NTNU – Trondheim
Norwegian University of
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Supply Chain Design under Uncertainty

Locating LNG distribution centers in an emerging market with uncertain demand

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Marine Technology
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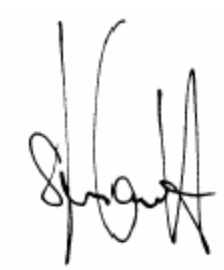
Background
Today, there is a growing demand for natural gas/LNG as an energy source, though there is considerable uncertainty related to future demand. According to the International Energy Agency, the global use of natural gas will increase by more than 50% from 2010, and will account for 25% of global fuel consumption by 2035.

Primary Objective
The overall objective of the thesis is to develop a model that can help decision makers to design a profitable supply chain for an uncertain future gas demand, focusing primarily on the facility location problem. A part of the objective will also be to examine the value of implementing the uncertainty into the model formulation.

Scope of work
The thesis shall presumably cover the following main points:

1. A brief presentation of the supply chain for natural gas, with a focus on LNG
2. Provide relevant literature on both deterministic and stochastic location analysis
3. Develop a deterministic facility location model.
4. Develop a stochastic facility location model.
5. Test the models on relevant data.
6. Discuss the models and results.
7. Find the value of the stochastic model
8. Analyze the sensitivity of the problem specific constraints and parameters

Implementation
Professor Stein Ove Erikstad will be the main supervisor from NTNU, and Professor Kjetil Fagerholt will be the co-supervisor. The work shall follow the guidelines made by NTNU for project work. The workload shall correspond to 30 credits, equivalent to one semester.



Stein Ove Erikstad
Professor/Main Supervisor

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ABSTRACT

According to the International Energy Agency, the global use of natural gas will increase dramatically in the next two to three decades. Due to factors ranging from national and international energy market regulations to availability of energy and economical growth, there is a high degree of uncertainty in these predictions concerning how the natural gas demand will develop in the future.

This thesis looks specifically at how to optimize the profit of a gas distribution company through development of distribution centers along the Norwegian coastline, given different scenarios for future demand. Both the amount of distribution centers to be constructed, their locations and capacity are considered. The distribution methods are limited to shipping between liquefaction plants and distribution centers, and subsequent truck transportation to end-customers. A deterministic model with one aggregated demand scenario and a stochastic model with three different scenarios are presented, implemented and compared.

Due to the high flexibility in the problem, where it is possible to expand and construct new distribution centers throughout the lifetime of the project, it is found that the difference in achieved profit between the stochastic and deterministic solution is insignificant in most cases. Only when the low or high demand scenarios are heavily weighted in the probability distribution does the use of a stochastic model become valuable in certain cases. Tests show that the usefulness of the stochastic model, compared with the deterministic model, increases when the flexibility decreases, and vice versa.

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SAMMENDRAG

Prognoser gitt av det internasjonale energibyrået viser at den globale bruken av naturgass vil øke dramatisk de neste tretti årene. Det er allikevel flere usikkerhetsmomenter knyttet til hvordan denne etterspørselen vil utvikle seg, der økonomisk vekst, tilgjengelige energikilder og nasjonale og internasjonale forskrifter innenfor energimarkedet er påvirkende faktorer.

Denne masteroppgaven handler om hvordan man kan optimalisere fortjenesten i et gassdistribusjonsselskap ved å opprette distribusjonssentere for LNG langs norskekysten, gitt ulike scenarier for den fremtidige etterspørselen. Både antall distribusjonssentre som skal bygges, geografisk plassering og størrelse er ukjente variabler som skal vurderes. Distribusjonsmetoden er begrenset til skipsfrakt mellom LNG-produksjonsanlegg og distribusjonssenter, og lastebiltransport fra distribusjonssenter til sluttkunder. Det er både utviklet en deterministisk modell med et gjennomsnittlig etterspørselsscenario og en stokastisk modell som tar hensyn til tre ulike etterspørselsscenarioer.

Grunnet den høye fleksibiliteten i problemet, i form av muligheten til å opprette og utvide distribusjonssentere, viser det seg at forskjellen mellom oppnådd profitt for en stokastisk og deterministisk løsning er begrenset i de fleste tilfeller. Tester viser likevel at det i enkelte tilfeller, der sannsynligheten er stor for enten et høyt eller lavt etterspørselsscenario, er gunstig å bruke en stokastisk modell.

Ved å endre på faktorene som avgjør modellens fleksibilitet, som for eksempel å korte ned på lede-tiden for opprettelse av distribusjonssentere, vil også nytteverdien av den stokastiske løsningen bli påvirket. En lavere fleksibilitet, i dette eksempelet høyere lede-tid, gjør den stokastiske løsningen mer verdifull, mens en høyere fleksibilitet gjør den mindre verdifull.

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PREFACE

This master thesis has been written during spring 2014 at the Department of Marine Technology, Norwegian University of Science and Technology (NTNU). The overall objective of the thesis is to develop a model that can help decision makers to design a profitable supply chain for an uncertain future gas demand, using operational analysis. The objective in the thesis is form by me, in cooperation with my supervisors. The work is a continuation of my project thesis from the autumn 2013.

Working on this master thesis has been rewarding for both my professional and personal development. To work that hard on one single task over a long period of time has given me personal insight, where one example is that my gut feeling always has told me when something is wrong and needs to be studied further. I have also learned that it is very difficult to discover minor errors in a complicated model when working alone.

I would like to thank my supervisor Professor Stein Ove Erikstad and co-supervisor Kjetil Fagerholt at the Department of Marine Technology and the Department of Industrial Economics and Technology Management respectively for all the guidance and valuable discussions throughout the last two semesters. I would also like to thank Morten Christophersen at Connect LNG for giving me a better understanding of the natural gas market. Lastly, I would like to thank Kristina Marki for providing linguistic guidance throughout the thesis.

Trondheim, June 9th 2014

Marius Kongsfjell Fekene

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1 INTRODUCTION

In 2011, The International Energy Agency (IEA) presented something they called "GAS Scenario", which states that the global use of natural gas will increase by more than 50% from 2010 and will account for 25% of global fuel consumption by 2035. This claim depends upon various factors such as international and national regulations in the energy market, the availability of energy and the global economical growth (IEA (2011)). Figure 1.1 show the global natural gas demand by scenarios, where different policies influence the demand. The figure below illustrates the global growth in demand, but the trend is also applicable for Norway. The demand in Norway has increased with 500% from 2004 to 2011, and is according to Haugland, Yttredal et al. (2013) expected to raise to over 1400% in 2016.

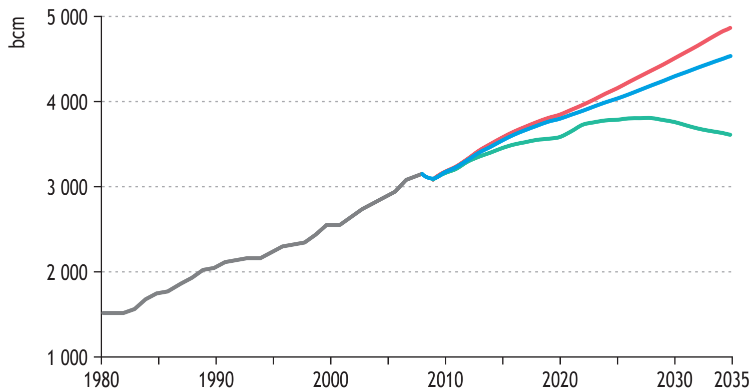

Figure 1.1 Worlds natural gas demand by scenario, IEA (2010)

There are two important observations to be made from this graph. Firstly, that the demand for natural gas could potentially increase a lot. Secondly, that the demand is uncertain and varies considerably between scenarios.

The current demand in countries such as Norway is generally satisfied by transporting the natural gas as Liquefied Natural Gas (LNG) from production location to the end-customer. This is probably a good transportation solution as long as the demand is at the present level, but this can change if the demand increases sharply.

An alternative transportation solution, given sharply increased demand, is to expand with an additional transportation step. Instead of transporting directly to end-customers, it is reasonable to transport large quantities to distribution centers, and then transport it further to the end-customer. In this way one can take advantage of economies of scale by transporting large quantities in one batch. Distribution centers will herby be referred to as import terminals.

In a large market, locating these import terminals represents a major challenge, considering the number of different variables that exist in the problem and the cost incurred by not locating terminals at the most profitable place. Imagine the complexity of this decision: you have a market where you have to decide the location, the quantity and capacity of these terminals, in addition to various companies to buy the gas from and hundreds of customers. This kind of location problem, with all its variables and parameters, can be optimized in an optimization model using proper assumptions and constraints.

The main scope in this thesis is to develop a model that can help decision makers to design a profitable supply chain for scenarios with an uncertain future gas demand, focusing on the location of potential import terminals. The idea is that this model shall be able to design a more profitable supply system for the uncertain future, better than the human gut feeling or simple spreadsheet calculations are able to design. A part of the scope will be to examine the value of implementing the uncertainty into the model formulation.

LNG transportation is defined as the transportation method, due to the relatively low demand of natural gas in Norway, the rough vegetation and the fact that customers are spread over a large geographical area. This will be presented in detail in the background chapter.

The thesis is structured concerning the main scope. Chapter 2 gives an understanding of the different components of the supply chain, while Chapter 3 provides literature relevant to the problem. Chapter 4 limits and defines the problem in written form, while Chapter 5 presents both a deterministic and stochastic mathematical formulation of the problem. Chapter 6 analyzes the problem and discusses and validates the model developed in Chapter 5. Chapter 7 presents a post analysis of the problem, where the sensitivity of problem specific constraints and parameters are tested. The concluding remarks are presented in Chapter 8, whilst suggestions for further work are presented in Chapter 9.

2 BACKGROUND

It is necessary to understand the supply chain of LNG in order to be able to model it. The focus in this chapter is to get the necessary understanding of natural gas, LNG and its supply chain. This information will be used to form the problem description, mathematical formulations and computation study in later chapters. The process of collecting information has been challenging, because of all the secrecy in the industry. The information presented in this chapter is therefore considered to be my understanding of the LNG supply chain.

2.1 NATURAL GAS

Sakmar (2013) describes natural gas as a "Bridge Fuel" which, despite its status as fossil fuel, acts as a step towards more usage of renewable energy sources. Natural gas is widely considered as a cleaner alternative to oil and coal and was in a official statement from the Norwegian Energy Committee (Stortinget (2001)), noted as an important step in the transformation of energy production and consumption in Norway. Natural gas consists primarily of methane and pollute far less than oil and coal. The numbers in Table 2.1 below show the emission relative to natural gas and confirm this statement.

Table 2.1 Air pollutants relative to natural gas, modified from Energy Information Administration (1999)

Pollutant	Symbol	Natural Gas	Oil	Coal
Carbon Dioxide	CO_2	1	1.4	1.8
Carbon Monoxide	CO	1	0.8	5.2
Nitric Oxide	NO_X	1	4.9	5
Sulfur Dioxide	SO_2	1	1122	2591
Particles	-	1	12	392
Mercury	Hg	0	0.007	0.016

All natural gas in Norway is retrieved from the Norwegian continental shelf, where pipelines export the gas to onshore processing facilities. Figure 2.1 shows the natural gas, represented by green lines, which is either re-distributed in pipelines as natural gas to end-customers or transformed to LNG. According to Taran Fæhn, Cathrine Hagem et al. (2013), two-thirds of all international gas trade goes via pipelines and is considered as the cheapest and safest form of transport for markets with high demand.

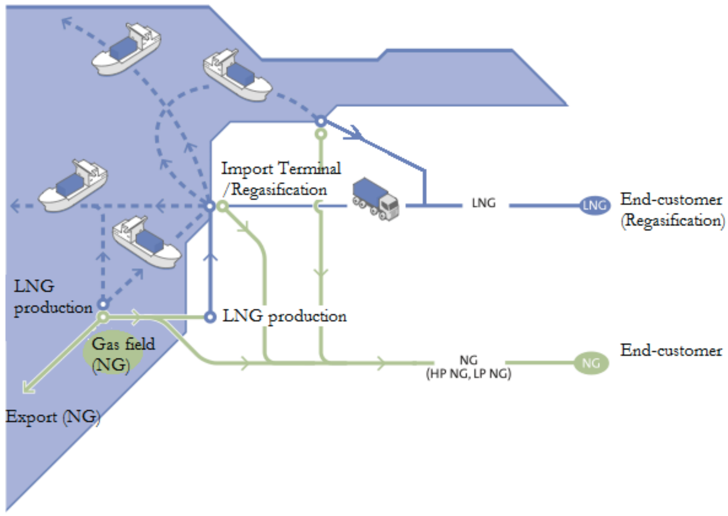

Figure 2.1 Natural Gas (NG) and LNG supply chain, modified from SINTEF, MARINTEK et al. (2002)

Norwegian gas production started in 1977 and has increased steadily in production volume ever since. Unlike oil production in Norway that peaked in 2000, the Norwegian gas production is still increasing. Norway produced over 114 millions Sm^3 in 2012, divided between 63 different fields, according to SSB (2013). This is 28% more than total oil production the same year and is equivalent to 717 billion barrels of oil.

2.2 LIQUEFIED NATURAL GAS

LNG is natural gas cooled to a temperature below -163°C. When the gas is cooled to this temperature the gas condenses into a liquid at atmospheric pressure and reduces its volume to 1/625 of the volume of natural gas, making it attractive to use in transportation. Several undesirable substances are removed from the gas before it is considered as LNG, a process called liquefaction. LNG is a clear, colorless and odorless liquid that is neither corrosive nor toxic.

A lot of the expected increase in consumption of LNG in Norway is due to the new emission restrictions for the European Emission Control Area (ECA) that will be introduced from 2015. ECA include the North Sea (south of 62 degrees latitude), the

English Channel and the Baltic Sea. One of the new emission restrictions applies to the maximum sulfur level in the fuel at 0.1%, which in practice corresponds to the purest distillate quality of marine diesel. LNG as fuel makes sure that you are below this limit.

In addition to the new emission restrictions, financial support from the NOx-foundation is a major reason for the increased consumption of LNG in Norway. Figure 2.2 show predicted LNG consumption in Norway until 2017, where the consumption has grown to approximately 425,000 ton LNG. The two green areas illustrate the share of consumption with financial support from the NOx-foundation.

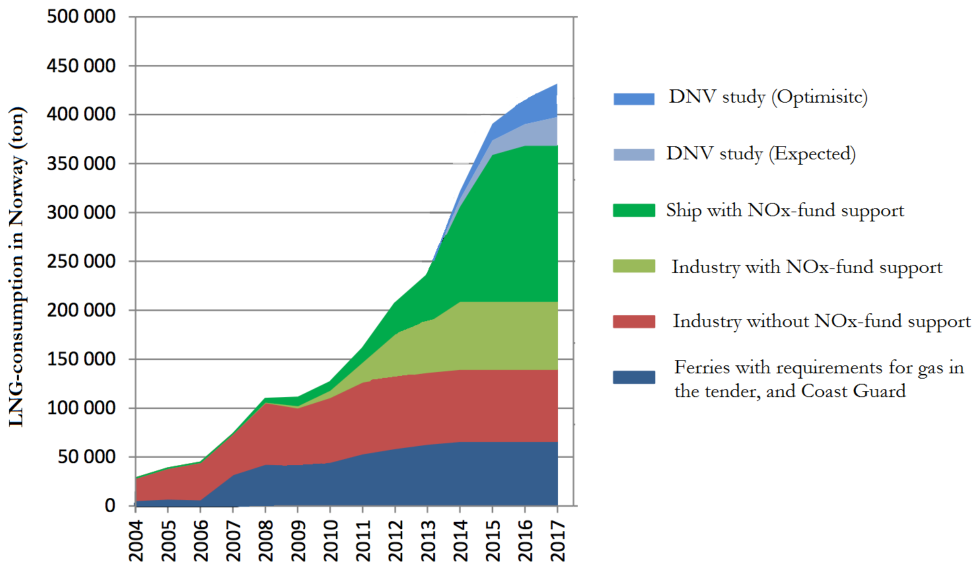

Figure 2.2 LNG consumption in Norway 2004-2017, modified from Haugland, Yttredal et al. (2013)

While reduction in volume is the key benefit for LNG, capital investment in infrastructure, distribution cost and energy loss is considered as the biggest challenges in the LNG market today. It is difficult to get someone to invest in LNG carriers (sea transport) as long as it does not exist import terminals, and vice versa. This problem is often illustrated in the industry by the "chicken or egg"-riddle. The distribution cost of LNG is more expensive than transporting other fossil fuels. The reason for this is that it requires more advanced technology and expertise to process LNG. The energy loss is also large when transferring LNG from one storage device to another, so fewer links in the supply chain is preferred.

According to Haugland, Yttredal et al. (2013), the Norwegian LNG market is organized in a way that prevents new operators to arise. New operators need to build their own infrastructure to compete in an existing operators area. This requires large extra capital cost, which in practice makes it impossible to compete against established operators. They are also stating that the LNG prices are confidential between sellers and byers in Norway, something that undermines the trust to LNG as an energy alternative. Table 2.2 presents the average LNG prices in Norwegian industry and mining from 2009 to 2011. These prices are higher than European gas prices. The gas prices will also vary between end-customers because of the variation in transportation distance and requested volume.

Table 2.2 Average retail price for gas in industry and mining, modified from SSB 2013

	2009	2010	2011
Liquefied Natural Gas [NOK/m^3]	1,753	2,277	2,233

2.3 THE SUPPLY CHAIN OF LNG

Christopher (2005) defines supply chain management as *"the upstream and downstream relationships with suppliers and customers in order to deliver superior customer value at less cost to the supply chain as a whole"*, where the supply chain represents the different processes of a product, from raw material to final delivery at end-customer.

Sakmar (2013) limits the LNG supply chain to involve the processes shown in Figure 2.3, where the natural gas arrives from a gas field to a LNG production facility, called liquefaction plant. The natural gas that arrives at the plant contains a variety of gases and liquids, including propane, water and oil that is removed in a process called gas treatment. The process of cooling the gas to LNG, called liquefaction, can start when all undesirable gases and liquids are removed. After the liquefaction process, the gas is stored and transported as LNG until it arrives at a regasification terminal where the LNG is converted back to natural gas. The LNG can either be transported directly to end-customers where the LNG is converted back to natural gas, or it can be transported via an import terminal where the gas is stored and re-distributed with trucks to end-customers before the liquid is converted back to gas.

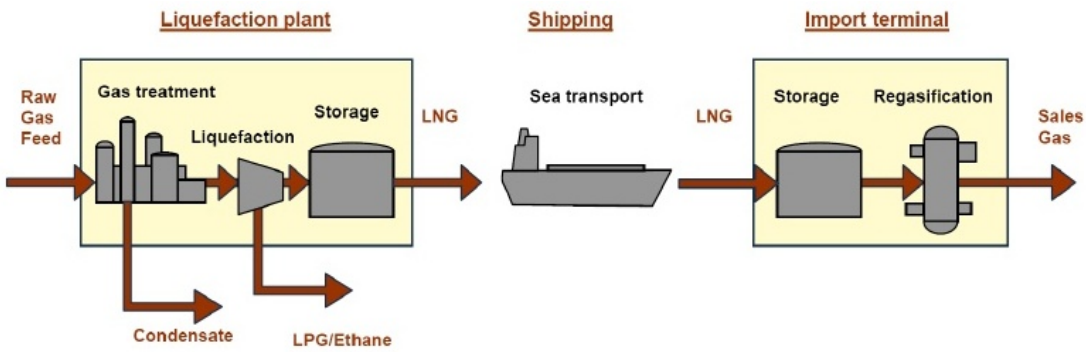

Figure 2.3 LNG Supply Chain, GIIGNL (2009)

The process above is a simple description of the LNG supply chain based on Sakmar (2013), where the gas is converted to LNG from natural gas and back again. The following subsections will go trough the different kinds of infrastructure that is needed to facilitate this LNG supply chain.

2.3.1 LIQUEFACTION PLANTS

There are currently four liquefaction plants in Norway. Table 2.3 show an overview of these facilities, their names, geographic location, owners and production capacity. The total LNG production in Norway is approximately 4.6 million tons per year, where Statoils plant in Hammerfest accounts for nearly all production. This is a very small amount compared to the overall LNG production in the world of approximately 279 million tons per year. There are 13 countries in the world that produce more LNG than Norway, according to Haugland, Yttredal et al. (2013). The table below shows that liquefaction plants can be owned by distribution companies such as Gasnor, Skangass and oil and gas companies such as Statoil.

Table 2.3 Liquefaction facilities in Norway, Haugland, Yttredal et al. (2013)

Name	Municipal	Owner	Production capacity [10^3 ton/year]
Snurrevarden	Karmøy	Gasnor	20
Kollsnes	Øygarden	Gasnor	120
Stavanger	Stavanger	Skangass	300
Melkøya	Hammerfest	Statoil et.	4,200

The production facility at Melkøya in Hammerfest is at the moment the only large-scale production facility in Europe and delivers most of its gas to customers in Spain and USA.

2.3.2 IMPORT TERMINALS

It is common that the cargo-owner, in this thesis referred to as gas distribution companies, is responsible for the capital investment and operation of the import terminals. The capital cost is not possible to standardize due to large variation in existing infrastructure. Capital cost for an import terminal can vary with 100 million NOK due to these variations, according to calculation done by ConnectLNG (2013). The capital cost consists of the storage capacity cost and LNG infrastructure cost, where LNG infrastructure consists of equipment such as jetty construction and pipelines. The storage cost varies with the size of capacity and the LNG infrastructure cost is approximately fixed. DMA (2011) points out that studies show a big economy of scale in the LNG terminal business, based on the non-linearity of storage capacity cost. A small import terminal with 700m^3 LNG storage capacity is for instance 1000% more expensive per cubic meter than a import terminal with 20,000m^3 LNG storage capacity (Lindfeldt (2011)).

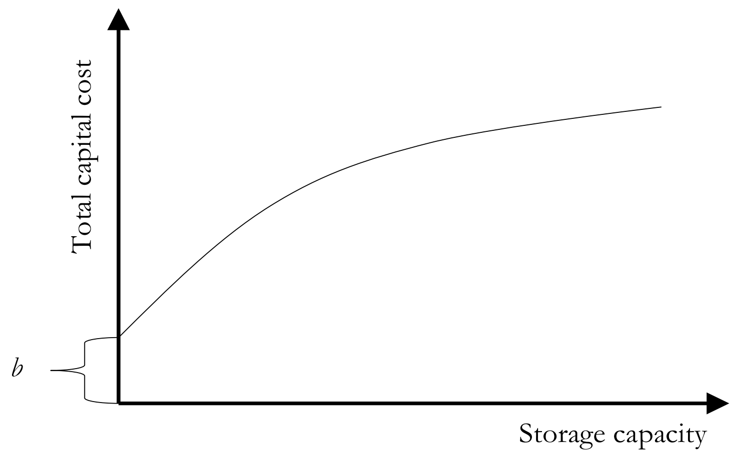

Figure 2.4 Import terminal capital cost

Figure 2.4 shows a simplification of the total capital cost for import terminals, where b represents the fixed LNG infrastructure cost. Economy of scale is clearly illustrates by the concave non-linear function.

It can take between two to four years to complete construction of an import terminal. The reason why this process is so time consuming is because of the variations in existing infrastructure and authorizations from the authorities. The lifetime for import terminals is estimated to 40 years, according to Lindfeldt (2011).

2.3.3 CUSTOMER TERMINALS

It exists more than 50 independent systems adapted to individual customer needs. All facilities receive LNG from either LNG carriers or trucks. It was registered four refueling facilities for ships along the Norwegian cost in 2011. Three of these were exclusively for the oil service bases. (Haugland, Yttredal et al. (2013))

These terminals are usually located at the end-customer. The customer terminal is in some situations shared between two companies. The storage capacity of these terminals varies between 100 m^3- 2000 m^3 and the construction time can take up to one year from planning start to finish. The construction cost is not large compared to the rest of the supply chain. Every end-user needs a costumer terminal to convert the LNG back to natural gas. (Rollefsen (2014))

2.3.4 SHIP TRANSPORT

Ship transport is particularly suitable in a distribution strategy where large volumes of LNG is transported from liquefaction plants to import terminals and further to end-customers with truck transport or for end-customers with significant demands and own import terminals that make them independent of further distribution (MARINTEK (2005)). Figure 2.5 shows the intersection between when it is profitable to transport natural gas in pipelines and as LNG by ship. A simple rule of thumb is that gas is transported as natural gas in pipelines when the volume is large and the distance is short and that the gas is transported as LNG when the volume is small and the distance is long.

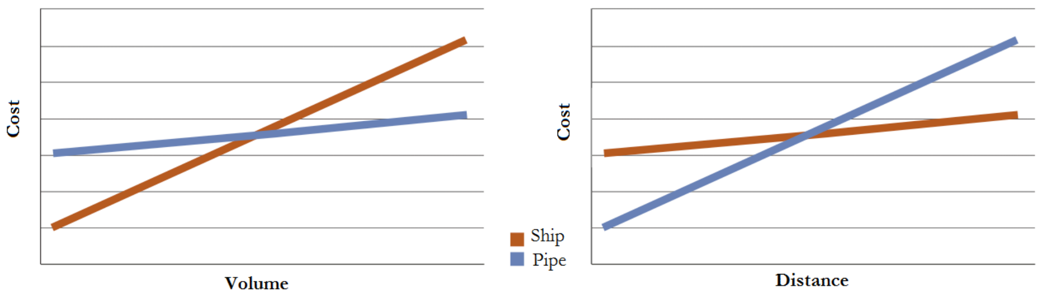

Figure 2.5 Pipelines versus ship transport, modified from SINTEF, MARINTEK et al. (2002)

Distribution of LNG with ships is a specialized market within ship transport. LNG carriers are characterized as ships with advanced tank design and cargo handling systems. According to SINTEF, MARINTEK et al. (2002), the market is characterized by few stakeholders, economy of scale, long-term freight agreements and a difficult second hand market.

SINTEF, MARINTEK et al. (2002) further argues that time chartering is the most common type of agreement within this type of shipping. This is an agreement between ship-owner and cargo-owner where cargo-owner dispose the ship over a given interval, usually long-term in LNG transportation. Time charter implies that cargo-owner determines the usage of the ship, while ship-owner provides operation and manning. The capital- and operational cost is covered through a fixed time charter rate. The cargo owner is in addition to the charter rate cost, paying for the bunker fuel and port costs. Strand (2013) has made a cost estimation sheet for cargo owners with the LNG carrier Coral Energy as example. This shows that the time charter cost generally is determined in advance due to long-term contracts and represents the major expense, while bunker fuel and port fees vary with the operation pattern. Simple calculations in the cost estimation sheet indicates a cost distribution where time charter rate cost counted for 61% of the cost, bunker cost counted for 23% and port cost counted for 16%.

2.3.5 TRUCK TRANSPORT

Distribution of LNG by truck is said to be cost efficient for regions with low demand or regions close to a liquefaction plant (MARINTEK (2005)). The LNG trucks are typically owned and operated by the gas distribution companies. Norways biggest gas distribution company, GASNOR, operate with two cost rates for truck distribution. Short round trips up to 150 km are charged with 30 NOK/km and long round trips up to 1000 km are charged with 18 NOK/km (Ameln (2014)). These transportation rates include capital cost and operational cost for the trucks. The transportation rates above assume fully utilization for each truck. The lifetime for a truck is estimated to 10-15 years (Ameln (2014)).

3 LITERATURE

This chapter presents contributions within the field of operational analysis, relevant for this problem. The problem outlined in the introduction is about locating import terminals given an uncertain future LNG demand. This problem can from an optimization point of view be considered as a facility location problem/location analysis with uncertainty. ReVelle and Eiselt (2005) states that *"The term Location Analysis refers to the modeling, formulation and solution of a class of problems that can best be described as siting facilities in some given space"*. The facility location literature can be divided into single-, two- and multi-echelon problems. The difference between these problems is depicted in Figure 3.1 to Figure 3.3, where the distribution flow is illustrated by production facilities (PF), distribution-centers (DC) and end-customers (EC). A single-echelon facility location problem has one transportation link, a two-echelon facility location problem has two transportation links and a multi-echelon facility location problem has more than two transportation links.

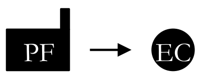

Figure 3.1 Single-echelon

Figure 3.2 Two-echelon

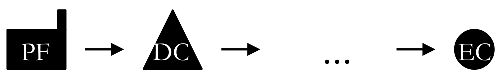

Figure 3.3 Multi-echelon

The chapter is divided into four parts, where the first part is about understanding a single-echelon facility location problem, by explaining the model in detail. The second part is about relevant deterministic location analysis literature for single-, two and multi-echelon problems. The third part is focused on how to incorporate the uncertainty into the model formulation. The last part is about relevant location analysis for single- and multi-echelon problems with uncertainty.

3.1 A FACILITY LOCATION PROBLEM

Lundgren, Rönnquist et al. (2010) formulates a single-echelon facility location problem as a problem of choosing a number of facilities m and from these, support a number of costumers n. Each facility i has a given capacity S_i and each costumer has a given demand D_j. Costs that are included in the problem are fixed capital cost F_i and a unit cost C_{ij} for transportation between facilities i and customers j. There are defined two different variables, $y_i = 1$ if facility i is constructed, and 0 otherwise. Variable x_{ij} is the number of units transported between facility i and customer j.

$$\text{Minimize} \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}x_{ij} + \sum_{i=1}^{m} F_i y_i \tag{3.1}$$

Subject to

$$\sum_{j=1}^{n} x_{ij} \leq S_i y_i \qquad i = 1,..,m \ (Supply) \tag{3.2}$$

$$\sum_{i=1}^{m} x_{ij} = D_j \qquad j = 1,..,n \ (Demand) \tag{3.3}$$

$$x_{ij} \geq 0 \qquad i = 1,..,m;\ j = 1,..,n \tag{3.4}$$

$$y_i \in \{0,1\} \qquad i = 1,..,m \tag{3.5}$$

Equation (3.1) describes the objective function of the facility problem. The objective function minimizes the total cost, where the total cost is divided into transportation cost and capital cost. Equation (3.2) is a capacity constraint that ensures that the transported quantity from a terminal does not exceed its given capacity. Equation (3.3) ensures that every costumer j receive its demand D_j. Equation (3.4) ensures non-negativity for variable x_{ij} and equation (3.5) ensures that y_i is binary.

Figure 3.4 illustrates the solution of a simple facility location problem, where the circles represents customer with a given demand D_j and the squares represents the potential facilities y_i with the given capacity S_i. It is assumed that all seven customers are demanding one unit and all potential facilities have a capacity of seven units. The solution in the figure shows that two facilities are constructed, although it is enough to construct one. This means that it is cheaper to construct two terminals and shorten the transportation distance, than construct one terminal and increase the transportation distance.

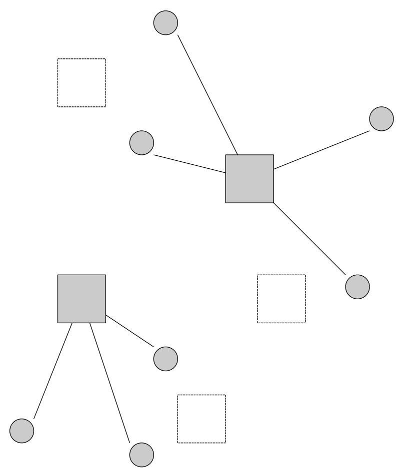

Figure 3.4 Single Echelon Facility Location Problem, example

The facility location problem above is a good example on how to formulate facility location problems, where transportation cost, capital cost, facility capacity and customer demand are considered. The number of transportation links is one major difference between this problem formulation and the problem in the thesis, where it is required transportation between liquefaction plants and import terminals and between import terminals and end customers.

3.2 LOCATION ANALYSIS UNDER CERTAINTY

3.2.1 SINGLE-ECHELON

Deterministic location analysis is, according to Owen and Daskin (1998), the most basic location analysis. With regard to deterministic, they mean problems that take constants and known quantities as input to make one single solution at one point in time. One of the first studies on location analysis was done by Weber (1909), where he tried to minimize the total distance between one single warehouse and several customers. Due to lack of computer power it took over fifty years until this field of study got attention again.

Cooper (1963) splits a general location problem into three known and three unknown values. The given values are customer location, demand and transportation cost for a given area. The values to be determined are the location, capacity and number of facilities. Cooper assumes that the facilities have no capacity limitation and that the transportation unit cost is independent of the total amount transported to each facility in his model.

Feldman, Lehrer et al. (1966) writes about non-convex warehouse location involving geographic location and size of facilities in a distribution system. The focus in his paper is the non-convexity of facilities due to economies of scale in construction and operation. The article develops a heuristic to solve this problem and make use of a concave function to represent the economies of scale. One of the important results in the paper is that the optimal size and location of the facilities are very sensitive with respect to the concave cost function.

ReVelle and Swain (1970) developed a model that designates *p* of *n* society as centers (facilities) for themselves and other communities. The objective function in the model minimizes the average distance each person has to travel to get to the *p* facility. The paper focuses mainly on the location of *p* number of facilities, but it also discusses ways to implement an indefinite number of facilities into the model. This can be done by removing the *p* facilities-restriction and add a binary variable and a cost for each facility in the objective function. Hakimi (1964) did something similar when he introduced the *P*-median problem by minimizing the total distance between costumers and their closest facility to find the optimum location of a "switching center" in a communication network and to locate the best place to build a police station in a highway system.

Current, Min et al. (1990) classifies model formulations in a location problem into different categories. Cost minimization is the first category, where the P-median problem and set-covering problem represent this category. The P-median problem formulates a way to find the location of P facilities by minimizing the demand-weighted transportation distance between the demanding nodes and the potential facility sites. This problem formulation is used to locate a variety of both public and private facilities. The set-covering problem minimizes the cost of locating facilities, given that all nodes are within an acceptable distance from minimum one facility. This problem formulation is applicable for problems such as locating fire stations or ambulances. The second category is demand-oriented formulations, where the objective is to optimize the demand served. The maximal covering problem is an example of this type of problem where the objective is maximizing the amount of demand covered. This forces the decision maker to prioritize his resources and is best suited as formulation in the public sector, where the goal often is to serve as many customers as possible. The third category is well suited for the problem in the thesis, where the objective is to maximize profit. A max profit objective function will for example consist of income from sale and costs consisting of capital and transportation costs. This model formulation has no requirement to fulfill a certain demand and serves only the customers that give positive profit.

3.2.2 TWO- AND MULTI-ECHELON

Tragantalerngsak, Holt et al. (2000) deals with the development of a branch and bound algorithm for the two-echelon, single-source, capacitated facility location problem (TSCFLP), where the objective is to serve all customers at minimum cost by locating both the potential facilities (production facilities) and potential depots (distribution centers). Each potential depot can only be served by one facility and each customer can only be served by one depot. The main focus in the paper is to develop a Lagrangian relaxation-based branch and bound algorithm to shorten the computational solution time.

Hinojosa, Puerto et al. (2000) are using Lagrangean relaxation and heuristic to solve a two-echelon multicommodity capacitated plant location problem. The models objective is to minimize the total cost for meeting all demand from every customer over a given time horizon, by locating both the potential production facilities and potential distribution centers. The computational study showed that the developed heuristics preformed well in a wide range of problems, measured by solution time and optimality gap.

Romeijn, Shu et al. (2007) developed a deterministic two-echelon problem, which considers inventory planning and supply chain network design. They treat uncertainty at the retailer by including a safety stock at both the retailer and the distribution center to achieve suitable service levels. They proposed to use column generation to deal with the exponentially large number of variables.

3.3 STOCHASTIC PROGRAMMING

Sensitivity analysis is used in deterministic linear programming models to study the robustness for a solution when data change, and can tell you how much data can change before the optimal solution change. But what if the data is uncertain and the solution is sensitive to changes? Deterministic what-if- and scenario analysis are used to handle these kinds of questions. But the solutions for these analysis are still solved with deterministic data and do not account for an uncertain future in their models. Stochastic programming on the other hand account for these issues in the modeling and is according to Midthun (2009) about decision making under uncertainty. King and Wallace (2012) states that the core in stochastic programming is about modeling what might happen and how to handle each and every situation, while deterministic models do not say anything about what to do when parameters are not as expected.

The facility location problem in Figure 3.4 illustrated a solution to a deterministic single-echelon facility location problem. In the example, we assumed a known customer demand. But what if the demand was uncertain? A deterministic way to adapt to this uncertainty is to solve for the worst-case scenario. This method will keep the costs down, but is a poor solution if a high demand scenario occurs. A stochastic way to adapt to the uncertainty is to solve the model with respect to all scenarios. The result will be a solution that has facilities well positioned against all scenarios, called a robust solution. Mulvey, Vanderbei et al. (1995) defines robust solutions as solutions for a system that is "close" to optimal for all scenarios of the input data.

One fundamental assumption in stochastic programming is that we know a probability distribution of the uncertain parameters. Midthun (2009) assumes that a joint probability distribution can be constructed as a discrete approximation, called scenario approach. According to Midthun (2009), this this approach assumes that there is a finite number of decisions that nature can make. Vanston Jr, Frisbie et al. (1977) describes a 12-step scenario generation technique to obtain scenario sets.

Higle and Wallace (2003) points out the importance of a more thoughtful approach to model development when faced with uncertainty in the demand. The model needs to capture the relationship between the point in time we make decisions and the times the demand is known. Stochastic programming with a recourse model is appropriate model formulation if one has to take a decision before the demand is known (Higle (2005)). The term "recourse" is according to Higle (2005) "*the opportunity to adapt a solution to the specific outcome observed*". A recourse decision will therefor come after new information about the uncertain parameters is known. Figure 3.5 below shows a classic sales example where stochastic programming and the recourse model are used. Stage 0 is where to decide the production quantity of a product. The first stage is where the information about the demand is revealed and one of the three demand scenarios is reality. It is important to distinguish between time periods and stages. While stages are where it is natural to commit decisions because of new information, time periods is a way to monitor the time.

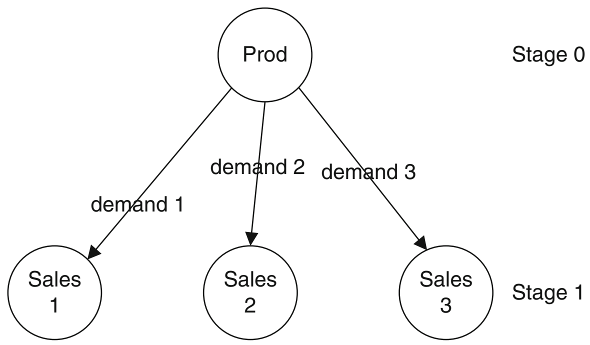

Figure 3.5 Scenario tree describing a stage structure, King and Wallace (2012)

Midthun (2009) summarizes the pros and cons for stochastic recourse models with two- and multiple stages. The pros for a two-stage structure are that you can add more details to the model and use many scenarios, due to the simplicity of the structure. The con is the rough representation of the information. The opposite is true for the multi-stage structure that represents the information in a good way but have problems with the algorithm and the solution time that grows exponentially.

Birge (1982) introduce a method to measure the value of solving stochastic programming instead of deterministic programming. The quantity is called value of the stochastic solution (VSS) and is shown in equation (3.6), where VSS is the value of the stochastic solution for a maximization problem, SP is the solution of the stochastic programming and EEV is the expected results of using expected value solutions. In a two-stage model with three different demand scenarios and equal probability, expected value solutions (EV) are the solution you get when you use the average demand to calculate a deterministic solution, and use this solution in the first stage of the stochastic model as fixed parameters and solve it for all demand scenarios in second stage. The EEV is then the average of these three solutions. The value of the stochastic solution increases with the size of the VSS. The deterministic solution is as good as the stochastic solution if the VSS reaches zero.

$$VSS = SP - EEV \tag{3.6}$$

While the VSS is a measure on how good the stochastic solution is, the value of perfect information $(EVPI)$ represent the loss of profit due to the presence of uncertainty (Birge and Louveaux (2011)). Equation (3.7) shows the $EVPI$ where WS is the wait-and-see solution where the calculation is done deterministically with perfect information. The $EVPI$ is a good measure when it is possible to reveal more accurate information, whilst the VSS is according to Birge (1982) is more pertinent for decision makers when its not possible to gather more information about the future.

$$EVPI = WS - SP \tag{3.7}$$

3.4 LOCATION ANALYSIS UNDER UNCERTAINTY

3.4.1 SINGLE-ECHELON

Snyder (2006) writes in her review that locations are generally first-stage decisions and the assigning of customers to facilities are second-stage, recourse decisions. The author points out that if both decisions happen in first stage, the model can be reduced to a deterministic model by replacing the uncertain parameter with its mean.

Louveaux (1986) study how to transform deterministic location models into two-stage stochastic models with recourse when uncertainty on parameters is introduced, including uncertainty on demand. Location and size of facilities are first-stage decisions, while the distribution of produced goods to the most profitable demand locations is the second-stage decision. He introduced a penalty variable in the demand constraints and objective function as a "slack" variable to unmet demand. It can be hard to define the penalty cost parameter since the cost of not meeting demand can be a lot more than just the loss in profit.

3.4.2 MULTI-ECHELON

Tsiakis, Shah et al. (2001) considers the design of a multiproduct and multi-echelon supply chain system, using the scenario approach to handle the uncertain demand. The warehouses and distribution centers has unknown locations, while the number of customer locations is fixed. The model is a mixed integer linear programming optimization problem, where the objective is to minimize the total cost of the network, taking both infrastructure and operating cost into account. Things to determine are the number, location and capacity of warehouses and distribution centers, the transportation links that needs to be established and the flow of materials. The authors point out that the computational complexity that arises when introducing uncertainty and time periods to the model.

Li, Armagan et al. (2011) have developed a stochastic optimization formulation that designs a multi-echelon natural gas production network that deals with product quality and uncertainty in the system. The uncertainty in the system is considered with a multi-scenario, two-stage stochastic recourse method. The first stage decision is about designing the infrastructure in the problem, while the second stage decision is about planning the operation of the system. This is a very complicated model formulation and is therefor solved with help from decomposition methods. Tomasgard, Rømo et al. (2007) are also studying the natural gas value chain, but are focusing on the uncertainty in demands and prices from a production company point of view. They use the scenario approach with a two-stage recourse formulation.

4 PROBLEM DESCRIPTION

This chapter describes the problem in the thesis, based on the information given in the background chapter. Necessary assumptions and definitions are presented and the objective in the problem is described by words at the end of the chapter. The problem description is a necessary step towards model development in the next chapter.

The problem is established from a "Gas Distribution Company" point of view, where the company is responsible for purchase, distribution and sale of natural gas. The revenue is defined as the difference between LNG purchase cost and selling price. The selling price is negotiable and can therefore vary between end-customers. The gas demand in the problem is considered uncertain and can vary between time periods and end-customers.

The problem for the "Gas Distribution Companies" is to determine whether and where to construct import terminals given an uncertain future demand, when the goal is to maximize profit. Based on the background chapter, the distribution method in this problem is restricted to LNG transportation with ships and trucks. Pipeline distribution is therefor excluded from the problem. The problem is further defined as a two-echelon distribution problem where the LNG is exclusively transported by ship from liquefaction plants to import terminals and re-distributed by truck to end-customers. Figure 4.1 below illustrates the stepwise distribution in the problem where liquefaction plants (LP) are the first step in the distribution chain that uses ship transportation (ST) to deliver LNG to the import terminals (IT). It is possible to expand the capacity of the import terminals if necessary, illustrated by terminal expansion (TE). The transportation between IT and customer terminals (CT) is done by truck transportation (TT).

Figure 4.1 shows three different kinds of plants/terminals for the problem. Liquefaction plants are assumed owned by the production companies and are therefor not considered as a cost in the problem. The number, location and production capacity for the liquefaction plants are given and the only cost associated with the plant is LNG purchase cost. Import terminals are assumed owned by the "Gas Distribution Companies". The quantity of the import terminals, their location and storage capacity are unidentified, as these are the parameters that the "Gas Distribution Company" aims to find. The storage capacity for import terminals can vary, where the cost is affected by economy of scale. Storage capacities, capital- and operational costs for the import terminals are given values. The capital cost can vary a lot with the location due to existing infrastructure and is therefore unique for all potential locations. Construction/lead-time for an import terminal is assumed to be equal for all potential locations. It should also be possible to expand the storage capacity at already constructed import terminals if the demand requires this, where the lead-time is assumed equal to the construction time for an import terminal. The customer terminals presented at the bottom of Figure 4.1 exist at every end-customer. The capital- and operational cost can therefore be included in the last transportation step.

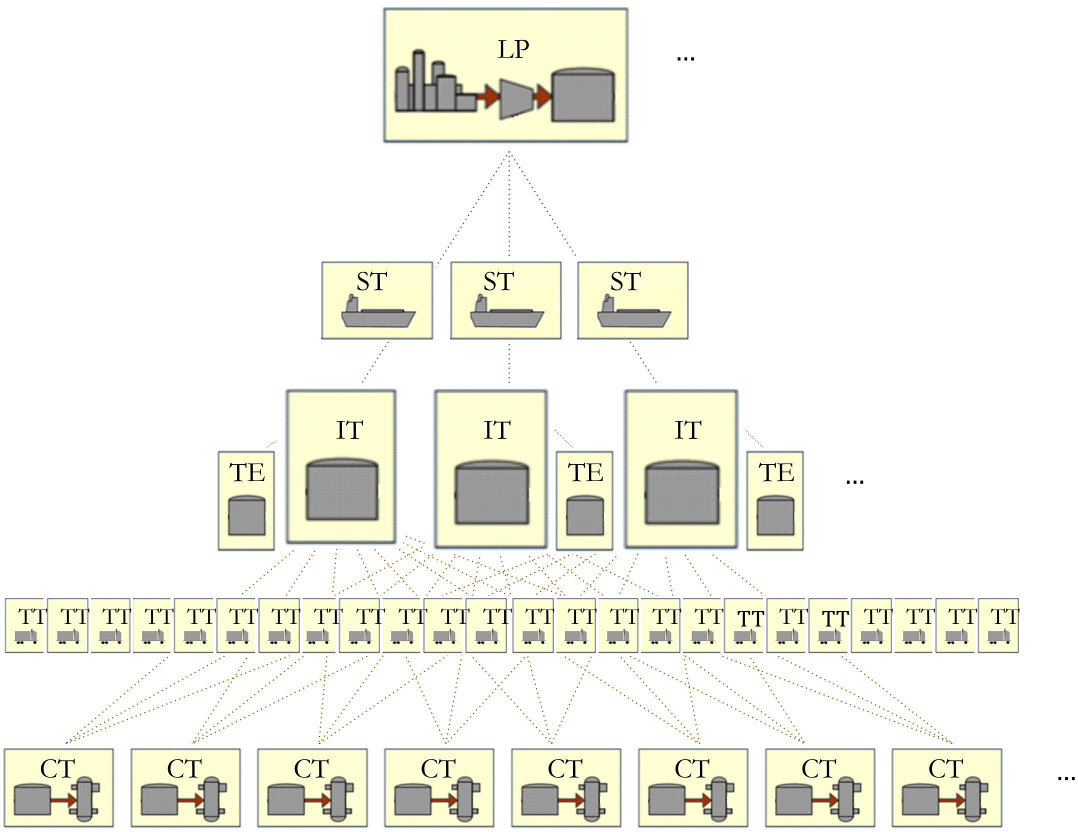

Figure 4.1 Problem description, LNG supply chain

The problem is about locating import terminals, which according to Meyr, Wagner et al. (2008) defines the problem as a long-term planning problem. By modeling this kind of planning problem, problems such as distribution planning and scheduling needs to be simplified in some aggregated level in order to make good decisions. The distribution methods in this problem is already mentioned and illustrated in Figure 4.1 where large volumes of LNG is distributed from the liquefaction terminal to import terminals and re-distributed in smaller volumes to the end-customers. Ship transport represents the first distribution step, where ships are chartered inn on long-term contracts over the whole period. Truck transport represents the second step, these vehicles are owned by the "Gas Distribution Company". It is assumed that ship transportation cost consists of time charter cost, fuel cost and port cost. Charter and fuel cost is both assumed linearly dependent on the distance, while port cost is fixed. The ship transportation cost is in addition affected by economy of scale, which corresponds to lower unit transportation cost by transporting large quantities. All costs associated with truck transportation are assumed included in the unit cost and there is no economy of scale in this transportation form, due to the small amount each truck is able to carry.

Almost every link in the supply chain is depending on a high capacity utilization to be profitable. Since the problem in this thesis is considered to be a long-term planning problem, capacity utilization is assumed optimal.

There exist several problem specific limitations and restrictions in addition to the definitions and assumption above. Firstly, in accordance to how the market works, the "Gas Distribution Company" can decide how much they are able to supply the end-customers. Secondly, it is determined that there not shall remain any LNG at the import terminal at the end of a time period, implying that the distribution flow must be equal within each time period. This is a fair assumption because each time period is sufficiently long.

The objective is to determine how many import terminals to construct, where these terminals should be located, how large they should be and when to start the construction. The construction of these terminals enables transportation of large volumes of LNG to lower the distribution cost. The revenue in the objective is, as already mentioned above, the difference between LNG purchase costs and selling price. The total cost of the system includes capital- and operational cost for import terminals, ship transportation costs, truck transportation costs and potential import terminal expansion costs.

5 MATHEMATICAL FORMULATIONS

This chapter will go trough both the deterministic and the stochastic mathematical formulation of the problem described in the previous chapter. The deterministic representation of the problem is modeled with the uncertain future demand for LNG as a given parameter. The stochastic formulation is modeled with the future demand as an uncertain parameter. The uncertainty is assumed discrete with a scenario planning approach, where each scenario represents a different demand situation. The model formulations are presented in compact form in appendix A and appendix B.

Both models are formulated as mixed integer programming models (MILP) and have a predefined discrete set of alternatives to locate the import terminals. This is a realistic assumption since there usually are a limited number of possible locations for the import terminals. Both models are also formulated with time periods. This is included in the formulation due to the nature of the problem, where assets in the distribution system have different lifetimes and the demand for LNG changes every year.

The models are based on Lundgren, Rönnquist et al. (2010) model formulation of the single-echelon facility location problem, presented in the literature chapter, with the necessary adjustments and extensions. One of the fundamental differences between our problem and Lundgren, Rönnquist et al. (2010) is the numbers of distribution steps. The problem in this thesis is a two-echelon facility location problem, with the first distribution step from liquefaction plant to the import terminal and the second distribution step from the import terminal to the end-customer.

5.1 DETERMINISTIC LOCATION MODEL

In this section, the deterministic formulation of the problem is presented. Sets, indices, decision variables and parameters are presented before the objective function and constraints are developed. Figure 5.1 illustrates an example with a given liquefaction plant capacity and given end-customer demand. Two out of three import terminals are constructed, and six out of eight end-customers are served.

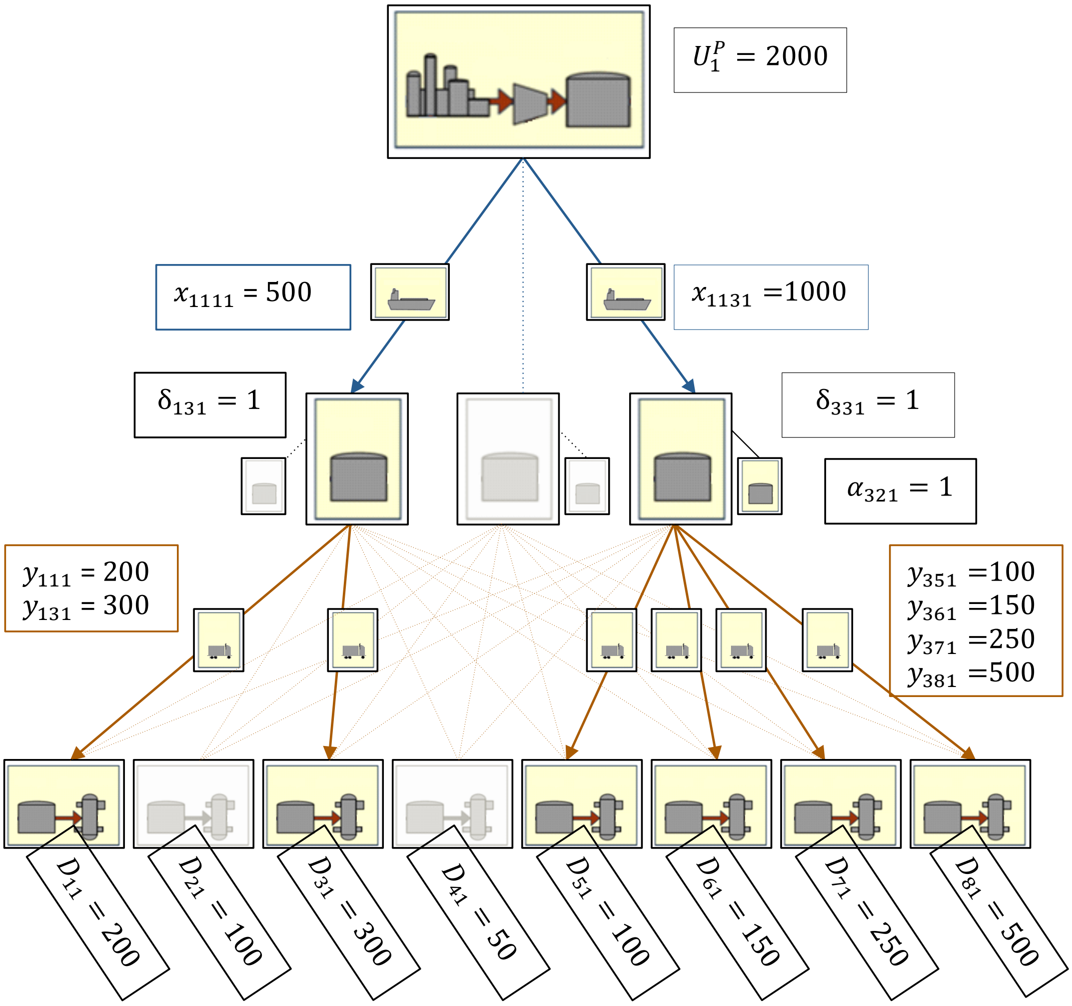

Figure 5.1 Decision variables and parameters with random values, deterministic formulation

Let $\mathcal{P}$ be the set of liquefaction plants, indexed by p, $\mathcal{I}$ be the set of import terminals, indexed by i and $\mathcal{J}$ be the set of end-customers, indexed by j. These three sets represent the three different locations where the LNG is distributed between in the system. The number of different sizes of import terminals is discretized, let $\mathcal{W}$ be the set of different import terminal storage capacities, indexed by w and $\mathcal{V}$ be the equivalent set of different storage expansion capacities, indexed by v. Let $\mathcal{F}$ be the set of transportation fares representing the economies of scale for ship transportation, indexed by f. Let $\mathcal{T}$ be the set of time periods, indexed by t. The distribution of LNG can start after $t \in \mathcal{T}: t > \bar{T}^L$, where $\bar{T}^L$ is the lead-time for new import terminals and import terminal expansions.

With respect to the variables in the problem, the model consists of two continuous decision variables, two binary decision variables and one auxiliary binary variable. Decision variables are variables that are included in the objective function, while the auxiliary variable is a model technical variable. The two continuous variables are as follows; let x_{fpit} be the amount of LNG distributed with fare f from liquefaction plant p to import terminal i for the time period t and y_{ijt} be the amount of LNG distributed from import terminal i to end-costumer j for the time period t. The two binary decision variables is defined as δ_{iwt} that get value 1 if import terminal i with storage capacity alternative w is constructed in time period t, elsewhere 0, and α_{ivt} that gets value 1 if import terminal i expands its storage capacity with alternative v in time period t, elsewhere 0. Let γ_{fpit} be the auxiliary binary variable that gets value 1 if shipping fare f is chosen from liquefaction plant p to import terminal i for the time period t.

The deterministic location model is hence:

$$\max z = \sum_{t \in \mathcal{T}} \sum_{j \in \mathcal{J}} \sum_{i \in \mathcal{I}} R_{jt} y_{ijt} \tag{5.0a}$$

$$- \sum_{t \in \mathcal{T}} \sum_{w \in \mathcal{W}} \sum_{i \in \mathcal{I}} C_{iwt} \delta_{iwt} \tag{5.0b}$$

$$- \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{I}} \sum_{p \in \mathcal{P}} \sum_{f \in \mathcal{F}} C_{fpit}^{PI} x_{fpit} \tag{5.0c}$$

$$- \sum_{t \in \mathcal{T}} \sum_{j \in \mathcal{J}} \sum_{i \in \mathcal{I}} C_{ijt}^{IJ} y_{ijt} \tag{5.0d}$$

$$- \sum_{t \in \mathcal{T}} \sum_{v \in \mathcal{V}} \sum_{i \in \mathcal{I}} C_{ivt}^{E} \alpha_{ivt} \tag{5.0e}$$

The objective function (5.0a) to (5.0e) represents a maximization of potential profit for a "Gas Distribution Company" given a deterministic demand of LNG. Expression (5.0a) represents the total revenue (sales price minus purchase price) in the distribution system where R_{jt} is the unit revenue of LNG transported to end-customer j in time period t. Expression (5.0b) represents expected cost for constructing and operating import terminals where C_{iwt} is the total capital- and operational cost for the entire evaluation period for an import terminal constructed in area i with capacity alternative w in time period t, the economy of scale for the import terminal is pre-defined in the cost parameter. Expression (5.0c) represents expected transportation cost from the liquefaction plants to the import terminals, where C^{PI}_{fpit} is the ship unit transportation cost with fare alternative f from liquefaction plant p to import terminal i in time period t. It should be noticed that the fare index f only applies for the transportation between liquefaction plant and import terminal due to economy of scale in ship transportation. The unit transportation cost is piecewise linearized in order to keep the model linear, where the different discrete fares lower the unit transportation cost per distance as the freight volume increases, illustrated in Figure 5.2. Expression (5.0d) represents expected transportation cost from the import terminals to the end-customers, where C^{IJ}_{ijt} is the truck unit transportation cost from import terminal i to end-customer j in time period t. The two different truck transportation fares, discussed in the previous chapter, is pre-defined in the cost parameter, based on the distance between import terminal and end-customer. Finally, expression (5.0e) represents expected cost for the expansion of an import terminal, where C^{E}_{ivt} is the total capital- and extra operational cost for the entire evaluation period when expanding capacity at import terminal i with storage capacity alternative v in time period t. The economy of scale for the different expansion options is pre-defined in the cost parameter.

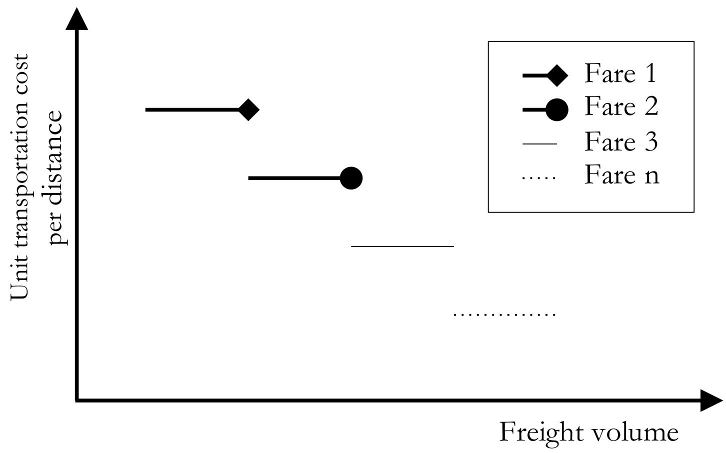

Figure 5.2 Ship transport fares

The problem is subjected to constraints (5.1) to (5.13):

$$\sum_{f\in\mathcal{F}}\sum_{i\in\mathcal{I}} x_{fpit} \leq U_{pt}^{P} \qquad p\in\mathcal{P}, t\in\mathcal{T}: t>\bar{T}^{L} \qquad (5.1)$$

$$\sum_{i\in\mathcal{I}} y_{ijt} \leq D_{jt} \qquad j\in\mathcal{J}, t\in\mathcal{T} \qquad (5.2)$$

$$\sum_{f\in\mathcal{F}}\sum_{p\in P} x_{fpit} = \sum_{j\in\mathcal{J}} y_{ijt} \qquad i\in\mathcal{I}, t\in\mathcal{T}: t>\bar{T}^{L} \qquad (5.3)$$

Constraints (5.1) to (5.3) are the transportation constraints in the problem. Constraints (5.1) ensure that it is not transported more LNG than produced from the different liquefaction plants to the different import terminals, where U_{pt}^{P} is the maximal production capacity for liquefaction plant p in time period t. Constraints (5.2) ensure that it is not transported more LNG to the end-customer than demanded, where D_{jt} is the demand for end-customer j in time period t. Both constraints make it possible to transport less than the maximum limit. Constraints (5.3) make sure that the amount of LNG transported from the liquefaction plants to the different import terminals equals the amount of LNG transported from the import terminal to the end-customers.

$$\sum_{f\in\mathcal{F}}\sum_{p\in P} x_{fpit} \leq \sum_{t'>\bar{T}^{L}}^{t}\sum_{w\in\mathcal{W}} W_w \delta_{i,w,t'-\bar{T}^{L}} + \sum_{t'>\bar{T}^{L}}^{t}\sum_{v\in\mathcal{V}} V_v\, \alpha_{i,v,t'-\bar{T}^{L}} \qquad i\in\mathcal{I}, t\in\mathcal{T} \qquad (5.4)$$

$$\sum_{t'>+\bar{T}^{L}}^{t}\sum_{w\in\mathcal{W}} W_w \delta_{i,w,t'-\bar{T}^{L}} \geq \sum_{t'>\bar{T}^{L}}^{t}\sum_{v\in\mathcal{V}} V_v\, \alpha_{i,v,t'-\bar{T}^{L}} \qquad i\in\mathcal{I}, t\in\mathcal{T} \qquad (5.5)$$

Constraints (5.4) to (5.5) are the storage capacity constraints. Constraints (5.4) enable distribution of LNG only to the areas that have constructed an import terminal. The allowable distribution amount is determined by the capacity alternative chosen and possible capacity gained by expanding the import terminal, where W_w is the storage capacity in one period for storage capacity alternative w and V_v is the extra storage capacity in each period if the import terminal is extended with storage expansion alternative v. Constraints (5.5) make it impossible to extend a terminal if it is not already constructed.

$$L_{f-1}\gamma_{fpit} \leq x_{fpit} \leq L_f\gamma_{fpit} \qquad f \in \mathcal{F}\backslash\{1\}, p \in \mathcal{P}, i \in \mathcal{I}, t \in \mathcal{T}: t > \bar{T}^L \qquad (5.6)$$

$$0 \leq x_{fpit} \leq L_f\gamma_{fpit} \qquad f = 1, p \in \mathcal{P}, i \in \mathcal{I}, t \in \mathcal{T}: t > \bar{T}^L \qquad (5.7)$$

$$\sum_{f \in \mathcal{F}} \gamma_{fpit} \leq 1 \qquad p \in \mathcal{P}, i \in \mathcal{I}, t \in \mathcal{T}: t > \bar{T}^L \qquad (5.8)$$

Constraints (5.6) to (5.8) are the restrictions that make the economy of scale for the ship transport possible. Constraints (5.6) provide the transportation between liquefaction plant and import terminal with the right fare in each time period based on the amount of transported LNG, where L_f is the threshold alternative f for different economy of scale alternatives. These constraints apply to all fare alternatives except alternative 1, where $f \in \mathcal{F}\backslash\{1\}$ ensures this. Constraints (5.7) work in the same way as the previous constraints, but only for fare alternative 1, where $f = 1$ ensures this. Constraints (5.8) ensure that one fare at the most is chosen between a given liquefaction plant and import terminal in each time period.

$$x_{fpit} \in \mathbb{Z}^+ \qquad f \in \mathcal{F}, p \in \mathcal{P}, i \in \mathcal{I}, t \in \mathcal{T}: t > \bar{T}^L \qquad (5.9)$$

$$y_{ijt} \in \mathbb{Z}^+ \qquad i \in \mathcal{I}, j \in \mathcal{J}, t \in \mathcal{T}: t > \bar{T}^L \qquad (5.10)$$

$$\gamma_{fpit} \in \{0,1\} \qquad f \in \mathcal{F}, p \in \mathcal{P}, i \in \mathcal{I}, t \in \mathcal{T}: t > \bar{T}^L \qquad (5.11)$$

$$\delta_{iwt} \in \{0,1\} \qquad i \in \mathcal{I}, w \in \mathcal{W}, t \in \mathcal{T} \qquad (5.12)$$

$$\alpha_{ivt} \in \{0,1\} \qquad i \in \mathcal{I}, v \in V, t \in \mathcal{T} \qquad (5.13)$$

Constraints (5.9) to (5.10) impose non-negativity and integrality to the respective variables, while constraints (5.11) to (5.13) impose the variables to binarity.

5.2 STOCHASTIC LOCATION MODEL

The stochastic model formulation is presented in this section, where the future LNG demands for end-costumers are considered as the only uncertain parameter. Since the lead-time for constructing new import terminals is long and new information regarding the LNG demand will be revealed by the time the constructed import terminals is ready for use, stochastic modeling with a recourse model is used. The recourse model is included to capture the revealing of new information about the demand situation after construction of import terminals is completed. The uncertain demand is dealt with by dividing the probability for different demand situations into discrete scenarios, using the scenario approach described in the literature chapter. Figure 5.3 shows an example of a situation with two different scenarios, where the size of the circles illustrates the quantity of the demand. Demand scenario 2 is consistently larger than scenario one.

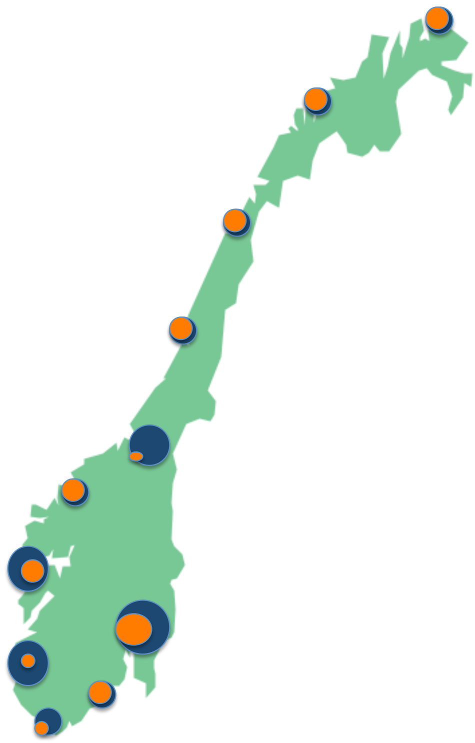

Figure 5.3 Demand scenario example

The formulation is a two-stage recourse model with time periods where both stages are about strategical decision-making. The first stage decision is whether and where to construct import terminals with given probabilities for different demand scenarios. It is assumed that new information about the demand is revealed after the lead-time for the constructed terminals in the first stage. It is now possible to make second stage decision on the basis of the new information about the demand. The second stage decisions can be to construct new import terminals, expand already existing terminals, do both or nothing. Figure 5.4 illustrates the scenario tree for the stochastic formulation with two stages, n possible scenarios and an undefined number of time periods, where the time periods between first and second stages are defined as the lead-time. Each scenario represents a demand situation for the end-customers. There is no need for nonanticipative constraints in the two-stage model because all the decision variables in first stage are without scenario index and are similar for all scenarios in second stage.

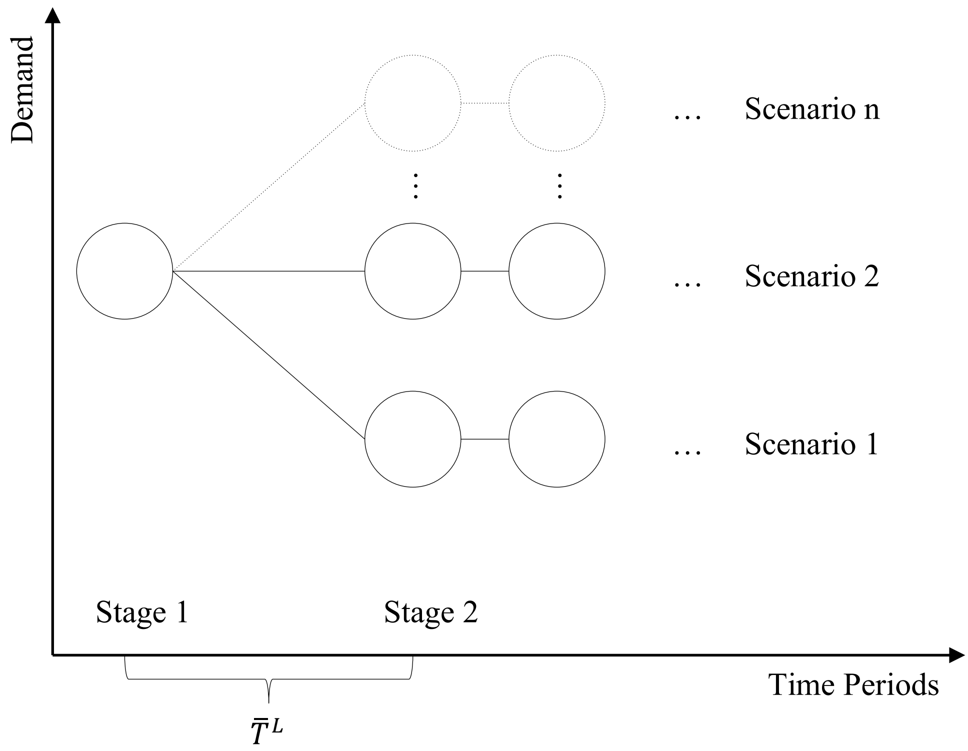

Figure 5.4 Scenario tree

The formulation of the stochastic model is based on the formulation of the deterministic model, and differs from the deterministic model by introducing scenarios and the opportunity to make recourse decisions. To make it easier to read the model, both the part of the model that has not changed and the new part of the model will be presented in the following sections.

Let $\mathcal{P}$ be the set of liquefaction plants, indexed by p, $\mathcal{I}$ be the set of import terminals, indexed by i and $\mathcal{J}$ be the set of end-customers, indexed by j. These three sets represent the three different locations where the LNG is distributed between in the system. The number of different sizes of import terminals is discretized, let $\mathcal{W}$ be the set of different import terminal sizes, indexed by w and $\mathcal{V}$ be the equivalent set of different storage expansion capacities, indexed by v. Let $\mathcal{F}$ be the set of transportation fares representing the economies of scale for ship transportation, indexed by f. Let $\mathcal{T}$ be the set of time periods, indexed by t. The distribution of LNG can start after $t \in \mathcal{T}: t > \bar{T}^L$, where $\bar{T}^L$ is the number of periods of lead-time for new import terminals and import terminal expansions. The number of import terminals is restricted to the once constructed in the first stage until $t \in \mathcal{T}: t \leq (\bar{T}^1 + \bar{T}^L)$ is broken, where $\bar{T}^1$ is the number of periods in first stage. The first time period where more import terminals can be ready to use are after $t \in \mathcal{T}^2: t > (\bar{T}^L + \bar{T}^1)$, where $\mathcal{T}^2 \subseteq \mathcal{T}$ is a subset of time periods in second stage. In addition to the already existing sets and indices, let $\mathcal{S}$ be the set of different discrete demand scenarios, indexed by s.

The number of variables has increased by one, and all variables, except one, are given the scenario index s. The problem still consist of two continuous decision variables, where x_{fpits} is the amount of LNG distributed with fare f from liquefaction plant p to import terminal i for the time period t and scenario s, and y_{ijts} is the amount of LNG distributed from import terminal i to end-customer j in time period t and scenario s. The delta-variables that represented the import terminals in the deterministic model are now divided into two delta-variables that represent import terminals in first and second stage of the model. Let δ_{iw}^{S1} be a first stage binary decision variable, that get value 1 if import terminal i with storage capacity alternative w is constructed, elsewhere 0, and δ_{iwts}^{S2} be a second stage binary decision variable that get value 1 if import terminal i with storage capacity alternative w is constructed in time period t and scenario s, elsewhere 0. The binary decision variable α_{ivts} gets value 1 if import terminal i expands its storage capacity with alternative v in time period t and scenario s, elsewhere 0. The last binary variable is the auxiliary binary variable γ_{fpits} that gets value 1 if shipping fare f is chosen from liquefaction plant p to import terminal i for the time period t and scenario s.

The stochastic location model is hence:

$$\max\ z\ = -\sum_{w\in\mathcal{W}}\sum_{i\in\mathcal{I}} C_{iw}^{S1}\delta_{iw}^{S1} \tag{5.14a}$$

$$+\sum_{s\in\mathcal{S}}\pi_s\left\{\sum_{t\in\mathcal{T}}\sum_{j\in\mathcal{J}}\sum_{i\in\mathcal{I}} R_{jt}y_{ijts}\right. \tag{5.14b}$$

$$-\sum_{f\in\mathcal{F}}\sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{I}}\sum_{p\in\mathcal{P}} C_{fpit}^{PI}x_{fpits} \tag{5.14c}$$

$$-\sum_{t\in\mathcal{T}}\sum_{j\in\mathcal{J}}\sum_{i\in\mathcal{I}} C_{ijt}^{IJ}y_{ijts} \tag{5.14d}$$

$$-\sum_{t\in\mathcal{T}}\sum_{w\in\mathcal{W}}\sum_{i\in\mathcal{I}} C_{iwt}^{S2}\delta_{iwts}^{S2} \tag{5.14e}$$

$$\left.-\sum_{t\in\mathcal{T}}\sum_{v\in\mathcal{V}}\sum_{i\in\mathcal{I}} C_{ivt}^{E}\alpha_{ivts}\right\} \tag{5.14f}$$

The objective function (5.14a) to (5.14f) represents a maximization of potential profit for a "Gas Distribution Company" given an uncertain future demand of LNG, where π_s is the probability that scenario s occurs. Expression (5.14a) represents expected cost for constructing and operating import terminals in first stage, where C_{iw}^{S1} is the total capital- and operational cost for the entire evaluation period for an import terminal constructed in area i with capacity alternative w in first stage, the economy of scale for the import terminal is pre-defined in the cost parameter. Expression (5.14b) represents the total revenue (sales price minus purchase price) in the distribution system where R_{jt} is the unit revenue of LNG transported to end-customer j in time period t. Expression (5.14c) represents expected transportation cost from the liquefaction plants to the import terminals, where C_{fpit}^{PI} is the ship unit transportation cost with fare alternative f from liquefaction plant p to import terminal i in time period t. It should be noticed that the fare index f only applies for the transportation between liquefaction plant and import terminal due to economy of scale in ship transportation. The unit transportation cost is piecewise linearized in order to keep the model linear, where the different discrete fares lower the unit transportation cost per distance as the freight volume increases, illustrated in Figure 5.2. Expression (5.14d) represents expected transportation cost from the import terminals to the end-customers, where C_{ijt}^{IJ} is the truck unit transportation cost from import terminal i to end-customer j in time period t. The two different truck transportation fares, discussed in the previous chapter, is pre-defined in the cost parameter, based on the distance between import terminal and end-customer. Expression (5.14e) represents expected cost for constructing and operating import terminals in second stage, where C_{iwt}^{S2} is the total capital- and operational cost for the entire evaluation period for an import terminal constructed in area i with capacity alternative w in time period t and second stage, where the economy of scale for the import terminal is pre-defined in the cost parameter. Finally, expression (5.14f) represents expected cost for the expansion of an import terminal, where C_{ivt}^{E} is the total capital- and extra operational cost for the entire evaluation period when expanding capacity at import terminal i for storage capacity alternative v in time period t. The economy of scale for the different expansion options is pre-defined in the cost parameter.

The problem is subjected to constraints (5.15) to (5.30):

$$\sum_{f\in\mathcal{F}}\sum_{i\in\mathcal{I}} x_{fpits} \leq U_{pt}^{P} \qquad p\in\mathcal{P}, t\in\mathcal{T}: t>\bar{T}^{L}, s\in\mathcal{S} \tag{5.15}$$

$$\sum_{i\in\mathcal{I}} y_{ijts} \leq D_{jts} \qquad j\in\mathcal{J}, t\in\mathcal{T}: t>\bar{T}^{L}, s\in\mathcal{S} \tag{5.16}$$

$$\sum_{f\in\mathcal{F}}\sum_{p\in P} x_{fpits} = \sum_{j\in\mathcal{J}} y_{ijts} \qquad i\in\mathcal{I}, t\in\mathcal{T}: t>\bar{T}^{L}, s\in\mathcal{S} \tag{5.17}$$

Constraints (5.15) to (5.17) are the key transportation constraints in the problem. Constraints (5.15) ensure that it is not transported more LNG from the different liquefaction plants to the different import terminals than produced, where U_{pt}^{P} is the maximal production capacity for liquefaction plant p in time period t. Constraints (5.16) ensure that it is not transported more LNG to the end-customer than demanded, where D_{jts} is the demand at end-customer j in time period t and scenario s. Both constraints make it possible to transport less than the maximum limit. Constraints (5.17) make sure that the amount of LNG transported from the liquefaction plants to the different import terminals equals the amount of LNG transported from the import terminal to the end-customers.

$$\sum_{f \in \mathcal{F}} \sum_{p \in P} x_{fpits} \leq \sum_{w \in \mathcal{W}} W_w \delta_{iw}^{S1} \qquad i \in \mathcal{I}, \; t \in \mathcal{T}: t \leq (\bar{T}^1 + \bar{T}^L), \; s \in \mathcal{S} \tag{5.18}$$

$$\sum_{f \in \mathcal{F}} \sum_{p \in P} x_{fpits} \leq \sum_{w \in \mathcal{W}} W_w \left(\delta_{iw}^{S1} + \sum_{t' > (\bar{T}^1 + \bar{T}^L)}^{t} \delta_{i,w,t'-\bar{T}^L,s}^{S2} \right) + \sum_{t' > (\bar{T}^1 + \bar{T}^L)}^{t} \sum_{v \in \mathcal{V}} V_v \, \alpha_{i,v,t'-\bar{T}^L,s} \qquad i \in \mathcal{I}, \; t \in \mathcal{T}^2: t > (\bar{T}^L + \bar{T}^1), \; s \in \mathcal{S} \tag{5.19}$$

$$\sum_{w \in \mathcal{W}} \left(\delta_{iw}^{S1} + \sum_{t \in \mathcal{T}} \delta_{iwts}^{S2} \right) \leq 1 \qquad i \in \mathcal{I}, s \in \mathcal{S} \tag{5.20}$$

$$\sum_{w \in \mathcal{W}} \left(\delta_{iw}^{S1} + \sum_{t' > (\bar{T}^1 + \bar{T}^L)}^{t} \delta_{i,w,t'-\bar{T}^L,s}^{S2} \right) \geq \sum_{t' > (\bar{T}^1 + \bar{T}^L)}^{t} \sum_{v \in \mathcal{V}} \alpha_{i,v,t'-\bar{T}^L,s} \qquad i \in \mathcal{I}, \; t \in \mathcal{T}^2: t > (\bar{T}^L + \bar{T}^1), \; s \in \mathcal{S} \tag{5.21}$$

Constraints (5.18) to (5.21) are considered as the storage capacity constraints. Constraints (5.18) ensure distribution of LNG only to the areas that have constructed an import terminal in first stage. The allowable distribution amount is determined by the storage capacity chosen, where W_w is the storage capacity in each period for storage alternative w. Constraints (5.19) acquire the function of the latter constraint, for second stage, where it is possible to construct new import terminals and expand already existing terminals. The constraints ensure that the lead-time is included in the calculations. V_v represents the extra storage capacity in each time period if the import terminal is extended with storage expansion alternative v. Constraints (5.20) make it impossible to construct more than one import terminal in an area. Constraints (5.21) make it impossible to extend a terminal if it is not already constructed.

$$L_{f-1} \gamma_{fpits} \leq x_{fpits} \leq L_f \gamma_{fpits} \qquad f \in \mathcal{F}/\{1\}, p \in \mathcal{P}, i \in \mathcal{I}, t \in \mathcal{T}: t > \bar{T}^L, s \in \mathcal{S} \tag{5.22}$$

$$0 \leq x_{fpits} \leq L_f \gamma_{fpits} \qquad f = 1, p \in \mathcal{P}, i \in \mathcal{I}, \; t \in \mathcal{T}: t > \bar{T}^L, s \in \mathcal{S} \tag{5.23}$$

$$\sum_{f \in \mathcal{F}} \gamma_{fpits} \leq 1 \qquad p \in \mathcal{P}, i \in \mathcal{I}, t \in \mathcal{T}: t > \bar{T}^L, s \in \mathcal{S} \tag{5.24}$$

Constraints (5.22) to (5.24) are the restrictions that make the economy of scale for the ship transport possible. Constraints (5.22) provide the transportation between liquefaction plant and import terminal with the right fare in each time period based on the amount of transported LNG, where L_f is the threshold alternative f for different economy of scale alternatives. This constraint applies to all fare alternatives except alternative 1, where $f \in \mathcal{F}\backslash\{1\}$ ensures this. Constraints (5.23) work in the same way as the latter constraints, but only for fare alternative 1, where $f \in \mathcal{F}: f = 1$ ensures this. Constraints (5.24) ensure that one fare at the most is chosen between a given liquefaction plant and import terminal in each time period.

$$x_{fpits} \in \mathbb{Z}^+ \qquad f \in \mathcal{F}, p \in \mathcal{P}, i \in \mathcal{I}, t \in \mathcal{T}: t > \bar{T}^L, s \in \mathcal{S} \tag{5.25}$$

$$y_{ijts} \in \mathbb{Z}^+ \qquad i \in \mathcal{I}, j \in \mathcal{J}, t \in \mathcal{T}: t > \bar{T}^L, s \in \mathcal{S} \tag{5.26}$$

$$\delta_{iw}^{S1} \in \{0,1\} \qquad i \in \mathcal{I}, w \in \mathcal{W} \tag{5.27}$$

$$\delta_{iwts}^{S2} \in \{0,1\} \qquad i \in \mathcal{I}, w \in \mathcal{W}, t \in \mathcal{T}^2, s \in \mathcal{S} \tag{5.28}$$

$$\alpha_{ivts} \in \{0,1\} \qquad i \in \mathcal{I}, v \in V, t \in \mathcal{T}^2, s \in \mathcal{S} \tag{5.29}$$

$$\gamma_{fpits} \in \{0,1\} \qquad f \in \mathcal{F}, p \in \mathcal{P}, i \in \mathcal{I}, t \in \mathcal{T}: t > \bar{T}^L, s \in \mathcal{S} \tag{5.30}$$

Constraints (5.25) to (5.26) impose non-negativity and integrality to the respective variables, while constraints (5.27) to (5.30) impose the variables to binarity.

6 COMPUTATIONAL STUDY

Both the mathematical models derived in chapter 5 are implemented in commercial software for operation analysis, presented in appendix D and appendix E. Xpress-IVE version 1.22.04 is used, where Xpress-Mosel was used as modeling language. This language makes it possible to formulate the software model close to the original model formulation, with only few changes. The solution method in Xpress-IVE is based on calculation techniques such as Simplex, "Branch and Bound" and valid inequalities. All optimization is solved on a HP computer with Intel(R) Xeon(R) 3.33GHz processor and 32 GB memory.

Figure 6.1 illustrates the different steps in the workflow where the data is calculated in excel before it is copied into a text-file that serves as input file to Xpress-IVE. The results are written to an output text-file after the problem is solved in Xpress-IVE.

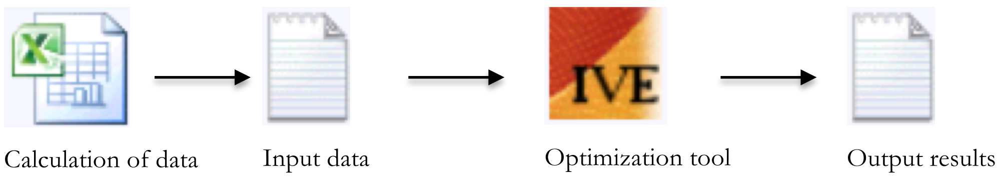

Figure 6.1 Workflow

The main scope in this chapter is to test the two models and examine the value of the stochastic solution. Assumptions and data gathering are described in section 6.1. The deterministic solution is presented in section 6.2 and the stochastic solution is presented in section 6.3. The value of stochastic solution is calculated in section 6.4 and the expected value of perfect information is calculated in section 6.5. Discussion of the results and the model are presented in section 6.6.

6.1 ASSUMPTIONS AND DATA GATHERING

Assumptions and data gathering for both the deterministic and stochastic model is presented in this section, where the differences between the models are clearly noted. Data presented in this section is provided by various sources and is gathered to validate the models in the best way possible. Where possible, assumptions have been made in an effort to minimize the computational time. All costs and revenues are calculated with a discount rate of 10% per year, which includes inflation and interest rate.

6.1.1 TIME PERIODS

The time periods in this problem is divided into years, due to the long lifetime of the problem. This means that all model parameters are given in annual sizes, such as demand, storage capacity and lead-time, where the currency is NOK and SI-units are used. The planning period is set to 13 periods, corresponding to 13 years. The number of planning periods is based on the desire to shorten the model running time in Xpress-IVE, the fact that value of money in the future is lower and the demand uncertainty one will meet in the future.

Alvarez, Tsilingiris et al. (2011) are considering how to include the residual value of ships that live beyond the finite planning horizon. One way to do this is to assign the residual value, called sunset-value to each ship. This value will, according to the authors, correspond to the estimated revenues that can be derived from the vessel throughout its remaining lifetime. The sunset-value in this problem is included using the method described above.

6.1.2 GEOGRAPHICAL DATA

The geographical area in the computational study is limited to Norway, illustrated in Figure 6.2. The blue squares in the figure represent the discrete locations of three liquefaction plants, where the locations is similar to three of the liquefaction plants presented in Table 2.3. The 16 yellow triangles represent potential discrete locations for import terminals. It is assumed that all potential import terminal locations are located in an area where there is an end-customer. The 24 end-customers are located at the red dots, representing an aggregated demand for each region. The merging of end-customers is done both to simplify the gathering of data and shorten runtime in Xpress-IVE.

Distance matrixes between liquefaction plants, import terminals and end-customers, attached in appendix C, are later used in the variable reduction and to calculate the unit transportation cost. It is worth noting that the topography varies and that the path is not necessarily a straight line between two points. The distance between almost every liquefaction plant and import terminal is provided by Voyage-calculator (2013). The ports that did not exist in database were measured using map. The distance between import terminals and end-customers are measured using Maps (2013). The fact that it can be shorter to travel across the country rather than along the coast is taken into account in the calculations.

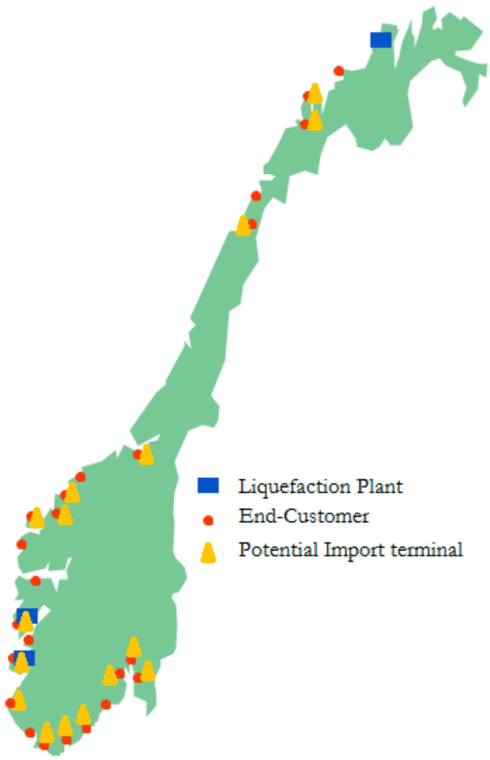

Figure 6.2 Geographical area

6.1.3 REVENUE

The revenue is defined as the difference between selling price and purchase price and is assumed to be equal for all end-customers. The selling price is obtained from SSB (2013), set to 2326 NOK/m^3, adjusted for the consumer price index. The purchase price is assumed to be 775 NOK/m^3, one third of the selling price, which corresponds to a revenue of 1551 NOK/m^3.

6.1.4 PLANTS AND TERMINALS

The end-customer terminals are neglected in the computational study, due to its low cost compared to all other elements in the calculation.

The production capacity for the liquefaction plants in Norway, presented in table 2.3, is insufficient to cover the future demand estimated in this thesis. It is hard to predict where the capacity will expand when the demand escalates, so all liquefaction plants are assumed to have a LNG production of 100 million m^3/year, which in practical terms means that there are no capacity restrictions for the liquefaction plants.

The import terminal cost consists of capital- and operational costs, where both costs are based on estimates from Lindfeldt (2011). The number of different import terminal sizes has been set to three. The size, capital cost, lifetime and lead-time are presented in Table 6.1, where one can see a significant economy of scale when the size of the terminal increases. The operational cost is set to 5% of the capital cost based on assumptions from MARINTEK (2005) .The cost distribution, lifetime and lead-time are assumed equal for all potential import terminals. It is estimated that the terminals are filled with LNG every second week, representing an annual capacity of 260,000 m^3/year, 520,000 m^3/year and 1,300,000 m^3/year, respectively.

Table 6.1 Import terminal data

Size (m^3)	Capital cost (MNOK)	Lifetime (Year)	Lead-time (Year)
10,000	375	40	3
20,000	440	40	3
50,000	640	40	3

The two different expansion opportunities for import terminals are presented in Table 6.2, where the extra operational cost per year is set to 5% of the capital cost. It is emphasized that expanding an import terminal at a later stage is approximately 30% more expensive than to make a larger terminal from construction start. The capital cost is still larger in Table 6.1, due to extra capital cost in infrastructure such as jetty construction. The size of the import terminal expansion represents an annual capacity of 130,000 m^3/year and 260,000 m^3/year, respectively.

Table 6.2 Import terminal expansion data

Size (m^3)	Capital cost (MNOK)	Lifetime (Year)	Lead-time (Year)
5,000	220	40	3
10,000	255	40	3

Both the import terminal cost and the import terminal expansion cost is presented in appendix C. Each cost element in the matrix represents the total capital- and operational cost for the entire evaluation period. One can notice from the matrix that the cost decreases with time, due to the discount rate and the decreasing number of years in operation. The sunset-value is included in the way described in section 6.1.1, where the residual value is included. An import terminal constructed in time period 6 has for instance a greater residual value than an import terminal constructed in time period 1.

6.1.5 TRANSPORTATION

Sea transportation unit cost is based on calculations done by SINTEF, MARINTEK et al. (2002), where the unit transportation cost from Hammerfest to Stavanger was calculated. All other sea transportation unit costs are calculated assuming a linear relationship between price and distance, with a minimum unit transportation cost of 10% of the cost from Hammerfest to Stavanger. A cost matrix with calculation assumptions is presented in appendix C, where the transportation unit cost from a liquefaction plant to a potential import terminal in the same region is assumed to be zero. Economy of scale is taken into account for ship transport by multiplying the unit transportation cost with a factor of 0.9 when transporting over 200,000 m^3/year and a factor of 0.8 when transporting over 400,000 m^3/year, to one single import terminal.

The distance between import terminals and end-customers determines truck transportation unit cost, based on Ameln (2014). It has been assumed that a truck with a capacity of 50 m^3 has a cost of 30 NOK per km travelled if the distance is lower than 75 km and 18 NOK per km if the distance is higher than 500 km. A linear price reduction is set up between these two distances, presented in Figure 6.3. The unit cost matrix is presented in appendix C. The sunset-value for truck transport investment is neglected because of its low present value after the planning horizon.

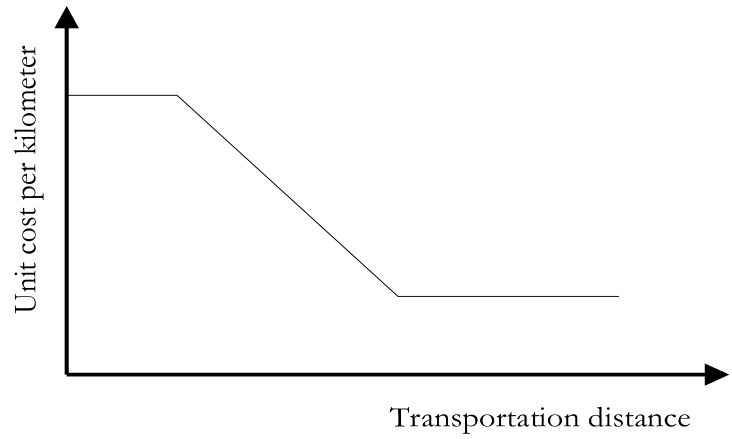

Figure 6.3 Unit cost distribution, truck transport

6.1.6 DEMAND

The demand in the deterministic part of the computational study is set to the weighted average of the different demand scenarios in the stochastic part of the study, where the demand is divided into three scenarios; low, normal and high. The high scenario is based on estimates done by MARINTEK (2005), where the demand is identified in different regions along the Norwegian cost in 2025. This scenario is highly uncertain and is a positive prediction of the future. This is the reason why the other two scenarios are lower. The lead-time for the import terminals ensures that no end-customers are served before time period 4. Due to this, the differences between each scenario are introduced at this time. At this initial time, scenario low is set to 40% of the high scenario and scenario normal is set to 70% of the high scenario. In later time periods, the low demand scenario becomes lower, while the demand grows slowly in the normal and high scenarios. The three scenarios and the weighted average are illustrated in Figure 6.4, where each time period represents the total demand for the different scenarios. The trend in demand is inspired by the demand predictions from Figure 1.1 in the introduction. Appendix C shows the demand for each end-customer for the different scenarios. The probabilities for the different scenarios are all set to one third.

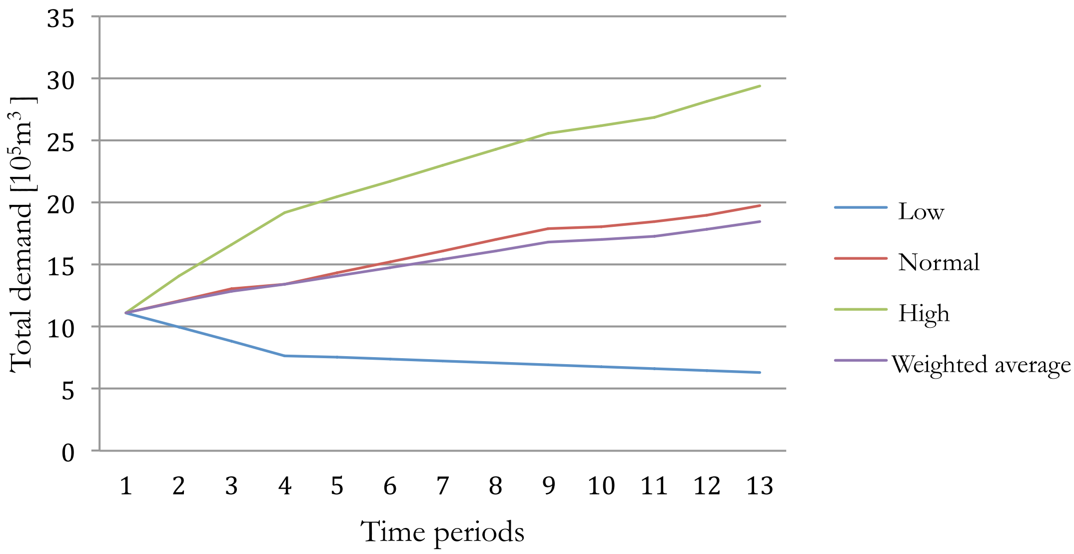

Figure 6.4 Demand scenarios

Figure 6.5 illustrates the different stages, scenarios and time periods in this implementation of the stochastic model, where information about the demand is revealed after three time periods, equivalent to $\bar{T}^L$ (lead-time). Step 1 is about locating import terminals under the uncertainty of three different scenarios. The demand is revealed in step 2, so this step is about designing the most optimal supply chain for each scenario with step 1 decisions as basis. The solution for the various scenarios is presented in the next subsections. It is important to emphasize that it is the location of import terminals in first stage that is the most important decisions in the stochastic solution, because this is the decision that needs to be taken first.

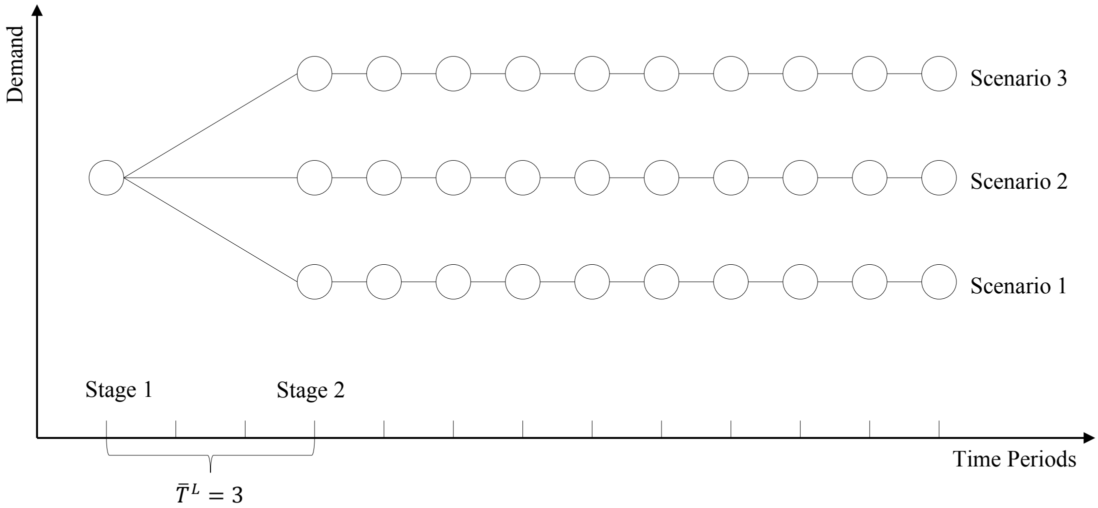

Figure 6.5 Stochastic model illustration for the computational study

6.1.7 VARIABLE REDUCTION

The variable reduction is implemented in the model to exclude unnecessary variables in the solution space, which may lower the solution time. Firstly, a large number of ship transportation variables x_{fpit} can be eliminated on the basis of the distance from liquefaction plant. The large capacity on the liquefaction plants makes it possible to eliminate all sea transportation variables (x_{fpit}) with a distance longer than 800 nautical miles (nmi). Secondly is it not necessary to construct import terminals that are not completed before the planning period is over, which means that all delta-variables with time index after time period 10 can be eliminated. Table 6.3 presents the difference between the numbers of variables before and after implementing the variable reduction. With this reduction, the numbers of constraints are reduced with approximately 30% for both the deterministic model and the stochastic model, which most likely will reduce the solution time.

Table 6.3 Variable reduction in the deterministic/stochastic model

Variable	Without variable reduction	With variable reduction
x_{fpit}/x_{fpits}	1440/4320	960/2880
$\delta_{iwt}/\delta_{iwts}^{S2}$	624/1440	480/1008
$\alpha_{ivt}/\alpha_{ivts}$	416/960	320/672
Total	2480/6720	1760/4560

6.2 DETERMINISTIC SOLUTION

The deterministic data gathered and justified in the previous section is run as input data in this section. The optimal solution was obtained after one minute, with a total profit of 8.94 billion NOK. The establishment of import terminals is presented in Figure 6.6 and Table 6.4, where the yellow triangles represent import terminals constructed in time period 1. The deterministic solution is to construct two import terminals in time period 1. The two import terminals are fully utilized in time period 13, something that makes the import terminals not able to serve all end-customers in time period 13.

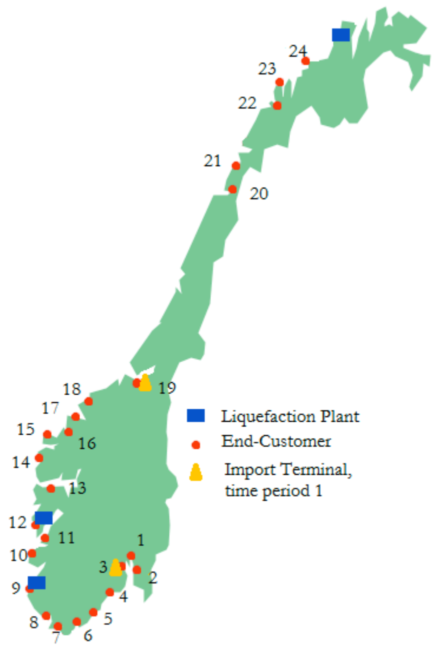

Figure 6.6 Deterministic solution

Table 6.4 Deterministic solution

Constructed import terminals (region)	Capacity (10^3m^3/year)	Construction start (time)	End-customers served (time periods)
3	1,300	1	1-14(4-13), 15(10-13), 16(8-13), 17(9-13), 18(12,13)
19	520	1	15(4-11), 16(4-8), 17(4-9,11), 18-24(4-13)

6.3 STOCHASTIC SOLUTION

This section presents the solutions obtained from the stochastic model. The input data is based on the same assumptions as the deterministic input, but differ due to the introduction of recourse opportunity and demand scenarios. The optimal solution was obtained after 22 minutes, with a total profit of 8.83 billion NOK.

6.3.1 SCENARIO 1

The establishment of import terminals is presented in Figure 6.7 and Table 6.5, where the yellow triangles represent import terminals constructed in first stage. The solution in scenario 1 is to not construct any new import terminals in addition to the three terminals constructed in first stage, due to the low demand in scenario 1. All end-customers from 1-24 are served 100% of their demand and no import terminals are fully utilized.

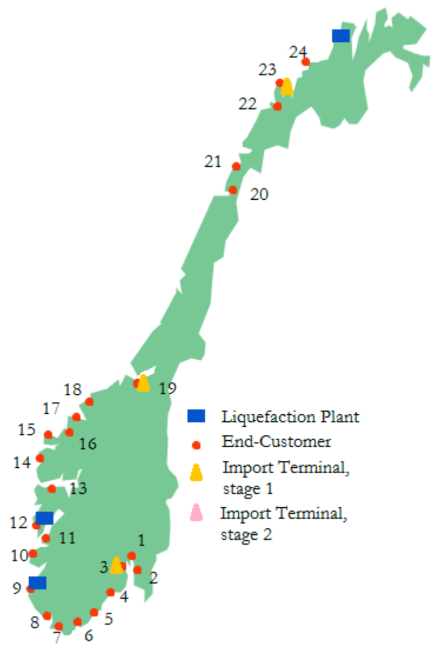

Figure 6.7 Stochastic solution, scenario 1

Table 6.5 Stochastic solution, scenario 1

Constructed import terminals (region)	Capacity (10^3m^3/year)	Construction start (time)	End-customers served (time periods)
3 (Stage 1)	1,300	1	1-13(4-13), 14(6-13)
19 (Stage 1)	520	1	13(4-5), 14(4-5), 15-20(4-13)
23 (Stage 1)	260	1	21-24(4-13)

6.3.2 SCENARIO 2

The establishment of import terminals is presented in Figure 6.8 and Table 6.6, where the yellow triangles represent import terminals constructed in first stage. The solution in scenario 2 is the same as the solution in scenario 1, where no import terminals are constructed in second stage. All end-customers from 1-24 are served 100% of their demand and import terminal 1 is fully utilized in time period 13.

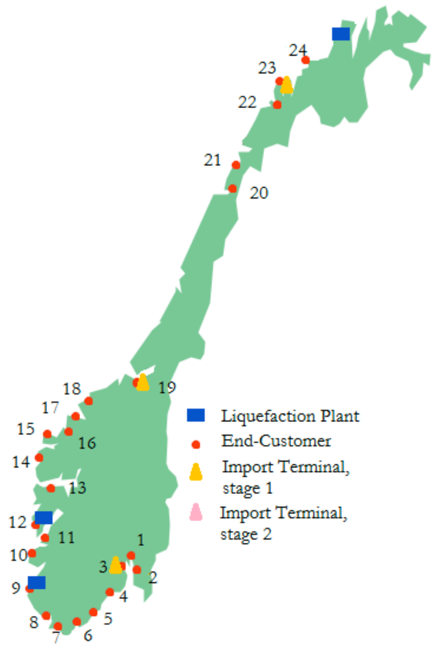

Figure 6.8 Stochastic solution, scenario 2

Table 6.6 Stochastic solution, scenario 2

Constructed import terminals (region)	Capacity (10^3m^3/year)	Construction start (time)	End-customers served (time periods)
3 (Stage 1)	1,300	1	1-13(4-13), 14(4,5,8-13)
19 (Stage 1)	520	1	13(6,7), 14(6-8,13), 15-20(4-13)
23 (Stage 1)	260	1	20(11-13), 21-24(4-13)

6.3.3 SCENARIO 3

The establishment of import terminals is presented in Figure 6.9 and Table 6.7, where the yellow triangles represent import terminals constructed in first stage, the pink triangle represent the import terminal constructed in second stage, time period 4 and the outline pink triangle represents import terminal expansion. The solution in scenario 3 is to construct one new import terminal in second stage, time period 4 and expand it further in time period 8. All import terminals are fully utilized in time period 13, something that makes the import terminals not able to serve all end-customers in time period 13.

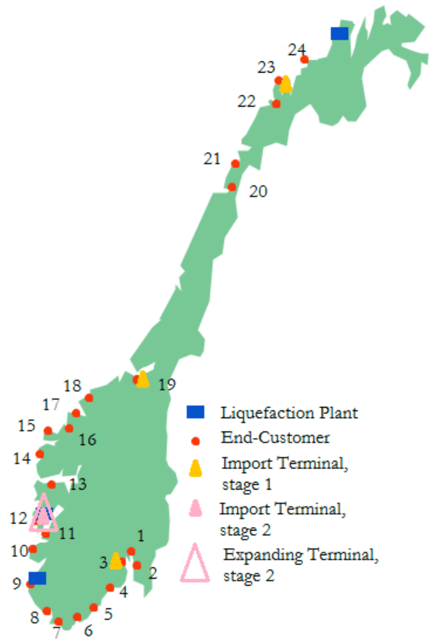

Figure 6.9 Stochastic solution, scenario 3

Table 6.7 Stochastic solution, scenario 3

Constructed import terminals (region)	Capacity (10^3m^3/year)	Construction start (time)	End-customers served (time periods)
3 (Stage 1)	1,300	1	1-6(4-13), 7(4-6,9,10), 8(4-6,10), 9(4-6), 10(4,5), 11(4,5), 12(4-6), 13(4,5), 14(4)
19 (Stage 1)	520	1	13(5,6), 14(5,6), 15(4-6), 16(4-6,9,10), 17(4-6), 18-20(4-13), 21(12)
23 (Stage 1)	260	1	20(4-6,10), 21-24(4-13)
12 (Stage 2)	520+260	4+8	6(7,8,11-13), 7(7-9,11-13), 8-18(7-13)

6.4 VALUE OF STOCHASTIC SOLUTION

You need both the value of the stochastic solution and the expected value for the three scenarios to calculate the value of stochastic solution (VSS), described in the literature chapter. The stochastic solution is summarized in Table 6.8 and the expected values are presented in Table 6.9. In addition to the overall profit, the number of constructed import terminals and expanded import terminals are displayed in the tables.

Table 6.8 Stochastic (SP) solution, overall profit: 8.83 billion NOK

		Constructed import terminals	Expanded import terminals
Stage 1		3	-
Stage 2	$s = 1$ Low	0	0
	$s = 2$ Normal	0	0
	$s = 3$ High	1	1

The overall profit for the different demands in Table 6.9 are calculated using the method described in the literature chapter, where the decisions in the deterministic solution for the first time period is used as first stage decisions in the stochastic model. The three different solutions below represent how good the deterministic solution (choice of import terminals) is for the various scenarios.

Table 6.9 Expected value (EV) solution

		Constructed import terminals	Expanded import terminals	Overall profit [billion NOK]
Stage 1		2	-	-
Stage 2	$s = 1$ Low	0	0	3.53
	$s = 2$ Normal	1	0	9.35
	$s = 3$ High	2	0	13.43

Using the overall profit listed in Table 6.9 the EEV is calculated to be 8.77 billion NOK. In comparison, the SP overall profit from Table 6.8 is 8.83 billion NOK. This gives a VVS of 0.06 billion NOK, which corresponds to a 0.68% increase.

6.5 EXPECTED VALUE OF PERFECT INFORMATION

The expected value of perfect information (EVPI) is the amount of money one would be willing to pay for absolutely correct information about the uncertain parameters. This information is of little value when it is not possible to obtain 100% correct information. There is nobody that can predict the future LNG demand with certainty, but the information can nevertheless tell something about what one would be willing to pay for better predictions.

The overall profit for the different demands in Table 6.10 are calculated using the method described in the literature chapter, where the wait-and-see solutions is calculated by using the three different scenarios as input in the deterministic model.

Table 6.10 Wait-and-see (WS) solution

	Constructed import terminals	Expanded import terminals	Overall profit [billion NOK]
$s = 1$ Low	2	0	3.76
$s = 2$ Normal	3	0	9.37
$s = 3$ High	4	1	13.89

Using the overall profit listed in Table 6.10 the WS is calculated to be 9.00 billion NOK. In comparison, the SP overall profit from Table 6.8 is 8.83 billion NOK. This gives a EVPI of 0.16 billion NOK, which corresponds to a 1.88 % increase.

6.6 DISCUSSION

The discussion is divided into two parts, where the solution from the computational study, VSS and EVPI are discussed in the first part. The second part concerns the model formulations.

6.6.1 RESULTS

From the comparison of the deterministic and stochastic solution one can observe that the deterministic solution gives higher profit than the stochastic solution. This makes sense considering that the stochastic solution is the optimal solution for three different scenarios, while the deterministic solution is the optimal solution for the average of these scenarios. The most important difference between the deterministic and the stochastic solution is the number of import terminals commenced in time period 1, because this is the decision you have to take in the first time period. The deterministic solution suggests starting construction of two import terminals while according to the stochastic solution, it is recommended to start construction of three import terminals in time period 1. The stochastic model suggests to construct three import terminals because the extra profit you achieve if the normal or high scenario occurs is greater than the loss if the low scenario occurs. The total capacity of the three import terminals constructed in time period 1 in the stochastic solution is 12,5% larger than the two terminals constructed in the deterministic solution.

It is worth mentioning that it seems like the solutions attempt to achieve economy of scale by transporting more than 200,000 m^3 or 400,000 m^3 per time period, which are 10 and 20 present cheaper per unit transported respectively. This is often the reason why the end-customers change the selection of import terminal throughout the time periods.
It is difficult to determine whether this affects the location of the import terminals or not, but the phenomena where goods are being transported from one location to another in large scale and almost back again as smaller deliveries is common in the supply chain industry. The post analysis tests the impact of the economy of scale on first stage decisions by removing the fare-alternatives.

A closer examination of scenario 3 in the stochastic solution shows that the two import terminals serve the same end-customers at the same time. The reason is simple: import terminal 3 is the preferred import terminal for end-customer 13, by means of unit truck transportation cost. The problem is that the utilization of import terminal 3 is 100% in time period 5. With an increasing demand, end-customer 13 needs to get the demand from import terminal 19 instead. Both import terminals serves end-customer 13 in time period 5, while the entire demand is served by import terminal 19 in time period 6. This argument applies in the same way to all the solutions.

The number of potential import terminals was limited to 16 different locations to decrease the computational solution time. The potential locations were located in areas with significant demands, which led to a distance of 257 nmi between potential locations 19 and 20. In a real world situation, it would be more accurate to evaluate possible import terminal locations and perhaps include potential locations without any demand, for example located a potential import terminal between 19 and 20.

A positive VSS means in theory that it is profitable to use a stochastic formulation when locating import terminals under uncertainty. But when the value is as low as 0.68%, you can consider this as equal to zero because of all the uncertainties in choice of parameters. This will in practice mean that the deterministic solution is as good as the stochastic solution in this case. But I will still describe the reason behind the positive VSS in the computational study. By only locating 2 import terminals in first stage in the deterministic solution, you are missing a lot of potential income if the high scenario occurs, due to lack of capacity at the import terminals. It will take three time periods (lead-time) before one has adapted the supply system to the large demand, and therefor missed potential profit. This gives the stochastic solution is a slightly better value.

The EVPI is low because the number of import terminals constructed in the first stage does not change much with different demand. While the stochastic solution suggests constructing three terminals in first stage, the deterministic wait-and-see solutions suggest constructing two, three and four terminals in first stage respectively. This makes the wait-and-see solutions slightly better in the low and high scenario, where there is a difference between the numbers of import terminals constructed.

6.6.2 MODEL

King and Wallace (2012) describe robustness as something that can withstand random events and flexibility as something that can accommodate those events. Both the deterministic and stochastic model formulations possess flexible characteristics in the way they can expand and construct new terminals if the demand changes. But the ability to withstand random events is more descriptive for the stochastic formulation because it optimizes its supply chain on the basis of several possible scenarios. The results from the computational study showed, however, that both models located their terminals at the same places, something that makes both models robust considering the different scenarios. Although this occurred in the computational study, this might differ in other situations. The difference between the deterministic and stochastic models will be examined further in the post analysis.

It seems like the flexibility keeps the difference between the deterministic and stochastic solution small. A test showed that the VSS increases to 8.94% by removing the opportunity to construct or expand terminals after time period 1. The example is not very applicable, but it shows that the VSS increases when the flexibility goes down, by means of freedom to choose when to construct.

The model formulation assumes that the lead-time for constructing import terminals is equal to the time it takes to expand an import terminal. In reality, it may be faster to expand than to construct new import terminals. Shorter lead-time for the expansion option would increase the flexibility in the model even more, and probably cause the VSS to decrease.

7 POST ANALYSIS

The post analysis is carried out to examine the sensitivity of various parameters, investigate the influence of the problem-specific constraints and test the computational time with respect to number of potential import terminals. The stochastic model is used as model formulation for the test instance.

7.1 INFLUENCE OF PROBLEM-SPECIFIC CONSTRAINT

The problem-specific constraints and indices differ from the simple facility location formulation presented in the literature chapter, and are therefor interesting to examine. These constraints and indices are included in the problem formulation in an attempt to make the model more accurate. The question is whether they do so or not.

7.1.1 FARES

Several end-customers became affected by economy of scale in ship transportation, according to the discussion in the computation study. An interesting question is whether it affects the location of the import terminals. Table 7.1 presents the comparison of the solution between stochastic model formulation with and without economy of scale in ship transport in first stage. Overall profit and solution time in Xpress decreased due to respectively more expensive ship transport and fewer variables. The number of constructed import terminals did not change, but the location of the three import terminals changed. Import terminal 19 is moved to location 12, which reduces the total ship transportation distance per time period with 433 nmi. A closer examination shows that the model with economy of scale are transporting more units to import terminal 3 to achieve economy of scale on that route. This forces import terminal 3 to serve more end-customers, which again causes the distance to the next import terminal to increase.

Table 7.1 Influence of economy of scale

	With economy of scale	Without economy of scale
Constructed import terminals in scenario 1	3, 19, 23	2, 12, 23
Overall profit [billion NOK]	8.83	8.77
Solution time to optimality [Sec.]	1270	938

It is realistic to assume that the relocation of import terminal 3 to location 2 is because of the relocation of import terminal 19 to 12. Import terminal 12 is now serving a lot of the end-customers import terminal 3 previously served. Import terminal 3 can now move closer to the big end-customers 1 and 2.

7.1.2 EXPANSION OPTION

The opportunity to expand the capacity of constructed import terminals is something that gives the model more flexibility in the way that you can accommodate increasing demand by expanding import terminals.

Table 7.2 presents the comparison between the stochastic solution with and without expansion options for scenario 3. The results show that the first stage decisions do not change if one removes the expansion option. The only difference is that import terminal 9 is constructed instead of expanding import terminal 12. The solution time has almost tripled without the expansion option, although the number of variables has decreased. The decrease in number of variables usually reduces the solution time for each node in the branch and bound tree and therefore also the total solution time, but the removal of the expansion option has obviously made the solution space bigger.

Table 7.2 Influence of expansion option

	With expansion option	Without expansion option
Constructed import terminals, stage 1	3, 19, 23	3, 19, 23
Constructed import terminals, stage 2	12	9, 12
Scenario 3, Expansion option	12	-
Overall profit [billion NOK]	8.829	8.828
Solution time to optimality [Sec.]	1270	3455

Although the elimination of the expansion option did not change the solution in first stage in this example, one cannot preclude that this cannot happen in another case with other parameters.

7.2 COST AND REVENUE

A sensitivity analysis has been carried out on the cost and revenue parameters in the computational study, where the goal is to examine how sensitive the construction of import terminals in first stage is for changes in these parameters. The analysis will examine the parameters from 50% to 150% of their original value with an increase of 25% for each step.

7.2.1 SHIP TRANSPORTATION

Figure 7.1 shows the stochastic solution of constructed import terminals in first stage with varying ship transportation unit cost, where all solutions suggest constructing three import terminals. All solution was solved to optimality.

The solutions in first stage show that the location of terminals changes with the level of ship transportation unit cost. The locations tend to increase the ship transportation distance when the cost is low, and decrease the distance when the cost is high.

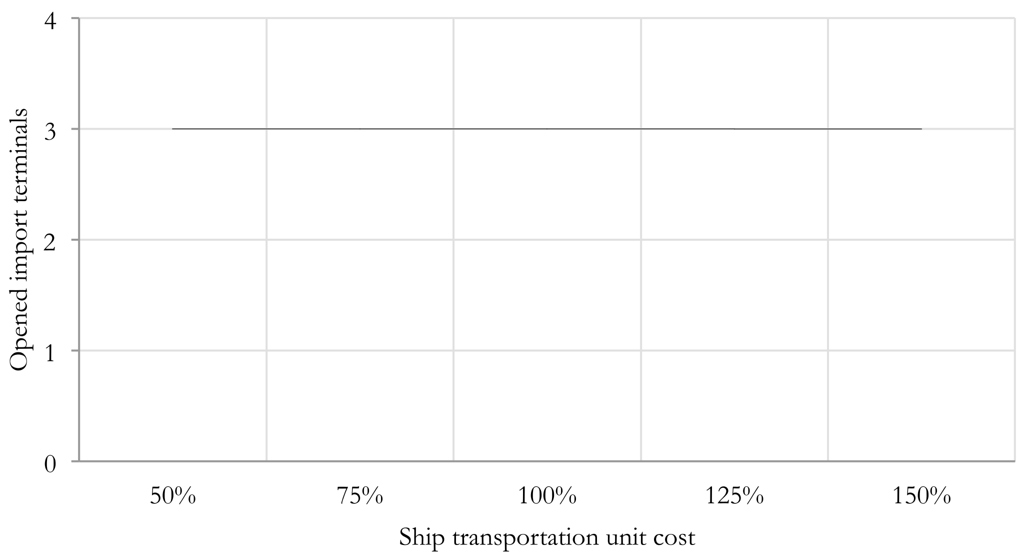

Figure 7.1 Ship transportation, cost sensitivity

By further examination of the ship transportation unit cost one can observe that the number of import terminals constructed in first stage do not change when the cost is set to 10% of the initial cost. It is realistic to assume that this is caused by the major capital cost that occurs when constructing new import terminals. Even when the unit cost is increased to 200%, the number of constructed terminals does not change. This is because the ship transportation is still cost efficient compared to truck transportation.

The result shows that the number of constructed terminals in stage is not sensitive for changes in ship transportation unit cost. The locations of the import terminals, on the other hand, are.

7.2.2 TRUCK TRANSPORTATION

Figure 7.2 shows the stochastic solution of constructed import terminals in first stage with varying truck transportation unit cost. The solutions with 50% and 125 % truck transportation unit cost were not solved to optimality, with an optimality gap on 0.33% and 0.27% respectively after 10,000 seconds. The "50% truck transportation unit cost"-solution constructed only two import terminals in first stage. The low truck transportation unit cost favors truck transportation over ship transportation and is therefor limiting the number of import terminals. From the "125% solution" to the "150% solution", the number of constructed import terminals increases by eight. This result is a consequence of too big truck transportation unit costs, where it is more cost efficient to transport the demanding units on ships and invest in import terminals.

Figure 7.2 Truck transportation, cost sensitivity

The results show that the number of constructed terminals in first stage is sensitive for large changes in truck transportation unit cost. The number of constructed import terminals increases sharply when truck transportation unit cost exceeds a certain limit.

7.2.3 CONSTRUCTION AND EXPANSION OF IMPORT TERMINALS

Figure 7.3 shows the stochastic solution of constructed import terminals in first stage with varying import terminal cost and as a result also expansion cost. All solution where solved to optimality, except for the solution with 150% increase in costs, which had an optimality gap of 0.86% after 10,000 seconds. According to the solution with 50% costs it would be optimal to construct four import terminals. The solutions with 75% - 150% costs suggest constructing three import terminals and the solution with 150% costs suggests two import terminals in first stage.

The solution implies that it is more cost efficient to construct one extra import terminal and increase the ship transportation when the import terminal cost is 50%. The opposite is true for the 150% increase in import terminal cost.

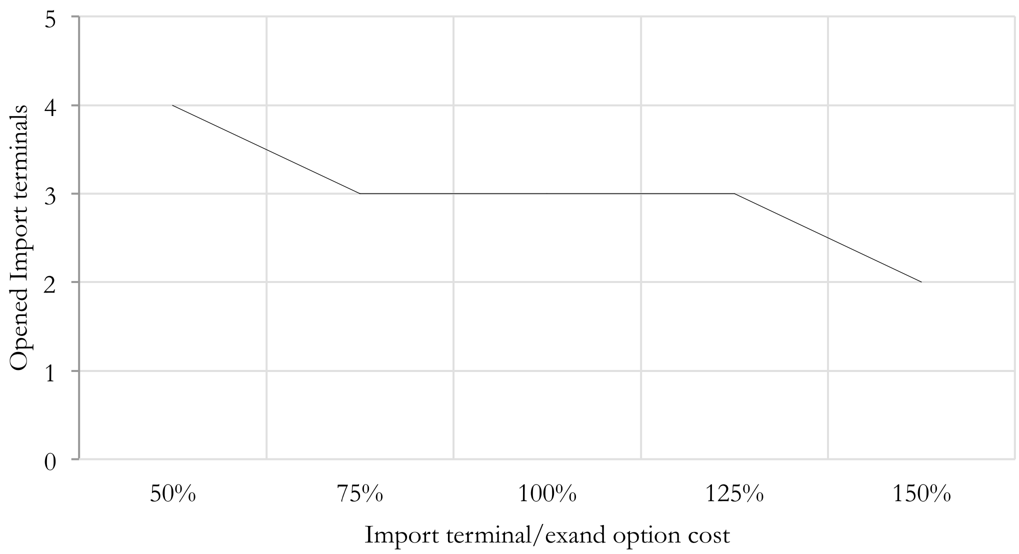

Figure 7.3 Construction and expansion of import terminals, cost sensitivity

The results show that the number of constructed import terminals in first stage is sensitive for large changes in import terminal/expand option cost, where number of constructed terminals increases when the price goes down and decreases when the price goes up.

7.2.4 REVENUE

Figure 7.4 show the stochastic solution of constructed import terminals in first stage with varying revenue. All solutions where solved to optimality.

It is interesting to note that the number of constructed import terminals in first stage stabilizes at three, independent of how large the revenue is. The solution is however more sensitive to a decrease in revenue. This is shown at 75% and 50% revenue where the solutions suggest to only constructing two import terminals. Further tests show that zero import terminals are constructed when the revenue is set to 20%, implying that the supply chain no longer is profitable. When the revenue is set to 200% of the original revenue, the solution still suggests constructing three import terminals.

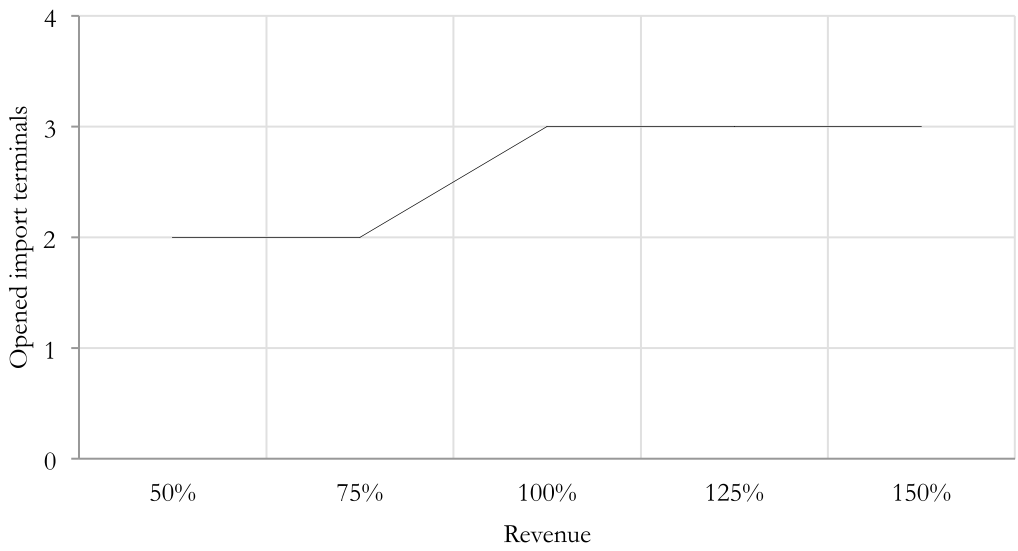

Figure 7.4 Revenue sensitivity

These results show that the number of constructed import terminals only is sensitive to a decrease in revenue.

7.3 PROBABILITY DISTRIBUTION

The computational study assumed an even probability distribution for the three different scenarios. Table 7.3 presents a solution summary with respect to various probability distributions, where both the deterministic and stochastic solution is presented with related solution time, optimality gap and VSS. All other parameters, except for the probability distribution, are equal to the ones presented in the computational study. The test instances are divided into four groups from 1-4, where all groups consist of three different instances. Capital letters L, N and H symbolize situations where the low, normal and high scenarios are weighted in the probability distribution. Maximal runtime is set to 10,000 seconds.

All test instances, except 2L show the general trend in the solutions, where the VSS is below 1.75%. For reasons similar to those given in section 6.6, these values can be considered equal to zero and thus equate stochastic and deterministic solution. Test instances 1L and 3H are examples of solutions where the deterministic and stochastic model are equal in the first time period, resulting in a 0% VSS. The test instances with a VSS between 0.06% and 1.68% has typically close to identical solutions for the deterministic and stochastic solutions, with only small differences such as different location of one of the constructed import terminals.

Table 7.3 Solution summary with respect to weighted probability distributions

Instance	Probability distribution (low, normal, high)	VSS [%]	Solution time [s]		Optimality gap [%]	
			Det.	Stoch.	Det.	Stoch.
1L	0.8, 0.1, 0.1	0.00	47	1,683	0	0
1N	0.1, 0.8, 0.1	0.59	543	5,189	0	0
1H	0.1, 0.1, 0.8	0.93	547	10,000	0	0.41
2L	0.7, 0.2, 0.1	5.59	186	3,974	0	0
2N	0.2, 0.7, 0.1	0.06	57	10,000	0	0.49
2H	0.1, 0.2, 0.7	0.17	663	1,349	0	0
3L	0.6, 0.2, 0.2	1.35	426	6,437	0	0
3N	0.2, 0.6, 0.2	0.64	404	1,546	0	0
3H	0.2, 0.2, 0.6	0.00	435	1,099	0	0
4L	0.5, 0.25, 0.25	1.68	186	10,000	0	0.98
4N	0.25, 0.5, 0.25	0.45	242	10,000	0	0.03
4H	0.25, 0.25, 0.5	1.46	140	1,586	0	0

Test instance 2L, however, differs from the other solutions with a VSS equal to 5.59%, which indicates that the stochastic model formulation is better to use than the deterministic model formulation in this case. The reason behind this result is that the import terminals constructed in the deterministic and stochastic solutions have different locations and capacities. Whilst the deterministic solution optimizes its supply chain on the basis of the weighted average of the scenarios, the stochastic solution optimizes its supply chain with 70% respect to the low scenario and 20% and 10% to the normal and high scenarios, respectively. This causes the stochastic solution to adjust its solution to the possibility of a normal and high scenario, making it more robust than the deterministic.

Figure 7.5 illustrates the VSS presented in Table 7.3 for the different test instances. Although the majority of the test instances have a low VSS, one can notice a trend in the solutions. The N-line, represented by the test instance where the normal scenarios are weighted in the probability distribution, shows a steady and low VSS. The H-line shows some tendencies to suggest that the results are to more than just noise, with a VSS at almost 1.5%. The L-line, in comparison, is far more unstable, with a VSS ranging between 0% and 5.59%.

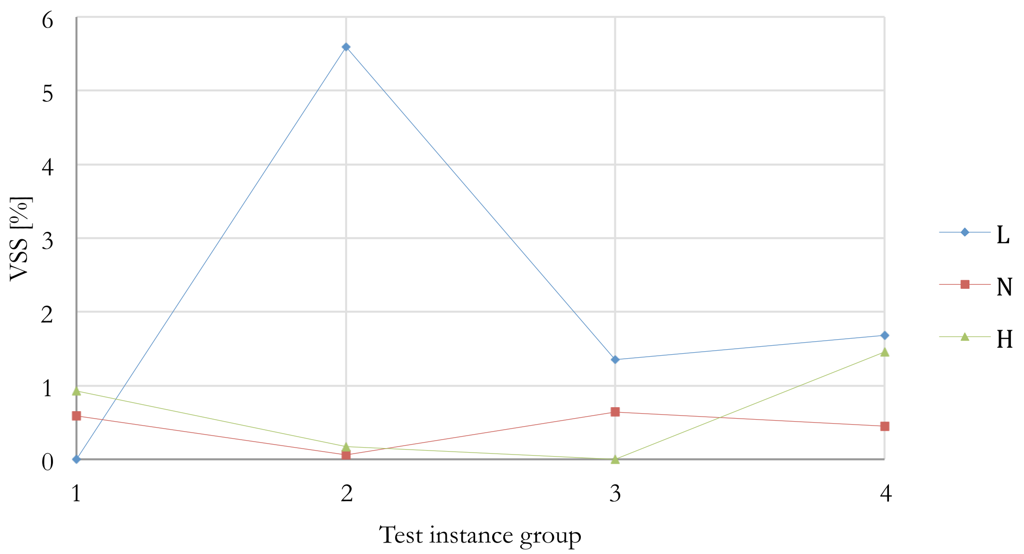

Figure 7.5 VSS with respect to weighted probability distribution

7.4 LEAD-TIME

The lead-time was set to 3 time periods in the computational study, representing both the duration of construction time for import terminals and the number of time periods between first and second stage.

By changing the lead-time to 2 time periods, tests showed that the VSS decreased to 0.14%. The reason why the VSS is decreases when the lead-time does the same is because the importance of designing a solution that works well in various scenarios decreases when the time it takes to adapt to the different scenarios declines. It is reasonable to assume that the opposite is true when the lead-time increases, where one is dependent on the solution in first stage for a longer time, before one can adapt to the new information.

The arguments above show that the need of a robust solution changes with the degree of flexibility, where the flexibility increases when the lead-time decreases.

7.5 SOLUTION TIME

The computational solution time is tested with respect to the number of potential import terminals included. The results are illustrated in Figure 7.6. The solution time was examined from 5 to 23 potential import terminal locations with an increase of 2 locations for each step. The results showed a steady increase in solution time from 5 (19 seconds) to 21 (48 minutes) potential locations. It is interesting to notice the major increase in solution time for 23 potential locations, where the model run was stopped after 24 hours with an optimality gap of 3.9%. This shows that the problem starts to get really complicated to solve with the computer used in the analysis. It is difficult to say whether the computer would manage to solve the problem, if given enough time, or if it simply has run out of memory.

It is realistic to assume that this analysis also can be related to increases in number of liquefaction plants or end-customers, where an increase in variables will make it difficult to solve the problem.

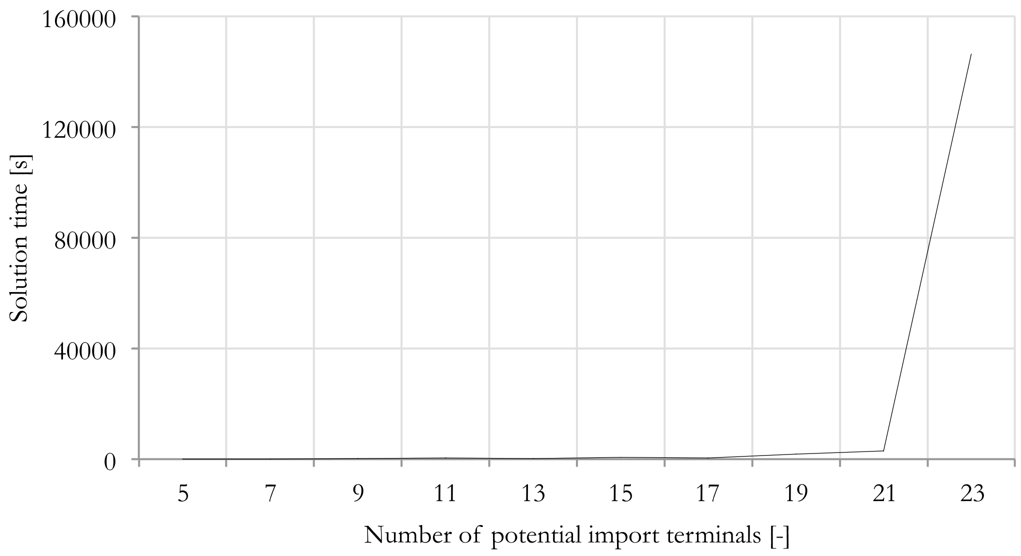

Figure 7.6 Solution time with respect to import terminals

8 CONCLUDING REMARKS

The primary objective in this master thesis was to develop a location model that can help decision makers to design a profitable supply chain for an uncertain future gas demand. Both a deterministic and stochastic model was developed and the results were compared by calculating the value of stochastic solution (VSS). Both models produce sensible results that correspond logically to expected solutions for the problem. Still, there are room for improvements both in the system size, test amount and information gathering.

The problem has proven to be flexible in the way that it can adapt to different situations by constructing and expanding import terminals throughout the project lifetime. This is the reason why the deterministic and stochastic solutions are close to each other through the computational study and the post analysis.

The VSS gives information about how valuable the stochastic solution is, compared to the deterministic one. Various tests have shown that the VSS is sensitive to changes in flexibility, such as changes in lead-time. A decrease in lead-time decreases the requirement for a robust supply chain and lowers the VSS, and vice versa for an increase in lead-time.

A lead-time of 3 time periods has proven to be one of the parameters that have kept the VSS low in the computational study and the post-analysis. More realistic model choices such as shorter lead-time for import terminal expansion options would contribute to an even lower VSS.

While the computational study assumed an even probability distribution and achieved a low VSS, the post analysis tested the model with a various number of uneven probability distributions. Based one these analysis, one can observe that the VSS tends to zero percent in situations where the normal scenario is weighted in the probability distribution, like in the computational study. The trend is less consistent in cases where a low or high scenario is weighted in the probability distribution, especially low. This implies that the stochastic model formulation can be of greater value when the probability distribution is weighted on the minimum or maximum scenarios.

Other observations from the post analysis show that cost parameters are not sensitive to changes in the number of import terminals constructed in first stage within the range of a 25% change in cost. Within this limit, only the location of the import terminals changes, where logical mechanisms such as more expensive truck transportation increase the sea transportation distance. The post analysis also showed that the first stage solution changed when the problem specific economy of scale constraints were removed. The elimination of the import terminal expansion option did not affect the first stage solution in the computational study, but this can be different in other cases.

9 FURTHER WORK

Stange (2008) points out that industry-specific challenges for distribution of LNG are high operational costs, big capital investment costs and high distribution costs. These kinds of conditions are forcing a streamlined supply chain and high utilization to secure profit. This thesis has focused on facilitating good supply chains by selecting the best locations for import terminals. A good next step would be to implement and use these strategic decisions to develop a model that can optimize the tactical and operational part of the planning process. Problem types such as fleet size and mix, in addition to inventory planning, are methods to streamline the supply chain and achieve high utilization. Fagerholt (1999) presents a solution method for deciding an optimal fleet in a liner-shipping problem with multiple trips per ship. Christiansen and Nygreen (1998) present a solution method for a ship-planning problem by combining a multi-pickup problem with time windows and an inventory model.

It could also be interesting to expand the model developed in this thesis to deal with both facility location and inventory planning. This would make the problem more complex and would require a lot more background research on the supply chain. The complexity would also require a heuristic algorithm to limit the solution space. The paper by Tsiakis, Shah et al. (2001), discussed in the literature chapter, combines inventory planning with facility location and develops such an algorithm.

The whole coastline of Norway was used as geographical area in the computational study, where 16 regions along the coast represent the end-customers. The demand was different in all regions, but varied in the same degree for the three different scenarios. It would be interesting to investigate the value of the stochastic solution in a study on a smaller geographical area with scenarios that had different variations for each end-customer.

The problem complexity in the thesis sat limitations to the number of variables in the computational study. As it emerges from the literature study, where the majority of multi-echelon papers used heuristic algorithms to solve the problem, it could be an interesting next step to implement this into the problem. This would help to solve the problem faster and enable the computational study to use more variables.

In addition to the possible next steps for my problem, there are a number of changes that can be done to the model or parameters to increase the value of the results. Firstly, one could introduce uncertainty to other parameters, such as the revenue. This would make the model more realistic, but probably also increase the solution time.

Secondly, one could decrease the number of time periods and extend the model to a three-stage model. A three-stage model would be an even more realistic approach to the real world as new information about the demand appears all the time, but the drawback is the major increase in solution time.

Thirdly, the amount of runs with different demand scenarios could be increased, to be able to draw more reliable conclusions.

Lastly, I will recommend others who are going to work with facility location to use coordinates to locate all kinds of potential and fixed facilities and customer. By using coordinates, Xpress-IVE will provide a graph, based on the coordinates, which show the results of the facility location.

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Appendix A

Deterministic Model in Compact Form

Sets	
$\mathcal{P}$	Set of liquefaction plants, indexed by p
$\mathcal{I}$	Set of import terminals, indexed by i
$\mathcal{J}$	Set of end-costumers, indexed by j
$\mathcal{W}$	Set of import terminal capacities indexed by w
$\mathcal{V}$	Set of import terminal expansion capacities indexed by v
$\mathcal{F}$	Set of transportation fares indexed by f
$\mathcal{T}$	Set of time periods indexed by t
Constants	
R_{jt}	Unit profit of LNG transported to end-customer j in time period t
C_{iwt}	Total CAPEX and OPEX for all time periods for import terminal i constructed with capacity alternative w in time period t
C_{ivt}^{E}	Total CAPEX and OPEX for all time periods for expanding import terminal i with capacity alternative v in time period t
C_{fpit}^{PI}	Unit transportation cost with fare f from liquefaction plant p to import terminal i, in time period t
C_{ijt}^{IJ}	Unit transportation cost from import terminal i to end-customer j, in time period t
D_{jt}	Consumer demand j, in time period t
W_{w}	Storage capacity w
V_{v}	Storage expansion capacity v
U_{pt}^{P}	Maximal production capacity for liquefaction plant p in time period t
L_{f}	Threshold f for different economy of scale alternatives
$\bar{T}^{L}$	Lead-time for import terminals and import terminal expansion
Variables	
x_{fpit}	Quantity delivered with fare f from liquefaction plant p to import terminal i in time period t
y_{ijt}	Quantity delivered from import terminal i to end-customer j in in time period t
δ_{iwt}	Get value 1 if distribution terminal i with capacity w is constructed in time period t, else 0
α_{ivt}	Get value 1 if terminal i expand its capacity with alternative v in time period t, else 0
γ_{fpit}	Get value 1 if shipping fare f is chosen from liquefaction plant p to import terminal i in time period t

$$\max z = \sum_{t\in\mathcal{T}}\sum_{j\in\mathcal{J}}\sum_{i\in\mathcal{I}} R_{jt}y_{ijt} - \sum_{t\in\mathcal{T}}\sum_{w\in\mathcal{W}}\sum_{i\in\mathcal{I}} C_{iwt}\delta_{iwt} - \sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{I}}\sum_{p\in\mathcal{P}}\sum_{f\in\mathcal{F}} C^{PI}_{fpit}x_{fpit} - \sum_{t\in\mathcal{T}}\sum_{j\in\mathcal{J}}\sum_{i\in\mathcal{I}} C^{IJ}_{ijt}y_{ijt} - \sum_{t\in\mathcal{T}}\sum_{v\in\mathcal{V}}\sum_{i\in\mathcal{I}} C^{E}_{ivt}\alpha_{ivt}$$

Subject to

$$\sum_{f\in\mathcal{F}}\sum_{i\in\mathcal{I}} x_{fpit} \le U^P_{pt} \qquad p\in\mathcal{P}, t\in\mathcal{T}: t > \bar{T}^L$$

$$\sum_{i\in\mathcal{I}} y_{ijt} \le D_{jt} \qquad j\in\mathcal{J}, t\in\mathcal{T}$$

$$\sum_{f\in\mathcal{F}}\sum_{p\in P} x_{fpit} = \sum_{j\in\mathcal{J}} y_{ijt} \qquad i\in\mathcal{I}, t\in\mathcal{T}: t > \bar{T}^L$$

$$\sum_{f\in\mathcal{F}}\sum_{p\in P} x_{fpit} \le \sum_{t'>\bar{T}^L}^{t}\sum_{w\in\mathcal{W}} W_w\delta_{i,w,t'-\bar{T}^L} + \sum_{t'>\bar{T}^L}^{t}\sum_{v\in\mathcal{V}} V_v\,\alpha_{i,v,t'-\bar{T}^L} \qquad i\in\mathcal{I}, t\in\mathcal{T}$$

$$\sum_{t'>+\bar{T}^L}^{t}\sum_{w\in\mathcal{W}} W_w\delta_{i,w,t'-\bar{T}^L} \ge \sum_{t'>\bar{T}^L}^{t}\sum_{v\in\mathcal{V}} V_v\,\alpha_{i,v,t'-\bar{T}^L} \qquad i\in\mathcal{I}, t\in\mathcal{T}$$

$$L_{f-1}\gamma_{fpit} \le x_{fpit} \le L_f\gamma_{fpit} \qquad f\in\mathcal{F}\backslash\{1\}, p\in\mathcal{P}, i\in\mathcal{I}, t\in\mathcal{T}: t > \bar{T}^L$$

$$0 \le x_{fpit} \le L_f\gamma_{fpit} \qquad f = 1, p\in\mathcal{P}, i\in\mathcal{I}, t\in\mathcal{T}: t > \bar{T}^L$$

$$\sum_{f\in\mathcal{F}} \gamma_{fpit} \le 1 \qquad p\in\mathcal{P}, i\in\mathcal{I}, t\in\mathcal{T}: t > \bar{T}^L$$

$$x_{fpit} \in \mathbb{Z}^+ \qquad f\in\mathcal{F}, p\in\mathcal{P}, i\in\mathcal{I}, t\in\mathcal{T}: t > \bar{T}^L$$

$$y_{ijt} \in \mathbb{Z}^+ \qquad i\in\mathcal{I}, j\in\mathcal{J}, t\in\mathcal{T}: t > \bar{T}^L$$

$$\gamma_{fpit} \in \{0,1\} \qquad f\in\mathcal{F}, p\in\mathcal{P}, i\in\mathcal{I}, t\in\mathcal{T}: t > \bar{T}^L$$

$$\delta_{iwt} \in \{0,1\} \qquad i\in\mathcal{I}, w\in\mathcal{W}, t\in\mathcal{T}$$

$$\alpha_{ivt} \in \{0,1\} \qquad i\in\mathcal{I}, v\in V, t\in\mathcal{T}$$

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APPENDIX B

STOCHASTIC MODEL IN COMPACT FORM

Sets	
$\mathcal{P}$	Set of production terminals, indexed by p
$\mathcal{I}$	Set of distribution terminals, indexed by i
$\mathcal{J}$	Set of consumers, indexed by j
$\mathcal{S}$	Set of demand scenarios indexed by s
$\mathcal{W}$	Set of Capacity indexed by w
$\mathcal{V}$	Set of expanded storage capacity indexed by v
$\mathcal{F}$	Set of fares indexed by f
$\mathcal{T}$	Set of time/periods indexed with t
$\mathcal{T}^1 \subseteq \mathcal{T}$	Subset of $\mathcal{T}$ in first stage
Parameters	
R_{jt}	Unit revenue of total volume sold to consumer
C_{fpit}^{PI}	Unit transportation cost f from production terminal p to distribution terminal i, in time t
C_{ijt}^{IJ}	Unit transportation cost from distribution terminal i to consumer j, in time t
C_{iw}^{S1}	Total CAPEX and OPEX for all periods for terminals built in stage 1 with capacity alternative w
C_{iwt}^{S2}	Total CAPEX and OPEX for terminals built in time t in stage 2 with capacity alternative w
C_{ivt}^{E}	Total CAPEX and OPEX for capacity expand alternative v in stage 2 and time t
L_f	Threshold f for different economy of scale alternatives
D_{jts}	Consumer demand j, in stage 2, time t and scenario s
W_w	Storage capacity alternative w
V_v	Storage capacity expand alternative v
U_p^P	Production capacity for production terminal p
π_s	Probability that scenario s occur in stage 2
$\bar{T}^1$	Number of periods in stage 1
$\bar{T}^L$	Number of periods of lead time for new terminals
Variables	
x_{fpits}	Quantity delivered with fare f from production terminal p to distribution terminal i in time t and scenario s
y_{ijts}	Quantity delivered from distribution terminal i to consumer j in in time t and scenario s
δ_{iw}^{S1}	Get value 1 if distribution terminal i with capacity alternative w is build in stage 1, else 0
δ_{iwts}^{S2}	Get value 1 if distribution terminal i with capacity alternative w is build in scenario s in stage 2 and time t, else 0
α_{ivts}	Get value 1 if terminal i expand its capacity with alternative v in stage 2, time t and scenario s, else o
γ_{fpits}	Get value 1 if shipping fare f is chosen from production terminal p to distribution terminal i in time t and scenario s

$$\max\ z = -\sum_{w\in\mathcal{W}}\sum_{i\in\mathcal{I}} C_{iw}^{S1}\delta_{iw}^{S1}$$
$$+\sum_{s\in\mathcal{S}}\pi_s\left\{\sum_{t\in\mathcal{T}}\sum_{j\in\mathcal{J}}\sum_{i\in\mathcal{I}} R_{jt}y_{ijts}\right.$$
$$-\sum_{f\in\mathcal{F}}\sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{I}}\sum_{p\in\mathcal{P}} C_{fpit}^{PI}x_{fpits}$$
$$-\sum_{t\in\mathcal{T}}\sum_{j\in\mathcal{J}}\sum_{i\in\mathcal{I}} C_{ijt}^{IJ}y_{ijts}$$
$$-\sum_{t\in\mathcal{T}}\sum_{w\in\mathcal{W}}\sum_{i\in\mathcal{I}} C_{wt}^{S2}\delta_{iwts}^{S2}$$
$$\left.-\sum_{t\in\mathcal{T}}\sum_{v\in\mathcal{V}}\sum_{i\in\mathcal{I}} C_{vt}^{E}\alpha_{ivts}\right\}$$

Subjected to

$$\sum_{f\in\mathcal{F}}\sum_{i\in\mathcal{I}} x_{fpits} \le U_{pt}^{P} \qquad p\in\mathcal{P}, t\in\mathcal{T}: t>\bar{T}^{L}, s\in\mathcal{S}$$

$$\sum_{i\in\mathcal{I}} y_{ijts} \le D_{jts} \qquad j\in\mathcal{J}, t\in\mathcal{T}: t>\bar{T}^{L}, s\in\mathcal{S}$$

$$\sum_{f\in\mathcal{F}}\sum_{p\in P} x_{fpits} = \sum_{j\in\mathcal{J}} y_{ijts} \qquad i\in\mathcal{I}, t\in\mathcal{T}: t>\bar{T}^{L}, s\in\mathcal{S}$$

$$\sum_{f\in\mathcal{F}}\sum_{p\in P} x_{fpits} \le \sum_{w\in\mathcal{W}} W_w\delta_{iw}^{S1} \qquad i\in\mathcal{I}, t\in\mathcal{T}: t\le(\bar{T}^{1}+\bar{T}^{L}), s\in\mathcal{S}$$

$$\sum_{f\in\mathcal{F}}\sum_{p\in P} x_{fpits} \le \sum_{w\in\mathcal{W}} W_w\left(\delta_{iw}^{S1}+\sum_{t'>(\bar{T}^{1}+\bar{T}^{L})}^{t}\delta_{i,w,t'-\bar{T}^{L},s}^{S2}\right) + \sum_{t'>(\bar{T}^{1}+\bar{T}^{L})}^{t}\sum_{v\in\mathcal{V}} V_v\,\alpha_{i,v,t'-\bar{T}^{L},s} \qquad i\in\mathcal{I}, t\in\mathcal{T}^{2}: t>(\bar{T}^{L}+\bar{T}^{1}), s\in\mathcal{S}$$

$$\sum_{w\in\mathcal{W}}\left(\delta_{iw}^{S1}+\sum_{t\in\mathcal{T}}\delta_{iwts}^{S2}\right)\le 1 \qquad i\in\mathcal{I}, s\in\mathcal{S}$$

$$\sum_{w\in\mathcal{W}}\left(\delta_{iw}^{S1}+\sum_{t'>(\bar{T}^{1}+\bar{T}^{L})}^{t}\delta_{i,w,t'-\bar{T}^{L},s}^{S2}\right)\ge \sum_{t'>(\bar{T}^{1}+\bar{T}^{L})}^{t}\sum_{v\in\mathcal{V}}\alpha_{i,v,t'-\bar{T}^{L},s} \qquad i\in\mathcal{I}, t\in\mathcal{T}^{2}: t>(\bar{T}^{L}+\bar{T}^{1}), s\in\mathcal{S}$$

$$L_{f-1}\gamma_{fpits}\le x_{fpits}\le L_f\gamma_{fpits} \qquad f\in\mathcal{F}/\{1\}, p\in\mathcal{P}, i\in\mathcal{I}, t\in\mathcal{T}: t>\bar{T}^{L}, s\in\mathcal{S}$$

$$0\le x_{fpits}\le L_f\gamma_{fpits} \qquad f=1, p\in\mathcal{P}, i\in\mathcal{I},\ t\in\mathcal{T}: t>\bar{T}^{L}, s\in\mathcal{S}$$

$$\sum_{f\in\mathcal{F}}\gamma_{fpits}\le 1 \qquad p\in\mathcal{P}, i\in\mathcal{I}, t\in\mathcal{T}: t>\bar{T}^{L}, s\in\mathcal{S}$$

$$x_{fpits}\in\mathbb{Z}^{+} \qquad f\in\mathcal{F}, p\in\mathcal{P}, i\in\mathcal{I}, t\in\mathcal{T}: t>\bar{T}^{L}, s\in\mathcal{S}$$

$$y_{ijts}\in\mathbb{Z}^{+} \qquad i\in\mathcal{I}, j\in\mathcal{J}, t\in\mathcal{T}: t>\bar{T}^{L}, s\in\mathcal{S}$$

$$\delta_{iw}^{S1}\in\{0,1\} \qquad i\in\mathcal{I}, w\in\mathcal{W}$$

$$\delta_{iwts}^{S2}\in\{0,1\} \qquad i\in\mathcal{I}, w\in\mathcal{W}, t\in\mathcal{T}^{2}, s\in\mathcal{S}$$

$$\alpha_{ivts}\in\{0,1\} \qquad i\in\mathcal{I}, v\in V, t\in\mathcal{T}^{2}, s\in\mathcal{S}$$

$$\gamma_{fpits}\in\{0,1\} \qquad f\in\mathcal{F}, p\in\mathcal{P}, i\in\mathcal{I}, t\in\mathcal{T}: t>\bar{T}^{L}, s\in\mathcal{S}$$

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APPENDIX C

INPUT DATA

Truck Transportation Distance Matrix (km)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	10	64	95	165	265	333	424	535	552	639	419	480	422	494	568	563	502	575	773	1249	1481	1920	2039	2340
2	64	10	30	101	201	268	359	470	486	573	449	558	486	437	616	627	557	629	827	1304	1535	1975	2094	2395
3	95	30	10	71	171	238	329	440	460	547	519	483	522	547	726	663	605	678	876	1352	1584	2023	2142	2443
4	265	201	171	100	10	68	159	270	299	386	472	505	674	693	872	917	761	834	1032	1508	1740	2179	2298	2599
5	333	268	238	167	68	10	91	202	233	320	406	439	608	755	934	979	822	895	1093	1569	1801	2240	2359	2660
6	424	359	329	258	159	91	10	111	166	252	338	373	542	710	888	933	912	985	1183	1659	1891	2330	2449	2750
7	552	486	460	393	299	233	166	77	10	87	173	214	383	550	729	774	666	739	937	1413	1645	2084	2203	2504
8	639	573	547	479	386	320	252	163	87	10	86	137	306	473	651	697	589	662	860	1336	1567	2007	2126	2427
9	480	558	483	407	505	439	373	283	214	137	120	10	169	336	515	560	454	526	724	1201	1432	1872	1991	2292
10	568	616	726	658	872	934	888	798	729	651	634	515	346	179	10	45	107	180	378	854	1086	1525	1644	1945
11	563	627	663	703	917	979	933	843	774	697	680	560	391	224	45	10	81	154	352	828	1060	1499	1618	1919
12	502	557	605	665	761	822	912	735	666	589	582	454	346	280	107	81	10	73	271	747	979	1418	1537	1838
13	773	827	876	935	1032	1093	1183	1006	937	860	853	724	617	551	378	352	271	198	10	476	708	1147	1266	1567
14	1249	1304	1352	1411	1508	1569	1659	1482	1413	1336	1329	1201	1093	1027	854	828	747	674	476	10	232	671	790	1091
15	1920	1975	2023	2082	2179	2240	2330	2153	2084	2007	2000	1872	1764	1698	1525	1499	1418	1345	1147	671	439	10	119	420
16	2039	2094	2142	2201	2298	2359	2449	2272	2203	2126	2119	1991	1883	1817	1644	1618	1537	1464	1266	790	558	119	10	301

Truck Transportation Unit Cost Matrix (NOK/m3)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	6	39	56	91	131	151	171	192	199	230	170	178	171	180	204	203	181	207	278	450	533	691	734	842
2	39	6	18	59	106	132	158	177	179	206	175	201	179	173	222	226	200	227	298	469	553	711	754	862
3	56	18	6	42	93	121	150	173	176	197	187	178	188	197	261	239	218	244	315	487	570	728	771	879
4	131	106	93	58	6	41	88	132	142	164	177	182	243	249	314	330	274	300	371	543	626	784	827	936
5	151	132	121	92	41	6	54	107	119	148	168	173	219	272	336	352	296	322	393	565	648	806	849	958
6	171	158	150	128	88	54	6	64	91	126	153	161	195	255	320	336	328	355	426	597	681	839	882	990
7	199	179	176	165	142	119	91	46	6	51	94	112	163	198	262	279	240	266	337	509	592	750	793	901
8	230	206	197	178	164	148	126	90	51	6	51	77	144	177	235	251	212	238	309	481	564	722	765	874
9	178	201	178	168	182	173	161	137	112	77	69	6	92	152	185	202	175	190	261	432	516	674	717	825
10	204	222	261	237	314	336	320	287	262	235	228	185	155	97	6	27	62	97	162	307	391	549	592	700
11	203	226	239	253	330	352	336	304	279	251	245	202	165	115	27	6	48	85	156	298	381	540	582	691
12	181	200	218	239	274	296	328	265	240	212	210	175	155	136	62	48	6	44	132	269	352	510	553	662
13	278	298	315	337	371	393	426	362	337	309	307	261	222	198	162	156	132	105	6	178	255	413	456	564
14	450	469	487	508	543	565	597	534	509	481	479	432	393	370	307	298	269	243	178	6	119	242	284	393
15	691	711	728	750	784	806	839	775	750	722	720	674	635	611	549	540	510	484	413	242	173	6	68	170
16	734	754	771	793	827	849	882	818	793	765	763	717	678	654	592	582	553	527	456	284	201	68	6	142

Ship Transportation Distance Matrix (nmi)

	1(1)	2(2)	3(3)	4(5)	5(6)	6(7)	7(9)	8(10)	9(12)	10(15)	11(16)	12(17)	13(19)	14(20)	15(23)	16(24)
1(9)	321	289	273	173	125	76	0	32	99	321	337	371	532	789	1023	1091
2(12)	420	388	372	272	224	175	99	67	0	222	238	272	433	690	924	992
3(25)	1613	1581	1565	1465	1417	1369	1293	1261	1194	972	956	922	761	504	270	202

Ship Transportation Unit Cost Matrix (NOK/m3)

	1(1)	2(2)	3(3)	4(5)	5(6)	6(7)	7(9)	8(10)	9(12)	10(15)	11(16)	12(17)	13(19)	14(20)	15(23)	16(24)
1(9)	79	74	71	53	45	36	0	28	40	79	82	88	117	162	203	215
2(12)	97	91	88	71	62	54	40	34	0	62	65	71	99	145	186	198
3(25)	308	302	299	282	273	265	251	246	234	194	192	186	157	112	70	58

-The unit cost estimated by SINTEF, MARINTEK et al. (2002) included the cost of four small import terminals, this is taken into account by subtracting 30% of the cost.
-15 % is added as port costs
-The unit cost is adjusted to the consumer price index

Demand - Scenario 1 (1000m³ LNG)

Time period	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	208733	118470	64171	78980	6347	62761	7052	14809	57825	8462	7757	60645	32438	7052	9873	16924	9167	28207	154434	41606	11283	74749	9873	10578
2	187269	106288	57573	70859	5694	56307	6327	13286	51879	7592	6959	54409	29103	6327	8857	15184	8225	25307	138554	37327	10123	67063	8857	9490
3	165661	94024	50930	62683	5037	49810	5597	11753	45893	6716	6156	48131	25745	5597	7835	13432	7276	22387	122567	33020	8955	59325	7835	8395
4	144053	81760	44287	54507	4380	43313	4867	10220	39907	5840	5353	41853	22387	4867	6813	11680	6327	19467	106580	28713	7787	51587	6813	7300
5	141172	80125	43401	53417	4292	42447	4769	10016	39109	5723	5246	41016	21939	4769	6677	11446	6200	19077	104448	28139	7631	50555	6677	7154
6	138291	78490	42515	52326	4205	41581	4672	9811	38310	5606	5139	40179	21491	4672	6541	11213	6074	18688	102317	27565	7475	49523	6541	7008
7	135410	76854	41629	51236	4117	40715	4575	9607	37512	5490	5032	39342	21043	4575	6405	10979	5947	18299	100185	26991	7319	48491	6405	6862
8	132529	75219	40744	50146	4030	39848	4477	9402	36714	5373	4925	38505	20596	4477	6268	10746	5821	17909	98054	26416	7164	47460	6268	6716
9	129648	73584	39858	49056	3942	38982	4380	9198	35916	5256	4818	37668	20148	4380	6132	10512	5694	17520	95922	25842	7008	46428	6132	6570
10	126767	71949	38972	47966	3854	38116	4283	8994	35118	5139	4711	36831	19700	4283	5996	10278	5567	17131	93790	25268	6852	45396	5996	6424
11	123886	70314	38087	46876	3767	37249	4185	8789	34320	5022	4604	35994	19253	4185	5859	10045	5441	16741	91659	24693	6697	44365	5859	6278
12	121005	68678	37201	45786	3679	36383	4088	8585	33522	4906	4497	35157	18805	4088	5723	9811	5314	16352	89527	24119	6541	43333	5723	6132
13	118124	67043	36315	44695	3592	35517	3991	8380	32723	4789	4390	34320	18357	3991	5587	9578	5188	15963	87396	23545	6385	42301	5587	5986

Demand - Scenario 2 (1000m³ LNG)

Time period	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	208733	118470	64171	78980	6347	62761	7052	14809	57825	8462	7757	60645	32438	7052	9873	16924	9167	28207	154434	41606	11283	74749	9873	10578
2	226716	128677	69700	85784	6893	68168	7659	16085	62806	9191	8425	65870	35233	7659	10723	18382	9957	30637	167739	45190	12255	81189	10723	11489
3	245371	139265	75435	92843	7461	68168	8290	17408	67974	9947	9119	71290	38132	8290	11605	19895	10776	33158	181541	48908	13263	87869	11605	12434
4	252093	143080	77502	95387	7665	68168	8517	17885	69837	10220	9368	73243	39177	8517	11923	20440	11072	34067	186515	50248	13627	90277	11923	12775
5	268900	152619	82668	101746	8176	68168	9084	19077	74492	10901	9993	78126	41788	9084	12718	21803	11810	36338	198949	53598	14535	96295	12718	13627
6	285706	162157	87835	108105	8687	68168	9652	20270	79148	11583	10617	83009	44400	9652	13513	23165	12548	38609	211384	56948	15444	102314	13513	14478
7	302512	171696	93002	114464	9198	68168	10220	21462	83804	12264	11242	87892	47012	10220	14308	24528	13286	40880	223818	60298	16352	108332	14308	15330
8	319318	181235	98169	120823	9709	68168	10788	22654	88460	12945	11867	92775	49624	10788	15103	25891	14024	43151	236252	63648	17260	114350	15103	16182
9	336124	190773	103336	127182	10220	68168	11356	23847	93116	13627	12491	97658	52236	11356	15898	27253	14762	45422	248687	66998	18169	120369	15898	17033
10	339486	192681	104369	128454	10322	68168	11469	24085	94047	13763	12616	98634	52758	11469	16057	27526	14910	45876	251174	67668	18351	121573	16057	17204
11	346275	196535	106456	131023	10529	68168	11698	24567	95928	14038	12868	100607	53813	11698	16378	28076	15208	46794	256197	69021	18718	124004	16378	17548
12	356664	202431	109650	134954	10845	68168	12049	25304	98805	14459	13254	103625	55427	12049	16869	28919	15664	48198	263883	71092	19279	127724	16869	18074
13	370930	210528	114036	140352	11278	68168	12531	26316	102758	15038	13785	107770	57645	12531	17544	30075	16291	50126	274438	73935	20050	132833	17544	18797

Demand - Scenario 3 (1000m³ LNG)

Time period	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	208733	118470	64171	78980	6347	62761	7052	14809	57825	8462	7757	60645	32438	7052	9873	16924	9167	28207	154434	41606	11283	74749	9873	10578
2	264098	149893	81192	99929	8030	79408	8922	18737	73162	10707	9814	76731	41042	8922	12491	21413	11599	35689	195397	52641	14276	94576	12491	13383
3	312116	177147	95954	118098	9490	93846	10544	22143	86464	12653	11599	90682	48504	10544	14762	25307	13708	42178	230923	62212	16871	111771	14762	15817
4	360133	204400	110717	136267	10950	108283	12167	25550	99767	14600	13383	104633	55967	12167	17033	29200	15817	48667	266450	71783	19467	128967	17033	18250
5	384142	218027	118098	145351	11680	115502	12978	27253	106418	15573	14276	111609	59698	12978	18169	31147	16871	51911	284213	76569	20764	137564	18169	19467
6	408151	231653	125479	154436	12410	122721	13789	28957	113069	16547	15168	118584	63429	13789	19304	33093	17926	55156	301977	81354	22062	146162	19304	20683
7	432160	245280	132860	163520	13140	129940	14600	30660	119720	17520	16060	125560	67160	14600	20440	35040	18980	58400	319740	86140	23360	154760	20440	21900
8	456169	258907	140241	172604	13870	137159	15411	32363	126371	18493	16952	132536	70891	15411	21576	36987	20034	61644	337503	90926	24658	163358	21576	23117
9	480178	272533	147622	181689	14600	144378	16222	34067	133022	19467	17844	139511	74622	16222	22711	38933	21089	64889	355267	95711	25956	171956	22711	24333
10	492182	279347	151313	186231	14965	147987	16628	34918	136348	19953	18291	142999	76488	16628	23279	39907	21616	66511	364148	98104	26604	176254	23279	24942
11	504187	286160	155003	190773	15330	151597	17033	35770	139673	20440	18737	146487	78353	17033	23847	40880	22143	68133	373030	100497	27253	180553	23847	25550
12	528196	299787	162384	199858	16060	158816	17844	37473	146324	21413	19629	153462	82084	17844	24982	42827	23198	71378	390793	105282	28551	189151	24982	26767
13	552204	313413	169766	208942	16790	166034	18656	39177	152976	22387	20521	160438	85816	18656	26118	44773	24252	74622	408557	110068	29849	197749	26118	27983

Demand - Aggregated scenario (1000m³ LNG)

Time period	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	208733	118470	64171	78980	6347	62761	7052	14809	57825	8462	7757	60645	32438	7052	9873	16924	9167	28207	154434	41606	11283	74749	9873	10578
2	226028	128286	69488	85524	6872	67961	7636	16036	62616	9163	8400	65670	35126	7636	10690	18327	9927	30544	167230	45053	12218	80942	10690	11454
3	241049	136812	74106	91208	7329	72478	8144	17101	66777	9772	8958	70035	37460	8144	11401	19545	10587	32574	178344	48047	13030	86322	11401	12215
4	252093	143080	77502	95387	7665	75798	8517	17885	69837	10220	9368	73243	39177	8517	11923	20440	11072	34067	186515	50248	13627	90277	11923	12775
5	264738	150257	81389	100171	8049	79600	8944	18782	73340	10733	9838	76917	41142	8944	12521	21465	11627	35775	195870	52769	14310	94805	12521	13416
6	277383	157433	85276	104956	8434	83402	9371	19679	76843	11245	10308	80591	43107	9371	13119	22490	12182	37484	205226	55289	14994	99333	13119	14057
7	290027	164610	89164	109740	8818	87204	9798	20576	80345	11758	10778	84265	45072	9798	13718	23516	12738	39193	214581	57810	15677	103861	13718	14697
8	302672	171787	93051	114525	9203	91006	10225	21473	83848	12270	11248	87939	47037	10225	14316	24541	13293	40902	223936	60330	16361	108389	14316	15338
9	315317	178964	96939	119309	9587	94808	10653	22370	87351	12783	11718	91612	49002	10653	14914	25566	13848	42610	233292	62850	17044	112917	14914	15979
10	319478	181326	98218	120884	9714	96059	10793	22666	88504	12952	11873	92821	49649	10793	15110	25904	14031	43173	236371	63680	17269	114408	15110	16190
11	324783	184336	99849	122891	9875	97654	10972	23042	89974	13167	12070	94363	50473	10972	15361	26334	14264	43890	240295	64737	17556	116307	15361	16459
12	335288	190299	103078	126866	10195	100813	11327	23787	92884	13593	12460	97415	52106	11327	15858	27186	14725	45309	248068	66831	18124	120069	15858	16991
13	347086	196995	106706	131330	10553	104360	11726	24624	96152	14071	12898	100843	53939	11726	16416	28142	15244	46904	256797	69183	18761	124294	16416	17589

Import terminal cost and import terminal expansion cost

	Time period	1	2	3	4	5	6	7	8	9	10	11	12	13
	Capacity alternative													
Terminal cost - Deterministic Model	1	324094657	283028473	245695578	211756583	180902951	152854195	127355326	104174535	83101090	63943412	-	-	-
	2	380271064	332086741	288282812	248461058	212259463	179348922	149430249	122231455	97505279	75026937	-	-	-
	3	553121547	483035260	419320453	361397902	308741037	260871160	217353089	177791207	141825860	109130090	-	-	-
Terminal expansion cost - Deterministic Model	1	190135532	166043371	144141406	124230529	106129731	89674461	74715124	61115727	48752639	37513468	-	-	-
	2	220384367	192459361	167072993	143994477	123014007	103940853	86601621	70838684	56508741	43481520	-	-	-
Terminal cost - Stage 1 - Stochastic Model	1	324094657	-	-	-	-	-	-	-	-	-	-	-	-
	2	380271064	-	-	-	-	-	-	-	-	-	-	-	-
	3	553121547	-	-	-	-	-	-	-	-	-	-	-	-
Terminal cost - Stage 2 - Stochastic Model	1	-	-	-	211756583	180902951	152854195	127355326	104174535	83101090	63943412	-	-	-
	2	-	-	-	248461058	212259463	179348922	149430249	122231455	97505279	75026937	-	-	-
	3	-	-	-	361397902	308741037	260871160	217353089	177791207	141825860	109130090	-	-	-
Terminal expansion cost - Stage 2 - Stochastic Model	1	-	-	-	124230529	106129731	89674461	74715124	61115727	48752639	37513468	-	-	-
	2	-	-	-	143994477	123014007	103940853	86601621	70838684	56508741	43481520	-	-	-

APPENDIX D

SOURCE CODE (DETERMINISTIC MODEL)

```
!This is an implementation of the deterministic model developed in the Master Thesis
!Created by Marius Kongsfjell Fekene, Spring 2014
!Norwegian University of Science and Technology

model deterministic_model

options explterm,   !require statment termination with ;
        noimplicit  !require all symbols to be declared before use

uses  "mmxprs";     !MIP (integer or mixed integer programming)

!..............................................................................................
!                                            Importing data file
!..............................................................................................
parameters
DataFile = 'InputDeterministicModel(C.LastChance).txt';
end-parameters

!..............................................................................................
!                                          Declaration of indices
!..............................................................................................
declarations
Production:     set of integer;
Distribution:   set of integer;
Consumer:       set of integer;
Capacity:       set of integer;
Expand:         set of integer;
Time:           set of integer;
Fare:           set of integer;
end-declarations

!..............................................................................................
!                                    Declaration of the amount of indices
!..............................................................................................
declarations
AmountP:    integer;
AmountD:    integer;
AmountCo:   integer;
AmountCa:   integer;
AmountV:    integer;
AmountT:    integer;
AmountF:    integer;
end-declarations

!..............................................................................................
!                                     Retrieves paramters from datafile
!..............................................................................................
initializations from DataFile
AmountP;
AmountD;
AmountCo;
AmountCa;
AmountV;
AmountT;
AmountF;
end-initializations

!..............................................................................................
!                                          Definition of indices
!..............................................................................................
Production      := 1.. AmountP;
Distribution    := 1.. AmountD;
Consumer        := 1.. AmountCo;
Capacity        := 1.. AmountCa;
Expand          := 1.. AmountV;
Time            := 1.. AmountT;
Fare            := 1.. AmountF;

!..............................................................................................
!                                          Finalizing of indices
!..............................................................................................
finalize(Production);
finalize(Distribution);
finalize(Consumer);
finalize(Capacity);
finalize(Expand);
finalize(Time);
finalize(Fare);

!..............................................................................................
!                                         Declaration of variables
!..............................................................................................
declarations
X:                   dynamic array (Fare,Production,Distribution,Time)    of mpvar;
Y:                   dynamic array (Distribution,Consumer,Time)           of mpvar;
Z:                   dynamic array (Fare,Production,Distribution,Time)    of mpvar;
```

```
Delta1:             dynamic array (Distribution,Capacity,Time)          of mpvar;
Alpha:              dynamic array (Distribution,Expand,Time)            of mpvar;

!.....................................................................................
!                                        Declaration of Parameters
!.....................................................................................
Revenue:            array (Time)                                of integer;
TransportCostPD:    array (Production,Distribution)             of integer;
TransportCostDC:    array (Distribution,Consumer)               of integer;
TerminalCost:       array (Capacity,Time)                       of integer;
DiscountR:          array (Fare)                                of real;
Threshold:          array (Fare)                                of integer;
ExpandCost:         array (Expand,Time)                         of integer;
Demand:             array (Time,Consumer)                       of integer;
CapacityDi:         array (Capacity)                            of integer;
CapacityEx:         array (Expand)                              of integer;
CapacityPr:         array (Production)                          of integer;
DistanceS:          array (Production,Distribution)             of integer;
DiscountF:          array (Time)                                of real;
tLead:                                                          integer;

!.....................................................................................
!                            Declaration of ObjectiveFunction and Constraints
!.....................................................................................
ObjectiveFunction:                                              linctr;
Constraint1:        array(Production,Time)                      of linctr;
Constraint2:        array(Consumer,Time)                        of linctr;
Constraint3:        array(Distribution,Time)                    of linctr;
Constraint4:        array(Distribution,Time)                    of linctr;
Constraint5:        array(Distribution,Time)                    of linctr;
Constraint6:        array(Fare,Production,Distribution,Time)    of linctr;
Constraint7:        array(Fare,Production,Distribution,Time)    of linctr;
Constraint67:       array(Fare,Production,Distribution,Time)    of linctr;
Constraint8:        array(Production,Distribution,Time)         of linctr;
end-declarations

!.....................................................................................
!                            Retrieves the rest of the paramters from datafile
!.....................................................................................
initializations from DataFile
Revenue;
TransportCostPD;
TransportCostDC;
TerminalCost;
DiscountR;
Threshold;
ExpandCost;
Demand;
CapacityDi;
CapacityEx;
CapacityPr;
tLead;
DistanceS;
DiscountF;
end-initializations

!.....................................................................................
!                                              Creation Variables
!.....................................................................................
forall (ff in Fare,pp in Production,dd in Distribution,tt in Time|tt>(tLead)) do
    if DistanceS(pp,dd)<800 then !varaibale reduction
        create(X(ff,pp,dd,tt));
    end-if
end-do

forall (dd in Distribution,cc in Consumer,tt in Time|tt>(tLead)) do
        create(Y(dd,cc,tt));
end-do

forall (ff in Fare,pp in Production,dd in Distribution,tt in Time|tt>(tLead)) do
    create(Z(ff,pp,dd,tt));
end-do

forall (dd in Distribution,kk in Capacity,tt in Time|tt<=(AmountT-tLead)) do  !variable reduction
    create(Delta1(dd,kk,tt));
end-do

forall (dd in Distribution,ee in Expand,tt in Time|tt<=(AmountT-tLead)) do  !variable reduction
    create(Alpha(dd,ee,tt));
end-do

!.....................................................................................
!                                         Creation of Binary Variables
!.....................................................................................
forall (dd in Distribution,kk in Capacity,tt in Time) do
    Delta1(dd,kk,tt) is_binary;
```

```
end-do

forall (ff in Fare,pp in Production,dd in Distribution,tt in Time) do
    Z(ff,pp,dd,tt) is_binary;
end-do

forall (dd in Distribution,ee in Expand,tt in Time) do
    Alpha(dd,ee,tt) is_binary;
end-do

!...............................................................................................
!                                           Objective function
!...............................................................................................
ObjectiveFunction:=
+ sum(tt in Time) (sum(cc in Consumer) (sum(dd in Distribution)
  Revenue(tt)*Y(dd,cc,tt)))                                                          !(5.0a)
- sum(tt in Time) (sum(kk in Capacity) (sum(dd in Distribution)
  TerminalCost(kk,tt)*Delta1(dd,kk,tt)))                                             !(5.0b)
- sum(tt in Time) (sum(dd in Distribution) (sum(pp in Production) (sum(ff in Fare)
  DiscountF(tt)*DiscountR(ff)*TransportCostPD(pp,dd)*X(ff,pp,dd,tt))))               !(5.0c)
- sum(tt in Time) (sum(cc in Consumer) (sum(dd in Distribution)
  DiscountF(tt)*TransportCostDC(dd,cc)*Y(dd,cc,tt)))                                 !(5.0d)
- sum(tt in Time) (sum(ee in Expand) (sum(dd in Distribution)
  ExpandCost(ee,tt)*Alpha(dd,ee,tt)));                                               !(5.0e)

!...............................................................................................
!                                              Constraints
!...............................................................................................
!Constraint 5.1
forall(pp in Production,tt in Time|(tt)>(tLead)) do
    Constraint1(pp,tt):=
            sum(ff in Fare) (sum(dd in Distribution) X(ff,pp,dd,tt)) <= CapacityPr(pp);
end-do

!Constraint 5.2
forall(cc in Consumer,tt in Time) do
    Constraint2(cc,tt):=
            sum(dd in Distribution) Y(dd,cc,tt) <= Demand(tt,cc);
end-do

!Constraint 5.3
forall(dd in Distribution,tt in Time) do
    Constraint3(dd,tt):=
             sum(pp in Production) (sum(ff in Fare) X(ff,pp,dd,tt))
             = sum(cc in Consumer) Y(dd,cc,tt);
end-do

!Constraint 5.4
forall(dd in Distribution,tt in Time|(tt)>tLead) do
    Constraint4(dd,tt):=
             sum(pp in Production)(sum(ff in Fare) X(ff,pp,dd,tt))
             <=(sum(kk in Capacity) (CapacityDi(kk)*(sum(ii in (tLead+1)..tt)Delta1(dd,kk,ii-tLead)))
             + sum(ii in (tLead+1)..tt) (sum(ee in Expand) CapacityEx(ee)*Alpha(dd,ee,ii-tLead)));
end-do

!Constraint 5.5
forall(dd in Distribution,tt in Time|(tt)>tLead) do
    Constraint5(dd,tt):=
             sum(kk in Capacity) (sum(ii in 1..tt) Delta1(dd,kk,ii))
             >=sum(ii in 1..tt) (sum(ee in Expand) Alpha(dd,ee,ii));
end-do


!Part 1 ofConstraint 5.6
forall(ff in Fare,pp in Production,dd in Distribution,tt in Time|(ff)>1) do
    Constraint6(ff,pp,dd,tt):=
            Threshold(ff-1)*Z(ff,pp,dd,tt)<= X(ff,pp,dd,tt);
end-do

!Part 1 of Constraint 5.7
forall(ff in Fare, pp in Production,dd in Distribution,tt in Time|(ff)=1) do
    Constraint6(ff,pp,dd,tt):=
            0 <= X(ff,pp,dd,tt);
end-do

!Part 2 of Constraint 5.6 and 5.7
forall(ff in Fare,pp in Production,dd in Distribution,tt in Time) do
    Constraint67(ff,pp,dd,tt):=
            X(ff,pp,dd,tt) <= Threshold(ff)*Z(ff,pp,dd,tt);
end-do

!Constraint 5.8
forall(pp in Production,dd in Distribution,tt in Time) do
    Constraint8(pp,dd,tt):=
            sum(ff in Fare) Z(ff,pp,dd,tt) <= 1;
end-do
```

```
!..............................................................................................
!                                    Maximization of objective function
!..............................................................................................
maximize(ObjectiveFunction);

!..............................................................................................
!                                             Writing Output
!..............................................................................................
fopen("Deterministic-result.txt", F_OUTPUT);

writeln('ObjectiveFunctionValue: ',getsol(ObjectiveFunction));

writeln('New Terminals: (',strfmt(sum(dd in Distribution)(sum(kk in Capacity) (sum(tt in Time)
        (getsol(Delta1(dd,kk,tt))))),1),'/',AmountD,')');

forall  (dd in Distribution,kk in Capacity,tt in Time|getsol(Delta1(dd,kk,tt)) > 0.1) do
        writeln('(T: ',tt,')','  Terminal ',dd,' ','(Ca: ',kk,')');
end-do

        writeln;

writeln('Expand Terminals: (',strfmt(sum(dd in Distribution)(sum(ee in Expand) (sum(tt in Time)
        (getsol(Alpha(dd,ee,tt))))),1),'/',AmountD,')');
forall  (dd in Distribution,ee in Expand,tt in Time|getsol(Alpha(dd,ee,tt))> 0.1) do
        writeln('(T: ',tt,')','  Terminal ',dd,' ','(Ca: ',ee,')');
end-do

    writeln;

writeln('From Production to Distribution:');
forall  (ff in Fare,pp in Production, dd in Distribution,tt in Time|
        getsol(X(ff,pp,dd,tt)) > 0.1) do
    write(tt,'  ',strfmt(getsol(X(ff,pp,dd,tt)),4),'    ' );
    writeln(pp, '   ',dd);
end-do

    writeln;

writeln('From Distribution to Consumer:');
forall  (dd in Distribution,cc in Consumer,tt in Time| getsol(Y(dd,cc,tt)) > 0.1) do
    write(tt,'  ',strfmt(getsol(Y(dd,cc,tt)),4),'   ' );
    writeln(dd, '   ',cc);
end-do

    writeln;

writeln('Part of Y served:');
forall  (dd in Distribution,cc in Consumer,tt in Time|(tt)>tLead and getsol(Y(dd,cc,tt))>0.9) do
    writeln(dd,' to ',cc,'  ','  (T:',tt,')   (',getsol(Y(dd,cc,tt)),')/(',getsol(Demand(tt,cc)),')'
end-do

fclose(F_OUTPUT);

end-model
```

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APPENDIX E

SOURCE CODE (STOCHASTIC MODEL)

```
!This is an implementation of the stochastic model developed in the Master Thesis
!Created by Marius Kongsfjell Fekene, Spring 2014
!Norwegian University of Science and Technology

model stochastic_model

options explterm,   !require statment termination with ;
        noimplicit !require all symbols to be declared before use

uses  "mmxprs";     !MIP (integer or mixed integer programming)

!...............................................................................................
!                                          Importing data file
!...............................................................................................
parameters
DataFile = 'InputStochasticModel(C.LastChance).txt';
RUNTIME = 10000;
end-parameters

setparam("XPRS_maxtime", RUNTIME);

!...............................................................................................
!                                          Declaration of indices
!...............................................................................................
declarations
Production:     set of integer;
Distribution:   set of integer;
Consumer:       set of integer;
Scenario:       set of integer;
Capacity:       set of integer;
Expand:         set of integer;
Time:           set of integer;
Fare:           set of integer;
end-declarations

!...............................................................................................
!                                   Declaration of the amount of indices
!...............................................................................................
declarations
AmountP:    integer;
AmountD:    integer;
AmountCo:   integer;
AmountS:    integer;
AmountCa:   integer;
AmountV:    integer;
AmountT:    integer;
AmountF:    integer;
end-declarations

!...............................................................................................
!                                   Retrieves paramters from datafile
!...............................................................................................
initializations from DataFile
AmountP;
AmountD;
AmountCo;
AmountS;
AmountCa;
AmountV;
AmountT;
AmountF;
end-initializations

!...............................................................................................
!                                          Definition of indices
!...............................................................................................
Production      := 1.. AmountP;
Distribution    := 1.. AmountD;
Consumer        := 1.. AmountCo;
Scenario        := 1.. AmountS;
Capacity        := 1.. AmountCa;
Expand          := 1.. AmountV;
Time            := 1.. AmountT;
Fare            := 1.. AmountF;

!...............................................................................................
!                                          Finalizing of indices
!...............................................................................................
finalize(Production);
finalize(Distribution);
finalize(Consumer);
finalize(Scenario);
finalize(Capacity);
finalize(Expand);
finalize(Time);
finalize(Fare);
```

```
!..............................................................................................................
!                                          Declaration of variables
!..............................................................................................................
declarations
X:                  dynamic array (Fare,Production,Distribution,Time,Scenario)  of mpvar;
Y:                  dynamic array (Distribution,Consumer,Time,Scenario)         of mpvar;
Z:                  dynamic array (Fare,Production,Distribution,Time,Scenario)  of mpvar;
Delta1:             dynamic array (Distribution,Capacity)                       of mpvar;
Delta2:             dynamic array (Distribution,Capacity,Time,Scenario)         of mpvar;
Alpha:              dynamic array (Distribution,Expand,Time,Scenario)           of mpvar;

!..............................................................................................................
!                                          Declaration of Parameters
!..............................................................................................................
Revenue:            array (Time)                                 of integer;
TransportCostPD:    array (Production,Distribution)              of integer;
TransportCostDC:    array (Distribution,Consumer)                of integer;
TerminalCost1:      array (Capacity)                             of integer;
TerminalCost2:      array (Capacity,Time)                        of integer;
DiscountR:          array (Fare)                                 of real;
Threshold:          array (Fare)                                 of integer;
ExpandCost:         array (Expand,Time)                          of integer;
DemandS1:           array (Time,Consumer)                        of integer;
DemandS2:           array (Time,Consumer)                        of integer;
DemandS3:           array (Time,Consumer)                        of integer;
CapacityDi:         array (Capacity)                             of integer;
CapacityEx:         array (Expand)                               of integer;
CapacityPr:         array (Production)                           of integer;
Probability:        array (Scenario)                             of real;
DistanceS:          array (Production,Distribution)              of integer;
DiscountF:          array (Time)                                 of real;
t1:                                                                 integer;
tLead:                                                              integer;

!..............................................................................................................
!                              Declaration of ObjectiveFunction and Constraints
!..............................................................................................................
ObjectiveFunction:                                                             linctr;
Constraint1:        array(Production,Time,Scenario)                         of linctr;
Constraint2:        array(Consumer,Time)                                    of linctr;
Constraint3:        array(Distribution,Time,Scenario)                       of linctr;
Constraint4:        array(Distribution,Time,Scenario)                       of linctr;
Constraint5:        array(Distribution,Time,Scenario)                       of linctr;
Constraint6:        array(Distribution,Scenario)                            of linctr;
Constraint7:        array(Distribution,Time,Scenario)                       of linctr;
Constraint8:        array(Fare,Production,Distribution,Time,Scenario)       of linctr;
Constraint9:        array(Fare,Production,Distribution,Time,Scenario)       of linctr;
Constraint89:       array(Fare,Production,Distribution,Time,Scenario)       of linctr;
Constraint10:       array(Production,Distribution,Time,Scenario)            of linctr;

end-declarations

!..............................................................................................................
!                              Retrieves the rest of the paramters from datafile
!..............................................................................................................
initializations from DataFile
Revenue;
TransportCostPD;
TransportCostDC;
TerminalCost1;
TerminalCost2;
DiscountR;
Threshold;
ExpandCost;
DemandS1;
DemandS2;
DemandS3;
CapacityDi;
CapacityEx;
CapacityPr;
Probability;
DistanceS;
DiscountF;
t1;
tLead;
end-initializations

!..............................................................................................................
!                                          Creation of Variables
!..............................................................................................................
forall (ff in Fare,pp in Production,dd in Distribution,tt in Time,ss in Scenario|tt>(tLead)) do
    if DistanceS(pp,dd)<800 then     !variable reduction
        create(X(ff,pp,dd,tt,ss));
    end-if
end-do
```

```
forall (dd in Distribution,cc in Consumer,tt in Time,ss in Scenario|tt>(tLead)) do
        create(Y(dd,cc,tt,ss));
end-do

forall (ff in Fare,pp in Production,dd in Distribution,tt in Time,ss in Scenario|tt>(tLead)) do
    create(Z(ff,pp,dd,tt,ss));
end-do

forall (dd in Distribution,kk in Capacity) do
    create(Delta1(dd,kk));
end-do

forall (dd in Distribution,kk in Capacity,tt in Time,ss in Scenario|
       (tt)>(t1)and (tt)<=(AmountT-tLead)   ) do  !variable reduction
    create(Delta2(dd,kk,tt,ss));
end-do

forall (dd in Distribution,ee in Expand,tt in Time,ss in Scenario|
       (tt)>(t1)and (tt)<=(AmountT-tLead)) do   !variable reduction
    create(Alpha(dd,ee,tt,ss));
end-do

!.........................................................................................
!                                          Creation of Binary Variables
!.........................................................................................
forall (dd in Distribution,kk in Capacity) do
    Delta1(dd,kk) is_binary;
end-do

forall (ff in Fare,pp in Production,dd in Distribution,tt in Time,ss in Scenario|tt>(tLead)) do
    Z(ff,pp,dd,tt,ss) is_binary;
end-do

forall (dd in Distribution,kk in Capacity,tt in Time,ss in Scenario|(tt)>(t1)) do
    Delta2(dd,kk,tt,ss) is_binary;
end-do

forall (dd in Distribution,ee in Expand,tt in Time,ss in Scenario|(tt)>(t1)) do
    Alpha(dd,ee,tt,ss) is_binary;
end-do

!.........................................................................................
!                                              Objective function
!.........................................................................................
ObjectiveFunction:=
- sum(kk in Capacity) (sum(dd in Distribution)   TerminalCost1(kk)*Delta1(dd,kk))         !(5.14a)
+ sum(ss in Scenario) Probability(ss)*(
    + (sum(tt in Time) (sum(cc in Consumer) (sum(dd in Distribution)
        Revenue(tt)*Y(dd,cc,tt,ss))))                                                     !(5.14b)
    - (sum(tt in Time) (sum(dd in Distribution) (sum(pp in Production) (sum(ff in Fare)
        DiscountF(tt)*DiscountR(ff)*TransportCostPD(pp,dd)*X(ff,pp,dd,tt,ss)))))         !(5.14c)
    - (sum(tt in Time) (sum(cc in Consumer) (sum(dd in Distribution)
        DiscountF(tt)*TransportCostDC(dd,cc)*Y(dd,cc,tt,ss))))                            !(5.14d)
    - (sum(tt in Time) (sum(kk in Capacity) (sum(dd in Distribution)
        TerminalCost2(kk,tt)*Delta2(dd,kk,tt,ss))))                                       !(5.14e)
    - (sum(tt in Time) (sum(ee in Expand) (sum(dd in Distribution)
        ExpandCost(ee,tt)*Alpha(dd,ee,tt,ss)))));                                          !(5.14f)

!.........................................................................................
!                                               Constraints
!.........................................................................................
!Constraint 5.15
forall(pp in Production,tt in Time,ss in Scenario|(tt)>(tLead)) do
    Constraint1(pp,tt,ss):=
            sum(ff in Fare) (sum(dd in Distribution) X(ff,pp,dd,tt,ss)) <= CapacityPr(pp);
end-do

!Constraint 5.16
forall(cc in Consumer,tt in Time) do
    Constraint2(cc,tt):=
            sum(dd in Distribution) Y(dd,cc,tt,1) <= DemandS1(tt,cc);
            sum(dd in Distribution) Y(dd,cc,tt,2) <= DemandS2(tt,cc);
            sum(dd in Distribution) Y(dd,cc,tt,3) <= DemandS3(tt,cc);
end-do

!Constraint 5.17
forall(dd in Distribution,ss in Scenario,tt in Time|(tt)>(tLead)) do
    Constraint3(dd,tt,ss):=
            sum(ff in Fare) (sum(pp in Production) X(ff,pp,dd,tt,ss))
            = sum(cc in Consumer) Y(dd,cc,tt,ss);
end-do

!Constraint 5.18
forall(dd in Distribution,tt in Time,ss in Scenario|(tt)<=(t1+tLead)) do
    Constraint4(dd,tt,ss):=
```

```
        sum(ff in Fare) (sum(pp in Production) X(ff,pp,dd,tt,ss))
        <= sum(kk in Capacity) CapacityDi(kk)*Delta1(dd,kk);
end-do

!Constraint 5.19
forall(dd in Distribution,tt in Time,ss in Scenario|(tt)>(t1+tLead)) do
    Constraint5(dd,tt,ss):=
        sum(ff in Fare) (sum(pp in Production) X(ff,pp,dd,tt,ss))
        <=  (sum(kk in Capacity) CapacityDi(kk)*(Delta1(dd,kk)
        + sum(ii in (tLead+t1+1)..tt) Delta2(dd,kk,ii-tLead,ss))
        + sum(ii in (tLead+t1+1)..tt) (sum(ee in Expand) CapacityEx(ee)*Alpha(dd,ee,ii-tLead,ss))
end-do

!Constraint 5.20
forall(dd in Distribution,ss in Scenario) do
    Constraint6(dd,ss):=
         sum(kk in Capacity) (Delta1(dd,kk) + sum(tt in Time) Delta2(dd,kk,tt,ss)) <= 1;
end-do

!Constraint 5.21
forall(dd in Distribution,tt in Time,ss in Scenario) do
    Constraint7(dd,tt,ss):=
         sum(kk in Capacity) (Delta1(dd,kk)
         + sum(ii in (tLead+t1+1)..tt) Delta2(dd,kk,ii-tLead,ss))
         >=sum(ii in (tLead+t1+1)..tt) (sum(ee in Expand) Alpha(dd,ee,ii-tLead,ss));
end-do

!Part 1 Constraint 5.22
forall(ff in Fare,pp in Production,dd in Distribution,tt in Time,ss in Scenario|
    (ff)>1 and (tt)>(tLead)) do
    Constraint8(ff,pp,dd,tt,ss):=
        Threshold(ff-1)*Z(ff,pp,dd,tt,ss)<= X(ff,pp,dd,tt,ss);
end-do
!Part 1 Constraint 5.23
forall(ff in Fare,pp in Production,dd in Distribution,tt in Time,ss in Scenario|
    (ff)=1 and (tt)>(tLead)) do
    Constraint9(ff,pp,dd,tt,ss):=
        0 <= X(ff,pp,dd,tt,ss);
end-do

!Part 2 Constraint 5.22 and 5.23
forall(ff in Fare,pp in Production,dd in Distribution,tt in Time,ss in Scenario| (tt)>(tLead)) do
    Constraint89(ff,pp,dd,tt,ss):=
        X(ff,pp,dd,tt,ss) <= Threshold(ff)*Z(ff,pp,dd,tt,ss);
end-do

!Constraint 5.24
forall(pp in Production,dd in Distribution,tt in Time,ss in Scenario|(tt)>(tLead)) do
    Constraint10(pp,dd,tt,ss):=
        sum(ff in Fare) Z(ff,pp,dd,tt,ss) <= 1;
end-do

!.....................................................................................................
!                                    Maximization of objective function
!.....................................................................................................
maximize(ObjectiveFunction);

!.....................................................................................................
!                                              Writing Output
!.....................................................................................................
fopen("Stochastic-result.txt", F_OUTPUT);

    writeln;

writeln('ObjectiveFunctionValue: ',getsol (ObjectiveFunction));

    writeln;

writeln('Open Terminals (stage 1):');

writeln('(',strfmt(sum(dd in Distribution)(sum(kk in Capacity)
        (getsol(Delta1(dd,kk)))),1),'/',AmountD,')');
forall  (dd in Distribution,kk in Capacity| getsol(Delta1(dd,kk)) = 1) do
        writeln('  Terminal ',dd,' ','(Ca: ',kk,')');
end-do

    writeln;

writeln('Stage 2 Decisions:');
writeln('Scenario 1:');

writeln('New Terminals: (',strfmt(sum(dd in Distribution)(sum(kk in Capacity) (sum(tt in Time)
        (getsol(Delta2(dd,kk,tt,1))))),1),'/',AmountD,')');

forall  (dd in Distribution,kk in Capacity,tt in Time,ss in Scenario|getsol(Delta2(dd,kk,tt,ss)) = 1)
        if (ss = 1) then
```

```
            writeln('(T:',tt,')',' Terminal ',dd,' ','(Ca: ',kk,')');
        end-if
end-do

    writeln;
writeln('Expand Terminals: (',strfmt(sum(dd in Distribution)(sum(ee in Expand) (sum(tt in Time)
        (getsol(Alpha(dd,ee,tt,1))))),1),'/',AmountD,')');
forall (dd in Distribution,ee in Expand,tt in Time,ss in Scenario|getsol(Alpha(dd,ee,tt,ss)) = 1) do
        if (ss = 1) then
            writeln('(T:',tt,')',' Terminal ',dd,' ','(Ca: ',ee,')');
        end-if
end-do

    writeln;
writeln('Scenario 2:');
writeln('New Terminals: (',strfmt(sum(dd in Distribution)(sum(kk in Capacity) (sum(tt in Time)
        (getsol(Delta2(dd,kk,tt,2))))),1),'/',AmountD,')');
forall (dd in Distribution,kk in Capacity,tt in Time,ss in Scenario|getsol(Delta2(dd,kk,tt,ss)) = 1)
        if (ss = 2) then
            writeln('(T:',tt,')',' Terminal ',dd,' ','(Ca: ',kk,')');
        end-if
end-do

    writeln;
writeln('Expand Terminals: (',strfmt(sum(dd in Distribution)(sum(ee in Expand) (sum(tt in Time)
        (getsol(Alpha(dd,ee,tt,2))))),1),'/',AmountD,')');
forall (dd in Distribution,ee in Expand,tt in Time,ss in Scenario|getsol(Alpha(dd,ee,tt,ss)) = 1) do
        if (ss = 2) then
            writeln('(T:',tt,')',' Terminal ',dd,' ','(Ca: ',ee,')');
        end-if
end-do

    writeln;
writeln('Scenario 3:');
writeln('New Terminals: (',strfmt(sum(dd in Distribution)(sum(kk in Capacity) (sum(tt in Time)
        (getsol(Delta2(dd,kk,tt,3))))),1),'/',AmountD,')');
forall (dd in Distribution,kk in Capacity,tt in Time,ss in Scenario|getsol(Delta2(dd,kk,tt,ss)) = 1)
        if (ss = 3) then
            writeln('(T:',tt,')',' Terminal ',dd,' ','(Ca: ',kk,')');
        end-if
end-do

    writeln;
writeln('Expand Terminals: (',strfmt(sum(dd in Distribution)(sum(ee in Expand) (sum(tt in Time)
        (getsol(Alpha(dd,ee,tt,3))))),1),'/',AmountD,')');
forall (dd in Distribution,ee in Expand,tt in Time,ss in Scenario|getsol(Alpha(dd,ee,tt,ss)) = 1) do
        if (ss = 3) then
            writeln('(T:',tt,')',' Terminal ',dd,' ','(Ca: ',ee,')');
        end-if
end-do

    writeln;

writeln('From Production to Distribution:');
writeln('Scenario 1:');
forall (ff in Fare,pp in Production, dd in Distribution,tt in Time,ss in Scenario|
        getsol(X(ff,pp,dd,tt,ss)) > 0.1) do
    if (ss = 1) then
        write(tt,'  ',strfmt(getsol(X(ff,pp,dd,tt,ss)),4),' ' );
        writeln(pp, '    ',dd);
    end-if
end-do

    writeln;

writeln('Scenario 2:');
forall (ff in Fare,pp in Production, dd in Distribution,tt in Time,ss in Scenario|
        getsol(X(ff,pp,dd,tt,ss)) > 0.1) do
    if (ss = 2) then
        write(tt,'  ',strfmt(getsol(X(ff,pp,dd,tt,ss)),4),' ' );
        writeln(pp, '    ',dd);
    end-if
end-do

    writeln;

writeln('Scenario 3:');
forall (ff in Fare,pp in Production, dd in Distribution,tt in Time,ss in Scenario|
        getsol(X(ff,pp,dd,tt,ss)) > 0.1) do
    if (ss = 3) then
        write(tt,'  ',strfmt(getsol(X(ff,pp,dd,tt,ss)),4),' ' );
        writeln(pp, '    ',dd);
    end-if
end-do

    writeln;
```

```
writeln('From Distribution to Consumer:');
writeln('Scenario 1:');
forall (dd in Distribution,cc in Consumer,tt in Time,ss in Scenario| getsol(Y(dd,cc,tt,ss)) > 0.1) do
    if (ss = 1) then
        write(tt,'  ',strfmt(getsol(Y(dd,cc,tt,ss)),4),'     ' );
        writeln(dd, '    ',cc);
    end-if
end-do

writeln('Scenario 2:');
forall (dd in Distribution,cc in Consumer,tt in Time,ss in Scenario| getsol(Y(dd,cc,tt,ss)) > 0.1) do
    if (ss = 2) then
        write(tt,'  ',strfmt(getsol(Y(dd,cc,tt,ss)),4),'     ' );
        writeln(dd, '    ',cc);
    end-if
end-do

writeln('Scenario 3:');
forall (dd in Distribution,cc in Consumer,tt in Time,ss in Scenario| getsol(Y(dd,cc,tt,ss)) > 0.1) do
    if (ss = 3) then
        write(tt,'  ',strfmt(getsol(Y(dd,cc,tt,ss)),4),'     ' );
        writeln(dd, '    ',cc);
    end-if
end-do

    writeln;

writeln('Part of Y served in Scenario 1:');
forall (dd in Distribution,cc in Consumer,tt in Time|(tt)>t1 and getsol(Y(dd,cc,tt,1))>0.9) do
    writeln(dd,' to ',cc,'  ',' (T:',tt,')    (',getsol(Y(dd,cc,tt,1)),')/(',
    getsol(DemandS1(tt,cc)),')' );
end-do

    writeln;

writeln('Part of Y served in Scenario 2:');
forall (dd in Distribution,cc in Consumer,tt in Time|(tt)>t1 and getsol(Y(dd,cc,tt,1))>0.9) do
    writeln(dd,' to ',cc,'  ',' (T:',tt,')    (',getsol(Y(dd,cc,tt,2)),')/(',
    getsol(DemandS2(tt,cc)),')' );
end-do

    writeln;

writeln('Part of Y served in Scenario 3:');
forall (dd in Distribution,cc in Consumer,tt in Time|(tt)>t1 and getsol(Y(dd,cc,tt,1))>0.9) do
    writeln(dd,' to ',cc,'  ',' (T:',tt,')    (',getsol(Y(dd,cc,tt,3)),')/(',
    getsol(DemandS3(tt,cc)),')' );
end-do

    writeln;

fclose(F_OUTPUT);


end-model
```

```
writeln('From Distribution to Consumer:');
writeln('Scenario 1:');
forall  (dd in Distribution,cc in Consumer,tt in Time,ss in Scenario| getsol(Y(dd,cc,tt,ss)) > 0.1) do
    if (ss = 1) then
        write(tt,'  ',strfmt(getsol(Y(dd,cc,tt,ss)),4),'     ' );
        writeln(dd, '   ',cc);
    end-if
end-do

writeln('Scenario 2:');
forall  (dd in Distribution,cc in Consumer,tt in Time,ss in Scenario| getsol(Y(dd,cc,tt,ss)) > 0.1) do
    if (ss = 2) then
        write(tt,'  ',strfmt(getsol(Y(dd,cc,tt,ss)),4),'     ' );
        writeln(dd, '   ',cc);
    end-if
end-do

writeln('Scenario 3:');
forall  (dd in Distribution,cc in Consumer,tt in Time,ss in Scenario| getsol(Y(dd,cc,tt,ss)) > 0.1) do
    if (ss = 3) then
        write(tt,'  ',strfmt(getsol(Y(dd,cc,tt,ss)),4),'     ' );
        writeln(dd, '   ',cc);
    end-if
end-do

    writeln;

writeln('Part of Y served in Scenario 1:');
forall  (dd in Distribution,cc in Consumer,tt in Time|(tt)>t1 and getsol(Y(dd,cc,tt,1))>0.9) do
    writeln(dd,' to ',cc,'  ',' (T:',tt,')   (',getsol(Y(dd,cc,tt,1)),')/(',
    getsol(DemandS1(tt,cc)),')' );
end-do

    writeln;

writeln('Part of Y served in Scenario 2:');
forall  (dd in Distribution,cc in Consumer,tt in Time|(tt)>t1 and getsol(Y(dd,cc,tt,1))>0.9) do
    writeln(dd,' to ',cc,'  ',' (T:',tt,')   (',getsol(Y(dd,cc,tt,2)),')/(',
    getsol(DemandS2(tt,cc)),')' );
end-do

    writeln;

writeln('Part of Y served in Scenario 3:');
forall  (dd in Distribution,cc in Consumer,tt in Time|(tt)>t1 and getsol(Y(dd,cc,tt,1))>0.9) do
    writeln(dd,' to ',cc,'  ',' (T:',tt,')   (',getsol(Y(dd,cc,tt,3)),')/(',
    getsol(DemandS3(tt,cc)),')' );
end-do

    writeln;

fclose(F_OUTPUT);


end-model
```