Laminate Strength and Failure Tsai-Wu Criterion

MER452: Composite Materials

Typical Failure Stresses (Mpa)

	Graphite Reinforced	Glass Reinforced
σ_1^c	-1250	-600
σ_1^t	1500	1000
σ_2^c	-200	-120
σ_2^t	50	30
τ_{12}^f	100	70

Failure Criteria

- von Mises Criteria – Isotropic Materials

$$\frac{1}{2}\cdot\left(\frac{1}{\sigma^{Y}}\right)^{2}\cdot\left[\left(\sigma_1-\sigma_2\right)^2+\left(\sigma_1-\sigma_3\right)^2+\left(\sigma_2-\sigma_3\right)^2\right]=1$$

$$F(\sigma_1,\sigma_2,\sigma_3)=1$$

$$F(\sigma_1,\sigma_2,\sigma_3)<1$$

- Von Mises Criteria – Orthotropic Mateials
  - Proposed by Hill

$$F\cdot\left(\sigma_1-\sigma_2\right)^2+G\cdot\left(\sigma_1-\sigma_3\right)^2+H\cdot\left(\sigma_2-\sigma_3\right)^2+2\cdot L\cdot\tau_{12}^2+2\cdot M\cdot\tau_{13}^2+2\cdot N\cdot\tau_{23}^2=1$$

$$F(\sigma_1,\sigma_2,\sigma_3,\tau_{23},\tau_{13},\tau_{12})=1$$

$$F(\sigma_1,\sigma_2,\sigma_3,\tau_{23},\tau_{13},\tau_{12})<1$$

Failure Criteria: Tsai-Wu

- Composite Materials – Plane Stress

$$F(\sigma_1,\sigma_2,\tau_{12}) = F_1\cdot\sigma_1 + F_2\cdot\sigma_2 + F_6\cdot\tau_{12} + F_{11}\cdot\sigma_1^2 + F_{22}\cdot\sigma_2^2 + F_{66}\cdot\tau_{12}^2 + 2\cdot F_{12}\cdot\sigma_1\cdot\sigma_2 + 2\cdot F_{16}\cdot\sigma_1\cdot\tau_{12} + 2\cdot F_{26}\cdot\sigma_2\cdot\tau_{12}$$

$$F(\sigma_1,\sigma_2,\tau_{12}) = 1$$

$$F(\sigma_1,\sigma_2,\tau_{12}) < 1$$

Constant Determination F_1, F_{11}

$$F_1 \cdot \sigma_1^T + F_{11} \cdot \left(\sigma_1^T\right)^2 = 1$$

$$F_1 \cdot \sigma_1^C + F_{11} \cdot \left(\sigma_1^C\right)^2 = 1$$

$$F_1 = \frac{1}{\sigma_1^T} + \frac{1}{\sigma_1^C}$$

$$F_1 = -\frac{1}{\sigma_1^T \cdot \sigma_1^C}$$

Constant Determination F_2, F_{22}

$$F_2 \cdot \sigma_2^T + F_{22} \cdot \left(\sigma_2^T\right)^2 = 1$$

$$F_2 \cdot \sigma_2^C + F_{22} \cdot \left(\sigma_2^C\right)^2 = 1$$

$$F_2 = \frac{1}{\sigma_2^T} + \frac{1}{\sigma_2^C}$$

$$F_2 = -\frac{1}{\sigma_2^T \cdot \sigma_2^C}$$

Constant Determination F_6, F_{66}

$$F_6 \cdot \tau_{12}^T + F_{66} \cdot \left(\tau_{12}^T\right)^2 = 1$$

$$F_6 \cdot \tau_{12}^C + F_{66} \cdot \left(\tau_{12}^C\right)^2 = 1$$

$$F_6 = 0$$

$$F_{66} = \left(\frac{1}{\tau_{12}^F}\right)^2$$

Constant Determination F_{16}, F_{26}

$$F_1 \cdot \sigma_1^* + F_{11} \cdot \left(\sigma_1^*\right)^2 + F_{66} \cdot \left(\tau_{12}^*\right)^2 + 2 \cdot F_{16} \cdot \sigma_1^* \cdot \tau_{12}^* = 1$$

$$F_1 \cdot \sigma_1^* + F_{11} \cdot \left(\sigma_1^*\right)^2 + F_{66} \cdot \left(\tau_{12}^*\right)^2 - 2 \cdot F_{16} \cdot \sigma_1^* \cdot \tau_{12}^* = 1$$

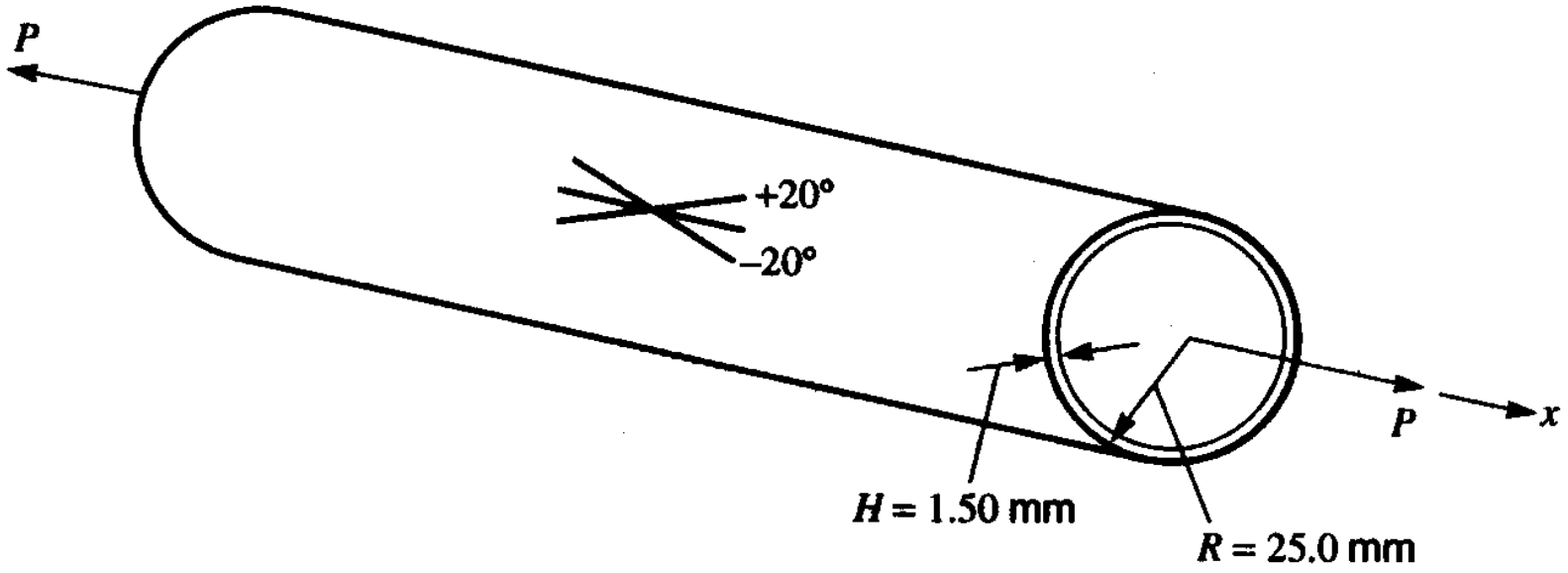
$$F_{16} = 0$$

$$F_{26} = 0$$

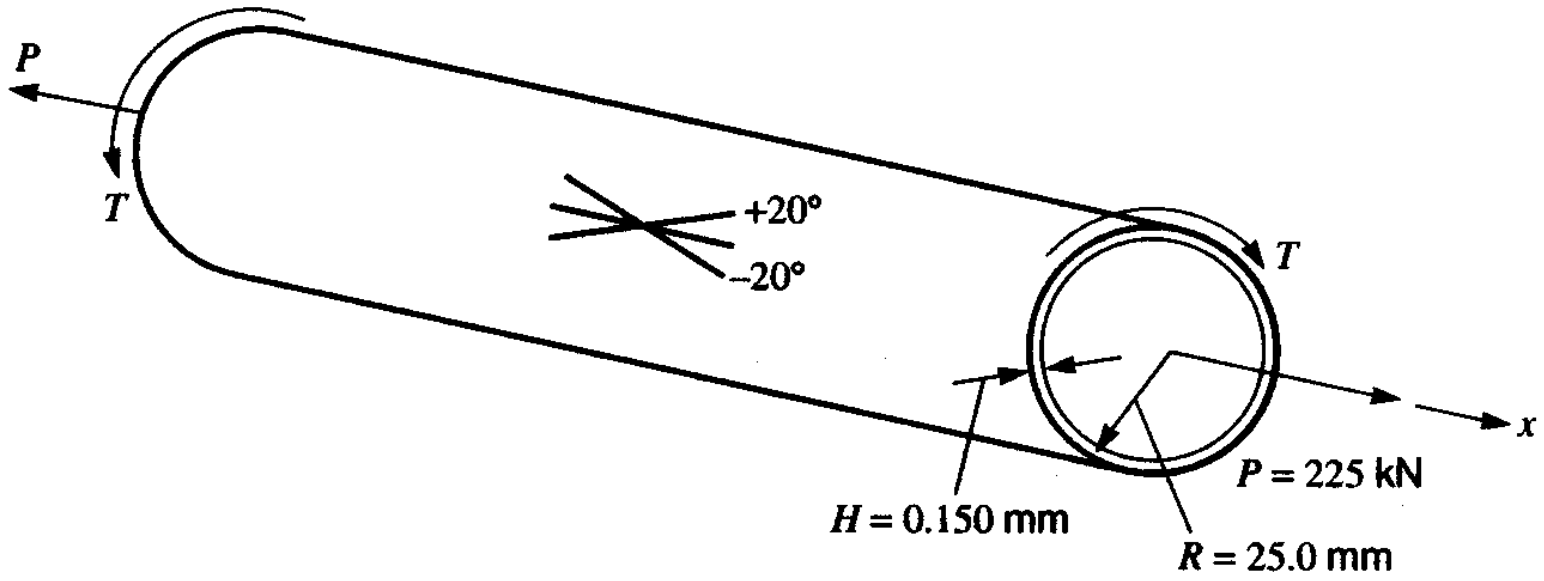
Constant Determination F_{12}

$$F_{12} = -\frac{1}{2} \cdot \sqrt{F_{11} \cdot F_{11}}$$

Example $[\pm 20/0_3]_s$



Example $[\pm 20/0_3]_s$



Example $[\pm 20/0_3]_s$