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Research paper

A modified failure criterion for transversely isotropic rocks

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A B S T R A C T

A modified failure criterion is proposed to determine the strength of transversely isotropic rocks. Mechanical properties of some metamorphic and sedimentary rocks including gneiss, slate, marble, schist, shale, sandstone and limestone, which show transversely isotropic behavior, were taken into consideration. Afterward, introduced triaxial rock strength criterion was modified for transversely isotropic rocks. Through modification process an index was obtained that can be considered as a strength reduction parameter due to rock strength anisotropy. Comparison of the parameter with previous anisotropy indexes in literature showed reasonable results for the studied rock samples. The modified criterion was compared to modified Hoek-Brown and Ramamurthy criteria for different transversely isotropic rocks. It can be concluded that the modified failure criterion proposed in this study can be used for predicting the strength of transversely isotropic rocks.

1. Introduction

The existing experimental evidence (Donath, 1964; Hoek, 1964; McLamore and Gray, 1967; Horino and Ellickson, 1970; Kwasniewski, 1993; Ramamurthy, 1993; Nasseri et al., 2003; Colak and Unlu, 2004; Karakul et al., 2010) indicates that most of sedimentary and metamorphic rocks, such as shale, slate, gneiss, schist and marble display a strong anisotropy of strength. Rocks flow and recrystallize under new tectonic stresses and form weak foliation planes. These planes of weakness (i.e. schistosity and foliation) affect the strength and deformational behaviors of rocks with orientation of applied stresses. Hence, these types of rocks usually exhibit some preferred orientation of fabric or possess distinct bedding planes, which result in transversely isotropic behavior on the macro-scale. Lo et al. (1986) stated that transversely isotropic behaviors of rocks such as elasticity, electrical conductivity and permeability are related to the both matrix and pore space distributions.

Although many attempts have been made in the past to describe the strength anisotropy of transversely isotropic rocks, no general methodology has emerged yet. The first attempt seems to be Jaeger's single weakness plane theory (Jaeger, 1960), where two independent failure modes, i.e., failure along the discontinuity and failure through intact material, were assumed to exist. The idealized distribution of triaxial strength predicted by Jaeger's theory is similar to that of planes in Fig. 1a. Throughout the paper, inclination angle β is the angle between direction of major principal stress and weakness plane. For those rocks displaying a discrete fabric (i.e., multiple weakness planes), the experimental results have shown that the strength varies continuously with β (Fig. 1b).

In order to reproduce the gradual variation of the strength, Jaeger (1960) postulated that the cohesion of rock material, within the plane inclined with respect to the weakness plane, was not constant but varied depending on the angle of inclination, whereas the friction angle was considered as constant. More recently, Hoek and Brown (1980) assumed that the strength parameters m and s in their well-known failure criterion are not constant but varied depending on the direction of weakness plane. However, although

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Abbreviations

β	Weakness plane orientation in relation to major loading direction
φ	Friction angle of rock
C	Cohesive strength of rock
R_c	Degree of strength anisotropy
E	Young's modulus
E_{max}, E_{min}	Maximum and minimum values of Young's modulus
UCS	Uniaxial Compressive Strength
$\sigma_{c\beta}$, σ_{cj}	UCS with anisotropy direction of β
A, D	Rock constants
β_{min}	Minimum angle of anisotropy
$\sigma_{c(90)}$	UCS perpendicular to the weakness plane
$\sigma_{c(min)}$	Minimum value of UCS commonly in a weakness plane
K_β	Strength anisotropy parameter for different orientation of weakness plane, β
m_i	Rock constant
σ_1, σ_3	Maximum and minimum principal stresses
A, B	Rock constants
r	Strength reduction factor
σ_{ci}	UCS of intact rock
α	Strength reduction parameter in the proposed criterion
$\sigma_{c\beta-pr}$	UCS predicted by modified criterion
$\sigma_{c\beta-lab}$	UCS from laboratory testing
α_j, B_j	Parameters in the Ramamurthy criterion as functions of anisotropy orientation j (in relation to major stress direction similar to β)
RMSE	Root mean square error
σ_i^t, σ_i^p	Tested and predicted values of σ_1 for the ith data

the values of m and s are selected based on the orientation of weakness planes, it should be noted that the formulation remains isotropic, so that it is doubtful whether the orientation of failure plane predicted by this approach is realistic. Another drawback of this approach, as well as the earlier one by Jaeger (1960), is the requirement that the dip direction of weakness planes should coincide with the direction of minor principal stress. Saroglou and Tsiambaos (2008) modified the Hoek-Brown criterion by testing some metamorphic rocks from Greece, and demonstrated that m and s are independent of anisotropy direction. In general, however, Jaeger (1960) and Hoek and Brown's works are of importance in that they showed that the failure criterion can be modified to take into account the anisotropy in strength properties. While the applicability of Hoek and Brown (H–B) approach is restricted, Nova (1980) extended the discussion on anisotropy to the true triaxial stress conditions. Amadei and Savage (1989) also analyzed the transversely isotropic strength of jointed rock having a single set of weakness planes in three-dimensional (3D) conditions. In that work, the intact rock strength is described by the H–B criterion, whereas the joint strength is modeled by the Coulomb criterion with zero cohesion. Although the variation of material properties with orientation was not directly considered, the authors showed that the strength of the jointed rock depends on the direction of weakness planes and the intermediate principal stress.

A large number of research papers were documented on strength anisotropy of rocks. For instance, Nasseri et al. (1996 and 1997) investigated the anisotropy on gneiss and schist. Ramamurthy et al. (1988, 1993) assessed the anisotropy of phyllites. Al-Harthi (1998) concentrated on the behavior of sandstones and Attewell and Sandford (1974) worked on shale and slate. Pomeroy et al. (1971) evaluated the strength anisotropy of coal. Allirote and Boehler (1970) focused on strength anisotropy of diatomite while Elmo and Stead (2010) assessed rock pillar anisotropy of limestone and Wardle and Gerrard (1972) studied on the strength

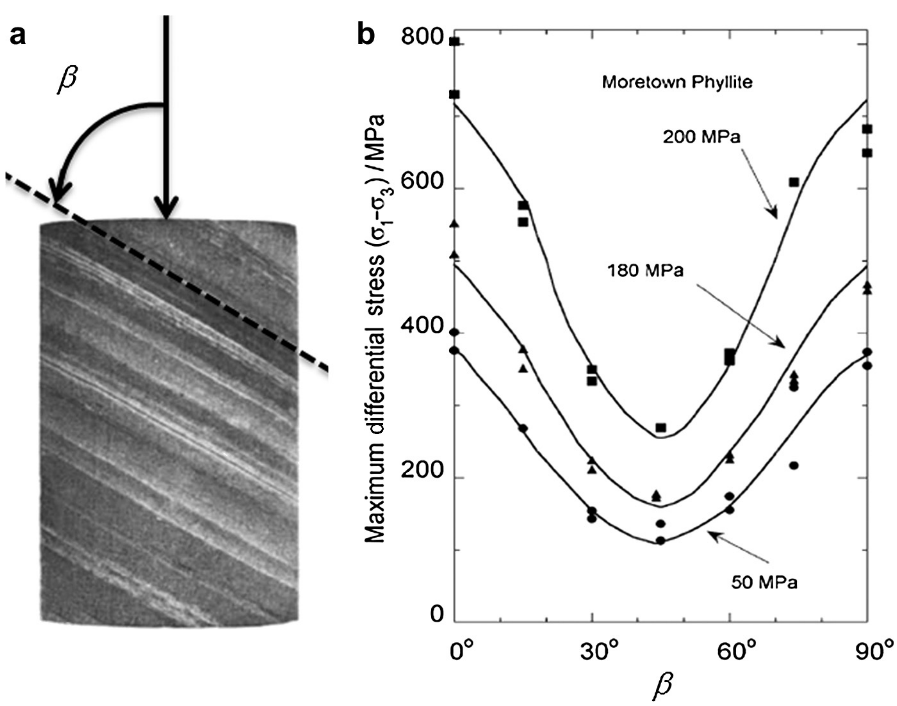

Figure 1. (a) Angle of weakness plane measured from major loading direction, (b) variation of differential stress at failure condition of triaxial compression test with respect to plane of weakness (after McLamore and Gray, 1967).

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anisotropy of layered rock and soil masses. Saroglou et al. (2004) studied anisotropic nature of some metamorphic rocks from Greece. In the entire works recently done, clearly stated that minimum strength of transversely isotropic rocks is at the critical weak plane of $45^\circ + \phi/2$, where ϕ is the friction angle of weakness plane. It was also concluded that variation of elastic rock parameters like Young's modulus, Poisson's ratio and tensile strength is similar to that of the ultimate strength (Read et al., 1987).

Nowadays, most of the rock engineering designs and structures are related to the transversely isotropic rocks with their particular properties. Stability analysis of these structures requires a representative failure criterion. Rafiai (2011) proposed a new empirical failure criterion for intact rock and rock masses under general condition of triaxial and polyaxial stresses. He showed that the criterion could predict the strength of rock over wide range of stresses with high accuracy.

To that end, in the present study an attempt is made to modify the proposed failure criterion (Rafiai, 2011) to be applicable in representing transversely isotropic rock strength in triaxial condition. Mechanical properties of slate from three case studies (S, G and Z) along with data documented by Saroglou and Tsiambaos (2008); Tien and Kuo (2001) and Zhang et al. (2009) were evaluated to make a comprehensive uniaxial and triaxial database for proposing a modified empirical criterion for transversely isotropic rocks. The results were compared with those given by the modified Hoek-Brown and Ramamurthy criteria for strength determination of transversely isotropic rocks.

2. Transversely isotropic rock strength database

To evaluate the behavior of transversely isotropic rocks under triaxial testing condition, a database containing results of both metamorphic and sedimentary rocks was collected which commonly show transversely isotropic behavior rather than igneous rocks. Slates S and G were obtained and tested in uniaxial and triaxial conditions in our laboratory. Seyedi (2005) conducted a complete triaxial and uniaxial test on the slate Z obtained from Zhave dam of Iran. In addition, the triaxial and uniaxial tests of gneisses A and B, schist, marble, limestone, sandstone and shale documented by Saroglou and Tsiambaos (2008); Tien and Kuo (2001) and Zhang et al. (2009) were taken into account to validate the findings. Table 1 shows the available data and ranges of uniaxial compressive strength, σ_c, major and minor principal stresses, σ_1 and σ_3 with respect to the anisotropy orientation β.

3. Sampling and preparation

The rock samples cored at different direction with respect to the plane of anisotropy (β) of 0°, 15°, 30°, 45°, 60° and 90°. Each sample was prepared according to ISRM suggested method (ISRM, 2007) with diameter of 54 mm and length to diameter ratio of 2–3. Ends of each sample were ground to be flat to ±0.01 mm and parallel to each other. The deviation in the diameter and undulation of the ends were less than 0.2 mm. The vertical deviation was less than 0.001 radian. Triaxial tests were carried out using multi-stage loading method (ISRM, 2007) and most of the samples failed in 5 to 15 min. Loading rate was adjusted to 0.5–1 MPa/s. In this method confining pressure was increased stage by stage manually as the axial pressure increases where at all times axial loads exceed confining pressure by no more than on tenth of the rock UCS until peak stress reached. Therefore, in this study, slate S, G and Z were tested with confining pressure ranges 3–35 MPa (Table 1). Fig. 2a,b shows those places where rock blocks were obtained and were transported to the laboratory for coring and preparation. Fig. 2c shows samples of slate Z prepared for triaxial testing.

Thin sections of the samples were prepared perpendicular to the foliations (Fig. 3), petrography analysis revealed that slate S is mainly consisted of quartz and meta-sandstone veins with very

Table 1
Triaxial datasets provided for different transversely isotropic rocks.

Rock type	No. of pair data	$\beta = 0^\circ$					$\beta = 15^\circ$					$\beta = 30^\circ$				
		σ_3		σ_1		σ_c	σ_3		σ_1		σ_c	σ_3		σ_1		σ_c
		Min	Max	Min	Max		Min	Max	Min	Max		Min	Max	Min	Max	
Slate S	47	0	30	33	220	50	0	15	25	70	20	0	18	15	90	8
Slate G	15	5	20	105	210	92	–	–	–	–	–	5	20	33	59	25
Slate Z	15	3	10	53	91	32	–	–	–	–	–	3	10	26	40	10
Gneiss A[a]	34	0	31	43	270	42	–	–	–	–	–	0	12	21	81	22
Gneiss B[a]	36	0	31	33	201	39	–	–	–	–	–	0	29	22	132	18
Schist[a]	39	0	31	58	228	–	0	31	58	160		0	31	52	179	–
Marble[a]	38	0	40	80	242	80	–	–	–	–	–	0	46	71	230	78
Sandstone[b]	25	0	60	110	249	100	0	60	80	248	68	0	60	75	247	62
Shale[c]	35	1	50	75	154	73	–	–	–	–	–	1	50	45	146	43
Limestone[c]	40	0	28	60	150	60	–	–	–	–	–	0	28	40	125	40

$\beta = 45^\circ$					$\beta = 60^\circ$					$\beta = 90^\circ$				
σ_3		σ_1		σ_c	σ_3		σ_1		σ_c	σ_3		σ_1		σ_c
Min	Max	Min	Max		Min	Max	Min	Max		Min	Max	Min	Max	
–	–	–	–	–	0	18	0	100	40	0	35	55	250	65
5	20	39	64	28	5	20	50	80	35	5	20	141	287	126
3	10	30	61	21	3	10	42	80	33	3	10	124	189	96
0	31	38	156	41	–	–	–	–	–	0	31	58	257	61
0	31	38	133	25	–	–	–	–	–	0	46	85	360	85
0	31	52	179	–	3.6	31	88	188	–	0	46	67	236	67
0	46	85	244	75	0	19	69	170	100	0	46	80	253	90
0	60	95	245	70	0	60	105	248	89	0	60	107	249	95
1	50	55	145	50	1	50	60	147	55	1	50	50	150	50
–	–	–	–	–	–	–	–	–	–	0	28	60	160	58

Note: All stresses are in MPa and angles in degree.
[a] Saroglou and Tsiambaos (2008).
[b] Zhang et al. (2009).
[c] Tien and Kuo (2001).

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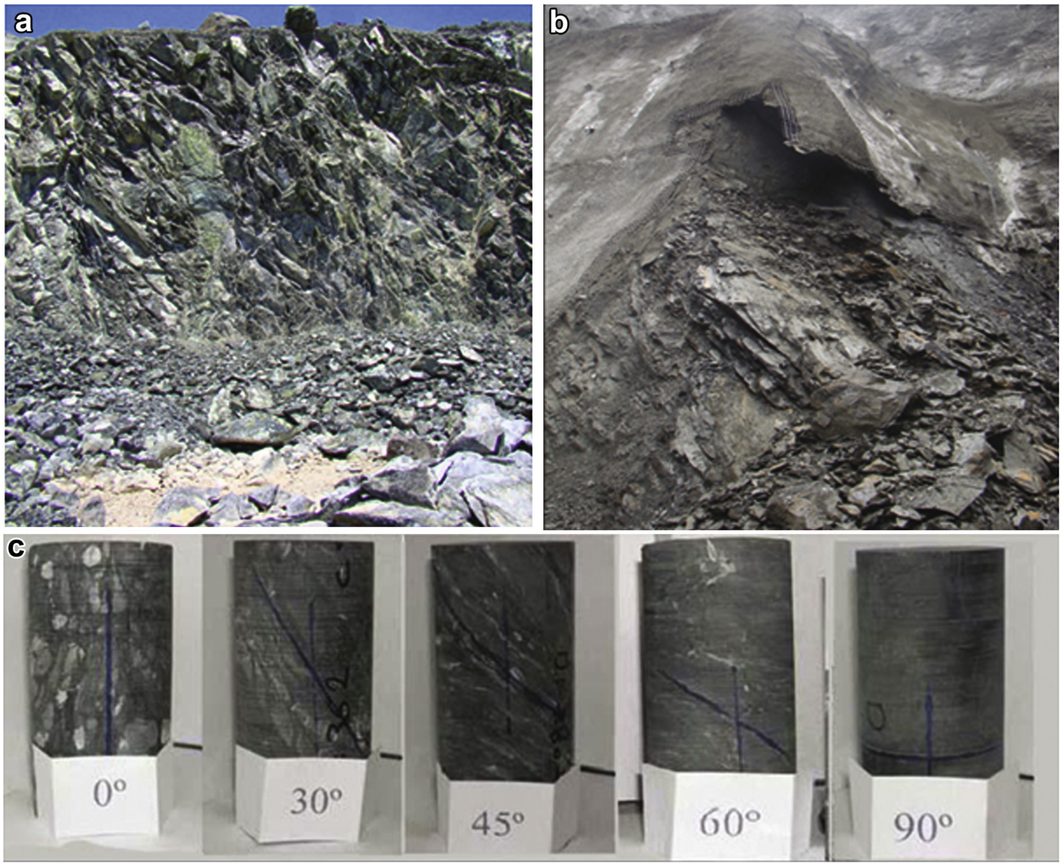

Figure 2. (a) Outcrop view of the slate at Golpayegan water tunnel used for obtaining slate G, (b) blocks of a collapsed berm in Sardasht dam used for obtaining slate S, (c) samples prepared from Zhaveh Dam site (Slate Z).

thin interbeddings of clay, shale, some organic detritus and volcanic ash while slate G contains mica and muscovite, and slate Z includes crystals of quartz and feldspar. Quartzitic slate S and Z were mainly made up of cryptocrystalline to fine grained flaky micaceous minerals, preferably oriented with fine-grained recrystallized quartz, which are in abundance. In addition, analyses showed that the preferred orientation (texture) of the quartz was almost parallel to the apparent direction of slate foliation.

4. Transversely isotropic behavior of the slate in UCS test

The most commonly used equation relating rock strength and direction of anisotropy was initially introduced by Jaeger (1960) and modified by Donath (1961). This equation is as follow:

$$\sigma_{c\beta} = A - D\cos 2(\beta - \beta_{\min}) \tag{1}$$

where β is the anisotropy orientation regarding the maximum loading, $\beta_{\min}$ is the angle of minimum UCS, A and D are constant parameters. To determine the values of parameters A and D, UCS data at the angles of weakness plane, 0°, 30° and 90°, is required. Hence, available uniaxial strength data (i.e. those data presented in Table 1) and Eq. (1) were used to determine the constants parameters A and D. Since parameter D is related to the strength anisotropy, value of this parameter represents the strength anisotropy effect. Generally, the variation of strength of intact rock in uniaxial and triaxial loading conditions with respect to the anisotropy orientation is defined as the "strength anisotropy" and its magnitude is representing the degree of anisotropy Eq. (2).

$$R_c = \frac{\sigma_{c(90)}}{\sigma_{c(\min)}} \tag{2}$$

where R_c is the degree of anisotropy, $\sigma_{c(90)}$ is the UCS perpendicular to the planes of anisotropy and $\sigma_{c(\min)}$ is the minimum value of σ_c commonly at $\beta = 30°{-}45°$. In addition, strength anisotropy can be represented in terms of Young's modulus as $E_{\max}/E_{\min}$, where $E_{\max}$ and $E_{\min}$, respectively, are the maximum and minimum values of Young's modulus in the transversely isotropic rocks (Amadei, 1996). Table 2 compares the degree of strength anisotropy in slates S, G and Z according to the above-mentioned.

According to the obtained ratios ($\geq$3) presented in Table 2, slates S, G and Z are categorized as the highly anisotropy rocks (Ramamurthy, 1993; Colak and Unlu, 2004). Fig. 4a,b shows the variation of UCS and Young's modulus of the slates G, S and Z with respect to anisotropy orientation β. It should be noted that the maximum strengths are obtained when the applied load is perpendicular to the foliation. However, minimum strengths of the slates are determined when the angle of foliation and applied load make an approximate degree of 30°.

As depicted in Fig. 4, variation of UCS and Young's modulus versus loading direction show a U-Shaped trend. There are actually many reasons explaining differences between values obtained for UCS and Young's modulus such as cohesion, friction and mineralogy of rocks. Hence, cohesive strength, C and friction angle, ϕ of slates G, S and Z were determined from linear portion of Mohr envelopes at $\beta = 0°$and $\beta = 90°$ as presented in Table 3, because behavior of rock in these directions is similar to that of the intact isotropic rock (Jaeger et al., 2007). It is obvious that slates G and S

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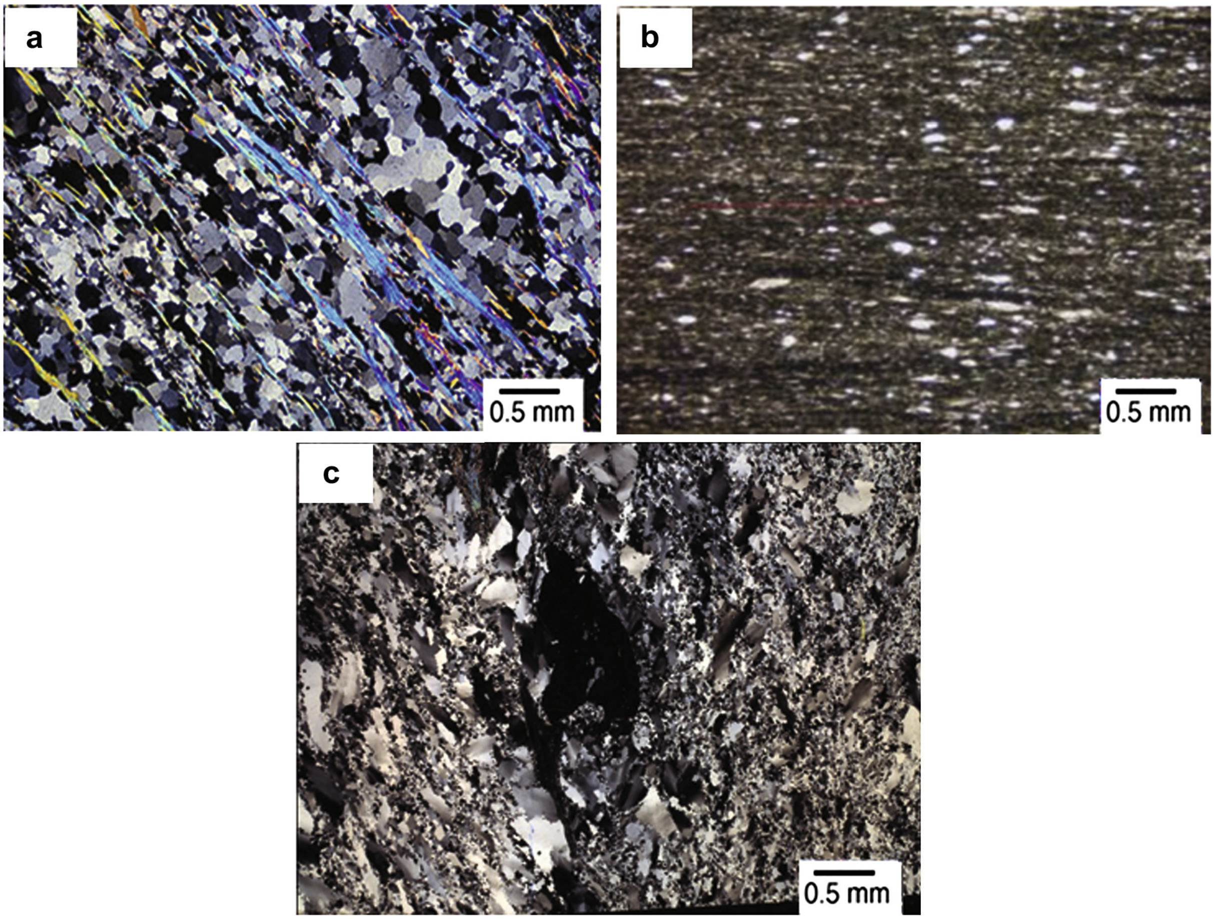

Figure 3. Thin sections of studied rock samples obtained perpendicular to the foliation, (a) slate G, (b) slate Z, (c) slate S.

mostly have the maximum and minimum values of the cohesive strength and friction, respectively.

It can be inferred from Table 3 that cohesive strength and friction are of the main reasons explaining different behaviors of slates tested. As for the first time McLamore and Gray (1967) investigated inconstant cohesion and friction angle with the loading orientation and proposed a failure criterion known as "the variable cohesive strength and friction theory". They found that cohesive strength and friction of transversely isotropic rocks were least amounts at angle of 30°–45° with respect to major loading direction.

5. Transversely isotropic behavior of different rock types in triaxial condition

5.1. Modified Hoek-Brown criterion

Saroglou and Tsiambaos (2008) modified the Hoek-Brown criterion (Hoek and Brown, 1980) by adding a strength anisotropy coefficient K_β, as follow:

Table 2
Strength anisotropy parameters in uniaxial compression test for slate.

Parameter	Slate S	Slate G	Slate Z
R_c	4.33	5.04	3.06
E_{max}/E_{min}	4.2	4.72	3.4
D	37.68	68.86	56.7
A	52.93	93.78	65.6

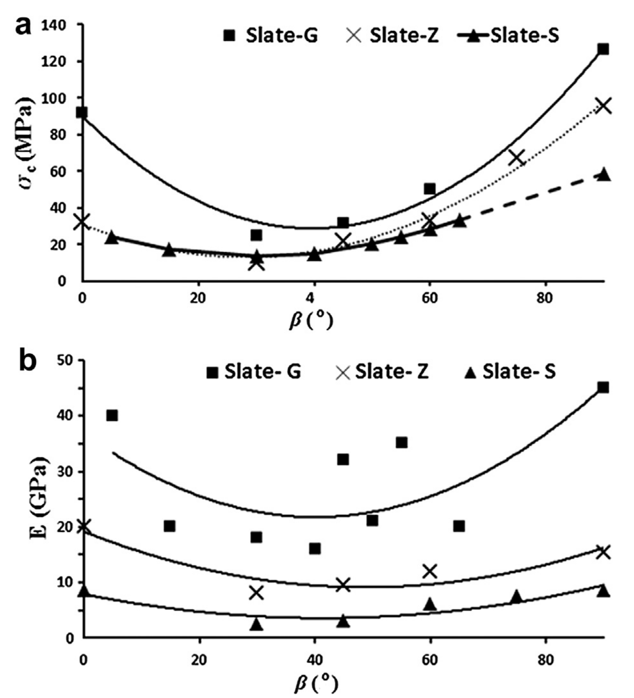

Figure 4. (a) The variation of UCS with degree of anisotropy, and (b) the variation of Young's modulus with degree of anisotropy.

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Table 3
Cohesive strength and friction angle of the slate G, S and Z.

	$\beta = 0°$		$\beta = 90°$	
	C (MPa)	φ	C (MPa)	φ
Slate G	15.45	47.7	17.52	53.7
Slate S	8	43.2	15.75	44.4
Slate Z	10.12	47.4	16.67	42.9

$$\sigma_1 = \sigma_3 + \sigma_{c\beta}\left(K_\beta m_i \frac{\sigma_3}{\sigma_{c\beta}} + 1\right)^{0.5} \quad (3)$$

where $\sigma_{c\beta}$ is the UCS at the anisotropy orientation β and K_β is the parameter of strength anisotropy. The intact rock parameter m_i varies from 4 for very fine weak rock like clay stone to 33 for coarse igneous light-colored rock like granite (Hoek, 1990). Saroglou and Tsiambaos (2008) also mentioned that the ratio of K_{90}/K_{30} can be considered as the strength anisotropy effect. They concluded that the parameter m_i is the characteristic of each rock and independent from loading direction. Fig. 5 shows the variation of K_β with the anisotropy orientation for slates.

However, the variation of K_β for slate S is different from others and has erratic pattern with angle β similar to its modulus variation in Fig. 4b. It may relate to the petrological properties of the slate S where presence of thin interbeddings of clay, shale, some organic detritus and volcanic ash may affect its mechanical properties.

Fig. 5 implies the strong relationship of K_β with anisotropy orientation of slates realized by Saroglou and Tsiambaos (2008). Hence, despite what Colak and Unlu (2004) expressed in their original paper, results of this study imply that m_i not varies with anisotropy direction. The same procedure as by Saroglou and Tsiambaos (2008) has been used to obtain m_i values so "by fitting the Hoek-Brown criterion to the triaxial data obtained at $\beta = 90°$, the value of m_i is determined where in this case $K_\beta = 1$, then the value of K_β can be obtained at other anisotropy angles". The values of m_i in the current study were obtained as 13.4, 12.1, 11.5, 24.6, 23.2, 9.5, 9.6, 17, 7.05 and 3.54 for slates S, G and Z, gneisses A and B, schist, marble, sandstone, limestone and shale, respectively.

5.2. Ramamurthy criterion

Ramamurthy et al. (1988) and Rao et al. (1986) proposed an empirical strength criterion to predict non-linear strength behavior of transversely intact isotropic rocks as follow:

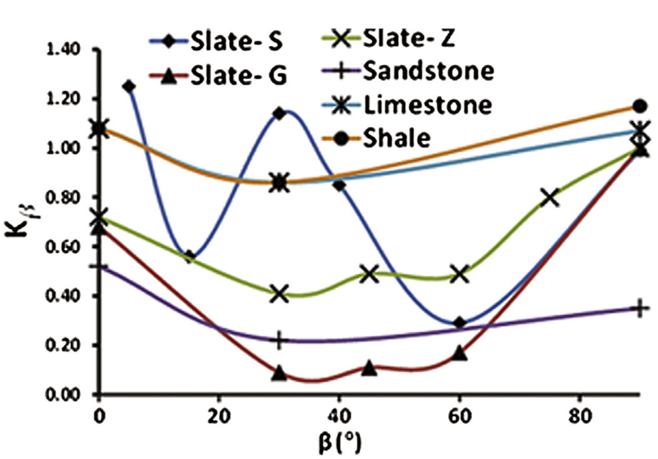

Figure 5. The variation of K_β parameter with anisotropy orientation.

$$\frac{(\sigma_1 - \sigma_3)}{\sigma_3} = B_j\left(\frac{\sigma_{cj}}{\sigma_3}\right)^{\alpha_j} \quad (4)$$

where σ_1 and σ_3 are the major and minor principal stresses, and σ_{cj} is the UCS at the particular anisotropy orientation β. Material strength anisotropy is taken into account here by defining the parameters α_j and B_j as the functions of anisotropy orientation as:

$$\frac{\alpha_j}{\alpha_{90}} = \left(\frac{\sigma_{cj}}{\sigma_{c90}}\right)^{1-\alpha_{90}}$$

$$\frac{B_j}{B_{90}} = \left(\frac{\alpha_{90}}{\alpha_j}\right)^{0.5} \quad (5)$$

where σ_{c90} is the UCS in $\beta = 90°$, and α_{90} and B_{90} are regarded as the values of α_j and B_j in $\beta = 90°$. In the current study, a few triaxial data at $\beta = 90°$ has resulted in obtaining parametersα_{90} and B_{90} from log-log plot of $(\sigma_1-\sigma_3)/\sigma_3$ and σ_{c90}/σ_3. Substituting the obtained parameters into Eq. (5), α_j and B_j can be calculated at any weakness planes.

5.3. A modified rock failure criterion for transversely isotropic rocks

5.3.1. Introduction

Rafiai (2011) proposed two rock failure criteria for isotropic rocks, which could be fitted to the polyaxial (true–triaxial) test data and triaxial test data. Because of the lack of true– triaxial data especially in the field of transversely isotropic rocks, in this study, just the triaxial failure criterion Eq. (6) has been used. The proposed empirical criterion is used for prediction of intact rock brittleness and ductility, and can be extended to rock mass strength. This original empirical failure criterion in triaxial loading condition is expressed as:

$$\frac{\sigma_1}{\sigma_{ci}} = \frac{\sigma_3}{\sigma_{ci}} + \left[\frac{1 + A(\sigma_3/\sigma_{ci})}{1 + B(\sigma_3/\sigma_{ci})}\right] - r \quad (6)$$

where σ_{ci} is the UCS of intact rock and A and B are constant parameters, depending on the properties of rock. The parameter r is the strength reduction factor indicating the extent to which the rock mass has been fractured. This parameter is considered equal to zero for intact rock and equal to one for heavily jointed rock masses.

To apply the failure criterion Eq. (6) for transversely isotropic rocks fitting procedure was conducted on the gathered database. As mentioned, the parameter r is considered equal to zero due to intact state of the rock. The results have shown that a new parameter as the strength reduction parameter should be taken into consideration for extending the generalization of Eq. (6) for transversely isotropic rocks. The modified criterion is as follow:

$$\sigma_1 = \sigma_3 + \sigma_{c\beta}\left[\frac{1 + A\left(\sigma_3/\sigma_{c\beta}\right)}{\alpha + B\left(\sigma_3/\sigma_{c\beta}\right)}\right] \quad (7)$$

where $\sigma_{c\beta}$ is the UCS of transversely intact isotropic rock at anisotropy orientation, α is the strength reduction parameter related to the rock anisotropy, and A and B are constants parameters.

5.3.2. Modified failure criterion in triaxial condition

Once the modified criterion was obtained, attempts were made to fit the modified criterion together with modified Hoek-Brown and Ramamurthy criteria to the transversely isotropic rocks in triaxial condition from Table 1. Two methods of fitting were used to fit the relations to the triaxial data. Simple linear regression was used to fit the modified Hoek-Brown and Ramamurthy criteria and

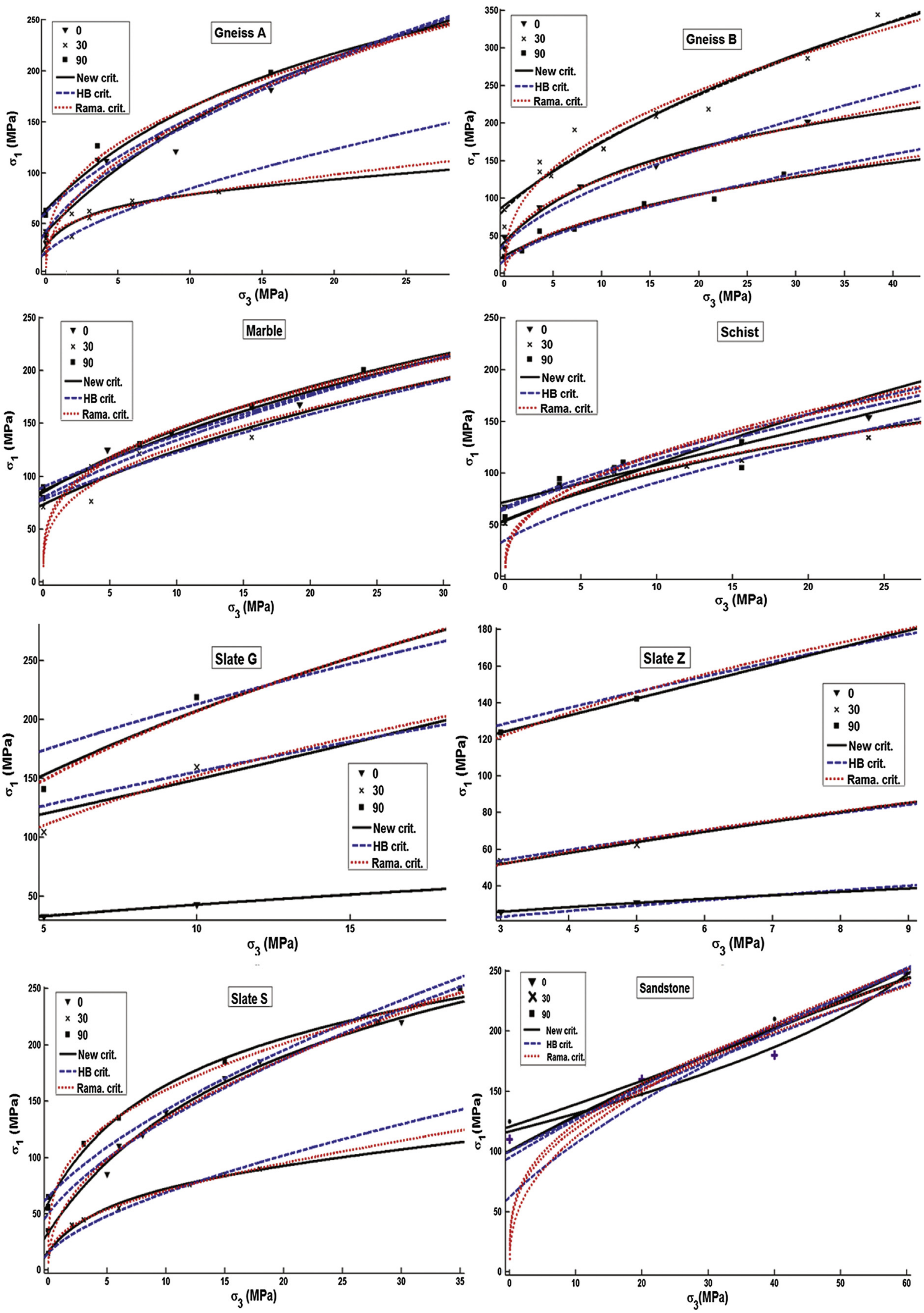

Figure 6. Comparison of failure envelopes of the new modified, modified Hoek-Brown and Ramamurthy criteria for different transversely isotropic rocks.

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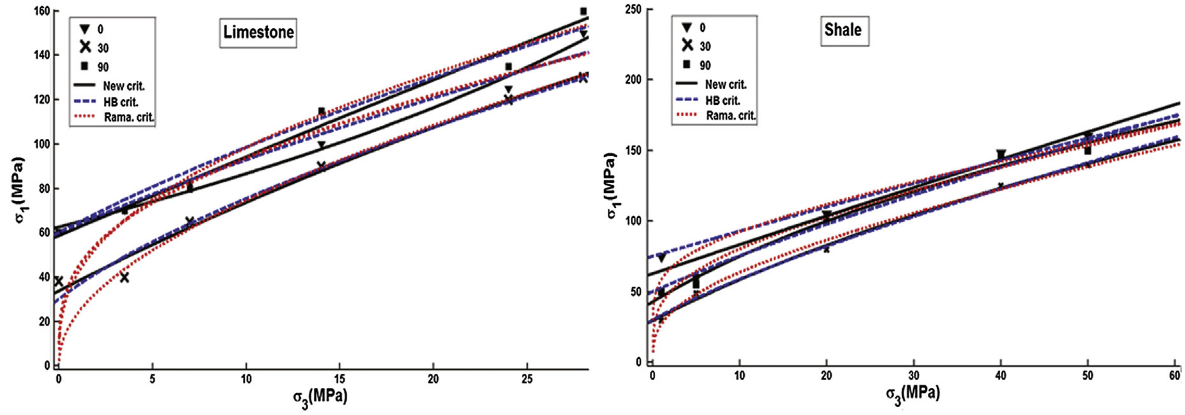

Figure 6. (*continued*).

non-linear regression was considered to fit the new modified criterion Eq. (7) using Matlab software (Matlab, 2009). Two algorithms of fitting, Levenberg-Marquardt and Trust-Region, were applied and the best correlation coefficient and Root Mean Square Errors (RMSE) were determined. Correlation coefficient and root mean square errors are criteria used for assessing the goodness of fit. To obtain constants of the modified triaxial criterion of Eq. (7) it can be re-written in the form

$$Z = AX - BY \tag{8}$$

where

$$X = \frac{\sigma_3}{\sigma_{c\beta}} \tag{9}$$

$$Y = \frac{\sigma_3}{\sigma_{c\beta}}\left(\frac{\sigma_1}{\sigma_{c\beta}} - \frac{\sigma_3}{\sigma_{c\beta}}\right) \tag{10}$$

$$Z = \alpha\left(\frac{\sigma_1}{\sigma_{c\beta}} - \frac{\sigma_3}{\sigma_{c\beta}}\right) - 1 \tag{11}$$

The values of A and B can be calculated as

$$A = \frac{\sum XY \sum YZ - \sum Y^2 \sum XZ}{\left(\sum XY\right)^2 - \sum X^2 \sum Y^2} \tag{12}$$

$$B = \frac{\sum X^2 \sum YZ - \sum XY \sum XZ}{\left(\sum XY\right)^2 - \sum X^2 \sum Y^2} \tag{13}$$

The generic acceptability of a rock failure criterion depends greatly on its application in wide range of rock mechanical tests. Fig. 6 compares the failure envelops of the modified criterion and those of modified Hoek-Brown and Ramamurthy criteria for different rock types at three different anisotropy orientations, $\beta = 0°, 30°, 90°$.

It can be seen that the new modified criterion is fitted to the triaxial data for transversely isotropic rocks rather than those of the modified Hoek-Brown and Ramamurthy criteria. The curvature of the new criterion envelope is quite appropriate and shows high non-linearity. The results of the analysis using the three criteria for transversely intact isotropic rocks are given in Table 4.

As given in Table 4, the proposed modified criterion is able to properly predict the triaxial test data with the correlation coefficient of more than 0.98. Since failure did not occur at the $\beta = 0°$ and $\beta = 90°$, in which the behavior of transversely intact isotropic rock is similar to intact isotropic rock (Jaeger, 1960), values of parameter α at these directions are near to one and the new modified criterion decreases to its original form for the intact isotropic rock.

To determine the ability of each criterion in predicting the strength of transversely isotropic rocks, RMSE was calculated. For the aim of present study, RMSE can be calculated as

$$\mathrm{RMSE} = \sqrt{\frac{\sum_{i=1}^{n}\left(\sigma_i^t - \sigma_i^p\right)}{n}} \tag{14}$$

where σ_i^t and σ_i^p are the tested and predicted values of σ_1 for the ith data, respectively and n is the number of data points. Fig. 7 compares the RMSE values of the new modified criterion with the other two ones.

As depicted in Fig. 7, the modified criterion shows reasonable RMSE value, which is lower and much better than those of the modified Hoek-Brown and Ramamurthy criteria. Hence, it can be concluded that highest correlation coefficient and lowest RMSE are associated with the modified criterion indicating its strength in predicting the behavior of the transversely isotropic rock. Furthermore, one additional way of assessing the accuracy of a criterion is measuring its ability to predict the rock uniaxial compressive strength. According to Table 4 given, the predicted UCS of proposed criterion, $\sigma_{c\beta-pr}$, is quite close to that of the laboratory test, $\sigma_{c\beta-lab}$.

5.3.3. *Strength reduction parameter of the modified criterion*

The results obtained from fitting the new modified on the triaxial data have shown that parameter α (i.e. the one presented in Table 4 as the strength anisotropy parameter) has a consistent relationship with β. It will be more obvious when we look at the value of α in $\beta = 0°$ and $\beta = 90°$ where parameter α is nearly equal to 1 and the modified criterion changes to its original form Eq. (6) for intact isotropic rock. Fig. 8 shows the variation of parameter α with anisotropy orientation β.

As shown in Fig. 8a,b, the parameter α decreases when the angle of anisotropy is between 30°–45°, which introduces it as a strength reduction parameter for transversely isotropic rocks.

Based on the above definitions the ratio of α_{90}/α_{30} is greater for the rocks with a high degree of anisotropy, R_c hence slate, gneiss and reduces significantly for the rocks with a low degree of anisotropy, marble, shale and limestone (Fig. 9). The value of α_{90} is the value of α in Eq. (7) when loading is perpendicular to the schistosity, equal to unity, and α_{30} is its value at the orientation of minimum strength, at $\beta = 30°–45°$. Fig. 9 shows the comparison between the three anisotropy indexes, R_c, α_{90}/α_{30} and K_{90}/K_{30} for the studied rock types.

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Table 4
Obtained parameters from fitting the new modified, the modified Hoek-Brown and the Ramamurthy criteria for different anisotropic rock types.

β	$\sigma_{c\beta-lab}$	H–B criterion		Ramamurthy criterion			New modified criterion				
		k_β	R^2	α_j	B_j	R^2	α	A	B	σ_{c-pr}	R^2
Gneiss A											
0°	39.4	1.79	0.98	0.57	6.2	0.87	1.11	17.5	2.15	40.6	0.97
30°	35.5	0.42	0.67	0.8	3.61	0.45	0.55	22.77	6.3	21	0.90
90°	66.5	1	0.97	0.67	4.58	0.73	1.02	17.08	3.31	61	0.98
Gneiss B											
0°	45.4	0.88	0.97	0.67	4.61	0.77	0.93	14.16	2.47	38	0.98
30°	23.4	0.59	0.96	0.6	4.5	0.90	0.3	5.38	0.63	19	0.97
90°	85.7	1.01	0.96	0.63	4.44	0.83	1	9.55	1.37	87	0.95
Marble											
0°	88.1	0.99	0.97	0.73	2.92	0.53	0.94	6.84	1.7	81	0.98
30°	76.1	0.91	0.96	0.71	2.73	0.73	0.85	7.15	1.92	77	0.96
90°	89.7	1	0.98	0.71	2.8	0.35	1.02	9.64	2.95	87	0.99
Schist											
0°	66	1.32	0.88	0.64	3.21	0.71	1.2	6.83	0.8	65	0.88
30°	25	0.77	0.83	0.73	3.68	0.62	0.4	4.61	0.87	27	0.91
90°	67	1.04	0.99	0.65	3.01	0.65	1.03	2.48	0.013	66	0.99
Slate G											
0°	92	0.68	0.9	0.56	4.08	0.98	1	4.05	0.5	92	0.94
30°	25	0.35	0.99	0.76	1.64	0.96	0.45	3.93	1.77	26	1
90°	126	1	0.88	0.54	5	0.95	1.5	25.56	5.42	125	0.97
Slate Z											
0°	32	0.73	0.98	0.58	4.01	0.95	1	6.82	0.79	31	0.99
30°	11	0.38	0.86	0.76	2.8	0.94	0.7	3.54	0.93	12	1
90°	96	1	0.97	0.66	3.96	0.97	1.002	8.3	0.3	95	1
Slate S											
0°	50	1.1	0.96	0.57	4.92	0.89	1.5	25.3	4.51	48	0.99
30°	15	0.93	0.96	0.69	4.56	0.91	0.49	7.55	0.68	15	0.99
90°	65	0.9	0.96	0.72	3.83	0.7	1.09	27.04	7.05	65.4	0.99
Sandstone											
0°	100	0.52	0.94	0.72	2.03	0.25	0.85	0.67	0.89	99	0.97
30°	62	0.22	0.84	0.73	2.97	0.12	0.61	1.75	0.3	62	0.99
90°	95	0.35	0.89	0.64	2.36	0.14	0.78	0.16	0.38	94	0.98
Shale											
0°	60	0.89	0.92	0.84	1.5	0.9	1.2	1.4	0.13	73	0.96
30°	45	0.72	0.99	0.68	2.51	0.99	0.85	2.72	0.49	31	0.99
90°	50	1.002	0.97	0.76	2	0.97	1.01	4.38	1.38	50	0.98
Limestone											
0°	60	1.08	0.96	0.72	2.33	0.35	0.96	0.3	−0.83	60	0.96
30°	45	0.86	0.97	0.54	3.54	0.81	0.89	3.4	0.35	28	0.98
90°	60	1.07	0.90	0.66	2.7	0.51	1.024	2.77	0.12	59	0.97

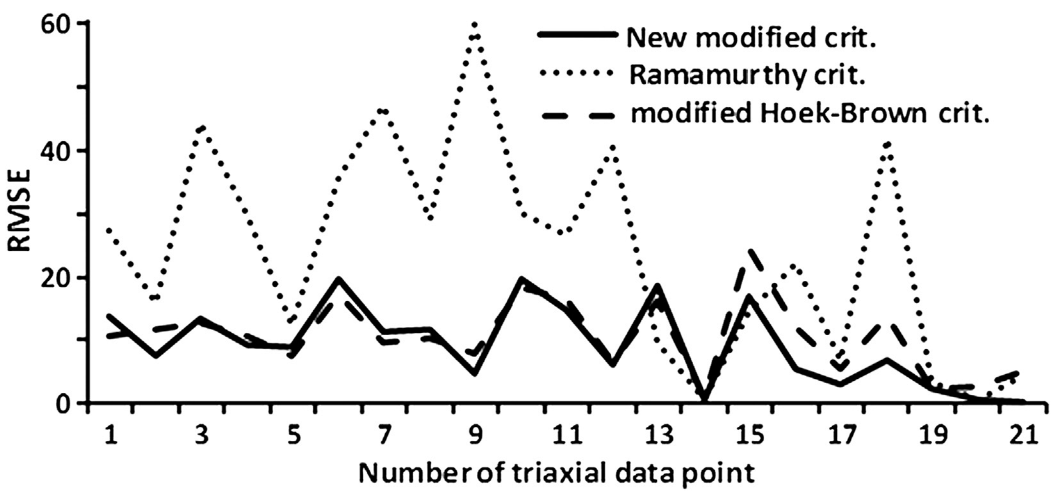

Figure 7. RMSE values calculated by fitting the new modified, modified Hook-Brown and Ramamurthy criteria to the triaxial data.

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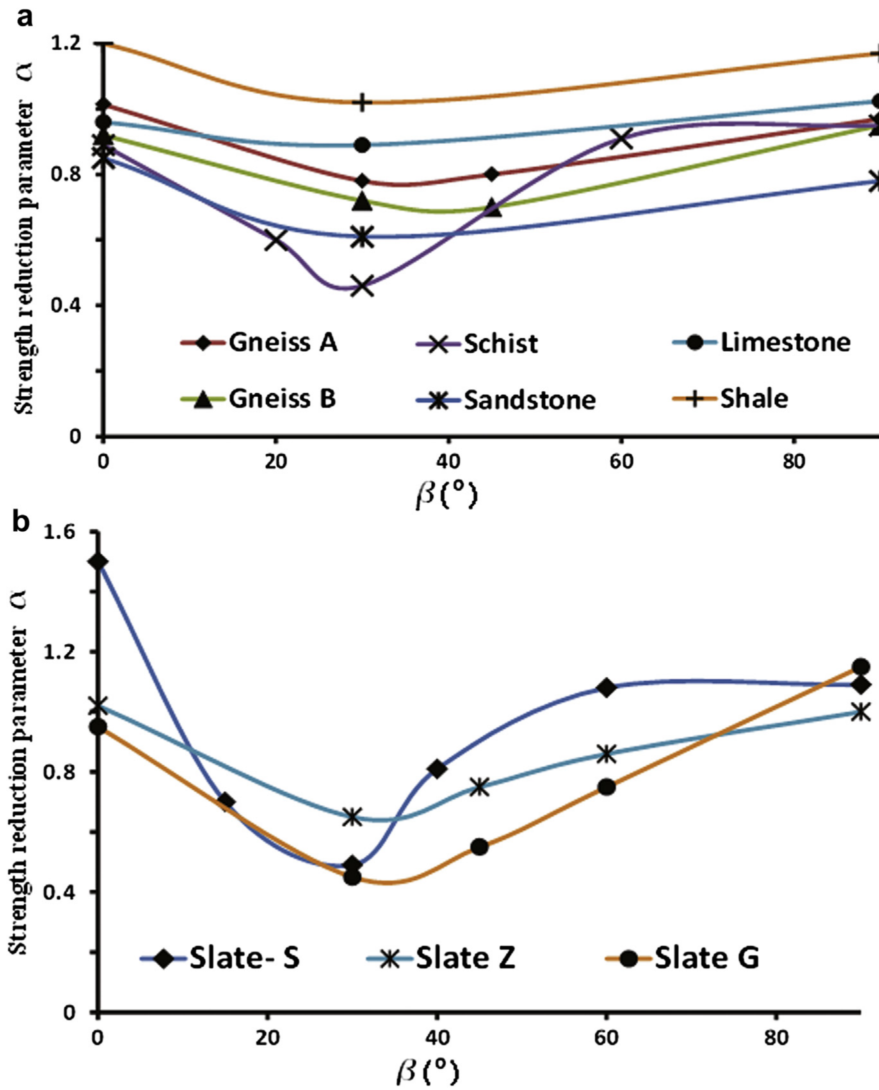

Figure 8. The variation of parameter α with the anisotropy orientation β for different rock types.

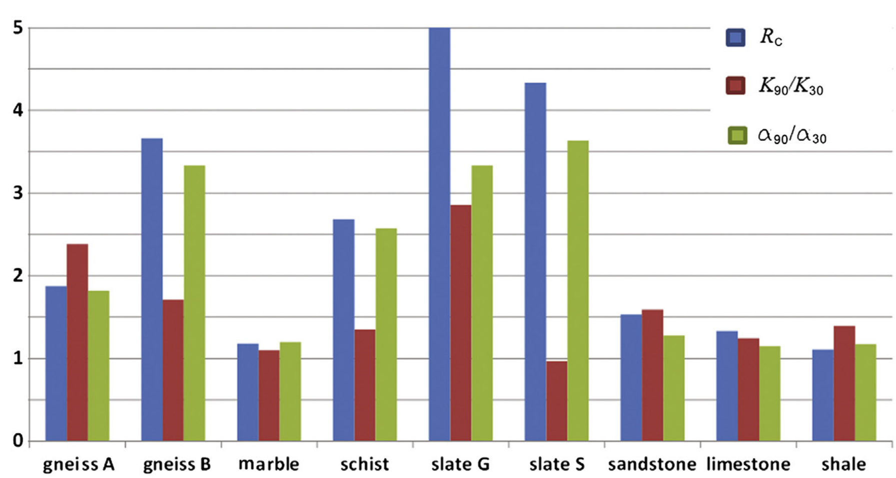

Figure 9. Comparison of strength anisotropy indexes using the triaxial test data.

Ramamurthy (1993) introduced the first classification on the degree of anisotropy, R_c for different rock types. However, the values of K_{90}/K_{30} (Saroglou and Tsiambaos, 2008) show less agreement with R_c as it seen in Fig. 9 the values of α_{90}/α_{30} are close to R_c for the current rock types. It can be concluded that the ratio of α_{90}/α_{30} shows a good representation of the degree of rock anisotropy.

6. Conclusion

A study on the mechanical behavior of the different transversely isotropic rocks obtained from different references is presented. A recently proposed rock failure criterion was modified to be usable for determining the strength of transversely intact isotropic rocks. Triaxial datasets for metamorphic and sedimentary rocks, which commonly show strength anisotropy, were gathered. Failure envelopes of the proposed criterion were compared to those of the modified Hoek-Brown and Ramamurthy criteria. The modified criterion was tested for triaxial test data of the transversely isotropic intact rocks and higher correlation coefficient and lower root mean square error relative to the well-known modified Hoek-Brown criterion and Ramamurthy criterion were obtained. It also can approximate the UCS of the transversely intact isotropic rocks, precisely. The parameter α involved in the proposed modified criterion shows a U-shaped relationship with orientation of anisotropy. Hence, it can be considered as the strength reduction parameter.

The modified criterion represents the behavior of transversely intact isotropic rocks as its original failure criterion, which can predict the behavior of intact isotropic rocks accurately. However, the modified criterion is limited to the strength prediction for intact anisotropic rocks and triaxial testing conditions. Further study is needed to extend the modified criterion for anisotropic rock masses and polyaxial testing conditions with emphasis on the effect of intermediate principal stress. It will be worthwhile somehow if the modified criterion could predict strength of transversely isotropic rocks in different directions of weakness planes with limited data in one direction e.g. perpendicular to the weakness planes.

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