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Robustness in supply chain management and design

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Master of Science in Physics and Mathematics
Submission date: June 2010
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Problem Description

The purpose it to examine network design problems with uncertainty in demand

1. Study network design problem with uncertainty in data and formulate a suitable optimization model.
2. Find data for a real case for heating plants at a forest biofuel company and the network design of the terminal structure.
3. Develop a suitable solution method based on robust optimization.
4. Analyze the solution approach and its potential use in practical planning.

Assignment given: 18. January 2010
Supervisor: Brynjulf Owren, MATH

Preface

We are two students at the Norwegian University of Science and Technology (NTNU), who will finish our Master of Science in Engineering degrees with this master thesis; Arnt Inge Enoksen at the Department of Industrial Economics and Technology Management and Martin Aspebakken Sværen at the Department of Mathematical Sciences. It has been carried out at NTNU between January and June 2010. The thesis is written together but will be seperately delivered at the different departments.

The thesis deals with robust design of forestry supply chains. Such supply chains may include a large amount of data and factors that need to be taken into consideration. We have developed optimization models based on deterministic, stochastic and robust optimization approaches. Such models may become very large and difficult to solve on regular computers, and we have therefore worked to reduce the size of the problems and to increase the solution times, by use of aggregation, preprocessing and a LP-relaxation based heuristics, that are all used and tested in this thesis.

We would especially like to thank our supervisor, Professor Mikael Rönnqvist, for his good advices and help with the thesis. We also wish to thank Patrik Flisberg (Skogforsk) for help with providing and interpreting data. We also wish to thank Skogforsk, Sveaskog and Stora Enso for the data they provided us.

Trondheim, June 9, 2010

Arnt Inge Enoksen   Martin A. Sværen

Abstract

We have studied a supply chain for delivering biomass to heating plants where there are uncertainty in demand. Design of such supply chains considers finding the optimal terminal structure and an optimal inventory policy. Such problems may become complex and difficult to solve manually. We have therefore developed optimization models based on three approaches in order to solve the problem; a deterministic optimization approach with preset required safety stock levels, a robust model and a stochastic model.

We have used these models to solve one test case and two large industrial cases. As fixed costs of opening terminal were unknown, the models were runned with a preset number of opened terminals. In order to solve the two industrial cases on a normal computer we needed to reduce the size of the problem. This was done by use of supplier and assortment aggregation, arc-removal, presolving and an LP-relaxation based heuristic. We have shown that if the proper aggregation approach is used, the use of aggregation could reduce the size of the problems considerably while only marginally reduce the quality of the solutions. The LP-relaxation based heuristic speeded up the solution times, but the solution quality may become poorer when the number of opened terminals is low.

We have found that robustness could be achieved by increasing the capacities in the supply chain, increasing inventories or by deciding on the optimal terminal structure. The stochastic and robust optimization approach returned higher objective values than the deterministic approach. The reason seemed to be the increased safety stock levels used in the deterministic approach.

The stochastic and robust solutions were evaluated by use of the value of stochastic solution (VSS), and value of robust solution (VRS). The two approaches returned solutions which performed better in the future than the deterministic model, and the models gave similar solutions. The stochastic model used less memory and less solution time.

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1 Introduction

In times of the ongoing global climate change debate, renewable energy sources have gained new popularity. Governments are now seeking opportunities to replace fossil fuels with renewable energy sources, such as wind energy, solar energy and biomass. One of the countries that have succeeded in this aim is Sweden. In the last three decades Sweden has undergone a change from depending on oil and fossil fuels for heating, to increasingly utilizing biomass and other renewable energy sources.

The motivation behind the aim of Sweden has been twofold. In the 80s, the goal was to minimize the country's oil dependency, but today, biomass is increasingly used as a means for reducing the country's emissions of climate gasses. District heating accounts for about 40 % of the heating market, and now more than 62 % of the utilized fuel in district heating is biomass[1]. Their use of biomass fuel is expected to grow the coming years. Of similar reasons, use of biomass is expected to grow in other countries as well, such as Austria, Denmark, Finland, Germany, Norway and USA.

The most popular bio fuels are mainly wood fuels, ethanol, black liquors and tall oil pitches. One of biomass' main advantages is that it is a very versatile energy source, generating not only electricity, but also heat and bio fuels that could be utilized in the transportation sector. It is also one of the few renewable energy sources that may be stored and generate energy on-demand.

The Swedish forest institute believe that harvest of primary forest fuel could be doubled or more, but is limited by an efficient logistic planning. [Flisberg et al., 2010]. Due to new extraction methods and economies of scale due to larger harvested quantities, production costs have decreased the last years. But prices have also remained low, despite price increases of competing fossil fuels. The challenge for the biomass fuel is therefore to improve the efficiency of the logistic processes in order to keep costs lower in order to compete against other fuel types.

Logistical costs have a great influence on the production costs. Optimization models have therefore been developed to increase the quality planning in the supply chains. The Forestry Research Institute of Sweden, Skogforsk, has developed several planning systems for use in forestry. This includes, FlowOpt, a decision support tool transportation planning, FuelOpt, a decision support tool for planning forest fuel logistics, and RuttOpt, a decision support tool for routing of logging trucks. These systems could be used for improving the logistics at a operational and a tactical level, and they could also be used for analyzing changes in the supply chain design.

[1] The Government Offices of Sweden, http://www.sweden.gov.se/sb/d/5745/a/19594, accessed 02.03.2010.

[Frisk and Rönnqvist, 2005] [Flisberg et al., 2010] [Andersson et al., 2007]

Bredström & Rönnqvist developed a model for solving tactical decisions in a biomass supply chain with one assortment. They did not consider the configuration of the terminals. They developed a robust optimization model for this problem, and they developed and tested an algorithm for solving LP-problems with uncertain parameters on this problem. [Bredström and Rönnqvist, 2008]

In the author's project thesis [Enoksen and Sværen, 2009], we developed a formulated a supply chain design problem to find optimal terminal structures. We also looked at the number of terminals and what level of safety stock that were needed in order to make supply chains robust under uncertainty in demand

There has been limited amount of work on supply chain design of forestry supply chains. Such supply chains often face uncertainty in demand. The challenge is to take this uncertainty into consideration when designing supply chains in a way that maximizes both robustness and cost efficiency. The design includes finding the optimal terminal structure and their inventory policy. Even though the FuelOpt model found in [Flisberg et al., 2010], mainly is designed for tactical decisions in a supply chain, it could also be used to analyze changes in the supply chain design. FuelOpt uses deterministic optimization, and uses preset required safety stock levels to handle uncertainty. However, uncertainty is only assumed and not modeled. This could results in a supply chain design and an inventory policy that perform well on paper, but could result in excess costs and difficulties in fulfilling demand as demand changes in the future.

An alternative is to use stochastic or robust optimization. These approaches take uncertainty into consideration. We have studied the biomass supply chain design problem, and developed optimization model based on all three approaches. These have been used to solve two real cases. We have also developed methods in order to solve the models on regular computers. The approaches and their potential use in practical planning are evaluated in the end of this thesis. When considering terminal inventory policy, we will limit the work to look at optimal inventory levels for the different terminal structures.

We will in Section 2 and 3 present the problem and theory about biomass supply chains. In Section 4 and 5 we formulate the optimization problem. Section 6 and 7 show how we have implemented our models and used different methods in order to reduce and solve the models. Our results are shown in Sections 8, 9 and 10 and we evaluate and discuss the models and results in Section 11.

2 Problem description

2.1 Description of supply chain

The biomasses we will consider are solid wood fuels that are extracted from forests. It is produced by planting trees that are later cut down. The logs are usually transported away once they are cut, while tops and branches usually remain in the forests. The logs or pulpwood could be utilized as timber, paper or biomass fuel, or other purposes. The leftovers could be harvested and used to produce energy as biomass. Up to a year later these leftover are therefore harvested, and processed to either wood chippings or other biomass assortments.

The processing of harvested wood into biomass is shown in Figure 1[2]. The processing occurs either in the catchment areas or at terminals. Wood chippings could be produced by chipping and drying the wood. The non-refined non-dried wood chippings usually have humidity as high as 55 %, and could only be burned in larger heating plants. As the non-dried wood chippings may rot due to humidity, it needs to be dried at suitable areas. As the wood is dried, the humidity is reduced to about 15 %, and could give a useful heat of 2000-2600 kWh/fm^3 dependent on wood type and humidity[3] [4]. Wood chippings could also be extracted from residue from industrial processes, but may require different processing to become useful for heating plants.

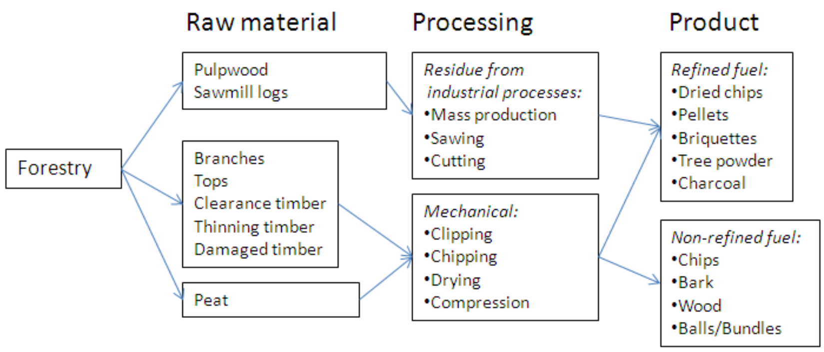


Figure 1: Processing of solid biomass fuels extracted from the forest

A biomass supply chain consists of suppliers, terminals and customers. The

[2]Based on a figure from ENOVA. `http://www.fornybar.no/file.axd?fileID=4`, accessed 30.5.2010, 13:20

[3]1 fm^3 = 1 solid cubic meters

[4]Helmer Belbo: "Harvesting and production of chippings", slides from Bioenergy conference in Molde, 28-29.4 2007. Extracted from `http://www.google.no/url?sa=t\&source=web\&ct=res\&cd=2\&ved=0CBkQFjAB&url=http%3A%2F%2Ffylkesmannen.no%2FSkog_og_landskap_Hc7ok685739ap.pdf.file&rct=j\&q=Innh%C3%B8stingsmetoder+og+produksjon+av&ei=hV_9S7KEFsX6-QbmhNHECw\&usg=AFQjCNF12NtAUDI_cj7vvhc7RadM-4NyQQ`, accessed 27.5.2010, 23:50

(a) Processing machinery (b) Cutting machinery

Figure 2: Machinery used in the gathering and processing of wood, pictures provided by Skogforsk

suppliers and the terminals are contracted to or owned by the respective company. Processing of assortments and building inventory could be performed at all of the parts of the supply chain, but inventory at the customers is usually not allowed.

The supply points could be of different sizes and supply. The processing is usually done by dedicated machinery. At the forest sites, different sized forwarder-mounted chippers, truck mounted chippers, bundlers and modified forwarders could be used for the processing, while large wood crushers could be found at the terminals, as shown in Figure 2. The harvested wood could also be sent directly to the demand points without any processing based on customer demand.

The terminals receive assortments from the suppliers, and may choose to store the assortment to fulfill later demand, or process assortments. Terminals are used for processing, and in most cases the greatest part of the processing machinery exists at the terminals. They are also used as transshipments points in order to utilize economies in scale when transporting to customers. The capacity of existing terminals could usually be expanded if needed.

Several types of systems may be used to transport the biomass to the heating plants. If the wood is chipped out in the forests, large chip trucks may be used to transport it directly to the customers. Container solutions, where a container is filled up in the forest, while another one is transported away, could also be used. If chipping is done at terminals or customers, logging trucks for clash bundles or special forest residue trucks may be used. One kind of truck combines the chipping and transportation of the chips by use of a mounted chipper. These systems may also be combined with train transport, as described in [Flisberg et al., 2010]. Transportation and processing capacities are limited by available work hours for the crews.

The wood is brought to district heating plants, where heat is generated by use of either surplus heat from industries or through combustion of biomass or other fuels such, oil or garbage. The plants distribute heat to industries and residents. Some systems may also produce electricity by cogeneration; combined generation of electricity and heat. Demand is given in assortment groups. This means that demand could often be fulfilled by more than one type of biomass assortment. One example is that assortment groups, such as saw logs, pulpwood, and forest residues could also be divided into subgroups based on qualities or dimensions, or that demand for wood chippings could be covered by chippings both from grot and logs. [Gunnarson, 2007]

2.2 Planning and decision process

Delivery of biomass fuel is based on the customers' demand. Each month the customers submit their orders. These orders are based on the demand for heat, which varies from month to month as the seasons change. But demand for heat would also vary due to temperature or weather changes, or for other reasons. The demand for biomass could therefore be hard to predict, and the true demand would only be known when the customers have submitted their orders.

At the start of the year, the supplying company and the customers agree on preliminary monthly volumes for the different biomass fuels. The fixed volume for the full year is fixed at this time. The customers are allowed to order quantities which lay within a contracted percentage above or under agreed monthly volume, but this deviation is to be cancelled out through the rest of the year in order to not deviate from the contracted annual volumes. There is therefore uncertainty in demand for the forest fuel supply chain, but this uncertainty is bounded and limited to upper and lower bounds. Also, one month ahead, the customers must specify the coming month's demand.

Planning levels are often divided in three: strategic, tactical and operational. [Chopra and Meindl, 2007]. Strategic decisions involve long-term decisions such as supply chain design, while tactical decisions decide on how to exploit the existing supply chain most efficient and operational decisions involve the day- to-day-planning, i.e. routing of trucks.

Tactical decisions, such as deciding on inventory and processing levels, are usually made by use of rolling horizon planning. The planning horizon is then divided into time periods, e.g. a year is divided into months, and information on coming demand and supply is used to plan inventory and processing for the coming months. However, as uncertainty may lead to changes in the used data for later months, the decisions

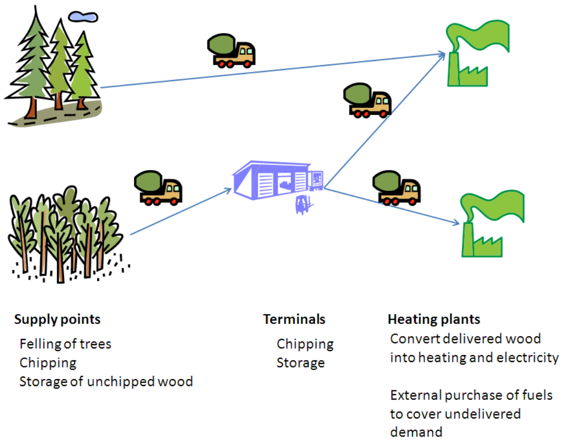


Figure 3: An overview of the biomass supply chain

regarding later months are not implemented. Only decisions regarding the time period close to when the planning is done is implemented. At the end of the month, information is updated and the planning is redone for the remaining months.

A strategic decision is the supply chain design. This could affect the supply chain profits and how well it handles sudden changes in demand. The design basically concerns the terminal structure, i.e. how many terminals to be used and where they are to be located. Inventory could be stored at the terminals in order to exploit transportation and processing capacities more efficient in high-demand periods. But inventory and terminals would incur costs, and terminals may also face investment costs. An optimal terminal structure with an appropriate inventory policy is therefore important in order to find the optimal balance between costs and robustness.

3 Designing biomass supply chains

The decision that is considered to have the most significance on a supply chain's performance, is its design. Supply chains are designed with a time horizon of several years, and such decisions are therefore regarded as strategic. These decisions involve the configuration of the chain, allocation of resources and location of processes [Chopra and Meindl, 2007]. A major challenge in supply chain design is to respond to changes in prices, demand and other uncertain conditions for the coming years. If facility are given inefficient locations, it could result in excess costs being incurred throughout the lifetime of the facilities, even though decisions on production plans, transportation options, inventory management, and information sharing are optimized in response to changing supply chain design. [Daskin and Berger, 2005]

Biomass supply chains have some features that separates them from other types of supply chains. They may have a high number of sources with small quantities of supply, demand for biomass are often subject to seasonality, the quality and supply of the raw materials may vary, and there may be a need for specialized transportation and storage of the raw materials. Also, the sources may in some time be non-active due to need to e.g. regrow forest. Design of biomass supply chains should make sure that there is supply to meet customers' demand the entire year, and that correct levels of biomass raw materials are delivered. [Fiedler et al., 2007] The costs in a biomass supply chain also differs from other supply chains, as the products often are of low value, and the dominant part of the costs are connected to the logistics of the biomass. Decreased logistic costs are therefore particular important in order to decrease biomass production costs.

According to [Fiedler et al., 2007], location of terminals should depend on:

- The position of catchment areas
- The processing equipment at the production site
- The existing and planned traffic infrastructure
- The infrastructural conditions and connection to traffic networks
- The inventory capacities at the production and the terminal sites
- The seasonal availability of biomass
- The fluctuations in demand
- Planning and controlling expenses

Design of biomass supply chain could therefore become a difficult and challenging task, as the number of factors to take into consideration is very large. Uncertainty will further increase this difficulty. For more information on biomass supply chains, the articles by Fiedler and Eksioglu discuss many of the challenges with biomass logistics, and particularly the balance between long-term strategic decisions and medium & short decisions on a tactical and operational level. [Fiedler et al., 2007] [Eksioglu et al., 2009]

3.1 Creating robust supply chains

As there is uncertainty in demand, we must ensure that the supply chain is robust. Robustness is defined as being capable to perform well against uncertain future conditions. [Snyder, 2003] Robustness considers external variations such as changes in demand or production costs, while reliability considers internal variations, such as if a terminal breaks down because of fire or a supplier shuts down. Even though planners are reluctant to consider robustness and reliability at design time,"large improvements in reliability and robustness can often be attained with only small increase in the cost of the supply chain network." [Daskin and Berger, 2005]

Robustness is usually handled through facility or inventory decisions or investment in capacity. Capacity investment would increase the supply chain's ability to respond to changes, while increased inventory would be used to buffer against variations in demand. Facility decisions, such as locations and capacities for plants or warehouses, could also serve to increase robustness. As such decisions often incurs investment costs, and often involve a long time horizon, they are hard to change on a short time horizon. [Enoksen and Sværen, 2009]

3.2 Terminal location structures

There are in principle two forms of terminal structures: a centralized with few terminals, and a decentralized one with several terminals. For the first case, the chipping would be centralized and cover a large area. In the second, chipping would be spread out over several terminals, and serve regional customers and suppliers.

One advantage of using a structure with a low number of terminals would be that costs associated with running the terminals are reduced. Safety stock levels could also be reduced due to risk pooling. Such a structure is however only feasible if transportation capacity is quite high.

One advantage of using a larger number of terminals, is that inventories may be situated closer to customers. As transport capacity is often limited, this increases robustness, as demand in peaking periods could be stored at the terminals, and the transportation capacity could therefore be more efficient utilized as distances to customers decreases. This increases the robustness of the supply chain, but increases costs. If processing capacity at the suppliers is restricted, it could also save transportation costs of locating the terminals close to supply points, as the wood demanded does not need to take unnecessary detours because of processing.

3.3 Inventory policy

Inventory is usually created in order to store finished product that are soon to be delivered, or for balancing transportation and processing capacities in the supply chain. The last point is true for the terminal inventory. It can be used used to store assortments closer to the customer in order to be able to deliver to the costumer in case of peaks in demand that the supply chain otherwise would not handle. We would therefore expect increased inventory levels if transportation or processing capacities are limiting. Inventory could also be used for gaining economies of scale in production and transportation.

Required safety stock is used to balance expected demand with uncertainty. One option is to have a fixed preset level of safety stock for all the time periods. This can be very inefficient as such levels at given time periods could be considerably higher than demand. In order to better handle demand variations, one option is to have a safety stock that follows the expected demand. [Flisberg et al., 2010] use a given level of the future expected demand. In communication with the authors we learned that they used 40 % of the expected demand in the next period as a level for the required safety stock.

One other option is to use a robust or stochastic optimization model to find optimal required safety stock levels by extracting the lowest inventory built by the models over the scenarios. The main trade-off in choosing safety stock levels is the increased robustness of the supply chain versus the costs this inventory induces.

3.4 Optimization approaches

3.4.1 Deterministic optimization

Deterministic optimization is the most common optimization approach. It uses the expected value of all the given data, and the solutions are therefore called the expected value solutions. This approach gives smaller data models as uncertainty in data is not included. In order to use a deterministic model in supply chain planning under demand uncertainty, a deterministic level of safety stock can be introduced in the model.

3.4.2 Stochastic optimization

If the future is hard to predict, and data may be uncertain, deterministic optimization could return solutions that would perform rather poorly in the future. If we have some kind of knowledge on how the future may develop we can use stochastic optimization. Stochastic optimization does a trade-off between different future scenarios of input data to find the best feasible solution. This solution would not be optimal in all future realizations of data, but would be the best solution given the uncertainty that surrounds the future.

Future realizations of data are usually modeled as scenarios, and those are assumed connected to a probability distribution. Variables are divided in two categories: anticipative and non-anticipative. Anticipative variables are used for decisions taken after data uncertainties have been revealed, while non-anticipative solutions are taken as first stage decisions before this uncertainty is revealed.

Stochastic optimization solutions could be evaluated by the use of the expected value of perfect information (EVPI), which is the difference between the wait & see-problem and the stochastic solution. The wait & see-problem corresponds to solving the stochastic optimization problem for all scenarios, but the non-anticipative variables are made anticipative, i.e. given a value for each scenario. This would evaluate the solution on how well it performs against all future realizations of data.

One other form of measurement used in stochastic optimization is the value of stochastic solution (VSS). VSS is calculated by first running the stochastic program, and then running the deterministic model and import the terminal structure and the required safety stock levels as input to the stochastic model. VSS is then found as the difference between the stochastic solution and the inputted deterministic solution. VSS gives information on when the stochastic models give better solutions than the deterministic models.

A disadvantage with stochastic optimization is that information on the correct parameters and information on the stochastic distribution functions may be difficult to provide. [Bredström and Rönnqvist, 2008] One approach for overcoming this problem is to replace the stochastic distribution functions in the recourse function with the sample mean,[Higle, 2005]. A prerequisite is that we have complete recourse, i.e. the recourse subproblem is feasible for all outcomes of earlier decisions.[Higle, 2005] This approach is dependent on the scenarios drawn, and is therefore affected by error. The stochastic optimization models could also become very large and difficult to solve.

3.4.3 Robust optimization

The purpose of robust optimization is to find solutions that are feasible for any possible future outcomes. Ben-Tal and Nemirovski use the term "conservativeness" about robust optimization-models as an indicator of how much the objective function worsens in order to gain robustness. They confirm this by a study showing that real world LP-problems can be severely affected by small perturbations of the data, while the robust optimization-methodology is used with success. [Ben-Tal and Nemirovski, 2002]

Robust optimization-problems can become very large and in some cases also NP-hard. In these cases [Ben-Tal and Nemirovski, 2002] suggests using an approximate robust model instead of the true one. As robust optimization-models do not find the optimal solution, using an approximation which might give a slightly worse solution, should not further deteriorate the solutions.

In this thesis, we will use the minimize worst regret method which maximizes the worst profit and hence all future profits will be larger than the solution of the robust optimization. Either discrete scenarios or continuous ranges could describe uncertain parameters. [Snyder, 2006] We assume that these scenarios or ranges contain the future worst case. And as long as this assumption holds in the future, the robust model will give solution that could be used in the future, and the profit will always be the same or better than the solution, as it is given for the worst case.

In order to evaluate how good the robust solutions are, we compare them to the deterministic solutions, and we introduce the value of the robust solution (VRS). In order to calculate the VRS we have used a methodology that is inspired by the method of the value of stochastic solution (VSS). We use the solution of the terminal structure given from the deterministic model as input in the robust model together with the deterministic required safety stock levels. We find the worst case solution for the deterministic terminal structure and compare that with the corresponding solution with the same number of terminals from the robust model, which will show how much better robust optimization is compared to the deterministic model in finding robust terminal structures that will perform good in the future.

3.5 Summary

The supply chain for delivering forest fuels to district heating plants consists of several steps. Roughly, it consists of several large catchment areas of different sizes, terminals for storage and processing, and heating plants that burn the wood. Our challenge is to design a supply chain for the biomass that is both robust and creates the largest profits. Two of the most important decisions for accomplishing these goals are the terminal structure and the terminals' inventory policy. As the supply chains may be very large and cover a large amount of suppliers customers, and potential terminal locations, optimization should in some cases be considered used as decision support, as the design problem could become difficult and complicated.

As the stochastic and robust optimization approaches behave better in the future we may expect them to provide more profits and more robust solutions than the deterministic approach with safety stock, but such problems may become very large. Solving these problems may take considerable more time and may in some cases become too large to be solved on regular computers. We will therefore try to compare these approaches and look at methods that could help solving such problems.

4 Modeling

In the forest supply chain network, there are three different types of nodes; the suppliers, terminals and demand points. Transportation of biomass assortments is modeled as flows between the nodes. The flow is allowed to go either directly from the suppliers to the demand points or via the terminals. Processing of the biomass into the different assortments is allowed at the suppliers and the terminals.

As the objective is to decide on which terminals to open, the problem could be classified as a facility location problem. Facility location problems choose which of a proposed list of facilities to open in order to service specified customer demands at minimum total costs.[Rardin, 2000] The problem could also be called a warehouse location problem. It could also be argued that the problem is a network design problem, but network design or fixed-charge network flow models decide which arcs to open, while facility location models decide which nodes of a network to open.

4.1 Arc formulation

We have chosen to use an arc formulation for the network design problem. An advantage by using this formulation is that we do not create variables for flow between nodes that are not naturally connected in real life, e.g. due to very long distances or obstacles such as rivers or mountains. However, this requires that all possible roads are defined in advance. The alternative would be to create flow variables between all nodes.

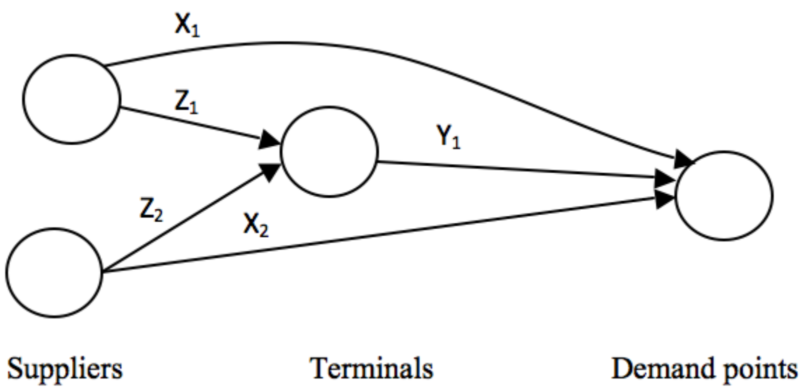


Figure 4: By using an arc formulation, all arcs are given its own name. In this case, the flow alternatives are either to ship directly from the suppliers to the customer, or via a terminal

We have defined each arc by its start and end point, and divided them in four types based on the type of nodes they cover, as shown in Figure 4. We could also have included time periods and truck type used, but chose not to do this as this could be handled by the use of sets. Arc flows are given by the energy supplied from the wood, given by MWh.

4.2 Assortments and assortment groups

A demand for a certain assortment group can be covered by a supply of different assortments. This allows more flexibility in the supply chain. In order to model this we use the subset H_g^G, as given in table 1, which is the assortments which could fulfill the demand of assortment group g. An example is that all wood chipping types belongs to an assortment group, consisting of the assortments of chip from logs and chips from from small branches. In some cases we need the inverse set, i.e. the set of groups the assortments could fulfill, $\mathcal{G}_h^H$.

4.3 Processing

The processing of wood take place at three different locations: at the catchment areas, at the terminals and on chipping machines mounted on combo trucks. In some cases processing may also take place at the demand points, but this is unusual and we have therefore chosen not to include this in our models. The processing could be modeled as a process where quantities of an assortment is sent in to a node, and exits the node as one or several forms of assortments.

Sometimes we may experience that some of the wood get lost in the processing or become by-products in the process that could be sold for profits. This is modeled with the use of the constant $f^c_{h'hn}$. Successful convertion from one assortment to one other could then be modeled by setting $f^c_{h'hn}$= 1, as this is the ratio of quantities that is successfully converted from assortment h' to assortment h.

4.4 Node balances

If we define a node for each combination of (i,h,t) where i is the node, h is an assortment and t is a time period, we get the node balance shown in figure 5.

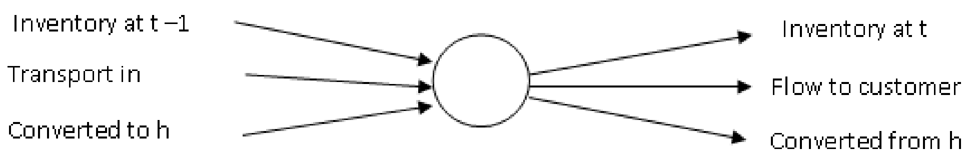


Figure 5: Node balance for a node defined for (i,h,t)

Table 1: Sets used in the models

Sets	
$\mathcal{G}$	Set of assortment groups, $g \in \mathcal{G}$
$\mathcal{H}$	Set of assortments, $h \in \mathcal{H}$
$\mathcal{I}$	Set of suppliers, $i \in \mathcal{I}$
$\mathcal{J}$	Set of demand points, $j \in \mathcal{J}$
$\mathcal{K}$	Set of trucks, $k \in \mathcal{K}$
$\mathcal{M}$	Set of terminals, $m \in \mathcal{M}$
$\mathcal{N}$	Set of machines, $n \in \mathcal{N}$
$\mathcal{T}$	Set of time periods, $t \in \mathcal{T}$
$\mathcal{R}^A$	Set of arcs between suppliers and terminals, $(i, m) \in \mathcal{R}^A$
$\mathcal{R}^B$	Set of arcs between suppliers and demand points, $(i, j) \in \mathcal{R}^B$
$\mathcal{R}^C$	Set of arcs between terminals and demand points, $(m, j) \in \mathcal{R}^C$
$\mathcal{R}^D$	Set of arcs between the terminals, $(m, m') \in \mathcal{R}^D$
Supporting sets	
$\mathcal{G}_h^H$	Set of groups the assortments could fulfill, $g \in \mathcal{G}_h^H$
$\mathcal{H}_g^G$	Set of assortments used to fulfill assortment group g, $h \in \mathcal{H}_g^G$
$\mathcal{H}^k$	Set of assortments which could be transported by truck type k, $h \in \mathcal{H}_k^K$
$\mathcal{K}_h^H$	Set of trucks which could transport assortment h, $k \in \mathcal{K}_h^H$
$\mathcal{K}^C$	Set of combotrucks, $k \in \mathcal{K}^C$
$\mathcal{N}_{ihh'}$	Set of machines associated with suppliers, that process wood from h to h', $n \in \mathcal{N}_{ihh'}$
$\mathcal{N}_{mhh'}$	Set of machines associated with terminals, that process wood from h to h',$n \in \mathcal{N}_{mhh'}$
$\mathcal{N}_k^C$	Set of combo machines, $n \in \mathcal{N}_k^C$

At the suppliers, the inventory balance could be split in two parts; free inventory, which is inventory that also could be used for other purposes than biomass fuel, and bought inventory, which is inventory meant to be used for this purpose, as shown in Constraints (4.1) and (4.2).

$$S_{iht} + l_{ih(t-1)}^{I-F} - l_{iht}^{I-F} - b_{iht}^{I} = 0, i \in \mathcal{I}, h \in \mathcal{H}, t \in \mathcal{T} \tag{4.1}$$

After the purchase of wood assortments, the wood is either processed, stored or transported away.

$$b_{iht}^{I} + l_{ih(t-1)}^{I-B} - \sum_{h' \in H} \sum_{n \in N_{ihh'}^{I}} v_{ihnt}^{IT} + \sum_{h' \in H} \sum_{n \in N_{ih'h}^{I}} f_{nh'h}^{c} v_{ih'nt}^{IT} - \sum_{k \in K_h^H} \sum_{j:(i,j) \in R^A} x_{ijkht}^{I-J} - \sum_{k \in K_h^H} \sum_{m:(i,m) \in R^B} x_{imkht}^{I-M} - l_{iht}^{I-B} = 0, i \in \mathcal{I}, t \in \mathcal{T}, h \in \mathcal{H} \tag{4.2}$$

Often a maximum level on the inventory of bought volumes at the suppliers is

Table 2: Variables used in the models- alternative

Variables	
v^{IT}_{ihnt}	Processing of assortment h at supplier i with machine n at time period t
v^{MT}_{mhnt}	Processing of assortment h at terminal m with machine n at time period t
x^{I-J}_{ijkht}	Flow from supplier i to demand point j with truck k of assortment h at time period t
x^{I-M}_{imkht}	Flow from supplier i to terminal m with truck k of assortment h at time period t
x^{M-J}_{jmkht}	Flow from terminal m to demand point j with truck k of assortment h at time period t
$x^{M-M}_{mm'kht}$	Flow from terminal m to terminal m' with truck k of assortment h at time period t
y_{jhgt}	Fulfillment of demand of assortment group g at demand point j and time period t
w_{jgt}	Unfulfilled demand of assortment group g at demand point j and time period t
l^{J}_{jht}	Inventory of assortment h at demand point j and time period t
l^{M}_{mht}	Inventory of assortment h at terminal m and time period t
l^{I-F}_{iht}	Free inventory of supply of assortment h at supplier i and time period t
l^{I-B}_{iht}	Bought inventory of assortment h at supplier i and time period t
b^{M}_{mht}	Bought assortments from terminal locations
b^{I}_{iht}	Bought supply of assortment h at supplier i and time period t
b^{J}_{jht}	Bought supply of assortment h at demand point j and time period t
s^{C}_{n}	Extra processing capacity contracted of machine n
b^{C}_{kt}	Extra transportation capacity contracted of truck type k at time period t
v_m	Binary variable to indicate if terminal m is open or not

used. This can be modeled by Constraint (4.3).

$$l^{I-B}_{iht} - u^{I}_{ih} \leq 0, i \in \mathcal{I}, t \in \mathcal{T}, h \in \mathcal{H} \tag{4.3}$$

For the terminals, the inventory balance consists of inventory, transport in, transport out, transport between terminals, assortment processing and assortments bought at terminal points.

$$\begin{aligned} & l^{M}_{mh(t-1)} + b^{M}_{mht} + \sum_{k \in K^H_h} \sum_{i:(i,m) \in \mathcal{R}^{\mathcal{B}}} x^{I-M}_{imkht} + \sum_{k \in K^H_h} \sum_{m':(m',m) \in \mathcal{R}^{\mathcal{D}}} x^{M-M}_{m'mkht} + \\ & \sum_{h' \in \mathcal{H}} \sum_{n \in \mathcal{N}^M_{mh'h}} f^{c}_{h'hn} v^{MT}_{mh'nt} - \sum_{h' \in H} \sum_{n \in \mathcal{N}^M_{mhh'}} v^{MT}_{mhnt} - \sum_{k \in K^H_h} \sum_{m':(m,m') \in \mathcal{R}^{\mathcal{D}}} x^{M-M}_{m'mkht} - \\ & \sum_{k \in K^H_h} \sum_{j:(m,j) \in \mathcal{R}^{\mathcal{C}}} x^{M-J}_{mjght} - l^{M}_{mht} = 0, m \in \mathcal{M}, h \in \mathcal{H}, t \in \mathcal{T} \end{aligned} \tag{4.4}$$

For the demand points we can model the node balance by Constraint (4.5). We have to convert assortments into assortment groups as demand is given in assortment groups. If we are not able to supply enough to fulfill demand, the remaining demand must be fulfilled by other sources of energy, e.g. oil or electricity.

$$\sum_{k \in K_h^H} \sum_{i:(ij) \in R^A} x_{ijkht}^{I-J} + \sum_{k \in K_h^H} \sum_{m:(m,j) \in R^C} x_{mjkht}^{M-J} + l_{jh(t-1)}^J - \sum_{g \in G_h^H} y_{jhgt} + b_{jht}^J - l_{jht}^J = 0, j \in \mathcal{J}, h \in \mathcal{H}, t \in \mathcal{T} \tag{4.5}$$

4.5 Combo truck activities

Combo truck are trucks that processes assortments by use of a mounted chipper, and then transport the chips to the demand points. The work performed each month is dependent on how far they travel, their driving speed, and the amount that are to be processed and delivered. This can be written as in Constraint (4.6), where capacity is given by available work hours for each month.

$$\sum_{i:(i,j) \in \mathcal{R}^A} \sum_{h \in \mathcal{H}_k^K} \frac{u_k^{KW} d_{ij}^D x_{ijkht}^{I-J}}{u_{kt}^{CH} u_{kt}^{CW}} + \sum_{i \in \mathcal{I}} \sum_{h \in \mathcal{H}} \sum_{h' \in \mathcal{H}} \sum_{n \in \mathcal{N}_k^C} f_{hh'n}^t v_{ihnt}^{IT} - u_{kt}^{K+} \leq 0, t \in \mathcal{T}, k \in \mathcal{K}^C \tag{4.6}$$

The combo truck has to work at a catchment area and transport all of the wood chippings it produces. This is given by Constraint (4.7).

$$\sum_{j:(i,j) \in \mathcal{R}^A} x_{ijkht}^{I-J} - \sum_{n \in \mathcal{N}_k^C} \sum_{h' \in \mathcal{H}} f_{h'hn}^c v_{ih'nt}^{IT} = 0, t \in \mathcal{T}, i \in \mathcal{I}, k \in \mathcal{K}^{\mathcal{C}}, h \in \mathcal{H}_k^K \tag{4.7}$$

4.6 Deciding on terminal structure

To decide on which terminals to open, we introduce the binary variable v_m for each potential terminal location. If opening a terminal provides economic value, it has to participate in the fulfillment of demand. We could therefore restrict the use of a terminal by restricting the outflow of the terminal. If transportation between terminals are to be allowed, we have modeled this by only allowing transport to go to and from opened terminals.

$$\sum_{j:(m,j) \in \mathcal{R}^{\mathcal{C}}} \sum_{h \in \mathcal{H}} \sum_{k \in \mathcal{K}} x_{mjkht}^{M-J} - u_m^{M+} v_m \leq 0, m \in \mathcal{M}, t \in \tag{4.8}$$

We can not have any required inventory levels on closed terminals. This is solved by the Constraints (4.9), where safety stock levels are given as assortment groups, due to demand being given by assortment groups.

$$\sum_{h \in \mathcal{H}_g^G} l_{mht}^{M} - u_{mgt}^{M1} v_m \geq 0, m \in \mathcal{M}, t \in \mathcal{T}, g \in \mathcal{G} \tag{4.9}$$

4.7 Objective function

The objective function consists of the maximization of revenues and subtraction of costs. Costs include transportation, inventory, processing, the purchase of biomass from suppliers and the terminal locations, and handle costs on the terminals.

$$\begin{aligned}
\max z = & \sum_{j \in \mathcal{J}} \sum_{g \in \mathcal{G}} \sum_{h \in \mathcal{H}_g^G} \sum_{t \in \mathcal{T}} p_{jgt} y_{jhgt} - \sum_{m \in \mathcal{M}} F_m v_m - \sum_{i,j:(i,j) \in \mathcal{R}^A} \sum_{k \in \mathcal{K}} \sum_{h \in \mathcal{H}_k^K} \sum_{t \in \mathcal{T}} c_{ijh}^{I-J} x_{ijkht}^{I-J} - \\
& \sum_{i,m:(i,m) \in \mathcal{R}^B} \sum_{k \in \mathcal{K}} \sum_{h \in \mathcal{H}_k^K} \sum_{t \in \mathcal{T}} c_{imh}^{I-M} x_{imkht}^{I-M} - \sum_{m,j:(m,j) \in \mathcal{R}^C} \sum_{k \in \mathcal{K}} \sum_{h \in \mathcal{H}_k^K} \sum_{t \in \mathcal{T}} c_{mjh}^{M-J} x_{mjkht}^{M-J} - \\
& \sum_{m,m':(m,m') \in \mathcal{R}^D} \sum_{k \in \mathcal{K}} \sum_{h \in \mathcal{H}_k^K} \sum_{t \in \mathcal{T}} c_{mm'h}^{M-M} x_{mm'kht}^{M-M} - \sum_{m \in \mathcal{M}} \sum_{h \in \mathcal{H}} \sum_{t \in \mathcal{T}} c_{mh}^{IM} l_{mht}^{M} - \sum_{i \in \mathcal{I}} \sum_{h \in \mathcal{H}} \sum_{t \in \mathcal{T}} c_{iht}^{B-I} b_{iht}^{I} - \\
& \sum_{i \in \mathcal{I}} \sum_{h \in \mathcal{H}} \sum_{t \in \mathcal{T}} c_{iht}^{I-I-B} l_{iht}^{I-B} - \sum_{i \in \mathcal{I}} \sum_{h \in \mathcal{H}} \sum_{h' \in \mathcal{H}} \sum_{n \in N_{ihh'}^{I}} \sum_{t \in \mathcal{T}} f_{n,h,h'}^{t} c_n^f v_{ihnt}^{IT} - \\
& \sum_{m \in \mathcal{M}} \sum_{h \in \mathcal{H}} \sum_{h' \in \mathcal{H}} \sum_{n \in \mathcal{N}_{mhh'}^{M}} \sum_{t \in \mathcal{T}} f_{n,h,h'}^{t} c_n^f v_{mhnt}^{MT} - \sum_{j \in \mathcal{J}} \sum_{h \in \mathcal{H}} \sum_{t \in \mathcal{T}} c_{jht}^{IJ} l_{jht}^{J} - \\
& \sum_{n \in \mathcal{N}} c_n^N s_n^C - \sum_{k \in \mathcal{K}} \sum_{t \in \mathcal{T}} c^C b_{kt}^C - \sum_{j \in \mathcal{J}} \sum_{g \in \mathcal{G}} \sum_{t \in \mathcal{T}} c_{jgt}^P w_{jgt}^P - \sum_{i \in \mathcal{I}} \sum_{h \in \mathcal{H}} \sum_{t \in \mathcal{T}} c^{IF} l_{iht}^{I-F} - \\
& \sum_{j \in \mathcal{J}} \sum_{h \in \mathcal{H}} \sum_{t \in \mathcal{T}} c_{jht}^{B} b_{jht}^{J} - \sum_{m \in \mathcal{J}} \sum_{h \in \mathcal{H}} \sum_{t \in \mathcal{T}} c_{mht}^{M} b_{mht}^{M} - \sum_{j:(m,j) \in \mathcal{R}^C} \sum_{k \in \mathcal{K}} \sum_{t \in \mathcal{T}} \sum_{h \in \mathcal{H}_k^K} c_{mh}^{H} x_{mjkht}^{M-J}
\end{aligned} \tag{4.10}$$

4.8 Decision stages

To be able to model the stochastic and robust optimization problem, we need to know the decision stages for the problem. The first-stage decisions would become non-anticipative variables, while other decisions would be dependent on which scenario it is a part of. For this problem we may model the decision stages as the following:

1. Decide on terminal structure and inventory policy, i.e. required safety stock levels.

2. Based on information on realizations of demand, decide on inventory levels, production levels, transportation etc. for each month.

When considering terminal inventory policy we will limit ourselves to look at optimimal inventory level that is needed for the different terminal structures

4.9 Compared to the model of Flisberg et.al.

Compared to the model in [Flisberg et al., 2010], the model we have presented looks specific on the terminal structure, while the other model tries to formulate the tactical supply chain planning problem. We have therefore formulated a model where the terminal structure is not given in advance. We have also removed the possibility of using train transport. As this decreases the flexibility in choosing the transportation mode that is most preferable, the answers given by our model may deviate. This have been done to simplify the model, and to focus more on the terminal location problem than the transportation. [Enoksen and Sværen, 2009] However, the flows are not meant to be implemented, and only serves to model the impact the terminal structure have on the rest of the supply chain.

We have also chosen to write the arcs only given by the nodes which they travel between, meaning that we have four sets of arcs. $\mathcal{R}^A$ for arcs between supplier and terminals, $\mathcal{R}^B$ for arcs between supplier and demand points, $\mathcal{R}^C$ for arcs between terminal and demand points and $\mathcal{R}^D$ for arcs between the terminals. To prevent inventories of wood chippings occurring in the catchment areas, we also have to introduce a constraint on the inventory capacities of the different assortments at the suppliers. This is given by the Constraint (4.3).

In the model by [Flisberg et al., 2010] they use indices for both assortments and assortment groups in all the transport and inventory variables. As assortments are what is actually transported and stored, we have chosen to use only this index in these variables. The use of assortment groups is limited to the demand points, as demand is given for assortment groups. Safety stock is also modeled differently, as we have a variable number of opened terminals.

Table 3: Constants used in the models

Parameter data

$p_{j,g,t}$	Selling price for assortment group g at demand point j and time period t
c_{jgt}^{IJ}	The inventory cost for assortment group g at demand point j and time period t
u_j^{J+}	The maximum inventory at demand point j
c_{jgt}^{P}	Costs for unfulfilled demand at demand point j for assortment group g and time period t
D_{jgt}	Demand for assortment group g for customer j in time period t
c_{iht}^{B-I}	The price for buying assortment h at supplier i and time period t
c_{iht}^{I-I-B}	The inventory cost for bought inventory of assortment h at supplier i and time period t
c^{IF}	The inventory cost for free/ non-bought inventory at suppliers
c_{mht}^{IM}	The inventory cost for assortment h at terminal m and time period t
u_{ih}^{I}	The maximum inventory at supplier i of bought inventory
S_{iht}	Increase in available supply for each time period and supplier. Unused supply is sent to free inventory
F_m	Fixed cost for opening terminal m
u_{mgt}^{M1-}	The minimum inventory for group g at terminal m and time period t
u_m^{M+}	The maximum inventory at terminal m
c_{mht}^{M}	Costs for assortments bought from terminal locations
S_{mh}^{M}	Available assortments that could be bought at terminal locations
SO_{mh}^{M}	Initial inventory at terminal m
c_{mh}^{H}	Terminal handle costs
c_n^f	The processing costs per hour for machine n
$f_{nh'h}^t$	The processing time for processing h' to h at machine n
$f_{nh'h}^c$	The processing factor for processing h' to h at machine n
u_n^{f+}	Processing capacity for machine n
c_n^N	Extra processing capacity costs for machine n
c_n^f	The processing costs per hour for machine
c_{ijh}^{I-J}	The cost for flow from supplier i to demand point j for assortment h
c_{imh}^{I-M}	The cost for flow from supplier i to terminal m for assortment h
c_{mjh}^{M-J}	The cost for flow from terminal m to demand point j for assortment h
$c_{mm'h}^{M-M}$	The cost for flow from terminal m to terminal m' for assortment h
u_k^{KW}	Scaling factor for transportation work
d_{ij}^D	Distance between supplier i and customer j
d_{im}^D	Distance between supplier i and terminals m
d_{mj}^D	Distance between terminals i and customer j
c^C	Extra transportation capacity costs
u_{kt}^{KC}	The total transport capacity of trucks of type k
u_{kt}^{K+}	The total transport capacity of combo trucks of type k
u_{kt}^{CH}	The average speed of a combo truck
u_{kt}^{CW}	The average load of a combo truck
u_{kt}^{K+}	Total capacity for all combo trucks

5 Models

5.1 Deterministic model

$$
\begin{aligned}
\max z = & \sum_{j\in\mathcal{J}}\sum_{g\in\mathcal{G}}\sum_{h\in\mathcal{H}_g^G}\sum_{t\in\mathcal{T}} p_{jgt}y_{jhgt} - \sum_{m\in\mathcal{M}} F_m v_m - \sum_{i,j:(i,j)\in\mathcal{R}^A}\sum_{k\in\mathcal{K}}\sum_{h\in\mathcal{H}_k^K}\sum_{t\in\mathcal{T}} c_{ijh}^{I-J} x_{ijkht}^{I-J} - \\
& \sum_{i,m:(i,m)\in\mathcal{R}^B}\sum_{k\in\mathcal{K}}\sum_{h\in\mathcal{H}_k^K}\sum_{t\in\mathcal{T}} c_{imh}^{I-M} x_{imkht}^{I-M} - \sum_{m,j:(m,j)\in\mathcal{R}^C}\sum_{k\in\mathcal{K}}\sum_{h\in\mathcal{H}_k^K}\sum_{t\in\mathcal{T}} c_{mjh}^{M-J} x_{mjkht}^{M-J} - \\
& \sum_{m,m':(m,m')\in\mathcal{R}^D}\sum_{k\in\mathcal{K}}\sum_{h\in\mathcal{H}_k^K}\sum_{t\in\mathcal{T}} c_{mm'h}^{M-M} x_{mm'kht}^{M-M} - \sum_{m\in\mathcal{M}}\sum_{h\in\mathcal{H}}\sum_{t\in\mathcal{T}} c_{mh}^{IM} l_{mht}^{M} - \sum_{i\in\mathcal{I}}\sum_{h\in\mathcal{H}}\sum_{t\in\mathcal{T}} c_{iht}^{B-I} b_{iht}^{I} - \\
& \sum_{i\in\mathcal{I}}\sum_{h\in\mathcal{H}}\sum_{t\in\mathcal{T}} c_{iht}^{I-I-B} l_{iht}^{I-B} - \sum_{i\in\mathcal{I}}\sum_{h\in\mathcal{H}}\sum_{h'\in\mathcal{H}}\sum_{n\in N_{ihh'}^I}\sum_{t\in\mathcal{T}} f_{n,h,h'}^t c_n^f v_{ihnt}^{IT} - \\
& \sum_{m\in\mathcal{M}}\sum_{h\in\mathcal{H}}\sum_{h'\in\mathcal{H}}\sum_{n\in\mathcal{N}_{mhh'}^M}\sum_{t\in\mathcal{T}} f_{n,h,h'}^t c_n^f v_{mhnt}^{MT} - \sum_{j\in\mathcal{J}}\sum_{h\in\mathcal{H}}\sum_{t\in\mathcal{T}} c_{jht}^{IJ} l_{jht}^{J} - \\
& \sum_{n\in\mathcal{N}} c_n^N s_n^C - \sum_{k\in\mathcal{K}}\sum_{t\in\mathcal{T}} c^C b_{kt}^C - \sum_{j\in\mathcal{J}}\sum_{g\in\mathcal{G}}\sum_{t\in\mathcal{T}} c_{jgt}^P w_{jgt}^P - \sum_{i\in\mathcal{I}}\sum_{h\in\mathcal{H}}\sum_{t\in\mathcal{T}} c^{IF} l_{iht}^{I-F} - \\
& \sum_{j\in\mathcal{J}}\sum_{h\in\mathcal{H}}\sum_{t\in\mathcal{T}} c_{jht}^B b_{jht}^J - \sum_{m\in\mathcal{J}}\sum_{h\in\mathcal{H}}\sum_{t\in\mathcal{T}} c_{mht}^M b_{mht}^M - \sum_{j:(m,j)\in\mathcal{R}^C}\sum_{k\in\mathcal{K}}\sum_{t\in\mathcal{T}}\sum_{h\in\mathcal{H}_k^K} c_{mh}^H x_{mjkht}^{M-J}
\end{aligned}
\tag{5.1}
$$

s.t.

$$l_{ih0}^{I-F} = 0, i \in \mathcal{I}, h \in \mathcal{H} \tag{5.2}$$

$$l_{ih0}^{I-B} = 0, i \in \mathcal{I}, h \in \mathcal{H} \tag{5.3}$$

$$l_{jg0}^{J} = 0, j \in \mathcal{J}, g \in \mathcal{G} \tag{5.4}$$

$$l_{mh0}^{M} - SO_{mh}^{M} = 0, m \in \mathcal{M}, h \in \mathcal{H} \tag{5.5}$$

$$S_{iht} + l_{ih(t-1)}^{I-F} - l_{iht}^{I-F} - b_{iht}^{I} = 0, i \in \mathcal{I}, h \in \mathcal{H}, t \in \mathcal{T} \tag{5.6}$$

$$b^{I}_{iht} + l^{I-B}_{ih(t-1)} - \sum_{h'\in H} \sum_{n\in N^{I}_{ihh'}} v^{IT}_{ihnt} + \sum_{h'\in H} \sum_{n\in N^{I}_{ih'h}} f^{c}_{nh'h} v^{IT}_{ih'nt} - \sum_{k\in K^{H}_{h}} \sum_{j:(i,j)\in R^{A}} x^{I-J}_{ijkht} -$$
$$\sum_{k\in K^{H}_{h}} \sum_{m:(i,m)\in R^{B}} x^{I-M}_{imkht} - l^{I-B}_{iht} = 0, i \in \mathcal{I}, t \in \mathcal{T}, h \in \mathcal{H} \tag{5.7}$$

$$l^{M}_{mh(t-1)} + b^{M}_{mht} + \sum_{k\in K^{H}_{h}} \sum_{i:(i,m)\in \mathcal{R}^{\mathcal{B}}} x^{I-M}_{imkht} + \sum_{k\in K^{H}_{h}} \sum_{m':(m',m)\in \mathcal{R}^{\mathcal{D}}} x^{M-M}_{m'mkht} +$$
$$\sum_{h'\in \mathcal{H}} \sum_{n\in \mathcal{N}^{M}_{mh'h}} f^{c}_{h'hn} v^{MT}_{mh'nt} - \sum_{h'\in H} \sum_{n\in \mathcal{N}^{M}_{mhh'}} v^{MT}_{mhnt} - \sum_{k\in K^{H}_{h}} \sum_{m':(m,m')\in \mathcal{R}^{\mathcal{D}}} x^{M-M}_{m'mkht} - \tag{5.8}$$
$$\sum_{k\in K^{H}_{h}} \sum_{j:(m,j)\in \mathcal{R}^{\mathcal{C}}} x^{M-J}_{mjght} - l^{M}_{mht} = 0, m \in \mathcal{M}, h \in \mathcal{H}, t \in \mathcal{T}$$

$$\sum_{t\in \mathcal{T}} b^{M}_{mht} \le S^{M}_{mh}, m \in \mathcal{M}, h \in \mathcal{H} \tag{5.9}$$

$$\sum_{k\in K^{H}_{h}} \sum_{i:(ij)\in R^{A}} x^{I-J}_{ijkht} + \sum_{k\in K^{H}_{h}} \sum_{m:(m,j)\in R^{C}} x^{M-J}_{mjkht} + l^{J}_{jh(t-1)} -$$
$$\sum_{g\in G^{H}_{h}} y_{jhgt} + b^{J}_{jht} - l^{J}_{jht} = 0, j \in \mathcal{J}, h \in \mathcal{H}, t \in \mathcal{T} \tag{5.10}$$

$$\sum_{i,m:(i,m)\in \mathcal{R}^{B}} \sum_{h\in \mathcal{H}^{K}_{k}} u^{KW}_{k} d^{D}_{im} x^{I-M}_{imkht} + \sum_{m,j:(m,j)\in \mathcal{R}^{\mathcal{C}}} \sum_{h\in \mathcal{H}^{K}_{k}} u^{KW}_{k} d^{D}_{mj} x^{M-J}_{mjkht} +$$
$$\sum_{i,j:(i,j)\in \mathcal{R}^{A}} \sum_{h\in \mathcal{H}^{K}_{k}} u^{KW}_{k} d^{D}_{ij} x^{I-J}_{ijkht} + \sum_{m,m':(m,m')\in \mathcal{R}^{\mathcal{D}}} \sum_{h\in \mathcal{H}^{K}_{k}} u^{KW}_{k} d^{D}_{mm'} x^{M-M}_{mm'kht} - \tag{5.11}$$
$$u^{KC}_{k} - b^{C}_{kt} \le 0, k \in \mathcal{K}, t \in \mathcal{T}$$

$$\sum_{i\in I} \sum_{h\in \mathcal{H}} f^{t}_{hh'n} v^{IT}_{ihnt} + \sum_{m\in \mathcal{M}} \sum_{h\in \mathcal{H}} f^{t}_{hh'n} v^{MT}_{mhnt} - u^{f+}_{n} - s^{C}_{n} \le 0, n \in \mathcal{N}, t \in \mathcal{T} \tag{5.12}$$

$$\sum_{(i,j)\in \mathcal{R}^{A}} \sum_{h\in \mathcal{H}^{K}_{k}} \frac{u^{KW}_{k} d^{D}_{ij} x^{I-J}_{ijkht}}{u^{CH}_{kt} u^{CW}_{kt}} + \sum_{i\in \mathcal{I}} \sum_{h\in \mathcal{H}} \sum_{h'\in \mathcal{H}} \sum_{n\in \mathcal{N}^{C}_{k}} f^{t}_{hh'n} v^{IT}_{ihnt} - u^{K+}_{kt} \le 0, t \in \mathcal{T}, k \in \mathcal{K}^{C} \tag{5.13}$$

$$\sum_{j:(i,j)\in\mathcal{R}^A} x^{I-J}_{ijkht} - \sum_{n\in\mathcal{N}^C_k}\sum_{h'\in\mathcal{H}} f^c_{h'hn} v^{IT}_{ih'nt} = 0, t\in\mathcal{T}, i\in\mathcal{I}, k\in\mathcal{K}^C, h\in\mathcal{H}^K_k \tag{5.14}$$

$$\sum_{h\in\mathcal{H}^G_g} y_{jhgt} + w_{jgt} - D_{jgt} = 0, j\in\mathcal{J}, g\in\mathcal{G}, t\in\mathcal{T} \tag{5.15}$$

$$\sum_{h\in\mathcal{H}} l^J_{jht} - u^{J+}_j \leq 0, j\in\mathcal{J}, t\in\mathcal{T} \tag{5.16}$$

$$\sum_{j:(m,j)\in\mathcal{R}^C}\sum_{k\in\mathcal{K}}\sum_{h\in\mathcal{H}^K_k} x^{M-J}_{mjkht} - u^{M+}_m v_m \leq 0, m\in\mathcal{M}, t\in\mathcal{T} \tag{5.17}$$

$$\begin{aligned}\sum_{m':(m,m')\in\mathcal{R}^D}\sum_{k\in\mathcal{K}}\sum_{h\in\mathcal{H}^K_k} x^{M-M}_{mm'kht} - u^{M+}_m v_m \leq 0, m\in\mathcal{M}, t\in\mathcal{T}\\ \sum_{m':(m',m)\in\mathcal{R}^D}\sum_{k\in\mathcal{K}}\sum_{h\in\mathcal{H}^K_k} x^{M-M}_{m'mkht} - u^{M+}_m v_m \leq 0, m\in\mathcal{M}, t\in\mathcal{T}\end{aligned} \tag{5.18}$$

$$\sum_{h\in\mathcal{H}^G_g} l^M_{mht} - u^{M1-}_{mgt} v_m \geq 0, m\in\mathcal{M}, t\in\mathcal{T}, g\in\in\mathcal{G} \tag{5.19}$$

$$l^{I-B}_{iht} - u^I_{ih} \leq 0, i\in\mathcal{I}, t\in\mathcal{T}, h\in\mathcal{H} \tag{5.20}$$

$$v_m \in \{0,1\}, m\in\mathcal{M} \tag{5.21}$$

$$\text{All variables} \geq 0 \tag{5.22}$$

Initial inventory for the different nodes are given by (5.2), (5.3), (5.4) and (5.5). The inventory balances at given by (5.7), (5.6), (5.8) and (5.10). The transport capacity is given (5.11), and processing capacity is given by (5.12). The combo truck is constrained by (5.13) and (5.14). Demand fulfillment is given by (5.15). The terminals has a limited capacity which is given by (5.17)and a required level of minimum inventory given by (5.19). The costumer and the suppliers may also have a maximum capacity on inventory given by (5.20) and (5.16). Transport between terminals only available between opened terminals in Constraint (5.18).

Customer inventory, extra transportation capacities, unfulfilled demand and fuels bought from other sources were not wanted, and the costs of utilizing such options were replaced by penalty costs. Penalty costs are used to indicate that the solution is not robust, i.e. it is not possible to fulfill demand.

5.2 Stochastic model

The two-stage decision structure in the biomass supply chain makes us able to model our problem as a two-stage recourse problem. We may in some cases lack knowledge on what kind of stochastic distribution that is connected to the realizations of demand. As all outcomes of the recourse problem are feasible, the sample mean could be used as replacement for the stochastic distribution. The difference from the deterministic model is that all variables, except the first stage variables, is given for each scenario and the objective function is given by 5.23. $h(\mathbf{v}, s)$ is the recourse function, s is the index for scenario ($s \in \mathcal{S}$), and $\mathbf{v}$ is the vector of the terminal variables.

$$\max z = E[h(\mathbf{v}, s)] - \sum_{m \in \mathcal{M}} F_m v_m \tag{5.23}$$

s.t.

$$v_m \in \{0, 1\}, m \in \mathcal{M} \tag{5.24}$$

5.3 Robust model

To model a robust optimization model, we first need to clarify which decisions that are the fist-stage decisions. In this case, it is the terminal structure. All variables, except the first stage variables, is given for each scenario and the objective function is given by 5.25, where $h(\mathbf{v}, s)$ is the objective function for each scenario and $\mathbf{v}$ is the vector of variables for the terminals. θ is the least profit for the given scenarios.

$$\begin{aligned} \max z = \theta - \sum_{m \in \mathcal{M}} F_m v_m \\ \theta < h(\mathbf{v}, s), s \in \mathcal{S} \end{aligned} \tag{5.25}$$

s.t.

$$v_m \in \{0, 1\}, m \in \mathcal{M} \tag{5.26}$$

6 Implementation

6.1 Generating demand scenarios

To model uncertainty in demand, demand scenarios were generated. A scenario represented a possible future demand realization for all customers and assortments in each time period except the first, as demand for the first month after new demand information has been submitted, could be regarded as rather certain. We generated scenarios where demand variations for each customer and assortments were correlated and uncorrelated. Correlated demand variations could be due to e.g. weather changes and demand for heat would then increase in every region.

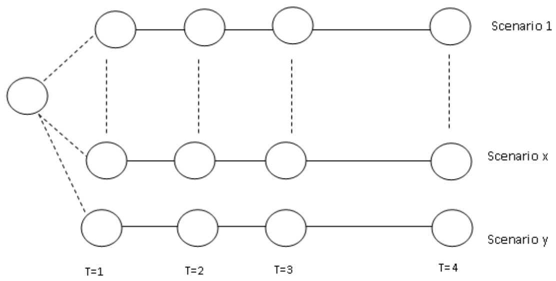


Figure 6: The scenario tree

Uncorrelated demand were generated by letting all demand for each customer in each time period except the last time period, vary within ± maximum allowed variance. The last period were set as the difference between the contract specified annual demand, and the sum of demand for all other periods. If the last period varied with more than the maximum variance, the process was repeated until a solution was found.

Correlated demand scenarios were generated by first generating demand base levels for all time periods for each scenario, where the base level was generated randomly and forced to be within the allowed demand variations and to be cancelled out throughout the year. Demand for each customer and assortment were then generated randomly by use of an even distribution with the demand base level as the middle-point and within the allowed demand deviations[5]. If the demand variations exceeded the annual contract, the whole process of finding demand base levels and variation for each month were repeated until a solution were found. This method was based on the approach used in [Bredström et al., 2010].

[5]If demand base level is db, the expected demand is ED and the maximum variance allowed is $mvar$, the demand for each customer and assortment can be found like this: if $db \geq 0$, then $a := mvar * ED$ and $b := ED * (2 * db - mvar)$. If $db \leq 0$, then $a := ED * (2 * db + mvar)$ and $b := -mvar * ED$. $Demand := ED + a - (a - b) * random$. The demand base level db is generated randomly earlier.

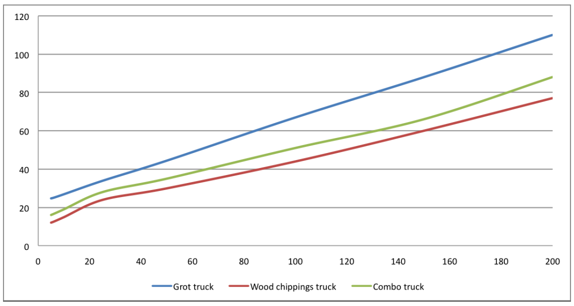


Figure 7: Transportation cost for different truck types

6.2 Creating transportation costs

Transportation costs were found a priori the running of the models, and were given as input data. Normally, these transportation costs are not linear as shown in Figure 7[6], as the marginal costs would be higher for shorter distances due to transferring trucks to starting points. We could use special ordered sets of type two to interpolate to find the costs. SOS2 is a set of variables where at most two of the variables can be non-zero, and these two variables need to be adjacent in the ordering given to the set. [Williams, 2007] For distances larger than the given values for interpolation, a new point were created by extrapolating the last two known values.

6.3 Terminal fixed costs

The fixed costs of opening a new terminal are usually not known before the preliminary work on the construction process of the terminal in started. These costs are dependent on several factors, such as the size of the terminal and existing infrastructure. It is therefore difficult to make decisions on how many terminals to open. One way to overcome this challenge, is to demand that the sum of opened terminals are equal to a given number, and then running the model where this number is given as input for all possible sum of terminals to open. This will result in a profit function of the sum of opened terminals. One can then decide on which terminals that are profitable to open. We introduce a constraint in order to enforce terminals to be open, as shown in Constraint 6.1. If one later finds that one of the terminals would be too expensive, one could demand that this terminal is to be closed in the model.

$$\sum_{m \in \mathcal{M}} v_m = \text{NoTerminals} \tag{6.1}$$

[6]The truck types and costs are from the Sveaskog case with aggregated assortments.

6.4 Required safety stock levels

Required safety stock levels are used for building inventory on open terminals in order to avoid stock-outs due to variations in demand. As in described in Section **??** we chose to set the sum of safety stock to 40 % of the demand for the coming period, except for the last period, when no inventory was needed. This gave a new constraint as seen in Constraint 6.2, which will replace Constraint 5.19. This is a nonlinear constraint, and needs to be linearized to be able to implement it in Xpress IVE. However, the sum of terminals used may be set a priori each running of the model, and the sum of terminals opened may then be replaced by a constant.

$$\sum_{h \in \mathcal{H}_g^G} l_{mht}^M - 0.4 \frac{\sum_j D_{jgt+1} v_m}{\sum_{m' \in \mathcal{M}} v_{m'}} \geq 0, m \in \mathcal{M}, g \in \mathcal{G}, t \in \{1..11\} \tag{6.2}$$

For the stochastic and robust models, we ran the models and used the lowest inventory values over the scenarios for each of the time periods and assortments as required safety stock levels.

6.5 Data structures in Xpress

The models were implemented in Xpress-IVE. In order to create general models we made new data structures using the records feature in Xpress. From the example below we see that each arc is given by a unique number in the arcset, and each arc has unique values for its attributes.

```
ARC_S_DC: array(ARCSET_S_DC:range) of record
                    Supplier: string ! Source of arc
                    DC: string ! Sink of arc
                    Distance: real ! The distance of the arc
                    Cost: array(TRUCKS) of real
                              ! The corresponding costs for each
                              ! truck type
end-record
```

6.6 Presolving

Some variables may not be needed and we would like to remover these variables in order to make the model smaller and reduce the solution time. This is called presolving or preprocessing, and could be done inside optimization solvers and at the modeling stage. [Ashford, 2007]

At the modeling stage, presolving can be done by utilizing the knowledge of the problem and the case in order to only create the variables that are needed. One example is that some trucks only could transport a given type of assortment. We therefore do not need to create variables for combinations of this truck types and other assortments, as these combinations cannot be used. In Xpress this is done by creating variables by use of dynamic arrays and only create the variables specified. In the modeling stage we can also remove constraints that are redundant, but this can also be done by the optimization solver. [Baricelli et al., 1998] [Ashford, 2007].

Inside the optimization solver there are several methods which is included in the presolving, and contribute in reducing the size of the model:

- Fixing of variables
- Tightening bounds
- Utilizing specialized constructs
- Adding cuts
- Removing unneeded variables
- Removing redundant constraints

These are in most cases very complex algorithms that utilize the input data and the structure of the model in order to reduce the problem size. One example is that when all terminals are forced to be open, the presolver should remove all terminal variables and replace them by integers.

6.7 Implementation process

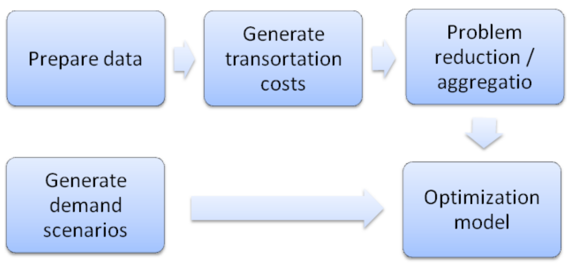


Figure 8: Flow chart of the implementation of our models

The implementation of the cases consisted of first preparing the case data, and adapting it to the format used by the model. Transport costs and demand scenarios were then generated based on these data. We then tried to run the model, and tested how many scenarios it could run before either a memory error occurred or the solution times became too high. Based on this information we had to try to reduce the size of the problem through presolving or use of solution methods. We then tried to run the model again, and continued this process of adjusting the model until it could handle a decent number of scenarios. An initialization script was written in Xpress IVE to make sure the models could be run automatically for different parameters. We have tried to summarize the implementation in Figure 8. Results were printed to data files. All of the cases were run on a computer with MS Windows XP SP3, Intel Core 2 Duo E6700 (2x2,6GHz) and 4GB DDR2 RAM.

7 Preprocessing, aggregation and LP-relaxtion based heuristics

For some cases, the problem may become too large to be solved on regular computers. In those cases, methods needs to be applied in order to be able to solve the problem. They can also be applied in order to decrease solution times or increase the number of scenarios the models can handle. We have looked for methods that seeks to reduce the size of the problems without deteriorating the solution quality. We will here present the ones we have employed.

The quality of the methods have been analyzed by comparing the solutions with solutions where no solutions methods are applied. We have tested the methods by using the terminal structure we get from testing the models by employing the methods, and inserting this into the original problem, and compare the objective values. In that way we will see the influence the methods have on terminal structures and supply chain profits. For the assortment aggregation and the removal of the longest arcs, a stochastic model for Case Sveaskog with 5 scenarios and 75 suppliers were used.

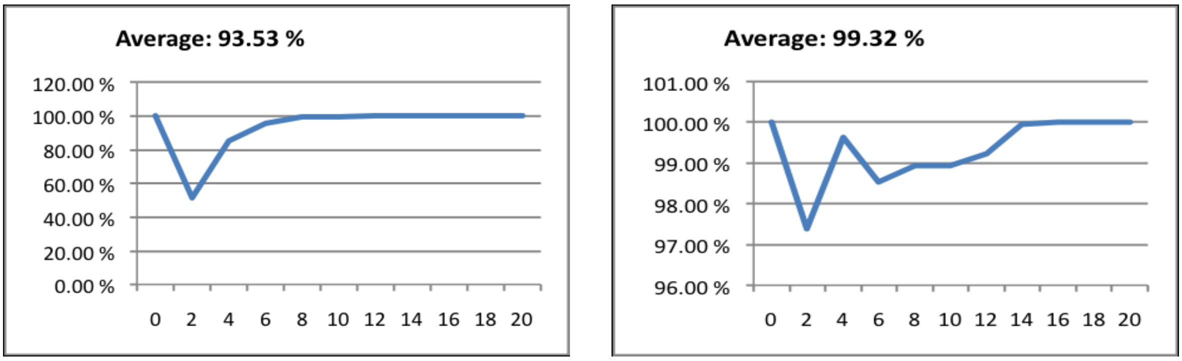
7.1 Aggregation of suppliers

The largest contribution to size of the problem came from the number of flow variables. Biomass supply chains usually contain a large amount of suppliers, which increases the number of arcs. As the uncertainty is on demand, we could exploit this by aggregating suppliers. We will compare two approaches for doing this. One approach is to use the name of the location of the supplier, and another is to utilize the distances between the suppliers. In both approaches, aggregated supply was placed on one of the existing suppliers.

7.1.1 Aggregation by name of location

In our cases, the name of each supplier was given as a number code; where the two first numbers were counties and the two next represented the municipality. By help of these numbers it was therefore possible to aggregate suppliers in each municipality or county.

This procedure does not take into consideration the distances between the different suppliers. Supply may therefore be modeled as moved further away from where it actually is located. Another drawback is that aggregated supply could be placed on suppliers that initially contained very little supply. Some supply points may also



(a) Aggregation within counties

(b) Aggregation within municipalities

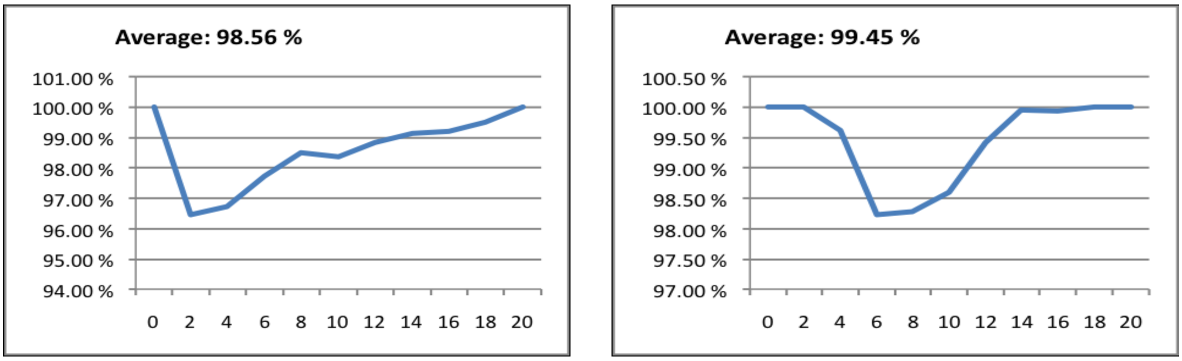
Figure 9: Deviation from the standard model when using a random supplier

lay closer to supply points in other municipalities than suppliers in its own. We can therefore risk that the solution quality may be reduced.

Aggregated supply could be placed on one of the existing supply points, and choosing this supplier could be done randomly. This approach is easy to implement manually. In our case, we used MS Excel 2007 Professional for this task. The solutions were evaluated by inserting the solved terminal structures in a deterministic problem without aggregation. For this aggregation method, the worst solutions only contributed to 51 % and 97 % of the optimal profits, as shown in Figure 9. The case contains 392 suppliers.

To improve this form for aggregation, we could have placed the supply at the largest supplier, or by placing it on the supplier located in the middle compared to the other suppliers. However, this requires that we know data for distance between the suppliers. Another possibility is to exploit the distances between the suppliers and the customers, as customers are the only nodes that we know will be used by the model. We can therefore place the supply at the supplier with the closest average distance to the customers. This method, however, neglects the fact that many suppliers rarely supply more than a single customer or a single terminal.

From Figure 10, we see that the worst solutions when aggregating over counties increased its performance from 51 % to 96 %. For the case with 158 suppliers, the performance is about equal to the previous approach, where supply is placed on a random terminal. A possible explanation is that increasing the new number of aggregated suppliers, decreases the flexibility in which existing suppliers is left after the aggregation, and the chosen suppliers could therefore end up being rather equal.



(a) Aggregation inside county (b) Aggregation inside municipality

Figure 10: Deviation from the exact solution when using the suppliers with the least average distance to the demand points

7.1.2 Aggregation by distance

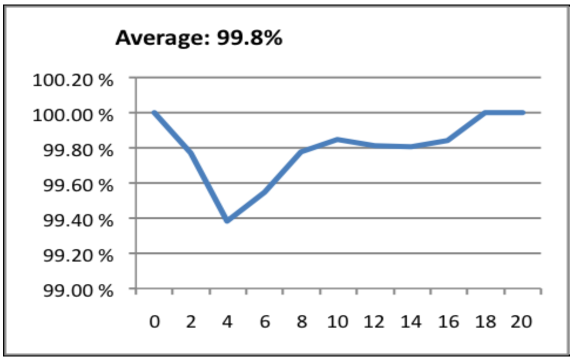
If distance data between the suppliers is available, it can be used for aggregating the suppliers which are located geographical close to each other. In that way we may prevent much of the supply changing location. We therefore also considered the size of the supply of the suppliers, and excluded suppliers from aggregation if the supply was larger than a preset parameter.

Suppliers were aggregated by gradually increasing the radius of chosen suppliers from zero up to given distance limit. We used a supply limit, which was the supply allowed on the supplier before it was excluded from the aggregation, and a distance limit. We used an algorithm with the following pseudo-code:

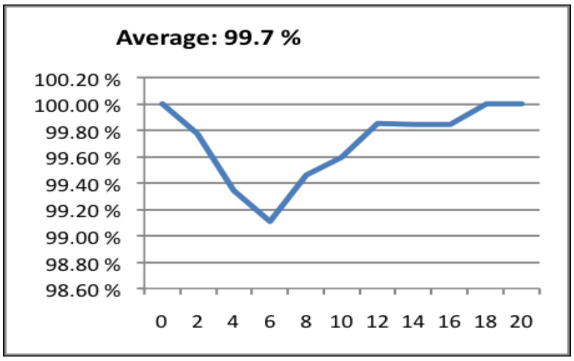
```
forall( ii in suppliers)
        if( supply(ii) > supplylimit)
                ii will not connect to others
        else
                start with ii
                while(connectedsupply(ii) < supplylimit)
                        find closest supplier  jj
                                if (within distancelimt and
                                        total supply within
                                        supplylimit)
                                        add supplier jj


end
```

The algorithm could be improved by choosing the order suppliers are selected as starting points for the aggregation more intelligently, by e.g. choosing the largest



(a) 60 suppliers



(b) 75 Suppliers

Figure 11: Deviation from the exact solution with the aggregation algorithm

supplier. This algorithm gave us the ability to create an infinite number of aggregations by changing supply and distance limit parameters. We therefore got the ability to adapt aggregations to better fit the data cases. The algorithm was easily implemented in Xpress. It was combined with an algorithm that wrote out new data to new files, by picking data from existing data files into new ones.

An overview of different aggregations with the different distance and supply limits can be seen in Table 4. Exact solution means that the terminal configurations found by the deterministic model is equal to the solution where no aggregation is applied.

Table 4: An overview of the different aggregations

Aggregation level	NoSuppliers	Distance limit	Supply limit	Exact solution
0	392	0	0	-
1	75	50	10000	No
2	60	100	10000	No
3	108	50	5000	Yes
4	317	200	200	Yes
5	199	20	2500	Yes

7.1.3 Comparing the supplier aggregations

Aggregation by distances between the suppliers performed considerably better than the two other approaches, and gave relatively good solutions for all of the aggregation made by using the algorithm, as shown in Table 4. There are only a few differences in which terminals the models open. This is most likely due to supply being moved away from where it actually is located. However, the change in objective value when evaluating the terminal structures in the full model were below 1 %.

7.2 Aggregating assortments

Almost all of the variables contains an index of type of assortment. By reducing the number of assortments, we could reduce the number of variables in the models considerably. However, an aggregation of assortments could lead to increase in capacities, as capacities for e.g. transportation and processing are given specific for each assortment. As terminals could be used to overcome capacity challenges, it could affect the solution quality.

To aggregate assortments, we manually added together the different capacities for the transport and processing. We had to create new costs and data for the assortments, and calculate new arc data due to changes in truck costs. This form of aggregation is therefore not as fast to implement as the aggregation by the distances between suppliers. We tested the Sveaskog case, and the difference in terminal structure contributed to less than 2.5 % change in profits. When we aggregate the assortments we may increase some of the capacities as they are no longer assortment specific, and this may lead to finding non-robust solutions.

7.3 Removing the longest arcs

We may assume suppliers in one part of the country would not deliver to customers in the opposite part of the country, and that terminals only want to serve suppliers and customers that are not located too far away. We may therefore remove the arcs with the longest distances between the nodes. This will reduce the number of variables.

We have implemented this by searching for the longest arcs for all the four arc sets, and then removing all arcs with distance above a given percentage of the largest arc distance in each category. We tested it on Case Sveaskog where the assortments were aggregated. Removing all arcs above 50 % and 80 % of the longest arc for all arc sets gave no changes in the terminal structure. The method resulted in a reduction of the average solution times on 56 % and 37 % respectively, as shown in Figure 12. When all arcs longer than 30 % were removed, the change in supply chain profits were below 2 %.

The arc removing should be dependent on the geography in the case. It would therefore be necessary to look into the distances for the problem before deciding on removal of arcs. For Case Stora Enso we could remove all arcs longer than 20 % of the longest arcs with no changes in the terminal structure.

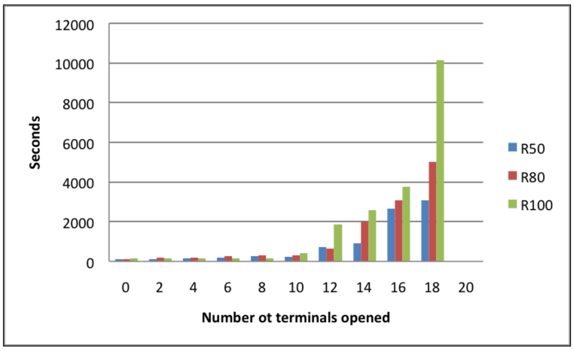


Figure 12: Solution time reduction when removing the longest arcs

7.4 LP-relaxation based heuristic

The model includes binary variables for every potential terminal location, and solving the problem by branch and bound could result in long solution times and memory problems as the branched LP problems for each terminal structure may take considerable time to solve, and several branches may be needed to find the optimal solution.

An advantage of having to run the models with a given number of terminals opened given a priori, is that we may replace the binary variables with continuous variables and demand that the terminal open variables be in the interval [0,1], and choose the terminals with highest variable values. This would considerably decrease solution times.

This heuristics was implemented by replacing the terminal binary variable v_m with continuous variables, and introducing the constraint $\sum_{m \in \mathcal{M}} \mathrm{v_m} = n$. We can then choose to open the n terminals with the highest variable value. These variables are then rounded up to 1, while all others are set to 0. We find the objective function value of the solution, by inputting the terminal structure to the problem. A flowchart for this heuristic is shown in Figure 13.

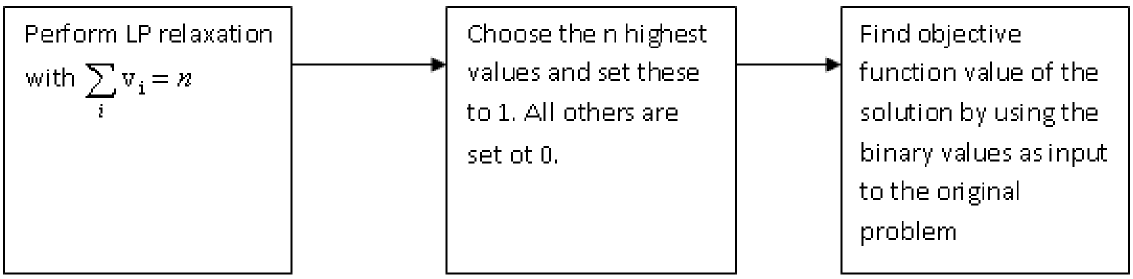
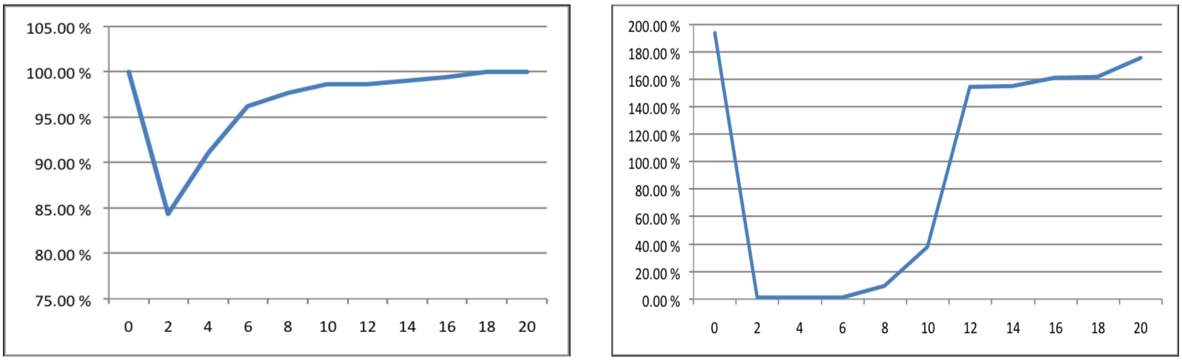


Figure 13: A flowchart for the LP-relaxation based heuristic

The heuristics was tested by running it on Case Sveaskog, and comparing the results with results from the deterministic model, where no solution methods are



(a) Deviation in objective value

(b) Deviation in solution time

Figure 14: Deviation for the LP-relaxation based heuristics from exact solution

used. The deviation is shown in Figure 14. As shown by Figure 14 the solution times were dramatically decreased for solutions where the number of opened terminals was low. When the problem becomes more relaxed with a higher number of allowed terminals, the solution times becomes higher than for the exact model. The reason is that the heuristics run the problem twice; one for finding the terminal structure and one for finding the objective function value.

For a low number of terminals, the heuristics provides solutions which are rather poor compared to the exact solution (85-95 %). When the number of terminals increases, the LP relaxation heuristic returns solutions which lay close to the exact solution ($\geq$ 95 %), as shown in Figure 14.

8 Test case

A test case was constructed to test the model. It consisted of a supply chain with ten suppliers, four customers, four potential terminal locations and two assortments. For the stochastic model we used 100 correlated demand scenarios. But due to higher solution time with the robust model, it was reduced to 50 with the robust model.

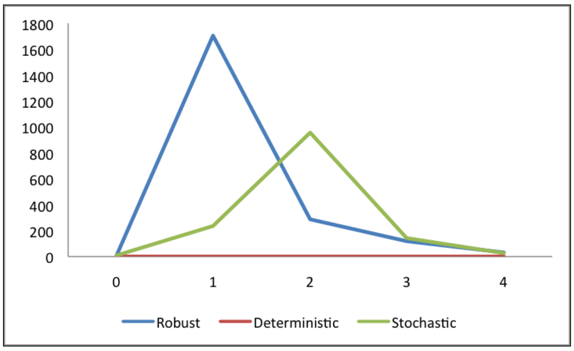
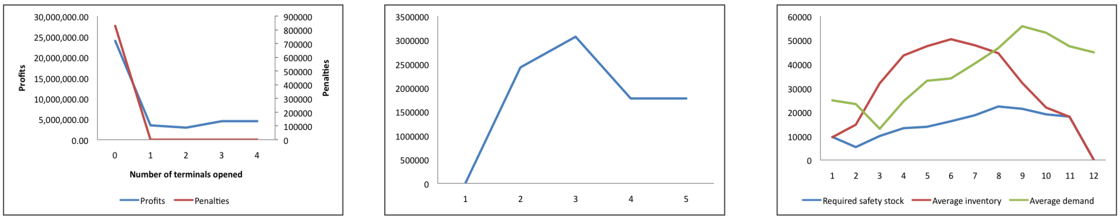


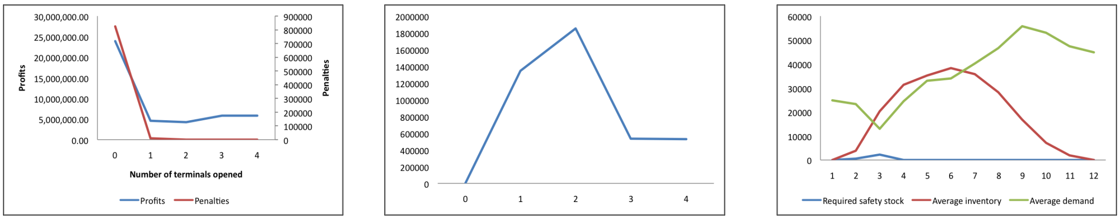
Figure 15: Solution times for our models for the Test case

The stochastic and robust solution times were 100 times higher than the deterministic model. All models build inventory to meet peaks in demand in later periods. The deterministic model does however build more inventory than the stochastic and the robust model. The stochastic and robust model show that required safety stock is required for period 3. These results are show in Figures 16, 17 and 18, where (a) shows how profits improves with the number of opened terminals, and (b) shows the total inventory cost of the opened terminals, and (c) compares the average inventory on each terminal with demand and its required safety stock levels when the number of terminals opened is two.



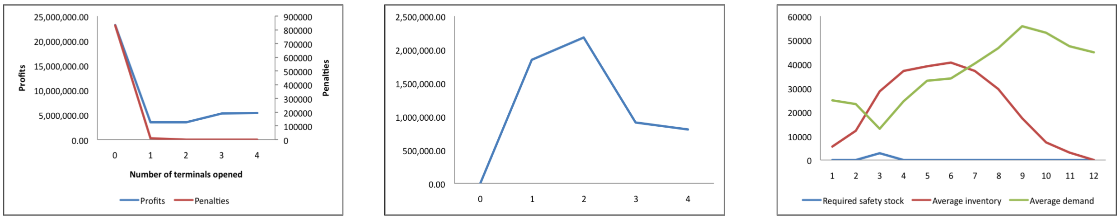
(a) The objective function (b) The inventory cost (c) Inventory compared to demand

Figure 16: Solutions of the test case for the deterministic model



(a) The objective function (b) The inventory cost (c) Inventory compared to demand

Figure 17: Solutions of the test case for the stochastic model



(a) The objective function (b) The inventory cost (c) Inventory compared to demand

Figure 18: Solutions of the test case for the robust model

9 Case Sveaskog

9.1 Case description

Sveaskog is Swedens largest forest owner and its leading supplier of timber and biofuel. In 2009 it had a turnover of 6 billion SEK, and had on average 928 employees[7]. Case Sveaskog considered an area in Sweden located in a square restricted by Stockholm and the Norwegian border, and Linköping and Gävle, as shown in Figure 19. It uses data from 2006 and 2007, and has a total demand of 620 MWh. It consists of 20 terminals, 21 demand points and 392 suppliers, and was originally used for testing the model developed by [Flisberg et al., 2010].

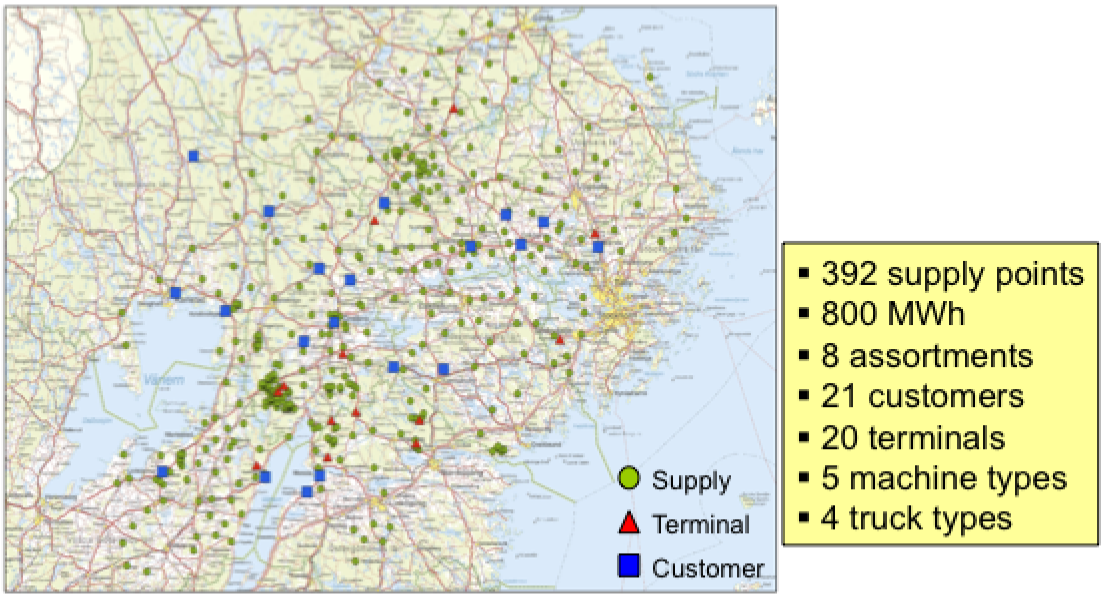
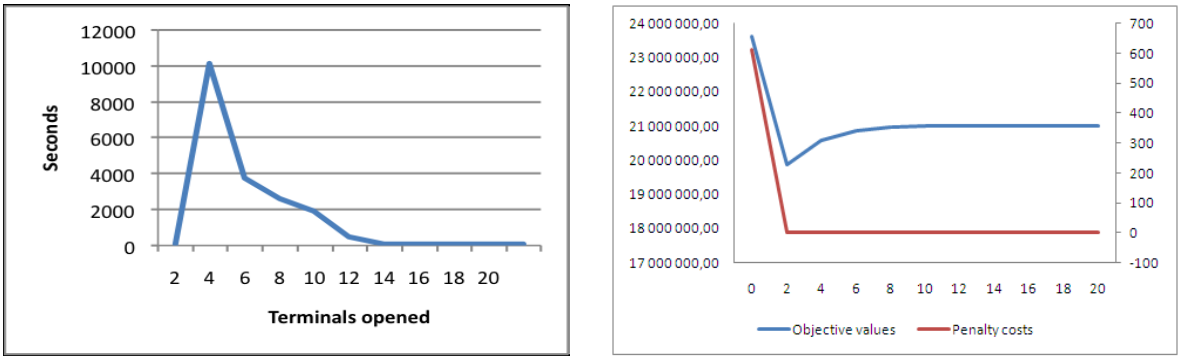


Figure 19: Map for Case Sveaskog

We received data for two data sets for this case; one where the assortments were aggregated from eight to two assortments, and one without any aggregation. For the non-aggregated version, the processing capacity at the terminals became specific to each terminal, which is opposite of the aggregated case. This implies that in this data set, the total processing capacity at the terminals increases with the number of terminals opened. Terminal-to-terminal transport were not included in the Sveaskog case.

[7]Numbers from: `http://www.sveaskog.se/Om-Sveaskog/Var-verksamhet/Foretagsfakta-2008/`



(a) Solution time

(b) Objective values

Figure 20: Solutions for the deterministic model with aggregation on assortments

9.2 Solving the case

We looked at which terminals which were opened if the sum of terminals opened were set a priori. The problem was run with no terminals opened and then increased by two opened terminals until all were opened. Scenarios were generated as correlated demand scenarios. The Sveaskog Case is a large case, and the solution times may therefore become high, and memory errors may occur because the problem either contains too much data, or because of memory difficulties in solving the problem, e.g. with the branch & bound tree. Hence, we needed to reduce the problem in order to be able to solve the problem with scenarios. We therefore aggregated the number of suppliers from 400 to 75 for the stochastic and robust model by use of the algorithm developed in Section 7.1.

9.3 Deterministic solution

9.3.1 Aggregated assortments

The problem was solved with no memory errors, but the solution time became as high as 19500 seconds. The solution times became very high for low number of terminals opened. A possible explanation is that the problem becomes more relaxed when the number of terminals exceeds 10. As shown in Figure 20, the objective value only changes marginally when the sum of opened terminals opened is above eight, as there are no costs connected to open terminals and the safety stock is divided on the opened terminals. It seems that the supply chain achieves flexibility in exploiting the supply chain as cost efficient as possible at this point. The solution incurred a penalty cost when no terminals were opened. The reason was that the processing capacity became too low as most of the processing capacity is located at terminals.

9.3.2 Non-aggregated assortments

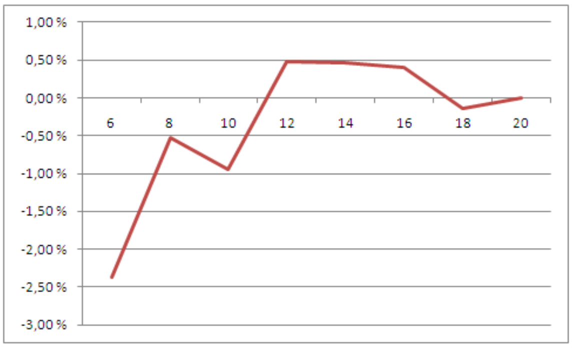
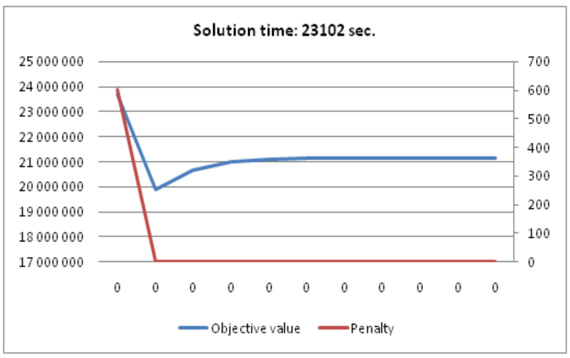


Figure 21: The difference between the solutions from the models with and without aggregation of assortments. Only solutions without penalties are included

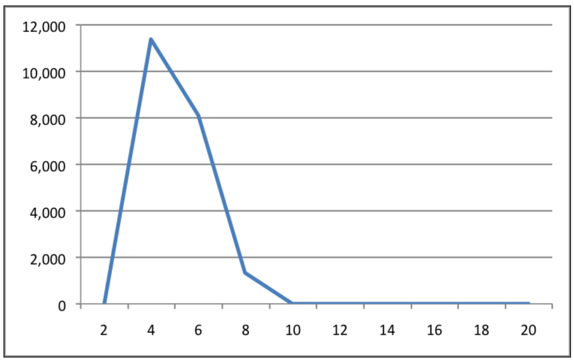
As shown in Figure 21, the difference in the objective function is small for the solutions with and without aggregation of assortments when there is no penalty. There are differences, but the difference is less than 2.5 %. The non-aggregated problem incurred penalties with a higher number of opened terminals than the aggregated. The reason is that the assortment aggregation ignores that some capacities are dependent on type of assortment. Total capacities in the supply chain therefore increase by use of the assortment aggregation.

9.4 Stochastic solution

The purpose of using a stochastic model is to take into consideration the uncertainty in demand. We were therefore interested in using as many scenarios as possible. For the assortment-aggregated case the maximum number of scenarios it could use before memory errors incurred, was two.



(a) The objective function



(b) The total inventory costs

Figure 22: Solution of the stochastic model with 5 scenarios

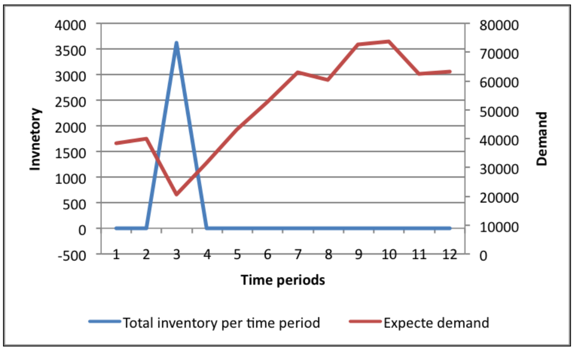


Figure 23: Average inventory for the stochastic solution with six terminals

As shown in Figure 22, the stochastic solution curve reaches the point where there are only marginal increases in the profits with six terminals opened. This is earlier than for the deterministic solution, but is due to the aggregation.

9.4.1 Inventory policy

The inventory policy for the terminals could be found by the stochastic model. Required safety stock values could be extracted from the stochastic model as the minimum inventory level over all scenarios.

The stochastic model builds inventory when the number of terminals are low. When the number of terminals exceeds 10, inventory is no longer needed, as shown in Figure 22. As shown in Figure 23, the stochastic model chooses to build inventory to meet higher demand in the next period for all scenarios. This is not permanent, as demand continues to increase, and it seems that it chooses to build inventory in order to reduce costs in earlier time periods when the demand is low and the possibility to efficiently exploit the transportation and processing resources is more likely.

9.4.2 A stochastic solution - a more detailed analysis

One of the solutions with the highest profits with the aggregated case, is the solution with twelve terminals opened. This solution is a feasible solution, as it does not need to breach any penalties for fulfilling demand.

9.4.2.1 Terminal structure The terminal structure could be drawn on a map as shown in Figure 24. Several of the opened terminals are located close to suppliers. The reason is probably that the transportation capacity is high enough to avoid being limiting on the case. The terminals receive 24.5 % of the volumes intended for the demand points, while the rest is transported directly from the suppliers.

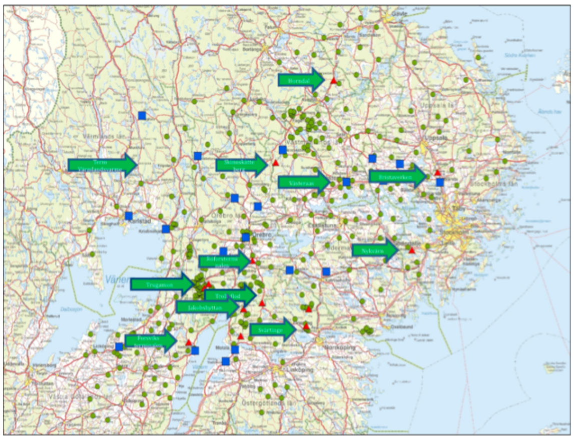


Figure 24: Map of the solution: Round green points are supply points, while square blue points are demand points and red triangles are existing terminals

9.4.2.2 Revenues and costs The highest costs are transport at 30% of total costs, processing 19% and purchased volumes from supply points at 51 %. Gross margin is 19.9 %. The processing costs are evenly distributed for terminals and supply points, while direct transport from suppliers to customers accounts for 66 % of the transportation costs.

9.4.2.3 Transportation, processing and inventory levels The terminals are used exclusively for processing. There are no chipped volumes transported in to any of the terminals, and only chipped volumes are transported out. For this solution, no inventory is found at the terminals. 80 % of the processing capacity at the suppliers, including the combo trucks, is used. The flow directly to the customers, is concurrent with the demand for unprocessed assortments plus the processing capacity at the suppliers. This indicates that terminals are only opened in order to exploit the processing capacities at terminals, and also that the supplier processing capacity is exploited at maximum.

The combo truck is used to supply customers where the distance from the supplier to the customer is small, and as extra chipping capacity where the distance between demand point and supplier is relatively small. The reason is that use of the combo truck is more expensive in transport and processing.

9.4.2.4 Suppliers 5 of the 75 suppliers are not used at all. Total supply is 10 % higher than the demand. One reason some suppliers are not used, could be that they are placed too far away from opened terminals and customers.

9.4.2.5 Flow of assortments By sending the terminal structure into the case where the assortments were not aggregated, we found that all demanded wood parts and pulpwood were sent directly from the suppliers to the customers. The demanded volumes of firewood and grot were sent directly from the supply points to customers, but larger volumes were sent to the terminals to be processed to chips. The suppliers process tree parts and grot into chippings and bunts of grot.

9.4.3 Solution with non-aggregated assortments

Solving the stochastic model when there was no aggregation of assortments was harder. As we wanted to compare the results with the results for when the assortments were aggregated, we aggregated the suppliers down to 75. In order to run the model without memory errors we used 3 scenarios and the LP-relaxation based heuristics, as discussed in Section 7.4.

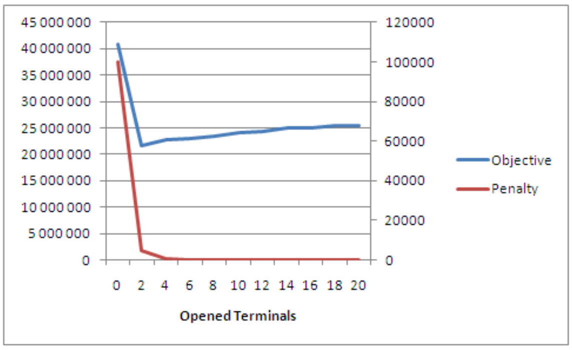


Figure 25: Solution of the stochastic model with non-aggregated assortments

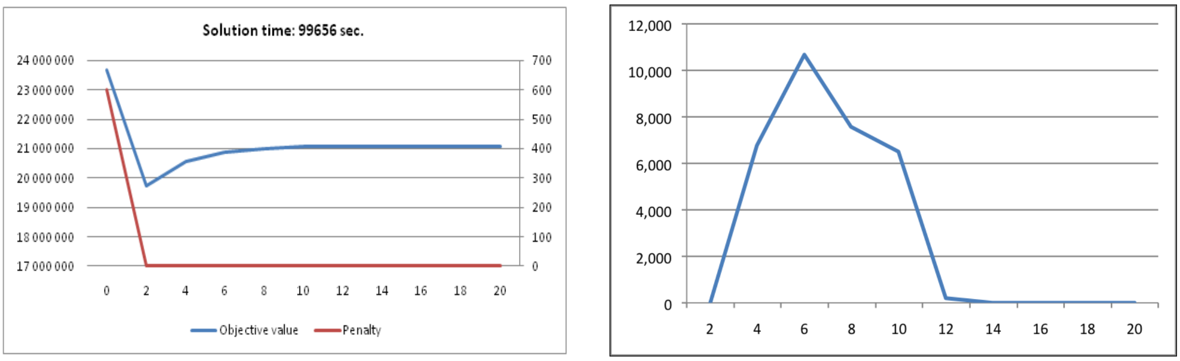
The objective value increased for all numbers of terminals opened. This did not happen when the assortments were aggregated. However, in the data provided for us, the processing capacity at the terminals was given for each terminal, i.e. the terminal capacity increased for each terminal opened. We therefore increased flexibility in the supply chain for each opened terminal.

The penalties are higher in this case compared to when the assortments were aggregated. With an increased number of assortments, and the transportation and processing capacities more restricting as they are given per assortment, we believe that it would become harder to exploit the resources in the model. This explains the higher penalty costs when there are no terminals opened.

It returned different terminal structures, and this is most likely due to a more detailed supply of the different assortments, and that some of the processing is required to be performed at terminals. The model does not return any clear recommendation on which terminal structure that would increase profits the most. This is however as expected, as there are no fixed costs connected to opening new terminals.

9.5 Robust solution

We wanted to run the robust optimization with as many demand scenarios as possible in order to take uncertainty into consideration. We discovered however that we could only use one scenario before the robust model incurred a memory error.



(a) The objective function (b) The total terminal inventory cost

Figure 26: Solution of the robust model with 5 scenarios

The robust model returns an infeasible solution when no terminals are opened, as the model is forced to invest in extra processing capacity at the suppliers. The best solution is found with 14 terminals opened. The robust solution curve reaches the point where there are only marginal increases in the profits with six terminals opened. This is earlier than for the deterministic solution. It might be explained by the aggregation. This curve might be used for finding maximum allowed marginal annual costs of opening terminals.

9.5.1 Inventory policy

The inventory policy for the terminals could be found by the robust model. The robust model builds inventory when the number of terminals are low. When the number exceeds 12, inventory is no longer needed, as shown in Figure 26. Required safety stock values could be extracted from the robust model as the minimum inventory level over all scenarios. The model rarely built inventory, and when it built inventory, it was only in some of the scenarios. We could therefore not find any values for the required safety stock above zero.

9.5.2 A robust solution - a more detailed analysis

We printed the solutions to data files, and these could be used to analyze the solutions. One of these solutions is a solution with fourteen terminals opened. This is one of the solutions with the largest profits. This solution is a feasible solution, as it does not need to breach any penalties for meeting demand.

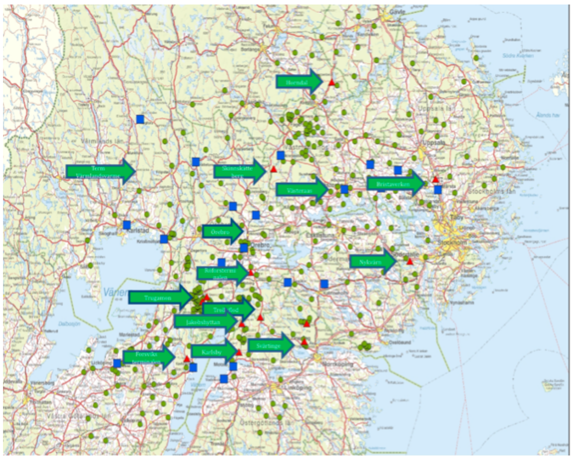


Figure 27: Map of the solution: Round green points are supply points, while square blue points are demand points and red triangles are existing terminals

9.5.2.1 Terminal structure The terminal structure could be drawn on a map as shown in Figure 27. Several of the opened terminals are located close to suppliers. This is a less robust, but a more cost efficient solution. The reason is probably that the transportation capacity is high enough to avoid being limiting on the case. The terminals receive 25 % of the volumes sent to the customers, while the rest is transported directly from the suppliers.

9.5.2.2 Revenues and costs The highest costs are the transportation costs at 30 % of total costs, bought volumes from the suppliers at 50 % and processing of the wood at 18%. Processing costs are evenly split between suppliers and customers, while transportation directly to demand points contributes to two thirds of the transportation costs. Gross margin is 19.8%.

9.5.2.3 Transportation, processing and inventory levels The total direct transport from suppliers to demand points is concurrent with the total demand of unprocessed assortments plus the processing capacity at the suppliers. This indicates that terminals are primarily used for exploiting the chipping capacity located at the terminals. Hence, everything that is possible to transport directly from a supplier to a customer is transported directly. The combo truck is used to supply customers where the distance from the supplier to the customer is small, and as extra chipping capacity where the distance between demand point and supplier is relatively small. This is because it is more expensive in transportation and processing.

9.5.2.4 Suppliers 5 of the 75 suppliers in the stochastic model are not used at all. Total supply is 10% higher than the demand. One reason some suppliers are not used, could be that they are placed too far away from opened terminals and customers.

9.5.2.5 Flow of assortments By sending the terminal structure into a problem where the assortments are not aggregated, we see that all demanded wood parts and pulpwood are sent directly from the suppliers to the customers. The demanded volumes of firewood and grot are sent directly from the supply points to customers, but larger volumes are sent to the terminals to be processed to chips. The suppliers process tree parts and grot into chippings and bunts of grot.

9.5.3 No aggregation of assortments

We wanted to run the model with no aggregation of assortments. We aggregated the number of suppliers down to 75 suppliers, used three demand scenarios, and by using the LP-relaxation based heuristics, as discussed in Section 7.4, and turning the presolver in Xpress[8] off to reduce memory usage, we managed to run the model.

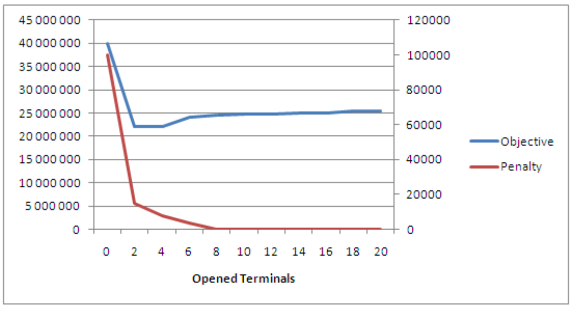


Figure 28: Solution of the robust model with non-aggregated assortments

The penalties were larger, and this could be explained by the increased number of assortments, which makes the processing and transportation capacities more restricting due to assortment-specific capacities. We see that we need over 6 terminals in order to have gain robustness in the supply chain by use of the robust solutions. The higher profits than the ones with aggregated assortments are due to different costs and prices.

9.6 Resource analysis

We were interested in looking at how the optimal terminal structure would change if some of the resources in the supply chain were changed. As for the aggregated assortment case, we have seen that the transportation capacity is not particular limiting. We therefore decided to look on the chipping capacities. We therefore

[8] The presolver is discussed in Section 6.6

tested the deterministic model with and without the combo truck, and for cases where the supplier or terminal processing capacities was lowered by ten percent individually and together. We found that if the combo truck is removed, it seems that it is optimal to open an extra terminal. The existence of a combo truck is also the factor which contributes the most on the supply chains profits. An explanation is the lower transportation costs for the combo truck, than for the grot truck, as well as the extra processing that originate from the combo truck. The processing capacity at the suppliers influences the profits, but this is not the case for the terminal processing capacity. This is due to the processing capacity in the assortment aggregated case is very high at the terminals.

The dual variables tell how much the objective function will change if the capacities belonging to each category of resources are increased. In the case where 14 terminals are opened by the deterministic model, we inputted the solved MIP solution into the deterministic LP model without safety stock levels, to study the dual variables. They showed that the two most valuable resources were the machine capacity of processing wood to wood chippings at the suppliers and the available work hours each month for each combo truck. In both cases, this would have decreased the need for transporting grot, which is the most expensive transportation form, and could avoid possible detours via terminals for delivering fuel to customers. In order to expand the capacity and the profit of the supply chain, one should therefore invest in processing capacity at the suppliers or more work hours for the combo trucks.

9.7 Discussion

The Sveaskog case proved to be a difficult case to solve due to memory errors when introducing scenarios and we needed to use some of the approaches discussed in Section 7 in order to get solutions from our models.

It is difficult to conclude on which terminal structure the Sveaskog case should use. This would be a trade off between fixed annual fixed of opening the terminals and the marginal profit associated with the terminals opened. The graphs we had presented could easily return these marginal profits.

The deterministic safety stock policy costs the Sveaskog supply chain SEK 99 000 each year, when the assortments are aggregated. The inventory returned from the robust model, as shown in figure 26, and the stochastic models, as shown in figure 22, indicates that changing the inventory policy could save some of these costs. The transportation and processing capacities are so high that there is no need to build as much inventory as the deterministic safety stock policy requires.

The terminal structure found by the stochastic solution deviated from the deterministic solution where no aggregation were used, and looked to a large degree similar to the terminal solutions found by the deterministic model with the same aggregation. However, the supplier aggregation could lead to deviations up 1 % in profits. This could for some solutions exceed what could be saved by changing the inventory policy.

We could therefore ask how much the terminal structure influences the supply chain profits. We compared the optimal terminal structures' profits with profits from opening closed terminals and closing the open ones for each solution. The differences between the solutions are shown in Figure 29. The difference is higher when the number of terminals opened is low, as the opened terminals have a greater influence on the profits. Profits could however be gained by using a close-to-optimum terminal structure for the Sveaskog supply chain.

Figure 29: The influence on profits by terminal structure with aggregated assortments

10 Case Stora Enso

10.1 Case description

Stora Enso is a Finnish-Swedish company, and one of the world's largest pulp and paper manufacturer. The Group has 27 000 employees and 88 production facilities in more than 35 countries worldwide[9]. The case consists of data from 2008, and has a total demand of 3055 MWh divided over four assortments. The supply chain consists of 81 terminals, 70 demand points and 1200 suppliers.

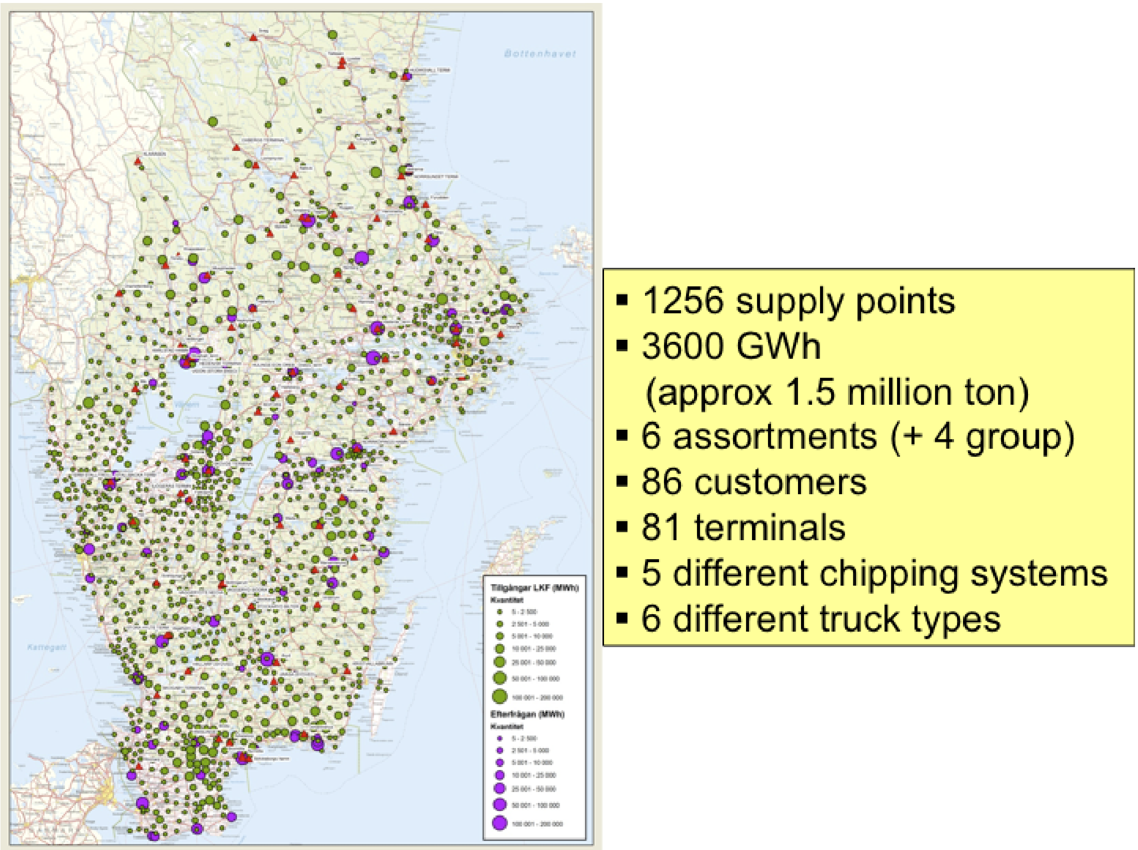


Figure 30: Map for Case Stora Enso

The Stora Enso case contained some differences compared to the Sveaskog case. It included inventory at existing terminal locations that could be bought and utilized. It also included terminal handling costs for the flow out of the terminals. One of the assortments could only be processed at the terminals. This required a different modeling of the problem. We now accepted flows going out and in of closed terminals, but prevented processing and inventory at the closed terminals. Constraint 5.17 were therefore replaced with Constraints 10.1 and 10.2, and we removed

[9]Stora Enso, `http://www.storaenso.com/about-us/stora-enso-in-brief/Pages/stora-enso-in-brief.aspx`, accessed 31.5.2010 09:40

Constraint 5.18.

$$\sum_{h\in\mathcal{H}} f^{t}_{hh'n} v^{MT}_{ihnt} \leq u^{f+}_{n} v_m, m \in \mathcal{M}, n \in \mathcal{N}^M, t \in \mathcal{T} \tag{10.1}$$

$$l^{M}_{mht} \leq u^{M0+}_{mht} v_m, m \in \mathcal{M}, h \in \mathcal{H}^R, t \in \mathcal{T} \tag{10.2}$$

Where $\mathcal{N}^M$is a set of processing machines at the terminals, and $\mathcal{H}^R$ is a set of assortments that could deteriorate due to humidity if left in the catchment areas too long, such as e.g. wood chippings. The initial inventory at the terminals could be bought and used in the rest of the supply chain. We needed to limit the use of this inventory, as shown in Equations 10.3 and 10.4. We did not allow supply at closed terminals to be transported to a supplier for processing.

$$\sum_{t\in\mathcal{T}} b^{M}_{mht} \leq InitInv^{M}_{mh}, m \in \mathcal{M}, h \in \mathcal{H} \setminus \mathcal{H}^R \tag{10.3}$$

$$b^{M}_{mh1} \leq InitInv^{M}_{mh}, m \in \mathcal{M}, h \in \mathcal{H}^R \tag{10.4}$$

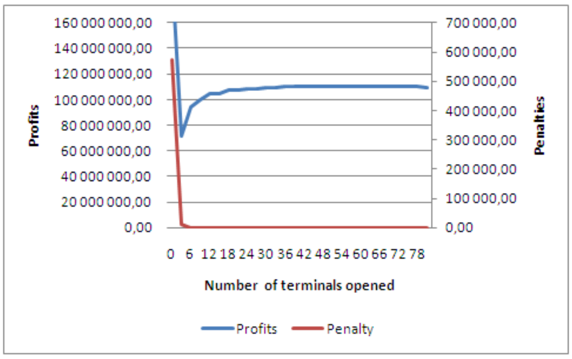
10.2 Solving the case

This case proved to be a challenge to solve. Due to the very large size of the problem, we had to reduce the problem considerably. We did this by first aggregating the number of assortments from 4 to 3, and then aggregating the number of suppliers from 1200 to 53. All arcs with distances larger than 40 % of the distance of its largest arcs were also removed. This made us able to run the stochastic and robust model with 3 scenarios. Due to the large number of binary variables, we had to use the LP relaxation heuristics to shorten the solution times.

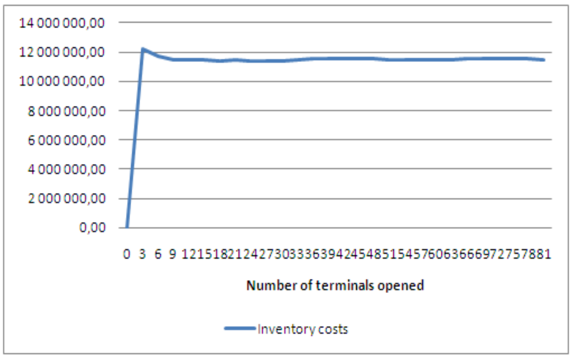
Redundant variables were also removed. Customer inventory and unfulfilled demand were removed, as these options are to be penalized. Flows between the nodes were only created for flows where there existed supply or demand for the given assortment. We also removed the constraint for the level of customer inventory, and turned the Xpress presolver off.

10.3 Deterministic solution

The deterministic solution used about 14 hours to be solved for all number of terminals from 0 to 81 with an increment of 3. The solutions incurred penalty costs when the number of terminals was three or fewer. As shown in Figure 31 the profits gradually stops increasing when the number of terminals reaches 30, and decreases



(a) The objective function



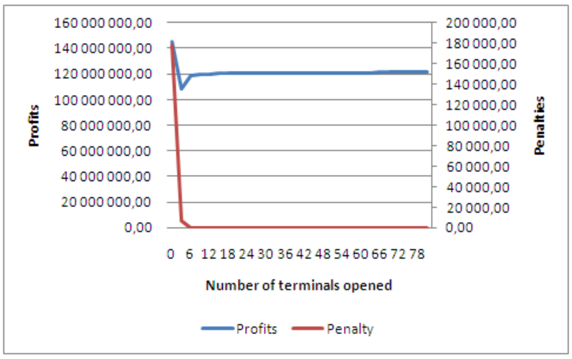
(b) The total terminal inventory cost

Figure 31: Solutions for the deterministic model of the Stora Enso case.

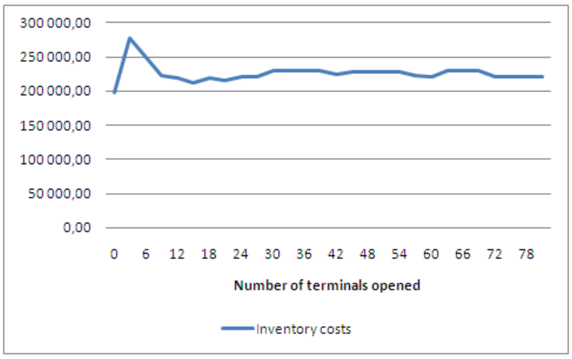
when it reaches 54. This is due to increased transportation costs for the transport of safety stock to terminals that may be located further away. The inventory costs are high for a low number of terminals, but quickly fall down to the levels for the required safety stock.

10.4 Stochastic solution

We used approximately 35 hours to solve the stochastic model with an interval of three terminals by use of the LP relaxation heuristics. For no opened terminals and three opened terminals the supply chain is not able to deliver volumes to fulfill demand.



(a) The objective function



(b) The total terminal inventory cost

Figure 32: Solutions for the stochastic model of the Stora Enso case.

A large part of the difference between the stochastic and the deterministic solution is due to reduced inventories due to the lack of preset safety stock levels, and reduced transportation costs. When the number of opened terminals increases, transportation costs decreases.

10.4.1 Inventory

The inventory levels are considerably lower than for the deterministic solutions. They only decreases slightly as the number of terminals increases. The costs include storage costs of assortments bought from closed terminals, which does not need to be brought to facilities for storage, but could be stored in the forests, e.g. grot. Increased use of such inventories is an explanation for why the inventory levels do not decrease more.

10.4.2 A stochastic solution - a more detailed analysis

One of the most profitable solutions is a terminal configuration with 27 opened terminals.

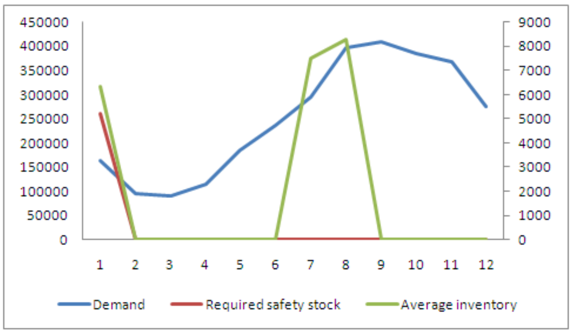


Figure 33: Safety stock levels for the stochastic solution with 27 opened terminals

10.4.2.1 Inventory policy According to the model, required safety stock is found for the first period. But this level also include initial inventory transported from closed terminals. On average, the model also builds inventory in order to handle demand peaks. However, this inventory differs dependent on the scenario. This shows that there exists a large degree of flexibility in how inventory is to be built for later periods.

10.4.2.2 Revenues and costs Among the highest costs is transportation at 18.5 %, processing at 28.0 % and purchasing of wood from suppliers at 46.8 %. Most of the activity in the supply chain is located at the suppliers, which contributes to 88.8 % of the processing costs and transports 90 % of the supply directly to the customers. Gross margin is 22.6 %.

10.4.2.3 Transportation, processing and inventory levels Of the demanded volumes, 68.3 % is processed at the suppliers, 15.5 % is processed at terminals and the last 15.7 % are sent directly from suppliers or terminal points without any processing. The combo trucks participate in this processing at the supply points. The combo truck delivers to customers that lay fairly close. The combo trucks processes 26.4 % of the processed volumes at the supply points. The usage of the combo truck is however more expensive, and the capacity of the combo truck is therefore only exploited fully in high-demand scenarios. It seems therefore that the combo truck could be used to increase robustness. All types of assortments are transported directly from suppliers to customers, and from terminals to customers, while chippings is not transported in to terminals. Even though all types of assortments are bought at terminal locations, only firewood is transported to other terminals. This is due to lower transportation costs for firewood compared to other assortments.

10.4.2.4 Suppliers There is 12.8 % higher supply than total demand, but 13.2 % of these volumes are located at potential terminal locations. 13.5 % of the volumes at the suppliers are not used.

10.4.2.5 Resources Processing capacities are exploited at maximum in high-demand periods. It is therefore profitable to invest in processing capacities for grot into chippings at suppliers, firewood into chippings at terminals and combo truck capacity. Capacities for trucks for chippings could also be increased. However, the extra capacity is only profitable in high-demand periods.

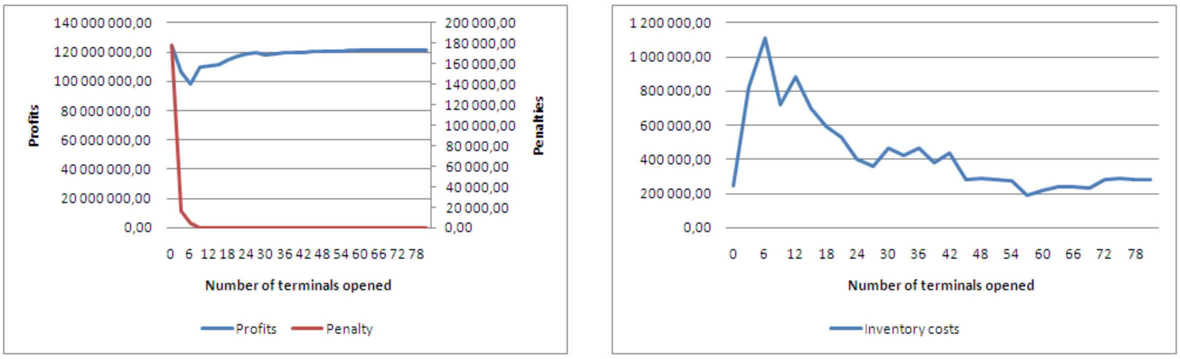
10.5 Robust solution

The robust model used three scenarios, and used approximately 30 hours by use of the LP relaxation heuristics. It incurs a penalty when the number of opened terminals is below six, as the supply chain would not be able to fulfill demand. The profits reach its peak at about thirty terminals, and remains at this point even with an increase in the number of opened terminals.

With an increased number of terminals, transportation costs decreases, more volumes are bought from terminal locations, and a larger part of the processing is done at the terminals. Compared to the deterministic solutions, there are considerably less inventory costs, lower transportation costs, and a larger part of the assortments are routed via terminals.

10.5.1 Inventory

The inventory levels decreases with an increase in the number of terminals. The deterministic model incurred a cost of 11.5 million for its inventory, and explains



(a) The objective function (b) The total terminal inventory cost

Figure 34: Solutions for the robust model of the Stora Enso case.

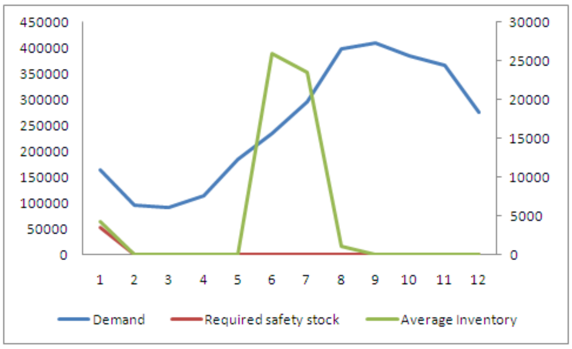


Figure 35: Safety stock levels for the stochastic solution with 27 opened terminals

why the robust solution increases the supply chains profits by 10 million, compared to the deterministic solutions. The increase in inventory at a higher number of opened terminals is due to increased use of buying volumes at terminal.

10.5.2 A robust solution - a more detailed analysis

One of the most profitable solutions is a terminal configuration with 27 opened terminals.

10.5.2.1 Inventory policy According to the model, required safety stock on the terminals is required in the first period. However, the value is found by also including transport in of initial inventories from closed terminals. On average, inventory is built in later periods to meet peaks in demand. Required safety stock is not required in these periods, and it seems that there is much flexibility in how this inventory could be built.

10.5.2.2 Revenues and costs Among the highest costs is transportation at 18.4 %, processing at 28.2 % and purchasing of wood from suppliers at 45.5 %.

Most of the supply is located at the suppliers, which contributes to 87.8 % of the processing costs and transports 90 % of the supply directly to customers. Gross margin is 22.4 %.

10.5.2.3 Transportation and processing Of the demanded volumes, 67.2 % is processed at the suppliers, 15.6 % is processed at the terminals and the last 17.2 % are sent directly from suppliers or terminal points without any processing. The combo trucks participate in this processing with the other machines at the supply points. The combo truck delivers to customers that lay fairly close. The combo trucks processes 22.9 % of the processed volumes at the supply points. The usage of the combo truck is however more expensive, and the capacity of the combo truck is therefore only exploited at maximum in high-demand periods. All assortments are transported from suppliers to customers, and from terminals to customers, but only grot and firewood are transported from suppliers to terminals. All types of assortments are bought at terminal locations, but only firewood is transported into open terminals from closed terminals. This is due to cheaper transport costs.

10.5.2.4 Suppliers There are 12.8 % higher supply than total demand, but 13.2 % of these volumes are located at potential terminal locations. 13 % of the volumes at the suppliers are not used.

10.5.2.5 Resources Some resources are used at maximum in high-demand periods. These are the truck capacities for chippings, chipping capacities at suppliers and capacities for processing firewood into chippings at terminals. According to the dual variables, it would be profitable to invest in all of these capacities.

10.6 Discussion

The number of terminals influences how well the supply chain performs. If less than six terminals are used, there will be too few terminals to be able to fulfill demand. The profit increases fade out as the supply chain reaches its desired flexibility to exploit the terminal structure as efficient as possible. This happens for the deterministic model at around 30 opened terminals, for the robust at 45 and for the stochastic at 51. This is due to the deterministic model preset safety stock levels, and the need to use terminals to achieve flexibility in the supply chain when uncertainty is introduced. The stochastic and robust solutions show that the deterministic safety stock levels are set too high, and considerable costs could be saved by changing the inventory policy.

Several approaches were used to reduce the problem to be able to run it on a regular computer. It is difficult to state how much the solution quality has been affected, and the terminal structure are probably not at the exact optimum. As only three scenarios were used, the robustness of the solutions could also be questioned. However, from a supply chain planner's view, the solutions may actually prove to be rather good, as they probably increases the profits compared to doing supply chain design manually. This is especially true if a planner chooses a manual found solution with a low number of terminals.

11 Discussion & applicability

11.1 Implementation and initialization of the models

The modeling and implementation of the supply chain design problem were time consuming and challenging, as the problem is complex and relatively large. An advantage of these problems and use of stochastic optimization, is that much work has already been done in this field. It was therefore not difficult to find relevant literature sources on these subjects. However, on robust optimization, there was less to be found. It was therefore more challenging to work with the robust model.

A challenge with supply chain design is that supply chains may be different, and require different modeling. For example would the Stora Enso case contain initial inventory at existing terminals which needs to be transported to other terminals if the terminal is to be closed. This means that we cannot develop models which would fit to all cases, but need to adapt the model and the implementation approach to each case.

The number of suppliers, customers, terminals and assortments may become quite high. This means that the problems may become very large. This leads to two challenges. One is that the memory on the computers may become too limiting. This means that much work and time is required for presolving and implementation of solution methods to reduce the size of the problems. It also means that the number of scenarios that could be used is limited. This could lead to less robust solutions. Secondly, errors in the implementation or data may occur. But as it could be hard to imagine how a solution would look, it is harder to discover these errors. A reference point or close cooperation with the company could have helped overcoming this problem, but in our case we did not have this. As the solution time also increases, time used for searching for errors would increase.

11.2 Analyzing solutions

The amount of information possible to extract from solving these cases could be very high and it could be easy to lose track of what information that is of interest. As the solution time could be very high for some cases, it is important to clarify this in advance. We printed the information we felt of interest to data files, which were written so they could be analyzed directly or by use of Microsoft Excel. The models could be used for analyzing the supply chains. Different terminal configurations could be inserted in the model, and data for the optimal exploitation of this solution could be printed for the analysis. The problem is then changed to a LP problem, and values for the dual variables and reduced costs could be extracted.

11.3 Modeling uncertainty

The uncertainty has been modeled with use of three or five demand scenarios, with correlated demand variations. In order to evaluate at how good this modeling of uncertainty was, additional correlated and uncorrelated demand scenarios were created, for 5 and 10 scenarios. We used computers from a cluster with considerably more memory than a normal computer in order to solve the models with additional scenarios [10]. It was tested on the assortment-aggregated Sveaskog case aggregated down to 75 suppliers by use of a stochastic model.

The terminal structure changed with how the uncertainty was modeled. For five scenarios, the stochastic model gave different terminal structures than the deterministic. For correlated demand scenarios, 37 terminals were shifted, while 39 were shifted for the uncorrelated case. 32 terminals were different when correlated scenarios were used compared to uncorrelated. The robust model made fewer changes. When the number of scenarios was increased to 10, all the models replaced over thirty terminals compared to the case of 5 scenarios. The change was largest for uncorrelated scenarios.

An increase in the number of scenarios would force the stochastic and robust model to adapt more to other changes in demand realizations, and would lead to more robust solutions. It seems therefore that an increase in the number of scenarios would have provided other and more robust solutions than the ones presented.

We have found that the type of correlation affects the terminal structure returned from the models. The correlation of the demand should depend on how the demand uncertainty actually is. This requires a proper analysis of the variations in demand. We have in the cases assumed a large degree of correlation as we assume that demand for heat to a large degree is correlated with temperature and weather.

11.4 Preprocessing, aggregation and the LP based heuristic

The two industrial cases were initially too large to be solved on regular computers, and a reduction of the problem size were needed. The variables which contributed the most to the size were the flow variables.

We aggregated both the assortments and the number of suppliers. The aggregation of assortments was effective as most of the variables in the problem were given for each assortment or assortment group. The aggregation of supply points reduced the number of arcs, variables and constraints for the assortments. The use of aggregation could however return solutions which are different from the non-aggregated

[10]The Solstorm cluster at Department of Industrial Economics and Technology Management, NTNU. See `https://solstorm.iot.ntnu.no/wordpress/`

case, as shown in Section 7.1 and 7.2. For assortment aggregation the deviation from its non-aggregated case was below 2.5 percent, and for the supplier aggregation the deviation from its non-aggregated case was below one percent. This show that use of aggregation could be an effective way of solving similar problems. However, supplier aggregation by use of the algorithm we have developed give better solutions than the assortment aggregation, and is also easier to implement and adapt to the given cases. Assortment aggregation could also increase the capacities in the supply chain, and could therefore return solutions that are not actually robust.

The LP-relaxation based heuristics reduced the solution times considerably. But as shown in Section 7.4, the terminal structures found by this heuristics could behave rather poorly, especially when the number of terminals is low. The use of the heuristics could also be improved. We could have reduced solution times by solving the terminal structure when all or none terminals were opened, but instead only evaluated the terminal structure. Variables and constraints for closed terminals could be removed for closed terminals in the model that evaluated terminal structures.

We also removed the arcs which had the longest distances. We discovered that this could reduce the solution times considerably. We removed other redundant and unnecessary variables, and chose to only create the variables which were explicitly needed. In Case Stora Enso, we did not create variables for e.g. flow of assortments to demand points which were not demanded, or flow variables for assortments which did not fit to the truck, and we removed the customer inventory capacity constraint and the variables for customer inventory and unfulfilled demand, as these variables had penalty costs attached to them, and other variables served to indicate if the supply chain design is robust or not.

We had to use all of these solution methods to solve the problems. It is not easy to evaluate how much the solution methods have influenced the solutions, and how far the solutions are from the global optimum. However, the alternative is to design the supply chain manually. As described in Section 3, there are many factors to consider, and uncertainty may increase the difficulty in finding good solutions. The result of such manual planning could therefore become rather bad. But use of decision support systems, such as the models we have developed, could help the planner in finding better solutions.

The constraints that restricted in- and outflows on closed terminals were in our original problem formulation written in a way that increased the size of the problems. But by writing the constraint different, by splitting the sum of flows into time periods and assortments, we managed to reduce the problem, and speed up solution times and decrease memory usage. u_m^{M+} could be replaced by a smaller value, however the Xpress presolver should normally do this.

Old formulation:

$$\sum_{j:(m,j)\in\mathcal{R}^C}\sum_{k\in\mathcal{K}}\sum_{h\in\mathcal{H}_k^K} x_{mjkht}^{M-J} - u_m^{M+}v_m \leq 0, m\in\mathcal{M}, t\in\mathcal{T} \tag{11.1}$$

New formulation:

$$\sum_{j:(m,j)\in\mathcal{R}^C} x_{mjkht}^{M-J} - u_m^{M+}v_m \leq 0, m\in\mathcal{M}, k\in\mathcal{K}, h\in\mathcal{H}_k^K, t\in\mathcal{T} \tag{11.2}$$

11.5 Comparing the optimization approaches

The models could be evaluated on solution times and how well the solutions perform. The expected value of perfect information (EVPI) is the difference between the wait-and- see problem solution and the stochastic solution, and is used to determine how much the solution deviates from a solution if all information were known. The value of stochastic solution (VSS) and value of robust solution (VRS) is used to evaluate how well the models behave when taking uncertainty into consideration. To evaluate how well the robust and the stochastic solutions perform under uncertainty, we have inserted the terminal structure and required safety stock found by the robust model into the stochastic model to find its stochastic counterpart, and vice versa. We have in this analysis, chosen to include the analysis of the Stora Enso case and the non-assortment aggregated Sveaskog case, as these two cases had limited capacities and showed the largest effects.

11.5.1 Implementation & running

Compared to the deterministic model, the robust and the stochastic model was more likely to incur memory errors, and required more work to implement them in a way that made it possible to run them. The robust model required the most computer capacity and incurred memory errors more often.

11.5.2 Inventory levels

Inventory is built in the models, and we have seen that inventory decreases with the number of opened terminals, e.g. Figure 16. The deterministic model builds more inventory than the stochastic and the robust model, and this is due to the preset required safety stock levels in the deterministic models. The stochastic and robust solutions show that a preset safety stock of 40 % of the next month's demand is too high, and should be reduced.

The stochastic and robust models builds inventory, but required safety stock levels above zero is not always found. It seems to mainly occur for earlier time periods, even though average inventory may be higher in all periods. It seems therefore that there exists flexibility in how to choose inventory levels for each time periods in order to build inventory to meet peaks in demand. In the models, this is dependent on the scenario drawn. Required safety stock levels could however also be needed for other purposes than balancing capacities, such as increasing the reliability in the supply chain. [11]

11.5.3 Terminal structures

When the number of opened terminals increases, transportation costs usually decreases. However, the robust solutions seem different than the stochastic ones. The robust model have higher costs for transportation from suppliers to terminals and higher processing costs at terminals. This indicates that the robust model exploits terminals more than the stochastic model. This trend was observed in both cases. The stochastic model returned the highest profits, and is mainly due to lower transportation costs. The deterministic model had higher inventories due to its preset inventory levels, and also increased transportation due to the inventory.

From Figure 36 we see that the deterministic model chooses terminals that lay closer to customers and suppliers than the two other models. The stochastic model and robust model often choose terminals that lay further away.

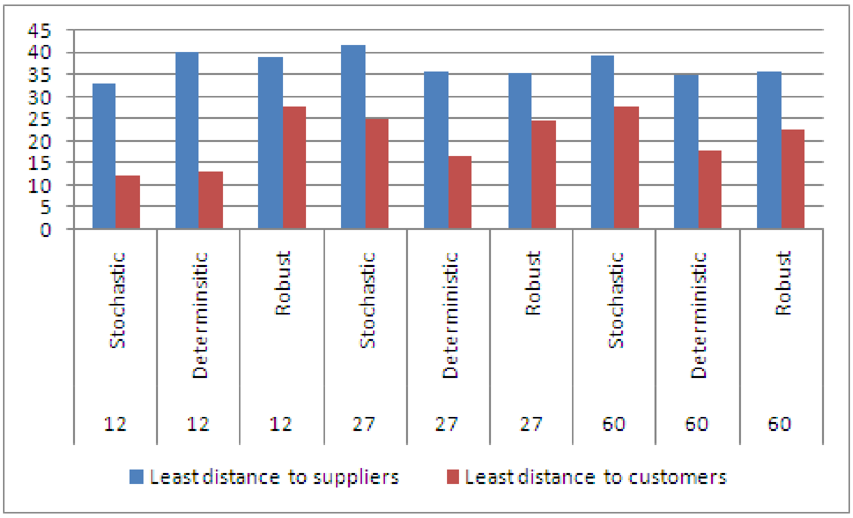


Figure 36: Average least distance between opened terminals and suppliers / customers with 12, 27 and 60 opened terminals

The number of times the terminals were opened was used to determine which factors that contributed to the opening of the given terminals for each model. For

[11] See Section 3.1.

the deterministic and robust model, demand factors had larger influence than supply factors, even though the cheaper initial inventory at the given terminal had a significant impact. The least distance to the closest customer had larger impact than the least distance to a supplier for the deterministic and robust models, and is due to terminals being located closer to customers than suppliers, and this is also a more robust configuration.

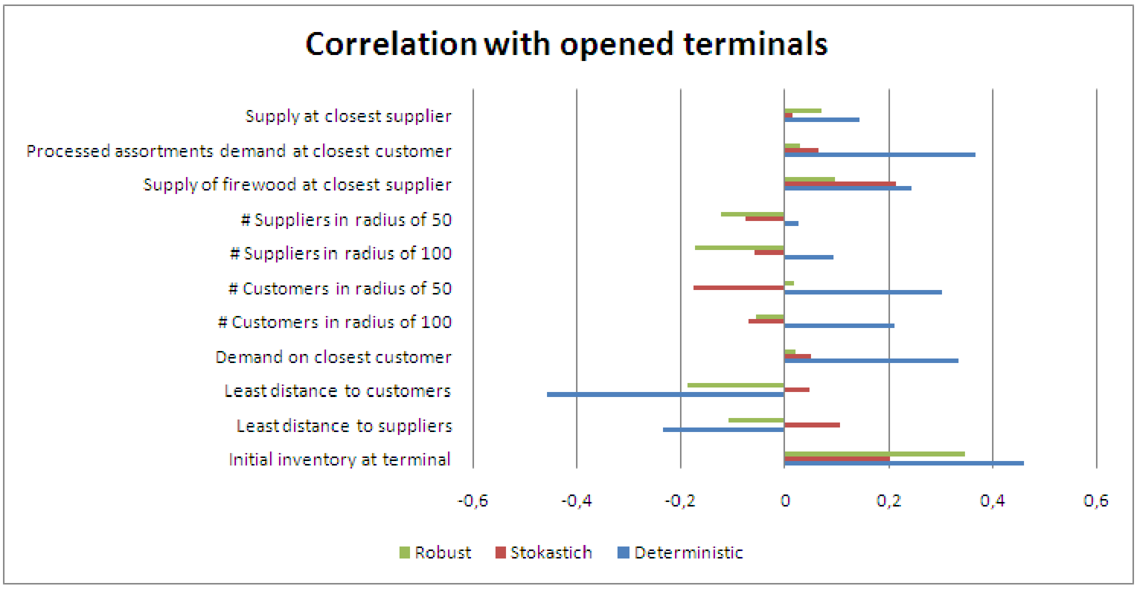
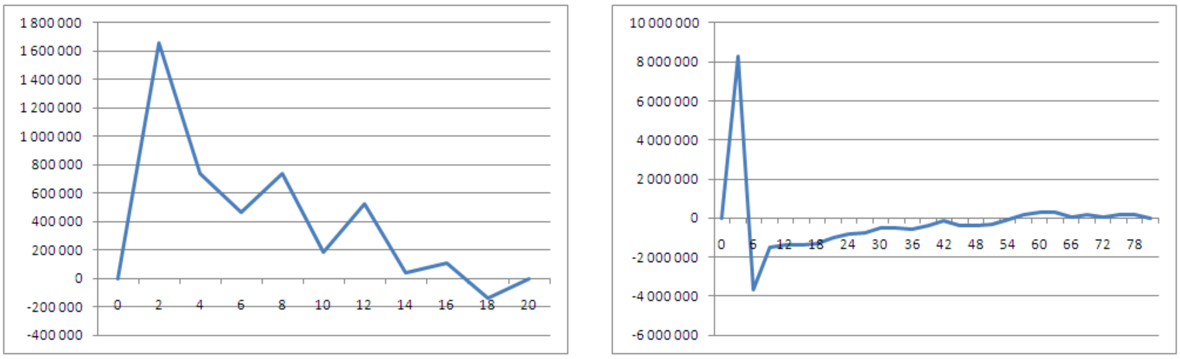


Figure 37: Correlation with opened terminals

The deterministic model correlated considerably more than the other models. It seems that the deterministic model chooses solutions where the terminals are located close to customers and suppliers, and chooses a cost-efficient solution. When uncertainty is introduced, the factors become less significant. A possible explanation is that the robust and stochastic solution chooses to open terminals that lay further away. This is due to the limited transportation capacity and in cases where customers far away from suppliers demands higher volumes in high-demand periods, the transportation capacity could become too limiting. The stochastic and robust models would therefore try to locate terminals in order to avoid this.

11.5.4 EVPI

EVPI is the difference between the wait and see problem and the solution of the stochastic problem, and explains how well the solutions perform in the future. An EVPI-value close to zero means that the inventory policy and terminal structure found is optimal also in the future. The wait- and see-problems is solved by use of the LP-relaxation based heuristics. The values could therefore be affected by error.



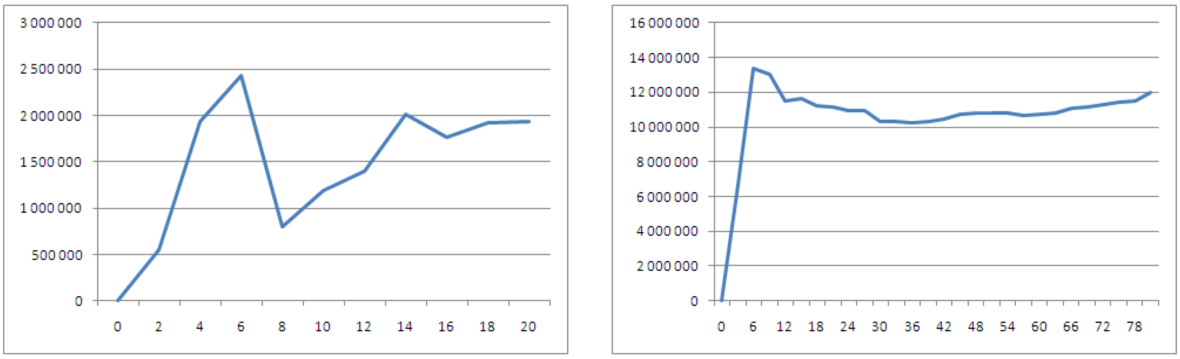
(a) Sveaskog. Penalty up to 4 terminals

(b) Stora Enso. Penalty up to 6 terminals

Figure 38: EVPI

The EVPI-values, as shown in Figure 38, show that if the number of terminals increases, the supply chain would perform closer to optimum in the future. For the Sveaskog case, there are penalties up to 4 opened terminals, and if the optimal future solution is chosen, there could be a maximum of 3 % increase in profits. The EVPI values should be positive, but as seen for the Stora Enso case, the values are mostly negative. The reason is most probably the LP relaxation based heuristics, as it provides poorer solutions when the number of terminals is low. It seems therefore that use of the LP relaxation should be avoided when calculating EVPI values. However, as the EVPI values are not higher, we may expect that the solutions for the Stora Enso case will perform close to a future optimum.

11.5.5 Value of stochastic solution



(a) Sveaskog. Penalty up to 6 terminals

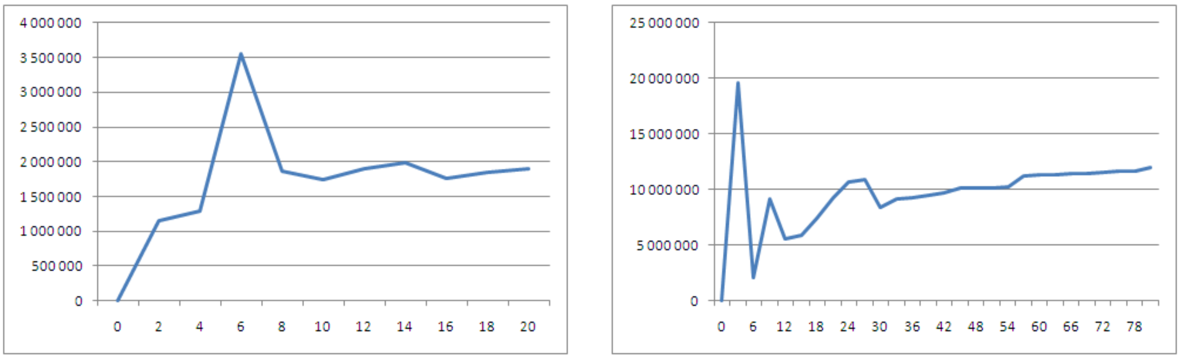
(b) Stora Enso. Penalty up to 3 terminals

Figure 39: VSS

The value of stochastic solution is the difference between the stochastic solution and the deterministic solution, and tells how well the deterministic solution performs when subject to uncertainty. The stochastic solution would in theory perform better in the future than the deterministic. For both cases, costs could be saved by use of stochastic optimization. The savings is approximately 10 %, and are therefore considerable. This is mainly due to reduced inventories as the deterministic inventory policy is replaced by the stochastic safety stock values.

For the Sveaskog case, the VSS values start to increase once the deterministic terminal structure becomes robust in the stochastic model. The reason is that the deterministic model could utilize its inventory to fulfill demand. But as the number of terminals increases, the need for this inventory decreases, and the VSS increases. For the Stora Enso case, the VSS values start to decrease once the deterministic terminal structure is robust in the stochastic model. This could be explained by the decreased marginal influence on supply chain profits as the number of terminals increases, and as the deterministic model would open terminals that lay further away only when the number of terminals increases. This incurs increased transportation costs to deliver their required safety stock.

11.5.6 Value of robust solution



(a) Sveaskog. Penalty up to 6 terminals

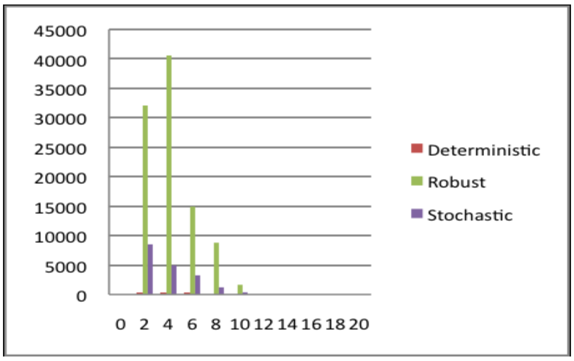
(b) Stora Enso. Penalty up to 3 terminals

Figure 40: VRS

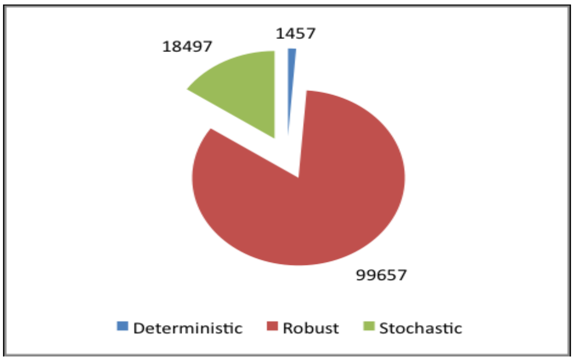
The value of robust solution is used to evaluate how well the solutions perform compared to the deterministic solution if a worst- case scenario should occur. In both cases the VRS is positive and contributes to savings of approximately 10 % by using the robust model compared to the deterministic model. This is mainly due to reduced inventory costs.

For the Sveaskog case, as shown in Figure 40, the VRS-values are almost constant once the deterministic solution becomes robust in the robust model. This is mainly due to the required safety stock of the deterministic model. For the Stora Enso case, the deterministic solutions perform badly for a low number of terminals opened, but gradually stabilize its performance. The VRS values increase as the need for the deterministic safety stock decreases.

11.5.7 Solution times



(a) Compared for different number of terminals opened



(b) Shown as parts of the total solution time

Figure 41: The solutions time of our models for Case Sveaskog with aggregated suppliers and assortments

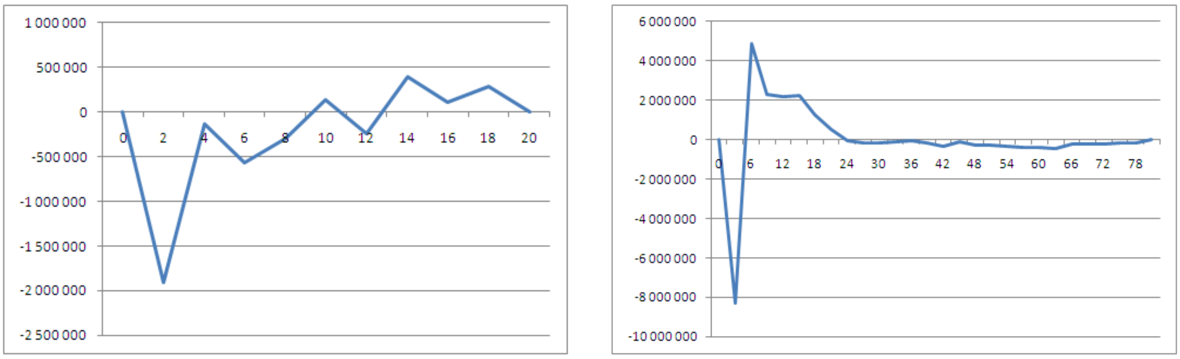
The solution times for the stochastic and the robust model were considerable larger than the deterministic model. To speed up the solution times, the LP-relaxation based heuristics was used. This reduced solution times considerably, as shown in Section 7.4, and the difference in the stochastic and robust solution times were reduced. The stochastic model used less memory than the robust model.

Case Sveaskog with aggregated assortments were the only problem we were able to solve without the LP-relaxation based heuristics. The solution times were lowest for the deterministic model as shown in Figure 41. The deterministic model is however considerably smaller than the robust and stochastic model. The stochastic model seems to solve in almost one fifth of the time the robust model uses. We see the same trends in solution time for all cases. The largest solution times is found when the numbers of terminals opened are low. An explanation is that the problems become more sensible to which terminals it opens with a small number of terminals opened.

11.5.8 Comparing the stochastic and the robust optimization approach

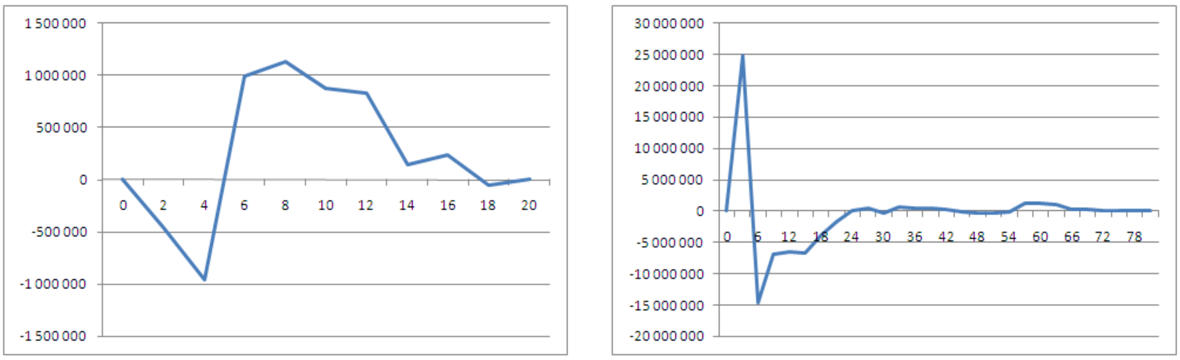
We have implemented the stochastic solution into the robust model and vice versa to evaluate how good the solutions perform under uncertainty. We would expect that the robust solutions would behave best in the robust model, and the stochastic model would behave best in the stochastic model. However, we have used a LP-relaxation based heuristics that could affect the quality of the solutions, i.e. what terminal structure that is found. As can be seen for the Sveaskog case, the robust solutions would actually perform better in the stochastic model, than the stochastic solutions for a low number of opened terminals. However, for the Stora Enso case, the trend is completely opposite, as shown in Figures 42 and 43. The stochastic solutions actually perform better than the robust solutions.

We expected the models to behave similar as we have a low number of scenarios, and as shown, the difference in behavior is below three percent. An explanation is that while the robust solution evaluates the terminal structures based on worst case scenarios and chooses the cheapest robust solution, the stochastic solution evaluates terminal structures based on all scenarios, and chooses the robust and expected cheapest solution. This explains the incremental increase in profits that we found for the stochastic model. As the solution times and memory usage is lower for the stochastic model, it also seems more attractive.



(a) Sveaskog. Penalty up to 4 terminals (b) Stora Enso. Penalty up to 6 terminals

Figure 42: VSS for the robust solution inputted to the stochastic model



(a) Sveaskog. Penalty up to 4 terminals (b) Stora Enso. Penalty up to 3 terminals

Figure 43: VRS for the stochastic solutions inputted to the robust model

12 Conclusion

We have implemented three optimization models to solve a supply chain design problem for finding optimal terminal structures and inventory policy for forestry biomass supply chains. A deterministic, a stochastic and a robust model have been implemented and used to solve a constructed test case and two large industrial cases.

In order to run the models for the large problems, we have developed methods to decrease solution times and memory usage. This is done by the use of data preprocessing, an LP-relaxation based heuristic, arc removal and aggregations of suppliers and assortments. We have especially studied supplier aggregation, and developed an algorithm that efficiently could create aggregations with only minor deviations in solution quality. The methods developed in this thesis is easy to implement and manipulate for further use on similar cases or models. The LP-relaxation based heuristic is effective to reduce solution times, but gives poorer solutions for a low number of opened terminals.

Costs could be saved by choosing an optimal terminal structure and inventory policy. These saving decline with the number of opened terminals. The value of stochastic solution and the value of robust solution show that taking uncertainty into consideration would save costs. The main contribution in savings is explained by changes in inventory policy, as the levels of safety stock used in the deterministic model are set too high. Robustness could be achieved by either increasing the number of terminals, inventory levels, processing and transportation capacity in the supply chain. Increasing the processing capacities at the suppliers seems to be an attractive investment.

The stochastic and robust models return solutions that corresponds to equal supply chain profits. Both models could both increase profits. However, the stochastic model solves in one fifth of the robust model and uses less memory. Use of the LP-relaxation based heuristics reduces the differences in solution times between the stochastic and robust model decreases substantially.

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Appendices

Xpress code

Xpress code used to solve the Stora Enso case is included on a CD attached with the printout. This includes; the stochastic and robust models, the LP-relaxation based heuristics, the supplier aggregation algorithm, generation of transportation costs, the removal of longest arcs, the scenario generator and an initializator used to initialize the models for different parameters.