Borehole Stability Analysis in Transversely Isotropic Rock using Anisotropic failure criterion

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ABSTRACT

This study demonstrates the importance of considering anisotropy in borehole stability of transversely isotropic rock. Both analytical and numerical methods are conducted in order to analyze the horizontal borehole stability in transversely isotropic rock. The data for transversely isotropic rock are based on laboratory experiments on shale. The boundary condition is assumed as in situ stress based on the stress ratio of Pohang, Korea. The critical plane approach is applied to describe the anisotropic Mohr-Coulomb failure criterion. A various scenarios with depth in transversely isotropic rock are considered in order to examine the borehole breakout.

Keywords: transversely isotropic, borehole stability, borehole breakout

1. INTRODUCTION

It is required to analyze the stress and displacement state around the borehole in important subsurface engineering applications such as Enhanced Geothermal Systems (EGS), petroleum, tunnels and mining. The presence of a borehole in a stressed subsurface rock formation alters the local principal stress directions and magnitudes around the borehole. Stresses around borehole are given by the classical Kirsch solution in isotropic elastic homogeneous rocks and this solution is widely used in practice. However, many boreholes are drilled in anisotropic formations due to the existence of stratified layers, schistosity, foliation and fractures. For example, shale whose importance is rising due to its role as caprock in CO_2 storage, EGS reservoir and shale gas production has shown the characteristics of transversely isotropic models. This rock anisotropy not only causes changes of stress concentration around borehole but also affects the direction of borehole breakout and fracture initiation. The stress and displacement analysis that do not consider the anisotropic behavior of rock can be erroneous in varying degrees and therefore, it is necessary to consider the anisotropy in borehole stability analysis.

In this study, both analytical and numerical methods were conducted in order to analyze the borehole stability in transversely isotropic rock. The data are based on laboratory experiments on Boryeong shale (Cho et al., 2012) and the boundary condition is based on the in situ stress in Pohang, Korea, where the first EGS activites is currently going on (Yoon et al., 2011). This study examinesthe borehole breakout using the anisotropic Mohr-Coulomb failure criterion considering strength anisotropy. The importance of considering anisotropy in borehole stability analysis is demonstrated in a wide variety of scenario with depth up to 5 km.

2. THEORIES

2.1 Analytic solution for transversely isotropic medium

A general solution for the stresses around a borehole in an anisotropic medium can be found by using the concept of Airy stress functions. The general expressions for the borehole-induced stresses

$\sigma_{induced}$ which can be superimposed onto the corresponding components of the far field in-situ stress tensor ($\sigma_{original}$) to get the borehole stress tensor ($\sigma_{borehole}$) are

$$\begin{aligned}
\sigma_{xx,borehole} &= \sigma_{xx,original} + \sigma_{xx,induced} \\
\sigma_{yy,borehole} &= \sigma_{yy,original} + \sigma_{yy,induced} \\
\tau_{xy,borehole} &= \tau_{xy,original} + \tau_{xy,induced} \\
\tau_{xz,borehole} &= \tau_{xz,original} + \tau_{xz,induced} \\
\tau_{yz,borehole} &= \tau_{yz,original} + \tau_{yz,induced} \\
\sigma_{zz,borehole} &= \sigma_{zz,original} + \sigma_{zz,induced}
\end{aligned} \tag{1}$$

The problem can be fully solved by finding solutions to the analytic functions by applying the correct boundary conditions in the far-field as well as on the borehole wall (Amadei, 1982; Ong, 1994). Especially, the degeneration of the general solution happens in the case of transversely isotropic medium with the plane of isotropy striking parallel or perpendicular to the hole axis. The detailed formulation for transversely isotropic medium is introduced on Lekhnitskii (1963), Amadei (1982) and Ong (1994).

2.2 Strength anisotropy and anisotropic Mohr-Coulomb failure criterion

Although many attempts have been made to describe the strength anisotropy of transversely isotropic rocks, no general methodology has emerged yet. In order to describe the strength characteristics of geological materials, critical plane approach has been proposed (Pietruszczak and Mroz, 2001). This method searches for a direction of failure plane that the value of failure function reaches maximum. A few studies analyzed anisotropic behavior or sedimentary rocks (Pietruszczak et al., 2002), structural masonry (Ushaksaraei and Pietruszczak, 2002), and transversely isotropic rock mass (Lee and Pietruszczak, 2008) using this methodology. The critical plane approach is applied to describe the anisotropic Mohr-Coulomb failure criterion in this study. A detailed methodology about the critical plane approach for anisotropic Mohr-Coulomb failure criterion is presented in Pietruszczak and Mroz (2001) and Lee and Pietruszczak (2008).

According to Lee and Pietruszczak (2008), a concept of fabric tensor is adopted to define the cohesion and friction angle in transversely isotropic rock. It is assumed that, on a plane having the unit normal n, the parameters c (cohesion) and ϕ (friction angle) can be defined in terms of distribution function, Ω (Lee and Choi, 2011),

$$\begin{aligned}
c &= c_0\left(1+\Omega_{ij}{}^{c} n_j n_i\right) \\
\phi &= \phi_0\left(1+\Omega_{ij}{}^{\phi} n_j n_i\right)
\end{aligned} \tag{2}$$

where c_0 and ϕ_0 is the orientation average of cohesion and friction angle, and Ω_{ij} is a traceless symmetric tensor describing the bias in the spatial distribution of c and ϕ with respect to the mean value. It should be noted that for an isotropic material the Ω_{ij} vanish, so that c and ϕ become constant.

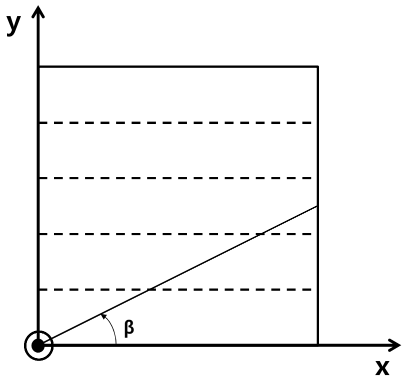


Figure 1. Local coordinate system (modified from (Lee and Choi, 2011))

In the case of transversely isotropic rock, the fabric tensor Ω_{ij} in local coordinate system (Figure 1) can be represented as following (Lee and Choi, 2011):

$$\Omega_{ij} = \begin{bmatrix} \Omega_0 & 0 & 0 \\ 0 & -2\Omega_0 & 0 \\ 0 & 0 & \Omega_0 \end{bmatrix} \tag{3}$$

So it simplifies to

$$\begin{aligned} c &= c_0\left(1+\Omega_0{}^c\left(1-3\cos^2\beta\right)\right) \\ \phi &= \phi_0\left(1+\Omega_0{}^\phi\left(1-3\cos^2\beta\right)\right) \end{aligned} \tag{4}$$

Equation (4) indicates the cohesion (c) and internal friction angle (□) on a plane that makes an angle β with the x-direction. The anisotropy in these two strength parameters, c and □, can be calculated considering the relative rotation between the principal stress coordinate and the principal material triad through the numerical triaxial test (Lee and Pietruszczak, 2008; Pietruszczak and Mroz, 2001).

2.3 Application of anisotropic Mohr-Coulomb failure criterion to horizontal borehole stability

Although there are studies that considered the borehole stability in transeversely isotropic rock, few studies applied the anisotropic Mohr-Coulomb failure criterion to borehole stability analysis. To demonstrate the performance of the anisotropic Mohr-Coulomb failure criterion, conventional triaxial tests on the samples having various inclinations of weakness plane were simulated and the resulting triaxial strength and dip angle of failure plane were discussed by Lee and Choi (2011). In order to analyze the borehole stability considering strength anisotropy in transversely isotropic rock, two strength parameters in Mohr-Coulmb failure criterion, i.e., c and □ are calculated with consideration of the relative angle in local coordinate system.

In local coordinate system (Figure 1) normal and shear stresses acting on a plane whose outward unit normal vector is rotated counterclockwise from the x-direction by an angle θ are (Jaeger et al., 2007):

$$\begin{aligned} \sigma &= \frac{1}{2}\left(\sigma_{xx}+\sigma_{yy}\right)+\frac{1}{2}\left(\sigma_{xx}-\sigma_{yy}\right)\cos 2\theta+\tau_{xy}\sin 2\theta \\ \tau &= -\frac{1}{2}\left(\sigma_{xx}-\sigma_{yy}\right)\sin 2\theta+\tau_{xy}\cos 2\theta \end{aligned} \tag{5}$$

Equations (4) and (5) are substituted into following Mohr-Coulomb failure function in transversely isotropic rock:

$$F_{MC} = \tau-\left(\tan\phi\right)\sigma-c \tag{6}$$

A positive value of F_{MC} indicates that the failure occurs. The occurrence of failure for each point on borehole domain can be determined using this method.

3. RESULTS AND DISCUSSION

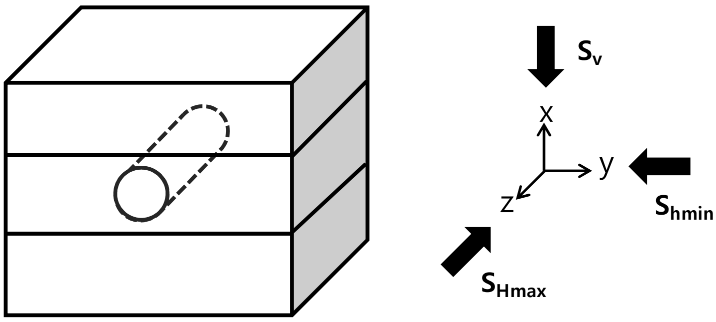
Elastic constants of Boryeong Shale (Cho et al., 2012) and anisotropic parameters of Green River Shale (McLamore and Gray, 1967) were used (**Table 1**). And the analytic study and numerical study using COMSOL was conducted in order to determine the stress states around a borehole domain as well as at borehole wall.

Table 1. Parameters used for borehole stability analysis

E	39.3 GPa	Boryeong Shale (Cho et al., 2012)
E□	19 GPa	
ν	0.18	
ν□	0.2	
G□	8.7 GPa	
c_0	45MPa	Green River Shale (McLamore and Gray, 1967)
□$_0$	20°	
Ω_0^c	0.12	
$\Omega_0^{□}$	0.1	

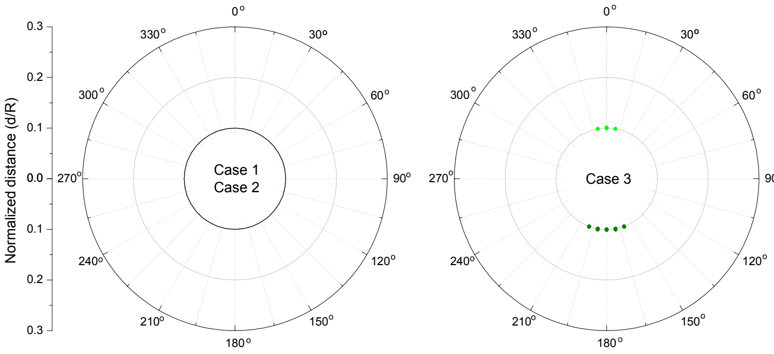
In order to examine the borehole breakout in a wide range of boundary conditions, failure analysis in transversely isotropic rock was carried out in each boundary condition (**Figure 2**). Those boundary conditions are calculated values based on Pohang region and the ratio of S_{Hmax} to S_{hmin} is set to be 4:3 (Chang et al., 2010). The range of stress ratio (K), which is a ratio of the horizontal principal stress to the vertical principal stress, is assumed to vary from 0.5 to 2. The vertical stress component is regarded as overburden weight.

The case of K=0.5 represents the normal stress regime ($S_V > S_{Hmax} > S_{hmin}$), the case of K=1 is the strike-slip stress regime ($S_{Hmax} > S_v > S_{hmin}$) and the case of K=2 is thrust stress regime ($S_{Hmax} > S_{hmin} > S_V$).



Depth	S_v (MPa)	K=0.5		K=1		K=2	
		S_{Hmax} (MPa)	S_{hmin} (MPa)	S_{Hmax} (MPa)	S_{hmin} (MPa)	S_{Hmax} (MPa)	S_{hmin} (MPa)
1000m	26.5	Case 1		Case 2		Case 3	
		11	16	23	30	46	60
3000m	79.5	Case 4		Case 5		Case 6	
		34	45.5	68	91	136	182
5000m	132.5	Case 7		Case 8		Case 9	
		57	75.5	114	151	227	303

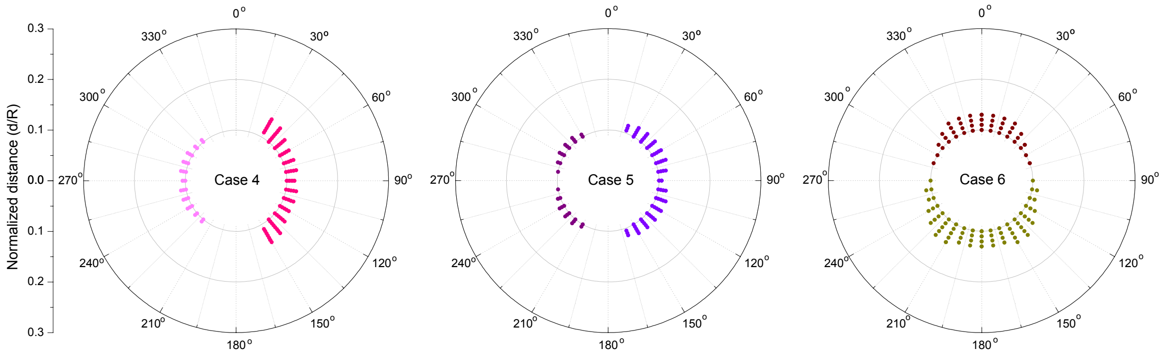
Figure 2. Boundary conditions for failure analysis in transversely isotropic rock



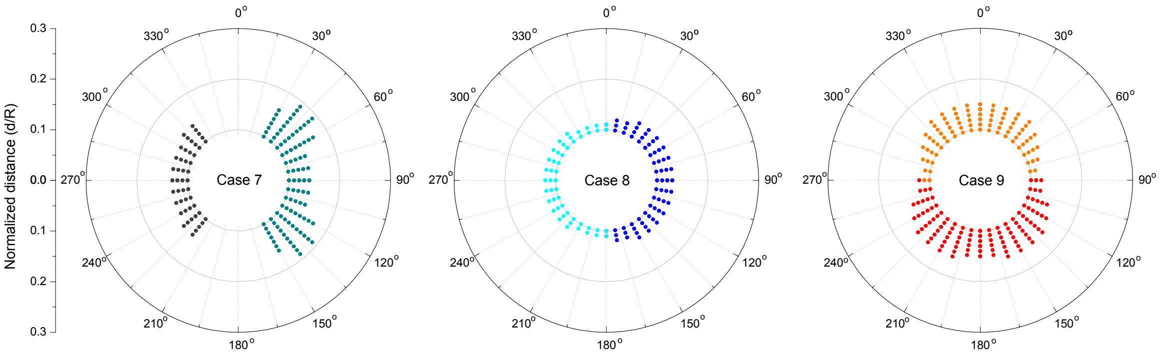
(a) Case 1 (1000m depth, K=0.5)
Case 2 (1000m depth, K=1)

(b) Case 3 (1000m depth, K=2)

No breakout



(c) Case 4 (3000m depth, K=0.5)   (d) Case 5 (3000m depth, K=1)   (e) Case 6 (3000m depth, K=2)



(f) Case 7 (5000m depth, K=0.5)   (g) Case 8 (5000m depth, K=1)   (h) Case 9 (5000m depth, K=2)

Figure 3. Borehole breakout analysis result according to the stress regime. The regions where borehole breakout occurs are indicated with dots. In each figure, analyses using isotropic failure criteria are shown in the left or top, and analyses considering anisotropic failure criteria are indicated in the right or bottom.

Figure 3 shows the region that borehole breakout is expected when the suggested anisotropic criterion considering strength anisotropy is applied (top or left) and strength anisotropy is not considered (bottom or right), i.e., isotropic failure criteria is applied (Ω_0^c and $\Omega_0^{\square}$ are zero).
In 1000m depth, when the normal (Case 1) and strike-slip stress regime (Case 2) are applied, there is no borehole breakout as shown in

Figure 3 (a). But in the thrust stress regime (Case 3) boundary condition, the narrow borehole

breakout region appears.
In 3000m and 5000m depth, the borehole breakout appears in the entire stress regime cases (
Figure 3 (c), (d), (e), (f), (g), (h)). But the shape of breakout is different case by case. The broad and flat breakout appears around the borehole wall in strike-slip (Case 5, 8) and thrust stress regime (Case 6, 9). But the shape of breakout changes in the normal stress regime (Case 4, 7). These breakout models have steeper edge in flank; the difference of boundary condition can be a reason for that. The ratio of applied boundary condition around the borehole, i.e., the ratio of x-direction stress and y-direction stress is different. In normal (Case 1, 4, 7), strike-slip (Case 2, 5, 8) and thrust stress regime (Case 3, 6, 9) stress ratio is 2.3, 1.2, and 0.6, respectively. When the maximum stress direction is perpendicular to the isotropic plane and the stress ratio (K) is bigger, the shape of borehole breakout changes.

Generally, the range of breakout considering the strength anisotropy is wider and thicker than that without considering. The main reason is the difference of failure criteria. When the anisotropic failure criterion considering strength anisotropy is applied, the possibility of failure increases. There are a lot of failure lines depending on cohesion and internal friction angle in transversely isotropic case and it depends on the orientation of weakness plane. But only one medium-valued failure line is considered in isotropic case.

Also, the size and shape of breakout zone in transversely isotropic rock depends on not only the failure criterion but also the boundary stress. The failure region becomes wider and thicker as the depth become deeper. The location of the borehole breakout is affected by the magnitude of the maximum principal stress (σ_1). Needless to say, the possibility of failure increases with the increase of the maximum principal stress.

4. CONCLUSIONS

In this study, the anisotropic Mohr-Coulomb criterion associated with critical plane approach was applied to predict the borehole failure in transversely isotropic rock. The size and shape of breakout zone in transversely isotropic rock depends on the failure criterion and boundary condition. The borehole stability analysis that does not consider the anisotropic behavior of rock can mislead to erroneous results and, therefore, it is necessary to consider the anisotropy in borehole stability analysis.

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