Random sets-based system for geotechnical soundings density estimation

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Abstract. The required number of soundings for geotechnical site investigation depends on different factors such as geology of the site, soil variability and the type of project to build. Indications on the number of soundings are given in codes of practice (Eurocode 7…) but they focus mainly on a minimal number which depends only on the area of the project.

Geotechnical investigation is a process conducted, in general, in two steps. The first step consists in collecting available information and executing a limited number of soundings on site, while the second step of investigation is based on the first and requires more soundings for soil testing. The preliminary information is expressed by using random sets of the influencing factors (soil variability, geology…). Intervals of values are proposed by experts (engineers) concerning the influence of each factor based on preliminary information from site (soil variability, geology, type of project). The expert proposes an interval of values based on his degree of belief.

Using Eurocode7 recommendations for site soundings, an "objective function" f(X) is constructed to rely "soil variability" to the number of soundings. It permits constructing the random set and obtain the number of soundings by unit area for each expert (engineer). The same reasoning is applied with other parameters such as geology and an inference system is obtained. Information is aggregated from the available parameters (Soil variability, Geology…) and the random sets computed from which upper and lower probabilities (probability boxes) are built. They permit optimizing the number of soundings to be carried out on site.

Keywords. Geotechnical Investigation, Soundings density, Random Sets, Upper and Lower probabilities, Focal element

1. Introduction

Geotechnical site investigation is a process conducted generally in two steps. One preliminary stage consisting in collecting available information and executing a limited number of soundings on site, and a second step of investigation based on the first stage using more soundings for soil testing. The optimal number of soundings is not known, it depends on number of factors such as geology of the site, soil variability and the type of project to build.

The engineer in charge of investigation should take into account all those parameters and preliminary information to decide on the density of soundings to be conducted in the second stage of investigation.

Cambefort (1980) noticed that there is no specific rule on the number of soundings for geotechnical investigations. If an arbitrary loose mesh of soundings used in the preliminary study shows that the project area is relatively homogeneous then this quantity is satisfactory. However, if the results of the preliminary study show erratic conditions, the site characterization requires more soundings.

Previous information on the site is generally given in form of geological and topographic maps and eventual results from adjoining sites. Engineer's judgment is important factor in this case.

Given certain preliminary information (soil variability, geology, type of project…) one can affect a "degree of belief" (O'Hagan, 2007) to the expert (engineer) about the density of boreholes. The more important the preliminary information the more significant will be the degree of belief on the density of soundings to carry out. The degree of belief is expressed as a subjective probability by experts (O'Hagan, 2007).

In a previous study a fuzzy inference system was proposed for this purpose (Boumezerane et al.,2011, Boumezerane et al. 2014). The use of random sets with one input parameter (soil variability) was introduced by Boumezerane and Belkacemi (2012). In the present work is presented the use of random sets to estimate intervals of soundings when two input parameters are taken into account (Soil variability, Geology).

2. Parameters influencing Geotechnical Investigations

There are no particular rules to follow on the planning of density of soundings for a given site (Magnan, 2000), it depends on preliminary information among which;

- Geologic context of the project area,
- Project type, and
- Soil variability.

2.1. Geology

The available information about site's geology is obtained using geological and topographical maps. A visit on site is necessary, it permits having a reliable idea about the visible ground and formations constituting the soil.

The degree of information (knowledge) depends mainly on the scale of geological maps used, on the quality of information available (rough or precise) and on the on-site engineer's judgement. Published geological maps are fundamental tools for any of the analysis; however details have to be investigated by more specific studies. The use of maps is essential to have a first idea on the geological formations constituting the site, their properties, as well as the possibilities of inadequate or adverse geological details. Clayton et al. (2005) recommend for geotechnical studies to use geologic maps in the scale 1/2500.

The spacing of soundings depends on the geology of the area and may vary from a site to another. It should be selected to intersect distinct geological characteristics of the project. Soundings should be situated to confirm the location of significant changes in subsurface conditions as well as to confirm the continuity of apparently consistent subsurface conditions (US Corps of Engineers, 1994).

2.2. Soil variability

The preliminary step of geotechnical investigation consists in few soundings which permit to have a rough idea about the variability of soil properties. The parameter "Soil Variability" is important for the engineer to decide on the number of required soundings in the second stage of site investigation. The soil variability is related to the number of different soil layers, their orientations and thickness. Average values of soil parameters can be obtained from different points of the site. For important variability of soil properties the density of soundings should be significant.

3. Random Sets Concept

A *random set*, sometimes also referred to as a *Dempster-Shafer structure* (Oberguggenberger, 2005) is given by finitely any subsets *Ai*, $i = 1, \ldots, n$ of a given set X, called the *focal sets*, each of which comes with a *probability weight* $mi = m(Ai)$, $\sum m(Ai) = 1$. An example of a random set is shown in Fig.1

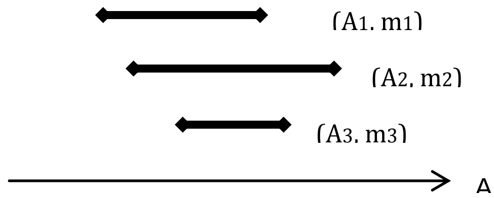

Figure 1. A random set.

In the Dempster-Shafer approach (Alvarez, 2008), the random set allows to define a degree of belief $\gamma(S)$ and a degree of plausibility $\eta(S)$, respectively, that the realizations of the parameter A lie in S by;

$$\gamma(S) = \sum_{Ai \subset S} m(Ai)$$

$$\eta(S) = \sum_{Ai \cap S \neq \emptyset} m(Ai)$$

The belief function $\gamma(S)$ or *Bel*, of a subset S is a set-valued function obtained through summation of basic probability assignments of subsets *Ai* included in S and the plausibility function $\eta(S)$, or *Pl*, of subset S is a set-valued function obtained through summation of basic probability assignments of subsets *Ai* having a non-zero intersection with S. They are envelopes of all possible cumulative distribution functions compatible with the data.

If one considers for instance a Dempster Shafer (D.S) structure which is formed by gathering the information provided by four different sources (e.g. books, experts, previous analysis, etc.) on the friction angle of some soil; each of those opinions will form one element Ai of the focal set A. Suppose that A = {A1= [20°,22°], A2 = [21°,23°], A3 = [18°,20°], A4 =[20°,25°]}. The basic mass assignment given to each of those focal elements will represent the importance of each of those opinions in our assessments. Suppose for example that ($m(A_1)=0.4$, $m(A_2)=0.2$, $m(A_3)=0.1$, $m(A_4)=0.3$; this means that we are giving to our first source of information the largest relevance (Alvarez, 2008).

4. Random sets and number of soundings

The idea underlying the use of random sets as a tool to estimate soundings density is supported by their ability to handle vague and uncertain information. The degree of belief an engineer could have given preliminary information is used to construct the upper and lower probabilities to estimate the number of soundings for geotechnical investigation. The calibration is done upon minimal number of soundings per surface as recommended by Eurocode7.

Preliminary information from site (soil variability, geology, type of project) helps engineers to have opinions concerning the soundings density. Each engineer can give an interval of values based on his degree of belief. His judgment is supported by available information. If "Soil Variability" obtained from preliminary soundings is "Very Important" for instance then he will propose an important number of soundings with a strong degree of belief.

How Soil Variability is quantified by experts? A scale between 1 and 10 is proposed representing intervals of "Very Low", "Low", "Medium" and "High" Variability. Eurocode 7 (ENV, 1997) recommends 1 sounding per 40x40m² as a minimum for an investigation. The degree 1 of soil variability corresponds to a "very low" variability. We consider this degree of variability necessitating the minimal number of soundings recommended by Eurocode7. The maximum number of soundings recommended by codes and some authors (Hunt, 2007) is given by 1 sounding / area of 15x15m². Globally the number of soundings per unit area of 40x40m² varies between 1 as a minimum and 6 as maximum, but it is possible to have more soundings if information is not enough.

5. Point to point approach

According to Eurocode7 (ENV, 1997) the minimum number of soundings is 1 for an area of 40x40m². This minimum number as explained before could be used for a "Very low" soil variability (which is comprised in the interval [0,2] on the scale). If the maximum number of soundings (6 to 7 / unit area 40x40m²) corresponds to a "High variability" (10 on the scale) one could argue a linear variation and construct an "objective function" to rely "soil variability" to the number of soundings. This function will permit to construct Upper and Lower probability boxes as a decision aid tool.

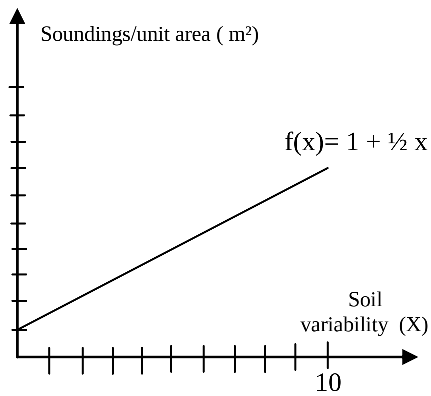

Figure 2. Objective function

On the other hand using the same reasoning an objective function is constructed for the input parameter "geology" using a linear variation. The minimum number of soundings as

recommended by Eurocode7 corresponds to a low “geology” (which is comprised in the interval [0,2] on the scale). The maximum number of soundings (6 to 7 / unit area 40x40m²) corresponds to a “High Geology” (10 on the scale).
The combination of the two parameters is carried out using a bilinear function of the form:

$$F(X,Y) = a.XY + b.X + C.Y + d \qquad (1)$$

Where X corresponds to « Soil variability » and Y corresponds to “Geology”.

Using the calibration from Eurocode7 and in the case of linear objective functions:

$$a = \frac{1}{20}, b = \frac{1}{2}, c = \frac{1}{2}, d = 1$$

Example

After having access to some information an expert is asked to give his degree of belief for “soil variability” and “geology” of a given site on a scale of 1 to 10, in the form:

$m_{12} = \frac{1}{2}, X_{12} = [1,5]$ and $m_{11} = \frac{1}{2}, X_{11} = [0,3]$
$m_{21} = \frac{1}{2}, Y_{21} = [2,7], m_{22} = \frac{1}{2}, Y_{22} = [2, 5]$

m_{ij} is the weight (degree of belief)

With the condition $m_{11} + m_{12} = 1$ and
$m_{21} + m_{22} = 1$

This information from the expert will be processed to estimate the number of soundings on site.
There are different possibilities of combining X and Y by using the product $m_{ij}.m_{ij}$ and A_{ij} surfaces are obtained each time. For example, A_{11} corresponds to the product $m_{11}.m_{21} = \frac{1}{4}$ with a surface comprised in $[0,3]$x$[2,7]$.
The image of A_{11} using $F(X,Y)$ will permit to calculate the limits and the probability (weight) of the focal set A_{11}.

$$F(A_{11}) = [\, \underset{(X,Y) \in A_{11}}{minF(X,Y), \qquad maxF(X,Y)} \,]$$

From which:

$minF(X,Y)$ for A_{11} given by $F(0,2) = 2$

$maxF(X,Y)$ for A_{11} given by $F(3,7) = \frac{99}{20}$

The image of A_{11} is then $\left[2, \frac{99}{20}\right]$ and its weight $m_{A11} = \frac{1}{4}$

Using the same approach the other focal sets are calculated:

$$m_{A12} = \frac{1}{4}, \; A_{12} = \left[2, \frac{17}{4}\right]$$

$$m_{A21} = \frac{1}{4}, A_{21} = \left[\frac{38}{20}, \frac{115}{20}\right]$$

$$m_{A22} = \frac{1}{4}, A_{22} = \left[\frac{38}{20}, \frac{95}{20}\right]$$

As one can notice $\sum_{ij} m_{Aij} = 1$ and the intervals of A_{ij} are situated on the axis “Number of soundings / unit area”.

The followed scheme to construct the random sets and probability boxes (Upper and Lower cumulative probabilities is shown in Figures 3,4.

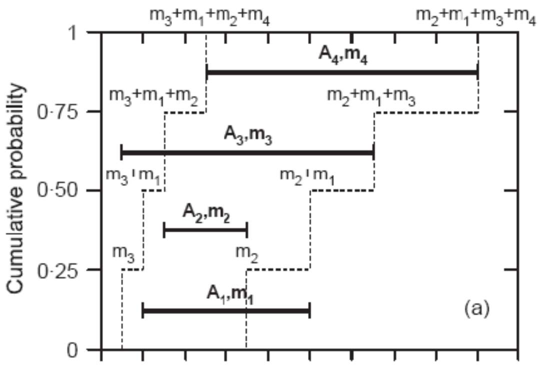

Figure 3. Construction of random sets

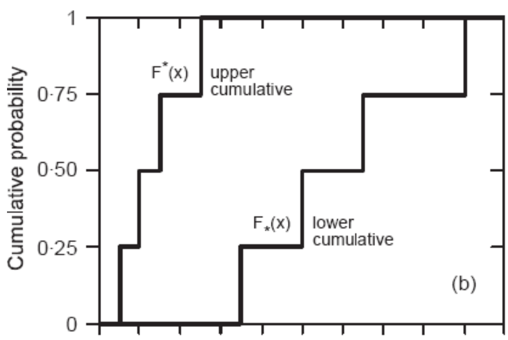

Figure 4. Construction of Upper and Lower probabilities

For the previous example, the following upper and lower probabilities are obtained, using the rule of stochastic mixture.

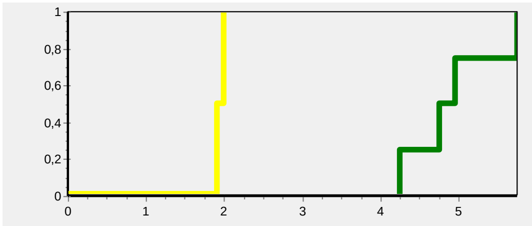

Figure 5. Construction of Upper and Lower probabilities vs Number of soundings

The suitable number of soundings is around 4 in this case. This example was run to illustrate the way how to use the random set-based approach.

When considering more than one expert an aggregation is necessary. According to Hall et al.(2004) when there are "n" alternative random sets describing some variable x, each one corresponding to an independent source of information (expert in this case) for each focal element A,

$$m(A) = \frac{1}{n}\sum m_i(A)$$

In the case when random sets (A*i*, *mi*) : *i* = 1,. . . , *n* from different sources do not contain the same focal elements a merged random set is obtained using union and *m*(*A*) is obtained from the previous equation.

There are other combination rules such as "Dempster rule", Yager's modified Dempster's rule, Inkagi's unified rule of combination, Dubois and Prade's rule and others (Sentz, 2002).

The Dempster's rule combines multiple belief functions through their basic probability assignments (m). The combination (joint) m_{12} is calculated from the aggregation of two pba (probability basic assignment) m_1 and m_2 as it follows:

$$m_{12}(A) = \frac{\sum_{B\cap C=A} m_1(B)m_2(C)}{1-K} \quad \text{when} \quad A \neq \emptyset$$

$$m_{12}(\emptyset) = 0$$

Where $K = \sum_{B\cap C=\emptyset} m_1(B).m_2(C)$

The result of aggregation is still a cumulative function of distribution which could be used as a tool for decision making.

6. Conclusion

The random set-based approach introduced here can be considered as a valuable tool to estimate the number of soundings for geotechnical investigations. Information gathered from an expert, and containing uncertainties can be expressed in form of intervals and degree of belief. The given example is shown just an illustration of the way the system works.

Combining information from two input parameters (Soil variability and Geology) and extracting its influence on the number of soundings according to codes of practice is a first step. Once the expert gives his judgment on information an objective function is used as a relation to the minimal number of soundings required by codes of practice (Eurocode7…). The construction of random sets is another step. After aggregation Upper and Lower probabilities are built, which gives the limits of the decision.

The proposed system needs to be run on real sites, with different experts opinions and then aggregate them together to obtain a suitable upper and lower probabilities for estimating the number of soundings on site

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