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On the Possibilities of Grid Shells

Conceptual design of an elongated grid shell

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Master of Science in Civil and Environmental Engineering
Submission date: June 2016
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Abstract

Shell structures has for a long time fascinated both architects and structural engineers alike, since interesting geometries as well as incredible structural soundness is possible to achieve with them. The analysis of shell structures is hard to carry out without the aid of numerical tools, especially if the geometry is very complex. These complex geometries may be defined by free-hand by an architect, or by other measures. Structurally optimized geometries can also be obtained for shells by utilizing different techniques; so-called form finding techniques. By utilizing such techniques, the freedom in shaping the structure now gets altered, because the shape itself is optimized automatically. The freedom lies in defining the architectural constraints, the boundary conditions and the load situation. This process is highly dependent on expertise from both an architect and a structural engineer, due to the creative and structural nature of the process. This thesis focuses on problems arising when utilizing form finding techniques for elongated shell structures, or more specifically; grid shell structures. Different “form found” geometries are explored with close collaboration with an architect in order to conceptualize a good structure. A more thorough analysis of a built grid shell by Steinar Hillersøy Dyvik and John Haddal Mork is first carried out in. The results from these analysis is then used as a basis for the conceptual design of the elongated grid shell. It was found that the desirable shell behaviour was reduced after the elongation, and the structure acted more like an arch. This led to the requirement of adding edge beams to the structure to increase its stiffness.

Sammendrag

Skallkonstruksjoner har lenge fascinert både arkitekter og bygningsingeniører, siden de muliggjør interessante geometrier i tillegg til sterke konstruksjoner. Det å analysere en skallkonstruksjon er vanskelig uten hjelp fra numeriske verktøy, spesielt dersom geometrien er svært kompleks. Disse komplekse geometriene kan oppstå når en arkitekt tegner skallkonstruksjonen for frihånd eller på andre måter. Det finnes også ulike metoder for å optimere geometrien til skallkonstruksjoner; såkalte ”form finding” metoder. Ved å benytte seg av slike metoder, vil friheten til å definere konstruksjonens form bli forandret, siden formen blir optimert automatisk. Friheten ligger nå i hvordan man definerer randbetingelsene og lastsituasjonen. Denne prosessen er derfor svært avhengig av ekspertise både fra arkitekt og ingeniør. Denne oppgaven tar for seg hvilke problemer som oppstår dersom man benytter seg av ”form finding” metoder for avlange skallkonstruksjoner, eller mer spesifikt; gitterskallkonstruksjoner. Ulike former ble utforsket i nært samarbeid med en arkitekt for å designe en god konstruksjon. Først ble en mer grundig analyse av et bygd gitterskall av Steinar Hillersøy Dyvik og John Haddal Mork utført. Deretter ble resultatet av denne analysen benyttet som et utgangspunkt for det konseptuelle designet av det avlange gitterskallet. Det kom frem at de gunstige skalleffektene ble redusert ved et avlangt gitterskall, og konstruksjonen oppførte seg mer som en bue. Dette førte til at kantbjelker måtte påføres konstruksjonen for å øke stivheten.

Preface

This paper is meant to serve as a concluding thesis for the master programme in structural engineering at the Norwegian University of Science and Technology.

As a part of their master's thesis, Steinar Hillersøy Dyvik and John Haddal Mork designed and built a kinematic grid shell in Trondheim, spring 2015. A kinematic grid shell is a type of shell which is assembled on a flat plane, and thereafter "bent" into the desired shape. The shape in which the grid shell bends into, is found by structurally optimizing the shape so that mainly membrane forces act on the structure. How this structure react to different loading is studied, both by tests on site and numerically. The grid shell stood as a $10 \times 10m$ temporary pavilion, and was therefore loaded to its ultimate failure load.

Shell structures, especially kinematic grid shell structures, quickly became very appealing to me when I first learnt about them. The extreme thinness and lightness that is possible to achieve, together with the interesting structural behaviour makes such shells intriguing to me. Since very few large scale grid shells exist in the world, it would be highly fascinating to assess the structural behaviour of them and investigate the possibilities and challenges they bring.

In addition, architecture, especially the interface between architecture and structural engineering, has always fascinated me, and I wanted to write my thesis while collaborating with an architect.

This research paper's main objective will be to examine the structural behaviour of the shell by Steinar and John, and use the result from this analysis to conceptualize a larger, elongated version of the shell. Secondly it serves as an exploration of conceptual design in which architects and engineers not only collaborate closely, but also simultaneous.

Since this thesis focuses on conceptual design, detailed design using building codes is not considered and no, or little, thought has been given to the design of connections or support details. It is also chosen, for simplicity, to only consider elastic, homogenous and isotropic material behaviour. Dynamic behaviour is also not considered, but the first eigenmodes and –frequencies are obtained.

I would especially like to thank my supervisor Anders Rønnquist from the department of structural engineering for informative and intriguing, academic conversations. In addition, I would like to thank professor Bendik Manum, Steinar Hillersøy Dyvik and John Haddal Mork from the department of architecture, senior scientist Nathalie Labonotte from SINTEF Building and Infrastructure and Marcin Luczkowski from the department of structural engineering. This paper could not be written without their insights and knowledge about architecture, FEM-modelling, form finding methods and optimization techniques.

Trondheim, 2016-06-10

Magnus Nilsen

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1 Introduction

1.1 Architects and engineers

In ancient and renaissance times, the architect and the structural engineer usually was the same person (although these labels were not used at the time). One prime example was the famous renaissance genius Filippo Brunelleschi [1]. He is most known for designing the dome for Santa Maria del Fiore cathedral in Florence, which was the longest spanning dome ever built at that time. Brunelleschi's success can be attributed, in no small degree, to his technical and mathematical capabilities, as well as his creative mind and knowledge about architecture. Other examples include Robert Hooke, Christopher Wren and Isidore of Miletus [1].

Today, architecture and structural engineering is separated into two different disciplines. It is in modern times a widespread misunderstanding that architects are the designers of a building from concept to detail, whereas the structural engineer only care for its stability [2]. In reality, it is the function of the building which defines the two roles. The architect could solely design a building if it is multifunctional in a social context, for example a family house where no engineer is needed. Similarly, an engineer could design a building alone if it serves a singular structural purpose, for example infrastructure such as bridges where no architect is typically needed. A high-rise building typically needs expertise from both [2].

Shell structures play a special, important role for engineers, because their shape directly defines their load-bearing behaviour. For thin shells this is crucial, where certain shapes can eliminate all bending forces and the shell exhibits membrane forces only, making the shell structure dramatically more efficient. But even though such optimized shapes initially doesn't leave much space for an architect (nor for the engineer's imagination), it is fortunately not unusual that architects and engineers collaborate strongly upon designing such structures [2].

To construct thin-shell geometries which exhibits bending forces only, one usually adopt so-called form finding techniques. When using such techniques, it is key that engineers and architects collaborate tightly. Firstly, because the shapes from form finding techniques are highly sensitive to architectural constraints, and secondly, because there are extremely numerous realizations of a given set of constraints and which one to choose should not be chosen according to structural efficiency alone [3]. This type of structures is therefore highly dependent on expertise from both architects and engineers, and it is these type of structures this thesis will focus on.

Figure 1.1: Mannheim Multihalle, interior

1.2 State of the art structures

When designing new structures, it is a good idea to examine similar structures from history. This chapter will study six different state of the art shell structures in which architects and engineers have cooperated tightly, and where innovative solutions to different challenges have been assessed.

Mannheim Multihalle

Mannheim Multihalle was constructed as part of the Mannheim Bundesgartenschau (federal garden exhibition) in 1975 [4]. The Bundesgartenschau is a government sponsored gardening show held every two years in a major city of West Germany. In essence, the Bundesgartenschau consists of a large, open park area redeveloped and landscaped in order to display new and unusual species of plants from various growers and nurseries around Germany.

In 1970, the city of Mannheim was chosen as the site for the 1975 exhibition, and the planning began immediately. It was decided that a multipurpose event hall should be built and a design competition was held in order to see which idea would serve the city best for its moment in the spotlight [4]. Several different design proposals were considered, but all of them was abandoned due to high material and construction costs. Eventually they asked Professor of Architecture, Frei Otto, to assist in the design of the project [5].

After discussions with Frei Otto, a lattice *gridshell* structure was chosen for the best design for achieving the open, airy nature they desired, while also fulfilling the structural purpose of the hall. Timber was used for the gridshell for both structural and aesthetical reasons. Aesthetically, it was light and fit with the theme of a garden show, and structurally, it was stiff enough to resist buckling, but flexible enough to be able to bend all members from an initially flat grid. When the whole grid structure is initially flat, and eventually bent and locked into place, it is called a kinematic grid shell (more on this in chapter 2.5).

The idea of a large timber gridshell was an innovative idea, but one still very rare in practice. Thus, very little was known about the detailed structural behaviour of these types of shells, and how that behaviour would affect the materials in the structure. The original engineers contracted resigned after stating that the structural calculations were too difficult. The clients then hired structural engineering firm Ove Arup and Partners, together with other important contributors (Buro Happold).

Kinematic grid shell structures rely upon carrying forces in membrane action only. As mentioned in chapter 1.1, in order to design such structures, one needs to adopt some sort of form-finding method. Computational analysis was still in its infancy when the Mannheim Multihalle was built, and it was more common to rely on physical models. Thus, the decision was made to build a scale "hanging chain" model in order to find the appropriate form which only exhibits membrane forces (Figure 1.2: Hanging chain model of Mannheim Multihalle.). The *hanging chain* model could not be used to design the structure alone, due to possible errors in the mechanical construction of the model as well as measurement errors. Therefore, a technique known as the *force density method* was used to further verify the structure (the force density method is described in more detail in chapter 1.6).

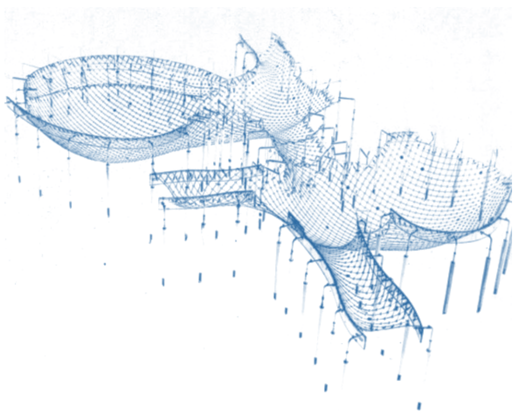


Figure 1.2: Hanging chain model of Mannheim Multihalle.

The architect had previously decided that the grid laths should be no larger than $50 \times 50 mm$, but this was shown not to be adequate. It was then decided that the grid should be composed of two layers, so that the second moment of area was large enough (although the shape is optimized to carry membrane forces only, wind loads and other skew loads will introduce bending forces on the structure). The problem then becomes that the structure will be too stiff to be able to shape it from a flat grid without introducing too high bending forces. The solution was then to drill elongated holes for the bolts, so that the two layers work almost independently of each other during erection. When the final shape was established, the bolts were tightened and the two layers were connected via so called shear blocks in order to obtained the desired stiffness for the structure.

The Multihalle is a structure where architects and engineers have cooperated really tightly, and it is created from rigorous mathematical investigations as well as practical form finding methods. The idea behind the structure was revolutionary for its time in the 1970’s and continues to serve as an inspiration today.

Figure 1.3: Downland Gridshell interior

Downland gridshell

The Weald and Downland Open Air Museum is a leading international centre for historic timber buildings. In their commission, the Museum wanted a modern structure, which would extend the lineage of the historic timber buildings into the 21st century. Edward Cullinan Architects won the commission, and Buro Happold was assigned as structural engineer.

The Downland Gridshell Building was the first timber gridshell to be constructed in the United Kingdom, and is regarded as an iconic building by both architects engineers [6]. The building was completed in 2002 and is a lightweight structure made of oak laths. To prepare the oak laths for use all defects were removed and the resulting pieces finger-jointed together into standard lengths of 6m. Six of these pieces were then joined to form 36m laths. The laths have a cross section of $50\times35mm$, with $1m$ spacing. Like Mannheim Multihalle, The Downland Gridshell is also a kinematic gridshell structure. The timber laths were bent into shape, and then locked by edge beams running along the sides of the whole building. Downland grid shell's grid is also doubly layered, but in addition consists of a triangulating timber bracing.

Where Downland Gridshell differ the most from Mahheim Multihalle, is in the hinge connections in the grid. In Downland, the laths are connected at the nodes of the grid with a patented system of steel plates and bolts, which is visible in the top left corner in Figure 1.3: Downland Gridshell interior.

The shape of the gridshell is primarily driven by stiffness requirements. At first glance, the gridshell might look like a barrel vault, but it has double curvature which generates geometric stiffness and is fundamental to its structural action in resisting asymmetric loads [7]. The form finding process used was a combination of physical modelling and computer simulations.

Under construction, the Downland Gridshell was bent into shape without relying on cranes or any other upward lifting force. The flat grid was built on top of a smaller platform, which then caused the initially flat grid to bend at the edges due to gravity. Then, the grid was pulled towards CLT edge beams and eventually locked into place.

Figure 1.4: Savill Garden Gridshell interior

The Savill Garden Gridshell

The Downland Gridshell got quite a lot of attention in the British architecture environment, and when the park area in Savill Garden in Berkshire submitted a commission for a new visiting centre, the architect Glenn Howell saw an opportunity to include a grid shell in his design. It would be the largest gridshell in England, measuring about $90\times25m$, and the first place where a gridshell is used as a roof construction. Once again, Buro Happold was assigned as the structural engineer.

The laths is again made up of wood, more precisely local larch with a characteristic strength of about $30\ N/mm^2$. The cross section of each lath is $80\times50mm$, and the grid spacing is $1m$.

The shell's geometry is defined analytically. The building's plan is defined as two intersecting circle sections, and the gridshell itself is defined by sinusoidal functions and parabolas. This made it easy for the architect and engineer to construct both a practical and aesthetically pleasing shape easily by modifying different parameters, but it does makes it harder to define a regular grid along the surface (more on this in chapter 2.2).

The main challenge with this building, is the flat geometry of the shell. This makes the roof act more like a slab than a shell, and is highly subjective to asymmetrical loading. Because of this, there was need for even larger spacing between the two grid layers, which again required larger shear blocks. This makes the roof too stiff to be bent like the previous mentioned shells, so they had to approach it differently. The solution was to first bend the bottom grid layer, and then assemble the shear blocks and the top layer on top of it. To lock the shape in place the whole grid is covered with CLT panels, which stiffens the shear deformations of the quadrilaterals.

Figure 1.5: The YAS Hotel, Abu Dhabi, exterior

The YAS Hotel

Until now, only timber grid shells have been studied. The YAS Hotel in Abu Dhabi, is a steel grid shell, and is the world's first hotel to have a Formula 1 race track built around it [8]. At the time it was built it was also the world's largest LED project. The main attraction is, however, the curvilinear gridshell covered with over 5 300 diamond shaped steel panels, containing nearly 5 000 LEDs.

The shape of the gridshell that surrounds the hotel, is not defined by form finding nor analytical expressions. It is a so-called free-form geometry, where the architect has taken complete liberty when defining curvatures and overall shape. This, like the Savill Garden Gridshell, makes it hard (sometimes impossible) to divide the whole shell surface into equal quadrilaterals. The YAS Hotel gridshell is not made up of equal quadrilaterals at all, and is a product of careful computational meshing of an already defined surface. This greatly increases production and construction costs compared to earlier mentioned gridshells, because each grid member must be tailored to fit at every point.

By choosing steel as the material for the gridshell, a kinematic approach is problematic. Because of steel's high Young's modulus, huge forces or very thin cross sections would have been necessary in order to bend the steel grid into the desired shape. On the plus side, by using curved members, no initial bending stresses are introduced to the finished structure and the shell can work in purer tension or compression. More on the differences between different types of gridshells is discussed in chapter 1.7.

Figure 1.6: One of the shell bridges that span the Cascara river in Madrid.

Cascara bridges

West 8 and MRIO architects was responsible for the master plan for the reclaimed river banks and several other areas around Madrid. There are three identical pedestrian bridges which spans the Cascara river. The “roof” of these bridges are dome-like and made of reinforced concrete, and does also bears the deck for the pedestrians. The deck is connected to the concrete shell by numerous thin vertical cables, which make the bridge appear open and lightweight.

The shell structure spans $41m$, and is about $8m$ high at the apex. The shape of the openings in the longitudinal direction appears to be a catenary or a parabola. Standard for arch bridges is that the arch is a parabola, since the main loads from the weight of the deck and traffic, are basically uniformly distributed loads, and the optimal curve for bearing such loads is the parabola (this is explained in chapter 2.3). The top part of the shell in this bridge is, however, “flattened” on top.

This shell is obviously not kinematic for several reasons. Firstly, the low tensile strength of concrete would not allow for such large bending deformations, and secondly, the shell is continuous which makes it way too rigid against shear deformations in order to produce any double curvature.

Figure 1.7: The bridge of peace, Tbilisi, Georgia

The Bridge of Peace

The Bridge of Peace is a pedestrian bridge in the capital of Georgia, Tbilisi, which spans $160m$. The pedestrian deck is suspended from a steel gridshell supported solely by the four supports on its two embankments. This shell is also not kinematic. The bending of the relatively large cross-section of the steel beams, would give rise to very large bending stresses. In addition, by inspecting the quadrilaterals the gridshell is made out of, one can see that not all of the quadrilaterals are of the same size, which means that the gridshell would display too much shear stiffness (see chapter 1.7).

It is not as easy to say if the shape has been form found, but good insights about shell geometry is nevertheless displayed. The cantilevering parts at the end of the bridge is “bent” upwards, which helps the structure to carry loads to the supports (see chapter 1.5), and the transverse beams seem to correspond to parabolas or catenaries.

1.3 Geometry and structural performance

When talking about structural performance, one could mean a number of things. One could talk about the structure's efficiency in terms of its material usage, or for example the costs, taking both material usage and construction costs into account. This part will mainly focus on the efficiency in terms of material usage, hereby named mechanical structural performance. It is challenging to propose a precise and rigorous definition of structural performance, but some good qualities of structures that perform well are:

- Minimal material usage
- Robust equilibrium solution
- Smooth flow of forces

The advantage of robust equilibrium solutions is that the structure is not prone to sudden stability failures, such as snap-through buckling or lateral torsional buckling. For example, if a structure has an "optimized" cross section at each point for a given load case, so that the value of stress is uniform along the whole structure, one could say that the structure has been optimized structurally in terms of material usage. But having a uniform stress distribution along the whole structure makes the structure equally likely to fail at every point, which could be very dangerous if the load is close to the design load.

A smooth flow of forces circumvents having stress concentrations, which gives rise to unreasonably high stresses at small spatial areas. Having a smooth force flow usually reduce the material usage, but by optimizing the stress distribution, one does not necessarily optimize the material usage.

Bending and membrane action
It was mentioned in the introduction that by eliminating bending forces, and having pure membrane (compression and tension) action in the structure, dramatically increase the efficiency of the given structure. The reasons behind this is discussed in more detail in this section. For simplicity let's compare two beams; one beam in pure bending about the z-axis (see Figure 2.1 for coordinate system definition), and another in pure compression.

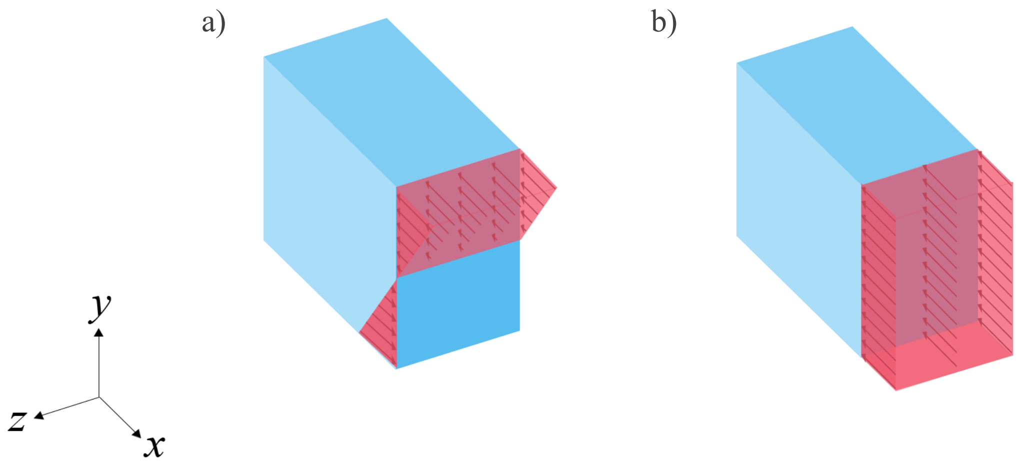


Figure 1.8: a) Beam in pure bending about the z-axis. b) Beam in pure compression

Both beams are made up of a perfect elastic, isotropic material, and consist of a uniform rectangular cross section with height h. The stress utilization at a distance y from the neutral axis is then given by:

$$\eta(y) = \left|\frac{\sigma(y)}{\sigma_0}\right|,$$

where $\sigma(y)$ is the stress in the beam at a distance y from the neutral axis and σ_0 is a reference stress value, for example the yield stress. In pure bending, the stress in the beam is given by (tension is positive):

$$\sigma_{bending}(y) = -\frac{2y}{h}\sigma_0,$$

where h is the total beam height.

For a pure compressional beam, the stress $\sigma_{compression}$ is simply equal to the reference stress σ_0 in magnitude. The two different stress utilizations then become:

$$\eta_{bending}(y) = \frac{2}{h}|y|$$

and

$$\eta_{compression}(y) = 1.$$

A more useful quantity is the average stress utilization over the entire cross section, which can be defined as:

$$\bar{\eta}(y) = \frac{1}{h}\int_{-h/2}^{h/2} \eta(y)\,dy = \frac{1}{h}\int_{-h/2}^{h/2} \left|\frac{\sigma(y)}{\sigma_0}\right| dy\,,$$

which for the pure bending beam gives:

$$\bar{\eta}_{bending}(y) = \frac{1}{h}\int_{-h/2}^{h/2} \frac{2}{h}|y|\,dy = \frac{1}{h^2}\left[\left(-\frac{1}{2}h\right)^2 + \left(\frac{1}{2}h\right)^2\right] = \frac{1}{2}.$$

The average stress utilization for the pure compressional beam, $\bar{\eta}_{compression}$, is obviously equal to one, since $\eta_{compression}$ does not depend on y. Therefore, one could say that, on average, a beam in pure compression (or tension) is twice as effective as a beam in pure bending. The same principle applies for thin plates and shells. Techniques for eliminating bending action is discussed in more detail in chapter 2.3 and 2.4.

Compression and tension

How a structure react differently in compression and tension, is highly dependent of the material used. Concrete, for example, has a very low tensile strength, which makes concrete structures perform better when in compression. On the other end of the spectrum, we have textile membranes or cables, which doesn't work in compression at all, but can be very strong in tension.

Generally, for a generic material which work similarly in compression and tension, like steel, structural members give rise to more robust equilibrium solutions when working in tension. This is because compressional forces may give rise to buckling instabilities, which requires larger members to take care of. One interesting family of structures which takes advantage of the different properties of tensional and compressional members are so-called tensegrity structures. When designing tensegrity structures, one identifies which parts of the construction that exhibits tension and which parts that exhibits compression. This can make aesthetically interesting structures which are highly effective.

Figure 1.9: The Millenium Dome, London by Sir Richard Rogers

The most notable example of a tensegrity structure is perhaps the largest spanning dome in the world, namely the Millenium Dome in London. Here, pillars of steel together with steel cables, bears the huge textile membrane which spans a whopping $365m$ [9]. The membrane is only a few millimetres thick, making the span-to-thickness ratio of the dome roughly one hundred of that of an egg. Some might argue that it is not a dome since the roof is not self-supporting, but nonetheless it stands out as an impressive structure. The structural engineers behind the dome is Ove Arup & Partners.

Curvature

A curved beam does not exhibit the same forces as a straight beam. To investigate this, a continuous, curved beam, whose shape is given by the height from the left support, $h(x)$, is considered (Figure 2.3).

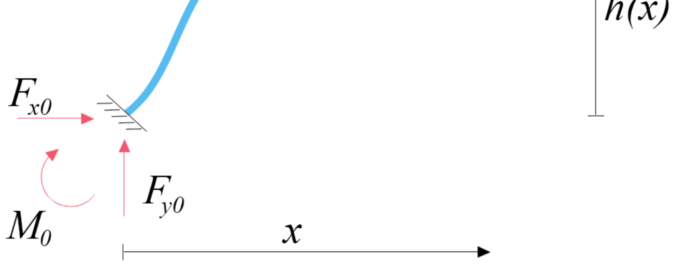


Figure 1.10: A curved beam given by the height function, h(x) subjected to forces as shown

The moment at a distance x from the support, $M(x)$, can be found by taking moment equilibrium about the point in which it acts:

$$M(x) = M_0 + F_{y0} \cdot x - F_{x0} \cdot h(x) - M_q(x)\,, \tag{1.1}$$

where $M_q(x)$ is the moment due to the external vertical load, which might depend on $h(x)$.

It is clear that the moment along the beam, $M(x)$, depends on the height function $h(x)$. It is notable that $M(x)$ is also dependent of the loading $q(x)$, which means that an optimal function $h(x)$ is load dependent. In chapter 1.5 and 1.6 it will be discussed how to choose $h(x)$ so that the moment $M(x)$ vanishes completely for a given load situation, and only membrane forces are left.

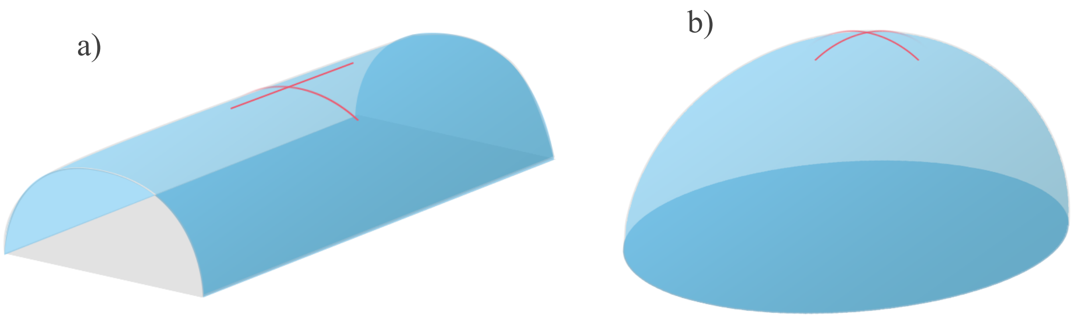


Figure 1.11: a) Singly curved shell in the form of a barrel vault. b) Doubly curved shell in the form of a dome-like structure

The same principles can be applied to curved shells, but here one needs to distinguish between singly curved and doubly curved shells. Singly curved shells, like barrel vaults, act similar to a curved beam (arches). Doubly curved shells on the other hand is much stiffer, since their so-called intrinsic geometry resists out of plane deformations [10]. The mathematical framework which describe this is the theory of differential geometry. Differential geometry, and its mathematical treatment, is way beyond the scope of this thesis, but qualitative descriptions of some of the ideas will hereby be given.

Asking the question whether a two-dimensional surface is singly or doubly curved, is related to asking the question if a given two-dimensional surface can be embedded in a plane (laid out flat) in three dimensions without straining the surface [11]. Imagine taking a flat piece of paper, and bend it like a barrel vault. This can easily be done without straining the paper. Now imagine taking the same piece of paper and bend it into the shape of a dome. This is not possible without straining the paper. Consequently, if a dome surface is given, it is not possible to lay it out flat. This is the reason why two-dimensional world maps cannot depict distances and areas of the earth precisely. If a given surface cannot be laid out flat, it is said that the surface displays *intrinsic* curvature.

A useful measure of the intrinsic curvature of a surface is the Gaussian curvature. It is defined at each point as the product of the principal curvatures at that point. The principal curvatures are illustrated in Figure 1.4 for a barrel vault and a dome-like structure. For a barrel vault, the curvature along the vault's longitudinal axis is zero, so the Gaussian curvature is also zero. For a hemisphere, the curvature along any direction is $1/r$, so the Gaussian curvature is $1/r^2$ along the entire surface.

In shell structures, the intrinsic curvature of a surface can be used to the structure's advantage. By exploiting surfaces which displays intrinsic curvature, the shell will be resistant to certain deformations. This is the reason why doubly curved surfaces are generally more robust than singly curved surfaces.

Even though shells are modelled as two-dimensional surfaces, the mathematical theory of shells introduce a sort of "fictitious" shell thickness which give rise to what we call bending stiffness. This will introduce strains in the shell even though the two-dimensional surface which defines the shell is unstrained. For example, if the piece of paper from earlier had been thicker, bending stresses would have been introduced when bending it into a barrel vault, since the inner and outer parts of the paper would be of different lengths. The same holds true for beams, but since beams are one-dimensional elements, they cannot have intrinsic curvature. This makes sense if we imagine an extremely thin beam, like a string, which can be shaped into any curve without introducing strains.

1.4 Freeform structures

Freeform, free-curved or sculptural structures are structures which are generated without particularly taking structural performance into consideration. They can be defined by pen or paper, or digitally, where they are often described by higher degree polynomials, like NURBS (Non-Uniform Rational Basis Splines). The shape generated is often inspired by organic forms, like hills and valleys.

One particularly interesting quality about freeform structures, is that they often tend to be stronger under asymmetrical loading than optimized shells. This is due to the fact that the shell is not optimized for a certain load configuration.

1.5 Funicular geometry

In contrast to freeform geometries, funicular geometries are geometries which are guided by the structural performance alone. As mentioned earlier, if the geometry of a structure is carefully chosen for a given load situation, no bending forces will act on the structure. Such a geometry is defined in this thesis as funicular, and only exhibits membrane forces.

Arches

A hanging chain is a typical example of funicular geometry. The chain is unable to carry any compression or bending forces, so a hanging chain, must carry all of its weight by tension. Robert Hooke postulated in 1676 that by inverting the shape of a hanging chain, one will obtain a geometry which only acts in compression. This was a beautiful idea, and is an extremely effective and intuitive way of finding an optimal shape for a compressive arch [3].

By considering equation (1.1) from chapter 1.3 again, we can investigate which constraints $h(x)$ must fulfil in order to make the bending moment $M(x)$ vanish everywhere:

$$M(x) = M_0 + F_{y0} \cdot x - F_{x0} \cdot h(x) - M_q(x) = 0 \,.$$

By rearranging and solving for $h(x)$ one obtains:

$$h(x) = \frac{1}{F_{x0}} \Big(M_0 + F_{y0} x - M_q(x) \Big) \,. \tag{1.2}$$

Constants are not relevant when trying to obtain the overall shape, so the height function $h(x)$ can be written on the form

$$h(x) = c_0 + c_1 x + c_2 M_q(x) \,. \tag{1.3}$$

The expression for the bending moment due to the external load $M_q(x)$ is given by:

$$M_q(x) = a(x) \int_0^x q(x)\, dx \,, \tag{1.4}$$

where $a(x)$ is the centroid of the load area between 0 and x, measured from x. The expression for $a(x)$ in this equation can be written as:

$$a(x) = x - \frac{\int_0^x x\, q(x)dx}{\int_0^x q(x)dx},$$

which inserted into equation (1.4) gives

$$M_q(x) = x\int_0^x q(x)dx - \int_0^x x\, q(x)dx\,.$$

Using integration by parts on the last term, the equation can be simplified to:

$$M_q(x) = x\int_0^x q(x)dx - \left(x\int_0^x q(x)dx - \iint_0^x q(x)dx^2\right) = \iint_0^x q(x)dx^2\,. \quad (1.5)$$

Let us now consider a uniform distributed load $q(x) = q$. This gives $M_q(x) = \frac{1}{2}qx^2$, which inserted into equation (1.3) gives:

$$h(x) = c_0 + c_1x + c_2x^2\,,$$

which is the mathematical description of a parabola, which is well known from literature to be the funicular curve for a uniformly loaded structure [12].

A beam only subjected to gravity, has a load situation which is dependent of its shape, because the steeper the curve, the more mass is present in a unit horizontal length (Figure 2.5). The load distribution $q(x)$ is therefore a function of the shape function $h(x)$. To obtain this relation, imagine a hanging chain, which is horizontal at $x = 0$.

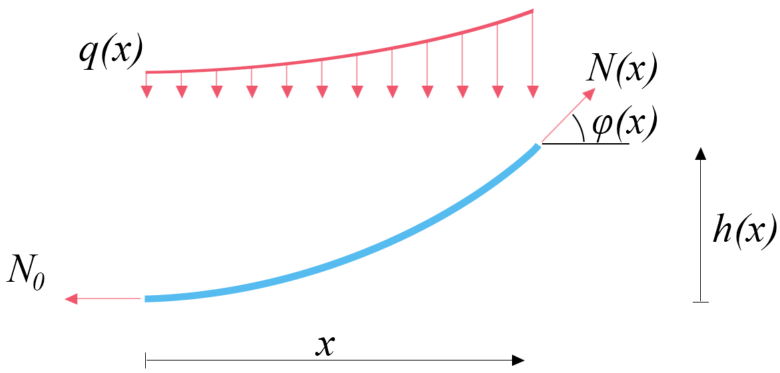


Figure 1.12.: A curved beam with distributed load due to gravity

An infinitesimal peace of the beam of length ds subjected to gravity, will feel a force $dF = \mu g\ ds$, where μ is the mass per unit length (assumed constant) and g is the acceleration of gravity. The same force can be described by the distributed load $q(x)$ as $dF = q(x)dx$. Since these must be equal, the distributed load as a function of the shape function $h(x)$ can be obtained:

$$dF = \mu g\ ds = \mu g\sqrt{dx^2 + dh^2} = \mu g\sqrt{1 + \left(\frac{dh}{dx}\right)^2}\ dx = q(x)dx\ .$$

Thus,

$$q(x) = \mu g\sqrt{1 + \left(\frac{dh}{dx}\right)^2}\ .$$

The contribution to the bending moment for this load then becomes (by equation (1.5)):

$$M_q(x) = \mu g \iint_0^x \sqrt{1 + \left(\frac{dh}{dx}\right)^2}\ dx^2\ .$$

inserting into equation (1.3) and differentiate with respect to x gives:

$$\frac{dh}{dx} = c_1 + c_2 \int_0^x \sqrt{1 + \left(\frac{dh}{dx}\right)^2}\ dx\ ,$$

which is known to be the differential equation for the catenary curve [13]. Solving this differential equation yields:

$$h(x) = a\left(\cosh\left(\frac{x}{a}\right) - 1\right),$$

where a is a constant. Both the catenary and the parabola is plotted in Figure 1.13: Comparison between a parabola (blue) and a catenary (red)., to illustrate the difference between the funicular curve for a uniformly distributed load and a load due to gravity.

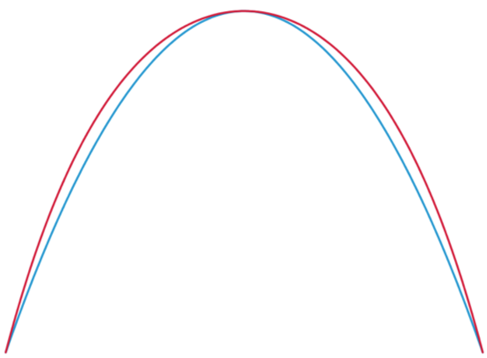


Figure 1.13: Comparison between a parabola (blue) and a catenary (red).

Shells

A barrel vault, which is continuously supported along the sides, is structurally similar to an arch. Therefore, a funicular geometry of a continuously supported barrel vault subjected to gravity is simply an "extruded" catenary. But when exploring funicular geometries for two-dimensional surfaces, one quickly finds that there exists an infinite number of funicular geometries given support conditions and a load situation [3]. When trying to find funicular geometries for more complex, doubly curved surfaces, the problem becomes much harder and closed form solutions usually don't exist for a given load situation. Therefore, alternative techniques must be exploited in order to find funicular geometries for shells.

The most popular technique for identifying funicular surface geometries, is by utilizing form-finding methods, which is discussed in more detail in chapter 2.4. In this chapter, a qualitative description of funicular shells will be given, in order to develop an understanding of what to expect from two-dimensional funicular geometries. Multiple scenarios are explored and discussed. For simplicity, only gravitational forces will be considered henceforth.

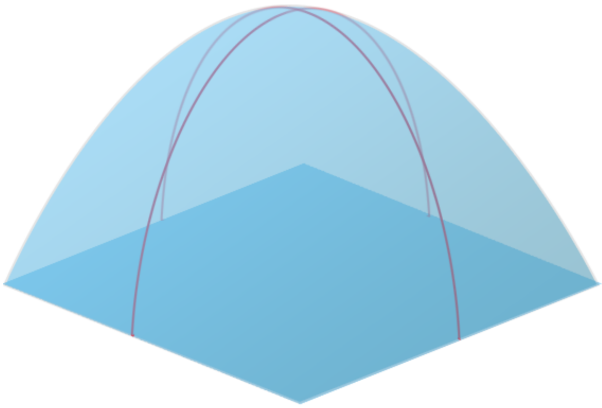


Figure 1.14: Doubly curved shell continuously supported on a square. Catenary curves are shown in red.

Consider a shell which is continuously pin-supported along all four sides of a square. In order to estimate a geometry which eliminates all bending forces in such a shell, one could imagine an infinite number of catenaries, all with a common apex, spanning from one side of the square to the opposing side, collectively constructing a surface. A similar technique could be employed for a shell supported on a circle, which was actually done by Christopher Wren in 1669 when designing the inner dome for the St. Paul's Cathedral in London [14].

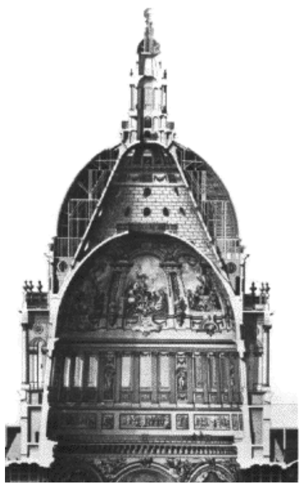


Figure 1.15: Cross section of St. Pauls Cathedral (Photo from wikipedia)

Now let's imagine what happens if the shell is only supported on three of the sides. By using the same technique as before, one could again imagine taking an infinite number of catenaries with a common apex, but now the "open" side itself will also work as a catenary (Figure 1.16: Doubly curved shell continuously supported on three sides of a square).

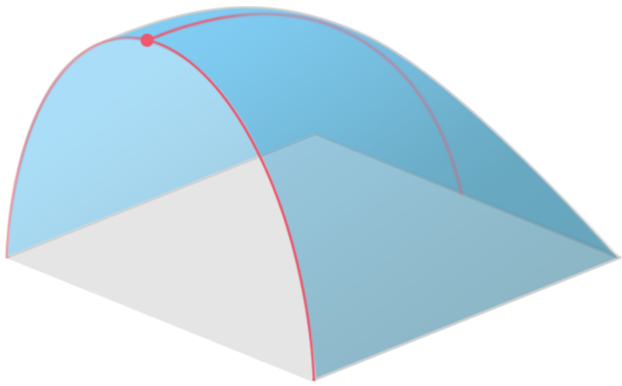


Figure 1.16: Doubly curved shell continuously supported on three sides of a square

Another approach, is to recall that a truncated catenary is still a funicular shape for a curved beam, and use the same logic to guess that a truncated funicular shell, is still funicular. Imagine now a vertical imaginary plane, that "cuts" through the surface like shown in the figure below. The former shell was a special case of a cut like this, where a rectangular shell was cut in equal halves.

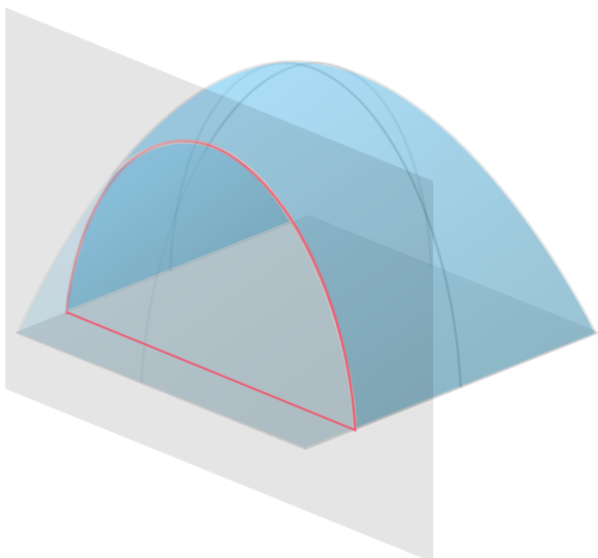


Figure 1.17: Doubly curved surface, truncated vertically

It is expected that both of these truncated shells should act funicular, but they have however lost some of their horizontal stiffness due to the non-supportive side. A possible remedy for this is to instead of cutting the surface through a plane, one can “tunnel” the surface itself, preserving more of the catenary-like geometry (Figure 1.18: A shell continuously supported on three sides, where the opening has been "tunnelled" through the surface.).

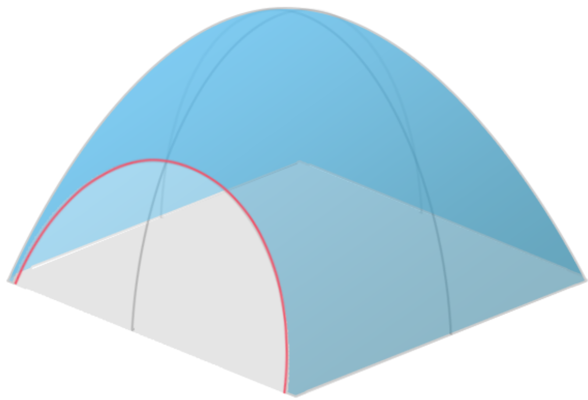


Figure 1.18: A shell continuously supported on three sides, where the opening has been "tunnelled" through the surface.

As mentioned earlier, there exists an infinite number of solutions to a funicular surface for a given set of boundary conditions and loads. One interesting family of solutions, are solutions where parts of the surface is cantilevering. We can examine some possible solutions by using the same three-sided boundary condition as earlier, but where the “free” side is cantilevering (Figure 2.12a).

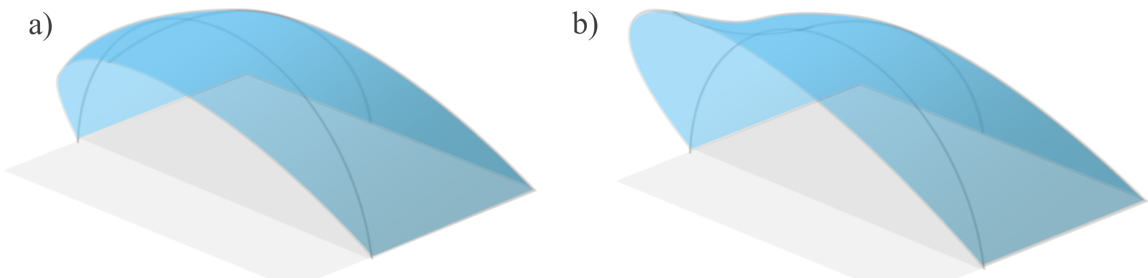


Figure 1.19: Cantilevered shell where the cantilevering part is loaded in a) tension and b) compression.

In this example, the cantilevering part is intuitively loaded in tension. It could still be a geometry absent of bending forces, but when designing for example concrete shells, it is desirable that the entire shell should be loaded in compression. This is possible to achieve by inverting the Gaussian curvature in the parts which is cantilevering, so that every place that initially was loaded in tension, is now loaded in compression (Figure 2.12b).

1.6 Form finding

In the previous chapter, some qualitative descriptions of different problems regarding funicular geometries were mentioned. In this chapter, more quantitative descriptions of different methods used to find funicular shapes is given. These methods are called *form finding* methods.

Physical form finding

The first idea of form finding was first presented by the English engineer and scientist Robert Hooke in 1676. Ha postulated that “As hangs the flexible line, so but inverted will stand the rigid arch” [3]. The idea was simple: invert the shape of the hanging chain, which by definition is in pure tension and free of bending, to obtain the equivalent arch that acts in pure compression. If the chain is loaded in different ways, for example by some weights, the principle still holds true. If the chain is only subjected to gravity, the shape of the curve will be that of a catenary, which was derived in chapter 2.3.

For shell surfaces, it is possible to use a similar approach. One could for example hang a piece of cloth, which also acts in pure tension, and invert it to obtain a surface in pure compression. The main disadvantage of using ordinary cloth, is that it has some shear stiffness, which makes the cloth “buckle” (fold or wrinkle) when loaded in shear. This reduces the possible solutions to surfaces which cannot exhibit in-plane shear forces. To overcome this disadvantage, one could for example use special materials with negligible in-plane shear resistance or make a net by hinge-connecting numerous strings together. By recalling the Mannheim Multihalle from chapter 1.2, the “hanging net” model in Figure 1.2. was used as a form finding tool, together with numerical form finding.

Numerical form finding stands in contrast to physical form finding in that it is a form finding technique which utilizes a computer to numerically obtain a funicular shape for a given structure and load situation. There exists multiple different numerical form finding methods, and a few of them is hereby described.

Force density method

The force density method is based on constructing a net of pin-connected bars of a given axial stiffness and initial length [15]. Its mathematical formulation gives rise to a system of linear equations, making the force density method a quick and effective form numerical finding method. The governing equations of the force density method are:

$$\boldsymbol{D}_N \boldsymbol{x}_N = \boldsymbol{p} - \boldsymbol{D}_F \boldsymbol{x}_F \ ,$$

where $\boldsymbol{D}_N$ and $\boldsymbol{D}_F$ are matrices which are dependent of the topology (how the bars are connected) and the force densities (axial forces per unit bar length) of the net. $\boldsymbol{x}_N$ is an unknown vector containing the position of all nodes of the net which is not fixed, $\boldsymbol{p}$ is a vector representing the forces at each node and $\boldsymbol{x}_F$ is a vector containing the position of all fixed nodes of the net.

The force density method allows, especially in the early stages of a project, the instant exploration of large number of alternative, feasible solutions [15]. It has been applied to the design of many built structures, like the Mannheim Multihalle which was presented in chapter 1.2.

Thrust network analysis

The thrust network analysis (TNA) method for form finding is appropriate for the form finding of compressive funicular shells, like concrete or brick shells. The concept of TNA is to generalize the two-dimensional thrust line theory to three dimensions. Thrust line analysis, together with graphic statics, has been used to find stable forms of compressive masonry arches. A thrust line is a line connecting the resultant axial forces in each cross section of a structure [16]. The three-dimensional version of a thrust line is called a *thrust network*.

In TNA, only vertical loads are considered, thus the equilibrium of the horizontal force components in the thrust network can be computed independently of the chosen external loading. This allows splitting the form finding process into two steps: first solve for equilibrium of the horizontal thrust, and secondly, solve for the heights of the nodes of the thrust network, based on the external vertical loads, boundary conditions and the obtained horizontal equilibrium.

TNA allows the full control of three-dimensional equilibrium, and thus the ability to *steer* the shape in a very intuitive and flexible manner. The design process starts by constructing a flat grid, and then a corresponding *force diagram* is constructed from that grid. This diagram can tell you something about the overall distribution of forces, and can be used to make better performing structures. A three-dimensional shape is then constructed out of the equilibrium of the force diagram.

Dynamic relaxation

Dynamic relaxation (DR) was invented by Alistair Day in 1965 and is a very popular form finding methods for gridshells (gridshells is discussed in more detail in the following chapter), and includes the effect of bending stiffness of the shell [17]. Summarized, the technique traces the motion of the structure through time under applied load.

The basis of the method is to trace step by step for small increments, Δt, the motion of each point is changed until the entire structure comes to rest in static equilibrium. During the form-finding process, the numerical values of axial, and bending stiffness are arbitrary since it is only their ratio that affect the shape. If the bending stiffness is zero, the resulting shape is that of an optimal two-dimensional surface embedded in three dimensions.

The DR formulation uses Newton's second law of motion. The residual force at node i in the x-direction at time t is

$$R_{ix}(t) = M_i \dot{v}_{ix}(t) \tag{1.6}$$

where $\dot{v}_{ix}$ is the acceleration at node i in direction x. It is the sum of all the forces acting on a node from the members connected to it and the applied loading. M_i is the lumped mas at node i.

Expressing the acceleration term in equation (1.6) in a first-order Taylor series expansion around time $t + \Delta t$ gives

$$v_{ix}(t + \Delta t) = v_{ix}(t) + \frac{R_{ix}(t)}{M_i} \Delta t$$

and hence the updated geometry is obtained as

$$x_i(t + \Delta t) = x_i(t) + v_{ix}(t + \Delta t) \cdot \Delta t\ .$$

Having obtained the complete, updated geometry, the new member forces can be determined to give the updated residuals. This process is continued, through each iteration, to trace the motion of the structure. But without any damping the solution will begin to oscillate. To prevent this, damping must be introduced. By introducing damping, the motion of the structure will oscillate with lower and lower amplitude, but never quite reaching static equilibrium. By introducing a tolerance criterion, the static equilibrium can be reached for all practical purposes.

In summary, the value of using DR as a form-finding tool, comes from its ability to interactively model variations in geometry due to changes in stiffness parameters.

1.7 Grid shells

There are broadly two different types of gridshells: kinematic (strained), and unstrained gridshells [5]. Strained gridshells are constructed by bending a lattice of beams or laths into a shell, which introduces bending stresses in the process of doing so. The main advantage of this type of gridshell is that a large number of similar elements can be assembled together into a flat grid, which greatly reduces the manufacturing costs compared to unstrained gridshells, where members and joints must be specially made to be suitable for its specific geometric location. The disadvantages are that bending stresses are introduced (which was shown in chapter 2.1 was undesirable), and a lot of time is spent on site constructing the gridshell. Unstrained gridshells can be constructed entirely at the manufacturer, which greatly reduces the time spent on site.

Kinematic gridshells

As mentioned earlier, the shear stiffness of a surface makes it harder to shape the surface into a desirable geometry. A particular elegant remedy for this issue, is to use kinematic gridshells. Kinematic gridshells are made up of a quadrilateral lattice of beams or laths, hinge-connected at their intersections. Since one quadrilateral of the lattice is made up of four hinge-connected members, it has no shear stiffness. This gives the quadrilateral two additional rigid body modes compared to a continuous, rectangular element (illustrated in Figure 2.13 on a Q4 plane stress element and Figure 2.14 on a gridshell quadrilateral).

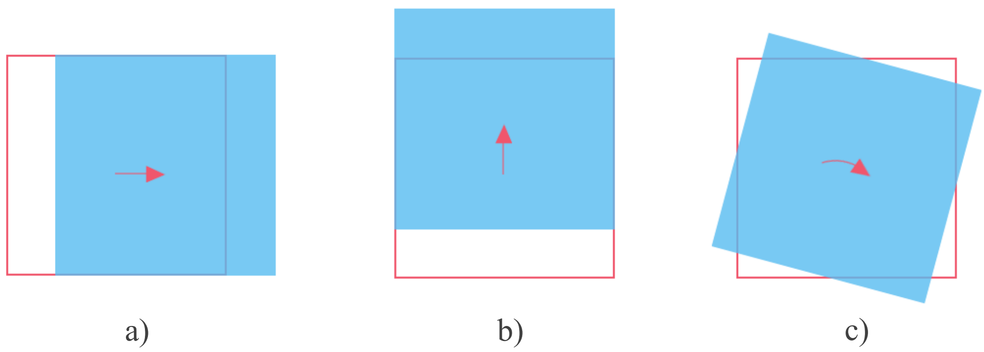


Figure 1.20: Rigid body modes of a Q4 plane stress element. a) and b) corresponds to translations, and c) to rotation.

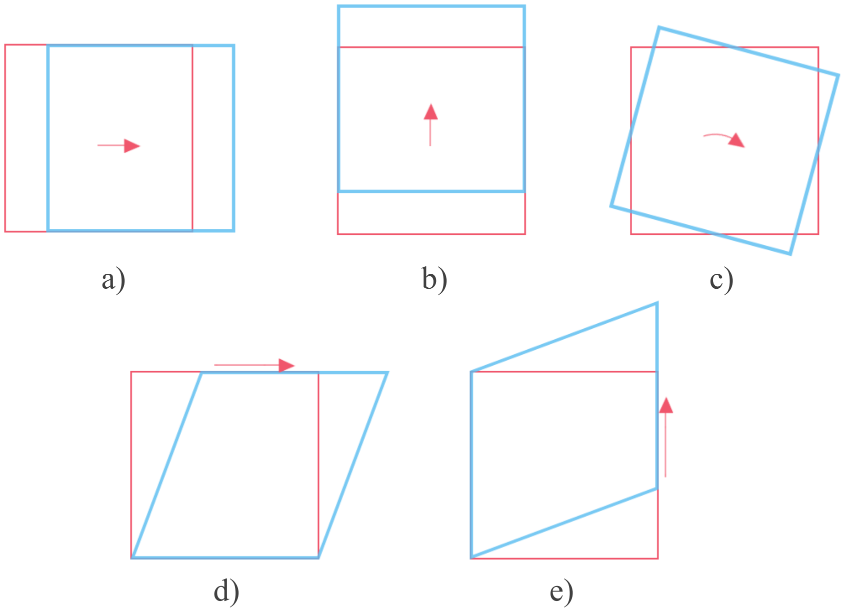


Figure 1.21: Rigid body modes of a plane gridshell quadrilateral. a) and b) corresponds to translations, c) to rotation, and d) and e) to shear deformation.

These additional rigid body modes, makes the gridshell much more flexible than a continuous shell, and is a key property for kinematic gridshells in order to shape the shell into the desired shape. But even if a quadrilateral gridshell can be used to find numerous shapes, not all geometries can be obtained. A sphere for example, cannot be divided into quadrilaterals, so it is not possible to form a perfect hemisphere from an initially flat gridshell.

These rigid body modes could also be the downfall of the shell. When the shell has obtained its final shape, it is still very flexible, and large deformations will occur when asymmetric or concentrated loads act on the structure. To overcome this issue, the rigid shear modes must again be eliminated *after* the shape has been obtained. At Mannheim Multihalle (see chapter 1.2), cross bracings made of steel cables were added to introduce shear stiffness to the grid's quadrilaterals, whereas at the Downland gridshell, diagonal timber laths were added. At the Savill Garden gridshell, the roof cladding itself provides shear stiffness to the gridshell.

The laths' layout of the flat gridshell, also affects the possible shapes. The lattice-like nature of a gridshell, basically gives rise to an orthotropic surface behaviour. This means that if the grid was laid out differently, the set of solutions of funicular shapes of the grid is altered.

Unstrained gridshells
For unstrained gridshells, the grid does not need to be made up of quadrilaterals, since the additional rigid body modes are only useful when the grid should be deformed into shape. In fact, the missing shear stiffness of quadrilaterals, makes it unsuitable for shell structures. However, if the quadrilaterals are irregular (for example if the four sides are of different lengths), some shear stiffness is introduced. Free-form gridshells, is usually not made up of shapes which can be defined by equal quadrilaterals, so irregular quadrilaterals are often used (like the YAS Hotel and the Bridge of Peace from chapter 1.2). Methods for making a quadrilateral grid from a free-form surface does exist, but is not discussed further in this thesis.

Grid optimization
How to optimize a grid, given a funicular surface, is an interesting question. This is most useful for unstrained gridshells, where the grid does not need to be made up of quadrilaterals. Topology optimization, the homogenization method and eigenshells, are all techniques used for optimizing a grid. Some effort was done to optimize the grid in this thesis, by first optimizing the thickness of a continuous shell, and then define a function which construct a grid from the optimized thickness distribution. The "optimized" grid was not used in any analysis, so is not discussed further.

2 Analysis

The analysis of shell structures with nontrivial geometries is fairly complex, and closed form solutions rarely exist [18]. Therefore, numerical models, and sometimes even scale models, plays an important role when designing shell structures.

This chapter will describe different analysis aspects when designing two different gridshell projects. The first, is a small pavilion built by two master students at NTNU, spring 2015, and the second is a conceptual exploration of the challenges that arises when a gridshell is elongated in one particular direction. The second gridshell uses results obtained from both numerical and physical tests of the former gridshell. The following chapter, chapter 3.1, describes the overall analysis procedure which is adopted for both gridshells.

2.1 General procedure

The applied analysis procedure which were adopted when analysing the two gridshells have certain common traits. They both rely heavily on the finite element method, and they both require a thorough understanding of the geometries involved.

Form finding
After the architectural constraints have been defined (i.e. sun conditions, maximum height, maximum span), the form finding process can begin. The constraints must be fulfilled to a certain extent when trying to obtain a funicular shape for the structure. Sometimes though, by fulfilling all constraints exactly, impractical solutions, which may exhibit unnecessary high stresses or deformations, can occur. Therefore, when utilizing form finding, one either must introduce *weights* to the architectural constrains, representing how important it is for the designer that these constraints are fulfilled, or describe the architectural constraints as mutable parameters, so that the architect can make qualified choices regarding the constraints. In these projects, *parametric modelling* is used, as this gives rise to a more dynamic collaboration between the architect and the structural engineer.

Form finding is a process that requires instant feedback when changing different architectural parameters. Therefore, the form finding technique used have to be quick and intuitive, so exploration of different is done effortlessly. At the same time, it has to represent stresses and be as accurate as possible. In both of the gridshells that is considered in this thesis, dynamic relaxation (DR) (see chapter

2.4) has been used, since it takes bending stresses of the laths into account under the form finding process. DR has been implemented in Rhino® using grasshopper. DR in grasshopper alone is not suitable for undertaking detailed analysis, thus after a suitable shape has been acquired using DR, the geometry must either be reproduced in or exported to a finite element software. Both shells in this thesis has been reproduced in Abaqus® after the shape has been acquired. This method is chosen mainly because when exporting the final geometry from the form finding process, the information regarding how it acquired this geometry is lost. This is most relevant for kinematic gridshells, where it is crucial to know if the shell can be built from a flat grid. In addition, the largest stresses may occur during the erection process, which makes it necessary to trace the motion of the grid from flat to curved.

Finite element modelling

When reproducing the gridshell in Abaqus, it would be highly cumbersome to model the gridshell by hand using the graphical user interface. This is because a gridshell consists of numerous nodes that has to be placed in the correct location, and then, beam elements with correct orientation must be defined between all nodes and finally, each node must be defined as a hinge connection. Therefore, it is much easier to define the gridshell by parameters. Nathalie Labonotte from SINTEF, made a python script in 2015 which produces a gridshell geometry in Abaqus given different parameters. The script correctly places all nodes, defines all beam elements with correct orientation, mesh and element type as well as defining each node as “elastic hinges”. It is highly timesaving to use this script, especially if the geometry has to be changed later on.

In Rhino, the laths of the flat gridshell all lie on the same plane, and the connection between them is a perfect hinge. In Abaqus, the gridshell is best modelled by putting the laths on top of each other. This is because a hinge in Abaqus requires two different nodes to be defined. This is also how gridshells usually is assembled on site as well. By putting the laths on top of each other, an additional level of asymmetry is introduced to the grid, since the bottom layer must be curved more than the top layer (in the case of a compressional shell), which could possibly change how the grid works.

Each connection that link perpendicular laths in the grid are modelled as perfect hinges in Rhino. In Abaqus however, the hinge-connection must have some stiffness in order to make the calculation converge. Without any stiffness, a small moment in the hinge would give rise to infinite rotations. The numerical values of the stiffness should be very small, or else they would consume too much elastic energy for the structure to function as designed. When the structure has reached its final shape, these connections are usually tightened in order to increase the stiffness of the structure. This could also be modelled numerically, but which value to use for the rotational stiffness of the hinge is not trivial. Therefore, the stiffness of the hinges is not changed in the following analyses (this is also a conservative assumption).

In kinematic gridshells, the structure is stiffened after the shell has got its desired shape. This change in the structure should be reproduced in some way in Abaqus. Abaqus do have a function called “model change”, in which it is possible to add or remove elements in a step. It is important that these elements should be stress-free when added (the elements will most likely have gotten stretched or compressed during erection).

Finite element analysis

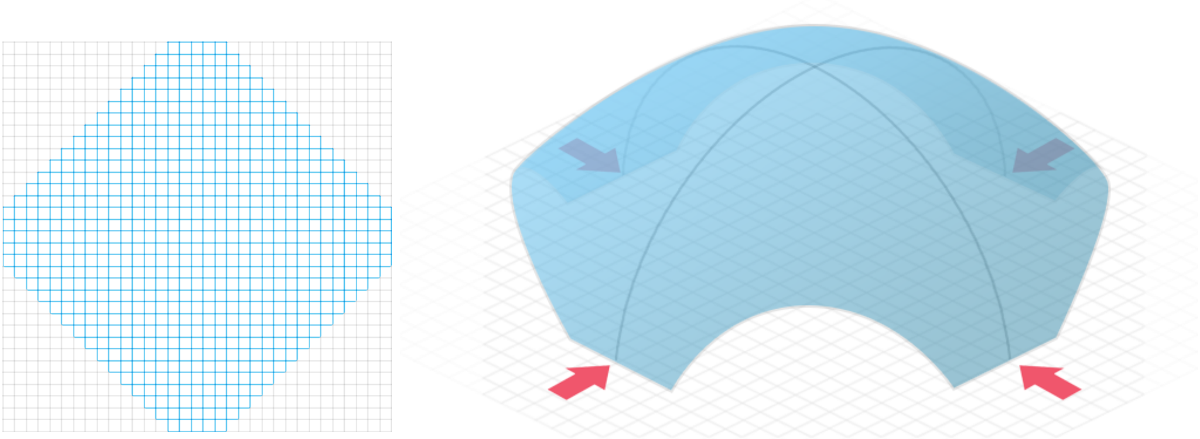
The analysis of a kinematic gridshell must usually be done in two consecutive steps: Firstly, from an initial flat grid, the given forces and boundary conditions erect the gridshell into a curved shell. Secondly, design loads are added to the curved gridshell. If, however, the bending stresses that arises during erection is small compared to the design stress of the material, these two steps may be investigated independently. In other words, consider the curved shell as stress-free, add the design loads to the structure, and add the resulting stresses to the bending stresses due to erection. In order to check whether this is a valid approximation, an investigation of the strain energy may be in place. If the strain energy behaves linearly in both steps, the approximation may be valid. If not, the two steps must be carried out successively. It should be noted that the erection process is highly geometrically non-linear because of the large deformations, so a non-linear analysis must be carried out. However, when the structure approaches its final geometry, the deformations become smaller compared to the loads, so the final structure may act linearly anyway.

The main concern in these analyses is stresses. Since the gridshell laths are made out of beam elements, it is possible to check the stresses due to axial, shear and bending forces in the laths. These forces are coupled to each other due to the geometric stiffness, so the stresses cannot in general be checked independently. However, since the laths are thin and the stresses are small compared to the deformations, shear stresses may be neglected and all stresses can be considered as normal stresses along each lath due to bending and axial forces. Also, by cross bracing or other similar techniques for stiffening the kinematic gridshell after assembly, all stresses in the stiffeners may be regarded as normal stresses. This simplifies the analysis compared to continuous shells, where the in-plane shear stresses must be considered and the direction of the normal stresses must be determined. If, however a continuous cladding is used for shear stiffening the gridshell (as in the Savill Garden Gridshell), this cladding must be considered as continuous.

Approximations
Dynamic properties are not studied in detail in these gridshells, but the first eigenmodes and the corresponding eigenfrequincies are calculated.

Loads of interest in these structures are a few of the realistic loads which may occur during the structure's lifetime. Only static loads will be considered, so wind and earthquake analysis is not carried out. In addition, temperature, humidity and soil conditions are also neglected.

Figure 2.2: Grid shell by Steinar and John. Concrete blocks and the diagonal laths that were used to fix the structure is visible.



a) b)

Figure 2.1: a) Flat grid projection as a part of a larger square. b) concept illustration

2.2 Grid shell by Steinar and John

Description

As a part of their master's thesis, Steinar Hillersøy Dyvik and John Haddal Mork designed and built a kinematic grid shell in Trondheim, spring 2015. The structure was made of wood and spanned $10\times10m$, with a maximum height of $4.5\ m$. When flat, the grid could be thought of as a truncated square with an orthogonal grid mesh (Figure 2.1a). The four shortest edges act as supports (Figure 2.1b), and the shape of the grid was obtained by digital form finding using dynamic relaxation.

When Steinar and John built the grid shell, they made a custom platform which was placed at midpoint under the grid and was used to lift the grid vertically. Subsequently, gravity pulled down the outer parts of the grid as the platform was lifted higher and higher. Eventually they forced each supporting edge to the desired position, and attached them to a corresponding concrete block (these concrete blocks are visible in Figure 2.2). In order to increase the in-plane shear stiffness, diagonal laths were added after the final shape was found (these laths are also visible in Figure 2.2).

Material and cross section

In 2015, several students at NTNU conducted a number of material tests on the timber laths that were used in this actual gridshell. Three- and four-point bending tests were carried out, and it resulted in an average Young's modulus value of about $10\,000\, N/mm^2$ and an average fracture strength of $55\, N/mm^2$. During the following analysis of this gridshell, these are the values that are used. Since most of the action in the gridshell are along the lath's grain orientation, a homogenous, elastic material model is implemented.

The cross section of the laths is rather complex, since both the position of the neutral axis and the height of the section is varying along each lath (Figure 2.3). This may lead to unfortunate stress concentrations where the cross section is thinner, due to a sudden change in geometry (as mentioned in chapter 1.3).

Figure 2.3: How the alternating lath design lead to a varying position of the neutral axis (red).

In addition, since the neutral axis is varying, additional bending moments will be introduced in the laths in the presence of axial forces. These bending moments can be estimated. If h is the height of each lath, and by examining Figure 2.3, it is evident that the maximum bending moment due to a horizontal axial load N is $1/2\, Nh$ (Figure 2.4). This effect is initially neglected during the analysis, but by first calculating the maximal axial forces due to straight beams, the accuracy of the approximation can be evaluated.

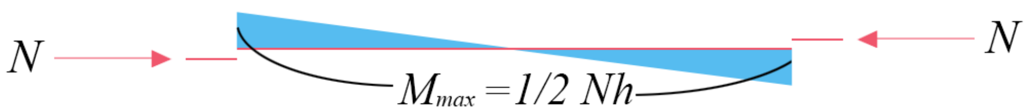


Figure 2.4: Bending moment due to the antisymmetric design of the beams

Analysis overview

History and experiments shows that the stresses which arise during construction of a kinematic grid shell, could be the largest stress values the structure is exposed to. Because of this, it is important to account for these stresses in a controllable and reliable manner. The construction process can be rather complicated to model numerically, since it is highly nonlinear. In addition, the loads which are forcing the structure in place can be difficult to predict in advance. When analysing this gridshell, two different approaches were studied:

- Applying an inverse gravity load on the flat grid, and

- introduce small perturbations, followed by fixed displacements of the supports.

Inverse gravity method

The inverse gravity method is interesting to investigate since this is what the architects used in Rhino when they originally ran the form finding analysis. First, the flat grid was defined and constraints representing the supports were added. The support constraints were defined by the architects to be curved. This was done because they felt like the curvature of the shell became more apparent. From a structural engineering viewpoint, it could also be beneficial for the structure, since this additional curvature may give the shell more doubly curvature near the supports (see chapter 1.3). However, this will most likely introduce additional bending moments in the support laths, so this should be taken into account.

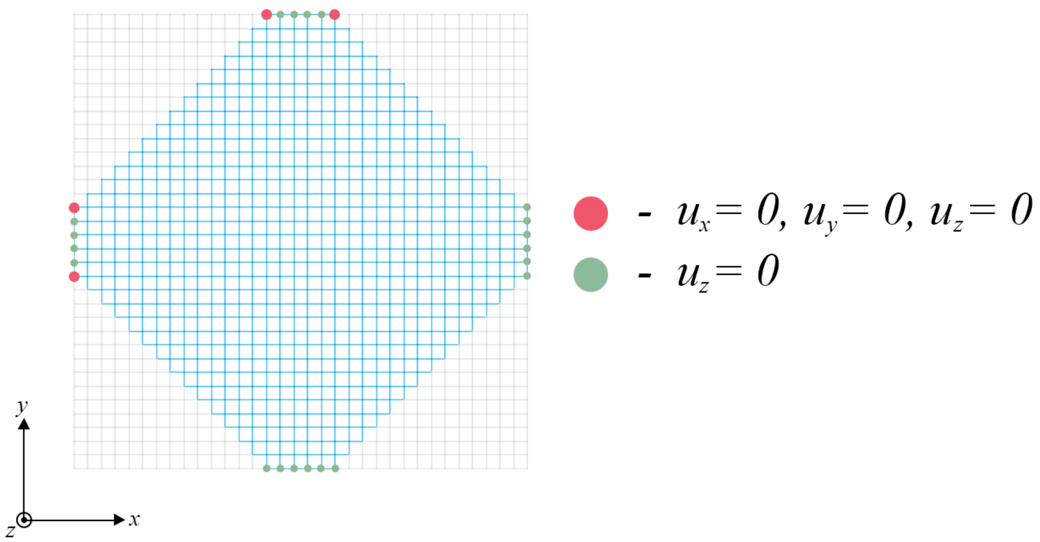


Figure 2.5: The initial support conditions when the inverse gravity method was used.

When modelled in Abaqus, the supports were defined a bit differently. Here, the outmost laths of the supports were “simply supported”, and the other ones were pinned to the ground (see Figure 2.5). This was done because it was desirable to investigate how the supports would naturally differ from a straight line during erection. The magnitude of the inverse acceleration of gravity was then adjusted until the structure obtained its designed height and width. Finally, the acceleration of gravity is reversed to its natural, downward-pointing direction. The result can be seen in Figure 2.6.

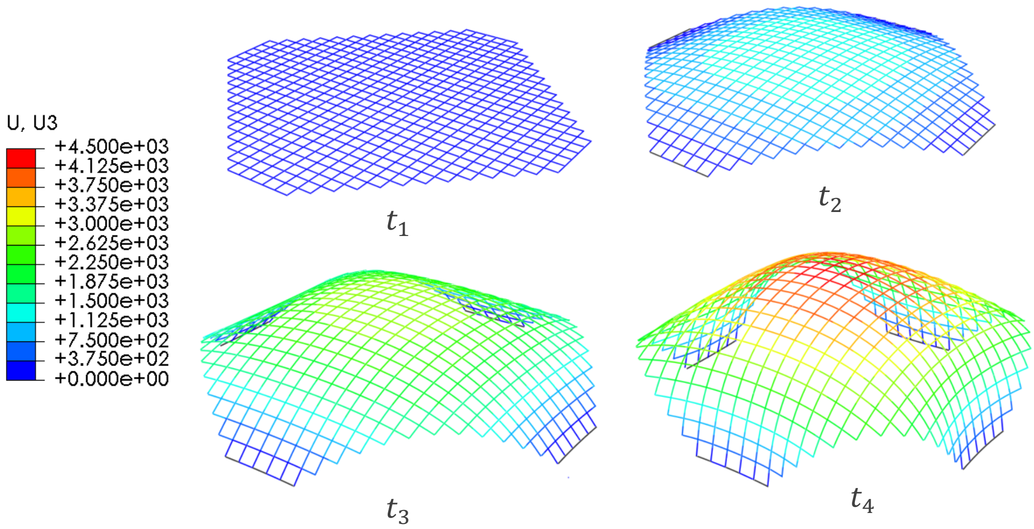


Figure 2.6: Different steps in the erection process, when the inverse gravity method is used.

It is evident that the edges of the support remain virtually straight by using this technique. One might add additional boundary conditions which displaces the supports into a curve, but this was not done here. It is important to understand that when using this method, there is no downward-pointing gravity during the erection. First after the shape is realized, the boundary conditions are locked in place and gravity is added. This may lead to an unrealistic solution, that might be very difficult to build in real life. The next method introduces a technique for obtaining the shape while gravity acts on the structure.

Buckling the grid

By displacing the supports to the correct position right away, the gridshell will intuitively “buckle” into shape. Some small perturbations must be introduced in the gridshell before the supports are displaced, or else the perfectly flat grid might buckle locally, or the incorrect global buckling mode might be activated. The prescribed displacements are only applied to the middle laths of the support, in order to get the curved support conditions that the architects designed (Figure 2.7). The result of this method can be seen in Figure 2.8.

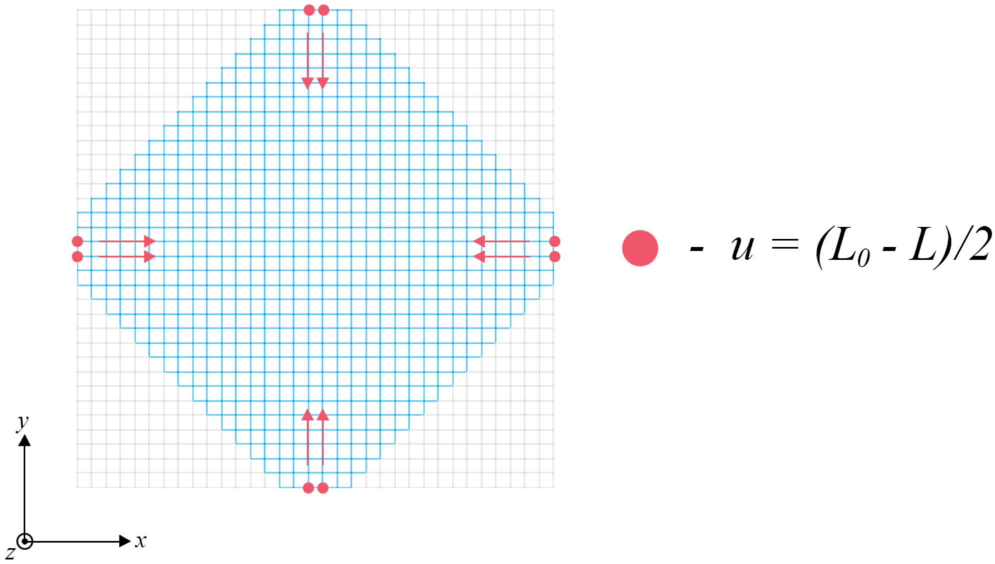


Figure 2.7: Diagram showing how the middle laths of each support gets displaced. L_0 is the gridshell's length when flat, and L is the span length after the desired shape is obtained

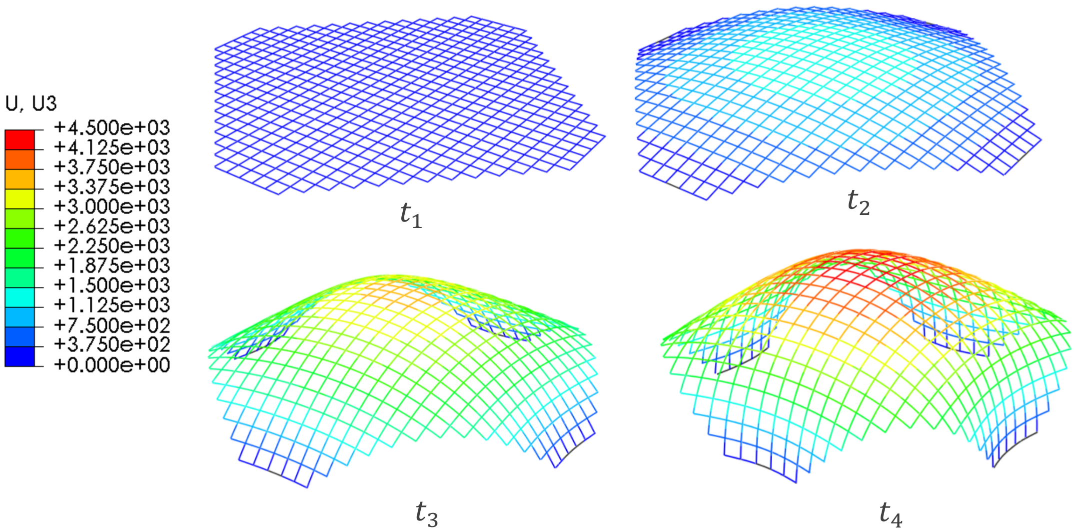


Figure 2.8: Different steps in the erection process, by displacing the supports into the desired position. The colours represent the vertical displacement measured in mm.

The main difference here, is that the supports now have obtained its curvature. By using this method, not only does the supports get the desired curvature but in addition, it is a more realistic model of the erection process, since gravity acts on the structure during the whole process. It is also much easier to obtain the desired result with different parameters because the displacements are prescribed, in contrast to the former method, where the inverse gravity must be fine-tuned in order to get the desired result. Therefore, this is the model that is used henceforward.

Stresses due to assembly and gravity

The first stress state to examine, are the normal stresses (first principal stresses) in the laths after assembly. It is expected that the maximum values of the normal stresses occur in the lath with the most curvature since the axial stress should be virtually uniform in each lath. The deformed structure coloured in with normal stress values is shown in Figure 2.9.

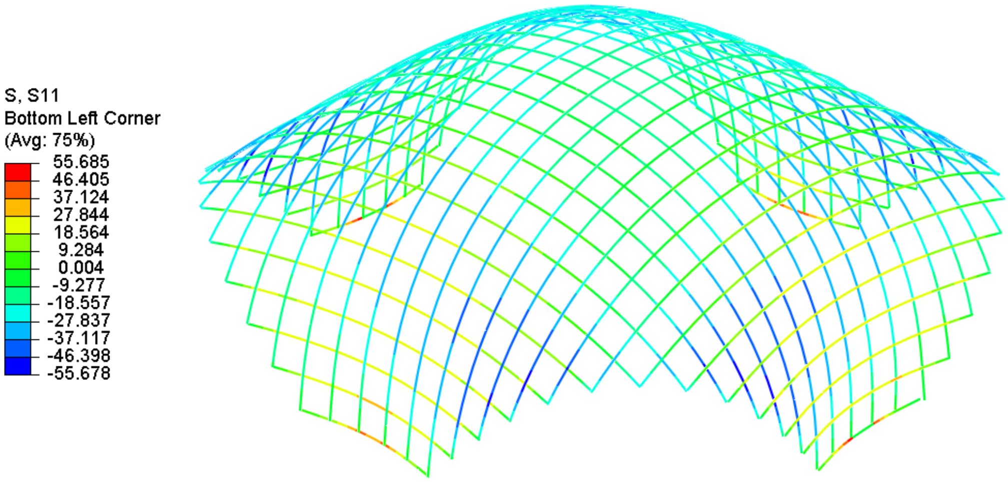


Figure 2.9: Plot of the normal stress distribution after erection. The stresses are measured in N/mm^2

It is notable that the highest normal stress values occur where the curvature appear to be the highest, which was expected. What is not so obvious is why the adjacent sides, which appear to have similar curvature, does not have the same stress values. This is probably due to the fact that the laths lie on top of each other, introducing an additional level of asymmetry to the structure. It is, however, bizarre that the section forces shown later are very symmetrical. In other words, the asymmetry in the stress distribution must be caused by something else than the section forces.

It is most alarming that already after assembly, the structure exceeds the fracture strength. This was actually noticed on site as well, since a lot of the laths broke while assembling them. Two possible remedies for this includes: use a stronger material, and utilizing a doubly layered gridshell, as was done in Mannheim Multihalle (see section 1.2). By adding diagonal stiffeners after erection, the structure may also be able to carry larger loads.

The next stresses of interest are the shear stresses (second principal stresses). Since the geometry of the gridshell should be funicular, it is expected that very little shear stresses due to bending moments should arise. Again, a plot showing the shear stresses is given in Figure 2.10.

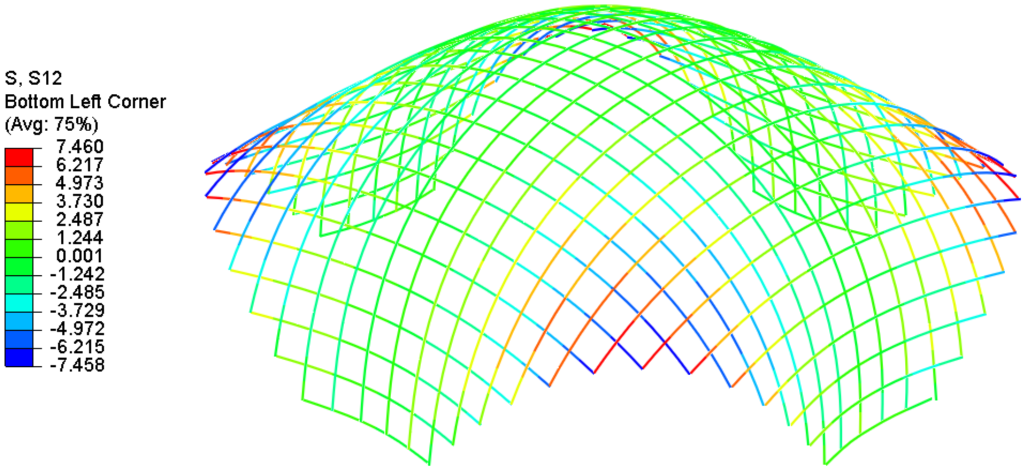


Figure 2.10: Deformed plot of the gridshell showing the shear stresses in N/mm^2

Apparently, some shear stresses arise at the cantilevering parts of the gridshell. One might first think that these stresses arises due to defects in the geometry that produces bending moments, but in reality, these shear stresses arise due to torsional deformation of the laths. These values are also alarming, since the shear strength of timber is usually taken to be about 5 N/mm^2. A magnified view of the torsional deformation can be seen in Figure 2.11.

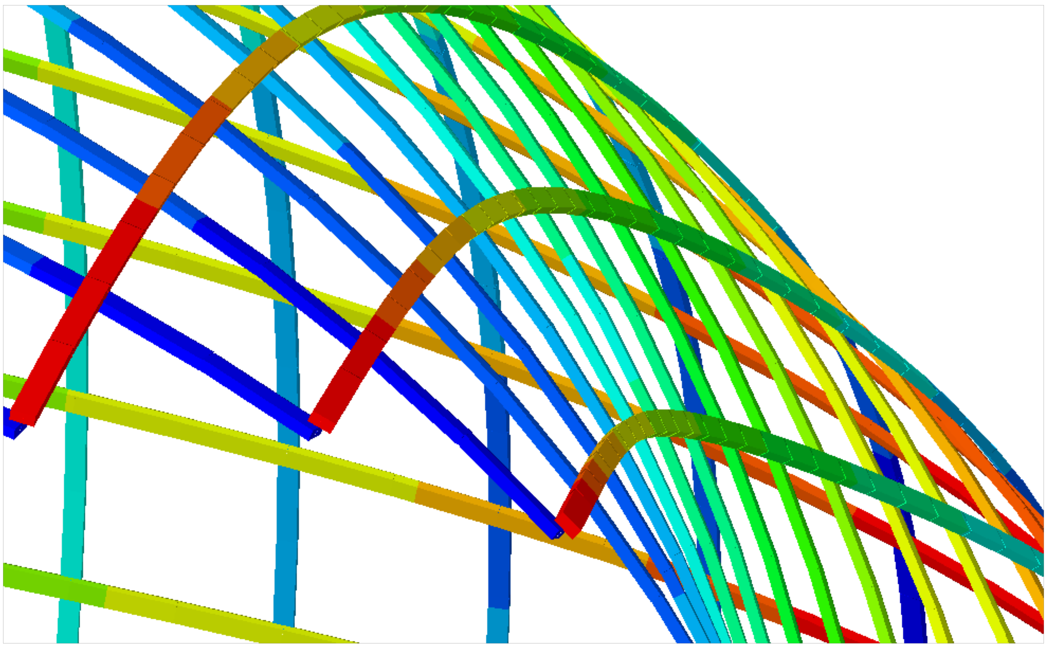


Figure 2.11: The rotation of the lath's cross section (torsion) gives rise to shear stresses.

To verify that it actually is torsion that produces these shear stresses, the torsional moment is shown in Figure 2.12. One possible remedy for these stresses is to attach diagonal stiffeners, which helps restrain the torsional rotation of the laths.

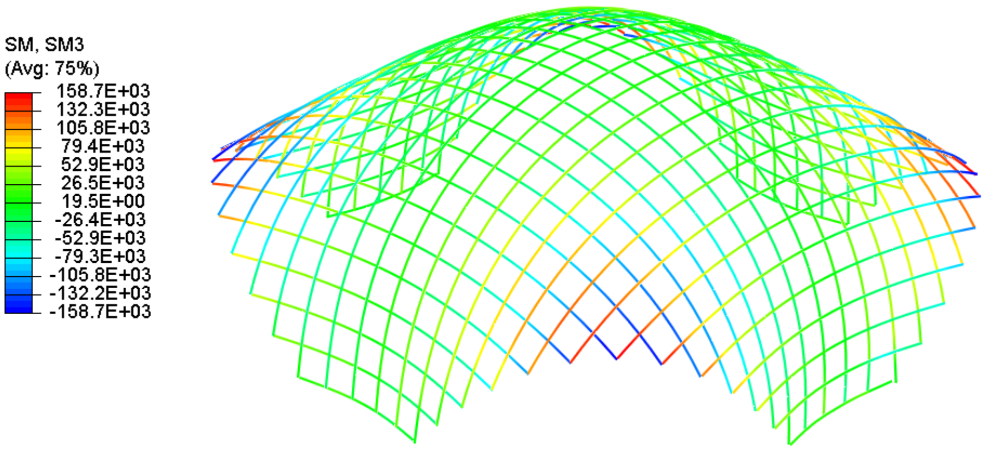


Figure 2.12: Contour plot of the torsional moment of the laths.

Bending moments

In addition to the torsional moment, it is also interesting to look at the bending moments. Since the laths are bent about two different axes, the bending moment about both the minor and major axes (assuming $b \times h = 48 \times 23mm$) are shown in Figure 2.13 and 2.14 respectively.

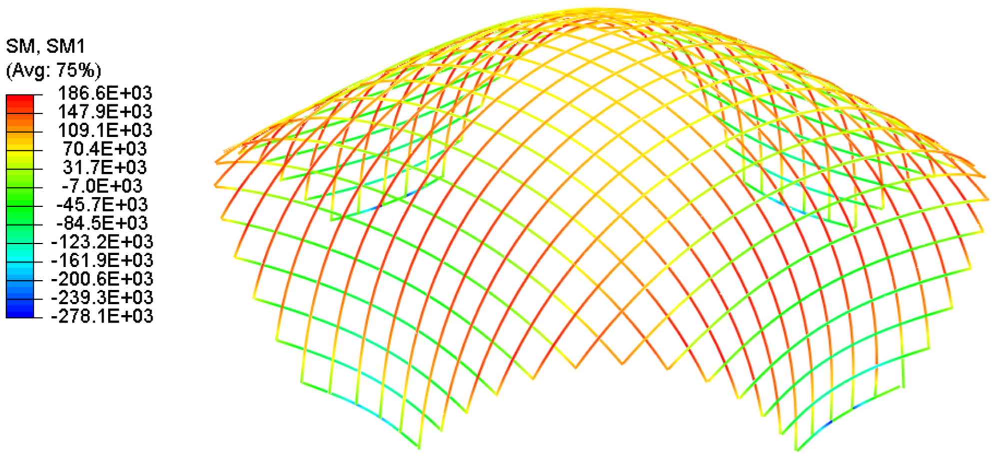


Figure 2.13: Contour plot of the bending moment about the lath's minor axes.

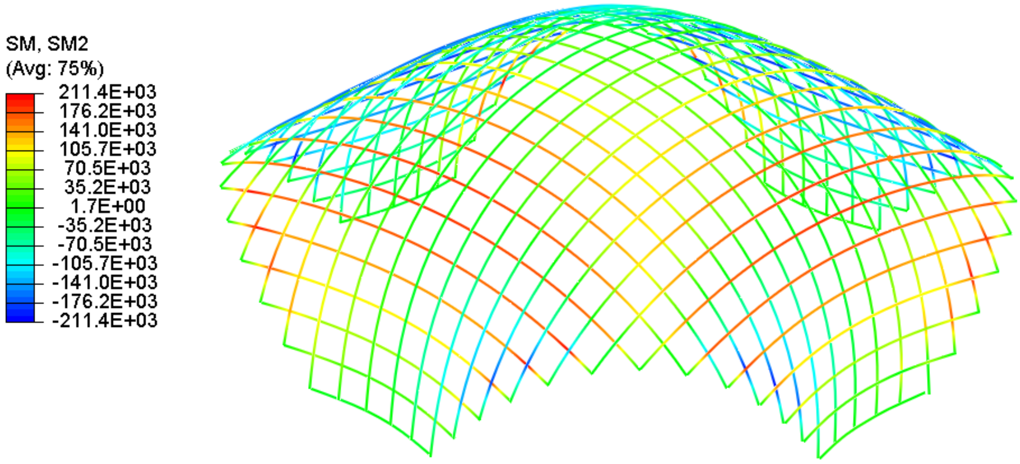


Figure 2.14: Contour plot of the bending moment about the lath's major axes.

The maximum bending moments appears on the support laths on Figure 2.13. The curvature imposed on the supports may therefore be too extreme. If the bending of the support lath is ignored, the largest bending moments apparently emerges in the "horizontal" laths (Figure 2.14). These bending moments gives rise to maximum bending stresses about $50\ N/mm^2$, which is fairly close to the failure strength. These laths however, does does not undergo as much axial compression as the vertical laths. Consequently, the total stresses in the vertical laths might be higher.

Stresses due to additional loading

In fall 2015, after the gridshell was built, numerous load tests were conducted on the gridshell. The tests were carried out by hanging weights on different parts of the gridshell. Subsequently, the deformation of the structure was measured by both a handheld laser scanner and a 3D point-cloud scanner.

The tests showed that the gridshell with cross bracings was remarkably effective when carrying symmetrical loads. When loading the gridshell with the maximum load that was available, it was loaded in 8 nodes with each weight weighing 90kg. The maximum deflection that was measured was a total deflection of about 86mm. This deflection was located on the apex of the gridshell, and was hardly noticeable to the naked eye.

When the cross bracing was removed, the same load gave a maximum deflection of about 300mm. On site, this deflection was quite noticeable and also amplified some asymmetries in the structure. These asymmetries probably have roots in an imperfect assembly, but could also be due to the fact that the structure began to contact a tree on one side of the gridshell.

In Abaqus, only the situation where the cross bracing was not present was studied. This was done in order to simplify the modelling process and because the small deflections measured was too small to do any decent numerical comparison with. Figure 2.15 shows the Abaqus model with cross section $b{\times}h = 48{\times}23mm$, and how that model deflected when subjected to the symmetric loading.

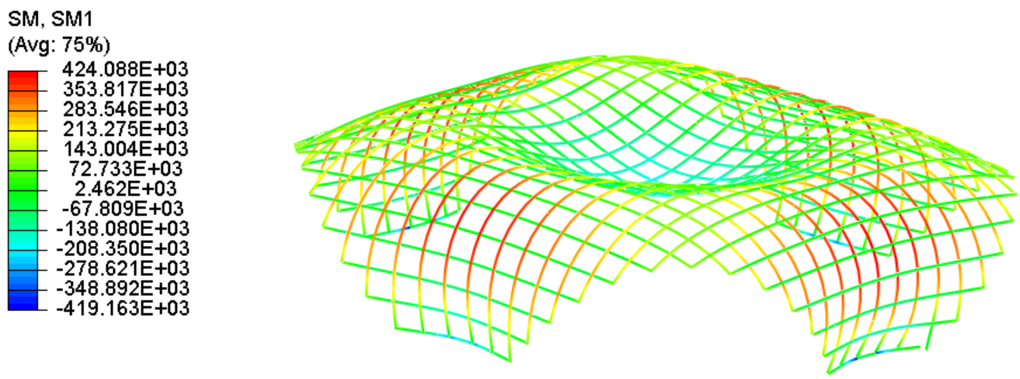


Figure 2.15: Deformation and bending due to symmetric loading by eight 90kg weights.

As is evident, the deformation in Abaqus exceeds 300mm quit a lot. The maximum deflection measured in Abaqus is about 2500mm. Presumably, the cross section is too small in Abaqus to carry these weights, consequently the cross section should perhaps be altered. If it is assumed that the cross section is $b{\times}h = 48{\times}48mm$ instead, the following deformation pattern is obtained (Figure 2.16).

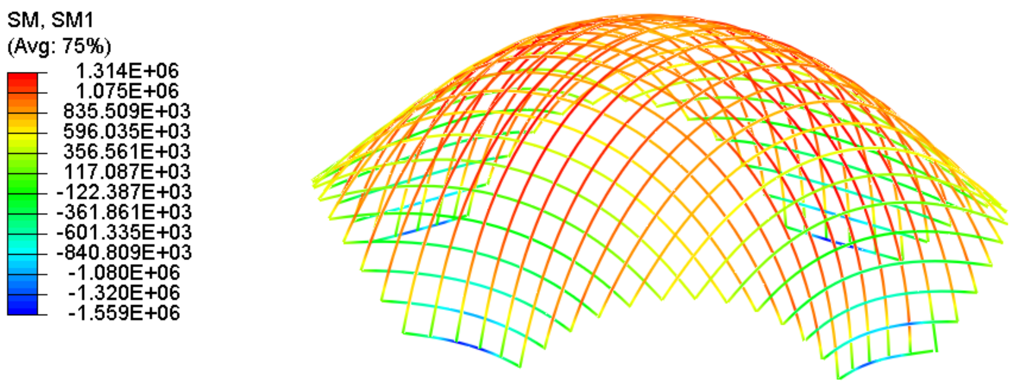


Figure 2.16: Deformation and bending due to symmetric loading, with $b{\times}h = 48{\times}48mm$ *laths*

Now, the deformation measured in Abaqus is about 150mm, which more accurately resemble the real structure. However, two problems now have appeared. Firstly, the bending moments emerging due to the erection of the gridshell are now three times higher. This gives rise to a bending stress of about $118\ N/mm^2$, which clearly would break the laths. Secondly, the “openings” of the shell, is too shallow, measuring about 1.5m above ground, which is $500mm$ lower than the real structure. This discrepancy could be due to the cross section of the laths, which act differently than expected since the two layers of the cross section, may have some complex interactions between them. Another possibility, is that since these tests were conducted 4 months after the gridshell was built, the timber may have undergone relaxation and sliding movement due to viscous material behaviour and friction, which may have changed the shell's structural performance.

During erection, the connections were fairly "loose", making the two layers of the gridshell act more independently. After the final shape was obtained, the connections were tightened, and the two layers acted more dependently after that. This might explain why the structure did not break during assembly, but at the same time is strong enough after assembly to resist heavy loads. To model this, a higher "effective cross section" may be used. By altering the cross section to $b \times h = 48 \times 40 mm$, the correct deflection is obtained. This is not a conservative assumption, but it is nonetheless interesting to think about what might be the cause of the differences between the numerical values and the measured values.

The more interesting situation is when the loading is asymmetrical. When loading the gridshell asymmetrically, one of the laths at the support failed after seven point loads of 90kg (See Figure 2.17). The failure appeared due to a combination of large bending moments and compression. The result from Abaqus, when the model is subjected to the same loading is seen in Figure 2.18.

Figure 2.17: The outmost lath of the support failed in bending under asymmetrical loading

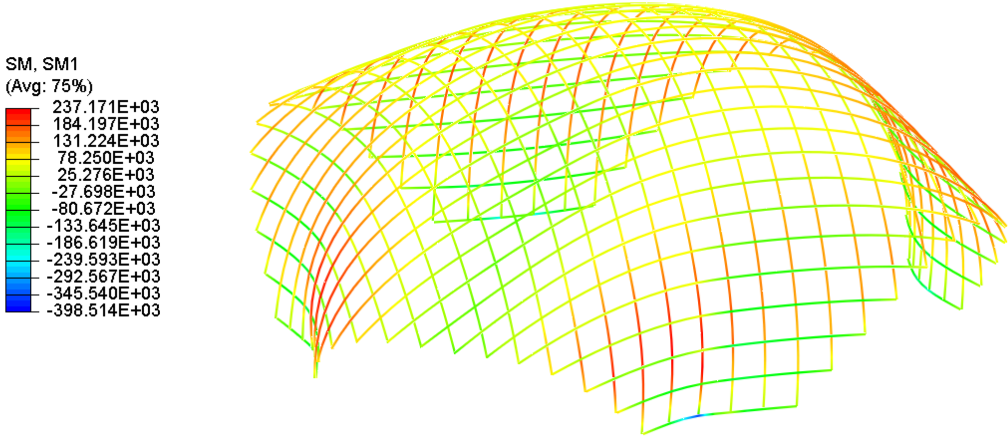


Figure 2.18: Deformation due to asymmetrical loading. Large bending moments near the supports are visible.

The figure shows the bending moments in the laths, and the largest values are consistent with where the failure actually happened. The large bending moment of the horizontal support laths is not particularly realistic, since the real structure is able to rotate this part without much bending arising. The large bending moment in the Abaqus model surface due to the way the boundary conditions are defined.

Eigenmodes and -frequencies

An examination of the first eigenmodes and –frequencies is interesting, because these properties give insight to the overall stiffness of the structure as well as which load situations that are most critical. The first two modes are similar modes in different planes, and give rise to the same frequency (Figure 2.19).

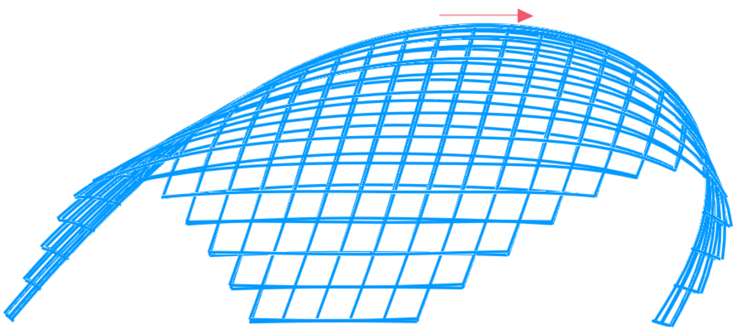


Figure 2.19: (Side view) The first and second eigenmodes corresponds to translation of the top part of the gridshell.

These mode's eigenvalues were calculated to be about $0.56\ Hz$, which is very low. The stiffness of the structure in the horizontal direction is in other words small compared to the vertical stiffness, which is not surprising given the shearing rigid body modes that are present (see chapter 1.7). The asymmetrical loading from before showed the same phenomena, where asymmetric loading gave rise to large deflections. By bracing the structure, the structure's stiffness increases, and so will the corresponding eigenfrequencies.

The third mode corresponds to a rotation of the top part of the gridshell, and displayed an eigenvalue of about $1.00\ Hz$ (Figure 2.20).

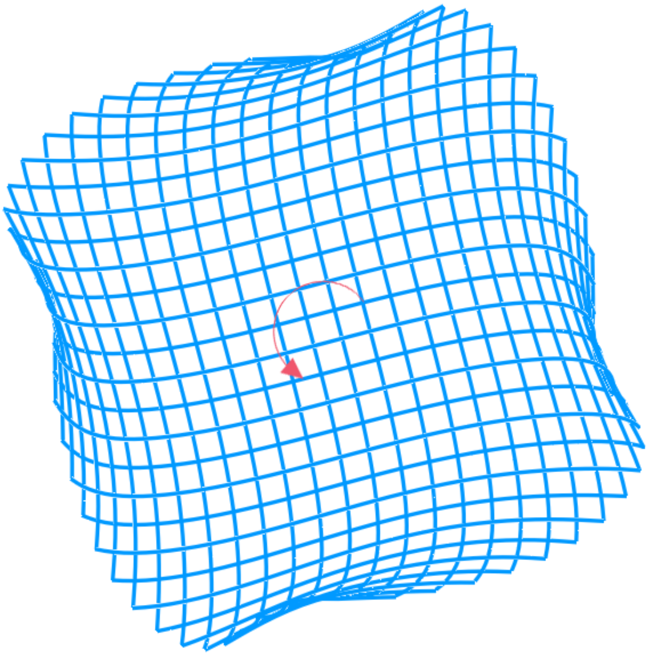


Figure 2.20: (Top view) The third mode corresponds to a rotation of the top part of the gridshell.

The frequency of this mode is also quite small. The same phenomenon is displayed, namely that the in-plane shear stiffness is very low. Again, bracings would have stiffened this mode as well, simply because it is the shearing that gives rise to the low frequency.

The fourth mode is particularly interesting, since it by inspection almost exclusively consists of shear deformations of the quadrilaterals (Figure 2.21). If the gridshell laths had lacked bending resistance, this mode would perhaps be the lowest mode. The bending resistance of each lath helps stiffen this mode.

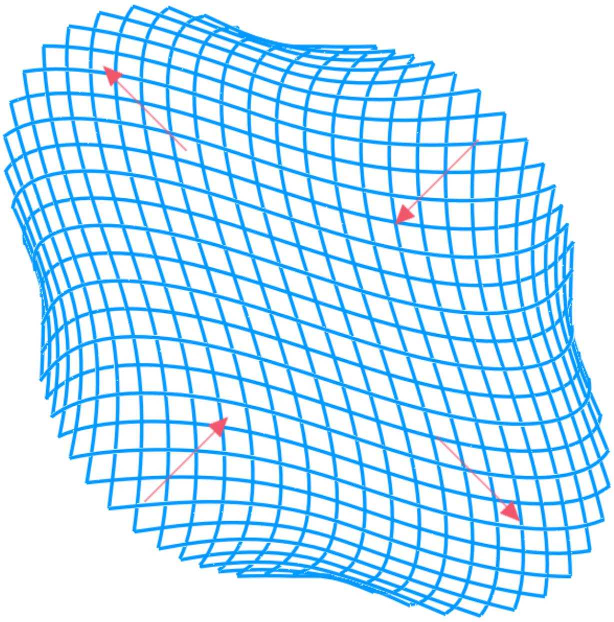


Figure 2.21: (Top view) The fourth mode corresponds to "squeezing" the top part of the gridshell.

The calculated eigenfrequency of this mode was found to be about $1.53\ Hz$. It is still a low frequency, but nevertheless three times larger than the two lowest frequencies. This mode would clearly benefit from bracing, as almost all motions consist of shearing the quadrilaterals in the grid.

Summary

To summarize, the gridshell works very well when subjected to symmetrical loads, and, by adding the bracing, the shell is stiffened a whole lot. It was seen that the first four eigenmodes were modes in which bracings would have increased the stiffness significantly. The problem arises when the loading is asymmetrical and the bracing is removed. High bending stresses appear near the supports, which introduces a possible failure mode. The experiment that was conducted on site showed that the structure failed in that area. In addition, the lath, not surprisingly, failed where the cross section was the smallest.

Together with Nathalie Labonette, John Haddal Mork, Steinar Hillersøy Dyvik, Anders Rønnquist and Bendik Manum, it was written a paper for the WCTE (World Conference of Timber Engineering) 2016 regarding snow loads on this gridshell. The paper concluded that the gridshell was not able to resist the (asymmetrically loaded) design snow loads in Trondheim. The full paper can be read in appendix 1.

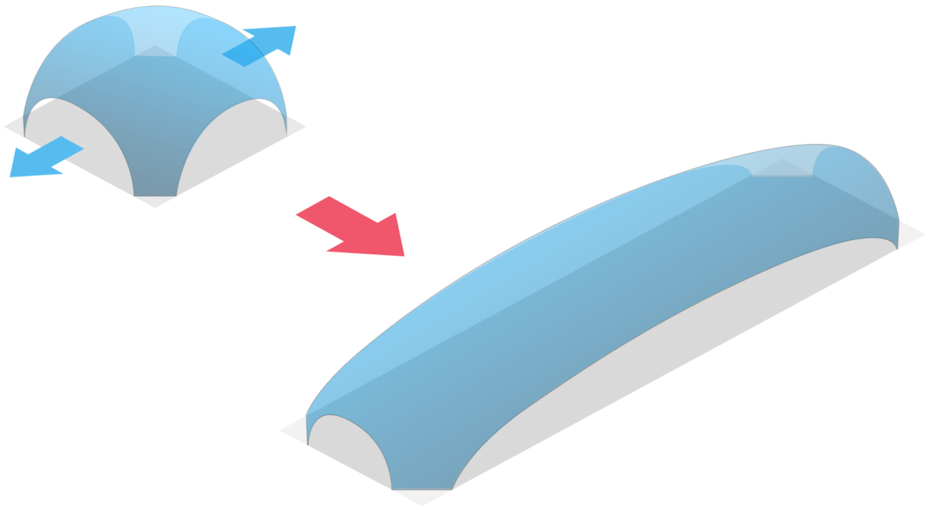


Figure 2.22: Elongating the square grid shell, concept illustration.

2.3 Elongated gridshell (gridshell bridge)

Description

After studying the gridshell built and designed by Steinar and John, thoughts around larger gridshell immediately surfaced. What happens with the structural performance if the size of the gridshell was much larger? Mannheim Multihalle (section 1.2) is perhaps the best example of a gridshell structure where the possibilities of large gridshell structures was addressed. Where the Mannheim gridshell differs the most from Steinar and John's gridshell, except for its size, is that the Mannheim gridshell is continuously supported almost around the whole structure, whereas at Steinar and John's gridshell, only a portion of the shell's total circumference acts as supports. This often leads to more interesting and surprising funicular geometries during the form finding, but also requires more load to be taken by the support laths.

My supervisor, Nils Erik Anders Rønnquist, proposed to study a gridshell structure which not only was large, but was way larger in one spatial direction than the other: an elongated gridshell. The gridshell should not be continuously supported, like that of Mannheim, because this would lead to a barrel vault-like funicular geometry. The basic idea was to take the gridshell that Steinar and John had designed, and "stretch" it in one direction (Figure 2.22). Rønnquist got the idea when studying the Cascara bridges in Madrid (Figure 1.6), and wondering how bridges like these would have looked like if they were gridshells. The cascara bridges do have some striking shell geometries, but the continuous concrete surface also absorbs a lot of light from the surrounding environment. The gridshell spans on the YAS Hotel in Abu Dhabi (Figure 1.5) and on the Bridge of Peace in Tbilisi (Figure 1.7) are perceived as lighter structures, and does not absorb as much light from the environment. Both the YAS Hotel and the Bridge of Peace are freeform shells, and is therefore not a direct product of form finding. On The Bridge of Peace, especially, it looks like the steel members have larger cross sections than they need to, and this could be avoided by a combination of having a denser grid (like on the YAS Hotel) and/or by utilizing form finding to make sure the geometry is funicular.

The question whether the elongated gridshell should be kinematic or not, will remain an open one, but it is initially assumed that the gridshell is kinematic. This is chosen because the gridshell by Steinar and john is kinematic, and it is desirable to investigate the effects of elongating this particular gridshell. In addition, when the gridshell is kinematic, the “meshing” (dividing the gridshell up into grids) is much easier because it is done on a flat surface. Some discussion about optimizing the grid will surface later on, and then the gridshell is no longer considered as kinematic.

Predictions

When the gridshell is elongated as shown in Figure 2.22, the structure will probably act more like an arch, and less like a shell. This means that the geometry perpendicular to the span will be of less importance, or in other words; the beneficial effects due to double curvature may vanish. The span-to-width ratio of the shell should therefore not be too extreme in order to preserve the shell behaviour.

It is expected that, due to the small supports compared to the size of the structure, that the beams near the supports will exhibit a lot of axial forces, especially under asymmetrical loading, which were the weakest parts of the previous shell as well. Since the structure would act more like an arch when elongated, it is expected that it is even more prone to asymmetrical loading. It will also probably be necessary to introduce “edge beams” in order to make the shell withstand heavier loads.

Numerical form finding

During the form finding process, tight cooperation between engineer and architect was desirable in order to, not only make a shell that is structurally sound, but also a shell that is aesthetically pleasing and fulfil the architectural constraints in a good way. In addition, Steinar as the architect, has experience with form finding from earlier, and he is able to obtain numerous funicular shapes very quickly given parametric boundary conditions and constraints. Dynamic relaxation, implemented in Rhino and Grasshopper, is the form finding method used.

When defining the boundary conditions, the Cascara bridges were used as a reference because it was not clear how the flat geometry should be defined in order to obtain a pleasing result. If one of the Cascara bridges had been laid out flat, the geometry would have been something like the illustration on Figure 2.23.

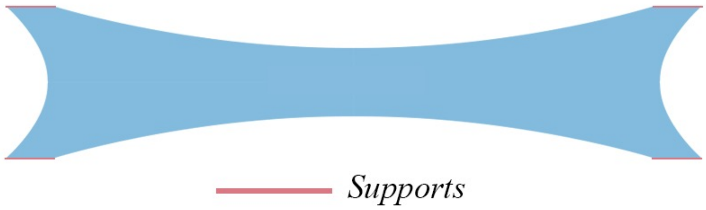


Figure 2.23: A flattened version of one of the Cascara bridges

Initially, we made a gridshell out of the same geometry, but due to the "meshed" nature of the flat grid shell, the curves became jagged which did not look particularly good. A simple, square grid plan was then proposed, keeping the same boundary conditions (Figure 2.24a). The geometry of this plan was expected to turn out like the geometry seen in Figure 2.24b.

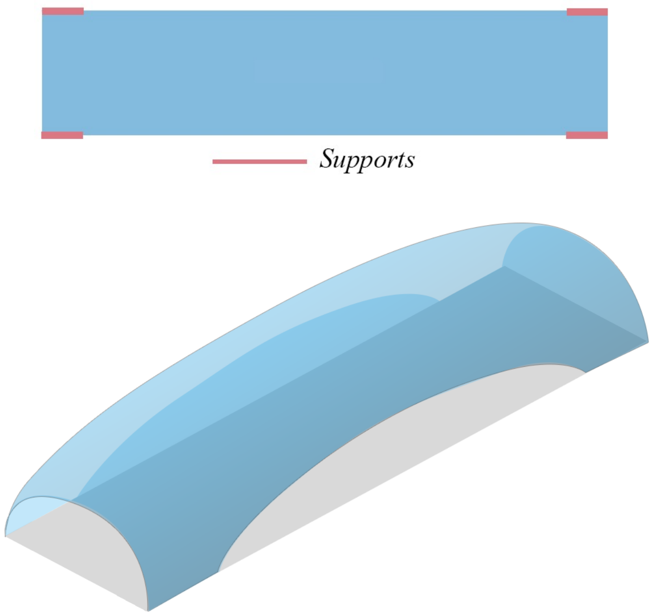


Figure 2.24: A square grid plan (a), and how its expected funicular geometry would look like (b)

From their previous gridshell, Steinar and John learned that the orientation of the grid alters the funicular geometry quite drastically because the directions of anisotropy changes. Different orientations of the flat grid were therefore tested in order to see how this affected the shape. In addition, he introduced a "curved" support line (like in the previous shell) in order to impose some double curvature.

First, we tried with a regular, orthogonal grid (Figure 2.25).

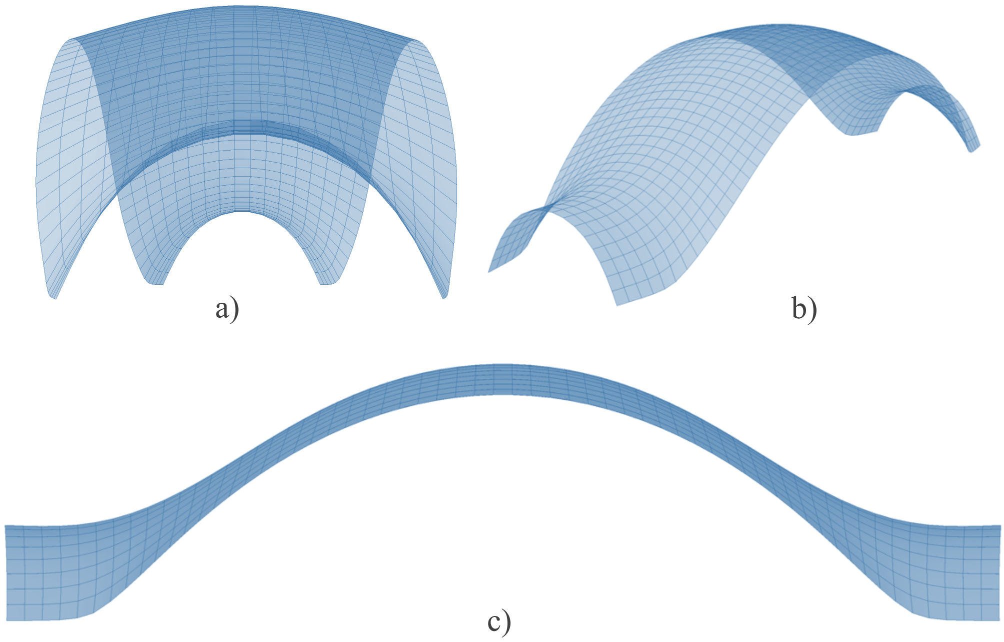


Figure 2.25: Solutions to the form finding process with an orthogonal grid mesh. a) Front view. b) Perspective view. c) Side view.

As can be seen, the openings at the ends act like simple barrel vaults, while the middle part acts more like a wide arch. It is interesting to see how the barrel-vault-like geometry merges into the arch, forming a sort of hybrid structure. There is an absence of double curvature in the middle part, which was expected. It is possible to impose some double curvature on the geometry by remembering that funicular geometry is dependent on the load situation. In other words, by introducing permanent loads to the structure, the following funicular geometry should change. If we now assume that the elongated shell structure should be a bridge, and the weight of the load is carried by cables, we can use this as our additional loading. Figure 3.26 shows a geometry where the middle part of the structure has been pulled down.

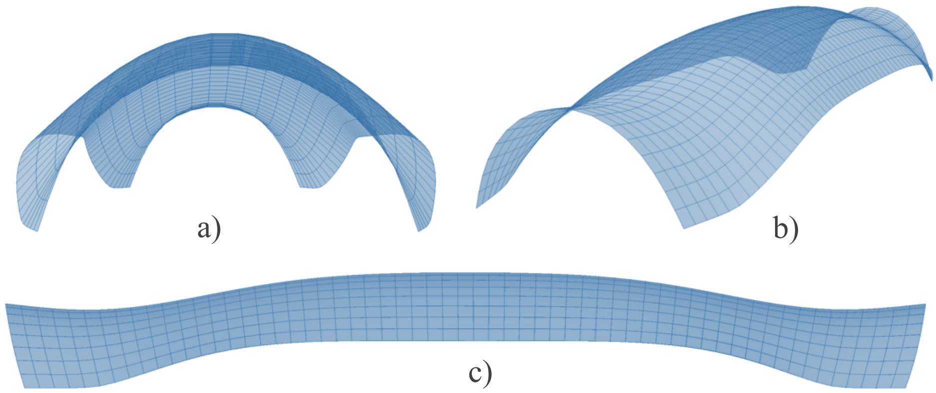


Figure 2.26: Solutions to the form finding process with an orthogonal grid together with edge loading. a) Front view. b) Perspective view. c) Side view

Now, the shell looks more like a long barrel vault where the middle part is supported higher above ground. The imposed loads naturally lower the total height of the bridge as well, which makes the bridge act like a barrel vault which is supported at the ends. This will probably lead to some longitudinal tension forces in the bottom part of the shell, and longitudinal compression forces in the top part (bending action). From chapter 1.3 it was shown that such action lowered the utilization grade of the structure, so it should be avoided. What is good with an orthogonal grid mesh is that the curves on the edges that arises during the form finding process is very smooth and visually pleasing. The main disadvantages are that a significantly amount of the shell is singly curved like a barrel vault, and that the flow of forces is not directed towards the supports. The forces must therefore change direction in order to get absorbed by the supports, which can be made possible by introducing edge beams.

By choosing a diagonal grid mesh instead, very different results are obtained. We first tried without pulling down the middle part of the shell, and acquired some troubling shapes (Figure 2.27).

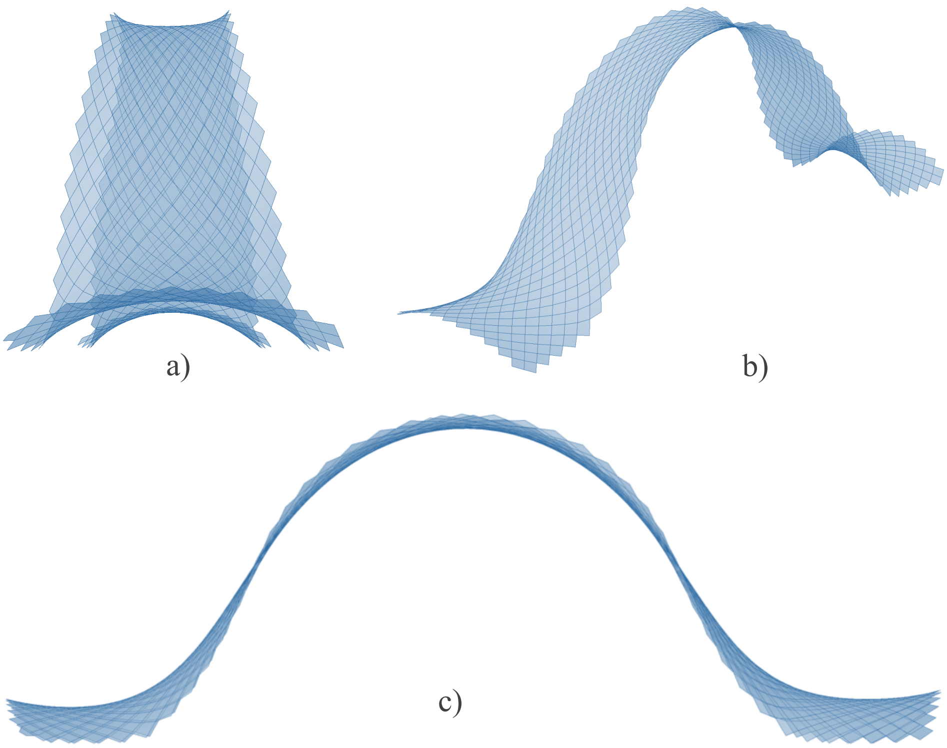


Figure 2.27: Solutions to the form finding process with a diagonal grid mesh. a) Front view. b) Perspective view. c) Side view

Due to the low shear resistance of the grid, the shell became extremely elevated in the middle part, which again led to that the parts near the supports became too shallow compared to the desired result. What is interesting, is that the edges on the middle part actually bend upwards. The gridshell was able to do this, again, because of the low shear stiffness in that direction. By again introducing forces resembling the cables of the bridge, a much more pleasing results was acquired (Figure 2.28).

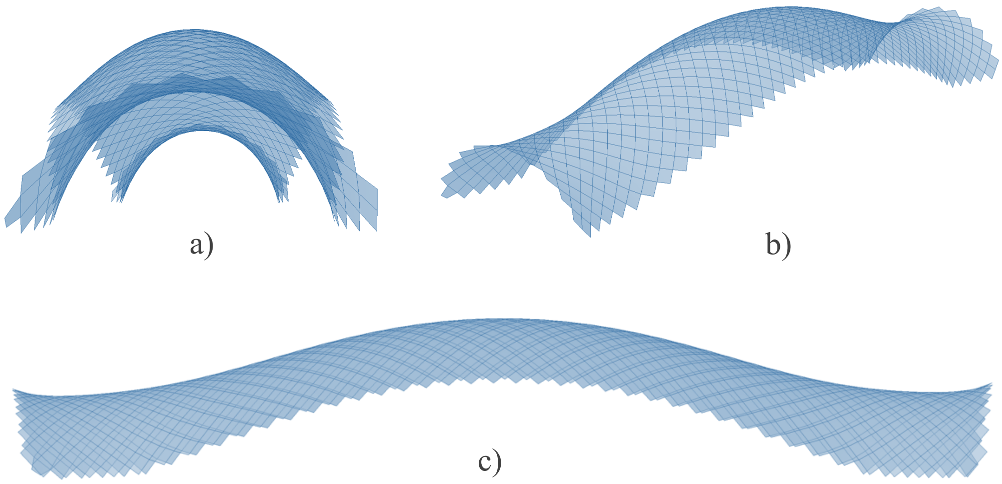


Figure 2.28: Solutions to the form finding process with a diagonal grid with edge loading. a) Front view. b) Perspective view. c) Side view

Now, the shell clearly has obtained its desirable double curvature as well as sufficiently high openings at the ends. Due to the diagonal layout, the flow of forces is also more directed towards the support, which helps smoothing out the stress distribution. The main downside with this shape is that the jagged edges obstruct the smooth curves obtained when using an orthogonal grid. A possible remedy is to introduce edge beams to smooth out the jagged lines as well as providing stiffness and additional strength to the structure. The shape obtained here is what will be used as a reference further on during the analysis.

Physical form finding

A quick physical "hanging chain" model was made to assess how the physical model differed from the numerical one (Figure 2.29). The physical form finding model showed that by pulling down the middle part of the shell, double curvature will be imposed. In addition, near the edge openings, the shell "bends" upwards as it did on the numerical model.

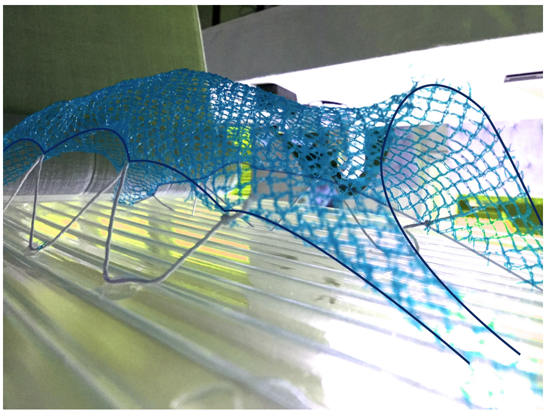


Figure 2.29: Physical form finding model (upside down) with edges outlined for clarity.

Preliminary analysis

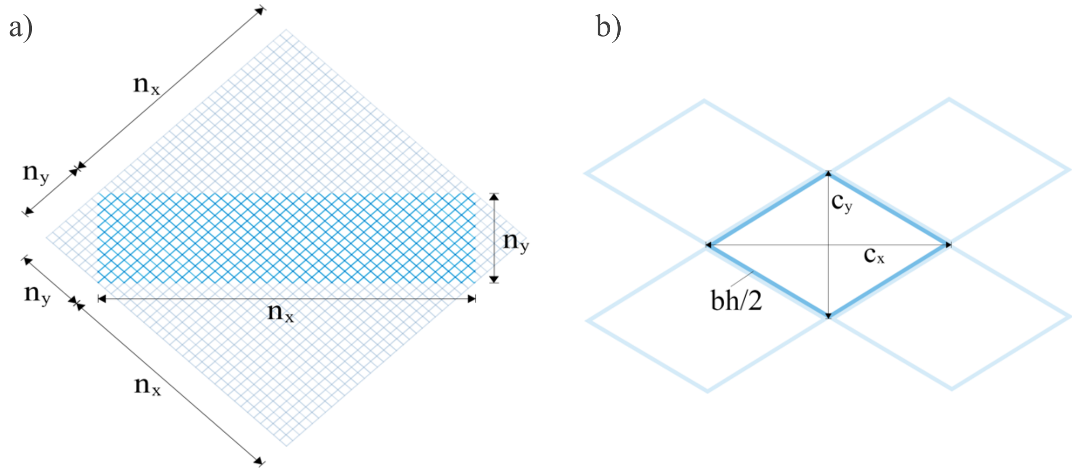


Figure 2.30: a) The flat grid as a part of a larger equilateral quadrilateral. n_x and n_y represent the number of quadrilaterals in the length and width of the grid respectively. b) Volume of each quadrilateral is given by considering half the width of the laths. Definitions of c_x and c_y.

In order to check whether the grid shell will realistically withstand the prescribed loads, it might be a good idea to do a quick, simple calculation of the compressive stresses at the supports. To achieve this, some assumptions are introduced for simplicity:

- All loads are distributed equally to each support, i.e. one quarter of the total load is taken by each support (non-conservative).
- The live load is uniform and in the vertical direction along the whole flat grid (conservative).
- Every lath at each support carries the same load (non-conservative).
- The stress in each lath should not exceed half of the compressive stress capacity (conservative).

Let N be the number of laths at each support. Then, the force F at each lath is given by:

$$F = \frac{1}{4N}\left(\rho g V_{grid} + q A_{load}\right) < \frac{1}{2} f b h \,,$$

where ρ is the density of wood, g is the gravitational constant, V_{grid} is the total grid volume, q is the live load, A_{load} is the area of which the load act, f is the compressive stress capacity and b and h is the width and height of each lath respectively.

Solving for N gives:

$$N > \frac{\rho g V_{grid} + q A_{load}}{2fbh}. \tag{2.1}$$

To obtain V_{grid} it is convenient to consider the flat grid as a part of a larger equilateral quadrilateral mesh as show in Figure 2.30a. The grid's total volume can be calculated as the sum of the volumes of each quadrilateral in the desired grid plus half the volume of the quadrilaterals which lie on the edges of this grid:

$$V_{grid} = V_{quads}\left(N_{quads} + \frac{1}{2}N_{quads,edge}\right).$$

The total number of quadrilaterals in the desired grid is obtained by taking the sum of all quadrilaterals and subtracting the parts which lies outside the desired grid:

$$N_{quads} = \left(n_x + n_y\right)^2 - 2\sum_{k=1}^{n_x} k - 2\sum_{k=1}^{n_y} k\,.$$

The two sums in the expression is the triangular number, which counts the number of objects in a discrete equilateral triangle. It can easily be shown that such a sum equals ${}^1\!/_2\, n(n+1)$, where n is the number of objects on each side of the equilateral triangle. Therefore

$$N_{quads} = \left(n_x + n_y\right)^2 - n_x(n_x + 1) - n_y(n_y + 1)\,.$$

The number of quadrilaterals which lie on the edges of the desired grid is obviously the circumference of the grid:

$$N_{quads,edge} = 2\left(n_x + n_y\right).$$

By considering the volume enclosed by the neutral axes of one quadrilateral (see Figure 2.30b), one will obtain the volume of each quadrilateral in the grid:

$$V_{quads} = 4 \cdot \frac{1}{2} bh \cdot \frac{1}{2}\sqrt{c_x^2 + c_y^2} = bh\sqrt{c_x^2 + c_y^2}\,.$$

The live loads may vary nontrivially in space, but for simplicity the load is considered uniform in space and acts along the whole flat grid, such that

$$A_{load} = n_x c_x n_y c_y\,.$$

Inserting into (2.1) gives:

$$N > \frac{\rho g V_{quads}\left(\left(n_x + n_y\right)^2 - n_x(n_x + 1) - n_y(n_y + 1) + n_x + n_y\right) + qA_{load}}{2fbh}$$

By expanding the square term and summing we get:

$$N > \frac{2\rho g bh\sqrt{c_x^2 + c_y^2} n_x n_y + q n_x c_x n_y c_y}{2fbh}\,.$$

By assuming the same timber as earlier we have $f = 55\ N/mm^2$, $\rho = 350\ kg/m^3$, $g = 9.81\ m/s^2$, $b = 48\ mm$, $h = 23\ mm$ and live load $q = 5kN/m^2$ we get:

$$N > \sim 40\,.$$

Clearly, 40 laths at each support would be way too dense. By learning from Mannheim Multihalle, a second grid layer could be introduced, which gives the approximate number of required laths to become 20. This means, firstly, that the load due to self-weight of the structure is negligible compared to the live load, and secondly, that the number of laths is still too big, requiring a dense grid. One solution to this problem is to use edge beams. If it is assumed that the edge beams are able to absorb half of the axial forces in each support, about 10 laths is needed.

When trying out different form finding shapes, it was found that if about 80% of the total length of the bridge is spanning, a satisfactory shape was found. Therefore, with 10 laths in each support and a total length of 100m, 80m of the bridge should be spanning, and 10m on each side acts as supports. In other words, the grid spacing in the longitudinal direction will be 1m. To obtain the grid spacing in the other direction, consider the following diagram (Figure 2.31).

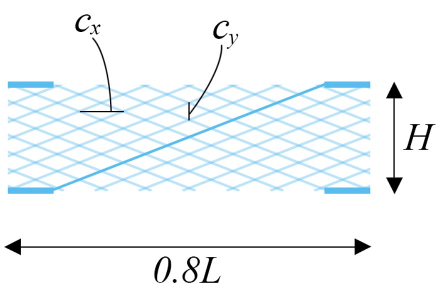


Figure 2.31: It was desirable that the relationship between c_x and c_y was such that a diagonal spans diagonally over to another support

By choosing that the diagonal lath (highlighted in the figure above) will span like shown, the grid spacing can be obtained by equal triangles:

$$\frac{H}{0.8L} = \frac{c_y}{c_x} \Rightarrow c_y = \frac{5}{4}\frac{H}{L}c_x \approx 0.3m\,.$$

Here, $H = 26m$, $L = 110m$ and $c_x = 1m$.

Numerical modelling
It was quickly seen that the analyses would have been too time consuming in Abaqus if the grid were to be modelled exactly. Therefore, only one sixth (1/6) of the laths in the grid was modelled (similar to what was done when modelling the Mannheim Multihalle). It is still believed that the end result would be the same, except that the section forces would be higher in each lath than they would be in reality. The script from Nathalie was again used to define the geometry of the gridshell

The same technique to “lift” the gridshell as was used in the previous gridshell, the buckling method, is used here as well. The boundary conditions are defined as shown in Figure 2.32.

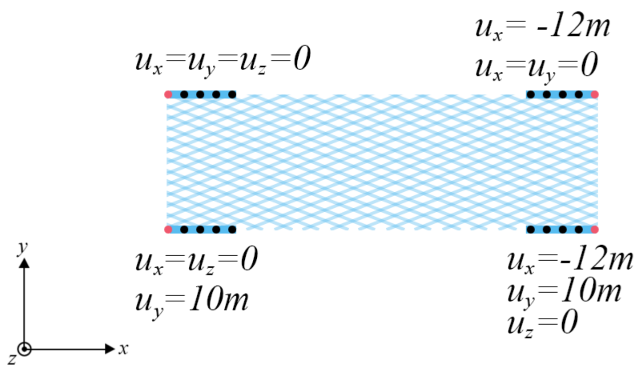


Figure 2.32: Boundary conditions. The values depicted correspond to the movement of the red dot. The black dots corresponds to $u_z = 0$.

Like on the previous shell, there is a mixture of imposed and "free" boundary conditions. The black dots on Figure 2.32 represents the "free" constraints and are chosen to be free in order to assess how the curve is formed during erecting the shell.

First, the structure after the erection without self-weight is considered. This is chosen because the analysis will become very unstable when gravity is enabled during the erection process. The acquired shape in Abaqus is shown in Figure 2.33.

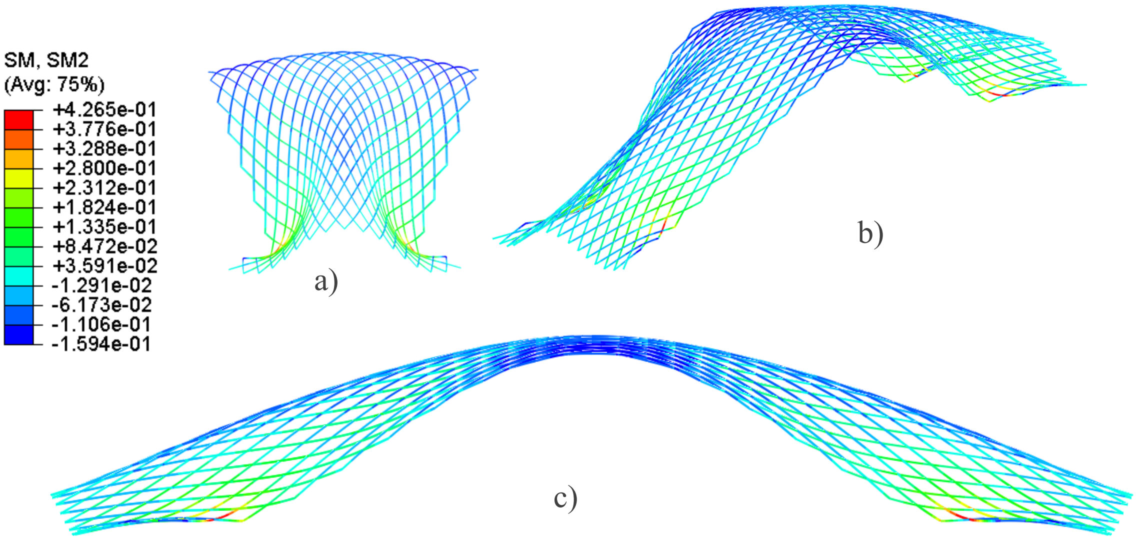


Figure 2.33: Solution in Abaqus after assembly (Bending moment plot).

It is evident that the middle part of the structure has not been pulled down significantly. It is also noticeable that it differs quite a lot compared to the solution from the form finding. This discrepancy is probably caused by the stiffness of the joint which are necessary in Abaqus in order to get a convergent solution. There are many more joints in this model than in the previous one, so the error is more apparent here. From this analysis it can be seen that it is a good idea to run the erection step in Abaqus, to see where it will differ from the form finding. The largest bending moments due to the curvature appear in one of the laths near the supports, which did not appear in the form finding model. It should be noted that these bending moments are very small, due to the sheer size of the structure compared to the small cross sections of the laths.

To get rid of these high curvatures proved to be a nontrivial task. In order to do so, the edge loading, representing the bridge cables, had to be introduced. In addition, supplementary spurious forces were defined near the supports to “force” the ends into a barrel vault. The final result can be seen in Figure 2.34.

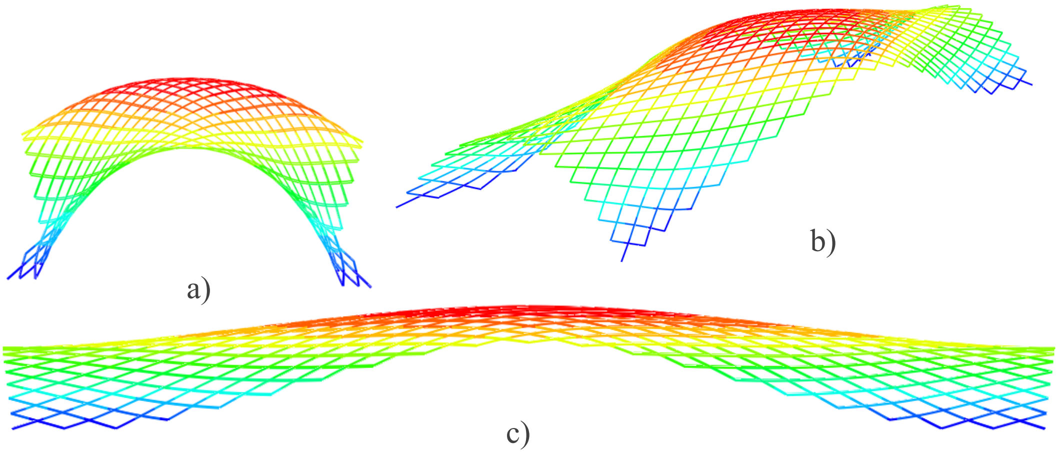


Figure 2.34: Solution in Abaqus with edge loading. (vertical displacement plot)

Now, the structure more closely resemble the geometry found by the form finding process earlier. The main differences are that the edges near the bridge’s openings does not “bend” upward as much, and the middle part is “flatter”. These dissimilarities occur due to the rotational stiffness of the joints in Abaqus, and also due to the fact that the grid layout is different (the angle between the crossing laths is less than 90°).

By examining the stress-strain history of different elements, some pointers about the nonlinearity of the erection process can be obtained (Figure 2.35). As can be seen, the behaviour is quite non-linear at first, but tends to behave more and more linearly. This is due to the fact that the structure becomes stiffer as the shell erects, which leads to less large, rigid motions and more elastic deformations.

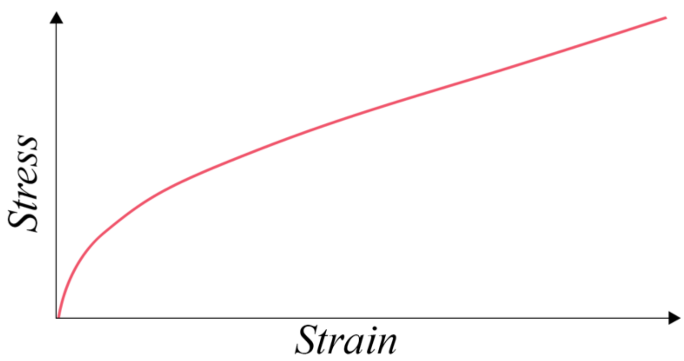


Figure 2.35: Typical stress-strain curve in the shell.

Bending moments

As mentioned, the bending moments that arises are very small, virtually negligible. However, it could be useful to examine where the structure has the most curvature. The moments are shown in Figure 2.36.

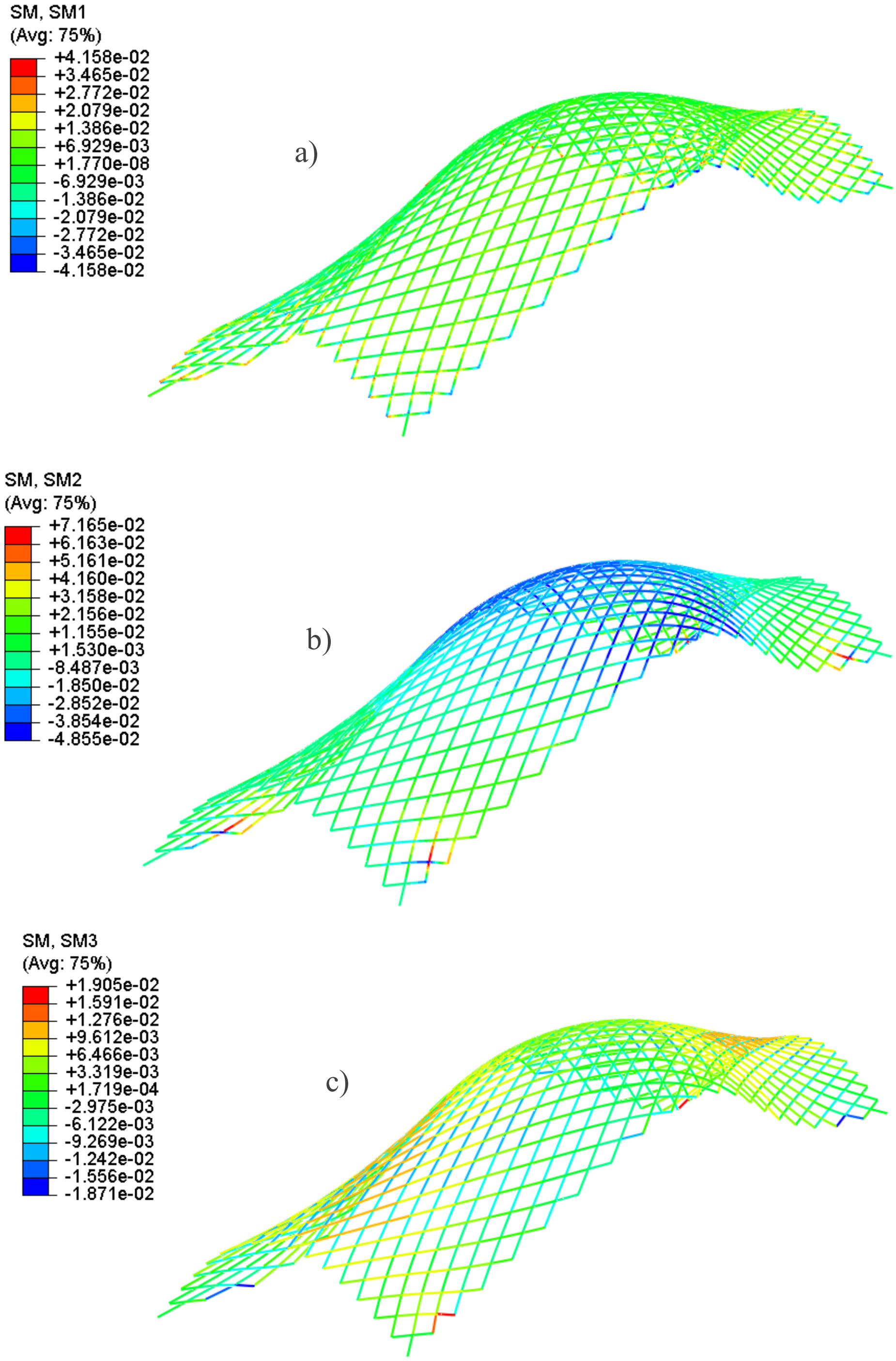


Figure 2.36.: Bending moments about the a) first principal axis, b) second principal axis and c) torsional moment

Even though these bending moments are small, it can be seen that the largest values accumulate near the supports, both in torsion and in bending. This, together with the fact that the normal stresses near the supports are high (from the preliminary analysis part), means that the supports will likely be the most vulnerable part of the structure.

Asymmetrical loading

The global stiffness of the structure, is expected to be low, due to the fact that the bridge acts more like an arch than a shell. It is therefore expected that the structure will be highly prone to asymmetric loading. One possible remedy for this, is to use a cable layout which distribute the asymmetrical loading evenly (given that the loading is on the road, and not on the shell itself). Another one is to introduce strong edge beams, which will help increase the overall stiffness.

Sadly, no convergent solution was able to be carried out to verify the expected response.

Cable layout

The layout of the cables is not studied in detail in this thesis, but some discussion is given henceforth. In the Cascara bridges in Madrid, the cables were densely laid out in order to use thinner cables which did not obstruct the view of the environment. The straight, vertical layout also minimized the obstruction. There are, however, more effective cable layouts to use for bridges. Per Tveit, a professor emeritus at the University of Agder, has studied cable layouts for suspension bridges extensively [19]. He has been studying network arch bridges, especially, and these all use a similar cable layout (Figure 2.35).

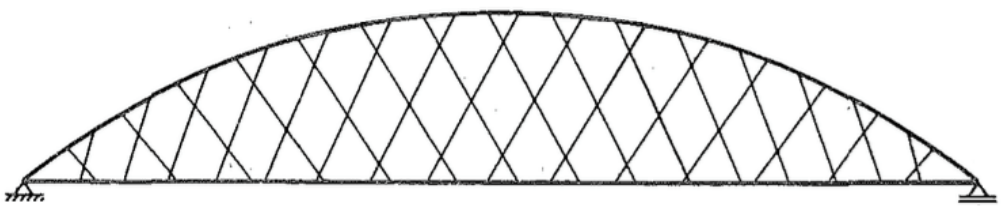


Figure 2.37: A network arch bridge, with its characteristic cable layout.

The main advantage of this type of layout, is that the bending under asymmetrical and concentrated load of the arch (which is the shell in this case) is greatly reduced. By utilizing the same cable layout in a shell bridge, the shortcoming under asymmetrical loading will very likely not be as evident. In addition, the cables can be thinner with this type of layout, providing the pedestrians with a better view from the bridge. It would be interesting to study the behaviour of the gridshell bridge with this layout, but this analysis is not carries out in this thesis.

Summary

In this shell, like the previous one, it is under asymmetrical loading in which the shell is weakest. Even more so in this one, since the shell acts more like an arch, which is known to be prone to asymmetrical loading. In other words, the double curvature of the shell becomes less important as the structure is elongated. One possible remedy for this, is to use the cable layout as studied by Per Tveit (Figure 2.35). This will make sure the asymmetrical loading is distributed evenly along the whole shell.

The in-plane shear stiffness of the shell can be greatly increased by the same techniques as used in previous gridshells, namely to introduce bracing. By using cables as cross bracing, the quadrilateral grid will be maintained, and the structure will appear lighter (as in Mahheim Multihalle). If edge beams are added, they also contribute to the in-plane shear stiffness as well, since they are placed parallel the diagonals of the grid.

If the grid is chosen to be regularly meshed, kinematic, made of timber, and consist of quadrilaterals, the density of the grid must be very high. A similar design as was used in Mannheim Multihalle was chosen, where the grid consists of small cross sections and two layers. This reduced the number of required laths on the supports, but first when combined with edge beams, the number of required laths near the supports were acceptable.

The edge beams also help stiffen the shell a lot, both under asymmetrical loading and under in-plane shear motion, and must be considered as a necessity. In addition to the structural purpose, it also serves an aesthetic purpose by hiding the “jagged” lines produced by the diagonal grid. One interesting question about the edge beams becomes; how much does the actual geometry of the shell carry the loads if the edge beams are there? If the shell itself could be extremely thin (like chicken wire) and the edge beams still is able to carry the loads, the shell geometry is irrelevant to the structural purpose of the bridge. On the other hand, it is known that shell behaviour carries loads more efficiently than beam behaviour, so there must exist a “sweet spot” in the span-to-width ratio where a shell is the better solution compared to primary bearing edge beams.

3 Discussion and concluding remarks

3.1 Form finding as part of conceptual design

Challenges and opportunities
Form finding is a great technique for designing good performing structures. Firstly, it gives the designer pointers to how the geometry of the structure should be defined in order to minimize the bending moments, and secondly, the designer can explore different interesting shapes in a quick and easy way. The shapes found by the form finding technique was found to be highly dependent on the imposed boundary conditions. If the architectural constraints of a given project are not rigorously defined, this gives rise to basically an infinite possible solutions of funicular shapes. Some natural question then arises; how should the designer pick these constraints? And if a given set of constraints gives rise to a well-defined solution, why should the designer continue the form finding process?

Defining constraints
When using form finding for the gridshell bridge, it quickly became clear that the shapes are highly dependent on the imposed constraints. The first shapes we obtained, did not look like anything we wanted or expected. Either, the openings of the bridge became too shallow, or, the shell became too flat. The design process then quickly changed from "optimizing" a shell geometry to altering the constraints, such that the result was satisfactory. Both engineering and architecture expertise was therefore necessary in order to design a good structure. To oversimplify, the architect had insight about some of the form-found shapes and how it could be altered, and the engineer had insights about how to change the constraints in order to fulfil the architect's input. This eventually led to a form that was not only visually pleasing, but also structurally sound.

FEM for form finding
Using Abaqus for form finding felt very tedious and unnecessary. If the designer must wait 30 minutes in order to get a proposition for a funicular geometry, the conceptual design phase would have taken too much time. By utilizing form finding techniques, such as dynamic relaxation, it is possible to get a realistic solution in real-time, and the designer can explore numerous shapes much more efficiently. After picking a particular shape from the form finding process, a more detailed and thorough analysis can be carried out in a FEM-software, like Abaqus. The problem then becomes how to get the funicular geometry over to Abaqus. One possibility is to simply export the final geometry, and carry out FEM-analysis on that. This completely ignores the erection process (for kinematic gridshells), which was

shown earlier to be a very important step. Another possibility is to derive the stresses directly from the curvature of the beams. From the previous analyses, it was shown that the stress state was more complex than so. Geometrical nonlinearities gave rise to different stresses than what would have been the case if the stresses arose solely by the current deformed configuration. This makes kinematic gridshell structures very interesting and complex to model numerically.

Form finding vs free forming

What would have been a very interesting experiment is to compare two gridshells; one in which its geometry is defined by a form finding process, and another, similar structure, which its geometry is drawn by free hand. It was mentioned in chapter 2 that "optimized" structures, such as those defined by form finding, are often prone to instabilities such as buckling, especially for loading in which the structure is not specifically optimized for. Free formed shells on the other hand, may have some geometry which is not specifically optimized for a specific load, and is therefore often more resistant to different load situations. A possible approach to get the best of both worlds could then be to use the form found model as a basis for further free formed design.

3.2 Architect and engineer cooperation

As mentioned earlier, designing a structure with help from the form finding technique, proved to be a process in which the architect and the engineer really benefits from cooperating with each other since it lies in a place with overlapping competence (Figure 4.1). Both roles play an important part: For the architect, since he by using form finding techniques are able to explore funicular shapes quickly, and for the engineer, because he is able to define constraints that gives rise to sensible structures. Since form finding is very quick and intuitive, the architect and engineer can cooperate simultaneous during the form finding process, and they can give each other instant feedback during the exploration of forms. This is essential during the form finding process, and makes it possible to design extraordinary structure, both architecturally and structurally.

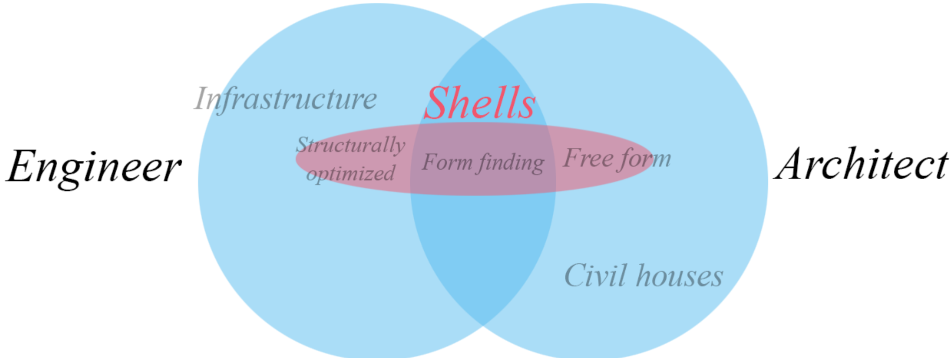


Figure 3.1: Venn-diagram showing intersecting shell competence for architects and engineers.

The architect, Steinar, had this to say about the collaboration:

> *"I think the collaboration have been nice, and would've liked to spend more time with the design and details on the bridge (materials and placement). I believe these problems would've altered several aspects of the design, and also provide us with some missing input, but this is maybe the architect's job?"*

3.3 Elongating the Square Grid Shell

By studying the square grid shell by Steinar and John, it was apparent that the shell worked very well under symmetrical loading. The diagonal bracings were also found to stiffen the structure quite a lot, which became very apparent when testing the shell under asymmetric loading. When the bracings were removed and asymmetric loading was present, the deformations were large and the shell eventually failed.

My supervisor, Anders Rønnquist, then asked the question: What happens if the gridshell gets elongated in a particular spatial direction? The question was interesting because very few structures were to be found where a gridshell has been elongated. The Downland Gridshell could be thought of as an elongated gridshell, but it is continuously supported along the sides. We wanted a gridshell that had a larger span. The Cascara Bridges in Madrid (See chapter 1.2) are more like the sort of shell structure we wanted to consider. These bridges are, however, not gridshells, but continuous concrete shells, which absorbs much of the light from the environment. The idea of a grid shell bridge was thus born. The architect, thought the grid shell bridge was a good idea architecturally, since it gives rise to both a slim and elegant form, and at the same time solve the structural purpose.

The analysis of the grid shell bridge proved to possess some challenges. When the shell is elongated, the structural behaviour becomes more arch-like and less shell-like, which give rise to instabilities when acted on by asymmetrical loading. By utilizing the cable layout studied by Per Tveit (see Figure 3.35), part of this shortcoming is expected to disappear. Another challenge was that due to the small support area, compared to the size of the structure, a large number of laths were required for the laths near the supports to not fail in compression. To overcome this issue, a double layered gridshell was proposed, together with edge beams in order to increase the area which is supporting the structure.

The edge beams also serve several other purposes: The thicker edge beams are able to absorb forces from the shell and direct them to the supports, and, because they are added along the grid’s direction without shear resistance, they increase the in-plane shear resistance of the shell. The main question regarding the edge beams is if the shell behaviour will become negligible if the edge beams are too strong. In other words, does it even matter if we used form finding to define a shape for the grid shell, if the beams added afterward carry most of the loads anyway?

3.4 Further Work

There are a lot more work that can be done in this area of research. For both of the gridshells that have been studied, there has not been any analyses regarding temperature loads, non-linear material behaviour, soil conditions, wind loads and earthquake loads. In addition, the building codes has been completely ignored.

When it comes to the elongated grid shell part, the bridge was only an exploration of one particular realization of an elongated gridshell. There are therefore many questions that still remain unanswered:

- How long should an elongated grid shell span before the desirable shell behaviour is negligible?
- How could the grid layout be optimized in order to maintain the required number of laths near the supports, and at the same time not be as dense where it is not required?
- How would the shell act if it had multiple spans?
- How could such a bridge actually be built?
- A complete analysis, where all laths are taken into account.
- Detailing, such as supports and connections.

The collaboration between the architect and the engineer could also be even more close than it has been in this work. By having the architect and the engineer close to each other at all times, all decisions could be taken together, which may produce even better structures.

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5 Appendix

Appendix 1: WCTE 2016 Conference paper

Appendix 2: IASS 2016 Conference paper

Appendix 1: WCTE 2016 Conference paper

WCTE 2016
World Conference on
Timber Engineering
August 22-25, 2016 | Vienna, Austria

EXPERIMENTAL AND NUMERICAL STUDY OF THE STRUCTURAL PERFORMANCE OF A TIMBER GRIDSHELL

Nathalie Labonnote[1], John Haddal Mork[2], Steinar Hillersøy Dyvik[3], Magnus Nilsen[4], Anders Rønnquist[5], Bendik Manum[6]

ABSTRACT: The structural performance of gridshell structures is strongly related to shape, but comparisons between design and construction shape are seldom reported. This paper documents the evolution of shape following the erection of a timber gridshell built in Norway. Structural performance was evaluated using experimental measurements, and was observed to be significantly affected by the mechanical characteristics of the connections between the different structural members. This study is expected to foster the development of innovative connection methods enabling adaptation of the timber gridshell concept to the design of permanent buildings that can withstand harsh climatic conditions.

KEYWORDS: timber construction, gridshell, FEA, Abaqus

1 INTRODUCTION

Shells play a special, singular role for engineers. Their shape directly derives from their flow of forces and defines their load-bearing behaviour and lightness [1]. If well-formed, concrete shells show no bending, but membrane forces only (axial compression and tension), permitting to save material by creating local employment. This usually results in very thin shells, such as the famous works of Felix Candela, among others Los Manantiales, built in Mexico city in 1957, which reveals a radical thickness of only 4 cm [2].

If regular holes are made in the shell, with the removed material concentrated into the remaining strips, the resulting structure is a *gridshell* [3]. The grid may have more than one layer, but the overall thickness of the shell is small compared to its overall span [4].

A *kinematic gridshell* differs from a regular gridshell in its construction process. A regular grid of slender laths is laid out flat; at each intersection point the members are connected by a hinge-type connector; finally the grid is shaped so that it takes up a doubly curved form. After erection of the lattice, bracing is introduced which triangulates the square grid, providing shear strength [5]. The prime benefit of a *kinematic gridshell* is the simplicity of the construction sequence.

Timber is among the most appropriate construction materials for these structures due to its lightweight, its small torsional thickness, its capacity to bend and its capacity to remain elastic. Timber members can be easily bent into shape due to their low bending stiffness. Moreover during the construction phase the members might be subjected to tighter radii of curvature than the ones they will have in their final state [6].

Very few large-scale ti*mber gridshell structures* exist worldwide. Frei Otto first developed this type of structure in 1972 for the Mannheim Multihalle in Germany [7]. Subsequently, in 2002, the Downland Museum was built in England by Buro Happolds [8], and was followed in 2005 by the Savill Garden Visitor Centre, also in England, designed by Buro Happolds and Glenn Howells [9].

Gridshells are complex structures, and several research groups are currently investigating ways of improving their design. Approaches include the use of composite materials [10], the optimisation of cross-section with respect to curvature [11], the development of innovative joint methods [12] and the development of more effective form-finding algorithms [13].

A major challenge facing designers is in achieving and maintaining the shape of the structure during its lifetime. Comparisons between design and construction shape are seldom reported. The same is true of creep and other time-related mechanisms. Since the structural performance of such structures is strongly related to shape, more knowledge of the evolution of construction shape is needed.

This study focuses on quantifying the structural performance of a timber gridshell pavilion over time. Several measurements were performed on the specimen

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construction after initial structural settlement. A comparison between experimental results and theoretical values derived from a finite element model is also discussed.

2 MATERIALS AND METHODS

2.1 THE TIMBER GRIDSHELL PAVILION

A four-metre high, ten-metre wide timber gridshell pavilion was erected in Trondheim, Norway, in June 2015 [14] (see Figure 1). The 23 x 48 mm grid members are organised in four layers. Timber members were all made knot-free, and are assumed to be of strength class C14.

The innovative modular design of this gridshell pavilion is based on 520 identical modules connected to each other. Details of this simple and effective solution called "segment lath" (see Figure 2 and Figure 3) can be found in the detailed study carried out by Haddal Mork and Hillersøy Dyvik [12].

The design procedure for form-finding was implemented via a particle-spring model. The form-finding process is largely inspired from the work of Pone et al. [15], using Grasshopper [16], a graphical algorithm editor tightly integrated with Rhino’s 3-D [17] modelling tools.

The gridshell was erected by means of a lifting tower, see Figure 4. The grid was assembled by starting with the centre module, leaving the bolts un-tightened. The anchor-points were pulled towards four already positioned foundations using cargo straps. After erection, the bolts were tightened and diagonal members of the same timber quality and cross-section were mounted to provide shear stiffness. In the rest of the study, diagonal members are referred to as "bracing(s)".

2.2 EXPERIMENTAL PROTOCOL

Three dimensional measurements of the full shape were performed with the distancemeter Leica 3D Disto [18]. Accuracy is expected to be of 1 mm at 10 m, but was assumed in this study to be 10 mm at 10 m, partly because of the reflective effect of bolts . Measurements were carried out on the gridshell during four test campaigns described in Table 1.

Loading was carried out without additional loading (Campaign 1), or with additional loading using buckets of water (Campaign 2) and weights (Campaigns 3 and 4). Measurements were carried out on the complete structural shape for all test campaigns. Additional measurements were carried out on selected members for test campaign 3: one diagonal stiffener and two transverse members, see Figure 5, Figure 6 and Figure 7.

Figure 1: *The built gridshell in Trondheim, Norway. Credits: Sophie Labonnote-Weber*

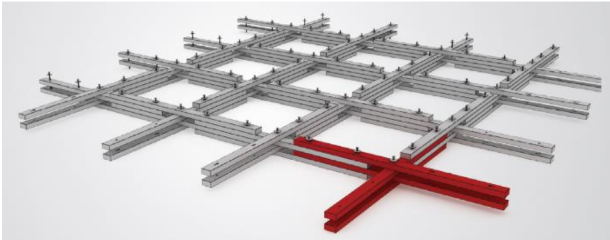

Figure 2: *The original "segment lath" modular solution, reproduced with permission from Haddal Mork and Hillersøy Dyvik [14]*

Figure 3: *The original modular solution, detail from the experimental gridshell. Credits: Sophie Labonnote-Weber*

Figure 4: *The lifting tower during erection. Credits: Steinar Hillersøy Dyvik and John Haddal Mork*

Table 1. *Loading protocol for test campaigns 1, 2, 3, and 4*

Test campaign		Test	Total load [kg]	Point loads [kg]	Diagonal members
1	June 2015	1.a	0	-	On
2	August 2015	2.a.	0	-	On
		2.b.	180	5	On
		2.c.	540	15	On
3	August 2015	3.a.	0	-	On
		3.b.	720	90	On
		3.c.	0	-	On
4	September 2015	4.a.	0	-	On
		4.b.	0	-	Removed
		4.c.	400	50	Removed
		4.d.	540	67.5	Removed
		4.e.	720	90	Removed
		4.f	0	-	Removed

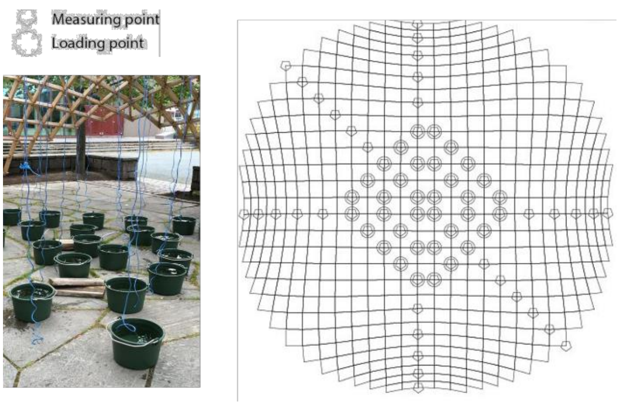

Figure 5. *Experimental protocol for test campaign 2*

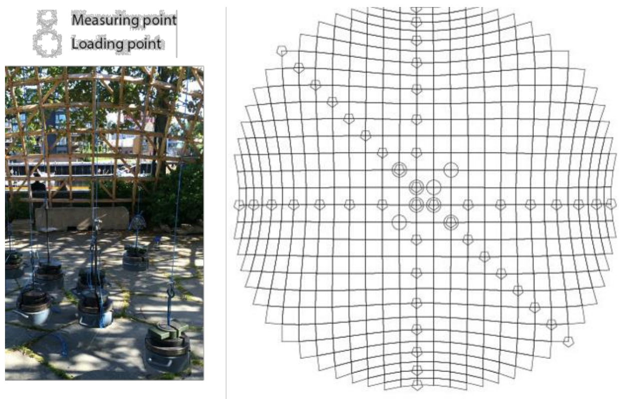

Figure 6. *Experimental protocol for test campaigns 3 and 4*

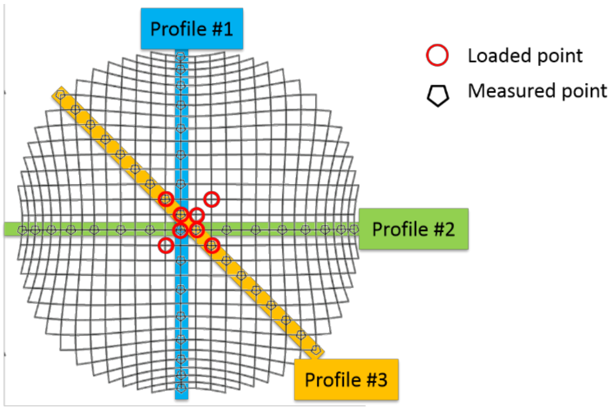

Figure 7: *Different profiles measured under Test campaign 3*

2.3 NUMERICAL ANALYSES

2.3.1 Features of the numerical model

Finite element analyses are performed using commercially available Abaqus software [19]. Given the complexity and the large number of connectors, the model geometry and its features are parametrically built using third party scripting provided by Python programming language [20].

Gridshell members are modelled as beam elements using the general two-node cubic interpolation B33 beam element. A mesh size of approximately 10 centimetres is selected and corresponds to a converging model.

The modular solution described in section 2.1 is modelled as the superposition of four layers of continuous beams. In order to represent the gaps in the built gridshell between each module (see Figure 8), the numerical continuous beams exhibit a regular distribution of a two different sections:

- A "hard" section implementing C14 strength class material properties, displayed in orange colour in Figure 8.
- A "soft" section implementing dummy isotropic properties with near-zero stiffness in order to model the gaps, displayed in yellow colour in Figure 8.

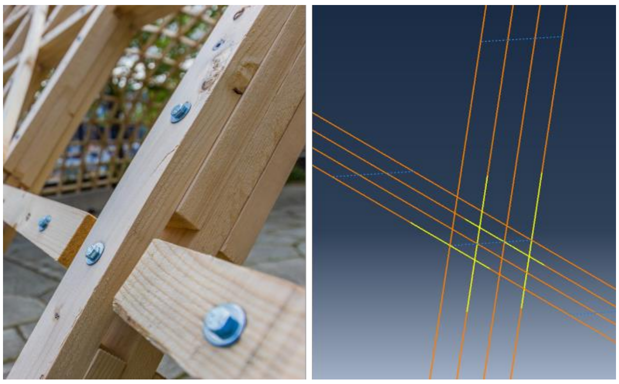

Figure 8: *Modelling of the modular solution with dummy sections (in yellow colour) to numerically represent the gaps between real timber beams (in orange colour). Connections are numerically implemented as constraints along the blue dotted lines.*

Material properties for C14 strength class are defined and implemented as transversely isotropic, following a linear-elastic behaviour. Material properties are adapted from Dahl [21] and are given in **Error! Not a valid bookmark self-reference.**.

***Table 2**: Material properties corresponding to C14 strength class ("1" applies to the longitudinal direction, "2" and "3" to the transversal directions)*

E1	7000 MPa
E2 = E3	230 MPa
$\upsilon 12$	0.39
$\upsilon 13$	0.49
υ *23*	0.64
G12 = G13	440 MPa
G23	30 MPa
ρ	350 kg/m^3

2.3.2 Connections

Real untight bolt connections between modules are parametrically implemented as numerical connectors, i.e. with numerical constraints between two points (see Figure 8). Total number of numerical connectors is 7464 with this method.

Connectors are of type cardan, which allows to implement in total six stiffness values: three translational stiffnesses and three rotational stiffnesses. For sensivity analysis purposes, three sets of values were implemented:

- A set "LOW" with one low rotational stiffness in the plane of the gridshell surface in order to represent the hinge connection: *D66* = 1.
- A set "MEDIUM" with medium rotational stiffness in the plane of the gridshell surface in order to represent friction in the hinge connection: *D66* = 100.
- A set "HIGH" with high rotational stiffness in the plane of the gridshell surface in order to represent a rotation-fixed connection: *D66* = 10000.

The three translational stiffnesses and the two remaining rotational stiffnesses are set to high values – in the order of 100000 - in order to model the behaviour of an untight bolt.

2.3.3 Loading and boundary conditions

The numerical gridshell is originally modelled as flat, following the real construction process. The numerical construction process is achieved in a non-linear static step by imposing the following displacements, see Figure 9 :

- Vertical displacement of the anchor points with a value equal to the planned height of the gridshell
- Lateral displacement of the anchor points towards the centre of the gridshell so that the floor area of the gridshell corresponds to the planned value
- Fixed translation of a selection of four points, corresponding to the four corners of the tower used for erection (see Figure 4).

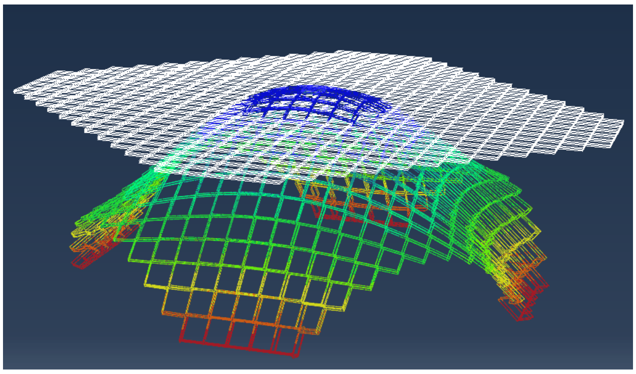

***Figure 9:** Shaping the gridshell in Abaqus: from the original flat gridshell (white colour) to the bent gridshell (coloured gradient of displacements).*

3 RESULTS AND DISCUSSION

3.1 Agreement between planned shape and built shape

The construction process imposes:

- the height of the gridshell, which is given by the final height of the lifting tower, and
- the floor area of the gridshell, which is given by the distance between the anchor points.

A good agreement is therefore observed between the planned shape and the built shape for these parameters.

However, comparison between the planned shape – calculated by the form-finding software described in section 2.1 – and the built shape some days after erection – measured in Test Campaign 1.a – shows substantial discrepancies for the arches of the gridshell, see Figure 10. Built arches are lower than their planned shape, sometimes as much as 40 cm.

Discrepancies tend to be smaller for areas leading to the arches, where again the gridshell members should be higher than they are in reality. As a whole, the built gridshell looks saggy compared to the planned gridshell.

Figure 10 emphasizes two distinct areas within the gridshell:

- a "strong cross"- which links the opposite anchor points and transfers loading down to the ground - shows a fair agreement with the planned shape, and
- a "weak cross" - which links the opposite arches - shows substantial discrepancies with the planned shape.

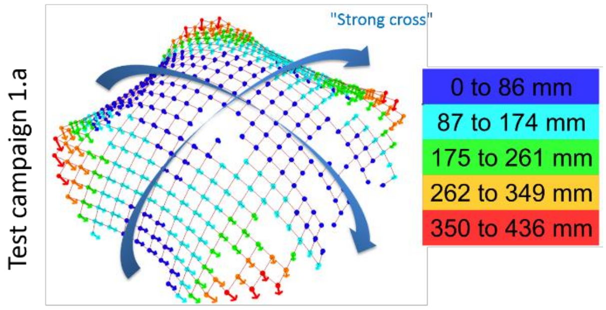

Figure 10: *Observed discrepancies between the planned shape and the built shape. Measurements are given in [mm].*

3.2 Sensitivity analysis of the built shape

The finite element analyses show that the final shape is highly dependent on the rotational stiffness of the connectors in the plane of the gridshell surface. The stiffer the connectors are, the more pronounced the arches will be (see Figure 11).

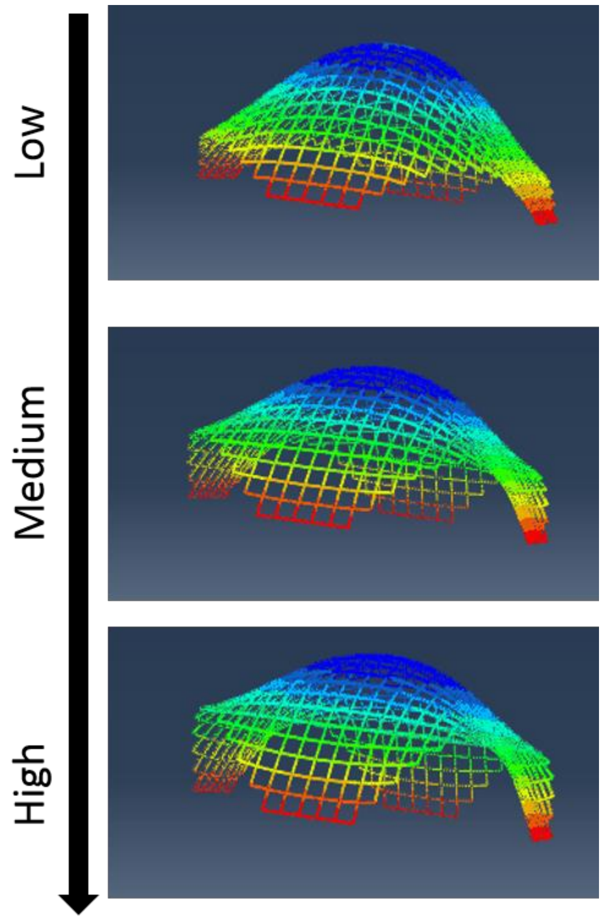

Figure 11: *Effect of the rotational stiffness of the connectors on the final shape.*

3.3 Structural performance with bracings

Test campaigns 2 and 3 are dedicated to evaluating the structural performance of the gridshell with bracings under various loadings, see Table 1.

The total load applied in Test campaign 2: 180 kg, and then 540 kg was not high enough to induce displacements larger than the precision of the measuring equipment. Results from test campaign 2b and 2c are therefore disregarded.

During Test campaign 3.b, the loading of the gridshell with a total load of 720 kg induces a substantial deflection of the top of the gridshell of almost 10 cm. Other affected areas include the weak cross that links opposite arches, to a lesser extent.

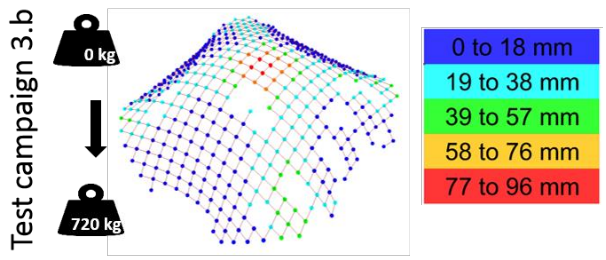

Figure 12: *Observed deflections during test campaign 3.b. Displacements are given in [mm].*

Profile results (see Figure 13) show an instantaneous deflection of about 10 cm for profile #1 and profile #2, and about 7 cm for profile#3. For all profiles, a residual deflection of about 2 cm is observed.

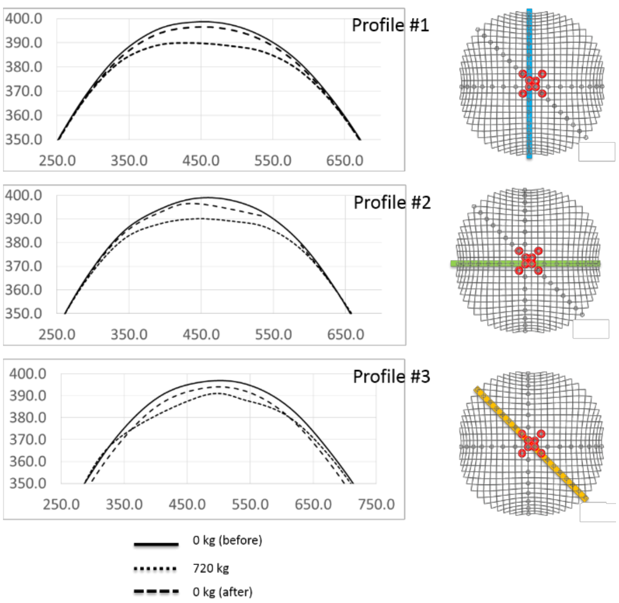

Figure 13: *Deflections observed under a 720 kg loading for three selected profiles. Distances and displacements are given in [cm].*

3.4 Structural performance without bracings

Test campaign 4 is dedicated to evaluating the structural performance of the gridshell without bracings under various loadings, see Table 1.

The observed deflections for the gridshell without bracings are almost four times larger than the ones observed for the gridshell with bracings. The "strong cross" is also substantially more affected: Figure 14 shows that the gridshell without bracing seems to bulge inwards because of lack of strength.

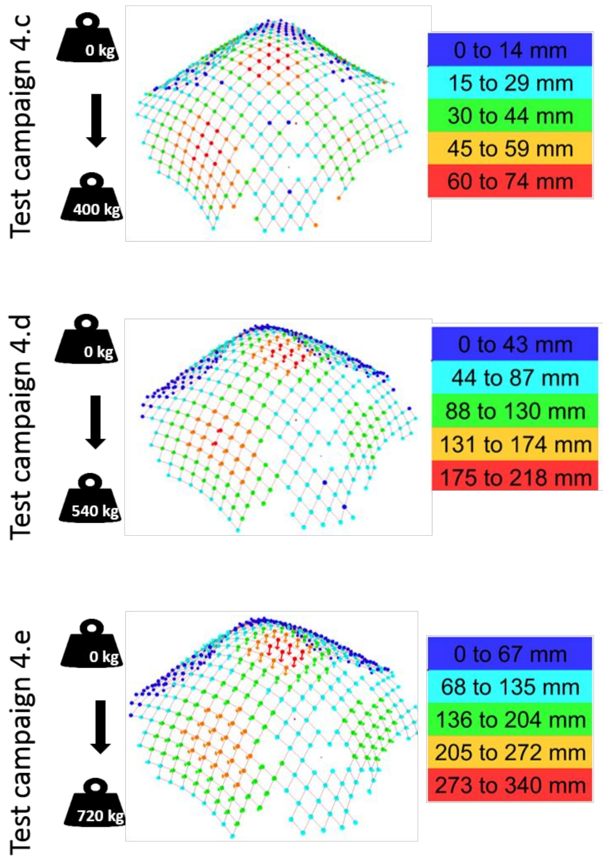

Figure 14: *Deflections observed under various loadings for the gridshell without bracing during test campaign 4. Displacements are given in [mm].*

3.5 Residual deflections after loadings

Residual deflections observed during test campaigns 3.c and 4.f are shown in Figure 15. Previous observations (see sections 3.3 and 3.4) are still valid:

- The gridshell with bracing is affected mainly along the "weak cross".
- The gridshell without bracing is mainly affected along the "strong cross".
- Residual deflections for the gridshell without bracing are about five times larger than for the gridshell with bracings.

These observations are also corroborated by the analysis of the deflected profiles for test campaign 3, see Figure 13.

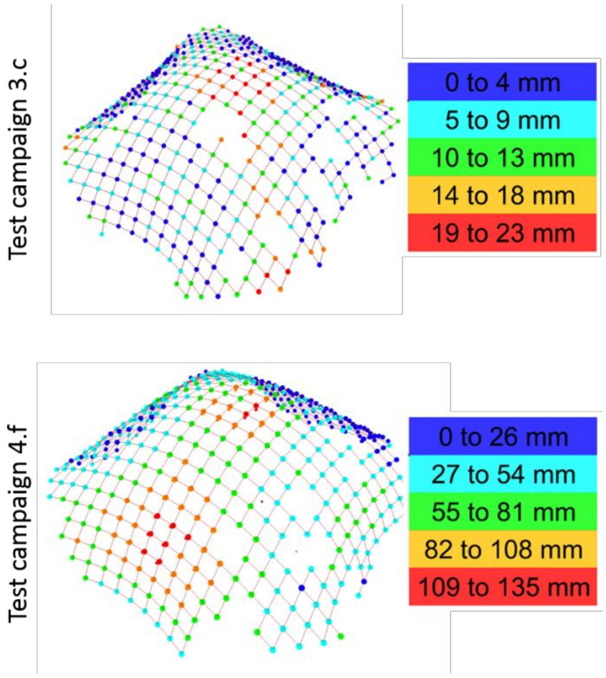

Figure 15: *Residual deflections observed after 720 kg loading for the gridshell with bracing (test campaign 3.c) and without bracing (test campaign 4.f). Displacements are given in [mm].*

3.6 Evolution of the gridshell with time

In general, the shape of the gridshell evolves with time, even without being submitted to loadings performed for scientific purposes.

Figure 16 shows for example settlement observed between June and August, without any (known) loading. Most affected areas are again the "weak cross" and the arches.

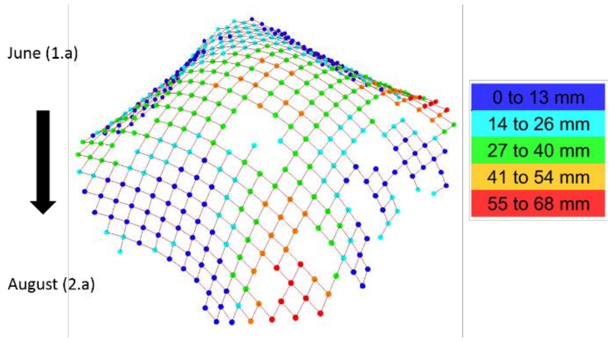

Figure 16: *Settlement observed between June and August without any known loading. Displacements are given in [mm].*

The cause of the instability of the timber gridshell is of high interest since its structural performance directly depends on it.

Possible external causes include settlement after erection and unknown climatic loadings (e.g. storm). Internal causes, i.e. those originating from the structure itself, may be related to the complex material properties of timber. Wood as a material and as structural element indeed exhibits a time-dependent behaviour as a result of moisture, temperature and loading conditions [22]. Those phenomena also take into account time-dependent deformation under constant load and relaxation under constant deformation. Wood creep is a well-studied phenomenon, but its effects are usually long-term: it is unlikely that creep is the cause of the settlement observed in a two-month period.

The design itself could then be the cause of the observed instability. A fundamental question is therefore to quantify the effects of a "non-perfect" shape onto the stability of a timber gridshell.

4 SUMMARY AND CONCLUSIONS

The following conclusions can be drawn:

- Two distinct areas of the gridshell show different behaviours: a "strong cross" which links the opposite anchor points and transfers loading to the ground, and a "weak cross" which links the opposite arches.
- The strong cross shows good agreement between planned and built shape, and is the most deflected area under loading (after the top area) for the gridshell without bracing.
- The weak cross shows substantial discrepancies between planned and built shape, and is the most deflected area under loading (after the top area) for the gridshell with bracing. It is also the area that is observed to be the most affected by settlement without known loading.
- Residual deflections are always observed after known loadings, and are several times higher for the gridshell without bracing compared to the gridshell with bracing.
- Shape of the arches is shown to depend on the rotational stiffness of the connectors.

Further work include:

- Upgrading the finite element model to the final topology of the gridshell, in order to take into account both the diagonal stiffeners and the change of connector properties: from rotation free to rotation fixed.
- Performing finite element analses with the final topology of the gridshell in order to evaluate whether the architectural pavilion meets the EUROCODE 5 requirements with respect to structural performance.
- Performing sensitivity analyses to qualify and possibly quantify the effect of connectors on the structural performance of the gridshell.

In conclusion, this study underlines the need for more precise modelling of the connections in order to gain precision in predicting the shape, and consequently predicting the structural performance of kinematic timber gridshells.

ACKNOWLEDGEMENTS

The authors gratefully acknowledge the financial support provided by Innovation Norway. Marius Bakkejord and Martin Kjellesvik are also acknowledged for their contributions as part of their Master's theses.

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Appendix 2: IASS 2016 Conference paper

Proceedings of the IASS Annual Symposium 2016
"Spatial Structures in the 21st Century"
26–30 September, 2016, Tokyo, Japan
K. Kawaguchi, M. Ohsaki, T. Takeuchi (eds.)

Modular Kinematic Timber Gridshell; a Simple Scheme for Constructing Advanced Shapes

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Abstract

This paper explains the construction scheme of a modular post formed gridshell. The scheme uses timber modules connecting laths of 900mm length into a 2-layer module. The scheme and the module are designed to efficiently handle both Form Finding, Fabrication and Assembly, and it is tested through the construction of a full scale pavilion.

Keywords: Timber post formed Gridshell, Timber Structure, Digital Form Finding, Kangaroo, Full scale prototype, Segment Lath

1. Introduction

Kinematic Gridshells are elegant and light structures that can create complex shapes by simple means. Gridshell structures have been constructed in various forms since Edmund Happold started experimenting with lattice roofs in the 1960s [1], and Frei Otto designed Mannheim Multihalle in 1975. The last decades, timber gridshells returned in the form of the Weald and Downland Museum (2002), Chiddingstone Orangery (2004) and the Savill Building (2006). More recently Sergio Pone and Sofia Colabella with the group at gridshell.it [3], have done several gridshell experiments on a smaller scale, and set the agenda for the development.

While buildings with simple shapes easily can be constructed by simple, repetitive procedures, advanced shapes often demand complex construction schemes. With new tools for both designing and manufacturing, the construction of gridshells have again resurrected. The used scheme uses both digital form finding with Grasshopper, and manufacturing with CNC milling. The work of Pone and Colabella has shown the potential in gridshells, with the construction of several structures the last decade, and modelling with Daniel Pikers Kangaroo are crucial for the form finding process.

However, the examples have all shown common issues, making it more complicated and time consuming than necessary to construct. Issues are found in all the three parts of: 1) Form Finding (Designing), 2) Fabrication (Production) and 3) Raising (Assembly).

This paper explains a kinematic gridshell construction scheme, simplifying the whole process. It is demonstrated through a 10x10x4m pavilion built in Trondheim, Norway, in 2015.

2. The gridshell in Trondheim

The built pavilion was done as a part of the diploma thesis in architecture at NTNU, Norwegian University of Science and Technology. The actual manufacturing and building took 10 days and was done by two persons. The shell has a setup of 28x28 nodes of a quadratic grid with c/c 500mm. It is a flat grid of 13.5m from foundation point to foundation point, with chamfered edges of 6 nodes width [fig.1]. The final shape has an 11.2 m outline, while the foundation edges are moved 2.12 m towards the centre. This created openings of 5.4m and heights of 2m in the openings and 4.1m in the centre. The construction stood for 6 months, and was tested for structural performance before disassembly. The structural performance will be described in another publication.

Photo 1
Photo of the gridshell pavilion built in Trondheim.

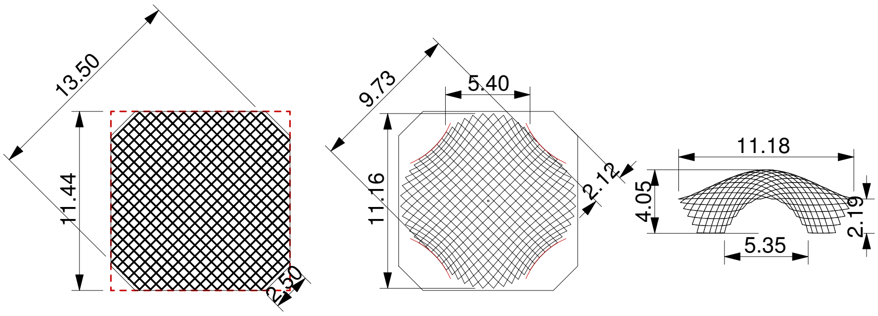

Figure 1
Dimensions of the planned gridshell pavilion built in Trondheim. The dimensions varied slightly in the final pavilion. Left: Size of the flat grid. Right: Sizes of the raised construction.

Kommentert [JHM1]: Supersexy!!! He vi ett bilde som likna opprisset? Hadde evt vore kult å hatt det som førstebilde

Blir evt der her?

3 Construction scheme

The scheme [fig.3] is greatly inspired by the work of Pone and Colabella [8] and the built pavilion is similar in both scale and techniques to many of their projects, and in the form finding tool (gfft) [8]. Through the explanation of the scheme, the aim is to highlight the improvements made in form finding, fabrication and raising.

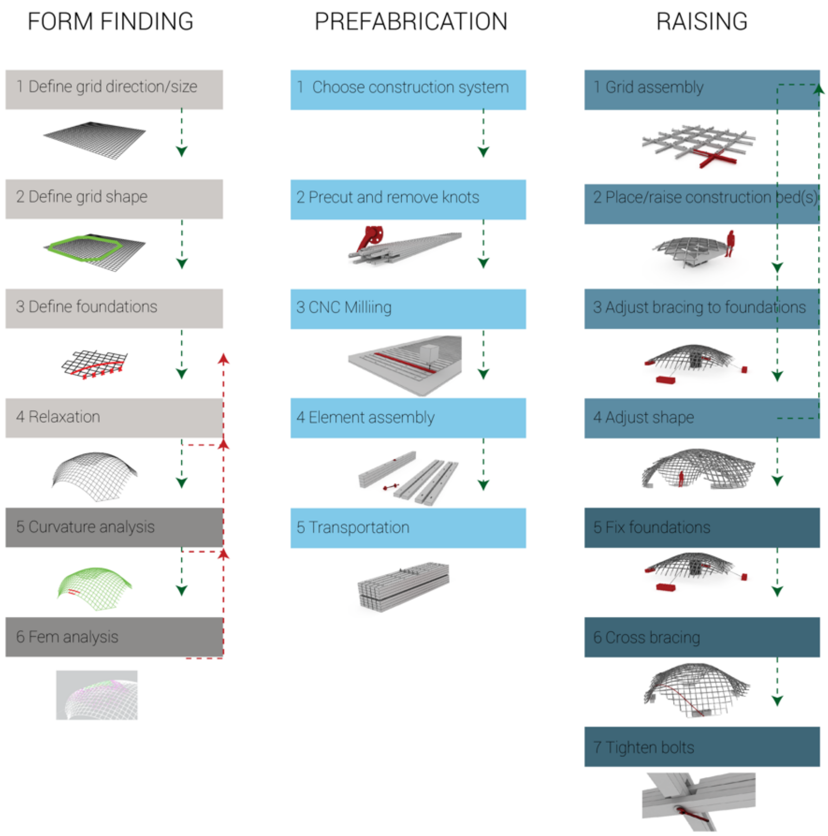

Figure 2
The construction scheme is divided into three parts; form finding, fabrication and raising. The form finding can be done without having decided construction principles, thus taking this decision can feed back information to the form finding.

3.1 Form finding

The shape is found by using dynamic relaxation with Grasshopper and Kangaroo. The grasshopper code, from now on referred to as the code, includes structural analyses taking into account material properties and curvature considerations.

Form finding setup

1. The code creates a grid with the desired grid direction (orthogonal or diagonal) and grid size. The number of nodes is picked approximately to give an outline for the next step.
2. Define the outer shape of the flat grid by drawing a curve with the desired shape within the nodes. The code recognizes the nodes inside the curve, and creates the grid according to them.
3. Extract the outer points form the grid, and assign the desired ones as anchor points for the construction. This should correspond with the actual foundation points later on.
4. Draw the anchor curve in rhino, to which the anchor points should be pulled towards, and connect them to the code. In the double symmetric alternative, one anchor curve is drawn and the rest are mirrored in position. A curved anchor curve gives better performance.
5. The form finding can start. The code is designed with four important physical conditions:
    a. There is an inverted gravity force, that pushes the grid upwards.
    b. There is a bending resistance between each lath, trying to keep them flat. This simulates the bending resistance in the timber laths.
    c. All the laths are defined as very strong springs. For the form finding work, the laths must be allowed to stretch a fraction of their length.
    d. The anchor curves have been assigned a pull force, pulling the anchor points towards the anchor curves.

Form adjustment

1. The shape can now be adjusted by moving and adjusting the anchor curve.
2. The code gives feedback on the curvature of the shell. It is possible to assign a timber type and quality, and the corresponding minimum curvature will be checked according to the model. The software code colorizes the laths curved over their capacity, and it is possible to adjust the shape is adjusted if needed.
3. The code is also connected to Karamba for structural analysis [6] [Fig. 6].

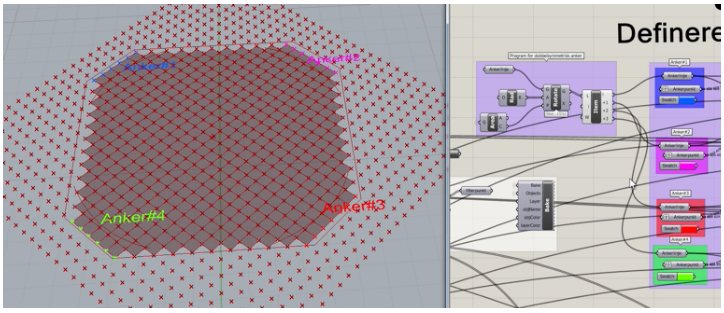

Figure 4
The form finding starts by deciding grid size and direction, and drawing an approximate outline of the flat grid. The anchor points are then extracted and connected to the code.

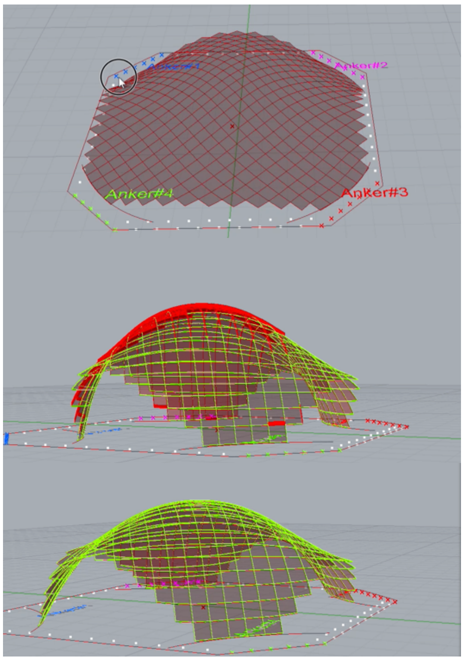

Figure 5
Further steps of the form finding. With a minimum curvature analysis based on material quality, the model can check if it is possible to construct the found shape.

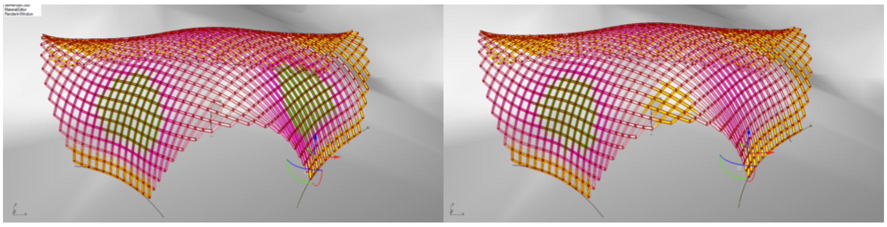

Figure 6
Structural performance testing with Karamba.

3.2 Construction method - The Segment Lath

A crucial development for this scheme is the design of the segment-lath [Fig. 6]. Instead of finger joining, lap joining or irregular extension systems, the exception has been made the rule.The grid is constructed with a two-layer module of 900mm laths. Each lath has five milled holes. One in the center, for the node, and two on each side, for weaving.The It can adapt to any grid size. The segment lath deals with many of the common challenges in a new way.

1. The segment-lath solves the issue of knot removal [Fig. 7]. The 900mm pieces can be cut manually for waste reduced removal of knots, or pre cut and then sorted for a more time efficient removal of knots. The wastage will be higher with larger grid sizes.
2. With overlapping as a rule, there is never an issue acquiring the correct length of the laths [Fig. 6]
3. The length of the overlap can vary, but generally, the shear block is included by default in this system [fig 8]. Shear blocks are usually added in between the two layers, to increase the total moment of inertia and thereby the stiffness. The height of the shear block depends on the actual lath heights, but the length can in theory be optimized for the grid to save material and weight.
4. As the center hole in each piece is circular, and the side holes are either circular or slotted. The top and bottom layer has slotted holes, allowing free sliding relative to the central layer.
5. Cross bracing is not solved as a part of the Segment Lath cross bracing is in this example is handled in the outer layer.
6. Transportation of the grid becomes efficient, as the modules get compact. In the built pavilion, the whole grid needed two euro pallets, or about 2m^3, for transport from workshop to the site. [Photo 2]

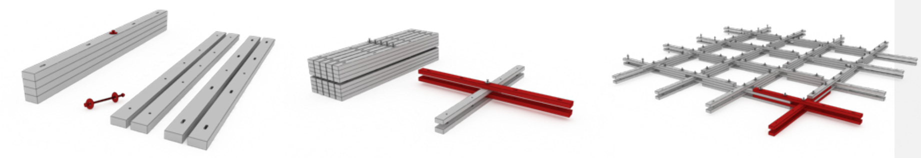

Figure 7
The segment lath. From module to assembly.

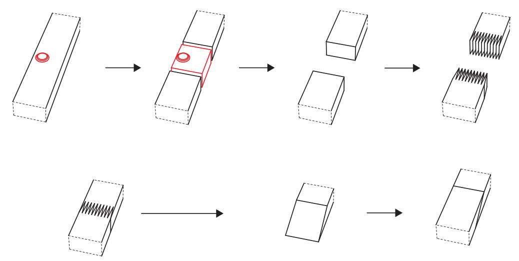

Figure 8
Knot removal in Saville building. The segment lath gives another solution to these issues.
Top: Knots were removed, and finger joint up to 6 meter lengths.
Bottom: In this case they were also extended from 6 to 36 meters using lap-joints.

Kommentert [BM2]: Exception ¿???
Uansett: for kort forklart. Se Johns paper og lån litt derfra

Kommentert [BM3]: forklar shear block ¿?

Kommentert [BM4]: språk

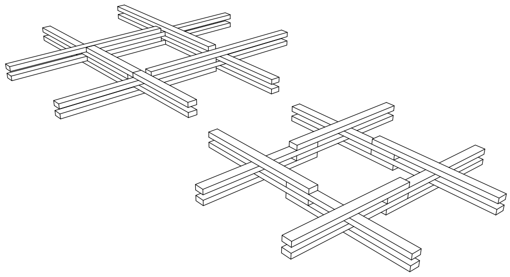

Figure 9
The overlap in the segment lath has the same function as the shear blocks. Same grid size can be archived with different overlaps.

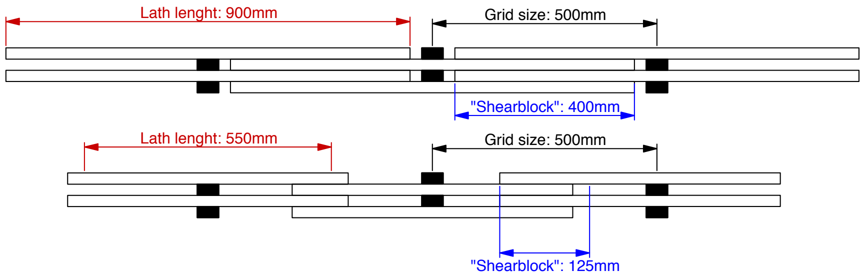

Figure 10
Top: Maximum lath length for the chosen grid size, as in the built pavilion.
Bottom: Different lath length, but same grid size.

Kommentert [BM5]: slutt med sentrert tekst!!

Kommentert [SHD6R5]: Trur tekst-stilen sentrerte seg sjøl for bildetekst, men kan endre på..

Photo 2
The segment lath makes transportation very compact. This image shows 50% of the modules in the built pavilion.

3.3 Raising

The erection of the gridshell was done by two persons in the pavilion. The process could have been done more efficiently with more hands, but for this scale, it was never necessary. The process from modules to assembly goes as follows.

1. The site is prepared with foundations placed in the correct positions. In addition to the grid-bed, which are the the only precision needed to start the raising of the structure. To get a precise form, it is also necessary to extract some reference positions, like the height on the top and next to the entrances
2. A grid-bed [Photo 3] is constructed to guide the construction in place. The grid bed has the same shape as the final grid at the same position. The bed should will be lifted during the construction process. In the built pavilion, it was placed in the centre. This method seems applicable to larger scale grid shells. In that case more than one bed could be necessary.
3. The grid bed is placed on a euro pallet. This allows a step-by-step lifting, adding one pallet of 100mm for each step. In the built pavilion, a manual pallet jack was used, but if the lifting is done gently, a motorized truck would be more efficient.
4. The manufactured parts are assembled into a grid of a manageable size, slightly larger than the grid-bed. It is then placed in position on top of the bed. [Fig. 9]
5. More elements are added sequentially to the grid, evenly around the center. Because the added elements contribute to a slight weight increase, it is easy to keep the shell in balance under assembly. [Fig. 10] The shell is connected to the foundations with straps during the raising in order to ensure stability.

Photo 3
The shape if the grid bed is taken directly from the 3D model, and cut to fit the final shape.

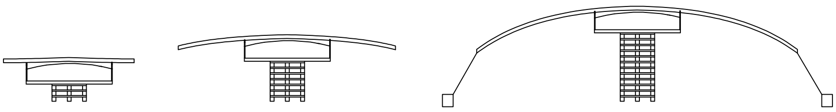

Figure 11
Adding pallets raises the shell evenly and will also help keeping a good working height. Gravity will help shaping the shell gradually as more weight is added. At a point, temporary bracing is added to keep the structure mode in balance.

Photo 4
The pallets can be lifted with a manual pallet jack, but in larger projects a fork lift is more relevant.

Photo 5
Connecting a new module done in a good working height. Too much tension can make the connecting more difficult.

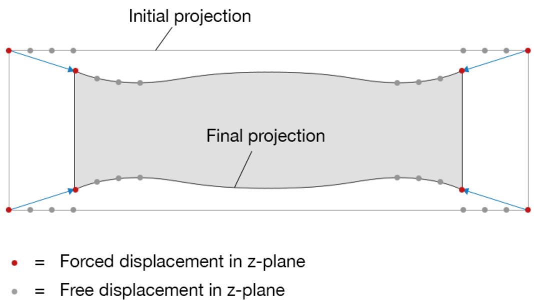

Figure 12
The curved shape of the anchor curve found using Abaqus

4. Discussion

- Further studies could prove that less overlap/shorter shear blocks can provide better structural performance.

- The geometry of the anchor-curves are critical to reach a good result. Straight lines seem to be the worst alternative. A form finding test in Abaqus, with anchors free in xy-plane also gave a curved shape. [Fig. 12] Allowing this shape in the final foundations are an important feature of kinematic gridshells.

- A notable found with Karamba is how the structural performance of the shape is significantly improved by fine-tuning the geometry of the foundations. [Fig. 5]

Kommentert [BM7]: Dette hører kaknskje mer hjemme i diskusjon / konklusjon enn her i beskrivevele??
Disse punktene burde kanskje skille mellom prinsippeiell metode og konkret urvikling av fromen i detalj???
Punkt 7 og 8 er jo viktige tema som gjerne kunne utdypes noe, mens de andre er mer beskrivende. ¿??

4. Conclusions

Kommentert [JHM8]: Fint å starte med: This paper set out to…. (gjenta problemstillinga di)

This paper set out to highlight possibilities in a modular kinematic gridshell construction scheme. We aimed to improve known challenges of kinematic gridshell construction.

- The Segment-Lath gave different solutions to many common issues with common gridshell construction, including knot removal, shear blocks, fabrication and transportability.

- The gridshell can get help in shaping from gravity by iteratively lifting of the shell during construction, giving a gentle shaping process.

- Our version of the form finding in the gfft is an efficient method for designing kinematic gridshells.

Acknowledgements

Kommentert [BM9]: Siden anders og jeg ikke er medforfatter (som det er helt ok at vi ikke er), mener jeg i all ubeskjedenhet at vi bør ha høy prioritet i acknowledegments, og vi bør oppgis med navn og fakultet/institutt – for å tydeliggjøre samarbeid mellom ark og ing på NTNU. Samarbeid med ingeniørstudentene bør også nevnes.

The authors highly acknowledge the supervision from Bendik Manum at Faculty of Architecture and Fine Arts, NTNU, Norwegian University of Science and Technology, Anders Rønnquist, Department of Structural Engineering, NTNU, Nathalie Labonnote, SINTEF Byggforsk and the collaboration with master students at Department of Structural Engineering.
The authors highly acknowledge the hospitality and sharing of knowledge by Sofia Colabella and Sergio Pone.
The authors are very grateful for the funding for this research provided by Innovasjon Norge.

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