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A failure criterion for transversely isotropic rocks

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Abstract

In this paper, a new failure criterion for the transversely isotropic rocks is presented. The new criterion is based on two distinct failure modes; one is the sliding mode where the failure is caused by sliding along the discontinuity, and the other is the non-sliding mode where the failure is controlled by the rock material and is not dependent on discontinuity. This failure criterion is defined with seven material parameters. The physical meanings of, and the procedures for determining, these parameters are described. Both the original Jaeger criterion and the extended Jaeger criterion are shown to be special cases of the proposed criterion. The accuracy and applicability of the proposed failure criterion are examined using the published experimental data. The data used cover various types of transversely isotropic rocks, different orientation angels and confining pressures. The predicted strength behaviors of the transversely isotropic rocks agree well with the experimental data from various investigators. The accuracy and applicability of the proposed empirical failure criterion are demonstrated in this paper.

1. Introduction

The constitutive laws and failure criteria of rock materials and rock masses are required in most rock engineering analyses that are based on solid mechanics. Due to the preferred fabric orientation or the existence of non-random discontinuity, anisotropic behaviors of rock masses should be fully accounted for in the analysis. Significant errors may be introduced into the analysis by treating the anisotropic rocks as the isotropic materials. Several types of rocks such as sedimentary rocks and metamorphic rocks may be transversely isotropic. Most foliated metamorphic rocks, such as schist, slates, gneisses, and phyllites, contain fabric with preferentially parallel arrangements of flat or long minerals. Metamorphism changes the initial fabric of rocks with the directional structure. Foliation induced by the non-random orientation of macroscopic mineral, parallel fracture or microscopic mineral plates, such as fracture cleavage, slaty cleavage, bedding cleavage, lepidoblastic schistosity, nematoblastic schistosity or lineation, leads to rock properties that are highly direction-dependent [1]. Stratified sedimentary rocks like sandstone, shale or sandstone–shale alteration often display anisotropic behaviors. The anisotropy may also be found in the isotropic rocks, such as granite and basalt, if cut by regular discontinuities [2,3].

Over the past several decades, many authors have devoted considerable efforts to the study of rock anisotropy, from both the theoretical and the experimental points of view. Many scholars have investigated mechanical properties of both nature and synthetic transversely isotropic rocks under varied confining pressures [4–14]. The shape of the curve of compression strength and the orientation angle (the angle between the discontinuity and the direction of major principal stress) are the most common representation of the nature of strength anisotropy. Most transversely isotropic rocks are found to have their maximal compression strength at an orientation angle $\beta = 0^\circ$ or 90°, and their minimal compression strength at an orientation angle in the range of 30–45°. As the confining pressure is increased, the anisotropic rocks become more ductile, and the effect of the strength anisotropy is usually reduced. Based upon the analysis of the shape of the anisotropy curve, Ramamurthy [15] classified the anisotropy of rocks into three groups, namely, U type, undulatory type, and shoulder type anisotropy (see Fig. 1).

Several scholars have developed failure criteria for the transversely isotropic rocks that can account for

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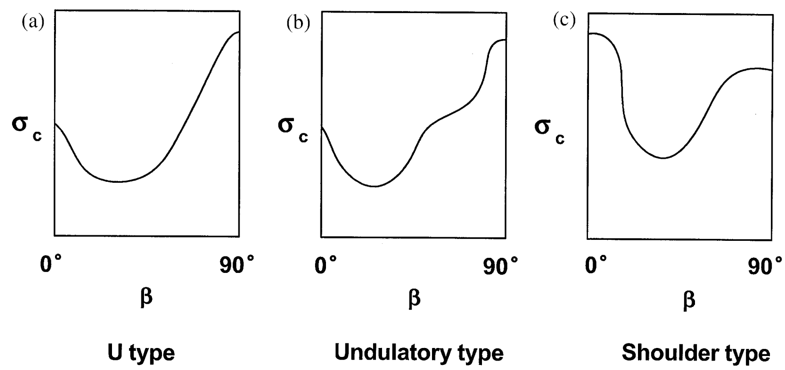

Fig. 1. (a–c) Classification of anisotropy for transversely isotropic rocks (after Ramamurthy [15]).

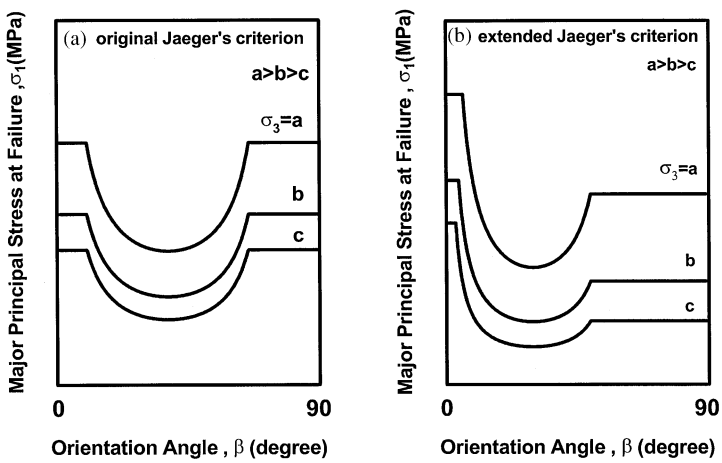

Fig. 2. Schematic view of strength variation versus β (a) original Jaeger's criterion (b) extended Jaeger's criterion.

the variation of compression strength with the orientation angles under various confining pressures. Jaeger [16] introduced an instructive analysis of the case in which the rocks contain well-defined, parallel discontinuity. This original Jaeger's criterion (Fig. 2a) used four parameters, and treated rocks as the isotropic materials when the sliding along discontinuity is prevented. Jaeger's criterion yields the same compression strength at $\beta = 0^\circ$ and 90°. However, the published experimental data show that in some rocks, the maximum strength occurs at $\beta = 0^\circ$, while in other rocks, it occurs at $\beta = 90^\circ$ [17–19]. In order to account for this discrepancy (possible strength discrepancy at $\beta = 0^\circ$and 90°), other researchers have modified Jaeger's criterion by adding two more parameters. Such modification is referred to herein as the extended Jaeger's criterion (Fig. 2b). Furthermore, Duveau and Shao [20] provided yet another modification by replacing the Mohr–Coulomb criterion with a non-linear model to express the strength along discontinuity. Their criterion used seven parameters to describe the failure strength for transversely isotropic rocks.

The strength criteria for the transversely isotropic rocks developed by McLamore and Gray [7], Hoek and Brown [21], and Ramamurthy [15] generally provide fairly accurate simulation of the experimental data. However, their approaches all require a wide range of tests and/or a considerable amount of curve fitting work. A more general criterion, expressed as a quadratic function for anisotropic materials, was proposed by Hill [22]. This criterion is an extension of von Mises' isotropic criterion. While von Mises' and Hill's criteria assume that the strength of the material is independent of hydrostatic stress and are suitable for metals and composite materials, they may not be directly applicable to geological materials because the strength behavior of most geological materials is dependent on the hydrostatic. Pariseau [23] and Cazacu et al. [24] extended Hill's criterion to account for the effect of the hydrostatic stresses. These criteria express the strength in terms of stress invariance, and they are applicable to truly 3-D stress cases and can easily be implemented numerically. Nova [25] proposed a generalized failure criterion for the transversely isotropic rocks under compression. Both Hill's and Nova's criteria [25] describe the continuous variation of strength with the orientation angle, which is referred to herein as the continuous model. However, the continuous model is not suitable for the shoulder and undulatory types of rocks, especially for the rocks cut by discontinuities.

In this paper, a new failure criterion is developed for transversely isotropic rocks based on the Jaeger's criterion [16] and the maximum axial strain theory. Here, the axial strain is calculated from the constitutive law of the transversely isotropic rocks [26]. In the proposed criterion, seven material parameters are used. The physical meanings of, and the procedures to determine, these parameters, are presented. The accuracy of this proposed model is demonstrated by examining the experimental data of various types of transversely isotropic rocks from the literature.

2. Failure modes of transversely isotropic rocks

In the development of a failure criterion, it is important to observe the failure modes of rock specimens with different orientation angles and under different confining pressures. An ideal failure criterion should be able to predict not only the state of stress at failure but also the failure mode. The failure mode of anisotropic rocks under triaxial compression is influenced by the orientation of the stresses, as well as the confining pressure. Hence, it is far more complicated than that of isotropic rocks. Many scholars [4,7,11] have described in detail the failure modes of the transversely isotropic rocks under various confining pressures. Jaeger [16] simplified the failure of transversely isotropic rocks into two modes: (1) sliding along the discontinuity, and (2) fracture through the rock materials. Jaeger's criterion is mainly based on the simplified assumption of failure modes described above. Recently, Tien and his colleagues [14,27–30] have developed a sample preparation technique for synthetic layered rocks. The overall mechanical properties of synthetic layered rocks are found to be very similar to those of transversely isotropic natural rocks.

Fig. 3 show synthetic, layered rock samples that are made of gypsum (white layers) and the mixture of gypsum and clay (brown layers). The strength and stiffness of white layers are higher than those of brown layers, while the brown layers are more ductile. The overall mechanical properties of synthetic layered rock are macroscopically transversely isotropic. The detailed sample preparation procedures, material mixing ratio and mechanical properties of the prepared samples have been documented by Tien et al. [27]. Fig. 3 shows a series of photos that depict the failure modes of the samples in triaxial compression tests. For samples with $\beta = 0^\circ$ or 90° and loaded without confining pressure,

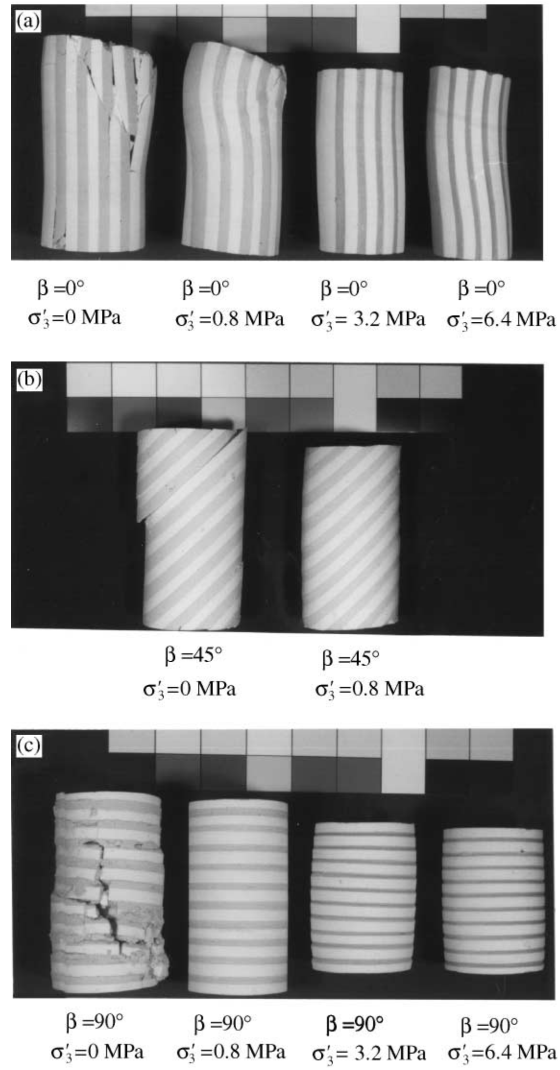

Fig. 3. Deformation characteristic of saturated synthetic layered rocks after triaxial test (a) $\beta = 0^\circ$ (b) 45° and (c) 90°.

fracture through both white and brown layers was observed. When they were loaded under confining pressures, behavior of ductile deformation (i.e., axial strain accumulation) was observed. For samples prepared at $\beta = 45^\circ$ and loaded without confining pressure, the failure mode was that of sliding along the discontinuity. When they were loaded under a confining pressure up to 0.8 MPa, the artificial layered rocks behave more like ductile materials, and the sliding mode was suppressed.

The axial strain has intimate relation with the orientation angle β. For example, the axial strain of samples with $\beta = 90^\circ$ is mainly due to the compression of brown layers (softer layers), whose thickness is the same as that of the white layers before the test. For samples prepared at $\beta = 0^\circ$, the white layers and the brown layers have the same axial strain, and thus the white layers will carry greater load than the brown layers since the stiffness of white layers is higher.

It is obvious from the above observations, the failure of transversely isotropic rocks may be divided into two failure modes: (1) sliding mode in which the plane discontinuity predominated, (2) non-sliding mode in which the material strength dominated. Jaeger treated the former failure mode by the theory of plane of weakness, which yielded fairly accurate and reasonable prediction of the strength. However, the plateau of constant strength at low values of β, or high values of β predicted by Jaeger's criterion is not always present in the experimental strength data when the sliding mode is prevented. This suggests that the assumption of rock as an isotropic material in Jaeger's criterion results in an oversimplified representation of strength, when the failure is controlled by the rock materials.

In the present study, the rock materials are considered as a transversely isotropic medium in an attempt to establish a more reasonable failure criterion to account for non-sliding mode.

3. Proposed failure criterion

3.1. Sliding along the discontinuity

Jaeger [16] derived the shear strength induced by sliding along the discontinuity. Applying the stress transformation to the loading shown in Fig. 4 yields the following equations:

$$\sigma_n = \tfrac{1}{2}(\sigma_1 + \sigma_3) - \tfrac{1}{2}(\sigma_1 - \sigma_3)\cos 2\beta \tag{1}$$

$$\tau = \tfrac{1}{2}(\sigma_1 - \sigma_3)\sin 2\beta \tag{2}$$

The major principal stress for sliding along the discontinuity

$$\sigma_{1(\beta)} = \sigma_3 + \frac{2(c_w + \sigma_3 \tan\phi_w)}{(1 - \tan\phi_w \tan\beta)\sin 2\beta} \tag{3a}$$

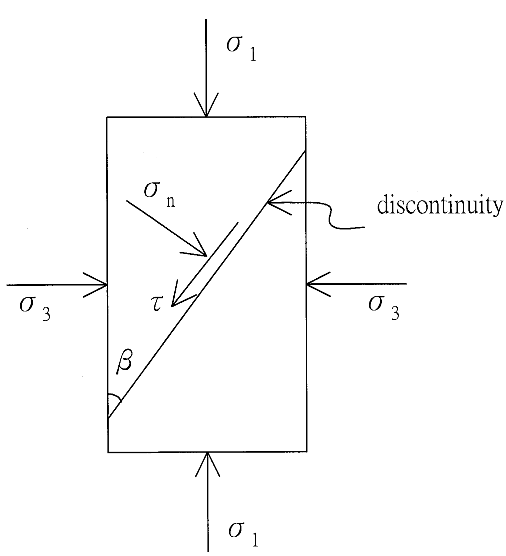

Fig. 4. Normal stress and shear stress on the discontinuity.

or

$$S_{1(\beta)} = \sigma_{1(\beta)} - \sigma_3 = \frac{2(c_w + \sigma_3 \tan\phi_w)}{(1 - \tan\phi_w \tan\beta)\sin 2\beta} \tag{3b}$$

where c_w and ϕ_w are the cohesion and friction angle of the discontinuity, and $S_{1(\beta)}$ is the major deviatoric stress at failure for the specimens with β.

3.2. Incorporation of the difference between $\sigma_{1(0^\circ)}$ and $\sigma_{1(90^\circ)}$

As the values of β approaching 0° or in the range of $(90° - \phi_w)$–90°, the sliding failure along the discontinuity will not occur. In such case, the strength of rocks is dominated by the rock materials and is independent of the discontinuity. The major principal stress at the failure of the specimen under a given confining pressure σ_3 must be controlled by the strength of the rock material. In the original Jaeger's criterion, the transversely isotropic rock is modeled as an isotropic material containing well-defined, parallel planes of weakness. The major principal stress at failure under a given confining pressure must be controlled by the strength of the isotropic material and will not vary with the orientation angle β. However, the constant strength at low values of β, or high values of β, predicted by the Jaeger's criterion is not supported by experimental data. Borecki and Kwasniewski [17] have collected dozens of values of $\sigma_{c(0^\circ)}/\sigma_{c(90^\circ)}$, which shows the ratio to be in range of 0.6–1.33. Thus, Jaeger's criterion is indeed an oversimplified representation of the strength of rock specimens whose failure is controlled by rock material.

To correctly reflect the difference between the strength of rocks at $\beta = 0°$ and 90°, the proposed criterion treats the rock material as transversely isotropic material. Thus, rocks at $\beta = 0°$ and 90° have different strength, and both are assumed to follow the failure criterion by Hoek and Brown [31]:

$$\sigma_{1(0^\circ)} = \sigma_3 + \left(m_{(0^\circ)}\sigma_3\sigma_{c(0^\circ)} + \sigma^2_{c(0^\circ)}\right)^{0.5} \tag{4a}$$

or

$$S_{1(0^\circ)} = \sigma_{1(0^\circ)} - \sigma_3 = \left(m_{(0^\circ)}\sigma_3\sigma_{c(0^\circ)} + \sigma^2_{c(0^\circ)}\right)^{0.5} \tag{4b}$$

$$\sigma_{1(90^\circ)} = \sigma_3 + \left(m_{(90^\circ)}\sigma_3\sigma_{c(90^\circ)} + \sigma^2_{c(90^\circ)}\right)^{0.5} \tag{5a}$$

or

$$S_{1(90^\circ)} = \sigma_{1(90^\circ)} - \sigma_3 = \left(m_{(90^\circ)}\sigma_3\sigma_{c(90^\circ)} + \sigma^2_{c(90^\circ)}\right)^{0.5} \tag{5b}$$

where $\sigma_{c(0^\circ)}$, $\sigma_{c(90^\circ)}$ are the uniaxial compression strength of rock samples at $\beta = 0°$ and 90°, respectively; $m_{(0^\circ)}$, $m_{(90^\circ)}$ are the m values in the Hoek and Brown criterion for the rock samples at $\beta = 0°$ and 90°, respectively.

Treating rocks at $\beta = 0°$ and $90°$ as two different materials is in some way similar to the extended Jaeger's criterion (Fig. 2b). However, in the latter Mohr–Coulomb criterion was used, while in the proposed criterion, the Hoek–Brown criterion is adopted. The rationale for adopting the Hoek–Brown criterion in the present study lies in the fact that the $\sigma_1 - \sigma_3$ relationship is generally nonlinear, particularly when the range of σ_3 under consideration is large. The Hoek–Brown criterion can fit the experimental data in both brittle and ductile regions better than the Mohr–Coulomb criterion does. Since both criteria require two parameters, using the Hoek–Brown criterion in the present study does not increase the number of parameters required in the proposed model.

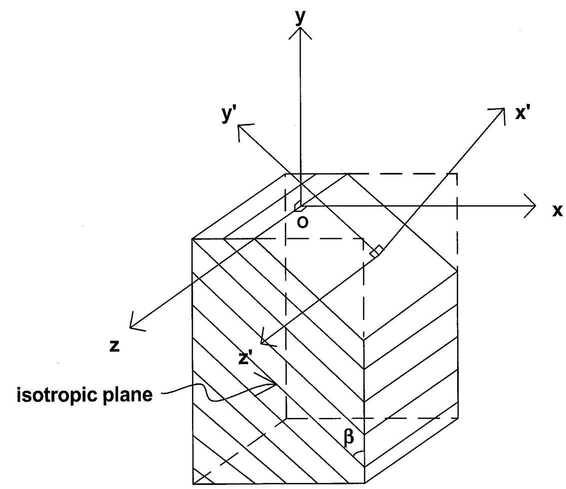

Fig. 5. Coordinate systems used for transformation law for material constants of a transversely isotropic medium.

3.3. Axial strain of transversely isotropic rocks

As mentioned before, ductile deformation due to axial strain accumulation is another important failure mode of transversely isotropic rocks, in addition to the failure mode of sliding along the discontinuity. The axial strain may be calculated using the theory of elasticity of an anisotropic medium. The constitutive laws of linearly elastic, transversely isotropic medium in the local coordinate system (x', y', z') takes this form (Fig. 5):

$$\begin{bmatrix} \varepsilon_{x'x'} \\ \varepsilon_{y'y'} \\ \varepsilon_{z'z'} \\ \gamma_{y'z'} \\ \gamma_{x'z'} \\ \gamma_{x'y'} \end{bmatrix} = \begin{bmatrix} \frac{1}{E} & \frac{-v}{E} & \frac{-v'}{E'} & 0 & 0 & 0 \\ \frac{-v}{E} & \frac{1}{E} & \frac{-v'}{E'} & 0 & 0 & 0 \\ \frac{-v'}{E'} & \frac{-v'}{E'} & \frac{1}{E'} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G'} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G'} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{2(1+v)}{E} \end{bmatrix} \times \begin{bmatrix} \sigma_{x'x'} \\ \sigma_{y'y'} \\ \sigma_{z'z'} \\ \tau_{y'z'} \\ \tau_{x'z'} \\ \tau_{x'y'} \end{bmatrix} \tag{6}$$

where E is the Young's modulus of rock in the direction parallel to the isotropic plane, E' the Young's modulus of rock in the direction perpendicular to the isotropic plane, G' the shear modulus for the plane normal to the isotropic plane, v the Poisson's ratio that characterized the transverse strain expansion in the isotropic plane due to a compressive stress in the same plane and v' the Poisson's ratio that characterized the transverse strain expansion in the isotropic plane due to a compressive stress in a direction normal to it.

The constitutive equations of the transversely isotropic medium in the global coordinate system (x, y, z), defined in Fig. 5, can be obtained by tensor transformation:

$$\begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} K_{11} & K_{12} & K_{13} & 0 & 0 & K_{16} \\ K_{12} & K_{22} & K_{23} & 0 & 0 & K_{26} \\ K_{13} & K_{23} & K_{33} & 0 & 0 & K_{36} \\ 0 & 0 & 0 & K_{44} & K_{45} & 0 \\ 0 & 0 & 0 & K_{45} & K_{55} & 0 \\ K_{16} & K_{26} & K_{36} & 0 & 0 & K_{66} \end{bmatrix} \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{bmatrix} \tag{7}$$

where the coefficients $K_{11}, K_{22}, \ldots, K_{66}$, are readily available in the literature [2,3,6]. The state of stresses at failure, when subjected to the triaxial loading, as shown in Fig. 6, can be decomposed into the hydrostatic and deviatoric stress components,

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \sigma_3 \\ \sigma_1 \\ \sigma_3 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \sigma_3 \\ \sigma_3 \\ \sigma_3 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \sigma_1 - \sigma_3 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \sigma_3 \\ \sigma_3 \\ \sigma_3 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ S_1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{8}$$

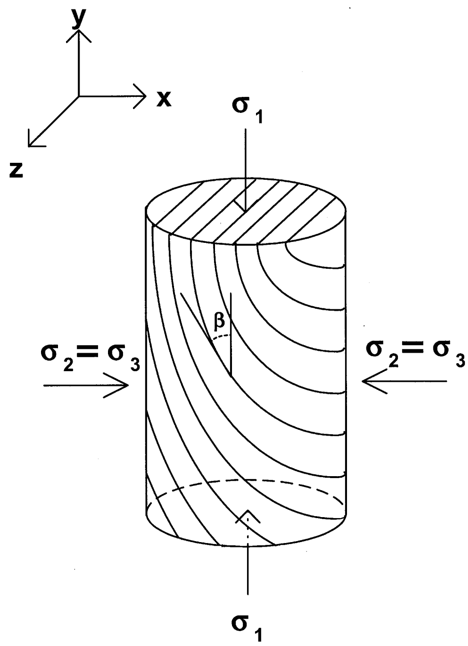

Fig. 6. Conventional triaxial test for transversely isotropic rock.

where

$$S_1 = \sigma_1 - \sigma_3. \tag{9}$$

The strain tensor during the application of the deviatoric stress can be obtained by

$$\begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} K_{11} & K_{12} & K_{13} & 0 & 0 & K_{16} \\ K_{12} & K_{22} & K_{23} & 0 & 0 & K_{26} \\ K_{13} & K_{23} & K_{33} & 0 & 0 & K_{36} \\ 0 & 0 & 0 & K_{44} & K_{45} & 0 \\ 0 & 0 & 0 & K_{45} & K_{55} & 0 \\ K_{16} & K_{26} & K_{36} & 0 & 0 & K_{66} \end{bmatrix} \begin{bmatrix} 0 \\ S_1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \tag{10}$$

Thus, the axial strain is

$$\varepsilon_{yy} = K_{22} S_1 \tag{11}$$

where

$$K_{22} = \frac{1}{E_y} = \frac{\cos^4\beta}{E} + \frac{\sin^4\beta}{E'} + \cos^2\beta\sin^2\beta\left(\frac{1}{G'} - \frac{2\nu'}{E'}\right) \tag{12}$$

3.4. Maximum axial strain criterion

The failure criteria for anisotropic materials may be categorized into three basic types: (1) stress dominated; (2) strain dominated (3) interactive [32]. The ductile deformation due to the axial strain accumulation is referred to herein as the strain-dominated criterion. To account for also the non-sliding mode in the proposed criterion, it is assumed that the failure occurs when the axial strain exceeds its maximum limiting value, ε_{yf} under a specific confining pressure. This failure criterion is referred to herein as the maximum axial strain criterion.

With the maximum axial strain criterion,

$$S_{1(\beta)} = E_y \varepsilon_{yf} \tag{13a}$$

$$S_{1(90^\circ)} = E_{(90^\circ)} \varepsilon_{yf} \tag{13b}$$

The value of axial strain at failure ε_{yf} is varied with different confining pressures and independent of orientation angle. This paper adopted Hooke's law to calculate the axial strains and the strength ratio of specimens with various orientation angles under a specified confining pressure. According to the coordinate system of Figs. 5 and 6,

$$E = E_{(0^\circ)} \tag{14a}$$

$$E' = E_{(90^\circ)} \tag{14b}$$

From Eqs. (12), (13a), and (13b),

$$\frac{S_{1(\beta)}}{S_{1(90^\circ)}} = \frac{1}{\left[\left(\cos^4\beta/E_{(0^\circ)}\right) + \left(\sin^4\beta/E_{(90^\circ)}\right) + \cos^2\beta\sin^2\beta\left(1/G' - 2\nu'/E_{(90^\circ)}\right)\right]E_{(90^\circ)}} \tag{15}$$

By introducing the strength ratio, k [17] and the transversal anisotropy parameter, n:

$$k = E_{(0^\circ)}/E_{(90^\circ)} = S_{1(0^\circ)}/S_{1(90^\circ)}, \tag{16}$$

$$n = \left(E_{(90^\circ)}/2G'\right) - \nu' \tag{17}$$

Eq. (15) becomes

$$\frac{S_{1(\beta)}}{S_{1(90^\circ)}} = \frac{\sigma_{1(\beta)} - \sigma_3}{\sigma_{1(90^\circ)} - \sigma_3} = \frac{k}{\cos^4\beta + k\sin^4\beta + 2n\sin^2\beta\cos^2\beta} \tag{18}$$

Eq. (18) represents the failure condition of the transversely isotropic rocks for the non-sliding mode. In this paper, it is assumed that the strength ratio is equal to the ratio of apparent Young's modulus at $\beta = 0^\circ$ over that at 90°. Thus, the variation of strength of the transversely isotropic rock with the orientation angle follows the same transformation rules of compliance. The use of the strength ratio to account for the variation of strength with respect to the orientation angle is a more direct approach. Besides, it is simpler and more precise to determine the strength of rock than to determine the elastic modulus and the value of axial strain at failure ε_{yf}.

Jaeger showed that Eq. (3) is the state of stress at failure for the sliding mode. This paper adopted Eqs. (3) and (18), which accounts for sliding mode and non-sliding mode, respectively, to describe the compressive strength for the transversely isotropic rocks.

The strength ratio k and transversal anisotropy parameter n, reflect the degree of anisotropy of the rock material. A special case of Eq. (18) is obtained by considering a case where $n = 1$:

$$\begin{aligned}\frac{S_{1(\beta)}}{S_{1(90^\circ)}} &= \frac{k}{\cos^4\beta + 2\sin^2\beta\cos^2\beta + k\sin^4\beta} \\ &= \frac{k}{(\cos^4\beta + \sin^2\beta\cos^2\beta + \sin^4\beta) + (k-1)\sin^4\beta} \\ &= \frac{k}{(\cos^2\beta + 2\sin^2\beta)^2 + (k-1)\sin^4\beta} \\ &= \frac{k}{1 + (k-1)\sin^4\beta} \end{aligned} \tag{19}$$

In the above special case, the term $(k-1)\sin^4\beta \rightarrow 0$, as β approaches zero. Thus, Eq. (19) can be further simplified:

$$S_{1(\beta)} = kS_{1(90^\circ)} = S_{1(0^\circ)}, \Rightarrow \sigma_{1(\beta)} \approx \sigma_{1(0^\circ)}. \tag{20}$$

On the other hand, as β approaches 90°, Eq. (19) becomes

$$\frac{S_{1(\beta)}}{S_{1(90^\circ)}} \approx \frac{k}{1 + (k-1)} = 1 \tag{21}$$

As can be seen from Eqs. (20) and (21), in the special case of $n = 1$, the compression strength $\sigma_{1(\beta)}$ varies little with the change in the orientation angle β if the angle is near 0° or 90°. As illustrated in Fig. 7, which is based on Eq. (19), there is practically no variation in the strength, when β is near 0° or 90°. Thus, when $n = 1$, the proposed model (Eq. (18)) yields approximately the same results as that obtained by the extended Jaeger's criterion.

In another special case, where $n = 1$ and $k = 1$, Eq. (18) becomes

$$S_{1(\beta)} = \frac{S_{1(90^\circ)}}{(\cos^2\beta + \sin^2\beta)^2} = S_{1(90^\circ)} = S_{1(0^\circ)} \tag{22}$$

Eq. (22) shows that the compressive strength is independent of the orientation angle β. Thus, the proposed model is reduced to the original Jaeger's criterion, as $n = 1$ and $k = 1$. It should be noted that k can be calculated by Eqs. (4), (5) and (16) and not a basic material parameter in the proposed criterion. The special case of $k = 1$ implies the same strength at $\beta =$ 0° and 90° (i.e., $m_{(0^\circ)} = m_{(90^\circ)}$, $\sigma_{c(0^\circ)} = \sigma_{c(90^\circ)}$.

4. Determination of the material parameters

The proposed criterion is based on two distinct failure modes, and thus, the model parameters of the proposed criterion can be categorized into two groups:

(1) Strength parameters of discontinuity (c_w and ϕ_w), related to the sliding failure mode,

(2) Strength parameters of rock material ($m_{(0^\circ)}$, $\sigma_{c(0^\circ)}$, $m_{(90^\circ)}$, $\sigma_{c(90^\circ)}$, n), related to the non-sliding failure mode.

The proposed criterion is a seven-parameter model. Therefore, seven experimental data points are required in order to determine these parameters. The material parameters of proposed model can be obtained by conducting triaxial tests for at least four orientation angles, say $\beta = 0°, 30°, 60°$, and 90°. In this section, the procedures for determining the material parameters that are required in the proposed model are presented.

4.1. Determination of c_w and ϕ_w

The parameters, c_w and ϕ_w, are the cohesion and friction of the discontinuity, respectively. As per Eq. (3a), $\sigma_{1(\beta)}$ is a function of β. By setting the derivative of $\sigma_{1(\beta)}$ with respect to β equal to 0, the orientation angle at which the minimum strength occurs is obtained

$$\tan\phi_w = \cot 2\beta_{\min} \tag{23}$$

$$\beta_{\min} = \frac{\pi}{4} - \frac{\phi_w}{2} \tag{24}$$

In principle, ϕ_w can be obtained from Eq. (24) if the orientation angle that corresponds to a minimum strength is determined. One possible approach to

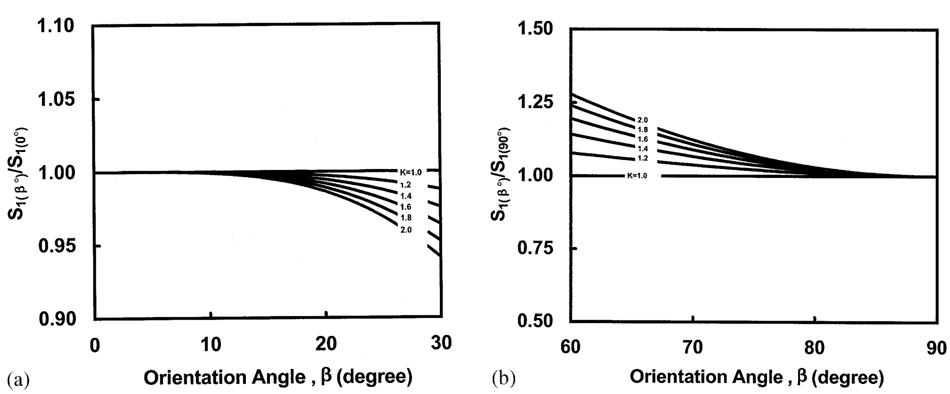

Fig. 7. Strength variation versus β for the case transversal anisotropy parameter, $n = 1$ (a) $0 \leq \beta \leq 30°$ (b) $60° \leq \beta \leq 90°$.

determining the shear strength parameters of discontinuity (c_w, ϕ_w) is conducting triaxial tests on specimens with $\beta = 20-50^\circ$, in which the sliding mode is expected. By transforming the stress state into the normal stress (σ_n) and the shear stress (τ) on the discontinuity using Eqs. (1) and (2), and then plotting of σ_n versus τ at failures, the parameters c_w (intercept) and ϕ_w (slope) can be determined.

It should be noted that the failure of specimens in the triaxial tests for determining c_w and ϕ_w must be of "sliding mode". As evidenced from the experiments and the prediction by the proposed criterion (see Sections 2 and 5), the sliding mode may be suppressed under high confining pressures (e.g. Fig. 3(b); Fig. 16), even for the specimens with $\beta = 30^\circ$ or 45°. Thus, any triaxial test for determining c_w and ϕ in which the specimen does not fail in the sliding mode should be discarded.

4.2. Determination of $m_{(0^\circ)}$, $\sigma_{c(0^\circ)}$, $m_{(90^\circ)}$, $\sigma_{c(90^\circ)}$

The four Hoek–Brown parameters of rock material ($m_{(0^\circ)}$, $\sigma_{c(0^\circ)}$, $m_{(90^\circ)}$, $\sigma_{c(90^\circ)}$) should be determined from triaxial tests on the rock specimens that are prepared with $\beta = 0^\circ$ and 90°, respectively. Hoek–Brown criterion is a two-parameter model. Thus, each set of triaxial tests should be conducted under at least two confining pressures. Hoek and Brown [33] provided guidance for selecting confining pressures and procedures for determining these material parameters. The test should be carried out over a range of confining pressures from zero to $0.25\sigma_{c(0^\circ)}$ (or $0.25\sigma_{c(90^\circ)}$), with eight equally spaced value of confining pressure [33].

The strength ratio, k under a specified confining pressure, can be expressed in terms of Hoek–Brown parameters ($m_{(0^\circ)}$, $\sigma_{c(0^\circ)}$, $m_{(90^\circ)}$ and $\sigma_{c(90^\circ)}$) according to Eqs. (4), (5), and (16). It should be noted that the value of strength ratio, k may vary slightly with confining pressure, thus k is not a basic material parameter in the proposed criterion.

4.3. Determination of transversal anisotropy parameter n

The transversal anisotropy parameter n is the unique new parameter introduced in the proposed criterion. It plays a critical role to describe the strength variation when the sliding failure cannot occur (usually in the range of $\beta = 0-10^\circ$ and $\beta = 60-90^\circ$). The transversal anisotropy parameter n can be determined by performing triaxial tests at $\beta = 60^\circ$ (or $\beta = 75^\circ$ alternatively) and 90°. Table 1a and b list the strength ratio at $\beta = 60^\circ$ ($S_{1(60^\circ)}/S_{1(90^\circ)}$) and $\beta = 75^\circ$ ($S_{1(75^\circ)}/S_{1(90^\circ)}$) calculated by Eq. (18). The values along diagonal lines in Table 1a and b are more or less constant. It can be explained in the following. The term $\cos^4\beta$ in Eq. (18) becomes negligible if β is in the range of 60–90° (for example, $\cos^4 60^\circ = 0.0625$, $\cos^4 75^\circ = 0.0045$). Thus, Eq. (18)

Table 1
Strength ratio caculated from Eq. (18)

	n				
k	1	$\sqrt{2}$	2	$2\sqrt{2}$	4
(a) Strength ratio at $\beta = 60^\circ$ ($S_{1(60^\circ)}/S_{1(90^\circ)}$) calculated from Eq. (18)					
1/2	0.696	0.572	0.457	0.356	0.271
$\sqrt{2}/2$	0.847	0.714	0.584	0.465	0.361
1	1.000	0.866	0.727	0.593	0.471
$\sqrt{2}$	1.147	1.019	0.879	0.737	0.602
2	1.280	1.164	1.032	0.890	0.744
(b) Strength ratio at $\beta = 75^\circ$ ($S_{1(75^\circ)}/S_{1(90^\circ)}$) calculated from Eq. (18)					
1/2	0.885	0.811	0.725	0.630	0.532
$\sqrt{2}/2$	0.949	0.887	0.813	0.726	0.631
1	1.000	0.951	0.889	0.814	0.727
$\sqrt{2}$	1.039	1.001	0.951	0.890	0.815
2	1.069	1.040	1.002	0.953	0.891

becomes approximately

$$\frac{S_{1(\beta)}}{S_{1(90^\circ)}} \approx \frac{k}{k\sin^4\beta + 2n\sin^2\beta\cos^2\beta} = \frac{1}{\sin^2\beta\left(\sin^2\beta + (2n/k)\cos^2\beta\right)} \tag{25}$$

In Eq. (25), the strength ratio $S_{1(\beta)}/S_{1(90^\circ)}$ is a function of n/k. It is, however, independent of k. Rewrite Eq. (18) in terms of n/k and let $k = 1$, the strength ratio becomes

$$\begin{aligned}\frac{S_{1(\beta)}}{S_{1(90^\circ)}} &= \frac{1}{\cos^4\beta + \sin^4\beta + 2(n/k)\sin^2\beta\cos^2\beta} \\ &= \frac{1}{\left(\cos^2\beta + \sin^2\beta\right)^2 + 2\left[(n/k) - 1\right]\sin^2\beta\cos^2\beta} \\ &= \frac{1}{1 + 2\left[(n/k) - 1\right]\sin^2\beta\cos^2\beta}\end{aligned} \tag{26}$$

For $\beta = 60^\circ$, Eq. (26) becomes

$$\frac{S_{1(60^\circ)}}{S_{1(90^\circ)}} = \frac{1}{1 + 0.375\left[(n/k) - 1\right]} \tag{27a}$$

For $\beta = 75^\circ$, Eq. (26) becomes

$$\frac{S_{1(75^\circ)}}{S_{1(90^\circ)}} = \frac{1}{1 + 0.125\left[(n/k) - 1\right]} \tag{27b}$$

Fig. 8 shows a plot of $S_{1(60^\circ)}/S_{1(90^\circ)}$ and $S_{1(75^\circ)}/S_{1(90^\circ)}$ versus the ratio n/k. This plot is based on Eq. (18) with $k = 1$, which is the same as Eqs. (27a), (27b). While not shown in this figure, the difference in the obtained curves using different k values is negligible, as implied by Eq. (25). Thus, the relationship between the strength ratio $S_{1(60^\circ)}/S_{1(90^\circ)}$ and $S_{1(75^\circ)}/S_{1(90^\circ)}$ and the parameter ratio n/k shown in Fig. 8 is valid for different k values.

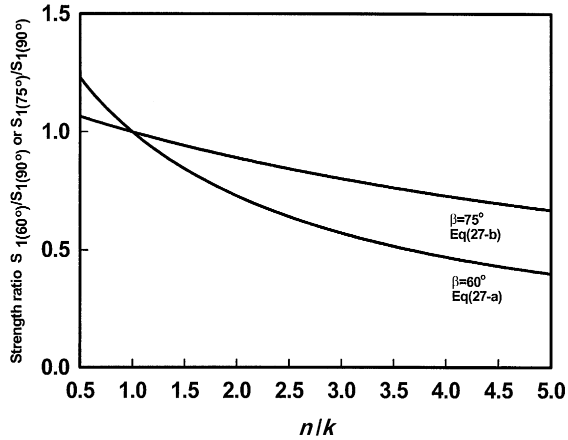

Fig. 8. Chart for determination of parameter n.

Table 2
Major principal stress $\sigma_{1(\beta)}$ at failure of Martinsburg slate tested by Donath [4] (unit: MPa)

	β						
σ_3 (MPa)	0°	15°	30°	45°	60°	75°	90°
3.5	128	51	22	43	75	128	194
10.5	162	82	44	63	102	160	241
35	272	134	87	107	150	216	335
50	355	172	129	150	194	284	410
100	530	286	230	260	315	456	600

Given the strength ratio $S_{1(60^\circ)}/S_{1(90^\circ)}$ (or $S_{1(75^\circ)}/S_{1(90^\circ)}$ alternatively), which may be obtained from triaxial tests at orientation angle $\beta = 60^\circ$ (or 75° alternatively) and 90°, the transversal anisotropy parameter n can be determined from Fig. 8 or Eqs. (27a), (27b). The procedure for determining the transversal anisotropy parameter n is described in the next section.

4.4. Complete example for determination of material parameters

Using the results of triaxial tests on Martingsburg slate performed by Donath [4], shown in Table 2, the above procedures for determining material parameters for the proposed failure criterion are illustrated below.

Step 1: Determine c_w and ϕ_w

The shear strength parameters for the discontinuity, c_w and ϕ_w, may be determined from a set of triaxial test in which sliding failure controls. Generally, the sliding mode occurs in the samples with $\beta = 20-50^\circ$. The best orientation angle to create a sliding mode is $\beta_{\min}$ defined in Eq. (24). The typical values of ϕ_w are in the range of 15–40°. Thus, the specimens with orientation angle $\beta = 15-45^\circ$ are suitable for triaxial tests to determine c_w and ϕ_w.

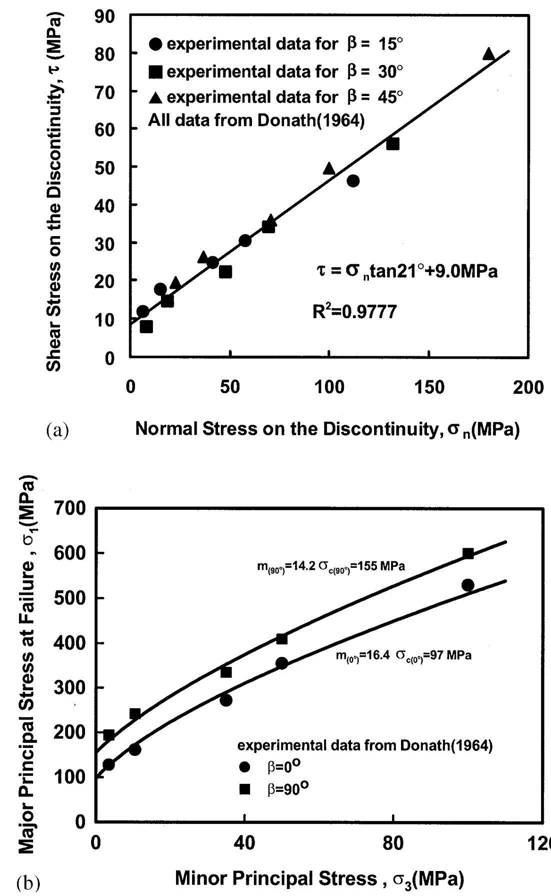

Fig. 9. Determination of material parameters of proposed failure criterion for Martinsburg slate test by Donath [4]. (a) strength parameters of discontinuity, (b) strength parameters of rock materials at $\beta = 0^\circ$ and 90°.

Using the triaxial test result of samples with $\beta = 15^\circ$, 30° and 45° and Eqs. (1) and (2), the shear stress and normal stress on the discontinuity can be obtained. At failure, the shear stress and normal stress on the discontinuity of Martinsburg slate are shown in Fig. 9a. The shear strength parameters for the discontinuity for Martinsburg slate are then obtained: $c_w = 9.0$ MPa and $\phi_w = 21^\circ$.

Step 2: Determine $m_{(0^\circ)}$, $\sigma_{c(0^\circ)}$, $m_{(90^\circ)}$, $\sigma_{c(90^\circ)}$

Fig. 9b shows the triaxial test data for sample with $\beta = 0^\circ$ and 90° from Table 2. The strength parameters of Hoek–Brown's criterion for Martinsburg slate with $\beta = 0^\circ$ and 90° are

$$m_{(0^\circ)} = 16.4, \quad \sigma_{c(0^\circ)} = 97\,\text{MPa}, \tag{28}$$

$$m_{(90^\circ)} = 14.2, \quad \sigma_{c(90^\circ)} = 155\,\text{MPa}. \tag{29}$$

The k value under a specified confining pressure can be determined from the four Hoek–Brown parameters. For example, The k values of Martinsburg slate under varied confining pressure are shown in Table 3. The k value varies slightly with confining pressures and is not a basic material parameter of proposed criterion.

Table 3
Transversal anisotropy parameter n of Martinsburg slate under various confining pressures

Parameters	σ_3= 3.5 MPa	σ_3= 10.5 MPa	σ_3= 35 MPa	σ_3= 50 MPa	σ_3= 100 MPa
Calculated $S_{1(0°)}$, MPa Eq. (4b)	122	162	255	298	410
Calculated $S_{1(90°)}$, MPa Eq. (5b)	178	217	318	366	494
$k = S_{1(0°)}/S_{1(90°)}$ Eq. (16)	0.69	0.74	0.80	0.81	0.83
Measured $S_{1(60°)} = \sigma_{1(60°)} - \sigma_3$, MPa Table 2	71.5	91.5	115	144	215
$S_{1(60°)}/S_{1(90°)}$ Measured $S_{1(60°)} = \sigma_{1(60°)} - \sigma_3$	0.40	0.42	0.36	0.39	0.43
n/k	5.0	4.7	5.7	5.1	4.5
n	3.5	3.5	4.6	4.1	3.7
Average n			3.9		

Table 4
Recommendation of orientation angles of specimens and number of tests for determining material parameters for the proposed criterion

β (deg)	No. of σ_3	Related material parameters
30	2	c_w, ϕ_w
0	2	$m_{(0°)}, \sigma_{c(0°)}$
90	2	$m_{(90°)}, \sigma_{c(90°)}$
60	1	n

Step 3: Determine the transversal anisotropy parameter n

The transversal anisotropy parameter n can be obtain from the strength ratio $S_{1(60°)}/S_{1(90°)}$ under a specified confining pressure. An example illustrating the procedures to determine the parameter n for Martinsburg slate based on experimental data from Donath [4] is listed in Table 2. First, substitute the Hoek–Brown's strength parameters in Eqs. (28) and (29) into Eqs. (4b) and (5b) to obtain $S_{1(0°)}$, $S_{1(90°)}$, and $S_{1(0°)}/S_{1(90°)}$. The experimental data of $S_{1(60°)}$ under varied confining pressure are taken from Table 2. The transversal anisotropy parameter n can then be determined by the strength ratio $S_{1(60°)}/S_{1(90°)}$ and Eq. (27a) or Fig. 8.

Table 3 shows that the obtained transversal anisotropy parameter n under different confining pressures. The obtained parameter n is approximately a constant, with an average of 3.9 in this case. Thus, the parameter n may be considered as a material parameter independent of confining pressure. To determine the transversal anisotropy parameters n, only one triaxial test of rock specimen with $\beta = 60°$ under a convenient confining pressure is required, although more tests to confirm the result is desired.

In summary, there are seven material parameters in the proposed criterion. These parameters can be determined by performing triaxial tests on a minimum of seven specimens at varied combinations of orientation angles and confining pressures, as shown in Table 4, although more tests to confirm is highly recommended.

5. Evaluation of the proposed criterion

To evaluate the capabilities of the proposed criterion, comparisons are made between experimental data taken from the literatures and the predictions obtained from the proposed criterion based on Eqs. (18) and (3).

The proposed failure criterion is examined by comparing model predictions with experimental data from the literature. Figs. 10–18 show these comparisons. The material parameters for each transversely isotropic rock, natural or artificial, are listed in Table 5. In each figure, the solid lines correspond to the predictions obtained from the proposed failure criterion, while data points represent experimental results. These rocks include slates, shales, limestone, and artificial layered rocks and exhibited three types of anisotropies, namely U type, shoulder type and undulatory type as defined by

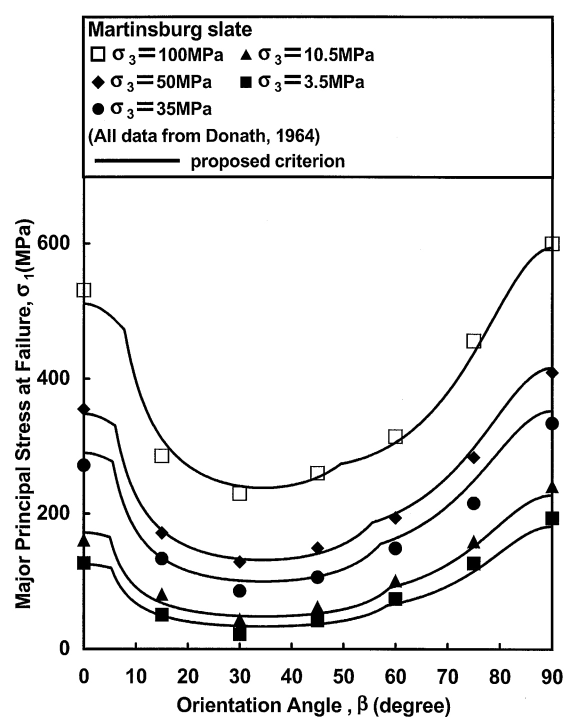

Fig. 10. Comparison of experimental data (after Donath [4]) and predicted failure strength of Martinsburg slate.

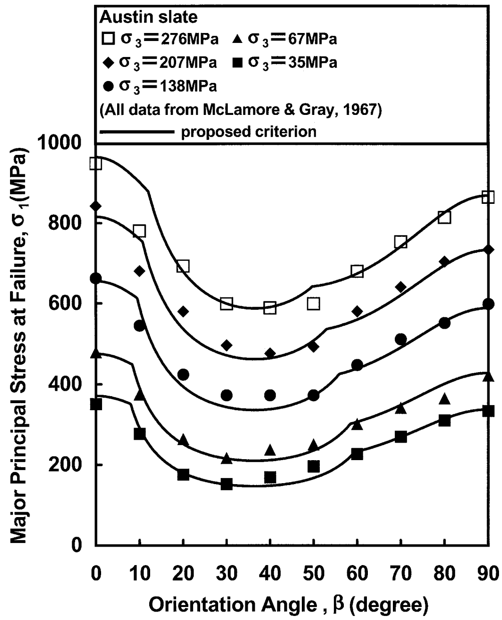

Fig. 11. Comparison of experimental data (after McLamore and Gray [7]) and predicted failure strength of Austin slate.

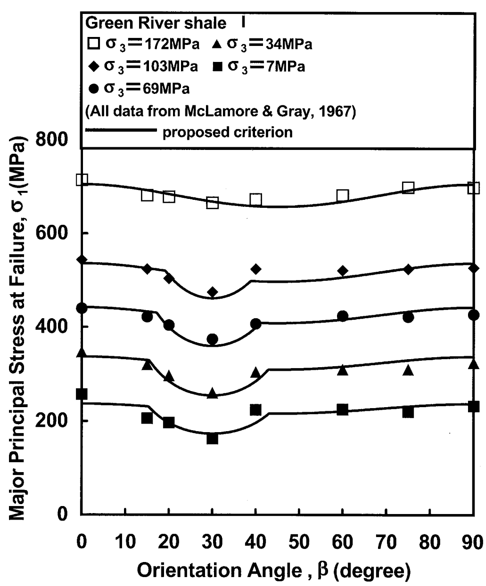

Fig. 12. Comparison of experimental data (after McLamore and Gray [7]) and predicted failure strength of Green River shale I.

Ramamurthy [15]. The proposed criterion is shown to be applicable to all types of anisotropies.

Duveau and Shao [20] adopted a nonlinear model (proposed by Barton for rock joint) to modify the Mohr–Coulomb criterion for discontinuity. Generally, a nonlinear model provides flexibility for describing the shear strength along discontinuity. However, it requires more parameters than does the linear Mohr–Coulomb criterion. In the present study, the linear Mohr–Coulomb criterion is adopted for its good balance of model simplicity with the accuracy.

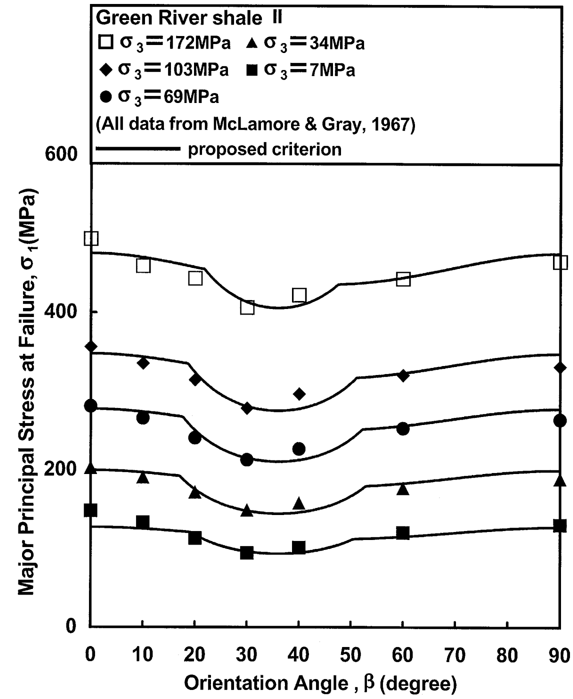

Fig. 13. Comparison of experimental data (after McLamore and Gray [7]) and predicted failure strength of Green River shale II.

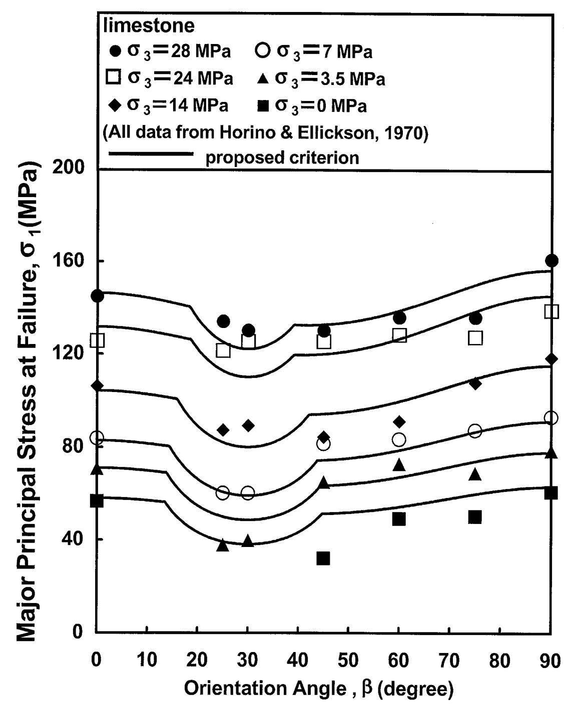

Fig. 14. Comparison of experimental data (after Horino and Ellickson [8]) and predicted failure strength of limestone.

The transversal anisotropy parameter n reflects the strength variation for the region where non-sliding failure occur. The value of n varies from 1.0 to 4.0 for most of the transversely isotropic rocks. When the value

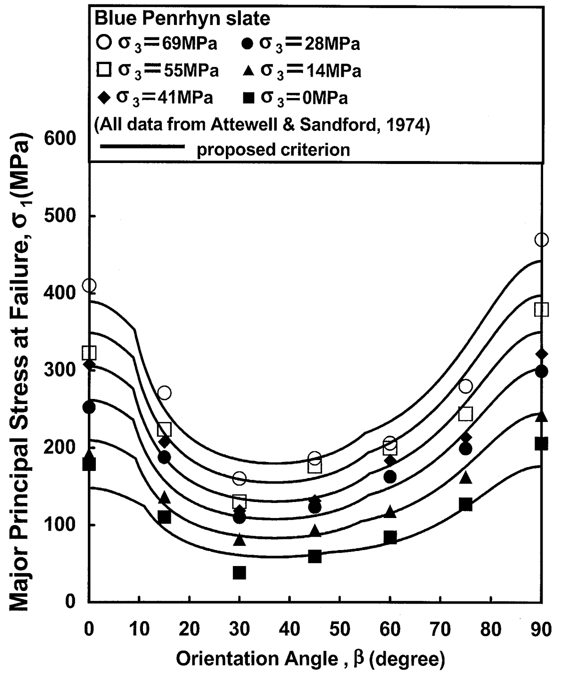

Fig. 15. Comparison of experimental data (after Attewell and Sandford [9]) and predicted failure strength of Blue Penrhyn slate.

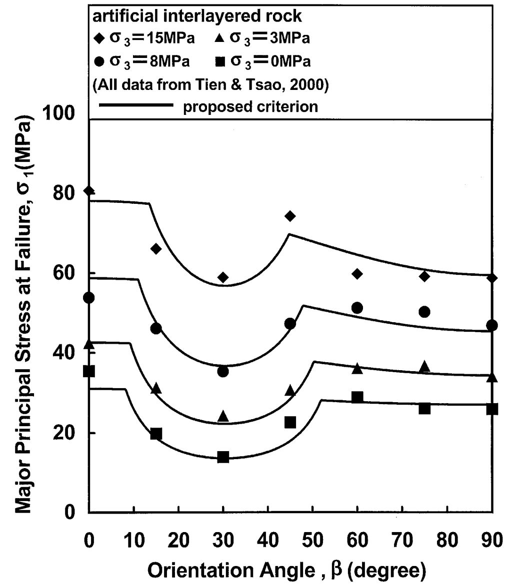

Fig. 17. Comparison of experimental data (after Tien and Tsao [14]) and predicted failure strength of artificial interlayered rock.

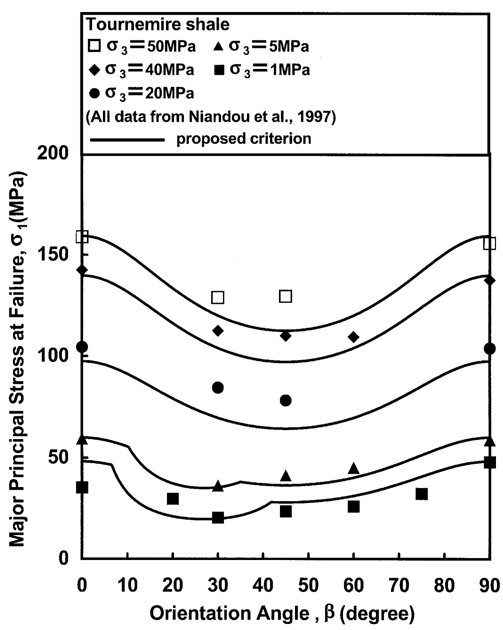

Fig. 16. Comparison of experimental data (after Niandou et al. [11]) and predicted failure strength of Tournemire shale.

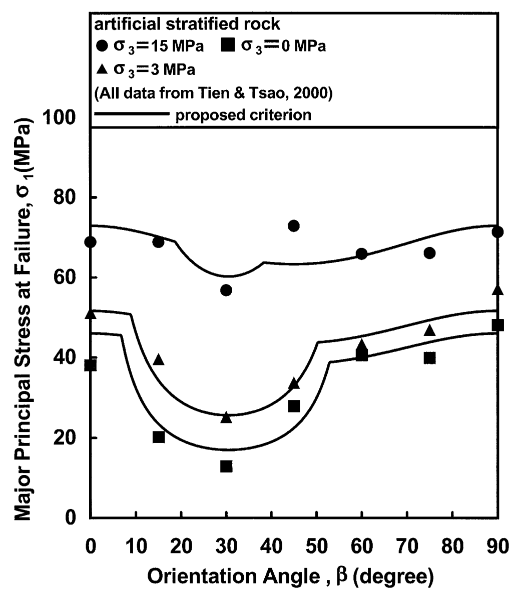

Fig. 18. Comparison of experimental data (after Tien and Tsao [14]) and predicted failure strength of artificial stratified rock.

of n is in the range of 1.0–2.0 (for example, Figs. 12–14, 17 and 18), the strength is roughly constant in the neighborhood of $\beta = 0^\circ$ or 90°. Those rocks may be classified into shoulder type anisotropy. As value of n increases, the region of "shoulder" disappear gradually, the strength variation around the neighborhood of $\beta = 0^\circ$ or 90° is more significant as shown in Figs. 10 and 15. The type of anisotropy for such rocks may be referred to as U type or undulatory type.

The failure criteria for anisotropic rocks can be categorized into two groups: (1) discontinuous models (e.g. Jaeger's criterion and extended Jaeger's criterion) and (2) continuous models (e.g. Pariseau's criterion, Cazacu et al. criterion), depending upon the continuous and discontinuous characteristics of the corresponding anisotropy [2]. Compared to the experimental observations, the discontinuous models predict relatively well the strength behavior of a rock cut by joint. However, the continuous models are more suitable for the

Table 5
Material parameters of the proposed criterion for various rocks

Rock	c_w (MPa)	ϕ_w (°)	$m_{(0°)}$	$\sigma_{c(0°)}$ (MPa)	$m_{(90°)}$	$\sigma_{c(90°)}$ (MPa)	n	Data source
Martinsburg slate	9	21	16.4	97	14.2	155	3.9	Donath [4]
Austin slate	31	17	6.0	249	4.6	234	2.7	McLamore and Gray [7]
Green River shale I	49	28	6.7	208	6.7	208	1.2	McLamore and Gray [7]
Green River shale II	29	18	4.4	106	4.4	106	1.3	McLamore and Gray [7]
Limestone	11	30	5.9	58	7.1	63	1.3	Horino and Ellickson [8]
Blue Penrhyn slate	22	16	7.9	148	8.9	177	3.7	Attewell and Sandford [9]
Tournemire shale	4	36	4.4	45	4.4	45	2.5	Niandou et al. [11]
Artificial interlayered rock	4	29	6.5	31	3.1	27	1.1	Tien and Tsao [14]
Artificial stratified rock	5	29	1.8	46	1.8	46	1.4	Tien and Tsao [14]

continuous rocks. The discontinuous models divide failure modes into the sliding and non-sliding modes. The sharp corner exists in the plot of failure stress as a function of orientation angle implies the transition point of two distinct failure modes. On the other hand, the continuous models treat the transversely isotropic rock as a continuous medium, ignoring the existence of the sharp corner and evading the failure mode problem.

Because the relationship between the failure stresses and the orientation angle of the transversely isotropic rocks obtained from the experiment is discrete, it is generally difficult, by the experimental approach *alone*, to identify whether the sharp corner exists or not. It is more meaningful to discuss this issue from both experimental and theoretical approaches, and by considering both strength variation and failure mode simultaneously. Whether a sharp corner exists depends on the rock type and the confining pressure. For an anisotropic rock that can be treated as a continuous medium at the sample scale, a continuous variation of strength with the orientation angle is expected. For the continuous rock or the discontinuous rock under high confining pressure, the effect of discontinuity is fully suppressed; the sharp corner is not significant. The phenomenon of suppression of discontinuity effect (or anisotropy) as the confining pressure increases has been identified by the experimental evidence [15,18]. Such phenomenon can also be accounted for by the newly developed criterion presented in this paper. The proposed criterion is a discontinuous model at lower confining pressure, and as confining pressure increases, it is gradually transformed into a continuous model. For example, as shown in Fig. 16, when $\sigma_3 > 20$ MPa, the proposed criterion (Eq. (18)) becomes a continuous model.

From the results shown in Figs. 10–18, the proposed failure criterion is shown to be able to accurately predict the compression strength of transversely isotropic rocks of various types, prepared at different orientation angels and under various confining pressures.

As a final note, the proposed criterion is a hybrid of the two well-known criteria in the field of rock mechanics, the Hoek–Brown and the Mohr–Coulomb criteria. Both the Hoek–Brown and the Mohr–Coulomb formulations are expressed in terms of major and minor principal stresses, neglecting the effect of the intermediate principal stress. Thus, the proposed criterion inherits this limitation. Further research to improve the proposed criterion considering three-dimensional stress conditions is worth undertaking.

6. Conclusions

1. A new failure criterion for the transversely isotropic rocks has been developed and presented. The new criterion is based on two distinct failure modes; one is the sliding mode where the failure is caused by sliding along the discontinuity, and the other is the non-sliding mode where the failure is controlled by the rock material and is not dependent on discontinuity.
2. The newly developed failure criterion consists of seven material parameters. They are the cohesion and the friction angle of the discontinuity (c_w, ϕ_w), Hoek–Brown's parameters ($\sigma_{c(0°)}$, $\sigma_{c(90°)}$, $m_{(0°)}$, $m_{(90°)}$) and the transversal anisotropy parameter (n). The physical meanings of, and the procedures for determining, these parameters are described.
3. When $n = 1$, the proposed failure criterion is very similar to the extended Jaeger's criterion. With additional condition that $k = 1$, which implies that $m_{(0°)} = m_{(90°)}$, $\sigma_{c(0°)} = \sigma_{c(90°)}$, the proposed criterion becomes the original Jaeger's criterion.
4. The predictions of the strength behaviors of various types of the transversely isotropic rocks with different orientation angels and under various confining pressures agree well with experimental data from various investigators. The accuracy and the versatility of the proposed failure criterion are demonstrated.

Acknowledgements

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