Invariant formulation of an elastic/viscoplastic constitutive equation for anisotropic rock

Formulation invariante d'un modèle de comportement élasto-viscoplastique d'une roche anisotrope

Invariante Formulierung eines elastisch/viskoplastischen Modells für anisotropes Gestein

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ABSTRACT: An invariant formulation of an elastic/viscoplastic model for an initially transversely isotropic rock is presented. To describe the nonlinearity of the elastic response, the elastic moduli are considered to be stress dependent. Furthermore, it is shown that in order to have a conservative elastic response, the elastic moduli may depend on stress only through the mean stress and a mixed stress-structural tensor invariant $tr\, M\, \sigma$, where $M = S_1 \otimes S_1$, S_1 being the axis of symmetry. The influence of the anisotropy on the irreversible response and strength is described by means of a fourth order symmetric anisotropic tensor A_{ijkl}, which is involved in the expressions of the flow rule, yield function, and failure criterion in the form of a transformed stress tensor: $\Sigma_{ij} = A_{ijkl}\, \sigma_{kl}$. The model adequacy is demonstrated by applying it to a sedimentary rock, Tournemire shale. The comparison between the model predictions and data is within the natural scatter of the data

RÉSUMÉ: On présente un modèle élasto-viscoplatique d'une roche initiallement isotrope transverse. Le modèle rend compte de la nonlinéarite de la réponse élastique. On montre que le choix des lois de variation des modules élastiques avec l'état de contrainte ne peut pas être arbitraire: afin que la réponse soit conservative les paramètres doivent s'exprimer que par des fonctions de la contrainte moyenne et de l'invariant $tr\, M\sigma$, ou $M = S_1 \otimes S_1$, S_1 étant l'axe de symmetrie. L'anisotropie du domaine plastique et de la rupture est décrite à l'aide d'un tenseur d'order quatre A, invariant par rapport à toute transformation orthogonale appartenant au groupe de symmetrie. A intervient dans les expressions de la surface de charge, de la régle d'écoulement et du critère du rupture sous le forme d'un tenseur de contrainte transformé $\Sigma_{ijj} = A_{ijkl}\, \sigma_{kl}$. Le modèle a été appliqué à une roche sedimentaire, l'argilite de Tournemire. La comparison entre les prédictions du modèle et les données expérimentales est satisfaisante.

ZUSAMMENFASSUNG: Eine invariante elasto-viskoplastische nicht asoziierte Gleichung für ein anfangs transversal isotropes Gestein wird vorgestellt. Gezeigt wird, wie die elastischen Moduli von Spannungszustand abhängen können. Die Anisotropie wird durch einen Tensor vierter Ordnung A_{ijkl} beschrieben. Er ist in den Ausdrücken für die Fliessregel, die Fliessfunktion und das Bruchkriterium durch Verwendung eines transformierten Spannungstensors enthalten: $\Sigma_{ij} = A_{ijkl}\, \sigma_{kl}$. Die Eignung des Modells wird demonstriert, indem es bei einem geschichteten Sedimentgestein, Tournemire Tonschiefer, angewendet wird.

1 INTRODUCTION

In many rocks, due to the existence of well-defined fabric elements such as bedding, layering, foliation or lamination planes, or due to the existence of linear structures, anisotropy can be important. The symmetries most frequently encountered are: transverse isotropy and orthotropy. By adopting both theoretical and experimental approaches, many authors have investigated the effect of the presence within the rock of pronounced anisotropic feature on strength. The experimental studies have been carried out mainly on cylindrical specimens subjected to axisymmetrical state of compressive stresses. It has been found that at zero or low degree of confinement the compressive strength varies significantly with the orientation angle β (β is the angle between the strata's planes and the direction of maximum compression). Most rocks undergo a significant reduction in strength when the anisotropic plane is aligned at about 30° to the axis of major principal stress (for most rocks $\beta \in (30^o, 45^o)$) while the maximum compressive strength occurs at an orientation angle β of 0^o or 90^o depending upon the rock type. As the amplitude of the confined pressure is raised the rock responds in a more ductile manner, and the effect of the strength anisotropy is usually reduced (Attewell and Sandford (1974); Chenevert and Gatlin (1965); Donath (1964, 1972); Niandou *et al* (1997), etc.). However, an increase in the confining pressure causes for all angles β an increase in the strength of most rocks. Failure criteria that accounts for the continuous variation of the compressive strength with orientation for transversely isotropic intact rock have been developed by several authors (e.g., McLamore and Gray (1967); Ramamurthy (1993)). These criteria are simple in concept and in expression and they provide good approximations for the strength under axi-symmetric loading conditions. However, they all require a wide range of tests and a large amount of curve fitting. The numerical implementation is generally difficult, these theories cannot be applicable to truly 3-D stress state. A more general approach was adopted by Pariseau (1972). Pariseau's criterion takes into account the possibility of unequal tensile and compressive strengths and describe the effect of the hydrostatic stress on strength. In the framework of the theory of invariance, a general theory of the flow and fracture of anisotropic solids was developed by Boehler (see Boehler (1987)). To describe the behavior of initially anisotropic sedimentary rocks a generalization of Cam-Clay model has been proposed by Nova (1986).

In this paper an invariant formulation of an elastic/viscoplastic non associated constitutive equation for initially transversely isotropic rock is presented. To match the experimental data for unloading and reloading cycles, the elastic moduli are considered to be stress-dependent. However, the laws of variation of the elastic moduli with the stress tensor σ cannot be arbitrary. We demonstrate that in order to obtain a conservative elastic response the moduli may depend on stress only through its first invariant $\alpha_1 = tr\,\sigma$ and the mixed invariant $\alpha_4 = tr\,M\sigma$, where $M = S_1 \otimes S_1$ and S_1 is the direction of transverse isotropy. For each loading level the limit of the elastic domain is given by a yield function whose expression is a priori unknown and is determined from data. The basic assumption adopted here is that the type of those expressions is a priori unknown and is determined from data. The structure of the yield function, flow rule, and short-term failure criterion by means of a constant fourth order anisotropic tensor. The adequacy of the model is demonstrated by applying it to a stratified sedimentary rock, Tournemire shale.

2 HYPERELASTIC NON-LINEAR MODEL

For most rocks the elastic moduli are stress dependent. Under the assumption of small deformations, in the elastic regime the behavior is thus described by a constitutive equation of the general form

$$\dot{\varepsilon} = B(\sigma)\dot{\sigma} \tag{1}$$

where ε is the strain tensor of small deformation, σ is the Cauchy stress tensor, $\boldsymbol{B}(\sigma)$ is linear in σ. In addition, $\boldsymbol{B}(\sigma)$ is symmetric and transversely isotropic. Let us define (S_1, S_2, S_3) the reference coordinate system associated with the material symmetries: S_1 is the symmetry axis, while (S_2, S_3) is the isotropy plane. Hence, $\boldsymbol{B}(\sigma)$ can be described in the reference Cartesian system (S_1, S_2, S_3) by five scalar smooth functions *a, b, u, w, t* : $D' \to R$, *i.e.*:

$$B_{ijrs} = a(\sigma)\,\delta_{ij}\,\delta_{rs} + b(\sigma)\left(\delta_{ir}\,\delta_{js} + \delta_{is}\,\delta_{rj}\right) + w(\sigma)\,\delta_{i1}\,\delta_{j1}\,\delta_{r1}\,\delta_{s1} + t(\sigma)\begin{pmatrix}\delta_{ir}\,\delta_{j1}\,\delta_{s1} + \delta_{jr}\,\delta_{s1}\,\delta_{i1} + \\ \delta_{js}\,\delta_{r1}\,\delta_{i1} + \delta_{is}\,\delta_{r1}\,\delta_{j1}\end{pmatrix} \tag{2}$$
$$i, j, r, s = 1, \cdots 3.$$

The full use of the hypothesis of transverse isotropy will also require that the scalar constitutive functions *a, b, u, w, t* depend on σ only through the scalar invariants

$$\alpha_1 = tr(\sigma),\ \alpha_2 = tr(\sigma^2),\ \alpha_3 = tr(\sigma^3),\quad \alpha_4 = tr(M\,\sigma),\ \ \alpha_5 = tr(M\,\sigma^2) \tag{3}$$

where $M = S_1 \otimes S_1$ and 'tr' denotes the trace operator. For the material to have a conservative elastic response, the constitutive equation (1) has to be hyperelastic. The constitutive equation (1) with $\boldsymbol{B}(\sigma)$ given by (2) is hyperelastic if and only if

$$\frac{\partial a}{\partial \sigma_{11}} = \frac{\partial a}{\partial \sigma_{22}} + \frac{\partial u}{\partial \sigma_{22}} = \frac{\partial a}{\partial \sigma_{33}} + \frac{\partial u}{\partial \sigma_{33}},\quad \frac{\partial a}{\partial \sigma_{12}} = \frac{\partial a}{\partial \sigma_{13}} = \frac{\partial a}{\partial \sigma_{23}} = 0,$$
$$\frac{\partial u}{\partial \sigma_{12}} = \frac{\partial u}{\partial \sigma_{13}} = \frac{\partial u}{\partial \sigma_{23}} = 0,\quad \frac{\partial a}{\partial \sigma_{22}} = \frac{\partial a}{\partial \sigma_{33}}, \tag{4}$$
$$b = const.,\qquad t = const.$$
$$\frac{\partial u}{\partial \sigma_{11}} = \frac{\partial u}{\partial \sigma_{22}} + \frac{\partial w}{\partial \sigma_{22}} = \frac{\partial u}{\partial \sigma_{33}} + \frac{\partial w}{\partial \sigma_{33}},$$
$$\frac{\partial w}{\partial \sigma_{12}} = \frac{\partial w}{\partial \sigma_{13}} = \frac{\partial w}{\partial \sigma_{23}} = 0.$$

(see Cazacu, 1998).
The restrictions (4) written in terms of engineering strengths are

$$\frac{1+\nu_{23}}{E_2} = C_1,\quad \frac{1}{G_{12}} = C_2,\ \frac{\nu_{23}}{E_2} - \frac{\nu_{21}}{E_2} = f(\alpha_1, \alpha_4),$$
$$\frac{\partial w}{\partial \alpha_1} = \frac{\partial f}{\partial \alpha_4},\quad \frac{\partial}{\partial \alpha_4}\left(-\frac{\nu_{23}}{E_2}\right) = \frac{\partial f}{\partial \alpha_1},\ w = w(\alpha_1 \alpha_4). \tag{5}$$

where $w = 1/E_1 + 1/E_2 + 2\nu_{21}/E_2 - 1/G_{12}$.
It can be seen that relations (5) impose severe limitations in the choice of constitutive functions describing the evolution of the elastic moduli with the stress state. Let us denote by G_{23} the shear modulus in the isotropy plane (S_2, S_3); thus $G_{23} = \dfrac{E_{23}}{2(1+\nu_{23})}$. Therefore, the choice of the laws of variation of the elastic moduli with the stress tensor σ cannot be arbitrary. The elastic response is conservative if and only if: (1) the shear moduli G_{12} and G_{23} are constant; (2) the elastic moduli E_1, E_2, ν_{23}, and ν_{21} depend on stress only through its first invariant $\alpha_1 = tr\,\sigma$, and the mixed stress-structural tensor invariant $\alpha_4 = tr\,M\sigma$. Let us note that if $w = 0$ the shear modulus G_{12} is expressed as a simple combination between the elastic moduli E_1, E_2 and the Poisson ratio ν_{21}.

Saint-Venant indicated such a possible relation between the elastic parameters of orthotropic bodies with the principal directions S_2 and S_3 in the form

$$\frac{1}{G_{ij}} = \frac{1}{E_i} + \frac{1}{E_j} + \frac{2\nu_{ji}}{E_j},\quad i, j = 1, 2\ or\ 3. \tag{6}$$

In transversely isotropic bodies with the direction S_1 normal to the plane of isotropy this gives the shear modulus G_{12} in the form (6). Most of the published experimental data on transversely isotropic rocks support the validity of Saint-Venant's approximation (6) (see the review of over 200 quasi-static and dynamic tests on transversely isotropic rock due to Worotnicki (1993)).

The data on Tournemire shale were obtained in Lille Mechanics Laboratory (Niandou, 1994; Niandou *et al.*, 1997). The experimental data set includes ultrasonic measurements and quasi-static triaxial compression tests with loading and unloading cycles. Various loading orientations and confining pressures were considered. At the macroscopic level this rock is characterized by a well-defined stratified structure. Ultrasonic measurements carried out on cubical specimens have shown that this rock exhibits intrinsic transverse isotropy. This type of anisotropy is conserved up to a high level of the deviatoric stress (i.e. the difference between the axial stress and the applied confining pressure) as shown by the triaxial compression test results. An experimental technique (see Cristescu (1989)), which permits a good separation of viscous effects from unloading was used. Thus, at different stress levels the rock was allowed to reach by a short-time relaxation a quasi-stable state, afterwards

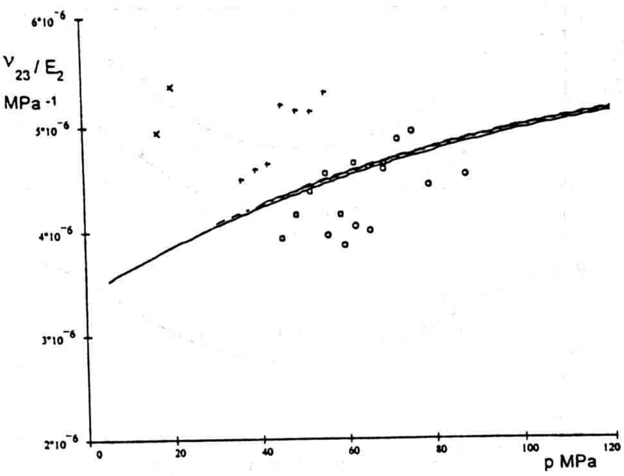

Figure 1. Comparison between theoretical and experimental variation of ν_{23}/E_2 as function of the mean pressure p (data after Niandou (1994))

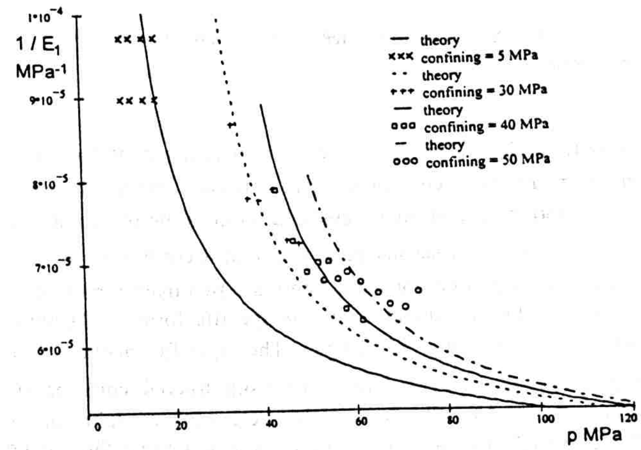

Figure 2. Comparison between theoretical and experimental variation of $1/E_1$ as function of the mean pressure (data after Niandou (1994))

unloading-reloading cycles were performed. In this way the hysteresis loops were practically eliminated permitting an accurate estimate of the elastic moduli from the unloading slopes. For Tournemire shale, we obtain the expressions of the elastic modulus E_2 and of the Poisson's ratio ν_{21} as function of the invariants α_1 and α_4. As an example, Fig. 1 shows the predicted evolution of ν_{23}/E_2 as a function of p in comparison with the data. From data in triaxial compression tests performed for several values of all elastic parameters were determined.

Figure 2 shows a comparison between the predicted evolution of $1/E_1$ as function of the mean stress p and data.

3 IRREVERSIBLE BEHAVIOR

The limit of the elastic domain is given by a yield function whose expression is a priori unknown and was determined from data. The basic assumption adopted in the formulation of the constitutive equation is that the type of anisotropy of the rock does not change during the deformation process. The anisotropy is thus described by a fourth order tensor A satisfying the usual symmetry conditions $A_{ijkl}=A_{jikl}=A_{klij}=A_{ijlk}$ and the general requirement of invariance under any orthogonal transformation belonging to the symmetry group of the material. Thus, in the structural frame (S_1,S_2,S_3), the truncated matrix of A is:

$$A=\begin{bmatrix} a & b & b & 0 & 0 & 0 \\ b & d & e & 0 & 0 & 0 \\ b & e & d & 0 & 0 & 0 \\ 0 & 0 & 0 & \dfrac{d-e}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \dfrac{c}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \dfrac{c}{2} \end{bmatrix} \tag{7}$$

where a, b, c, d and e are constants. In the present paper the tensor A is assumed to be constant: it does not depend on time nor on deformation. It is involved in the expression of the flow rule, of the yield function, and of the failure criterion in the form of a transformed stress tensor defined by:

$$\Sigma_{ij}=A_{ijkl}\ \sigma_{kl} \tag{8}$$

The 5 independent components of the anisotropic tensor A are identified using strength data in conjunction with an anisotropic short-term failure criterion of the form:

$$\frac{3}{2}tr(\Sigma')^2-\frac{m}{3}tr\Sigma-1=0. \tag{9}$$

where m is a material constant, Σ' is the deviator of the second order tensor Σ (Cazacu *et al.*,1998). Using (7) and (8), we can express the invariants of Σ in terms of the stress invariants and of the mixed invariants of σ and M

$$\begin{aligned} tr\,\Sigma &= (a+b+d-e)\alpha_4+(b+d+e)\alpha_1 \\ tr\left(\Sigma'^2\right) &= AA(\alpha_4)^2+BB(\alpha_1)^2+CC\alpha_1\alpha_4+DD\alpha_5+EE\alpha_2 \end{aligned} \tag{10}$$

where α_i are defined by (3) and AA, BB, CC, DD, and EE are algebraic combinations of the coefficients a, b, c, d, e and m. The first invariant of Σ is linear in σ and thus can be thought as a generalization to transversely isotropic conditions of α_1 ,the mean stress. The second invariant of the transformed tensor deviator is a quadratic homogeneous function of σ which reduces to $J_2=(1/2)tr(\sigma')^2$ for $a=d=c=1$, $b=e=0$. For isotropic conditions, this criterion reduces to the Mises-Schleicher criterion

$$3J_2+(\sigma_C-\sigma_t)\alpha_1-\sigma_t\sigma_C=0$$

In the structural system (S_1,S_2,S_3), the Anisotropic Mises-Schleicher (AMS) criterion writes:

$$\begin{aligned} &d_1\sigma_{11}+d_2(\sigma_{22}+\sigma_{33})+D_{11}\left(\sigma_{11}^2\right)+D_{22}\left(\sigma_{22}^2+\sigma_{33}^2\right)+ \\ &2D_{12}\,\sigma_{11}(\sigma_{22}+\sigma_{33})+2D_{23}\,\sigma_{22}\,\sigma_{33}+D_{44}\sigma_{23}^2 \\ &+D_{55}\left(\sigma_{12}^2+\sigma_{13}^2\right)=1 \end{aligned} \tag{11}$$

The coefficients in (10) are algebraic combinations of the independent components of the stress tensor A and m; their expressions in terms of the engineering strengths are:

$$\begin{aligned} &d_1=\frac{1}{xc}-\frac{1}{xt}, \quad D_{11}=\frac{1}{(xc)(xt)}, \quad d_2=\frac{1}{yc}-\frac{1}{yt}, \\ &D_{22}=\frac{1}{(yc)(yt)}, \quad D_{44}=4D_{22}-D_{11}, \\ &D_{12}=-\frac{1}{2}D_{11}, \quad D_{23}=-D_{22}+\frac{D_{11}}{2}, \quad D_{55}=\frac{1}{R^2}, \end{aligned} \tag{12}$$

In (9) and throughout the text the compressive strengths are taken positive, xc and $(-xt)$ are the uniaxial compressive and tensile strengths in the S_1-direction, yc and $(-yt)$ are the uniaxial compressive and tensile strengths in the S_2-direction (i.e. along any direction belonging to the strata planes), R is the shear strength in the (S_2, S_3) plane. In the three-dimensional space of the principal stresses the failure surface (9) is an elliptic paraboloid for any orientation θ of the principal stresses system with respect to the system associated with the material symmetries (θ is the angle between the maximum stress axis and the S_1 axis). The distance between the origin O, and the point where the hydrostatic axis intersects the paraboloid is $\sqrt{3}(\sigma_T)$ where

$$\sigma_T = \frac{1}{2(1/yc - 1/yt) + (1/xc - 1/xt)} \qquad (13)$$

is the hydrostatic tensile strength of the material (independent of θ). Thus, the AMS criterion is able to model hydrostatic stress induced failure. It predicts that the application of multiaxial tensile stresses on rock reduces the value of the failure strength, i.e. the predicted value of the hydrostatic strength is less than the uniaxial tensile strength in any direction. Indeed, from (13) follows that if the condition $2xt < yc$ is satisfied, as it is the case with most rocks, then $|\sigma_T| < yt$. Similarly, if condition $2yt < yc$ is satisfied then $|\sigma_T| < xt$. This is in contrast with most existing criteria that intoduce "tension cuttoffs" by postulating that failure occurs when the major principal stress is equal to the uniaxial tensile strength. The intersections of the AMS failure surfaces with the octahedral plane demonstrates the ability of the criterion to describe the directional character of the strength of transversely isotropic material under general loading conditions. The AMS criterion involves a few number of parameters that are directly expressible in terms of the engineering strengths (see (12)). For Tournemire shale, the mean arithmetic value of xc is of 48 MPa, whereas yc = 50 MPa. No tensile tests were available; we have assumed xt = 3.92 MPa, and yt = 4.1 MPa, the estimate being based on tensile strengths of oily shales reported in the literature (Lama and Vutukuri (1978)). Since for rocks shear tests are very difficult to perform and to interpret R has been estimated by least square fit using the test results at a confining pressure of 50 MPa and θ= 30°, 45°, 60° and 90.° Figure 3 shows the variation of the peak axial stress σ_a with the orientation θ for several confining pressures. The solid lines correspond to the predictions of the AMS criterion, while the experimental points are represented by symbols. The comparison with the data is successful in the whole. The influence of the confining pressure on strength is well described although only the test results at p_c = 50 MPa were used for the determination of R. For Tournemire shale, we suppose that the observed irreversibility in behavior is due to transient creep only. Thus, the flow rule is assumed to be of the following general form:

$$\dot{\varepsilon}_{VP} = \eta\left\langle 1 - \frac{W^I(t)}{H(\sigma)}\right\rangle(\Sigma + U(\sigma)I) \qquad (14)$$

where $H(\sigma)$ is the yield function, $H(\sigma)=W^I$ (t) defines the equation of the stabilization boundary ($\dot{\varepsilon}^{VP} = 0$ and $\dot{\sigma} = 0$). The work hardening parameter is the irreversible stress work per unit volume, W^I defined by

$$W^I(T) = \int_0^T \sigma(t) : \dot{\varepsilon}^{VP}(t)dt \qquad (15)$$

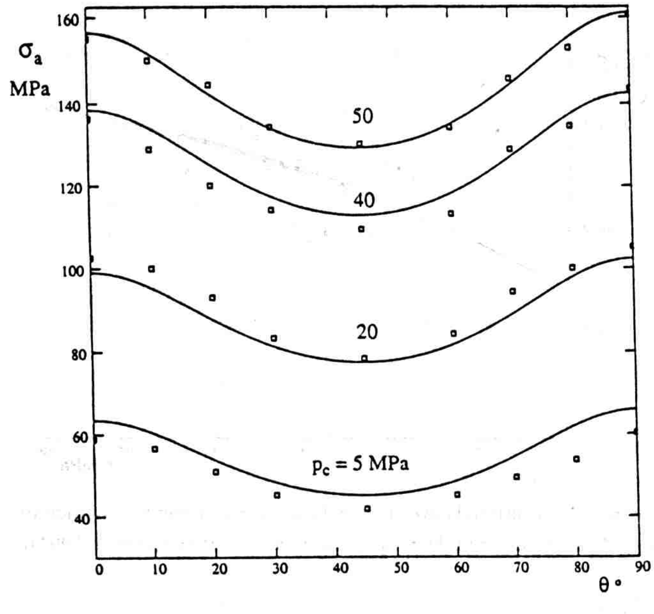

Figure 3. The AMS criterion applied to Tournemire shale (data after Niandou (1994))

Further, in (14) $U(\sigma)$ is a scalar valued function of the stress tensor σ, I is the second order identity tensor. The sum between $U(\sigma)I$ and the transformed tensor Σ defines the orientation of $\dot{\varepsilon}_{VP}$ while η is a viscosity parameter that is considered to be in a first approximation constant. There are two main topics to be addressed: the determination of the specific form of the yield function $H(\sigma)$ and of $U(\sigma)$. The specific mathematical expression of $H(\sigma)$ is determined from triaxial compression data by computing the irreversible stress work at stabilization. Since in such tests the first stage is hydrostatic and the second one is deviatoric, it was assumed that $H(\sigma)$ is the sum of two terms,

$$H(\sigma) = H_h(\sigma) + H_d(\sigma,\tau) \qquad (16)$$

such that $H_d(\sigma,0) = 0$, σ stands for mean stress while τ stands for the octahedral shear stress. Computing W^I in hydrostatic and deviatoric tests we get

$$H\ (\sigma,M) = c_o \sin(\omega\sigma/\sigma_o + \varphi) + c_o + B_1 tr\Sigma + B_2\left(\frac{3}{2}tr(\Sigma')^2\right) \qquad (17)$$

where c_o = 0.38 MPa, ω=0.15, φ = 283° and σ_o = 1 MPa and B_1 and B_2 may depend on the mean pressure and the octahedral shear stress. H is a scalar valued isotropic function of σ and M and thus satisfies automatically the combined objectivity-symmetry requirements. The shapes of the yield surfaces H =0.24 MPa, together with the corresponding failure surfaces for several orientations θ are shown in Figure 4. As expected, the model exhibits only compressibility. The anisotropy is well described: for the same value of the hardening parameter W^I yielding takes place at lower values of the octahedral stress for θ = 0° than for θ = 45° and θ = 90°, respectively. Similarly, we determine for Tournemire shale

$$U(\sigma,\tau) = u_{1h}\sigma^2 + u_{2h} + \left[n_1 + n_2\left(\sigma - \tau/\sqrt{2}\right)\right]\tau + \left[m_1 + m_2\left(\sigma - \tau/\sqrt{2}\right)\right]\tau^2 \qquad (18)$$

where $u_{1h}=1.098\cdot10^{-4}$ MPa^{-1}, $u_{h2}=0.025$ MPa, $m_1 = 0.026$ MPa^{-1}, $m_2 = -2.6\cdot10^{-5}$ MPa^{-3}, $n_1 = -0.0012$, $n_2 = 3.579\cdot10^{-5}$ MPa^{-1}. As an example in Figure 5 is shown a comparison between the model predictions and data obtained in a hydrostatic test. The anisotropy of deformation under isotropic stress conditions is well reproduced. In Figure 6 we present the simulations of a triaxial compression test for one sample orientation. The triaxial test is identified by two numbers as follows (orientation of the bedding plane)/(confining pressure). The overall prediction is reasonable. Particularly, the rock anisotropy is clearly and correctly described for several other orientations as well.

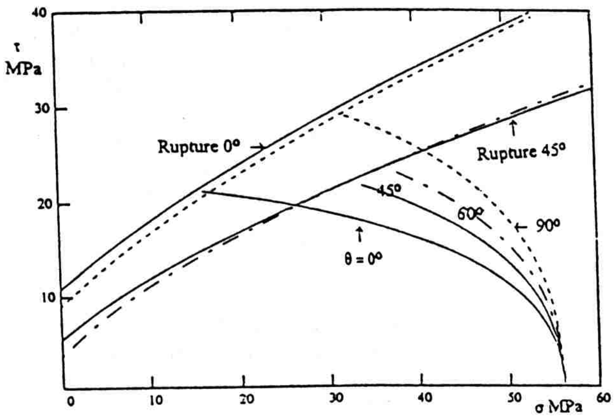

Figure 4. Predicted yield loci and failure surfaces for different sample orientations θ

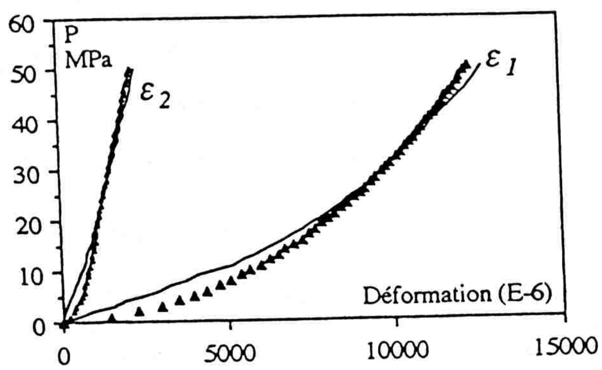

Figure 5. Comparison between model prediction and hydrostatic compression test data

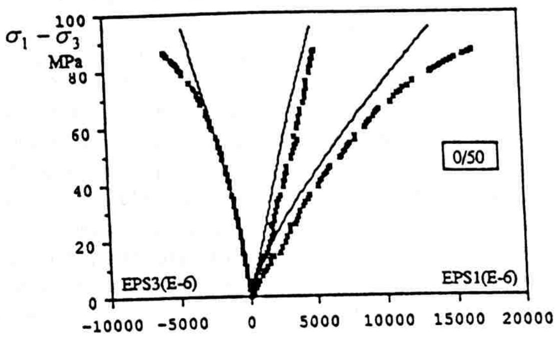

Figure 6. Comparison between model prediction and triaxial compression test data for one orientation.

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