

# An Anisotropic Shear Strength Model for Cyclic Accumulated Plastic Strain of Overconsolidated Clay

**Fang Cai**

*PhD Candidate*

*Department of Civil and Transport Engineering, Norwegian University of Science  
and Technology*

*e-mail: [fang.cai@ntnu.no](mailto:fang.cai@ntnu.no)*

**Gudmund Reidar Eiksund**

*Professor, Department of Civil and Transport Engineering, Norwegian University  
of Science and Technology*

*e-mail: [gudmund.eiksund@ntnu.no](mailto:gudmund.eiksund@ntnu.no)*

**Gustav Grimstad**

*Professor, Department of Civil and Transport Engineering, Norwegian University  
of Science and Technology*

*e-mail: [gustav.grimstad@ntnu.no](mailto:gustav.grimstad@ntnu.no)*

**Hans Petter Jostad**

*Professor, Norwegian Geotechnical Institute, NGI, Oslo, Norway*

*e-mail: [Hans.Petter.Jostad@ngi.no](mailto:Hans.Petter.Jostad@ngi.no)*

## ABSTRACT

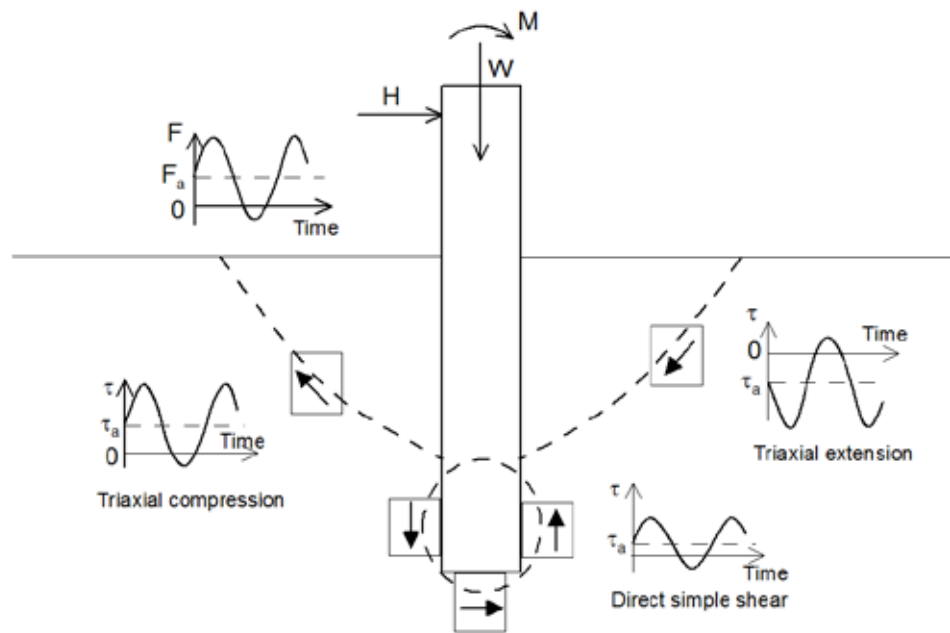
Many geotechnical problems involve calculation of the permanent deformation of overconsolidated clay under undrained cyclic loading. There are many existing methods in the literature which have been made for predicting cumulative plastic deformation for soils under cyclic loading. This paper presents the development of an anisotropic shear strength model for undrained cyclic accumulated plastic strain of overconsolidated clay. Effects of cyclic stress ratio, average stress ratio and number of stress cycles are considered in the model. In addition, the model also differentiate the cyclic behaviour of clay between active (A) and passive (P) modes of loading. Test data obtained from a series of NGI cyclic triaxial tests are employed to verify the model. Comparisons between predicted and experimental results show the validity and good applicability of the anisotropic model. For general 3D stress conditions the model is extended into a modified Tresca formulation which can be implemented into finite element codes for the prediction of permanent deformation of offshore foundation under cyclic loading.

**KEYWORDS:** Cyclic; accumulated plastic strain; anisotropy; overconsolidated clay.

## INTRODUCTION

Offshore foundations are subjected to combinations of static and cyclic loads from wind and waves. For foundations on clay the loading conditions during a storm can be considered as undrained. Monopiles are a frequently used foundation type for offshore wind turbines. Due to

strict requirements regarding accumulated deformations models for predicting the soil response during cyclic loading are needed for these foundation types. The stress path in the soil surrounding a monopile foundation under cyclic loading is complicate. However, the shear stress paths along a potential failure surfaces can be simplified as shown in Fig. 1. This stress path simplification philosophy was proposed by Lambe (1967) and Bjerrum (1973) originally for the applications under monotonic loading, but is here used for cyclic loading conditions. Thereafter, the cyclic behavior for different stress paths can be characterized by standard cyclic triaxial and DSS tests.

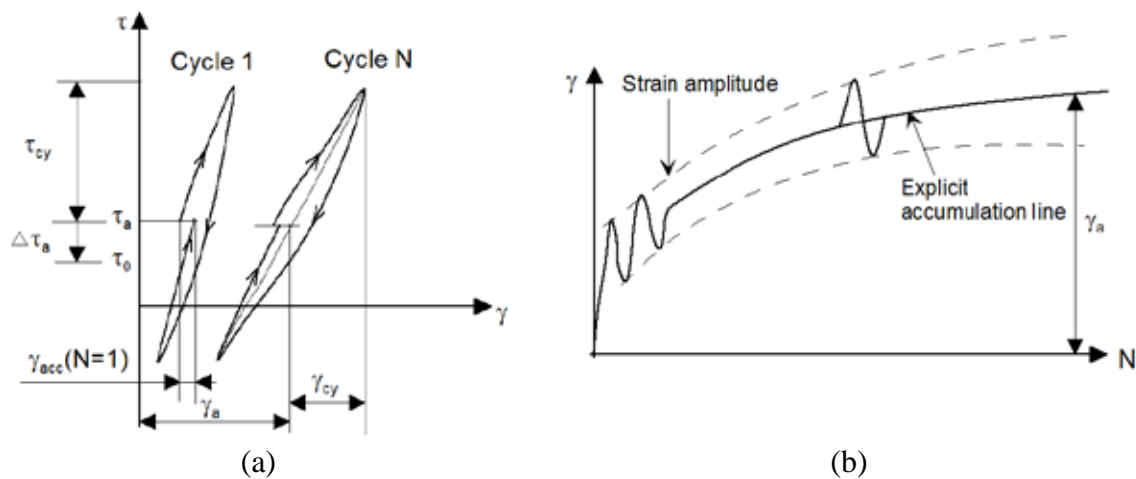


**Figure 1:** Simplified stress path along a potential failure surface (Jostad 2013)

Standard cyclic test procedures (Andersen et al. 1988 and Vucetic 1988) are proposed and a considerable amount of standard cyclic tests are carried out to investigate the behaviors of soils under cyclic loading. Excessive pore pressure is observed under undrained cyclic loadings (Matasovic et al. 1995 and Andersen 2004). The increase of excessive pore pressure can lead to low effective stresses and eventually to a loss of the overall stability. Besides, due to cyclic loading, an accumulation of irreversible strains is generated (Li et al. 1996 and Guo et al. 2013). It can endanger the long-term serviceability of structures especially when the displacement tolerance of the structures is small. As the cyclic behavior of clay depends strongly on OCR, this paper focuses on the prediction of the accumulation plastic shear strain of overconsolidated clay under undrained conditions.

A summation of small residual strains is developed under cyclic loadings. In computational sense, two computational strategies, implicit or explicit strategy, have been proposed for the prediction of the accumulated plastic strain due to cyclic loading. With the implicit method, each cycle is calculated by many strain increments. The fact that the stress loops are not perfectly closed results in accumulated plastic strain, as shown in Fig. 2(a). However, as inevitable cumulative numerical errors occur with an application of a large number of cycles (Niemunis et al. 2005), the practicability of this method is limited by the number of cycles.

With an explicit method, the strain accumulation is calculated directly as a function of the number of cycles. This concept is similar to visco-plastic models where plastic strains develop as function of time instead of number of stress cycles. This method will not be sensitive to cumulative numerical errors. Therefore, the explicit concept is adopted to develop the model proposed in this paper.



**Figure 2:** (a) Stress–strain curve during cyclic loading (Adapted from Andersen 2004); (b) Explicit accumulative plastic strain (Adapted from Niemunis etc. 2005);

Various equations have been proposed for predicting the accumulative plastic strain in soil. Among these equations, the most commonly used one is the power model (Monismith et al. 1975 and Knutson et al. 1977) as following:

$$\gamma_{acc} = A \cdot N^b \tag{1}$$

where  $\gamma_{acc}$  is accumulated plastic strain,  $N$  is the number of cyclic stress applications and  $A$  is the accumulated plastic strain for the first cycle. Parameters  $A$  and  $b$  are dependent on soil type, soil properties, average stress state, cyclic shear stress amplitude, loading direction, strain rate, etc.

An explicit accumulated plastic strain model for sand under high cycle number is proposed by Niemunis et al (2005). In this model, anisotropic effects during settlement are accounted for by a tensorial formulation. However, a large number of parameters and an extensive amount of laboratory tests are required. In addition, a cyclic multi-dimensional simple shear device was developed to obtain information of the anisotropic soil behaviour (Wichtmann et al. 2006).

Besides, a model together with a calculation procedure for analyses of cyclic and permanent displacements and capacity of foundations subjected to a combination of permanent and cyclic undrained loading, with direct input of cyclic triaxial and DSS contour diagrams for a given equivalent number of cycles (as shown in Figure 3), was presented by Jostad and Andersen (2009).

In order to incorporate accumulated plastic shear strain explicitly into an elastoplastic model, the constitutive relation can be expressed as following

$$d\sigma = D \cdot (d\varepsilon - d\varepsilon^{acc} - d\varepsilon^{pl}) \tag{2}$$

where  $\varepsilon^{acc}$  is the accumulated plastic strain,  $\varepsilon^{pl}$  is the plastic strain developed under monotonic loading and it can be calculated with the NGI-ADP shear strength model (Grimstad et.al 2012) as following

$$\sqrt{H(w_a) \cdot \hat{J}_{2a}} - \kappa_a \cdot \frac{s_u^A + s_u^P}{2} = 0 \quad (3)$$

$$d\varepsilon^{pl} = d\lambda_{pl} \cdot \frac{\partial Q_{pl}}{\partial \sigma} \quad (4)$$

where  $H(w_a)$  is a term to approximate the Tresca criterion,  $\hat{J}_{2a}$  is modified second stress invariant,  $\kappa_a$  is stress path dependent hardening parameter,  $s_u^A$  and  $s_u^P$  are undrained shear strengths in plane strain active and passive loading respectively, and  $Q_{pl}$  is the flow potential for plastic strain under monotonic loading.

For triaxial tests, equation (2) can be reformulated as

$$d\tau_a = G \cdot (d\gamma - d\gamma_{acc} - d\gamma_{pl}) \quad (5)$$

where  $\tau_a$  is the average stress,  $G$  is the elastic shear modulus,  $\gamma_{acc}$  is the cyclic accumulated plastic shear strain developed under cyclic loading, and  $\gamma_{pl}$  is plastic shear strain developed under monotonic loading.

The accumulated plastic strain  $\varepsilon^{acc}$  in equation (2) can be related to average stresses by another hardening relation.

In the formulation presented in this paper, the relation between average stress and accumulated plastic strain is investigated. The average shear stress is expressed as a function of accumulated plastic shear strain, cyclic shear stress and number of cycles. Anisotropic effects are included as the cyclic behaviors of overconsolidated clay in active (A) loading mode and in passive (P) loading mode are different. The implementation procedure will be presented in a companion paper later.

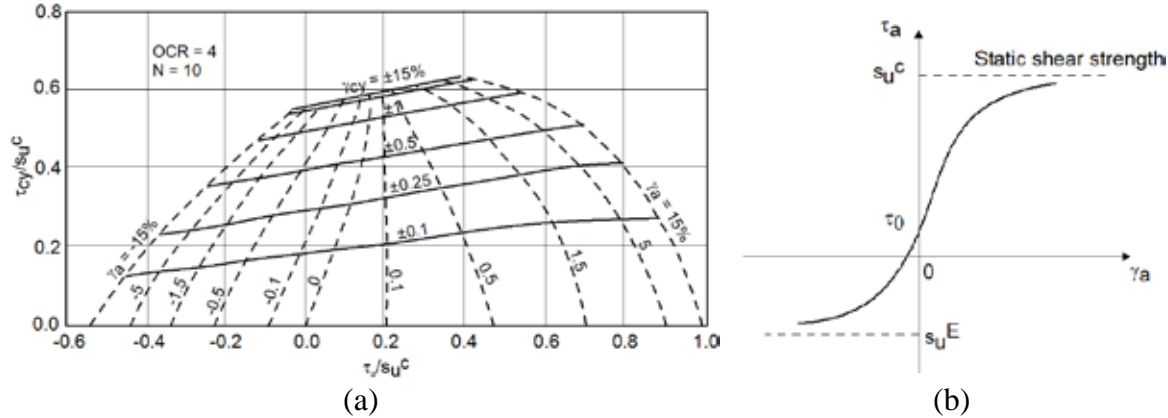
This paper is organized as follows: in Section 2, the relation of hardening for accumulated plastic strain is formulated and explained. An equation to describe the relation between accumulation plastic shear strain and number of cycles is proposed in Section 3. In Section 4, the model is verified through the comparisons between test data and calculated results. Material parameters are determined by curve fitting with least square method. Section 5 presents the formulation of the modified Tresca formulation for the finite element implementation.

## CYCLIC HARDENING RELATION FOR ACCUMULATED PLASTIC STRAIN

Contour diagrams are proposed by Andersen et al. (1988) to present cyclic test results. In these diagrams, cyclic and average shear strains are shown as functions of cyclic and average shear stresses for a given number of cycles. An example of the contour diagram for Drammen clay with OCR 4 after 10 cycles is shown in Fig. 3(a).

The undrained compression strength  $s_u^c$  of clay increases with the preconsolidation stress  $\sigma'_{vpc}$ . The SHANSEP procedure was proposed by Ladd and Foott (1974) and Ladd et al. (1977) for the static characterization of NC and OC clays. In this procedure, all measured stresses were normalized with  $s_u^c$  or  $\sigma'_{vpc}$  prior to further analyses. This procedure is also used here for cyclic loading. Therefore, the cyclic shear stress  $\tau_{cy}$  is divided by  $s_u^c$ , and the normalized parameter

$\tau_{cy}^* = \tau_{cy}/s_u^c$  is called cyclic stress ratio; The average shear stress  $\tau_a$  is divided by  $s_u^c$ , the normalized parameter  $\tau_a^* = \tau_a/s_u^c$  is called average stress ratio, and initial shear stress  $\tau_0$  is divided by  $s_u^c$ , and the normalized parameter  $\tau_0^* = \tau_0/s_u^c$  is called initial stress ratio.



**Figure 3:** (a) Cyclic contour diagram for Drammen clay with OCR 4 after N=10 (Adapted from Andersen et al. 1988); (b) Stress – strain curve under monotonic triaxial tests;

The contours presented in Fig. 3(a) show information about the average and cyclic shear strains developed under cyclic loading. The stress-strain curve under monotonic loading can also be constructed from the data in the diagrams as shown in Fig. 3(b). Thereafter the accumulated plastic strain can be calculated with the following equation

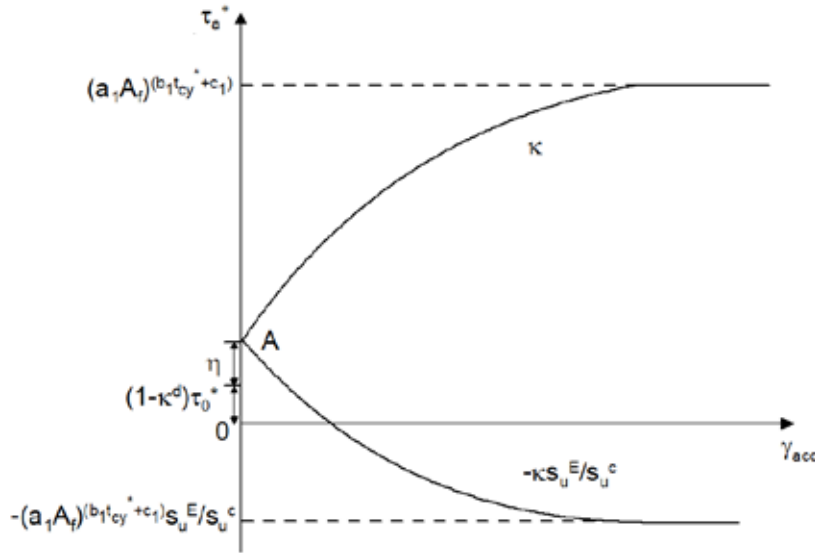
$$\gamma_{acc} = \gamma_a - \gamma_{ep} \tag{6}$$

where  $\gamma_{acc}$  is the accumulated plastic shear strain,  $\gamma_a$  is average shear strain and  $\gamma_{ep}$  is the elastoplastic strain developed under monotonic loading.

It is indicated from a series of NGI cyclic triaxial test results that average shear strain  $\gamma_a$  depends on the average shear stress  $\tau_a$ , the cyclic shear stress  $\tau_{cy}$  and cycle number N. Based on the correlations found in the contours, the cyclic hardening equation can be decomposed into two parts: triaxial compression part and triaxial extension part. Point A is the division point between triaxial compression and triaxial extension as shown in Fig.4.

$$\tau_a^* = \begin{cases} (1 - \kappa^d) \cdot \tau_0^* + \eta + \kappa, & \text{Triaxial compression} \\ (1 - \kappa^d) \cdot \tau_0^* + \eta - \kappa \cdot \frac{s_u^E}{s_u^c}, & \text{Triaxial extension} \end{cases} \tag{7}$$

where  $s_u^E$  is the undrained triaxial extension strength,  $s_u^c$  is the undrained triaxial compression strength,  $d$  is a power parameter and  $\tau_0^*$  is the initial stress ratio. A state variable  $\eta$  is introduced to account for the translation of point A caused by cyclic loading. State variable  $\kappa$  represents cyclic strength hardening.



**Figure 4:** Stress–strain curves for cyclic triaxial compression and extension tests.

Phenomenological laws are put forward for the cyclic translation  $\eta$  and the cyclic hardening  $\kappa$  respectively. In cyclic triaxial tests,  $\eta$  is observed to be a function of the cycle number and cyclic stress ratio. It grows with both cycle number and cyclic stress ratio. The value of  $\eta$  is 0 if there is no cyclic loading. The hardening parameter  $\kappa$  is observed to be a function of the cycle number, the cyclic stress ratio and the accumulated plastic shear strain. For given cycle number and cyclic stress ratio,  $\kappa$  increases with accumulated plastic shear strain, and for a given accumulated plastic strain, it reduces with the increase of cycle number and cyclic stress ratio.

$$\kappa = \begin{cases} (a_1 \cdot \gamma_{acc})^{(b_1 \cdot \tau_{cy}^* + c_1)} \cdot N_{eq}^{(-d_1 \cdot \tau_{cy}^*)}, & \gamma_{acc} < \gamma_p^f \\ (a_1 \cdot \gamma_p^f)^{(b_1 \cdot \tau_{cy}^* + c_1)} \cdot N_{eq}^{(-d_1 \cdot \tau_{cy}^*)}, & \gamma_{acc} \geq \gamma_p^f \end{cases} \quad (8)$$

$$\eta = \frac{a_2 \cdot \tau_{cy}^*}{b_2 + \tau_{cy}^*} \cdot N_{eq}^{c_2} \quad (9)$$

Where  $\gamma_p^f$  is the failure (peak) plastic shear strain and  $N_{eq}$  is the equivalent number of cyclic stress applications at a constant cyclic shear stress amplitude level (Andersen et al. 1992).  $a_1, b_1, c_1, d_1, a_2, b_2$  and  $c_2$  are parameters which are dependent on soil properties and soil physical state. They can be determined by curve fitting to the results from cyclic laboratory tests.

## RELATION BETWEEN ACCUMULATED PLASTIC STRAIN AND CYCLE NUMBER

Based on equation (1), a formulation for the relation between accumulated plastic strain and cycle number is proposed.

$$\gamma_{acc} = A \cdot N_{eq}^{\left(\frac{d_1 \cdot \tau_{cy}^*}{b_1 \cdot \tau_{cy}^* + c_1}\right)} \quad (10)$$

Thus, the rate form of the accumulated plastic strain can be expressed as

$$\dot{\gamma}_{acc} = A \cdot \left( \frac{d_1 \cdot \tau_{cy}^*}{b_1 \cdot \tau_{cy}^* + c_1} \right) \cdot N_{eq} \left( \frac{d_1 \cdot \tau_{cy}^* - b_1 \cdot \tau_{cy}^* - c_1}{b_1 \cdot \tau_{cy}^* + c_1} \right) \quad (11)$$

where  $\dot{\gamma}_{acc} = \frac{d\gamma_{acc}}{dN_{eq}}$ .

Rearranging equation (8) into equation (11) gives

$$\dot{\gamma}_{acc} = \begin{cases} A \cdot \left( \frac{d_1 \cdot \tau_{cy}^*}{b_1 \cdot \tau_{cy}^* + c_1} \right) \cdot \left( \frac{\kappa}{(a_1 \cdot \gamma_{acc})^{(b_1 \cdot \tau_{cy}^* + c_1)}} \right)^{\left( \frac{b_1 \cdot \tau_{cy}^* + c_1 - d_1 \cdot \tau_{cy}^*}{d_1 \cdot \tau_{cy}^* \cdot (b_1 \cdot \tau_{cy}^* + c_1)} \right)}, & \gamma_{acc} < \gamma_p^f \\ \infty, & \gamma_{acc} \geq \gamma_p^f \end{cases} \quad (12)$$

Alternatively, equation (12) can be rewritten into equation (13) in order to express  $\kappa$  as a function of the accumulated plastic strain rate.

$$\kappa = (a_1 \cdot \gamma_{acc})^{(b_1 \cdot \tau_{cy}^* + c_1)} \cdot \dot{\gamma}_{acc} \left( \frac{d_1 \cdot \tau_{cy}^* \cdot (b_1 \cdot \tau_{cy}^* + c_1)}{b_1 \cdot \tau_{cy}^* + c_1 - d_1 \cdot \tau_{cy}^*} \right) \cdot A \left( \frac{d_1 \cdot \tau_{cy}^* \cdot (b_1 \cdot \tau_{cy}^* + c_1)}{d_1 \cdot \tau_{cy}^* - b_1 \cdot \tau_{cy}^* - c_1} \right) \cdot \left( \frac{b_1 \cdot \tau_{cy}^* + c_1}{d_1 \cdot \tau_{cy}^*} \right)^{\left( \frac{d_1 \cdot \tau_{cy}^* \cdot (b_1 \cdot \tau_{cy}^* + c_1)}{b_1 \cdot \tau_{cy}^* + c_1 - d_1 \cdot \tau_{cy}^*} \right)} \quad \text{when } \gamma_{acc} < \gamma_p^f \quad (13)$$

Also, with the insertion of equation (10), equation (8) and equation (9) can be expressed as

$$\kappa = \begin{cases} (a_1 \cdot A)^{(b_1 \cdot \tau_{cy}^* + c_1)}, & \gamma_{acc} < \gamma_p^f \\ (a_1 \cdot A_f)^{(b_1 \cdot \tau_{cy}^* + c_1)}, & \gamma_{acc} \geq \gamma_p^f \end{cases} \quad (14)$$

$$\eta = \begin{cases} \left( \frac{a_2 \cdot \tau_{cy}^*}{b_2 + \tau_{cy}^*} \cdot \left( \frac{\gamma_{acc}}{A} \right)^{\left( \frac{(b_1 \cdot \tau_{cy}^* + c_1) \cdot c_2}{d_1 \cdot \tau_{cy}^*} \right)} \right), & \gamma_{acc} < \gamma_p^f \\ \left( \frac{a_2 \cdot \tau_{cy}^*}{b_2 + \tau_{cy}^*} \cdot \left( \frac{\gamma_p^f}{A_f} \right)^{\left( \frac{(b_1 \cdot \tau_{cy}^* + c_1) \cdot c_2}{d_1 \cdot \tau_{cy}^*} \right)} \right), & \gamma_{acc} \geq \gamma_p^f \end{cases} \quad (15)$$

where  $A_f$  is the value of  $A$  when accumulated plastic strain  $\gamma_{acc}$  reaches the specified failure strain  $\gamma_p^f$ .

## MODEL VERIFICATION

Cyclic triaxial tests data of overconsolidated clays from Drammen (Andersen et al. 1988) and Moum (Andersen et al. 1989) are used for the model verification. The details about test procedures are elaborated in the corresponding references.

### Drammen Clay

Drammen clay is a marine clay with plasticity index  $I_p = 27\%$  and clay content of 45-55% (Bjerrum 1967). Before the cyclic tests, the "undisturbed" samples were consolidated to a vertical effective stress  $\sigma'_{vpc}$  of 392 kPa. No lateral strain was allowed during consolidation. The overconsolidated clay is here created by unloading the samples to smaller vertical effective stress  $\sigma'_{vc}$ .

The database developed by Andersen et al. (1988) for Drammen clay with OCR 4 is used to demonstrate the applicability of the proposed model. Several tests are selected for the calibration of the parameters required by the model. The selected tests are listed in Table 1. Least square

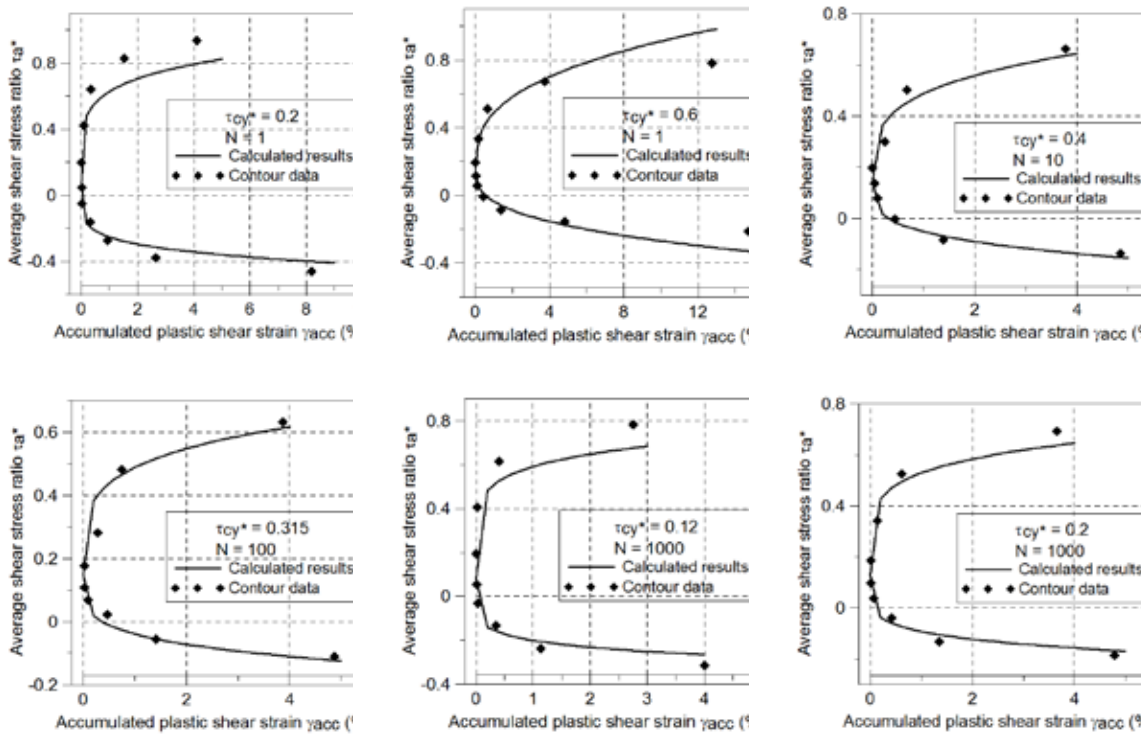
method is employed for the parameter calibration. The obtained parameters are shown in Table 2. Fitted results are plotted together with test results in Fig. 5.

**Table 1:** Tests for the calibration of model parameters

Cycle number	Cyclic stress ratio $\tau_{cy}^*$
1	0.2
1	0.6
10	0.4
100	0.315
1000	0.12
1000	0.2

**Table 2:** Model parameters for Drammen clay with OCR = 4

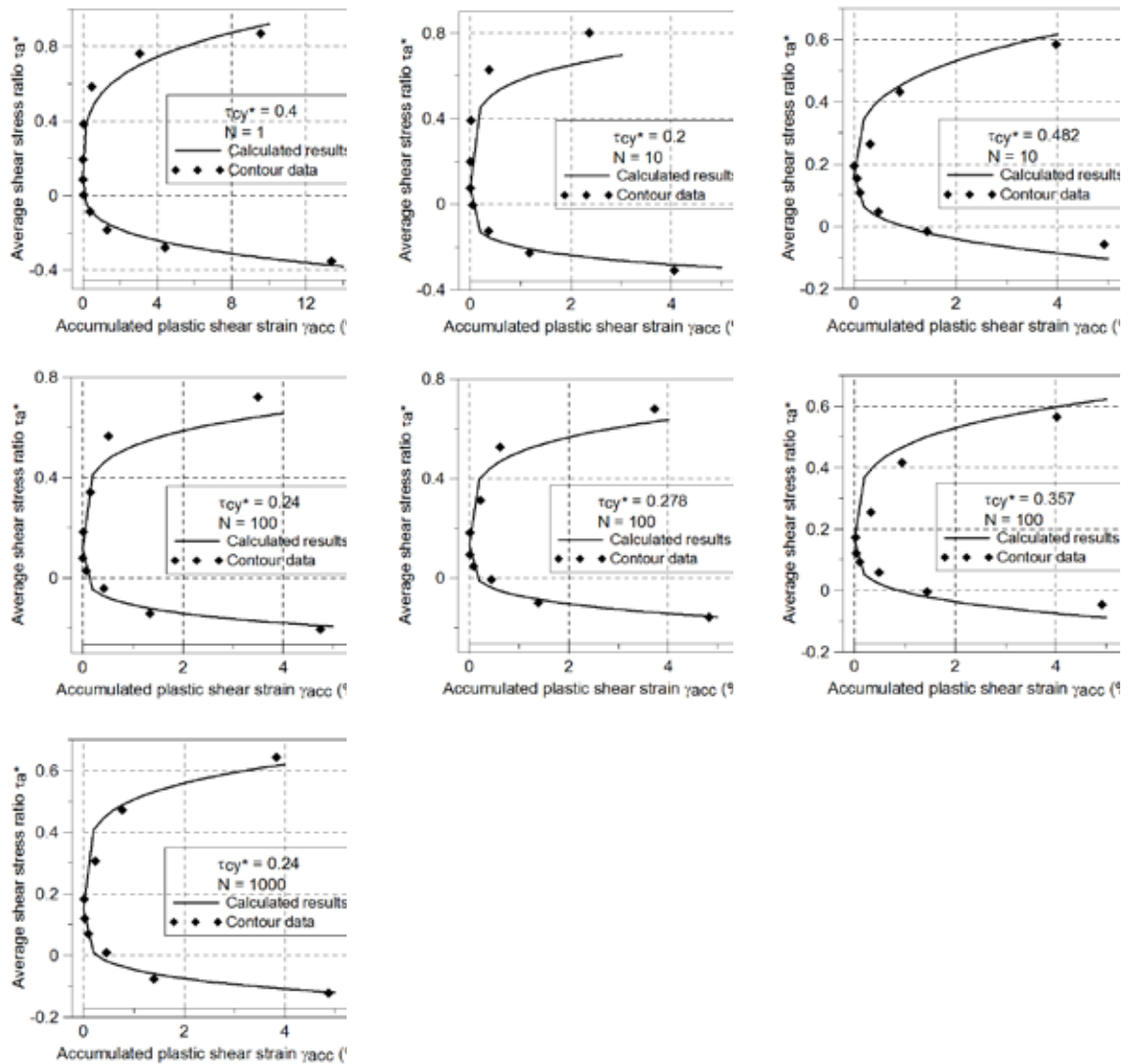
$a_1$	$b_1$	$c_1$	$d_1$	$a_2$	$b_2$	$c_2$	$d$
4.6	0.42	0.1	0.25	0.6	1.81	0.11	20



**Figure 5:** Fitted results of selected tests for Drammen clay with OCR = 4

It is shown in the graphs that the parameters for the model are well fitted. Secondly, the model with calibrated parameters is used to predict the average shear stress vs. accumulated plastic strain curve for other cyclic stress ratios and cycle numbers. The comparisons between experimental and calculated results are plotted in Figure 6. It shows that the model give good predictions for the accumulated plastic strain also for these cases.





**Figure 6:** Comparisons between calculated results and test data for Drammen clay with OCR = 4

### *Moum Clay*

The second clay was obtained from Moum, in the southeastern part of Norway. After being reconstituted and reconsolidated, the clay was: salt content 12.5 g/L (14.0 g/L), liquid limit 50.9% (48.0%), plasticity index 28.5% (25.1%), and clay content 45% (45%)  $< 2\mu$  (Dyvik et al. 1989).

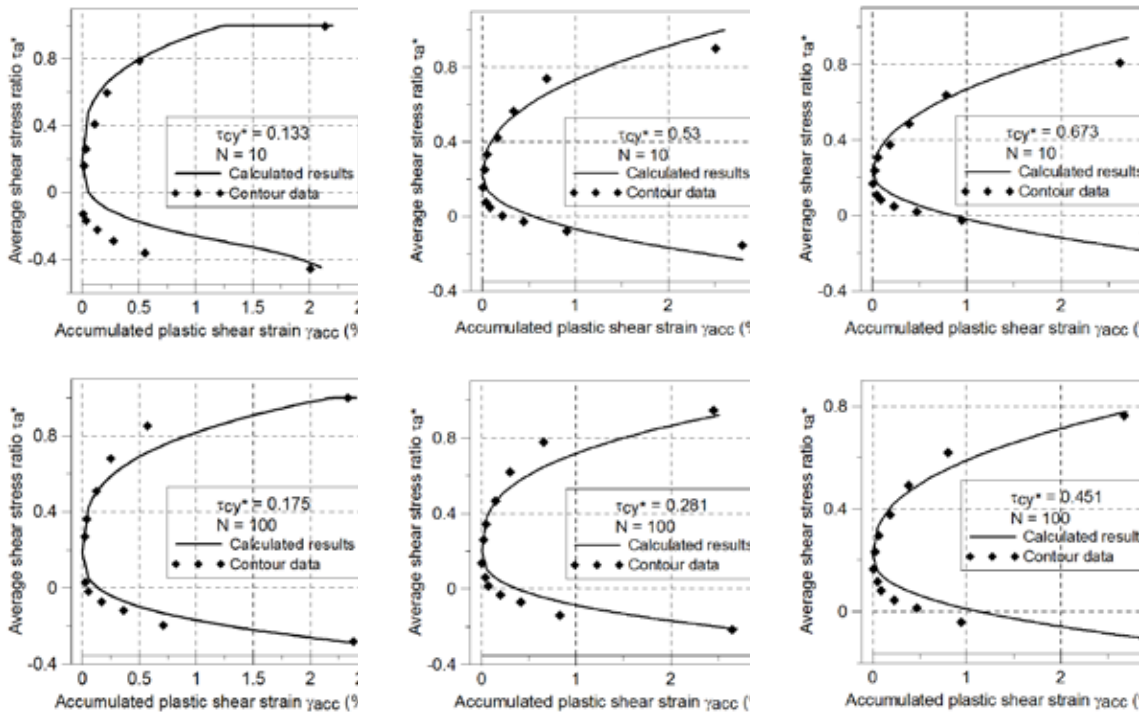
The database compiled by Andersen et al. (1989) for Moum clay with OCR = 3.4 is also used for the verification of the model. Tests selected for the parameters calibration with least square method are listed in Table 3. The calibrated values of the parameters for Moum clay OCR 3.4 are shown in Table 4. Fitted results are plotted together with test data in Figure 7.

**Table 3: Tests for the calibration of model parameters**

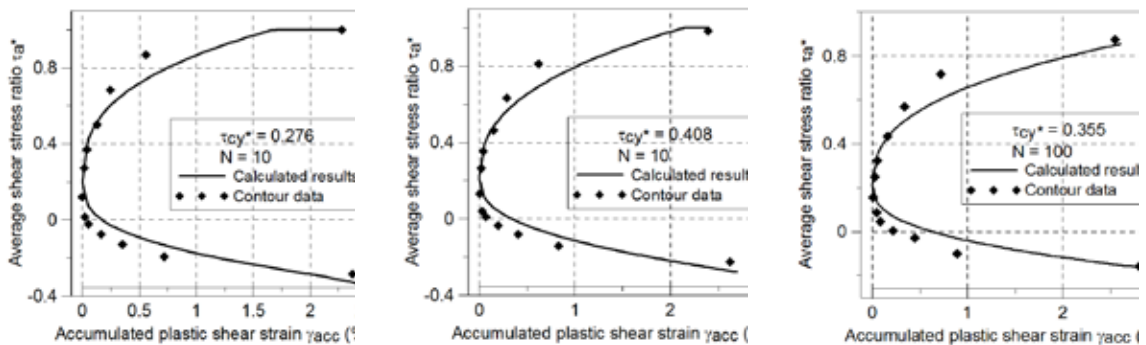
Cycle number	Cyclic stress ratio $\tau_{cy}^*$
10	0.133
10	0.53
10	0.673
100	0.175
100	0.281
100	0.451

**Table 4: Model parameters for Moum clay**

$a_1$	$b_1$	$c_1$	$d_1$	$a_2$	$b_2$	$c_2$	$d$
64.6	0.32	0.27	0.39	0.14	0.21	0.01	20



**Figure 7: Fitted results of selected tests for Moum clay OCR 3.4**



**Figure 8: Comparisons between calculated results and test data for Moum clay with OCR 3.4**

It is shown from the graphs that the parameters for the model are rather well fitted. The predicted average shear stress vs. accumulated plastic strain curve for other cyclic stress ratios and cycle numbers are plotted in Figure 8.

From the comparisons between calculated results and test data, it is shown that the predictions from the model fit the test data for Moum clay OCR3.4 rather well.

## MODEL FORMULATION

The model proposed in the previous sessions is shown to be suitable for cyclic triaxial load cases. In order to make use of it for different stress conditions, the model should be formulated for a general stress state. The model formulation is presented here in the following steps: starting with '1D' anisotropy in cyclic triaxial test condition; Thereafter, the formulation is extended to full 3D stress state with modified Tresca formulation.

### Tresca criterion

The Tresca criterion is widely used in geotechnical engineering to represent isotropic undrained shear strength. It can be represented as a hexagonal prism in three-dimensional principal total stress space.

$$F = \tau - \kappa \cdot s_u = \sqrt{J_2} \cos\theta - \kappa \cdot s_u = 0 \tag{16}$$

Where  $J_2$  is the second deviatoric stress invariant,  $\theta$  is the Lode angle,  $\kappa$  is the strength hardening parameter, and  $s_u$  is the isotropic undrained shear strength.

#### 1D Model Presentation

The Tresca criteria can be modified to account for the difference of undrained shear strength in compression and extension (Grimstad et al. 2012). Thus, Tresca criteria is reformulated to fit equation (3) with the following form

$$F = \left| \tau_a - \eta \cdot s_u^c - \kappa \cdot \frac{s_u^c - s_u^E}{2} - (1 - \kappa^d) \cdot \tau_0 \right| - \kappa \cdot \frac{s_u^c + s_u^E}{2} = 0 \tag{17}$$

#### The Model in 3D Stress Space

To use the model for the general stress condition, a modified deviatoric stress vector is introduced as following

$$\begin{bmatrix} \hat{S}_{xx} \\ \hat{S}_{yy} \\ \hat{S}_{zz} \\ \hat{S}_{xy} \\ \hat{S}_{xz} \\ \hat{S}_{yz} \end{bmatrix} = \begin{bmatrix} \sigma'_{xx} + \eta \cdot \frac{2}{3} \cdot s_u^c + \kappa \cdot \frac{1}{3} \cdot (s_u^c - s_u^E) - (1 - \kappa^d) \cdot \sigma'_{x0} - \hat{p} \\ \sigma'_{yy} + \eta \cdot \frac{2}{3} \cdot s_u^c + \kappa \cdot \frac{1}{3} \cdot (s_u^c - s_u^E) - (1 - \kappa^d) \cdot \sigma'_{y0} - \hat{p} \\ \sigma'_{zz} - \eta \cdot \frac{4}{3} \cdot s_u^c - \kappa \cdot \frac{2}{3} \cdot (s_u^c - s_u^E) - (1 - \kappa^d) \cdot \sigma'_{z0} - \hat{p} \\ \tau_{xy} \\ \tau_{xz} \cdot \frac{s_u^c + s_u^E}{2 \cdot s_u^{DSS}} \\ \tau_{yz} \cdot \frac{s_u^c + s_u^E}{2 \cdot s_u^{DSS}} \end{bmatrix} \tag{18}$$

where a modified mean stress is defined accordingly as

$$\hat{p} = p' - (1 - \kappa^d) \cdot p'_0$$

$p'_0$  is the initial mean stress.

Therefore the modified Tresca criterion can be expressed as

$$F = \sqrt{H(\omega)} \cdot \hat{J}_2 - \kappa \cdot \frac{s_u^c + s_u^E}{2} = 0 \quad (19)$$

where  $\hat{J}_2$  is modified second deviatoric stress invariant

$$\hat{J}_2 = -\hat{s}_{xx}\hat{s}_{yy} - \hat{s}_{xx}\hat{s}_{zz} - \hat{s}_{yy}\hat{s}_{zz} + \hat{s}_{xy}^2 + \hat{s}_{xz}^2 + \hat{s}_{yz}^2$$

The term  $H(\omega)$  is introduced as below to approximate the Tresca criterion:

$$H(\omega) = \cos^2\left(\frac{1}{6} \cdot \arccos(1 - 2\omega)\right)$$

where

$$\omega = \frac{27}{4} \cdot \frac{\hat{J}_3^2}{\hat{J}_2^3}$$

And  $\hat{J}_3$  is the third deviatoric stress invariant.

$$\hat{J}_3 = -\hat{s}_{xx}\hat{s}_{yy}\hat{s}_{zz} + 2\hat{s}_{xy}\hat{s}_{yz}\hat{s}_{xz} - \hat{s}_{xx}\hat{s}_{yz}^2 - \hat{s}_{yy}\hat{s}_{xz}^2 - \hat{s}_{zz}\hat{s}_{xy}^2$$

For the further implementation of the model, cyclic stress ratio should be calculated individually and used as an input parameter for the calculation of the accumulated deformation of the overconsolidated clay. And as the accumulation of the strain will be predicted directly due to a package of cycle numbers, number of cycles will also be needed as input for the calculation. In reality, a storm is composed of loads with varying amplitudes and periods, so in order to make use of the laboratory test results which are gained with one constant cyclic shear stress amplitude throughout each test, the determination of an equivalent number of cycles for irregular cyclic load histories is also needed.

## CONCLUSION

This paper presents a cyclic hardening formulation of accumulated plastic strain for overconsolidated clays. The effects of average shear stress, cyclic shear stress and cycle number on the accumulated plastic strain are accounted for in the model. The anisotropic effects of overconsolidated clays are also included in the model. A function between accumulated plastic strain and cycle number is proposed and incorporated into the model. The cyclic shear stress amplitude which also is input to the model, needs to be calculated by another suitable model. The comparisons between calculated results and test data indicate that the model predicts the test data rather well. Finally, the model is extended to a general 3D stress state by using a modified Tresca formulation. This model may then be implemented into a finite element program.

## ACKNOWLEDGMENT

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