Design and comparison of adaptive estimators for Under-Balanced Drilling

Amirhossein Nikoofard*, Tor Arne Johansen, and Glenn-Ole Kaasa

***Abstract*—Real time knowledge of total mass of gas and liquid in the annulus and geological properties of the reservoir is very useful in many active controllers, fault detection systems and safety applications in the well during petroleum exploration and production drilling. Sensors and instrumentation can not measure the total mass of gas and liquid in the well directly and they are computed by solving a series of nonlinear algebraic equations with measuring the choke pressure and the bottom-hole pressure. This paper presents different estimator algorithms for estimation of the annular mass of gas and liquid, and production constants of gas and liquid from the reservoir into the well during Under Balanced Drilling. The results show that all estimators are capable of identifying the production constants of gas and liquid from the reservoir into the well, while the Lyapunov based adaptive observer gives the best performance comparing with other methods when there is a significant amount of noise.**

***Index Terms*—Under Balanced Drilling(UBD), Nonlinear observer, Lyapunov stability and Unscented Kalman Filter.**

I. Introduction

In petroleum drilling operations, the drilling fluid (mud) must be pumped down through the drill string toward the drill bit by the mud pump (see Figure 1). The annulus is sealed with a rotating control device (RCD), and the mud exits through a controlled choke valve, allowing for faster and more precise control of the annular pressure. The mud carries cuttings from the drill bit to the surface. In conventional (over-balanced) drilling, or Managed Pressure Drilling (MPD), the pressure of the well must be kept greater than pressure of reservoir to prevent influx from entering the well. But in the UBD operation, the hydrostatic pressure of the well must be kept greater than pressure of collapse and less than pressure of reservoir

$$p_{coll}(t, x) < p_{well}(t, x) < p_{res}(t, x) \tag{1}$$

at all times t and positions x. Since hydrostatic pressure of the well is intentionally lower than the reservoir formation pressure, influx fluids (oil, free gas, water) from the reservoir are mixed with rock cuttings and mud fluid in the annulus. Therefore, modeling of the UBD operation should be considered as multiphase flow. Different aspects of modeling relevant for UBD have been studied in the literature [1]–[4]. Estimation and control design in MPD has been investigated by several researchers [5]–[11]. However, due to complexity of multiphase flow dynamic in the UBD operation, there are a few studies on the estimation and control of the UBD operation [1], [12]–[14]. Nygaard et al. [12] compared and evaluated the performance of the extended Kalman filter, the ensemble Kalman filter and the unscented Kalman filter to estimate the states and production index in UBD operation. Lorentzen et al. [13] designed an ensemble Kalman filter to tune the uncertain parameters of a two-phase flow model in the UBD operation. In Nygaard et al [14], a finite horizon nonlinear model predictive control in combination with an unscented Kalman filter was designed for controlling the bottom-hole pressure based on a low order model developed in [1], and the unscented Kalman filter was used to estimate the states, and the friction and choke coefficients.

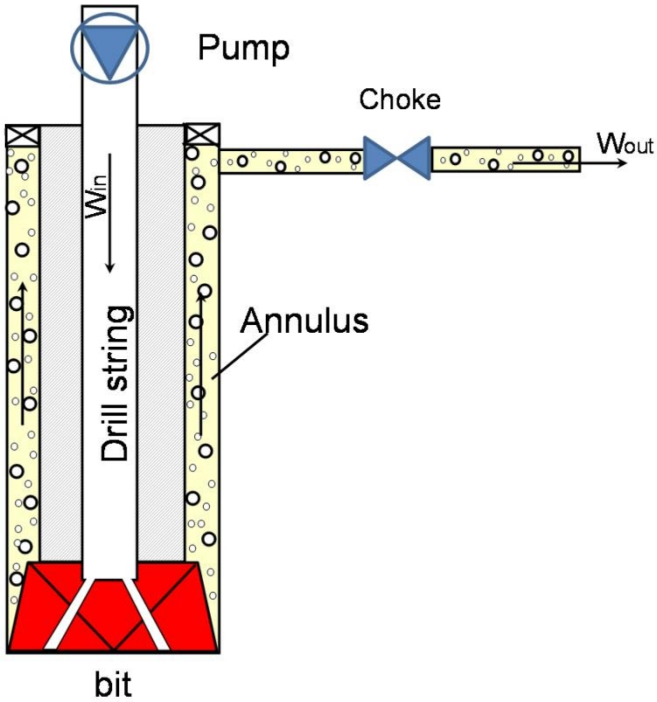

Figure 1. Schematic of an UBD system

State and parameter estimation of linear and nonlinear dynamical systems is one of the main topics in the control theory. The adaptive observer was proposed by Carrol et al. in 1973 [15]. The Lyapunov based adaptive observer is generally used to design a Luenberger type observer for the state while

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appropriate adaptive law to estimate the unknown parameters [16]–[19]. If the observability and persistency excitation condition is satisfied, then both the state and the parameter estimation will converge to their true values. The original Kalman filter based on linear model was developed to estimate both state and parameter of the system usually known as an augmented Kalman filter. Several Kalman filter techniques have been developed to work with non-linear system. The unscented Kalman filter has been shown to have a better performance than other Kalman filter techniques for nonlinear system in same cases [20], [21].

Since total mass of gas and liquid in the well could not be measured directly, they are estimated by solving a series of nonlinear algebraic equations. In these equations the choke and the bottom-hole pressures are known from the measurements. It is assumed to transmit the bottom-hole pressure (BHP) readings continuously to the surface through a telemetry pipe with a pressure sensor at the measurement while drilling (MWD) tool [22]. Some geological properties of the reservoir such as production constants of gas and liquid from the reservoir might vary and could be uncertain during drilling operations. This paper describes the design of Lyapunov-based adaptive observer, recursive least squares and joint unscented Kalman filter for estimating the total mass of gas and liquid in the annulus and geological properties of the reservoir during UBD operation. These designs are based on a nonlinear two-phase fluid flow model. The performance of these methods is evaluated for the case of pipe connection operations where the main pump is shut off and the rotation of the drill string and the circulation of fluids is stopped. The adaptive estimators are compared to each other in terms of speed of convergence, sensitivity of noise measurement, and accuracy.

This paper consists of following sections: Section II presents a low-order model based on mass and momentum balances for UBD operation. Section III explains some popular method for simultaneously estimating the states and model parameters of a nonlinear system from noisy measurements. In Section IV, the simulation results are provided for state and parameter estimation. Finally, the conclusions are presented in Section V.

II. Mathematical Modeling

Modeling of Under-balanced Drilling (UBD) in an oil well is a challenging mathematical and industrial research area. Due to existence of multiphase flow (i.e. oil, gas, drilling mud and cuttings) in the system, the modeling of the system is very complex. Multiphase flow can be modeled as a distributed (infinite dimension) model or a lumped (finite dimension) model. A distributed model is capable of describing the gas-liquid behavior along the annulus in the well. In this paper, a Low-order lumped (LOL) model is used. The lumped model considers only the gas-liquid behavior at the drill bit and the choke system. This modeling method is very similar to the two-phase flow model found in [1], [23]. Some important simplifying assumptions of the LOL model are listed as below:

- Ideal gas behavior
- Simplified choke model for gas, mud and liquid leaving the annulus
- No mass transfer between gas and liquid
- Isothermal condition and constant system temperature
- Constant mixture density with respect to pressure and temperature.
- Liquid phase considers the total mass of mud, oil, water, and rock cuttings.

The simplified LOL model equations for mass of gas and liquid in an annulus are derived from mass and momentum balances as follows

$$\dot{m}_g = w_{g,d} + w_{g,res}(m_g, m_l) - \frac{m_g}{m_g + m_l} w_{out}(m_g, m_l) \tag{2}$$

$$\dot{m}_l = w_{l,d} + w_{l,res}(m_g, m_l) - \frac{m_l}{m_g + m_l} w_{out}(m_g, m_l) \tag{3}$$

where m_g and m_l are the total mass of gas and liquid, respectively. The liquid phase is considered incompressible, and ρ_l is the liquid mass density. The gas phase is compressible and occupies the space left free by the liquid phase. $w_{g,d}$ and $w_{l,d}$ are the constant mass flow rate of gas and liquid from the drill string, $w_{g,res}$ and $w_{l,res}$ are the mass flow rates of gas and liquid from the reservoir. The total mass outflow rate is denoted by w_{out}.

The total mass outflow rate is calculated by the traditional valve equation

$$w_{out} = K_c Z \sqrt{\frac{m_g + m_l}{V_a}} \sqrt{p_c - p_{c0}} \tag{4}$$

where K_c is the choke constant. Z is the control signal to the choke opening, taking its values on the interval $(0, 1]$. The total volume of the annulus is denoted by V_a. p_{c0} is the constant downstream choke pressure (atmospheric). The choke pressure is denoted by p_c, and derived from ideal gas equation as follows

$$p_c = \frac{RT}{M_{gas}} \frac{m_g}{V_a - \frac{m_l}{\rho_l}} \tag{5}$$

R is the gas constant, T is the average temperature of the gas, and M_{gas} is the molecular weight of the gas. The flow from the reservoir into the well for each phase can be modeled by the linear relation with the pressure difference between the reservoir and the well. The mass flow rates of gas and liquid from the reservoir into the well are given by

$$w_{g,res} = K_g(p_{res} - p_{bh}) \tag{6}$$

$$w_{l,res} = K_l(p_{res} - p_{bh}) \tag{7}$$

where p_{res} is the known pore pressure in the reservoir. K_g and K_l are the production constants of gas and liquid from the reservoir into the well, respectively. Finally, the bottom-hole pressure is given by the following equation

$$p_{bh} = p_c + (m_g + m_l)g\sin(\Delta\theta) + \Delta p_f \tag{8}$$

where Δp_f is the friction pressure loss in the well, g is the gravitational constant and $\Delta\theta$ is the average angle between gravity and the positive flow direction of the well. Reservoir parameters could be evaluated by seismic data and other geological data from core sample analysis. But, local variations of reservoir parameters such as the production constants of gas and liquid may be revealed only during drilling. So, it is valuable to estimate the reservoir parameters while drilling is performed [12].

III. Estimation Algorithm

In this section, first an adaptive observer to estimate states and parameters in UBD operation is derived. Then, the recursive least squares method and joint unscented Kalman filter are presented for same problem. The total mass of gas and liquid in the well are outputs of the plant. Since the total mass of gas and liquid in the well could not be measured directly, they are computed by solving a series of nonlinear algebraic equations with measuring the choke pressure and the bottom-hole pressure. The measurements and inputs of LOL model can be summarized in Table I. The production constant of gas (K_g) and liquid (K_l) from the reservoir into the well are unknown and must be estimated. K_g and K_l are defined by θ_1 and θ_2, respectively.

Table I
Measurements and inputs of LOL model

Variables	Measurement/Input
Choke pressure (p_c)	Measurement
Bottom-hole pressure (p_{bh})	Measurement
Drill string mass flow rate of gas ($w_{g,d}$)	Input
Drill string mass flow rate of liquid ($w_{l,d}$)	Input
Choke opening (Z)	Input

A. Lyapunov-based adaptive observer

A full-order adaptive observer for the system (2)-(3) is

$$\dot{\hat{m}}_g = w_{g,d} + \hat{w}_{g,res}(m_g^c, m_l^c) - \frac{m_g^c}{m_g^c + m_l^c} w_{out}(m_g^c, m_l^c) + l_1(m_g^c - \hat{m}_g) \tag{9}$$

$$\dot{\hat{m}}_l = w_{l,d} + \hat{w}_{l,res}(m_g^c, m_l^c) - \frac{m_l^c}{m_g^c + m_l^c} w_{out}(m_g^c, m_l^c) + l_2(m_l^c - \hat{m}_l) \tag{10}$$

where

$$\hat{w}_{g,res} = \hat{\theta}_1(p_{res} - p_{bh}) \tag{11}$$

$$\hat{w}_{l,res} = \hat{\theta}_2(p_{res} - p_{bh}) \tag{12}$$

and l_1,l_2 have to be chosen positive. $\hat{m}_g$ and $\hat{m}_l$ are estimates of states m_g and m_l, $\hat{\theta}_1$ and $\hat{\theta}_2$ are estimates of parameters θ_1 and θ_2. m_g^c and m_l^c are calculated by measurements of the choke pressure and the bottom-hole pressure and an inversion of the equations (5) and (8)

$$m_l^c = \frac{1}{1 - \frac{p_c M_{gas}}{RT\rho_l}} \left(\frac{p_{bh} - p_c - \Delta p_f}{g\sin(\Delta\theta)} - \frac{p_c M_{gas} V_a}{RT}\right) \tag{13}$$

$$m_g^c = \frac{p_c M_{gas}(V_a - \frac{m_l^c}{\rho_l})}{RT} \tag{14}$$

Defining the state estimation errors $e_1 = m_g - \hat{m}_g$ and $e_2 = m_l - \hat{m}_l$, the error dynamics can be written as follows

$$\dot{e_1} = (\theta_1 - \hat{\theta}_1)(p_{res} - p_{bh}) - l_1 e_1 \tag{15}$$

$$\dot{e_2} = (\theta_2 - \hat{\theta}_2)(p_{res} - p_{bh}) - l_2 e_2 \tag{16}$$

with $\tilde{\theta}_1 = \theta_1 - \hat{\theta}_1$, $\tilde{\theta}_2 = \theta_2 - \hat{\theta}_2$. Let the Lyapunov function candidate for adaptive observer design be defined as

$$V(e, \tilde{\theta}) = \frac{1}{2}(q_1 e_1^2 + \ q_2 e_2^2 + \ \tilde{\theta}_1^2 + \ \tilde{\theta}_2^2) \tag{17}$$

where q_1 and q_2 are positive tuning parameters. It is easy to check that $V(e, \tilde{\theta})$ is positive definite decrescent. From (15) and (16), the time derivative of $V(e, \tilde{\theta})$ along trajectory of the error dynamics is

$$\dot{V}(e, \tilde{\theta}) = - \ l_1 q_1 e_1^2 - l_2 q_2 e_2^2 + \ \tilde{\theta}_1 \ [q_1(p_{res} - p_{bh})e_1 + \dot{\tilde{\theta}}_1] \\ \tilde{\theta}_2 \ [q_2(p_{res} - p_{bh})e_2 + \dot{\tilde{\theta}}_2] \tag{18}$$

In order to force the term in the brackets to zero, the adaptation laws are derived:

$$\dot{\hat{\theta}}_1 = \ -\dot{\tilde{\theta}}_1 = \ q_1(p_{res} - p_{bh})e_1 = q_1 \eta e_1 \tag{19}$$

$$\dot{\hat{\theta}}_2 = \ -\dot{\tilde{\theta}}_2 = \ q_2(p_{res} - p_{bh})e_2 = q_2 \eta e_2 \tag{20}$$

with $\eta = p_{res} - p_{bh}$. The adaptation laws can be implemented by using $e_1 = m_g^c - \hat{m}_g$ and $e_2 = m_l^c - \hat{m}_l$. This gives time derivative of $V(e, \tilde{\theta})$

$$\dot{V}(e, \tilde{\theta}) = - \ l_1 q_1 e_1^2 - l_2 q_2 e_2^2 \leq 0 \tag{21}$$

which implies that all signals $e_1, e_2, \tilde{\theta}_1, \tilde{\theta}_2$ are bounded. From (15,16) and $e_1, e_2, \tilde{\theta}_1, \tilde{\theta}_2 \in \mathcal{L}_\infty$, $\dot{e_1}, \dot{e_2}$ are bounded. It follows by using Barbalat's lemma that e_1, e_2 converge to zero. If η is persistently exciting, i.e, $\int_t^{t+T} \eta^2(\tau) \ d\tau \geq \alpha$ for some $\alpha, T > 0$ and $\forall t \geq 0$, then the parameter estimates will converge to their true values [16]. Thus according to theorem 4.9 in [24], the adaptive observer system is globally asymptotically stable if the persistency excitation condition is satisfied. Note that bottom-hole pressure has some variation during pipe connection. Therefore, pipe connection procedure and noise of the system could satisfy persistence exciting condition.

B. Recursive Least Squares

The LOL model based on equations (2)-(3) can be represented by a discrete explicit scheme given by

$$x_k = f(x_{k-1}, \theta) + q_k \tag{22}$$

$$y_k = h(x_k) + r_k \tag{23}$$

$$h(x_k) = [m_g^c, m_l^c]^T \tag{24}$$

where $q_k \sim N(0, Q_{k-1})$ is the zero mean Gaussian process noise, and $r_k \sim N(0, R_k)$ is the zero mean Gaussian measurement noise. The system equations in (22)-(24) could be represented as follows

$$y_k = \phi_{k-1}\theta + \psi_{k-1} + r_k + q_k \tag{25}$$

$$\theta = [K_g \ , \ K_l] \tag{26}$$

$$\psi_{k-1} = \begin{pmatrix} w_{g,d} - \frac{m_g^c}{m_g^c+m_l^c} w_{out}(m_g^c, m_l^c) \\ w_{l,d} - \frac{m_l^c}{m_g^c+m_l^c} w_{out}(m_g^c, m_l^c) \end{pmatrix} \tag{27}$$

where ϕ_k is a regressor. The update equation for the estimation parameter $\hat{\theta}$ at step k is given by RLS method as follows

$$\hat{\theta}_k = \hat{\theta}_{k-1} + K_k(y_k - \phi_k\hat{\theta}_{k-1} - \psi_{k-1}) \tag{28}$$

where K_k is a matrix to be determined which is called the estimator gain matrix. The update equations for the estimation-error covariance P and the estimator gain matrix K at step k are given by RLS method as follows

$$K_k = P_{k-1}\phi_k[I + \phi_k^T P_{k-1}\phi_k]^{-1} \tag{29}$$

$$P_k = [I - K_k\phi_k^T]P_{k-1} \tag{30}$$

The initial estimation-error covariance P is usually chosen as $P(0) = \alpha I$ with large values for α, because this leads to high correction vectors and therefore to fast convergence.

C. Joint Unscented Kalman Filter

The Unscented Kalman Filter (UKF) was introduced in [25]–[27]. The main idea behind the method is that approximation of a Gaussian distribution is easier than an arbitrary nonlinear function. The UKF estimates the mean and covariance matrix of estimation error with a minimal set of sample points (called sigma points) around the mean by using a deterministic sampling approach known as the unscented transform. The nonlinear model is applied to sigma points instead of a linearization of the model. So, this method does not need to calculate explicit Jacobian or Hessian. More details can be found in [20], [21], [26], [27].

Two common approaches for estimation of parameters and state variables simultaneously are dual and joint UKF techniques. The dual UKF method uses another UKF for parameter estimation so that two filters run sequentially in every time step. At each time step, the state estimator updates with new measurements, and then the current estimate of the state is used in the parameter estimator. The joint UKF augments the original state variables with parameters and a single UKF is used to estimate augmented state vector. The joint UKF is easier to implement [21], [27].

Using the joint UKF, the augmented state vector is defined by $x^a = [X, \theta]$. The state-space equations for the the augmented state vector at time instant k is written as:

$$\begin{bmatrix} x_{1,k} \\ x_{2,k} \\ \theta_{1,k} \\ \theta_{2,k} \end{bmatrix} = \begin{bmatrix} f_1(X_{k-1}, \theta_{1,k-1}) \\ f_2(X_{k-1}, \theta_{2,k-1}) \\ \theta_{1,k-1} \\ \theta_{2,k-1} \end{bmatrix} = f^a(X_{k-1}, \theta_{k-1}) \tag{31}$$

m_g and m_l are denoted by x_1 and x_2, respectively.

IV. Simulation Results

The parameter values for the simulated well and reservoir are given in Table II. These parameters are used from the offshore test of WeMod simulator [28]. WeMod is a high fidelity drilling simulator developed by the International Research Institute of Stavanger (IRIS). The measurements have been synthetically generated by using the LOL model described in Section II, adding the normal distributed noise to the process and measurement results. The augmented process covariance noise matrix used in this plant model and the joint state-parameter estimation is

$$Q = diag[10^{-1}, 10^{-1}, 10^{-4}, 10^{-4}]$$

Table II
Parameter Values for well and reservoir

Name	LOL	Unit
Reservoir pressure (p_{res})	270	bar
Collapse pressure (p_{coll})	255	bar
Friction pressure loss (Δp_f)	10	bar
Well total length (L_{tot})	2300	m
Well vertical depth (L)	1720	m
Drill string outer diameter (D_d)	0.1397	m
Annulus volume (V_a)	252.833	m^3
Annulus inner diameter (D_a)	0.2445	m
Liquid flow rate ($w_{l,d}$)	44	Kg/s
Gas flow rate ($w_{g,d}$)	5	Kg/s
Liquid density (ρ_l)	1475	$\mathrm{Kg/m^3}$
Gas average temperature (T)	25	°C
Average angle ($\Delta\theta$)	0.8448	rad

UKF parameters are determined empirically. The parameter values for UBD model, adaptive observer, RLS and UKF are summarized in Table III.

Table III
Parameter Values for model and estimators

Parameter	Value	Parameter	Value
K_g	5×10^{-6}	L	4
K_l	5×10^{-5}	K_c	0.013
q_1	2×10^{-9}	l_1	0.09
q_2	5×10^{-10}	l_2	0.3
$P_{RLS}(0)$	100 I	κ	0
β	2	α	0.5

The time-step used for discretizing the dynamic model and adaptive estimator was 5 seconds. This time-step is chosen for reducing computation time. The initial values for the estimated and real states and parameters are as follows

$$x_1 = 5446.6, \quad x_2 = 54466.5, \quad \theta_1 = 5, \quad \theta_2 = 5$$

$$\hat{x}_1 = 4629.6, \quad \hat{x}_2 = 46296.2, \quad \hat{\theta}_1 = 8, \quad \hat{\theta}_2 = 3$$

The scenario in this simulation is as follows, first the drilling in a steady-state condition is initiated, then at t = 10 min the main pump is shut off to perform connections procedure. The rotation of the drill string and the circulation of fluids are stopped for 10 mins. Next after making the first pipe connection at t = 20 min the main pump and rotation of the drill string are restarted. Then at t=52 min the second pipe connection procedure is started, and is completed after 12 mins. Two different simulations with low and high measurement noise covariances are performed. To compare the three estimators, production constant of gas and liquid from the reservoir into the well during the UBD operation are estimated by each of the estimators. In the first simulation, the total mass of gas and liquid measurements are corrupted by zero mean additive white noise with the following covariance matrix

$$R = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$$

Figures 2 and 3 show the measured and estimated total mass of gas and liquid, respectively. The estimation of the production constants of gas and liquid from the reservoir into the well are shown in Figures 4 and 5, respectively. In estimation of production constants of gas and liquid from the reservoir into the well, RLS has a very fast convergence rate, about 30 seconds or less. But, UKF and adaptive observer take much longer and it is in the order of minutes.

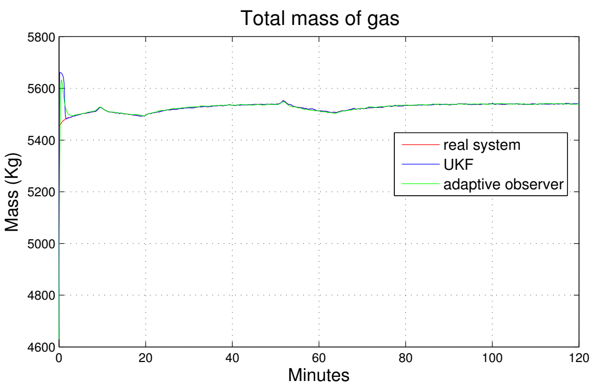

Figure 2. Total mass of gas with the low measurement noise covariance

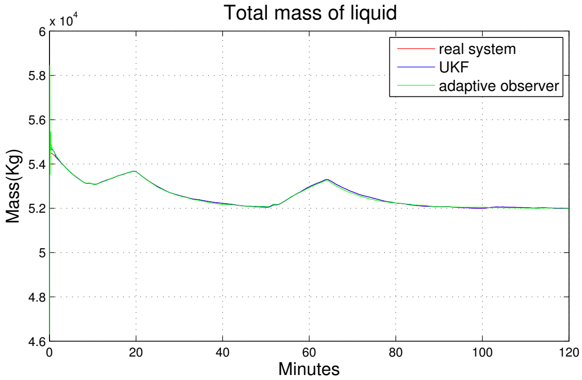

Figure 3. Total mass of liquid with the low measurement noise covariance

In second simulation, the total mass of gas and liquid measurements are corrupted by zero mean additive white noise

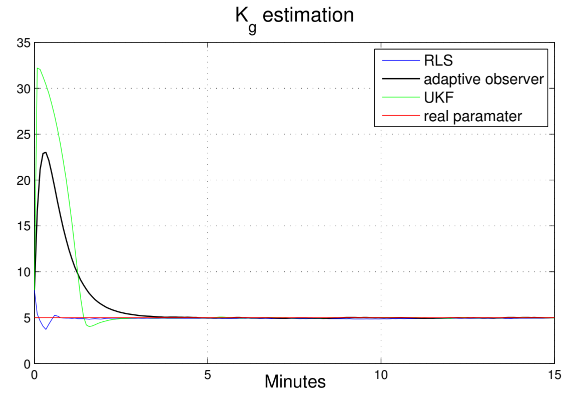

Figure 4. Production constant of gas with the low measurement noise covariance

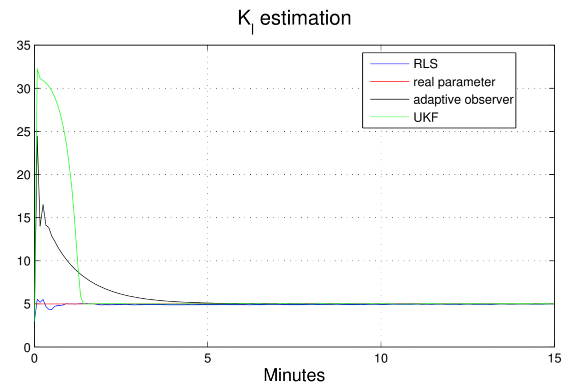

Figure 5. Production constant of liquid with the low measurement noise covariance

with the following covariance matrix

$$R = \begin{bmatrix} 50 & 0 \\ 0 & 50 \end{bmatrix}$$

Figures 6 and 7 show the measured and estimated total mass of gas and liquid, respectively. According to Figures 6 and 7, the adaptive observer is a very sensitive to noise measurement. This sensitivity is due to the fact that adaptive observer uses measurement instead of estimates in the adaptive observer equations. The estimation of the production constants of gas and liquid from reservoir into the well are illustrated in Figures 8 and 9, respectively. In estimation of the production constants of gas and liquid from the reservoir into the well, adaptive observer has a good convergence rate, about 2 minutes, but the UKF takes almost 20 minutes. The RLS method has a reasonable convergence rate but has offset due to use of nonlinear regressor.

In this paper, performance of these adaptive estimators is evaluated through the root mean square error (RMSE) metric. The RMSE metric for adaptive observer, RLS and UKF in two cases are summarized in Table IV. According to the RMSE metric table, RLS has the better performance than other methods for parameter estimation while it is lower measurement noise covariance. But, the adaptive observer

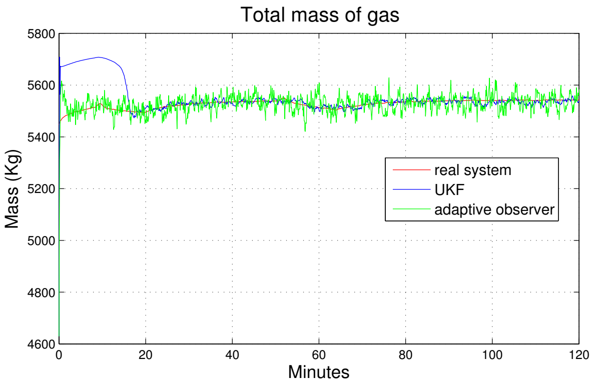

Figure 6. Total mass of gas with the high measurement noise covariance

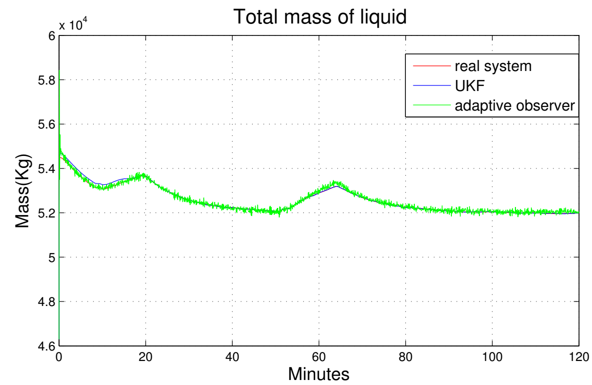

Figure 7. Total mass of liquid with the high measurement noise covariance

Table IV
RMSE METRIC

Method	R	x_1	x_2	θ_1	θ_2
Adaptive observer	0.5	27.482	243.03	1.304	0.900
UKF	0.5	28.018	216.04	2.066	2.199
RLS	0.5	–	–	0.073	0.097
Adaptive observer	50	39.070	256.83	1.425	0.911
UKF	50	70.412	225.49	9.925	8.813
RLS	50	–	–	2.981	3.084

has better performance than the other methods for parameter estimation while the measurement noise covariance is high.

V. Conclusions

This paper presents an adaptive observer, recursive least squares and joint UKF for estimating states and parameters in under-balanced drilling operations. The low-order lumped model presented here only captures the major phenomena of the UBD operation. Simulation results demonstrate satisfactory performance of adaptive observer, RLS method and joint UKF for state and parameter estimation during pipe connection procedure with low measurement noise covariance. The Lyapunov-based adaptive observer shows better convergence than the other methods with high measurement noise covariance. However, for parameter estimation with low measurement noise covariance RLS has a better performance

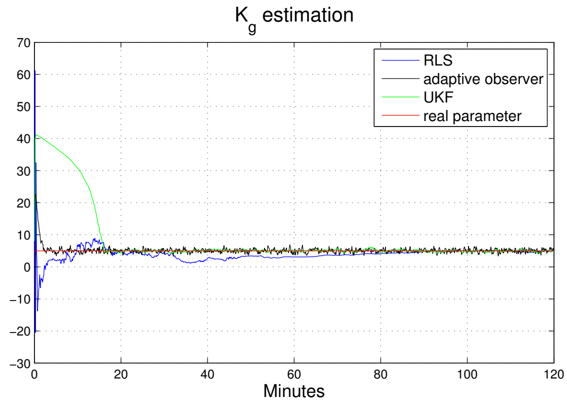

Figure 8. Production constant of gas with the high measurement noise covariance

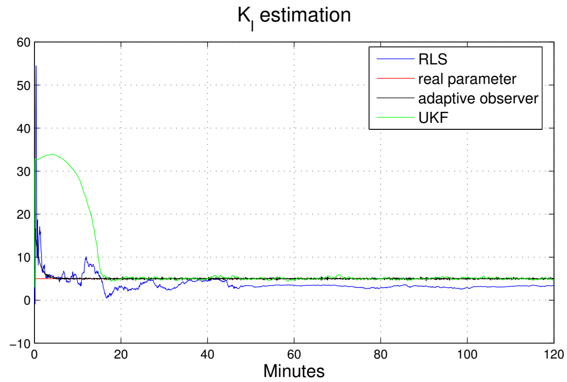

Figure 9. Production constant of liquid with the high measurement noise covariance

than other aforementioned methods.

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