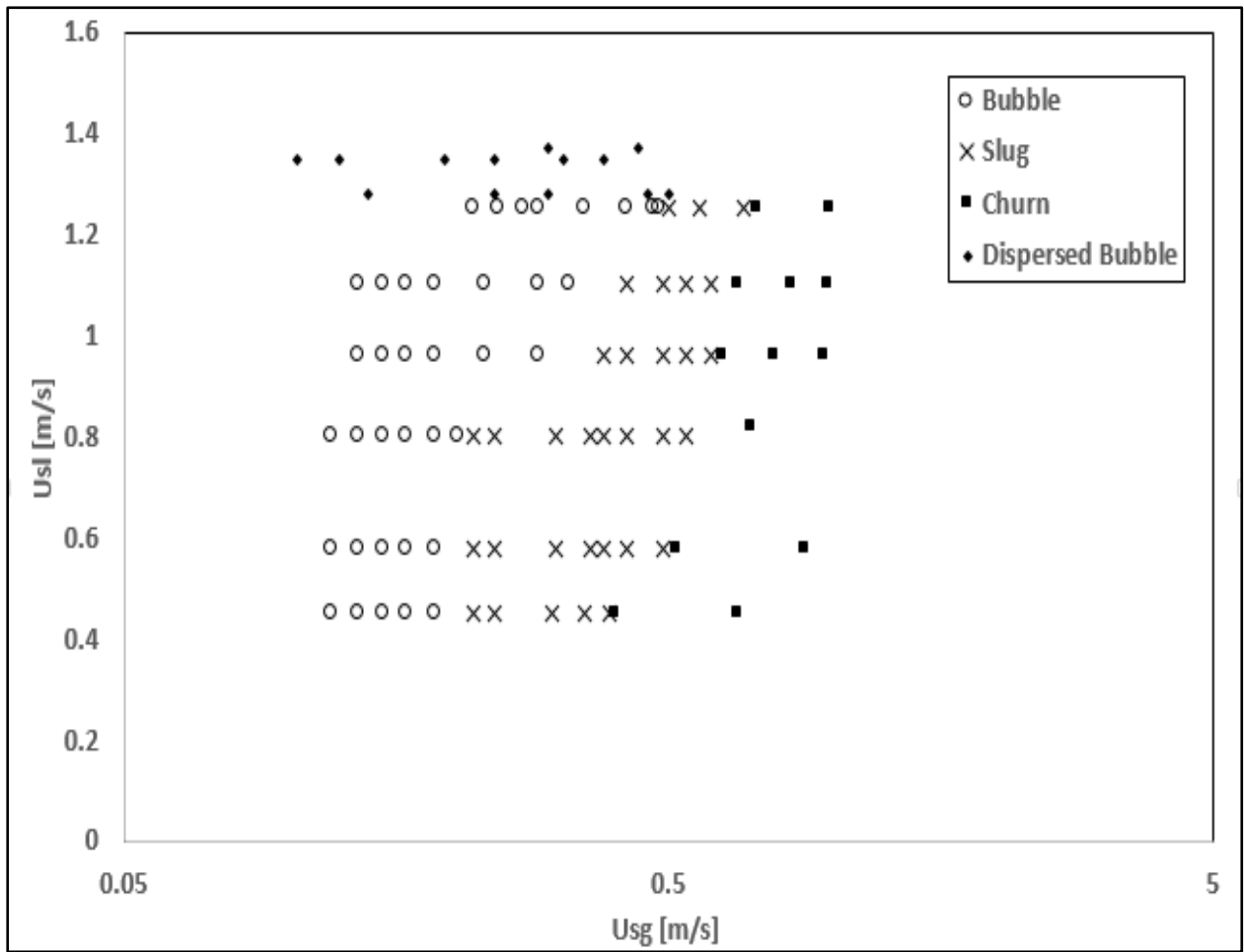*Figure 4.11. Air-Oil Flow Pattern map for inclination of 30 degrees*

Figure 4.12. Air-Flow Pattern map for inclination of 45 degrees

5. CONCLUSION

In this research, single phase and two phase experimental investigation was carried out in an annulus test section with an outer pipe of inner diameter 90 mm and an inner rod of outer diameter of 30 mm.

Single phase flow experiments are conducted to measure pressure drop in the annulus for laminar and turbulent flow regions with the working fluid as oil and water respectively. The pressure drop measurements were compared with predicted pressure drop calculations based on the evaluation of several equivalent diameters. It was observed that the prediction of the pressure drop based on the hydraulic diameter was an under-prediction of -56% and -34% for laminar and turbulent flow when compared with the experimental measurements. Hence, the hydraulic diameter concept is not sufficient to model the pressure drop and measure the friction factor in the annulus test section. However, it was observed that the predictions of pressure drop with the use of the Knudsen diameter mentioned in this work performed very well with an ARE of -6% and 5% for laminar and turbulent flow respectively. The analysis of the single phase friction factor also showed that the measured friction factor is in good agreement with the predicted friction factor using the Haaland correlation.

Two phase flow experiments were also conducted using air-oil and air-water mixtures and flow pattern maps, with superficial velocities of gas and liquid as coordinates, are plotted based on the visual observation of flow regimes. The flow pattern maps for air-water and air-oil two phase flows for test section inclination angles of 15, 30, 45 and 60 degrees to the horizontal are presented in *Figure 4.6 – Figure 4.12*. For air-water, dispersed bubbles are observed in all cases of inclinations at a superficial velocity of water about 1.4 m/s. It was also observed that the slug region on the flow pattern map also appears to reduce with increasing inclinations. This was the same also for air-oil mixtures. It was also observed that there were earlier flow regime transitions occurring for air-oil mixture in comparison with air-water mixtures as result of much higher viscosity of oil.

6. RECOMMENDATIONS FOR FURTHER STUDIES

This research involved carrying out experiments for single phase and two phase gas-liquid flow in an annulus test section with outer diameter of inner rod of 30mm and inner diameter of outer pipe of 90mm. It has been established that the experimental results are in agreement with some theoretical models. Indeed, further studies need to be carried out to fully investigate the accuracy of these models as well as the results and conclusions in this study. It is also possible to examine new correlations which can perform better in comparison to experimental data. However, recommendations that could be considered for study include:

1. Creating a better entry section into the test section to nullify the riser slugging that occurs in the present set up. The test section is lifted above the ground level as high as 1.2m to obtain horizontal flows. This prevents the observation of the actual two phase flow regimes due to slugging prior to entry of the test section for several combinations of air and water flow rates. For this reason, flow patterns were not observed for horizontal flows as they would be inaccurate.

2. Installing pressure transducers with higher range for higher pressure drop measurements. The limitation of the range of the pressure transducers prevents the pressure drop measurements of flows in annuli at angles higher than 15 degrees to the horizontal due to the effect of the higher gravitational pressure drop with increasing angles.

3. Pressure taps should also be installed for pressure drop measurements for two phase flows as they would have more accurate and reliable measurements in comparison to the existing pressure transducers.

4. The possibility of having water and oil pumps with a higher capacity. This will enable a wider range of flow rates for oil and water so as to enable better investigation e.g. Transition regimes for single phase flows.

5. The use of high speed cameras would be useful to capture the flow regimes for much higher flow rates of air, water and oil.

6. There is a limitation regarding the maximum angle which can be achieved with the inclinable annulus test section. At the moment, it is about 75 degrees. The possibility of making adjustments to the present annulus test section so as to have a fully vertical situation should be considered.

7. The design of the annulus by mounting the inner solid rod with screws as 'spikes' onto the walls of outer pipe has proved quite useful but has its limitations. For much higher flow rates of air, oil and water, it is observed that inner rod shifts or moves laterally. As such, an adjustment to the design should be considered.

8. This research work only investigates flow in a concentric annulus but does not investigate the effect of eccentricity on the flow through an annulus. This can be considered for further studies. However, this will require a better design or adaptation of the existing concentric annulus set-up.

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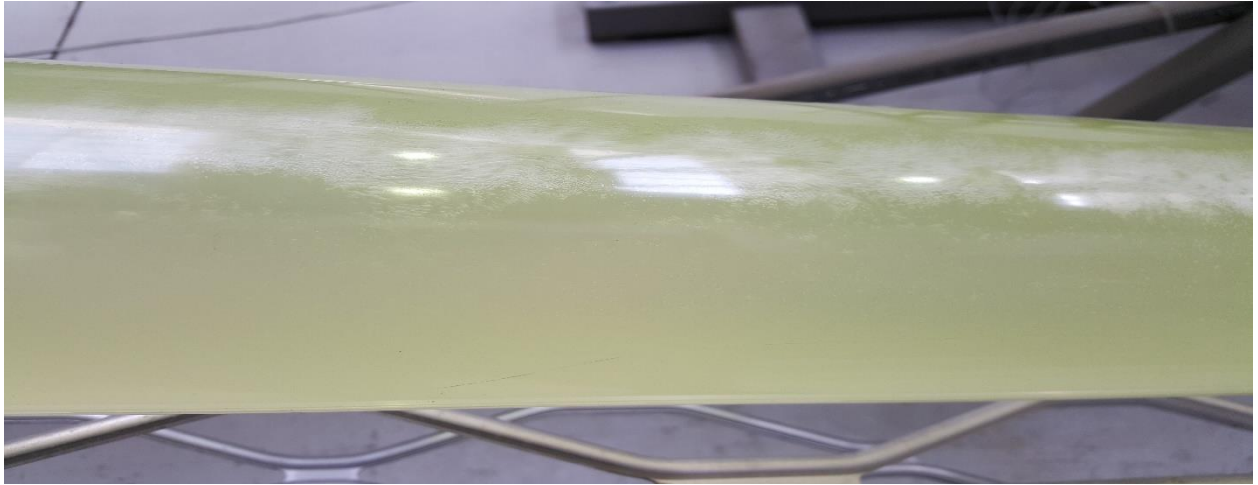
APPENDIX

*Appendix Table 1. Data for single phase flow with oil as working fluid*

Mass flow rate of oil (kg/h)	Measured pressure drop (over a length of 1.73 m) (Pa)
3994.04	147.9
4357.9	163.3
4702.97	214.86
5101.57	234.12
5419.68	258.22
5729.48	269.29
6069.25	281.97
6406.5	293.07
6755.35	299.06
7000.11	310.21
7287.21	321.17
7515.34	329.4
7872.42	344.02
8151.32	354.7
8457.81	367.56
8784	377.82
9106	400.83
9390	425.15
9758	499.99
10012	525.66

Appendix Table 2. Data for single phase with water as working fluid

| Volumetric low rate of water (l/s)	Measured pressure drop (over a length of 1.73 m) (Pa)
4.17	271.77
4.51	306.79
4.85	342.94
5.17	378.57
5.47	416.67
5.804	451.72
6.14	496.95
6.46	545.92
6.75	588.44
7.1	633.77
7.09	682.8
7.34	728.03
7.63	776.38
7.93	826.42
8.25	879.98
8.83	936.61
9.1	988.1
9.37	1036.45
9.65	1088.56
9.94	1145.1

Appendix Figure 1. Dispersed Bubble Flow (Aerial view) (AIR-WATER) (15 degrees inclination, U_{sl} = 1.35m/s, U_{sg} = 0.14 m/s)
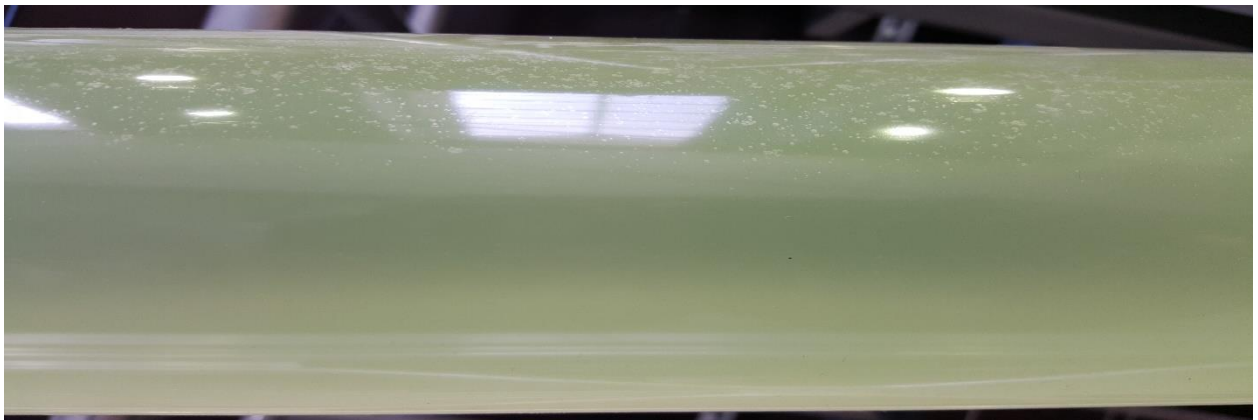
Appendix Figure 2. Dispersed Bubble Flow (AIR-WATER) (15 degrees inclination, U_{sl} = 1.35m/s, U_{sg} = 0.14 m/s)
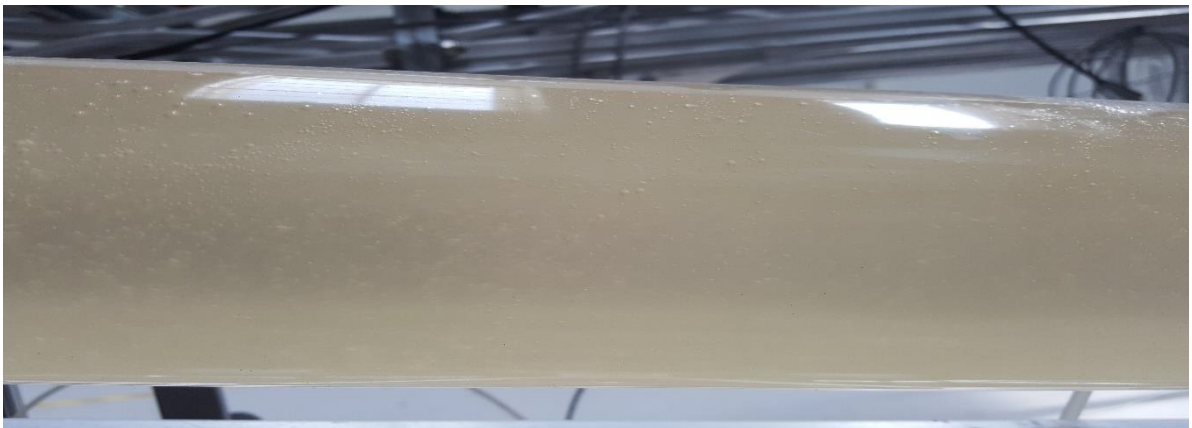
Appendix Figure 3. Bubble Flow (AIR-WATER) (15 degrees inclination, U_{sl} = 0.96 m/s, U_{sg} = 0.17 m/s)
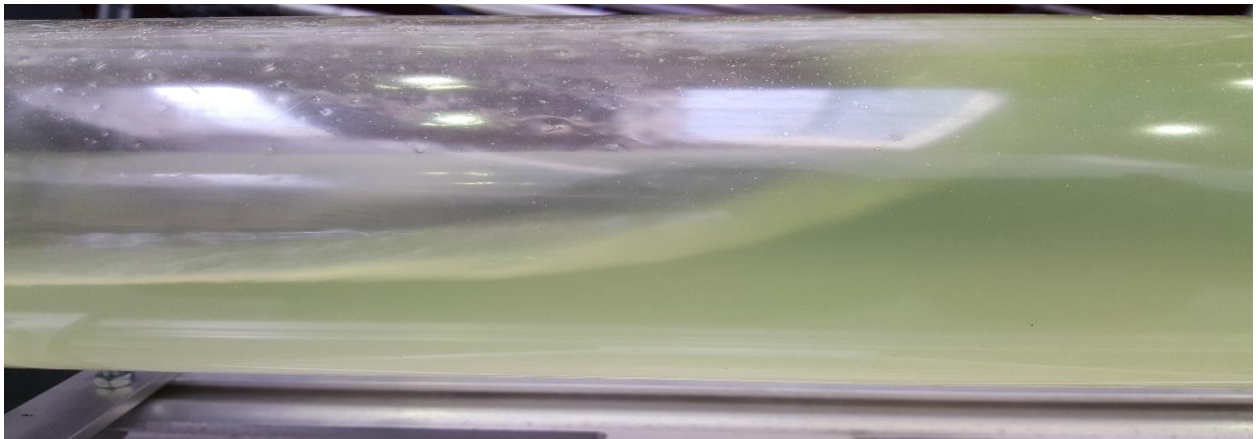
Appendix Figure 4. Slug Flow (AIR-WATER) (15 degrees inclination, U_{sl} = 0.96 m/s, U_{sg} = 0.33 m/s)
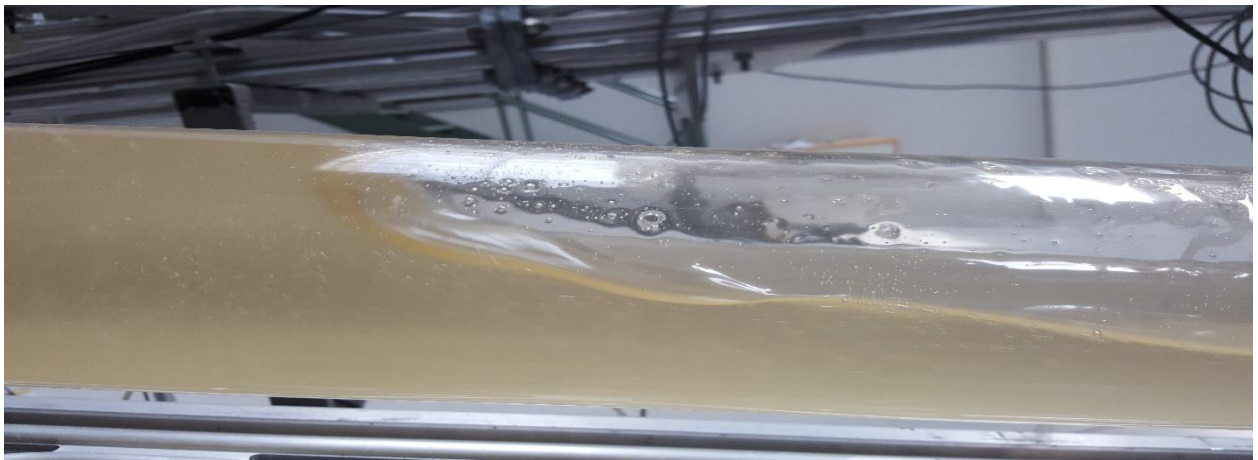
Appendix Figure 5. Churn Flow (AIR-WATER) (15 degrees inclination, U_{sl} = 1.1 m/s, U_{sg} = 0.82 m/s)

Appendix Figure 6. Dispersed Bubble Flow (AIR-OIL) (15 degrees inclination, U_{sl} = 1.28 m/s, U_{sg} = 0.14 m/s)

Appendix Figure 7. Bubble Flow (AIR-OIL) (15 degrees inclination, U_{sl} = 0.58 m/s, U_{sg} = 0.11 m/s)

Appendix Figure 8. Slug Flow (AIR-OIL) (15 degrees inclination, U_{sl} = 0.84 m/s, U_{sg} = 0.36 m/s)

Appendix Figure 9. Churn Flow (AIR-OIL) (15 degrees inclination, U_{sl} = 0.96 m/s, U_{sg} = 0.78 m/s)