NTNU – Trondheim
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RISER CONCEPTS FOR DEEP WATERS

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Abstract

Oil and gas exploration and production activities in deep and ultra deep waters in hostile environments necessitates the need to develop innovative riser systems capable of ensuring transfer of fluids from the seabed to a floating vessel and vice versa, with little or no issues with respect to influences of environmental loads and vessel motions.

The design of the riser system must focus on different types of loading and load effects than for traditional water-depth. A variety of different riser concepts are proposed, both with respect to geometric shape and selection of materials.

In the last few years, steel catenary risers have been a preferred riser solution for deep-water field developments due to its simple engineering concept, cost effective, flexibility in using different host platform and flexibility in geographical and environmental conditions.

In this report, a case study considering a steel catenary riser operating in 1000 m water depth was conducted. The riser was subjected to extreme environmental conditions and static and dynamic response analyses were performed by the computer program RIFLEX.

Last, parametric study is carried out to investigate the effects of parameter variation based on some parameters like current profiles, mesh density, wall thickness and so on. These parameters have significant effect on the structural response, especially in the touch down region.

Preface

This report with title “RISER CONCEPTS FOR DEEP WATERS” is the result of my Master‘s thesis for 5th year students at The Norwegian University of Science and Technology (NTNU) at the Department of Marine Technology the spring of 2012.

I would like to thank my advisor, Professor Bernt J.Leira at the Department of Marine Technology, NTNU for excellent guidance and deep insight on the topic. I am also grateful to Elizabeth Passano at Marintek for helping me on RIFLEX program.

Trondheim, Norway
June 11, 2012
Halil Dikdogmus

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for
Stud. Techn. Halil Dikdogmus

RISER CONCEPTS FOR DEEP WATERS

Stigerør-system for dypt vann

Future exploration and development of oil and gas resources will to a large extent face the challenge of water-depths exceeding 1000m. This implies that design of the systems which connect the subsea installations to the surface floater will need to focus on other loading and load-effect regimes than for more traditional water-depths.

A variety of different riser concepts are proposed, both with respect to geometric shape and selection of materials. Accordingly, methods for calculation of loads and load effects for the different configurations need to be both advanced and accurate, and generally numerical solution methods are required.

The candidate shall address the following topics:

1. Possible geometric shapes of the riser system shall be described (Vertical, simple and multiple catenary solutions). Various types of relevant materials for such applications are discussed (Steel, titanium, fiber-composites, flexible pipes). The candidate shall also summarize design procedures for critical types of end components (e.g. flex- and ball-joints, bend-stiffeners).

2. Methods for computation of load-effects along the riser shall be summarised. A brief summary of relevant guidelines for design of such systems shall also be given with focus on the Ultimate and Fatigue Limit States (ULS and FLS).

3. For a specific riser system, static and dynamic response analyses are to be performed. The type of system to be considered is specified by the supervisor based on discussion with the candidate. The computer program RIFLEX can be applied.

4. Parametric studies are performed for the selected riser system to the extent that time permits based on discussion with the supervisor. Both fatigue damage accumulation and extreme response (i.e. ULS and FLS) are to be considered.

The work scope may prove to be larger than initially anticipated. Subject to approval from the supervisor, topics may be deleted from the list above or reduced in extent.

In the thesis the candidate shall present his personal contribution to the resolution of problems within the scope of the thesis work.

Theories and conclusions should be based on mathematical derivations and/or logic reasoning identifying the various steps in the deduction.

The candidate should utilise the existing possibilities for obtaining relevant literature.

The thesis should be organised in a rational manner to give a clear exposition of results, assessments, and conclusions. The text should be brief and to the point, with a clear language. Telegraphic language should be avoided.

The thesis shall contain the following elements: A text defining the scope, preface, list of contents, summary, main body of thesis, conclusions with recommendations for further work, list of symbols and acronyms, references and (optional) appendices. All figures, tables and equations shall be numbered.

The supervisor may require that the candidate, in an early stage of the work, presents a written plan for the completion of the work. The plan should include a budget for the use of computer and laboratory resources which will be charged to the department. Overruns shall be reported to the supervisor.

The original contribution of the candidate and material taken from other sources shall be clearly defined. Work from other sources shall be properly referenced using an acknowledged referencing system.

The thesis shall be submitted in 3 copies:

- Signed by the candidate
- The text defining the scope included
- In bound volume(s)
- Drawings and/or computer prints which cannot be bound should be organised in a separate folder.

Supervisor: Professor Bernt J. Leira

Deadline: June 11th 2012

Trondheim, January 16th, 2012

Bernt J. Leira

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Nomenclature

Greek Characters

α_c	parameter accounting for strain hardening and wall thinning
α_{fab}	fabrication factor
σ_e	Von Mises equivalent stress
σ_1, σ_2, σ_3	Principal stresses
σ_a	Basic allowable combined stress
σ_y	Material minimum yield strength
ρ_i	density of the internal fluid
γ_A	load effect factor for accidental
γ_E	load effect factor for environmental
γ_F	load effect factor for functional

Symbol

A_e	external cross-sectional area
A_i	internal cross-sectional area
C_a	Added mass coefficient
C_d	Drag coefficient
C_m	Inertia coefficient
H_s	Significant Wave Height
P_a	Net allowable external design pressure
P_b	burst resistance
P_c	Predicted collapse pressure
P_d	design pressure differential
P_e	External pressure
T_{ed}	Design effective tension
T_{eA}	Effective tension from accidental loads
T_{eE}	Effective tension from environmental
T_{eF}	Effective tension from functional
T_p	Wave peak period

Abbreviations

ALS	Accidental Limit State
API	American Petroleum Institute
DNV	Det Norske Veritas
FLS	Fatigue Limit state
FPSO	Floating Production, Storage, and Offloading Unit
FPU	Floating Production Unit
JONSWAP	Joint North Sea Wave Project
LRFD	Load Resistance Factor Design
SCR	Steel Catenary Riser
RAO	Response Amplitude Operator
TDP	Touch Down Point
TTR	Top Tensioned Riser
ULS	Ultimate Limit State
WSD	Working Stress Design

CHAPTER 1 INTRODUCTION

1.1 Background

Deep sea is the newest and most exciting frontier of the offshore construction industry. The discovery of giant fields for oil and gas in deep water has presented a major challenge to the industry, resulting in remarkable developments in the way of equipment, procedures, instrumentation, and remote operations.

Advance riser technologies are one of important key elements for future oil and gas field development. Engineers are still trying to produce the most economical design and friendly technical solution of risers. There are a number of riser configurations that have been used in deepwater field such as flexible riser, steel catenary riser and hybrid riser.

Riser systems can form a significant proportion of the development costs of floating production systems, which are increasingly being considered for current and future field developments. Steel catenary risers offer a low cost alternative to conventionally used rigid and flexible risers on floating platforms and can also provide economic riser design solutions for fixed platforms. Therefore, in this report, a steel catenary riser concept will be considered in order to perform static and dynamic anaysis.

1.2 Scope

This report is the result of my Master’s thesis. Chapter 2 describes the different riser concepts with the differences due to its geometric shape (vertical risers, catenary riser), material (steel, titanium and composites) and end details (flex joints, bending stiffeners etc.) for deepwater field and harsh environments.

Chapter 3 explains the different loads which need to be considered for the static and dynamic analysis of the riser system. Chapter 4 describes the global response analysis of riser system. In chapter 5, DNV-OS-F201 Dynamic Riser and API RP 2RD Design of Risers for Floating Production Systems and Tension-Leg Platforms are provided as design codes for design of riser system. Chapter 6 provides a brief overview of the computer program RIFLEX.

In Chapter 7 , case study for a steel catenary riser is performed and the effects of parameter variation based on some parameters are investigated. Chapter 8 provides the conclusions from the study.

CHAPTER 2 DEEPWATER MARINE RISER SYSTEM

2.1 Introduction

A riser system is essentially conductor pipes connecting floaters on the surface and the wellheads at the seabed.

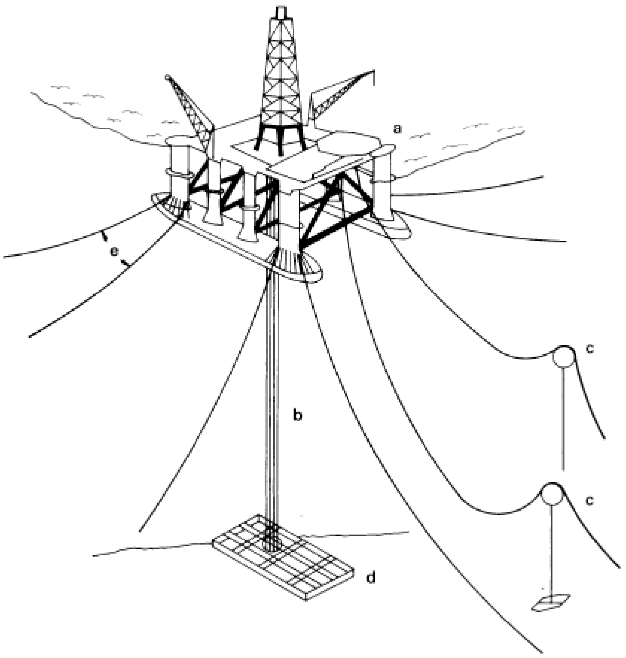

Figure Typical semi-submersible based floating production system: a – surface platform; b – multi-tube vertical drilling and production riser; c – flexible production risers; d – sea bed template; e – catenary moorings

Riser system is a key element in providing safety in all phases from drilling, completion/workover, production/injection to export. Main function of riser is to transport fluids or gas from seabed to a host platform. Additional functions of riser according to area of application are provided as follows [API, 1998]:

- Conveys fluid between the wells and the floater for production and injection risers.
- Export fluid from floater to pipeline for export riser.
- Guide drilling or workover tools and tubulars to and into the wells for drilling and workover riser.

Applications of riser system vary according to water depth and environmental conditions. The design of riser system for deepwater field is obviously more challenging than shallow water. Deepwater riser systems have been extensively applied at Gulf of Mexico, Brazil and West of Africa. In terms of environmental conditions, those locations are considered as benign to mild environmental condition. When it comes to North Sea, Norwegian Sea or Barent Sea, the environmental condition becomes harsh. It is predicted that the future oil and gas development will move to deepwater and harsh environments.

2.2 Marine Riser System

Typical elements of a riser system are [API, 1998]:

- Riser body: metal pipe or flexible pipe
- System Interfaces: top interface and bottom interface

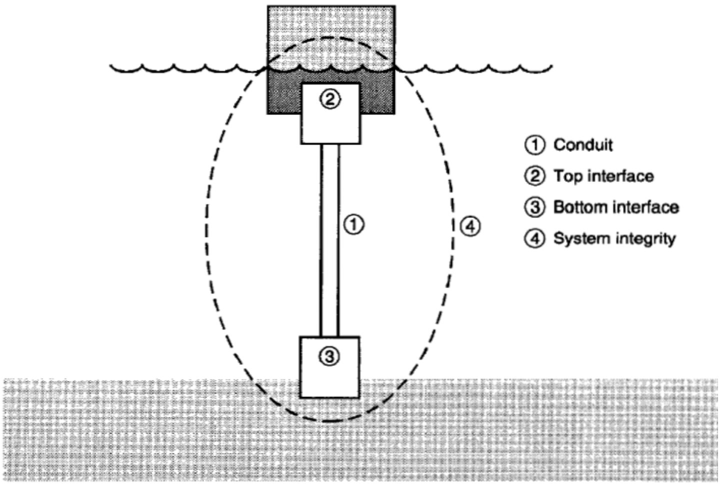

Figure 2-1 Essential Functional Elements of a Riser System

The riser system is in the interface between a static structure at the bottom interface and the dynamic floater structure at the top interface. The dynamic behaviour of floater at the surface is the main challenge for riser system design. This is the main reason for next categorizing of riser system according to the ability of riser system to cope with floater motion [DnV, 2001]:

- Top tensioned riser
- Compliant riser

Hybrid riser is the combination of tensioned and compliant risers.

2.2.1 Top Tensioned Riser

The riser in Top Tensioned Risers (TTRs) concept is supported in the floater by providing top tension force in order to maintain acceptable vertical movement. The horizontal motions of the floater induce stresses in the riser base and at the top end near the flex/keel joints. Typical TTRs applications can be seen from Figure 2.2.

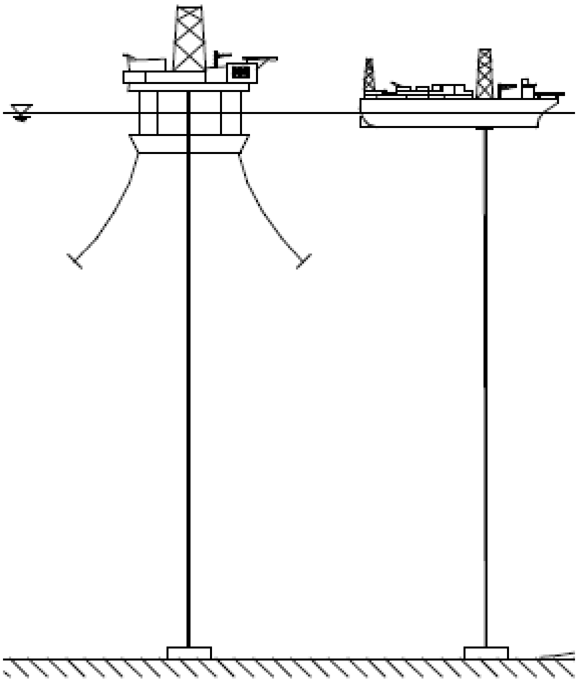

Figure 2-2 Top Tensioned Riser

TTRs are applied for dry tree production facilities such as SPARs or tension leg platforms (TLPs). SPARs and TLPs have small heave motion which is desirable for TTR concept. To some extent, semi-submersibles can also be considered as host platform for TTRs by incorporating separate heavy compensation system to account for the floater motions. Generally, TTR can be used for drilling, production, injection and export riser.

The TTR runs directly from the subsea well to the vessel deck where a surface tree is located. Tension is applied to the riser by either buoyancy cans or deck mounted hydro-pneumatic tensioners. For spars, the installation of buoyancy cans is a complex, costly and time-consuming process. Risers tensioned using hydro-pneumatic tensioners on spars or TLPs are less complex, and take less time to install in comparison to using buoyancy cans.

For deepwater application, the riser top tension requirements become significant to support riser weight and prevent bottom compression. The increase in riser tension affects the size of the tensioning system, the buoyancy requirements, as well as the size of the flex-joints or stress joints. In addition, harsh environments will give significant movement on the floaters and TTR itself. Therefore, at some level of combination between water depth and environmental conditions, TTR becomes technically unfeasible and uneconomical.

2.2.2 Compliant Riser

Compliant riser provides flexibility to cope with floater motions. Configurations of compliant riser are formed such that it could absorb floater motions without having additional equipment e.g. heave compensation system. The design flexibility to have high dynamic resistance allows compliant riser to work on deeper water depth and harsher environments. Compliant risers are mainly applied as production, export and injection risers. It can be applied to wide variations of floater such as TLPs, Semi-submersibles, and Ships.

Different compliant riser configurations were discussed by Bai and Bai (2005). Compliant risers can be installed in a number of different configurations. Riser configuration design shall be performed according to the production requirement and site-specific environmental conditions. Configuration design drivers include factors such as water depth, host vessel access/hang-off location, field layout such as number and type of risers and mooring layout, and in particular environmental data and the host vessel motion characteristics.

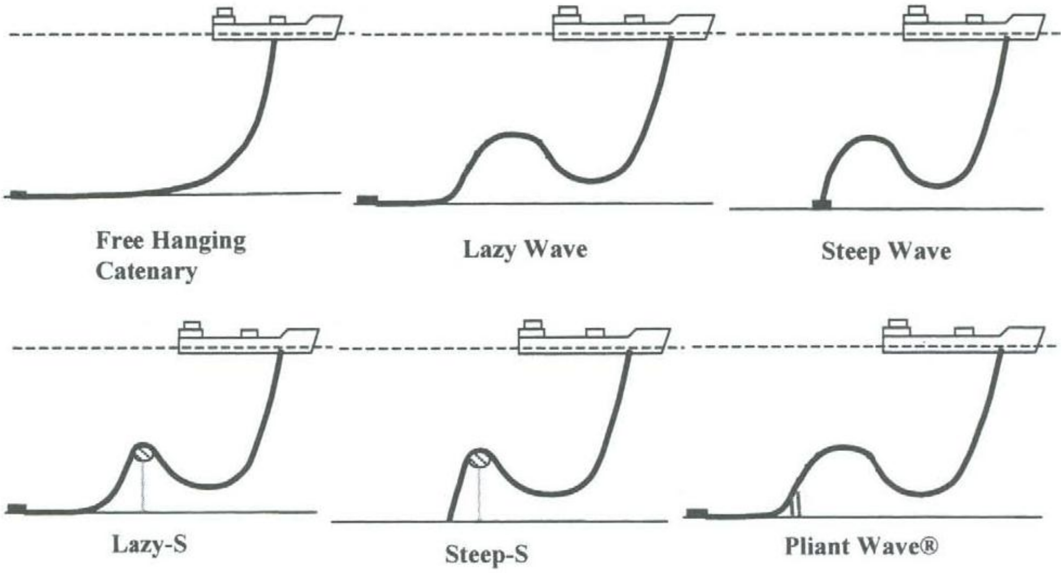

Figure 2-3 Standard Compliant Riser Configurations

• Free Hanging Catenary

The free hanging catenary riser is widely used in deep water. This configuration does not need heave compensation equipment, when the riser is moved up and down together with the floater, the riser is simply lifted off or lowered down on the seabed. In deeper water the top tension is large due to the long riser length supported, to reduce the size of the top tensioner buoyancy modules could be clamped to the top end of the riser. The surface motion is directly transferred to the Touch Down Point (TDP), this means that the failure mode could be overbend or compression at the TDP. The most severe motion is heave from the first order vessel motion.

• Lazy Wave and Steep Wave

For these configurations, buoyancy and weight are added along some length of the riser to decouple the vessel motions from the touchdown point of the riser. Lazy waves are preferred to steep waves because they require minimal subsea infrastructure. However, while lazy waves are prone to configuration alterations if pipe content density changes during the riser’s lifetime, steep wave risers are able to maintain their configuration even if the riser content density changes.

• Lazy S and Steep S

In these configurations, there is a subsea buoy which is either a fixed buoy (fixed to a structure at the seabed) or a buoyant buoy. The addition of the buoy removes the problem associated with the touchdown point. The subsea buoy absorbs the tension variation induced by the floater, and the touchdown point eventually experiences only little or no tension variations. In case of large vessel motions, a lazy-S might still result in compression problems at the riser touchdown, leaving the steep-S as a possible alternative.

Due to the complex installation procedure of ‘S’ configurations, they are considered only if catenary and wave configurations are not suitable for a particular field. A lazy-S configuration requires a mid-water arch, tether and tether base, while a steep-S requires a buoy and subsea bend stiffener.

- **Pliant Wave**

This configuration is almost like the steep wave configuration where a subsea anchor controls the touchdown point i.e. the tension in the riser is transferred to the anchor and not to the touchdown point. This configuration is able to accommodate a wide range of bore content densities and vessel motions without causing any significant change in configuration and inducing high stress in the pipe structure. However, due to complex subsea installation that is required, it would be required only if a simple catenary, lazy wave or steep wave is not viable.

2.2.3 Hybrid Riser

As field developments target deeper and deeper water, hybrid riser towers (HRTs) have become one of the solutions investigated systematically at bid stage. This is due to the capability of hybrid riser towers to accommodate the requirements for large diameter risers, reduced load on FPSO, demanding flow assurance requirements, and robust layout for later development phases.

A hybrid riser comprises a lower vertical steel section (hybrid tower) under tension, and an upper catenary section of flexible pipe (jumper). A buoyancy tank is located below the main wave zone at the upper end of the tower section, and the jumper is connected from the top of the tower or buoyancy tank to the floater. The tower section not only serves as a conduit for the reservoir fluid, but also as tendons to the buoyancy tank. The hybrid riser arrangement is shown in figure 2.4 below.

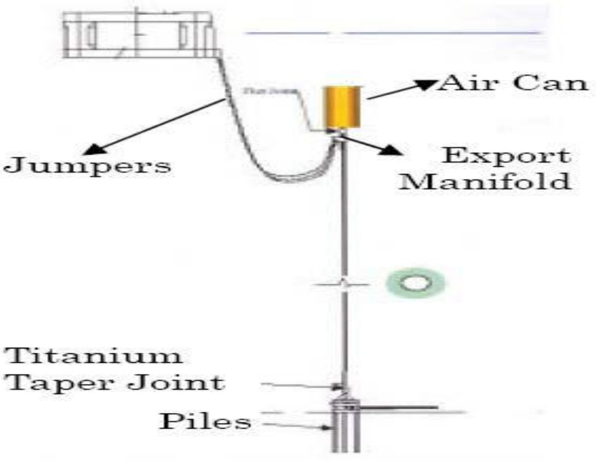

Figure 2-4 Buoyant Free Standing Riser

The hybrid riser combines the best qualities of vertical steel and flexible risers into one system. Using vertical steel through most of the water depth keeps cost per unit length to a minimum, while using flexible riser on the upper section enables the system to be compliant and cater for large vessel motions. This helps to reduce dynamic motions over a large part of the riser, meaning the tower section as well as the buoyancy tank will see little dynamics, with most of the motions taken in the jumper.

Free standing hybrid risers can be deployed both in bundle and single line arrangements.

- **Bundle Hybrid Riser (BHR)**

Bundle hybrid riser consists of a number of smaller diameter steel pipe strings and umbilicals that are grouped together, usually around a buoyant structural core pipe.

- **Single Hybrid Riser (SHR)**

Single hybrid riser consists of a concentric pipe-in-pipe vertical steel riser section. For typical offset hybrid riser system, SHR is also known as Single Line Offset Risers (SLORs). Further development from SLOR is Grouped-SLOR which consists of aligned group of single riser. This collectively constrains riser movement and eliminates the risk of clashing.

2.2.4 Steel Catenary Riser

In ultra-deep water, riser systems become increasingly technically challenging and comprise a major part of the overall field development costs. Large external pressures and high production temperatures in these great depths cause traditional flexible solutions to run into weight, temperature and cost problems. However, steel pipes do not have these temperature limits (SBM Atlantia, 2011).

Steel Catenary Riser (SCR) is one of direct alternative to flexible riser. It may be used at larger diameters, higher pressures and temperatures and may be produced more easily. SCR can be suspended in longer lengths, removing the need for mid-depth buoys. Steel lines are cheaper than flexible and may be used in greater water depths without a disproportionate increase in cost. At the seabed, the need of riser base, stress joint or flex joint have been eliminated. This reduces the complexity of riser system and cost savings are made as a result of simplified riser system. SCR arrangement is shown in figure 2.5 below.

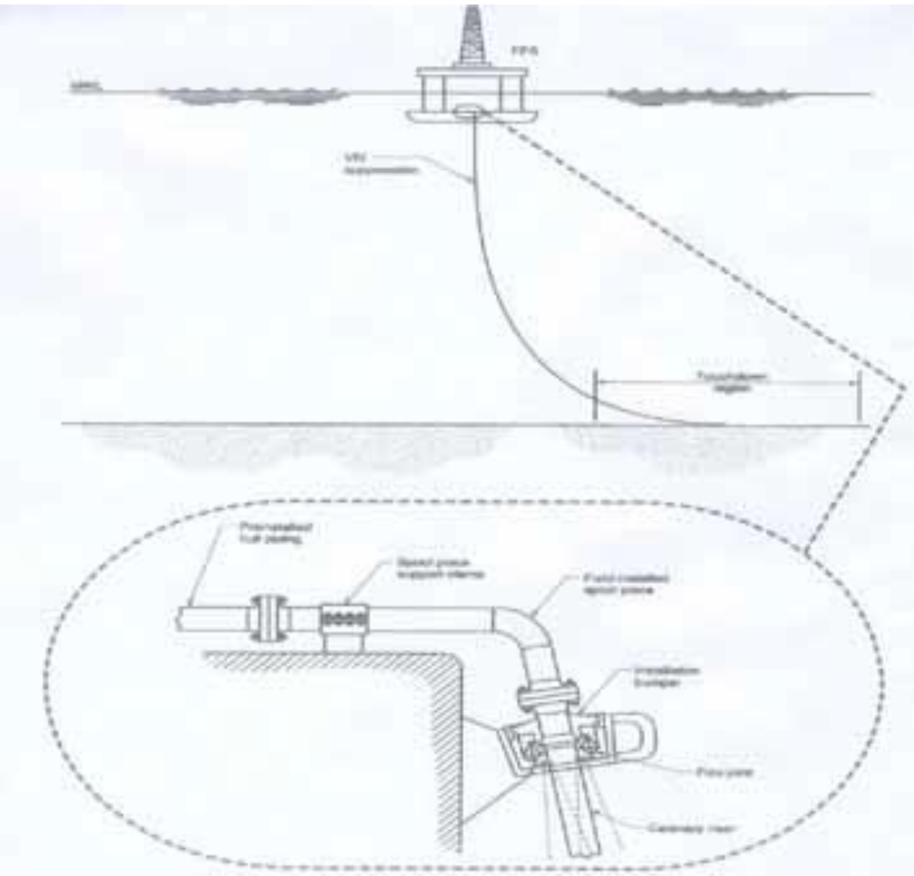

Figure 2-5 Schematic of Steel Catenary Riser

However, SCRs are very sensitive to environmental loading. Large heave and surge motion from host platform due to harsh environment results in buckling issues at touch down point. As the host platform moves, the lengths of pipe between the supports change. This makes the seabed touchdown point shift, hence moving the point of maximum curvature up and down along the length of pipe at the seabed. As a result, at touch down area, pipe is subject to maximum and almost zero curvature, making the region highly sensitive to fatigue damage. Vortex induced vibration due to current in deepwater application is another issue for SCR design.

These issues push the industry to develop new SCR solutions such that it could cope in deepwater and harsh environments. There are many studies have been done by the industry with regards to SCR optimization design.

2.3 Riser Materials

Materials for riser systems shall be selected with due consideration of the internal fluid, external environment, loads, temperatures (maximum and minimum), service life, temporary/permanent operations, inspection/ replacement possibilities and possible failure modes during the intended use. The selection of materials shall ensure compatibility of all components in the riser system. According to material selection, risers can be divided into:

- Rigid riser
- Flexible riser

Steel pipes have traditionally been applied for conventional water depths. Titanium and composite pipes are suggested for deep water applications in order to keep the top tension requirement at an acceptable level. Steel risers with buoyancy modules attached can alternatively be applied for deep water.

The required system flexibility for conventional water depths is normally obtained by arranging flexible pipes. In deep-water, it is however also possible to arrange metallic pipes in compliant riser configurations.

2.3.1 Metallic Pipe

Traditionally low carbon steel has been the principle material for most risers systems. Typical material grades are X60, X65 or X70. Other materials like Aluminum, Titanium alloys are also used for deep water applications.

For severe conditions like ultra deepwater, high temperature, high pressure applications, Titanium has been considered for the riser application. Titanium may offer several benefits relative to steel for some of these configurations. This is due to a low modulus of elasticity (half that of steel) implying a higher degree of flexibility. Furthermore, the yield stress is typically higher than for steel and the specific weight is much lower (about half the steel weight).

2.3.2 Flexible Pipe

A flexible pipe is a pipe with low bending stiffness and high axial tensile stiffness, which is achieved by a composite pipe wall construction. Basically, there are two types of flexible pipes, unbonded and bonded, composed of helical armoring layers and polymer sealing layers.

An unbonded pipe typically consists of a stainless-steel internal carcass for collapse resistance, an extruded-polymer fluid barrier, a carbon-steel interlocked circumferential layer for internal pressure loads (pressure armor), helically wound carbon-steel tensile-armor layers for axial strength, and an extruded watertight external sheath. The structure of a non-bonded flexible pipe is illustrated in Figure 2.6. And a bonded pipe typically consists of several layers of elastomer either wrapped or extruded individually and then bonded together through the use of adhesives or by applying heat and/or pressure to fuse the layers into single construction. Figure 2.7 shows an example of typical bonded pipe.

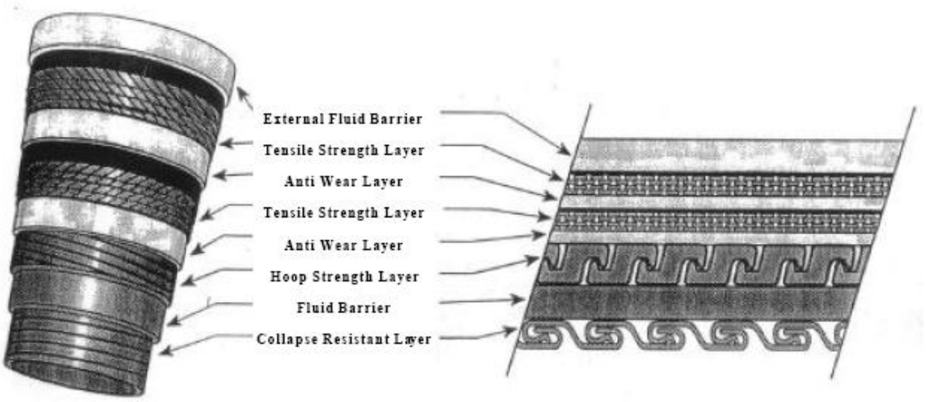

Figure 2-6 Nonbonded Flexible Pipe

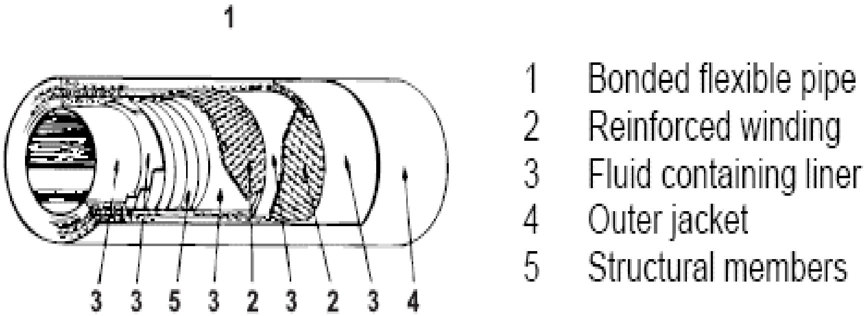

Figure 2-7 Bonded Flexible Pipe

2.4 Riser Components

The components of a riser system must be strong enough to withstand high tension and bending moments, and have enough flexibility to resist fatigue, yet be as light as practicable to minimize tensioning and floatation requirements.

Detailed descriptions of some riser components are given below:

- **Flex Joints**

Flex joint is provided at the top region of SCR in order to minimize bending moment. Flex joint which consists of alternating layers of metal and elastomeric materials allows angular deflections at top connection of riser [API, 1998]. For deepwater application, the design of flex joint shall consider the effect of high top tension and tension ranges for fatigue design [Bai, 2005]. The description of flex joint is shown in figure 2.8.

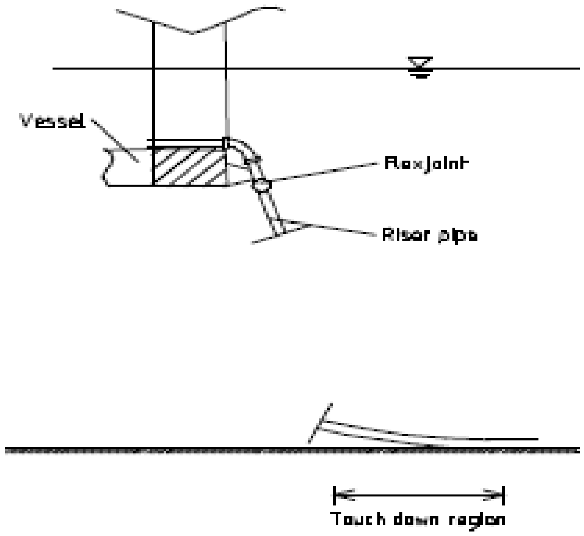

Figure 2-8 Flex Joint Description

- **Tapered Stress Joints**

Taper stress joint is used to provide a transition member between rigidly fixed or stiffer sections of the production riser and less stiff sections of the production riser. This is used to reduce local bending stresses and to provide flexibility at the riser end.

• Ball Joints

Ball joints consist of a ball and matching socket housing that join two pipe segments. Where required, fluid flow can be maintained through the ball joint by a sliding seal between the ball and socket. Shear and tension loads can be transferred across the joint with a minimal bending moment.

• Bend Stiffeners

Bend stiffeners are used to increase and distribute the pipe bending stiffness in localized areas when subjected to anticipate bending moments that would otherwise be unacceptable, the increased stiffness reduces curvature and hence strain in the pipe layers. A typical application of bend stiffeners is at the top of dynamic risers, where they provide a continuous transition between the flexible pipe, with its inherent low bending stiffness, and the metal end fitting, which is very stiff. Bend stiffeners are often made of a polymeric molded material surrounding the pipe and attached to the end fitting, see Figure 2.9.

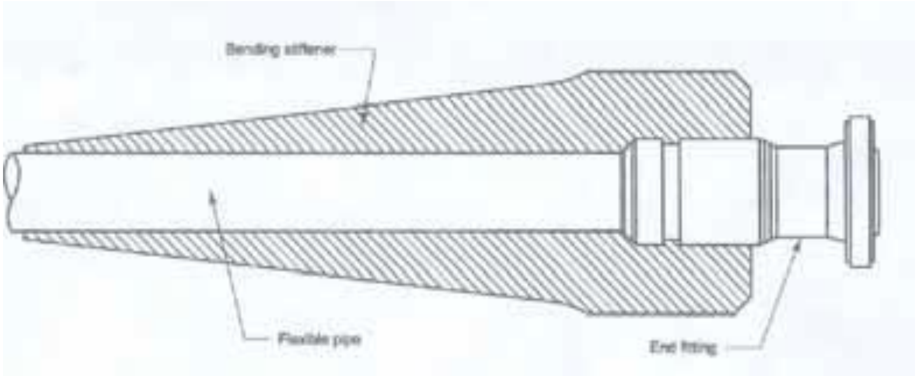

Figure 2-9 Bend Stiffeners

• Bending Restrictor

Bend restrictors are used to support a flexible pipe over free spans where there is the possibility of damaging the pipe structure because of over bending. Typical applications are at wellhead connectors, J-tube exits and rigid pipe crossovers. The bend restrictor normally consists of interlocking half rings that fasten together around the pipe so that they do not affect the pipe until a specified bend radius is reached, at which stage they lock.

CHAPTER 3 RISER DESIGN LOADS

The prediction of different loads is a key component for the riser system, either to determine design loads or to calculate vessel motions. API RP 2RD and DNV-OS-F201 classify the loads to be considered in the design of riser systems as follows:

- Functional and Pressure Loads
- Environmental Loads
- Accidental Loads

Functional loads are loads that are a consequence of the system's existence and use without consideration of environmental or accidental effects. Environmental loads are those imposed directly or indirectly by the ocean environment. Accidental loads are those resulting from unplanned occurrences.

3.1 Functional and Pressure Loads

Functional loads are defined by dead, live and deformation loads occurring during transportation, storage, installation, testing, operation and general use, while pressure loads are loads strictly due to combined effect of hydrostatic internal and external pressures. The functional and pressure are described below:

- Weight of riser, subsurface buoy, contents, and coating
- Internal pressure due to contents, and external hydrostatic pressure
- Nominal top tension
- Buoyancy
- Vessel constraints
- Weight of marine growth

Dead loads are loads due to the weight in air of principal structures (e.g. pipes, coating, anodes, etc.), fixed/attached parts and loads due to external hydrostatic pressure and buoyancy calculated on the basis of the still water level.

Live loads are loads that may change during operation, excluding environmental loads, which are categorized separately. Live loads will typically be loads due to the flow, weight, pressure and temperature of containment.

Deformation loads are loads due to deformations imposed on pipelines and risers through boundary conditions such as reel, stinger, rock berms, tie-ins, seabed contours, constraints from floating installations (risers), etc.

3.1.1 Effect of Marine Growth

Marine growth may accumulate and is to be considered in the design of risers. The highest concentrations of marine growth will generally be seen near the mean water level with an upper bound given by the variation of the daily astronomical tide and a lower bound, dependent on location, of more than 80 meters. Estimates of the rate and extent of marine growth may be based on past experience and available field data. Particular attention is to be paid to increases in hydrodynamic loading due to the change of:

- External pipe diameter;
- Surface roughness;
- Inertial mass;
- Added weight

3.2 Environmental Loads

Environmental loads are defined as loads imposed directly or indirectly by environmental phenomena.These are:

- Wave loads
- Current loads
- Vessel motions
- Seismic loads
- Ice loads
- Wind loads

Only the first three types of environmental loads are included in the analysis in this report.

3.2.1 Waves

Wind driven surface waves are a major source of dynamic environmental forces on the risers. Such waves are irregular in shape, can vary in length and height, and can approach the riser from one or more directions simultaneously.

Wave conditions may be described either by a deterministic design wave or by stochastic methods applying wave spectra. The selection of appropriate wave theories depends on the actual application and links to assumptions used for adjacent structures e.g. floater motion transfer function. Normally, linear wave theory combined with Wheeler Stretching should be considered in addition to disturbed kinematics if relevant.

3.2.2 Current

Current is a major contributor to both the static and dynamic loading on risers. However the relative importance of current loading increases with increasing water depth. The current velocity and direction profile is in general composed of wind driven and tide driven components. In some cases, the velocity and direction profile may also include contribution from:

- Oceanic scale circulation patterns
- Loop/eddy currents
- Currents caused by storm surge
- Internal waves

The vector sum of all current components at specified elevations from the seafloor to the water surface describes the current velocity and direction profile for the given location. The current velocity and direction normally do not change rapidly with time and may be treated as time invariant for each sea state. These profiles may be established using a velocity profile formulation based on site specific data or recognized empirical relationships.

3.2.3 Floater Motions

Forced floater motions are defined as displacements imposed on the riser due to motions of the surface floater. These forced displacements may be introduced at several elevations on the riser depending on type of floater (e.g Semi, TLP, Spar, Ship). These displacements will increase the bending stress in the riser, which may be critical in some cases. Floater motions needed for riser design are:

- Static offset (horizontal).
- Wave frequency motions (horizontal and vertical).
- Low frequency motions.

- Set down.

3.3 Accidental Loads

These are loads to which the riser may be subjected in case of abnormal operations, incorrect operation or technical failure. They typically result from unplanned occurrences. These include:

- Partial loss of station keeping capability
- Small dropped objects
- Tensioner failure
- Fires and explosions
- Flow-induced impact between risers
- Vessel impact

CHAPTER 4 GLOBAL ANALYSIS

Global analysis of the riser is performed to evaluate the global load effects on the riser. In order to evaluate the performance of the riser, the static configuration and extreme response of displacement, curvature, force, and moment from environmental effects should be calculated in the global analysis.

The global analysis includes two aspects: static analysis and dynamic analysis. The static analysis can determine the equilibrium configuration of the system under weight, buoyancy, and drag force. Additionally, it can also provide a starting configuration for dynamic analysis. In most cases, the static equilibrium configuration is the best starting point for dynamic analysis. The dynamic analysis is a time simulation of the motion of the model over a specified period of time, starting from the position derived by the static analysis. The environment defines the conditions to which the objects in the model are subjected, and it consists of the current, waves and seabed.

4.1 Static Analysis

The purpose of the static analysis is to establish an equilibrium position of the riser by subjected the riser to static loads. The equilibrium configuration obtained from the static analysis forms the basis for subsequent Eigen value and dynamic analysis. There are different methods to establish the static equilibrium of the riser system, such as catenary solution, FE solution. But the choice of the solution method is completely system dependent.

4.1.1 Static Catenary Analysis

In the catenary analysis the initial static equilibrium is established by using the standard catenary equations. In most riser system concepts the bending stiffness will have only a minor effect on the overall, static configuration. This fact together with the fact that catenary configuration are easy to calculate offer a convenient way of obtaining a fairly accurate static equilibrium configuration with minimum computational effort.

Shooting method is used to compute the static equilibrium configuration of a composite single line with boundary conditions specified at both ends. Mathematically the problem is a two point boundary value problem. If all boundary conditions are specified at one end, the configuration is uniquely determined. This reduces the problem to an initial value problem and the static configuration can be found by catenary computations, element by element, starting at the end with all boundary conditions specified.

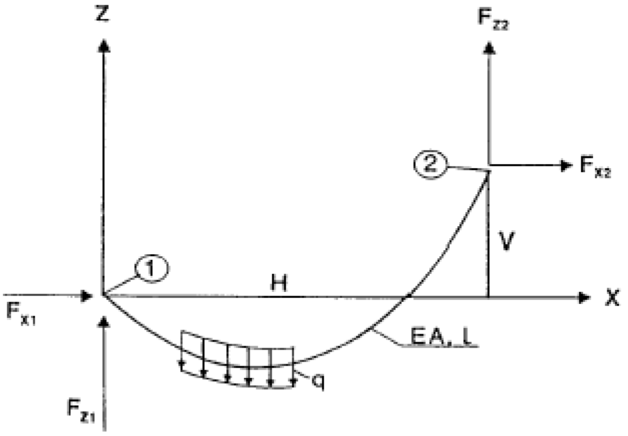

Figure 4-1 Definition of terms in classic catenary equations

The classic catenary equations are applied to compute the coordinates and force components at 2nd element end in the local element system:

$$
\begin{aligned}
F_{x2} &= -F_{x1} \\
F_{z2} &= -F_{z1} + ql \\
H &= -F_{x1}\Big[\frac{l}{EA} + \frac{1}{q}\ln\left(\frac{F_{z2}+T_2}{T_1-F_{z1}}\right) \\
V &= \frac{l}{2EAq}\left(T_2^2 - T_1^2\right) + \frac{1}{q}\left(T_2 - T_1\right) \\
l_d &= l + \frac{l}{2EAq}\left[F_{z2}T_2 + F_{z1}T_1 + F_{x1}^2\ln\left(\frac{F_{z2}+T_2}{T_1-F_{z1}}\right)\right] \\
&\text{Where } T_1 = \sqrt{F_{x1}^2 + F_{z1}^2};\ T_2 = \sqrt{F_{x2}^2 + F_{z2}^2}
\end{aligned}
\tag{4.1}
$$

4.1.2 Static Finite Element Analysis

Static riser analyses are normally performed using a nonlinear FE approach. Following standard FE terminology, it is convenient to distinguish between the following basic loading components:

- volume forces
- specified forces

- prescribed displacements
- displacement dependant forces

The purpose of the static Finite Element analysis is to determine the nodal displacement vector so that the complete system is in static equilibrium. The equation governing the static equilibrium of a riser system can be expressed as:

$$R^S(r) = R^E(r) \tag{4.2}$$

Where:

r : nodal displacement vector

$R^S(r)$: internal structural reaction force vector

$R^E(r)$: external force vector

Both the internal reaction forces and external loading will in general be nonlinear functions of the nodal displacement vector. In most cases it is not possible to solve the Eq.4.2 by analytical methods and the static equilibrium is often found numerically by applying one of the following solution techniques:

- Incremental or stepwise procedures
- Iterative procedures
- Combined methods

- **Incremental Methods**

Incremental methods provide a solution for the non-linear problem by a stepwise application of the external loading. For each step the displacement increment, Δr is determined by Eq.4.3. The total displacement is obtained by adding displacement increments. The incremental stiffness matrix K_I is calculated based on the known displacement and stress condition before a new load increment is applied and is kept constant during the increment.

$$K_I(r)\Delta r = \Delta R^E \tag{4.3}$$

This method is called Euler-Cauchy method. For load increment No.(m+1) is may be expressed as

$$\Delta R^E(r_{m+1}) = R^E(r_{m+1}) - R^E(r_m)$$
$$\Delta(r_{m+1}) = K_I(r_m)\Delta R^E(r_{m+1}) \quad (4.4)$$
$$r_{m+1} = r_m + \Delta r_{m+1}$$

With the initial condition $r_0 = 0$. In this way the load may be incremented up to the desired level. The illustrated method for a single degree of freedom system is shown in Figure 4.2.

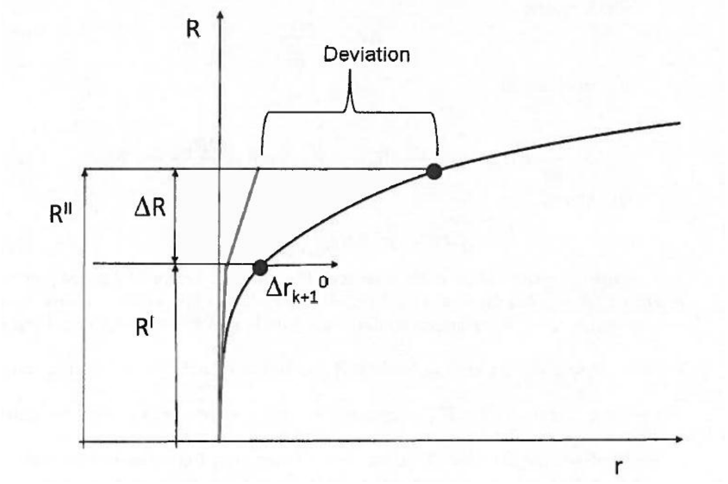

Figure 4-2 Euler-Cauchy Incremental Method

This method will not fulfill total equilibrium but accuracy can be increased by reducing the load increment. Improvement of the method is accomplished by performing an equilibrium correction so that global equilibrium is restored. The force imbalance vector, $R(r_m)$ at the m'th incremental load step is expressed as:

$$R_{eq}(r) = R_m^E(r) - R_m^S(r) \quad (4.5)$$

The unbalanced forces are accounted by adding the force imbalance vector to the next load increment where r_{m+1} is calculated and external loads are hence reduced. Formally the method is expressed as:

$$\Delta R(r_{m+1}) = R(r_{m+1}) - R(r_m)$$
$$R_{eq}(r) = R_m^E(r) - R_m^S(r) \quad (4.6)$$
$$\Delta r_{m+1} = K_I(r)^{-1}\Delta R(r_{m+1}) - K_I(r_m)^{-1} = K_I(r_m)^{-1}\left[\Delta R(r_{m+1}) + R_{eq}\right]$$
$$r_{m+1} = r_m + \Delta r_{m+1}$$

• Iterative Methods

The Newton-Raphson is the most commonly applied method for nonlinear problems due to its quadratic convergence rate offered by this procedure. The iterative algorithm used in this method is:

$$r_{n+1} - r_n = \Delta r_{n+1} = K_I^{-1}(r_n)(R - R^s(r_n))$$
$$r_{n+1} = r_n + K_I^{-1}(r_n)(R - R^s(r_n)) \tag{4.7}$$

The basic principle for this method is illustrated in Figure.4.3 for a single degree of freedom system.

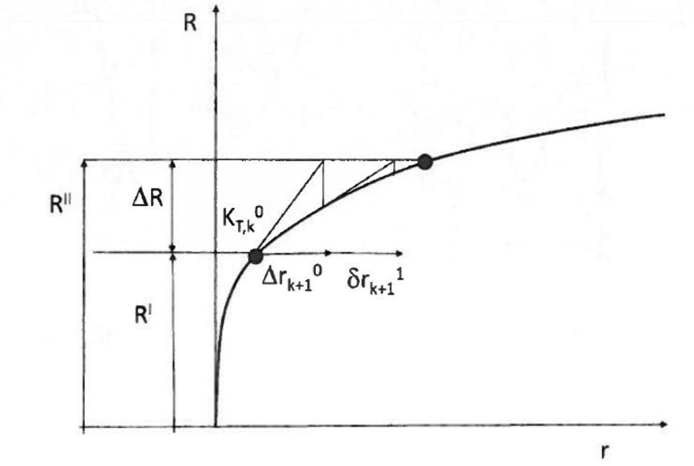

Figure 4-3 Newton-Raphson Iterative Method

• Combined Methods

In combined methods, both incremental and iterative methods are combined together during each load increments. For each load step incremental method is applied first and an iterative method is applied to obtain the equilibrium.

4.2 Dynamic Analysis

The dynamic analysis primarily represents the analysis of the riser response to the combined action of the wind, wave, and current. The starting point for dynamic simulation is the static equilibrium configuration. Dynamic simulation considers the RAO of the vessel over a specified period of time.

The dynamic analysis is necessary to ascertain the safe operational envelope of the drilling or production facility. It is used to calculate the maximum excursions of the riser displacements and stresses for a given sea state. This decides the limits of the environmental conditions beyond which the operations are stopped and the system is disconnected.

There are several methods for dynamic analysis of marine risers by using finite elements. A structure exposed to a dynamic load will have the following forces acting:

- Inertia forces
- Damping forces
- External, time-dependent forces

According to d'Alemberts principle the inertia forces can be added to the other force components. The equilibrium of these forces in a system with viscous damping and forced oscillations can be described by the equation of dynamic equilibrium:

$$M\ddot{r} + C\dot{r} + Kr = Q(t) \tag{4.8}$$

Where:
r = Matrix of nodal displacement vectors
$\dot{r}$ = Matrix of nodal velocity vectors
$\ddot{r}$ = Matrix of nodal acceleration vectors
Q(t) = Matrix of external, time-dependent forces
K = System stiffness matrix
C = System damping matrix
M = System mass matrix

There are several ways to solve the dynamic equilibrium equation, and a few relevant methods are briefly discussed in the following sections.

4.2.1 Frequency Domain Analysis

Solving the dynamic equilibrium in the frequency domain requires a linear relationship between the loads and the response which implies that all system matrices are constant. The main advantage with frequency domain analysis is that all the response spectra, and hence all statistical information, can be found directly. According to linear theory a load with frequency ω will give a response at the same frequency and the response amplitude will be proportional to the load amplitude. For systems where the nonlinear coupling between axial tension and lateral stiffness is known to be small and there are no other nonlinearities significantly affecting the responses, frequency domain analysis is appropriate. A traditional frequency domain solution of the dynamic equilibrium equation can be expressed by:

$$r(\omega) = [K + i\omega C - \omega^2 M]^{-1} R \tag{4.9}$$

where r and R are complex response and load vectors respectively. The frequency domain result will always give a description of a Gaussian process and consequently the nonlinearities in the hydrodynamic loading must be overcome by linearization of the quadratic drag term in Morison equation. An important issue is to what extent a nonlinear load response will give a non-Gaussian loading. Some uncertainties will therefore be present concerning the representation of hydrodynamic loading in the frequency domain.

4.2.2 Time Domain Analysis

The dynamic equilibrium equation can be solved numerically by step-by-step integration in the time domain. The time period that is to be solved is divided into intervals, and the dynamic equilibrium is found for each time step. The displacement and the velocity are found by integrating the acceleration twice for each time step. There are different methods for describing how the acceleration varies over the interval. Examples are constant initial acceleration, constant average acceleration or linear acceleration.

One method is the Newarks β −family. The displacement and velocity for the next time step is found from Taylor expansions with parameters β and γ. In this case β = ¼ is used to give constant average acceleration, and is unconditionally stable. This is the trapeze method (Euler-Gauss) formulated for problems of second order. γ = ½ , ensures no artificial damping.

This method is very useful in analysing nonlinear systems, as it can give a quite accurate representation of nonlinear behavior. The mass, and therefore the inertia forces, are assumed to be constant. Elastic and damping forces can be nonlinear functions of respectively displacement and velocity. The elastic forces must be calculated from the stiffness state in the elements.

By linearization of the damping matrix and the stiffness matrix the incremental matrices can be found. In the time integration the displacement and the velocities can only be found by iterations at each time step. If the stiffness and the damping matrix are found for each iteration the method is called Newton-Rhapson. This is time consuming, and by only updating them once or a few selected times, time is saved and the method is called modified Newton-Rhapson. Although the rate of convergence is slower, it is usually faster than true Newton-Rhapson.

4.2.3 Eigenvalue Analysis

The eigenfrequencies and mode shapes are factors that are important for the understanding of the dynamics of a structure. To obtain this information the free vibration equation, where damping and external force is set to zero, is used (Larsen C. M., 2008):

$$Mr + K\ddot{r} = 0 \tag{4.10}$$

Assuming that the solution has of the following form

$$r = \hat{r} sin\omega t \tag{4.11}$$

Where $\hat{r}$ is the eigenvector, ω is the eigenfrequency and t is time. The free vibration equation can then be transformed to an eigenvalue equation:

$$(K - \omega^2)\hat{r} = 0 \tag{4.12}$$

For a system of N degrees of freedom the solution has N eigenvalues $(\omega_1^2, \omega_2^2, \dots, \omega_N^2)$ and corresponding N eigenvectors $(\hat{r}_1, \hat{r}_2, \dots, \hat{r}_N)$. These represent the frequencies and mode shapes for free vibration. The eigenfrequencies of a system should not be equal or close to the frequencies of external loads. For a marine riser this means that the eigenfrequencies of the riser should not be in the range where wave frequencies with significant energy occur, if that happens large motions will be induced (Larsen C. M., 2008).

CHAPTER 5 DESIGN GUIDELINES

5.1 Safety Classes

The requirements to the structural safety of the riser system are made dependent upon the consequences caused by a failure. The consequences are grouped into:

- risk to life
- environmental pollution, and
- political and economical consequences.

Three safety classes are introduced as Table 5.1, i.e. low, normal and high. The selection of safety class for riser depends on:

- the fluid category of the riser content
- the location of the part of the riser that is being designed
- whether the riser is in its operating phase or in a temporary phase

Table 5-1 Normal Categorization of Safety Class

Riser operating condition	**Riser content**			
	Non flammable incompressible fluids and gas at atmospheric pressure		Flammable fluids or pressurised gas.	
	Location		Location	
	Near	Far	Near	Far
Temporary with no pipeline/well access	Low	Low	Low	Low
In-service with pipeline/well access	Normal	Low	High	Normal
NOTE 1 Location Near and Far refer to the separation distance from manned floater/platform. The extent of location class Near should be based on appropriate risk analysis.				

5.2 Limit States

The riser system, or part of the riser system, is considered not to satisfy the design requirements when it exceeds a state, beyond which it infringes one of the criteria governing its performance or use. This state is called a limit state. Limit states are divided into the following categories:

• **Serviceability Limit State (SLS)** requires that the riser must be able to remain in service and operate properly. This limit state corresponds to criteria governing the normal operation (functional use) of the riser.

Serviceability limit states are most often associated with determination of acceptable limitations to normal operation. Exceeding a SLS shall not lead to failure and an ALS shall be defined in association with exceedance of SLS. In addition, the frequency and consequences of events after exceeding an SLS shall be evaluated. Such events will typically be controlled by maintenance/inspection routines and by implementation of early warning or fail-safe type systems in the design.

• **Ultimate Limit State (ULS)** requires that the riser must remain intact and avoid rupture or fracture, but not necessarily be able to operate.

This limit state generally corresponds to the maximum resistance to applied loads. For operating condition this limit state corresponds to the maximum resistance to applied loads with 10^{-2}annual exceedance probability.

• **Fatigue Limit State (FLS)** results from excessive fatigue crack growth or Miner damage under cyclic loading. The FLS is an ULS due to effect of cyclic loads.

The riser system shall have adequate safety against fatigue within the service life of the system. All cyclic loading imposed during the entire service life, which have magnitude and corresponding number of cycles large enough to cause fatigue damage effects, shall be taken into account. Temporary phases like transportation, towing, installation, running and hang-off shall be considered.

All critical sites for anticipated crack initiation for each unique component along the riser shall be evaluated. These sites normally include welds and details that causes stress concentrations.

• **Accidental Limit State (ALS)** corresponds to the ultimate failure of the riser due to accidental loads and/or local damage with loss of structural integrity and rupture. The ALS ensures that local damage or accidental loads do not lead to complete loss of integrity or performance of the riser.

Accidental loads shall be understood as loads to which the riser may be subjected in case of abnormal conditions, incorrect operation or technical failure. Accidental loads typically results from unplanned occurrences.

5.3 Riser Design Methods

Risers are subjected to various types of loads and deformations that range from the routine to the extreme or accidental. The purpose of risers design is to design a riser system that can withstand load effects throughout its expected lifetime.

The design is safe if the resistance is more than response and the ratio of response over resistance shall be less than acceptance criteria or allowable factor. Safety factor shall be incorporated in design check in order to account for various uncertainties due to natural variability, inaccuracy in analysis procedures and control of load effects and uncertainties in structural resistance.

There are two methods to establish acceptance criteria in structural design. One method is often referred to as Working Stress Design (WSD) where one central safety factor is used for each limit state to account for uncertainties from response and resistance. Another approach is referred to as Load and Resistance Factor Design (LRFD) where partial safety factor is applied for each load effect and resistance. In riser systems design, WSD is provided in API-RP-2RD; meanwhile LRFD is provided in DnV-OS-F201.

5.3.1 WSD code – API-RP-2RD

The working stress design method is a design format where the structural safety margin is expressed by one central safety factor or usage factor for each limit state. In other words, the possible uncertainties in load effects and resistance are accounted for by a single usage factor.

5.3.1.1 Stresses

In marine riser system, the pipe is considered to be plain pipe due to its axisymmetric geometry. The principal stresses for plain pipe are in the axial, hoop and radial directions. Transverse shear and torsion are negligible for plain round pipe.

The three principal stresses are calculated to form a combined stress, called Von Mises equivalent stress and defined by the following equation:

$$\sigma_e = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1 + \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} \tag{5.1}$$

Where:

σ_e = Von Mises equivalent stress

$\sigma_1, \sigma_2, \sigma_3$ = principal stresses

According to API, 1998 section 5.2.3, the design criteria of WSD for plain pipe is:

$$(\sigma_p)_e < C_f \sigma_a \tag{5.2}$$

Where:

$(\sigma_p)_e$ = Equivalent von Mises stress where the principal stresses consist of primary membrane stresses.

σ_a = Basic allowable combined stress, $\sigma_a = C_a \sigma_y$

C_a = allowable stress factor, C_a =2/3

σ_y = material minimum yield strength

C_f = design case factor

= 1.0 (normal operating)

= 1.2 (extreme)

= 1.5 (survival)

The usage factor which is calculated by considering allowable stress factor and design case factor are provided in table 5.2.

Table 5-2 Usage Factors in API RP 2RD

Load combination	**Normal operating**	**Extreme**	**Survival**
Functional plus environmental	0.67	0.8	1.0

5.3.1.2 Deflections

The purpose of limiting deflection is to prevent high bending stresses or large riser curvatures. Moreover, deflections shall be controlled to prevent clashing between risers.

5.3.1.3 Hydrostatic Collapse

In deepwater application, hydrostatic pressure is high. Excessive external pressure may result in collapse failure. Consequently, riser tubular shall have resistance to collapse during installation or operation. According to API, the design criteria for collapse pressure is given by the following equation:

$$P_a \leq D_f P_c \tag{5.3}$$

Where

P_a = Net allowable external design pressure
P_c = Predicted collapse pressure
D_f = Design factor
= 0.75 for seamless or Electric Resistance Welded (ERW) API pipe
= 0.60 for (DSAW) internally cold expanded API pipe

5.3.1.4 Collapse Propagation

Impact or excessive bending due to tensioner failure is one of the sources that initiate the collapse at the pipe. The buckle will propagate and travel along the pipe until external pressure drops due to change in properties of pipe. Therefore, in order to prevent collapse initiation and propagation, thicker pipe shall be used or buckle arrestor is provided at some critical region.

The design criterion to prevent collapse propagation is provided in the following equation:

$$P_d < D_p P_p \tag{5.4}$$

Where

P_d = design pressure differential
P_p = predicted propagation pressure
D_p = design factor
= 0.72

These criteria are applied to demonstrate metal tubular that used in FPS risers will not collapse under external hydrostatic pressure.

5.3.1.5 Fatigue

A design criterion for fatigue is provided in the following equation [API]:

$$\sum_i SF_i D_i < 1.0 \tag{5.5}$$

Where

D_i = the fatigue damage ratio for each phase of loading
SF_i = associated safety factor

5.3.2 LRFD code – DnV-OS-F201

DNV-OS-F201 says the fundamental principle of the load resistance factored design method is to verify that factorised design load effects do not exceed factored design resistance for any of the considered limit states. Some of the failure modes associated with limit states include bursting, collapse, propagating buckling for ultimate limit state; fatigue failure for fatigue limit state; failure caused by accidental loads directly, or by normal loads after accidental events (damage conditions) for accidental limit state, and so on.

5.3.2.1 Serviceability Limit State

In this limit state, riser is subject to operating loads and shall remain functional. For typical export or import riser, there are some limits that have to be satisfied:

- Risers do not deflect by more than certain limits
- During riser installation, a weather limitation shall be set to avoid riser interference
- Out-of-roundness tolerance of the pipe shall be set to avoid premature local buckling. According to DnV, out-of-roundness tolerance from fabrication of the pipe shall be limited to 3.0%.

$$f_0 = \frac{D_{max} - D_{min}}{D_0} \leq 0.03$$

- Other serviceability limits may be determined to limit the degradation of riser coatings and attachments or for allowances due to wear and erosion

5.3.2.2 Ultimate Limit State

Load controlled conditions are emphasized on this design check. Pipe members subjected to pressure (collapse and bursting) and combined loading criteria (pressure and external loads) are the scope for ULS.

- **Bursting**

Bursting failure of the pipe occurs due to internal overpressure. Along the riser, top-end is the critical area for bursting where the external hydrostatic pressure is minimal and there is internal fluid pressure.

According to DnV, a criterion for pipe resistance to bursting failure at all cross section is provided in the following equation:

$$(p_{li} - p_e) \leq \frac{p_b(t_1)}{\gamma_m \gamma_{sc}} \tag{5.6}$$

Where:

p_{li} = Local incidental pressure: the maximum expected internal pressure with a low annual exceedance probability. Normally the incidental surface pressure, p_{inc} is taken 10% higher than the design pressure, p_d:

$p_{li} = p_{ld} + 0.1.p_d$

Where:

p_{ld} = local internal design pressure
$p_{ld} = p_d + \rho_i .g.h$

Where:

p_d = design pressure; for riser type export/import riser from/to pipeline, design pressure is maximum export/import pressure during normal operations.
ρ_i = density of the internal fluid
h = height different between the actual location and the internal pressure reference point
g = acceleration of gravity

p_e = External pressure

$p_b(t)$ = burst resistance

$$p_b(t) = \frac{2}{\sqrt{3}} \frac{2t}{D-t} min(f_y; \frac{f_u}{1.15}) \qquad (5.7)$$

The nominal wall thickness is given by:

$t_{nom} = t_1 + t_{corr} + t_{fab}$

The minimum required wall thickness for a straight pipe without allowances and tolerances is given by:

$$t_1 = \frac{D}{\frac{4}{\sqrt{3}\gamma_m\gamma_{sc}(p_{li}-p_e)} \, min(f_y; \frac{f_u}{1.15})} \qquad (5.8)$$

• **System Hoop Buckling (Collapse)**

Collapse failure of the pipe occurs due to external overpressure. Along the riser, lower part of riser is the critical area for collapse where external hydrostatic pressure is maxima.

According to DnV, a criterion for pipe resistance to collapse failure at all cross section is provided in the following equation:

$$(p_e - p_{min}) \leq \frac{p_c(t_1)}{\gamma_{sc}\gamma_m} \qquad (5.9)$$

Where:

p_{min} = minimum internal pressure; p_{min} is the local minimum internal pressure taken as the most unfavourable internal pressure plus static head of the internal fluid.
For installation p_{min} equals zero. For installation with water-filled pipe, p_{min} equals p_e.
p_c = resistance for external pressure (hoop buckling)

$$(p_c(t) - p_{el}(t))(p_c^2(t) - p_p^2(t)) = p_c(t)p_{el}(t)p_p(t)f_0 \frac{D}{t} \qquad (5.10)$$

Where:

p_{el} = elastic collapse pressure (instability) of a pipe

$$p_{el}(t) = \frac{2E(\frac{t}{D})^3}{1-v^2} \tag{5.11}$$

p_p = plastic collapse pressure

$$p_p(t) = 2\frac{t}{D}f_y\alpha_{fab} \tag{5.12}$$

Where:

α_{fab} = fabrication factor

f_0 = the initial ovality, i.e. the initial departure from circularity of pipe and pipe ends.

• Propagating Buckling

Local buckle on the pipe may possibly occur due to system failure such as tensioner failure during installation. The local buckle will propagate until external pressure drops due to change in pipe properties. In order to design the local buckle will not propagate, following criterion shall be satisfied:

$$(p_e - p_{min}) \leq \frac{p_{pr}}{\gamma_c\gamma_{sc}\gamma_m} \tag{5.13}$$

Where:

γ_c = 1.0 if no buckle propagation is allowed

= 0.9 if short distance buckle propagation is allowed.

p_{pr} = resistance against buckling propagation

$$p_{pr} = 35f_y\alpha_{fab}(\frac{t_2}{D})^{2.5} \tag{5.14}$$

Where

$t_2 = t_{nom} - t_{corr}$

Normally, propagating buckling criterion results in significantly thicker wall requirement compared to other criteria. Consequently, the design will be too conservative if this criterion has to be satisfied. In practice, designer would let the propagating limit to be exceeded and consequently, buckle arrestor shall be provided over the critical region. This method would save significant amount of riser weight and cost.

After riser system is designed to withstand both internal pressure and external pressure, riser is then checked for combination between axial load, bending moment, and pressure.

- **Combination Loading**

Combination between bending moment, effective tension and net internal overpressure shall be designed to satisfy the following equation:

$$[\gamma_{sc}\gamma_m]\left[\left(\frac{|M_d|}{M_k}\sqrt{1-\left(\frac{p_{ld}-p_e}{p_b(t_2)}\right)^2}\right)+\left(\frac{T_{ed}}{T_k}\right)^2\right]+\left(\frac{p_{ld}-p_e}{p_b(t_2)}\right)^2 \leq 1 \qquad (5.15)$$

Where:

M_d = design bending moment

$$M_d = \gamma_F M_F + \gamma_E M_E + \gamma_A M_A \qquad (5.16)$$

Where:

M_F, M_E, M_A = Bending moment from functional, environmental, accidental loads respectively.
γ_F, γ_E, γ_A = load effect factor for functional, environmental, accidental respectively.
M_k = the (plastic) bending moment resistance

$$M_k = f_y \alpha_c (D - t_2)^2 t_2 \qquad (5.17)$$

Where α_c is a parameter accounting for strain hardening and wall thinning.

$$\alpha_c = (1-\beta) + \beta \cdot \frac{f_u}{f_y}$$

$$\beta = \begin{cases} (0.4 + q_h) & \text{for } D/t_2 < 15 \\ (0.4 + q_h)(60 - D/t_2)/45 & \text{for } 15 < D/t_2 < 60 \\ 0 & \text{for } D/t_2 > 60 \end{cases}$$

$$q_h = \begin{cases} \dfrac{(p_{ld} - p_e)}{p_b(t_2)} \dfrac{2}{\sqrt{3}} & \text{for } p_{ld} > p_e \\ 0 & \text{else} \end{cases}$$

T_{ed} = Design effective tension

$$T_{ed} = \gamma_F T_{eF} + \gamma_E T_{eE} + \gamma_A T_{eA} \qquad (5.18)$$

T_{eF}, T_{eE}, T_{eA} = Effective tension from functional, environmental, accidental loads respectively.

Note: Normally A load is not considered simultaneously in global analyses.

The effective tension, T_e:

$$T_e = T_w - p_i A_i + p_e A_e \tag{5.19}$$

Where:

T_w = true wall tension
p_i = internal (local) pressure
p_e = external (local) pressure
A_i = internal cross-sectional area
A_e = external cross-sectional area
T_k = the plastic axial force resistance

$$T_k = f_y \alpha_c \pi (D - t_2) t_2 \tag{5.20}$$

Combination between bending moment, effective tension and net external overpressure shall be designed to satisfy the following equation:

$$(\gamma_{sc}\gamma_m)^2[(\frac{|M_d|}{M_k}) + (\frac{T_{ed}}{T_k})^2]^2 + (\gamma_{sc}\gamma_m)^2[\frac{p_e - p_{min}}{p_c(t_2)}]^2 \leq 1 \tag{5.21}$$

5.3.2.3 Accidental Limit State

ALS represents excessive structural damage as a consequence of accidents which affect the safety of the structure, environment and personnel. A design of riser shall maintain the structural integrity such that it will not be impaired during any accidental event or within a certain period of time after the accident. In riser design, these are some sources that may lead to accidental event:

- Fires and explosions
- Impact/collisions
- Hook/snag loads
- Loss of mooring line
- Earthquake, tsunamis, iceberg.

A simplified design check with respect to accidental load is shown in Table 5.3 below

Table 5-3 Simplified Design Check for Accidental loads

Prob. of occurrence	Safety Class Low	Safety Class Normal	Safety Class High
$> 10^{-2}$	Accidental loads may be regarded similar to environmental loads and may be evaluated similar to ULS design check		
$10^{-2} - 10^{-3}$	To be evaluated on a case by case basis		
$10^{-3} - 10^{-4}$	$\gamma_c = 1.0$	$\gamma_c = 1.0$	$\gamma_c = 1.0$
$10^{-4} - 10^{-5}$	*	$\gamma_c = 0.9$	$\gamma_c = 0.9$
$10^{-5} - 10^{-6}$	*		$\gamma_c = 0.8$
$< 10^{-6}$	*Accidental loads or events may be disregarded		

For the case of probability of occurrence 10^{-4}, a total safety factor of 1.1 as standard practice industry suggested and consistent with safety class normal according to table 5.2.

5.3.2.4 Fatigue Limit State

The FLS design is carried out to ensure that the structure has an adequate fatigue life.
The fatigue assessment methods may be categorized into:

- Methods based on S-N curves
- Methods based on fatigue crack propagation

In this section, fatigue criterion according to S-N curves is discussed. The fatigue criterion in provided in the following equation:

$D_{fat}DFF \leq 1.0$

Where:

D_{fat} = Accumulated fatigue damage (Palmgren-Miner rule)

DFF = Design fatigue factor, see table 5.4

Table 5-4 Design Fatigue Factors, DFF

Safety class		
Low	Normal	High
3.0	6.0	10.0

CHAPTER 6 RIFLEX

6.1 Introduction

RIFLEX is a tailor-made and advanced tool for static and dynamic analysis of slender marine structures. It represents state-of-the-art technology for riser analysis suitable for flexible, metallic or steel catenary riser applications. In addition, RIFLEX is an efficient program system for hydrodynamic and structural analysis of slender marine structures.

There are some advantages by using RIFLEX program to analyze slender structure:

- Extremely efficient and robust non-linear time domain formulation applicable for irregular wave analysis
- High flexibility in modeling, enabling analysis for a wide range of structures

A numbers of simple or complex marine systems can be analyzed by this program such as flexible risers, top tensioned risers, metallic catenary risers, mooring lines, TLP tendons, loading hoses, umbilicals, towing lines, pipe laying, seismic cables, and fish farming systems.

RIFLEX can perform various analyses of riser system:

- Static Analysis
- Static Parameter Variation Analysis
- Dynamic Time Domain Analysis including eigenvalue analysis
- Frequency Domain Analysis

Basic results are:

- Nodal point coordinates
- Curvature at nodal points
- Axial force
- Bending moment
- Shear force
- Torsion

6.2 Structures of RIFLEX Computer Program

According to RIFLEX-User manual, the program consists of different modules for performing different tasks:

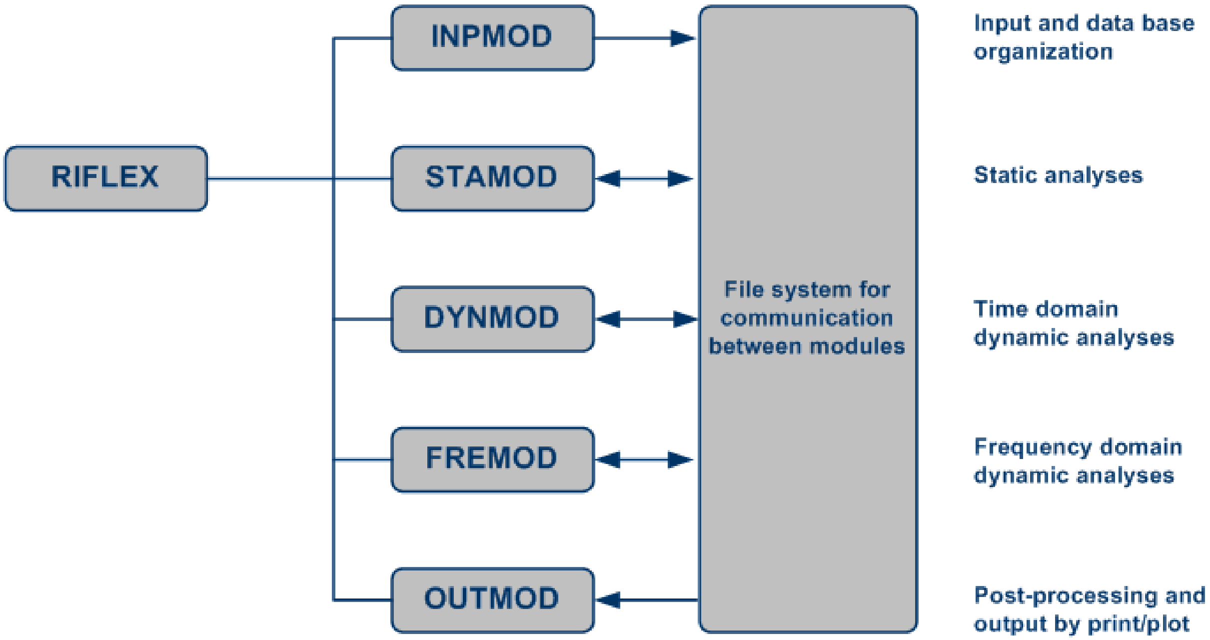

Figure 6-1 Structure of the Computer Program

- **INPMOD** Module: This module reads the input data required for describing the riser system. Once the INPMOD has been run, several analyses can be performed by the other modules.

- **STAMOD** Module: This module establishes the static equilibrium configuration of the riser system which will form the basis for subsequent dynamic and Eigen value analysis.

- **DYNMOD** Module: This module performs Eigen value and Time domain analysis based on the static configuration established by the STAMOD.

- **FREMOD** Module: This module is designed as a modular program which carries out frequency domain analyses with stochastic linearization of the quadratic Morison drag term.

- **OUTMOD** Module: This module performs post processing of results carried out by STAMOD and DYNMOD. This also provides files for viewing the results in the graphical interface.

• **PLOTMOD** Module: This is a graphical interface developed for viewing the results from static and dynamic analysis. This module can read only the files which are generate by the OUTMOD.

6.3 Riser System Modelling (INPMOD)

In the computer program, the riser system can be specified in different ways. In one method the riser can be defined with a general system called arbitrary riser (AR) system. In this system, the user has to specify the riser structural properties including all the boundary conditions. In order to simplify the system, there are some standard systems available which will also reduce the computational effort during analysis. The different standard systems in the computer program are listed below:

• **Single risers**

SA- Seafloor to Surface Vessel, One-Point Seafloor Contact. The riser is suspended between two defined points. The lower is fixed while upper end is connected to the support vessel. The system is used to model steep wave, steep S and jumper flexible riser configurations.

SB- Seafloor to Surface Vessel, Seafloor Tangent. This system has additional features from previous system:

- Seafloor tangent boundary condition
- Buoyancy guide at one point

Seafloor contact is modeled by bilinear stiffness. This stiffness is dicretized and implemented as springs at the nodal points that may touch seafloor. This system models simple catenary, lazy wave and lazy S configurations.

SC - Free Lower End, Suspended from Surface Vessel. This group is characterized by a free lower end, all degrees of freedom being specified at the upper end. This configuration represents typical installation phases.

SD – Free Upper End. Single line system is connected to seafloor at lower end and with free upper end. There are some examples of this system such as buoyed riser, loading system, etc.

- **Connected risers**

CA - Parallel risers with cross connection

CB - Branched riser system with common lower point

6.3.1 Line and Segment Description

A line is basically a linear structural element between two supernodes with specified boundary conditions. Each line is composed of different segments with homogeneous cross-section properties. These segments are to be used for finite element discretization for each element. It is also possible for internal fluid flow component in the line. The line specification can be shown in figure 6.2.

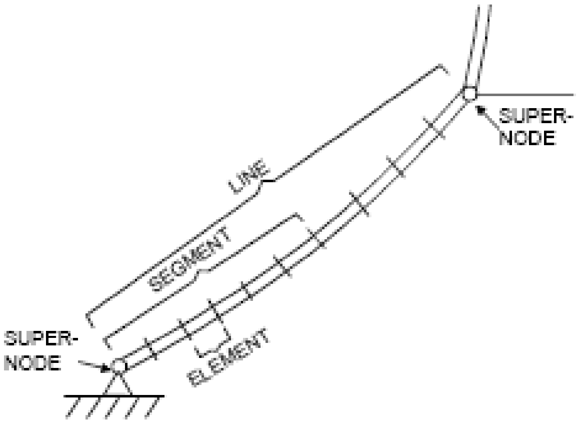

Figure 6-2 Line Specification

Simple steel catenary riser basically comprises of one single line with two supernodes. It is important to define sufficient length of segment for dynamic analysis in order to avoid any instability in analysis process.

6.3.2 Component Description

A component with elementary description of the mechanical properties is identified by a numerical identifier called component type number. Those components available in RIFLEX are:

1. Cross sectional component: mechanical properties of the component (Axial, bending and torsional stiffness). For simple catenary riser without any devices attachment, component is simply described as pipe cross section (CRS0).

2. Nodal component: used to model the submerged buoys, clump weight, etc.

3. Special component: used for modeling elastic contact forces between lines and for modeling of tensioner mechanism.

6.3.3 Seafloor Contact Modeling

As riser is partly resting on the bottom, seafloor contact is important to be modeled in order to establish proper riser-soil interaction. Horizontal contact with the seafloor is modelled independently in the axial and lateral directions. Contact is initially modelled with linear springs. Sliding will occur when an axial or lateral spring force reaches the friction force value. Springs will be reinstated if the line starts sliding in the opposite direction, or if the friction force increases and is greater than the spring force. The seafloor friction forces are calculated as friction parameter multiplied by vertical forces, and applied against the axial and lateral displacement. The seafloor spring stiffness and seafloor friction constants are given by the user.

6.3.4 Load Modeling

Waves can be defined either as regular or irregular waves. Several model spectra to describe irregular wave are build-in in the program such as Pierson-Moskowitz, Jonswap, etc. Wave spectral parameters are specified by significant wave height and period. The current speed and direction is assumed to be constant with time and can be arbitrarily defined by specifying the velocity and direction at any given water depth.

Hydrodynamic loading is modeled by means of drag and inertia force coefficient in longitudinal and transverse directions. Both quadratic and linear drag terms can be included. Force motions at vessel attachment points are modeled either by specifying vessel motion transfer function or by specifying motion amplitudes and phase angles directly.

6.4 Static Analysis (STAMOD)

In this analysis, the riser static equilibrium is established by incorporating volume forces (weight and buoyancy),prescribed displacements, specified forces (e.g. applied top tension), and displacement dependent forces (current loading).

There are three different methods to achieve riser static equilibrium; catenary method, finite element method, or combination of catenary and finite element method (CATFEM). The outputs of STAMOD are:

- Static XZ, YZ and XY configuration
- Static Forces
- Static Bending moment
- Static curvature

The outputs from STAMOD give initial impression of riser system analysis and will be used as input for dynamic analysis.

6.5 Dynamic Analysis (DYNMOD)

The purpose of these analyses is to study the influence of support vessel motions as well as of direct wave induced loads on the system.

The following types of dynamic analyses are included:

1. Eigenvalue analysis.
2. Harmonic (periodic) excitation.

- Forced displacements (harmonic) at one or more specified nodes
- Regular waves

3. Irregular excitation.

- Stochastic, stationary excitation due to support vessel motions and irregular waves
- Transient excitation. Special options available to simulate release or rupture, slug flow, time dependent current and external force variations

The outputs from DYNMOD are:

- Displacement envelope curves
- Force envelope curves
- Moment envelope curves
- Curvature envelope curves

6.6 Frequency Domain Analysis (FREMOD)

This module performs the dynamic analysis in frequency domain. The frequency domain analysis is based on the linearized dynamic equilibrium equation at static equilibrium position by application of stochastic linearization of the hydrodynamic loading.

6.7 Output Analysis (OUTMOD)

The post-processing module OUTMOD has two main purposes:

- Generate result printout from the INPMOD, STAMOD and DYNMOD modules
- Prepare a plot file (IFNPLO) for later use by the plot module (PLOMOD)

OUTMOD is the last step of RIFLEX analysis and the outputs from this analysis are used to evaluate the SCR performance. Some important outputs are Von Mises stress for strength design check and fatigue life for fatigue analysis check.

CHAPTER 7 CASE STUDY

7.1 Introduction

SCRs have become a preferred riser concept in almost every new deepwater field development and have been designed and installed on TLP, SPAR, Semi-submersible and FPSO. In this study a steel catenary riser is connected to the sea bed from a semi-submersible. A global nonlinear response analysis is performed on the steel catenary riser using the finite element computer program RIFLEX. The main purpose of this study is to give an initial assessment of the overall static and dynamic behavior of the specific riser system when subjected to extreme environmental loads.

7.2 SCR Strength Analysis Methods

According to DnV, the purpose of global riser system analyses is to describe the overall static and dynamic structural behaviour due to the system exposing to a stationary environmental loading conditions. The output of global riser system can be grouped into the following categories:

- Resulting cross-sectional forces (effective tension, bending moments, torsional moment)
- Global riser deflections (curvature, elongation, angular orientation)
- Global riser position (co-ordinates, translations, distance to other structures, position of touch down point on seafloor, etc)
- Support forces at termination to rigid structures (resulting force and moments)

In order to derive all necessary output in global riser analyses, the following sections describe the stepwise analysis methods.

7.2.1 Static Analysis

The main purpose of riser static analysis is to establish the equilibrium profile of riser under the combined effects of self-weight, buoyancy, extreme vessel offset and current. The basic loading components in static analysis are categorized into [DnV]:

- Volume forces (weight and buoyancy);
- Prescribed displacements (displacement of terminal points from stress free to specified positions)

- Specified forces (e.g. applied top tension), and
- Displacement dependent forces (current loading)

- **Volume Forces**

Volume forces of steel catenary riser are derived under the combined effect of selfweight, buoyancy, hydrostatic and internal fluid pressure. Figure 6.1 shows how the equilibrium of a segment of curved pipe under the combination of volume forces components can be equated to equilibrium under equivalent effective parameters by introducing equal and opposite pressures over the end faces of the segment.

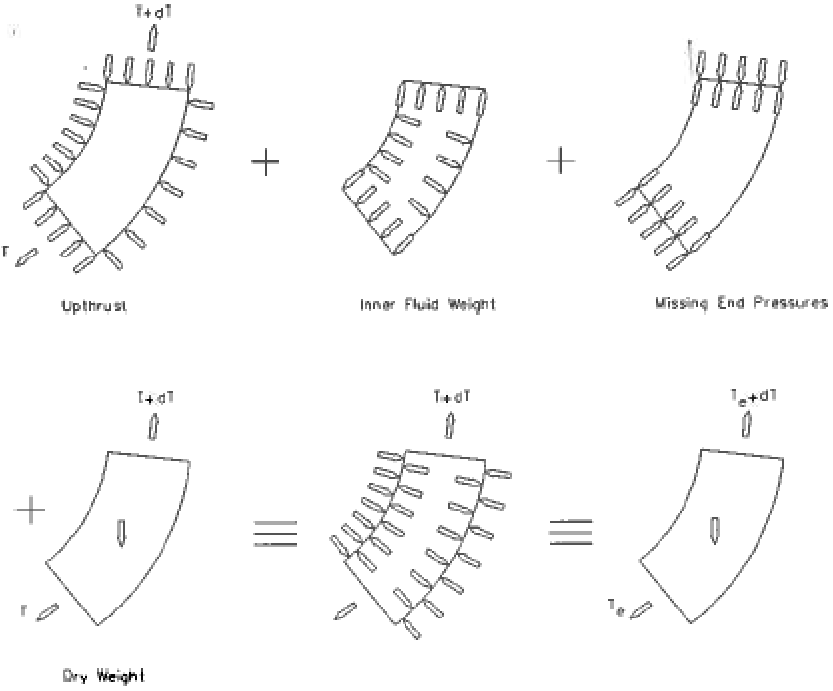

Figure 7-1 Effective weight and tension

The formulae for effective weight and tension are:

$$W_{eff} = \gamma_s A_s + \gamma_i A_i - \gamma_0 A_0 \tag{7.1}$$

$$T_{eff} = T_t + P_0 A_0 - P_i A_i - \rho_i u_i^2 A_i \tag{7.2}$$

Where:

γ	weight density
A	area
P	pressure
T	tension
ρ	mass density

and subscripts:

i	internal
o	external
s	structural
t	true

• Prescribed Displacement

Volume forces and specified displacement from stress free to final position of terminal points are included in the catenary start solution. This iterative approach on boundary condition will give deviations between specified translating boundary condition i.e. x and y- coordinate and boundary conditions computed by the catenary analysis.

Further, specified boundary conditions for rotations at the supports will not be satisfied by the catenary analysis due to neglection of bending stiffness. The final position is therefore found by application of prescribed displacements from catenary solution to specified positions.

• Specified Forces (nodal point load)

Specified forces are used for the system where additional forces are applied to riser.
One example is top tension force which is applied at the top end of riser in order to keep the riser in tension in any cases. It is normally applied for Top-tensioned riser system. In SCR system, tension force of riser relies on its submerged weight. No additional tensioner is applied to SCRs. Therefore, in this particular case, specified forces are not considered.

• Displacement dependent forces (current loading)

For completion of equilibrium static configuration, steady current is applied in order to consider the relative magnitude of these forces in comparison to the effective weight.

In order to perform static riser analyses, a nonlinear finite element approach is normally performed. The FE approach for SCR static configuration are analyzed in the order of volume forces (1)-prescribed displacement (2)-displacement dependent forces (4). Figure 7.2 below shows the sequence of establishing equilibrium static configuration.

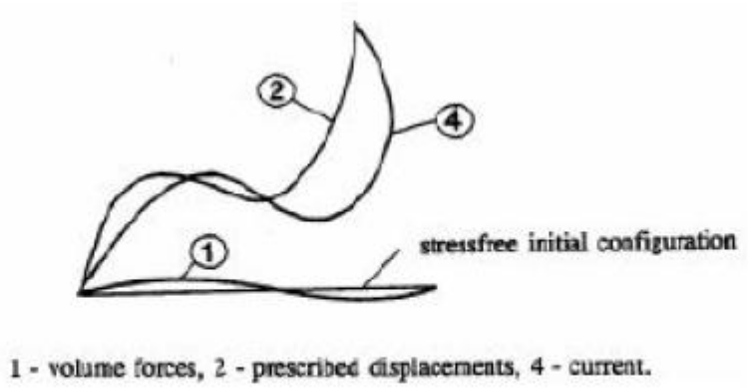

Figure 7-2 Default increment loading sequence

The static riser profiles are derived for near, mean, far locations of the host platform. The static profiles give an idea of the behaviour of riser under dynamic analysis. It is important to establish smooth transition at touch down area to prevent any excessive dynamic bending moment.

7.2.2 Dynamic Analysis

Steel Catenary Riser is the system that exposed to highly dynamic behaviour due to environmental loading from wind, waves and current. Actions induce loads in a SCR both directly through wave and current action on each segment of riser and indirectly through motion of suspension points on a floating vessel.

Furthermore, in slender marine structures such as riser, there are some nonlinear effects that must be considered in dynamic analyses [DnV]:

- Geometric stiffness (i.e. contribution from axial force to transverse stiffness).
- Nonlinear material properties.
- Hydrodynamic loading according to the generalized Morison equation expressed by relative velocities.
- Integration of loading to actual surface elevation.
- Contact problems (bottom contact, riser collision, vessel/riser,etc)

In addition, nonlinearities will be decisive for the statistical response characteristics for systems exposed to irregular loading [DnV]. Therefore, it is important to treat all the nonlinear effects in a correct way according to design criteria. Table 7.1 below is the available techniques for dynamic finite element analysis:

Table 7-1 Dynamic Finite Element Analysis Techniques

Non-linear time domain	Linearized time domain	Frequency domain
Non-linearities are evaluated at each time step and incorporated directly in the dynamic solution	Linearization of the dynamic equilibrium equations (stiffness, damping, inertia)- i.e. structural linearization	Linearization of stiffness, damping, inertia and external forces at static equilibrium position- i.e. structural and load linearization
Stiffness, damping, inertia and external forces are updated at each step	Stiffness, damping, inertia matrices are kept constant.	Stiffness, damping, inertia, and external forces matrices are kept constant
Give a good representation of a possible non-Gaussian response	Give a good representation if hydrodynamic loading is the major nonlinear contribution	Give a good representation of a possible Gaussian response
Application to the systems that undergo large displacements, rotations or tension variations or in situations where description of variable touch down location or material nonlinearities are important	Application to the tensioned risers with moderate transverse excursions	Not recommended for extreme response prediction. Main application to fatigue calculations and long-term response statistics to identify design conditions to be applied in time domain analyses

According to table comparison given above, for SCR strength analysis in deepwater field and harsh environmental conditions where large displacement, rotations and sensitivity in touch down area, non-linear time domain is best suited. Extremely time consuming in this method can be reduced by varying length of segment along different region of riser and time step.

7.3 Design Parameters

The properties of the components of a riser system, the behaviour of adjoining facilities such as the floating platform, and the environmental conditions of the area of deployment of the riser system determine the performance of the riser system. All these form the basic input parameters for a typical riser design and analysis operation. An accurate knowledge of these parameters is therefore important before commencing any riser system analysis.

7.3.1 Environmental Data

The water depth considered in this study is 1000m, with sea water density, ρ_{sw}, is 1025 kg/m^3. This is a typical water depth for deepwater areas of the North Sea.

The extreme sea state typical to the North Sea location is modelled by irregular waves. It is desirable to design a riser system such that it is able to withstand extreme sea states with a low probability of exceeding its 100-year response value. It is therefore common to design riser systems to be able to withstand different combinations of wind, waves and currents yielding the same return period of 100 years in conformity with standards such as NORSOK N-003, API RP 2RD, and DNV-OS-F201.For this study, the following sea state is considered:

100-year sea state:
- Significant wave height, H_s 15 m
- Corresponding wave peak period, T_p 16 sec

The sea state is modelled as a wave spectrum with energy distributed over a range of frequencies. The spectrum that typifies the North Sea condition is the JONSWAP (Joint North Sea Wave project) spectrum and this will be deployed in this study.

The current velocities vary with depth being maxima at the surface and vary with directions. The current flow and wave directions are assumed to be in the same direction as the vessel offset as the most critical loading conditions generally occur when these actions are in the plane of the catenary (DNV-OS-F201, 2010).

The 10-year return period current profile typical to the North Sea location is considered in this study. This is presented in Table 7-2.

Table 7-2 Current Data

Water Depth (m)	**10-year Current (m/s)**
At surface	0.93
-50	0.68
-300	0.47
-1000	0.00

7.3.2 Soil-Riser Interaction

According to Bai and Bai (2005), when the portion of a riser in contact with the seabed is subjected to oscillatory motion, there is complex interaction between the motion of the riser and the seabed. This forces the riser into the soil, thereby increasing the soil resistance. Soil-riser interaction is commonly modelled by use of friction coefficients and linear soil stiffness.

The soil-riser interaction parameters used in this study are as follows:

- Transverse friction stiffness 0.5
- Normal stiffness 600 Kpa

7.3.3 Hydrodynamic Coefficients

Hydrodynamic coefficients which consist of drag coefficient (C_D) and inertia coefficient (C_M) are used for calculating wave and current forces impose to structure. Common practice for this type of marine structures is to apply a C_D in the range of 0.7-0.8. However, in this work a C_D of 0.9 is used to include a possible increase in external diameter from marine growth. No marine growth was explicitly included. The inertia coefficient C_M used in this analysis is 2.0.

7.3.4 Vessel Data

As the oil and gas explorations are moving into deeper water and harsher environment, the use of semi-submersibles as floating production unit have increased. The slow drift motion of Semi-submersible can be 10-15% of water depth. Relatively high vessel offset for semi-submersible results in challenge for SCR design because the riser tension at the vessel becomes too great as the vessel drifts away from the touch down point (far load case) or the bending stresses near the seabed become too great as the vessel drifts towards the touch down point (near load case).

As deep water and harsh environment have been selected in this study, and looking at the trend of using semi-submersibles as floating unit for future SCR applications, attention is focused on SCRs attached to semi-submersible vessels.

The static vessel offset in connection with the extreme response analysis is 10% of water depth for intact mooring and 12% for one mooring line failure condition. Offsets to near and far side are considered in this study. Maximum offset is considered as 100m. Offset to transverse direction is not considered because only one single riser is studied. In order to consider dynamic excitation from wave frequency floater motions, RAOs of semi-submersible are applied to all steps of design analyses.

7.3.5 Upper End Termination

According to Song and Stanton (2007), terminating a steel catenary riser at a floater requires usage of a hang-off system. In general, three hang off systems have been used; they are flex joint, tapered stress joint (TSJ), and pull tube. Selection of any of these hang-off systems depends on its functional requirements in terms of required angular deflection, steel catenary riser size, and expected top tension. The most commonly used hang-off system is the flex joint due to its ability to better accommodate variations in riser performance characteristics.

For this study, the top end of the riser is assumed to be equipped with a flex joint attached to the riser termination point. Thus in the global analysis, the top end is modelled as pinned; in other words it was free to rotate.

7.4 Structural Modelling

In order to keep the SCR design as simple and economical as possible, a simple free hanging catenary shape is considered. The riser upper end is connected to a semisubmersible and the bottom end is connected to subsea flowline. The SCR is hanging from outside the pontoon as shown in figure 7.3.

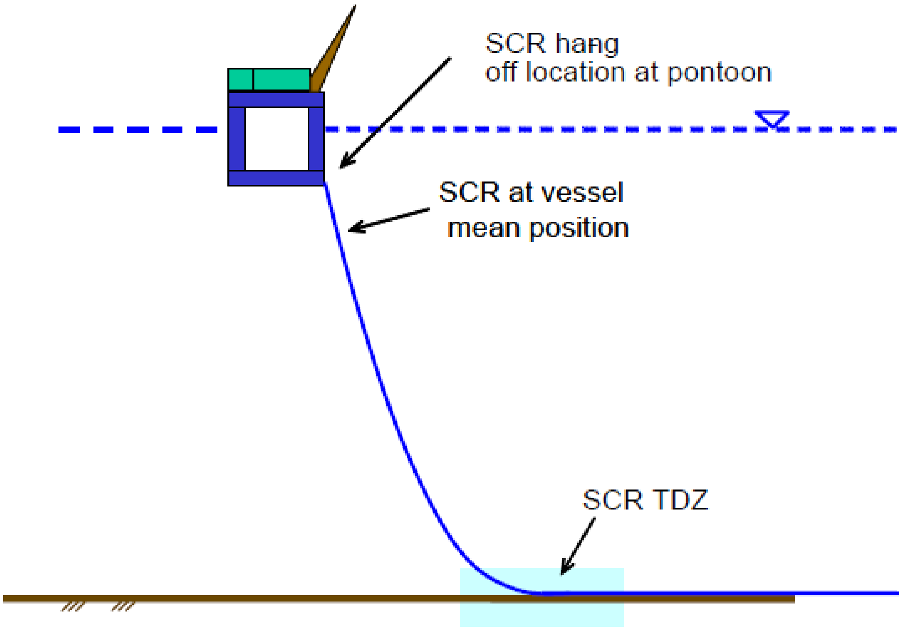

Figure 7-3 Riser Hang-off from Pontoon

The riser is placed 20m from the centre of the vessel and 20m below the mean sea level. Top angle of 15^o is defined in vessel mean position. In this study, the riser with inner diameter of 10 inch and uniform wall thickness of 22 mm is considered. It is an X65 carbon steel pipe with density 7850 kg/m^3.The whole riser was wrapped normal coating with thickness of 100 mm and density of 8000 kg/m^3.

Material properties for X65 carbon steel are provided as follows:

- Yield stress at 20^o C SMYS = 448 MPa
- Steel Young's modulus E = 207000 MPa

The riser is modeled as an arbitrary system with a line consisting of 510 beam elements with varying length between 2m and 10m.The total riser length is 2350m. The main concern in deciding total length of riser is to accommodate riser configuration for both near and far platform offset. Riser is segmented for finite element analysis. The purpose of this is to obtain an adequate representation of riser behaviour. Therefore, it is important to define sufficient riser segment at some critical regions such as at top-end, sag-bend and touch down area. Too long riser segment in highly dynamic region will cause instability results.

7.5 SCR Strength Analysis

SCR strength analysis is performed to check the extreme response of SCR under static and dynamic structural behavior. The strength analyses are performed for both near and far position vessel offsets with 100-year wave and 10-year current.

7.5.1 Analysis Procedure

The riser configuration is developed by satisfying the ULS design conditions. The basic configurations are obtained by performing nonlinear dynamic response analysis using the dynamic analysis program RIFLEX. In this case study, the following principle and assumptions in the static and dynamic analysis are applied:

- Three different models are prepared according to vessel offset: mean, near and far.
- The SCR is modeled by FEM principles using discrete beam elements.
- Static riser configuration is established by considering volume forces (weight and buoyancy), specified displacement and current forces.
- Nonlinear dynamic response analysis is used applying irregular waves. The riser configuration and tension are calculated at each time step by an iterative procedure and the dynamic response of the riser system is estimated using the Newmark-βmethod, with constant average acceleration algorithm.
- The dynamic analysis was performed for both 0° and 180° wave directions (in plane with the riser configuration). The lateral load case is not critical for riser dimensioning, so is not considered.

7.5.2 Static Analysis

The analysis results with regard to the riser configurations, effective tensions and static bending moments for the near, mean and far position are discussed and shown in figure 7.4, 7.5, 7.6 respectively.

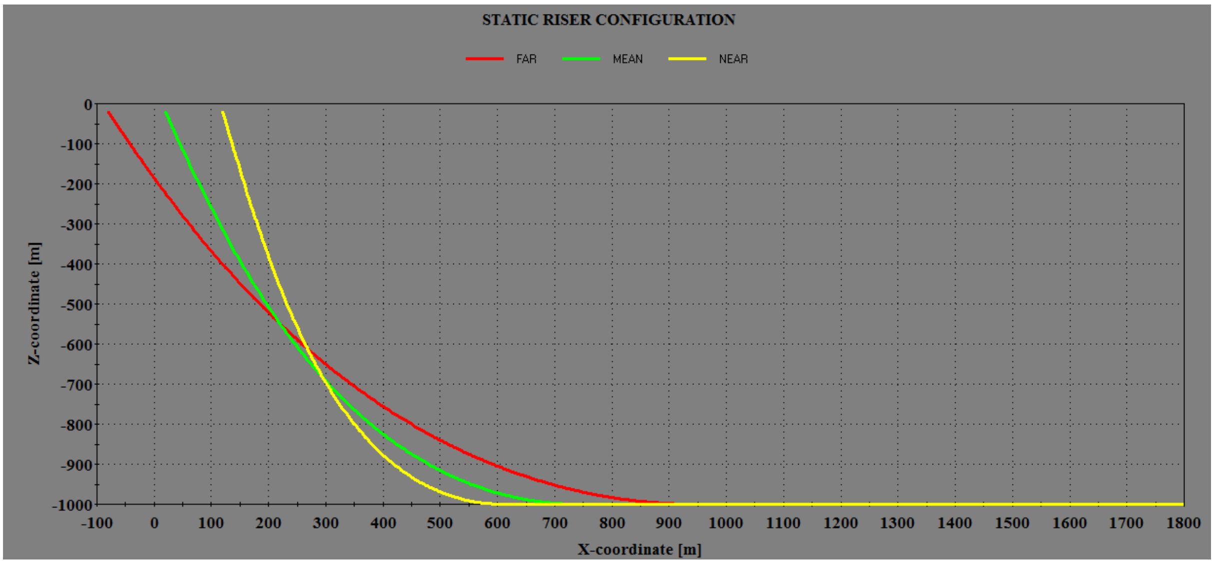

Figure 7-4 Static Riser Configuration

From the static riser configuration, vessel offsets have significant effect on riser configurations. An important design consideration with respect to SCR strength design is the significant change in location of TDP from near load case to far load case. This may result in significantly different dynamic behaviours. Further, it should be kept in mind that the increased water depth will increase the vessel offset which will be the problem for ultra deepwater development.

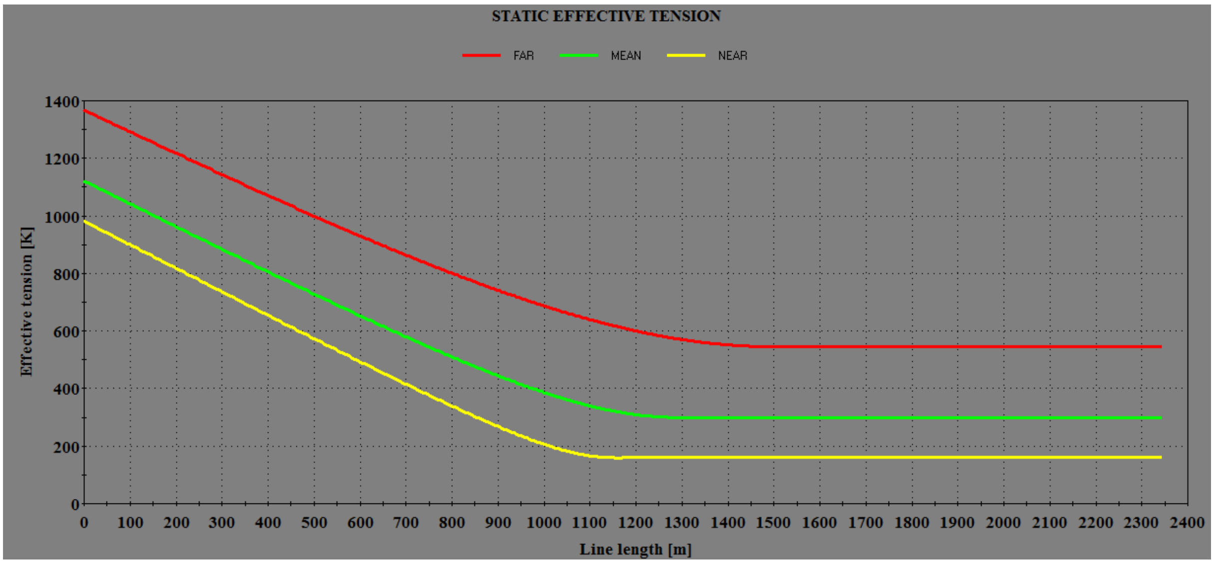

Figure 7-5 Static Effective Tension

The static effective tension for configurations is shown in Figure 7.5. From the figure, it can be observed that the effective tension forces are highest at the top end since the whole submerged weight of the riser is carried here. Static tension force is simply a function of suspended riser length. When the floater is in the far position, it has the highest static tension due to its longest suspended length compared to mean and near case.

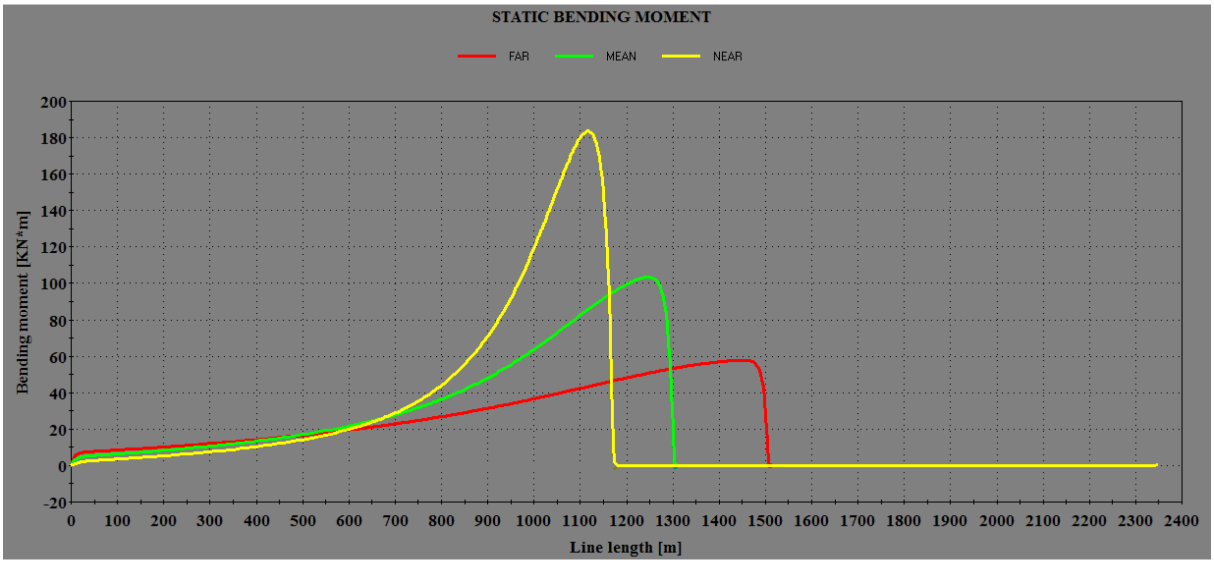

Figure 7-6 Static Bending Moment

The static configuration for the steel catenary riser is the result of tension at the top and maximum bending stress at the touch down point.

In general, the maximum bending moments is of greater significance than the maximum effective tension, and will be the governing design parameter in most cases. This is because the riser has far greater capacity to withstand axial loads than lateral loads.

Figure 7.6 shows static bending moment for all load cases. From the figure, one distinctive peaks stand out for each load case. The highest peak is found in the touch down region. The bending moments in the top region is considerably smaller than that in the touch down region. At the floater-riser interface the bending moments is zero due to the flex joint allowing the riser to rotate freely in all directions.

In near load case, the distance from host platform to TDP is the closest. This results in small sag-bend curvature i.e. high static bending moment as shown in figure 7.6. On the other hand, with high sag-bend curvature for far load case, the static bending moment is significantly low.

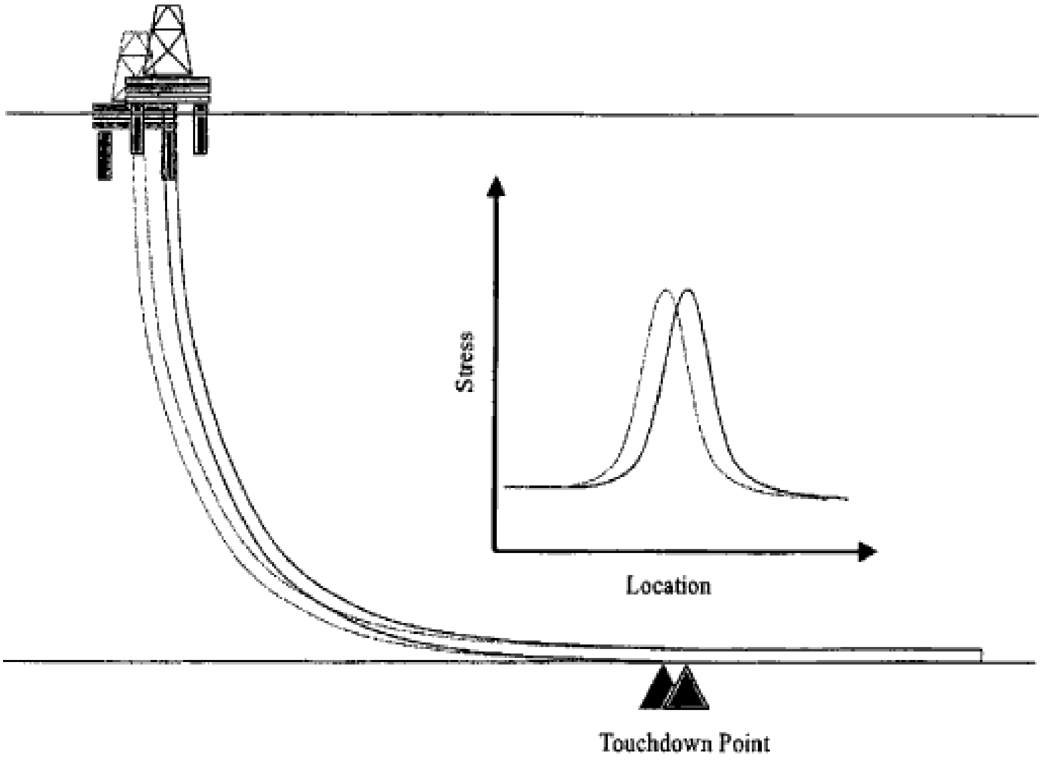

Figure 7-7 Illustration of the Movement of the TDP for an SCR

7.5.3 Dynamic Analysis

Nonlinear time domain analysis with irregular waves is performed to analyze dynamic response of SCR. The duration of time domain dynamic analysis is 3 h (10,800 s). The results from dynamic analyses are effective tension force and dynamic bending moment.

As it is seen in figure 7.5, far load case gives the highest force for total effective tension. Therefore, the envelope effective tension for far load case is considered as shown in figure 7.8.

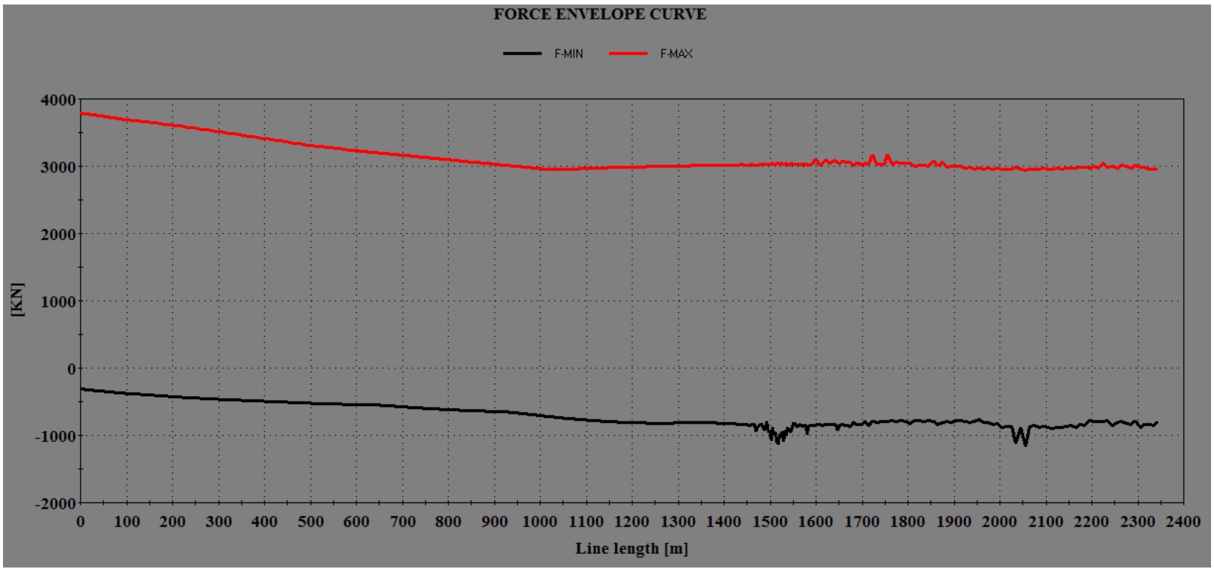

Figure 7-8 Dynamic Tension Force

From the figure, it can be seen that the minimum tension force is negative at TDP and it is not expected for SCR. Therefore, it is important to optimize the riser configuration and avoid any compression at TDP.

As it is seen in figure 7.6, near load case gives the highest bending moment compared to mean and far load case. Therefore, the envelope bending moment curve for near load case is considered as shown in figure 7.9.

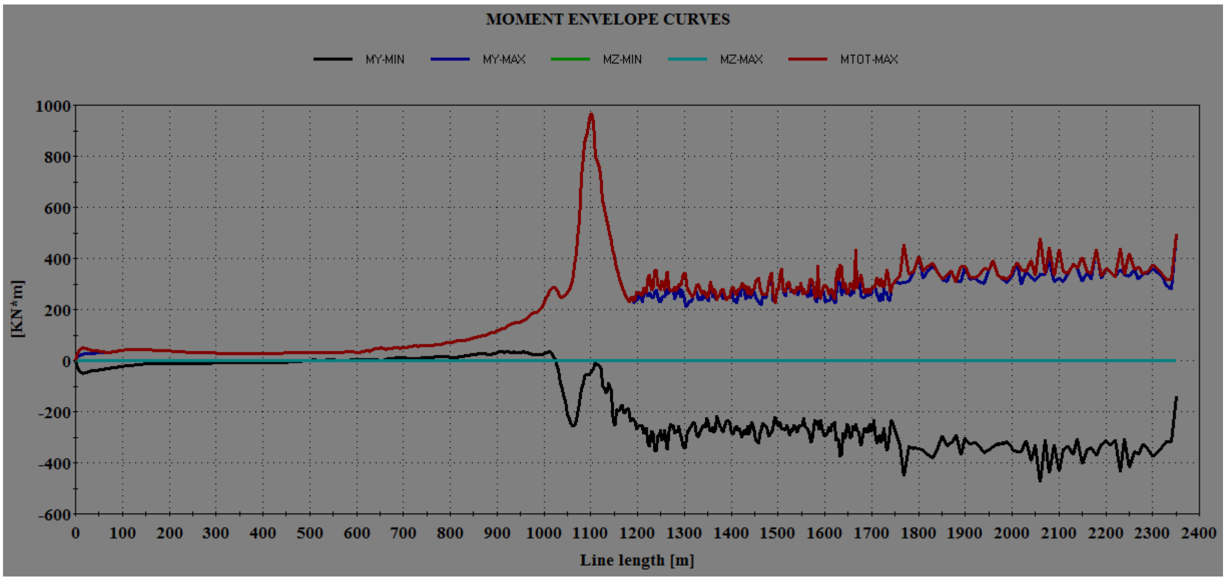

Figure 7-9 Dynamic Bending Moment

In figure 7.9, it can be analyzed that there is only one peak of bending moment, which is located at TDP. Furthermore, the pipe on the seafloor is unstable, indicated by the curly curve after TDP for near load case.

7.6 Sensitivity Study

As mentioned above, structural response of the riser system is a function of several parameters, including the offset/floater’s position, current directions, damping coefficient, marine growth, soil characteristics, etc. A detailed parameter study is thus generally required in order to determine the characteristic load effect.

In this study, the effects of various parameters on the structural performance of the steel catenary risers will be studied. The parameters to be studied here are as follows:

•Offset/Floater’s Position
•Current Profiles
•Damping Coefficient
•Mesh Density
•Wall thickness (WT)
•Internal diameter (ID)

7.6.1 Offset/Floater's Position

The maximum offset is considered as 10% of water depth and a smaller increase of the offset with 25 m from the near position is given without variation of current and wave directions. The results with regard to the riser configurations, effective tensions and bending moments are shown in figure 7.10, 7.11 and 7.12.

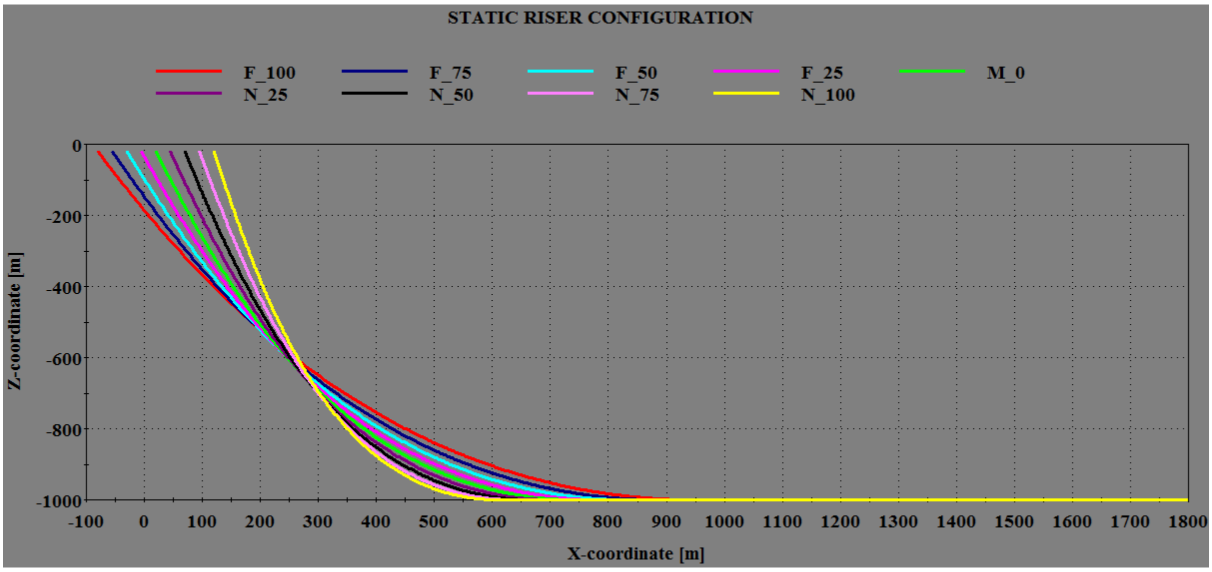

Figure 7-10 Riser Configurations with respect to Vessel Offset

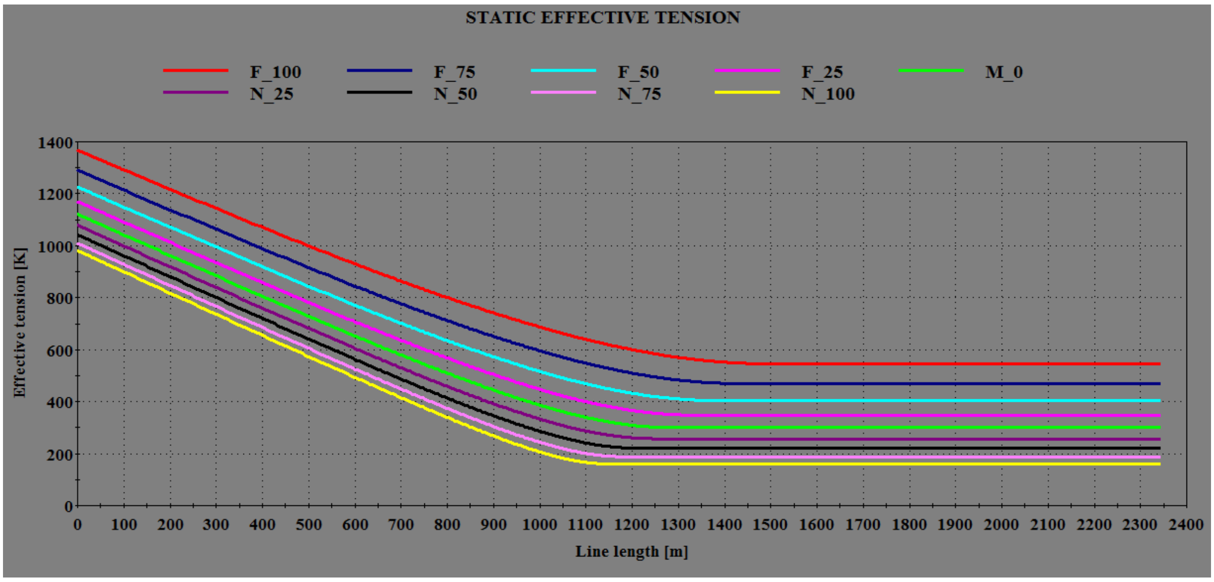

Figure 7-11 Effective Tensions with respect to Vessel Offset

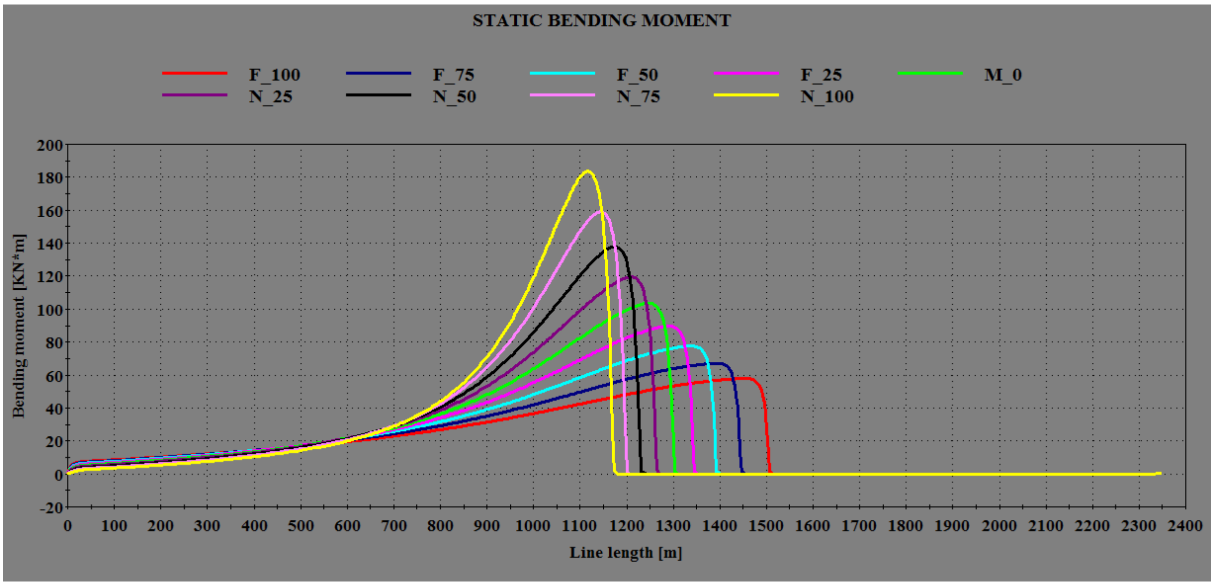

Figure 7-12 Bending Moments with respect to Vessel Offset

As mentioned before, the maximum bending moment occurs in the touch down region. When the floater is in the nearest position, the worst condition with the largest bending moment and the smallest effective tension can be obtained.

7.6.2 Current Profiles

The maximum bending moments and effective tension forces with current directions 0^0, 180^0 for mean load case are shown in Figure 7.13 and 7.14.

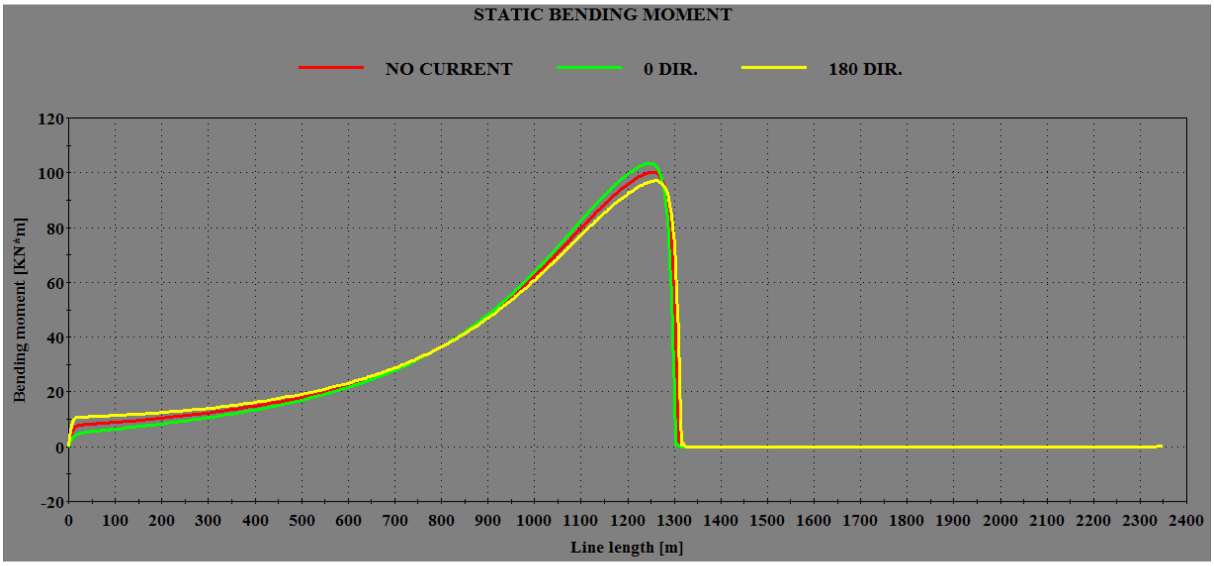

Figure 7-13 Bending Moments with respect to Current

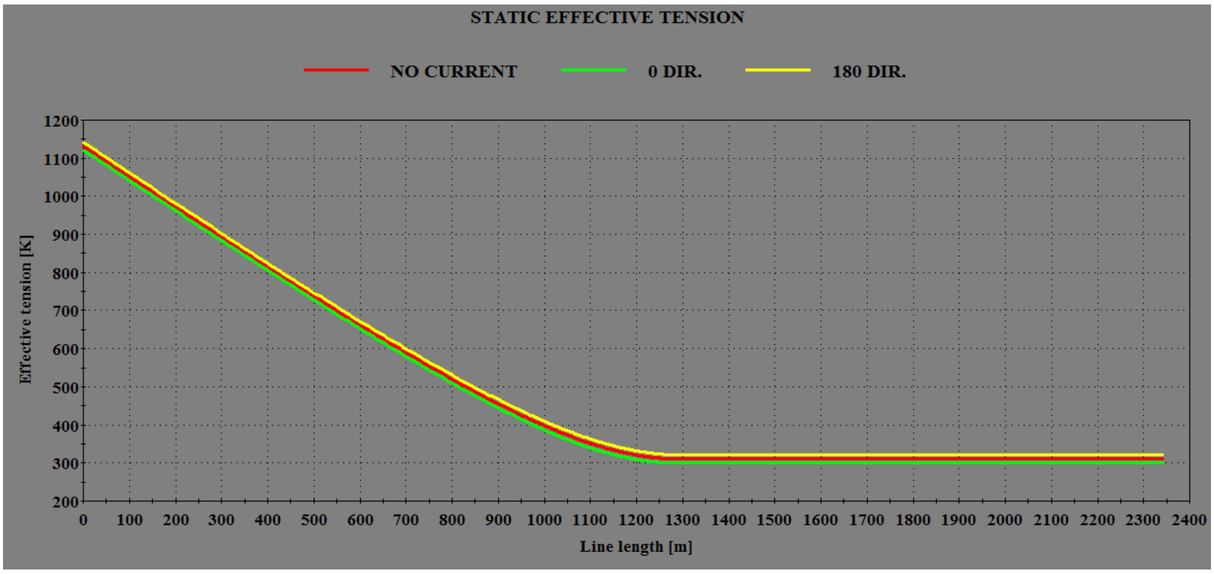

Figure 7-14 Effective Tensions with respect to Current

From the results shown in Figure 7.13 and 7.14, current profiles have significant effect on the bending moments of the riser system, especially in the touch down region. Current will increase the bending moment in the touch down region when it is in the positive direction of the x-axis. The bending moment is significantly decreased when the current is in the negative direction of the x-axis.

7.6.3 Damping Coefficient

The effect of structural damping is investigated by using different a_2 values for mean load case, see figures 7.15 and 7.16.

SCR base case

•ID	0.254 m
•WT	0.022 m
•Top hang off angle	15°
•Vessel offset	0 m, mean position
•Simulation length	1800 s
•Time step	0.1 s
•Damping coefficient	a_2=0.0127, 0.03, 0.5 and 1.

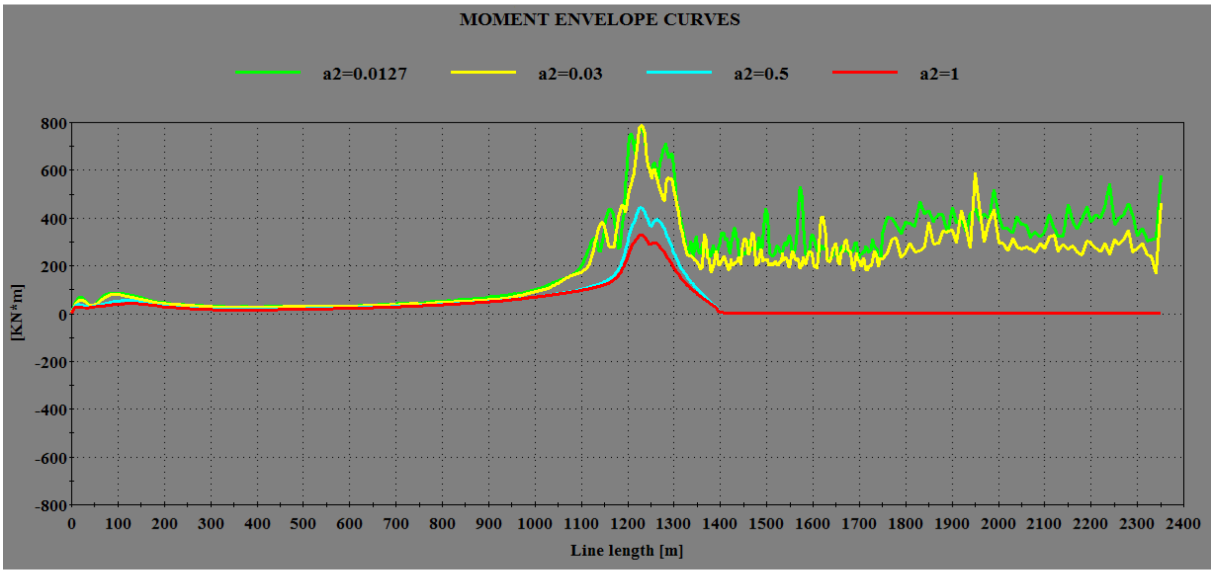

Figure 7-15 Moment Envelope Curves with respect to Damping Coefficient

From the figure , it can be seen that structural damping has a significiant effect on the structural response of the steel catenary riser. Bending moment in the touch down region can be reduced by increasing structural damping. It should be kept in mind that unrealistic high values like $a_2 = 0.5$ and 1.0 will influence the results too much.

The peak values of moment envelope curves within a length of about 250 m along the riser are shown in Figure 7.16.

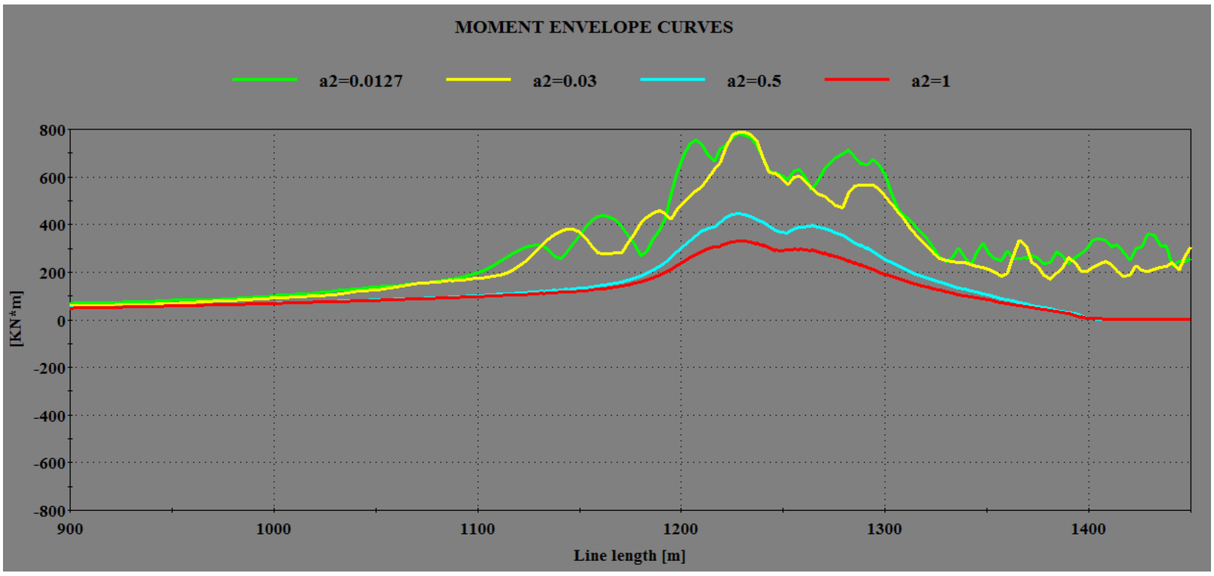

Figure 7-16 Peak values of Moment Envelope Curves

7.6.4 Mesh Density

In this section, static and dynamic results with respect to variation of mesh size are investigated. The riser is designed with beam elements. A set of 6 different mesh sizes is used for comparison, varying from 3 m to 10 m. The results are presented in Figure 7.17, 7.18, 7.19 and 7.20.

SCR base case

•ID	0.254 m
•WT	0.022 m
•Top hang off angle	15°
•Vessel offset	0 m, mean position
•Simulation length	1800 s
•Time step	0.1 s
•Mesh density	3, 4, 5, 6, 8 and 10 m

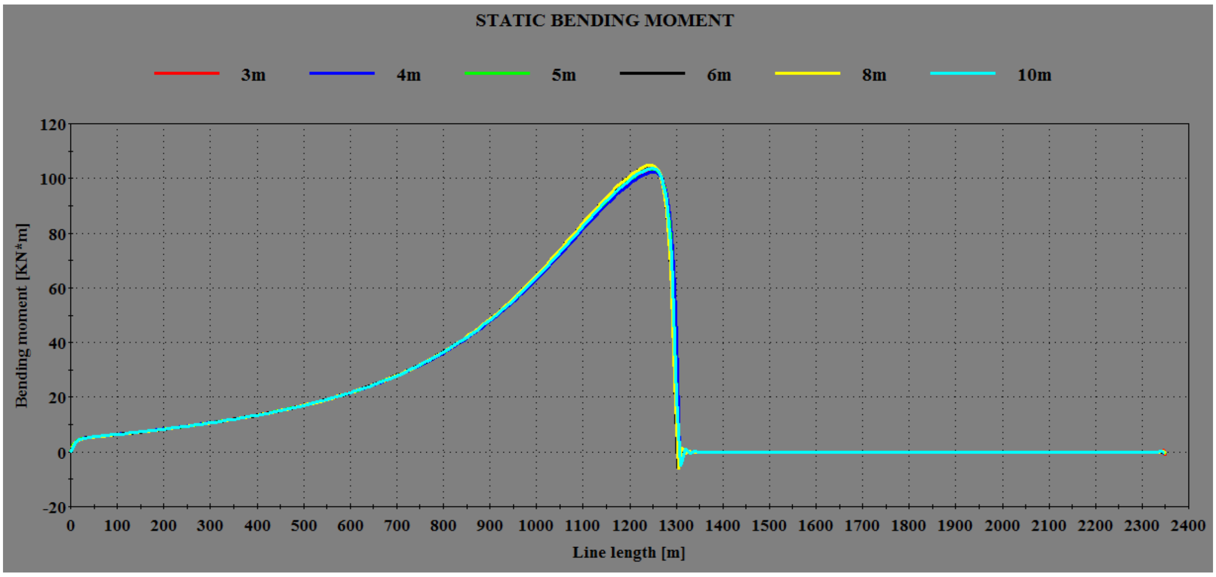

Figure 7-17 Bending Moments with respect to Mesh density

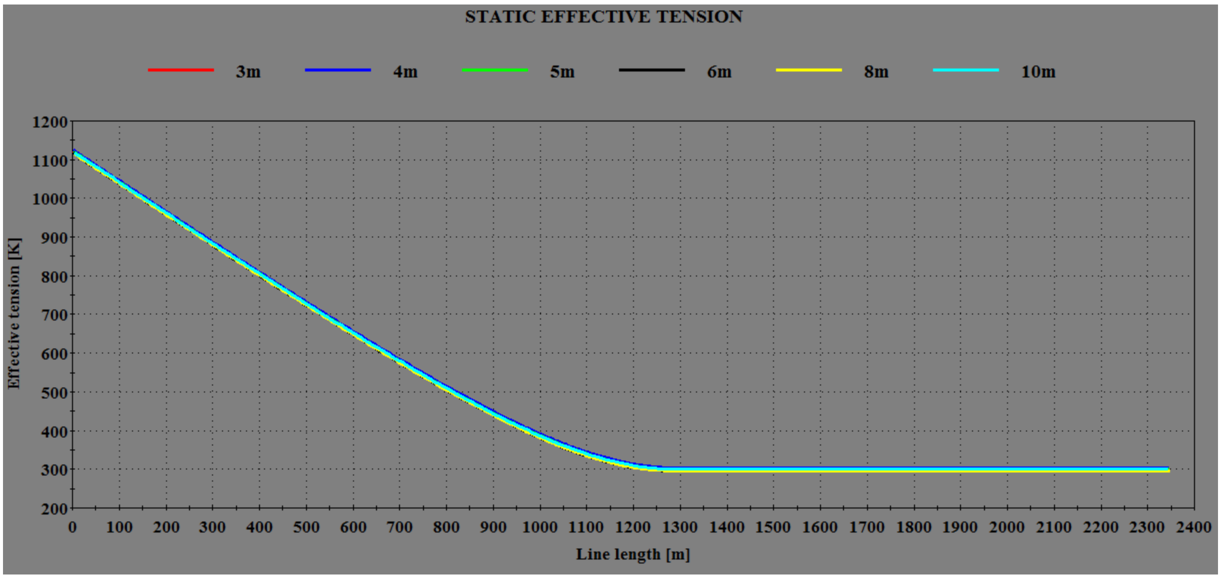

Figure 7-18 Effective Tensions with respect to Mesh density

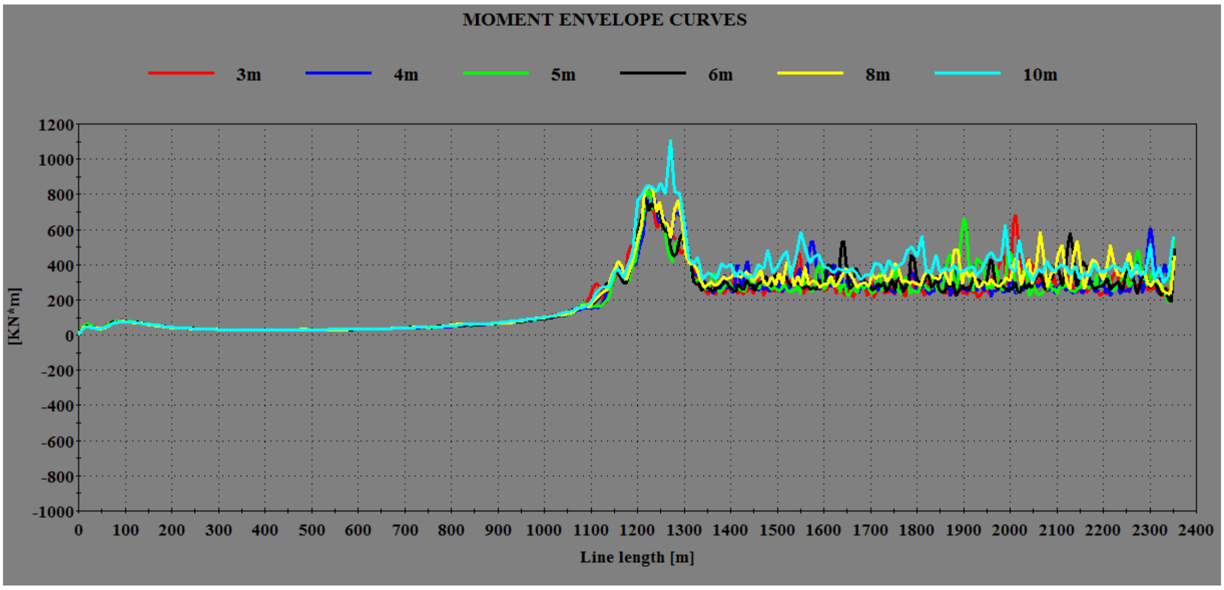

Figure 7-19 Moment Envelope Curves with respect to Mesh density

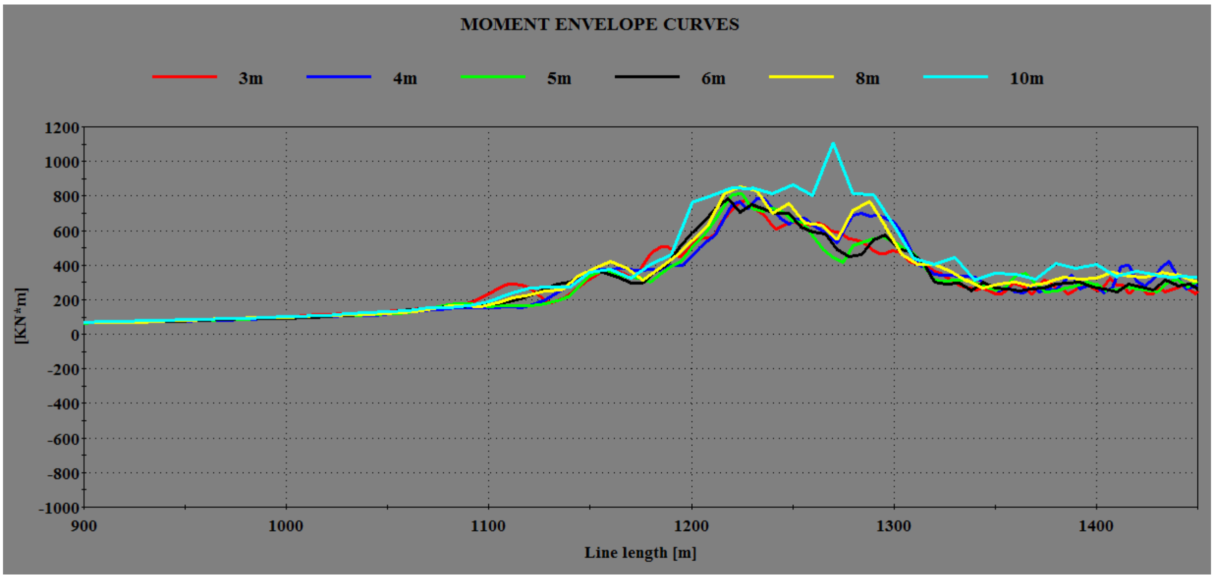

Figure 7-20 Peak values of Moment Envelope Curves

Smaller mesh density may lead to better results or more CPU time while larger mesh density may lead to unstable results for both in static and dynamic analysis. Bending moment seems more sensitive to mesh density than static tension force. Figures 7.17and 7.20 show that mesh sizes of 3 m to 5 m provided stable results for both static and dynamic analysis.

7.6.5 Wall thickness (WT)

Wall thickness sizing has significant effect for the system response. The riser with various WT is analyzed. Static and dynamic results are shown in Figures 7.21, 7.22, 7.23 and 7.24.

SCR base case

•ID	0.254 m
•Top hang off angle	15°
•Vessel offset	0 m, mean position
•Simulation length	1800 s
•Time step	0.1 s
•WT	0.018, 0.020, 0.022, 0.024 and 0.026 m

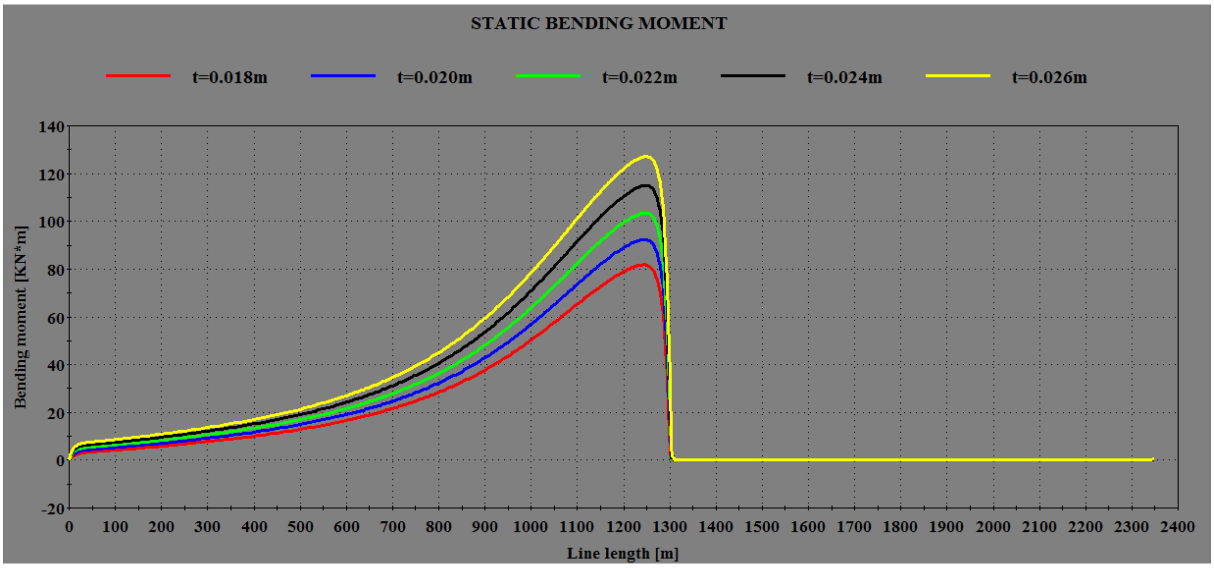

Figure 7-21 Bending Moments with respect to WT

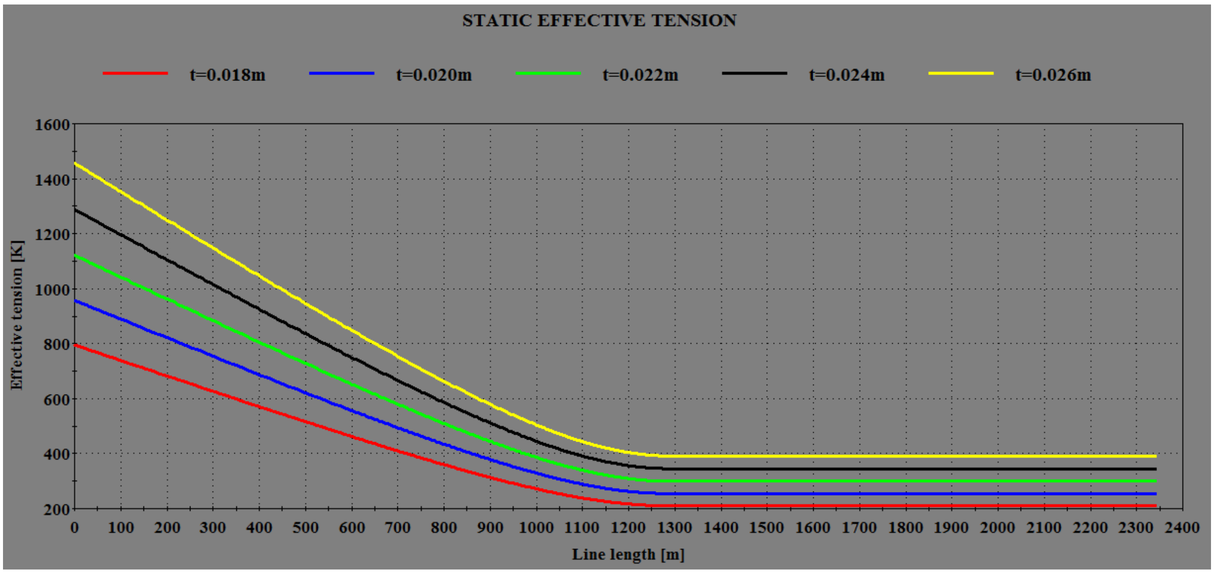

Figure 7-22 Effective Tensions with respect to WT

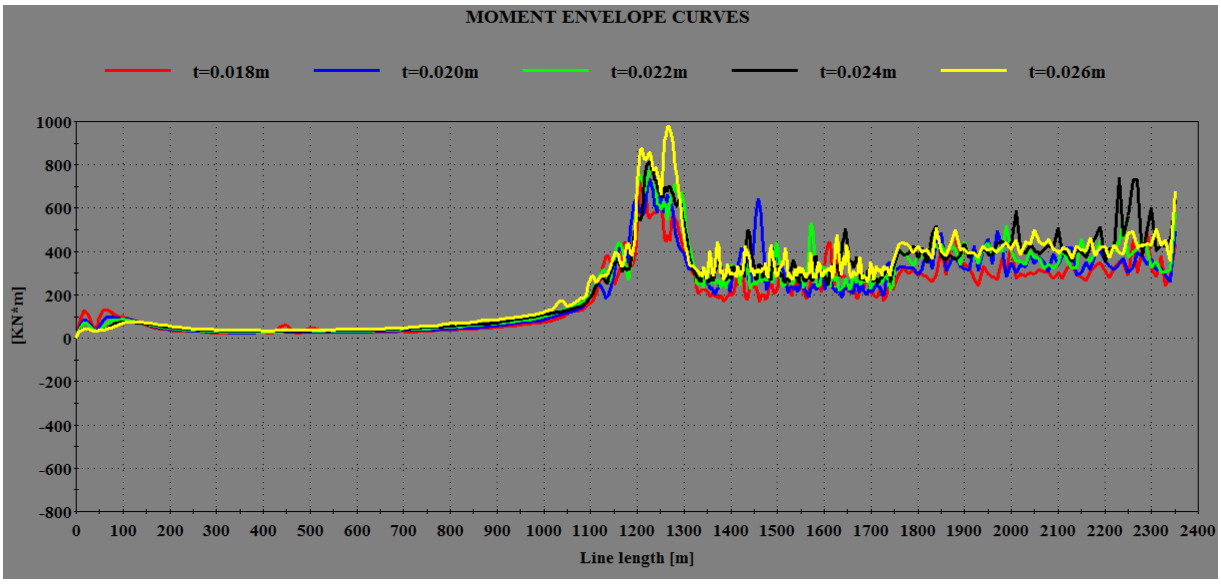

Figure 7-23 Moment Envelope Curves with respect to WT

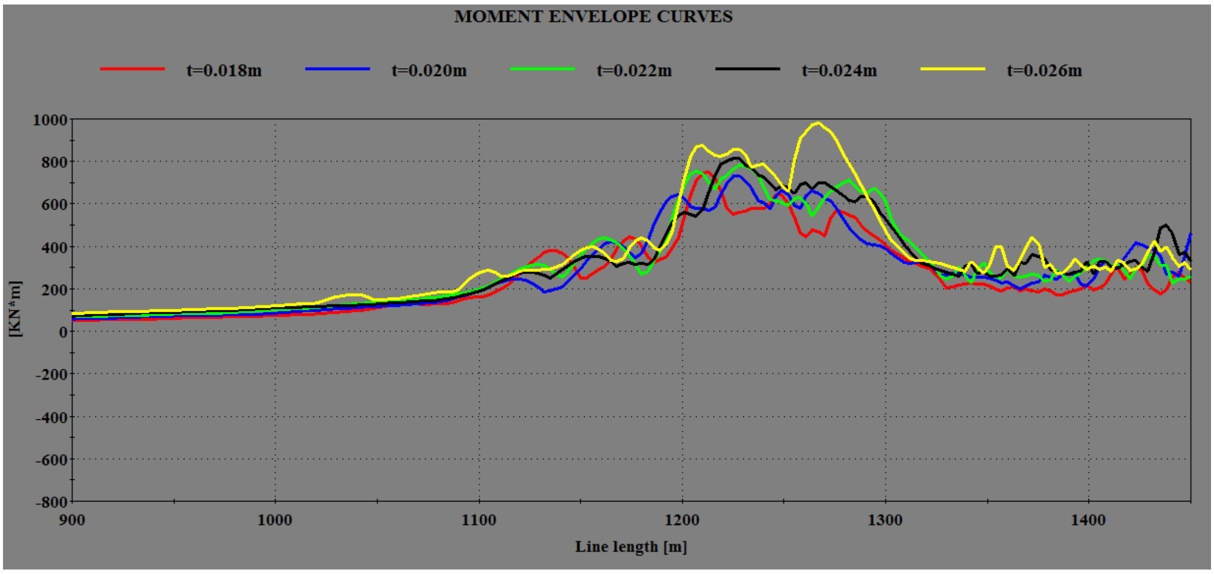

Figure 7-24 Peak values of Moment Envelope Curves

As shown in Figure 7.22, thicker wall increases static tension force, which has positive effect on buckling prevention. However, with the increase of riser wall thickness, both dynamic and static bending moments are increased. Hence, if one has not a buckling problem, wall thickness increase is not proposed.

7.6.6 Internal Diameter (ID)

Diameter is one of the critical parameters for the system response. A steel catenary riser base case with different IDs is investigated. Static and dynamic results are presented in Figures 7.25, 7.26, 7.27 and 7.28.

SCR base case

•WT	0.022 m
•Top hang off angle	15°
•Vessel offset	0 m, mean position
•Simulation length	1800 s
•Time step	0.1 s
•ID	6, 8, 10, 12 and 14 inch

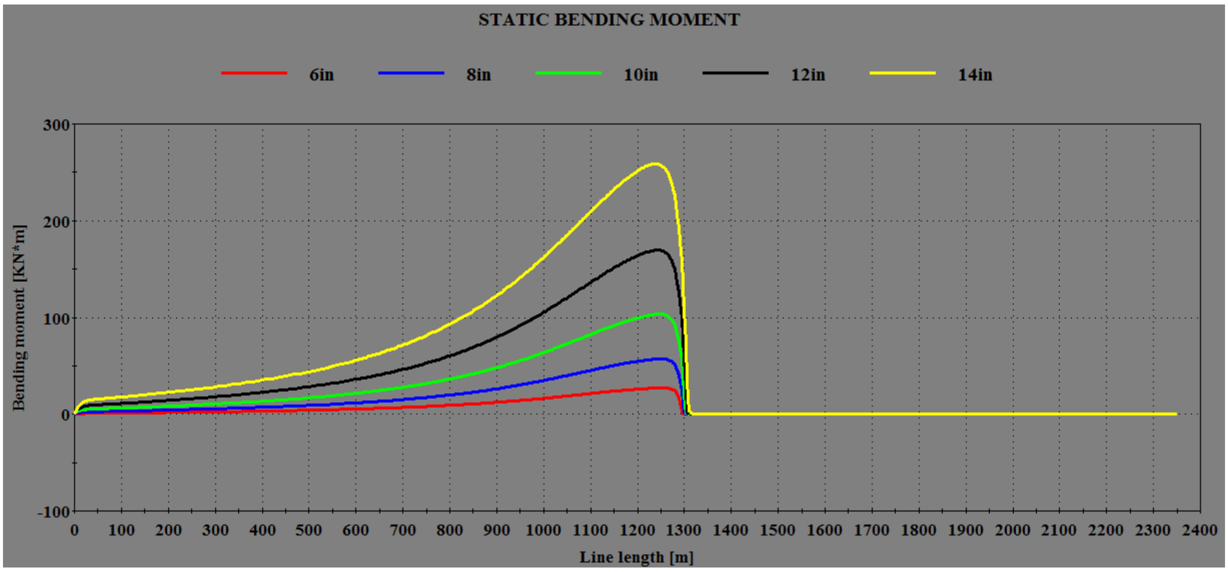

Figure 7-25 Bending Moments with respect to ID

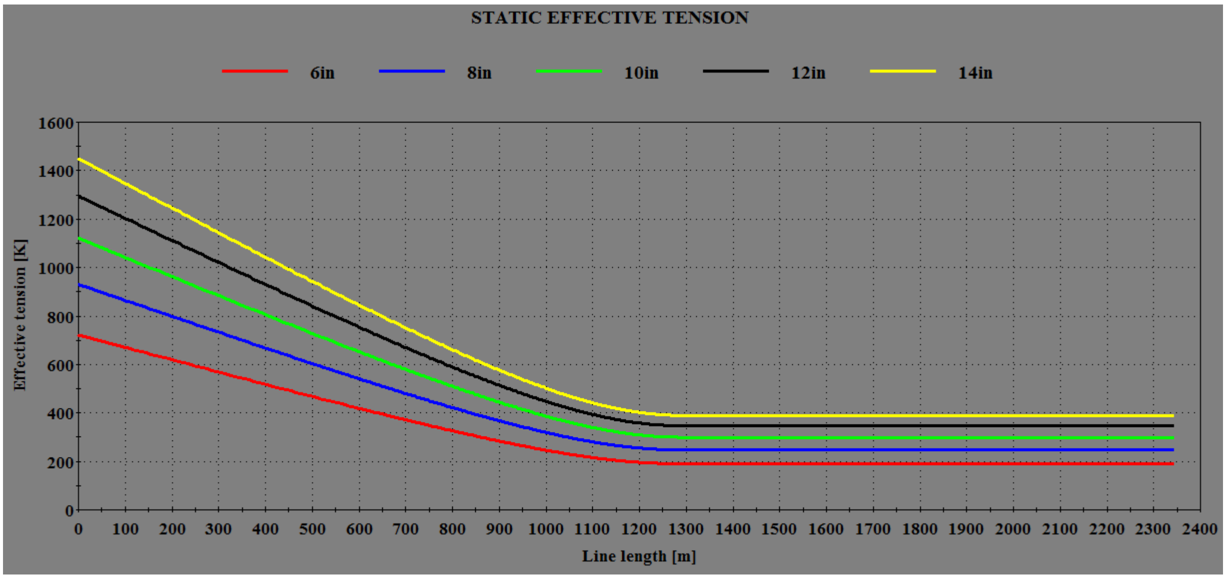

Figure 7-26 Effective Tensions with respect to ID

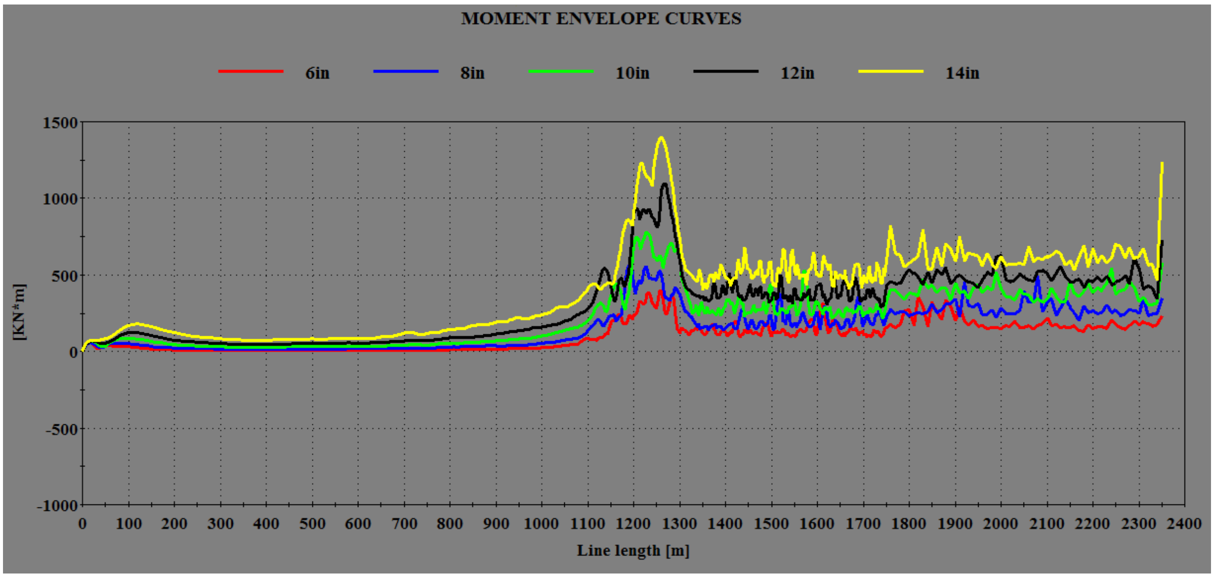

Figure 7-27 Moment Envelope Curves with respect to ID

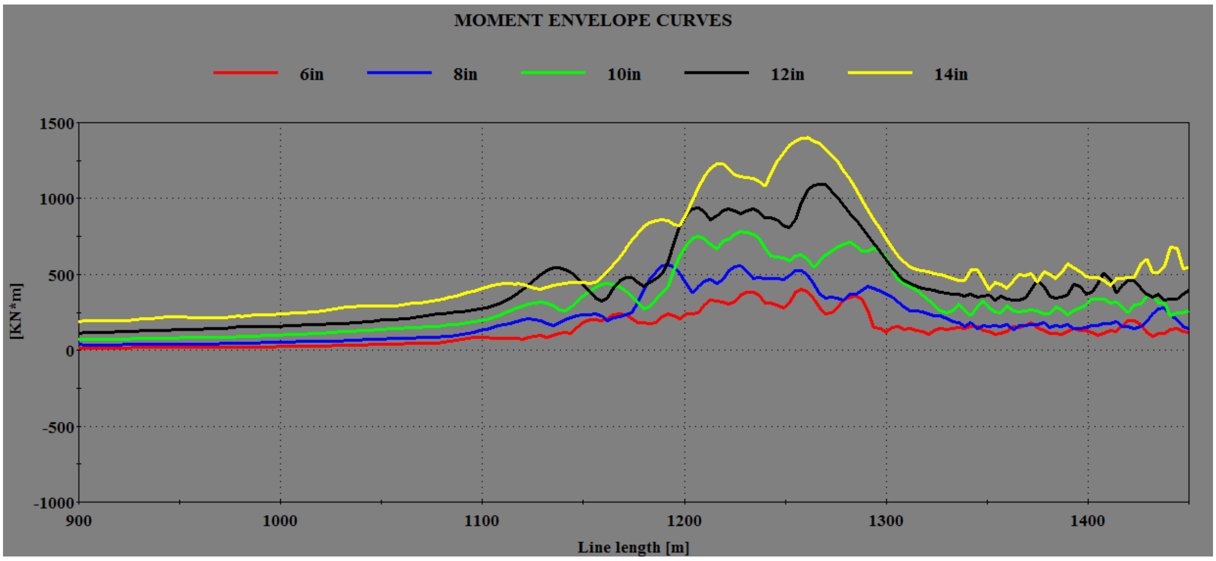

Figure 7-28 Peak values of Moment Envelope Curves

As the external hyrostatic pressure increases with water depth, the riser diameter is thus important to resist the external pressure. With the increase of riser internal diameter, the static and bending moments are increased as shown above. The peak values of moment envelope curves are concentrated within a length of 100 m along the riser.

CHAPTER 8 CONCLUSION

The trend in the offshore oil and gas industry over recent years has been a rapid move towards ultra deepwater developments. A direct result of this trend has been an equally rapid evolution of the technology surrounding marine risers. The result is the development of a number of new riser concepts, including top tensioned riser, flexible risers, hybrid risers and steel catenary risers, and the emergence of titanium as a material with enormous potential for use offshore in a wide range of applications.

A variety of different riser concepts are proposed, both with respect to geometric shape and selection of materials. In the last few years great attention is being given to steel catenary risers because they offer an interesting alternative to flexible risers for oil and gas production in deep waters. Such risers also have a potential benefit when used in high temperature and high pressure application even in shallow water conditions.

The design of the riser system must focus on different types of loading and load effects than for traditional water-depth. Accordingly, methods for calculation of loads and load effects for different configurations need to be both advanced and accurate, and generally numerical solution methods are required. The report also describes some essential design code requirements to be fulfilled in a riser design activity. This is followed by design analysis of the riser system.

This report presents the results of a study performed to develop steel catenary riser configurations for a semi-submersible in 1000 m water depth and severe environment in the Norwegian Sea. The study shows that the bending moment is the governing design parameter and the critical area is located near the touch down zone.

The simple catenary is generally highly stressed near the touchdown point at the seabed. In near load case, the distance from host platform to TDP is the closest. This results in small sag-bend curvature i.e. high static bending moment. The bending moments in the top region is considerably smaller than that in the touch down region.

In this report, a detailed analysis was done by conducting sensitivity studies to understand the contribution of parameters such as the floater position, current profiles, wall thickness, internal diameter and damping coefficient to the performance of the riser system in a typical

North Sea environment. These parameters have significant effect on the structural response, especially in the touch down region.

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16. Song, R. and Stanton, P. (2007): Deepwater Tieback SCR: Unique Design Challenges and Solutions. Paper prepared for presentation at the Offshore Technology Conference, Houston, USA.

APPENDIX A RIFLEX PROGRAM FILES

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'10" ID RISER

'JUNE 11,2012.

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'FLOATER:SEMI

'STRUCTURE MODELLING:AR SYSTEM

'CREATED BY:HALIL DIKDOGMUS

'=========================================================

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'FLOATER:SEMI
'STRUCTURE MODELLING:AR SYSTEM
'CREATED BY:HALIL DIKDOGMUS
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APPENDIX B WAVE SPECTRUM

There are several spectrum formulas that are used in the design of offshore structures. The most commonly used wave spectra are the Pierson-Moskowitz model, Bretschneider or ITTC two parameter spectrum, JONSWAP model, and the Ochi-Hubble spectrum model. The JONSWAP model is often used to model North Sea waves as it gives a good representation of the typical waves found at the location. In this appendix, the governing equation for the model is provided.

JONSWAP

The JONSWAP (Joint North Sea Wave Project) spectrum is often used to describe coastal waters where the fetch is limited. The governing equation for the spectrum is given as:

$$S(\omega) = \alpha g^2 \omega^{-5} exp(-1.25(\frac{\omega}{\omega_p})^{-4})\gamma^{(-(\omega-\omega_p)^2/2\sigma^2\omega_p^2}$$

where:

ω Angular wave frequency $= \frac{2\pi}{T_\omega}$

T_ω Wave period

T_p Peak wave period

T_z Zero up-crossing wave period $\frac{T_p}{T_z} = 1.407(1 - 0.287 ln\gamma)^{1/4}$

ω_p Angular spectral peak frequency $= \frac{2\pi}{T_p}$

g Acceleration due to gravity

α $5.058(1 - 0.287 ln\gamma)\frac{H_S^2}{T_p^4}$

σ Spectral width parameter

$= 0.07\ for\ \omega \leq \omega_p$

$= 0.09\ for\ \omega \geq \omega_p$

γ Peakedness parameter

$= 1.0\ for$ $T_p \geq 5\sqrt{H_s}$

$$= \exp\left(5.75 - 1.15\frac{T_p}{\sqrt{H_s}}\right) for \qquad 3.6\sqrt{H_s} \leq T_p < 5\sqrt{H_s}$$

$$= 5.0\ for \qquad T_p < 3.6\sqrt{H_s}$$