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A Study on the Combination Resonance Response of a Classic Spar Platform

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Abstract: The sum type combination resonance response of a classic spar platform is studied under a long periodic regular wave in this paper. The nonlinear differential equations for heave and pitch are established considering the time-varying incident wave elevation which is regarded as a parametric excitation. The first order steady-state response is solved by the method of multiple scales when the incident wave frequency approaches the sum of heave natural frequency and pitch natural frequency. The heave and pitch responses of a practical spar platform are obtained by solving the bifurcation equation using a numerical method. It is observed that the large amplitude sub-harmonic motion of heave and pitch mode are tripped when the wave height exceeds a certain value. The result shows that the spar platform may exhibit complicated dynamic behaviors when the sum type combination resonance occurs and indicates that the available damping plays a very important role in suppressing the instable motion responses.

Keywords: Classic spar platform, parametric excitation, sub-harmonic motion, sum type combination resonance.

1. INTRODUCTION

A deep-draft spar platform is regarded as a competitive alternative structure for deepwater oil field development. Typical natural periods of the spars deployed in the Gulf of Mexico are relatively long, 160s for surge, 60s for pitch and 28s for heave, due to the small waterplane area compared to the submerged volume. Hence the spar platforms are not usually excited vertically. As a result, the spar platform exhibits excellent motion characteristics even in rough seas and hence steel catenary risers can be adopted with the help of the buoyancy can (Choi, 2006). Cavaleri and Mollo-Christensen (1981) studied the damping plate and found out how the presence of a large horizontal plate at the bottom affected the wave response of spars. The calculations show that the addition of a damping plate decreases heave response for short waves but increases the response for very long waves. Rijken and Niedzwecki (1996) compared and discussed geometric and dynamic characteristics of different spars.

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Figures 1, 4, 15, 16 appear in color online: http://jvc.sagepub.com

Ran et al. (1998) investigated the nonlinear coupled responses of a moored spar in random waves in both time and frequency domains. Tahar et al. (2002) conducted a coupled dynamic analysis of a classic spar with mooring and riser systems, and showed that neglecting the portion of risers inside the spar hull results in an over-estimation of the pitch responses. Zhang et al. (2007, 2008) presented a numerical study on the hydrodynamic behavior of a cell-truss spar platform and conducted model tests, they showed that coupling the mooring system and riser with the vessel motion typically results in a reduction in extreme motion responses.

However, the heave motion is largely amplified at resonance due to small damping, and large heave motions can affect the restoring moment of pitch. It starts to vary with time in phase with the heave motion. In this situation, Mathieu-type stability must be examined (Haslum and Faltinsen, 1999). Rho et al. (2002, 2003, 2004) used the method of Multiple scales to analyze the coupled system of heave and pitch of a classic spar model with and without mooring lines. They carried out experiments with 1/400 scale spar model in 2-dimensional wave tank, and confirmed the unstable pitch motion when the heave natural frequency is twice the pitch natural frequency. The jumping phenomenon is observed in pitch response when the wave frequency is near the heave natural frequency. It was also found that the mooring system has little effect on stabilizing the nonlinear unstable motion. Koo et al. (2004) discussed the Mathieu-type instability of a spar platform with moorings and risers. They found that when the spar exhibits the Mathieu instability, the spar experiences lock-in phenomena in pitch motion. Depending on the amount of available damping, Mathieu instability may or may not occur. The Mathieu stability diagram shows that increasing pitch damping suppresses the Mathieu instability problem. The results showed that mooring line and riser buoyancy-can effects play an important role in the Mathieu instability analysis of a spar platform through increasing damping/shifting pitch natural period. Hong et al. (2005) conducted model tests under various regular wave conditions in Samsung Ship Model Basin. In the model tests, the spring-wire lines had the same equivalent horizontal system stiffness. It was observed that unstable pitch motions occurred when the periods of encounter wave were close to the heave resonant period and twice the pitch natural periods.

There has been much research on the dynamic responses of Spar platform in the heave resonant waves, but few studies have been done on other types of combination resonance. In this work, the nonlinear coupled equation of heave and pitch motions in combination resonant waves are analyzed. The method of Multiple scales is used to determine the first order steady periodic solution of both responses when the wave frequency gets close to the sum of heave natural frequency and pitch natural frequency in the case with mooring lines. We found that a large amplitude sub-harmonic motion of heave and pitch mode are tripped when the wave height exceeds a certain value.

2. COUPLED EQUATIONS FOR HEAVE AND PITCH

In this paper, the heave and pitch motions of a spar platform are considered only in regular waves. The coupled nonlinear equations of motion can be written as:

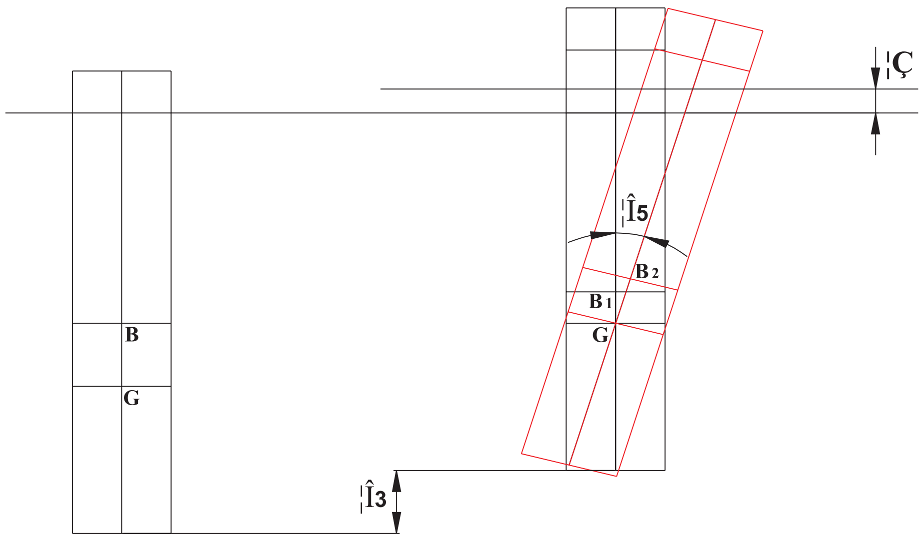


Figure 1. Geometry for heave and pitch of the Spar platform.

$$\begin{cases} \quad (m+m_{33})\ddot{\xi}_3 + B_3\dot{\xi}_3 + \rho g A_w\left(\xi_3 - \dfrac{\xi_5^2}{2}H_g - \eta(x,t)\right) = \bar{F}_3\cos(\Omega t+\theta_1) \\ \quad (I+I_{55})\ddot{\xi}_5 + B_5\dot{\xi}_5 + \Delta\left(\overline{GM} - \dfrac{1}{2}\xi_3 + \dfrac{\xi_5^2}{4}H_g + \dfrac{1}{2}\eta(x,t)\right)\xi_5 \\ = \quad \bar{F}_5\cos(\Omega t+\theta_2)\,. \end{cases} \tag{1a}$$

Where

m, m_{33}: Body mass and body's added mass, respectively
I, I_{55}: Pitch moment and added moment of inertia, respectively
B_i: Radiation damping
$\bar{F}_i$: Wave exciting force
$\overline{GM}$: Metacentric height
$\eta(x,t)$: Wave elevation
Δ: Total displacement
H_g: Distance from the center of gravity to the undisturbed water surface

According to the linear wave theory, the wave elevation $\eta(x,t)$ can be expressed as:

$$\eta(x,t) = \eta_0\cos(\Omega t - kx) \tag{1b}$$

Where

k: wave number ($k = 2\pi/L$ where L is the wave length)
η_0: wave amplitude
Ω: wave frequency

In this paper, we focus on the dynamic response of a spar platform in long period waves, in this situation k is so little that it can be neglected. The wave elevation $\eta(x,t)$ can be rewritten as:

$$\eta(x,t) = \eta_0 \cos \Omega t. \tag{1c}$$

Substituting equation (1c) into equation (1a) and neglecting higher order nonlinear terms, we get

$$\begin{cases} \ddot{\xi}_3 + 2\bar{\mu}_3\dot{\xi}_3 + \omega_{30}^2\xi_3 - a_3\xi_5^2 - b_3\eta_0\cos\Omega t = F_3\cos(\Omega t + \theta_1) \\ \ddot{\xi}_5 + 2\bar{\mu}_5\dot{\xi}_5 + \omega_{50}^2\xi_5 - a_5\xi_3\xi_5 + a_5\xi_5\eta_0\cos\Omega t = F_5\cos(\Omega t + \theta_2). \end{cases} \tag{1d}$$

Where

$$\begin{aligned} \bar{\mu}_3 &= \frac{B_3}{2(m+m_{33})}, \quad \omega_{30}^2 = \frac{\rho g A_w}{m+m_{33}}, \quad F_3 = \frac{\bar{F}_3}{m+m_{33}}, \quad a_3 = \frac{\rho g A_w H_g}{2(m+m_{33})}, \\ b_3 &= \frac{\rho g A_w}{m+m_{33}}, \quad \bar{\mu}_5 = \frac{B_5}{2(I+I_{33})}, \quad \omega_{50}^2 = \frac{\Delta\overline{GM}}{I+I_{33}}, \quad F_5 = \frac{\bar{F}_5}{I+I_{55}}, \\ a_5 &= \frac{\Delta}{2(I+I_{55})}. \end{aligned}$$

Because a_3 and a_5 have the same sign, it is convenient to eliminate them from equation (1d) by defining new dependant variables and amplitudes of the forcing before considering forced oscillations. We put

$$\begin{cases} \xi_3 = \dfrac{1}{a_5}x_3 \\ \xi_5 = \dfrac{1}{\sqrt{a_3a_5}}x_5. \end{cases} \tag{1e}$$

After calculating the wave exciting forces, we found that $\theta_1 \approx 0$ when the wave period is long, it means that in low frequency incident waves, the heave wave exciting force and wave elevation are almost synchronous. For simplicity, we assume that $\theta_1 = 0$, and let

$$\begin{cases} F_{31} = a_5\left(b_3\eta_0 + F_3\right) \\ F_{51} = \sqrt{a_3a_5}F_5 \\ D = a_5\eta_0. \end{cases} \tag{1f}$$

Substituting equation (1d) and equation (1e) into equation (1c), we get

$$\begin{cases} \ddot{x}_3 + 2\bar{\mu}_3\dot{x}_3 + \omega_{30}^2 x_3 - x_5^2 = F_{31}\cos\Omega t \\ \ddot{x}_5 + 2\bar{\mu}_5\dot{x}_5 + \omega_{50}^2 x_5 - x_3 x_5 + D x_5 \cos\Omega t = F_{51}\cos\left(\Omega t + \theta_2\right). \end{cases} \tag{1g}$$

3. MOTION OF SPARS IN COMBINATION RESONANT WAVES

The coupling effect between the heave and pitch motions becomes particularly significant when the heave and pitch natural frequencies are commensurable or nearly commensurable ($\omega_{30} \cong 2\omega_{50}$). Then, the so-called internal resonance between two modes of the system occurs. If the excitation amplitude is larger than the critical value, very strong nonlinear interaction between two modes occurs (Nayfeh et al., 1973). In order to study the combination resonance, the first order periodic solution of both modes are solved when the wave frequency gets close to the sum of heave natural frequency and pitch natural frequency, we let

$$\begin{cases} \bar{\mu}_3 = \varepsilon\mu_3 \\ \bar{\mu}_5 = \varepsilon\mu_5 \\ D = \varepsilon d \\ F_{31} = \varepsilon f_3 \\ F_{51} = \varepsilon f_5. \end{cases} \tag{2}$$

Substituting equation (2) into (1f), we get

$$\begin{cases} \ddot{x}_3 + \omega_{30}^2 x_3 = -2\varepsilon\mu_3\dot{x}_3(t) + x_5^2 + \varepsilon f_3\cos\Omega t \\ \ddot{x}_5 + \omega_{50}^2 x_5 = -2\varepsilon\mu_5\dot{x}_5(t) + x_3 x_5 + \varepsilon d x_5\cos\Omega t + \varepsilon f_5\cos(\Omega t + \theta_2). \end{cases} \tag{3}$$

And then we will obtain first order steady-state response of equation (3) by using the method of Multiple scales. According to this method, the different time scales is introduced and defined as

$$T_n = \varepsilon^n t. \tag{4}$$

We assume x_3 and x_5 have expansions of the form

$$\begin{cases} x_3(t;\varepsilon) \sim \varepsilon x_{31}(T_0, T_1, \ldots) + \varepsilon^2 x_{32}(T_0, T_1, \ldots) + \ldots \\ x_5(t;\varepsilon) \sim \varepsilon x_{51}(T_0, T_1, \ldots) + \varepsilon^2 x_{52}(T_0, T_1, \ldots) + \ldots. \end{cases} \tag{5}$$

The derivatives are transformed according to

$$d/dt = D_0 + \varepsilon D_1 + \varepsilon^2 D_2 + \dots \tag{6}$$

$$d^2/dt^2 = D_0^2 + 2\varepsilon D_0 D_1 + \varepsilon^2 \left(D_1^2 + 2D_0 D_1\right) + \dots. \tag{7}$$

Where $D_n = \partial/\partial T_n$.

Substituting equation (6) and equation (7) into equation (3) and equating coefficients of equal powers of ε, we obtain.

Order ε

$$\begin{cases} D_0^2 x_{31} + \omega_{30}^2 x_{31} = f_3 \cos \Omega t \\ D_0^2 x_{51} + \omega_{50}^2 x_{51} = f_5 \cos(\Omega t + \theta_2). \end{cases} \tag{8}$$

Order ε^2

$$\begin{cases} D_0^2 x_{32} + \omega_{30}^2 x_{32} = -2D_0 \left(D_1 x_{31} + \mu_3 x_{31}\right) + x_{51}^2 \\ D_0^2 x_{52} + \omega_{50}^2 x_{52} = -2D_0 \left(D_1 x_{51} + \mu_5 x_{51}\right) + x_{31} x_{51} - d x_{51} \cos \Omega t. \end{cases} \tag{9}$$

The solutions to equation (8) can be written in the form

$$\begin{cases} x_{31} = A_{31}(T_1 \dots) \exp(i\omega_{30} T_0) + B_3 \exp(i\Omega T_0) + cc \\ x_{51} = A_{51}(T_1 \dots) \exp(i\omega_{50} T_0) + B_5 \exp[i(\Omega T_0 + \theta_2)] + cc. \end{cases} \tag{10}$$

Where

$$\begin{cases} B_3 = \dfrac{f_3}{2(\omega_{30}^2 - \Omega^2)} \\ B_5 = \dfrac{f_5}{2(\omega_{50}^2 - \Omega^2)} \end{cases} \tag{11}$$

and cc represents the complex conjugate. A_{31} and A_{51} are determined by satisfying the solvability conditions at the next level of approximation.

Substituting equation (10) into equation (9), we have

$$\begin{aligned} D_0^2 x_{32} + \omega_{30}^2 x_{32} &= (-2i\omega_{30})(D_1 A_{31} + \mu_3 A_{31}) \exp(i\omega_{30} T_0) - 2i\Omega\mu_3 B_3 \exp(i\Omega T_0) \\ &+ A_{51}^2 \exp(2i\omega_{50} T_0) + B_{51}^2 \exp[2i(\Omega T_0 + \theta_2)] \\ &+ 2A_{51} B_5 \exp[i(\Omega T_0 + \omega_{50} T_0 + \theta_2)] + A_{51}\bar{A}_{51} \\ &+ 2B_5 \bar{A}_{51} \exp[i(\Omega T_0 - \omega_{50} T_0 + \theta_2)] + B_{51}^2 + cc \end{aligned} \tag{12}$$

$$
\begin{aligned}
D_0^2 x_{52} + \omega_{50}^2 x_{52} \;=\;& (-2i\omega_{50})(D_1 A_{51} + \mu_5 A_{51}) \exp(i\omega_{50} T_0) - 2i\Omega B_5 \exp[i(\Omega T_0 + \theta_2)] \\
+\;& A_{31} A_{51} \exp[i(\omega_{30} + \omega_{50}) T_0] + A_{31} B_5 \exp[i(\omega_{30} T_0 + \Omega T_0 + \theta_2)] \\
+\;& A_{31} \bar{A}_{51} \exp[i(\omega_{30} - \omega_{50}) T_0] + B_3 A_{51} \exp[i(\omega_{50} + \Omega) T_0] \\
+\;& B_3 B_5 \exp[i(2\Omega T_0 + \theta_2)] + B_3 \bar{A}_{51} \exp[i(\Omega - \omega_{50}) T_0] \\
+\;& B_5 \bar{A}_{31} \exp[i(\Omega T_0 - \omega_{30} T_0 + \theta_2)] + B_3 B_5 \exp(i\theta_2) \\
-\;& \frac{1}{2} d \left\{ A_{51} \exp[i(\omega_{50} + \Omega) T_0] + B_5 \exp[i(2\Omega T_0 + \theta_2)] \right. \\
+\;& \left. \bar{A}_{51} \exp[i(\Omega - \omega_{50}) T_0] + B_{51} \exp(i\theta_2) \right\} + cc. \qquad (13)
\end{aligned}
$$

We will only solve one term in the expansion, consequently, the T_n for $n \geq 2$ are considered constants. For convenience, we let ε be a measure of the amplitude of the response and consider it to be small compared with unity. To express the nearness of $\omega_{30} + \omega_{50}$ to Ω, and the nearness of $2\omega_{50}$ to ω_{30}, we introduce two detuning σ_1 and σ_2, so that

$$
\begin{cases}
\Omega = \omega_{30} + \omega_{50} + \varepsilon\sigma_1 \\
\omega_{30} = 2\omega_{50} - \varepsilon\sigma_2.
\end{cases} \qquad (14)
$$

Substituting equation (14) into equation (13), the solvability conditions (the conditions for the elimination of secular terms) can be written as

$$
\begin{cases}
\quad (-2i\omega_{30})(D_1 A_{31} + \mu_3 A_{31}) + A_{51}^2 \exp(i\sigma_2 T_1) \\
+ \; 2B_{51} \bar{A}_{51} \exp[i(\sigma_1 T_1 + \theta_2)] = 0 \\
\quad (-2i\omega_{50})(D_1 A_{51} + \mu_5 A_{51}) + A_{31} \bar{A}_{51} \exp(-i\sigma_2 T_1) \\
+ \; B_{51} \bar{A}_{31} \exp[i(\sigma_1 T_1 + \theta_2)] = 0.
\end{cases} \qquad (15)
$$

Let

$$
\begin{cases}
A_{31} = \dfrac{1}{2} \alpha_3(T_1) \exp\left[i\beta_3(T_1)\right] \\
A_{51} = \dfrac{1}{2} \alpha_5(T_1) \exp\left[i\beta_5(T_1)\right].
\end{cases} \qquad (16)
$$

And

$$
\begin{cases}
\gamma_1 = 2\beta_5 - \beta_3 + \sigma_2 T_1 \\
\gamma_2 = -\beta_3 - \beta_5 + \sigma_1 T_1 + \theta_2.
\end{cases} \qquad (17)
$$

Where $\alpha_n\left(T_1\right)$ and $\beta_n\left(T_1\right)$ are real. Substituting equation (16) into equation (17) and separating the result into real and imaginary parts, we obtain

$$\begin{cases} \omega_{30}\alpha_3' + \omega_{30}\mu_3\alpha_3 - \dfrac{1}{4}\alpha_5^2 \sin\gamma_1 - B_5\alpha_5 \sin\gamma_2 = 0 \\ \omega_{30}\alpha_3\beta_3' + \dfrac{1}{4}\alpha_5^2 \cos\gamma_1 + B_5\alpha_5 \cos\gamma_2 = 0 \\ \omega_{50}\alpha_5' + \omega_{50}\mu_5\alpha_5 + \dfrac{1}{4}\alpha_3\alpha_5 \sin\gamma_1 - \dfrac{1}{2}B_5\alpha_3 \sin\gamma_2 = 0 \\ \omega_{50}\alpha_5\beta_5' + \dfrac{1}{4}\alpha_3\alpha_5 \cos\gamma_1 + \dfrac{1}{2}B_5\alpha_3 \sin\gamma_2 = 0. \end{cases} \tag{18}$$

Where $\alpha_i' = d\alpha_i/dt$, $\beta_i' = d\beta_i/dt$.

For the steady-state response, we let $\alpha_3' = \alpha_5' = \gamma_1' = \gamma_2' = 0$. We can obtain the first order perturbation solution of equation (3) by getting α_3, α_5, γ_1 and γ_2 using numerical method.

$$\begin{cases} x_{31} = \varepsilon\alpha_3 \cos\left[\dfrac{1}{3}\left(-\gamma_1 - 2\gamma_2 + 2\theta_2\right) + \dfrac{2}{3}\Omega t\right] + \dfrac{\varepsilon f_3}{\omega_{30}^2 - \Omega^2}\cos\Omega t \\ x_{51} = \varepsilon\alpha_5 \cos\left[\dfrac{1}{3}\left(\gamma_1 - \gamma_2 + \theta_2\right) + \dfrac{1}{3}\Omega t\right] + \dfrac{\varepsilon f_5}{\omega_{50}^2 - \Omega^2}\cos\left(\Omega t + \theta_2\right). \end{cases} \tag{19}$$

The heave and pitch responses in sum type combination resonant waves can be expressed in the following form

$$\begin{cases} \xi_3 = \dfrac{1}{a_5}\left\{\alpha_3 \cos\left[\dfrac{1}{3}\left(-\gamma_1 - 2\gamma_2 + 2\theta_2\right) + \dfrac{2}{3}\Omega t\right] + \dfrac{F_3}{\omega_{30}^2 - \Omega^2}\cos\Omega t\right\} \\ \xi_5 = \dfrac{1}{\sqrt{a_3 a_5}}\left\{\alpha_5 \cos\left[\dfrac{1}{3}\left(\gamma_1 - \gamma_2 + \theta_2\right) + \dfrac{1}{3}\Omega t\right] + \dfrac{F_5}{\omega_{50}^2 - \Omega^2}\cos\left(\Omega t + \theta_2\right)\right\}. \end{cases} \tag{20}$$

4. WAVE EXCITING FORCES

Hong et al. (2005) conducted model tests under various regular wave conditions in Samsung Ship Model Basin. In the model tests, the spring-wire lines have the same equivalent horizontal system stiffness. In order to illustrate the result, we use parameters of this Spar in the present study. The main dimensions and characteristics of the model are summarized in Table 1 (Hong et al., 2005).

Heave and pitch natural periods with mooring lines are (Hong et al., 2005)

$$T_{\text{heave}} = 29.2 \text{ s}, \quad T_{\text{pitch}} = 57.2 \text{ s}.$$

Table 1. Main particulars of the spar model.

	Unit	Prototype
Length	m	212.9
Diameter	m	37.2
Draft	m	198.1
W_total	ton	215872.2
KB_total	m	99.1
KG_total	m	89
GM	m	10.08
Water_depth	m	1018
kyy_total	m	29.2
Freeboard	m	15.1
KFairlead	m	90.8

While the heave and pitch damping ratios are

$$\varsigma_{\text{heave}} = 0.012, \quad \varsigma_{\text{pitch}} = 0.019.$$

The damping ratio is defined as follows

$$\varsigma = B_e/B_{cr} = \frac{B_e}{2\,(I + A)\,\omega_0}. \tag{21}$$

In this paper, only linearized wave exciting forces are taken into account, and the linearized heave wave exciting force and pitch wave exciting moment are calculated in the wide range of wave periods. So the amplitudes of heave wave exciting force $\bar{F}_3$ and pitch wave exciting moment $\bar{F}_5$ can be expressed as

$$\begin{cases} \bar{F}_3 = K_3\,(\Omega)\,\eta_0 \\ \bar{F}_5 = K_5\,(\Omega)\,\eta_0. \end{cases} \tag{22}$$

The linearized wave exciting forces for heave and pitch under the unit wave amplitude are shown in Figures 2 and 3.

5. NUMERICAL EXAMPLE

The natural heave frequency is $\omega_{30} = 0.216$ rad/s, and the pitch natural frequency is $\omega_{50} = 0.109$ rad/s, so the wave frequency Ω is closed to to be 0.325 rad/s.

From Figures 4 and 5, we can see that when the wave frequency is equal to the sum natural frequency of the two modes, with the increase of wave height, the amplitude curves exhibit the typical bifurcation phenomena. The amplitudes of the two modes will not increase linearly when the wave height exceeds a certain value.

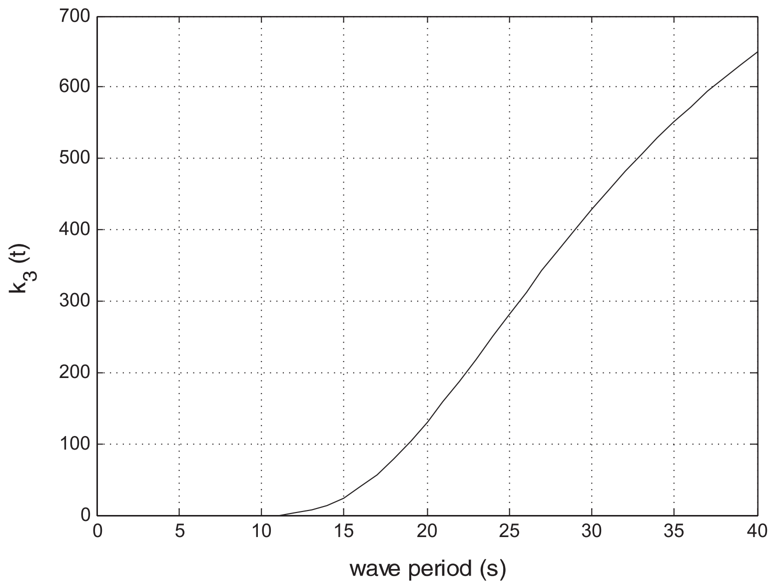


Figure 2. Heave wave exciting force K_3.

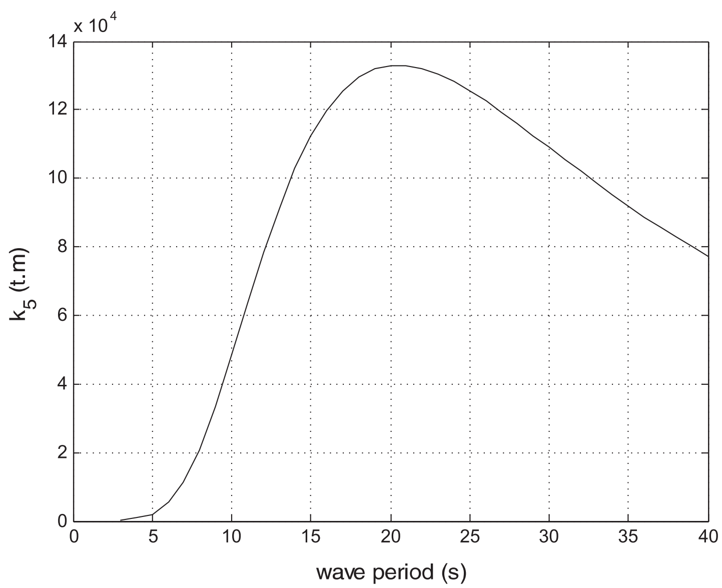


Figure 3. Pitch wave exciting moment K_5.

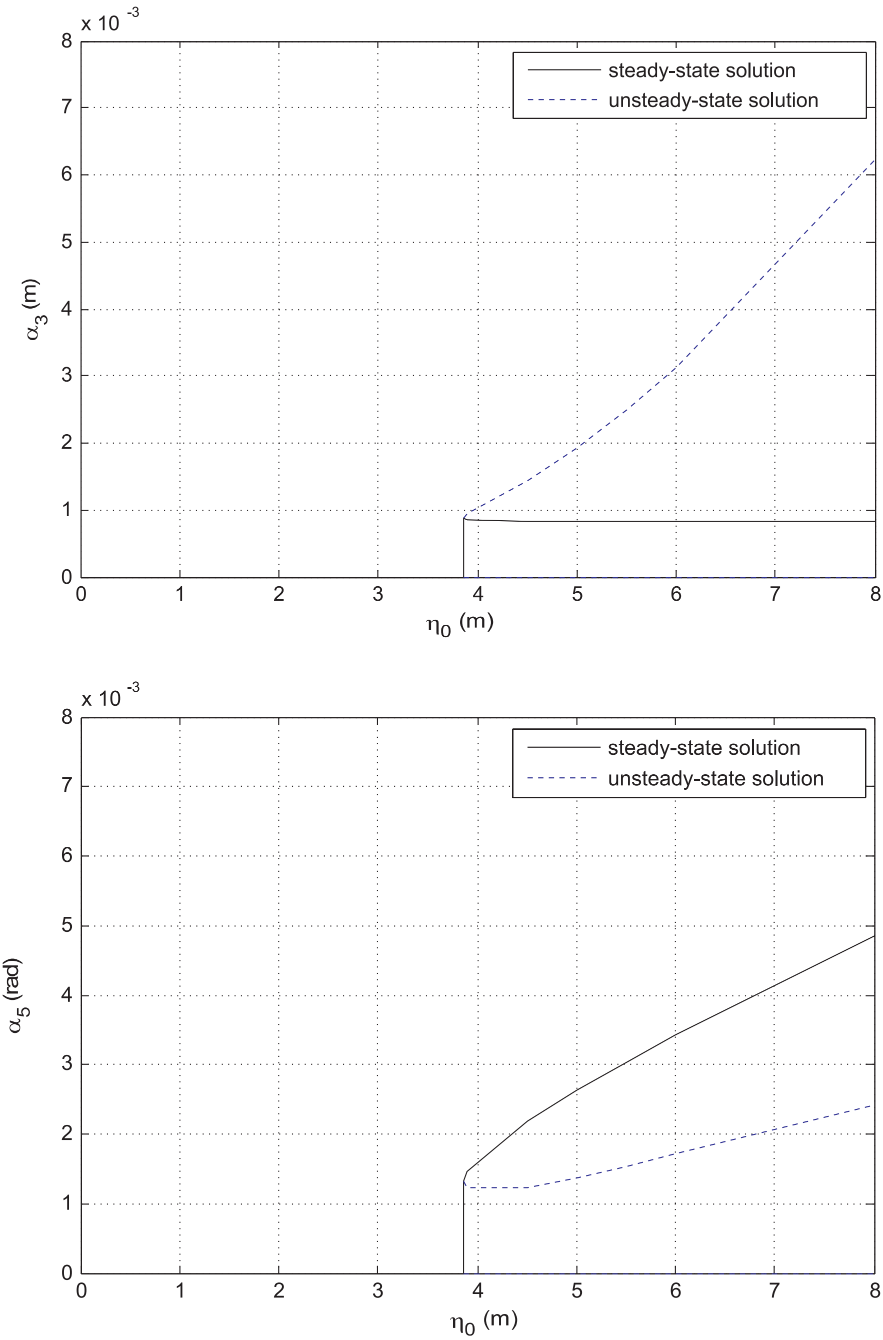
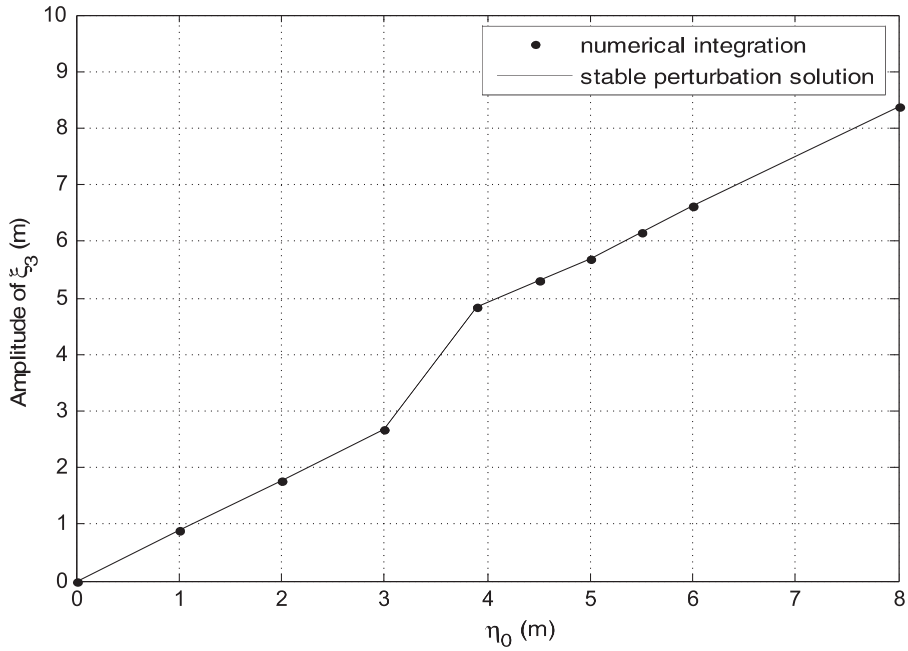


Figure 4. Amplitudes of α_3 and α_5 when $\varsigma_{\text{heave}} = 0.012$, $\varsigma_{\text{pitch}} = 0.019$, $\Omega = 0.325$ rad/s.



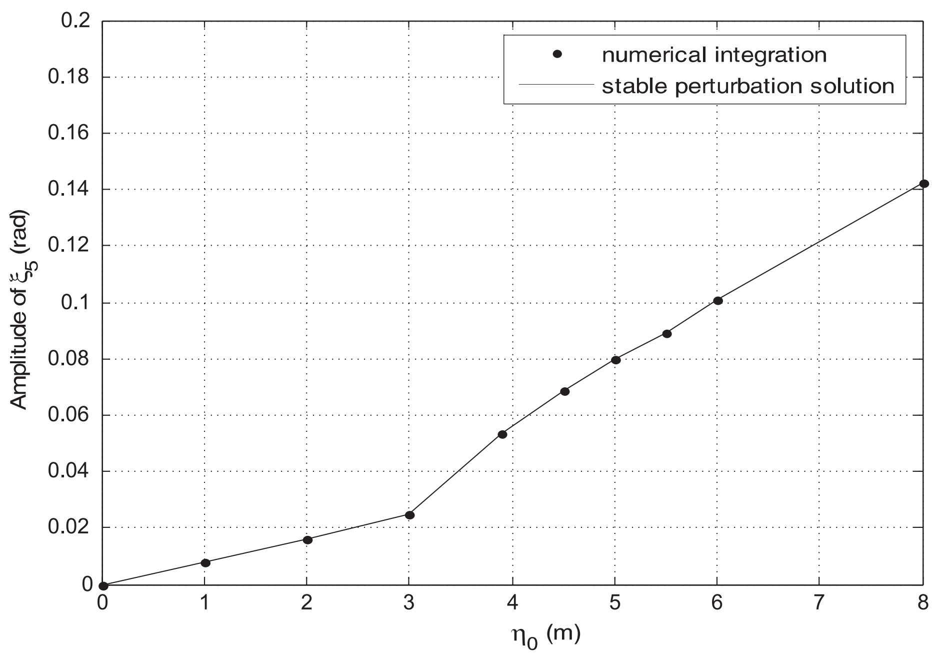
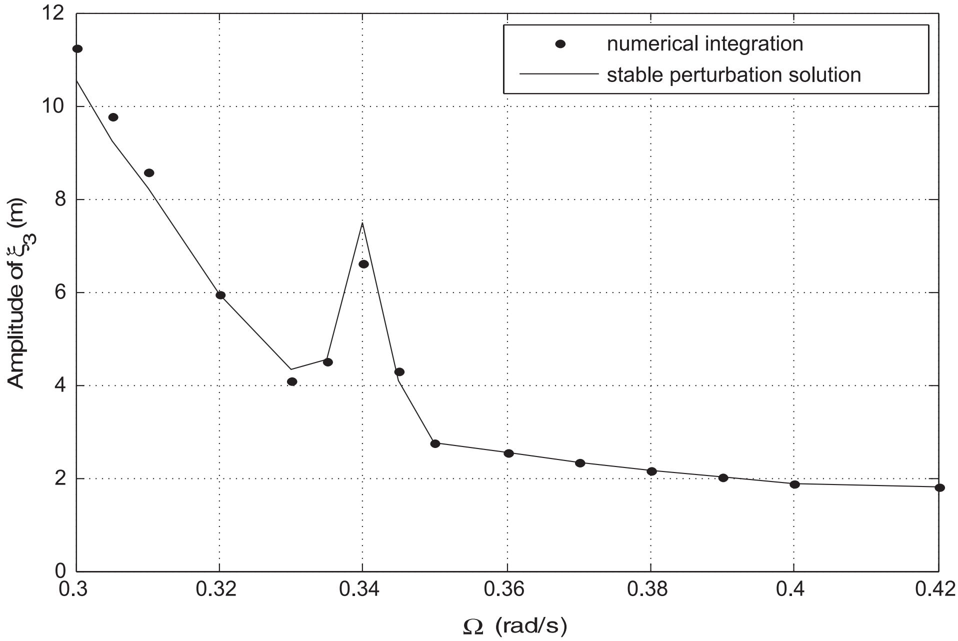


Figure 5. Amplitudes of heave and pitch when $\varsigma_{\text{heave}} = 0.012$, $\varsigma_{\text{pitch}} = 0.019$, $\Omega = 0.325$ rad/s.



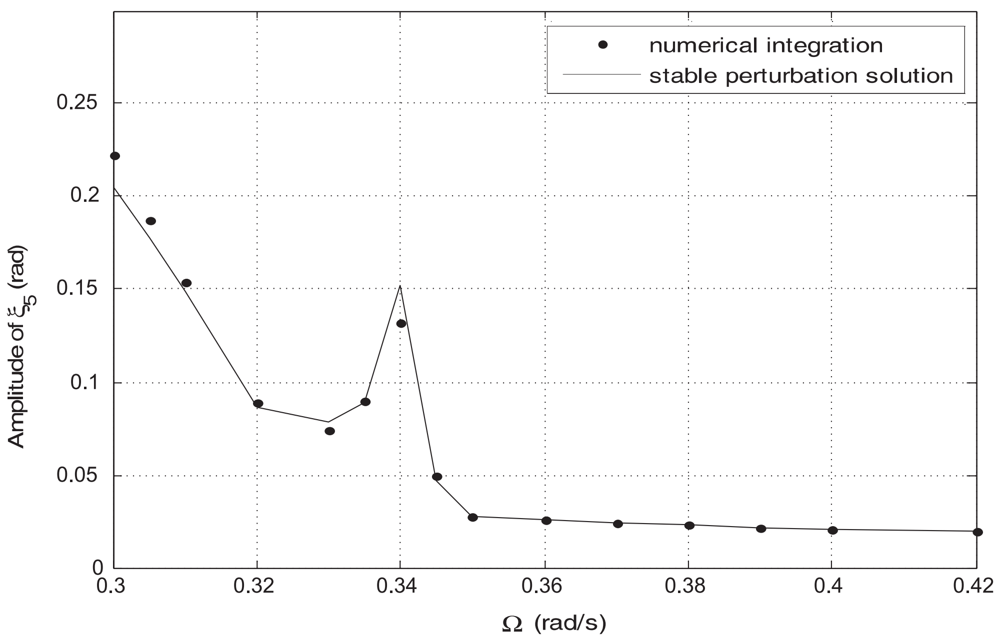


Figure 6. Amplitudes of heave and pitch when $\varsigma_{\text{heave}} = 0.012$, $\varsigma_{\text{pitch}} = 0.019$, $\eta_0 = 4$ m.

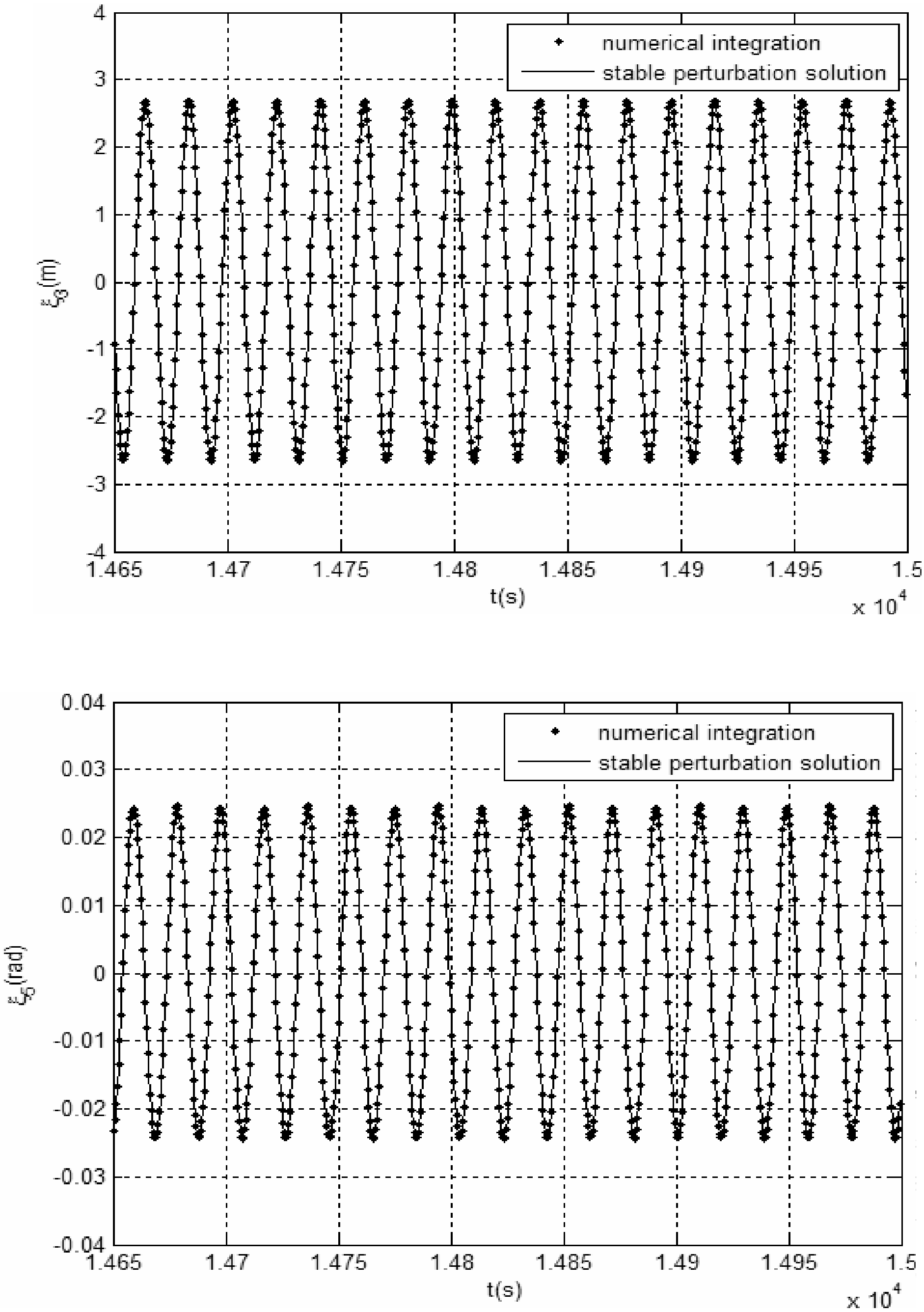


Figure 7. Time histories of heave and pitch when $\eta_0 = 3$ m, $\Omega = 0.325$ rad/s.

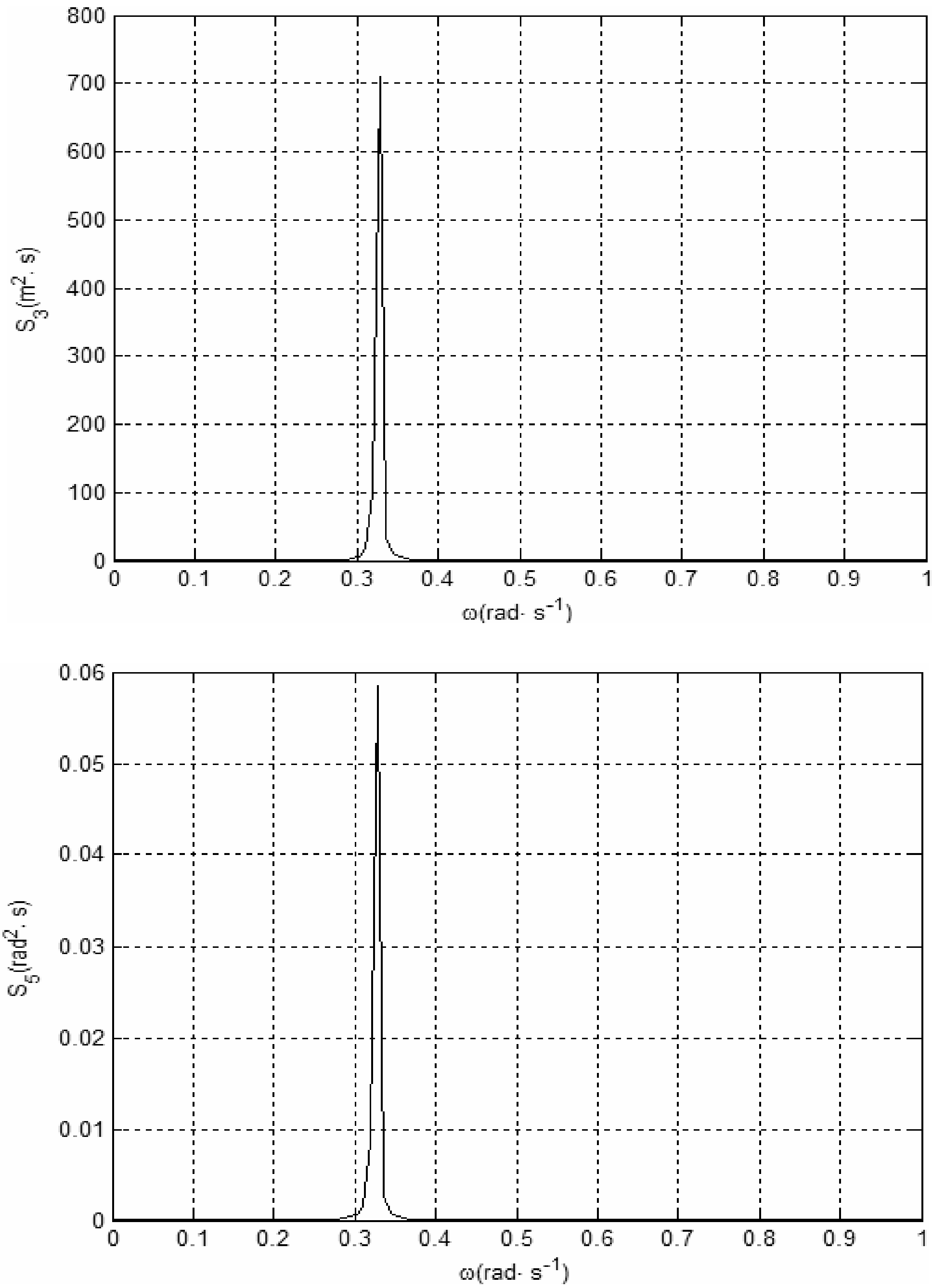
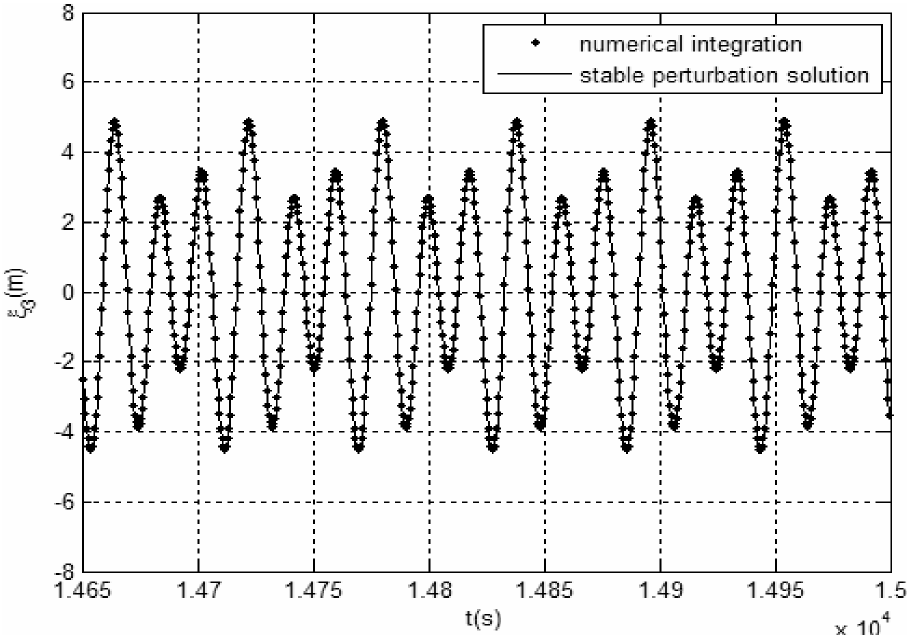


Figure 8. Power spectrum of heave and pitch when $\eta_0 = 3$ m, $\Omega = 0.325$ rad/s.



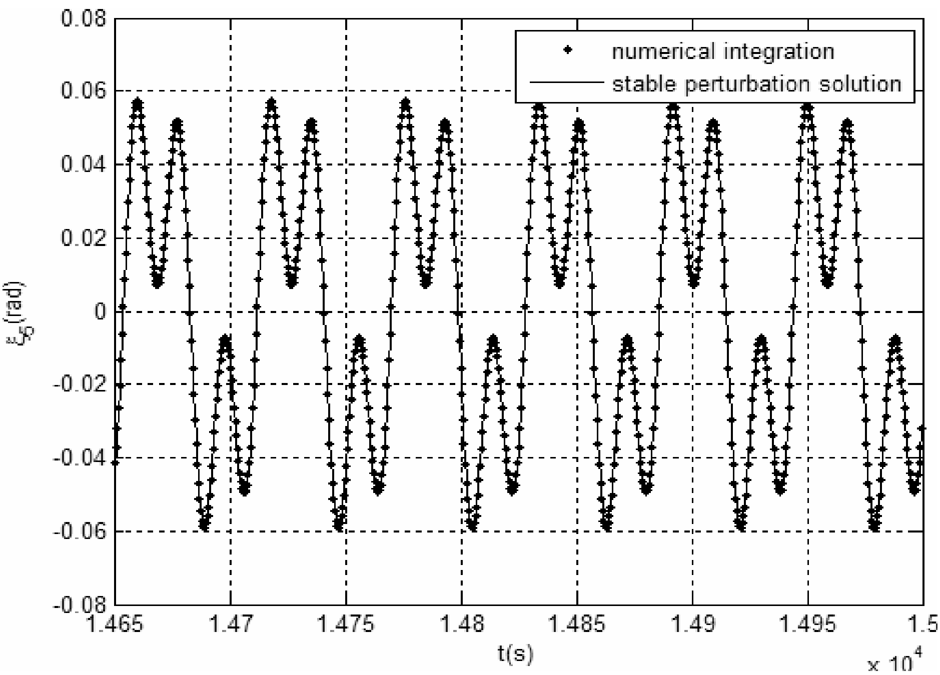


Figure 9. Time histories of heave and pitch when $\eta_0 = 4$ m, $\Omega = 0.325$ rad/s.

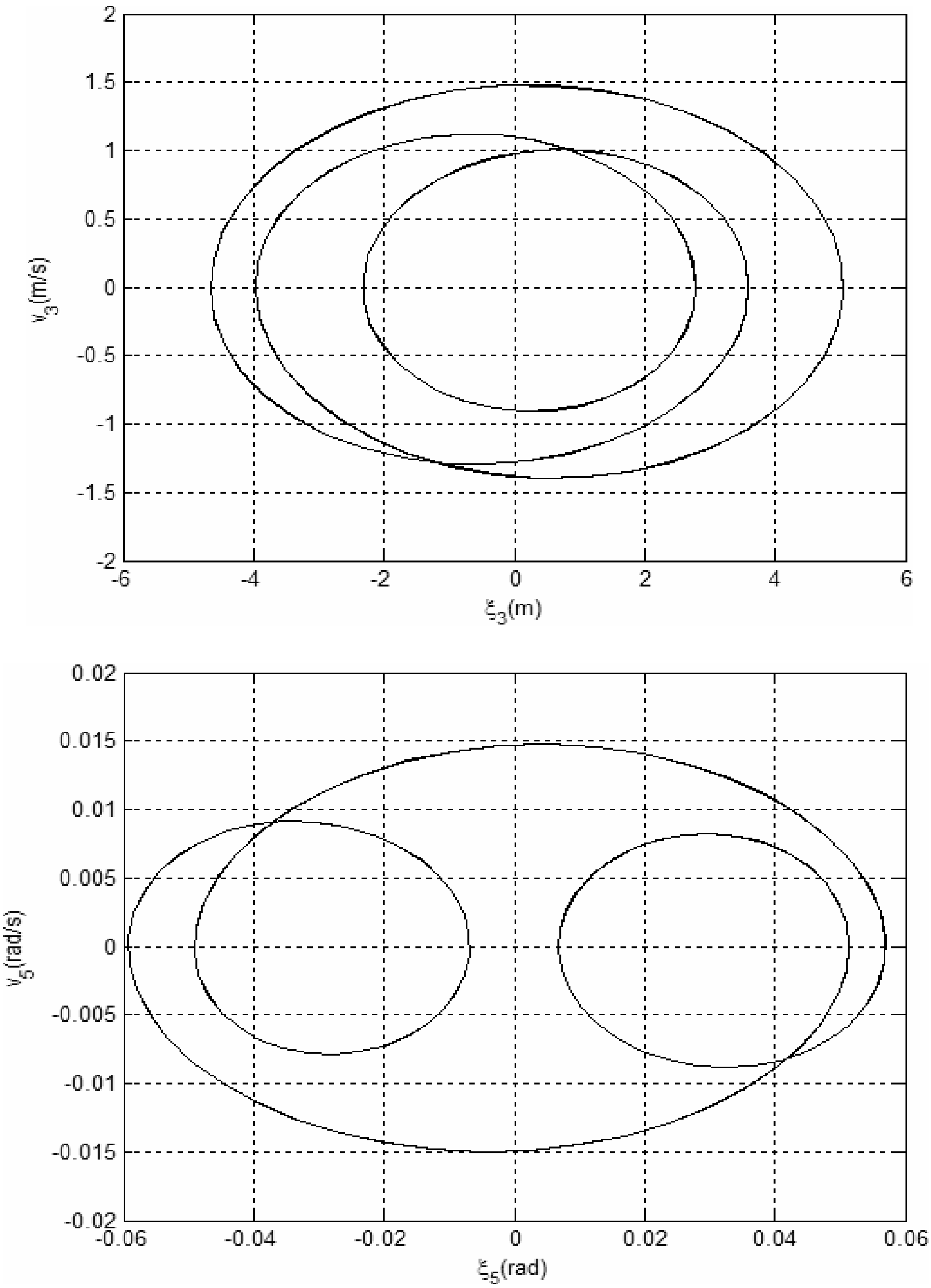


Figure 10. Heave and pitch phrases when $\eta_0 = 4$ m, $\Omega = 0.325$ rad/s.

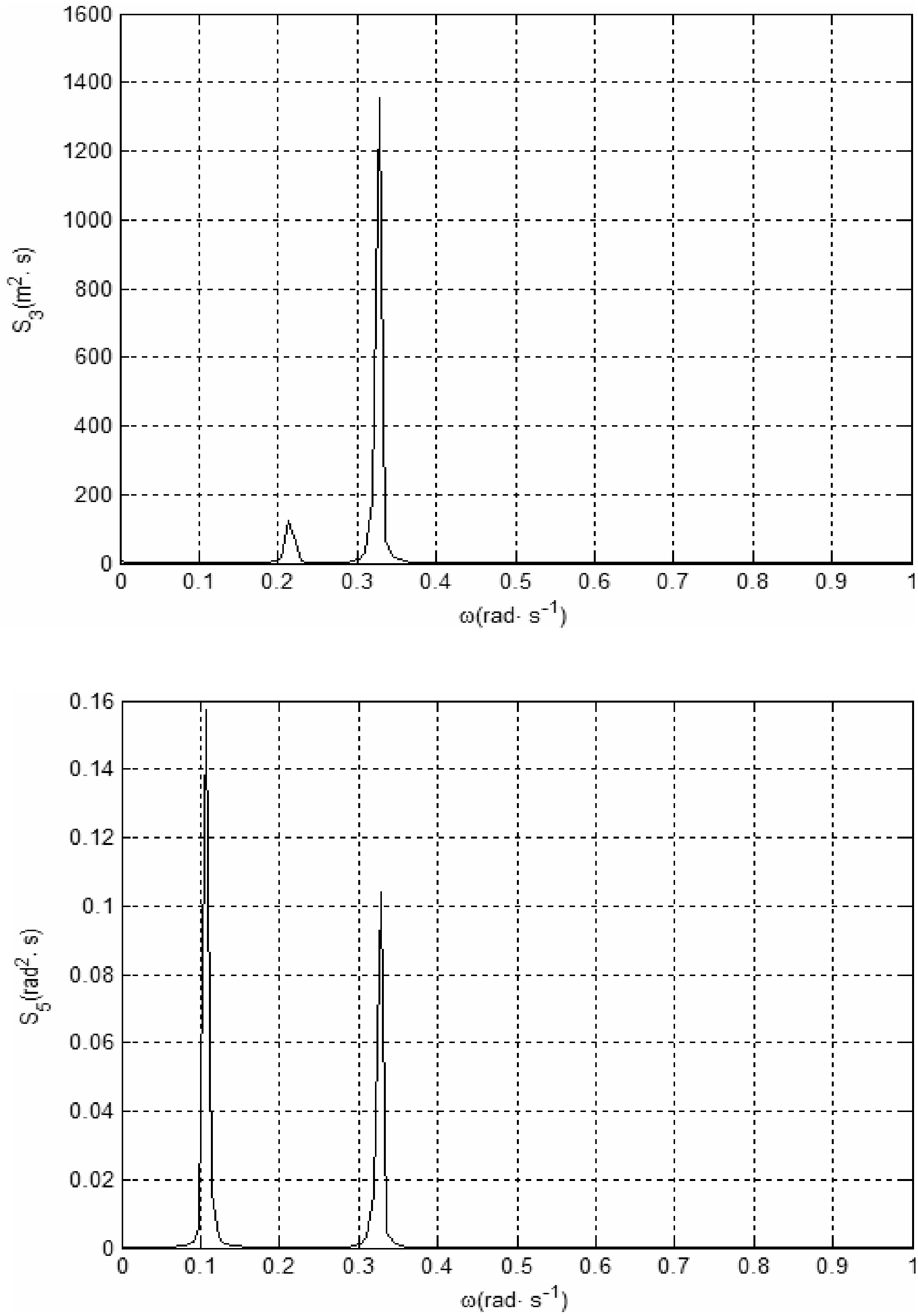
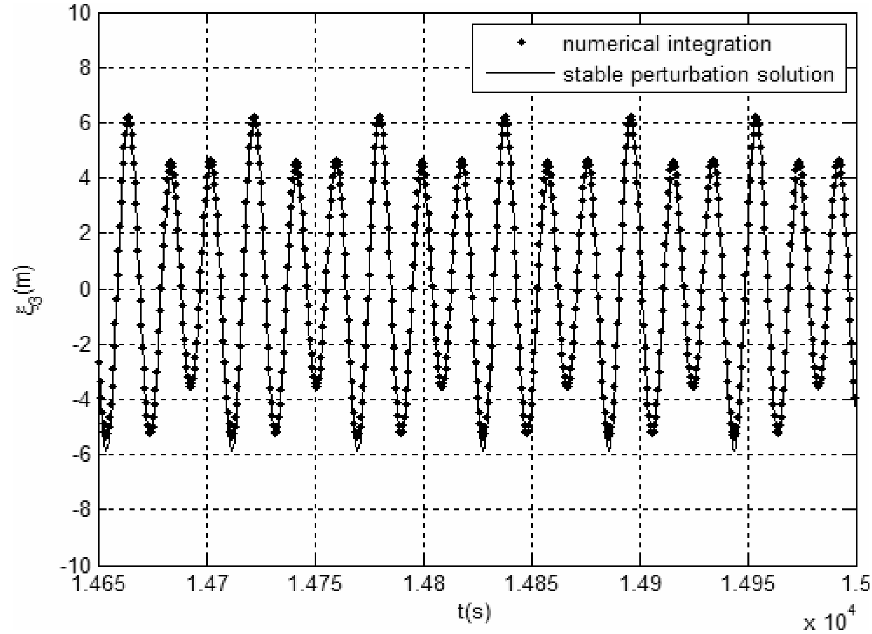


Figure 11. Power spectrum of heave and pitch when $\eta_0 = 4$ m, $\Omega = 0.325$ rad/s.



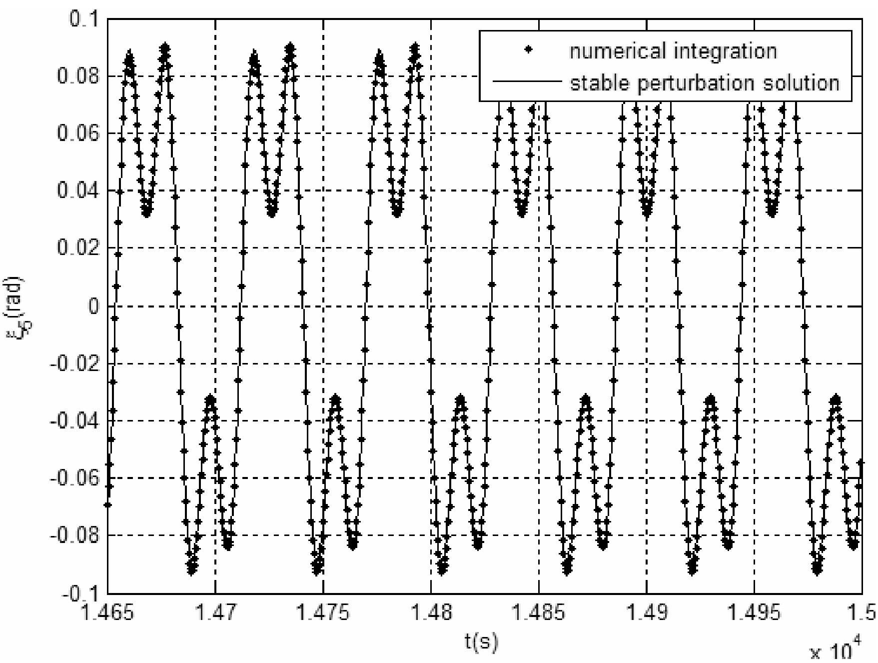


Figure 12. Time histories of heave and pitch when $\eta_0 = 5.5$ m, $\Omega = 0.325$ rad/s.

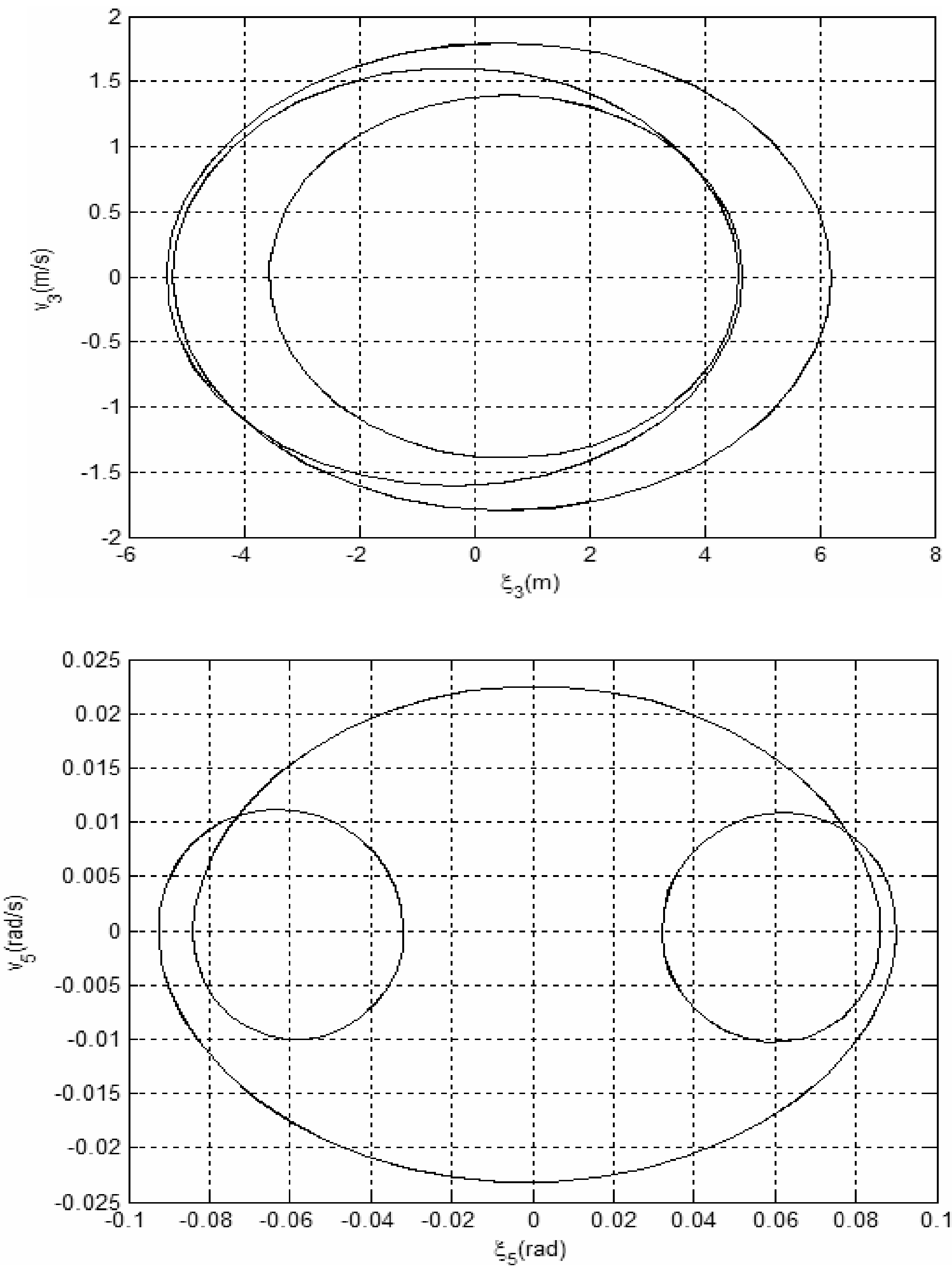


Figure 13. Heave and pitch phrases when $\eta_0 = 5.5$ m, $\Omega = 0.325$ rad/s.

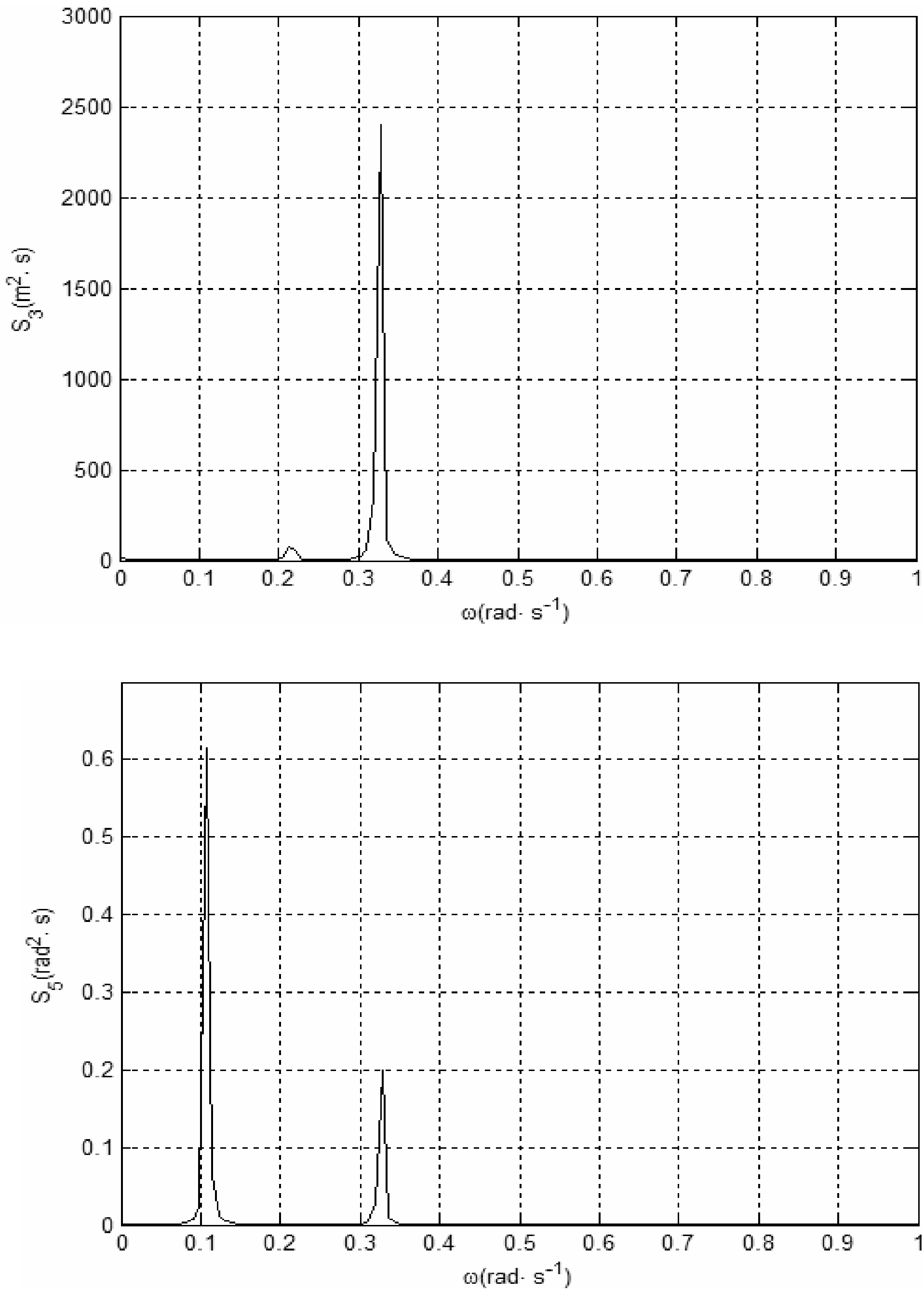


Figure 14. Power spectrum of heave and pitch when $\eta_0 = 5.5$ m, $\Omega = 0.325$ rad/s.

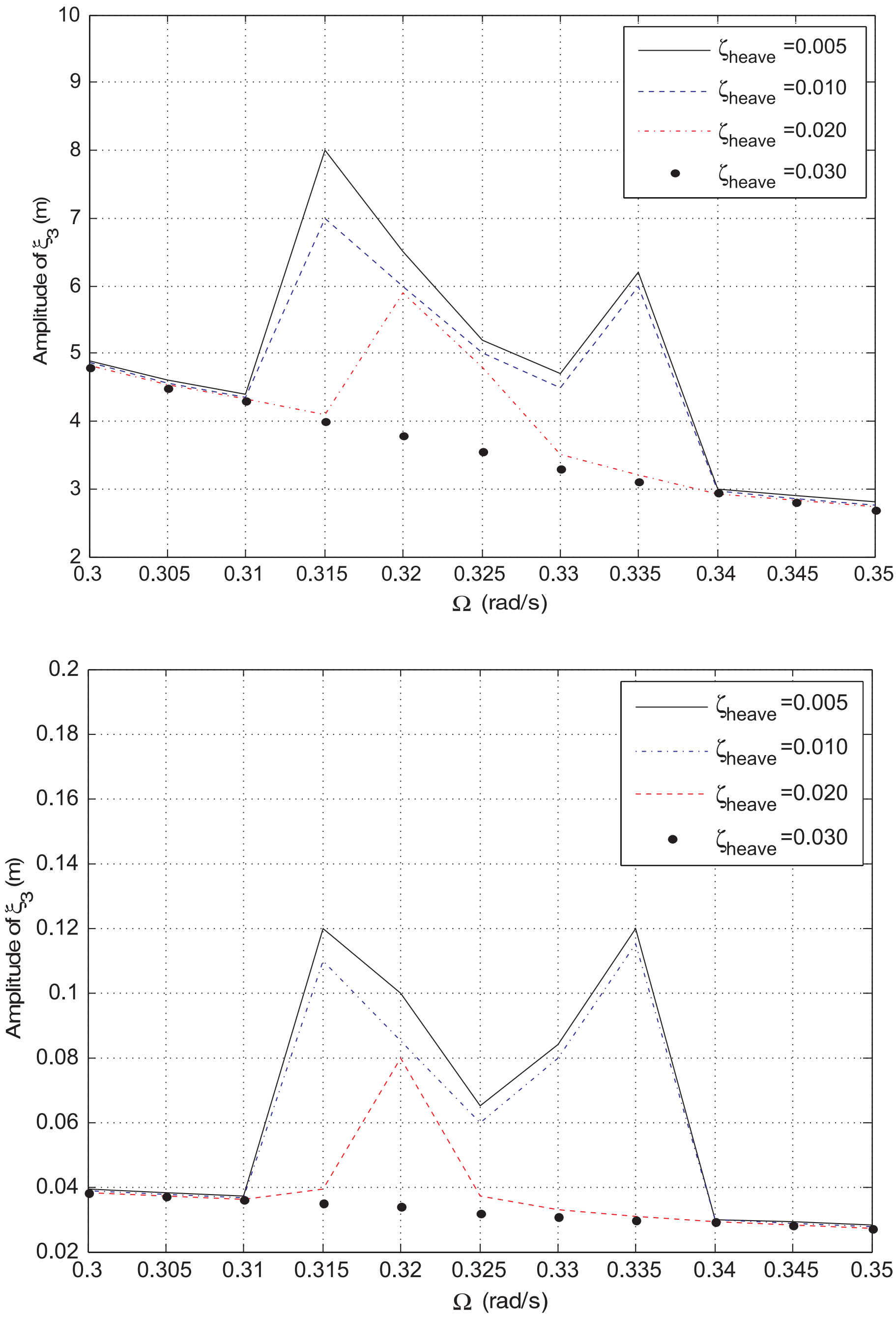
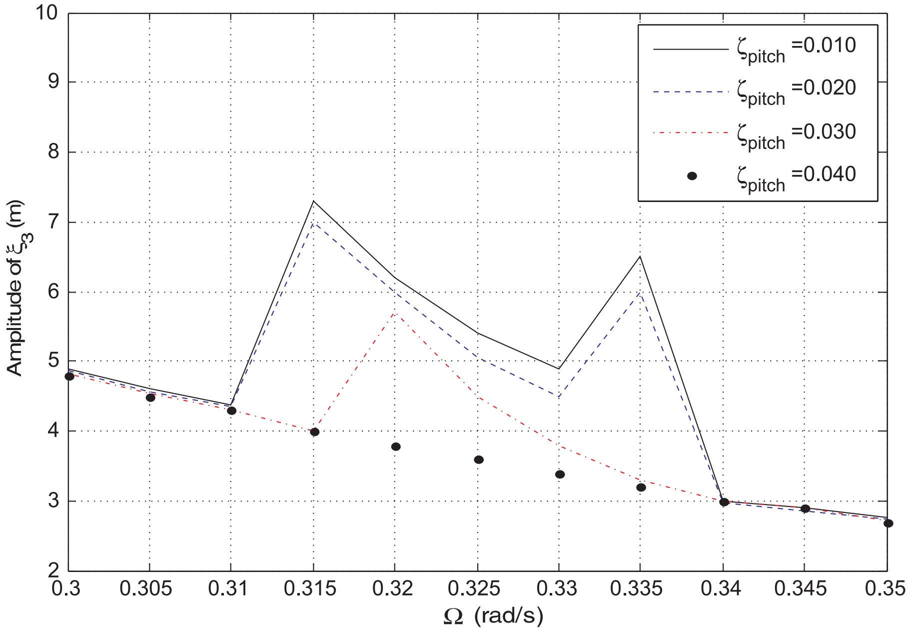


Figure 15. Amplitudes of heave and pitch when $\eta_0 = 4$ m, $\varsigma_{\text{pitch}} = 0.019$.



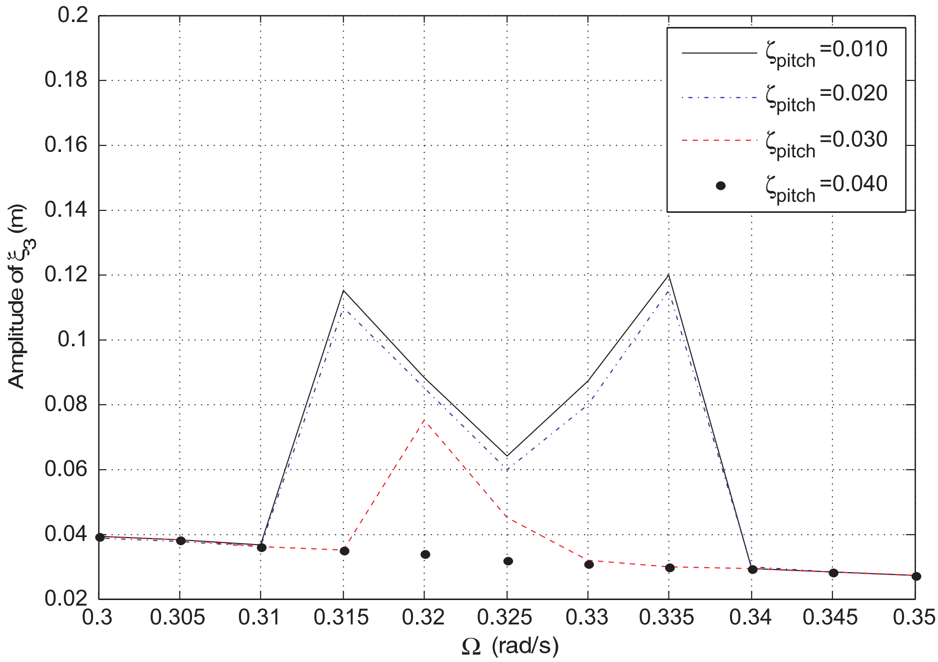


Figure 16. Amplitudes of heave and pitch when $\eta_0 = 4$ m, $\varsigma_{\text{heave}} = 0.012$.

6. CONCLUSIONS

Due to heave and pitch coupling of the spar platform in a nonlinear manner, the heave and pitch restoring coefficients affect each other, and this may cause some kinds of resonant response which endanger the spar platform. When the spar exhibits the sum type combination resonance, the spar experiences pitch motion in large amplitude.

Depending on the amount of available damping, combination resonance may or may not occur. The results of case studies can be summarized as follows:

(1) The simulation results show that coupled nonlinear motions may occur when the wave frequency get close to the sum of heave and pitch natural frequency. At this time, the spar experiences steady multiple frequency vibrations in both heave and pitch motion, the parametric excitation of wave elevation has little effect on it, and the amplitudes of the two modes will not increase linearly with the increase of wave height.

(2) When the spar exhibits sum type combination resonance, the sub-harmonic motion of two modes (especially in pitch modes) in large amplitude are tripped. The amplitude of 2/3 sub-harmonic components in heave motion remains constant, while the amplitude of 1/3 sub-harmonic components in pitch motion grows quickly compared to the wave frequency component, and it is possible to produce a large pitch amplitude. The ratio 2:1 of heave natural frequency to pitch natural frequency and strong coupling nonlinear factors are the main cause for it.

(3) The available damping plays a very important role in this coupled analysis of a spar platform so that the combination resonance can be suppressed efficiently through increasing heave or pitch damping. Some measures, such as adding more spiral strake and the damping plate to increase heave/pitch damping or to shift natural frequencies, are possible to reduce the combination resonance response.

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