Constitutive modelling

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Elastic/viscoplastic constitutive equation for anisotropic shale

O.Cazacu, J.F.Shao & J.P.Henry
Lille Mechanics Laboratory (URA, CNRS), Villeneuve d'Ascq, France

N.D.Cristescu
University of Florida, Gainesville, Fla., USA

ABSTRACT : This study is aimed at formulating a general elastic/viscoplastic non associated constitutive equation for a stratified sedimentary rock : Tournemire shale. This rock can be idealized as transversely isotropic, the plane of isotropy being the bedding plane. The dependence of the elastic moduli on the stress state is expressed in terms of the invariants of the stress tensor. The viscoplatic criterion, the flow rule and a short-term failure criterion are expressed in a general invariant form. Finally the model is checked against data.

1 INTRODUCTION

Isotropic rocks seldom occur in nature. Oriented internal structures such as oriented crystallographic axis, grains fissuration, cracks, result on the macroscopic level in a directional mechanical response of rocks. Thus, the design and stability analysis of underground excavations requires a good knowledge of the mechanical behavior of such rocks.Generally, the experimental investigations were performed on transversely isotropic rocks (Donath 1972, McLamore&Gray 1969, Allirot and Boehler 1979, Niandou 1993, etc.). These investigations led to theories able to predict the dependence of strength on θ - the angle between the normal to the isotropic plane and the direction of the major principal stress.

A suitable framework allowing to describe both yielding and failure of anisotropic rocks with the required generality and pertinence was provided by the theory of representation of tensor functions. Invariant formulation of the perfect plastic behavior with application to stratified cohesive materials as well as for materials with internal friction have been developed by Boehler and Sawczuk (1977) and Boehler(1978,1987) using irreductible representations of tensor functions. To describe the behavior of initially anisotropic sedimentary rocks, a generalization of the Cam-Clay model has been proposed by Nova (1986) following the pioneering works of Hill (1950) and Olszack & Urbanowschi (1956).

In this paper we present an elastic /viscoplastic non associated constitutive equation for a stratified sedimentary rock Tournemire shale. From the experimental investigation conducted at Lille Mechanics Laboratory (URA CNRS No 1441) on this shale (Niandou 1994), we can conclude that the variation of the mechanical characteristics with respect to the oriented structure of the material may be considered as continuos. The initial anisotropy is taken into account in the structure of the yield function, in that of the flow rule and of the short term failure criterion by means of a constant anisotropic fourth order tensor. No a priori assumption is made concerning the existence of a viscoplastic potential. A procedure to determine the constitutive parameters from the data is shown. Finally, a comparison between model prediction and experimental results obtained in several triaxial tests on specimens of different orientations with respect to the major principal stress direction is presented.

2 STRUCTURE OF THE CONSTITUTIVE EQUATION

A material is regarded as being orthotropic at any point in a preferred reference configuration if there exist three orthogonal planes such that the mechanical properties of the material are symmetrical with respect to each of these planes. If one of the symmetry planes is isotropic the medium is said to be transversely isotropic. A good example of the above is Tournemire shale which may be idealized as homogeneous but transversely isotropic material, the plane of anisotropy being the bedding plane. We will define as structural reference frame, the coordinate system (S_1,S_2,S_3) where one axis, say S_1, coincides with the symmetry axis and consequently the isotropy plane is parallel to (S_2,S_3) (see Figure 1). Accordingly, the group of material symmetries consists of all the rotations about S_1 and the symmetries with respect to the isotropy plane (S_2,S_3) .

Thus :

$$g = \{ Q \in O(3) | \, Q(S_1) = S_1 \; or \; Q(S_1) = -S_1 \} \qquad (1)$$

or equivalently,

$$g = \left\{ Q \in O(3) | \, QMQ^t = M \right\}$$

with $M = S_1 \otimes S_1$. **M** is the second order structural tensor and O(3) denotes the full orthogonal group.

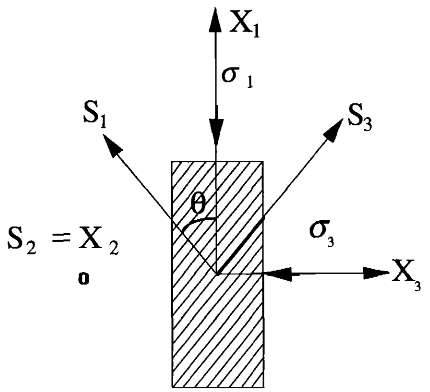

Figure 1 : Definition of the orientation θ

The instantaneous elastic response of the material is described by Hooke's law. The reference configuration used is the actual one, in both laboratory tests analysis and *in situ* underground excavations. Thus, stresses and strains have a relative meaning (Cristescu, 1989). In the coordinate system (S_1,S_2,S_3) the stress- strain relation is :

$$\begin{vmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{vmatrix} = \begin{vmatrix} \frac{1}{E_1} & -\frac{\nu_{21}}{E_2} & -\frac{\nu_{21}}{E_2} & 0 & 0 & 0 \\ -\frac{\nu_{12}}{E_1} & \frac{1}{E_2} & -\frac{\nu_{23}}{E_2} & 0 & 0 & 0 \\ \frac{\nu_{12}}{E_1} & -\frac{\nu_{23}}{E_2} & \frac{1}{E_2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2(1+\nu_{23})}{E_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{12}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{12}} \end{vmatrix} \begin{vmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{31} \\ \sigma_{12} \end{vmatrix} \qquad (2)$$

where E_1 is the Young modulus in the S_1 direction E_2 is the corresponding modulus in any direction in the isotropic plane, ν_{12}, ν_{21}, ν_{23} are the Poisson's ratios and G_{12} is the shear modulus in the (S_1,S_2) plane. As for most geomaterials, the static elastic moduli obtained during compression tests on Tournemire shale, depend on the stress state. As an example, we present the experimental values of E_1 on a grid in the deviatoric plane (σ,τ) (see Figure 2). σ represents the mean stress and τ the octhaedric stress defined by :

$$\begin{cases} \tau = \frac{1}{3}\left(2(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_1\sigma_2 - \sigma_1\sigma_3 - \sigma_2\sigma_3)\right)^{1/2} \\ \\ \sigma = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3) \end{cases} \qquad (3)$$

where σ_i are the principal stresses.

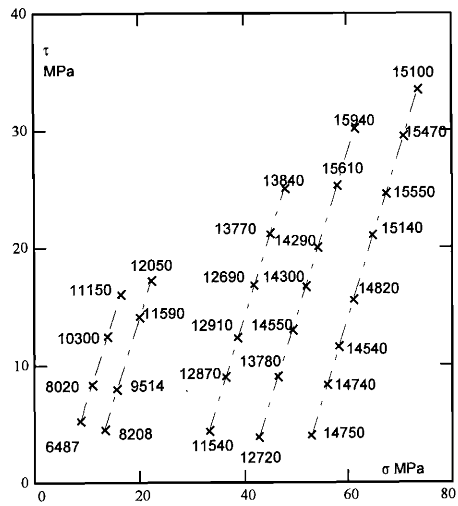

Figure 2 : Variation of E_1 with the stress state

In Figure 2, the interrupted lines show the loading paths followed in the Kármàn tests performed at θ=0°. The symbols correspond to the stress levels at which partial unloadings were performed.The results presented on the grid show that for a given confining pressure the modulus values increase with increasing τ ($\tau = \sqrt{2}(\sigma_1 - \sigma_3)/3$, for a Kármàn test) and at high pressures it seems that E_1 tend towards a constant limit value. We can also note that E_1 is influenced by the confining pressure. Since, for the 0° tests the load is applied at right-angles to the strata the modulus increase may be attributed to the compaction of these planes. However, as pressure increases and most of the pores are closed, E_1 approaches a constant limiting value.The tests were done such that the loading-unloading cycles are located at the same deviatoric level τ for different values of the confining pressure. Thus, we can analyze independently the dependence of E_1 on the stress invariants σ and τ. For τ =constant, E_1 increases with increasing σ. An exponential empirical law is proposed :

$$E_1 = E_{1\infty} - \alpha \cdot exp(-b\sigma) \tag{4}$$

$E_{1\infty}$ is supposed to be constant ($E_{1\infty}$=15900 MPa) The numerical values of b and α are determined for each level τ = constant by least-squares fit. It was found that b is essentially constant (b =0.0313 MPa^{-1}) while α depends on τ.The following approximation matches the data for Tournemire shale :

$$\alpha(\tau) = E_{1\alpha}\tau^2 + \frac{E_{1\beta}}{\tau + E_{1\delta}} \tag{5}$$

where $E_{1\alpha}$= 0.09 MPa^{-1}, $E_{1\beta}$ = MPa^2 and $E_{1\delta}$= 16 MPa. Concerning the other elastic moduli : the values of ν_{12} appear to increase with the applied deviatoric stress ; ν_{21} and ν_{23} seem also to increase slightly with the increase of the deviatoric stress. The Poisson's ratios ν_{21} and ν_{23} can be considered to be function of the mean stress only, while ν_{12} seems to be influenced by both σ and τ. The Young's modulus E_2 can be supposed to be a function of the mean stress only. We propose the following expressions :

$$\begin{cases} E_2 = 45000 - 2.35 \cdot 10^4\, exp(-0,0147\sigma) \\ \nu_{21} = 0.75 - 0.501\, exp(-0.0058\sigma) \\ \nu_{23} = 0.19 - 0.084\, exp(-0.0216\sigma) \end{cases} \tag{6}$$

The symmetry of the the compliance tensor lapses because the ratios ν_{12}/E_1 and ν_{21}/E_2 are quite distinct at low pressures but are close at higher pressures (see Niandou, 1994). For simplicity reasons, in our calculations we will suppose that: $\nu_{12} / E_1 = \nu_{21} / E_2$..

In principle, it is possible to measure a shear modulus during a compression test. However, there are practical difficulties to realize this, as the transversal strain gauges do not provide sufficient accurate information (these gauges are strongly influenced by the sliding of one plane over the other). For the 45° orientation G_{12} ranges between 2000 and 3000 MPa ; while at 30° is between 10000 and 11000 MPa. The variation of G_{12} as a function of the anisotropy orientation led us to question the validity of the shear coefficient estimated in this way . It was thus decided to calculate G_{12} using Saint Venant's approximation (Lechnitschii, 1963) :

$$\frac{1}{G_{12}} = \frac{1}{E_1} + \frac{1 + 2\nu_{21}}{E_2} \tag{7}$$

It can be shown that (7) means that G_{12} is equal to the shear modulus G_{45} for shear along the conjugated planes inclined at 45° and 135° angles to the plane of isotropy, and that furthermore the shear moduli on conjugate planes inclined at any ϕ and ϕ+90° angles to the plane of isotropy are equal to G_{45} (see Lekhnitskii 1963). Most of the published experimental data support the validity of Saint-Venant approximation (see Batugin and Nirenburg, 1972) but with major exceptions (see Worotnicki ,1993).

The limit of validity of Equation (2) will be given by a yield function whose expression is a priori unknown and will be determined on the basis of the experimental data. The basic assumption of our model is that the material is characterized by a fixed transverse isotropy i.e. the type of anisotropy does not change during the deformation process. The anisotropy is taken into account by making use of a fourth order tensor **A** which satisfies the usual symmetry conditions :

$$\boldsymbol{A}_{ijkl} = \boldsymbol{A}_{jikl} = \boldsymbol{A}_{klij} = \boldsymbol{A}_{ijlk} \tag{8}$$

and the general requirement of invariance under any orthogonal transformation which belongs to the symmetry group **g**. $\mathbf{A}_{ijkl}$ is supposed to be constant : it does not depend on time nor on the deformation. It is involved in the expression of the flow rule, of the yield function and of the failure criterion in the form of a transformed stress tensor Σ defined by :

$$\Sigma_{ij} = \boldsymbol{A}_{ijkl}\, \sigma_{kl} \tag{9}$$

Rather then making simplified assumptions concerning the form of the tensor **A**, as done by Boehler and Sawczuck (1970,1977,1978) we propose to retain all the five components of **A** as independent strength parameters. Therefore, in the structural frame (S_1,S_2,S_3), the truncated matrix of **A** is :

$$A = \begin{vmatrix} a & b & b & 0 & 0 & 0 \\ b & d & e & 0 & 0 & 0 \\ b & e & d & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{d-e}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{c}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{c}{2} \end{vmatrix}, \tag{10}$$

where *a, b, c, d* and *e* are material constants.

In conjunction with an anisotropic failure criterion (Cazacu and Cristescu 1995, Cazacu 1995) we determine the independent components of A from the strength characteristics of the rock in uniaxial and triaxial compression. The proposed criterion is expressed in the general invariant form :

$$\frac{3}{2}tr(\Sigma')^2 - \frac{m}{3}tr(\Sigma) - 1 = 0, \tag{11}$$

where m is a material constant, $\Sigma'$is the deviator of Σ This criterion is the generalization of Stassi's isotropic criterion to transversely isotropic conditions.

Unlike many other criteria, (11) is able to describe hydrostatic stress induced failure.We suppose that irreversibility is due to transient creep only. Thus the flow rule will be written in the form :

$$\dot{\varepsilon}^I = k\left\langle 1 - \frac{W^I}{H(\sigma)}\right\rangle(\Sigma + U(\sigma)I) \tag{12}$$

$H(\sigma)$ is the yield function with $H(\sigma) = W^I$ the equation of the stabilisation boundary ($\dot{\varepsilon}^I = 0$ and $\dot{\sigma} = 0$).W^I is the irreversible stress work per unit volume which is used as a work-hardening parameter:

$$W^I(T) = \int_0^T \sigma(t)\,\dot{\varepsilon}^I(t)dt \tag{13}$$

Further, in (12) $U(\sigma)$ is a scalar function and Σ the transformed stress tensor.The sum of these terms defines the orientation of $\dot{\varepsilon}^I$. k is a viscosity parameter which is supposed to be constant. However, for most rocks the viscosity coefficient may depend on stress or strain and maybe on a damage parameter accounting for the history of microcracking to which the rock was subjected. The bracket in (12) is the Macauley bracket used to denote the positive part of a function. There are two main topics to be addressed : the determination of the yield function $H(\sigma)$ and that of $U(\sigma)$.

3 DETERMINATION OF THE YIELD FUNCTION

The yield function $H(\sigma)$ is determined in two stages which correspond to the two stages of a standard triaxial compression test. Since in such tests the first stage is hydrostatic and the second one is deviatoric, we assume that $H(\sigma)$ is the sum of two terms :

$$H(\sigma) = H_h(\sigma) + H_d(\sigma) \tag{14}$$

such that $H_d|_{\sigma_1=\sigma_2=\sigma_3} = 0$. Thus, for hydrostatic conditions the yield function reduces to H_h which is supposed to depend on the mean stress only. The procedure to determine H_h is the following :

• First, from hydrostatic creep data calculate the irreversible hydrostatic stress work :

$$W_V(T) = \int^T \sigma(t)\dot{\varepsilon}_v^I(t)dt, \tag{15}$$

where $\dot{\varepsilon}_v^I$ is the irreversible volumetric rate of deformation.

• Then, plot the obtained values of $W_V(T)$ at stabilisation as a function of the mean stress σ. The following formula for H_h matches well the data :

$$H_h(\sigma) = c_0 \sin(\omega\,\sigma/\sigma_0 + \varphi) + c_1 \tag{16}$$

where : $c_0 = 0.38$ MPa, $c_1 = 0.377$ MPa, $\omega = 0.15$, $\phi = 283°$ and $\sigma_0 = 1$MPa.

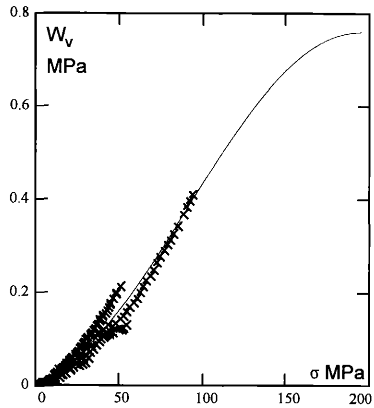

Figure 3 : Irreversible stress work versus pressure for Tournemire shale (x - experimental data)

Further we suppose that H_d is of the form :

$$H_d(\sigma, \mathbf{M}) = B_1 tr\Sigma + B_2\left(\frac{3}{2}tr\Sigma'^2\right) \tag{17}$$

where Σ' is the the deviator of Σ. In (17) B_1 and B_2 may depend on σ and τ. Using (8), (9) and (10) one can express the invariants of Σ in terms of σ and **M** as follows :

$$\begin{cases} tr\Sigma = (a+b-d-e)tr\mathbf{M}\sigma + (b+d+e)tr\sigma \\ tr(\Sigma')^2 = A(tr\mathbf{M}\sigma)^2 + B(tr\sigma)^2 + \\ C(tr\sigma)(tr\mathbf{M}\sigma) + D(tr\mathbf{M}\sigma^2) + \alpha^2 tr\sigma^2 \end{cases} \tag{18}$$

where :

$$\begin{cases} A = x^2 + 2\alpha x + 4\beta x + 3u^2 + 2xu + 4\beta u + 2\beta^2 \\ B = y^2 + 3v^2 + 2yv + 2\alpha v \\ C = 2xy + 2\alpha y + 4\beta y + 6uv + 2(xv + yu) + 2u\alpha + 4\beta v \\ D = 2\beta^2 + 4\beta\alpha \end{cases} \tag{19}$$

and

$$\begin{cases} x = a + d - 2c - 2b \\ y = b - e, \\ \alpha = d - e, \\ \beta = c - (d - e) \\ u = \frac{1}{3}(2b - 2e - a + d) \\ v = \frac{1}{3}(2e - b - d) \end{cases} \tag{19)'$$

From equations (16) to (19) follows that H is a scalar valuated isotropic function of σ and $\mathbf{M}$ and thus the combined objectivity-symmetry requirement is fulfilled.The invariants $tr\mathbf{M}\sigma$ and $tr\mathbf{M}\sigma^2$ account for the directional character of the response of the rock when subjected to a mechanical loading.They make precise the orientation of the principal stress directions with respect to the structural frame (S_1,S_2,S_3). Concerning the functions B_1 and B_2 involved in the expression of H_d, these are determined from data obtained in the deviatoric part of triaxial tests performed on specimens at θ=90° for several values of the confining pressure. For a given confining pressure, B_1 and B_2 are determined with the least square method from the equation :

$$B_1\left[tr\Sigma(\sigma,\tau,90) - tr\Sigma(\sigma,0,90)\right] +$$

$$\tfrac{3}{2}B_2\left[tr\Sigma'^2(\sigma,\tau,90) - tr\Sigma'^2(\sigma,0,90)\right] - W'(T) = 0 \tag{20}$$

where :

$$W'(T) = \int_{T_H}^{T}\left(\frac{3}{\sqrt{2}}\tau\right)\dot{\varepsilon}_1^I\,dt + \int_{T_H}^{T}\sigma_3\dot{\varepsilon}_v^I\,dt \tag{20)'$$

T_H represents the beginning of the deviatoric part of the test. It was found that B_1 depends on the confining pressure and can be approximated with :

$$B_1(\sigma_3) = b_{11} + b_{12}\sigma_3^2 \tag{21}$$

where b_{11} = -0.08688, b_{12} = $-9.714\cdot10^{-5}$ MPa^{-2}.
The coefficient B_2 can be considered to be constant : B_2 = 0.0065MPa^{-3}. Since $\sigma_3 = \sigma - \tau/\sqrt{2}$, it follows that the expression of the yield function in terms of invariants is :

$$H := \left[b_{11} + b_{12}(\sigma - \frac{\tau}{\sqrt{2}})^2\right]tr\Sigma + B_2\left(\frac{3}{2}tr\Sigma'^2\right) +$$

$$c_0\sin(\omega\frac{\sigma}{\sigma_o} + \varphi) + c_1 \tag{22}$$

Thus, the yield function is completely determined (obviously not in a unique way).The shapes of the yield surfaces for various values of θ (θ = 0°, 60°, 45° and 90°) are shown in Figure 4. Also, the corresponding failure surfaces calculated using the anisotropic failure criterion (11) are represented. As expected the model exbits compressibility only. The material responses for θ=0° and θ=90° are quite close and it is just the behavior around 45° that departs significantly from that at 0° and 90° (mainly what concerns failure).The anisotropy is well described : for the same value of W yielding takes place at lower values of τ for 45° than for 0° and at even higher values for 90°.

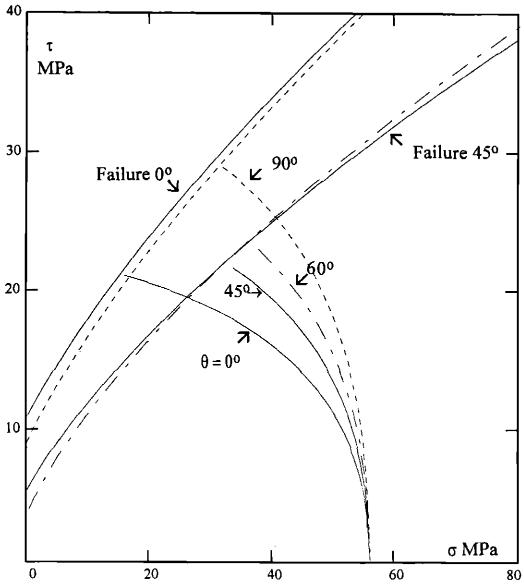

Figure 4 : Predicted yield loci and failure surfaces for different sample orientations θ

4 DETERMINATION OF U(σ)

We suppose that U(σ) is the sum of two terms :

$$U = U_h(\sigma) + U_d(\sigma,\tau) \tag{23}$$

such that $U_d(\sigma,0)$=0.The first term, $U_h(\sigma)$ can be determined in hydrostatic tests in conjunction with the formula :

$$U_h(\sigma) = \frac{\dot{\varepsilon}_2(a+2b) - \dot{\varepsilon}_1(b+d+e)}{\dot{\varepsilon}_1 - \dot{\varepsilon}_2}\sigma \tag{24}$$

where $\dot{\varepsilon}_1^I$ is the rate of the irreversible deformation in the S_1 direction and $\dot{\varepsilon}_2^I$ is the rate of the irreversible deformation in the bedding plane. For Tournemire shale the following approximation for the variation of U_h with σ applies :

$$U_h(\sigma) = u_{1h}\sigma^2 + u_{2h} \qquad (25)$$

where $u_{1h} = 1.098 \cdot 10^{-4} MPa^{-1}$ et $u_{2h} = 0.025 MPa$. $U_d(\sigma,\tau)$ is determined in the deviatoric part of triaxial tests performed at several confining pressures (σ_3=5,40 and 50 MPa) on specimens at $\theta=90°$.The following formula is used in conjunction with the data :

$$U_d(\sigma,\tau) = \frac{\dot{\varepsilon}^I_{3//}(\Sigma_{11}) - \dot{\varepsilon}^I_1(\Sigma_{22})}{\dot{\varepsilon}^I_1 - \dot{\varepsilon}^I_{3//}} - U_h(\sigma) \qquad (26)$$

$\dot{\varepsilon}^I_{3//}$ is the rate of the transversal irreversible deformation parallel to the bedding while $\dot{\varepsilon}^I_1$ is the axial irreversible deformation (see Figure 5).We found that U_d can be approximated by :

$$U_d(\sigma,\tau) = \left[n_1 + n_2\left(\sigma - \frac{\tau}{\sqrt{2}}\right)\right]\tau + \left[m_1 + m_2\left(\sigma - \frac{\tau}{\sqrt{2}}\right)\right]\tau^2 \qquad (27)$$

where m_1=0.026 MPa^{-1}, $m_2=-2.6 \cdot 10^{-5} MPa^{-3}$, n_1= -0.0012, $n_2=3.579 \cdot 10^{-5} MPa^{-1}$.

5 COMPARISON BETWEEN CALCULATED AND EXPERIMENTAL RESULTS

The model has been checked against experimental data obtained in hydrostatic and standard triaxial compression tests. Figure 5 shows a comparison between model prediction and experimental results in a hydrostatic test. The anisotropy of deformation under isotropic conditions seems to be well described by the model. As an example we also present the stress-strains curves obtained for two sample orientations. The position of the gauges for $\theta \neq 0°$ is shown in Figure 6. Each triaxial test is identified by two numbers as follows : (Orientation of the bedding plane θ) / (Confining pressure (MPa)) (Figures 7).Let us note that these tests have not been used for the identification of the constitutive parameters. Although for the ε_{3p} strain component the quantitative agreement is not so good, the overall prediction of the model is reasonable. Particularly, the rock anisotropy is clearly and correctly described.

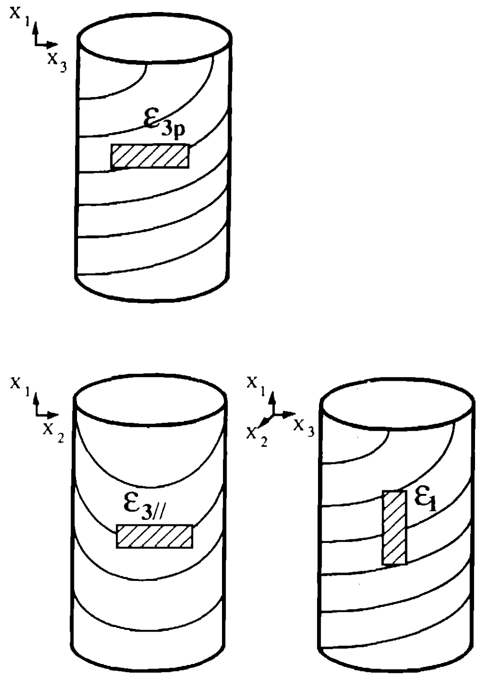

Figure 5 : Position of the gauges for $\theta \neq 0°$

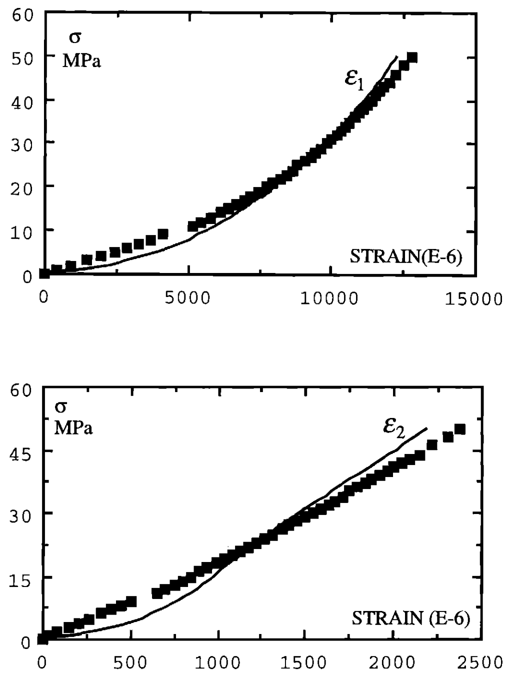

Figure 6 : Hydrostatic compression test

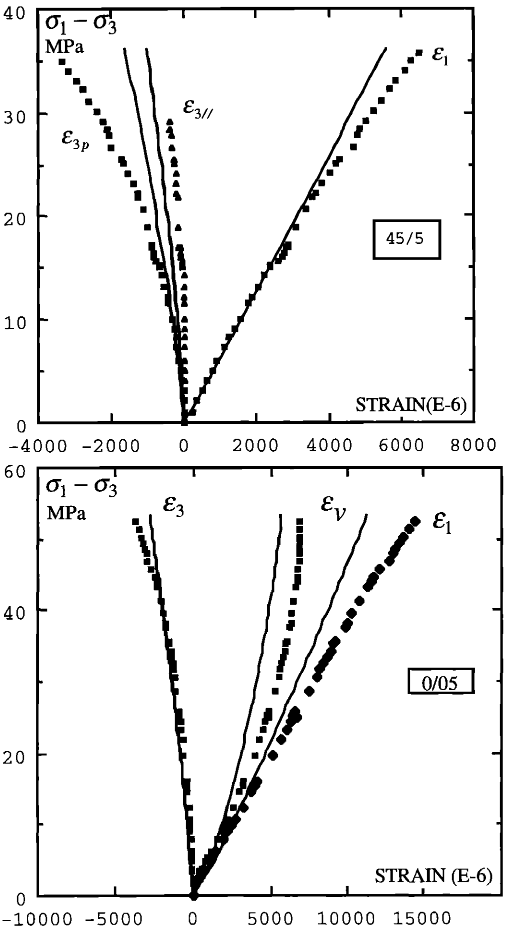

Figure 7 : Stress-strain curves in triaxial tests

CONCLUSIONS

This paper deals with the formulation of an elastic/viscoplastic constitutive equation for transversely isotropic rock. The proposed model describes reasonably well some important features of the behavior of anisotropic rocks subjected to a hydrostatic compression. So far the validity of the theory has been checked in triaxial compression tests only. An experimental program is presently under way in order to check whether or not the model is versatile enough to describe the material behavior for more complex loading paths such as : proportional tests and long-term triaxial creep tests. It will allow us to improve the model, if necesarry. In the future the model will be used for the stability analysis of underground openings.

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