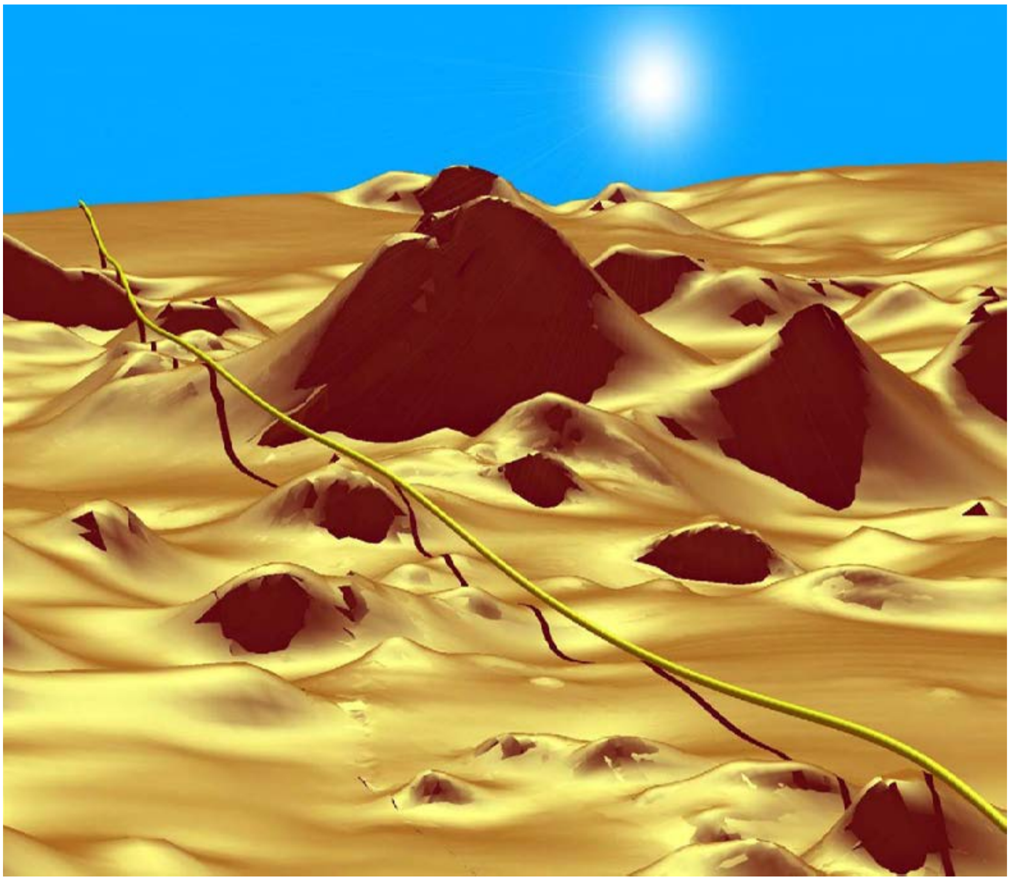
Master thesis Spring 2011

Fatigue of Pipelines resting on Uneven Seabed



Student name: JIN XING
Student number:717094

Preface

This report is written in order to cover the works that have been performed by the writer to accomplish the task of the Master thesis in spring 2011.

Thanks very much to my supervisor Prof. Svein Sævik, I really got a lot of advices and help during the whole procedure. Thanks a lot to Joachim Taby on the using of SIMLA software for the project. I also got a lot of help from Michael Macke who works in DNV Pipeline Technology Department, he give me lots of suggestions when I use FATFREE to calculate the fatigue damage of the pipe, and thanks to Mr Olav Fyrileiv to support me the FATFREE software. I really learned and practiced much during the whole procedure of writing this master thesis.

Marintek , NTNU, Trondheim
June 2011

Student name: Xing Jin

Signature: _______________ _

Symbols

Abbreviation

CF	Cross flow
IL	In line
FEM	Finite element method
VIV	Vortex induced vibration

Roman symbols

$\left(\frac{A}{D}\right)_{IL/CF}$	Amplitude ratio for IL or CF
C	Damping coefficient matrix
C_a	Added mass coefficient
C_d	Drag coefficient
C_l	Lift coefficient
D	Pipeline diameter
E	Elastic modulus
K	Stiffness matrix
F	Force, New parameter for ranking of active frequencies
F_D	Drag force
F_l	Lift force
F_D'	Oscillating drag force
F_L'	Oscillating lift force
f	Frequency
f_{osc}	Oscillation frequency
$f_{n,in}$	Eigen frequency for in-line direction
$f_{n,cf}$	Eigen frequency for cross-flow direction
f_v	Vortex shedding frequency in units of hertz
$\hat{f}$	Non-dimensional frequency
g	acceleration of gravity
I	Area moment of inertia
KC	Keulegan-Carpenter number for oscillatory flow
k	Characteristic size of roughness
L	Span length
M	Mass matrix, Number of measurement points
m	mass
$\bar{m}$	mass ratio
N	Integer number
R	Radium, Excitation load matrix
Re	Reynolds number

St	Strouhal number
T	Tension, Eigen period, Time
t	time
U	Velocity
U_N	Current flow velocity
z	spatial parameter

Greek Symbols

ν	Viscosity parameter
ω	frequency
$\omega(t)$	Modal weights
φ	Known mode shape
φ_n	Known mode shape of mode n
φ_n''	Second spatial differential of known mode shape
P	density
κ	curvature
ε	strain
Ψ	Known Mode shape

Abstract

Subsea pipelines represent the most cost effective way of transporting oil and gas from the subsea field to the market. A large network of subsea pipelines has therefore been installed both at the Norwegian continental shelf an elsewhere. In the near-shore areas of Norway, the seabed is irregular and pipeline free-spans are unavoidable. This in combination with significant current action, may cause vortex induced vibration (VIV) and fatigue in the pipeline welds. This project focus on studying the fatigue performance of free-spanning pipelines using a combination of FEM analysis and the DnV recommended practices.

Keywords: Free span pipeline; Vortex induced vibration; Fatigue damage

Work scope

The thesis work will continue the project work and is to be carried out as follows:

1. Literature study, including pipeline technology in general i.e. pipeline manufacture & installation, pipeline design procedures, pipe-soil interaction, seabed current models, pipeline failure modes and associated design criteria. This is also to include the techniques used to perform pipeline response analysis during installation and operational phases (ensuring that the relevant design criteria are met) including non-linear finite element methods to obtain the relevant equilibrium configuration and modal analysis techniques to calculate fatigue damage from VIV. Perform a detailed study of the response models available in DnV RP –F105 Free Spanning Pipelines due to VIV.

2. Familiarize with the computer code SIMLA, define a relevant free-spanning scenario for a selected pipeline including terrain data, pipeline diameter and thickness, coating, flow characteristics and environmental conditions. Establish a free-span model in SIMLA and perform static analysis including all phases of pipeline behaviour, i.e. installation, water-filling, hydrostatic testing, dewatering and operation. Perform eigen-mode analysis for 3 in-line and 3 cross-flow modes.

3. Use FATfree which is developed by DNV that based on inputs in terms of wave and current statistics, uses the eigen-modes found in item 2 and the procedure proposed in DnV RP F105 to calculate the fatigue damage along the pipeline due to VIV.

4. Perform a case study using SIMLA and the FATfree to calculate the fatigue damage. This is to be based on an assumed long term distribution of current velocities and wave data, the eigen-modes found from SIMLA and considering the contribution from VIV. Assume a water depth of 100 m.

5. Conclusions and recommendations for further work.

Content

Summary

This report will be carried out by the guidance of the given topic, and doing some literature study on pipeline technology.

First, it will give a brief introduction of a relevant free span scenario for a selected pipeline, which includes the terrain data, pipeline dimensions and environmental conditions, which contains the current and wave data, and also decide the damping ratio and safety factors to use in this project.

Find the dynamic differential equation of the pipeline span, which contains the theoretical foundation and numerical procedures to analyze the pipeline.

Establish this free span model in SIMLA and perform static analysis in following sections, and perform eigen mode analysis for 3 in-line and 3 cross-flow modes by using of SIMLA. Find the eigen frequencies for these three mode in two directions, respectively.

Then find the methods of how to calculate the fatigue damage caused by VIV, by using of DNV software FATFREE.

At last, give a summary on the conclusions from research findings. Also used to recommend further works following the work performed during this thesis.

Illustrations

1. INTRODUCTION

When a part of a subsea pipeline is suspended between two points on an uneven seabed, it is always referred to as a free span pipeline. They are often installed on irregular seabed when on-bottom pipelines from off-shelf fields climb onto the continental shelf, it may also be found closer to the coast when crossing rough topography.

The pipeline structure will have to stand complicate environmental forces caused by soil, current and waves. One of the main risk factors is fatigue failure due to ocean current and wave loading. If a free span is exposed to a current flow, vortex-induced vibrations (VIV) of the hanging part of the pipeline may occur. These vibrations may lead to unacceptable fatigue damage in pipeline.

The span evaluation is compliant with the design principles in DNV-RP-F105 in this study. Based on the DNV code, the study of a free spanning pipeline includes both response and force models. The response model is based on a Vortex Induced Vibration (VIV) amplitude response where the VIV is caused by vortex shedding across the pipeline.

In the free span section, there are two directions of modes. One is parallel to the current flow, which is called “in-line” direction, the presence of drag and lift effects are observed. The other one is “cross-flow” direction, vortex induced vibrations (VIV) forces and self-weight are usually present. From previous investigations, it has shown that vortex induced vibrations are very important element in the reduction of life-time service due to fatigue.

Both in-line and cross-flow VIV can be current induced or and wave-induced. The “combined” velocity is obtained from both current and wave velocities before it goes into the fatigue calculation, as shown in Section 4.1.5 in DNV RP-F105. The pipe may also experience fatigue damage and local over-utilization due to direct waves, typically in shallow water. The influencing factors in VIV and direct wave loading assessment are:

- Pipe size, weight, and geometry;
- Additional weight such as content, insulation, and concrete coating if applicable;
- Current and wave parameters;
- Static and dynamic seabed soil stiffness;
- Span shoulder geometry;

Residual lay tension;
Operational conditions such as temperature and internal pressure.

The objective of this report is to use the computer software SIMLA to establish a relevant free span model for a selected pipeline including terrain data, pipeline diameter and thickness, coating and flow characteristics and environmental conditions, and then perform static analysis including all phases of pipeline behavior, i.e. installation, water-filling, hydrostatic testing, dewatering and operation, then again analyze the eigen mode for 3 in-line and 3 cross-flow modes. For a given long term distribution of current velocities, use a new program called FatFree and the eigen-modes which got from SIMLA to calculate the fatigue damage based on the procedure proposed in the DnV Free-Span Recommended Practice.

SIMLA

SIMLA is a computer program for simulation of umbilical structures. It allows for both nonlinear static and dynamic analysis. In both cases the time domain is used to describe the load histories and the analysis sequence. Since then a lot of features have been implemented such as new element types and non-linear time domain dynamics, but SIMLA has no capability to handle vortex induced vibrations.

FATFREE

FATFREE is a Microsoft Excel VBA spreadsheet developed by DNV for design and (re-)assessment of submarine pipeline spans in compliance with DNV-RP-F105 “Free Spanning Pipelines”. FATFREE calculates the fatigue life capacity due to combined direct wave action and in-line Vortex Induced Vibrations (VIV) and Cross-Flow VIV.

In addition, simplified ULS design checks in terms of peak stress and equivalent stress due to combined static and dynamic actions are provided.

2. DEFINE A RELEVANT FREE-SPANNING SCENARIO

2.1 Environmental conditions

The environmental conditions are determined with general reference to the pipeline design basis in [*DNV-OS-F101]*. This is done in order to model realistic environmental conditions for pipelines that are located in the North Sea. The environmental data to be used in the assessment of the long-term distributions shall be representative for the particular geographical location of the pipeline free span. The conditions are mainly dominated by current flow at the free span level, it may have components from tidal current, wind induced current, storm surge induced current and density driven current.

2.1.1 Field Location

There are large areas in Norwegian waters with very uneven seabed. The deeper parts of the North Sea have numerous pock marks caused by slow seepage of underground gas. Large areas, particularly outside mid-Norway and northern Norway, are criss crossed by deep iceberg plough marks. Sandy beaches on the Norwegian coast are scarce and not located in areas where land falls are wanted. The near-shore area is in general rocky and very uneven.

DNV divides the free spans into four different categories depending on the ratio between the length and the outer diameter of the pipeline *[DNV-RP-F105 2006, p9]*. For my case, it deals primarily with the third free span, which response is dominated by combined beam and cable behavior. Relevant for free spans at uneven seabed in temporary conditions, the natural frequencies sensitive to boundary conditions, effective axial force(including initial deflection, geometric stiffness) and pipe "feed in". A typical uneven seabed has been selected in order to obtain a wide range of span lengths giving high fatigue damage. The soil property is medium stiff clay, then from RP-F105 table 3-1 the on-bottom roughness is 5.0E*06.

2.1.2 Water depth

It is assumed that the pipeline is located at relatively deep water corresponding to a pipeline near an offshore platform. The design water depth of H=100m is taken at the Norwegian continental shelf.

2.1.3 Current conditions

The current is described by a 3 parameter Weibull current distribution of the 10 minute average current measurement at 3 meters above the seabed. The current velocity u_C=0.5m/s.

The current that affects the pipeline is determined by assuming that the velocity profile is polynomial and can be formulated as:

$$u(z) = \frac{8}{7} u_c \cdot \left(\frac{z}{h}\right)^{\frac{1}{7}} \tag{2.1}$$

Where

- u is the current velocity affecting the pipe [m/s]
- z is the vertical coordinate with origin at the water surface [m]
- h is the water depth [m]
- u_c is the basic current parameter [m/s]

The velocity profile which uses a $1/7^{th}$ –power profile is generally in good agreement with a logarithmic formulation of the velocity profile. However, it is however that the logarithmic velocity profile tends to be more accurate because the seabed roughness is included as an additional parameter in this formulation. In case of an even seabed without significant roughness, the $1/7^{th}$ –power profile will underestimate the flow velocity near the seabed. The basic current parameter have been reduced due to boundary layer interaction, and it varies according to the return period of the design wave.

The current velocity is statistically described by a Weibull distribution as:

$$F_{U_{ref}}\left(U(z_{ref})\right) = 1 - \exp\left(-\left(\frac{U_{ref} - \gamma_{ref}}{\alpha_{ref}}\right)^{\beta_{ref}}\right) \tag{2.2}$$

Where γ_{ref}, α_{ref}, β_{ref} are Weibull parameters. The current velocity at a given depth $U(z_{ref})$istransferred to current velocity at pipe level.Where

The Weibull distribution parameters are linked to the statistical moments (μ: mean value, σ: standard deviation, δ: skewness) as follows:

$$\mu = \alpha\Gamma\left(1 + \frac{1}{\beta}\right) + \gamma \tag{2.3}$$

$$\sigma = \alpha\sqrt{\Gamma\left(1 + \frac{2}{\beta}\right) - \Gamma\left(1 + \frac{1}{\beta}\right)^2} \tag{2.4}$$

$$\delta = \left(\frac{\alpha}{\sigma}\right)^3 \left(\Gamma\left(1+\frac{3}{\beta}\right) - 3\Gamma\left(1+\frac{1}{\beta}\right)\Gamma\left(1+\frac{2}{\beta}\right) + 2\Gamma\left(1+\frac{1}{\beta}\right)^3\right) \quad (2.5)$$

$\Gamma$is the Gamma function defined as : $\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt$ (2.6)

Distribution parameters for an assumed distribution e.g. Weibull, are established using e.g. 3 equations (for 1, 10 and 100 year) with 3 unknowns (α, βandγ). This is, in principle, always feasible but engineering judgement applies as defining return period values inappropriately can lead to an unphysical Weibull pdf.

For a Weibull distributed variable the return period value is given by:

$$x_c = \alpha(\ln(N))^{1/\beta} + \gamma \quad (2.7)$$

2.1.4 Wave data

The wave data is taken from [Sea loads on ships and offshore structuresTable2.2] written by O.M.Faltinsen, see appendix A. It is a representative data for the northern North Sea, and shows a joint frequency of significant wave height and spectral peak period. The wave velocity u_W can be given as:

$$u_W = \frac{\pi H}{T} \frac{\cosh(2\pi h/\lambda)}{\sinh(2\pi h/\lambda)} \cos(k_W x - \omega t) \quad (2.8)$$

Where

H=2m is the wave height,
T=10s is the wave period,
d=100m is the water depth,
$k_W = \frac{\omega^2}{g} = 0.04$ is the wave number,
$\lambda=2\pi/k_W$=157 is the wavelength,
h=2m is the distance of pipeline to the seabed,
$\omega = 2\pi/T$=0.628 is the wave frequency.

From the above data, we can find the maximum wave velocity in 100m water depth is 0.072m/s.

2.2 Pipeline Data

The pipeline input parameters are defined as below, see Table 1

Table 1 Pipeline input parameters

Input parameters	Symbol	Magnitude	Unit
Outer pipe diameter	D	0.343	[m]
Wall thickness	t_{steel}	0.0265	[m]
Steel density	ρ_{steel}	7850	[kg/m^3]
Structural radius	R	0.14025	[m]
Young's modulus-Steel	E	$2.1 \cdot 10^5$	[MPa]
Poisson's ratio-Steel	ν_{steel}	0.3	
Expansion coefficient-Steel	α_{steel}	$1.17 \cdot 10^5$	[1/C^0]
Radial drag coefficient	C_D	1.0	
Radial added mass coefficient	C_M	2.29	
Pipe free span length	L	50	[m]
Dry mass per unit	M_D	207	[Kg]
Submerged mass per unit	M_S	112	[Kg]
Corrosion coating thickness	t_{coat}	0.018	[m]
Corrosion coating density	ρ_{coat}	1300	[kg/m^3]

Establish the free span model in SIMLA, and the scenario describes as follow:

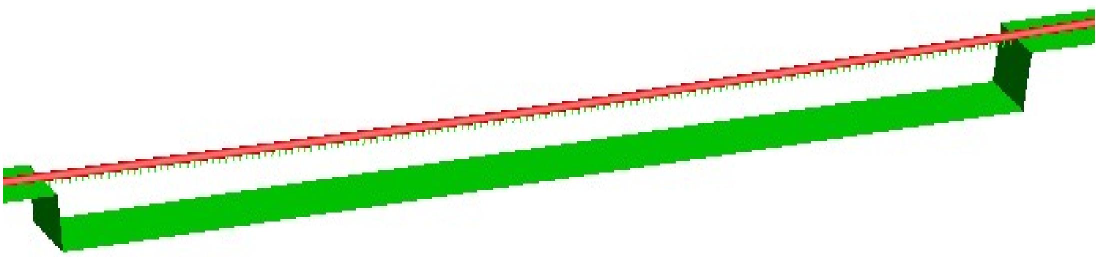


Figure 1 Free span model established in SIMLA

2.3 Damping data

Damping is an important aspect when determining the dynamic behavior of a structural system. Damping of a pipeline free-span arises from the following sources where some typical numbers for the damping ratios can be found from [DNV-RP-105 2006, P35, P23]:

- Structural damping: ζ_{str}=0.005
- Soil damping: ζ_{soil}(in-line)=0.009
  ζ_{soil}(cr-flow)=0.008
- Hydrodynamic damping: ζ_{h}=0.00

The structural material damping increases due to internal friction forces of the pipe material. Since the pipe is welded together, the steel in the pipeline does not contribute with much damping. However, the coating contributes with some damping – partly because of the friction between the grains in the concrete and partly because of the friction between the steel and the coating.

The vertical dynamic stiffness$K_V$is defined as $K_V = \Delta F_V/\Delta\delta_V$, where ΔF_V is the dynamic vertical force between pipe and soil per unit length of pipe, and $\Delta\delta_V$ is the associated vertical displacement of the pipe, measured relative to the static position of the pipe.The lateral dynamic stiffness K_L is defined as $K_L = \Delta F_L/\Delta\delta_L$, where ΔF_L is the dynamic horizontal force between pipe and soil per unit length of pipe, and $\Delta\delta_L$ is the associated horizontal displacement of the pipe.

The static vertical stiffness$K_{V,S}$ is a secant stiffness representative for penetration conditions such as during installation and erosion and during development of free spans. The static vertical stiffness $K_{V,S}$ is defined as$K_{V,S} = R_V/v$, where R_V is the static vertical soil reaction per unit length of pipe and v is the vertical penetration of the pipe required to mobilize this reaction.

The soil at the boundaries of the pipeline has a damping effect in the form of geometrical damping due to wave propagation through the soil and material damping due to friction between the grains. The main parameters for estimation soil damping are to consider the type of soil: Is if a sand type or a clay type and is it a soft or stiff soil. In this case, the soil is stiff clay, thus we use this option for soil properties in FATFREE. Then the following parameters will be given as: $K_V = 9.937E + 06$, $K_L = 6.868E + 06$, $K_{V,S} = 1.300E + 06$.

2.4 Safety zones and Safety factors

The primary functional state that is investigated in this project is the operational state of the pipeline. The safety factors that have been used in this project correspond to “normal” safety class and are taken from [DNV-RP-F105]. The free span is categorized as well defined, which means spans where important span characteristics like span length, gap and effective axial force are determined/measured, site specific soil conditions and a long-term description of the environmental conditions exist. Table 2 shows the safety factors that are used in this project.

Table 2 Safety factors corresponding to "standard" safety class

Description of factor	Symbol	Magnitude
Utilisation factor	η	0.5
Safety factor on damping	γ_k	1.15
Safety factor on inline natural frequency	$\gamma_{f,IL(inline)}$	1.10
Safety factor on cross-flow natural frequency	$\gamma_{f,CF(cr-flow)}$	1.10
Safety factor on stress range	γ_S	1.30
Safety factor on response model on-set value	$\gamma_{on,IL}$	1.10
Safety factor on response model on-set value	$\gamma_{on,CF}$	1.20
Correction factor on stress range	Ψ_R	1.00

3. Dynamic Differential Equation Of The Pipeline Span

3.1 Theoretical foundation

Based on Hamilton principle, we established the dynamic differential equation of free span pipeline under the interaction of internal flow and external environmental loads. Based on small deflection beam theory, constraint-equivalent method is adopted to deal with the constraint functions from the shoulder parts of the pipe. The lateral deformation and natural frequency of the pipeline are then evaluated.

We establish a typical free span pipeline model as shown below. Assuming the internal flow in the pipeline is ideal and incompressible, the energy equation according to the Hamilton principle can be expressed as:

$$\int_{t_2}^{t_1} \delta(E_k - E_p)\, dt + \int_{t_2}^{t_1} \delta W_{nv} dt = 0 \tag{3.1}$$

Where E_k is the kinetic energy, E_p is the potential energy, W_{nv} is the power of non-conservative forces on virtual displacement, and t_1 and t_2 denote the time period for observation.

Denoting the length of pipeline element as l, the kinetic energy of pipeline element can be expressed through the integration along the length as:

$$E_k = \frac{1}{2}\int_0^l \left[m_p\left(\dot{X}^2 + \dot{Y}^2 + \dot{Z}^2\right) + I\rho_p \dot{\theta}^2\right] ds + \frac{1}{2}\int_0^l m_1\left[(\dot{X} + c)^2 + (\dot{Y} + Y'c)^2 + (\dot{Z} + Z'c)^2\right] ds \tag{3.2}$$

Where

$m_p$is the pipeline quality of unit length;
$m_1$is the internal fluid quality of unit length;
X, Y and Z are the displacement of pipeline in the x, y, z directions, respectively;
I is the moment of inertia about pipeline of unit length;
$\theta$is the twist angle of pipeline;
$\rho_p$is the density of internal fluid to the wall of pipeline;
$\dot{X}$, $\dot{Y}$, $\dot{Z}$ and $\dot{\theta}$ denote the differential with respect to time t, respectively;
X', Y', Z' and θ' denote the differential with respect to the coordinates, respectively.

The potential energy can be expressed as:

$$E_p = \int_0^l \left[\frac{EA_p}{2}(X')^2 + \frac{GJ_p}{2}(\theta')^2 + \frac{EJ}{2}(Y'')^2 + \frac{EJ}{2}(Z'')^2\right] ds \qquad (3.3)$$

Where E is the elastic modulus, G is the shear modulus, A_p is the cross-sectional area of pipeline, J_p is the polar moment of inertia of cross-section, and J is the shaft moment of inertia of cross-section.

Ignoring the internal pressure of the pipeline, the power of non-conservative forces on virtual displacement can be expressed as:

$$\delta W_{nc} = -\int_0^l \mu(\dot{X}\delta X + \dot{Y}\delta Y + \dot{Z}\delta Z)\, ds \qquad (3.4)$$

Where μ is the damping coefficient.

Substituting Eqs. (3.2)-(3.4) into Eq. (4.1), and considering $\delta\dot{X} = \frac{\varphi}{\varphi t}(\delta X)$, $\delta X' = \frac{\varphi}{\varphi s}(\delta X)$, the dynamic differential equation of pipeline element can be written as:

$$\delta \int_{t_1}^{t_1} \int_0^l \frac{1}{2}\{(m_p + m_1)(\dot{X}^2 + \dot{Y}^2 + \dot{Z}^2) + I\rho_p\dot{\theta}^2 + m_1c^2 + m_1[2c(Y'\dot{Y} + Z'\dot{Z} + \dot{X}) + c^2(Y'^2 + Z'^2)] - (EA_pX'^2 + EJX''^2 + EJZ''^2 + EJ_p\theta'^2) - \mu(\dot{X}X + \dot{Y}Y + \dot{Z}Z)\}dsdt = 0 \qquad (3.5)$$

3.2 Numerical Procedure

Divide the pipeline into several nodes (661 in this case), and select the displacement interpolation matrix of node as **N**, the displacement of the pipeline can be expressed as:

$$\varphi = \mathbf{N}\varphi^{\mathrm{e}} \qquad (3.6)$$

Where $\varphi^{\mathrm{e}} = [\mathrm{X} \quad \mathrm{Y} \quad \mathrm{Z} \quad X' \quad Y' \quad Z']^{\mathrm{T}}$ is the node displacement vector.

Substituting Eq. (4.6) into Eq. (4.5), the element's dynamic equation becomes:

$$\boldsymbol{M^e}\ddot{\boldsymbol{\varphi}}^{\boldsymbol{e}} + \boldsymbol{C^e}\dot{\boldsymbol{\varphi}}^{\boldsymbol{e}} + \boldsymbol{K^e}\boldsymbol{\varphi^e} = \boldsymbol{f^e} \qquad (3.7)$$

Where,

$\boldsymbol{M^e}$ is the element mass matrix, including the pipeline mass and fluid mass;
$\boldsymbol{C^e}$ is the element damping matrix incited by fluid-solid coupling;
$\boldsymbol{K^e}$ is the element stiffness matrix, including the elastic stiffness matrix and stiffness matrix of fluid-solid coupling;
$\boldsymbol{f^e}$ is the element loading vector.

The element loading vector caused by horizontal load of the interaction of wave and current can be expressed as:

$$\boldsymbol{f}^e = \int_0^l f(s)\boldsymbol{\Phi} ds \tag{3.8}$$

Where f(s) is the horizontal load of the interaction of wave and current, and $\boldsymbol{\Phi}$ is the node's interpolation vector.

For simplification of computation, Rayleigh damping matrix is used to approximate the real damping matrix.

$$\boldsymbol{C}^e = \alpha \boldsymbol{M}^e + \beta \boldsymbol{K}^e \tag{3.9}$$

$$\alpha = \frac{2\xi\omega_1\omega_3}{\omega_1+\omega_3},\ \beta = \frac{2\xi}{\omega_1+\omega_3} \tag{3.10}$$

Where ω_1 *and* ω_3 are the first-order and third-order natural frequency of the span, and ξ is the damping ratio.

Assume that the wave and current all pass the pipeline in the y direction. The horizontal load of span under the interaction of wave and current can be written as:

$$f(s) = \frac{1}{2}\rho_f D(u_w + u_c)^2 C_L \cos(\omega_s t) + \frac{1}{2} C_D \rho_f D u_w |u_w| + (C_M - 1)\rho_f \frac{\pi D^2}{4}\frac{\delta u_w}{\delta t} \tag{3.11}$$

Where

C_L is the current lift coefficient,
C_D is the wave drag coefficient,
C_M is the wave inertia coefficient,
ω_s is the vortex-induced frequency ($\omega_s = 2\pi Sr u_c/D$, Sr is the Strouhal number),
ρ_f is the density of seawater,
D is the external diameter of pipeline,
u_c is the current velocity,
u_w and $\frac{\delta u_w}{\delta t}$ are the wave velocity and acceleration, respectively. They can be given as:

$$\mathrm{u_W} = \frac{\pi H}{T}\frac{\cosh(2\pi h/\lambda)}{\sinh(2\pi h/\lambda)}\cos(\mathrm{k_W x} - \omega t) \tag{3.12}$$

$$\frac{\delta u_w}{\delta t} = \frac{2\pi^2 H}{T^2}\frac{\sinh(2\pi h/\lambda)}{\sinh(2\pi h/\lambda)} sin(\mathrm{k_W x} - \omega t) \tag{3.13}$$

The dynamic equation of pipeline span can be then numerically solved under proper boundary conditions.

The curvature can be found as follow:

$$\omega_{,xx} = \mathbf{N}_{\omega_{,xx}}^{\mathrm{T}} \cdot \mathbf{\Psi} \tag{3.14}$$

$$\upsilon_{,xx} = \mathbf{N}_{\upsilon_{,xx}}^{\mathrm{T}} \cdot \mathbf{\Psi} \tag{3.15}$$

$$\mathbf{N}_{\omega_{,xx}}^{\mathrm{T}} = [0\ 0\ \frac{12\xi-6}{\ell^2}\ 0 - \frac{6\xi-4}{\ell}\ 0\ 0\ 0\ \frac{-12\xi+6}{\ell^2}\ 0 - \frac{6\xi-2}{\ell}\ 0] \tag{3.16}$$

$$\mathbf{N}_{\upsilon_{,xx}}^{\mathrm{T}} = [0\ \frac{12\xi-6}{\ell^2}\ 0\ 0\ 0\ \frac{6\xi-4}{\ell}\ 0\ \frac{-12\xi+6}{\ell^2}\ 0\ 0\ 0\ \frac{6\xi-2}{\ell}] \tag{3.17}$$

$$\mathbf{\Psi} = [u_1\ v_1\ \omega_1\ \theta_{x1}\ \theta_{y1}\ \theta_{z1}\ u_2\ v_2\ \omega_2\ \theta_{x2}\ \theta_{y2}\ \theta_{z2}]^T \tag{3.18}$$

$$\boldsymbol{\xi} = [\mathbf{0}\ \mathbf{1}] \tag{3.19}$$

$$\Delta\sigma_{ij} = 2E \cdot A_i[-R_j sin\theta \mathbf{N}_{\omega_{,xx}}^{\mathrm{T}} \mathbf{\Psi} - R_j cos\theta \mathbf{N}_{\upsilon_{,xx}}^{\mathrm{T}} \mathbf{\Psi}] \tag{3.20}$$

$$A = \kappa \cdot R \cdot E \tag{3.21}$$

We can get the eigen shape deflection from SIMLA, and based on above equations, we can find the unit diameter stress amplitude in both in-line and cross-flow directions.

4. STATIC ANALYSIS

4.1 FE Modeling

The pipeline is modeled using 2-node pipe elements. We established 660 elements, which mean 661 nodes to model the free span scenario. The element size used in the model can be 1OD as a start based on DNV-RP-F105. The FE model can be single pipe with or without concrete coating, depending on project requirements. In this project it is a single pipe without concrete coating, only with corrosion protects coating.

FE modeling of the span analysis is divided into two phases: static and dynamic (modal). In the static phase the sag deflection under the operating conditions, after the pipeline is laid on the seabed, is determined. In this phase, soil-pipe interaction is modeled using node-to-surface contact. In the dynamic phase, the natural frequencies and corresponding mode shapes are resolved and springs are used to model the interaction between soil and pipe. The dynamic phase is a linearised procedure that indicates linear effects, and any nonlinearity such as plasticity and friction are ignored in the dynamic phase even if these effects have been included in the static contact model.

4.2 Static analysis

The static analysis includes only the functional loads that may give rise to insignificant dynamic amplification of the response. After establishing the free span model in SIMLA, start to perform static analysis including all phases of pipeline behavior as follow:

- Empty pipeline;
- Water-filling pipeline;
- Hydrostatic testing;
- Dewatering and operation.

Key parameters and relationships to be deducted are mainly about relationship between lateral deflection and axial force of span and associated stresses and sectional forces and moments in pipe wall. Static shape and VIV of free span pipeline just show in Fig 4.

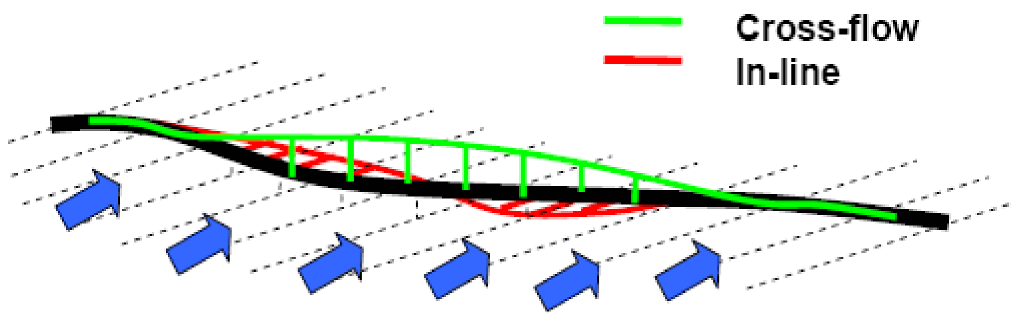


Figure 2 Static shape and VIV of free span pipeline

During the whole procedure, we consider the following loads which act on the free span pipeline: current and wave loads, external pressure and gravity, internal pressure, temperature load used to scale axial force in principle, concentrated nodal loads from DNV RP-F111 recommendation. We use TIMECO command in SIMLA to define the analysis as a function of time, the process can be described as follow:

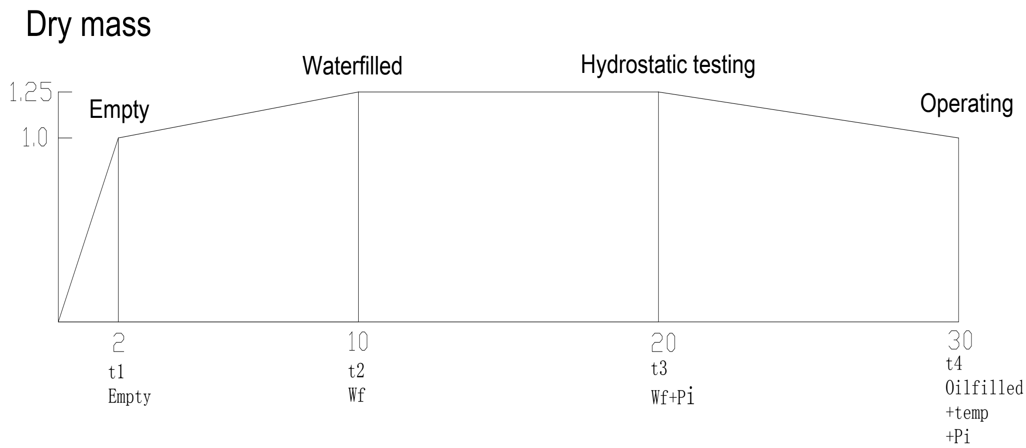


Figure 3 Loading procedure varying with time

4.2.1 Empty pipeline

Only dry mass load, current loads and concentrated loads work on the pipeline.

1. Axial force

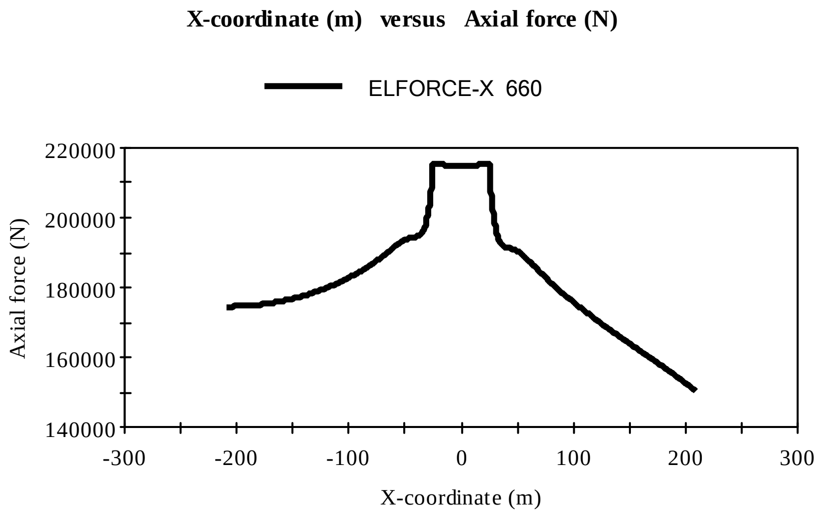


Figure 4 Axial force for empty pipeline

2. Moment

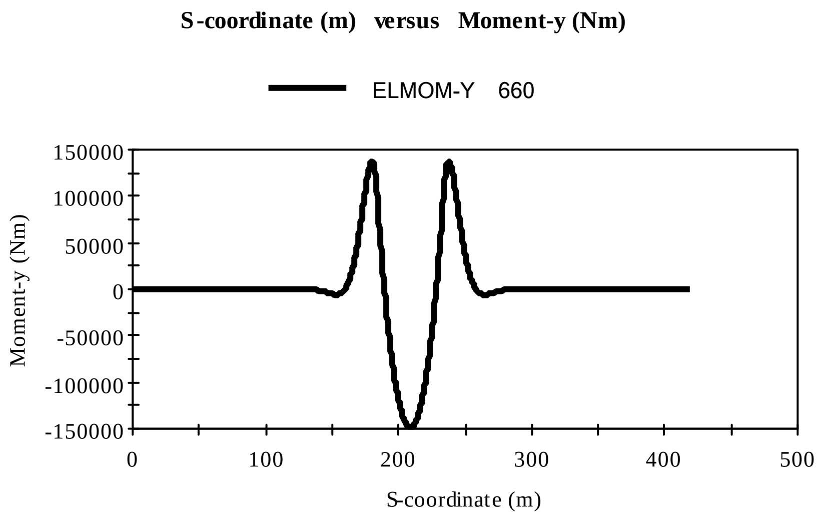


Figure 5 Moment for empty pipeline

3. Displacement

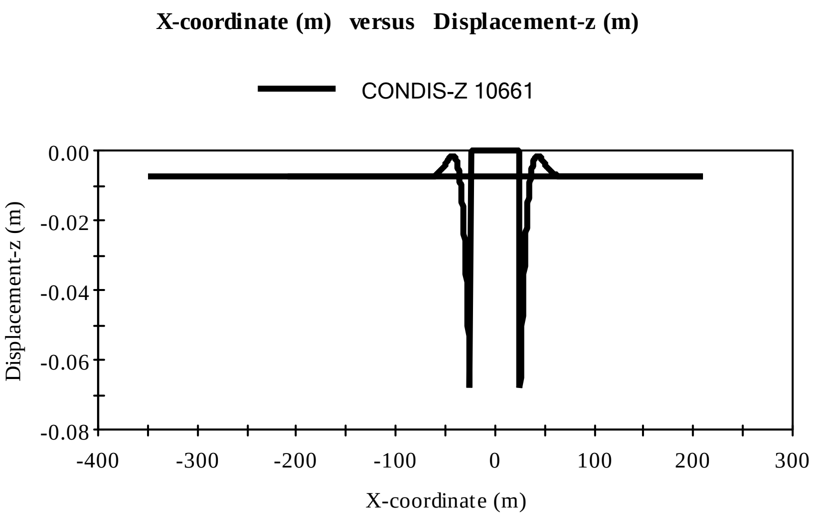


Figure 6 Displacement for empty pipeline

4. Rotation

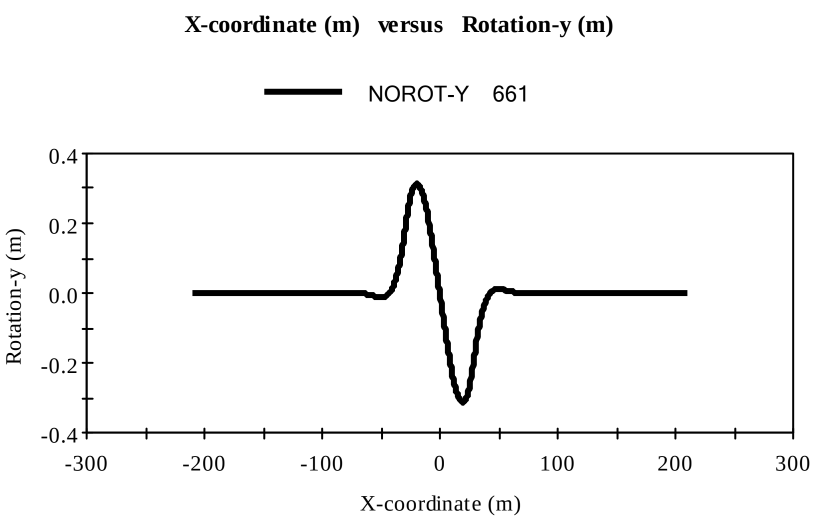


Figure 7 Rotation for empty pipeline

4.2.2 Water-filling pipeline

When the water filling procedure, there are still those three loads act on the free span. Obviously the gravity loads are highest for the water-filling pipeline, but the added submerged weight may cause additional seabed contact, thus eliminating or reducing some free spans.

1. Axial force

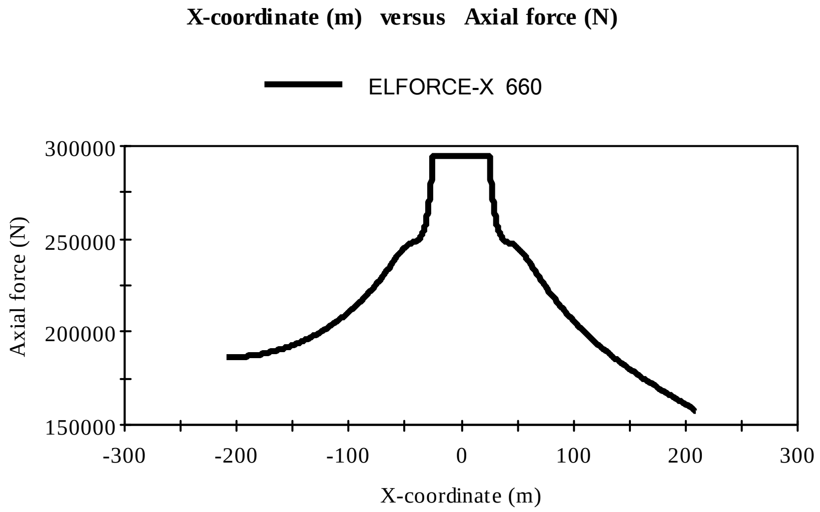


Figure 8 Axial force for water-filling pipeline

2. Moment

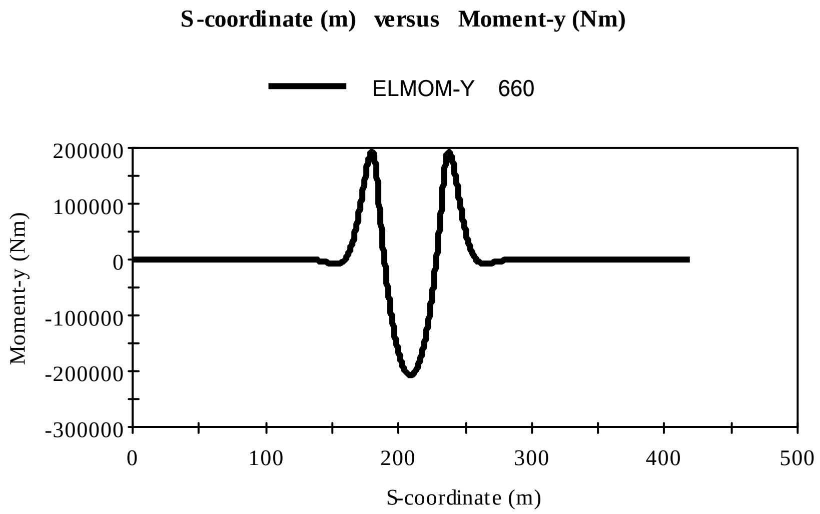


Figure 9 Moment for water-filling pipeline

3. Displacement

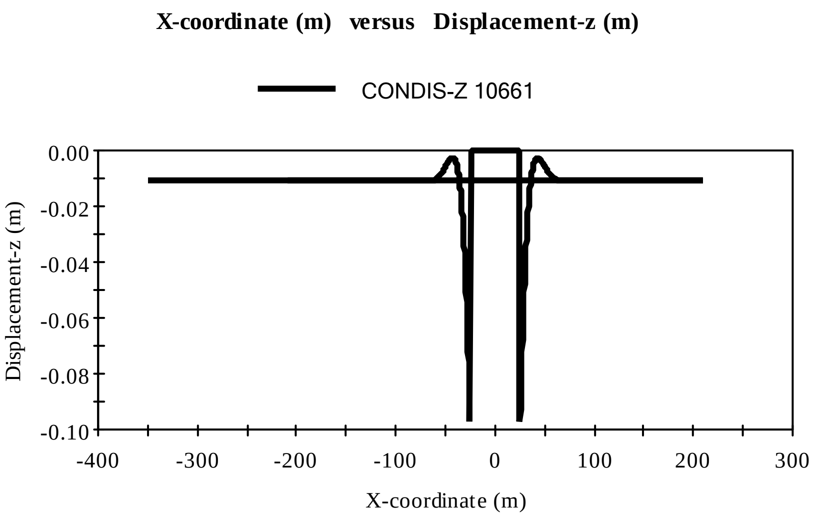


Figure 10 Displacement for water-filling pipeline

4. Rotation

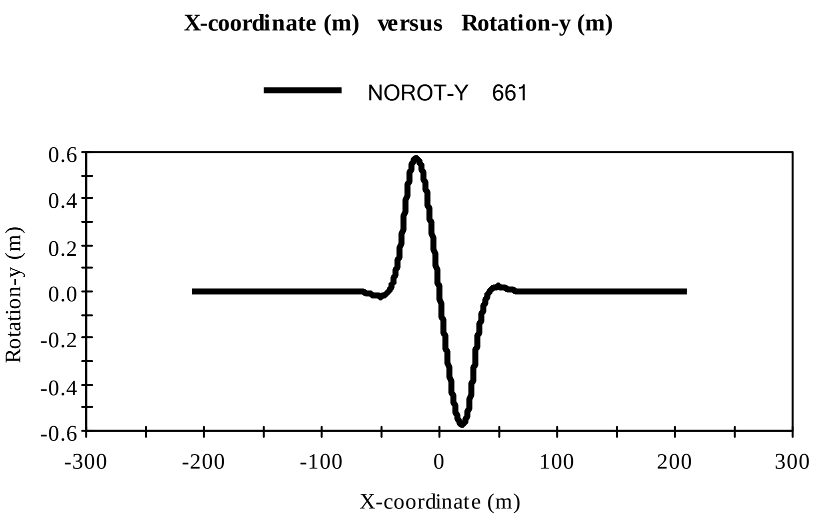


Figure 11 Rotation for water-filling pipeline

4.2.3 Hydrostatic testing

When all construction activities have been carried out, the final integrity of the installed pipeline is documented by hydrostatic testing. This requires that the pipeline be water-filled, the seawater is normally used for this purpose. Seawater is pumped into pipelines through a simple water winning arrangement that includes filtering and sometimes treatment of the seawater with providing the plough with two sets of shears. In the testing period, besides those three loads acting on the empty pipeline, the external pressure load also work on it.

1. Axial force

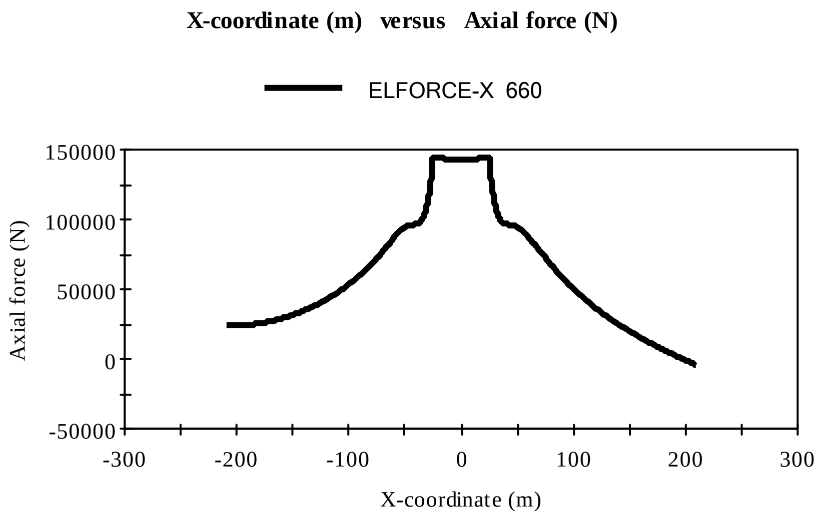


Figure 12 Axial force for Hydrostatic testing

2. Moment

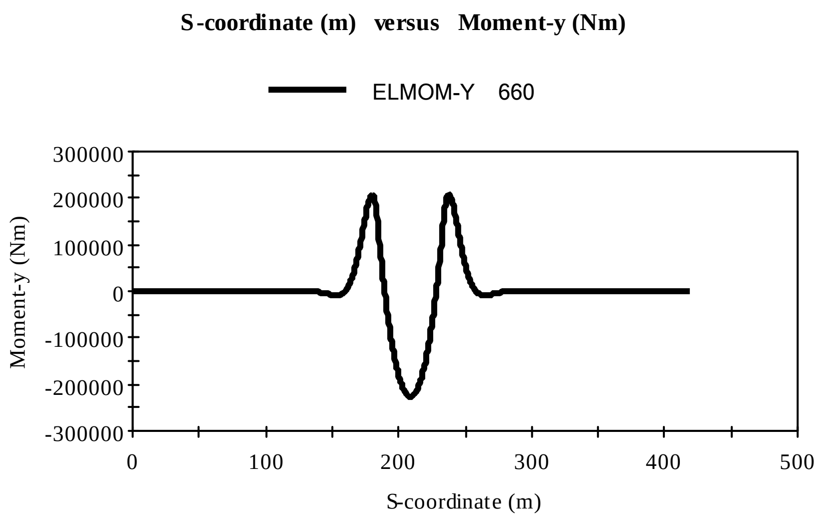


Figure 13 Moment for Hydrostatic testing

3. Displacement

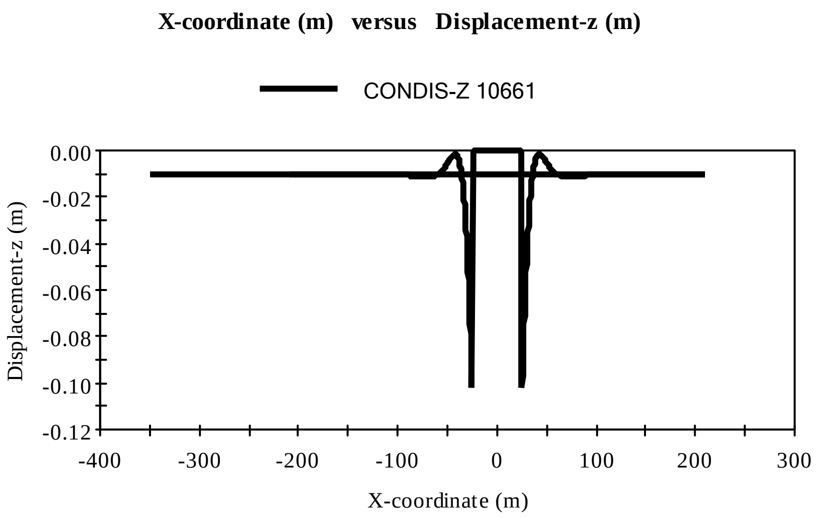


Figure 14 Displacement for Hydrostatic testing

4. Rotation

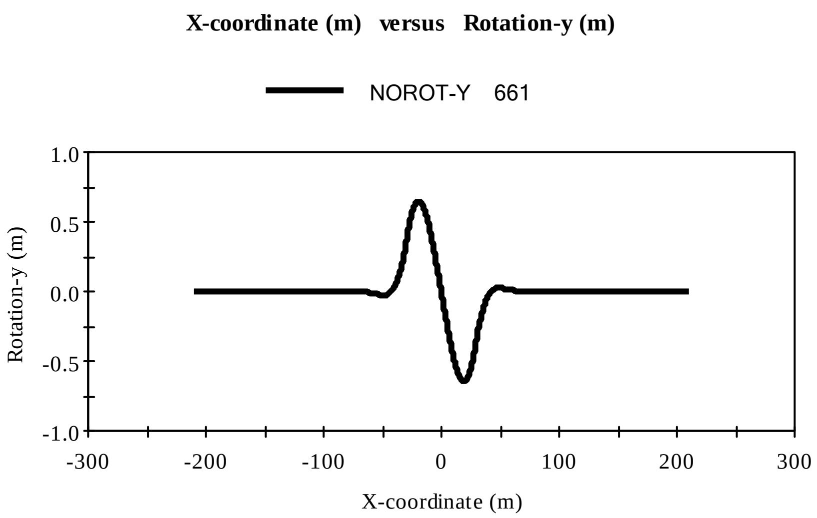


Figure 15 Rotation for Hydrostatic testing

4.2.4 Operating

When perform operating procedure, the pipeline is filled with oil instead of water, the temperature load and external pressure act on the pipeline.

1. Axial force

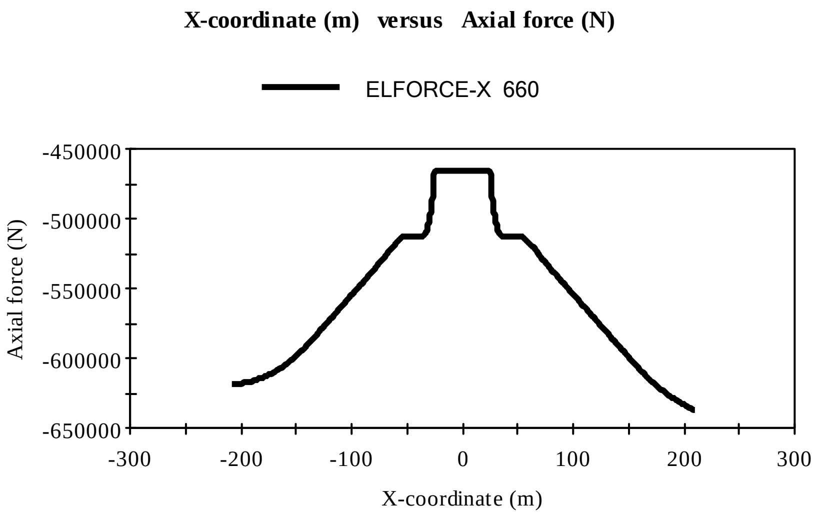


Figure 16 Axial force for Operating

2. Moment

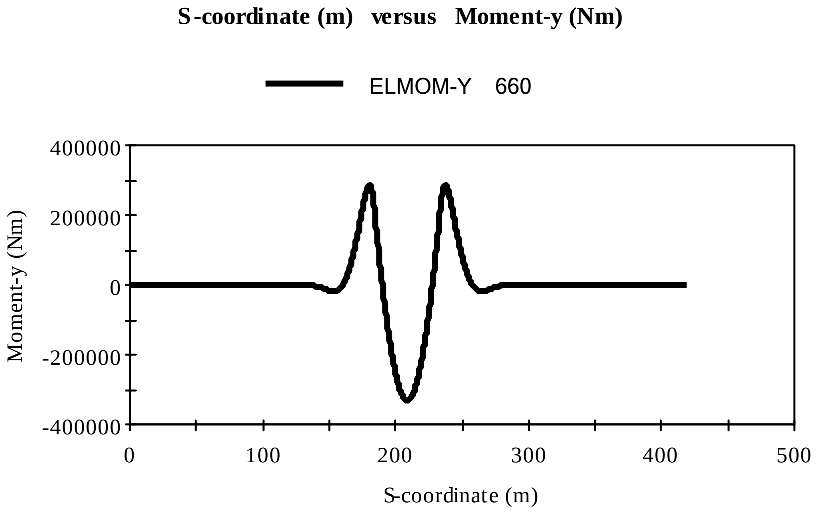


Figure 17 Moment for Operating

3. Displacement

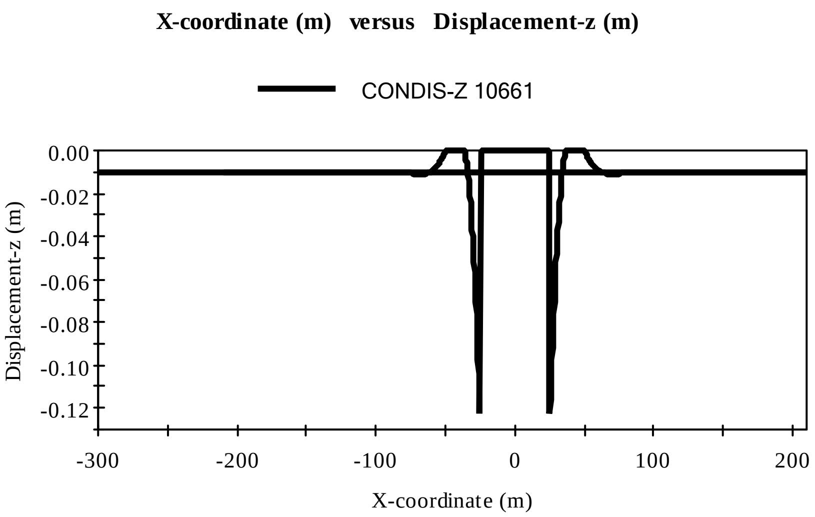


Figure 18 Displacement for Operating

4. Rotation

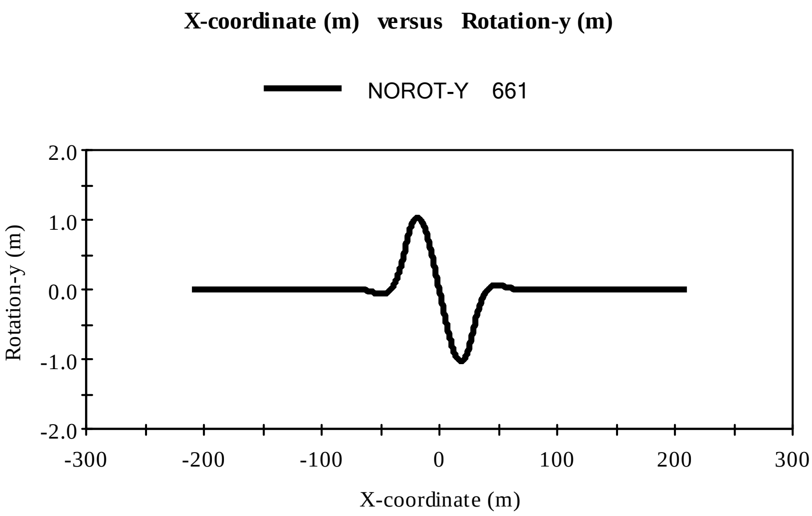


Figure 19 Rotation for Operating

5. EIGEN MODE ANALYSIS

The free span pipeline is a dynamic structure, which have well defined natural frequencies and modes. Such a structure is susceptible to amplified response when exposed to cyclic loads having frequency close to the natural frequency. Therefore, it requires an eigenvalue analysis of the free span for determination of natural frequencies and modal shapes. As the eigenvalue analysis is a linear analysis a consistent linearization of the problem must be made.

The analysis should account for the static equilibrium configuration, and the linearised stiffness of the soil shall be taken into account the correct properties of the soil. Special attention must be paid to the definition of the axial stiffness of the soil, as the results of the eigenvalue analysis in the vertical plane are very much affected by this axial stiffness. Where only the suspended span is modeled, the boundary conditions imposed at the ends of the pipeline section should represent the correct pipe-soil interaction and the continuity of the entire pipe length. The influence of the added mass as a function of seabed clearance has to be considered when calculating natural frequencies.

5.1 First Eigen Mode

Cross-flow

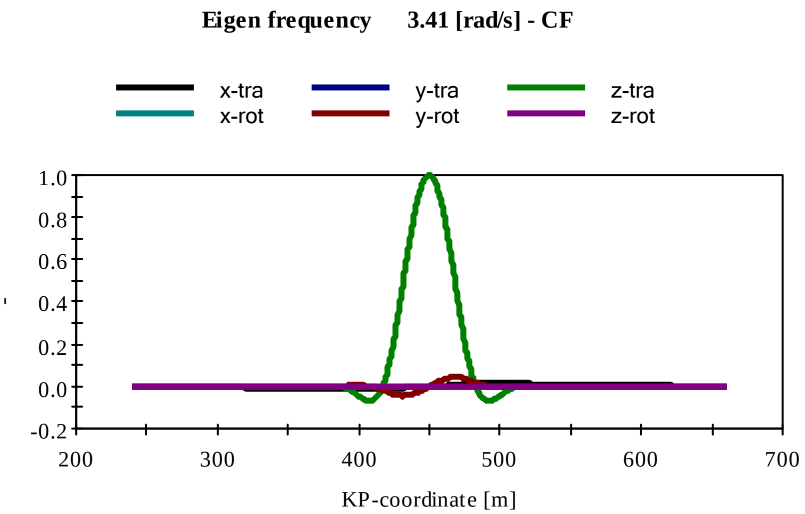


Figure 20First eigenmode at cross-flow

In-line

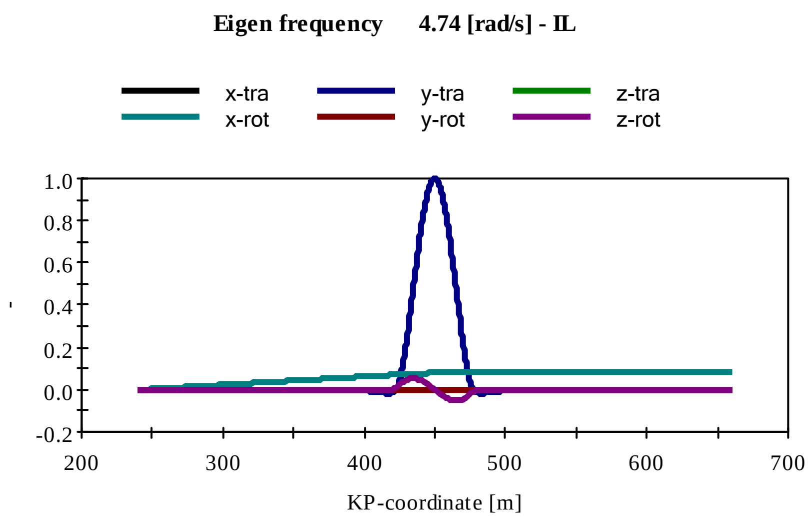


Figure 21 First eigenmode at in-line

5.2 Second Eigen Mode

Cross-flow

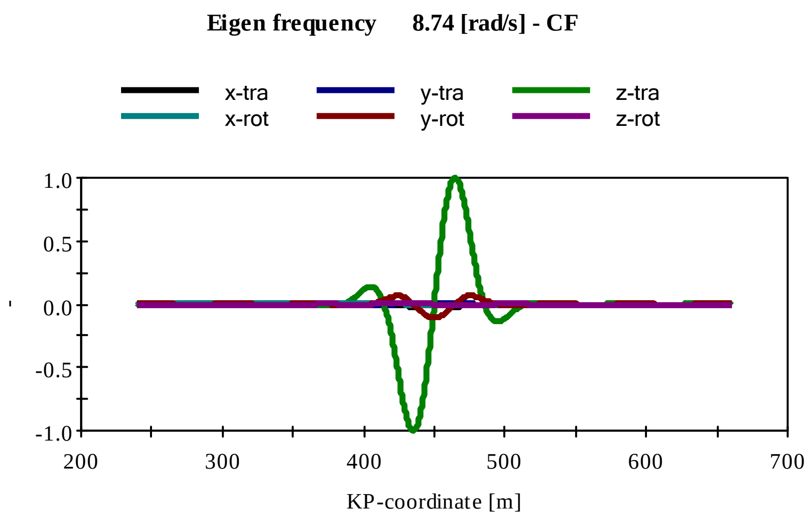


Figure 22 Second eigenmode at cross-flow

In-line

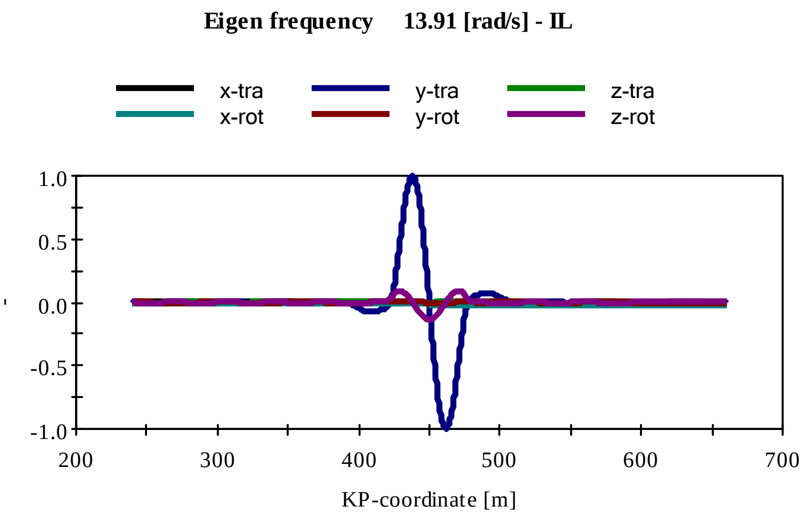


Figure 23 Second eigenmode at in-line

5.3 Third Eigen Mode

Cross-flow

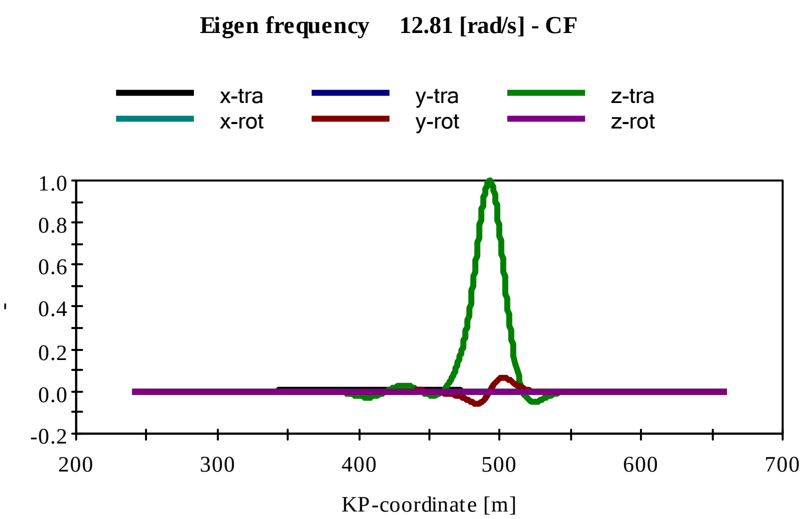


Figure 24 Third eigenmode at cross-flow

In-line

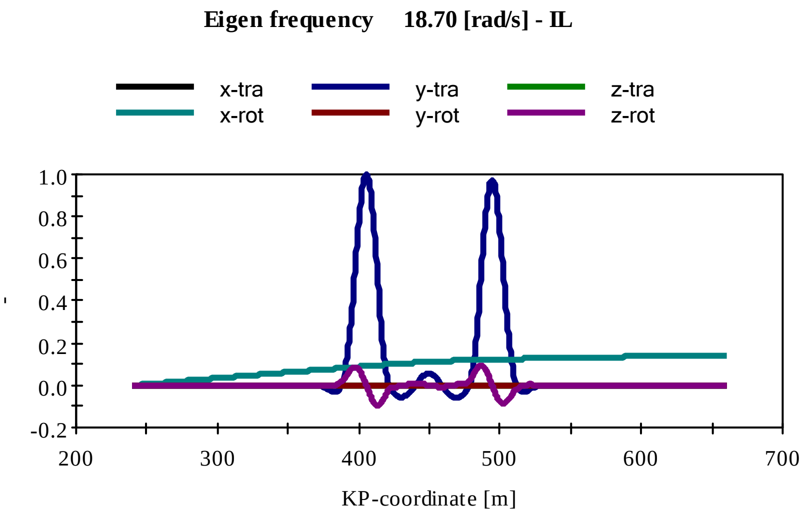


Figure 25 Third eigenmode at in-line

5.4 Results analysis

Table 3 show the value of natural frequencies of the first three inline and cross flow modes. These data are calculated based on the pipeline configuration for operation load condition. For these natural frequencies, no significant VIV damage will occur in the present bottom current velocity range.

Table 3 Eigen frequecies for 3 Eigenmodes in two directions

Hz	1stEigenmode	2nd Eigenmode	3rdEigenmode
Cross-flow	0.54	1.39	2.04
In-line	0.75	2.21	2.98

A pipe will start to oscillate in-line with the flow when the vortex shedding frequency is about one-third of the natural frequency of a pipe span. Lock-in may occur when the vortex shedding frequency is half of the natural frequency. As the flow velocity increases further, the cross-flow oscillation begins to occur and the vortex shedding frequency may approach the natural frequency of the pipe span. Amplified responses due to resonance between the vortex shedding frequency and natural frequency of the free span may cause fatigue damage.

6. VORTEX INDUCED VIBRATIONS OF PIPELINE

Typical steps required for VIV analysis in a deterministic method are outlined below:

a) Determine natural frequency. For the real case, a finite element method needs to be applied.
b) Estimate a frequency distribution of velocity.
c) For each level of current velocity, determine the response amplitude.

Amplitude response models are empirical models providing the maximum steady state VIV amplitude response as a function of the basic hydrodynamic and structural parameters. These response models are empirical relations between the reduced velocities defined in terms of the still-water natural frequency and the non-dimensional response amplitude.

The response model is based on a Vortex Induce Vibration (VIV) amplitude response where is the VIV is caused by vortex shedding across the pipeline. There are two types of VIV to consider: in-line and cross-flow oscillation, which occur with lateral and vertical vibration, respectively. Both in-line and cross-flow VIVs can be current induced or and wave-induced. The “combined” velocity is obtained from both current and wave velocities before it goes into the fatigue calculation, as shown in Section 4.1.5 in DNV RP-F105.

In the response models, in-line and cross-flow vibrations are considered separately. Damage contributions from both first and second in-line instability regions in current dominated conditions are implicit in the in-line model. Cross-flow induced additional in-line VIV resulting in possible increased fatigue damage is considered in an approximate way. Cross-flow induced in-line VIV is relevant for all reduced velocity ranges where cross-flow VIV occurs.

6.1 Introduction to vortex induced vibrations

6.1.1 Vortex shedding

Vortex shedding is a result of the basic instability which exists between the two free shear layers released from the separation points at each side of the cylinder into the down stream flow from the separation points. These free shear layers roll-up and feed vorticity and circulation into large discrete vortices which form alternately on opposite sides of the generating cylinder. At a certain stage in the growth cycle of the individual vortex, it becomes sufficiently strong to draw the other free shear layer with its opposite signed vorticity across the wake. This action cuts off further supply of vorticity to the vortex which ceases to grow in strength and is subsequently shed into the downstream flow. The process then repeats itself on the opposite side of the wake resulting in regular alternate vortex shedding, see figure 26, JP Kenny (1993).

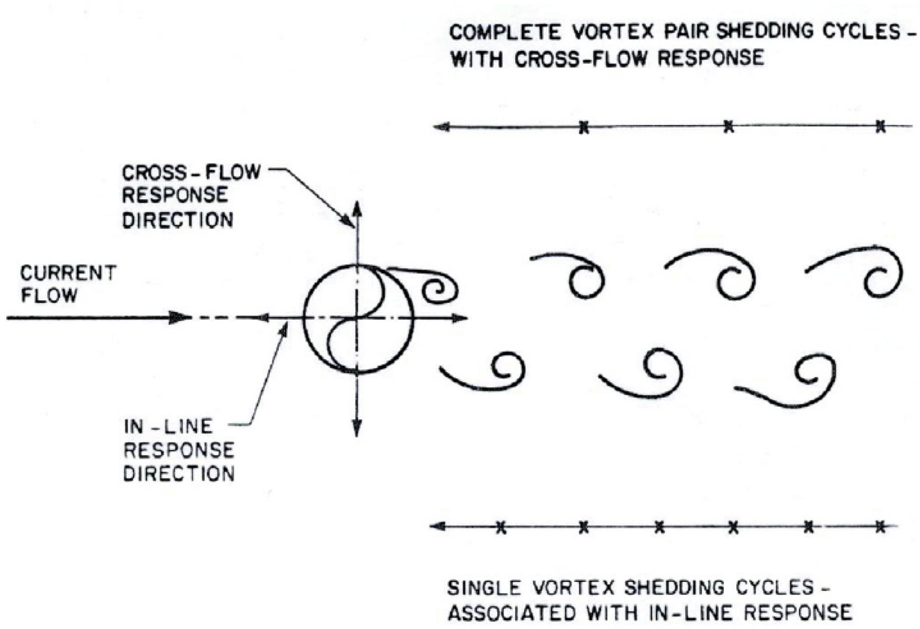


Figure 26 Staggered alternate vortex shedding - in-line and cross-flow response, JP Kenny (1993)

6.1.2 Flow field

The relation between the pressure gradient and acceleration in an incompressible inviscid fluid is demonstrated in Bernoulli equation

$$\rho\frac{\partial\phi}{\partial t} + p + \frac{1}{2}\rho U^2 + \rho g z = \text{constant} \qquad (6.1)$$

Where U is the fluid velocity, p is the pressure, ρ is the fluid density, g is acceleration of gravity and z is the height. For a stationary flow the above equation reduces to a more simple equation since the first term is equal to zero, $\frac{\partial\phi}{\partial t} = 0$ and the last term, $\rho g z$ is the static pressure and also in comparison with the other terms is neglected, then

$$p + \frac{1}{2}\rho U^2 = \text{constant} \qquad (6.2)$$

The tangential velocity of particle on a stationary cylinder in uniform flow can be given by using potential theory as follow:

$$u_t = -2U\sin\theta \qquad (6.3)$$

Here U is the free stream velocity and θ is the angle on the cylinder (see Figure 3). Therefore, by using Equation 3.1, the pressure distribution on the cylinder surface is given by

$$p = \frac{1}{2}\rho U^2 - 2\rho U^2 \sin^2\theta + p_0 \qquad (6.4)$$

Using this equation shows the pressure distribution is symmetrical. Furthermore, the integrating of pressure around the cylinder surface will result to a net force equal to zero and confirm the so called d'Alembert's paradox which states that a body in an inviscid fluid, i.e.$\nabla^2\phi = 0$ has zero drag.

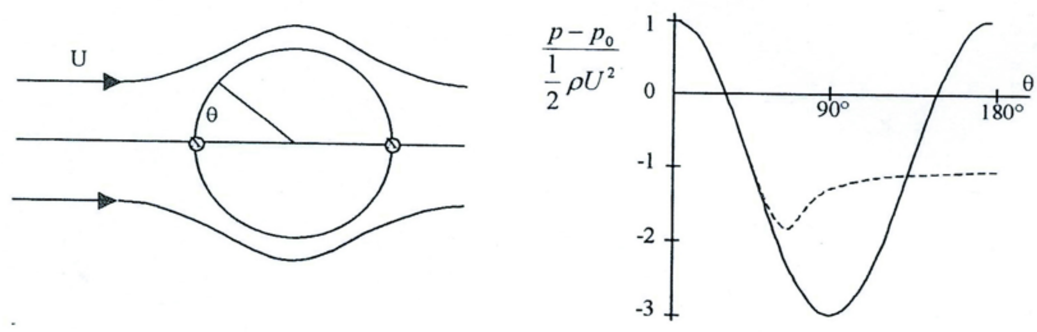


Figure 27 Flow and pressure distribution around a circular cylinder. (Pettersen 1999)

Consideration of the flow as a potential flow or irrotational flow neglecting the viscous shear is not the actual situation that normally is found in reality. Flow retardation due to viscous action close to the surface of the cylinder leads to the development of a boundary layer on it.

6.2 Dimensionless parameters

Many researchers have been working on the area of flow around circular cylinder and vortex induced vibration over many years. They have used different parameters and put different meaning into them. However, it is important to define these parameters precisely. We try to summarize and define these parameters in this section. Those parameters divided into three categories. The first group parameters in the categories are flow parameters and the other two are related to the interaction between the fluid and the structures named as structure and interaction parameters respectively.

6.2.1 Flow parameters

The velocity properties of the flow are described in this subsection. These parameters are related to the fluid properties.

Reynolds number, R_e

The *Reynolds number* classifies dynamically similar flows, i.e. flows which have geometrical similar streamlines around geometrical similar bodies, and defined as the ratio between the inertia forces and friction forces acting on a body.

$$R_e = \frac{U \cdot D}{\nu} \tag{6.5}$$

Where *U* is the free stream velocity, *D* is a characteristic dimension of the body around which the fluid flows (in the case of a cylinder is its diameter) and$\nu$is thekinematic viscosity coefficient of the fluid. The maximum flow velocity, *U* is used for the oscillatory flow.

Keulegan-Carpenter number, *KC*

The *Keulegan-Carpenter* number describes the harmonic oscillatory flows, as e.g.in waves. If the flow velocity, *U* is written as$U = U_M \sin(\omega t)$, the KC-number isdefined as:

$$KC = \frac{U_M T}{D} = \frac{2\pi A}{D} \tag{6.6}$$

Where U_M is the maximum flow velocity during one period, T is the period of oscillations and A is the oscillation amplitude of the oscillating cylinder. For the constant flow KC corresponds to a very high number.

The frequency parameter, β

The frequency parameter is defined as the ratio of Reynolds number over Keulegan-Carpenter number so that

$$\beta = \frac{R_e}{KC} = \frac{D^2}{\nu T} \tag{6.7}$$

Where T is the period of oscillation. β parameter represents the ratio of diffusionrate through a distance $\delta$to the diffusion rate through a distance D. δ is defined as the boundary layer thickness.

Turbulence intensity, I

Turbulence intensity parameter is dimensionless and used to describe the fluctuations in the mean incoming flow as follow:

$$\frac{u_{rms}}{U_{mean}} \tag{6.8}$$

Where u_{rms} is the root mean square (rms) of the velocity fluctuations, i.e. rmsof $u(t) = U(t) - U_{mean}$.

Shear fraction of flow profile

Current profiles can be non-uniform. The amount of shear in the current profile describes the shear fraction as:

$$\frac{\Delta U}{U_{max}} \tag{6.9}$$

Where ΔU is the variation of velocity over the length of the current profile and can be written as $U_{max} - U_{min}$ where U_{max} is the maximum flow velocity which can be occurred in the current profile.

6.2.2 Structural parameters

These parameters represent the properties of the body, geometry, density and damping. These parameters are related to the structure properties.

Aspect ratio

The aspect ratio is about the geometric shape of the structure which for a cylinder defined as the length of the cylinder over the diameter:

$$\frac{L}{D} = \frac{length}{width} \tag{6.10}$$

Where L is the length and D is the cylinder diameter.

Roughness ratio

The roughness ratio describes the surface of the body and defined as:

$$\frac{k}{D} \tag{6.11}$$

Where k is a characteristic dimension of the roughness on the surface of the body.

One of the parameter affects the friction of a body is roughness. The changes in the friction cause the change in the boundary layer and make it more turbulent and affect on the vortex shedding process since this process is highly depending upon the separation process on the body surface.

Mass ratio

The mass ratio for a structure is defined as the ratio of the cylinder mass per unit length, m, overρD^2as:

$$\frac{m}{\rho D^2} \tag{6.12}$$

In literature, the mass is applied with or without including added mass but in the present work the added mass is not included in the mass ratio.

Specific gravity

The specific gravity is used to describe the ratio of the structural mass per unit length to the displaced fluid mass per unit length as:

$$\frac{m}{{}^4\!/_{\pi}\rho D^2} \tag{6.13}$$

Like the mass ratio, also in this case the added mass is not included in the massper unit length.

Damping ratio

The damping ratio for a given mode is the ratio of the linear damping coefficient to its critical value as follow:

$$\zeta_n = \frac{c_n}{2m_n\omega_n} \tag{6.14}$$

Where c_n is the n'th damping coefficient, ω_n is the corresponding natural frequencyand $m_n$is the corresponding mass to $\omega_n$and the actual restoring force k_n. Thisratio is usually referred to the *structural damping*.

Damping ratio or damping factor may also be defined in terms of the energy dissipation by a vibrating structure:

$$\zeta = \frac{energy\ dissipation\ per\ cycle}{4\pi * total\ energy\ of\ structure} \tag{6.15}$$

Where $2\pi\zeta$ is the natural logarithm of the ratio of the amplitudes of any two successive cycles in free decay.

6.2.3 Interaction parameters

The interaction parameters are related to the interaction between the structure and the fluid.

Non-dimensional amplitude, A_y/D

The in-line response is important for studying vortex induced vibration. This response is non-dimensional and defined as:

$$\frac{A}{D} \tag{6.16}$$

It is important to note that this parameter can also be defined for cross-flow vibration in the same way.

Reduced velocity, U_r

Path length per cycle for steady vibrations can be defined by the distance the undisturbed flow is traveling during one cycle, *U/f*. The reduced velocity is the ratio of the path length per cycle to the model width as follow:

$$U_r = \frac{path\ length\ per\ cycle}{model\ width} = \frac{U}{f_n D} \tag{6.17}$$

Where the f_0 is the natural frequency in still water. Moe and Wu (1990) proposed reducedvelocity with the natural frequency in air which is called *nominal reduced velocity* and also with true vibration frequency which is called *true reduced velocity*.

True reduced velocity, U_{true}

The true reduced velocity is defined as

$$U_{true} = \frac{U}{f_{osc} D} \tag{6.18}$$

Added mass is known to vary with varying flow velocity. Consequently, the observed frequency of the cylinder changes and can be different from the natural frequency of the system in still water. In fact, the observed frequency, f_{osc} is a compromisebetween the natural frequency of the cylinder in still water, f_0 and the vortexshedding frequency for a fixed cylinder, f_{v_0}.

Non-dimensional vibration frequency, $\hat{f}$

The Non-dimensional vibration frequency is used to define the condition for a cylinder with forced motions. This parameter is the inverse of the true reduced velocity parameter.

$$\hat{f} = \frac{f_{osc} D}{U} \tag{6.19}$$

The Strouhal number, St

The *Strouhal number* is a non-dimensional parameter and is defined as

$$St = \frac{f_{v0} D}{U} \tag{6.20}$$

There is an almost constant relation between the vortex shedding frequency for a fixed cylinder, f_{v_0}, and the ambient velocity divided by the cylinder diameter, *U/D*. The proportionality constant of this relation is called the Strouhal number.

6.2.4 Hydrodynamic parameters

Reduced velocity, V_R

The reduced velocity is defined as:

$$V_R = \frac{U_c + U_w}{f_n D} \tag{6.21}$$

Where

f_n Natural frequency for a given vibration mode;
U_c Mean current velocity normal to the pipe;
U_w Significant wave-induced flow velocity;
D Outer pipe diameter.

The current flow velocity ratio, α

The current flow velocity ratio is defined by:

$$\alpha = \frac{U_c}{U_c + U_w} \tag{6.22}$$

The stability parameter, K_s

The stability parameter is representing the damping for a given modal shape is given by:

$$K_s = \frac{4\pi m_e \zeta_T}{\rho D^2} \tag{6.23}$$

Where

ρ Water density

m_e Effective mass

ζ_T Total modal damping ratio, which comprises structural damping ζ_{str}, soil damping ζ_{soil}, hydrodynamic damping ζ_h.

6.3 Structure response parameters

6.3.1 Basic theory

The differential equation for a freely vibrating beam may be written as:

$$EI\frac{\partial^4 v}{\partial x^4} + S_{eff}\frac{\partial^2 v}{\partial x^2} + m_e\frac{\partial^2 v}{\partial t^2} = 0 \tag{6.24}$$

Where v is the lateral deflection, x the axial co-ordinate, t is time, S_{eff} is the effective axial force, EI is the bending stiffness and m_e is the effective mass of the beam per length (including mass of content and hydrodynamical added mass).

The classical solution of this equation involves separation of variables assuming the solution having a form as:

$$v(x,t) = \varphi(x) \cdot Y(t) \tag{6.25}$$

Where $\varphi(x)$ describes the mode shape and Y(t) describes the time variation.

Now, two ordinary differential equations are obtained:

$$\ddot{Y} + \omega^2 Y = 0 \tag{6.26}$$

$$\varphi'''' + \frac{S_{eff}}{EI}\varphi'' - \frac{m_e}{EI}\omega^2\varphi = 0 \tag{6.27}$$

Where ω is the undamped, angular frequency of the beam.

The solution in terms of fundamental natural frequency for a vibrating beam with axial force becomes:

$$f_0 = C_1\sqrt{\frac{EI}{m_e L^4}\cdot(1 + C_2\cdot\frac{S_{eff}}{P_E})} \quad (6.28)$$

Where L is the span length, $P_E = \frac{\pi^2 EI}{L^2} = 1.89E + 5$ is the Euler buckling force, and C_1, C_2 are the boundary condition coefficients. It will be varies for different span lengths and soil conditions for real free spanning pipelines. Note that the $\frac{S_{eff}}{P_E}$ term becomes negative when the effective axial force is in compression since P_E is defined as positive.

6.3.2 RP-F105 expressions

The following approximate response quantities based on the effective span length concept are given in DNV-RP-F105.

The expression for the lowest natural frequency reads:

$$f_0 \approx C_1\cdot\sqrt{1 + CSF}\sqrt{\frac{EI}{m_e L_{eff}^4}\cdot(1 + C_2\cdot\frac{S_{eff}}{P_E} + C_3(\frac{\delta}{D})^2)} \quad (6.29)$$

Where δ is the static/steady-state deflection at span midpoint. Hence, the geometrical stiffening effect due to sagging is accounted for by the C_3 term. For the horizontal direction C_3 normally equals zero. Note that P_E is now based on effective span length. In this case, there is no concrete coating, thus CSF=0.

The modal (dynamic) stress for one diameter maximum deflection amplitude is derived from the mode shape and is given by:

$$A_{IL/CF} = C_4(1 + CSF)\frac{D\cdot(D_s - t)\cdot E}{L_{eff}^2} \quad (6.30)$$

Where D_s is the steel pipe diameter and D is the outer pipe diameter (including any coating). T is the steel pipe wall thickness and C_4 is a boundary condition coefficient.

The static bending moment may be estimated by:

$$M_{static} = C_5\frac{q\cdot L_{eff}^2}{(1+C_2\cdot\frac{S_{eff}}{P_E})} \quad (6.31)$$

Where q represents the loading, i.e. the submerged weight of the pipe in the vertical (cross-flow) direction or the drag loading in the horizontal (in-line) direction. This expression is based on the static moment of a span with an approximate correction due to the effective axial force.

The boundary condition coefficients C_1 to C_6 are given in Table 6-1 from DNV RP F105 for different support conditions. The present formulation for the effective span length implies that the values for single spans on elastic supports (seabed) approach the fixed-fixed values, as the effective span length becomes equal to the real span length.

Note that different effective span lengths apply in the horizontal and vertical directions and for the static and dynamic response quantities in case of different soil stiffness.

The approximate response quantities specified in this section may be applied for free span assessment provided:

- Conservative assumptions are applied with respect to span lengths, soil stiffness and effective axial force;
- The span is a single span on a relatively flat seabed, i.e. the span shoulders are almost horizontal and at the same level;
- The symmetrical mode shape dominates the dynamic response (normally relevant for the vertical, cross-flow response only). Here the following limits apply:

  $L/D_s < 140$

  $\delta/D < 2.5$

  Note that these are not absolute limits; the shift in cross-flow response from the symmetrical to the unsymmetrical mode will depend on the sagging and the level-ling/inclination of the span shoulders. In cases where a shift in the cross-flow response is considered as likely, the structural response of the span should be assessed by using FE analysis including all important aspects;

- Bar buckling is not influencing on the response, i.e.

  $C_2 S_{eff}/P_E > -0.5$

6.3.3 Functional loads

The functional loads which shall be considered are:

- Weight of the pipe and internal fluid
- External and internal fluid pressure

- Thermal expansion and contraction
- Residual installation forces

Response calculations must account for the relevant sequence of load application, if important.

The stiffness of the pipeline consists of material stiffness and geometrical stiffness. The geometrical stiffness is governed by the effective axial force, S_{eff}. This force is equal to the true steel wall axial force, N_{tr}, with corrections for the effect of external and internal pressures:

$$S_{eff} = N_{tr} - p_i A_i + p_e A_e \tag{6.33}$$

Where

N_{tr} "True" steel wall axial force
p_i Internal pressure
p_e External pressure
A_i Internal cross section area of the pipe
A_e External cross section area of the steel pipe

The effective axial force in a span is difficult to estimate due to uncertainties in operational temperature and pressure, residual lay tension and axial force relaxation by sagging, axial sliding (feed-in), lateral buckling, multi-spanning and significant seabed unevenness. All these effects should be considered and taken into account if relevant. The most reliable method to estimate the effective axial force is use of non-linear FE analysis.

While for a totally restrained pipe the following effective axial force applies (if pipe considered thin-walled):

$$S_{eff} = H_{eff} - \Delta p_i A_i (1 - 2\nu) - A_s E \Delta T \alpha_e \tag{6.34}$$

Where

H_{eff} Effective lay tension
Δp_i Internal pressure difference relative to laying, see DNV-OS-F101
A_s Pipe steel cross section area
ΔT Temperature difference relative to laying
α_e Temperature expansion coefficient, may be temperature dependent

Using the expression for totally restrained pipe given above may lead to over-conservative fatigue results for pipelines on very uneven seabed with several long spans and for pipelines experiencing lateral buckling/snaking. In such cases the structural response quantities must be based on refined, non-linear FE analyses.

From the definition in SIMLA, we can get the results as follow:

Table 4 Functional loads

Parameter	Magnitude	Unit
Euler buckling force P_E	$1.89 \cdot 10^5$	[N]
Effective lay tension H_{eff}	$1.5 \cdot 10^5$	[N]
Internal pressure p_i	113	[Bar]
Temperature difference ΔT	10	[°C]
Temperature expansion coefficient α_e	$1.17 \cdot 10^{-5}$	[$°C^{-1}$]
S_eff/P_E	-3.22	
Effective axial force S_{eff}	$-6.45 \cdot 10^5$	[N]

6.4 Method for VIV response calculation

A free span pipeline is assumed to respond at one discrete frequency identified as an eigenfrequency with added mass valid for the given flow condition. The response may be calculated by using finite elements and the frequency response method. The equation of dynamic equilibrium may be written:

$$\mathbf{M\ddot{r}} + \mathbf{C\dot{r}} + \mathbf{Kr} = \mathbf{R} \tag{6.35}$$

The external loads will in this case be harmonic, but loads at all degrees of freedom are not necessarily in phase. It is convenient to describe this type of load pattern by a complex load vector with harmonic time variation:

$$\mathbf{R} = \mathbf{X}e^{i\omega t} \tag{6.36}$$

The response vector will also be given by a complex vector and a harmonic time variation. Hence we have:

$$\mathbf{r} = \mathbf{x}e^{i\omega t} \tag{6.37}$$

By introducing the hydrodynamic mass and damping matrices dynamic equilibrium can now be expressed as:

$$-\omega^2(\mathbf{M_S} + \mathbf{M_H})\mathbf{x} + i\omega(\mathbf{C_S} + \mathbf{C_B})\mathbf{x} + \mathbf{Kx} = \mathbf{X} \tag{6.38}$$

The damping matrix $\mathbf{C_S}$ represents structural damping and will normally be proportional to the stiffness matrix. $\mathbf{C_B}$ contains damping terms from pipe/seafloor interaction. A simple matrix with elements on the main diagonal for vertical displacement has been applied in the present study.

Elements in the excitation vector $\mathbf{X}$ are always in phase with the local response velocity, but a negative lift coefficient will imply a 180 degrees phase shift and hence turn excitation into damping. Since the magnitude of the lift coefficient depends on the response amplitude (cfr. Figure 5), an iteration is needed to solve the equation. Note that the response frequency is fixed during this iteration.

The iteration will identify a response shape and amplitude that gives consistency between the response level, lift coefficients and the local flow condition. The mode shape corresponding to the selected response frequency is used as an initial estimate for the response vector only.

The VIV response is calculated by the frequency response method. However, iteration is necessary, the reason is the load depends on the response, and also hydrodynamic damping. The iteration, if successful, yields response amplitudes at the element nodes consistent with the loading. The frequency response method is not able to take into account all the nonlinearities that occur in connection with free spans. At present, we just focus on the effect of directional distribution of the current speed and reduced lift from inclined flow. A linear response model is deemed sufficient for investigating these properties.

We assume that the pipe responds to all possible in-line VIV response frequencies simultaneously, irrespective of the presence of any cross flow response. The fatigue damage is thus calculated by adding the contributions from all possible in-line frequencies and from any cross flow frequencies that are also present.

7. Fatigue analysis

The pipeline crossing is evaluated to determine fatigue damage due to vortex induce vibrations (VIV) in the operating condition. As the vortex shedding frequency of approaches the pipeline natural frequency, the free-span begins to resonate and can cause rapid pipeline failure.

The VIV fatigue analysis includes several factors such as soil-pipe interactions, modeling of environmental loads, structural response analyses, etc. In this case, we use SIMLA to simulate soil-pipe interactions and structural responses. Then the Fatfree worksheet is used to calculated the fatigue life capacity due to eigen frequencies and stresses based on the SIMLA analysis results in in-line and cross-flow directions.

The following steps were performed to obtain the VIV fatigue damage of the pipeline:

- In-line and cross-flow eigen frequencies and mode shapes are calculated for the free span pipeline with SIMLA software.

- Fatigue life damages are calculated with entered eigen frequencies and unit stress amplitudes in the FATFREE worksheet and then fatigue life can be given according to corresponding fatigue damages.

Dynamic loads from wave action, vortex shedding, etc. may give rise to cyclic stresses, which may cause fatigue damage to the pipe wall, and ultimately lead to failure. The fatigue analysis should cover a period that is representative for the free span exposure period, and fatigue calculations should only be applied to the pipeline conditions that are of such duration that noticeable damage may occur. Fatigue calculations are therefore normally neglected for the hydrotesting conditions.

The fatigue damage from vortex induced vibrations (VIV) should be calculated, including as a minimum:

- Dynamic effects when determining stress ranges;
- Calculation of the number of cycles in a representative number of stress ranges;
- Calculation of fatigue damage according to the Palmgren-Miner accumulation law;
- Determination of the number of cycles to failure using a suitable S-N curve.

The stress ranges to be used in the fatigue analysis may be found using two different methods:

- Applying an external load to the free span (load model).
- The stress ranges are determined using the normalized response amplitudes for a given flow situation (response model), appropriately scaled to the real free span.

Both methods may be applied to a wide range of flow conditions, and the use of one particular method is primarily determined by practical reasons or by the quality of the appropriate model for the actual case. Appropriate response models may be found in DNV RP F105 Free spanning pipelines, which recommend that the following flow conditions be considered:

- Cross-flow VIV in steady current and combined wave and current;
- In-line motion due to cross-flow VIV;
- In-line VIV in steady current and current dominated flow.

7.1 Fatigue assessment procedure

The fatigue assessment is performed by using FATFREE which is a software developed by DNV and supported by excel and visual basic application to extract results from the modal analysis and calculated the damage to the pipeline in temporary and operational phases. For the free span, the following procedure is adopted:

1. For each of the reduced velocities associated with each mode shape, the maximum non-dimensional response amplitudes, A_Y/D and A_Z/D, are determined as per the response models given by DNV RP F105.

2. The actual in-line and cross-flow displacement amplitudes are calculated along the span;

3. The pipeline curvatures ($\kappa(x) = 1/\rho(x)$) at each nodal location along the span, are then calculated based on the actual displacement amplitudes using a finite difference algorithm;

4. The in-line and cross-flow bending stress ranges at each nodal location along the span is determined from linear bending theory. The combined in-line and cross-flow VIV induced stress ranges due to multi-modal behavior of the free span is calculated using the response models got from SIMLA.

5. Fatigue damage is calculated for all nodes along the span, based on multi-modal response we have already got, and using the accumulation law of Palmgren-Miner. This implies replacing the actual stress range distribution by a histogram with a number, I, of constant amplitude stress range blocks, with corresponding stress ranges Si, the fatigue damage is then calculated as:

$$D_{fat} = \sum \frac{n_i}{N_i} \tag{7.1}$$

Where

D_{fat} Accumulated fatigue damage
n_i Number of stress cycles with stress range, Si
N_i Number of cycles to failure at stress range, Si (defined by the S-N curve)

The summation is in principle performed over all stress cycles in the design life, and the stress cycles Si (number and magnitude) may be calculated using a load model, through integration of the equation of motion, or through the application of a response model.

6. Fatigue criterion is checked to verify the free span capacity to resist VIV during the expected design life.

The fatigue damage due to in-line and cross-flow VIV is calculated based on DNV RP F105, the following fatigue criterion which is limited to stress cycles within the elastic ranges can be used for subsea free spanning pipeline assessment. The fatigue criterion can be formulated as:

$$\eta \cdot T_{life} \geq T_{exposure} \tag{7.2}$$

It is clear that the fatigue design life capacity must be longer than the exposure duration. The relationship between the fatigue design life capacities, exposure time and fatigue damage is:

$$D_{fatDamage} = \frac{T_{exposure}}{T_{life}} \cdot \eta \tag{7.3}$$

7.1.1 In-line VIV fatigue assessment for multi-mode response

The subsea pipeline, for free span and multi-mode scenario, in-line VIV fatigue life calculation procedures can be summarized as below based on DNF-RP-F105:

- Gather are the input data, including the pipeline design/operation data, soil data and environment data.

- Calculate the still water in-line eigen frequencies $f_{i,il-still}$ and associated mode shapes (i=1,2,3) by SIMLA.

- Calculate the inline VIV induced stress range for each mode $S_{i,il}(x)$ (for i=1,2,3) at the mid span at the current velocity V_k (mark k=1 for this step and the range for k)

$$S_{in} = 2A_{in} \cdot (A_Y/D) \cdot \Psi_{\alpha,in} \cdot \gamma_s \qquad (7.4)$$

Where A_{in} can be calculated directly from the eigenmode values got from SIMLA and A_Y/D can be obtained based on the in-line response model is shown below in Figure 28

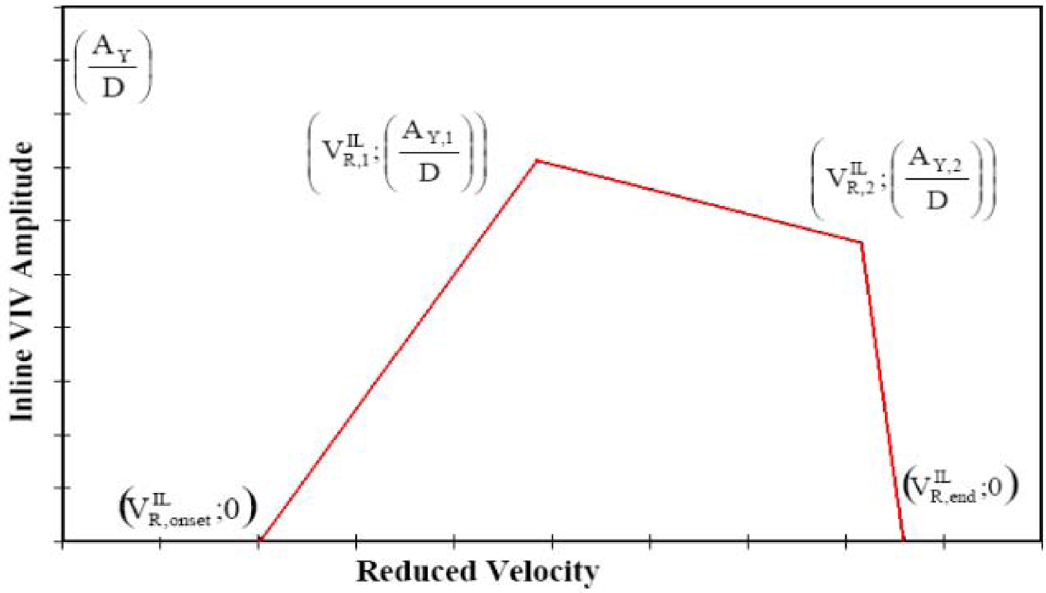


Figure 28 In-line response model generation principle

- Determine the final list of the actively participating modes by eliminating inconsequential modes at the mid span location which gives the maximum stress.

- Renumber the actively participating modes as mode i and the relevant still water in-line eigen frequencies $f_{i,il-still}$.

- Calculating the competing reduction factor α_i for each mode by the following rules:

Define $\alpha_{1.0} = \alpha_{n,n+1} = 1$ and:

$$\alpha_{i,i-1} = \begin{cases} 0.5 & \text{if } f_{i,il-still} < 2f_{i-1,il-still} \text{ and } S_{i,il}(x) < S_{i-1,il}(x) \\ 1 & \text{else} \end{cases} \tag{7.5}$$

$$\alpha_{i,i+1} = \begin{cases} 0.5 & \text{if } f_{i+1,il-still} < 2f_{i,il-still} \text{ and } S_{i+,il}(x) < S_{i,il}(x) \\ 1 & \text{else} \end{cases} \tag{7.6}$$

Then the α_i can be obtained by:

$$\alpha_i = \alpha_{i,i-1} \cdot \alpha_{i,i+1} \quad \text{(for i=1,2,3)} \tag{7.7}$$

- The pure inline VIV stress range for each mode $S_{i,il}(x)$ at the mid span can be considering the competing reduction factor.

- Find out the potential cross-flow induced inline VIV frequency as:

$$f_{i,in-cf} = \min(|f_{i,in-still} - 2f_{i,cf-RES}|) \tag{7.8}$$

Where $f_{i,cf-RES}$ is the frequency for the dominant cross-flow mode.

- Calculate the dominant cross-flow induced i^{th} inline VIV stress range by:

$$S_{i,il-cf}(x) = 2 \cdot 0.4 \cdot A_{i,in}(x) \cdot (A_{zDom}/D) \cdot \Psi_{\alpha,in} \cdot \gamma_s \tag{7.9}$$

- Calculate the stress for the mode that is potentially oscillated by the cross-flow induced in-line mode.

$$S_{i,il}(x) = \max[S_{i,il}(x), S_{i,il-cf}(x)] \tag{7.10}$$

- Calculate the combined stress by:

$$S_{comb,in}(x) = \sqrt{\sum_{i=1}^{n} f_{i,in} \frac{S_{i,il}(x)}{S_{comb,in}(x)}} \tag{7.11}$$

- Calculate the combined stress associated cycle counting frequency by :

$$f_{cyc,in}(x) = \sqrt{\sum_{i=1}^{n} f_{i,in} \frac{S_{i,il}(x)}{S_{comb,in}(x)}} \tag{7.12}$$

The counting frequency for the inline modes is obtained by:

$$f_{i,in} = \begin{cases} 2 \cdot f_{i,cf-RES} & \text{cross-flow induced inline mode} \\ f_{i,in-still} & \text{pure inline modes} \end{cases} \tag{7.13}$$

- Calculate the fatigue damage due to inline VIV at current velocity V.

$$D_{fat-in-k}(x) = f_{cyc,in}(x) \cdot (\frac{S_{comb,in}(x) \cdot SCF}{MPa})^{m(x)} \cdot \frac{P_k}{\bar{a}(x)} \quad (7.14)$$

Where SCF is the stress concentration factor, P_k is the current flow probability at Vk by weibull distribution, $m(x)$ is the fatigue exponent by S-N curve and $\bar{a}(x)$ is Characteristic fatigue strength constant.

- Calculate the inline VIV fatigue life:

$$D_{fat-in}(x) = \sum D_{fat-in-k}(x) \quad (7.15)$$

$$D_{fat-in}(x) = \max(D_{fat-in}(x)) \quad (7.16)$$

$$T_{fat-life-in} = \frac{\eta}{D_{fat-in}} \quad (7.17)$$

7.1.2 Cross-flow VIV fatigue assessment for multi-mode response

The subsea pipeline, for free span and multi-mode scenario, cross-flow VIV fatigue life calculation procedures can be summarized as below based on DNF-RP-F105:

- Gather are the input data, including the pipeline design/operation data, soil data and environment data.
- Calculate the still water cross-flow eigen frequencies $f_{i,cf-still}$ and associated mode shapes (i=1,2,3) by SIMLA.
- Find out the dominant cross-flow mode I for the free span at flow velocity V_k (mark k=1 for this step and the range for k), i.e. the cross-flow mode with the largest A_{Zi}/D value (A_{ZDom}/D) predicted from the response model for velocity V_k, then the “weak” and “negligible” cross flow mode can be determine by:

$$i = \begin{cases} \text{“weak”} \quad \text{for} \frac{A_{Zi}}{D} \geq 10\% \frac{A_{ZDom}}{D} \\ \text{“negligible”} \quad \text{for} \frac{A_{Zi}}{D} < 10\% \frac{A_{ZDom}}{D} \end{cases} \quad (7.18)$$

- Calculate the stress induce by the cross-flow mode at the mid span location for each cross-flow mode I by the following fomula:

$$S_{i,cf}(x) = \begin{cases} 1 \cdot 2A_{i,cf} \cdot \left(\frac{A_{Zi}}{D}\right) \cdot R_k \cdot \gamma_s & i = \text{"Dominant"} \\ 0.5 \cdot 2A_{i,cf} \cdot \left(\frac{A_{Zi}}{D}\right) \cdot R_k \cdot \gamma_s & i = \text{"weak"} \\ 0.0 & i = \text{"negligible"} \end{cases} \quad (7.19)$$

Where $A_{i,cf}(x)$ is obtained by SIMLA eigen mode analysis and $\frac{A_{Zi}}{D}$ can be based on the Cross-flow response model is shown in Figure 29.

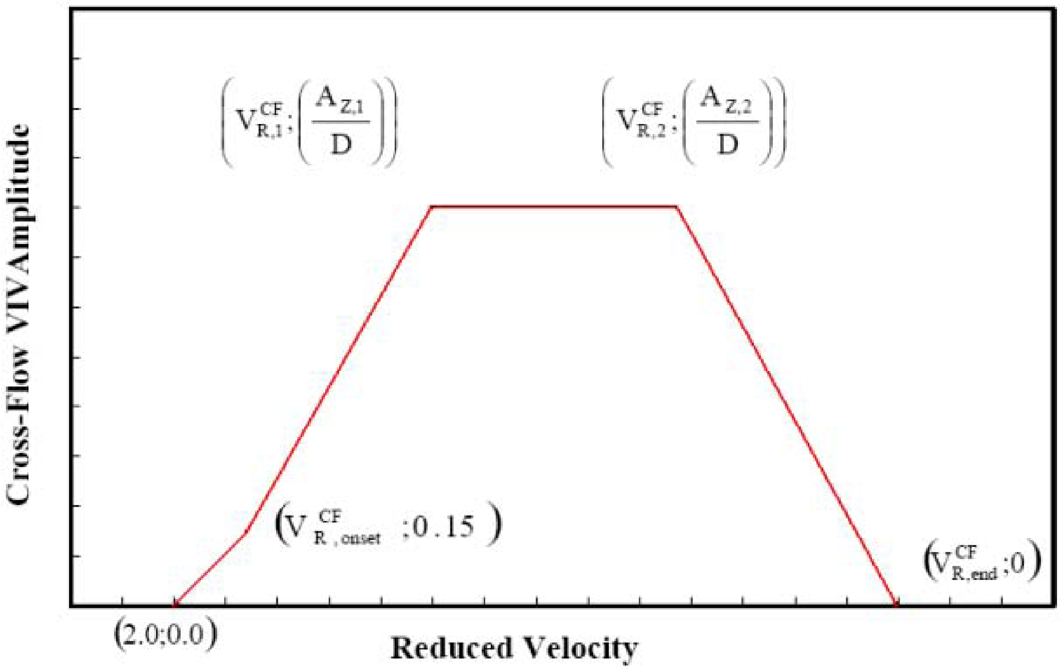


Figure 29 Cross-flow response model generation principle

- Calculate the combined cross-flow induce stress at the mid span by the following formula:

$$S_{comb,CF}(x) = \sqrt{\sum_{i=1}^{n}(S_{i,cf}(x))^2} \tag{7.20}$$

- Calculate the cycle counting frequency for this combined cross-flow induced stress at the mid span by the following formula:

$$f_{cyc,cf}(x) = \sqrt{\sum_{i=1}^{n} f_{i,cf}\frac{S_{i,cf}(x)}{S_{comb,cf}(x)}} \tag{7.21}$$

Where:

$$f_{i,cf} = \begin{cases} f_{i,cf-RES} & \text{for } i = \text{"Dominant"} \\ f_{i,cf-still} & \text{for } i = \text{"weak"} \\ 0 & \text{for } i = \text{"negligible"} \end{cases} \tag{7.22}$$

$$f_{i,cf-RES} = f_{i,cf-still}\sqrt{\frac{\frac{\rho_S}{\rho}+C_a}{\frac{\rho_S}{\rho}+C_{a,cf-RES}}} \tag{7.23}$$

$C_{a,cf-RES}$ value can be referred to the Figure below from DNV-RP-105:

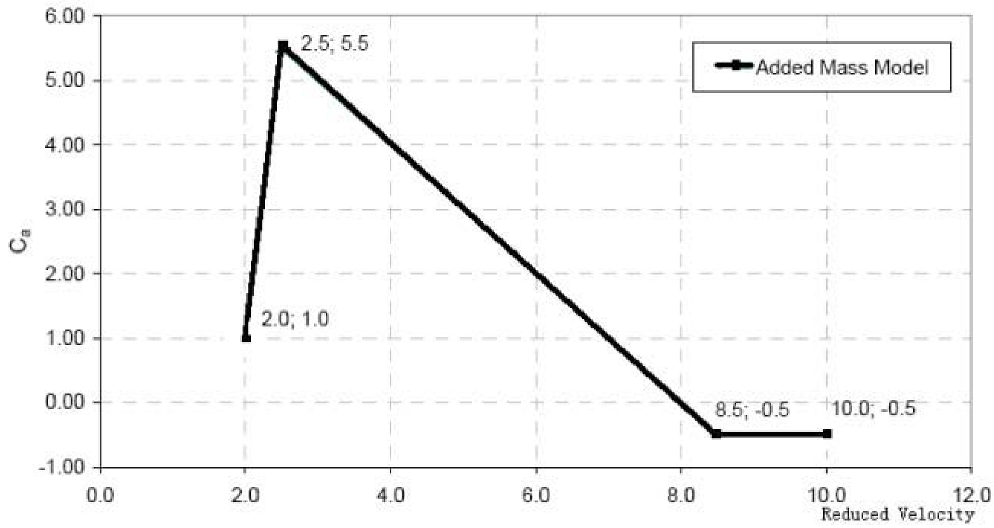


Figure 30 C_(a,cf-RES) as a function of reduced velocity

- Calculate the fatigue damage due to cross-flow VIV at the current velocity Vk.

$$D_{fat-cf-k}(x) = f_{cyc,cf}(x) \cdot \left(\frac{S_{comb,cf}(x) \cdot SCF}{MPa}\right)^{m(x)} \cdot \frac{P_k}{\bar{a}(x)} \quad (7.24)$$

- Calculate the cross-flow VIV fatigue life:

$$D_{fat-cf}(x) = \sum D_{fat-cf-k}(x) \quad (7.25)$$

$$D_{fat-cf}(x) = \max(D_{fat-cf}(x)) \quad (7.26)$$

$$T_{fat-life-cf} = \frac{\eta}{D_{fat-cf}} \quad (7.27)$$

7.2 S-N curves and safety factors

7.2.1 S-N curve selection

The S-N curve is a convenient and efficient method for pipeline fatigue analysis, where the S-N data are usually determined by fatigue test. The S-N curves (material constants m and C) may be determined from:

- Dedicated laboratory test data;
- Fracture mechanics theory;
- Accepted literature references like DNV RP C203 “Fatigue Strength Analysis of Offshore Steel Structures”.

If detailed information is not available, the S-N curves given in DNV RP C203 Fatigue strength analysis of offshore steel structures may be used, corresponding to cathodically protected carbon steel pipelines.

The S-N curves may be determined from a fracture mechanics approach, using an accepted crack growth model with an adequate (presumably conservative) initial defect hypothesis and relevant stress state in the girth welds. Considerations should be given to the applied welding and NDT specifications.

A stress concentration factor (SCF) may be defined as the ratio of hot spot stress range over nominal stress range. The hot spot stress is to be used together with the nominated S-N curve.

Stress concentrations in pipelines are due to eccentricities resulting from different sources. These may be classified as:

- Concentricity, i.e. difference in pipe diameters;
- Difference in thickness of joined pipes;
- Out –of –roundness and centre eccentricity.

In this case, we use the S-N curves given from DNV-RP-C203. The S-N curve must be applicable for the material, construction detail, location of the initial defect (crack initiation point) and corrosive environment. The basic principles in DNV RP C203 apply.

According to the latest version of DNV offshore steel structure code DNV-RP-C203, the basic design S-N curve is given as:

$$\log N = \log \bar{a} - m \log \Delta\sigma \tag{7.28}$$

Where N is the predicted number of cycles to failure for stress, $\Delta\sigma$ is the stress range, m is the negative inverse slope of S-N curve $\log \bar{a}$ is the intercept of log N-axis by S-N curve, it is given by the following equation:

$$\log \bar{a} = \log a - 2s \tag{7.29}$$

Where a is the constant relating to mean S-N curve, s it the standard deviation of $\log N$.

The number of cycles to failure is defined by an S-N curve of the form:

$$N = C(S)^{-m} = \begin{cases} \overline{a_1} \cdot S^{-m_1} & S > S_{SW} \\ \overline{a_2} \cdot S^{-m_2} & S < S_{SW} \end{cases} \tag{7.30}$$

Where

N - Number of cycles of failure at stress range S

S - Stress range based on peak-to-peak response amplitudes

m -Fatigue exponent (the inverse slope of the S-N curve)

C -Characteristic fatigue strength constant defined as the mean-minus-two-standard-deviation curve (in (MPa)m)

The constants m and C may change for a given S-N curve when the number of cycles exceeds a certain threshold value, typically 10^6or10^7.

The fatigue life can be calculated based on S-N curve under the assumption of linear cumulative damage by Palmgren-Miner and can be found from the following equation:

$$D_{fat} = \sum \frac{n_i}{N_i} = \frac{1}{a} \sum_{i=1}^{k} n_i (\Delta\sigma_i)^m \quad (7.31)$$

Where a is the intercept of the design S-N curve with the log N axis, m is the negative inverse slope of the S-N curve, k is the number of stress blocks.

For specific subsea pipeline, the S-N curve can be illustrated as Figure 28.

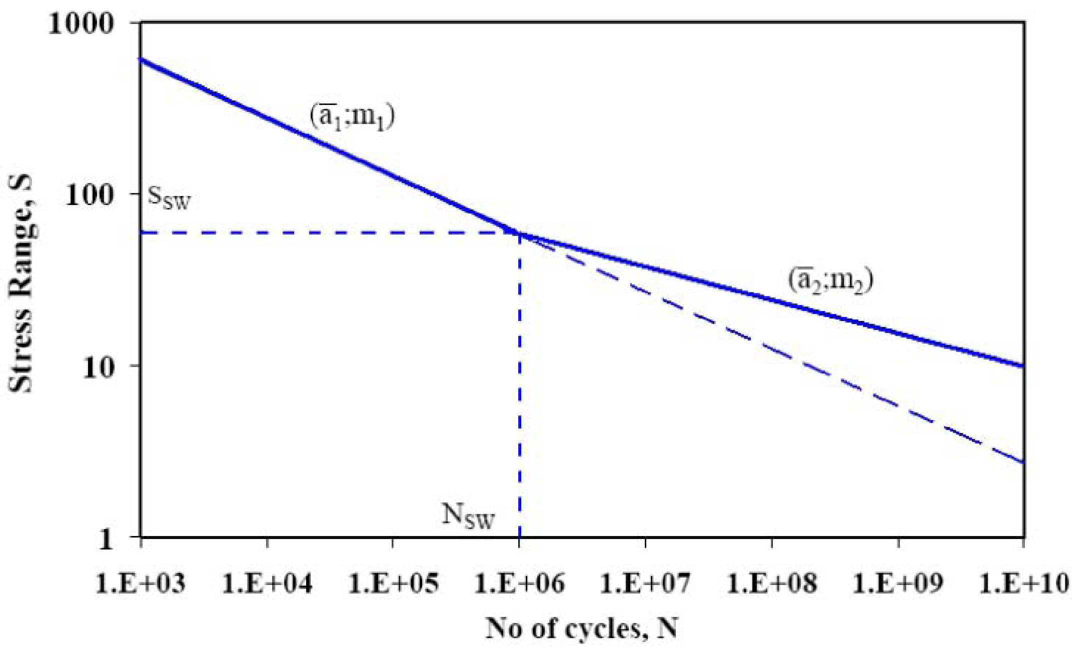


Figure 31 Typically two slope S-N curve

7.2.2 Safety factor

The allowable fatigue damage ratio depends on the safety class, and the values recommended by DNV RP F105 are stated in the following table.

Table 5 Allowable damage ratio for fatigue

Safety class	Low	Normal	High
α_{fat}	1.0	0.5	0.25

It should be noted that these factors are applied together with other partial safety factors in DNV RP F105.

The accumulated fatigue damage of the pipeline is incurred during the following phases:

Installation (typically pipe-laying);
On the seabed (empty and/or water-filled);
Operation.

To ensure a reasonable fatigue life in the operational phase it is common industry practice to assign no more than 10% of the allowable damage ratio to the two temporary phases (as mentioned above, hydro-testing is normally neglected).

7.2.3 Wave and current data directionality

During the fatigue calculation, the wave and current magnitude and direction are required to be 0. However, the direction information may not always be available. Therefore, the following conservative assumption of direction combination of wave and current is adopted:

- The current and wave-induced flow components at the pipe level are statistically independent.

- The current and wave-induced flow components are assumed co-linear. This implies that the directional probability of occurrence data for either waves or current (the most conservative with respect to fatigue damage) must be used for both waves and current.

If the directions of both currents and waves are not available, we can assume they are perpendicular to the pipeline in the analysis. However, if the direction of only one phenomenon is available, we have to define the same directionality for both phenomena. This is due to how the current and wave statistics are intended. They are independent, but in order to depict most accurately, the probability density function should be joint, i.e. there should be a three dimensional matrix of probabilities associated with current velocity (*Uc*), wave period (*Tp*) and wave height (*Hs*) for each direction. Therefore, the probability density functions are interpreted as simultaneous, i.e. the wave data and current data are assumed to act in the same direction at all times, as such wave and current are assumed locked to each other's direction.

7.3 Results analysis

From the calculation of FATFREE, we can get the following results:

Structural modeling intermediate results

STRUCTURAL MODELLING INTERMEDIATE RESULTS					
Static Stress [MPa]		Transfer values		Areas [m^2]	
σ_h	54,5	EI_{steel}	4,80E+07	A_i	0,05067
σ_N	-7,0	m_e	343	A_{steel}	0,02335
$\sigma_{M,cr}$	358,3	q	1499	$A_{coating}$	0,01838
$\sigma_{M,in}$ (100y)	12,9	S_{eff}	-6,45E+05	$A_{concrete}$	0,00000
		C_a	1,00	A_e	0,09240
		CSF	0,00		
		ρ_s/ρ	2,61		

Figure 32 Structural modelling intermediate results

Damage distribution vs Hs

Table 6 Damage distribution vs Hs

Damage distribution versus Hs			
RM (In-Line)	FM (In-Line)	Comb.(In-Line)	Cross-Flow
0,00	0,00	0,00	0,00
0,01	0,01	0,01	0,00
0,02	0,03	0,02	0,01
0,03	0,06	0,03	0,02
0,05	0,10	0,05	0,05
0,08	0,14	0,08	0,09
0,12	0,15	0,12	0,13
0,15	0,14	0,15	0,16
0,16	0,12	0,16	0,17
0,14	0,10	0,14	0,15
0,10	0,07	0,10	0,11
0,07	0,05	0,07	0,07
0,03	0,03	0,03	0,04
0,01	0,01	0,01	0,01
0,00	0,00	0,00	0,00

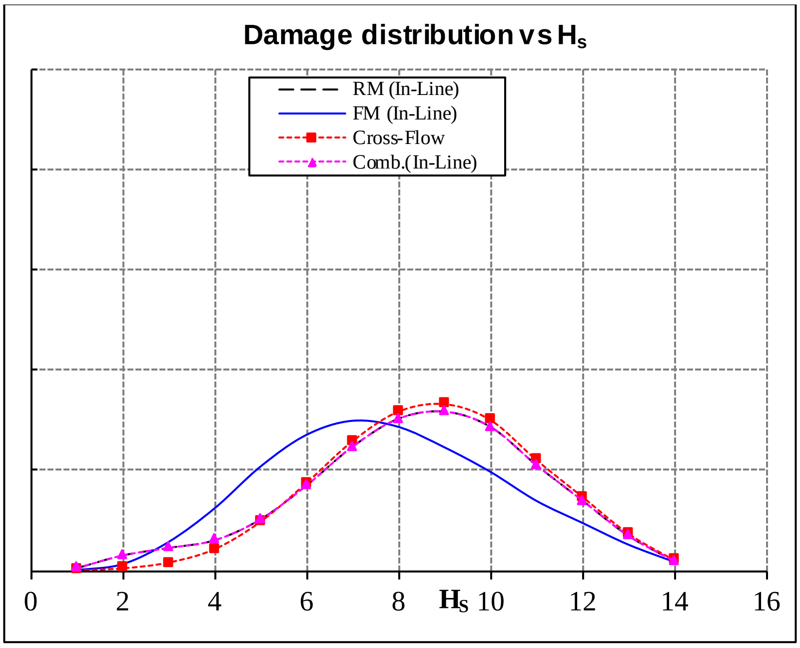


Figure 33 Damage distribution vs Hs

Mean value over direction and period

Table 7 Graphic presentation of results

Graphic presentation of results			
H_S	V_R*10	α*20	KC
1,00	6,02	18,91	0,26
2,00	6,96	16,81	0,94
3,00	8,48	14,25	2,08
4,00	10,53	11,73	3,66
5,00	13,06	9,51	5,66
6,00	16,07	7,67	8,10
7,00	19,55	6,21	10,99
8,00	23,46	5,09	14,35
9,00	27,78	4,24	18,15
10,00	32,48	3,59	22,42
11,00	37,33	3,10	26,89
12,00	42,97	2,67	32,35
13,00	47,60	2,40	36,53
14,00	54,06	2,09	43,00

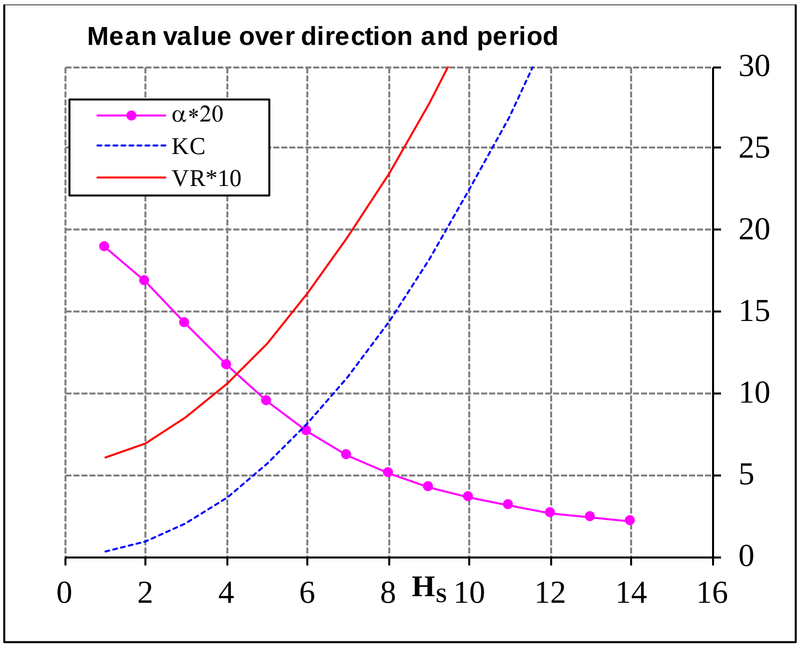


Figure 34 Mean value over direction and period

PDF for current using omnidirectional values

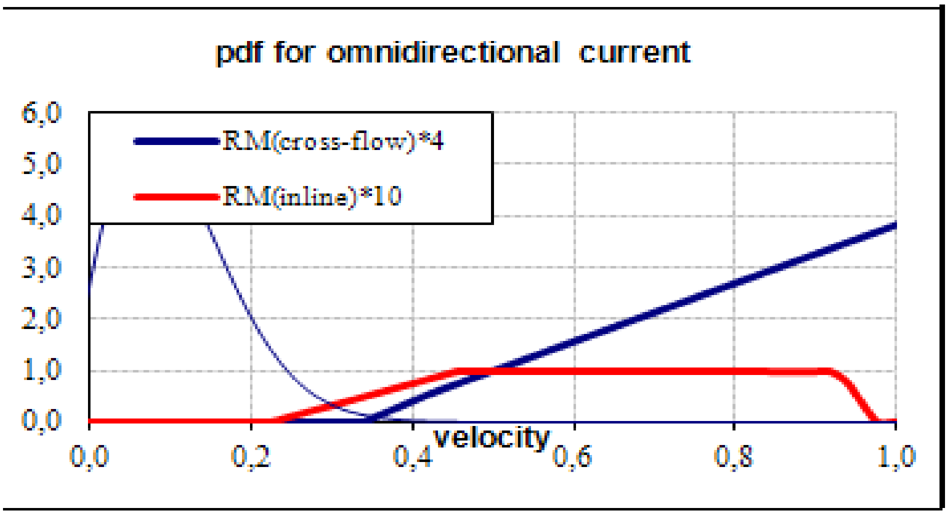


Figure 35 pdf for omnidirectional current

Dynamic Stress

DYNAMIC STRESS [MPa]					
Cross-flow			Inline		
	Peak	Von Mises		Peak	Von Mises
σ_x(1 year)	4,2	399,6	σ_x(1 year)	5,3	67,7
σ_x(10 year)	7,9	403,3	σ_x(10 year)	7,4	71,8
σ_x(100 year)	11,2	406,6	σ_x(100 year)	8,3	75,5

Figure 36 Dynaimic stress result

Fatigue life

FATIGUE LIFE		
In-line (Response Model)		1,82E+03 yrs
Cross-Flow		3,94E+03 yrs

Figure 37 Fatigue life

Finally, we get the fatigue life of the free span pipeline caused by vortex induced vibration, which the design life is 50 years. We can see from the figure that the in-line direction is more vulnerable than cross-flow direction.

8. SENSITIVITY STUDY

It is known that the free span pipeline fatigue damage has relation to many factors, such as the environment loads, seabed profile, free span length, seabed soil properties, pipeline residual laying tension, etc. Thus, comprehensive sensitivity analyses can help to determine which factors will exert great influence on the pipeline fatigue damage.

In this project, we will study different span length and soil properties as main factors to perform some VIV fatigue sensitivity analyses.

In-line and Cross-flow VIV fatigue damage is calculated for different span length which is presented in Table 8. As it is shown, fatigue damage increases by increasing the span length, which manifests that, in certain range; long pipeline free spans can be relatively more vulnerable than shorter ones. When the span length is short, the in-line VIV dominant and has caused certain damage to the pipe, however, with the increasing of the free span length, the cross-flow VIV can cause larger fatigue damage compared to the in-line VIV.

Table 8 Fatigue life for different span lengths sensitivity analysis

Case No.	L/D ratio	Free-span length (m)	In-line VIV fatigue life (year)	Cross-flow VIV fatigue life (year)
1	146	50	1,82E+03	3,94E+03
2	187	64	1,92E+02	2,22E+03
3	233	80	1,61E+02	2,04E+02

Dynamic soil stiffness for different soil type and fatigue life are presented in table 9 and 10 with increasing dynamic soil stiffness, the in-line VIV fatigue life is increased; however, cross-flow VIV fatigue life decreased. The in-line VIV can cause larger fatigue damage than the cross-flow VIV, thus more attention should be paid to the in-line VIV fatigue damage for the pipeline VIV check. The in-line and cross-flow pipeline fatigue life are fairly significant for 50 years of design life, thus an appropriate pipeline design can help the pipeline survive even the seabed soil stiffness is quite different.

Table 9 Dynamic soil stiffness for sensitivity analysis

Soil type	Vertical dynamic K_V (N/m/m)	Lateral K_L (N/m/m)	Vertical static $K_{V,S}$ (N/m/m)
Stiff clay	9,937E+06	6,868E+06	1,300E+06
Loose sand	1,962E+07	1,476E+07	2,500E+05

Medium sand	2,709E+07	2,050E+07	5,300E+05

Table 10 Fatigue life for soil stiffness sensitivity analysis

Soil type	In-line VIV fatigue life (year)	Cross-flow VIV fatigue life (year)
Stiff clay	1,82E+03	3,94E+03
Loose sand	3,33E+03	3,56E+03
Medium sand	5,76E+03	3,04E+03

9. Conclusions and recommendations

VIV free span assessment is influenced by many different factors, such as pipe diameter, pipe surface roughness, wall thickness, span length and current velocities , which are parameters to control the VIV. However, fatigue damage is a main factor which can cause reduction of the pipeline life time; fatigue design becomes an indispensible aspect for the subsea pipeline design especially when the seabed is extreme uneven and the water depth is deep.

From the analysis by using FATFREE, we find that for free span pipeline fatigue damage assessment, only the first modes are either dominant or participating modes, the modes with very high frequencies tend to cause very little damage to the pipeline. It is not very common to use the multi-mode + direct mode shape input option when use FATFREE to calculate the fatigue life.

We can see from the sensitivity analysis, the length of free-span plays a crucial role in cumulative fatigue damage of pipelines laid on free-span sections. The interaction between the wave and current induced loads in shallow water region is much more pronounced for offshore pipelines in deep water regions, while the assumption in current practice neglects the wave effect in deep water regions.

Proper and sound design of the pipeline can help to prevent the pipeline from free span fatigue damage even the environment and seabed condition is harsh and disadvantaged.

Further work should be done about wave and current combination use to obtain more comprehensive and generalized conclusions of free span and multi-mode fatigue assessment. VIV induced fatigue damage in SIMLA can be developed in the future.

APPENDIX

Appendix A Wave data

Table 2.2. *Joint frequency of significant wave height and spectral peak period. Representative data for the northern North Sea*

Significant wave height (m) (upper limit of interval)	Spectral peak period (s)																			
	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	21	22	Sum
1	59	403	1061	1569	1634	1362	982	643	395	232	132	74	41	22	12	7	4	2	2	8636
2	9	212	1233	3223	5106	5814	5284	4102	2846	1821	1098	634	355	194	105	56	30	16	17	32 155
3	0	8	146	831	2295	3896	4707	4456	3531	2452	1543	901	497	263	135	67	33	16	15	25 792
4	0	0	6	85	481	1371	2406	2960	2796	2163	1437	849	458	231	110	50	22	10	7	15 442
5	0	0	0	4	57	315	898	1564	1879	1696	1228	748	398	191	84	35	13	5	3	9118
6	0	0	0	0	3	39	207	571	950	1069	885	575	309	142	58	21	7	2	1	4839
7	0	0	0	0	0	2	27	136	347	528	533	387	217	98	37	12	4	1	0	2329
8	0	0	0	0	0	0	2	20	88	197	261	226	138	64	23	7	2	0	0	1028
9	0	0	0	0	0	0	0	2	15	54	101	111	78	39	14	4	1	0	0	419
10	0	0	0	0	0	0	0	0	2	11	30	45	39	22	8	2	1	0	0	160
11	0	0	0	0	0	0	0	0	0	2	7	15	16	11	5	1	0	0	0	57
12	0	0	0	0	0	0	0	0	0	0	1	4	6	5	2	1	0	0	0	19
13	0	0	0	0	0	0	0	0	0	0	0	1	2	2	1	0	0	0	0	6
14	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Sum	68	623	2446	5712	9576	12 799	14 513	14 454	12 849	10 225	7256	4570	2554	1285	594	263	117	52	45	100 001

Figure 38 Wave data

Appendix B-Seabed data

```
-300     0   -200.00   0 0 1
 -25     0   -200.00   0 0 1
 -24.9   0   -202.00   0 0 1
  24.9   0   -202.00   0 0 1
  25     0   -200.00   0 0 1
 300     0   -200.00   0 0 1
```

Appendix C-SIMLA code

```
HEAD DNV-RP-F111 RECOMMENDATIONS - Height = 1.0 m - Velocity = 3 m/s
HEAD 420 m Pipeline - 1100 m warpline    Units: N and m
#------------------------------------------------------------
# CONTROL DATA:
#------------------------------------------------------------
#          maxitndimisolvrnpointipriconrgacciprocirestp
CONTROL  100   3   1    16    1   1e-5  9.81  restart  2
#
#            Lumped mass   alfa1   alfa2   HHT-alfa parameter
DYNCONT   2               0.0     0.0    -0.05
#
#           Scaling factor
VISRES    integration 1 sigma-xx
#
# PULLOVER RESULTS (DYNPOST)
DYNRES_E    2    330    1   1
DYNRES_E    2    330    1   2
DYNRES_E    2    330    1   3
DYNRES_E    2    331    2   1
DYNRES_E    2    331    2   2
DYNRES_E    2    331    2   3
DYNRES_N    1    330    2
DYNRES_N    1    332    2
DYNRES_N    1    331    2
DYNRES_N    1    331    3
DYNRES_E    2    330    1   6
DYNRES_E    2    331    2   6
#
# PULLOVER RESULT CHECK
```

```
DYNRES_E   2   329   1  1
DYNRES_E   2   329   1  2
DYNRES_E   2   329   1  3
DYNRES_E   2   332   2  1
DYNRES_E   2   332   2  2
DYNRES_E   2   332   2  3
DYNRES_E   2   301   1  2
DYNRES_E   2   301   1  3
DYNRES_E   2   360   2  2
DYNRES_E   2   360   2  3
DYNRES_E   2   1     2  1
DYNRES_E   2   100   1  1
DYNRES_E   2   560   1  1
DYNRES_E   2   660   1  1
DYNRES_N   2   331   2
DYNRES_N   2   331   3
DYNRES_N   1   301   2
DYNRES_N   1   301   3
DYNRES_N   1   360   2
DYNRES_N   1   360   3
#
#--------------------------------------------------------
# Analysis time control:
#   Empty pipeline--------------------------------------------------------
#         tdtdtvidtdy  dt0    type    hla control
TIMECO 2.0 1.0   1.0   1.0 201.0 static NOHLA auto go-on ener 30 5 1e-5
#
#   Water filling
TIMECO 10.0 1.0 1.0   1.0 201.0 static NOHLA
#
#   Hydrostatic testing
TIMECO 20.0 1.0 1.0   1.0 201.0 static   NOHLA
#
#   Dewatering
TIMECO 25.0 1.0 1.0   1.0 201.0 static   NOHLA
#
#   Operating
TIMECO 30.0 1.0 1.0   1.0 201.0 static   NOHLA
#--------------------------------------------------------
# NODE INPUT:
```

```
#-------------------------------------------------------
# PIPELINE
NOCOOR coordinates    1      -210.0     0.0      -199.835
                     151     -60.0      0.0      -199.835
                     511      60.0      0.0      -199.835
                     661     210.0      0.0      -199.835
#sea
NOCOOR coordinates 20001   -350.000    -100.000     0.000
                   20011    350.000    -100.000     0.000
repeat 11 11 0.0 20.0 0.0
#
# PIPELINE
ELCON    statoilpipe1 pipe31 pipemat1 1 1 2 repeat 660 1 1
#
ELORIENT coordinates     1      0.0    300.0    -399.15
                        660     0.0    300.0    -399.15
# SEA BED
#            groupeltysurfID       ID    n1          n    j k
ELCON   pipeseabed     cont126   cosurf1   10001   1 repeat   661 1 1
#                                ID                  txtytz
ELORIENT eulerangle        10001              0.000    0.000      0
                              10661              0.000    0.000      0
# SEA SURFACE
#        group   elty   material   ID     n1     n2      n3      n4
ELCON   sea1   sea150   seamat   20001 20001 20002   20013   20012
repeat 10 1 1 repeat 10 10 11
#
#-------------------------------------------------------
# SEA BED SURFACE DATA:
#-------------------------------------------------------
#            name       data file        nlin kp0   x0   y0   fi route_ids
COSURFPR cosurf1   "Myseabed.txt"   1     0.0    0.0 0.0   0   100
#           route id              kp1            kp2      soiltype
COSUPR             100              -10000.0      10000.0          soil1
#-------------------------------------------------------
# CONTACT INTERFACE DATA:
#-------------------------------------------------------
#          groupnmasternameslavename  is1 isnistxistyistzmaxitigap
CONTINT pipeseabed statoilpipe1 cosurf1     1    660 2.00    2.0 0.00    60       1.0
#
```

```
CONTINT sea1                    sea1                    statoilpipe1
#
#--------------------------------------------------------
# ELEMENT PROPERTY INPUT:
#--------------------------------------------------------
#           name      type   rad      thCDrCdtCMrCMtwdwsODpODwrks
ELPROP statoilpipe1 pipe 0.14025 0.0265 1.0 0.0 2.29 0.0 207 112 0.343 0.343 0.5
#--------------------------------------------------------
# LOAD INPUT:
#--------------------------------------------------------
# Current and wave loads:
#         name     x1     y1       x2     y2      icurihist
SEALO     sea1 -4000      0         0      0        100     400
                      0      0   24000     0         100     400
#
#         no      depth     curr      fi
CURLOAD 100 global     0        0.50     1.57
                        -100       0.50     1.57
                        -500       0.50     1.57
                       -5000       0.50     1.57
#
#        seagrp type       wav hist   x0      y0 phi     T    H     D     Phase
WAVELO sea1     REGULAR 100 500 1667.27 0    2.437 10 2.0 2200 0
#
#--------------------------------------------------------
# External pressure and gravity:
#--------------------------------------------------------
#         PRESHIST GRAVHIST
PELOAD    150        100
#
#--------------------------------------------------------
# Internal pressure:
#--------------------------------------------------------
#         HIST      ELNR1     P1    ELNR2     P2
PILOAD    600        1          11.3e6      660      11.3e6
#
#--------------------------------------------------------
# TEMPERTURE LOAD USED TO SCALE AXIAL FORCE IN PIPELINE:
#--------------------------------------------------------
#         HIST      E1        T1         E2     T2
```

```
TLOAD     700       1        10.0     660    10.0
#
#--------------------------------------------------------
# DNV RP-F111 POINT LOAD RECOMMENDATIONS
#--------------------------------------------------------
CLOAD 800 1 661 150000
#
#--------------------------------------------------------
# Boundary condition data:
#--------------------------------------------------------
#          Locnode    dir
#PIPEENDS
BONCON GLOBAL   1         1
BONCON GLOBAL   1         2
BONCON GLOBAL   1         4
BONCON GLOBAL   661     1
BONCON GLOBAL   661     2
#SEA ELEMENTS
BONCON GLOBAL 20001       1
REPEAT      121 1
BONCON GLOBAL 20001       2
REPEAT      121 1
BONCON GLOBAL 20001       3
REPEAT      121 1
#--------------------------------------------------------
#CONSTRAINTS:
#--------------------------------------------------------
#--------------------------------------------------------
# HISTORY DATA:
#--------------------------------------------------------
#         no    istpfac
#DRY MASS HISTORY
THIST     100        0.0        0.0
                     2.0        1.0
                     10.0       1.25
                     20.0       1.25
                     30.0       1.20
#EXTERNAL PRESSURE
THIST     150        0.0        0.0
                     2.0        1.0
```

```
                    10.0      1.0
                    20.0      1.0
                    30.0      1.0
#INTERNAL PRESSURE
THIST     600       0.0       0.0
                    2.0       0.0
                    10.0      0.0
                    20.0      1.25
                    30.0      1.0
#
#temperature  LOAD HIST.
THIST 700 0.0      0.0
            2.0      0.0
10.0    0.0
20.0    0.0
30.0    1.0
# curload
THIST_r 400      1.0 2.0 rampcos 0.0
# waveload
THIST 500      0.0 0.0
          1000.0 0.0
# cload
THIST 800      0.0 1.0
               30.0 1.0
#---------------------------------------------------------
#    Material data:
#---------------------------------------------------------
# SEA
MATERIAL seamat       sea         1026.0
#LINEAR PIPE MATERIAL
#            name         type    poisstalfatecondheatc beta eaeiyeizgitemgm
MATERIAL  pipemat1  linear  0.3    1.1e-5    50       800      0    6.60e9  2.06e8  2.06e8
1.57e8 2.1e11 8e10
# SOIL
#              name            type            mux       muymutxxna meyna mezna metx name
y2name
#Reference penetration of 0.065 m in SIMLA is set equivalent to 20% of OD in real
life
MATERIAL soil1       r_contact     1.0 1.0 0.0    soilxsoilysoilzsoilrx
0.065    hat coulomb-userdefined
```

```
#
MATERIAL soilxepcurve    1      0.00       0.00
                                     0.001      0.599
                                     0.1        0.600
                                     100.0      0.601
#
MATERIAL        hat hycurve          -100.0     -0.0
                                     -1.031     -0.0
                                     -1.03      -40
                                     -0.515     -222
                                     -0.128     -560
                                     -0.064     -950
                                     -0.00776   -0.0
0.00776   0.0
                                     0.064      950
                                     0.128      560
                                     0.515      222
                                     1.03       40
                                     1.031      0.0
                    100.00     0.0
#
MATERIAL soilyepcurve    1      0.00       0.00
                                     0.001      0.599
                                     0.1        0.600
                                     100.0      0.601
#
MATERIAL soilzhycurve         -1000      -150e6
                                     0.0        0.0
                                     1000       0.0
#
MATERIAL soilrxhycurve      -1000.0       0.0
                                  1000.0        0.0
```

Appendix D-SIMPOST code

```
# global nodal plot
#------------------------
#           .raf  prefix      .mpf  prefix        Legend  x                  x-res.  Legend  y
y-res. No 1 No 2 X-fac Y-fac
GNPLOT   "DNV-H1S3"  "statconf-xz"  "X-coordinate  (m)"  X-COR  "Z-coordinate  (m)"
Z-COR   1   661   1        1
#
# global element plot
#---------------------
#            .raf  prefix       .mpf  prefix                  Legend  x        x-res.Legend  y
y-res.     El 1      El 2 X-fac Y-fac
GLPLOT "DNV-H1S3"    "statconf-ax"          "X-coordinate(m)" X-COR "Axial force(N)"
ELFORCE-X 1         660      1        1
GLPLOT      "DNV-H1S3"      "statconf-elmom-y"          "S-coordinate(m)"      E-COR
"Moment-y(Nm)"          ELMOM-Y      1         660      1        1
GLPLOT        "DNV-H1S3"       "statconf-condis-z"       "X-coordinate(m)"       X-COR
"Displacement-z(m)" CONDIS-Z    10001 10661 1        1
GnPLOT "DNV-H1S3" "statconf-depangle" "X-coordinate(m)" X-COR "Rotation-y(m)"
NOROT-Y      1         661      1 57.29577
#
#Eigen mode analysis
#          .raf  prefix  .mpf  prefix       Legend  x                x-res.  KPstrtKpend  LOADST
NMODES ROUGH
VIVFAT  "DNV-H1S3"  "eigenmode-viv"  "KP-coordinate[m]"  K-COR  240.0   360.0  30
6        MARIN
```

Appendix E-Direct mode shape input

Direct mode shape input						
Number of discrete points	30					
	Mode 1	Mode 1	Mode 2	Mode 2	Mode 3	Mode 3
X-coordinate	Inline	Cross-flow	Inline	Cross-flow	Inline	Cross-flow
X	Y_{IL1}/Y_{MAXIL1}	Y_{CF1}/Y_{MAXCF1}	Y_{IL2}/Y_{MAXIL2}	Y_{CF2}/Y_{MAXCF2}	Y_{IL3}/Y_{MAXIL3}	Y_{CF3}/Y_{MAXCF3}
400,0063	-0,00362	-0,04575	-0,04661	0,11566	0,76857	-0,02099
402,0064	-0,0052	-0,05502	-0,05834	0,13126	0,90408	-0,02384
406,0066	-0,00878	-0,06751	-0,07488	0,14009	0,99586	-0,02646
412,0069	-0,01391	-0,05454	-0,07354	0,06335	0,64159	-0,02004
414,007	-0,0152	-0,03768	-0,06694	0,00747	0,44979	-0,01535
422,007	-0,00405	0,11902	-0,00202	-0,38353	-0,00786	0,00892
426,0065	0,07153	0,26038	0,20071	-0,64971	-0,04349	0,02031
430,0057	0,23337	0,43018	0,5551	-0,87943	-0,05612	0,02728
434,0048	0,4425	0,60561	0,87212	-0,99502	-0,04562	0,02737
438,0041	0,65701	0,76574	1	-0,95136	-0,01665	0,02027
440,0039	0,75429	0,83439	0,96687	-0,86632	0,0012	0,01457
442,0038	0,83938	0,89291	0,86668	-0,74217	0,01887	0,008
448,0038	0,99077	0,994	0,24425	-0,19492	0,05353	-0,01122
449,6706	0,99998	1	0,02296	-0,01765	0,05508	-0,01481
450,0039	1	1	-0,0218	0,01805	0,0549	-0,01537
452,0041	0,98733	0,9916	-0,28621	0,23018	0,05043	-0,01737
454,0041	0,95324	0,96901	-0,52969	0,43153	0,04057	-0,01668
460,0039	0,73957	0,82359	-0,97609	0,88352	-0,0093	0,00575
462,0037	0,64067	0,75342	-0,99799	0,96163	-0,02699	0,0211
464,6699	0,49794	0,64782	-0,92535	1	-0,04636	0,04852
466,003	0,425	0,59129	-0,85052	0,99109	-0,05321	0,06547
472,0016	0,13075	0,32898	-0,34028	0,75475	-0,05312	0,17487
476,001	0,01592	0,17307	-0,05502	0,49298	-0,02654	0,29131
484,0008	-0,01606	-0,01768	0,05891	0,05178	0,28968	0,68341
488,001	-0,01369	-0,05665	0,07129	-0,07114	0,65616	0,89527
490,0011	-0,01207	-0,06553	0,07396	-0,1085	0,8165	0,96563
492,6679	-0,00967	-0,06854	0,0724	-0,13583	0,95106	1
494,0013	-0,00843	-0,06694	0,06933	-0,14089	0,97474	0,98891
498,0015	-0,00486	-0,05356	0,05235	-0,12905	0,86337	0,84975
500,0016	-0,00332	-0,04412	0,04108	-0,11264	0,72405	0,73538

Appendix F-Current profile

Return period values for ULS design Check		
1 (m/s)	10 (m/s)	100 (m/s)
0,442	0,502	0,557

Turbulence intensity; I_c	0,01
Measurement ref. Height; z_r [m]	3,0
On-bottom roughness, z_0 [m]	5,0E-06
Number of discrete directions	12
Number of discrete current measurements (max 20)	15
Time between independent current events [hour]	1
Reduction factor R_c	0,96

Uc Weibull pdf

Direction relative to geographic N	Sector probability	Weibull parameters F(x)=1-exp(-((x-γ)/α)^β) Shape (β)	Scale (α)	Location (γ)	Statistics mean value	CoV	Return period (years) 1 (m/s)	10 (m/s)	100 (m/s)
Omni	1	1,868	0,150	-0,028	0,105	0,702	0,46	0,52	0,58
0	0,126	1,596	0,118	-0,010	0,096	0,706	0,39	0,47	0,54
30	0,066	1,273	0,117	-0,013	0,096	0,897	0,49	0,63	0,75
60	0,06	1,450	0,114	-0,016	0,087	0,833	0,39	0,48	0,57
90	0,044	1,586	0,098	-0,015	0,073	0,777	0,29	0,35	0,42
120	0,045	1,490	0,084	-0,010	0,066	0,781	0,27	0,34	0,40
150	0,045	1,764	0,124	-0,022	0,089	0,728	0,32	0,39	0,45
180	0,035	1,632	0,111	-0,014	0,085	0,734	0,31	0,38	0,45
210	0,044	1,540	0,105	-0,013	0,082	0,764	0,32	0,40	0,47
240	0,053	1,750	0,133	-0,024	0,094	0,738	0,35	0,43	0,49
270	0,071	1,846	0,150	-0,031	0,103	0,730	0,38	0,45	0,52
300	0,201	2,959	0,240	-0,076	0,138	0,569	0,40	0,44	0,48
330	0,21	2,387	0,170	-0,033	0,118	0,570	0,36	0,41	0,45

Appendix G-Wave profile

Return period values for ULS design Check			
	1	10	100
H_S	11,43	13,45	15,35
T_p	15,57	16,36	17,02

Peakedness parameter i Wave Spectrum γ;	0,00
Wave Speading Constant	8,0
Number of discrete directions	4
Number of discrete H_s values (<20)	15
Number of discrete T_p values (<20)	19
Time between independent sea-states [hour]	3
Reduction factor R_D	0,95

Scatter Hs-Tp

direction	omni	E[H_s]	CoV	σ	δ	κ	Shape (β)	Scale (α)	Location (γ)	H_s (1 year)	H_s (10 year)	H_s (100 year)
sector probability	1,00	3,176	0,499	1,584	1,153	4,693	1,434	2,464	0,938	11,43	13,45	15,35

H_S \ T_p	3,0	4,0	5,0	6,0	7,0	8,0	9,0	10,0	11,0
1	5,9E-04	4,0E-03	1,1E-02	1,6E-02	1,6E-02	1,4E-02	9,8E-03	6,4E-03	4,0E-03
2	9,0E-05	2,1E-03	1,2E-02	3,2E-02	5,1E-02	5,8E-02	5,3E-02	4,1E-02	2,9E-02
3	0,0E+00	8,0E-05	1,5E-03	8,3E-03	2,3E-02	3,9E-02	4,7E-02	4,5E-02	3,5E-02
4	0,0E+00	0,0E+00	6,0E-05	8,5E-04	4,8E-03	1,4E-02	2,4E-02	3,0E-02	2,8E-02
5	0,0E+00	0,0E+00	0,0E+00	4,0E-05	5,7E-04	3,2E-03	9,0E-03	1,6E-02	1,9E-02
6	0,0E+00	0,0E+00	0,0E+00	0,0E+00	3,0E-05	3,9E-04	2,1E-03	5,7E-03	9,5E-03
7	0,0E+00	0,0E+00	0,0E+00	0,0E+00	0,0E+00	2,0E-05	2,7E-04	1,4E-03	3,5E-03
8	0,0E+00	0,0E+00	0,0E+00	0,0E+00	0,0E+00	0,0E+00	2,0E-05	2,0E-04	8,8E-04
9	0,0E+00	0,0E+00	0,0E+00	0,0E+00	0,0E+00	0,0E+00	0,0E+00	2,0E-05	1,5E-04
10	0,0E+00	0,0E+00	0,0E+00	0,0E+00	0,0E+00	0,0E+00	0,0E+00	0,0E+00	2,0E-05
11	0,0E+00	0,0E+00	0,0E+00	0,0E+00	0,0E+00	0,0E+00	0,0E+00	0,0E+00	0,0E+00
12	0,0E+00	0,0E+00	0,0E+00	0,0E+00	0,0E+00	0,0E+00	0,0E+00	0,0E+00	0,0E+00
13	0,0E+00	0,0E+00	0,0E+00	0,0E+00	0,0E+00	0,0E+00	0,0E+00	0,0E+00	0,0E+00
14	0,0E+00	0,0E+00	0,0E+00	0,0E+00	0,0E+00	0,0E+00	0,0E+00	0,0E+00	0,0E+00
15	0,0E+00	0,0E+00	0,0E+00	0,0E+00	0,0E+00	0,0E+00	0,0E+00	0,0E+00	0,0E+00

12,0	13,0	14,0	15,0	16,0	17,0	18,0	19,0	21,0	22,0
2,3E-03	1,3E-03	7,4E-04	4,1E-04	2,2E-04	1,2E-04	7,0E-05	4,0E-05	2,0E-05	2,0E-05
1,8E-02	1,1E-02	6,3E-03	3,6E-03	1,9E-03	1,1E-03	5,6E-04	3,0E-04	1,6E-04	1,7E-04
2,5E-02	1,5E-02	9,0E-03	5,0E-03	2,6E-03	1,4E-03	6,7E-04	3,3E-04	1,6E-04	1,5E-04
2,2E-02	1,4E-02	8,5E-03	4,6E-03	2,3E-03	1,1E-03	5,0E-04	2,2E-04	1,0E-04	7,0E-05
1,7E-02	1,2E-02	7,5E-03	4,0E-03	1,9E-03	8,4E-04	3,5E-04	1,3E-04	5,0E-05	3,0E-05
1,1E-02	8,9E-03	5,8E-03	3,1E-03	1,4E-03	5,8E-04	2,1E-04	7,0E-05	2,0E-05	1,0E-05
5,3E-03	5,3E-03	3,9E-03	2,2E-03	9,8E-04	3,7E-04	1,2E-04	4,0E-05	1,0E-05	0,0E+00
2,0E-03	2,6E-03	2,3E-03	1,4E-03	6,4E-04	2,3E-04	7,0E-05	2,0E-05	0,0E+00	0,0E+00
5,4E-04	1,0E-03	1,1E-03	7,8E-04	3,9E-04	1,4E-04	4,0E-05	1,0E-05	0,0E+00	0,0E+00
1,1E-04	3,0E-04	4,5E-04	3,9E-04	2,2E-04	8,0E-05	2,0E-05	1,0E-05	0,0E+00	0,0E+00
2,0E-05	7,0E-05	1,5E-04	1,6E-04	1,1E-04	5,0E-05	1,0E-05	0,0E+00	0,0E+00	0,0E+00
0,0E+00	1,0E-05	4,0E-05	6,0E-05	5,0E-05	2,0E-05	1,0E-05	0,0E+00	0,0E+00	0,0E+00
0,0E+00	0,0E+00	1,0E-05	2,0E-05	2,0E-05	1,0E-05	0,0E+00	0,0E+00	0,0E+00	0,0E+00
0,0E+00	0,0E+00	0,0E+00	0,0E+00	1,0E-05	0,0E+00	0,0E+00	0,0E+00	0,0E+00	0,0E+00
0,0E+00	0,0E+00	0,0E+00	0,0E+00	0,0E+00	0,0E+00	0,0E+00	0,0E+00	0,0E+00	0,0E+00

Appendix H-PDF for current using omnidirectional values

uc	pdf	RM(inline)*10	RM(cross-flow)*4
-0,008	2,066	0,000	0,000
0,011	3,559	0,000	0,000
0,030	4,624	0,000	0,000
0,050	5,259	0,000	0,000
0,069	5,492	0,000	0,000
0,088	5,378	0,000	0,000
0,108	4,999	0,000	0,000
0,127	4,440	0,000	0,000
0,146	3,787	0,000	0,000
0,166	3,111	0,000	0,000
0,185	2,469	0,000	0,000
0,204	1,896	0,000	0,000
0,223	1,410	0,000	0,000
0,243	1,018	0,080	0,000
0,262	0,713	0,162	0,000
0,281	0,486	0,244	0,000
0,301	0,322	0,326	0,000
0,320	0,207	0,408	0,000
0,339	0,130	0,490	0,005

0,359	0,079	0,572	0,133
0,378	0,047	0,654	0,261
0,397	0,027	0,736	0,390
0,417	0,016	0,818	0,518
0,436	0,009	0,900	0,634
0,455	0,005	0,982	0,743
0,475	0,002	0,984	0,852
0,494	0,001	0,984	0,962
0,513	0,001	0,984	1,071
0,532	0,000	0,983	1,180
0,552	0,000	0,983	1,290
0,571	0,000	0,983	1,399
0,590	0,000	0,982	1,508
0,610	0,000	0,982	1,617
0,629	0,000	0,982	1,727
0,648	0,000	0,982	1,836
0,668	0,000	0,981	1,945
0,687	0,000	0,981	2,055
0,706	0,000	0,981	2,164
0,726	0,000	0,980	2,273
0,745	0,000	0,980	2,382
0,764	0,000	0,980	2,492
0,784	0,000	0,979	2,601
0,803	0,000	0,979	2,710
0,822	0,000	0,979	2,820
0,841	0,000	0,979	2,929
0,861	0,000	0,978	3,038
0,880	0,000	0,978	3,148
0,899	0,000	0,978	3,257
0,919	0,000	0,977	3,366
0,938	0,000	0,784	3,475
0,957	0,000	0,374	3,585
0,977	0,000	0,000	3,694
0,996	0,000	0,000	3,803
1,015	0,000	0,000	3,913
1,035	0,000	0,000	4,022
1,054	0,000	0,000	4,131
1,073	0,000	0,000	4,240
1,093	0,000	0,000	4,350
1,112	0,000	0,000	4,459
1,131	0,000	0,000	4,568

References

[1]Pipelines and Risers (Yong Bai 2001)

[2] Free spanning pipelines, Recommended practice, DNV-RP-F105, Det Norske Veritas, Høvik, Norway

[3] Submarine pipeline systems, Offshore standard DNV-OS-F101, Det Norske Veritas, October 2007, Norway

[4] Mikael W. Braestrup, Design and Installation of Marine Pipelines, Blackwell Science Ltd, 2005

[5] Xu T. Fatigue of free spanning pipeline. Report to J P Kenny A/S, Tao Xu and Associates, Antioch,CA 94509, 1997.

[6] Kenny JP A/S. Force model and in-line fatigue of free-spanning pipelines in wave dominantsituations. Internal Document No. 8500.70.

[7] Mørk KJ, Fyrileiv O. An introduction to DNV guideline for free spanning pipelines.Proceedings ofOMAE98, 1998.

[8] Rune Yttervik, Carl M. Larsen, Gunnar K. Furnes. Fatigue from vortex-induced vibrations of free span pipelines using statistics of current speed and direction. OMAE2003-37223, June 8-13, 2003, Cancun Mexico.

[9] SveinSævik, Ole David Økland,GroSagliBaarholm and Janne K.Ø. Gjøsteen. SIMLA Version 3.14.0User Manual, August 20, 2010

[10] Sævik, Sævik. Simla - Theory Manual.Revised as of 2008-06-06, MARINTEK, Trondheim, Norway.

[11] Giertsen, Taby, Økland. SIMLAQuick Start User Guide, Revision 02 / 2010-08-20

[12] H.S. Choi. Free spanning analysis of offshore pipelines, 2000, Department of Naval Architecture and Ocean Engineering, Pusan National University, Pusan 609-735, South Korea.

[13] DNV (2005), “Recommended Practice DNV-RP-C203 Fatigue Design of Offshore Steel Structures”, Det Norske Veritas.

[14] DNV (2003), FATFree user manual, 2003-0511[S].

[15] Jams WANG, F.Steven WANG, Gang DUAN and Paul JUKES. VIV analysis of pipelines under complex span conditions. J P Kenny, Inc, 15115 Park Row, 3rd Floor, Houston, TX, 77084 USA.

[16] Ai Shang-mao and SUN Li-ping. The effect of functional loads on free spanning pipeline's VIV response. Harbin 150001. China

[17] Carl M. Larsen, Kamran Koushan, Elizabeth Passano. Frequency and Time Domain Analysis of Vortex Induced Vibrations For Free Span Pipelines. Draft paper, OMAE 2002-28064.

[18] Huang Xiao-guang, Xu Jin-quan. Numerical Procedure for Static and Dynamic Analysis of Fluid-Conveying Submarine Pipeline Span on Linear Elastic Seabed. Shanghai Jiaotong University, Shanghai 200240, China.

[19] Xu Jishang, Li Guangxue, Juan J. Horrillo, Yang Rongmin and CAO Lihua. Calculation of Maximum Allowable Free Span Length and Safety Assessment of the DF1-1 Submarine Pipeline. Ocean University of China, Science Press and Springer-verlag Berlin Heidelberg 2010.

[20] Kosar Rezazadeh**,** Liyun Zhu, Yong Bai, Liang Zhang. Fatigue Analysis of Multi-spanning Subsea Pipelines. OMAE2010-20847

[21] Tore Soreide and Gunnar Paulsen, Finn Gunnar Nielsen. Parameter Study of Long Free Spans.

[22] James Wang, Jianfeng Xu, Paul Jukes. Cost-Effective Span Analysis Methodology For Different Pipeline Applications, OMAE2010-20570.

[23] Ali Valipour, Mehrdad Zoroufi, Abbas Yeganeh Bakhtiary and Reza Valipour. Wave and current induced comulative fatigue damage assessment for offshore pipeline on free-span. OMAE2008-57317