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Comparison of the Svee and Thoma Stability Criteria for Mass Oscillations in Surge Tanks

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M.Sc. THESIS IN HYDRAULIC ENGINEERING

Candidate: Mr. Eirik Leknes

Title: Comparison of the Svee and Thoma Stability Criteria for Mass Oscillations in Surge Tanks

1. Background

Surge tanks are applied in hydropower plants to reduce the pressure forces during acceleration of the water, and to enable speed governing of the turbines. Reduction of the pressure forces is reduces the necessary strength of the structural components and thereby the costs. Speed governing enables hydropower plants to contribute to maintain a sufficient frequency in the power grid. In Norway, all hydropower plants above 10 MW are required to run with speed governing and to contribute to controlling the grid frequency.

Speed governing influences the mass oscillations in the surge tank, and may cause an instability. If the surge tank is too small, the speed governing can result in amplification of the mass oscillations and cause dangerous water levels and pressures. Thoma (1910) was the first to describe this phenomenon, and derive an equation to calculate the necessary size of the surge tank. However, Thoma's equation is based on simplifications and neglect factors such as the velocity head of the water, and the variable turbine efficiency during oscillations. To enable more accurate design and calculation of surge tanks, Svee (1972) derived a new equation which considers the velocity head of water and the variable turbine efficiency. This equation was proven to significantly reduce the necessary size of the surge tank in the hydropower project Paulo-Afonso III in Brazil. However, Svee's equation has not become standard in application. More work is necessary to validate the equation derived by Svee, and compare it with the Thoma equation.

2. Main questions for the thesis

The thesis shall cover, though not necessarily be limited to the main questions listed below.

2.1 Literature and desk study

The candidate shall carry out a literature study of design of hydropower waterways with surge tanks. Further, relevant theory for the design and calculation of surge tanks shall be studied. Finally, the candidate shall investigate and describe the theoretical approaches for evaluation of mass oscillation stability.

2.2 Main tasks

The candidate must find available background material such as former studies, textbooks and scientific papers on mass oscillation stability. Related to this material the following must be carried out:

1 Present a literature review of earlier works on mass oscillation stability.
2 Presentation of the different equations for mass oscillation stability.
3 Perform a parameter sensitivity analysis for the Thoma and Svee stability criteria.
4 Perform a stability analysis of mass oscillation in a case-study hydropower plant.
5 Establish a 1D simulation model of the case-study.

6. Perform simulations of stable and unstable conditions.
7. Compare the simulation results with the theoretical values.
8. Proposals for future work
9. Give an oral presentation in a seminar

3 Supervision and data input

Professor Leif Lia and PhD candidate Kaspar Vereide will supervise and assist the candidate, and make relevant information available.

Discussion with and input from colleagues and other research or engineering staff at NTNU is recommended. Significant inputs from other shall be referenced in a convenient manner.

The research and engineering work carried out by the candidate in connection with this thesis shall remain within an educational context. The candidate and the supervisors are therefore free to introduce assumptions and limitations which may be considered unrealistic or inappropriate in a contract research or a professional context.

4 Report format, references and contract

The master contract must be signed not later than 15. January. The report should be written with a text editing software, and figures, tables, photos etc. should be of good quality. The report should contain an executive summary, a table of content, a list of figures and tables, a list of references and information about other relevant sources. The report should be submitted electronically in B5-format .pdf-file in DAIM, and three paper copies should be handed in to the institute.

The executive summary should not exceed 450 words, and should be suitable for electronic reporting.

The Master’s thesis should be submitted with the time according to DAIM.

Trondheim 14. January 2015

Leif Lia
Professor
Department of Hydraulic and Environmental Engineering
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Abstract

This thesis discusses two different criteria for the stability of surge tanks in hydropower plants. In 1910, the first stability criterion was derived by Dieter Thoma. This criterion sets a minimum cross-sectional area for a surge tank to give stable conditions for the power plant. The criterion was put into use, but was found to give inaccurate results, and later modified. In 1970, Hallbjørn Roald Svee derived a second stability criterion, improving on the inaccuracy by including several factors that Thoma had left out. Svee's criterion was written as a reply to bad results in using the Thoma criterion for the stability of a surge tank in Brazil. However, after its use by Svee himself, the criterion has not been put into widespread use. The derivation of the two criteria are presented side by side, where both criteria have been rewritten to a modern form and non-explained steps from the original sources are derived.

To test the criterion, three studies were carried out. A parameter sensitivity analysis, a case study of Kvinen power plant and a one-dimensional numerical simulation on the same power plant, comparing the Thoma and Svee criteria. For the parameter sensitivity study, most of the results show the Svee criterion giving a smaller surge tank area than the Thoma criterion. A smaller surge tank is easier and less expensive to construct. The only parameter giving an opposite result is the turbine efficiency. For a system operating on a load higher than the best efficiency point, the Svee criterion gives a larger area. The area of the tunnels in the system is found to be the parameter most influential on the area of a stable surge tank.

The case study shows similar results, smaller surge tanks for Svee except in high load cases. The numerical simulation shows slow dampening of the mass oscillations for a system dimensioned after the Thoma criterion, and unstable oscillations for the Svee criterion. This should imply that the Svee criterion gives a surge tank design that is unstable, but the limitations of a numerical simulation suggests a physical model test should be performed.

Samandrag

Oppgåva tar for seg to forskjellege stabilitetskriterier for massesvingingar i opne svingekammer i vasskraftverk. I 1910 vart det første stabilitetskriteriet utleda av Dieter Thoma. Kriteriet setter eit minimumstverrsnitt for svingekammeret, slik at kraftverket oppnår reguleringsstabilitet. Kriteriet vart tatt i bruk, men det viste seg å gi unøyaktige resultat, og vart seinare omarbeida. I 1970 utleda Hallbjørn Roald Svee eit nytt stabilitetskriterium. Dette skulle forbetre nøyaktigheita ved å ta med fleire faktorar som Thoma hadde utelatt. Svee's kriterium vart skreve som ei utgreiing rundt dårlege resultat i samband med eit svingekammer i eit vasskraftverk i Brasil. Svee brukte sjølv det nye kriteriet i ei rekkje arbeid, men det ikkje i bruk per i dag. Utleiinga av dei to kriteria er tatt med i sin heilheit, oppdatert og utvida for å gi betre forståing.

Tre studiar vart laga for å teste Svee's kriterium. Ein parameterstudie, eit dømestudie av Kvinen kraftverk og ein numerisk simulering, også av Kvinen kraftverk. I alle tre studia blei Svee og Thoma sine kriterier satt opp mot kvarandre. Parameterstudien viste at for dei fleste kombinasjonar av parameter for eit kraftverk, så vil Svee's kriterium gi eit mindre areal i svingekammeret enn Thoma. Dette gjer Svee's kriterium interessant, då eit mindre svingekammer er enklare og billigare å byggje. Arealet av tunnelane i kraftverket er den parameteren som har mest å seie for størrelsen på svingekammeret. Den einaste parameteren som gir eit anna resultat, er turbinverknandsgraden. Om ein kjører turbinen på ei vassføring høgare enn bestpunktet, vil Svee's kriterium gi eit større areal enn Thoma.

Dømestudien gir liknande resultat. I den numeriske modelleringa gir svingekammer med areal etter Thoma ei sakte demping av massesvingingane, mens Svee gir eit areal på svingekammeret som gir ustabile, aukande svingingar. Dette kan tyde på at Svee's kriterium ikkje gir eit stabilt kraftverk, men dette må undersøkast nærare i eit modellforsøk.

Preface

This thesis is written as the final requirement for the Masters degree in Civil Engineering at the Norwegian University of Science and Technology, NTNU.

The work submitted was done at the Department of Hydraulic and Environmental Engineering, NTNU, Trondheim. Supervisors have been PhD Kaspar Vereide and Professor Leif Lia. The idea for the Thesis is to be credited to Kaspar, as it is in close connection with his research and doctoral thesis on air cushion surge tanks. Professors Torbjørn Nielsen and Bjørnar Svingen at the Department for Energy and Process Engineering, NTNU, have also given valuable contributions. Wolfgang Richter at the Technical University Graz is commended for excellent supervision on field trip to Graz.

A large thanks to my two supervisors, Kaspar and Leif, for encouragement, guidance and inspiration. Your time, discussions and attitude has helped me greatly, and will be with me in my coming life as a young professional. A special thanks to Kaspar and Wolfgang for organizing a great field trip, giving me an introduction to the field of hydraulics in academia.

Thanks to my family for good feedback and encouraging words. Thanks also to the other students at the department, for a good work environment with quiz, cake, and shenanigans. To my study group through this five-year program, Troll-ing, it has been an honor working with you all.

Eirik Leknes

Eirik Leknes

Trondheim 09.06.16

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List of symbols

Symbol	Unit	Description
L_T	m	Length of tunnel
A_T	m^2	Cross sectional area of tunnel
L_S	m	Length of pressure shaft/draft tube
A_S	m^2	Cross sectional area of pressure shaft/draft tube
A_{ST}	m^2	Cross sectional area of surge tank
M	$m^{1/3}$	Mannings number
R	m	Hydraulic radius
Q	m^3/s	Discharge through system
Qopt	m^3/s	Optimal discharge (best efficiency point)
H_0	mWc	Gross head
z	mWc	Head loss in tunnel system
Z_0	mWc	Constant value of z
Δz	mWc	Variable value of z
H_{eff}	mWc	Effective head $H_0 - Z_0$
H_{mom}	mWc	Momentary head $H_0 - Z$
V	m^3	Volume of water
ρ	kg/m^3	Density of water
g	m/s^2	Acceleration of gravity
γ	N/m^3	Specific weigh of water
F	N	Force
v	m/s	Water velocity
c	m/s	Speed of sound in water
t	s	Time
P	W	Turbine output
m	kg	Mass
α		Loss coefficient
η		Turbine efficiency

1. Introduction

This chapter gives background information and presents the scope of project. An introduction to the concepts of grid stability, water hammer, surge tanks and mass oscillations is given, presented from the fundamental equations of hydraulics. A historical literature review on the development of surge tank stability is also presented.

1.1. Hydropower and grid stability

Hydropower is the power source that is easiest to regulate and change. In an electricity grid, there needs to be a balance between power produced and power consumed. Since electricity is not stored in the grid, a sudden change in either production or consumption will lead to a change of the frequency in the grid. The grid frequency is 50 Hz in all of Europe. The stability of the grid frequency is crucial for stable and continuous operation of electric components. Especially small electronics such as computer or mobile phones will have their lifespan shortened by large variations in frequency. To ensure a stable grid, the Norwegian national grid, Statnett SF, require all hydropower plants over 10MW to have speed governing (Statnett SF, 2012). Speed governing is done through a device known as a governor, that connects to the grid and regulates the speed of the turbine to ensure a stable frequency at all times. The governor changes the guide vanes or nozzles to increase or reduce the amount of power produced. To ensure a stable grid, it is important that the water in the hydropower system quickly responds to changes in the flow. This is done by introducing a surge tank to the tunnel system. For the power plant to be stable, the surge tank has to be of a proper size.

1.2. Project scope

For the surge tank to operate stable, a criterion is set, governing its size in relation to the other dimensions of the power plant. The Thoma criterion for surge tank stability is in common use by hydropower engineers worldwide. However, Norwegian engineer Hallbjørn Roald Svee proposed a different stability criterion in his 1970 thesis. This criterion could reduce the needed cross-sectional area of the surge tank, reducing construction costs. This criterion is not in widespread use. This thesis hopes to change that, validating the criterion and comparing it to the Thoma criterion.

1.3. Scope of work

This thesis will present two competing theories and criteria for surge tank stability. A historical review of surge tank research, as well as derivation of the criteria will be included. A comparison in three parts will show a parameter sensitivity study, a case study and a numerical modelling simulation. The thesis will not take into account the larger picture of dynamic design of surge tanks. Special cases with multiple surge tanks in a system and throttling will not be taken into account, as it does not directly influence the stability criteria.

1.4. Fundamental equations

1.4.1. Continuity equation

The continuity equation looks at inflow and outflow from a control volume (CV), through its outer surface called the control surface (CS). For a total review on the equation and its derivation, see (Wylie & Streeter, 1993)In general form it can be written as:

$$\frac{d}{dt}\int_{CV}\rho dV+\int_{CS}\rho|\vec{v}|d\,\vec{A}=0 \tag{1}$$

The first term of the equation describes the net inflow through the control surface. This is then said to be equal to the accumulation of mass in the control volume. In its simplest form, the continuity equation states that all mass flowing in through the surfaces of an object is equal to the change in mass of that object. This follows logically from the principle of conservation of mass. The first term can be rewritten to be the total inflow minus the total outflow of mass. By also assuming a finite number of inlets and outlets from the control volume, the following can be derived:

$$\frac{d}{dt}m_{CV}+\sum_{CS}\frac{dm_0}{dt}-\sum_{CS}\frac{dm_i}{dt}=0 \tag{2}$$

1.4.2. Momentum equation

The momentum equation is an adaption of Newton’s second law to fluids. (Wylie & Streeter, 1993) shows a total derivation of the equation, but included her is a simplified form:

$$\sum F=\frac{d}{dt}\int_{CV}\vec{v}\rho dV+\int_{CS}\vec{v}\rho d\,\vec{A} \tag{3}$$

Here, the first term on the right denotes the change in momentum, the second term denotes the change in mass. Together, the continuity and momentum equations are used to describe different phenomenon connected to hydraulic transients.

1.4.3. Rotating mass and penstock time constant

The equations showed in this part are all taken from (Guttormsen, 2006)In a hydropower system, the rotating mass time Ta is calculated as the time it takes to accelerate the generator from zero to normal speed with full load torque.

$$T_a = \frac{GD^2}{4}\frac{\omega_0^2}{P_{max}} \tag{4}$$

The penstock time constant T_w describes the time it takes to accelerate the water in the headrace tunnel from zero to Q_0 under the influence of H_0. The sum of the tunnel lengths divided by the cross-sectional area gives an idea that shorter, wider tunnels will give a shorter Tw.

$$T_w = \frac{Q_0}{gH_0}\sum\frac{L}{A} \tag{5}$$

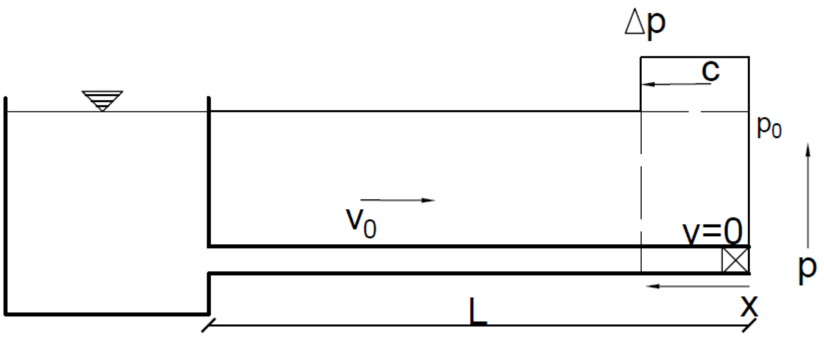
If Tw is larger than Ta, the water in the penstock will not accelerate fast enough to feed the turbine and generator spinning up. This will lead to negative pressure in the tunnels and water column separation. Ta is rather uninfluenced by power plant design and size, for large power plants it is often 6. For a stable operation, a criterion is set at

$$\frac{T_a}{T_w} > 4 \tag{6}$$

For conservatism, Tw<1 is usually used as a criterion for most power plants.

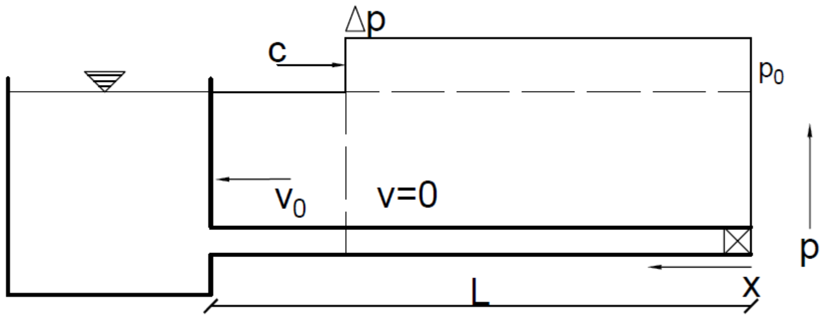
1.5. Water Hammer

Water hammer is a concept that involves water compressibility. For most intents, water is regarded as incompressible. This property of water means a change of flow in a closed system will form a pressure wave, compressing the water along the system, travelling at the speed of sound. A pressure wave interacts with boundaries between different materials. At the interface, the wave is reflected, travelling back the way it came from. In a closed tunnel system, boundaries are found at free water surfaces. A change in flow can occur by the opening or closing of valves, or start and stop of pumps or turbines. These changes can be abrupt and absolute, or slow and minute. For any change in flow, a pressure wave will propagate from the point of change, to the nearest free water surface. At the free water surface, the wave will be reflected, and a negative pressure wave of the same magnitude will propagate back to the point of origin. Figure 1 shows this cycle of pressure waves in a simple system.



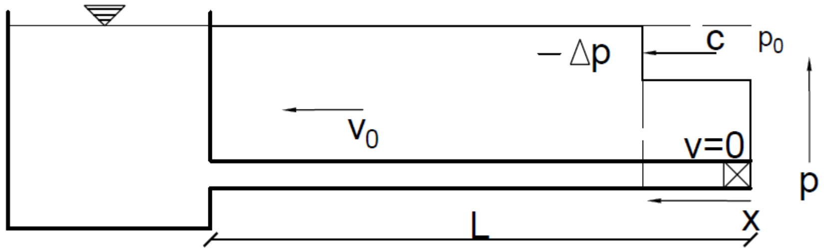
A 0<t<L/c

At the time of valve closing, water is flowing to the right. It stops, sending a positive pressure wave to the left up to the tank. When the pressure wave reaches the tank, all the water in the pipe has zero velocity.



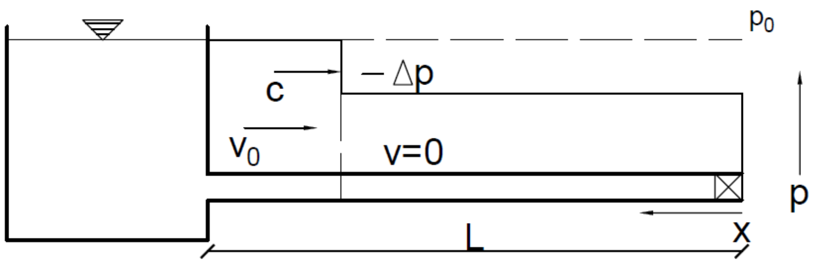
B L/c<t<2L/c

After hitting the tank, the pressure wave travels back towards the valve, and water starts flowing to the left. At t=2L/c, the pressure in the pipe is normalized, and water flows back to the tank.



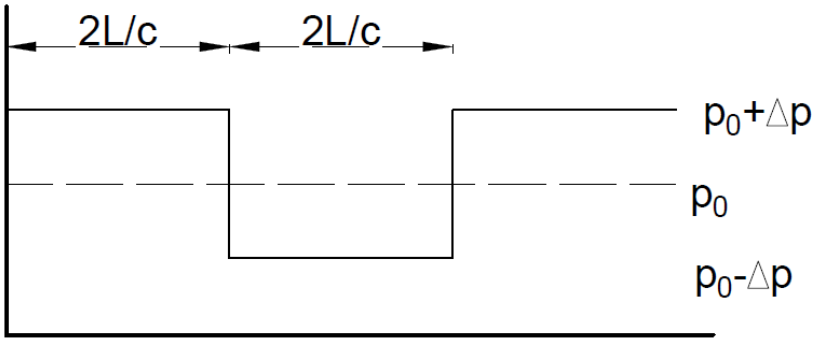
C 2L/c<t<3L/c

Since the valve is still closed, no water can flow to the left, leaving a negative pressure. This pressure wave travels back up to the tank. At t=3L/c, all the water in the pipe is again at rest, and under negative pressure.



D 3L/c<t<4L/c

At t=3/Lc, water starts flowing out of the tank, to equalize the negative pressure. At t=4L/c, the situation is back to the initial conditions at part A, and the cycle starts over.



E

This graph shows the pressure at the valve in relation to time. The form is idealized, in reality the pressure on the second oscillation will be smaller.

Figure 1 The cycles of water hammer oscillations.

The pressure waves will be dampened out by friction in the tunnels, and will eventually die out. The friction of the tunnel is dependent on the cross-sectional area and the roughness. The magnitude of the pressure wave is derived from the momentum equation. This result is taken from (Guttormsen, 2006)

$$\Delta p = v\rho c \frac{T_r}{T_w} \tag{7}$$

Where

$$T_r = \frac{2L}{c} \tag{8}$$

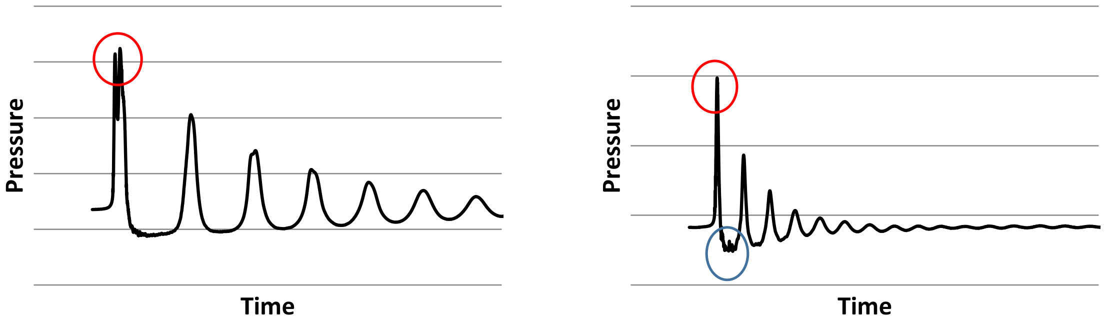
Because the water hammer wave is a dampened one, the first oscillation is the largest. The maximum pressure at the first oscillation is governed by equation (7). Since both the density, speed of sound and approach time are locked, if the resulting pressure is too high handle, other parameters must be changed. Changing the velocity of the water would mean making a larger tunnel, which would be very costly. The easiest is to reduce the penstock time constant by reducing the distance from the turbine to the nearest free water surface. This leads to the introduction of the surge tank.

1.6. The surge tank and mass oscillations

A surge tank or surge shaft is a vertical shaft or tank connected to the pressure tunnel, close to the turbine. This reduces the penstock time constant and therefore the maximum water hammer pressure. This ensures a stable output from the turbine, and a stable electricity grid. In the event of a total stop of the turbine, the surge tank also reduces the pressure on the steel components near the turbine. The surge tank also provides storage of a volume of water available for acceleration of the turbine, leaving the water in the tunnel enough time to accelerate. However, the introduction of a free surface leaves to free surfaces in the tunnel system, opening for mass oscillations.

Figure 2 shows mass oscillations and water hammer in a system with and without surge tank. The data is from a laboratory test of surge tanks at the Hydraulic institute at Technical University Graz, Austria, collected on a field trip. The system consists of a reservoir with a valve connected to a pressure gauge, and a surge tank that can be connected and disconnected. Looking at 2a, the initial pressure is higher than in 2b. 2 a shows the water hammer, with its extreme value circled in red. With no surge tank, the system only experiences the effect of water hammer, but it takes a while before it dampens out. In b, two wave structures are superposed. The first large oscillation shown in red, and the smaller ones shown in blue, is the water hammer. The amplitude of the water hammer decreases fast when comparing to a system without a surge tank. The remaining oscillations shown is the mass oscillation.

These have a longer frequency than the water hammer, but get dampened out by friction relatively quick.

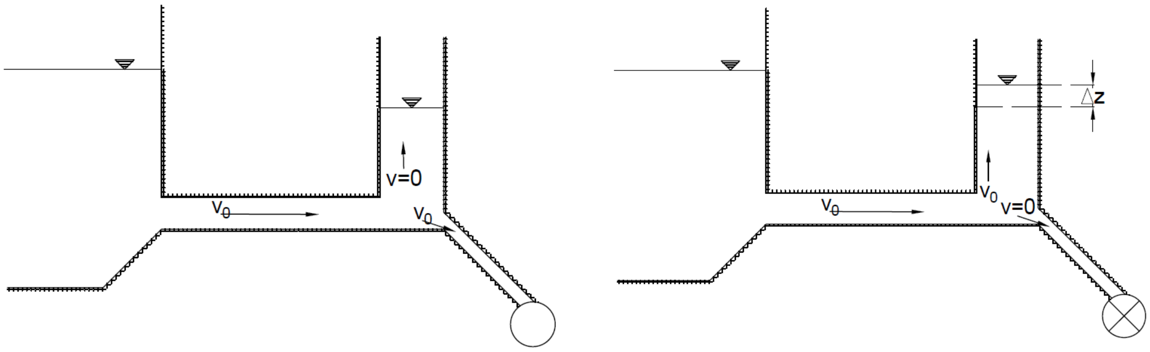


A No surge tank connected

B Surge tank connected

Figure 2: Water hammer and mass oscillations measured at the valve for a system with and without a surge tank. The red circles show the maximum water hammer amplitude; the blue circle shows the small oscillations of the water hammer.

Mass oscillations or U-tube oscillations is a phenomenon where water is treated as incompressible, and surge from one end of a u shaped tube to the other, between two or more free water surfaces (Guttormsen, 2006). Figure 3 shows a complete shutoff of the turbine in a simple hydropower system. When the turbine stops, the water travelling to the right in the tunnel will flow into the surge tank, increasing the water level.



A Power plant running

B Power plant stopped, water flowing to the surge tank

Figure 3: A system shutdown scenario showing the oscillations in the surge tank.

The mass oscillations lead to a new form of instability for the governor, tunnel and grid system. For stable operation, it must be ensured that the mass oscillations are dampened, and will die out by the friction in the tunnel system. The friction on the tunnel walls is the dampening, and is dependant on water velocity, cross-sectional area and roughness of the tunnels. To ensure a stable operation of the power plant, a stability criterion is set where the friction is sufficient to dampen the oscillations.

1.7. Surge tank stability

1.7.1. Surge tank design

The stability criterion is the first parameter used for design of a surge tank, and is the basis for the design calculations (Guttormsen, 2006). Other factors governing the size of a surge tank is the maximum pressure allowed in the tunnel, and the maximum up- and downsurge (Leknes, 2015). The dynamic design process of a surge tank includes a lot of iterations, and often the final design is far from the stability criterion, with expanded chambers, throttles and differential effects. The importance of the stability criterion can often be lost in the process. Since the criterion is the starting point of the design process, if it is not correct, the whole design process will be based on false pretenses.

1.7.2. Thoma criterion

In 1910, Dieter Thoma delivered his Ph. D. thesis on surge tanks at the Technischen Hochschule Munchen, in Munich, Germany (Thoma, 1910). Surge tanks were already known, but Thoma's thesis is often seen as the genesis of surge tank research. In the thesis the concepts of grid stability, water hammer and mass oscillations are discussed, and a criterion for the stability of surge tanks is derived. As mentioned (Statnett SF, 2012) all power plants in Norway over 10MW must be equipped with governors. In 1910, the effect of the governor was starting to become an issue, and Thoma's thesis is in reply to the challenges at a power plant in the Ruhr valley in Germany, where the speed governing lead to unstable oscillations. Air chambers, which had been used as stabilizers in pipe systems previously, was found to give unstable oscillations, and it was hypothesized that a simple surge tank would work in the same way as a large air chamber. The dangers of air being pulled into the tunnel on a downsurge is also discussed. The height of the surge tank as well as the water in the pressure shaft is neglected, stating the modest dimensions of these two water volumes. Thoma's stability criterion balances the power of the water in the tunnel with the water in the surge tank. A turbine with a rigid speed governor is set as the basis for the calculations. The criterion takes into account the friction in the tunnel, with a roughness factor, length and area to describe the tunnel.

The thesis discusses additional factors; a lag in the governor, permanent nonuniformity of regulation and variability. The common denominator for all these parameters is that they are thought by Thoma to have a positive impact of the system stability, therefore giving the stability criterion a sense of conservatism. The variance in both turbine efficiency and head losses are not taken into account, because they are thought to cancel each other out. On the other side, Thoma suggests building surge tanks larger than the criteria, as it is regarded as a minimum.

In 1955, a translation of Thoma's thesis was presented by the US Army Corps of Engineers (Thoma, 1955).

1.7.3. Further development of Thoma

In the early days of hydropower development, the tunnel systems were relatively simple, and as the systems became more complex, the phenomenon's connected to surge tanks was further investigated. German, English and Norwegian engineers were hard at work in the period from 1910 through the second world war, developing different designs of surge tanks, and encountering more complex scenarios than what Thoma's criterion could accommodate. Charles Jaeger, Armin Schoklitsch, Josef Frank and Fredrik Vogt are among the more notable researchers on the topic.

Armin Schoklitsch was an Austrian physicist, vital for the development of the hydraulic laboratory at Technichschen Hochschule Graz. He also invented a method for graphical solution of mass oscillations in surge tanks (Hveding, 1946). This was a good leap forward, as solving these equations is a tall order without any computer aids. This method is discussed by Vidkunn Hveding in his master thesis at the Norwegian Technical Institute in Trondheim under and immediately after the second world war. This shows that the method was taught in occupied nations during the war. Hvedings thesis also discuss the efforts of Frank and several other researchers, showing that over the years, there has been many competing criteria and theories on surge tank stability. After the war, Schoklitsch, who was an SS officer, fled to Argentina, and continued his work on hydraulic structures.

Charles Jaeger was a Swiss hydropower engineer recognized for his work as a professor of rock mechanics and hydropower, both in Switzerland, the UK and USA. He found discrepancies in Thoma's method, questioning the stability by show of examples that do not follow the criterion, that are stable, and power plants with unstable conditions that follow Thoma's criterion (Jaeger, 1960). Jaeger proposes several solutions. A variable safety factor is proposed, varying on the magnitude of the maximum surge height divided by the net head. The value of this safety factor will fall between 1.01 and 1.1. A simpler solution is to use a constant safety factor, proposed to be between 1.5 and 1.8 (Jaeger, 1949). Standard practice today is to use a safety factor of 1.5, without any mention of where it stems from and what it represents (Guttormsen, 2006). With the safety factor, this criterion is named modified Thoma. Other parts of Jaegers work discuss a system with several surge tanks, how they will influence each others stability (Jaeger, 1958) (Jaeger, 1953). It states, among other things, that the vertical shaft closest to the turbine, be it a creek intake or a surge tank, will act as the primary surge tank, and must be designed as such. The period of the mass oscillations is said to be the most important factor when designing a hydropower system with several surge tanks.

1.7.4. Svee criterion

In 1972 Hallbjørn Roald Svee delivered his Ph.D. thesis at the Technichschen Hochschule Graz, in Graz, Austria. He noticed a large cap between the Thoma criterion and data from his hydraulic laboratory test on the surge tanks at the Paulo Afonso III power plant in Brazil (Gegenleithner, et al., 2015). He realized that Thoma had neglected the velocity head, which was problematic for the Paulo Afonso III, having a very short tunnel (Svee, 1970). His thesis presents the derivation of a new, more complex stability criteria. With it, he significantly reduced the size of the surge tanks needed for Paulo Afonso III, from 900 m^2 to 270 m^2. A smaller surge tank is easier and less expensive to build, making this project more economically feasible. The thesis also shows several model test using the criterion for dimensioning surge tanks.

Although his thesis deals with surge tank stability, Svee is most noted for his invention of the air cushion surge tank (Svee, 1972b). The air cushion surge tank uses pressurised air inside the surge tank to act as a dampener, placing the surge tank close to the turbine, to give small mass oscillations and a stable system. This allows the construction of a directly inclined tunnel, reducing losses and costs. 10 air cushion surge tanks have since been built in Norway, and the design has been exported to China and Vietnam (Vereide, et al., 2015).

Svee's new stability criteria was never put into standard application (Vassdrags- og havnelaboratoriet, 1972a). He published several workbooks on it in Norwegian, but never published a more thorough report on the implications of it in English (Svee, 1972a) (Svee, 1972c). Possibly, the hydropower industry and other researches saw no need to fix something, namely Thoma's criterion, that was working just fine with the inclusion of the safety factor. Possibly, the criterion was considered too complex. Whatever the reason, hydropower engineers to this day still use the Thoma criterion when doing the initial design of a surge tank. Svee used his own research for dimensioning several surge tanks, but ultimately, other designs were chosen (Vassdrags- og havnelaboratoriet, 1972b).

The Svee criterion takes into account many of the parameters mentioned both by Thoma and other researchers. The headrace and tailrace are treated to two different criteria. In the headrace, the velocity head is added, to improve stability, while in the tailrace it is subtracted, reducing stability. A separate part takes the variable efficiency of the turbine into account. In most scenarios, it gives a cross-sectional area somewhere between that of Thoma and modified Thoma. Svee's thesis also discuss new surge tank designs, and their influence on stability, as well as showing examples of how his criterion can be applied, giving a smaller surge tank. Like the Thoma criterion, Svee's thesis make the assumption of a perfectly governed turbine, with a rigid governor. Provided this is true, the Svee criterion should give results closer to reality, as several concepts excluded in Thoma, are included here.

2. Stability criteria

This chapter presents the derivation of the two stability criteria, Thoma and Svee. The derivation is relatively complex, and is in some parts expanded on from the original format to provide a better understanding. The equations of the criteria are then compared side by side.

2.1. Thoma

This part is based on Thoma's thesis (Thoma, 1910). The nomenclature in the derivation of this criteria has been updated to meet the current common nomenclature. It has also been changed from horsepower to watt as the output unit for power, giving room to include ρ, the density of the water. Several other minor adjustments give the derivation a more modern form. Consider figure 4. A standard hydropower plant is seen, with a surge tank in the headrace. A rigid, perfect governor is controlling the output from the turbine.

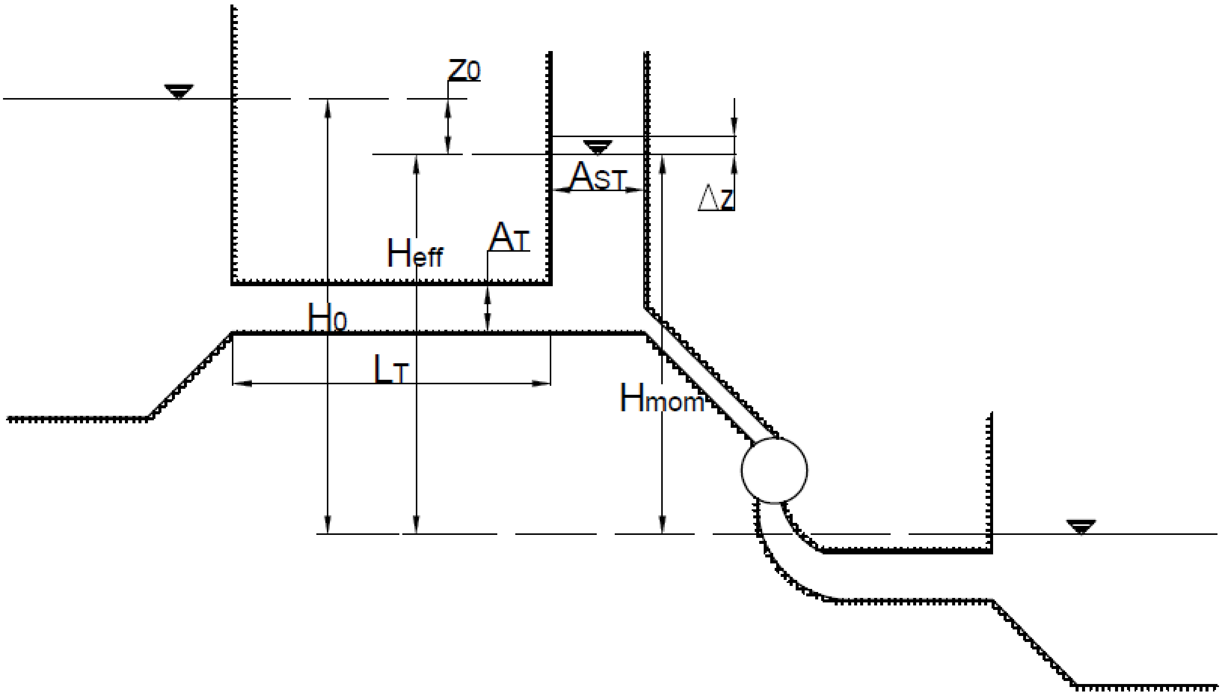


Figure 4: Hydropower plant with parameters as defined by Thoma

Starting off with the power equation for a turbine and penstock. The power is governed by the discharge, momentary head and physical parameters:

$$P = QH_{mom}\rho g\eta \quad (9)$$

Discharge from the turbine is expressed:

$$Q = \frac{P}{H_{mom} g \rho \eta} \tag{10}$$

Combining factors to make a substitution gives a second equation for discharge:

$$\frac{P}{\rho g \eta} = J \tag{11}$$

$$Q = \frac{J}{H_{mom}} = \frac{J}{H_0 - z} \tag{12}$$

Equating the friction in the tunnel to the head loss gives:

$$\frac{L}{g}\frac{dv}{dt} = z - \alpha v^2 \tag{13}$$

Next, consider the volume of water leaving or entering the surge tank:

$$A_{ST}\frac{dz}{dt} = Q - A_T v \tag{14}$$

Substituting from (12) gives:

$$A_{ST}\frac{dz}{dt} = \frac{J}{H_0 - z} - A_T v \tag{15}$$

(13) and (15) now form the two principal equations in the problem. A_{ST} is dependant on z, J on v. To solve for the stability criteria, on needs to find the boundary conditions where (13) and (15) are equal. Thoma shows that this can be found either using the theory of infinitesimal oscillations, or assuming constant output. Looking at the case where A_{ST} and J are constant, meaning constant tank area and constant turbine output gives a better understanding of the problem, without giving any large deficiency in accuracy.

Defining basic parameters:

$$z = z_0 + \Delta z \quad , \quad v = v_0 + \Delta v \tag{16}$$

Inserting (16) into (13) and (15) transforms the two equations:

$$\frac{L_T}{g}\frac{d\Delta v}{dt} = z_0 + \Delta z - \alpha(v_0 + \Delta v)^2 \tag{17}$$

$$\frac{A_{ST}dz}{dt} = Q - A_T(v_0 + \Delta v) \tag{18}$$

In (17), z_0 is the head loss in the tunnel, and can be said to be equal to $\alpha\ v_0^2$. Therefore:

$$L_T\,\frac{d\Delta v}{dt} = \Delta z - 2\alpha\Delta v v_o \tag{19}$$

In (18), $A_T\ v_0 = Q$, therefore:

$$A_{ST}\,\frac{d\Delta z}{dt} = -A_T\Delta v \tag{20}$$

Differentiating with respect to t gives:

$$A_{ST}\,\frac{d^2\Delta z}{dt^2} = -A_T\,\frac{d\Delta v}{dt} \tag{21}$$

dΔv/dt in (21) is substituted from (18) and Δv from (19), and rearranging gives:

$$A_{ST}\,\frac{d^2\Delta z}{dt^2} = -A_T\,\frac{g}{L_T}\,(\Delta z - 2\alpha v_0\Delta v) \tag{22}$$

$$A_{ST}\,\frac{d^2\Delta z}{dt^2} = -\frac{A_T g}{L_T}\left(\Delta z + 2\alpha v_0\,\frac{A_{ST}}{A_T}\frac{d\ \Delta z}{dt}\right) \tag{23}$$

$$\frac{d^2\Delta z}{dt^2} + \frac{2g\alpha v_0}{L_T}\frac{d\ \Delta z}{dt} + \frac{A_T g}{A_{ST}L_T}\Delta z = 0 \tag{24}$$

This second order differential equation can be solved to find Δz:

$$r = \frac{-\frac{2g\alpha v_0}{LT} \pm \sqrt{\left(\frac{2g\alpha v_0}{LT}\right)^2 - \frac{4A_T g}{A_{ST}L_T}}}{2} \tag{25}$$

$$r = -\frac{g\alpha v_0}{LT} \pm \sqrt{-\frac{A_T g}{A_{ST}L_T} + \frac{g^2\alpha^2 v_0^2}{L_T^2}} \tag{26}$$

The roots are shown to be complex, giving (23) a solution of the form:

$$\Delta z = e^{\lambda t}(C_1 \sin qt + C_2 \cos qt) \tag{27}$$

Where:

$$\lambda = -\frac{g\alpha v_0}{L_T} \tag{28}$$

$$q = \sqrt{\frac{A_T g}{A_{ST} L_T} - \frac{g^2 \alpha^2 v_0^2}{L_T^2}} \tag{29}$$

Since all the components of λ are positive, λ will always be negative. This means the oscillations will always be dampened. Considering the steady state where the head loss is constantly equal to z, the discharge can be said to be:

$$Q = \frac{J}{H_{eff}} \tag{30}$$

Given a slight disturbance, a sinusoidal wave as (26), the effective head can be represented by:

$$H_{mom} = H_{eff} + a\ \cos qt \tag{31}$$

The governor will now adjust the output from the turbine giving a new output of J_1:

$$Q = \frac{J_1}{H_{mom}} = \frac{J_1}{H_{eff} + a \cos qt} \tag{32}$$

For Q to be constant, this means that:

$$mean\ value\ of \frac{J_1}{H_{eff} + a \cos qt} = \frac{J}{H_{eff}} \tag{33}$$

Or:

$$\frac{q}{2\pi} \int_0^{\frac{2\pi}{q}} \frac{J_1}{H_{eff} + a \cos qt}\ dt = \frac{J}{H_{eff}} \tag{34}$$

Carrying out the integration gives, according to Thoma (1910):

$$J_1 = J\frac{\sqrt{H_{eff}^2 - a^2}}{H_{eff}} \tag{35}$$

This result cannot be reproduced using any known method, and is assumed correct on the authority of Thoma. (35) is not elaborated on in Thoma (1910), but expanding this expression to a Taylor series, then neglecting all parts with a/H_{eff} in higher orders, gives:

$$J_1 = J\sqrt{1 - \frac{a^2}{Heff^2}} \tag{36}$$

$$J_1 = J\left(1 - \frac{a^2}{2H_{eff}^2}\right) \tag{37}$$

If J is the energy supplied to the surge tank, and B_1 is the energy withdrawn from the surge tank, the average amount of energy remaining in the surge tank at any time is:

$$J - J_1 = J\frac{a^2}{2H_{eff}^2} \tag{38}$$

The oscillatory energy is equal to the weight of the water volume raised above normal level, multiplied by the distance of its centre of gravity from normal level, or:

$$E = A_{ST}a\frac{a}{2} \tag{39}$$

(37) indicates that on average, the change in energy in the surge tank is:

$$\frac{dE}{dt} = J\frac{a^2}{2H_{eff}^2} \tag{40}$$

(38) indicates this change in energy to be:

$$\frac{dE}{dt} = A_{ST}a\frac{da}{dt} \tag{41}$$

This gives:

$$J\frac{a^2}{2H_{eff}^2} = A_{ST}a\frac{da}{dt} \tag{42}$$

The integration of this part is not shown in Thoma (1910), but is elaborated here:

$$\frac{J}{2H_{eff}^2 A_{ST}} = \frac{1}{a}\frac{da}{dt} \tag{43}$$

$$\int \frac{J}{2H_{eff}^2 A_{ST}} dt = \int \frac{1}{a}\ da \tag{44}$$

$$a = a_0\ e^{\frac{J}{2A_{ST}H_{eff}^2}t} \tag{45}$$

We now have two differential equations with roots:

$$\frac{dE}{dt} = A_{ST}a\frac{da}{dt} \rightarrow\ a = a_0\ e^{\frac{J}{2A_{ST}H_{eff}^2}t} \tag{46}$$

$$\frac{d^2\Delta z}{dt^2} + \frac{2g\alpha v_0}{L_T}\frac{d\ \Delta z}{dt} + \frac{A_T g}{A_{ST}L_T}\Delta z = 0 \rightarrow\ \Delta z = e^{\lambda t}(C_1 \sin qt + C_2 \cos qt) \tag{47}$$

$$\lambda = -\frac{g\alpha v_0}{L_T}\ and\ q = \sqrt{\frac{A_T g}{A_{ST}L_T} - \frac{g^2\alpha^2 v_0^2}{L_T^2}} \tag{48}$$

In order to determine the boundary state, the change in head loss is set equal to the amplitude of the sinusoidal wave. This relates back to figure 4, showing that the water level changes in the surge tank, forming a sinusoidal wave, corresponds to the variable part of the head loss:

$$\Delta z = a \tag{49}$$

Δz is the root of (23), while a is the root of (41). The roots are found in (26) and (44):

$$a_0\ e^{\frac{J}{2A_{tank}H_{eff}^2}t} = e^{\lambda t}(C_1 \sin qt + C_2 \cos qt) \tag{50}$$

For (23), the complex root in (26) can only have one solution for λ, meaning the root of (23) is (27), giving:

$$-\frac{g\alpha v_0}{L_T}+\frac{J}{2A_{ST}H_{eff}^2}=0 \tag{51}$$

If we consider that $v_0 = Q_0/A_T = J/H_{eff}\,A_T$ we get:

$$\frac{2g\alpha A_{ST}H_{eff}}{A_T L_T}=1 \tag{52}$$

Giving the criterion:

$$A_{ST}\geq\frac{A_T L_T}{2g\alpha H_{eff}} \tag{53}$$

This equation is well known by all hydropower engineers, used in the initial stages of dimensioning all surge tanks. Henceforth, it will be named the original Thoma criterion, or AThoma. Most engineers will be familiar with a different with a different version, using a safety factor of 1,5.

$$A_{ST}\geq 1{,}5\,\frac{A_T L_T}{2g\alpha H_{eff}} \tag{54}$$

This criterion will hence forth be known as the modified Thoma criterion, or AThoma'

2.2. Svee

This part is based on Svee's thesis (Svee, 1970). Nomenclature has been updated to current standards, and some parts altered to give an understanding more in touch with current design standard. Start by examining a surge tank in the headrace tunnel. The system is governed by 3 general equations:

$$\vec{F}dt = (\vec{m}v) \tag{55}$$

$$Q = A_{ST}\frac{dz}{dt} + A_T v \tag{56}$$

$$\eta\, Q\left(H_0 - z + \frac{v^2}{2g}\right) = Constant \tag{57}$$

Where (55) shows that the force in the system is equal to the force of the water. (56) shows discharge through the tunnel to be discharge in the tunnel and the area and the water height difference in the surge tank. (57) shows that the output from the turbine, the product of the discharge, efficiency and net head, is kept constant. Figure 5 shows the general layout of a simple hydropower plant, with a surge tank in the headrace.

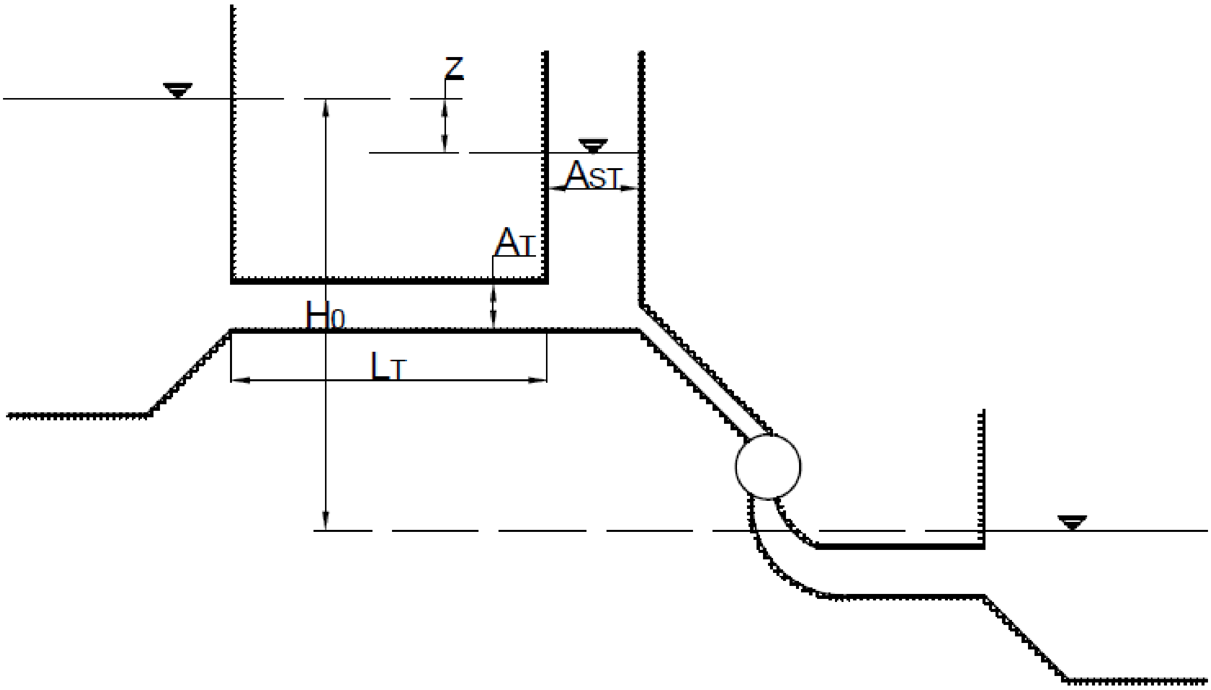


Figure 5: Hydropower plant with as parameters as defined by Svee

The turbine is governed by a perfect, rigid governor, ensuring equation (57) is always satisfied. The parameters v, z, Q, and η can be expanded or retracted as shown:

$$v = v_0 + \Delta v, \qquad z = z_0 + \Delta z, \qquad Q = Q_0 + \Delta Q, \qquad \eta = \eta_0 + \Delta\eta \tag{58}$$

Starting with equation (55), expansion of the terms gives:

$$F = m\frac{dv}{dt} + v\frac{dm}{dt} \tag{59}$$

$$F = \rho\, L_T\ A_T\frac{dv}{dt} - v\,\rho\, A_{ST}\frac{dz}{dt} \tag{60}$$

$$F = \rho\, L_T\ A_T\frac{d(\Delta v)}{dt} - v_0\,\rho\, A_{ST}\frac{d(\Delta z)}{dt} \tag{61}$$

In the tunnel two forces are working opposite each other. K denotes the force implied by gravity, and R the friction forces. K can be explained from figure 6:

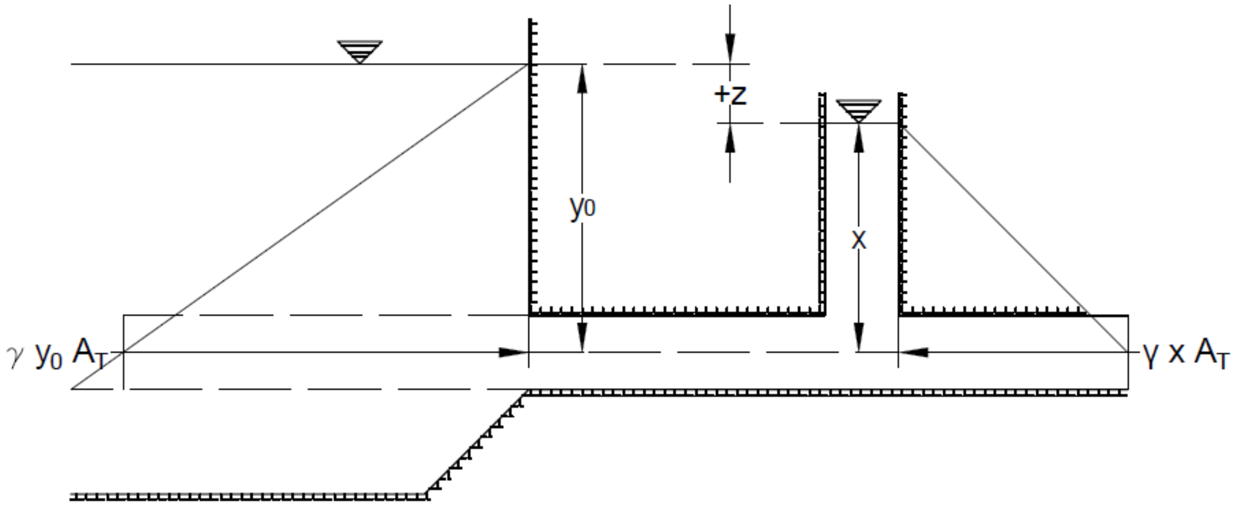


Figure 6: Forces in the tunnel for a running turbine

$$K = \gamma x A_T - \gamma y_0 A_T \tag{62}$$

$$K = \gamma A_T z \tag{63}$$

The friction force is dependent on the velocity and friction in the tunnel:

$$R = \alpha\, v^2\, \gamma\, A_T \tag{64}$$

The total force in the tunnel is expressed as:

$$F = K - R \tag{65}$$

In the steady state, these two forces are equal:

$$K_0 - R_0 = 0 \tag{66}$$

In the event of a surge they are noted as:

$$F = (K_0 + \Delta K) - (R_0 + \Delta R) \tag{67}$$

Where:

$$\Delta K = \gamma A_T \Delta z \tag{68}$$

$$\Delta R = 2\, \alpha\, v_0\, \Delta v\, \gamma\, A_T \tag{69}$$

These forces are shown in figure 7:

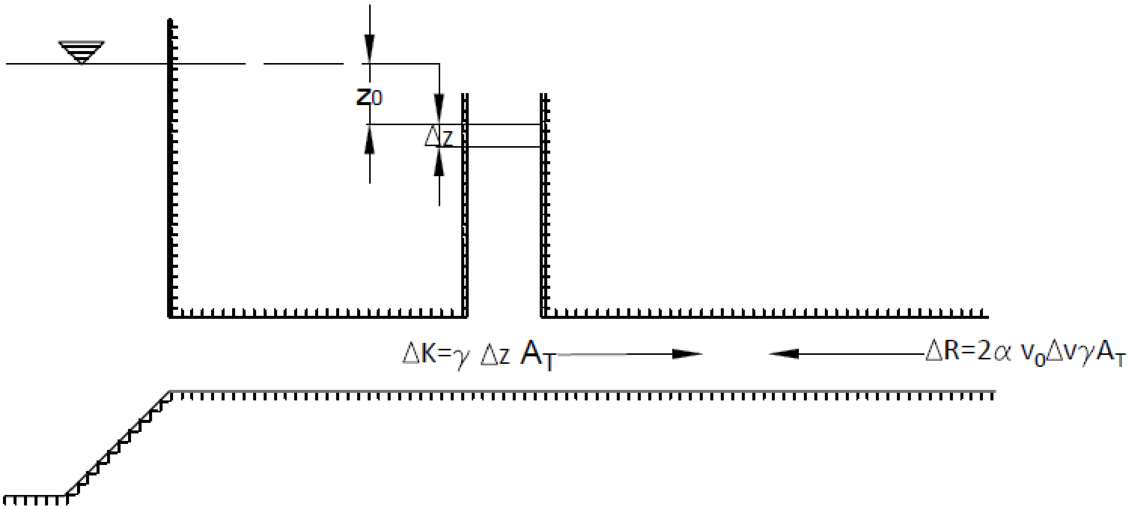


Figure 7: Forces in the tunnel in the event of a surge

Together, this gives:

$$F = \gamma\ A_T\ \Delta z - 2\ \alpha\ v_0\ \Delta v\ \gamma\ A_T \tag{70}$$

$$F = \gamma\ A_T\ (\Delta z - 2\ \alpha\ v_0\ \Delta v\ ) \tag{71}$$

Comparing (71) and (61) as expressions for F, the force in the system gives:

$$\gamma\ A_T\ (\Delta z - 2\ \alpha\ v_0\ \Delta v\ ) = \ \rho\ L_T\ \ A_T \frac{d(\Delta v)}{dt} - v_0\ \rho\ A_{ST} \frac{d(\Delta z)}{dt} \tag{72}$$

By introducing the relationship ρ/γ = 1/g:

$$\frac{L_T}{g} \frac{d(\Delta v)}{dt} = \Delta z - 2\ \alpha\ v_0\ \Delta v + \frac{v_0 A_{ST}}{g A_T}\ \frac{d(\Delta z)}{dt} \tag{73}$$

This equation relates the forces in the tunnel to the forces in the surge tank. Going back to (56), redefining with the parameters from (58) gives:

$$Q_0 + \Delta Q = A_{ST} \frac{d(z_0 + \Delta z)}{dt} + A_T (v_0 + \Delta v) \tag{74}$$

Where:

$$\frac{d(z_0 + \Delta z)}{dt} = \frac{d(\Delta z)}{dt} = \Delta z \tag{75}$$

$$Q_0 = v_0\ A_T \tag{76}$$

Reforming gives:

$$\Delta Q = A_{ST} \frac{d(\Delta z)}{dt} + A_T \Delta v \tag{77}$$

This equation relates the change in discharge to the flow in the tunnel. Going back to (57), redefining with the parameters from (58) gives:

$$(\eta_0 + \Delta\eta)\ (Q_0 + \Delta Q)\ \left\{H_0 - z_0 - \Delta z + \frac{1}{2g}\ (\ v_0 + \Delta v)^2\right\} = \ \eta_0\ Q_0\ \left(H_0 - z_0 + \frac{v_0^2}{2g}\right) \tag{78}$$

Expanding and eliminating parts of higher order, (68) can be rewritten as:

$$\{\eta_0\, Q_0 + \eta_0 \Delta Q + Q_0\, \Delta\eta\}\left\{H_0 -\ z_0 - \Delta z + \frac{v_0^2}{2g} + \frac{2v_0 \Delta v}{2g}\right\} = \eta_0 Q_0 \left\{H_0 - z_0 + \frac{v_0^2}{2g}\right\} \tag{79}$$

Rearranging gives:

$$-\eta_0\, Q_0\, \Delta z + \eta_0\, Q_0\, \frac{2v_0\, \Delta v}{2g} + \eta_0 \left(H_0 - z_0 + \frac{v_0^2}{2g}\right) \Delta Q + Q_0\, \left(H_0 - z_0 + \frac{v_0^2}{2g}\right) \Delta\eta = 0 \tag{80}$$

Introducing a parameter ζ:

$$\zeta = \left(H_0 - z_0 + \frac{v_0^2}{2g}\right) \tag{81}$$

Substitution with ζ in (80 gives:

$$-\eta_0\, Q_0\, \Delta z + \eta_0\, Q_0\, \frac{2v_0 \Delta v}{2g} + \eta_0\, \zeta\, \delta Q + Q_0\, \zeta\, \Delta\eta = 0 \tag{82}$$

Rearranging gives:

$$\eta_0\, \zeta \left\{\delta Q + \frac{Q_0}{Q_0} \Delta\eta\right\} - \eta_0\, Q_0\, \Delta z + \eta_0\, Q_0\, \frac{2v_0\, \Delta v}{2g} = 0 \tag{83}$$

$$\zeta\, \left(1 + \frac{Q_0\, \Delta\eta}{\eta 0\, \Delta Q}\right) \Delta Q -\ Q_0\, \Delta z + \frac{Q_0\, v_0}{2g} \Delta v = 0 \tag{84}$$

This opens for another factor to be substituted:

$$E = \left(1 + \frac{Q_0\, \Delta\eta}{\eta 0\, \Delta Q}\right) \tag{85}$$

Rewriting (84) to:

$$E\, \delta Q - \frac{Q_0}{\zeta} \Delta z + \frac{Q_0\, v_0}{\zeta\, g} \Delta v = 0 \tag{86}$$

Recapitulating, the equations (55), (56) and (57) have been reformed to equations (73), (77) and (86):

$$\vec{F}dt = (\vec{m}v) \rightarrow \frac{L_T}{g}\frac{d(\Delta v)}{dt} = \Delta z - 2\,\alpha\, v_0\,\Delta v + \frac{v_0 A_{ST}}{g A_T}\,\frac{d(\Delta z)}{dt} \tag{73}$$

$$Q = A_{ST}\frac{dz}{dt} + A_T v \rightarrow \Delta Q = A_{ST}\frac{d(\Delta z)}{dt} + A_T\,\Delta v \tag{77}$$

$$\eta\, Q\,\left(H_0 - z + \frac{v^2}{2g}\right) = Constant \rightarrow E\,\Delta Q - \frac{Q_0}{\zeta}\Delta z + \frac{Q_0\, v_0}{\zeta\, g}\Delta v = 0 \tag{86}$$

These are the same three equations governing the oscillations. Rewriting (86) gives:

$$\Delta Q = \frac{Q_0}{\zeta\, E}\Delta z - \frac{Q_0\, v_0}{\zeta\, E\, g}\Delta v \tag{87}$$

Comparing (87) and (77) as expressions for ΔQ gives:

$$\frac{Q_0}{\zeta\, E}\Delta z - \frac{Q_0\, v_0}{\zeta\, E\, g}\Delta v = A_{ST}\frac{d(\Delta z)}{dt} + A_T\Delta v \tag{88}$$

$$\left\{A_T + \frac{Q_0\, v_0}{\zeta\, E\, g}\right\}\Delta v = \frac{Q_0}{\zeta\, E}\Delta z - A_{ST}\frac{d(\Delta z)}{dt} \tag{89}$$

Using the relationship $Q_0 = v_0\, A_T$, (89) can be rewritten:

$$A_T\left\{1 + \frac{v_0^2}{\zeta\, E\, g}\right\}\Delta v = \frac{v_0\, A_T}{\zeta\, E}\Delta z - A_{ST}\frac{d(\Delta z)}{dt} \tag{90}$$

Another substitution:

$$B = A_T\left\{1 + \frac{v_0^2}{\zeta\, E\, g}\right\} \tag{91}$$

Transforming (90) into an equation for Δv:

$$\Delta v = \frac{A_T\, v_0}{\zeta\, E\, B}\,\Delta z - \frac{A_{ST}\, d(\Delta z)}{B\, dt} \tag{92}$$

Differentiating this part is done on Newton notation in (Svee, 1970). Using standard Leibniz notation, differentiating this equation gives:

$$\frac{\mathrm{d}(\Delta v)}{dt} = \frac{A_T\ v_0}{\zeta\ E\ B}\frac{\mathrm{d}(\Delta z)}{dt} - \frac{A_{ST}}{B}\frac{\mathrm{d}^2(\Delta z)}{dt^2} \tag{93}$$

Combining equations (73), (77), (92) and (93) gives:

$$\frac{L_T}{g}\left\{\frac{A_T v_0}{\zeta\ E\ B}\frac{\mathrm{d}\Delta \mathrm{z}}{dt} - \frac{A_{ST}}{B}\frac{\mathrm{d}^2(\Delta z)}{dt^2}\right\} = \Delta z - 2\ \alpha\ v_0\left\{\frac{A_T\ v_0}{\zeta\ E\ B}\Delta z - \frac{A_{ST}}{B}\frac{\mathrm{d}(\Delta z)}{dt}\right\} + \frac{v_0\ A_{ST}}{gA_T}\frac{\mathrm{d}(\Delta z)}{dt} \tag{94}$$

Rearranged this gives the second order differential equation:

$$\frac{L_T}{g}\frac{A_{ST}}{B}\frac{\mathrm{d}^2(\Delta z)}{dt^2} + \left\{\frac{2\alpha\ v_0^2\ A_{ST}}{B} + \frac{v_0\ A_{ST}}{g\ A_T} - \frac{L_T\ A_{ST}\ v_0}{g\ \zeta\ E\ B}\right\}\frac{\mathrm{d}(\Delta z)}{dt} + \left\{1 - \frac{2\alpha\ v_0^2\ A_T}{\zeta\ E\ B}\right\}\Delta\ z = 0 \tag{95}$$

This equation describes the mass oscillations in the surge tank. For the equation to give dampened oscillations, all the roots must be positive. This means the surge tank and tunnel system has to satisfy the three following equations:

$$\frac{L_T A_{ST}}{g\ B} > 0 \tag{96}$$

$$2\ v_0\ \alpha\frac{A_{ST}}{B} + \frac{v_0\ A_{ST}}{gA_T} - \frac{L_T\ A_T v_0}{g\ \zeta\ E\ B} > 0 \tag{97}$$

$$1 - \frac{2\ \alpha\ v_0^2\ A_T}{\zeta\ E\ B} > 0 \tag{98}$$

For satisfying (96), L_T, A_{ST} and g are all strictly positive, B is therefore the important parameter.

$$B > 0 \rightarrow\ A_T\left\{1 + \frac{v_0^2}{\zeta\ E\ g}\right\} > 0 \tag{99}$$

Rearranging gives:

$$\zeta > \frac{-v_0^2}{E\ g} \tag{100}$$

Expanding ζ and E and rearranging gives:

$$H_0 - z_0 + \frac{v_0^2}{2g} > \frac{-v_0^2}{E\ g} \tag{101}$$

$$H_0 > z_0 - \frac{v_0^2}{2g} - 2\frac{v_0^2}{2g}\ E^{-1} \tag{102}$$

$$H_0 > z_0 - \frac{v_0^2}{2g} - 2\frac{v_0^2}{2g}\left\{1 + \frac{Q_0\ \Delta\eta}{\eta 0\ \Delta Q}\right\}^{-1} \tag{103}$$

Since the gross head is always larger than the head losses, this criterion is always satisfied. The second equation to be satisfied is (97), which can be rewritten to:

$$A_{ST}\left\{2\frac{\alpha v_0}{B} + \frac{v_0}{g\ A_T}\right\} > \frac{L_T\ \ A_T\ v_0}{g\ \zeta\ E\ B} \tag{104}$$

$$A_{ST}\left\{\frac{2\ \alpha\ v_0\ g + v_0\ B}{g\ B\ A_T}\right\} > \frac{L_T\ \ A_T\ v_0}{g\ \zeta\ E\ B} \tag{105}$$

Assuming, by criterion (96) B>0:

$$A_{ST} > \frac{L_T\ \ A_T^2}{(2\ \alpha\ g\ A_T + B)\ \zeta\ E} \tag{106}$$

Expanding B, ζ and E, then rearranging gives:

$$A_{ST} > \frac{L_T\ A_T^2}{\left(2\ \alpha\ g\ A_T + A_T\left(1 + \frac{v_0^2}{\zeta\ E\ g}\right)\right)\ \zeta\ E} \tag{107}$$

$$A_{ST} > \frac{L_T\ A_T}{2\ g\ \left(\alpha + \frac{1}{2g}\right)\left(H_0 - z_0 + \frac{v_0^2}{2g}\right)\ \left(1 + \frac{Q_0\ \Delta\eta}{\eta 0\ \Delta Q}\right)\ + 2\frac{v_0^2}{2g}} \tag{108}$$

The last criterion, (98) can be rewritten to:

$$\zeta\ E\ B > 2\ \alpha\ v_0^2\ A_T \tag{109}$$

Expanding B gives:

$$\zeta\ E\ A_T\left(1+\frac{v_0^2}{\zeta\ E\ g}\right) > 2\ \alpha\ v_0^2\ A_T \tag{110}$$

$$\zeta\ E+\frac{v_0^2}{g} > 2\ \alpha\ v_0^2 \tag{111}$$

Assuming head loss z_0 is equal to $\alpha\ v_0^2$:

$$\zeta\ E > 2\ \left(z_0-\frac{v_0^2}{2g}\right) \tag{112}$$

Expanding ζ and E gives:

$$H_0-z_0+\frac{v_0^2}{2g} > 2\ \left(z_0-\frac{v_0^2}{2g}\right)\left(1+\frac{Q_0\ \Delta\eta}{\eta 0\ \Delta Q}\right)^{-1} \tag{113}$$

$$H_0 > \left(z_0-\frac{v_0^2}{2g}\right)\left\{1+2\left(1+\frac{Q_0\ \Delta\eta}{\eta 0\ \Delta Q}\right)\right\}^{-1} \tag{114}$$

This criterion states that the gross head must be larger than the head loss multiplied with the losses in the turbine. In practice, all power plants will fulfill this criterion, leaving the following criterion:

$$A_{ST} \geq \frac{L_T\ A_T}{2\ g\ \left(\alpha+\frac{1}{2g}\right)\left(H_0-z_0+\frac{v_0^2}{2g}\right)\left(1+\frac{Q_0\ \Delta\eta}{\eta 0\ \Delta Q}\right)+2\frac{v_0^2}{2g}} \tag{115}$$

This criterion will henceforth be known as the original Svee criterion for headrace, or ASvee.

2.2.1. Headrace vs tailrace

This total derivation has assumed a surge tank in the headrace. For a surge tank in the tailrace, the velocity head will decrease the stability, rather than contributing to it. For a surge tank in the tailrace, equation (115) is converted to this form:

$$A_{ST} \geq \frac{L_T\ A_T}{2\ g\ \left(\alpha+\frac{1}{2g}\right)\left(H_0-z_0-\frac{v_0^2}{2g}\right)\left(1+\frac{Q_0\ \Delta\eta}{\eta 0\ \Delta Q}\right)-2\frac{v_0^2}{2g}} \tag{116}$$

Neither of the two stability criteria, (115) and (116), take into account the other surge tank. Many power plants have a surge tank both in the headrace and the tailrace. The equations assume that the two surge tanks can be super positioned, giving stable conditions both in the headrace and tailrace.

(Svee, 1972a) shows that in the event where the two surge tanks have similar oscillation periods, they can influence each other. The paper suggests using equations for the periods to cope with this. Today, this effect is often studied in a numerical or physical model, rather than by the use of equations.

2.2.2. Other effects

In his thesis, Svee notes that there are several effects not taken into account in (115), such as throttling the surge tank, or throttling the tunnel itself. Another topic is the influence of the pressure shaft on the stability of the surge tank (Svee, 1972a). For the case of a throttling of the surge tank, there is no change in the stability criteria. However, a throttling of the headrace tunnel will introduce a Venturi effect, giving a large transient loss, overshadowing other factors. In the event that a headrace tunnel has varying areas, Svee suggested to sum the length divided by area for each part. Creek intakes are also mentioned; these improve the stability of the system.

Many of the concepts presented in this paper are not relevant today, or cannot be generalised to form parts for a comparison. A special interest is seen in the influence of the headrace shaft. In early hydropower plants, the pressure shaft was very short, and was mentioned as negligible by (Thoma, 1910). As pressure shaft grew longer, it needed to be included. To take into account the destabilising effect of the headrace shaft, Svee multiplies the original stability criterion with a factor (Svee, 1972a):

$$\frac{1+\frac{L_S A_T}{L_T\, A_S}}{1-3\,\frac{z_S}{H_0-z_0}} \tag{117}$$

In this factor, the length, cross-sectional area and head loss in the headrace shaft is introduced. For a surge tank in the tailrace, the analogue case is taking the draft tube and tunnel length from the turbine to the surge tank into account. This transforms the equations (115) and (116):

$$A_{ST} \geq \frac{L_T\, A_T}{2\,g\,\left(\alpha+\frac{1}{2g}\right)\left(H_0-z_0+\frac{v_0^2}{2g}\right)\,\left(1+\frac{Q_0\,\Delta\eta}{\eta 0\,\Delta Q}\right)+2\,\frac{v_0^2}{2g}}\,\frac{1+\frac{L_S A_T}{L_T\, A_S}}{1-3\,\frac{z_S}{H_0-z_0}} \tag{118}$$

$$A_{ST} \geq \frac{L_T\, A_T}{2\,g\,\left(\alpha+\frac{1}{2g}\right)\left(H_0-z_0-\frac{v_0^2}{2g}\right)\,\left(1+\frac{Q_0\,\Delta\eta}{\eta 0\,\Delta Q}\right)-2\,\frac{v_0^2}{2g}}\,\frac{1+\frac{L_S A_T}{L_T\, A_S}}{1-3\,\frac{z_S}{H_0-z_0}} \tag{119}$$

Equation (118) will hence forth be known as the modified Svee criterion, or ASvee'

2.3. Comparison of criteria

Looking at just the headrace surge tank, the two stability criteria state:

$$A_{ST} \geq 1{,}5\,\frac{A_T L_T}{2g\alpha H_{eff}} \tag{54}$$

$$A_{ST} \geq \frac{L_T\,A_T}{2\,g\,\left(\alpha+\frac{1}{2g}\right)\left(H_0-z_0+\frac{v_0^2}{2g}\right)\,\left(1+\frac{Q_0\,\Delta\eta}{\eta 0\,\Delta Q}\right)+2\,\frac{v_0^2}{2g}}\;\frac{1+\frac{L_S A_T}{L_T\,A_S}}{1-3\,\frac{z_S}{H_0-z_0}} \tag{118}$$

The Thoma criterion is much simpler, more factors are included in the Svee criterion. Svee's criterion also changes depending on if the surge tank is in the headrace or the tailrace. Many of the factors used in the criteria are composite factors, they rely on several underlying factors. Using the Manning number for friction, and a tunnel cross section where $R=0.265\sqrt{A}$, the Thoma and Svee criteria can be broken down to:

$$A_{ST} \geq 1{,}5\,\frac{L_T A_T}{2\,g\left(\frac{L_T}{M^2\left(0{,}0265\,\sqrt{A_T}\right)^{\frac{4}{3}}}\right)\left(H_0-\left(\frac{L_T}{M^2\left(0{,}0265\,\sqrt{A_T}\right)^{\frac{4}{3}}}\,\left(\frac{Q}{A_T}\right)^2\right)\right)} \tag{120}$$

And

$$A_{ST} \geq \frac{L_T A_T}{2\,g\left(\frac{L_T}{M^2\left(0{,}0265\sqrt{A_T}\right)^{\frac{4}{3}}}+\frac{1}{2g}\right)\left(H_0-\left(\frac{L_T}{M^2\left(0{,}0265\sqrt{A_T}\right)^{\frac{4}{3}}}\,\left(\frac{Q}{A_T}\right)^2\right)+\frac{\left(\frac{Q}{A_T}\right)^2}{2g}\right)\left(1+\frac{Q_0}{\eta_0}\frac{\Delta\eta}{\Delta Q}\right)+2\,\frac{\left(\frac{Q}{A_T}\right)^2}{2g}} \cdot \frac{1+\frac{L_S A_T}{L_T A_S}}{1-3\,\frac{\left(\frac{L_S}{M^2\left(0{,}0265\sqrt{A_S}\right)^{\frac{4}{3}}}\,\left(\frac{Q}{A_S}\right)^2\right)}{H_0\left(\frac{L_T}{M^2\left(0{,}0265\sqrt{A_T}\right)^{\frac{4}{3}}}\,\left(\frac{Q}{A_T}\right)^2\right)}} \tag{121}$$

This expansion shows that there is a real need for the parameter sensitivity study, as the criteria are too complex to evaluate quantitatively from looking at the expressions. The two criteria show two different eras of surge tank research, both stemming from a doctoral thesis at a major hydraulic institute. The evolution of hydropower plants, giving more complex tunnel systems, has lead to development of many different surge tank designs, while for stability criteria, a lot has been written, but Thoma’s criterion is still in use, over a hundred years after its conception. Given that the initial assumptions for the derivation are correct, Svee’s criterion should give a surge tank area closer to the physical boundary for stability, because it takes more factors into account.

3. Method

This chapter presents three methods of comparing the two criteria, and lists the data used, special cases and hypotheses. The purpose of the thesis is to compare the stability criteria against each other. For a hydropower system, each criteria calculates a surge tank area, and these are compared in a parameter sensitivity study, a case study and a numerical simulation study.

3.1. Parameter sensitivity study

For comparing the stability criteria against one another, a set of sample parameters were set up for a hypothetical hydropower system. Six parameters were taken into account: Lengths and areas of tunnels and shafts, in addition to gross head and nominal discharge. These six parameters define all but two of the characteristics of a hydropower plant. The roughness of the tunnel is also an important property, represented by the Mannings roughness factor. This is seldom a parameter that can be varied, as it is determined by the rock and tunnelling method used. Most Norwegian hydropower plants have unlined rock tunnels and a steel lined pressure shaft and draft tube. 35 and 80 is used as Mannings number for rock and steel lined tunnel, respectively. Turbine efficiency is the last property, and will be discussed separately. The parameters chosen for the parameter sensitivity study are listed below.

Table 1: Parameters and ranges for sensitivity study

Parameter	Property	Unit	Default value	Range	
L_T	Tunnel Length	m	10000	8000	12000
A_T	Tunnel Area	m^2	40	30	50
L_S	Pressure shaft Length	m	400	300	500
A_S	Pressure shaft Area	m^2	20	10	30
H_0	Gross head	mWc	400	300	500
Q	Nominal discharge	m^3/s	50	30	70

The parameter study was done in 2 parts. First, a headrace system was considered. A comparison was done between the Thoma and Svee criteria, dividing between the original and the modified versions of the Svee criteria. Then the same process was repeated for the tailrace, dividing between the original and modified Svee criteria. Both the original and modified Thoma criteria were used for comparison. The two sets of criteria compared were thus:

Table 2: The criteria for the headrace and tailrace systems

System	Svee criterion	Thoma criterion
Headrace	(118) and (115)	(53) and (54)
Tailrace	(119) and (116)	(53) and (54)

3.1.1. Turbine efficiency

Turbine efficiency is a parameter that will influence the stability of the power plant. If the turbine runs at a discharge under the best efficiency point, Q_{opt}, an increase in load will lead to a small change in discharge. This leads to an increase in efficiency, making the turbine adjust to the new power demand very quickly. This means that a system in this case will need less area in the surge tank, as it is more stable. If, however, the turbine is already running at the best efficiency point, an increase in discharge will lead to a drop in efficiency, and the turbine governor will not reach its demand, leading to another small increase in discharge. A system operating with this load will therefore need a larger area in the surge tank, as it is less stable than a system operating on a load lower than the best efficiency point. This concept is very interesting because of the change seen in the Nordic electricity market. Most large power plants in Norway were built during a period of monopoly, therefore designed for optimal efficiency, running on or close to the best efficiency point at all times. Today, with a free market, the prices can change rapidly, making it beneficial to run power plants on higher loads, giving less efficiency. If this efficiency has a significant impact on the stability of the power plant, that could be a potential problem. Figure 8 shows an efficiency curve of a typical Francis turbine, with varying η for different partial loads. For a Pelton or Kaplan turbine, the curve will take a different shape.

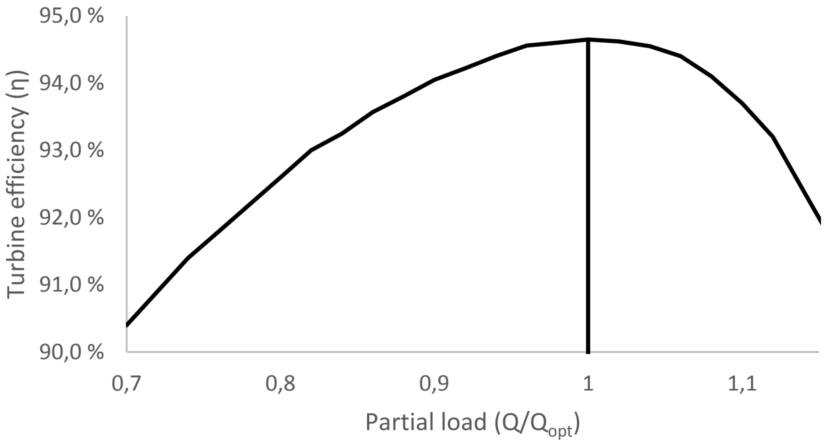


Figure 8: Turbine efficiency curve of a Francis turbine, shown by partial load

To take the changes in efficiency into account, a parameter analysis was run for the turbine efficiency alone. The default values from table 1 was used, and the load was varied from 70 % to 115 % of the load at the best efficiency point. This gives a range of discharge from 35 m^3/s to 58 m^3/s (Norconsult, 2005).

3.1.2. Assumptions

Since the criteria presented are relatively complex, making a good prediction of the results of the parameter sensitivity analysis is tough. However, a few key points can be deducted.

- Both the Thoma criteria make no distinction between the headrace and the tailrace, the Svee criteria do. Respectively, criterion (115) and (118) pertaining to the headrace and criterion (116) and (119) pertaining to the tailrace. The difference between these two is small, considering the order of magnitude of the water velocity in the tunnel, which is the one part in the criteria that changes its sign.
- The influence of the pressure shaft or draft tube will give a larger area. The parts taking this into account, (117), cannot be less than 1, therefore equations (118) and (119) will always give equal or larger valuesthan equations (115) and (116).
- Criteria (53) and (54), original and modified Thoma, will have a constant difference, since they are constant multiplications of each other. A hypothesis is that all of the Svee criteria will fall in between the two Thoma criteria.
- A special case will present itself when looking at the turbine efficiency. Here, the two Svee criteria will show a large difference, even giving a larger area than modified Thoma. This will be the only time the modified Thoma criterion will give a smaller area than the modified Svee criterion.

3.2. Case study

To complement the parameter sensitivity study, a case study is set up. The case to be studied was Kvinen Power plant, a power plant in the Sira-Kvina hydropower scheme in southern Norway. In the actual power plant, there are several creek intakes, and the headrace surge tank is built in connection to one of these (Sira-Kvina Kraftselskap, 1983). The tailrace surge tank is built with a large chamber on top of a narrow shaft. As the system is rather complex, a simplification was done, and the following parameters were used as input:

Table 3: Parameters for the case study

Parameter	Property	Unit	Value
H0	Gross head	mWc	116
Q	Nominal discharge	m^3/s	77
L_T1	Headrace tunnel length	m	4611
A_T1	Headrace tunnel area	m^2	48
L_S1	Pressure shaft length	m	317
A_S1	Pressure shaft area	m^2	13,2
L_T2	Tailrace tunnel length	m	6200
A_T2	Tailrace tunnel area	m^2	48
L_S2	Draft tube length	m	36
A_S2	Draft tube area	m^2	28
M_T	Mannings number tunnels	$m^{1/3}$	35
M_S	Mannings number pressure shaft and draft tube	$m^{1/3}$	80

For the headrace and tailrace system, all four criteria, the original and modified Thoma, and original and modified Svee, will give a surge tank area. To include the concept of varying turbine efficiency, three load cases were tried. First, the turbine is assumed to always be operation on the best efficiency point, meaning that the turbine efficiency is neglected. Two scenarios looking at a case with reduced and increased load is then studied. In these scenarios, the turbine is running at a load higher or lower than the load corresponding to the best efficiency point. That means the turbine efficiency will vary, giving a smaller or larger surge tank area for the two Svee criteria. A conceptual sketch of Kvinen power plant with parameters from table 3 is shown in figure 9.

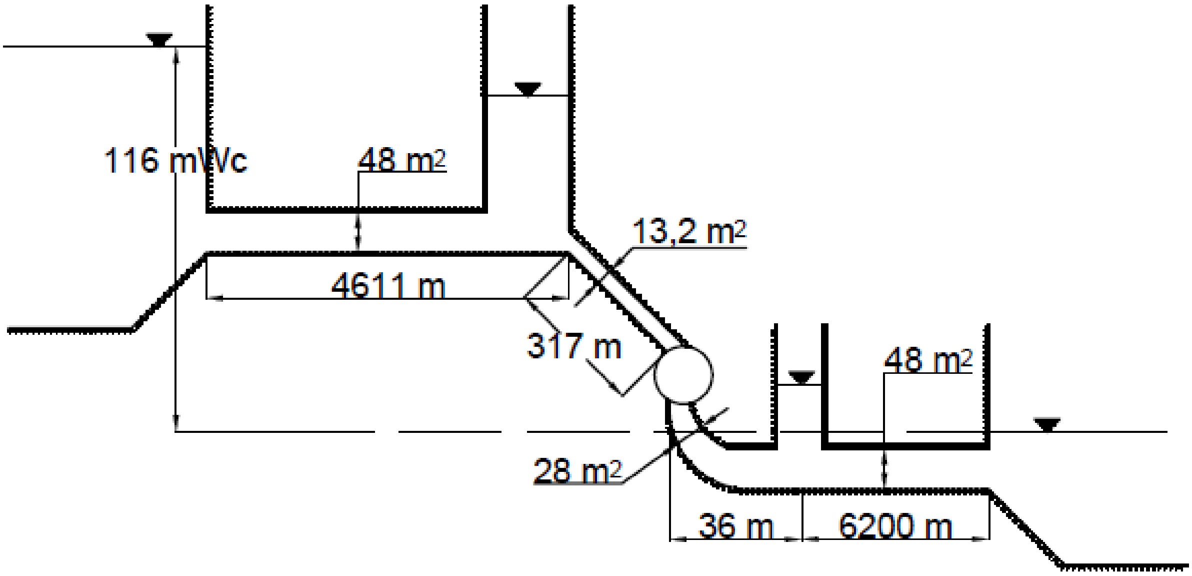


Figure 9: Concept sketch of Kvinen power plant, with dimensions used

The simplification means that the results of the case study cannot be directly compared to the current situation at Kvinen power plant. Kvinen has relatively long tunnels compared to the gross head. For power plants of a similar configuration, the case study will give valuable input to how the surge tank areas given by the different criteria interact.

3.3. Numerical simulation

The parameter sensitivity study and the case study only show how the criteria interact, nothing is mentioned about the actual stability of the surge tanks. Therefore, it was decided to do a numerical simulation to show the difference in stability between the criteria. A model of Kvinen power plant was built up in LVTrans, a numerical simulation software for transient pipe analysis. LVTrans uses the LabVIEW platform, representing each part of the hydropower system with a single block, with input for parameters. Data from the case study was used, and four systems were analyzed, using the surge tank areas given by the four criteria. LVTrans uses the characteristic method to solve the momentum and continuity equation for the tunnel system. For a closer look at this, see Svingen (2007) and Wylie & Streeter (1993). In short, the characteristic method is a method for solving partial differential equations by transformation to particular total differential equations. These are then solved numerically, for a small part of the tunnel. The calculations are repeated for very small time steps, allowing the user to observe the changes in the flow though the tunnel system.

In the numerical simulation, the Darcy-Weisbach friction factor is used, while most hydropower calculations use the Mannings number. To transition between Darcy-Weisbach friction and Mannings number the following equation is found in Guttormsen (2006):

$$f = \frac{8g}{M^2 R^{\frac{1}{3}}} \tag{122}$$

This gives a Darcy-Weisbach friction factor of 0.052 for the tunnels and 0.0109 for the pressure shaft and draft tube, corresponding to the values in table 3. The Svee compensation for different loads, as used in the case study, was left out for simplicity.

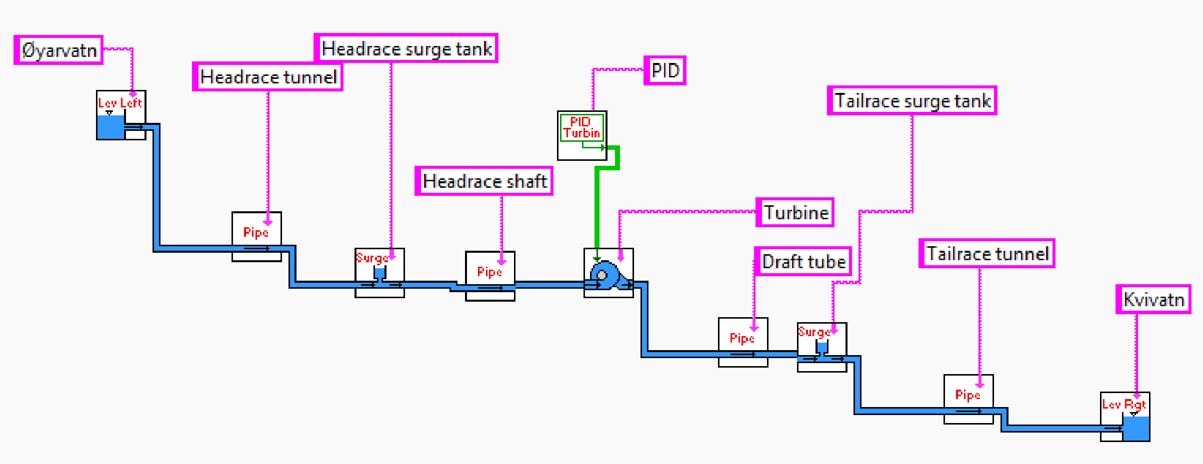


Figure 10: Kvinen power plant model in LVTrans

Both of the derivations of the criteria shown in chapter 2 have an assumption of perfect governing. In a perfect governor, the output from the turbine is kept constant. In a real power plant, this cannot be achieved, and LVTrans therefore uses a PID-governor, as found in most power plants. A PID-governor governs the speed of the turbine to try to match the demands of the grid, changing the position of the guide vanes after a set of rules for the turbine, so that the most stable operation is ensured. This means that LVTrans cannot be used to accurately model the physical assumptions made in the criteria. A definitive conclusion will therefore be difficult to draw, but the model gives a reasonable input to pointing out where a stability problem might occur. The testing method is to run the power plant at 90% load, then ramping it up to full load, and recording the mass oscillations in the surge tank, looking for instability. All large power plants operating on the Norwegian grid must be able to perform such a ramp-up test to be allowed to connected to the grid.

4. Results

This chapter presents the results from the parameter sensitivity study, the case study and the numerical simulation. For the parameter sensitivity study, only the headrace results are included. Since the results for a headrace surge tank is very similar to those in the tailrace, only the former is shown in this part. For tailrace results, see appendix A. In the case study and numerical simulation, Kvinen power plant is used as the case to be studied.

4.1. Parameter sensitivity study

The results are presented as seven charts, showing the variation of the results of the different criteria per parameter. In all the charts, the cross-sectional area of the surge tank is measured on the y-axis. The x-axis measures the change in each individual parameter. This part will compare the original and modified Thoma criteria with the original and modified Svee criteria for a headrace surge tank, criteria (115), (118), (53), and (54).

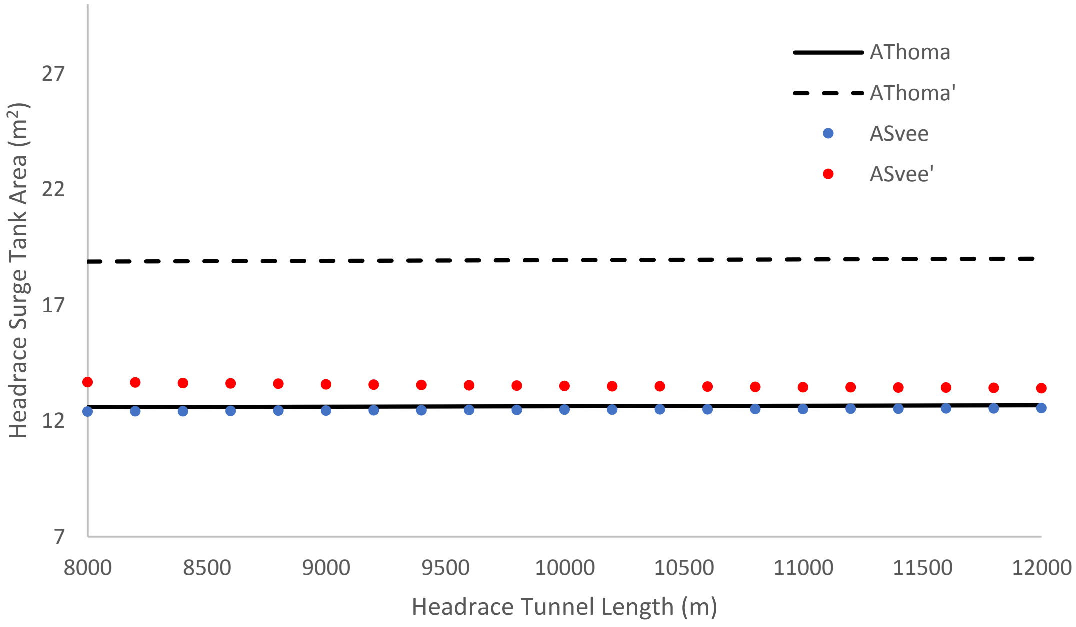


Figure 11: Headrace surge tank area as function of headrace tunnel length, for the four criteria

Figure 11 shows that none of the criteria vary significantly with increasing tunnel length.

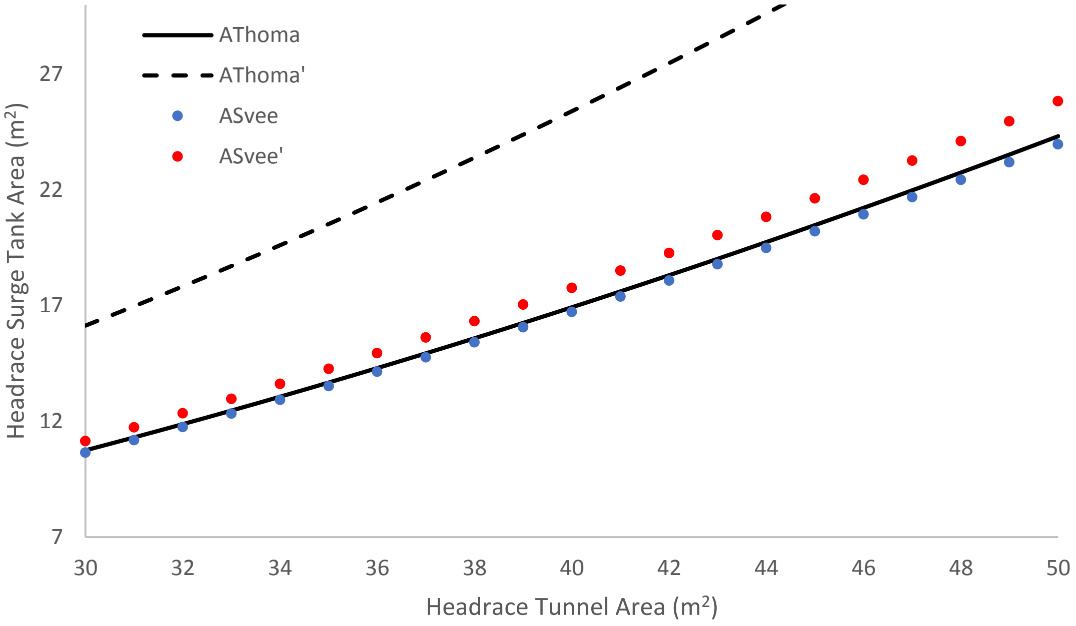


Figure 12: Headrace surge tank area as function of headrace tunnel area, for the four criteria

Figure 12 shows a large increase of surge tank area with increasing headrace tunnel area.

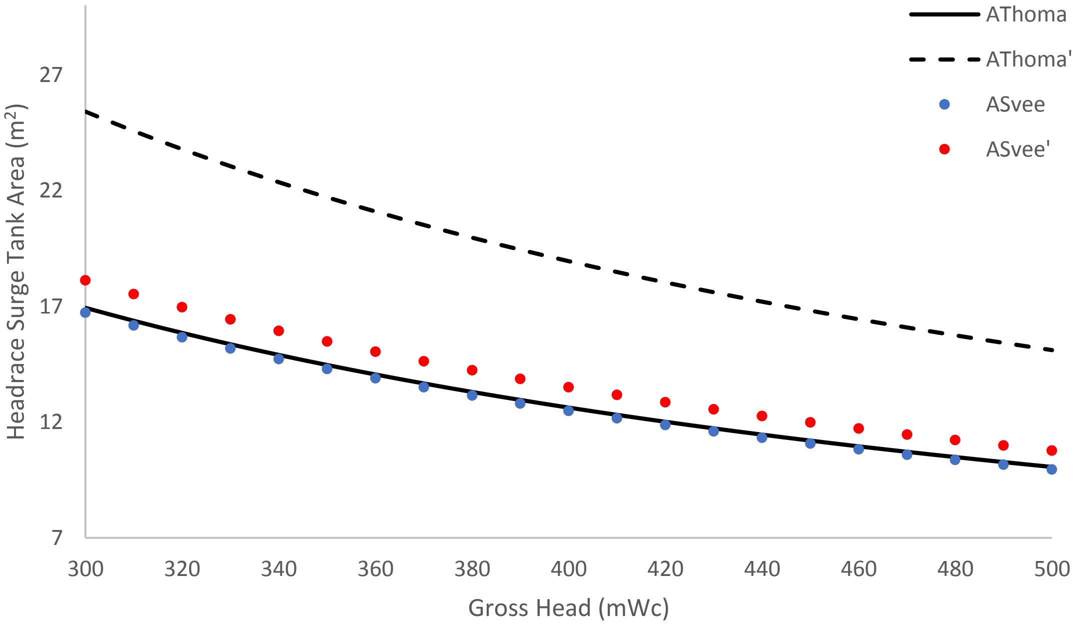


Figure 13: Headrace surge tank area as function of gross head, for the four criteria

Figure 13 shows a decrease in surge tank area with increasing gross head.

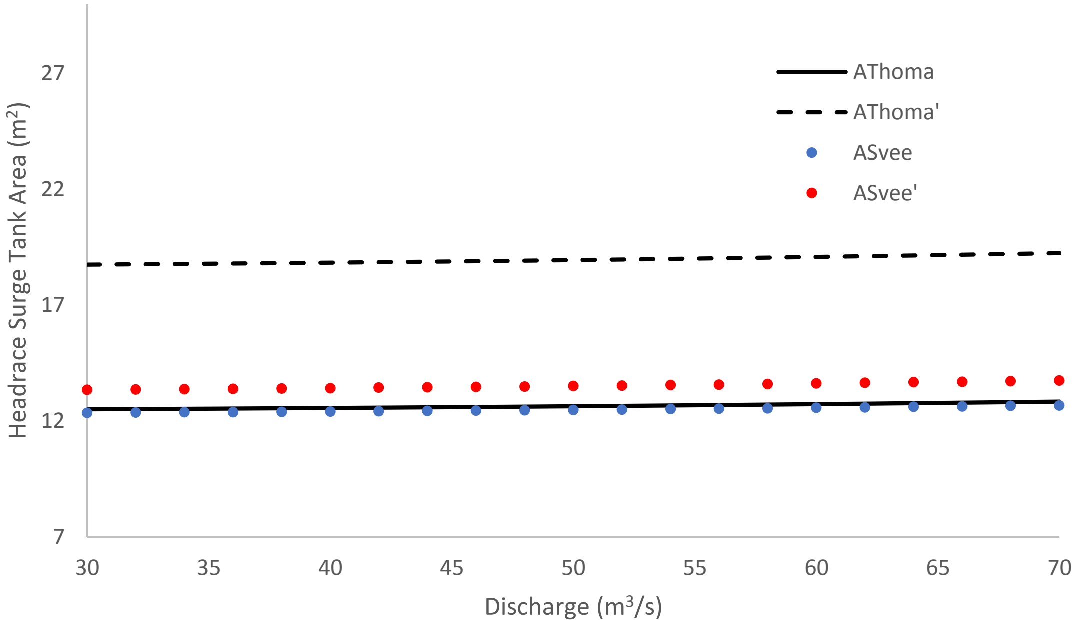


Figure 14: Headrace surge tank area as function of discharge, for the four criteria

Figure 14 shows a very slight increase in surge tank area with increasing discharge.

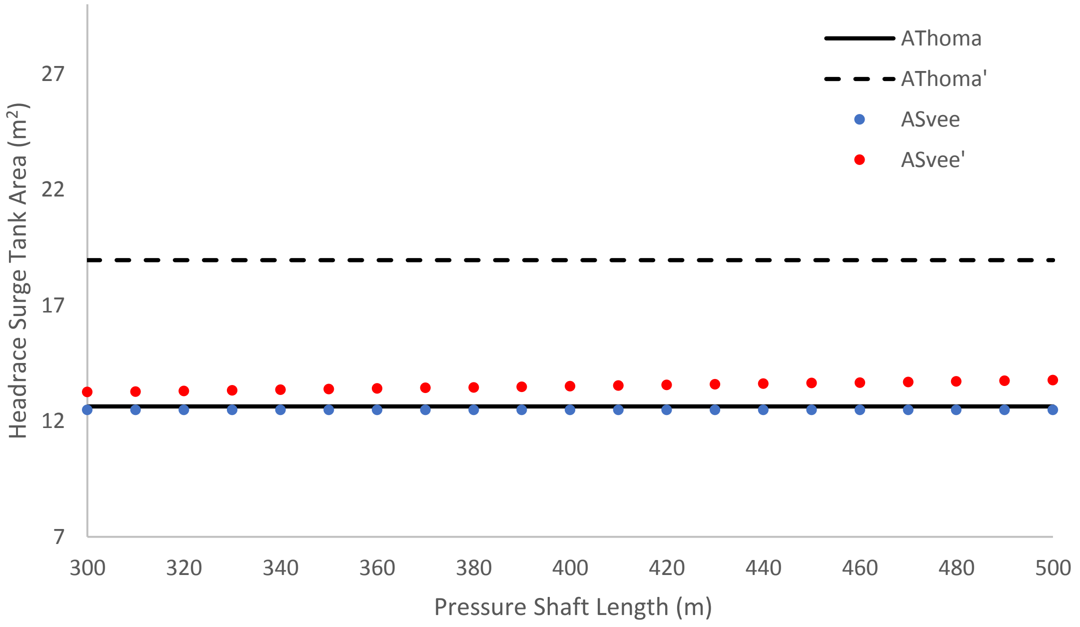


Figure 15: Headrace surge tank area as function of pressure shaft length, for the four criteria

Figure 15 shows that while the original Svee criterion and both the Thoma criteria are unaffected, the surge tank area given by the modified Svee criterion increases with increasing pressure shaft length.

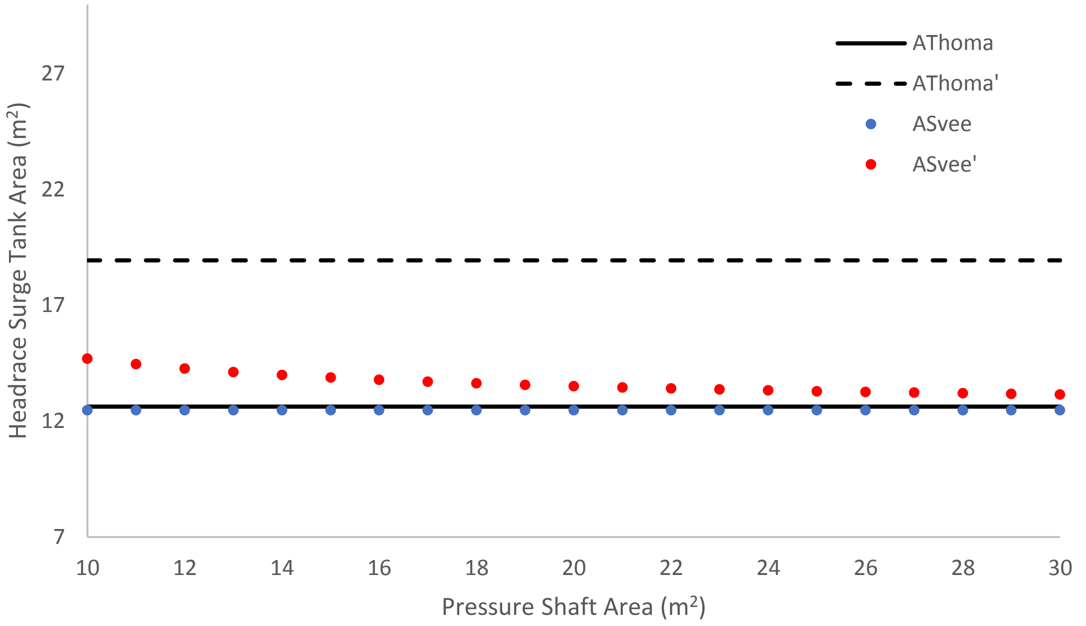


Figure 16: Headrace surge tank area as function of pressure shaft area, for the four criteria

Figure 16 shows that while the original Svee criterion and both the Thoma criteria are unaffected, the surge tank area given by the modified Svee criterion decreases with increasing pressure shaft area.

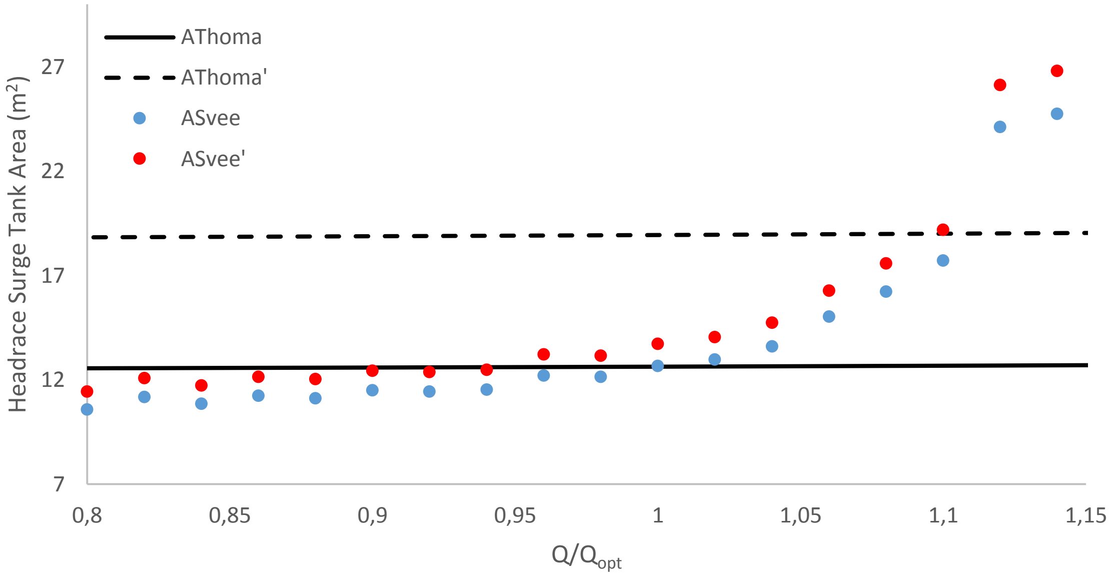


Figure 17: Headrace surge tank area as function of varying load, for the four criteria

Figure 17 shows that while the Thoma criteria are unaffected by the change in load, increasing load gives an increase in the surge tanks area given by both the Svee criteria.

To show the data in another format, two tables are presented here. One shows the surge tank areas resulting from the four criteria for a system where all the parameters are set to their initial value, in the middle of their respective ranges. A second table shows the change in surge tank area for each criteria, organized by the change in the parameters.

Table 4: Surge tank areas resulting from the four criteria for a system with initial default values

	Thoma	Thoma'	Svee	Svee'
Headrace surge tank area (m^2)	12.63	18.95	12.47	13.50
Tailrace surge tank area (m^2)	12.63	18.95	12.48	13.51

Table 5: Changes in the results of the criteria for the different parameter changes.

Parameter	Parameter change %	Thoma change %	Svee change %	Svee' change %
Length of headrace tunnel	20	0.3	0.5	-0.7
	-20	-0.3	-0.6	1.2
Area of headrace tunnel	25	43.6	43.4	45.4
	-25	-36.5	-36.4	-37.3
Gross head	25	-20.3	-20.3	-20.3
	-25	34.1	34.0	34.2
Discharge	40	1.6	1.6	1.8
	-40	-1	-1	-1.2
Pressure shaft length	+25	0	0	1.9
	-25	0	0	-1.9
Pressure shaft area	+20	0	0	-2.6
	-20	0	0	8.8
Load	+15	0.6	95.2	95.4
	-20	-0.8	-18.2	-18.3

4.2. Case study

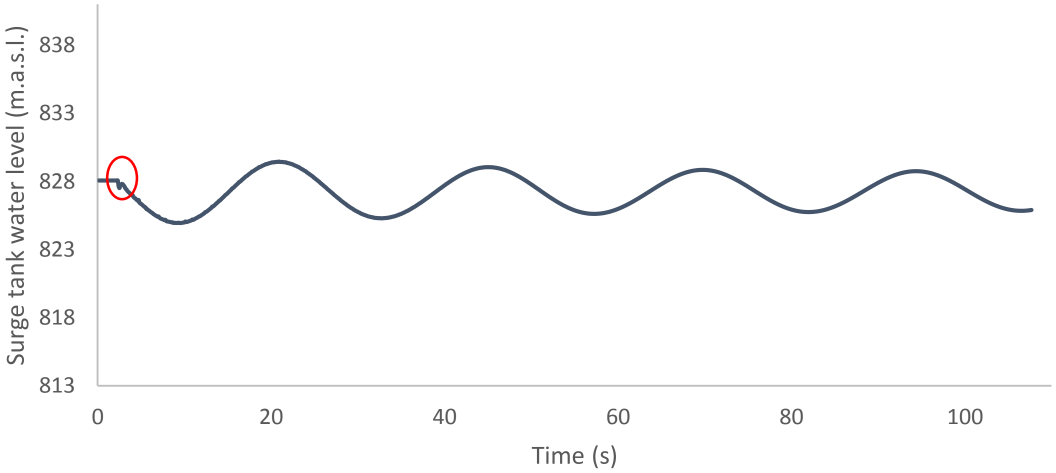
The case study of Kvinen power plant gives cross-sectional surge tank areas for the original and modified Thoma, and the original and modified Svee criterion. The results are shown for three distinct load cases, with the turbine running lower, higher or on the best efficiency point. Table 6 presents this data.

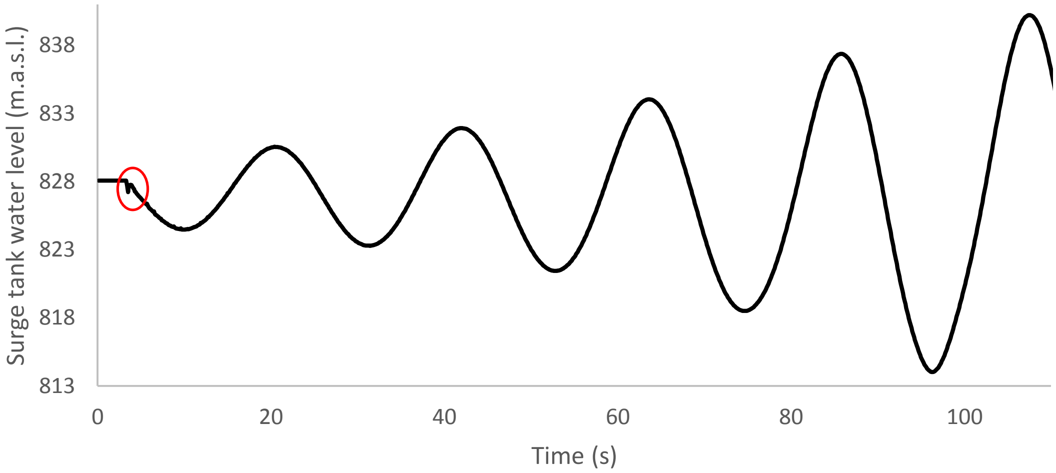
Table 6: Results of the four stability criteria when applied to Kvinen power plant

	Headrace			Tailrace		
Q/Qopt	1	0.83	1.13	1	0.83	1.13
AThoma (m^2)	60.4	60.4	60.4	61.2	61.2	61.2
AThoma' (m^2)	90.6	90.6	90.6	91.8	91.8	91.8
ASvee (m^2)	58.6	52.9	80.4	59.8	54.0	82.1
ASvee' (m^2)	76.8	69.5	105.6	60.4	54.6	83.0

4.3. Numerical simulation

These two graps shows oscillations in the headrace surge tank in a system with modified Thoma and modified Svee criterion. The small deviation of the water level when the oscillations start, is the water hammer effect. The numerical simulation shows the water level in the surge tank for a load increase from 90 % to 100 %. The water level in the surge tank at the start of the load increase is 828 meters above sea level.





5. Discussion

This chapter discusses the results from the 3 studies, separately. Data is systemized, comparisons are made and assumptions commented on. For the parameter sensitivity study, the discussion focuses on the headrace, with reference to Appendix A showing similar results in the tailrace system. For the case study and numerical modeling, both the headrace and the tailrace are taken into account.

5.1. Parameter sensitivity study

The results of the parameter sensitivity study were predicted in some extent in chapters 2.3 and 3.1.1. As stated in chapter 4.1, the results for the tailrace system gives results very similar to the headrace. Because of the difference in the Svee criteria for headrace and tailrace, the resulting surge tank area is smaller in the tailrace. This shows that the velocity head adds stability in the tailrace, while it decreases stability in the headrace. The two Thoma criteria does not take this part into account. The comments on the results of the parameter sensitivity study are made for a headrace surge tank, but are completely analogous for a tailrace surge tank unless otherwise stated. Table 4 shows the surge tank areas given when all the parameters are set to their original value, in the middle of their respective ranges. Table 7 shows these values relative to each other, with the modified Thoma criterion set as 100 %, because it gives the largest area in all but one case.

Table 7: Surge tank areas for the different criteria as relative values, where the modified Thoma criterion is set as 100 %

	Thoma %	Thoma' %	Svee %	Svee' %
Headrace surge tank area (m^2)	67	100	66	71
Tailrace surge tank area (m^2)	67	100	66	71

At a glance, one can see that because it does not take the pressure shaft into account, the original Svee criterion actually gives a smaller area than the original Thoma criterion. This is surprising, as the original Thoma criterion is long abandoned for being unsafe, needing a safety factor (Jaeger, 1949). The difference in results for the Svee criteria between the headrace and tailrace is just as predicted in chapter 3.1.2. The one parameter giving a larger area for the Svee criteria is increasing the load. This is also as predicted. The modified Svee criterion gives an area of the surge tank that falls between the original and modified Thoma criterion. This is reminiscent of earlier work, introducing a variable safety factor on the original Thoma criterion (Jaeger, 1960). Table 5 shows how the criteria react to changes in the different parameters. A few key points on each parameter is presented.

Tunnel length

The tunnel length is the largest number included in the calculations, 10000 meters in the default setting. It has to change a lot to make any difference in the surge tank area. For all the criteria, the longer tunnel gives more water that needs to be accelerated and decelerated, requiring a larger surge tank. The Thoma and original Svee criteria both give larger areas for longer tunnel, while the modified Svee criterion actually gives a smaller area, caused by the fact that the difference between the pressure shaft and tunnel length change.

Tunnel area

The area of the tunnel is a volatile parameter, showing changes in surge tank area as high as 45 % for a 25 % increase in the tunnel area. Physically, the area of the tunnel influences the water velocity, volume and pressure. The volatility can be seen in the equations for the criteria as the tunnel area is multiplied with the tunnel length, which, as stated above, is the largest number in the calculations. Increasing the tunnel area gives enlarged surge tank areas for all the criteria, but slightly more so for the modified Svee criterion than the Thoma criteria.

Gross head

Increasing the gross head gives a lower surge tank area, and vice versa. The results of the original Thoma and original Svee criteria give very similar results, only varying because of Svee's inclusion of the velocity head. With the high gross head shown here, the magnitude of the velocity head is relatively insignificant. In a tailrace system, the velocity head would work the opposite way, giving a larger change in the Svee criteria. The resulting changes in the modified Svee criterion differ more from the results from the Thoma criteria, this is simply because the small value of the velocity head gets multiplied with the parameter for the pressure shaft, increasing it slightly. The change of increasing the head is smaller than that of and equal decrease. This is not caused by a physical concept, only the size of the number compared to the other factors.

Discharge

The discharge has the largest parameter change, varying 40 %. The resulting change it gives for the surge tank area is small, but shows that an increased discharge leads to a slightly larger surge tank. For the two Thoma criteria, the results increase by only 1.6 %, the original Svee criterion a little less, at 1.57 %. The increase in discharge increases the head loss, and this influences the Thoma criteria more than the Svee criteria because of Svee's inclusion of the velocity head, giving higher net head. Again, in a tailrace system the effect of the velocity head would go the opposite way, giving a larger increase for the original Svee criterion. The results of the modified Svee criterion gives a change of 1.8 %, this increase is caused by the increased head loss in the pressure shaft. Reducing the discharge gives analogous results, but the actual numbers vary, caused by the nonlinear nature of the head loss as a function of discharge.

Pressure shaft

Only the modified Svee criterion is influenced by the pressure shaft. For a power plant with long pressure shafts or draft tubes, this effect is significant. Changing the length and area of the pressure shaft works exactly opposite of changes in the tunnel. A long, narrow tunnel will give a smaller surge tank, a long, narrow pressure shaft will give a larger surge tank. This is because a long narrow pressure shaft will give a larger head loss in the pressure shaft, with the same head loss in the tunnel. This result is in tune with the assumptions in chapter 3.1.2.

Load scenarios

The different load scenarios were initially thought to only influence the results of the Svee criteria. But, since the discharge changes, the results of the Thoma criteria exhibits the same changes already seen for changes in discharge. This change in results is also present in the two Svee criteria, but is insignificant next to the difference caused by the difference in efficiency as a function of load. As described in chapter 3.1.1, when the load is higher than the best efficiency point, it is more difficult to control the power plant, because the governor and guide vanes have to compensate for the change in efficiency when regulating output. For the case with a load lower than the best efficiency point, the opposite is true. For instance, if the turbine is running at 90 % load, and the load is reduced to 89 %, the efficiency will go down, making the needed movement of the guide vanes smaller. For a load 15 % higher than the best efficiency point, the change in the results for the two Svee criteria is 95 %, almost a twofold increase in surge tank area. This is the most extreme change in the whole parameter study. For a load 20 % below the best efficiency point, a decrease in the surge tank area for the Svee criteria of 18 % is seen. This almost gives a one to one relationship between the decrease in load and the decrease in surge tank area. This result is very close to the assumption in chapter 3.1.2, but the assumption disregarded the influence of the discharge on the Thoma criteria.

Summary

Except from the load and pressure shaft length, all the parameter curves are a variant of a parable with the minimum or maximum value at zero. This means that there is no “best” number to aim for, the smallest and largest surge tank areas are always found at the extreme end of each parameter. There is no way to design for an “optimal minimum” surge tank. The design process of a hydropower plant is dynamic, and a common design paradigm is to design the surge tanks to fit the power plant, instead of fitting the power plant to the surge tanks. This means that decreasing the area of the tunnel to get a smaller surge tank is not a valid procedure. The smaller surge tank would not make up for the increased head loss in the tunnel.

The load parameter is somewhat of an exception to this, because a power plant can be designed to run at a fixed load. If a power plant is built for industrial production, efforts are made to have it run at the best efficiency point of the turbine at all times, reducing the energy wasted and keeping a steady supply to the industry. In this case, the load will not vary much, and the surge tank can be reduced in size if the Svee criterion for surge tank stability is used. Many power plants in Norway are built to this specification. However, with a more volatile energy market, it is not economically viable to always run the turbine on the best efficiency point. This means that the change in stability caused by load change cannot be ignored.

Generally, a system with long, large tunnels, low head, high discharge and load give the largest surge tanks, and vice versa. This can be seen looking at figures 11-17. A system with large tunnels, higher discharge and low head favors the two Svee criteria over the Thoma criteria, giving the largest decrease in surge tank area. The different load scenarios are still the most important parameter governing the distance between the Thoma and Svee criteria.

5.2. Case study

The case study tests the stability criteria in a single hydropower plant. Kvinen power plant is a unique plant, and the data cannot be used for a large arrays of different power plants. For a power plant with long tunnels and short shafts, the case study gives good predictions on what surge tank areas the different criteria will give. Conducting several more case studies would give an even better understanding of how the criteria interact in different hydropower plants. Kvinen power plant is a moderate head plant at 116 meters. The moderate head is coupled with very long tunnels, totalling 10000 meters. Higher head automatically gives better stability, looking at both the Thoma and Svee criteria. Looking at the results in table 6, the results of the two Thoma criteria are almost unchanged by the change in load, as also seen in chapter 5.1. The two Svee criteria give a smaller area than the modified Thoma criterion, except for the situation with higher than nominal load. Building on this, a new table can be formed, setting the area of the modified Thoma criterion as the maximum value, 100 %, table 8 shows how the areas given by the other criteria relate to it.

Table 8: Results of the four different stability criteria when applied to Kvinen power plant. The results are shown relative to the modified Thoma criterion, which is set as 100 %.

	Headrace			Tailrace		
Q/Qopt	1	0.83	1.13	1	0.83	1.13
AThoma %	67	67	67	67	67	67
AThoma' %	100	100	100	100	100	100
ASvee %	64.6	58.4	88.8	65.1	58.9	89.5
ASvee' %	84.8	76.7	116.6	65.8	59.5	90.4

For the headrace, the result of the modified Svee criterion is significantly larger than that of the original Svee criterion. For the tailrace, the difference between areas given by the original and modified Svee criteria is less than 1 %. This is due to the very short draft tube, when compared to the very long tailrace tunnel. In the headrace, the pressure shaft has a more moderate length compared to the headrace tunnel. This implies that the modification of the Svee criterion is correct, as it scales well with the relative length of the pressure shaft and draft tube. This difference carries over to the different load cases, showing that a higher than nominal load has a much larger impact in the headrace surge tank, simply because the original number without the load change is larger. The original Svee criterion gives an area smaller than that of the original Thoma citerion, further strengthening the hypothesis that the modified version is more correct (Jaeger, 1960).

For the headrace, the modified Svee criterion gives results, falling between the area of the original and modified Thoma criterion. For the tailrace, the subtraction of the velocity head gives the result of the original Svee criterion a minor increase in size, but because of the short draft tube, even the modified Svee criterion gives a smaller area than the original Thoma criterion. This implies that the system will be unstable with this surge tank area (Jaeger, 1960). Looking at the increased load case, the modified Svee criterion lies between the original and modified Thoma criteria, implying that factoring in varying load is important.

5.3. Numerical simulation

Figure 18 shows Kvinen power plant with surge tanks according to the modified Thoma criterion, figure 19 according to the modified Svee criterion. Nothing other than the surge tanks is changed from figure 18 to 19, they both have the layout shown in figure 10. By equation (7), the water hammer pressure is equal in the two scenarios. The amplitude of the mass oscillations is totally dependant on the cross sectional area of the surge tank. All hydropower plants are unique, Kvinen is unique in the sense that both the headrace and the tailrace tunnels are very long compared to the gross head. This means that the water has a very long way to go, and the period of the mass oscillations from the reservoirs to the surge tank is long. The pressure shaft and draft tube are very short relative to this, giving a large difference in the period length from surge tank to surge tank compared to the surge tanks to the reservoir.

Figure 18, based on the modified Thoma criterion shows stable conditions, but the dampening of the oscillations is very slow. This means that the power plant will be very difficult to control, responding very slowly to changes in load and frequency. The Norwegian national grid, Statnett, has a set of control responses that a power plant needs to keep, and with this design, Kvinen power plant most likely would not meet the requirements set for hydropower plants on the Norwegian grid (Statnett SF, 2012). The surge tanks for the modified Svee criterion are 15 % and 44 % smaller than the modified Thoma criterion, in the headrace and tailrace, respectively. This leads to unstable oscillations. Instead of being dampened as needed, or even continuous, the mass oscillations actually increase. This is a very undesirable condition, as the governor will change the position of the guide vanes very fast, accelerating and decelerating the turbine and generator repeatedly, possibly wearing it out. In this situation, the power output is much to variable to supply the grid, and the power plant would have to be taken off the grid, rendering it unusable. The original Thoma and Svee criteria give smaller areas than that of the modified Svee criterion, and give even more adverse results. It is chosen to focus on the modified Svee criterion, as it gives the most promising results.

Looking back at the case study, the difference in the modified Svee criterion for headrace and tailrace is caused by the difference in length of the pressure shaft and draft tube. It is not due to the difference in the criterion for headrace and tailrace surge tanks, as this would mean the tailrace surge tank being even larger. If it was, the difference could be seen also in the original Svee criterion. The test of stability was done by increasing the load from 90 % to full load. This means the efficiency of the turbine would change, increasing, as the turbine is designed to have maximum efficiency at full load. According to the Svee criteria, this should lead to added stability, because some of the increase in demand is accounted for by the increased efficiency. Obviously, a smaller area than the one used would give even larger oscillations. It is possible that because the turbine accelerates and decelerates, the surge tank must always take into account the changes in efficiency, both up and down. This implies that the worst case scenario, with the values for a higher than regular load should be used.

The instability of the surge tanks given by the modified Svee criterion is surprising, as all the work conducted by Svee assumes the Svee criterion to be stable (Svee, 1970). Kvinen power plant is also known to operate safely in real life. One factor influencing the results is mentioned in chapter 3.3, dealing with the accuracy of the numerical simulation. The system is simplified, and the LVTrans software operates with a different governor than is assumed in the both the derivations of the stability criteria. Another factor is that Kvinen power plant seems to be a relatively unstable power plant, looking at the oscillations for a surge tank with a modified Thoma area.

The aspect of resonance is not discussed in this thesis. It is possible that the positioning of the surge tanks in this simplified model cause the mass oscillations to resonate from one surge tank to the other. This would increase the demand for a larger surge tank.

6. Conclusions and further work

6.1. Benefits

For most power plants, the modified Svee criterion gives a smaller area than the modified Thoma criterion. In extreme cases, the area given by the modified Svee criterion will be as large as that of the modified Thoma criterion, or even larger when accounting for a load higher than normal. The surge tank stability criterion is an important factor in building the tunnel system of a hydropower plant. In designing a surge tank, the designs can be complex, with several chambers, air cushions or throttles. Still, most surge tanks have a narrow shaft connecting these systems to the rest of the tunnels, built according to the modified Thoma criterion. If the cross-sectional area of this part of the surge tank can be reduced, the construction costs would decrease. For dimensioning an air cushion surge tank, the stability criterion is the first input to figuring out the volume of the air cushion. Decreasing it will give a smaller overall volume in the air cushion. The advantages of a reduced surge tank area are many, but to use the modified Svee criterion, it must be proved to produce a stable surge tank.

6.2. Implications

The purpose of this thesis is to compare the Thoma and Svee criteria for surge tank stability. Through the thesis, a total of four criteria has been presented; The original and modified Thoma, the original and modified Svee. Modifications are also made to Svee's criteria regarding the difference between a headrace and a tailrace surge tank. Looking back at results from (Jaeger, 1960) some surge tanks with an area smaller than what is given by the original Thoma criterion are shown to be unstable, while others are stable.

The parameter sensitivity study shows how the different criteria interact when changing the input parameters, showing that the area of the tunnel and the different load scenarios are the parameters that changes the surge tank area the most. The area of original Svee criterion is found to fall just around that of the original Thoma criterion, while the modified Svee criterion shows results somewhere between the results of the original and modified Thoma criteria. The largest differences between the Svee and Thoma criteria can be seen for a variation of the load, as well as change in the tunnel area.

In the case study of Kvinen power plant, where the draft tube is very short, the modification done to the Svee criterion makes very little difference. In a system with a longer draft tube or pressure shaft, the area of the modified Svee criterion will fall somewhere between that of the original and modified Thoma criteria. When a variation of load is not taken into account, the case study shows the modified Svee criterion giving a smaller area than the original Thoma criterion.

For the numerical simulation, the surge tank area given by the modified Thoma criterion is showed to give slowly dampened oscillations in the surge tank, making a slow responding power plant that would not be able to stabilize the grid. This shows that even with a surge tank based on the modified Thoma criterion, the power plant is very difficult to control, due to long tunnels combined with a moderate head. The surge tank areas given by the modified Svee criterion are shown to give increasing oscillations, rendering the power plant unusable and unsafe. Both these results are surprising, due to the fact that Kvinen power plant is in stable operation today. There are a lot of uncertainties and inaccuracy in the numerical model, including rigid governor theory, resonance and different load scenarios. At this time a conclusion based on the numerical simulation cannot be made as to whether or not the modified Svee criterion gives a surge tank that can considered to be stable.

6.3. Further work

Designing a surge tank is a complex process. Several more case studies with numerical modelling should be done to reveal if the modified Svee criterion can be used to make a stable system. Ultimately a physical model should also be used to test the criterion. Hydraulics is one of few fields of engineering where large scale physical modelling is still conducted. A physical model can take several factors into account that a numerical model cannot. Through a physical model, the modified Svee criterion can be validated and accepted for use by hydropower engineers.

The idea of several surge tanks in the same system should be investigated further. Different methods exists to calculate the area of a headrace surge tank influenced by a tailrace surge tank or another headrace surge tank (Jaeger, 1958) (Svee, 1972a) (Vassdrags- og havnelaboratoriet, 1972a). A good way to do this would be to take Svee's Norwegian workbooks, update the calculations and validate them. This could then be published in English, giving access to a broader audience. Svee's workbooks also take into account creek intakes, air cushion surge tanks and limits for the up- and downsurge. This, combined with a good numerical model, would be great tools for an engineer when designing a surge tank.

The cost benefits of the Svee criteria must be calculated, showing companies and engineers the benefits of taking the Svee criteria into practice. This should be carried out for several different power plants and surge tank designs. Together with a validation of the stability of the Svee criteria, this could make newer power plants more cost-efficiency, by reducing excess surge tank area.

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Appendix A

Parameter sensitivity study for a system with a surge tank in the tailrace.

This part shows the results for the parameter sensitivity study for the tailrace. The original and modified Thoma criteria are compared to the original and modified Svee criteria, criteria (116), (119), (53), and (54). The y-axis measures the cross sectional area, while the x-axis measures the deviation in the parameters.

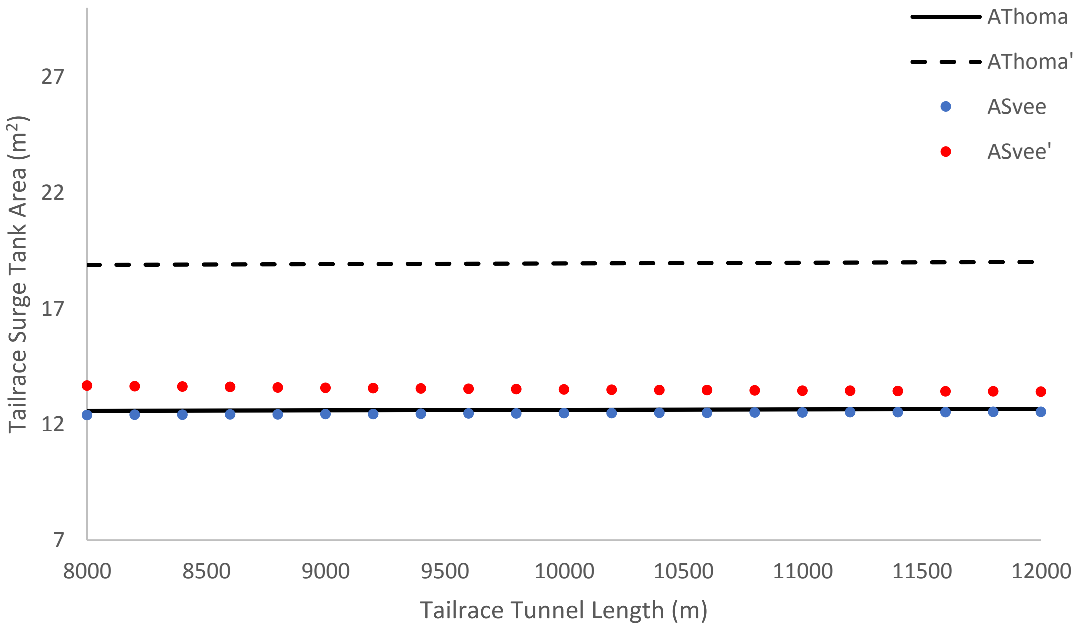

Figure 20: Tailrace surge tank area as function of tailrace tunnel length, for the four criteria

Figure 20 shows that none of the criteria vary significantly with increasing tunnel length.

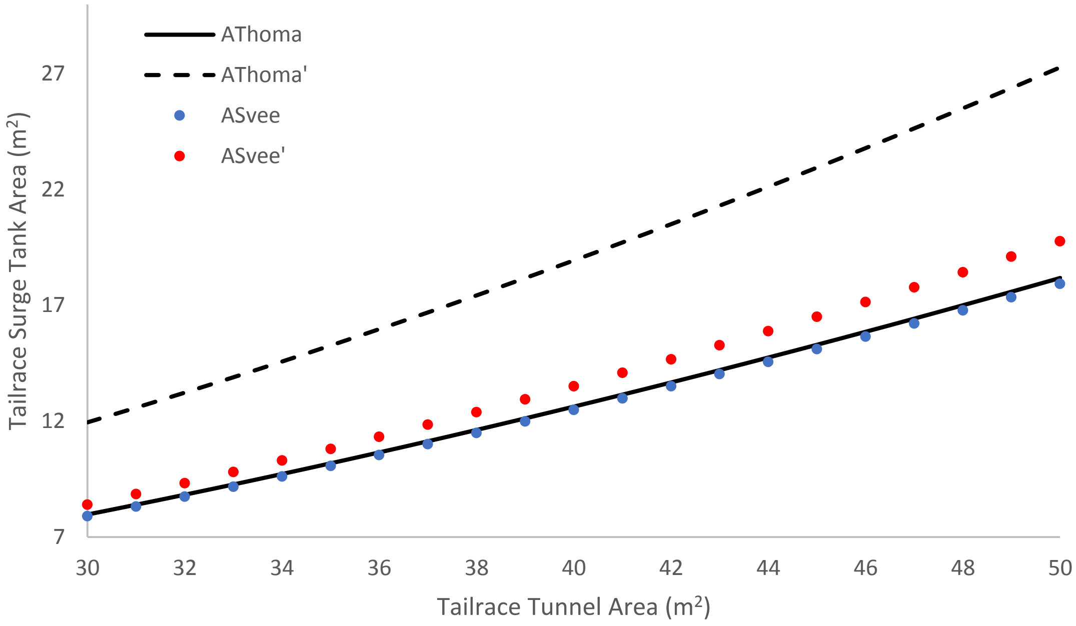

Figure 21: Tailrace surge tank are as function of tailrace tunnel area, for the four criteria

Figure 21 shows a large increase of surge tank area with increasing tailrace tunnel area.

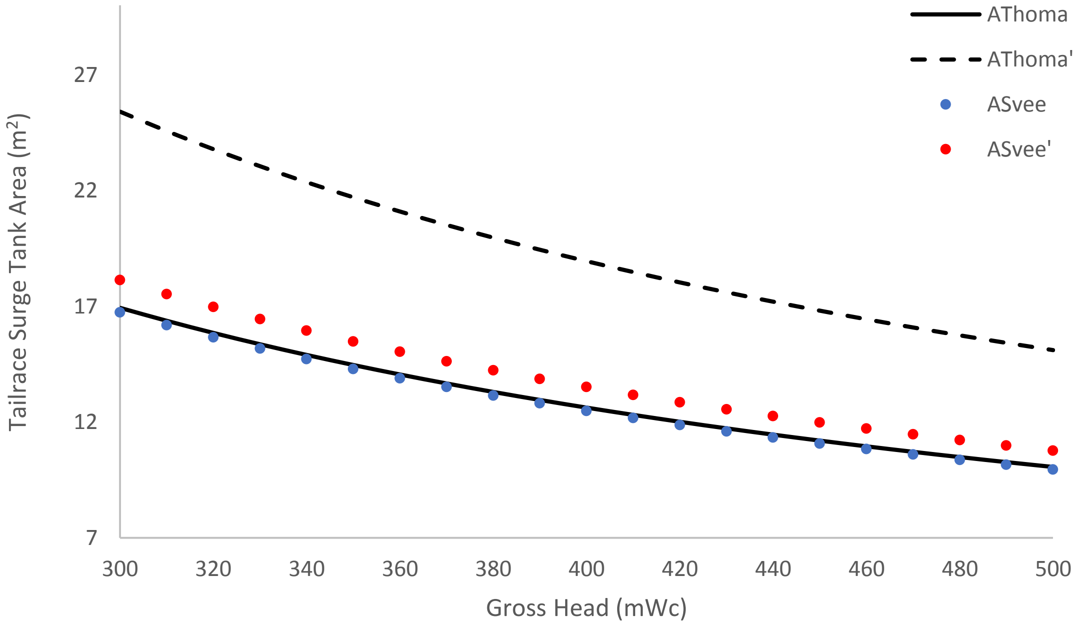

Figure 22: Tailrace surge tank area as function of gross head, for the four criteria

Figure 22 shows a decrease in surge tank area with increasing gross head.

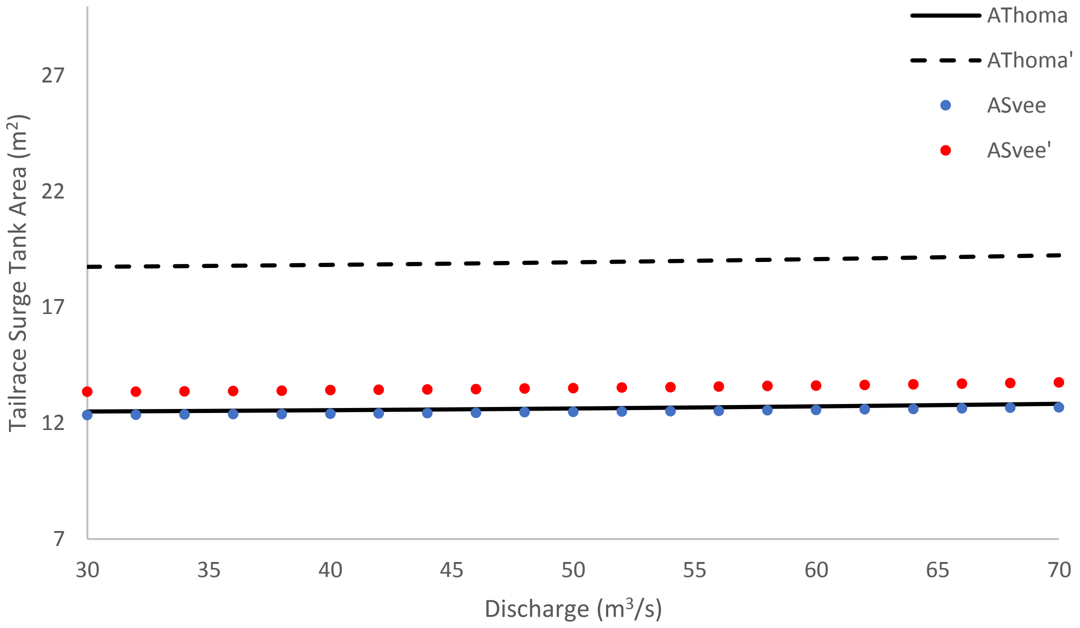

Figure 23: Tailrace surge tank area as function of discharge, for the four criteria

Figure 23 shows a very slight increase in surge tank area with increasing discharge.

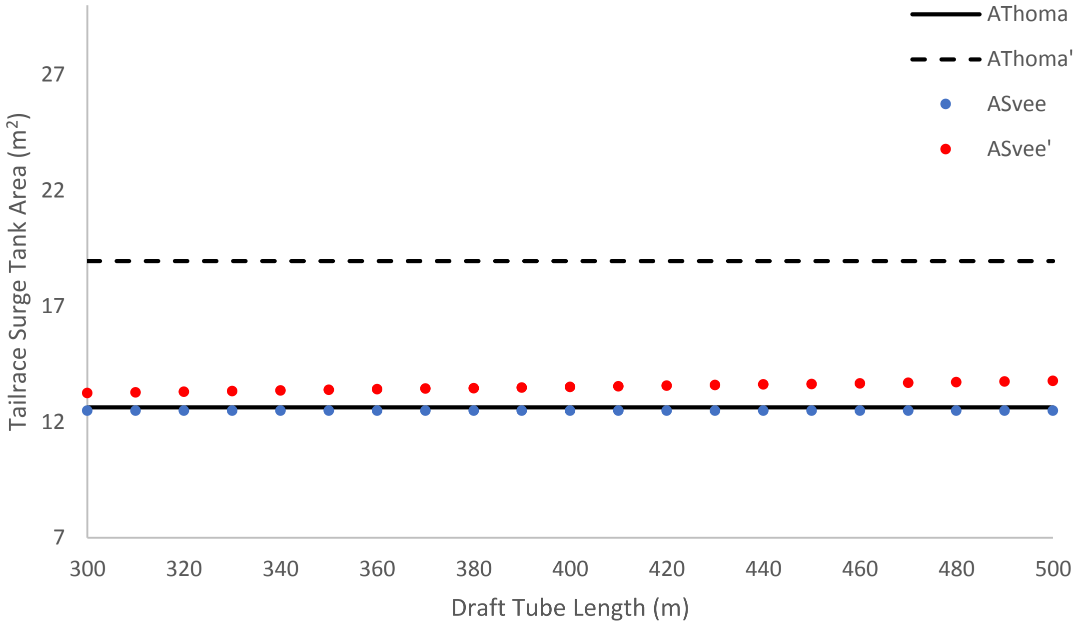

Figure 24: Tailrace surge tank area as function of draft tube length, for the four criteria

Figure 24 shows that while the original Svee criterion and Thoma criteria are unaffected, the surge tank area given by the modified Svee criterion increases with increasing draft tube length.

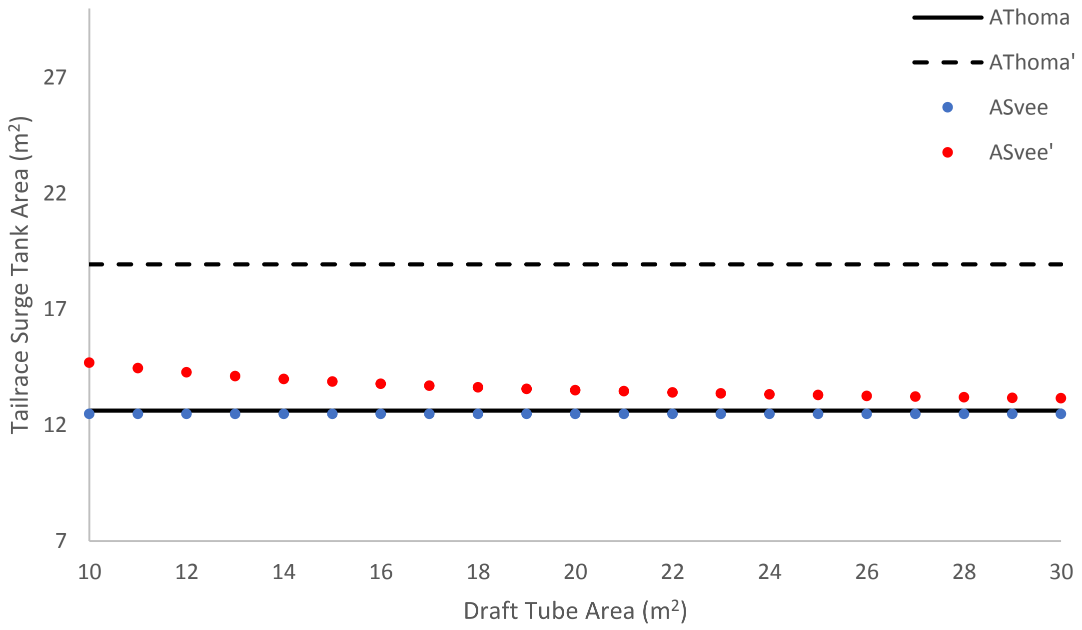

Figure 25: Tailrace surge tank area as function of draft tube length, for the four criteria

Figure 25 shows that while the original Svee criterion and Thoma criteria are unaffected, the surge tank given by the modified Svee criterion decreases with increasing draft tube area.

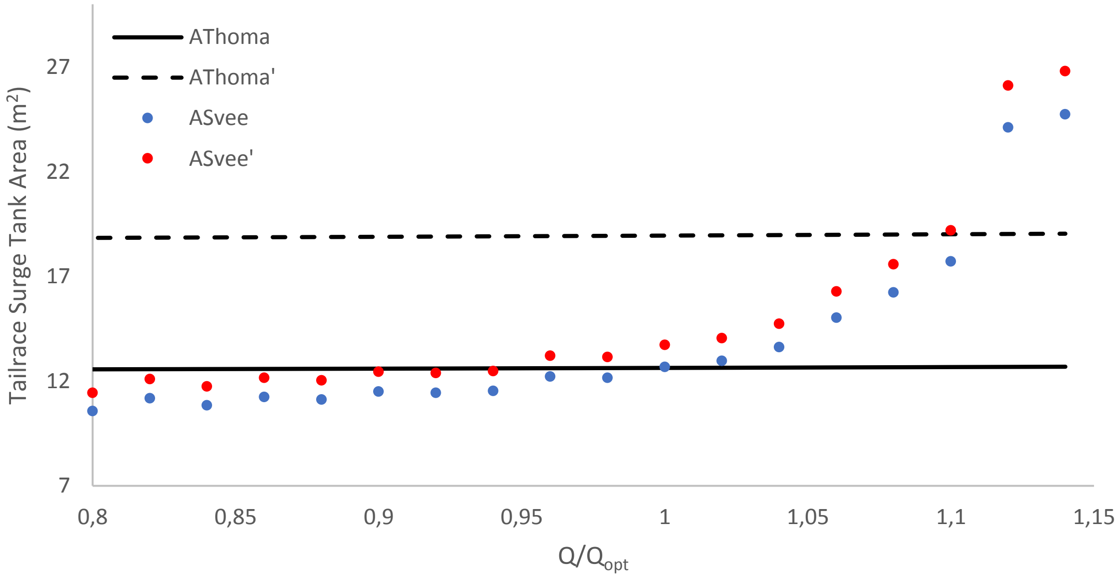

Figure 26: Tailrace surge tank area as function of varying load, for the four criteria

Figure 26 shows that while the Thoma criteria are unaffected by the change in load, increasing load gives an increase in surge tank area given by both the Svee criteria.

The changes seen in the figures are summarized in table 9.

Table 9: Changes in the results of the criteria for the different parameter changes

Parameter	Parameter change %	Thoma change %	Svee change %	Svee' change %
Length of tailrace tunnel	20	0.3	0.5	-0.7
	-20	-0.3	-0.6	1.2
Area of tailrace tunnel	25	44	43.7	46.4
	-25	-36.9	-36.7	-37.9
Gross head	25	-20.3	-20.3	-20.3
	-25	34.1	34.0	34.2
Discharge	40	1.6	1.6	1.8
	-40	-1	-1	-1.2
Draft tube length	+25	0	0	1.9
	-25	0	0	-1.9
Draft tube area	+20	0	0	-2.6
	-20	0	0	8.8
Load	+15	0.6	95.2	95.3
	-20	-0.8	-18.2	-18.3