Time-DomainSimulation of MarineStructures in IrregularSeas:
Second Order Wave

NTNU Norwegian University of Science and Technology

Michael Binsar Lubis (michaebl@stud.ntnu.no)

Supervisors: Professor Jørgen Amdahl, Professor II Sverre Haver

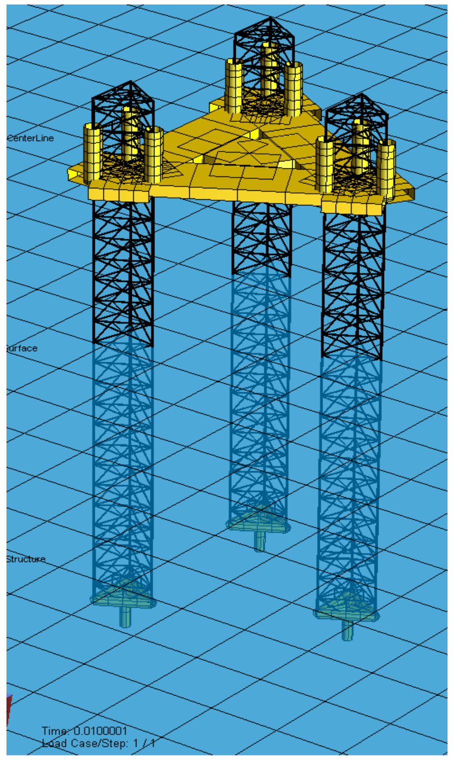
Introduction

For time-domain simulation, simulating 2nd order irregular wave produces better accuracy than 1st order irregular wave though it requires larger computational time. Therefore, strategies to reduce computational time is analyzed. However, the second order model is firstly compared to Forristal distribution and 5th Stokes wave to verify the model.

Objectives

- ✓ Compare the empirical distribution of sea surface from 2nd order irregular wave with Forristal wave crest distribution
- ✓ Compare the 2nd order irregular wave kinematics with Stokes 5th wave kinematics
- × Compare the quasistatic response of 2nd order irregular wave with Stokes 5th wave response
- × Analyze the dynamic response of 2nd order irregular wave
- × Observe several strategies to reduce the computational time: spool-to-extreme method, linear-to-extreme method, combination 1st order and 2nd order kinematics along the z-coordinate, calculating the wave kinematics on coarser grid.

FEM Jack-Up CJ62



Legs are represented by detail finite element model. For 1st order sea:

- **Cd**=1.15 and **Cm**=1.2 for z≤ 2m
- **Cd**=1.15 and **Cm**=1.6 for z>2m

While for 2nd order sea:

- **Cd**=1.05 and **Cm**=1.2 for z≤ 2m
- **Cd**=0.65 and **Cm**=1.6 for z>2m

Model

Second order surface correction(ζ_2)is calculated by equation 1[1].

$$\zeta_2 = \overbrace{\frac{1}{4}\sum_{i=1}^{N}\sum_{j=i}^{N}\zeta_{a1,i}\zeta_{a1,j}\left(\frac{D_{ij}^{-}-(k_ik_j+R_iR_j)}{\sqrt{R_iR_j}}+(R_i+R_j)\right)cos(\psi_i-\psi_j)}^{different-frequency} + \underbrace{\frac{1}{4}\sum_{i=1}^{N}\sum_{j=i}^{N}\zeta_{a1,i}\zeta_{a1,j}\left(\frac{D_{ij}^{+}-(k_ik_j-R_iR_j)}{\sqrt{R_iR_j}}+(R_i+R_j)\right)cos(\psi_i+\psi_j)}_{sum-frequency} \quad (1)$$

$$k_{ij}^{-} = |k_i - k_j| \;;\; k_{ij}^{+} = |k_i + k_j| \;;\; \psi_i = k_i x - \omega_i t + \varepsilon_i$$

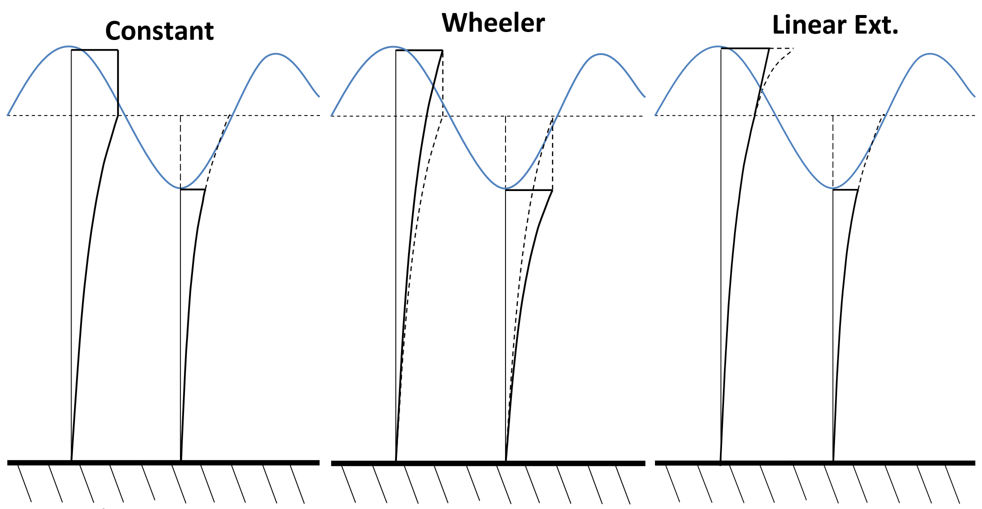
$$R_i = k_i \tanh(k_i d)$$

$$D_{ij}^{-} = \frac{(\sqrt{R_i}-\sqrt{R_j})[\sqrt{R_j}(k_i^2-R_i^2)-\sqrt{R_i}(k_j^2-R_j^2)] + 2(\sqrt{R_i}-\sqrt{R_j})^2(k_ik_j+R_iR_j)}{(\sqrt{R_i}-\sqrt{R_j})^2 - k_{ij}^{-}\tanh(k_{ij}^{-}d)}$$

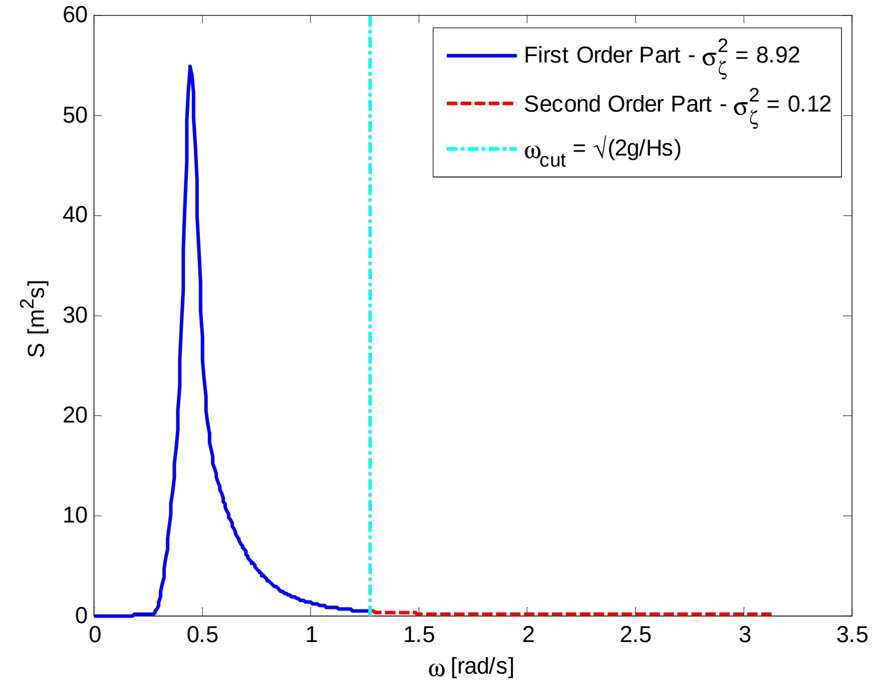
$$D_{ij}^{+} = \frac{(\sqrt{R_i}+\sqrt{R_j})[\sqrt{R_j}(k_i^2-R_i^2)+\sqrt{R_i}(k_j^2-R_j^2)] + 2(\sqrt{R_i}+\sqrt{R_j})^2(k_ik_j-R_iR_j)}{(\sqrt{R_i}+\sqrt{R_j})^2 - k_{ij}^{+}\tanh(k_{ij}^{+}d)}$$

The second order kinematic is determined by differentiating the 2nd order potential velocity (ϕ_2):

$$\phi_2 = \overbrace{\frac{1}{4}\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{g^2\zeta_{a1,i}\zeta_{a1,j}}{\omega_i\omega_j}\frac{cosh\left(k_{ij}^{-}(z+d)\right)}{cosh(k_{ij}^{-}d)}\frac{D_{ij}^{-}}{\omega_i-\omega_j}\sin(\psi_i-\psi_j)}^{different-frequency} + \underbrace{\frac{1}{4}\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{g^2\zeta_{a1,i}\zeta_{a1,j}}{\omega_i\omega_j}\frac{cosh\left(k_{ij}^{+}(z+d)\right)}{cosh(k_{ij}^{+}d)}\frac{D_{ij}^{+}}{\omega_i+\omega_j}sin\,(\psi_i+\psi_j)}_{sum-frequency} \quad (2)$$



The 1st order kinematics along z-coordinate are determined by constant stretching (left), Wheeler stretching (middle) and linear extrapolation (right)[2], while for 2nd order wave kinematics, only Wheeler and linear extrapolation are used. A cut-off frequency (ω_{cut}) is introduced when Wheeler (ω_{cut}=4ω_p)[3] and linear extrapolation (ω_{cut} =$\sqrt{2g/H_s}$)[4] are applied to 2nd order kinematics.



The irregular seas are established from JONSWAP wave spectrum. Significant wave height (Hs) and spectral peak period (Tp) are taken from metocean analysis. In equation 1 and 2, Establishing 2nd order irregular wave requires the harmonic component from 1st order irregular wave. For 1st order irregular wave, the harmonic amplitudes (ζ_{a1}) are determined randomly utilizing Rayleigh distribution with σ_i^2 = s(ω_i) $\Delta\omega$[5]. Harmonic frequency(ω) is established by equidistance interval whileharmonic phase is determined by a random number which is uniformly distributed between 0 and 2π.

Dividing a 3-hour simulations is into 9 20-minutes simulations increases the freqeuncy interval. As a result, the required components decrease from 10,800 to 1,200 for 3-hour simulation with $\Delta t = 0.5s$. Futhermore introducing ω_{cut} decreases the required number component from 1,200 to 220.

(Current) Conclussion

- Below sea surface, 2^{nd} order horizontal velocity from is smaller than 1^{st} order's due to 2^{nd} order difference term
- Introducing the ω_{cut} changes sea surface process from broadbanded to narrowbanded which decreases the deviation on lower tail CDF
- Linear extrapolation on 1^{st} order kinematics greatly overestimates the horizontal velocity while Wheeler stretching on 2^{nd} order kinematics underestimates the result.
- For baseshear and overturning moment, Wheeler stretching on 2^{nd} order kinematics underestimates the result while linear extrapolation on 2^{nd}order kinematics produces result close to 5^{th} Stokes result.

Reference

[1] Sharma, J.N. and Dean, R.G.(1981) "Second-Order Directional Seas and Associated Wave Forces,"*Society of Petroleum Engineers Journal 4, pp. 129-140*

[2] SINTEF Marintek (2010) *USFOS Hydrodynamics: Theory Description of Use Verification*, Trondheim: SINTEF Marintek

[3] Johannessen, T.B. (2008), "On the Use of Linear and Weakly Nonlinear Wave Theory in Continuous Ocean Wave Spectra: Converge with Respect to Frequency,"*Proc.27th OMAE Conf.*, Estoril, Portugal

[4] Stansberg, C.T. (1998) "Non-Gaussian Extremes in Numerically Generated Second-Order Random Waves in Deep Water,"*Proc. 8.th ISOPE Conf., Vol III, Montreal, Canada, pp 103-110*

[5] Tucker M.J.; Challenor, P.G. and Carter, D.J.T. (1984) "Numerical Simulation of A Random sea: A common Error and Its Effect upon Wave Group Statistic," *Applied Ocean Research, 1984, Volume 6, No.2, pp118-122*

[6] Edvardsen, K. (2015) *Forces on Simplified Offshore Structures According to Different Wave Models*, Master Thesis, University of Stavanger, Stavanger

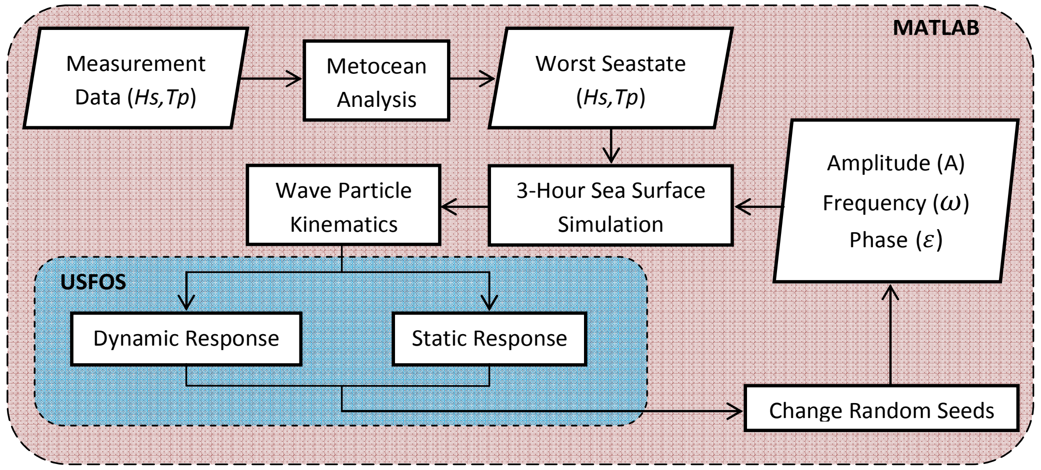
[7] Forristall, G. (2000) "Wave crest distributions: observations and second-order theory",*Journal of Physical Oceanography, 30, pp. 1931-1943 (2000)*

Acknowledgement

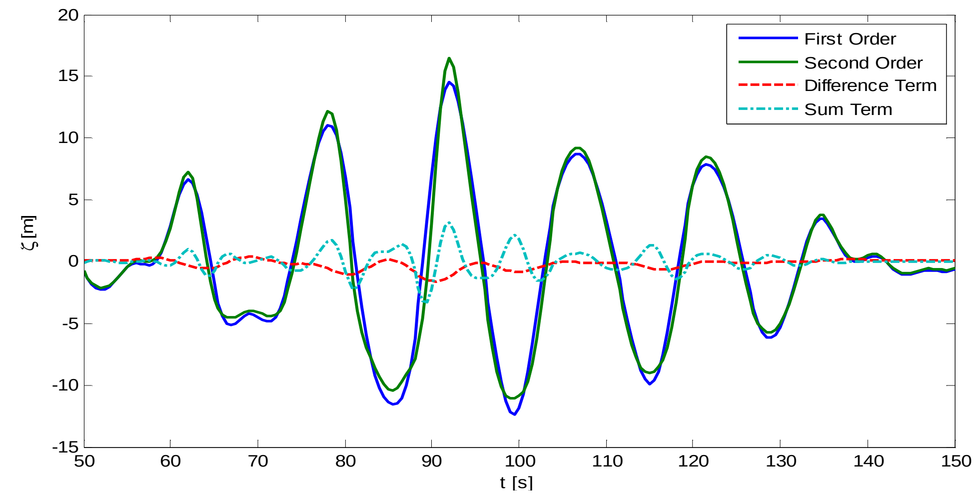
Thank you to Prof. Amdahl and Prof. Haver forguiding me on2nd order wave andfor modeling and simulating Jack-Up Platform in USFOS.

Simulation

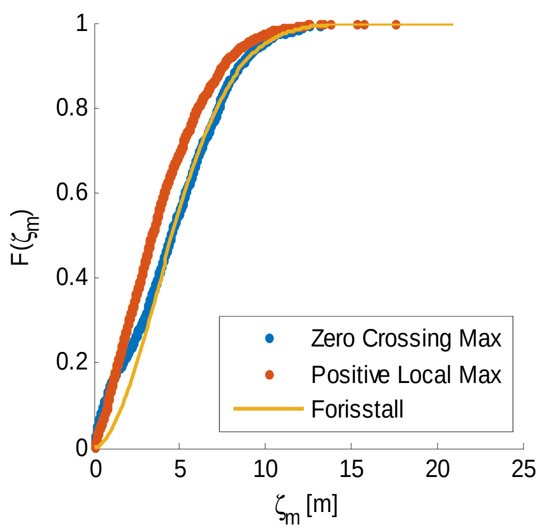
The simulation is based on flowchart:

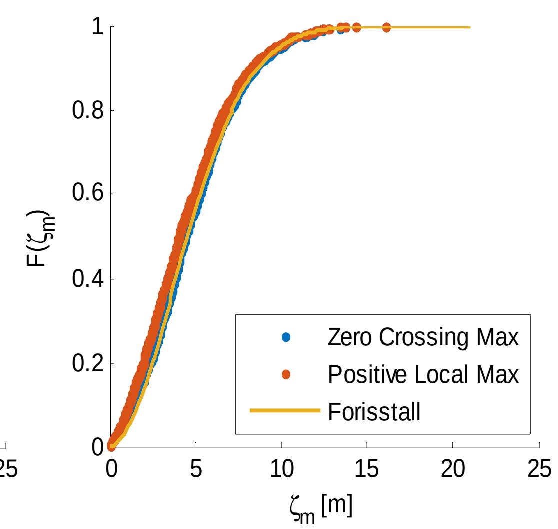


For verification purposes, Hs=14.9m and Tp=15.8s, which is the seastate on previous thesis by Evardsen[6]. The result of second order surface:

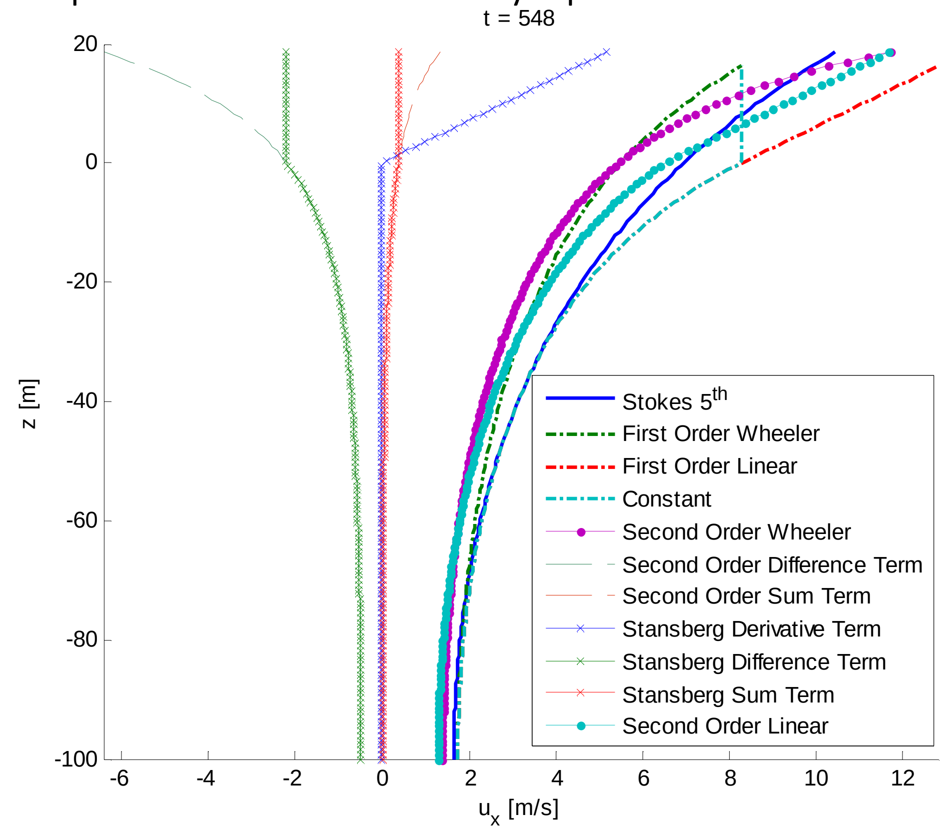


The local maximums and global maximums are gathered and sorted to create the empirical CDF. The empirical CDF is compared with Forristal crest distribution[7]. High frequency components induce deviation in the lower tail (left figure). However,introducing a ω_{cut} changes the sea surface process from broadbanded into narrowbanded (right figure) and significantly decreases the lower tail deviation.

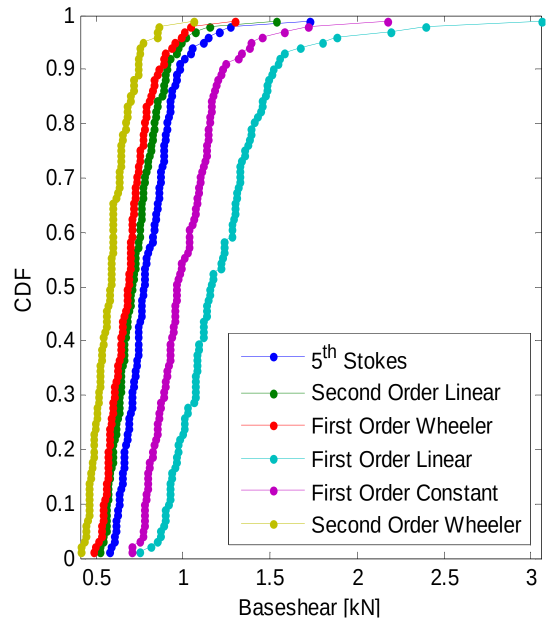


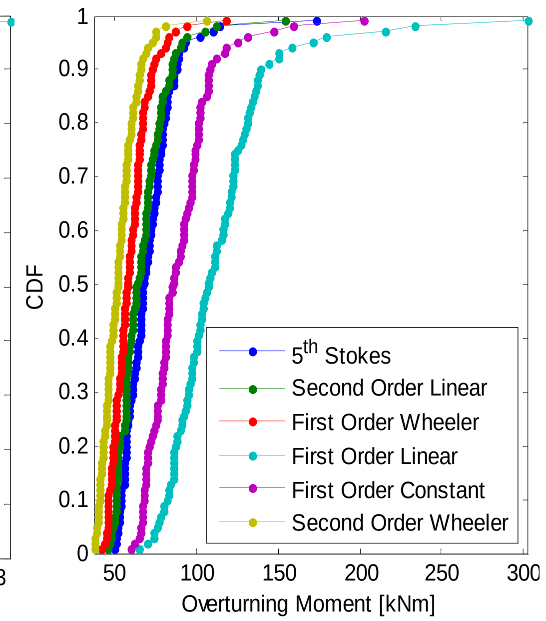


The comparisson of horizontal velocity is presented as:



The linear extrapolation on 2^{nd} order wave produces the kinematics that agree well with 5^{th} Stokes. However, linear extrapolation on 1^{st} order wave greatly overestimates the result. The comparison of quasistatic baseshear (left) and overturning moment (right) on sinlge cylinder are presented as:





It seems applying Wheeler stretching on 2^{nd} order wave will underestimates the static baseshear and overturning moment. In addition, the linear extrapolation on 1^{st} order wave greatly overestimates the result. The analysis will be continued with Jack-up model.From metocean analysis, it found that the H_{100} = 24.8m, C_{100} = 15.5m, T_{mean} = 14.6s.