

Performance modelling of optical packet switched networks with the Engset traffic model

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Abstract: Stochastic processes have been widely employed in order to assess the network layer performance of Optical Packet Switched (OPS) networks. In this paper we consider how the Engset traffic model may be applied to evaluate the blocking probability in asynchronous bufferless OPS networks. We present two types of the Engset traffic model, i.e. the Engset lost calls cleared traffic model and the Engset overflow traffic model. For both traffic models, the time-, call-, and traffic congestion are derived. A numerical study shows that the observed blocking probability is dependent on the choice of traffic model and performance metric.

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1. Introduction

Optical Packet Switching (OPS) has emerged as a promising Wavelength Division Multiplexing (WDM) based optical core network architecture for the future Internet. By avoiding electronic-optical (O/E) conversions in the switching nodes, OPS networks can

provide transport services with less cost and higher data transparency, compared with today's point-to-point WDM networks. These benefits will become increasingly evident, as electronics cannot accommodate the explosive traffic growth expected in the future Internet without relying on complex and expensive cascading switch constructions [1-3]. However, in order to have a commercial deployment of OPS, several advancements in enabling technology must be made, such as fast switching matrices and all-optical wavelength converters [3].

OPS networks may operate in asynchronous or slotted mode. In slotted OPS [4], fixed-sized packets arrive at a core switch in synchronized time-slots, where complex synchronizers compensate for delay variations occurring between packets. In asynchronous OPS [5], packets can arrive at a core switch at any instant, and there is no synchronization between the input ports. This paper will focus on asynchronous OPS networks.

A crucial issue in OPS networks is packet loss at the network layer [6]. In asynchronous OPS, such packet loss occurs due to contention when a packet is destined for wavelength currently busy transmitting another packet. Without any contention resolution mechanisms, the arriving packet must be dropped. However, in order to resolve such contentions, the wavelength-, time-, and space domain may be utilized [6]:

- **Wavelength domain:** Contending packets are converted to idle wavelengths on the same fibre using wavelength converters. This technique does not cause additional delay, nor reordering of the packets.
- **Time domain:** Contending packets are delayed and scheduled for transmission a later point in time when the wavelength is (hopefully) available. This technique results in an additional delay and may result in reordering of the packets.
- **Space domain:** Contending packets are transmitted on the same wavelength, but on a different idle output fibre, which leads to another node than originally intended. Hence, the packets may traverse non-optimal paths toward its destination. This technique results in a potential large extra delay and possible reordering of packets.

As shown in [6], both the wavelength- and time domain can effectively reduce the blocking probability in asynchronous OPS networks. However, since utilizing the time domain implies the use of Fibre Delay Lines (FDL) or electronic Random Access Memory (RAM) [7], the switch complexity and hardware cost increases. Also, due to the added delay from the buffers, packets may experience an increased end-to-end delay and re-ordering, which is unfavorable for e.g. high quality streaming services and TCP connections [8]. Regarding the space domain, the authors of [6] show that a limited reduction in the blocking probability can be obtained. Hence, due to the drawbacks and limitations associated with both the time- and space domain, we further focus on an optical packet switch architecture that utilizes either the wavelength domain for contention resolution, or no contention resolution mechanism at all. In such bufferless OPS architectures, the most significant QoS parameter is the blocking probability [6-8], which results from network layer contention.

Analytical models based on stochastic processes are a powerful tool to assess the network layer performance in OPS networks. The basis for these models have been developed by teletraffic scientists decades ago [9-11], but are now being renewed and put in the context of OPS [12-13]. In order to evaluate the blocking probability at an output port in asynchronous bufferless OPS, both the Erlang and Engset traffic model have been employed [12]. In particular, the Erlang traffic model assumes that packets arrive from ∞ input wavelengths, and the blocking probability can be calculated using the Erlang-B formula [11]. In the Engset traffic model, packets arrive to an output port in an optical packet switch from a finite number of input wavelengths, and the blocking probability can be obtained using either the time-, call- or traffic congestion. As the Engset traffic model assumes a finite number of input wavelengths, it is more accurate than the Erlang traffic model [12], and will be the focus in the rest of this paper.

The major aim in this paper is to present two basic types of the Engset traffic model, suitable for asynchronous OPS networks, i.e., the Engset lost calls cleared traffic model (Engset LCC) and the Engset overflow traffic model (Engset OFL). For each traffic model, we present analytical expressions for the time-, call-, and traffic congestion. Although parts of these models have been presented in earlier works [12], as best to our knowledge, a complete overview and comparison of these models and accompanied performance metrics in the context of OPS has not been presented before. We believe that the results presented in this paper are of great importance in order to better understand how the choice of traffic model and performance metric impacts the observed blocking probability in asynchronous OPS networks.

The rest of the paper is organized as follows: Section 2 presents the optical packet switch architecture and the basics regarding the Engset traffic model. The Engset LCC and the Engset OFL is presented in Sections 3 and 4, respectively. A numerical evaluation of the traffic models can be found in Section 5, followed by a conclusion in Section 6.

2. Optical packet switch architecture and the general Engset traffic model

We consider a generic asynchronous blocking-free optical packet core switch with F input and output fibres, as illustrated in Fig. 1. Each fibre provides W wavelengths by using WDM. Due to the asynchronous mode of operation, packets can arrive to the switch on the input wavelengths at any instant. When a packet arrives to the switch, the header is converted to the electronic domain to be processed by the control module, while the payload is delayed in the optical domain using input FDLs. Based on the destination information extracted from the header, the control module decides which output fibre and wavelength the arriving packet is switched to, and configures the switch fabric accordingly. We assume a uniform traffic pattern regarding arrival intensities and routing probabilities.

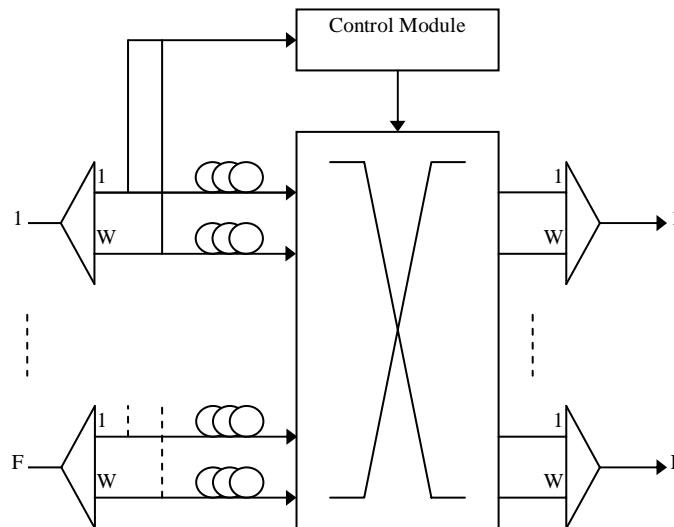


Fig. 1. A generic optical packet switch with F input/output fibres, and W wavelengths per fibre.

Further, we consider two different wavelength domain contention resolution architectures:

- **Non wavelength converter scenario (NWC):** There are no wavelength converters in the switch, which means that packets arriving on a certain input wavelength must leave the switch on the same output wavelength (but possibly on a different fibre pair). In this case, since we assumed a uniform traffic pattern, we can restrict our study to consider a single

output wavelength, denoted as the “tagged” output wavelength. The tagged output wavelength may receive packets from $S=F$ input wavelengths.

- **Full output wavelength converter scenario (FOWC):** Variable-input, variable-output wavelength converters are placed at each output wavelength. This means that packets can freely be converted from any input wavelength to any output wavelength. In this case, since we assumed a uniform traffic pattern, we can restrict our study to consider a single output fibre (consisting of W output wavelengths), denoted as the “tagged” output fibre. The tagged output fibre may receive packets from $S=FW$ input wavelengths.

Let the term ‘tagged output port’ denote either a tagged output wavelength or a tagged output fibre depending on whether we are considering the NWC or the FOWC scenario, respectively. The tagged output port consists of N output wavelengths, and may receive packets from S input wavelengths. Table 1 summarizes the parameters S and N for the two scenarios.

Table 1. Overview of the number of input/output wavelengths in the NWC and FOWC scenario.

	Number of input wavelengths (S)	Number of output wavelengths at the tagged output port (N)
NWC scenario	F	1
FOWC scenario	FW	W

We now apply the Engset traffic model [8] to the considered asynchronous optical packet switch described above. S independent and identical input wavelengths (sources) generate packets to a tagged output port consisting of N output wavelengths (servers). Each input wavelength behaves according to Pure Chance Traffic Type II (PCT-II), where the holding times for the idle and busy states are negative exponentially distributed with means $1/\lambda$ and $1/\mu$, respectively. This is illustrated in Fig. 2.

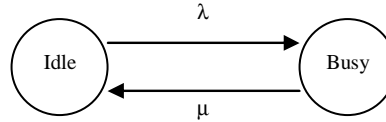


Fig. 2. A state diagram of an input wavelength, which changes between two states. In the idle state, no packets are arriving on the input wavelength, while in the busy state, the input wavelength is transmitting a packet to the tagged output port. The holding times are negative exponentially distributed.

When an input wavelength generates a packet, the input wavelength moves to the busy state and the generated packet seizes a wavelength at the tagged output port, if there is a wavelength available. When the packet has completed transmission, the input wavelength returns to the idle state, and starts to generate a new packet. Packets generated to a tagged output port when all the output wavelengths are busy, are lost. The offered traffic per idle input wavelength is $\beta=\lambda/\mu$, and the offered traffic per input wavelength is $\alpha=\beta/(1+\beta)$. Hence, the total offered traffic to the tagged output port is $A_T=S\alpha$, and the normalized system load is defined as $A=A_T/N=\alpha$. The peakedness of the Engset traffic model is given as [11]:

$$Z = \frac{\sigma^2}{A_T} = \frac{S\alpha(1-\alpha)}{S\alpha} \quad (1)$$

where σ^2 is the variance. Note that we can express the peakedness as a function of A_T and A as follows:

$$Z = 1 - \frac{A_T}{S} = 1 - \frac{AN}{S} = 1 - \frac{A}{F} \quad (2)$$

Note that:

$$\frac{\partial Z}{\partial F} = \frac{A}{F^2} > 0 \quad (3)$$

$$\frac{\partial^2 Z}{\partial F^2} = -\frac{2A}{F^3} < 0 \quad (4)$$

We observe that for the Engset traffic model we have $0 < Z < 1$. As a comparison, the Erlang traffic model [11] has a constant peakedness equal to $Z=1$. In this paper, we investigate two types of the Engset traffic model, i.e., the Engset LCC and the Engset OFL:

- In the **Engset LCC**, when an input wavelength generates a packet that sees no free output wavelengths at the tagged output port (i.e., a congested system), the input wavelength will immediately return to the idle state and start to generate a new packet.
- In the **Engset OFL**, when an input wavelength generates a packet that sees no free output wavelengths at the tagged output port, the input wavelength will stay in the busy state until the generated packet has completed “transmission”. The packet will never be served by the tagged output port, even though output wavelengths may become available during the packet service time.

Note that in both Engset LCC and Engset OFL, packets arriving to a congested system are lost. The only difference between the two traffic models is how the input wavelength that generated the lost packet behaves.

In order to measure the blocking probability in the Engset traffic model, three different performance metrics may be used [11]:

- **Time congestion:** The relative share of the time the tagged output port is congested. This is the operator perceived QoS.
- **Call congestion:** The relative share of packets that arrive when the tagged output port is congested. This is the user perceived QoS.
- **Traffic congestion:** The relative share of the offered traffic that is not carried by the tagged output port. This is the system perceived QoS.

We will use the term ‘blocking probability’ when referring to the time-, call-, and traffic congestion as whole. Further in this paper, we aim to present analytical expressions for the blocking probabilities for the Engset LCC (Section 3) and the Engset OFL (Section 4). Note that both the Engset LCC and the Engset OFL may be applied to the NWC and FOWC scenario.

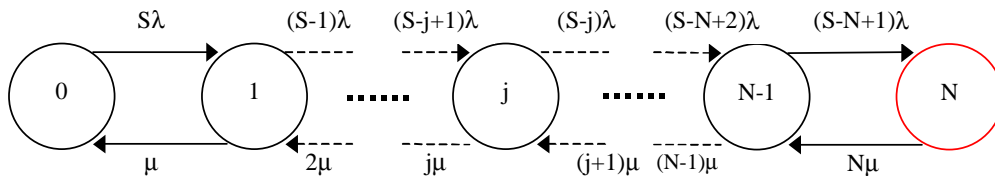


Fig. 3. State transition diagram of the Engset LCC. The states denote the number of busy wavelengths at a tagged output port. The tagged output port is congested in the red state.

3. The Engset lost calls cleared traffic model (Engset LCC)

A state transition diagram for the Engset LCC is depicted in Fig. 3. The states correspond to the number of busy wavelengths at a tagged output port, e.g., in state (j), j wavelengths at a tagged output port are busy. Since packets may arrive from S input wavelengths, the total arrival intensity to the tagged output port is dependent on the state and is given as $(S-j)\lambda$ when the tagged output port is in state (j). In the case of a packet arrival, the state changes from (j) to (j+1), if there are available wavelengths at the tagged output port. When a packet has

completed transmission, the state changes from (j) to (j-1). In state (N), marked red in Fig. 3, the tagged output port is congested, and all arrivals will be dropped without altering the state. This means that the input wavelength that generated the lost packet immediately returns to the idle state and starts to generate a new packet.

Let Q_j be the probability of being in state (j). From cut operations between state (j) and (j+1), we obtain the following balance equations:

$$Q_j (S - j) \lambda = Q_{j+1} (j + 1) \mu \quad (0 \leq j \leq N - 1) \quad (5)$$

By utilizing that $\sum_{j=0}^N Q_j = 1$, we can express the probability of being in state (j) as [11]:

$$Q_j = \frac{\binom{S}{j} \beta^j}{\sum_{k=0}^N \binom{S}{k} \beta^k} \quad (6)$$

The time congestion is probability of finding the tagged output port congested, i.e. the probability of being in the state marked red in Fig. 3:

$$E_{LCC}(N, S, \beta) = Q_N \quad (7)$$

Here, Q_N is obtained from Eq. (6). The call congestion is the relative share of arrivals generated when the tagged output port is congested:

$$B_{LCC}(N, S, \beta) = \frac{Q_N (S - N) \lambda}{\sum_{j=0}^N Q_j (S - j) \lambda} = E_{LCC}(N, S - 1, \beta) \quad (8)$$

Note that the call congestion equals the time congestion with one less input wavelength. This is known as the arrival theorem [11]. The traffic congestion is given as share of traffic not carried by the tagged output port:

$$C_{LCC}(N, S, \beta) = \frac{A - Y}{A} = \frac{S\alpha - \sum_{j=0}^N j Q_j}{S\alpha} = \frac{F - 1}{F} E_{LCC}(N, S, \beta) \quad (9)$$

Here, $A = S\alpha$ is the offered traffic, and Y is the carried traffic. Note that the relation between C_{LCC} and E_{LCC} has been taken from [11]. For the Engset LCC we always have that $E_{LCC}(N, S, \beta) = B_{LCC}(N, S + 1, \beta) > B_{LCC}(N, S, \beta) > C_{LCC}(N, S, \beta)$ [11].

4. The Engset overflow traffic model (Engset OFL)

A state transition diagram for the Engset OFL is illustrated in Fig. 4. Here, state (i,j) denotes the number of output wavelengths at the tagged output port currently busy transmitting packets (i), and the number of input wavelengths that have generated a blocked packet and still remains in the busy state (j). As long as there are free wavelengths at the tagged output port, a packet arrival brings the tagged output port from state (i,j) to (i+1,j). When the tagged output port is congested (marked red in Fig. 4), packets generated by the input wavelengths are lost, and the state changes from (i,j) to (i,j+1), since the input wavelength should be in the busy state until the packet has completed transmission. For instance, when there is a packet arrival to the tagged output port in state (N,0), one of the input wavelengths will move to the busy state until the generated packet has completed "transmission", i.e., the system changes from state (N,0) to state (N,1). Ultimately, in state (N,S-N), all wavelengths at the tagged output port are busy, which means that N input wavelengths are busy transmitting packets that

are currently served by the tagged output port, and $S-N$ input wavelengths are busy transmitting packets that have been dropped due to contentions. When a packet has completed transmission, the tagged output port moves from state (i,j) to either state $(i-1,j)$ or $(i,j-1)$, depending on whether the completed packet initially was accepted or dropped, respectively. Let $Q_{i,j}$ denote the probability for being in state (i,j) .

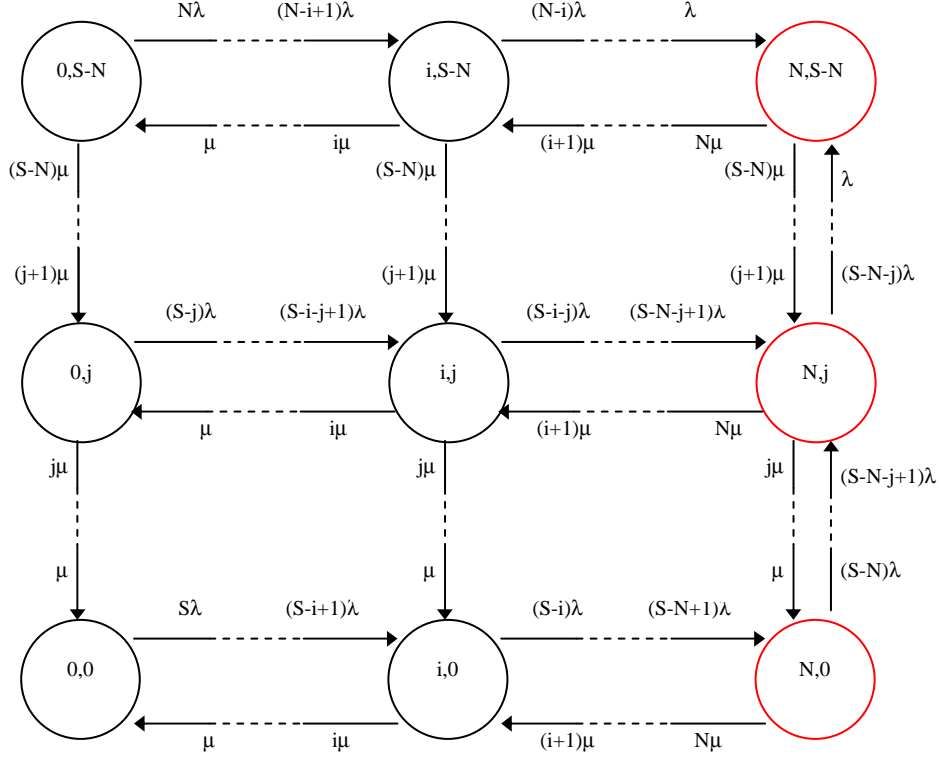


Fig. 4. State transition diagram of the Engset OFL. State (i,j) denotes the number of output wavelengths at the tagged output port currently busy (i), and the number of input wavelengths transmitting a packet that has been dropped (j). The tagged output port is congested in the red states.

We define the unit step function as:

$$u(k, L) = \begin{cases} 0 & k = L \\ 1 & k \neq L \end{cases} \quad (10)$$

From Fig. 4, we obtain the balance equations from node operations:

$$\begin{aligned} Q_{i,j} & \left((i+j)\mu + (S-i-j)\lambda \right) = \\ & Q_{i-1,j} \left((S-i-j+1)\lambda \right) \cdot u(N-i, N) + \\ & Q_{i+1,j} (i+1)\mu \cdot u(i, N) + \\ & Q_{i,j-1} (S-N-j+1)\lambda \cdot (1-u(i, N))u(S-N-j, S-N) + \\ & Q_{i,j+1} (j+1)\mu \cdot u(j, S-N) \end{aligned} \quad \begin{matrix} (0 \leq i \leq N) \\ (0 \leq j \leq S-N) \end{matrix} \quad (11)$$

$$\sum_{i=0}^N \sum_{j=0}^{S-N} Q_{i,j} = 1 \quad (12)$$

We can solve the equation set using e.g. numerical tools such as Matlab [14]. We obtain the time congestion by summing over all the states where the tagged output port is congested, which are marked red in Fig. 4:

$$E_{OFL}(N, S, \beta) = \sum_{j=0}^{S-N} Q_{N,j} \quad (13)$$

The call congestion is obtained by considering the relative share of arrivals when the tagged output port is congested:

$$B_{OFL}(N, S, \beta) = \frac{\sum_{j=0}^{S-N} Q_{N,j} (S - N - j) \lambda}{\sum_{i=0}^N \sum_{j=0}^{S-N} Q_{i,j} (S - i - j) \lambda} \quad (14)$$

The traffic congestion is given as follows:

$$C_{OFL}(N, S, \beta) = \frac{A - Y}{A} = \frac{S\alpha - \sum_{i=0}^N \sum_{j=0}^{S-N} i Q_{i,j}}{S\alpha} \quad (15)$$

For the Engset OFL, we have that $B_{OFL}(N, S, \beta) = C_{OFL}(N, S, \beta)$, which has been confirmed by the numerical evaluation presented in the next section.

5. Numerical evaluations

This section presents a numerical evaluation of the Engset LCC and the Engset OFL. Figures 5 and 6 show the blocking probability as a function of the normalized system load (A) for the NWC and FOWC scenario, respectively. Figures 7 and 8 show the blocking probability as a function of the number of input/output fibres (F) for the NWC and FOWC scenario, respectively. Among the considered performance metrics, we regard the C_{OFL} (and thus the B_{OFL}) to be the most accurate measure of the blocking probability. This is because the Engset OFL takes into account that an input wavelength that generated a packet that is lost due to contention, will remain in the busy state until the packet has completed transmission. Furthermore, regarding Engset OFL, since the time congestion considers only the share of time the output port is congested, it fails to capture certain crucial effects, as discussed below. From Figs. 5-8, we observe the following major findings:

- We observe that $E_{LCC} \geq B_{LCC} \geq C_{LCC}$ and $E_{OFL} \geq B_{OFL} = C_{OFL}$ for all parameter settings.
- As C_{OFL} is the most accurate measure for the blocking probability, we see that C_{LCC} tends to underestimate the blocking probability, while E_{LCC} , B_{LCC} , and E_{OFL} tend to overestimate the blocking probability.
- As illustrated in Figs. 7 and 8, the blocking probabilities C_{LCC} , B_{LCC} , and C_{OFL} increase as the number of input/output fibres (F) increases. This is expected, since an increase in the parameter F leads to an increased variance regarding arrivals to the tagged output port, which in turn leads to an increased blocking probability. This can be seen from Eq. (3). However, in Fig. 7, we see that this effect is not captured by neither E_{LCC} nor E_{OFL} . Hence, the time congestion is not an adequate performance metric for asynchronous bufferless OPS without wavelength conversion.
- The increase in the blocking probability regarding C_{LCC} , B_{LCC} , and C_{OFL} diminishes as the parameter F increases. This is because the variance increases less as F increases, as seen from Eq. (4).

- The blocking probabilities converge as the parameter F increases. Although not shown explicitly in Fig. 7, we have that, e.g., $E_{LCC} \approx B_{LCC} \approx C_{LCC} \approx E_{OFL} \approx C_{OFL} \approx 0.444$ when $F=10000$. Hence, the choice of traffic model and performance metric has greater impact on the observed blocking probability in switches with a small number of input/output fibres than in switches with a high number of input/output fibres.

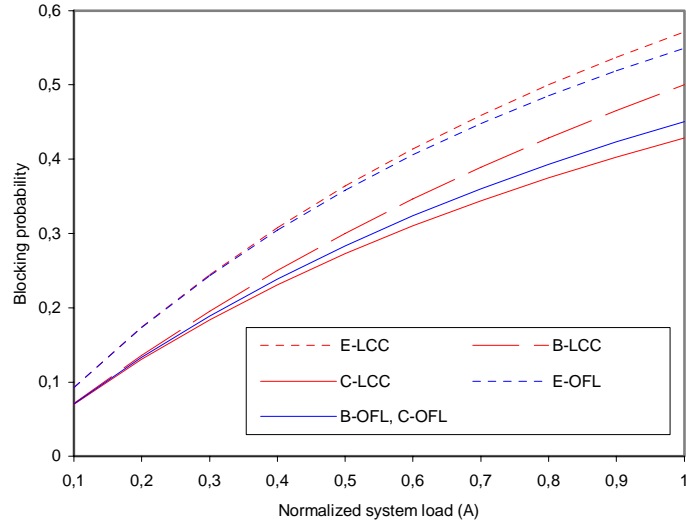


Fig. 5. The blocking probability as a function of the normalized system load (A) at a tagged output wavelength. $F=4$.

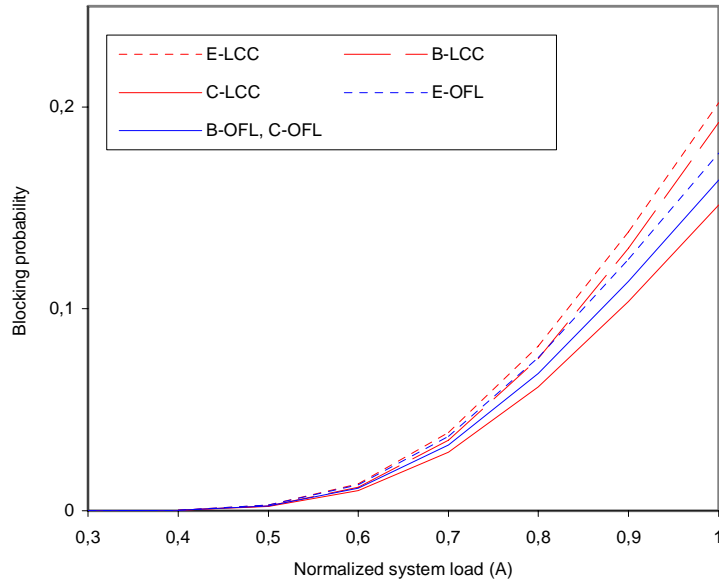


Fig. 6. The blocking probability as a function of the normalized system load (A) at a tagged output fibre. $F=4$, $W=16$.

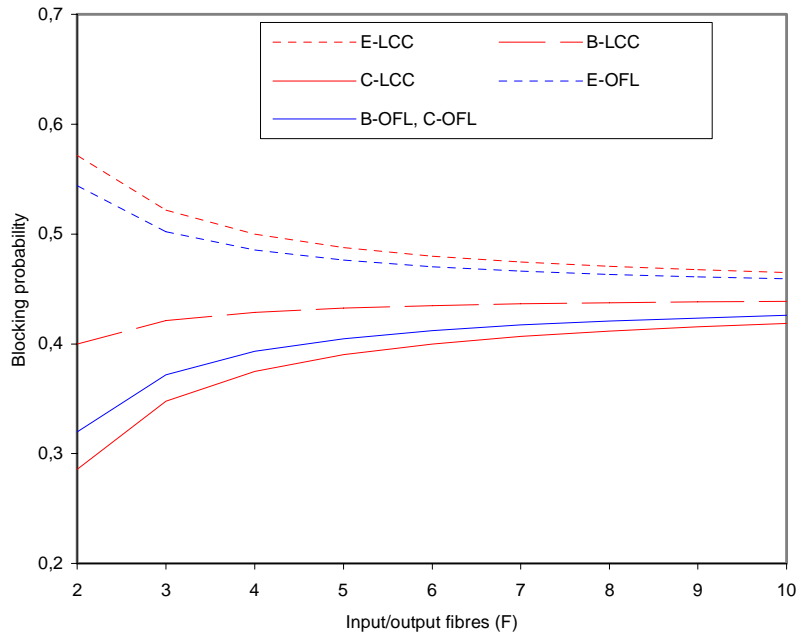


Fig. 7. The blocking probability as a function of the number of input/output fibres (F) at a tagged output wavelength. $A=0.8$.

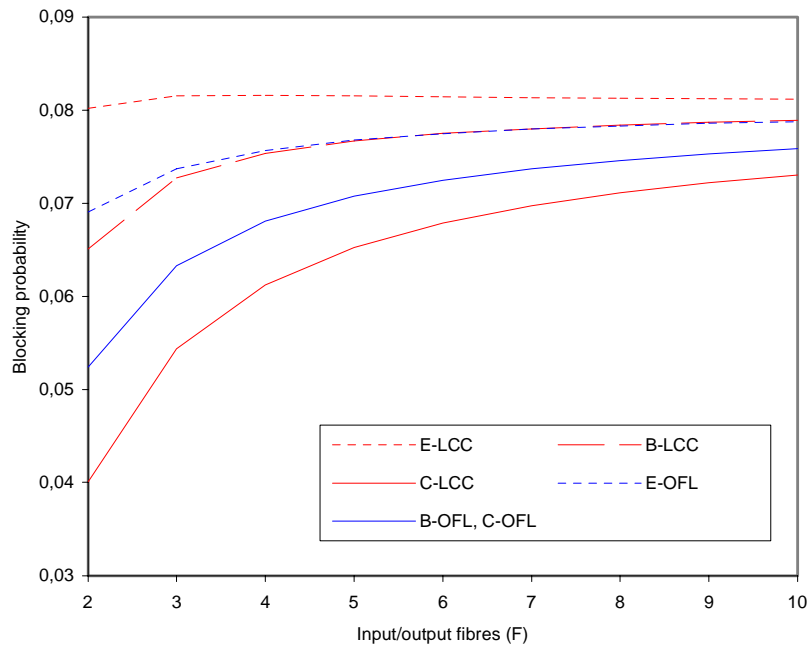


Fig. 8. The blocking probability as a function of the number of input/output fibres (F) at a tagged output fibre. $A=0.8$, $W=16$.

6. Conclusions

This paper has presented two types of the Engset traffic model, i.e., the Engset LCC and the Engset OFL. For both traffic models, the time-, call-, and traffic congestion have been derived. A numerical evaluation reveals that there is a small, but non-neglectable difference in the blocking probability, depending on the choice of traffic model and performance metric. Future research should consider how asymmetric traffic can be modelled using the Engset LCC and Engset OFL.

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