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Nonlinear Resonant Response of Deep Draft Platforms in Surface Waves

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ABSTRACT

To minimize body motions, floating marine structures are often designed with natural frequencies far away from the spectrum of ocean waves. Such design considerations led to a class of deep draft caisson vessels (DDCV or spars). Even so, large resonant responses may still be generated by excitation from nonlinear interactions of waves with body motions. Past experiments indicated that a DDCV experiences large-amplitude heave and pitch resonant motions when the incident wave frequency is much larger than the heave and pitch natural frequencies. Such resonant motions are not predicted by classical theories without considering nonlinear effects. This nonlinear mechanism has received little attention because of the complex nonlinear wave-body dynamics involved. In this work, we investigate nonlinear wave-wave and wave-body interaction effects on dynamic instability of such marine structures.

We first perform a linear stability analysis of the wave-frequency body motion. From the analysis, we find that at certain incident wave frequencies the body motion is unstable with natural heave and pitch motions growing exponentially with time by taking energy from the incident wave through nonlinear wave-body interactions. The condition for the occurrence of instability and the key characteristic features of unstable natural heave and pitch motions, predicted by the analysis, agree well with the experimental measurement and our full-nonlinear numerical simulations. As time-domain fully nonlinear numerical simulations are computationally expensive, we further develop an approximate time-domain analytic model, by including the second-order body nonlinearity only, for predicting the onset of instability and ultimate response of DDCVs in both regular and irregular waves. We use this model to systematically investigate the dependence of unstable motions on frequency detuning, damping, body geometry, and wave parameters.

1. INTRODUCTION

In marine engineering, floating vessels and structures are designed with minimum responses to the action of ocean waves for the purpose of safe operation. On the other hand, floating wave energy absorbers are designed to operate with maximum motions in response to waves for effective energy extraction. Understanding of fundamental mechanisms and basic characteristics of resonant responses and instabilities of a floating body in surface waves is of critical importance for the design and operation of such structures.

In a study of the global motion of a deep draft caisson vessel (DDCV), the experiments (conducted in the wave basin of the Offshore Technology Research Center by ExxonMobil in 1998) showed that when the incident (regular) wave period approached to ~22 seconds, the DDCV underwent, in addition to the wave-frequency motions, large-amplitude natural frequency heave (with period ~29 s) and pitch (with period ~99 s) motions. The observed resonant heave and pitch motions of the DDCV are not predicted by the classical linear wave theory.

In an analogy to parametric resonant roll motion of a ship, Haslum & Faltinsen [1] attributed such coupled resonant heave and pitch motions to the effect of the Mathieu (sub-harmonic) instability due to hydrostatic coupling of heave and pitch motions. In the above experimental observation, however, the incident wave frequency, natural heave frequency, and natural pitch frequency do not match with the condition of Mathieu instability ([2], [3] and [4]).

In this work, we investigate this problem in the context of general nonlinear wave-wave and wave-body interactions. Linear instability analyses are carried out to understand the fundamental mechanism for the occurrence of unstable coupled heave-pitch resonant motions of floating structures in waves and to study the dependencies of the growth rate of unstable motions on physical parameters. Fully nonlinear numerical simulations using a highly efficient high-order boundary element method are performed to verify the analysis and to understand the roles and importance of various nonlinear interactions involved. Based on the nonlinear studies, we develop a simplified analytic model with the inclusion of dominant interactions for the prediction of the onset and evolution of the unstable motions. The model prediction is compared with the experimental data, and is then used to investigate the dependencies of unstable motions on frequency detuning, wave amplitude, damping of the system, and irregular sea states.

2. STABILITY ANALYSES

We consider the global motion of a DDCV platform in response to the action of uni-directional ocean surface waves. For simplicity, we neglect the effect of wave motions in the moon-pool and model the DDCV platform as a truncated vertical circular cylinder with a closed bottom. We define a right-handed coordinate system *o-xyz*, which is fixed with respect to the mean position of the cylinder with the *x*-axis pointing in the direction of wave propagation and *z* positive upwards. The origin of the system (*o*) is in the plane of the undisturbed free surface. Under the action of surface waves, the cylinder may experience surge, heave, and pitch motions only. We denote the surge and heave displacements by η_1 and η_3, respectively, and the angle of pitch rotation by η_5. Note that in the present study, we use the center of gravity of the cylinder (*G*) as the center of rotation. For reference, the coordinate system and the translational and angular displacement conventions are shown in Figure 1.

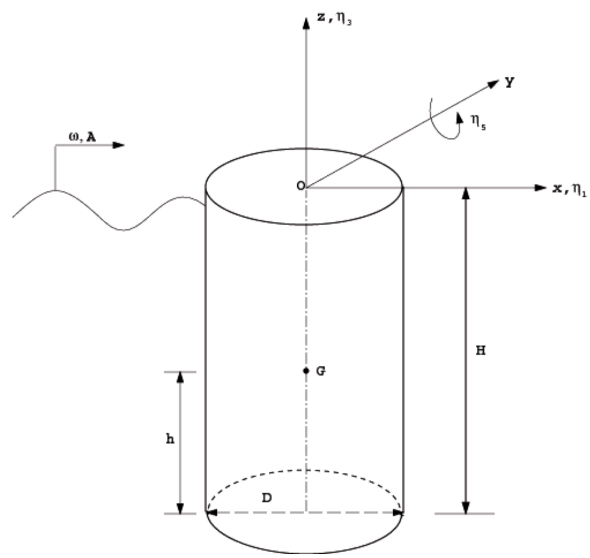


Figure 1. Definition sketch of a floating truncated vertical circular cylinder in a regular wave.

Let the base flow to be the one corresponding to an incident wave with frequency ω and amplitude A. Clearly, the base flow is the superposition of the incident, diffracted, and radiated waves. For simplicity, we use the linear solution of the wave-body interaction problem as the base flow solution.

Thus the base flow quantities (including body responses) have a frequency ω and are linearly proportional to A.

Let the resonant heave and pitch motions be $\eta_{3n}(t) = \mathrm{Re}\{\zeta_{3n} \exp(i\omega_{3n}t)\}$ and $\eta_{5n}(t) = \mathrm{Re}\{\zeta_{5n} \exp(i\omega_{5n}t)\}$, where ω_{3n} (ω_{5n}) is the natural heave (pitch) frequency, and ζ_{3n} (ζ_{5n}) the complex amplitude of resonant heave (pitch) motion that is assumed to be small initially. The objective here is to examine whether small resonant heave and pitch motions are stable or not due to interactions with the base flow. This can be addressed through a standard stability analysis.

By examining the frequency combinations in the interaction between the resonant heave/pitch motion with the base flow, we find that the system might be unstable under the condition of $\boldsymbol{\omega = \omega_{3n} + \omega_{5n}}$. Under this condition, we carry out a linear stability analysis to obtain the ordinary differential equations (ODEs) governing the disturbed resonant heave and pitch motions. This is done by superimposing the base flow potentials with the disturbance flow potentials and retaining all cross products of the respective first order terms in the free surface and body boundary conditions as well as that in the Bernoulli equation. The details of the analysis can be found in [5] and [6]. These equations can be expressed in the following symbolic form:

$$(m + A_{33})\ddot{\eta}_{3n} + B_{33}\dot{\eta}_{3n} + C_{33}\eta_{3n} = \mathrm{Re}\{kA\zeta_{5n}T_3 e^{i\omega_{3n}t}\} \quad (1)$$

$$(I_{55} + A_{55})\ddot{\eta}_{5n} + B_{55}\dot{\eta}_{5n} + C_{55}\eta_{5n} = \mathrm{Re}\{kA\zeta_{3n}T_5 e^{i\omega_{5n}t}\} \quad (2)$$

Where m (I_{55}) is the mass (moment of inertia) of the cylinder, A_{33} (A_{55}) heave (pitch) added mass coefficient, B_{33} (B_{55}) linear heave (pitch) damping coefficient, C_{33} (C_{55}) heave (pitch) hydrostatic restoring coefficient, and T_3 (T_5) represents the interaction coefficient between resonant pitch (heave) motion with the base flow for the resonant heave (pitch) motion. The above equations indicate that the interaction between the resonant pitch motion $\eta_{5n}(t)$ and the base flow (at frequency ω) gives an excitation to the heave motion at the resonant heave frequency ω_{3n}. Similarly, the interaction between the resonant heave motion $\eta_{3n}(t)$ and the base flow gives an excitation to the pitch motion at the resonant pitch frequency ω_{5n}. These interactions are represented by the complex transfer functions T_3 and T_5, which are dependent on the wave frequency, body motions at the wave frequency, and phases of the incident wave and initial disturbed body motions. Without loss of generality in illustrating the mechanism of instability, the coupling between surge and pitch is neglected in (2).

The coupled equations (1) and (2) can be combined to give:

$$\begin{aligned}&\frac{d^4\zeta_{5n}}{dt^4} + [i2(\omega'_{3n}+\omega'_{5n}) + B'_{33}+B'_{55}]\frac{d^3\zeta_{5n}}{dt^3} + \\ &[(i2\omega'_{3n}+B'_{33})(i2\omega'_{5n}+B'_{55}) + i\omega'_{3n}B'_{33} + \\ &i\omega'_{5n}B'_{55}]\frac{d^2\zeta_{5n}}{dt^2} + [i\omega'_{5n}B'_{55}(i2\omega'_{3n}+B'_{33}) + \\ &i\omega'_{3n}B'_{33}(i2\omega'_{5n}+B'_{55})]\frac{d\zeta_{5n}}{dt} - \\ &[\omega'_{3n}B'_{33}\omega'_{5n}B'_{55}+(kA)^2T']\zeta_{5n} = 0\end{aligned} \quad (3)$$

where $\omega'_{3n} = \omega_{3n}/\omega$, $\omega'_{5n} = \omega_{5n}/\omega$, $B'_{33} = B_{33}/(m+A_{33})\omega$, $B'_{55} = B_{55}/(I_{55}+A_{55})\omega$, and $T' = T_3T_5/[(m+A_{33})(I_{55}+A_{55})\omega^4]$. Note that similar equation can be obtained for the resonant heave motion ζ_{3n}. The fourth-order ODE, (3), can be solved with the solution expressed in the general form:

$$\zeta_{5n}(t) = \alpha_1 e^{\gamma_1 t} + \alpha_2 e^{\gamma_2 t} + \alpha_3 e^{\gamma_3 t} + \alpha_4 e^{\gamma_4 t} \quad (4)$$

where the coefficients γ_i, $i=1,2,3,4$ are the solutions of the resulting fourth-order polynomial equation. In general, these coefficients are functions of ω'_{3n}, ω'_{5n}, B'_{33}, B'_{55}, kA, and T'. Clearly, if the real part of any of γ_i, $i=1, 2, 3, 4$ is positive, the resonant motions are unstable with their amplitudes, ζ_{3n} and ζ_{5n}, growing with time exponentially. The value of real[γ_i] is the growth rate of the i-th mode. In general, the coefficients α_i, $i=1, 2, 3, 4$ are determined by the initial conditions of the problem.

For illustration, we take the measurement of a 1:70 scaled DDCV model in regular wave experiment as an example. The prototype cylinder has a diameter of D = 37 m, a draft of H = 198 m, and a radius of pitch gyration R = 75 m. The center of gravity of the cylinder is located at a distance h = 95 m from the keel of the cylinder. For this cylinder, we have ω_{3n} = 0.217 rad/s (with the period $T_{3n} \cong$ 29 s) and ω_{5n} = 0.063 rad/s (with the period $T_{5n} \cong$ 99 s). From the condition $\omega = \omega_{3n} + \omega_{5n}$, we consider the incident wave with ω = 0.28 rad/s (with period T $\cong$ 22 s). We note that for such a structure with deep draft and relatively low natural heave and pitch frequencies, the linear wave damping is negligibly small. For this cylinder, we find that in general there are two values of γ that are positive for any values of kA and T'. This indicates that the cylinder's response to the action of an incident wave of $\omega = 0.28$ rad/s is unstable and small disturbances will grow at natural heave and pitch frequencies.

In Figure 2, we show a contour of the maximum growth rate real[γ] as a function of real and imaginary parts of the parameter $(kA)^2T'$. The growth rate is positive and increases with the magnitude of $(kA)^2T'$.

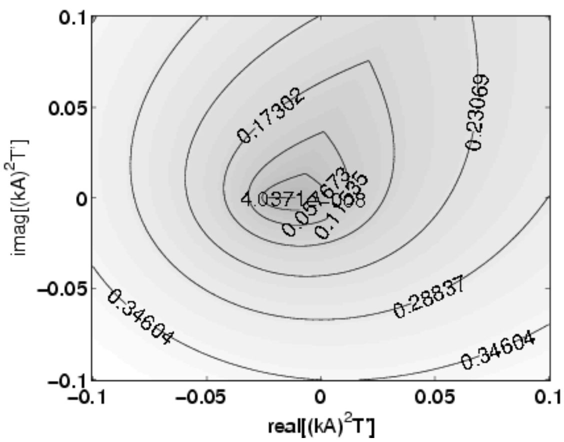


Figure 2. Contour of the maximum growth rate, real[γ], as a function of $(kA)^2T'$.

In Figure 3, we plot the four growth rates as a function of the incident wave steepness kA for a fixed $|T'| = 10$ with a phase of 45°. Two of them are positive and the other two are negative. The growth rate is not a linear function of kA as would be expected from (1) or (2). It has a nonlinear dependence on kA due to nonlinear coupling between heave and pitch motions.

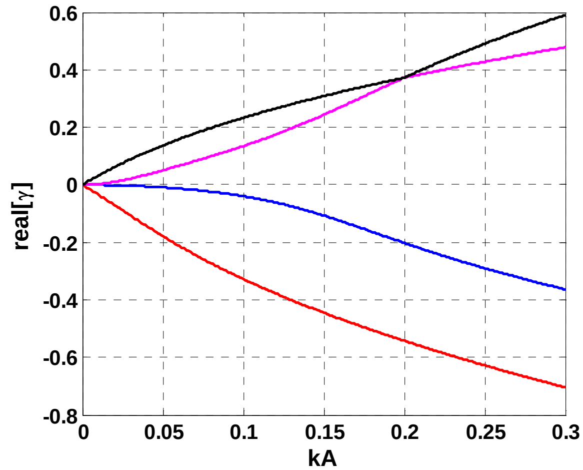


Figure 3. Growth rates real[γ_i], $i = 1, 2, 3, 4$ as a function of incident wave steepness kA for a fixed $|T'| = 10$ with a 45° phase.

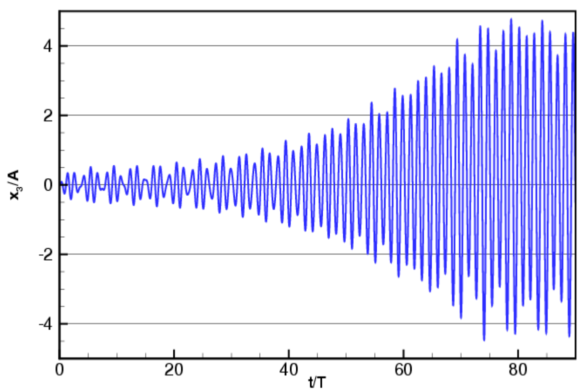
The dependence of the growth rates on other variables such as the phase of the incident wave and initial disturbances, natural heave and pitch frequencies, body geometry, and damping of the system can also be obtained similarly. More results can be found in [6].

3. FULLY NONLINEAR SIMULATIONS

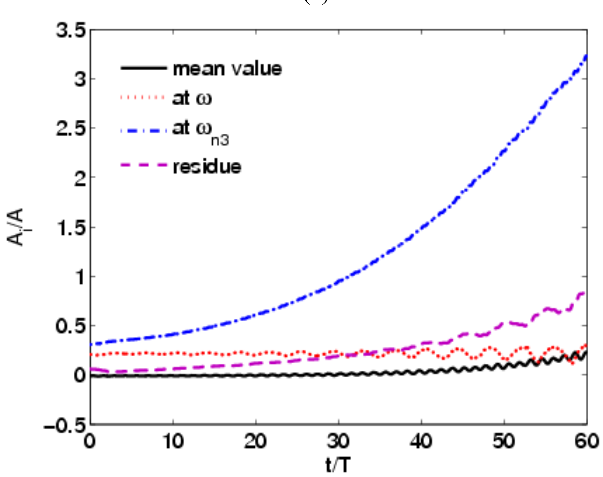
In this section, we apply direct numerical simulations to investigate the instability identified in the above. Fully nonlinear numerical simulations are performed using the efficient PFFT-QBEM method, which is described in [7]. The same cylinder geometry is considered. The incident wave has a wave steepness $kA = 0.02$ and a frequency $\omega = 0.28$ rad/s. In practice, the small disturbances in resonant heave and pitch motion are always there due to the presence of small amplitude long waves in the wave spectrum or nonlinear wave-wave and wave-body interactions. We perform a long-time simulation of fully nonlinear interactions of the floating cylinder with the incident wave field including nonlinear coupling of different modes of the body motion.

Figure 4a plots the normalized time history of the heave motion obtained by the fully nonlinear simulation. The normalized harmonic amplitudes of the heave motion are plotted as a function of time in figure 4b. The results show that the heave motion is unstable with the amplitude at the resonant frequency growing exponentially with time. The heave motion at the incident wave frequency is relatively small and does not grow with time. The similar results for the pitch motion are shown in Figure 5. The pitch motion is also unstable with the amplitude of the resonant pitch motion growing with time exponentially. After the unstable heave and pitch motions are developed to certain amplitudes, they stop growing as fully nonlinear interactions among the incident, radiated and diffracted wave fields and different modes of the body motions balance the instability effect.

Various fully nonlinear simulations are performed to study the dependence of the instability on incident wave steepness, initial phases of the disturbances, frequency detuning as well as body geometry effects. These results are discussed in detail in [6].

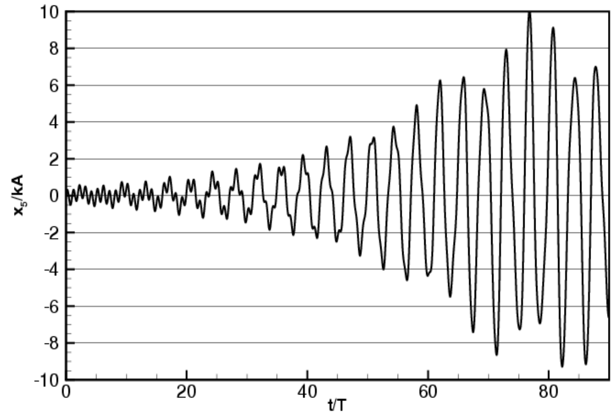


(a)

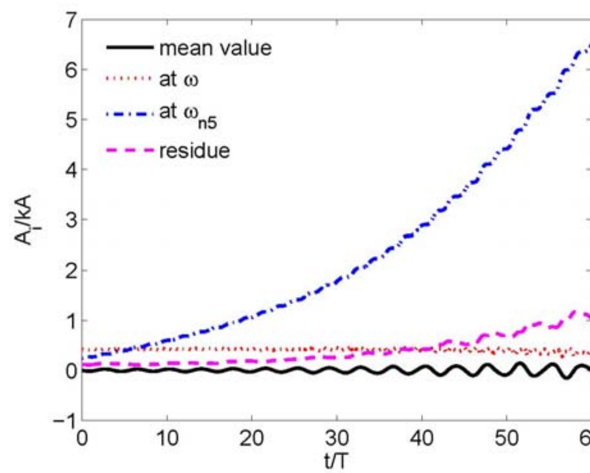


(b)

Figure 4. Time histories of the (a) heave motion, and (b) harmonic motion amplitudes of the cylinder in waves, obtained by the fully-nonlinear simulation.



(a)



(b)

Figure 5. Time histories of the (a) pitch motion, and (b) harmonic motion amplitudes of the cylinder in waves, obtained by the fully-nonlinear simulation.

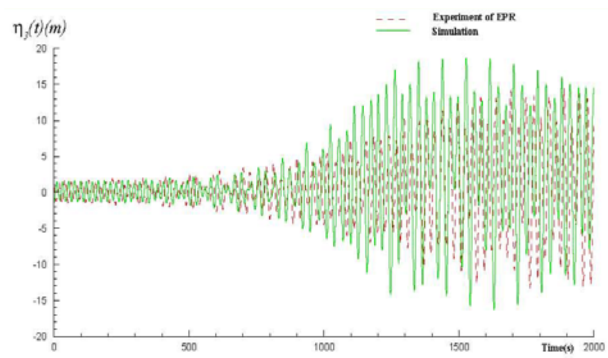
4. APPROXIMATE MODEL

While fully nonlinear simulations can account for full interactions between the base flow and the disturbances in resonant heave and pitch motions, they are computationally expensive especially when the growth rate is relatively weak. Thus, it is necessary and useful to develop a simplified model that can capture the key interactions for the study of the basic characteristics of the instability with a range of physical parameters and for the prediction of the cylinder motion including such instability effect under various ocean environmental conditions.

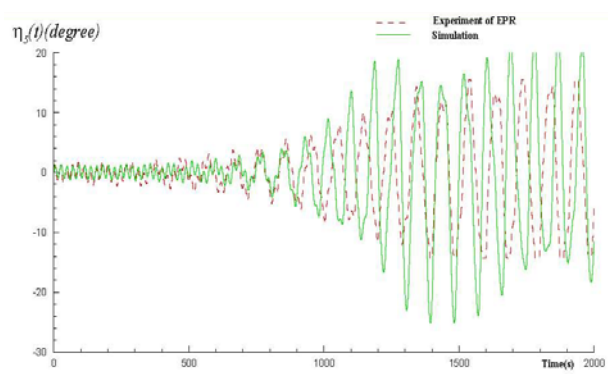
Since the natural heave and pitch frequencies of the DDCV are relatively low and the incident wave frequency for the occurrence of the instability is also quite low, we thus apply long-wave approximations to obtain the base flow solution and the hydrodynamic coefficients of the body at resonant heave and pitch frequencies. The equation of motion for heave is the same as (1). The equation of motion for pitch is similar to (2), with the coupling terms with the surge motion included. Since at such low frequencies, the linear wave damping is negligibly small, viscous damping is of importance in practice. We use a simple model based on Morison's formula to include the viscous damping effect in the simplified model. The key is to obtain the heave and pitch excitations due to interactions between the base flow and the resonant heave and pitch motions. Since the resonant heave and resonant pitch frequencies are low, the radiated waves due to such motions are relatively small. We thus neglect the effect of the interaction between the base flow and the radiated waves on the free surface at these frequencies. All other interactions between the base flow and the resonant heave/pitch motions (in particular on the body surface) are included. The excitations due to such interactions can be obtained in a relatively simple form, which can be found in [6]. Once the formula for the excitations is obtained, the coupled equations of motion for heave and pitch can be solved easily using any time integration scheme such as the fourth-order Runge-Kutta (RK4) approach.

Figure 6 shows the comparison between the simulation result by the simplified model and the experimental measurement of ExxonMobil for the time history of heave and pitch motions of the cylinder in regular waves. In addition to the wave-frequency oscillation, the heave response of the cylinder contains a component at the natural frequency. Both the prediction and experiment show that in the initial stage of the wave-cylinder interaction (t < 200 s), the wave frequency oscillation is dominant over the natural frequency motion. As the interaction continues, the natural frequency motion increases rapidly while the wave frequency oscillation remains almost unchanged in magnitude.

Similarly to the heave motion, the pitch motion also consists of the wave frequency and natural frequency oscillations. The natural period of the pitch motion in the simulation result is slightly smaller than that in the experiment (88s versus 99s). This is due to the fact that in the simulation, we consider the pitch motion with respect to the center of gravity of the platform while in the experiment the center of pitch rotation might be lower than the center of gravity. The added moment of inertia is thus smaller in the simulation than in the experiment, which leads to smaller natural period in the simulation. Overall comparisons indicate that the simulation results (with the chosen damping parameters) in heave and pitch motions properly reflect the instability effect and gives a satisfactory prediction of the growth of the unstable resonant heave and pitch motions.



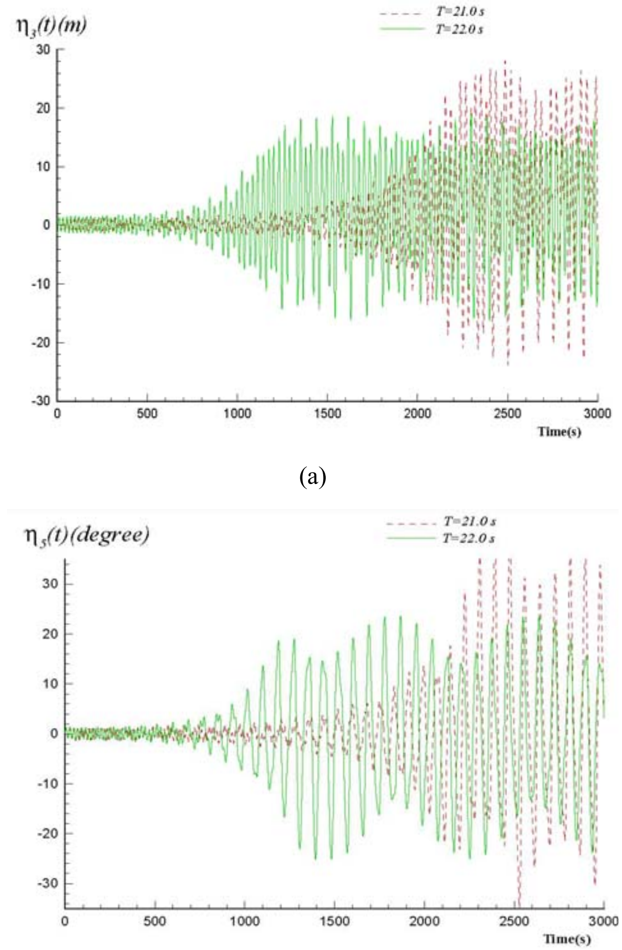
(a)



(b)

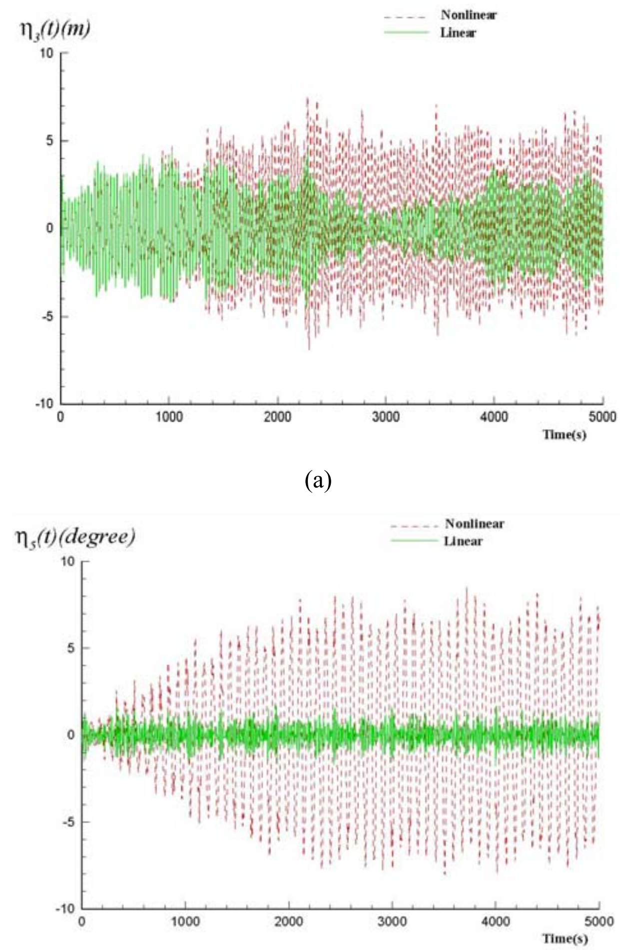
Figure 6. Comparisons between the simplified model prediction (—) and the experimental measurement (– – –) for the time histories of (a) heave $\eta_3(t)$, and (b) pitch $\eta_5(t)$ motions of the cylinder. In the simulation, B_{33} = 4% critical damping and B_{55} = 5% critical damping are used for heave and pitch motions and quadratic damping with C_D = 1.0 is used for surge motion. (Regular incident wave amplitude A = 6.1 m and period T = 22 s)

To investigate the effect of the incident wave period upon the resonant heave and pitch responses, we perform computations with a fixed incident wave amplitude A = 6.1 m and varying the wave period T. Figure 7 compares the results obtained with T = 21 s and T = 22 s. Clearly, as T is away from the 22 s, the growth of the unstable resonant heave and pitch responses with time becomes slower due to detuning effect so that longer time is required to obtain the steady state.



(b)

Figure 7. Time history of the (a) heave $\eta_3(t)$ and (b) pitch $\eta_5(t)$ motions of the cylinder obtained using the simplified model with the incident wave period T = 22 s (—) and 21 s (– – –). In the simulation, B_{33} = 4% critical damping and B_{55} = 5% critical damping for heave and pitch motions, quadratic damping with C_D = 1.0 for surge motion, and the incident wave amplitude A = 6.1 m are used.



(b)

Figure 8. Time history of the (a) heave $\eta_3(t)$ and (b) pitch $\eta_5(t)$ motions of the cylinder in irregular waves T_p = 18.0 s and $H_{1/3}$ = 12.5 m obtained using the simplified model. The plotted are the linear solution (—) and the nonlinear solution (– – –). In the simulation, quadratic dampings with C_{Dv} = 2.0 and C_{Dh} = 0.5 are used.

To study the unstable coupled heave-pitch response of the cylinder in irregular waves, as an example, we consider a uni-directional random sea state given by the modified Pierson-Moskowitz spectrum. We assume a 100-year sea state with a significant wave height $H_{1/3}$ = 12.5 m and a peak wave period T_p = 18 s. Figure 8 plots the time history of the heave and pitch motions of the cylinder. For comparison, the linear solution without the inclusion of nonlinear heave-pitch coupling effects is also shown. Resonant pitch response is obtained with the maximum value of $|\eta_5|$ near 8^o at the steady state. For the heave motion, the unstable resonant heave motion due to nonlinear instability effect is not apparent in this sea state. As $H_{1/3}$ increases, such as design storms in Celtic Sea ($H_{1/3}$ = 16.8 m, T_p = 18.7 s) and Shetland Sea ($H_{1/3}$ = 18.0 m, T_p = 18.2 s), stronger unstable resonant heave and pitch motions are obtained. In the Golf of Mexico ($H_{1/3}$ = 12.5 m, T_p = 14.6 s) however, unstable motions were not observed in both laboratory measurements and computations since the wave energy in the neighborhood of the critical period T = 22 s is negligibly small.

5. CONCLUSIONS

Through stability analyses and fully nonlinear simulations, we identify that the coupled heave-pitch resonant motions of the DDCV in waves are resulted from the second-order difference-frequency interactions between surface waves and body motions. We believe that the Mathieu instability is not the cause for the occurrence of the coupled heave-pitch resonant responses of the platform. The amplitudes of the resonant responses of the platform and the frequency bandwidth for the occurrence of such resonance depend critically on the incident wave amplitude and the (viscous) damping of the system. In general, larger wave amplitude and/or smaller damping lead to larger resonant responses and wider (resonance) frequency bandwidth. The coupled heave-pitch resonance of the DDCV may also occur in irregular waves depending on the peak wave period, the significant wave height of the spectrum and the natural heave and pitch periods. The identified resonance mechanism is general and can also be applied to other types of platforms such as FPSO's and TLP's with lightly damped modes of global motions.

6. ACKNOWLEDGEMENTS

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