NTNU – Trondheim
Norwegian University of
Science and Technology

Stiffened plates - Reduced Stress Method

Beregninger for platefelt - Alternativ metode

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Submission date: July 2013
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Abstract

Comparison of Effective width method and Reduced stress method is performed based on design procedures given in NS EN 1993-1-5

The two methods lead to exactly equal critical buckling stress and strength for uniaxially loaded plates and no significant difference is observed between hand calculated results and results of the numerical analysis.

Analysis of biaxially loaded plates showed that results of Reduced stress method and that of finite element analysis are very close for square plates, but not for rectangular plates. Reduced stress method gives 10 % - 20 % lower buckling stress and 6 % - 14 % lower strength values compared with results from finite element analysis.

Hand calculations show that both methods lead to equal critical buckling stress for stiffened plates. But Reduced stress method gives a very conservative strength results. It was found to be about 10 % difference between the two methods when applied on relatively narrow and long stiffened plates. Strength results based on Effective width method for stiffened plates are generally lower than results of numerical analysis. The difference is especially very large for both short and wide plates, since effect of column-like buckling is taken into consideration. Hence the interpolation equation used in the design process is not good enough to capture the real behavior of such stiffened plates

Sammendrag

Sammenligning av Effektiv bredde metoden og Redusert spenningsmetoden er gjort basert på dimensjoneringsregler gitt i NS EN- EN 1993-1-5.

Begge metodene gir lik knekkspenning og kapasitet for en- aksialt belastede, uavstivede plater. Dessuten vises det ingen signifikante forskjeller mellom håndberegnede resultater og numeriske analyser.

Analysen av bi – aksialt belastede plater viser at Redusert spennings metode og finite element metode samsvarer veldig godt for kvadratiske plater, men ikke for rektangulære plater. Redusert spennings metode gir 10 % - 20 % mindre knekkspenning og 6 % - 14 % mindre kapasitet i forhold til analysen som er gjort med Abaqus.

For avstivede plater gir begge håndberegningsmetodene lik knekkspenning. Men Redusert spenningsmoden gir en veldig konservativ kapasitet. Det ble observert at Redusert spenningsmetoden gir ca. 10 % mindre kapasitet for smale og lengre avstivede plater.

Effektiv beredde metoden gir generelt mindre kapasitet sammenlignet med resultatet fra den numeriske analysen. Avviket er spesielt stort for relativ korte eller breie plater, siden effekten av søyle-lik knekking er tatt i betrakting. Dette viser at interpolasjons likningen som er brukt i beregningene er ikke god nok til å fange oppførselen av avstivede plater

1. Bakgrunn

Plater med stivere inngår i store bjelker, broer, beholdere, skip og andre større konstruksjoner. Omfattende beregningsregler er gitt stål- og aluminiumsstandarden. Beregning av oppførsel og kapasitet til plater med stivere er godt egnet for simuleringer med datamaskinprogrammer, og beregningsresultater herfra kan gi god støtte til den eksisterende erfaringen på området og forståelsen av de reglene som er stilt opp. I Eurocode 3, del 1-5, er det beskrevet en alternativ metode "Reduced stress method" som skal undersøkes, og sammenlignes med de vanlige beregningsreglene for platefelt. Oppgaven omfatter litteraturundersøkelse, sammenligninger av regler, og numeriske simuleringer for utvalgte geometrier av avstivede plater. Arbeidet kan benytte resultater og eksempler fra tidligere studentarbeider for platefelt med stivere, og eksempler fra utførte konstruksjoner fra ulike prosjekter.

2. Gjennomføring

Oppgaven kan gjennomføres med følgende elementer:

- Redegjøre for beregningsreglene for uavstivede og avstivede plater med aksialbelastning.
- Se spesielt på den alternative metoden med «Reduced Stress Method» (NS EN 1993-1-5), og forklare dens teoretiske bakgrunn. Stikkord her er elastisk knekking, både lokal knekking (plate mellom stivere) og global knekking (platen med stivere knekker sammen), og modeller for kapasitetsberegninger.
- Gjøre håndberegninger med utvalgte konstruksjonseksempler for uavstivede og avstivede plater (platefelt), og sammenligne kapasiteten funnet med de ulike metodene.
- Etablere FE modeller av platefelt, med modellering av realistiske materialegenskaper og vanlig forekommende størrelser for formfeil/imperfeksjoner, og finne platefeltenes kapasitet. Sammenligning med håndberegninger.
- Konklusjoner vedrørende «Reduced Stress Method». Fordeler og ulemper. Er metoden et nyttig alternativ i dimensjoneringsarbeidet for en praktisk konstruksjon?

Kandidaten kan i samråd med faglærer velge å konsentrere seg om enkelte av punktene i oppgaven, eller justere disse.

Kommentar:

Opprinnelig var målet av oppgaven å fokusere på alternativ metoden som også er kalt Redusert spenningsmetode. Men det viste seg at det ikke var lett å finne relevant litteratur som er skrevet på norsk eller engelsk.

Veideler har kontaktet Dr. –Ing. Braun, som har forsket mye på dette området, og spurt om relevant litteratur. Som svar fikk vi at Redusert spennings metoden er brukt mye i Tyskland og derfor er mye av litteraturen skrevet på tysk. Svar av Braun er vist under.

Etter diskusjon med veileder, ble målet av oppgaven endret til sammenligning av dimensjoneringsmetodene gitt i NS-EN 1993 -1-5.

……………………. mailsvar fra Braun………………………..
Benjamin Braun <braun@spacestructures.de>
to Arne, derik, me

Hello Arne,

the reduced stress method is a German approach so that almost everything is published in German. It evolved since the 1950s and has been already incorporated in former standards DIN 4114 and DIN 18800 in a similar way.

In principle relevant literature is referenced in Sections 2.3.4.2 and 2.3.2 of my thesis. Further useful information with examples can be found in this book
http://www.amazon.de/18800-Teil-Beuth-Kommentar-Erlauterungen/dp/3433014043

If you look especially for the calibration of alpha-cr I recommend the thesis of Christian Müller, RWTH Aachen, see reference [84] in my thesis. Unfortunately it has been only available as printed copy as far as I know.

To get a broader perspective I recommend to look also into the background of shell buckling. There are some interesting publications by Thomas Winterstetter, Universität Essen, which address also topics of the reduced stress method for plate buckling.

Hope this helps.
BR Benjamin

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Preface

This thesis has been written as a final part of my study at Norwegian university of science and technology - Department of structural engineering.

The report focuses on design methods of thin steel plates given on Euro code-3-1-5. In Norway Reduced stress method is hardly used method, even it is not a commonly lectured topic at NTNU.

It was my intention to increase the familiarity of Reduced stress method by comparing it with the Effective width method. Simple calculations are performed on regular plates to show the difference of the two methods and how they are applied. I hope this report will turn out to be very useful for anybody who is interested in this topic.

At last I would like to express my appreciation and gratitude to my adviser, associate professor Arne Aalberg for his encouragement, suggestions and guidance.

July 2013, Trondheim

Abdirakib Mohammed Derik

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1. Introduction

Structures composed of thin plates members are widely used in steel constructions. It becomes increasingly a common practice to use thin plates in construction of bridges, ships and other marine structure. Higher aesthetic value, economic reasons and the availability of advanced productions techniques make the use of thin plates ideal. But thin walled steel plates are very susceptible to buckling.

Unlike columns a compressed plate can carry loading after it has buckled. Since the buckled part usually loses its stiffness, stresses will concentrate along the supported edges leading to a nonlinear stress distribution pattern.

It is inconvenient to deal with the nonlinear stress distribution in a practical design situation. Actual stress condition is usually idealized to an equivalent uniformly distributed stress situation which makes the design process easier. NS-EN 1993-1-5 provides two alternative methods of treating slender plated structure which are; The Effective width method and the Reduced stress method. Both alternatives help simplify the actual nonlinear stress situation to an equivalent uniformly distributed stress condition.

Effective width method involves reduction of cross section area of plated structures subjected to buckling. Reduced stress method also simplifies the actual stress distribution to an idealized form, but instead of reducing area, it limits the stress level. In other words, the nonlinearly distributed stress is replaced by a uniform average stress level.

The common design method used in Norway is the Effective width method. The Reduced stress method is hardly used as a design tool.

The thesis will focus on comparing the two methods to shed a light on differences, advantages and disadvantages of the two methods.

The original aim of the study was to mainly concentrate on Reduced stress method, especially on its background information, development of its design criteria …etc. But unfortunately it was not possible to find a relevant literature written in English or Norwegian. Most of the literature related to this topic was written in German or other languages. Therefore the aim of the thesis is modified to comparison of the two alternative methods based on the design criteria given in NS-EN 1993-1-5.

To study the difference between the two methods both stiffened and unstiffened rectangular plates are used. Both uniaxial loading and biaxial loading are applied to unstiffened plates; whereas only uniaxial loading is applied to the stiffened plates.

Critical buckling stress and design strength of plates with selected dimension are manually calculated based on the two methods. All plates which are hand calculated are also analyzed using the multipurpose soft ware, Abaqus.

2. Literature review

2.1. Braun (2010)

Braun (2010) studied stability of plates under combined loading. He especially focused on buckling of steel plates under biaxial compression and I-girder webs with transverse patch loading combined with shear stress.

Biaxially loaded plates are analyzed based on the Reduced stress method given by NS-EN 1993-1-5. A large discrepancy was observed between results of Reduced stress method and that of proven rules of DIN 1889-3 as shown on Figure 2-1. Furthermore NS-EN 1993-1-5 does not provide any interaction formula when combined transverse patch loading and shear stress are acting on I-girder webs.

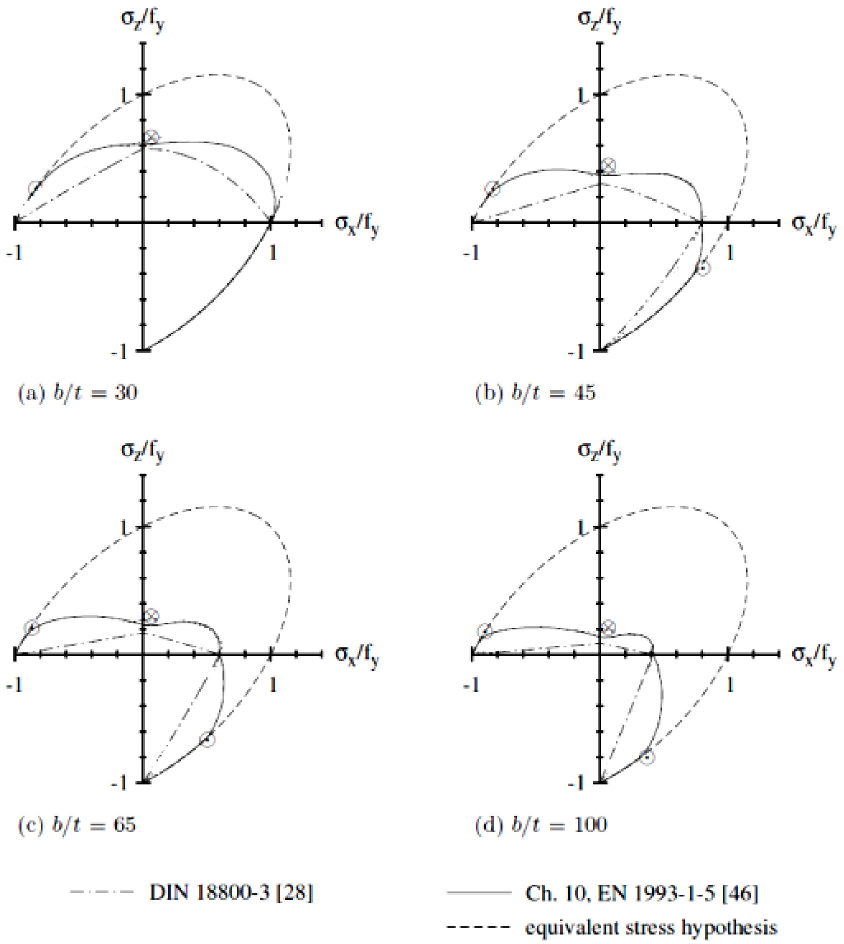


Figure 2-1 Comparison of Reduced stress method and DIN 18800-3, Braun (2010)

The main objective of the study was to improve these limitations of the current rules of the NS-EN 1993-1-5 and provide a more suited interaction equation.

Previous experimental works on biaxially loaded plates were restudied by using nonlinear finite element analysis. The main parameters focused during the finite element analysis were:

- Slenderness, $\frac{b}{t}$ ratio $= 30, 45, 65$ and 100
- Panel aspects ratio $\alpha = 1$ and 3
- Imperfection shape and amplitude
- Edge boundary conditions, both in-plane and rotational

Simulation results were compared with results of the Reduced stress method and the following interaction equation is proposed to describe the stability behavior of biaxially loaded plates.

$$\left(\frac{\sigma_{x,Ed}}{\sigma_{x,Rd}}\right)^2 + \left(\frac{\sigma_{z,Ed}}{\sigma_{z,Rd}}\right)^2 - \rho_x \cdot \rho_z \left(\frac{\sigma_{x,Ed} \cdot \sigma_{z,Ed}}{\sigma_{x,Rd} \cdot \sigma_{z,Rd}}\right) \leq 1$$

The study of I-girder web under combined shear and patch loading is not relevant for this thesis and will not be discussed further.

2.2. Master thesis: Sandstad, K (2004)

Kathrin Sandstad (2004) restudied a stiffened plate which was previously studied by B. Johansson and R. Maquoi (2002). In separate articles both Johansson and Maquoi (2002) presented capacity of longitudinally stiffened plates. Their results have shown that the strength curve dropped down and then raised up as the length of the plate panel was increased as shown on Figure 2-2.

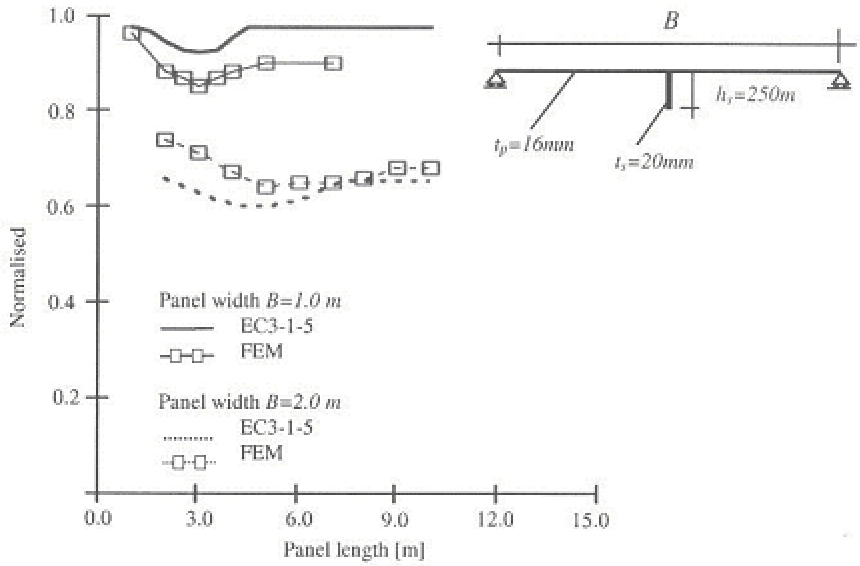


Figure 2-2 Normalized resistance of plate with single stiffener, Sandstad(2004)

Sandstad argued that it is a strange that shorter plates have lower capacity than longer plates, when all other factors are kept constant. She studied a longitudinally stiffened plate with a single stiffener.

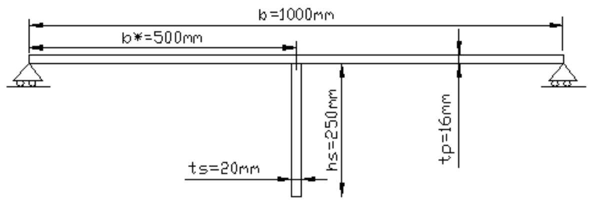


Figure 2-3 Plate geometry studied, Sandstad (2004)

She analyzed a single stiffened plate using Abaqus soft ware. Width and thickness were kept constant but length was varied from 1 m to 6 m. The plate was modeled with shell elements while the stiffener was modeled using two nodded beam elements.

Imperfection was introduced to the nonlinear analysis using mode shapes from buckling analysis.

Results of Sandstad (2004) didn't showed, the typical strength drop which was previously reported by Maquoi (2002) and B. Johansson as the length of the plate was increased.

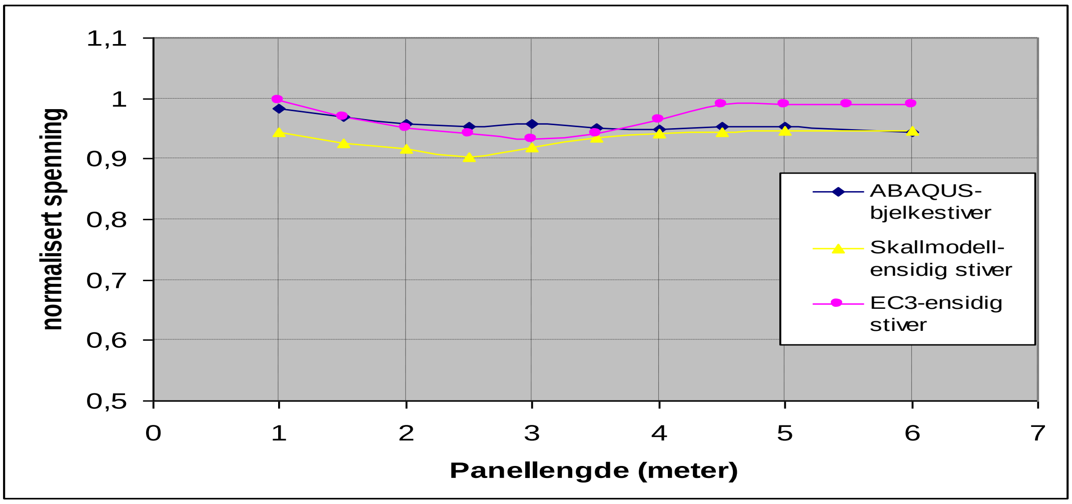


Figure 2-4 Comparison of strength analysis, Sandstad (2006)

2.3. Master thesis: Kleppe (2006)

Kleppe (2006) studied buckling and strength of steel plates using the rules of Eurocode 3-1-5, Eurocode -9-1-1 and the multipurpose finite element analysis soft ware, Abaqus.

Both unstiffened and stiffened plates were considered and studied. Both width and length of the plates were let to vary. This was done to see the effect of plate-like and column-like buckling.

Numerical analysis was performed by modeling the plates with shell elements, where as stiffeners were modeled using two nodded beam elements. Both linear buckling analysis and numerical strength analysis were performed.

A number of plate dimensions both unstiffened and stiffened were analyzed manually and numerically and results of both methods were compared.

For unstiffened plates, NS-EN-1993-1-5 states that a column-like buckling should be considered if the aspects ratio, $\alpha = (a/b) < 1$, but Kleppe (2006) has showed that consideration of column-like buckling is only necessary when $\alpha = (a/b) < 0{,}644$.

Moreover it was observed that strength results obtained by interpolating between column and plate-like buckling didn't match with results of the numerical analysis.

3. Elastic buckling of plates

The need to optimize materials and reduce self weight has led to an increased use of thin steel plates. Thin plates are commonly used in bridges, offshore, ship structures and aerospace structures. But the downside is, thin plates are very susceptible to buckling when loaded in compression. Buckling is a stability problem which causes a sudden out of plane deformation of the structure.

Unlike columns plates can carry loading beyond the buckling point due to redistribution of compressive stress. For a uniaxially loaded rectangular plate, it is the central longitudinal strip that buckles first. Since the buckled central strip has lost its stiffness, stresses will be redistributed along the supported edges. The plate will continue to carry the loading until the supported edges reach yielding stress.

The general critical buckling stress is expressed as:

$$\sigma_{cr} = k_\sigma \cdot \frac{\pi^2 E}{12(1-\upsilon^2)} \cdot \left(\frac{t}{b}\right)^2 \qquad \textbf{(3.1)}$$

Where:

k_σ = buckling coefficient

t = thickness

b = width of the plate

υ = Poisson's ratio

Analytical derivation of the above equation may involve a very complex expression which could not be easily solved. But for regular geometries, a sufficiently accurate critical buckling stress could be determined based on differential equation of plate buckling.

3.1. Differential equation of plate buckling

Differential equation of plate buckling is derived based on the following assumptions:

- The plate is considered to be homogeneous, isotropic and linear elastic material (i.e. Hook's law is applicable).
- The plate has a perfect geometry
- Small deformations
- Thin plate : the thickness of the plate is very small compared to its width and length
- The plate has a constant thickness

Consider membrane forces, N_x ,N_y and N_{xy} expressed as force per unit length are applied to an isolated infinitesimal plate element with dimensions, d_x and d_y as showing on the Figure 3-1 below. The plate element is given an infinitesimal deformation, w, in the vertical direction and equilibrium of forces is established using the deformed configuration.

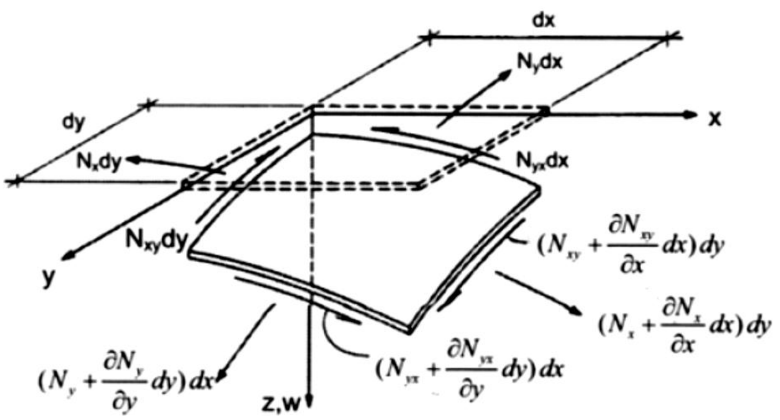


Figure 3-1 Membrane forces on deformed plate element dx.dy (Larsen, 2010)

All membrane forces shown on Figure 3-1 will attain a vertical component in Z-direction, due to rotation of forces relative to XY-plane as the plate deforms. Refer to Figure 3-2 and Figure 3-3 taken from Larsen (2010).

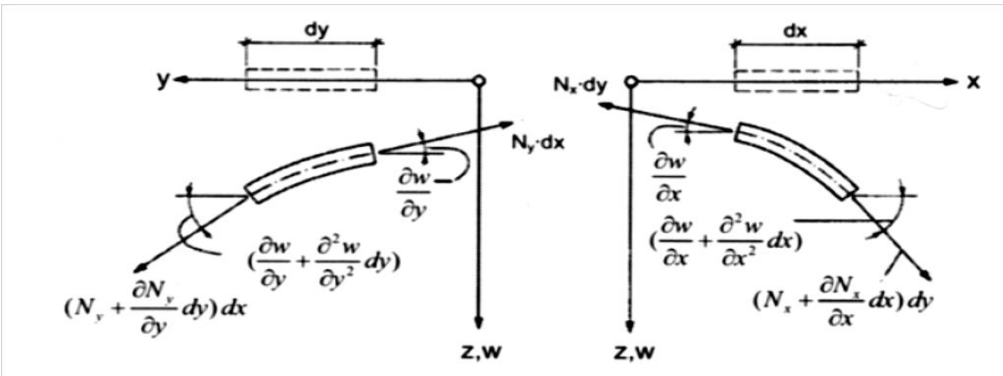


Figure 3-2 N_x **and** N_y **on a deformed plate element (Larsen, 2010)**

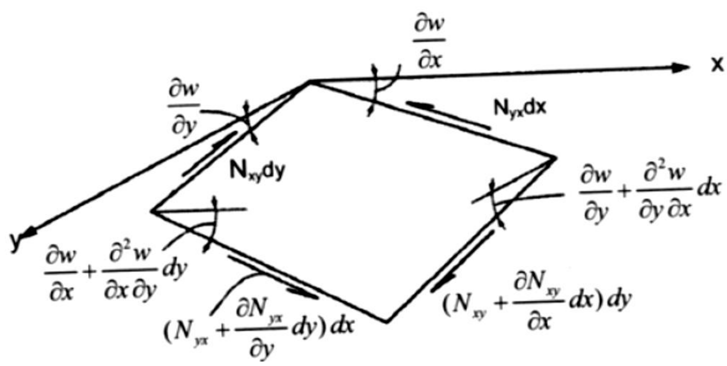


Figure 3-3 In-plane shear forces on a deformed plate element, (Larsen, 2010)

Now let's calculate the equilibrium conditions:

1) Equilibrium in the X- direction
$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{yx}}{\partial y} = 0 \qquad (3.2)$$

2) Equilibrium in Y-direction
$$\frac{\partial N_y}{\partial y} + \frac{\partial N_{xy}}{\partial x} = 0 \qquad (3.3)$$

3) Equilibrium in Z-direction: different forces contribute to equilibrium in the Z-direction:

Vertical component of N_x :

$$\left(N_x d_y + \frac{\partial N_x}{\partial x} dxdy\right)\left(\frac{\partial w}{\partial x} + \frac{\partial^2 w}{\partial x^2} dx\right) - N_x d_y \left(\frac{\partial w}{\partial x}\right)$$
$\Rightarrow$ If higher order term is omitted,

the expression is simplified as: $\Rightarrow \left(N_x \frac{\partial^2 w}{\partial x^2} + \frac{\partial N_x}{\partial x}\frac{\partial w}{\partial x}\right) d_x d_y$

Vertical component of N_y :

$$\left(N_y d_x+\frac{\partial N_y}{\partial y}d_y d_x\right)\left(\frac{\partial w}{\partial y}+\frac{\partial^2 w}{\partial y^2}d_y\right)-N_y d_x\left(\frac{\partial w}{\partial y}\right)$$ $\Rightarrow$ by omitting the higher order term,

the expression is simplified as: $\Rightarrow \left(N_y\frac{\partial^2 w}{\partial y^2}+\frac{\partial N_y}{\partial y}\frac{\partial w}{\partial y}\right)d_y d_x$

Vertical component of N_{xy} :

If the higher order term is neglected, the contribution of shear forces to the vertical equilibrium will be: $\Rightarrow \left(2N_{xy}\frac{\partial^2 w}{\partial x\partial y}+\frac{\partial N_{xy}}{\partial x}\frac{\partial w}{\partial y}+\frac{\partial N_{xy}}{\partial y}\frac{\partial w}{\partial x}\right)d_x d_y$

The sum of all vertical components of N_x, N_y and N_{xy} is considered to be equal to a uniformly distributed equivalent force, $q_{ekv}(x,y)$ acting in the Z-direction.

$$\left(N_x\frac{\partial^2 w}{\partial x^2}+\frac{\partial N_x}{\partial x}\frac{\partial w}{\partial x}\right)d_x d_y+\left(N_y\frac{\partial^2 w}{\partial y^2}+\frac{\partial N_y}{\partial y}\frac{\partial w}{\partial y}\right)d_y d_x+$$
$$\left(2N_{xy}\frac{\partial^2 w}{\partial x\partial y}+\frac{\partial N_{xy}}{\partial x}\frac{\partial w}{\partial y}+\frac{\partial N_{xy}}{\partial y}\frac{\partial w}{\partial x}\right)d_x d_y=q_{ekv}(x,y)d_x\,d_y$$

By rearranging we get:

$$\left(N_x\frac{\partial^2 w}{\partial x^2}+N_y\frac{\partial^2 w}{\partial y^2}+2N_{xy}\frac{\partial^2 w}{\partial x\partial y}\right)d_x d_y+\left(\frac{\partial N_x}{\partial x}+\frac{\partial N_{xy}}{\partial y}\right)\frac{\partial w}{\partial x}d_x d_y+\left(\frac{\partial N_y}{\partial y}+\frac{\partial N_{xy}}{\partial x}\right)\frac{\partial w}{\partial x}d_x d_y$$
$$=q_{ekv}(x,y)d_x\,d_y$$

From equations (3.2)and (3.3) above: $\frac{\partial N_x}{\partial x}+\frac{\partial N_{yx}}{\partial y}=0$ and $\frac{\partial N_y}{\partial y}+\frac{\partial N_{xy}}{\partial x}=0$

Then we get:

$$\left(N_x\frac{\partial^2 w}{\partial x^2}+N_y\frac{\partial^2 w}{\partial y^2}+2N_{xy}\frac{\partial^2 w}{\partial x\partial y}\right)=q_{ekv}(x,y) \qquad (3.4)$$

The uniformly distributed equilibrium force $q_{ekv}(x,y)$ can be determined by performing bending analysis and deriving the differential equation for plate bending.

Consider a uniformly distributed force; q(x, y) is applied to a rectangular plate as shown on Figure 3-4 below.

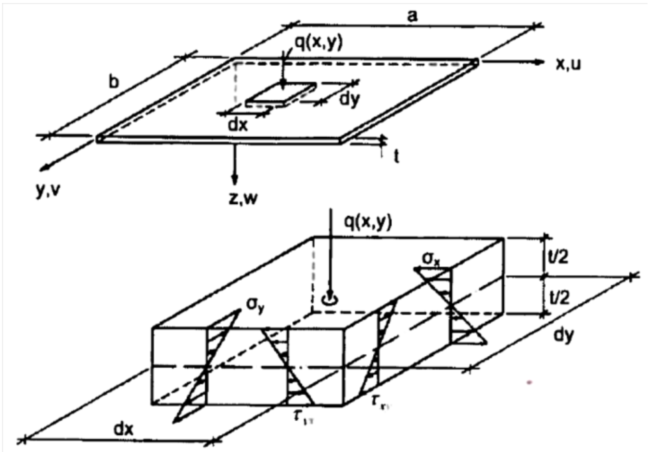


Figure 3-4 Stresses on rectangular plate loaded in bending, (Larsen, 2010)

Differential equation of plates in bending is given on eq. (3.5) shown below. For detailed derivation of the equation refer to Larsen,(2010).

$$\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2 \partial y^2}+\frac{\partial^4 w}{\partial y^4}=\nabla^4=\frac{q}{D} \quad (3.5)$$

Where: $D=\dfrac{Et^3}{12(1-\upsilon^2)}$

By inserting equation (3.4) into equation (3.5) we get:

$$\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2 \partial y^2}+\frac{\partial^4 w}{\partial y^4}=\nabla^4=\frac{\left(N_x\dfrac{\partial^2 w}{\partial x^2}+N_y\dfrac{\partial^2 w}{\partial y^2}+2N_{xy}\dfrac{\partial^2 w}{\partial x \partial y}\right)}{D}$$

Hence, the differential equation of plate buckling will be: (3.6)

$$\nabla_w^4 = \frac{1}{D}\left(N_x \frac{\partial^2 w}{\partial x^2} + N_y \frac{\partial^2 w}{\partial y^2} + 2N_{xy} \frac{\partial^2 w}{\partial x \partial y}\right)$$

3.2. Critical buckling load of a uniaxially loaded rectangular plate

Critical buckling force of longitudinally loaded plate shown on Figure 3-5 is determined by using the differential equation (3.6).

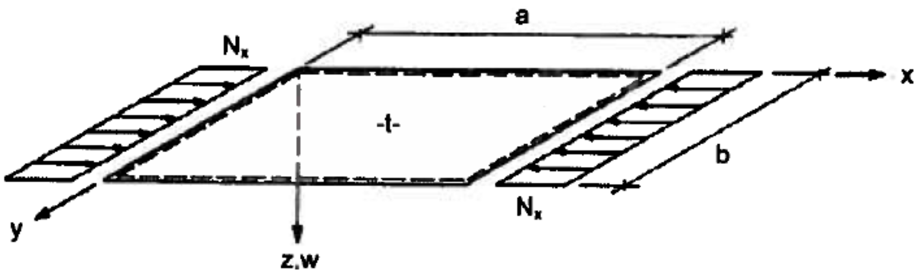


Figure 3-5 Simply supported plate with uniaxial loading (Larsen, 2010)

The differential equation for plate buckling:

$$\nabla_w^4 = \frac{1}{D}\left(N_x \frac{\partial^2 w}{\partial x^2} + N_y \frac{\partial^2 w}{\partial y^2} + 2N_{xy} \frac{\partial^2 w}{\partial x \partial y}\right) \text{, but } N_y \text{ and } N_{xy} = 0 \text{,}$$

$$\Rightarrow \nabla_w^4 + \frac{1}{D}\left(N_x \frac{\partial^2 w}{\partial x^2}\right) = 0$$

Since the plate is simply supported: $w = w,_{xx} = 0$ at x=0 & x=a (these are boundary conditions which should be satisfied).

According Larsen (2010), the following function will satisfy the boundary condition for the above plate.

$$w(x,y) = w_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \tag{3.7}$$

Inserting equation (3.7) into the differential equation, we get:

$$\left[\pi^4\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right) - \frac{N_x}{D}\left(\frac{m\pi}{a}\right)^2\right] w_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} = 0$$

Since $w_{nm} \neq 0$, for non-trivial solution, the expression in the parenthesis should be equal to zero.

$$\Rightarrow \left[\pi^4 \left(\frac{m^2}{a^2} + \frac{n^2}{b^2} \right) - \frac{N_x}{D} \left(\frac{m\pi}{a} \right)^2 \right] = 0 \qquad \Rightarrow N_x = N_{cr} = \frac{\pi^2 D}{b^2} \left(m\frac{b}{a} + \frac{n^2}{m}\frac{a}{b} \right)^2$$

where $D = \dfrac{Et^3}{12(1-\upsilon^2)}$

Substituting for flexural rigidity, D and dividing by thickness of the plate, t:

$$\sigma_{cr} = \left(m\frac{b}{a} + \frac{n^2}{m}\frac{a}{b} \right)^2 \frac{\pi^2 E}{12(1-\upsilon^2)} \left(\frac{t}{b} \right)^2 \qquad \textbf{(3.8)}$$

$$k_\sigma = \left(m\frac{b}{a} + \frac{n^2}{m}\frac{a}{b} \right)^2 \qquad \textbf{(3.9)}$$

Where:

n = no. of half sine-waves in the transverse direction

m = no. of half sine-waves in the longitudinal direction

b = width of the plate

a= length

t=thickness

$$\sigma_e = \frac{\pi^2 E}{12(1-\upsilon^2)} \left(\frac{t}{b} \right)^2 \qquad \textbf{(3.10)}$$

$$\sigma_{cr} = k_\sigma \cdot \sigma_e \qquad \textbf{(3.11)}$$

The Euler stress, σ_e depends only on material properties and is always constant for any given plate, but the buckling coefficient, k_σ depends on the loading situation, boundary conditions and plate geometry.

As stated on Larsen, (2010) the minimum critical buckling stress is only of interest for stability analysis. Critical buckling stress of a plate is minimum, when it buckles in a single half sine-wave in the transverse direction (i.e. When n =1 in equation (3.9) above).

Number of half sine-wave buckles in the longitudinal direction, m is equal to aspects ratio, α provided that α is an integer number.

The relationship between stress coefficient, k_σ and aspect ratio, α for longitudinally loaded, simply supported regular plate is shown on Figure 3-6 .

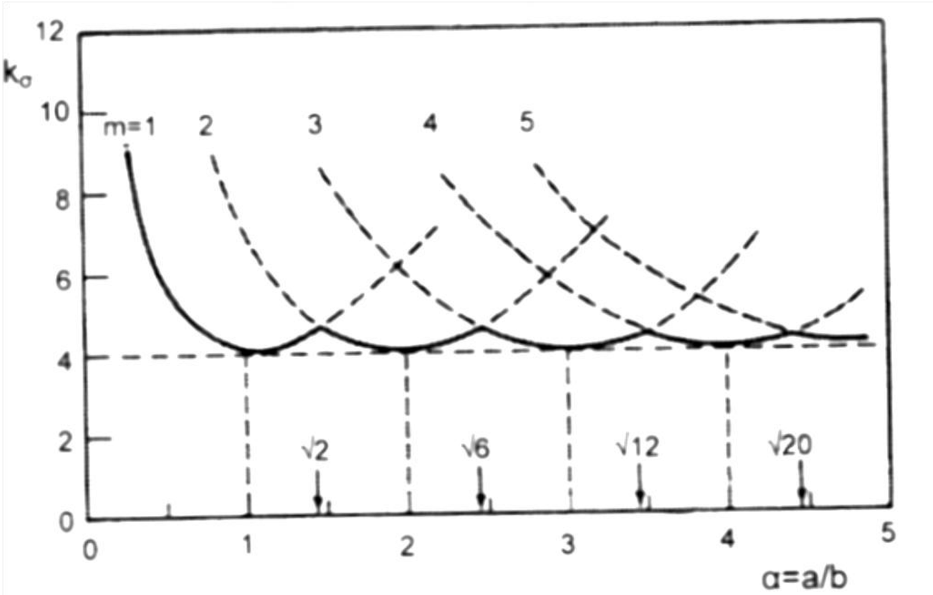


Figure 3-6 Stress coefficient, k_σ for simply supported plate with longitudinal loading (Larsen,2010)

In case of practical design, k_σ can be evaluated as:

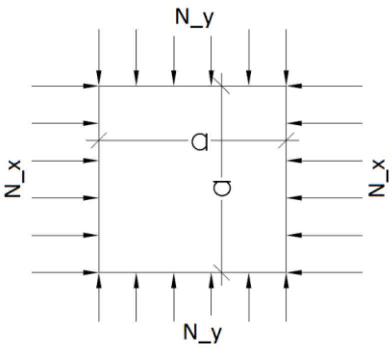
$$k_\sigma = 4 \quad \text{for } \alpha = \frac{a}{b} \geq 1$$

$$k_\sigma = \left(\alpha + \frac{1}{\sigma}\right)^2 \quad \text{for } \alpha = \frac{a}{b} < 1$$

3.3. Critical buckling load of biaxially loaded square plate

Analytical calculation of critical force for biaxial loading condition is only possible for square plates. Biaxially loaded Rectangular plates could only be analyzed numerically.

Critical buckling force for simply supported square plate shown could be calculated based on equation (3.6) given previously.



Her we assume $N_X = N_Y$ [N/mm]

Equal width and length = a square plate.

Equation (3.6) given as: $\nabla^4_w = \frac{1}{D}\left(N_x \frac{\partial^2 w}{\partial x^2} + N_y \frac{\partial^2 w}{\partial y^2} + 2N_{xy}\frac{\partial^2 w}{\partial x \partial y}\right)$ but

$N_{xy} = 0$ and $N = N_x = N_y$

$$\Rightarrow \nabla^4_w = \frac{1}{D}N\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right), \quad \nabla^4_w = \frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}$$

$$\Rightarrow \left(\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right) + \frac{1}{D}N\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) = 0 \quad (3.12)$$

As it was done for uniaxially loaded plates, we need a displacement function which could at least satisfy the boundary conditions.

The displacement function which satisfies the differential equation (3.12) and the required boundary conditions could be:

$$w(x,y) = \sum_{m=1} \sum_{n=1} w_{nm} \sin\frac{m\pi x}{b} \sin\frac{n\pi y}{b} \quad (3.13)$$

Where: m and n are number of half sine-wave buckles in the longitudinal and transverse direction respectively.

Differentiating equation (3.13) and inserting into the equation (3.12), we get:

$$\sum_{m=1}\sum_{n=1}\left[\frac{m^4\pi^4}{b^4}+2\frac{m^2n^2\pi^4}{b^4}+\frac{n^4\pi^4}{b^4}-\frac{12(1-\upsilon^2)N\pi^2}{Et^3b^2}\left(m^2+n^2\right)\right]w_{mn}\sin\frac{m\pi x}{b}\sin\frac{n\pi y}{b}=0 \quad \textbf{(3.14)}$$

$w_{nm} \neq 0 \quad \Rightarrow$ For nontrivial solution, the expression in the brackets should be equal to zero.

$$\Rightarrow \frac{m^4\pi^4}{b^4}+2\frac{m^2n^2\pi^4}{b^4}+\frac{n^4\pi^4}{b^4}-\frac{12(1-\upsilon^2)N\pi^2}{Et^3b^2}\left(m^2+n^2\right)=0$$

$$\Rightarrow N_{cr}=\frac{Et^3b^2}{12(1-\upsilon^2)\pi(m^2+n^2)}\left(\frac{m^4\pi^4}{b^4}+2\frac{m^2n^2\pi^4}{b^4}+\frac{n^4\pi^4}{b^4}\right)$$

According to Jones, (2006) critical force, N_{cr} is minimum when $m=n=1$, i.e. single half sine-wave buckle for both directions. Then the critical buckling force for the square plate will be:

$$N_{cr}=\frac{Et^3}{6(1-\upsilon^2)}\left(\frac{\pi}{b}\right)^2 \quad \textbf{(3.15)}$$

3.4. Cross sectional classes

NS-EN-1993-1-5 classifies plate cross sections depending on risk of buckling when loaded in compression. Slender cross sections may buckle long before the loading reach yielding stress, while others can carry compressive stress until plastic failure occurs. Based on the slenderness ratio, plate cross sections are classified in to the following four classes:

- Class 1: are usually compact cross sections with low value of slenderness ratio. Such cross sections can form plastic hinges and can carry the loading until plastic failure occurs.

- Class 2: this cross section is relatively compact and able to carry the loading into plastic zone. But it's not able to form plastic hinges with enough rotational capacity.
- Class 3: the stress in outer most fiber of the steel member can reach yield stress, but the cross section has a reduced capacity of plastification due to risk of buckling.
- Class 4: cross sections with high level of slenderness belong to this class. Such cross section buckles long before compressive stresses reach yielding value.

NS-EN-1993-1-5, table 5.2 helps determine cross sectional class of plated structures based on slenderness ratio, loading situation and support condition. If a cross-section is made up of multiple plates, like box profiles each and every individual member should be evaluated to determine its cross sectional class. Then the cross-section is classified according to least favorable class of its compression members as it is stated on NS-EN 1993-1-1: 5.5.2(6).

For cross sectional class 1-3, reduction of capacity to resist pure axial compression is done due to risk of global buckling, where as for class-4 load carrying capacity is limited due to risk of both local and global buckling.

3.5. Behavior of stiffened plates

Slender plates are usually stiffened to increase buckling strength. When compressive load applied to a stiffened plate reaches a critical level, the stiffened plate will buckle locally or globally.

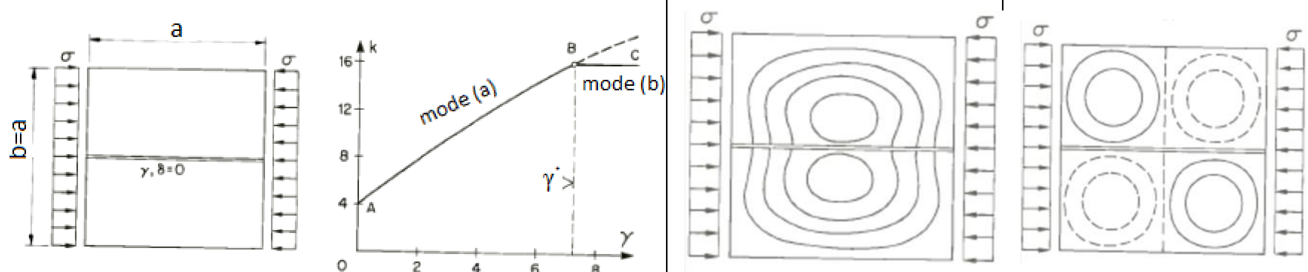
- Local buckling: during loading the stiffeners will stay straight and the plate panels between stiffeners will buckle. This occurs usually when the stiffeners are much stronger than the plate. If the web of the stiffener is very thin and high, the stiffener itself may buckle with the plate.
- Global buckling mode: the stiffener buckles together with plating. This happens when the stiffeners have small bending stiffness.

Increasing the rigidity of stiffeners will generally increase the stiffness of the whole stiffened plate until a certain limit is reached. After that limit, rigidity of stiffeners will not have any stabilizing effect on the plate. This situation is best described based

Figure 3-7, taken from Dubas, (1986).

As shown

Figure 3-7(b), buckling coefficient, k_σ of the plate increases until the rigidity of the stiffener reaches a certain limit, γ^*. Increasing rigidity of stiffener over that limit will not necessarily increase buckling strength of the plate, since the plate become susceptible to local buckling. At lower level of stiffener rigidity, the plate and stiffener buckle together (i.e. global buckling happens)



(a)

(b)

(c) buckling mode-a

Global buckling

(d) buckling mode-b

(local plate buckling)

Figure 3-7 Effect of increasing rigidity of stiffener on plate buckling mode

4. Resistance of plates against pure compression

A thin slender plate structure usually buckles before the compressive stresses reach its yielding stress. But the plate can carry loads long after it has buckled. Buckled plates will attain nonlinear form of stress distribution, since stresses concentrate along edges which are forced to remain straight due to the boundary condition.

Figure 4-1 Pattern of real stress distribution for uniaxially loaded plate

It is unpractical to deal with this nonlinear distribution of compressive stress when designing plated structures and design codes usually apply simplified design methods. NS-EN 1993-1-5 provides two alternative verification methods when designing plated structures.

- Effective width method (also called effective cross section method)
- Reduced stress method

4.1. Effective width method

As it is described above, real distribution of stresses of a buckled plate is nonlinear. For practical design, it is not easy to deal with such stress pattern. Therefore the real stress pattern is replaced with two constant stress blocks which are equivalent to the real stress situation. These two constant blocks are distributed over a reduced width of the plate denoted as effective width, b_{eff} as shown on Figure 4-2.

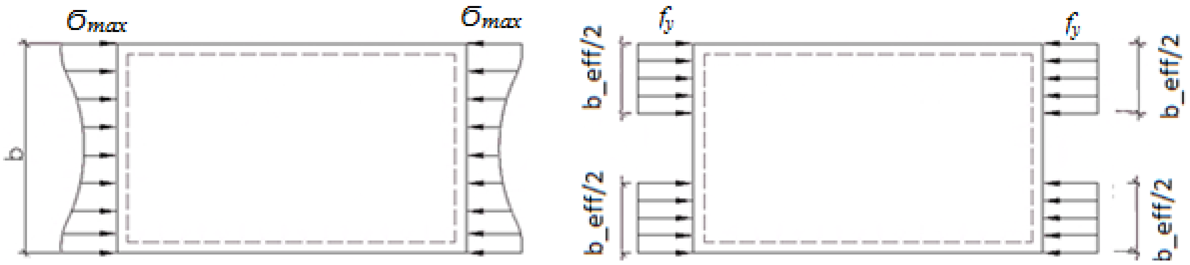


Figure 4-2 Real and simplified stress distribution pattern

The Effective width method is based on the hypothesis of Karman. Karman has proposed to replace the real plate, with a fictitious model plate having same boundary conditions as the real plate but with a reduced width, b_{eff} . According to Karman, effective width for simply supported uniaxially loaded rectangular plate is calculated as shown on eq. (4.1) below.

$$b_{eff} = b\left(\sqrt{\frac{\sigma_{cr}}{f_y}}\right) \Rightarrow b_{eff} = b\left(\frac{1}{\bar{\lambda}_p}\right) \quad \text{where } b = \text{width of the real plate} \qquad (4.1)$$

Karman assumed that the strength of the plate is reached when the stress level of the fictitious model plate reach yield stress.

The assumption of Karman is very close to the simplifications used in the design code, but its main weakness is, it was totally based on mathematical formulations and effect of imperfection was not considered. Karman's assumption was later modified to include the effect of imperfection.

The Effective width method given in NS-EN 1993-1-5 is only applicable for plates which satisfy the following geometrical requirements:

- The plate geometry should be rectangular with parallel flanges.
- Stiffener should be applied either in the longitudinal direction or transverse direction, but not diagonally.
- Thickness of the cross section should be uniform.
- Flange induced buckling should not occur.

4.1.1. Overview of design procedure: Effective width method

In the design process, calculation of cross sectional class should be done first, to determine whether the plate is susceptible to local buckling or not. Only plates belonging to cross sectional class 4 are prone to local buckling.

Stiffened and unstiffened plates are treated in slightly different ways when designing according the Effective width method given on NS-EN 1993-1-5.

A. Design procedure for unstiffened plates

The design procedure for unstiffened plates is summarized on the following table.

Table 4-1 Design procedure for unstiffened plate in compression

Step no.	Description	Equations used	Reference
1	Determination of cross sectional class: if the structure is made up of multiple plates each and every member plate should be considered	Table 5.2	NS-EN 1993-1-1
2	Aspects ratio, α	$\frac{a}{b}$	Chapter 8.2, Larsen, (2010)
3	Determine buckling coefficient, k_σ	Fig.8. 9	Chapter 8.2, Larsen, (2010)
4	Plate slenderness, $\bar{\lambda}_p$	$\bar{\lambda}_p = \left(\frac{\frac{\bar{b}}{t}}{28{,}4.\varepsilon\sqrt{k_\sigma}} \right)$	NS-EN 1993-1-5:eq (4.3)
5	Critical plate like buckling, $\sigma_{cr,p}$	$\sigma_{cr,p} = k_\sigma \cdot \sigma_E$	Chapter 8.2, Larsen, (2010)

6	Determination of reduction factor, ρ	For internal element if $\bar{\lambda}_p > 0{,}673$ $\rho = \frac{\bar{\lambda}_p - 0{,}055(3+\psi)}{\bar{\lambda}_p^2}$ $\rho = 1 \quad \text{if } \bar{\lambda}_p \le 1$ For outstand element if $\bar{\lambda}_p > 0{,}748$ $\rho = \frac{\bar{\lambda}_p - 0{,}188}{\bar{\lambda}_p^2}$ $\rho = 1 \quad \text{if } \bar{\lambda}_p \le 0{,}748$	NS-EN 1993-1-5:eq (4.2) and eq.(4.3)
7	Now calculate effective area, $A_{c,eff}$	$A_{c,eff} = \rho.A_c$	NS-EN 1993-1-5:eq (4.1)
8	Design strength, N_{Rd} will be:	$N_R = A_{c,eff}\frac{f_y}{\gamma_{m1}}$	NS-EN 1993-1-1:eq (6.11)
*For wide plate with aspects ratio, $\alpha < 1$ column like buckling should be checked			
9	First calculate critical column buckling, $\sigma_{cr,c}$	$\sigma_{cr,c} = \frac{\pi^2 E t^2}{12(1-\upsilon^2)a^2}$	NS-EN 1993-1-5:eq (4.8)
10	Determine relative column slenderness, $\bar{\lambda}_c$	$\bar{\lambda}_c = \sqrt{\frac{f_y}{\sigma_{cr,c}}}$	NS-EN 1993-1-5:eq (4.10)
11	Determination of reduction factor due to column like buckling, χ_c	$\chi_c = \frac{1}{\phi + \sqrt{\phi^2 - \bar{\lambda}_c^2}}$	NS-EN 1993-1-1:eq (6.49)
12	Interpolation between column-like and plate-like buckling	$\rho_c = (\rho - \chi_c)\xi(2-\xi) + \chi_c$	NS-EN 1993-1-5: eq (4.13)
13	Final design strength	$N_{Rd} = \rho_c \cdot A_c\left(\frac{f_y}{\gamma_{m1}}\right)$	NS-EN 1993-1-1:eq (6.11)

B. Design procedure for stiffened plates

Plates with only one or two longitudinal stiffeners are treated as column on elastic foundation when calculating the critical buckling stress of the plate. But for plates with more than three longitudinal stiffeners, an equivalent orthotropic plate is considered by smearing the stiffeners smoothly over the plate.

Generally design procedure of stiffened plates is similar to that of unstiffened plates, but reduction of area is done in a slightly different way.

The total plate area is subdivided into two parts which are:

- Area close to the supported edges which is 100 % effective and no reduction is needed.
- Remaining area denoted as A_c, which is susceptible for both local and global buckling. The area A_c is shown on Figure 4-3 taken from Eurocode-3-1-5.

The total effective area of the compressed stiffened plate will be:

$$A_{c,eff} = \rho_C A_{c,eff,loc} + \sum b_{edge,eff} t \qquad (4.2)$$

Local effective area, $A_{c,eff,loc}$ shown on Figure 4-3 is calculated based on local effective width of sub panels, b_{eff}.

For a rectangular plate with a uniform compression, effective width of a sub panel is calculated as:

$$b_{eff,i} = \rho_{loc,i} . b_i$$

Where:

b_i = width of sub panel "i"

$\rho_{loc,i}$ = local reduction factor for subpanel "i"

The local reduction, $\rho_{loc,i}$ factor is calculated exactly the same way as reduction factor, ρ for unstiffened plates given on Table 4-1 above.

After the effective widths of all subpanels are determined, the effective local area is calculated as:

$$A_{c,eff.loc} = \sum b_{eff,i} t = t \sum \rho_{loc,i} b_i \quad (4.3)$$

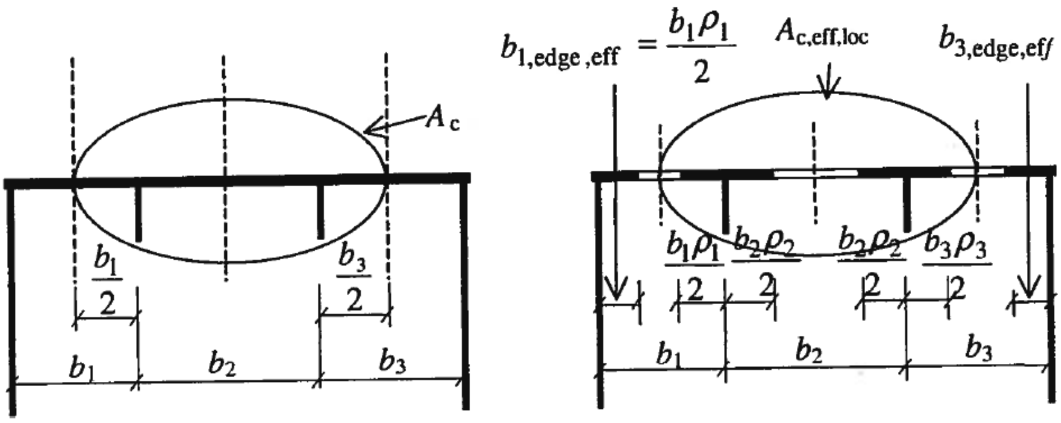


Figure 4-3 Stiffened plate in compression

The next step of the procedure will be calculation of relative slenderness of stiffened plate, $\bar{\lambda}_p$:

$$\bar{\lambda}_p = \sqrt{\frac{A_{c,eff,loc}}{A_c}\frac{f_y}{\sigma_{cr,p}}} \quad (4.4)$$

Now critical plate like buckling stress $\sigma_{cr,p}$ is needed to be determined. For plates with only one or two longitudinal stiffeners critical buckling stress, $\sigma_{cr,p}$ is done by considering the plate as a column on an elastic foundation. All relevant equations are given on Annex A.2 of NS-EN 1993-1-5. Plates with more than three longitudinal stiffeners are treated as equivalent orthotropic plates. Plate-like buckling stress, $\sigma_{cr,p}$ is determined using the equations given on Annex A.1 of NS-EN 1993-1-5.

The area is once more reduced due to risk of global buckling, and the global reduction factor is calculated as follow:

$$\rho_p = \frac{\bar{\lambda}_p - 0{,}055(3+\psi)}{\bar{\lambda}_p^2} \quad \text{for plate with uniform compression} \tag{4.5}$$

Column like buckling should be checked based on the equations given on section 4.5.3 of NS-EN 1993-1-5.

As it was done for unstiffened plate, plate-like and column-like buckling of stiffened plates should be interpolated using eq.(4.13) of the Eurocode. Then the final strength of the stiffened plate is calculated as:

$$N_{Rd} = \left(\rho_c \cdot A_{c,eff.loc} + \sum b_{edge,eff} t\right) \frac{f_y}{\gamma_{m1}} \tag{4.6}$$

4.2. Reduced stress method

NS-EN-1993-1-5 provides the Reduced stress method as an alternative method to determine the stress limits for both stiffened and unstiffened plates. As opposed to the Effective width method, the Reduced stress method assumes a linear stress distribution until first plate element buckles. If the structure is made up of multiple plate parts, the plate part which buckles first will govern the resistance of the entire cross section. Reduced stress method does not assume post critical strength of the entire cross section, which means the method does not consider load shedding from highly stressed to less stressed plate elements.

The difference between the Reduced stress method and the Effective width method becomes very clear, if the cross section is made up of multiple plate parts. For a cross section with only single plate element, the Reduced stress method gives equal results to that of Effective width method.

According section (10) of NS-EN 1993-1-5, for unstiffened and stiffened plate panels subjected to combined stresses $\sigma_{x,Ed}$, $\sigma_{z,Ed}$ and τ_{Ed}, class 3 section properties may be assumed, when equation (4.7) is fulfilled.

$$\frac{\rho.\alpha_{ult,k}}{\gamma_{m1}} \geq 1 \qquad (4.7)$$

Where:

$\alpha_{ult,k}$ = the minimum load amplifier for the design loads to reach the characteristic value of resistance of the most critical point of the plate.

ρ = the reduction factor depending on the plate slenderness, $\bar{\lambda}_p$ to take account of plate buckling.

The reduction factor, ρ could be either determined by taking the minimum value of ρ_x ,ρ_z and χ_w according clause 10(5a) of the Eurocode, or a value interpolated between them.

Instead of using a single buckling curve as equation (4.7), strength of the plate could be also verified by using clause 10(5a) of NS-EN-1993-1-5.

$$\left(\frac{\sigma_{x,Ed}}{\rho_x.f_y/\gamma_{m1}}\right)^2+\left(\frac{\sigma_{z,Ed}}{\rho_z.f_y/\gamma_{m1}}\right)^2-\left(\frac{\sigma_{x,Ed}}{\rho_x.f_y/\gamma_{m1}}\right).\left(\frac{\sigma_{z,Ed}}{\rho_z.f_y/\gamma_{m1}}\right)+3\left(\frac{\tau_{Ed}}{\chi_w.f_y/\gamma_{m1}}\right)^2 \leq 1 \qquad (4.8)$$

Reduction factors ρ_x and χ_w are determined based on equations on clauses 4.5.4(1) and 5.3(1).

It is important to mention the existence of disagreement between NS-EN 1993-1-5 and Beg et al. (2010) on how to calculate, ρ_z . According to the Eurocode, ρ_z could be determined by using the clause 4.5.4(1), but Beg et al.(2010) states that determination of ρ_z in that way may lead to a wrong results. Beg et al. (2010) instead recommend clause B.1 (3) of NS-EN 1993-1-5.

Table 4-2 Reduction factors and corresponding clause in the Euro code to be used

Type of loading	Reduction factor	Recommended clause in Eurocode-3-1-5
Longitudinal stress	ρ_x	Clause 4.5.4(1)
Transverse stress	ρ_z	Clause B.1(3)
Shear stress	χ_w	Clause 5.3(1)

To calculate the reduction factors given in the Table 4-2, plate slenderness, $\bar{\lambda}_p$ is needed, and it is calculated as:

$$\lambda_p = \sqrt{\frac{\alpha_{ult,k}}{\alpha_{cr}}} \quad (4.9)$$

Where: $\alpha_{ult,k}$ is defined previously

α_{cr} is the min. load amplifier for the design loads to reach the critical load of the plate under the complete stress field.

Bear in mind that in the Effective width method the plate slenderness, $\bar{\lambda}_p$ and the column slenderness, $\bar{\lambda}_c$ are calculated separately from different equations in the Euro code. But in the case of Reduced stress method, the slenderness value stated in equation (4.9) is used for both plate-like and column-like buckling.

For simple and regular cross sections the critical load amplifiers, α_{cr} and $\alpha_{ult,k}$ can be calculated manually, but in most practical situations they are determined by finite element method, FEM. This is one of the advantages of the Reduced stress method, since for a complex structure α_{cr} and $\alpha_{ult,k}$ can be easily extracted from a computer soft ware.

If Misses yield criteria is applied, the load amplifier $\alpha_{ult,k}$ can be calculated as shown on below.

$$\alpha_{ult,k} = \frac{f_y}{\sigma_{eq,Ed}} \quad (4.10)$$

Where: $\sigma_{eq,ED} = \sqrt{\sigma^2_{x,Ed} + \sigma^2_{z,Ed} - \sigma_{x,Ed} \cdot \sigma_{z,Ed} + 3 . \tau^2_{Ed}}$

f_y = yield stress

Calculation of α_{cr} manually for multiple loading conditions is challenging, but it could be easily determined by using soft ware like EBplate or Abaqus.

$$\alpha_{cr} = \frac{1}{\frac{1+\psi_x}{4.\alpha_{cr,x}} + \frac{1+\psi_z}{4.\alpha_{cr,z}} + \sqrt{\left(\frac{1+\psi_x}{4.\alpha_{cr,x}}\right) + \frac{1+\psi_z}{\alpha_{cr,z}} + \frac{\psi_x}{2.\alpha_{crx}^2}, + \frac{\psi_z}{2.\alpha_{cr,z}^2} + \frac{1}{\alpha_{cr,\tau}^2}}} \quad \textbf{(4.11)}$$

Where: ψ_x, ψ_z, are stress ratios along longitudinal and transverse edges. Such ratios could be determined based on table 4.1 & 4.2 NS-EN 1993-1-5.

$$\alpha_{cr,x} = \frac{\sigma_{cr,x}}{\sigma_{x,Ed}}, \quad \alpha_{cr,z} = \frac{\sigma_{cr,z}}{\sigma_{z,Ed}}$$

$$\alpha_{cr,\tau} = \frac{\tau_{cr}}{\tau_{Ed}}$$

4.2.1. Step by step design procedure: Reduced stress method

When design loads are verified the following procedure could be followed to verify the resistance of the plate.

A. Calculate the α_{cr}:

α_{cr} could be determined by hand calculation based on the eq. (4.11). Critical elastic buckling stresses, $\sigma_{cr,x}$, $\sigma_{cr,z}$ and τ_{cr} are determined exactly the same way as it is done for Effective width method. Suitable software like EBplate could be also used to determine α_{cr}.

B. Determination of $\alpha_{ult,k}$:

If the design loads are known, $\alpha_{ult,k}$ can be easily determined using the eq. (4.10) above.

C. Determine $\bar{\lambda}_p$ using the eq.(4.9)

D. Determination of the reduction factor:

The reduction factor is determined using Table 4-2 above. It is also possible to used the generalized equation given on Annex B.1 (3) of NS-EN 1993-1-5:

$$\rho = \frac{1}{\phi_p + \sqrt{\phi_p^2 - \bar{\lambda}_p}} \quad \textbf{(4.12)}$$

Where:

ρ = generalized reduction factor

$$\phi_p = 0,5\left(1 + \alpha_p\left(\bar{\lambda}_p - \bar{\lambda}_{p0}\right) + \bar{\lambda}_p\right)$$

$$\bar{\lambda}_p = \sqrt{\frac{\alpha_{ult,k}}{\alpha_{cr}}}$$

The Eurocode do not clearly state that clause B.1 (3) could be used for regular plates, but it is used in the COMBRI design manual (2008) as an alternative method.

E. Determination of loading resistance:

Resistance of the plate can be verified by using either eq. (4.7) or eq. (4.8).

Table 4-3 Effective width method versus Reduced stress method

No.	Effective width method	Reduced stress method
1	Is mainly described in section 4 of NS-EN-1993-1-5, but also includes section 5-7 for a complete design process.	Is described on section 10 of NS-EN-1993-1-5
2	Applicable only for steel structures with regular geometry. It is preferably used for I- and box-girder cross sections.	Is applicable for all form of geometries , such as plates with non orthogonal stiffeners, members with non parallel flanges, and webs with openings (both regular or irregular)
3	The actual nonlinear stress distribution is idealized by reducing the real width to an effective width	The actual nonlinear stress distribution of compressed plates is idealized to a uniform stress distribution
4	The cross section is allowed to carry loading until all member elements reach yielding stress.	Load shedding from highly stressed member to less stressed member is not considered.
5	Effect of load combinations must be separately verified using appropriate load interaction equations given in the Euro code.	Interaction between different loading conditions is taken care of by using the von Misses criterion. Hence the strength of the cross section is determined by single verification step.

5. Simulation program

Buckling and strength analysis will be carried on both unstiffened and stiffened plates. Hand calculations based on Effective width method and Reduced stress method will be done on selected plate dimensions. At last a finite element analysis using the multipurpose soft ware, Abaqus will be performed on all hand calculated plates.

All plates are simply supported and no other form of boundary condition is considered. Description of plate dimensions and loading conditions will be given on the following tables.

There is no practical laboratory tests involved in this project, and all plate dimensions and loading are theoretically assumed.

5.1. Uniaxially loaded unstiffened plates

Calculations will be performed using two forms of unstiffened plates:

1. Unstiffened plates with constant width and varying length
2. Unstiffened plates with varying width and constant length

Table 5-1 uniaxially loaded, unstiffened plate dimensions: constant width and varying length

No.	Plate size (width X length) [mm]	Thickness [mm]
1	1500 X 1500	20
2	1500 X 2000	20
3	1500 X 3500	20
4	1500 X 5000	20
5	1500 X 6000	20

Table 5-2 uniaxially loaded, unstiffened plate dimensions: constant length and varying width

No.	Plate size (width X length) [mm]	Thickness [mm]
1	2000 X 5000	30
2	3000 X 5000	30
3	4000 X 5000	30
4	5000 X 5000	30
5	6000 X 5000	30

5.2. Biaxially loaded unstiffened plates

Both hand calculation and finite element analysis will be performed on a biaxially loaded square and rectangular plate. Four different biaxial load combinations will be examined. Longitudinal load is considered to be the dominant load and will be kept constant. Description of plate dimensions and load combinations are given on the following tables.

Table 5-3 Load combinations for biaxially loaded plates

No.	Longitudinal loading [N/mm]	Transverse loading[N/mm]
Combination 1	1000	250
Combination 2	1000	500
Combination 3	1000	750
Combination 4	1000	1000

Load combinations on Table 5-3 will be applied to a square and rectangular plate. Buckling and strength analysis will be carried out using Euro code-3-1-5 rules and finite element analysis.

Table 5-4 biaxially loaded plate dimensions

Plate type	Width [mm]	Length [mm]	Thickness [mm]
Square	2000	2000	20
Rectangular	2000	4000	20

5.3. Stiffened plates

Buckling and strength analysis will be done based on Reduced stress method, Effective width method and finite element analysis.

Uniformly distributed uniaxial loading will be applied to rectangular plates. Plates are stiffened longitudinally with double sided flat stiffeners. Plate and stiffener dimensions are described below.

Table 5-5 Stiffener dimension

Type of stiffener	Total height [mm]	Thickness [mm]
Flat and double sided	160	12

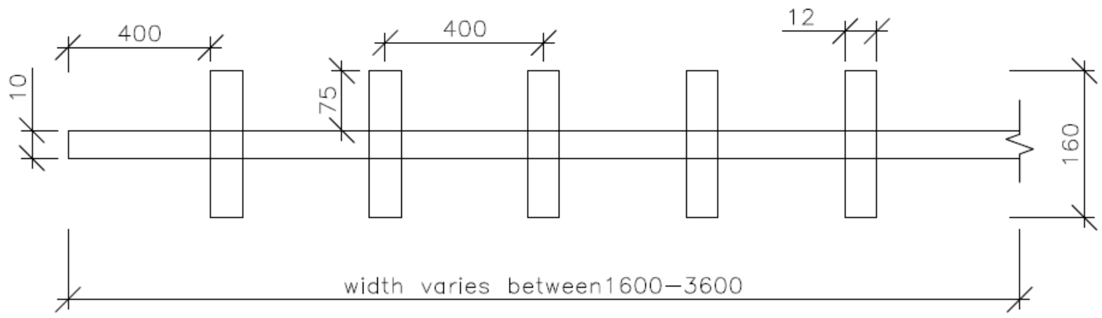


Figure 5-1 Typical cross section of stiffened plate

Table 5-6 Stiffened plate dimensions with varying width

No.	Plate size (width X length)	Thickness	No. of stiffeners
1	1600X7000	10	3
2	2000X7000	10	4
3	2400X7000	10	5
4	2800X7000	10	6
5	3200X7000	10	7
6	3600X7000	10	8

Table 5-7 Stiffened plates with constant width and number of stiffeners

No.	Plate size (width X length)	Thickness	No. of stiffeners
1	2000X2000	10	4
2	2000X3000	10	4
3	2000X4000	10	4
4	2000X5000	10	4
5	2000X6000	10	4
6	2000X7000	10	4
7	2000X8000	10	4
8	2000X9000	10	4
9	2000X1000	10	4
10	2000X12000	10	4

5.4. Abaqus modeling

All plates are modeled as three dimensional, deformable, planar shell in Abaqus/Cae. In the property module the section is defined as type: Shell/continuum Shell, homogeneous.

Material property:

Generally accepted values of Young's modulus, and Poisson's ratio for steel are used.

Since no laboratory test is carried out, with advisor's consent the yield stress is assumed to be $355 \frac{N}{mm^2}$. Moreover, it is considered a $5\,MPa$ hardening per $1\,\%$ increment of plastic strain.

Summary of material properties:

E= 210000 MPa and $\upsilon = 0,3$

True stress [MPa]	Plastic strain
355	0
380	0,05

Relationship between true and engineering stress/strain:

$$\sigma_{engineering} = E \cdot \varepsilon_{engineering} \quad \text{and} \quad \sigma_{true} = \sigma_{engineering}(1+\varepsilon_{engineering})$$

$$\varepsilon_{true}^{pl} = \ln(1+\varepsilon_{engineering}) - \frac{\sigma_{true}^{e}}{E}$$

Using the above relations a material property curve shown on Figure 5-2 is calculated and shown

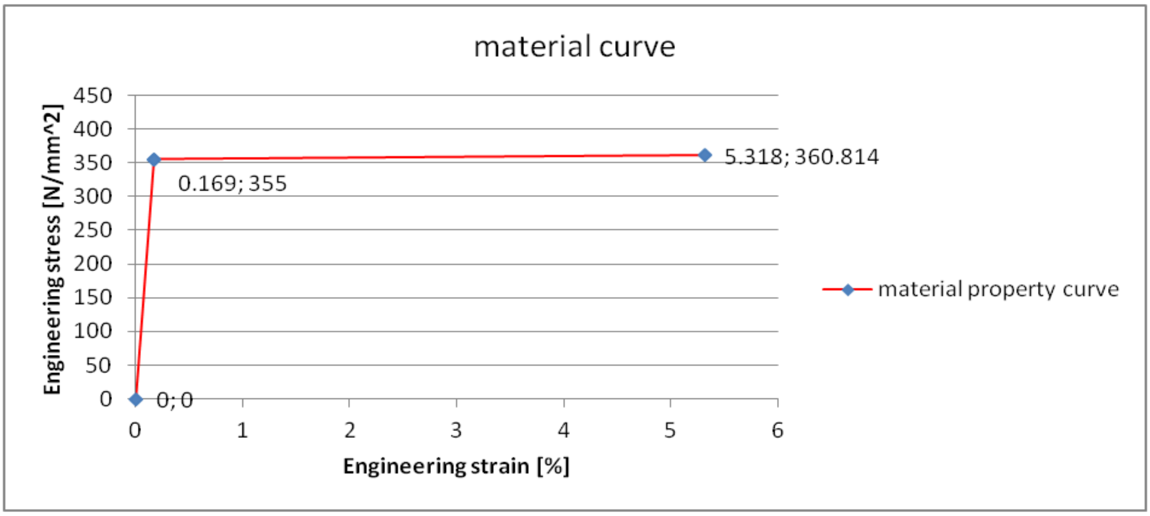


Figure 5-2 Theoretical material curve used in the material analysis

Element type and Meshing:

A shell element type, S4R is used for all plate models. For stiffeners a 2-node linear beam element, B31 is used. The beam elements have same cross section properties as that of real stiffeners.

Most plates are meshed with approximate global size of 50 mm X 50 mm, but a finer mesh will be considered if relevant. Beam elements are modeled with approximate global size of 25 mm X 25 mm.

Imperfection:

Imperfection is defined by using buckling mode shapes of the linear analysis. First linear buckling analysis is performed on a perfect geometry. Then buckling mode shape with lowest critical force is introduced into the nonlinear analysis as an imperfection. This is achieved by the help of *Imperfection command shown below.

*<u>*IMPERFECTION</u>, FILE=results_file, STEP=step, NSET=name*

With advisor's consent, imperfection amplitude is considered to be: $\frac{b}{500}$ (b = width of the plate). Only lowest buckling mode shape is introduced as an imperfection, since lowest buckling mode is assumed to provide the most critical imperfection.

5.4.1. Modeling uniaxially loaded, unstiffened plates

In practice simply supported plates could behave in two ways:

- Loaded edge will deform
- Loaded edge remains straight

To simulate this property, boundary conditions are applied into two different ways, denoted as "MYK" and "RETT" type boundary conditions.

**"MYK" = is a Norwegian word meaning "soft"*

**"RETT" = is a Norwegian word meaning "straight"*

Depending on how the simple support is applied, unstiffened plates are categorized in to two categories such as:

A. Plates with "MYK" type boundary conditions
B. Plates with "RETT" type boundary conditions

A) Plates with "MYK" type boundary conditions:

"MYK" type boundary condition is used to simulate plated structures, where loaded edges are expected to deform during loading. The Simple supports are provided at the middle of the plate and a uniformly distributed axial load is applied on both ends, see Figure 5-3.

I) Step:

For linear buckling analysis step, procedure: Linear perturbation ----> Buckle is used.

For nonlinear strength analysis, steps procedure: General -----> Static, Riks is used.

II) Loading:

For both linear buckling and nonlinear analysis a shell edge load of 1000 N/mm is applied from both ends as shown on Figure 5-3.

III) Boundary conditions:

Displacement out of plane (U3) is fixed along all edges. Translation in the longitudinal directions is avoided by fixing two points against movement in the X-directions (U1). To avoid rotation of the whole model, a single point is fixed against translation in transverse direction (U2). See Figure 5-3.

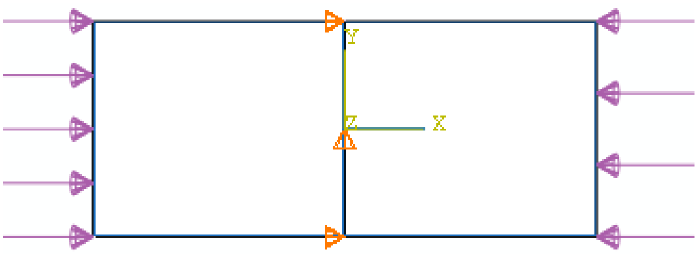


Figure 5-3 "MYK" type: boundary conditions and load application

B) Plates with "RETT" type boundary condition:

As shown on Figure 5-5, longitudinal translation is avoided by fixing the edge opposite to the loaded end. In addition the loaded edge is modeled to say straight during loading. In other words, all nodes along the loaded edge are constrained to have exactly equal displacement.

In most practical situations loaded edges stay straight during loading, and rules of Eurocode-3-1-5 are developed by assuming loaded edges stay straight during loading.

I) *Equation constraint:

All nodes along the loaded end will be assigned to have exactly the same translation in the longitudinal direction (i.e. U1). This is achieved by creating two sets of nodes, such as EDGE-NODES and RP (Reference point).

RP (Reference point): is a single node, at the middle of the loaded edge. During linear analysis, a concentrated force (CF1) is directly applied at RP, but for nonlinear analysis an imposed displacement is applied at RP.

EDGE-NODES: are all nodes on the loaded edge, except the RP.

By using *equation constraint, the two node sets will have exactly equal displacement in the longitudinal direction. Therefore loaded edge will stay straight during loading. *equation constraint created between the two node sets, is shown on Figure 5-4

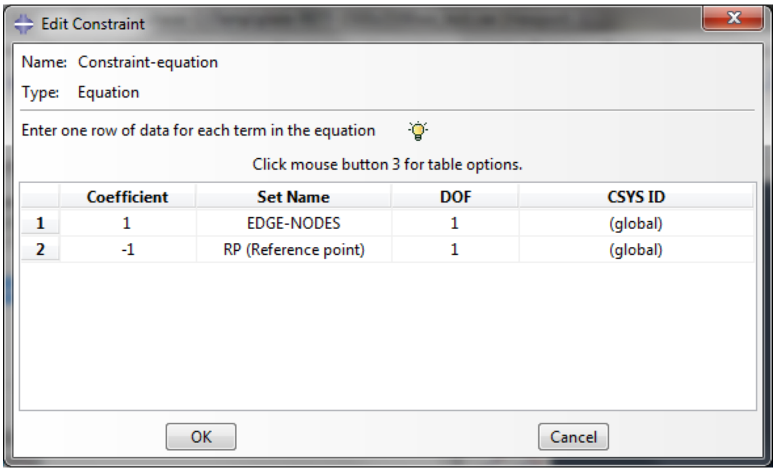


Figure 5-4 Equation constraint in Abaqus

II) STEP:

For linear buckling analysis step, procedure: Linear perturbation ----> Buckle is used

For nonlinear strength analysis, step procedure: General -----> Static, General is used

III) Loading:

For linear buckling analysis: Concentrated force (CF1) of magnitude 1000 N is applied longitudinally on one end. The force is applied at RP (reference point), see Figure 5-5.

For nonlinear analysis: Loading is applied as displacement driven. In other words strength it is determined by imposing longitudinal displacement at the RP (reference point). Applied displacements vary from 5 mm to 15 mm depending on plate dimensions.

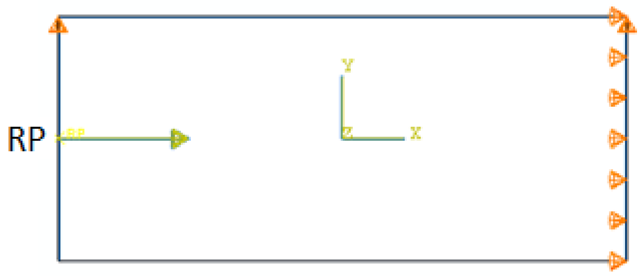


Figure 5-5 "RETT" type: boundary conditions and load application

IV) Boundary conditions:

All edges are fixed against out of plane displacement (i.e. U3). All nodes along the edge opposite to the loaded end are fixed against longitudinal translation (U1). To avoid free rotation of the whole model in space, two corner nodes are fixed against displacement in the transverse direction (U2), see Figure 5-5.

5.4.2. Modeling of biaxially loaded plates

Uniformly distributed load is applied for both linear buckling and nonlinear analysis. Load is only applied to one of the two opposite sides as shown on Figure 5-6. The magnitudes of the loads are given on Table 5-3.

Loaded edges are kept straight by using *equation constraint as discussed above, but in this case two *equation constraints are used.

To apply the *equation constraint successfully, node sets should be created and named. How node sets are named is explained below, using loaded edges AB and BD on Figure 5-6 as an example:

i. For loaded edge, AB two sets are created: RP1 (reference point 1) as one set and all other nodes on the same edge as another set. Using *equation constraint, all nodes on edge AB, will be assigned to have same displacement along x-direction.

ii. For loaded edge, BD, two sets are also created. RP2 (reference point 2) as one set and all other nodes on edge BD as another set. By using *equation constraint, all nodes of the two sets will have exactly equal displacement in transverse direction.

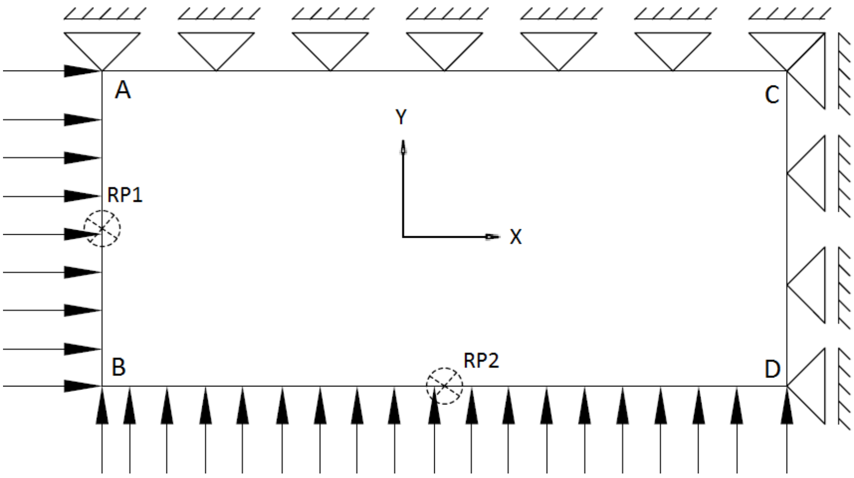


Figure 5-6 Biaxial load application and boundary conditions (in X - & Y-directions)

Linear perturbation /buckle step is used for linear buckling analysis, while STATIC-RIKS step is used for nonlinear analysis.

In case of boundary conditions, all edges are fixed against out of plane displacement (U3). As shown on Figure 5-6 above longitudinal edge AC is fixed against transverse direction, while the short edge, CD is fixed against longitudinal displacement.

5.4.3. Stiffened plates

Stiffened plates are model in similar way as unstiffened plates with "RETT" type boundary condition, but stiffeners are added to the model. Cross section of a stiffened plate is shown on Figure 5-1.

Stiffeners are modeled as Beam elements, and then attached to the plate part by the help of *TIE constraint in Abaqus.

*TIE constraint is in the Interaction module of Abaqus, and helps to fuse two regions together.

6. Hand calculations

Hand calculations are performing using the two alternative design methods given in the Euro code-3-1-5. Hand calculations of uniaxially loaded unstiffened plates, biaxially loaded plates and uniaxially loaded stiffened plates will be presented on separate subtopics.

6.1. Uniaxially loaded unstiffened plates

Critical buckling force and ultimate strength of unstiffened plates are calculated based on the two alternative methods given in the NS-EN-1993-1-5 (i.e. Effective width method and Reduced stress method).

Calculation work sheet is shown on the next page.

Rectangular plate:

Geometry:

a = length of the plate

b = width of the plate

$a := 5000mm$

$b := 1500mm$

$t := 30mm$

Material data

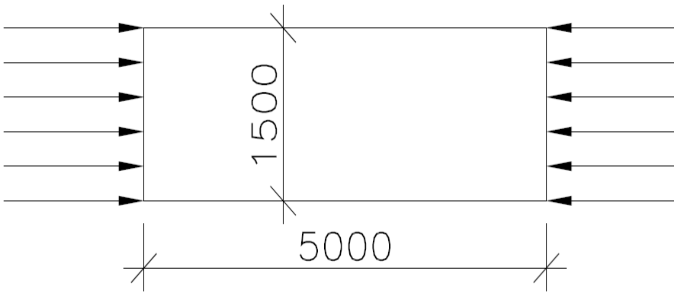
$E := 2.1 \cdot 10^5 MPa$

$f_y := 355MPa$

$\nu := 0.3$

$$\varepsilon := \sqrt{\frac{235MPa}{f_y}} = 0.814$$

Schematic drawing of plate with Loading



A: Using effective width method:

1) Cross-section check

NS-EN-1993-1-1: table 5.2:

$$\frac{b}{t} > 42 \cdot \varepsilon \rightarrow$$

Indicate the cross section is class 4

2) Aspect ratio, α

$$\alpha := \frac{a}{b} = 3.333$$

This leads to m=3 from fig.8.9, (Larsen, 2010)

$m := 3$

$$k_\sigma := \left(m \cdot \frac{b}{a} + \frac{1}{m} \cdot \frac{a}{b}\right)^2 = 4.045$$

OR k_σ can be calculated as:

$$k_{\sigma.alt} := \min\left[\left(\frac{1}{\alpha} + \alpha\right)^2, \left(\frac{2}{\alpha} + \frac{1}{2} \cdot \alpha\right)^2, \left(\frac{3}{\alpha} + \frac{1}{3} \cdot \alpha\right)^2, \left(\frac{4}{\alpha} + \frac{1}{4} \cdot \alpha\right)^2, \left(\frac{5}{\alpha} + \frac{1}{5} \cdot \alpha\right)^2\right] = 4.045$$

3) Slenderness, λ_p:

$$\lambda_p := \frac{\left(\frac{b}{t}\right)}{28.4 \cdot \varepsilon \cdot \sqrt{k_\sigma}} = 1.076$$

4) Reduction facto, ρ_p

$\psi := 1$ Uniform compression

$$\rho_p := \begin{cases} \dfrac{\lambda_p - 0.055(3+\psi)}{\lambda_p^2} & \text{if } \lambda_p > 0.673 \\ 1 & \text{otherwise} \end{cases} = 0.739369$$

5) Effective length/area:

$A_c := b \cdot t = 0.045 m^2$ — $A_{.c}$ is total area

$A_{c.eff} := \rho_p \cdot A_c = 0.033 m^2$ — According NS-EN-1993-1-5, eq.(4.1)

6) Critical plate like buckling, $\sigma_{cr.p}$:

$$\sigma_E := \frac{\pi^2 \cdot E}{12 \cdot (1-\nu^2)} \cdot \left(\frac{t}{b}\right)^2 = 75.92\, MPa$$

σ_{Ed} = reference buckling stress/Euler buckling stress

$\sigma_{cr.p} := \sigma_E \cdot k_\sigma = 307.064\, MPa$

$N_{cr.p} := \sigma_{cr.p} \cdot A_c = 13817.9\, kN$

N.B: for this specific plate there is no need of
calculating effect of column like buckling since $\alpha > 1$. But to make the work-sheet general, plate-like buckling and column-like buckling are calculated", and an interpolation equation will be used.

7) Design stress, $\sigma_{c.Rd.p}$:

$\sigma_{c.Rd.p}$ = indicate the strength of the plate when only "plate-like" effect is considered.

$$N_{c.Rd.p} := A_{c.eff} \cdot \frac{f_y}{\gamma_{m1}} = 11811.4\, kN$$

$\gamma_{m1} := 1$ — $\gamma_{.m.1}$ is equated this way, since results will be compared with results from Abaqus

$$\sigma_{c.Rd.p} := \frac{A_{c.eff}}{A_c} \cdot \frac{f_y}{\gamma_{m1}} = 262.476\, MPa$$

8) Colum buckling: Column- like buckling should be considered, if it is relevant

$$\sigma_{cr.c} := \frac{\pi^2 \cdot E \cdot t^2}{12 \cdot (1-\nu^2) \cdot a^2} = 6.833\, MPa$$

NS-EN-1993-1-5 eq.(4.8)

9) Relative column slenderness,λ_c will be:

$$\lambda_c := \sqrt{\frac{f_y}{\sigma_{cr.c}}} = 7.208$$

NS-EN-1993-1-5 e.q (4.10)

$$\alpha_c := 0.2$$

NS-EN-1993-1-1 e.q (6.49)

$$\phi := 0.5\cdot\left[1 + \alpha_c\cdot\left(\lambda_c - 0.2\right) + \lambda_c^{\,2}\right] = 27.213$$

N S-EN-1993-1-5: 4.5.3(5)

$$\chi_c := \frac{1}{\phi + \sqrt{\phi^2 - \lambda_c^{\,2}}} = 0.019$$

$$N_{Rd.søyle} := \chi_c\cdot A_c\cdot\frac{f_y}{\gamma_{m1}} = 298.8\,kN$$

10) Interpolation between plate-like and column-like buckling:

$$\rho_c := \left(\rho_p - \chi_c\right)\cdot\xi\cdot(2-\xi) + \chi_c = 0.739$$

$$\xi := \begin{cases} \frac{\sigma_{cr.p}}{\sigma_{cr.c}} - 1 & \text{if } 0 \le \frac{\sigma_{cr.p}}{\sigma_{cr.c}} - 1 \le 1 \\ 1 & \text{if } \frac{\sigma_{cr.p}}{\sigma_{cr.c}} - 1 > 1 \\ 0 & \text{otherwise} \end{cases}$$

$$\sigma_{Rd} := \rho_c\cdot\frac{f_y}{\gamma_{m1}}$$

$$N_{Rd} := \rho_c\cdot A_c\cdot\frac{f_y}{\gamma_{m1}} = 11811.4\,kN$$

$\xi = 1$ $\Rightarrow \xi=1$ indicate column-like buckling
has no effect to this plate.

2: REDUCED STRESS METHOD

We need to assume a value to the design stress, $\sigma_{x.Ed}$, and then perform an iterative process until we get the maximum value which makes equation (10.1) of NS-EN-1993-1-5 true.

$$\sigma_{x.Ed} := \sigma_{c.Rd.p}$$

A) Calculation of minimum load amplifiers, $\alpha_{ult.k}$ and α_{cr}

$$\alpha_{ult.k} := \frac{f_y}{\sigma_{x.Ed}} = 1.353$$

$$\sigma_{cr.p.2} := k_\sigma \cdot \frac{\pi^2 \cdot E}{12 \cdot \left(1 - \nu^2\right)} \cdot \left(\frac{t}{b}\right)^2 = 307.064\,\text{MPa}$$

$$\alpha_{cr} := \frac{\sigma_{cr.p.2}}{\sigma_{x.Ed}} = 1.17$$

NS-EN-1993-1-5:10(3)

$$\lambda_{p2} := \sqrt{\frac{\alpha_{ult.k}}{\alpha_{cr}}} = 1.075$$

λ_{p2}= slenderness of the plate- in this method it is just to differentiate from the symbol used in the previous method

λ_p from effective width method is almost equal to λ_{p2}.

B) Calculation of the reduction factors, ρ

$$\rho_2 := \frac{\lambda_{p2} - 0.055(3 + \psi)}{\lambda_{p2}^{\,2}} = 0.74$$

NS-EN-1993-1-5:10(2)

$\rho_p = \rho_2$ we get equal reduction factors

$$\rho_2 \cdot \frac{\alpha_{ult.k}}{\gamma_{m1}} = 1$$

ok!

NS-EN-1993-1-5:10(2)

$$N_{c.Rd.2.} := A_c \cdot \sigma_{x.Ed} = 11811.4\,\text{kN}$$

$N_{c,Rd.2}$= design axial force based on Reduced stress method.

$$\sigma_{c.Rd.2} := \sigma_{x.Ed} = 262.476\,\text{MPa}$$

$\sigma_{c,Rd.2}$ = design strength based on Reduced stress method

6.1.1. Results based on Effective width method

Table 6-1 Critical buckling force, N_{cr} and ultimate strength, N_{Rd} for plate with thickness

No.	Plate size [mm] (width x length)	N_{cr} [kN]	N_{Rd} [kN]
1	1500 X 1500	4049,1	5672,7
2	1500 X2000	4394	5870,5
3	1500 X 3500	4146	5729,5
4	1500 X 5000	4094.2	5699,3
5	1500 X 6000	4049.1	5672,7

Plate No.1 (1500X1500) and plate No.2 (1500 X 6000) show equal buckling force and ultimate strength. This is because they have equal buckling coefficient, k_{σ} and the cross sectional area is constant for all plates. Recall that critical buckling force depends on cross sectional area and buckling coefficient.

Similar calculations were also performed by increasing the thickness (t= 30 mm), while keeping the length constant.

Table 6-2 Critical buckling force, N_{cr} and ultimate strength, N_{Rd} for plate with constant length, and t=30 mm

No.	Plate size [mm] (width x length)	N_{cr} [kN]	N_{Rd} [kN]
1	2000 X 5000	10593,7	12683,8
2	3000 X 5000	7062,5	13459,6
3	4000 X 5000	5384	13951,4
4	5000 X 5000	4099,7	13864,5
5	6000 X 5000	3531,2	14235,5

6.1.2. Results based on Reduced stress method

Critical buckling force, N_{cr} and ultimate strength, N_{Rd} are again calculated for the same plate dimensions as above, but now we use Reduced stress method. Results based on Reduced stress method become exactly equal to that of Effective width method.

Table 6-3 N_{cr} and N_{Rd} for plates with t=20, based on Reduced stress method

No.	Plate size [mm]	N_{cr} [kN]	N_{Rd} [kN]
1	1500 X 1500	4049.1	5672,7
2	1500 X2000	4394	5870,5
3	1500 X 3500	4146	5729,5
4	1500 X 5000	4094,2	5699,3
5	1500 X 6000	4049,1	5672,7

The two methods gave equal buckling and strength results, since single plate panels are used. For a single plate there is no difference between the two methods.

For Reduced stress method: we simplify the stress distribution to an average and constant value: $N_{Rd} = A \cdot (\rho \cdot f_y)$

For Effective stress method: we reduce the area to an effective area

$N_{Rd} = (\rho \cdot A) \cdot f_y$

Therefore the two method lead to same results $\Rightarrow A \cdot (\rho \cdot f_y) = (\rho \cdot A) \cdot f_y$

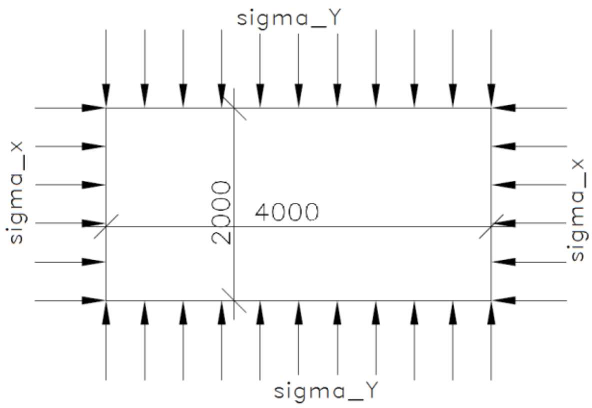
6.2. Results of biaxially compressed plates

A square plate (2000X2000) and a rectangular plate (2000X4000) of thickness, t=20 mm were loaded biaxially. Four different loading combinations are applied to each plate as shown on Table 5-3.

The effective width method is not applicable for plates with biaxial loading, therefore only Reduced stress method is applied to calculate the critical buckling stress, σ_{cr} and design strength, σ_{Rd}.

All hand calculations are done using MathCAD soft ware. A typical example showing all calculation steps is given below.

Biaxially loaded plate 2000X4000, t=20 mm



Geometry:

$a := 4000mm$

$b := 2000mm$

$t := 20mm$

Material data

$E := 2.1 \cdot 10^5 MPa$

$f_y := 355MPa$

$\nu := 0.3$

$$\varepsilon := \sqrt{\frac{235MPa}{f_y}} = 0.814$$

Stress:

$\sigma_{Ed.x} := 71.34MPa$

$\sigma_{Ed.x}$ = is determined by iteration.

$\sigma_{Ed.y} := \sigma_{Ed.x} = 71.34\,MPa$

$$\sigma_{Ed.eq} := \sqrt{\sigma_{Ed.x}^2 + \sigma_{Ed.y}^2 - \sigma_{Ed.x} \cdot \sigma_{Ed.y}} = 71.34\,MPa$$

$$\sigma_{E.x} := \frac{\pi^2 \cdot E}{12 \cdot (1 - \nu^2)} \cdot \left(\frac{t}{b}\right)^2 = 18.98\,MPa$$

Euler stress in the longitudinal direction

$$\sigma_{E.y} := \frac{\pi^2 \cdot E}{12 \cdot (1 - \nu^2)} \cdot \left(\frac{t}{a}\right)^2 = 4.745\,MPa$$

Euler stress in the transverse direction

1) Aspect ratio:

m = is the minimum number of half waves. Extracted manually from fig.8.9, (Larsen,2010)

$m := 2$

$$\alpha := \frac{a}{b} = 2 \qquad K_{\sigma.x} := \left(m \cdot \frac{b}{a} + \frac{1}{m} \cdot \frac{a}{b}\right)^2 = 4$$

$$K_{\sigma.y} := \left(\frac{b}{a} + \frac{a}{b}\right)^2 = 6.25$$

2) Critical stresses:

$\sigma_{cr.x} := K_{\sigma.x} \cdot \sigma_{E.x} = 75.92\,MPa$

$\sigma_{cr.y} := K_{\sigma.y} \cdot \sigma_{E.y} = 29.656\,MPa$

3) Minimum load amplifiers according NS-EN-1993-1-5:10(2) & (3)

$\alpha_{ult.k} := \frac{f_y}{\sigma_{Ed.eq}} = 4.976$

$\psi_x := 1$

$\psi_y := \psi_x$

$\alpha_{cr.x} := \frac{\sigma_{cr.x}}{\sigma_{Ed.x}} = 1.064$

$\alpha_{cr.y} := \frac{\sigma_{cr.y}}{\sigma_{Ed.y}} = 0.416$

4) Equation (10.6) of the NS-EN-1993-1-5 gives us:

$$\eta := \frac{1+\psi_x}{4\cdot\alpha_{cr.x}} + \frac{1+\psi_y}{4\cdot\alpha_{cr.y}} + \left[\left(\frac{1+\psi_x}{4\cdot\alpha_{cr.x}} + \frac{1+\psi_y}{4\cdot\alpha_{cr.y}}\right)^2 + \frac{1-\psi_x}{\left(2\cdot\alpha_{cr.x}\right)^2}\cdot\frac{1-\psi_y}{\left(2\cdot\alpha_{cr.y}\right)^2}\right]^{\frac{1}{2}} = 3.345$$

$\alpha_{cr} := \frac{1}{\eta} = 0.299$

5) The slenderness value will be:

$\lambda_p := \sqrt{\frac{\alpha_{ult.k}}{\alpha_{cr}}} = 4.08$

NS-EN-1993-1-5:10(3)

6) Reduction factors :

$\rho_x := \frac{\lambda_p - 0.055\left(3+\psi_x\right)}{{\lambda_p}^2} = 0.232$

NS-EN-1993-1-5:4.4(2)

$\rho_{.x}$ = is reduction factor for longitudinal loading

Note:

NS-EN-1993-1-5:10(5a) indicate that,ρ_y (reduction factor for transverse loading) should be determined in a similar way as $\rho_{.x}$. But Beg et al. (2010) highly recommend eq. (B.1) under Annex B.

$$\lambda_{po} := 0.8$$

$$\alpha_p := 0.34$$

$$\Phi := \frac{1}{2}\cdot\left[1 + \alpha_p\cdot\left(\lambda_p - \lambda_{po}\right) + \lambda_p\right] = 3.098$$

$$\rho_y := \frac{1}{\Phi + \sqrt{\Phi^2 - \lambda_p}} = 0.184$$

NS-EN-1993-1-5: B1 (3) and table B1

Reduction factors due to column like-buckling:

A*: longitudinal direction:*

Due to high aspect ratio, column -like buckling is not relevant for the investigated plates

B: Transverse direction

Since a generalized buckling curve is used (i.e.NS-EN-1993-1-5: B1(3)), there is no need of calculating the column-like buckling effect specifically.

$\gamma_{m.1} := 1$ Since the calculation results will be compared with that of Abaqus, γ_{m1} is let to be equal to one. This is true for all hand calculations.

By iteration the maximum loading situation which satisfy equation (10.5) is taken as the design load

From NS-EN-1993-1-5:eq. (10.5):

$$\left[\frac{\sigma_{Ed.x}}{\left(\rho_x\cdot\frac{f_y}{\gamma_{m.1}}\right)}\right]^2 + \left[\frac{\sigma_{Ed.y}}{\left(\rho_y\cdot\frac{f_y}{\gamma_{m.1}}\right)}\right]^2 - \left(\frac{\sigma_{Ed.x}}{\rho_x\cdot\frac{f_y}{\gamma_{m.1}}}\right)\cdot\left(\frac{\sigma_{Ed.y}}{\rho_y\cdot\frac{f_y}{\gamma_{m.1}}}\right) = 1$$

OK!

$$\sigma_{x.Rd} := \sigma_{Ed.x} = 71.34\,\text{MPa}$$

$$\sigma_{y.Rd} := \sigma_{Ed.y} = 71.34\,\text{MPa}$$

Maximum design load which satisfy eq. (10.5) of Eurocode-3-1-5, is assumed to be the strength of the plate.

Calculated results of biaxially loaded plates with different load combinations are given in the following two tables. These results will be later compared with results of numerical analysis.

Table 6-4 Calculated $\sigma_{cr,x}$ and $\sigma_{cr,y}$ of biaxially loaded plate: 2000X2000, t=20 mm

No.	Loading type [N/mm]	α_{cr}	$\sigma_{cr,x}$ [N/mm^2]	$\sigma_{cr,y}$ [N/mm^2]
1	Longitudinal = 1000 Transverse = 250	0,423	60,700	15,175
2	Longitudinal = 1000 Transverse = 500	0,384	50,611	25,306
3	Longitudinal = 1000 Transverse = 750	0,383	43,394	32,546
4	Longitudinal = 1000 Transverse = 1000	0,4	37,920	37,920

Where:
α_{cr} = minimum load amplifier for design loads to reach elastic critical buckling
$\sigma_{cr,x}$ = elastic buckling stress - longitudinal direction
$\sigma_{cr,y}$ = elastic buckling stress - transverse direction

Table 6-5 Calculated $\sigma_{cr,x}$ and $\sigma_{cr,y}$ for biaxially loaded plate: 2000X4000, t=20 mm

No.	Loading type [N/mm]	α_{cr}	$\sigma_{cr,x}$ [N/mm^2]	$\sigma_{cr,y}$ [N/mm^2]
1	Longitudinal = 1000 Transverse = 250	0,3637	46,300	11,575
2	Longitudinal = 1000 Transverse = 500	0,307	33,248	16,624
3	Longitudinal = 1000 Transverse = 750	0,294	25,960	19,470
4	Longitudinal = 1000 Transverse = 1000	0,299	21,330	21,330

Ultimate strength is also calculated for the same plates and load combinations as those used in the buckling analysis. A detailed description of plate dimensions and loading combinations are given on Table 5-3 and Table 5-4.

Table 6-6 Ultimate strength based on Reduced stress method: Biaxially loaded plate-2000x2000, t=20 mm

No.	Loading type [N/mm]	$\sigma_{Rd,x}$ [N/mm^2]	$\sigma_{Rd,y}$ [N/mm^2]
1	Longitudinal = 1000 Transverse = 250	143,500	35,875
2	Longitudinal = 1000 Transverse = 500	131,800	65,900
3	Longitudinal = 1000 Transverse = 750	113,300	84,975
4	Longitudinal = 1000 Transverse = 1000	94,800	94,800

Where:

$\sigma_{Rd,x}$ = ultimate strength – longitudinal direction

$\sigma_{Rd,y}$ = ultimate strength – transverse direction

Table 6-7 Ultimate strength based on Reduced stress method: Biaxially loaded plate-2000X4000, t=20 mm

No.	Loading type [N/mm]	$\sigma_{Rd,x}$ [N/mm^2]	$\sigma_{Rd,y}$ [N/mm^2]
1	Longitudinal = 1000 Transverse = 250	127,300	31,825
2	Longitudinal = 1000 Transverse = 500	108,300	54,150
3	Longitudinal = 1000 Transverse = 750	88,300	66,250
4	Longitudinal = 1000 Transverse = 1000	71,340	71,340

6.3. Results of stiffened plates

Longitudinally stiffened plates with flat stiffeners are uniaxially compressed. Critical buckling load and ultimate strength of plates are hand calculated based on the two alternative methods given in NS-EN-1993-1-5.

A typical calculation work sheet showing how the calculations are performed is also prepared. Since the calculation took many pages, it is preferred to present it as an appendix. Refer to appendix D.

6.3.1. Results of the stiffened plates based on effective width method

Plates with increasing number of stiffeners (i.e. varying width) and plates with increasing length are calculated. Plate and stiffener dimensions are shown on sub topic 5.3.

Critical buckling stress and ultimate strengths will be treated in a separate sub topics to avoid confusion.

6.3.1.1. Calculated buckling stress and buckling force based on Effective width method

Column- buckling stress/force and plate -like buckling stress/force are presented on Table 6-8 and Table 6-9 to show how buckling behavior varies as dimension changes.

Table 6-8 Critical buckling force/stress of rectangular plates with increasing number of stiffeners

No	Plate size(mm)	siffener no.	Area,total [mm^2]	Sigma_cr,c [N/mm^2]	N_cr,c [kN]	sigma_cr,p [N/mm^2]	N_cr,p [kN]
1	1600X7000	3	21760	29.589	643.8566	111.293	2421.7357
2	2000 X7000	4	27680	29.504	816.6707	72.04	1994.0672
3	2400 X7000	5	33600	29.504	991.3344	50.764	1705.6704
4	2800 X 7000	6	39520	29.504	1165.998	40.704	1608.6221
5	3200 X 7000	7	45440	29.504	1340.662	35.794	1626.4794
6	3600 X 7000	8	51360	29.504	1515.325	33.168	1703.5085

Where:

$\sigma_{cr,c} / N_{cr,c}$ = critical buckling stess / force : only column − like behavior considered

$\sigma_{cr,p} / N_{cr,p}$ = critical buckling stress / force : only plate − like behavior considered

Critical buckling stress, due to column-like behavior, $\sigma_{cr,c}$ is equal for all plate dimensions. This is because of σ_{cr} depends on parameters like, $A_{sl,1}$, length and $I_{sl,1}$ which are constant for all plate dimension in the table above.

As width of plates increase the effect of plate-like behavior decreases while the effect of column- like behavior increases. This is as expected, since column- like buckling effect gets larger as width of the plate increases.

On Table 6-8 length of plates is kept constant, while width increases. To see the effect of increasing length, a similar calculation as above is performed. It is done by keeping the width and number of stiffeners constant, but varying the length of the plate. Results are presented on Table 6-9.

Table 6-9 Calculated critical buckling stress and force for both column-like and plate-like behavior

No.	Plate size [mm]	No. of Stiffeners	$\sigma_{cr,c}$ [N/mm^2]	$N_{cr,c}$ [kN]	$\sigma_{cr,p}$ [N/mm^2]	$N_{cr,p}$ [kN]
1	2000 X 2000	4	361,424	10 004,2	320,105	8860,506
2	2000 X 3000	4	160,633	4 446,3	152,269	4214,806
3	2000 X 4000	4	90,356	2 501,1	98,026	2713,360
4	2000 X 5000	4	57,828	1 600,7	77,856	2155,054
5	2000 X 6000	4	40,158	1 111,6	72,138	1996,780
6	2000 X 7000	4	25,504	706,0	72,04	1994,067
7	2000 X 8000	4	22,589	625,3	72,04	1994,067
8	2000 X 9000	4	17,848	494,0	72,04	1994,067
9	2000 X 10000	4	14,45697	400,2	72,04	1994,067
10	2000 X 12000	4	10,04	277,9	72,04	1994,067

Where:

$\sigma_{cr,c} / N_{cr,c}$ = critical buckling stess / force : only column – like behavior considered

$\sigma_{cr,p} / N_{cr,p}$ = critical buckling stress / force : only plate – like behavior considered

Generally critical buckling force of the plate decreases as its length increase. Besides plate-like buckling will become the dominant buckling behavior as length is increased.

According calculation rules of NS-EN-1993-1-5, critical buckling stress will be constant, when certain limit of plate length is reached. Increasing plate length over that limit will not have any effect on critical buckling force.

As it can be seen on the results given on Table 6-9 and Figure 6-1, buckling stress, $\sigma_{cr,p}$ will reach its minimum value at plate length of 7000 mm and remain constant even though length is increased.

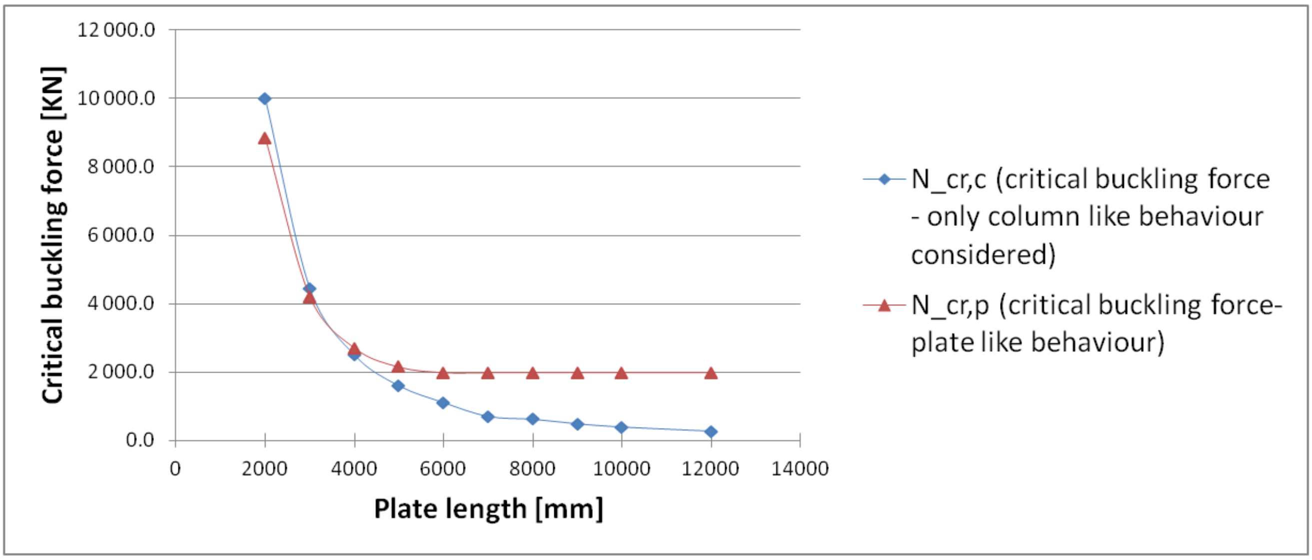


Figure 6-1 Comparison between $N_{cr,c}$ and $N_{cr,p}$ as a function of plate length increases

6.3.1.2. Ultimate strength of stiffened plate based on effective width method

On sub chapter 6.3.1.1 results of critical buckling stresses are thoroughly discussed. In this sub topic discussions will be confined only on ultimate strengths calculated based on Effective width method.

Ultimate strengths are calculated based on calculation rules given in section 4 of Euro code-3-1-5. Plate and stiffener dimensions are given on Table 5-6 and Table 5-7.

Strength of same plate dimension will be later calculated based on Reduced stress method and the results of the two methods will be compared.

Table 6-10 Ultimate strength of longitudinally stiffened plates with increasing number of stiffeners

No.	Plate size [mm]	No. of stiffeners	$N_{Rd,p}$ [kN]	$N_{Rd,interaction}$ [kN]
1	1600X7000	3	4149,846	4149,846
2	2000 X7000	4	4453,873	4453,873
3	2400 X7000	5	4676,351	4463,902
4	2800 X 7000	6	4970,873	3866,711
5	3200 X 7000	7	5344,954	3428,083
6	3600 X 7000	8	5771,034	3181,306

Where:

$N_{Rd,p}$ = Ultimate strength of the plate, when only plate- like behavior is considered.

$N_{Rd,interaction}$ = Ultimate strength interpolated between plate-like and column- like behavior.

As the width increases will $N_{Rd,interaction}$ decreases, since it is calculated based on interpolation eq.(4.13) of NS-EN-1993-1-5 and the effect of column-like behavior become very significant as width of the plate is increased.

Table 6-11 Calculated $N_{Rd,p}$ and $N_{Rd,interaction}$ for longitudinally stiffened plate with varying length

No	Plate size [mm]	No. of stiffeners	$N_{Rd,p}$ [kN]	$N_{Rd,interaction}$ [kN]
1	2000 X 2000	4	7156,35	5269,884
2	2000 X 3000	4	5672,295	3663,606
3	2000 X 4000	4	4918,813	3129,348
4	2000 X 5000	4	4566,295	3594,235
5	2000 X 6000	4	4455,806	4353,831
6	2000 X 7000	4	4453,873	4453,873
7	2000 X 8000	4	4453,873	4453,873
8	2000 X 9000	4	4453,873	4453,873
9	2000 X 10000	4	4453,873	4453,873
10	2000 X 12000	4	4453,873	4453,873

Table 6-11 shows that $N_{Rd,p}$ and $N_{Rd,interaction}$ become equal for all lengths above 7000 mm. This is because of the column -like effect become negligible as the stiffened plate length increases. Furthermore the plate strength will remain constant after its length has reached certain limit. This phenomenon is further explained below.

$\sigma_{cr,p} = k_\sigma \cdot \sigma_E$, where: k_σ is buckling coefficient

σ_E is Euler buckling: it is constant for the plate dimension examined.

$$k_\sigma = \frac{2\left(\left(1+\alpha^2\right)^2 + \gamma - 1\right)}{\alpha^2(\psi+1)(1+\delta)} \quad \text{if } \alpha \le \sqrt[4]{\gamma}$$

$$k_\sigma = \frac{4\left(1+\sqrt{\gamma}\right)}{(\psi+1)(1+\delta)} \quad \text{if } \alpha > \sqrt[4]{\gamma}$$

$$\text{where: } \alpha = \frac{a}{b} \ge 0{,}5$$

$$\gamma = \frac{I_{sl}}{I_p} \quad \text{and} \quad \delta = \frac{\sum A_{sl}}{A_p}$$

When length of the plate reach 7000 mm, $\alpha > \sqrt[4]{\gamma}$ and buckling coefficient will only depend on cross sectional properties, γ , δ which are independent of plate length.

6.3.2. Results of longitudinally stiffened plates based on Reduced stress method

Stiffened plates discussed on sub topic 6.3.1, are also analyzed based on Reduced stress method.

For uniaxial loading critical buckling stress, σ_{cr} is calculated the same way as it is done for Effective width method. In other words there is no difference between the two methods when calculating critical buckling stress for uniaxially loaded stiffened plates.

6.3.2.1. Ultimate strength of longitudinally stiffened plates using Reduced stress method

On the previous topic strength of stiffened plates was determined based on Effective width method. In this topic results calculated based on Reduced stress method are presented for the same plates.

Table 6-12 Ultimate strength, $N_{Rd,r}$ of longitudinally stiffened plates using Reduced stress method

No	Plate size [mm]	No. of stiffener	Area, total [mm^2]	$N_{Rd,r}$ [kN]
1	1600X7000	3	21760	3792,401
2	2000 X7000	4	27680	3987,872
3	2400 X7000	5	33600	3919,178
4	2800 X 7000	6	39520	3212,712
5	3200 X 7000	7	45440	2651,798
6	3600 X 7000	8	51360	2304,38

Where: $N_{Rd,r}$ = ultimate strength based on Reduced stress method.

One of the best merits of Reduced stress method is, computer soft ware could be utilized to perform the calculation. This is especially very important when performing calculations on complex plate geometries. In this project all plates have regular geometry and simple loading conditions which can be easily hand calculated. But as an alternative way EBplate soft ware is used to determine the critical buckling stress.

Table 6-13, shows critical buckling stresses, $\sigma_{cr,EBplate}$ determined by using EBplate soft ware. In addition the table shows resulting ultimate strength, $N_{Rd,r}$.

Table 6-13 Linear buckling stress, $\sigma_{cr,EBplate}$ and the ultimate strength based on Reduced stress method, $N_{Rd,r}$

No.	Plate size [mm]	No. stiffener	$\sigma_{cr,EBplate}$ [N/mm^2]	$N_{Rd,r}$ [kN]
1	1600X7000	3	138,36	4160,207
2	2000 X7000	4	76,892	4104,972
3	2400 X7000	5	54,137	4032,134
4	2800 X 7000	6	44,043	3312,092
5	3200 X 7000	7	38,933	2723,583
6	3600 X 7000	8	36,099	2352,853

There is no significant difference between the calculated ultimate strengths, $N_{Rd,r}$ on Table 6-12 and Table 6-13. The reason could be, plate geometries and loading conditions considered are relatively simple, so that both hand calculation and EBplate soft ware give very close critical buckling stress values.

Ultimate strength based on Reduced stress method is also calculated for stiffened plate dimensions given on Table 5-7. Hand calculated critical buckling stress, $\sigma_{cr,p}$ used to determine plate strengths is presented on Table 6-9. It should be recalled that a column-like buckling behavior is also considered according NS-EN-1993-1-5: 10(5a).

Table 6-14 Ultimate strength, $N_{Rd,r}$ based on Reduced stress method: Stiffened plate with varying length.

No	Plate size [mm]	No. Stiffeners	$\sigma_{cr,p}$ [N/mm^2]	$N_{Rd,r}$ [kN]
1	2000 X 2000	4	320,105	4634,529
2	2000 X 3000	4	152,269	2811,178
3	2000 X 4000	4	98,026	2402,224
4	2000 X 5000	4	77,856	3062,197
5	2000 X 6000	4	72,138	3888,257
6	2000 X 7000	4	72,04	3987,885
7	2000 X 8000	4	72,04	3987,885
8	2000 X 9000	4	72,04	3987,885
9	2000 X 10000	4	72,04	3987,885
10	2000 X 12000	4	72,04	3987,885

Ultimate strength, $N_{Rd,r}$ become constant after the length of the plate reach a certain limit, since critical buckling stress, $\sigma_{cr,p}$ will become independent of plate length. This phenomenon was also seen for Effective width method. To see the reason behind, refer to discussion given under subtopic 6.3.1.2.

6.3.3. Comparison of ultimate strength calculated based on Effective width and Reduced stress methods.

Strength results based on the two alternative design methods are presented separately on the previous tables. Now comparison of these strengths will be performed. Bear in mind that stiffened plates are grouped in to two categories, which are:

- Plates with increasing number of stiffeners and varying width
- Plates with constant number of stiffeners and varying length

Table 6-15 Comparison of ultimate strength calculated based on the two alternative methods: Plates with varying width

No.	Plate size(mm)	No. stiffener	$N_{Rd,eff,w}$ [kN]	$N_{Rd,r}$ [kN]	Difference [kN]	Difference [%]
1	1600X7000	3	4149,846	3792,401	357,445	9 %
2	2000 X7000	4	4453,873	3987,872	466,001	10 %
3	2400 X7000	5	4463,902	3919,178	544,724	12 %
4	2800 X 7000	6	3866,711	3212,712	653,999	17 %
5	3200 X 7000	7	3428,083	2651,798	776,285	23 %
6	3600 X 7000	8	3181,306	2304,38	876,926	28 %

Where:

$N_{Rd,eff,w}$ = Ultimate strength based on effective width method

$N_{Rd,r}$ = Ultimate strength based on Reduced stress method

The difference between $N_{Rd,eff,w}$ and $N_{Rd,r}$ is around 10 % for the narrow plates. This difference increases as the width of plates is increased. The cause of the difference is mainly due to the fact that Effective width method allows load shedding among member plates of the cross section. In other words plates are allowed to continue to carry loading, until all members of the cross section reach yielding. In case of Reduced stress method the cross section is as strong as its weakest link. No load shedding is allowed in Reduced stress method.

It is also interesting to see that the difference between $N_{Rd,eff,w}$ and $N_{Rd,r}$ increases as width of the plates increase. A possible cause of this could be the role of column-like buckling. Both methods apply column -like behavior when relevant. It seems consideration of column-like behavior has stronger effect on Reduced stress method leading to very conservative results. Bear in mind that use of interpolation function gives lower strength values for wider plates, regard less of the method used.

Comparison of ultimate strengths for plate dimensions with varying length is presented on the following table.

Table 6-16 Comparison of hand calculated $N_{Rd,r}$ and $N_{Rd,eff,w}$: Stiffened plate with increasing length

No.	Plate size [mm]	No.of Stiffeners	$N_{Rd,r}$ [kN]	$N_{Rd,eff,w}$ [kN]	Difference [kN]	Difference [%]
1	2000 X 2000	4	4634,529	5269,884	635,355	12 %
2	2000 X 3000	4	2811,178	3663,606	852,428	23 %
3	2000 X 4000	4	2402,224	3129,348	727,124	23 %
4	2000 X 5000	4	3062,197	3594,235	532,038	15 %
5	2000 X 6000	4	3888,257	4353,831	465,574	11 %
6	2000 X 7000	4	3987,885	4453,873	465,98772	10 %
7	2000 X 8000	4	3987,885	4453,873	465,98772	10 %
8	2000 X 9000	4	3987,885	4453,873	465,98772	10 %
9	2000 X 1000	4	3987,885	4453,873	465,98772	10 %
10	2000 X 12000	4	3987,885	4453,873	465,98772	10 %

Where: $N_{Rd,eff,w}$ = Ultimate strength based on effective width method

$N_{Rd,r}$ = Ultimate strength based on Reduced stress method

The difference between $N_{Rd,r}$ and $N_{Rd,eff,w}$ is larger for relatively shorter plates. The reason could be the column-like behavior influences the calculations in different manner. Recall that for Effective width method, $\bar{\lambda}_c$ is used for column slenderness, while $\bar{\lambda}_p$ is used as column slenderness in the case of reduced stress method.

Effective width method: $\bar{\lambda}_c = \sqrt{\frac{f_y}{\sigma_{cr,c}}}$ column slenderness used when calculating χ_c.

Reduced stress method: $\bar{\lambda}_p = \sqrt{\frac{\alpha_{ult,k}}{\alpha_{cr}}} = \sqrt{\frac{f_y}{\sigma_{cr,p}}}$ column slenderness used when calculating χ_c.

As the length of the plate increases the difference between the two strengths stabilizes to be 10 %. Effect of column-like buckling will be insignificant when length increased over certain limit, and the source of this 10 % difference is due to the fact that, Reduced stress method do not allow load shedding between member plates of the cross section.

A graphical presentation of the difference between strengths calculated based on Reduced stress method and Effective width method is given on the Figure 6-2.

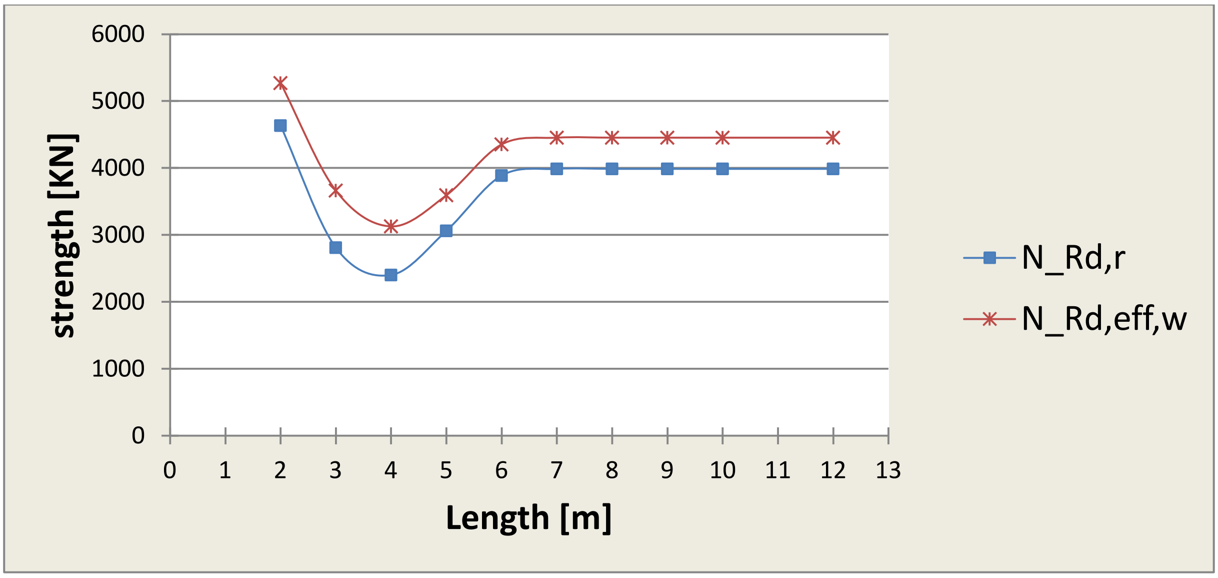


Figure 6-2 Comparisons between $N_{Rd,r}$ (strength based on Reduced stress method) and $N_{Rd,eff,w}$ (strength based on Effective width method).

7. Results of the Finite element analysis

As it is stated on the sub topic 5.4, a finite element analysis is carried out on unixally loaded unstiffened plates, biaxially loaded plates and longitudinally stiffened plates. Detailed description of plate dimensions, material properties, meshing, imperfection, loading and boundary conditions are presented on chapter 5.

7.1. Effect of boundary conditions on uniaxially loaded unstiffened plates

Plates with "MYK" type boundary condition and "RETT" type boundary conditions are analyzed by Abaqus. Both linear buckling force and ultimate strength are determined for each plate. It should be noted that all other conditions are kept constant and only boundary conditions are changed, between "MYK" type and "RETT" type.

"MYK" type boundary condition: loaded edges are free to deform and not kept straight. Loading is also applied longitudinally on both ends. The plate is supported on three points at the middle of the plate model.

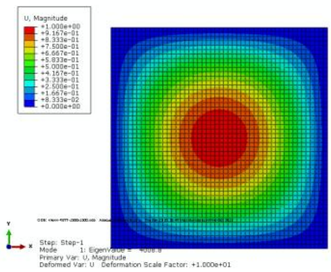
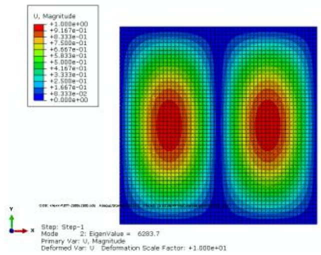
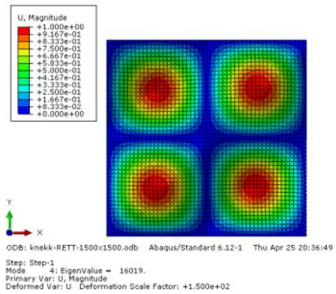
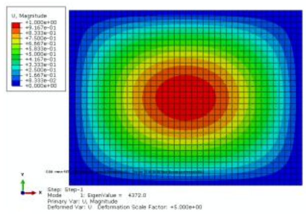
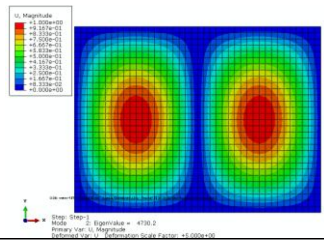
"RETT" type boundary condition: the loaded edge is kept straight by using the function *equation in the interaction module of the Abaqus soft ware.

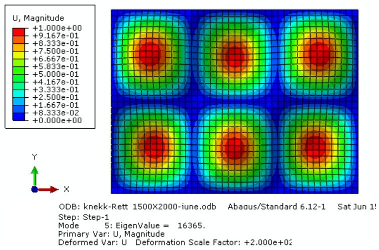
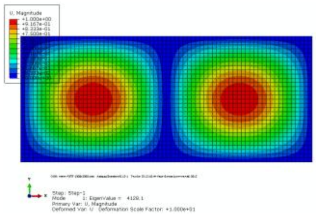
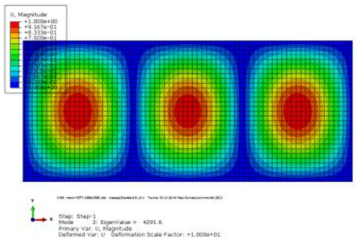
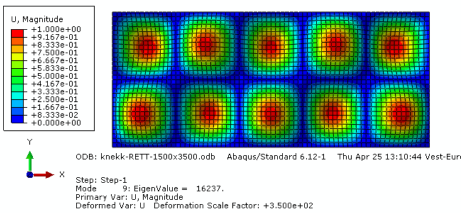
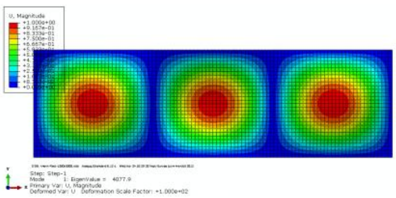
Both "MYK" type and "RETT" type boundary conditions are forms of simple support and allow free rotation at the support (i.e. rotational degrees of freedom are not fixed). The only difference between them is how and where the simple supports are applied. For further description of "MYK" and "RETT" type boundary conditions refer to chapter 5.4.1.

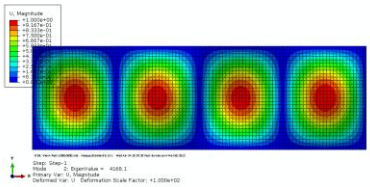
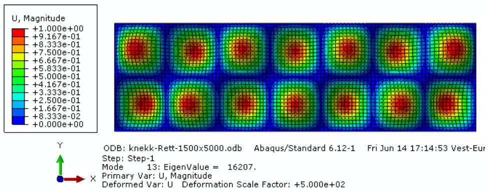
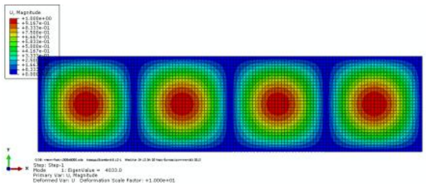
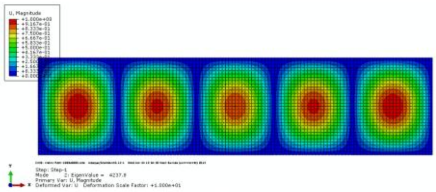
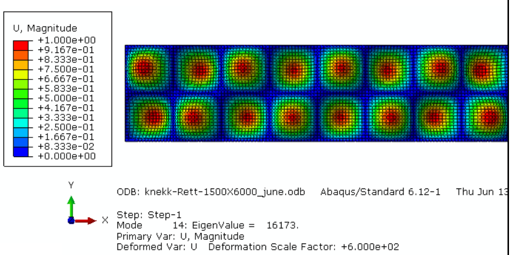
7.1.1. Buckling of plates with "RETT" type boundary condition

Linear buckling analysis of longitudinally loaded unstiffened plates with "RETT" type boundary condition is performed using Abaqus. Plots of the lowest global and local buckling modes are presented on Table 7-1.

Table 7-1 Linear buckling modes for longitudinally load plates with "RETT" type boundary condition

No	Plate size (width X length)	Buckling mode nr.	Figure	N_cr-abaqus
1	1500x1500	Mode 1		4008,8kN
		Mode 2		6283,7kN
		Mode 4		16019kN
2	1500x2000	Mode 1		4372,2kN
		Mode 2		4730,2kN

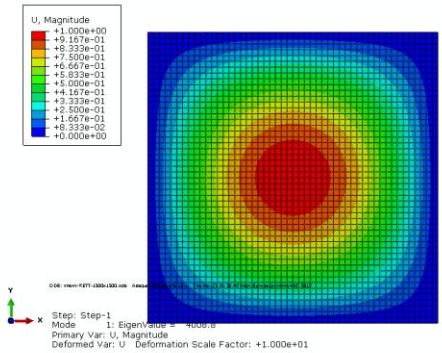
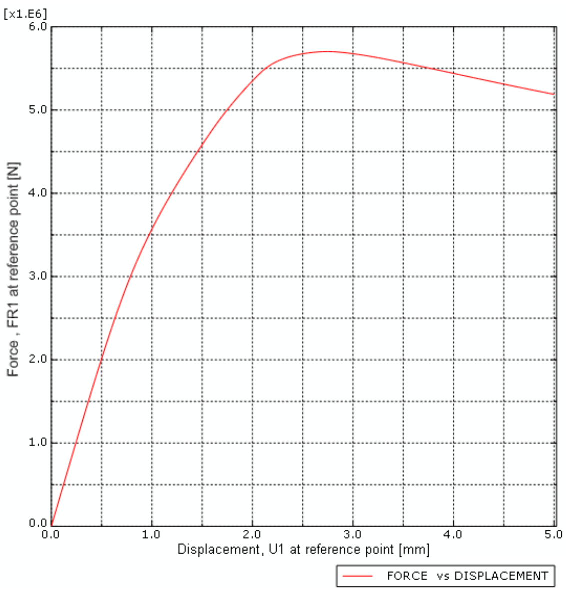
		Mode 5		16365kN
3	**1500x3500**	Mode 1		4128,1kN
		Mode 2		4291,6kN
		Mode 9		16237kN
4	**1500x5000**	Mode 1		4077,9kN

		Mode 2		4168,1kN
		Mode 13		16207kN
5	1500x6000	Mode 1		4033kN
		Mode 2		4237,8kN
		Mode 14		16173kN

7.1.2. Ultimate strengths of plates with "RETT" type of boundary condition

Ultimate strength of uniaxially loaded unstiffened plates with "RETT" type of boundary condition are numerically analyzed. Strength curve are presented on Table 7-2. Furthermore, the type of buckling mode used to introduce imperfection and imperfection amplitude are explained in the same table.

Table 7-2 Ultimate strength of plates with "RETT" type boundary conditions

<table>
<tr><th>Plate no.1</th><th>Plate 1500x1500, t=20</th><th>Remarks</th></tr>
<tr><td></td><td>
</td><td>Buckling mode 1 is used to introduce imperfection to the analysis.
The load is applied as a concentrated force at RP (midpoint of the left vertical edge). See Figure 5-5.
Applied load = 1000 N as a point load.</td></tr>
<tr><td></td><td>
</td><td>The force is extracted by imposing a displacement, U1 at longitudinal direction.
The imposed displacement was applied at RP (reference point), which is at the middle of the left vertical edge.
Max. force = 5702 kN
Displacement, U1 at max. force = 2,75 mm
Imperfection amplitude =3 mm</td></tr>
</table>

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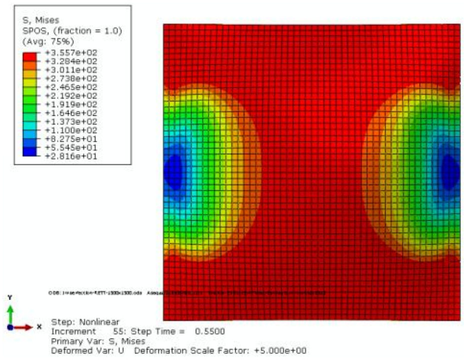
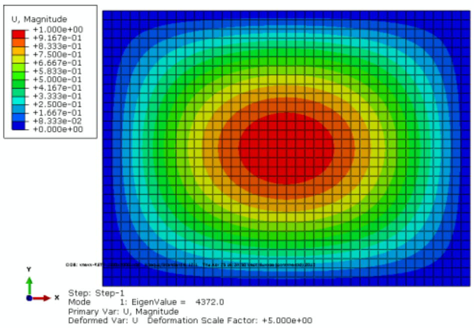
<td>
</td>
<td>The plot show stress pattern at maximum loading.
Since short edges are constrained to stay straight, a central horizontal plate strip buckle first and most of strength will be concentrated along the longitudinal edges.
On this plot most part of the model has reached yielding, But yielding has started first around corners.</td>
</tr>
<tr>
<td>Plate no.2</td>
<td>Plate 1500x2000, t=20</td>
<td>Remarks</td>
</tr>
<tr>
<td></td>
<td>
</td>
<td>Buckling mode 1 was used to introduce imperfection to the analysis.
The load is applied as a concentrated force at RP (midpoint of the left vertical edge).
See Figure 5-5
Applied load = 1000 N as a concentrated load.</td>
</tr>
<tr>
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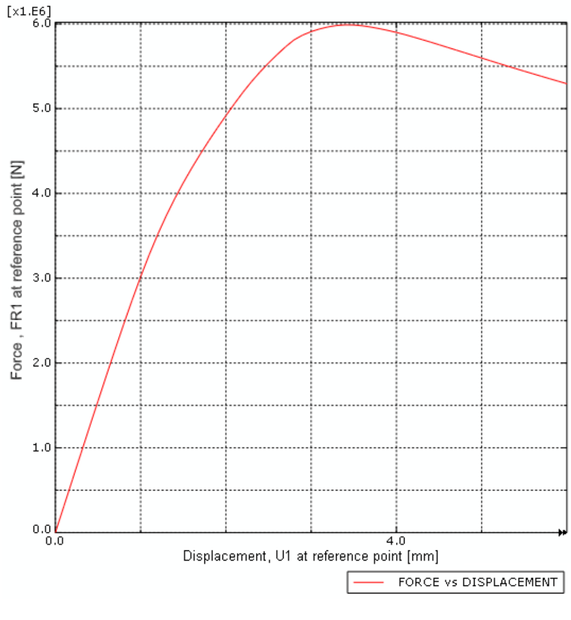
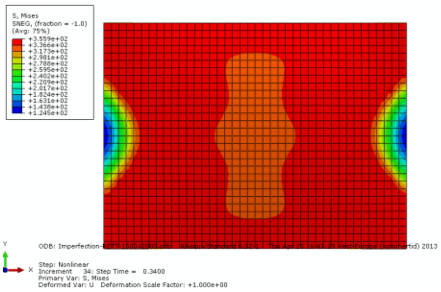
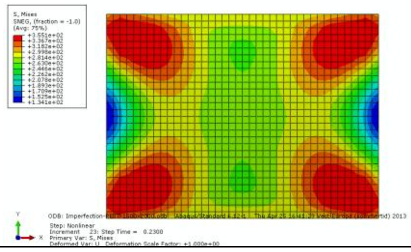
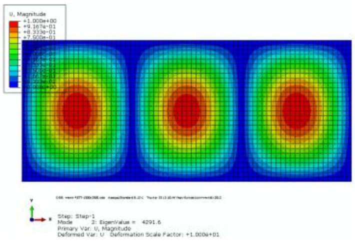
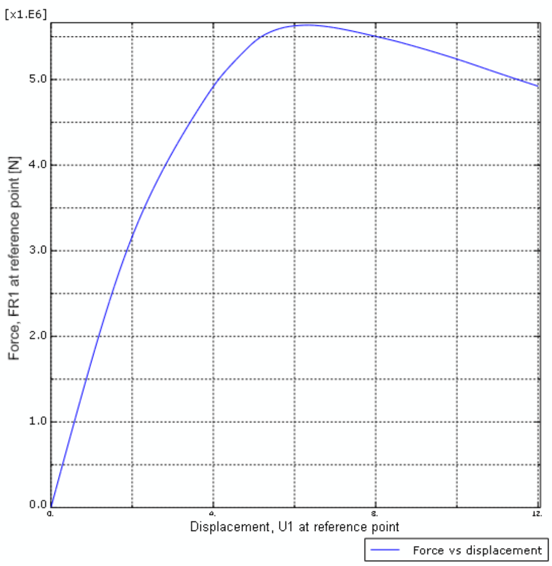
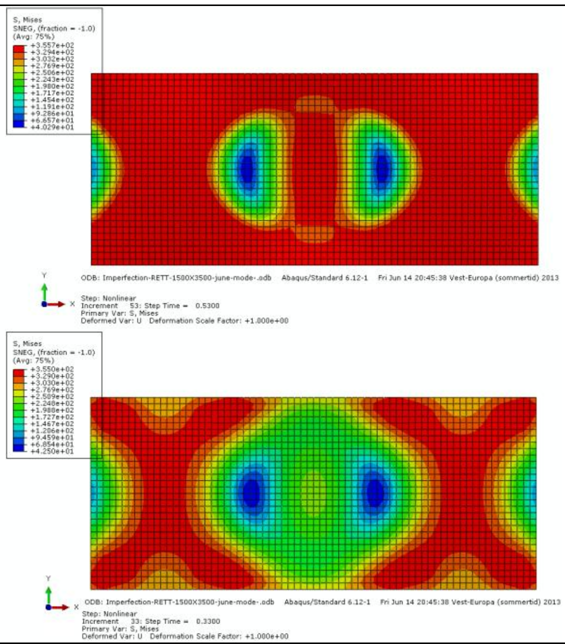
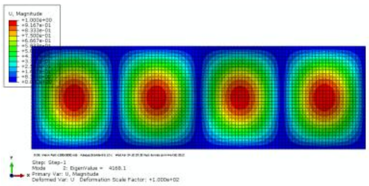
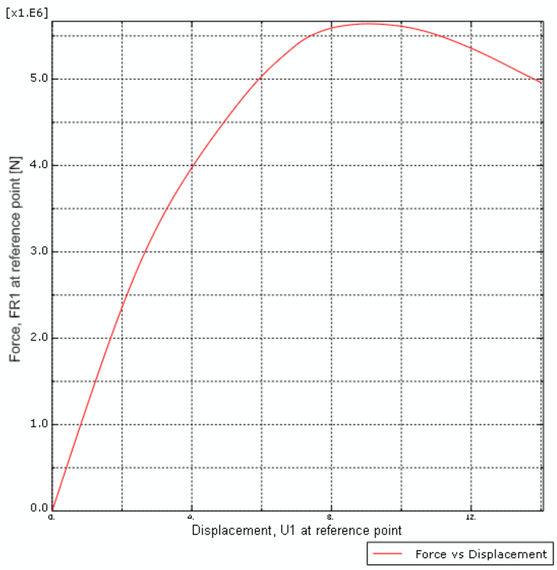
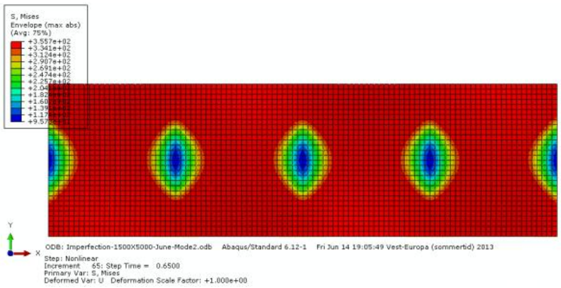
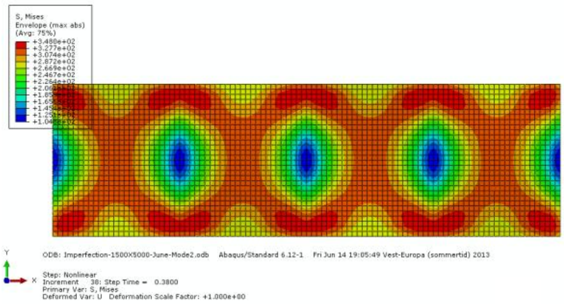
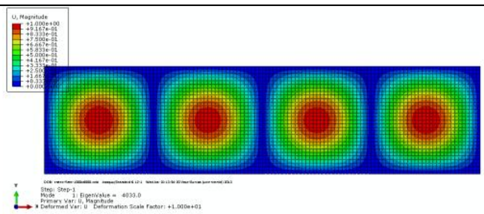
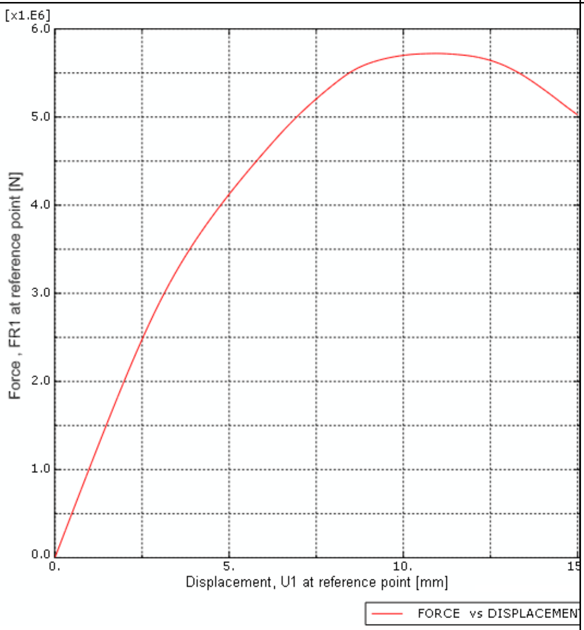
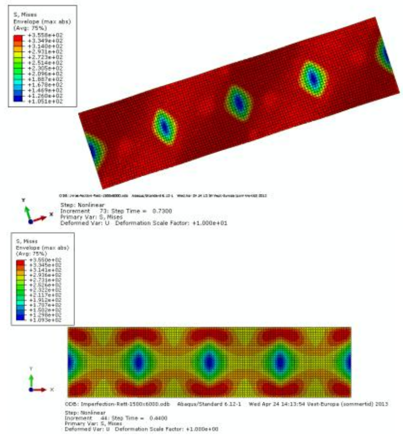
<td>
</td>
<td>The force is extracted by imposing a displacement, U1 at longitudinal direction.
The imposed displacement was applied at reference point, which is at the middle of the left vertical edge.
Max. force = 5988 kN
Displacement, U1 at max. force = 3,4 mm

Imperfection amplitude = 3 mm</td>
</tr>
</table>

		Stress pattern at the maximum loading. Though at this loading stage most part is red colored, yielding has started near the corners and spread along the longitudinal edges. This confirms the assumption of the Effective width method used in NS-EN-1993-1-5
		Stress pattern at the onset of yielding. Yielding has started at the corners.
Plate no.3	**Plate 1500x3500, t=20**	**Remarks**
		Buckling mode 2 is used to introduce imperfection to the analysis.
		The force is extracted by imposing a displacement, U1 at longitudinal direction. The imposed displacement was applied at reference point, which is at the middle of the left vertical edge. Max. force = 5635,6 kN Displacement, U1 at max. force = 6,36 mm Imperfection amplitude= 3 mm

		These two plots show stress pattern at two different loading stages. <u>Plot on the top</u>: stress pattern at maximum loading. <u>Plot under</u>: stress pattern at the onset of yielding. As already mentioned yielding starts at corners, but type of imperfection introduced has an effect on it.
Plate no. 4	**Plate 1500 X 5000, t=20**	**Remarks**
		Buckling mode 2 is used to introduce imperfection to the analysis. Concentrated load of 1000 N is applied at the reference point
		The force is extracted by imposing a displacement, U1 at longitudinal direction. The imposed displacement was applied at reference point, which is at the middle of the left vertical edge. Max. force = 5640 kN Displacement, U1at max. force = 9,10 mm Imperfection amplitude used = 3 mm

		Stress pattern at maximum loading.
		Stress pattern just before yielding stress is reached. (stress = 350 N/mm^2). Generally stress concentrations occur along supported edges. But the plot shows the imperfection mode introduced has an effect on the pattern of stress distributions.
Plate no. 5	**Plate 1500 X 6000 , t=10**	**Remark**
		Buckling mode 1 was used to introduce imperfection to the analysis.
		The force is extracted by imposing a displacement, U1 at longitudinal direction. The imposed displacement was applied at reference point, which is at the middle of the left vertical edge. Max. force = 5721 kN Displacement, U1 at max. force = 10,95 mm Imperfection amplitude =3 mm

		Stress pattern during loading. Stress plot top : stress pattern at maximum loading Stress plot bottom: stress pattern at the onset of yielding.

7.1.3. Buckling force of plates with "MYK" type boundary condition

Buckling analysis was also performed for unstiffened plates with "MYK" type boundary conditions. Both buckling force and pattern in each mode is exactly the same as that of plates with "RETT" type boundary condition.

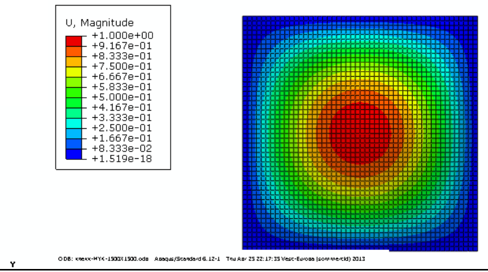
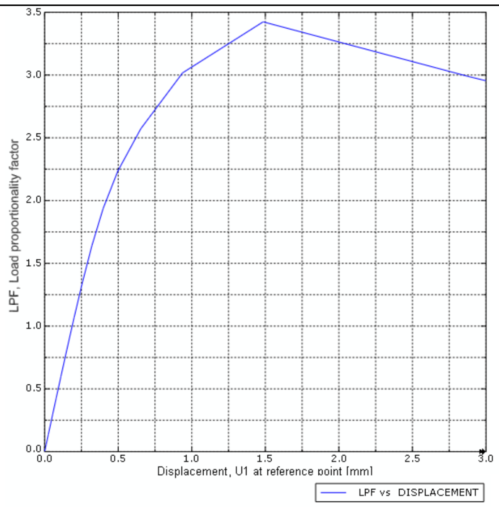
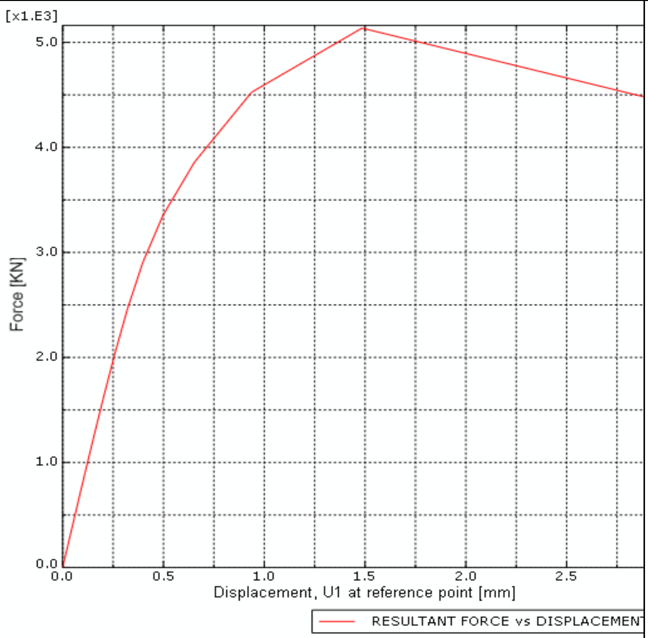
The reason why plates with "MYK" type and "RETT" type boundary have exactly equal buckling force and pattern is, in linear analysis original configuration is taken as a reference during computation of forces.

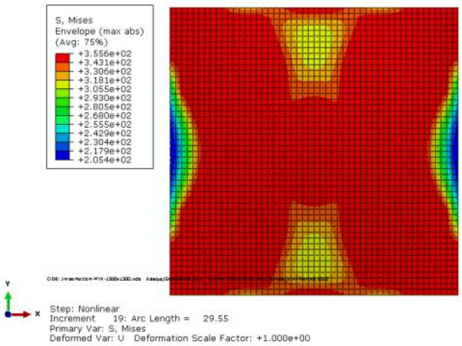
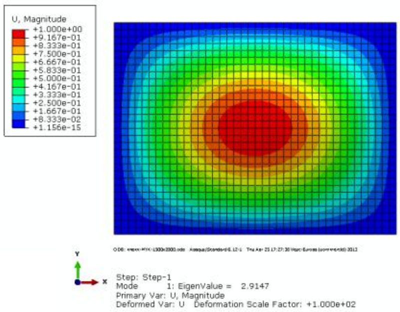
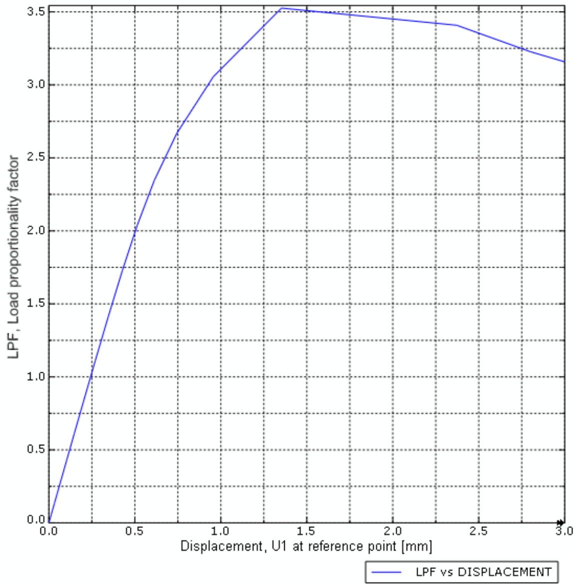
The lowest global and local buckling modes of plates analyzed are shown on Table 7-1.

7.1.4. Strengths of plates with "MYK" type boundary conditions.

Plates with "MYK" type boundary conditions are numerically analyzed to determine their strengths and strength curves are shown on Table 7-3.

Table 7-3 Ultimate strengths of unstiffened plates with "MYK" type boundary condition

No.	Plate 1500x1500 (a=1500mm and b=1500)	Remarks
1		This is a buckling mode 1, which is used to introduce imperfection to the nonlinear strength simulation. A distributed load of 1000 N/mm is applied from both ends
		The graph shows LPF versus Displacement. Displacement, U1 is taken from the reference point. Load applied = 1000 N/mm LPF_max = 3,42362 U1 at LPF_max = 1,485 mm Imperfection amplitude = 3 mm
		The resultant force is derived from LPF (load proportionality factor) as follow: F =LPF*(1000 N/mm)*b b= is width of the plate F_max =5135 kN

		Stress plot at maximum loading.
2	**Plate 1500x2000 (a=1500mm and b=2000) , t=20**	**Remarks**
		This is the buckling mode 1, used to introduce geometric imperfection. A distributed load of 1000 N/mm is applied from both ends.
		The graph shows LPF versus Displacement. U1 Displacement, U1 is taken from the reference point. Reference point is a point at the middle of left loaded edge. Load applied = 1000 N/mm LPF_max = 3,52554 U1 at LPF_max = 1,353 mm Imperfection amplitude =3 mm

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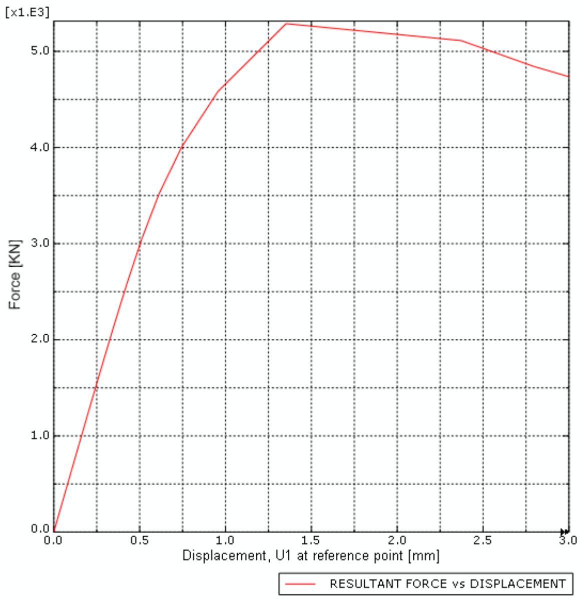
<td>
</td>
<td>Max force (axial strength) is derived from the LPF.

F =LPF*(1000 N/mm)*b

b= is width of the plate

F_max =5288 kN</td>
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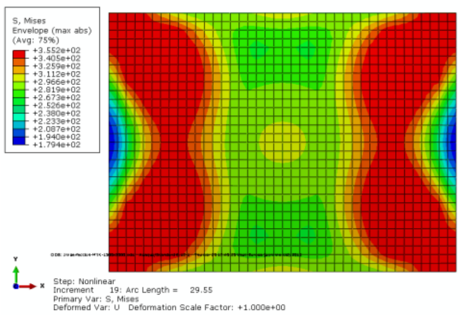
<tr>
<td></td>
<td>
</td>
<td>Stress plot of the plate at maximum loading.

It is worth noting, the pattern of the stress plot. The short ends are not restricted in both X- and Y-direction, which leads to higher stress concentration around the two ends of the plate.

As opposed to this, if the short edges were constrained to remain straight, the stress would have been concentrated along the longitudinal edges. See plate no.2 (i.e. plate 1500X2000) on Table 7-2.</td>
</tr>
<tr>
<td>3</td>
<td>Plate 1500 X 3500, t=20</td>
<td>Remarks</td>
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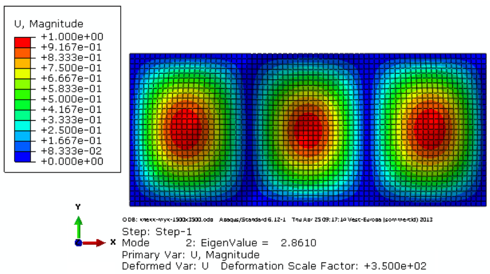
<td>
</td>
<td>Buckling mode 2, used to simulating the geometric imperfection.

Distributed load of 1000 N/mm is applied from both ends.</td>
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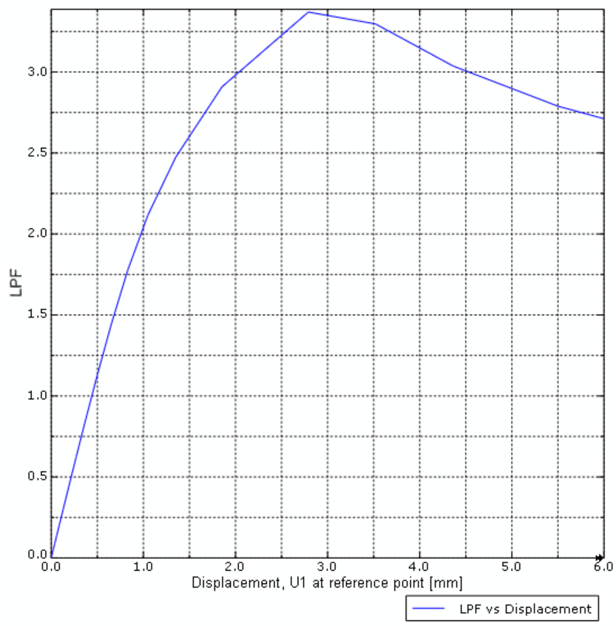
<td>
</td>
<td>The graph shows LPF vs. Displacement, U1 (i.e. longitudinal direction)

Displacement, U1 is taken from the reference point, which is the midpoint of the left loaded edge of the plate
Load applied = 1000 N/mm

LPF_max = 3,37

U1 at LPF_max = 2,791 mm

Imperfection amplitude =3 mm</td>
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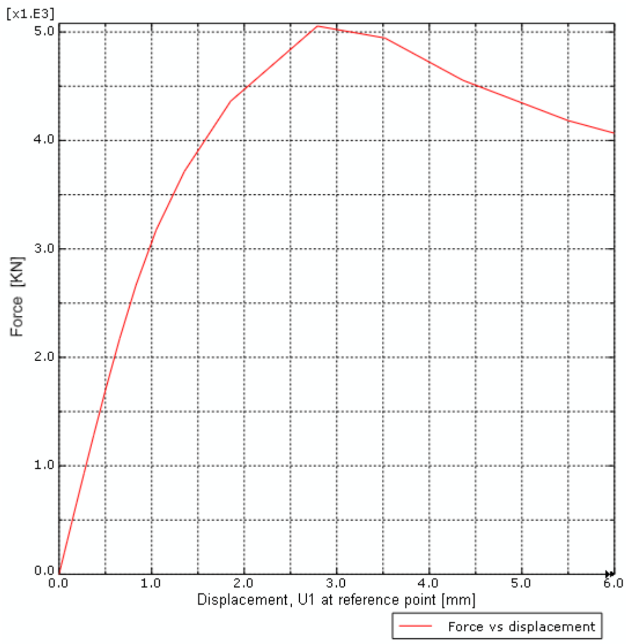
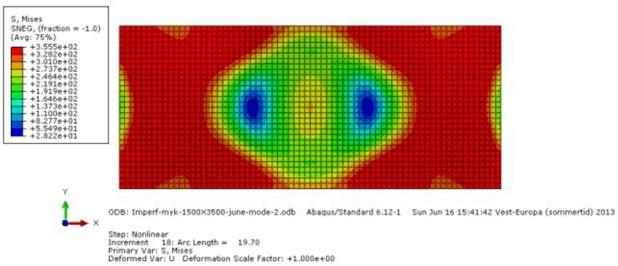
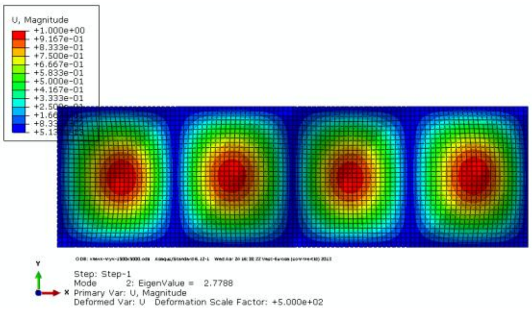
<td>
</td>
<td>Max force (axial strength) is derived from the LPF.

F =LPF*(1000 N/mm)*b

b= is width of the plate

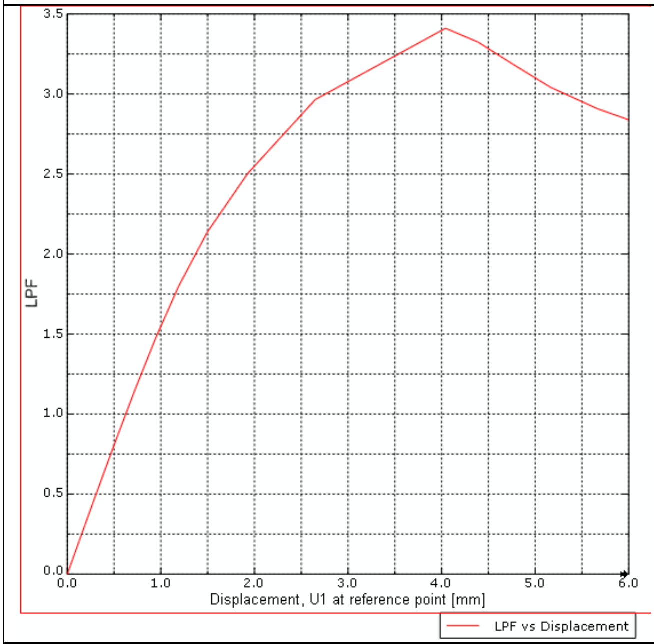
F_max =5054,8 kN</td>
</tr>
<tr>
<td>
</td>
<td>Stress pattern at maximum loading.

Now as it is late stage of loading, the plate has yielded all around except at the central region.</td>
</tr>
<tr>
<td>4</td>
<td>Plate 1500 X 5000 , t=20</td>
<td>Remarks</td>
</tr>
</table>



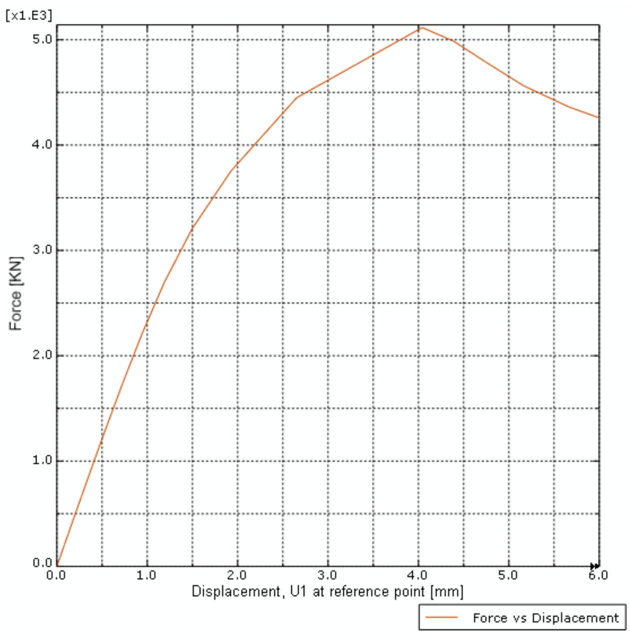
Buckling mode 2 was used to introduce imperfection to the analysis.

A distributed load of 1000 N/mm was applied on both ends, during buckling analysis



Distributed load is applied on both ends.
Load applied = 1000 N/mm
LPF_max = 3,41
U1 at LPF_max = 4,042 mm
Displacement, U1 is taken from the reference point, which is the midpoint of the left edge of the plate

Imperfection amplitude =3 mm

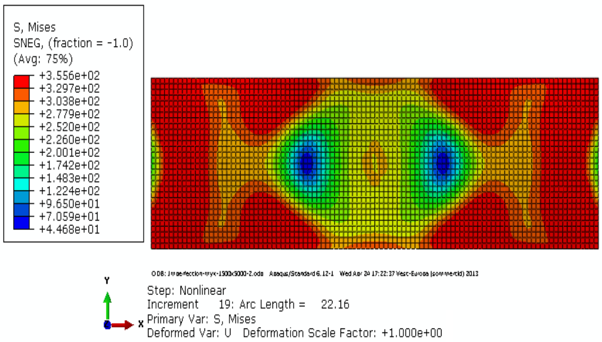


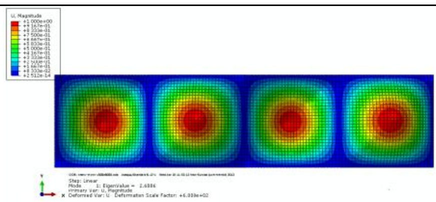
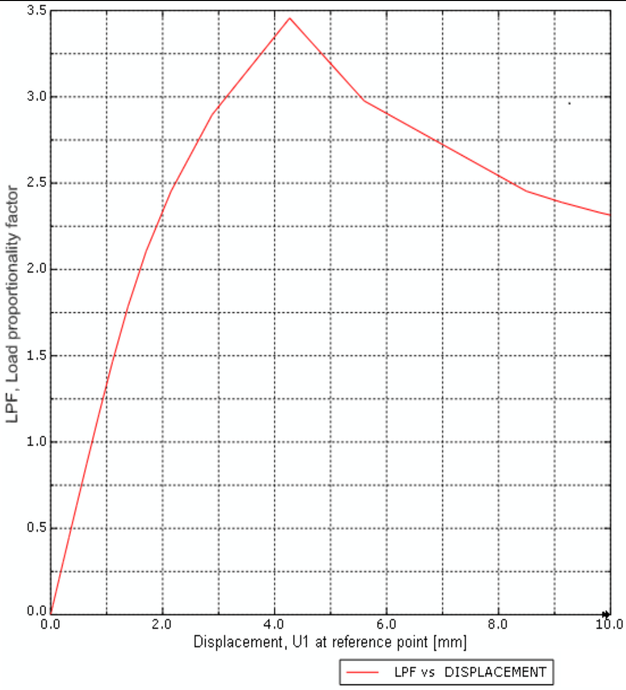
Resultant force is derived from the LPF. See the graph above.

Force = LPF*(1000 N/mm)*b

b= width of the plate

Max. axial force = 5116 kN,

		Stress pattern at maximum axial loading. Since short edges are not forced to stay straight, yielding occur first near short ends. The plot shows stress pattern in the late stage of loading (at max. strength). It shows a large area of the plate has reached yield stress. But it was the area near the short edges that has shown sign of yielding first.

5	Plate 1500X6000, t=10	Remarks
		Buckling mode-1 was used to introduce imperfection to the analysis.
		Distributed load is applied on both ends. Displacement, U1 is taken from the reference point, which is the midpoint of the left edge of the plate Load applied = 1000 N/mm LPF_max = 3,45616 U1 at LPF_max = 4,262 mm Imperfection amplitude = 3 mm

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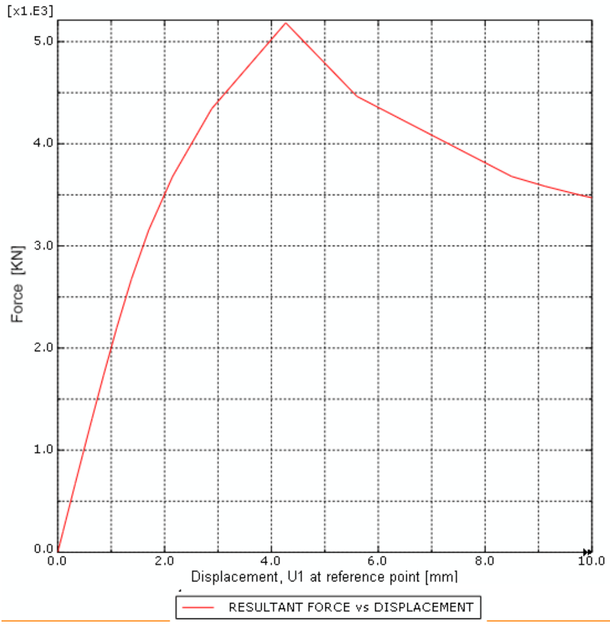
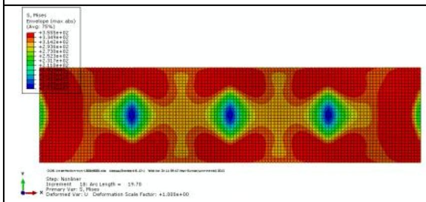
<td>
</td>
<td>Resultant force is derived from LPF-curve above.

Distributed load applied on both ends = 1000 N/mm

Force = LPF*(1000 N/mm)*b

Maximum axial force =5184,24 kN</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Stress pattern at maximum loading.
For the plate 5000X6000, t = 20</td>
</tr>
</table>

7.1.5. Comparison of plates with "MYK" type and "RETT" type boundary conditions.

Critical buckling of plates remained the same regardless of type boundary condition used. But there is a significant difference between strength of the plates depending on how the boundary condition is applied to the model.

Comparison of ultimate strength of plates with "MYK" type and "RETT" type boundary conditions is summarized on Table 7-4.

Table 7-4 Comparison of ultimate strength of plates with "MYK" and "RETT" type boundary conditions

No.	Plate size [mm]	Strength "MYK" type [kN]	Strength "RETT" type [kN]	Difference [kN]	Difference [%]
1	1500 X 1500	5135,4	5702,3	566,9	9,9 %
2	1500 X2000	5288,3	5988,1	699,8	11,7 %
3	1500 X 3500	5054,8	5635,6	590,8	10,5 %
4	1500 X 5000	5116	5640	524	9,3 %
5	1500 X 6000	5184,2	5721,5	537,3	9,4 %

Strength results on Table 7-4, show an average difference of 10 % between plates with "MYK" and "RETT" type boundary conditions. Plates with "MYK" type boundary conditions are softer, since the loaded edges are not forced to remain straight and can deform. Plates with "RETT" type of boundary conditions are stronger, since the boundary condition forces the loaded edges to stay straight.

7.2. Numerical analysis of biaxially loaded unstiffened plates

Biaxially loaded Plate dimensions given on Table 5-4 are analyzed by using Abaqus. Hand calculated results of the same plates are presented on subtopic 6.2.

Analysis of both linear buckling and ultimate strength is performed. Buckling mode plots and strength curves are given as an appendix on subtopics A.1 and A.2 . Only resulting buckling forces and ultimate strength are presented below.

Critical buckling force of square plate (2000X200, t=20) and rectangular plate (2000X4000, t=20) with different combinations of biaxial loading is presented as follow. Plots of buckling modes are given as an appendix on Table A- 1 and Table A- 2.

Table 7-5 Critical buckling stress of biaxially loaded plate 2000X2000, t=20

No.	Loading type [N/mm]	λ : Lowest buckling mode	$\sigma_{cr,x,abaqus}$ [N/mm^2]	$\sigma_{cr,y,abaqus}$ [N/mm^2]
1	Longitudinal = 1000 Transverse = 250	1,20830	60,415	15,104
2	Longitudinal = 1000 Transverse = 500	1,00690	50,345	25,173
3	Longitudinal = 1000 Transverse = 750	0,86308	43,154	32,366
4	Longitudinal = 1000 Transverse = 1000	0,75520	37,760	37,760

Table 7-6 Critical buckling stress of biaxially loaded plate 2000X4000, t=20

No.	Loading type [N/mm]	λ : for lowest buckling mode	$\sigma_{cr,x,abaqus}$ [N/mm^2]	$\sigma_{cr,y,abaqus}$ [N/mm^2]
1	Longitudinal = 1000 Transverse = 250	1,18390	59,195	14,800
2	Longitudinal = 1000 Transverse = 500	0,78914	39,457	19,729
3	Longitudinal = 1000 Transverse = 750	0,59182	29,591	22,193
4	Longitudinal = 1000 Transverse = 1000	0,47343	23,672	23,672

Where :

λ = eigen value from buckling analysis

$\sigma_{cr,x,abaqus}$ = critical buckling stress – logitudinal direction

$\sigma_{cr,y,abaqus}$ = critical buckling stress – transverse direction

Biaxially loaded, square plate (2000X2000, t = 20) and rectangular plate (2000X4000, t = 20) are numerically analyzed to determine their strength and results are given on Table 7-7 and Table 7-8. Strength curves extracted from the numerical analysis are given as an appendix on Table A- 3 and Table A- 4.

Table 7-7 Ultimate strength of biaxially loaded plate 2000X2000, t=20

No.	Loading type [N/mm]	Imperfection amplitude [mm]	$\sigma_{Rd,x,abaqus}$ [N/mm^2]	$\sigma_{Rd,y,abaqus}$ [N/mm^2]
1	Longitudinal = 1000 Transverse = 250	4	155,950	38,987
2	Longitudinal = 1000 Transverse = 500	4	131,765	65,882
3	Longitudinal = 1000 Transverse = 750	4	120,271	90,203
4	Longitudinal = 1000 Transverse = 1000	4	104,528	104,528

Table 7-8 Ultimate strength of biaxially loaded plate 2000X4000, t=20

No.	Loading type [N/mm]	Imperfection amplitude [mm]	$\sigma_{Rd,x,abaqus}$ [N/mm^2]	$\sigma_{Rd,y,abaqus}$ [N/mm^2]
1	Longitudinal = 1000 Transverse = 250	4	147,665	36,916
2	Longitudinal = 1000 Transverse = 500	4	114,072	57,036
3	Longitudinal = 1000 Transverse = 750	4	91,271	68,453
4	Longitudinal = 1000 Transverse = 1000	4	76,026	76,026

Where:

$\sigma_{Rd,x,abaqus}$ = ultimate strength – longitudinal direction

$\sigma_{Rd,y,abaqus}$ = ultimate strength – transverse direction

7.3. Numerical analysis of uniaxially loaded stiffened plates

Hand calculations of longitudinally stiffened plates were presented on the previous chapters. Results of the finite element analysis for the same plates will be presented on the subsequent subtopics. Plots of buckling modes and strength curves are given on appendix B and C.

7.3.1. Results of linear buckling analysis: stiffened plates.

Plates naturally tend to buckle to a mode shape which requires minimum energy, and critical buckling force is considered to be the minimum force required to deform the plate in certain mode shape.

Results of the linear buckling analysis are tabulated below. These results will be later compared with hand calculated results given on Table 6-8 and Table 6-9. Plots of the lowest buckling modes are presented as an appendix on Table B- 1 and Table C- 1.

Table 7-9 Critical buckling forces, $N_{cr,abaqus}$ for stiffened plates with increasing number of stiffener

No	Plate size(mm)	No. of stiffeners	$N_{cr,abaqus}$ [kN]
1	1600X7000	3	3012,2
2	2000 X7000	4	2148,4
3	2400 X7000	5	1845,2
4	2800 X 7000	6	1767,8
5	3200 X 7000	7	1796,1
6	3600 X 7000	8	1879,7

Similarly buckling analysis is carried out for stiffened plate with constant width but varying length. Results are given on Table 7-10 below.

Table 7-10 Critical buckling, $N_{cr,abaqus}$ of stiffened plate with varying length

No	Plate size(mm)	No. of stiffeners	$N_{cr,abaqus}$ [kN]
1	2000 X 2000	4	10 309,7
2	2000 X 3000	4	4 940,0
3	2000 X 4000	4	3 130,8
4	2000 X 5000	4	2 415,2
5	2000 X 6000	4	2 160,8
6	2000 X 7000	4	2 148,4
7	2000 X 8000	4	2 285,9
8	2000 X 9000	4	2 529,2
9	2000 X 10000	4	2416,1
10	2000 X 10000	4	2161,6

7.3.2. Ultimate strength of uniaxially loaded, stiffened plates

Stiffened plate dimension given on Table 5-6 and Table 5-7 are numerically analyzed using Abaqus. Results of linear buckling analysis for the plates are given on Table 7-9 and Table 7-10.

On the following tables only resulting strengths are presented. Strength curves are presented as an appendix on Table B- 2 and Table C- 2.

7-11 Numerically determined strength, $N_{Rd,abaqus}$ of plates with increasing number of stiffeners

No	Plate size[mm]	No. stiffeners	Imperfection amplitude [mm]	$N_{Rd,abaqus}$ [kN]
1	1600X7000	3	3,2	4 249,6
2	2000 X7000	4	4	4 326,0
3	2400 X7000	5	4,8	4 510,4
4	2800 X 7000	6	5,6	4 695,8
5	3200 X 7000	7	6,4	4 858,5
6	3600 X 7000	8	7,2	5 006,8

Table 7-12 Numerically determined strength, $N_{Rd,abaqus}$: Stiffened plate with increasing length

No	Plate size[mm]	No. stiffeners	Imperfection amplitude [mm]	$N_{Rd,abaqus}$ [kN]
1	2000 X 2000	4	4	6 339,8
2	2000 X 3000	4	4	4 576,9
3	2000 X 4000	4	4	4 172,4
4	2000 X 5000	4	4	4 110,6
5	2000 X 6000	4	4	4 189,6
6	2000 X 7000	4	4	4 326,0
7	2000 X 8000	4	4	4 452,6
8	2000 X 9000	4	4	4 580,7
9	2000X10000	4	4	4 110,9
10	2000X12000	4	4	4 190,1

8. Comparison of hand calculated and Abaqus results

Hand calculations based on Reduced stress method and Effective width method are performed for both unstiffened plates and stiffened plates. A finite element analysis using Abaqus is also carried out for the same plates. Comparisons will be done between hand calculations and Abaqus results in the following subtopics.

8.1. Uniaxially loaded unstiffened plates

Hand calculated results based on both the Effective width method and Reduced stress method are presented on subtopic 5.1. It has been shown for uniaxially stressed unstiffened plates both hand calculation methods lead to exactly equal crititical buckling stress and ultimate strength for all plate dimensions.

In case of finite element analysis, unstiffened plates are categorized depending on how the boundary conditions are modeled in Abaqus. The two categories are denoted as, plates with "RETT" type and "MYK" type boundary condition. Only results of plates with "RETT" type boundary conditions will be compared with hand calculated results. The reason is that, rules of Eurocode-3-1-5 are developed by assuming loaded edges remain straight during loading.

8.1.1. Comparison of critical buckling forces: Hand calculations versus Abaqus results

A comparison of results of hand calculation and Abaqus ("RETT" type boundary condition) is given below. In the Table 8-1 results of both design methods are denoted as, $N_{cr,hand}$ since they are equal.

Table 8-1 comparison of hand calculated critical buckling force, $N_{cr,hand}$ and Abaqus results, $N_{cr,abaqus}$

No.	Plate size [mm]	$N_{cr,hand}$ [kN]	$N_{cr,abaqus}$ [kN]	Difference [kN]	Difference [%]
1	1500 X 1500	4049,1	4008,8	-40,3	-1,0 %
2	1500 X2000	4394	4372	-22	-0,5 %
3	1500 X 3500	4146	4128,1	-17,9	-0,4 %
4	1500 X 5000	4094,2	4077,9	-16,3	-0,4 %
5	1500 X 6000	4049,1	4033	-16,1	-0,4 %

There is a quite close agreement between hand calculated buckling force, and result of linear buckling analysis using Abaqus.

8.1.2. Comparison of hand calculated and numerically determined strengths

For uniaxially loaded unstiffened plates both Reduced stress and Effective width methods give equal strength values, which will be denoted as $N_{Rd,hand}$ in the following table. These results will be compared with numerically determined strengths, using "RETT" type boundary conditions. Description of plates with "RETT" type boundary condition is given in subtopic 5.4.1.

Table 8-2 Comparison of hand calculated strength, $N_{Rd,hand}$ and numerically determined strength, $N_{Rd,abaqus}$

No.	Plate size(mm)	$N_{Rd,hand}$ [kN]	$N_{Rd,abaqus}$	Difference [kN]	Difference [%]
1	1500 X 1500	5672,7	5702,3	29,6	1 %
2	1500 X2000	5870,5	5988,1	117,6	2 %
3	1500 X 3500	5729,5	5849	119,5	2 %
4	1500 X 5000	5699,3	5804	104,7	2 %
5	1500 X 6000	5672,7	5721,5	48,8	1 %

Hand calculated strength, $N_{Rd,hand}$ and numerical determined strength, $N_{Rd,abaqus}$ show close agreement.

8.2. Biaxially loaded unstiffened plates

Hand calculated critical buckling stress and strength will be compared with results from finite element analysis. Hand calculations are performed based on Reduced stress method, since Effective width method does not apply for multiple loading situation. Hand calculated results are given on Table 6-4 and Table 6-5.

Table 8-3 Comparison of critical buckling stress based on Reduced stress method, $\sigma_{cr,x,r}$ and results of Abaqus analysis, $\sigma_{cr,x,abaqus}$

Plate -2000X2000, t=20 mm					
No.	**Loading combination [N/mm]**	**$\sigma_{cr,x,r}$ [N/mm^2]**	**$\sigma_{cr,x,abaqus}$ [N/mm^2]**	**Difference [N/mm^2]**	**Difference [%]**
1	Longitudinal = 1000 Transverse = 250	60,7	60,415	-0,285	-0,5
2	Longitudinal = 1000 Transverse = 500	50,611	50,345	-0,266	-0,5
3	Longitudinal = 1000 Transverse = 750	43,394	43,154	-0,240	-0,6
4	Longitudinal = 1000 Transverse = 1000				
Plate -2000 X 4000, t=20 mm					
No.	**Loading combination [N/mm]**	**$\sigma_{cr,x,r}$ [N/mm^2**	**$\sigma_{cr,x,abaqus}$ [N/mm^2]**	**Difference [N/mm^2]**	**Difference [%]**
1	Longitudinal = 1000 Transverse = 250	46,300	59,195	12,895	22
2	Longitudinal = 1000 Transverse = 500	33,248	39,457	6,209	16
3	Longitudinal = 1000 Transverse = 750	25,960	29,591	3,631	12
4	Longitudinal = 1000 Transverse = 1000	21,330	23,672	2,342	10

Reduced stress method does not clearly provide a way to calculate critical buckling stress, other than what is given as a definition to α_{cr}.

α_{cr} is defined as the minimum load amplifier for the design loads to reach the critical load of the plate under a complete stress field. Based on this definition of α_{cr}, the following relationship could be formulated:

$$\alpha_{cr} = \frac{\sigma_{cr}}{\sigma_{Ed}} \Rightarrow \sigma_{cr} = \alpha_{cr} \cdot \sigma_{Ed} \qquad \text{where} \qquad \begin{array}{l} \sigma_{cr} = \text{critical buckling stress} \\ \sigma_{Ed} = \text{design stress load} \end{array}$$

Since this project is a theoretical study, the design load, σ_{Ed} is not known in advance. Its value is found by carrying out an iteration scheme until equation (10.5) of Eurocode-3-1-5 is satisfied.

Critical buckling stress, σ_{cr} calculated based on the above relationship gives quite good result for uniaxial loading, but show certain weakness for biaxially compressed plates. In particular there is a quite large discrepancy between hand calculated values and Abaqus results for biaxially loaded rectangular plate. This shows aspect ratio has a role, which may not be captured in the above relationship. Thus a thorough study of background calculations and origin of α_{cr} is required. Unfortunately it is not possible to carry out such study, due to lack of relevant literature written in English or Norwegian.

Generally it could be concluded that Reduced stress method lack an explicit way of calculating critical buckling stress, especially for multiple loading situations. The indirectly deducted formula, $\sigma_{cr} = \alpha_{cr} \cdot \sigma_{Ed}$, lead to conservative results.

Table 8-4 Comparison of longitudinal ultimate strengths, $\sigma_{Rd,x,r}$ and $\sigma_{Rd,x,abaqus}$: Biaxially loaded plates

Plate -2000X2000, t=20					
No.	Loading combination [N/mm]	$\sigma_{Rd,x,r}$ [N/mm^2]	$\sigma_{Rd,x,abaqus}$ [N/mm^2]	Difference [N/mm^]	Difference [%]
1	Longitudinal = 1000 Transverse = 250	143,500	155,950	12,450	8 %
2	Longitudinal = 1000 Transverse = 500	131,800	131,765	-0,035	0 %
3	Longitudinal = 1000 Transverse = 750	113,300	120,271	6,971	6 %
4	Longitudinal = 1000 Transverse = 1000	94,800	104,528	9,728	9 %
Plate 2000 X 4000, t = 20					
No.	Loading combination [N/mm]	$\sigma_{Rd,x,r}$ [N/mm^2]	$\sigma_{Rd,x,abaqus}$ [N/mm^2]	Difference [N/mm^]	Difference [%]
1	Longitudinal = 1000 Transverse = 250	127,300	147,665	20,365	14 %
2	Longitudinal = 1000 Transverse = 500	108,300	114,072	5,772	5 %
3	Longitudinal = 1000 Transverse = 750	88,300	91,271	2,971	3 %
4	Longitudinal = 1000 Transverse = 1000	71,340	76,026	4,686	6 %

8.3. Uniaxially loaded stiffened plates

Results of hand calculations (both Reduced stress method and Effective width method) will be now compared with results obtained based on finite element analysis.

8.3.1. Comparison of critical buckling stresses

It was previously stated that both Reduced stress method and Effective with method lead to exactly same critical buckling force. Such hand calculated values are compared with Abaqus results on Table 8-5.

Table 8-5 Comparison of hand calculated critical buckling force, $N_{cr,hand}$ with $N_{cr,abaqus}$

No	Plate size [mm]	No. of stiffeners	$N_{cr,hand}$ [kN]	$N_{cr,abaqus}$ [kN]	Difference [kN]	Difference [%]
1	1600X7000	3	2421,7	3012,2	590,5	20 %
2	2000 X7000	4	1994,1	2148,4	154,3	7 %
3	2400 X7000	5	1705,7	1845,2	139,5	8 %
4	2800 X 7000	6	1608,6	1767,8	159,2	9 %
5	3200 X 7000	7	1626,5	1796,1	169,6	9 %
6	3600 X 7000	8	1703,5	1879,7	176,2	9 %

Table 8-5 above shows an average difference of 9 % between hand calculated results and results of finite element analysis. One of the reasons could be stiffened plates with more than two longitudinal stiffeners are treated as an equivalent orthotropic plates by smearing stiffeners smoothly over the plate. This is just an approximate approach and could lead to a lower buckling force and strength.

The difference is worse for narrow plate (1600X7000) with only three stiffeners. Since the stiffeners are smeared over the plate, the two stiffeners which are nearest to the longitudinal edges will miss some of their stiffness when hand calculated.

Table 8-6 Comparison of $N_{cr,hand}$ and $N_{cr,abaqus}$: Stiffened plate with varying length

No	Plate size [mm]	No. of Stiffeners	$N_{cr,hand}$ [kN]	$N_{cr,abaqus}$ [kN]	Difference [kN]	Difference [%]
1	2000 X 2000	4	8 860,5	10 309,7	1 449,2	14 %
2	2000 X 3000	4	4 214,8	4 940,0	725,2	15 %
3	2000 X 4000	4	2 713,4	3 130,8	417,4	13 %
4	2000 X 5000	4	2 155,1	2 415,2	260,1	11 %
5	2000 X 6000	4	1 996,8	2 160,8	164,0	8 %
6	2000 X 7000	4	1 994,1	2 148,4	154,3	7 %
7	2000 X 8000	4	1 994,1	2 285,9	291,8	13 %
8	2000 X 9000	4	1 994,1	2 529,2	535,1	21 %
9	2000 X 10000	4	1 994,1	2 416,1	422,0	17 %
10	2000 X 12000	4	1 994,1	2 161,6	167,5	8 %

Comparison of the buckling results on Table 8-6, show that there is also a significant difference between hand calculated values and the linear critical buckling force determined by finite element analysis. Similar reason could be given as for plates with increasing number of stiffeners on Table 8-5. Therefore results based on equivalent orthotropic considerations may not reflect the behavior of stiffened plate exactly.

The difference between hand calculated critical forces and Abaqus results on Table 8-6 is largest, for plate length of 9000 mm. this is due to two reasons:

- Hand calculated values become constant after plate length is increased over certain limit, but Abaqus results continue to vary slightly. See the blue curve on Figure 8-1.
- Abaqus analysis: Buckling force tend to increase just before the length limit, at which buckling mode shape changes from single half- sine-wave to two half-sine-waves.

In our case buckling mode changes from single half-sine -wave to two half-sine-waves when length is increased from 9000 mm to 10000 mm. Refer to buckling mode plots given in the appendix.

Comparison of hand calculated buckling force and Abaqus results are illustrated in Figure 8-1.

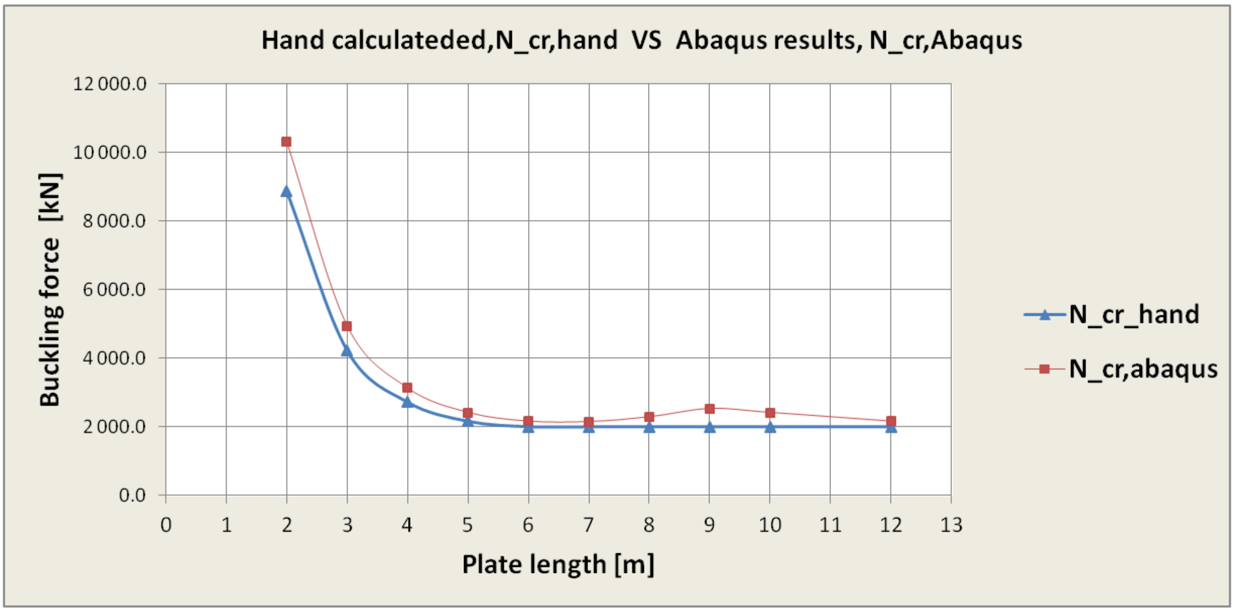


Figure 8-1 Comparison of critical forces: Hand calculated versus Abaqus results

8.3.2. Comparison of ultimate strengths: Longitudinally stiffened plate

Comparison of results of Reduced stress method and Effective width method was previously made on Table 6-15 and Table 6-16. Now each method will be compared with results of the numerical analysis.

Table 8-7 Comparison of ultimate strength based on Effective width method, $N_{Rd,eff,w}$ and results of numerical analysis, $N_{Rd,abaqus}$

No.	Plate size [mm]	No. of stiffeners	$N_{Rd,effe,w}$ [kN]	$N_{Rd,abaqus}$ [kN]	Difference [kN]	Difference [%]
1	1600X7000	3	4149,846	4 249,6	99,754	2 %
2	2000 X7000	4	4453,873	4 326,0	-127,873	-3 %
3	2400 X7000	5	4463,902	4 510,4	46,498	1 %
4	2800 X 7000	6	3866,711	4 695,8	829,089	18 %
5	3200 X 7000	7	3428,083	4 858,5	1430,417	29 %
6	3600 X 7000	8	3181,306	5 006,8	1825,494	36 %

Comparison of results shows $N_{Rd,eff,w}$ is very close to the results of the numerical analysis, $N_{Rd,abaqus}$ for relatively narrow plates. But the difference widens as the width increases. Strength of plates determined based on Effective width method is a result of an interpolation between plate-like and column- like behavior. As the width of plates increases, hand calculated plate strength decreases due to increased effect of column- like buckling. Therefore Effective width method fails to describe the real strength for wider plate.

Results of stiffened plate strength based on Reduced stress method is worse than that of Effective width method since Reduced stress method does not consider post buckling strength of all member plates of the cross section. Recall that previous comparison between these two methods has revealed that strength calculated based on Effective width method is on average 10 % higher than results of Reduced stress method.

Table 8-8 Comparison of strength based on Reduced stress method, $N_{Rd,r}$ and results of numerical analysis, $N_{Rd,abaqus}$

No.	Plate size [mm]	No. stiffeners	$N_{Rd,r}$ [kN]	$N_{Rd,abaqus}$ [kN]	Differenc e [kN]	Difference [%]
1	1600X7000	3	3792,401	4 249,6	457,2	11 %
2	2000 X7000	4	3987,872	4 326,0	338,1	8 %
3	2400 X7000	5	3919,178	4 510,4	591,2	13 %
4	2800 X 7000	6	3212,712	4 695,8	1 483,1	32 %
5	3200 X 7000	7	2651,798	4 858,5	2 206,7	45 %
6	3600 X 7000	8	2304,38	5 006,8	2 702,4	54 %

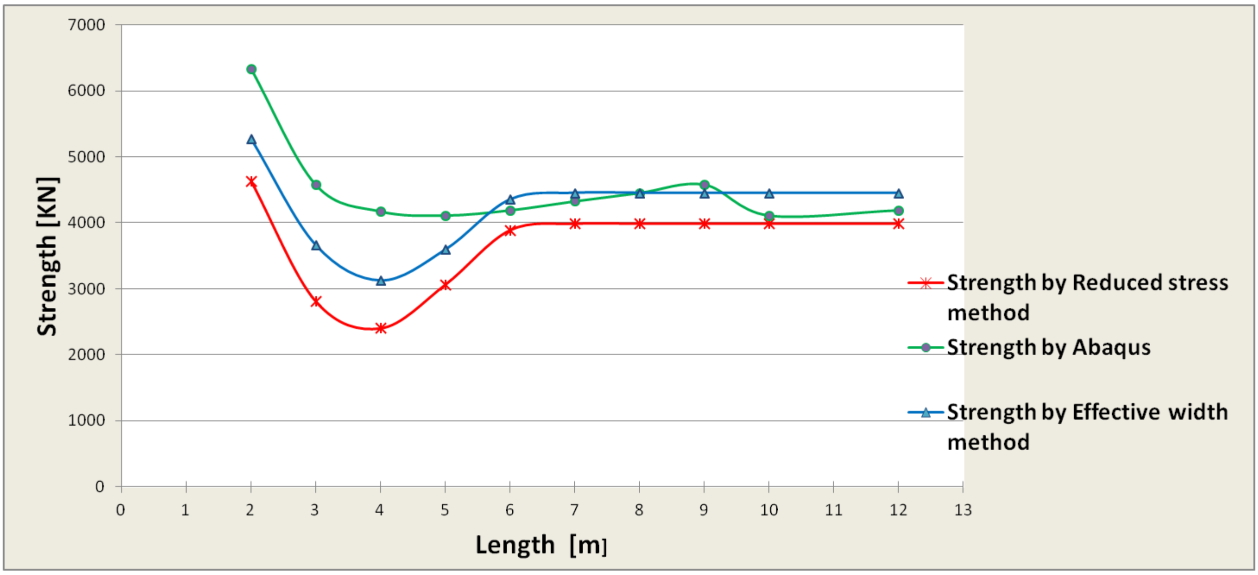


Figure 8-2 Plate with four longitudinal stiffeners, strength as a function of length

As shown on the Figure 8-2 strength curves based on Reduced stress method and Effective drops down in the interval 2 m to 4 m. This shows the interpolation equation used is not good enough to describe the behavior of the plate dimensions in this interval. As the length increases both curves rise up, since the effect of column-like buckling gets lesser.

9. Conclusion

For uniaxially loaded plates both Reduced stress method and the Effective width method showed exactly same critical buckling stress and strength. This is because of only single plate panel is used. In real life most plated structures are made up of multiple panels which could have different thickness. In such case the two methods are expected to produce different results.

In case of finite element analysis performed on uniaxially loaded unstiffened plate, no significant difference was found between hand calculated results and Abaqus results.

Hand calculations on biaxially loaded plates were done only based on Reduced stress method, since Effective width method is not applicable for multiple loading situations. Buckling analysis performed on biaxially loaded square plate showed no significant difference between results of Reduced stress method and Abaqus. But for biaxially loaded rectangular plate, the analysis showed quite large difference between hand calculated critical buckling stress and that of Abaqus. This shows Reduced stress method may fail to describe the behavior of biaxially loaded plates with higher aspect ratio.

Reduced stress method and Effective width method showed exactly equal critical buckling stress when applied on uniaxially loaded stiffened plates. Such hand calculated results are found to be quite lower than result of Abaqus. There is about 10 % - 15 % difference between hand calculated critical buckling stress of stiffened plates and result of finite element analysis done by Abaqus.

Strength analysis of stiffened plates showed that Reduced stress method is very conservative when compared to Effective width method. This is mainly due to the fact that Reduced stress method does not allow load shedding between member plates of a cross section. Effective width method allows load shedding between plate panels and lead to a better strength results.

Strength results based on Effective width method for stiffened plates are generally lower than results of numerical analysis. The difference is especially very large for both short and wide plates, since effect of column-like buckling is taken into consideration. Hence the interpolation equation used in the design process is not good enough to capture the real behavior of such stiffened plates.

Generally Effective width method gives better results for longitudinally stiffened regular plate dimensions with uniaxial loading condition. But Reduced stress method can help to make a fast judgment about strength of a plate cross section, if the minimum load amplifiers, α_{cr} and $\alpha_{ult,k}$ could be determined using a computer soft ware.

Reduced stress method is the only alternative method when dealing with irregular plate shapes, plates with arbitrarily arranged stiffeners, plate with large cut outs ... etc.

Recommendations for future study:

This thesis concentrated primarily on axially loaded rectangular plates. Slender plate structures loaded in bending and shear are also very common. Study of moment and shear loaded plates using Reduced stress method would have been very interesting.

Literature about the back ground work of Reduced stress method is hardly available in Norway. Therefore a comprehensive study on this subject would have been very important.

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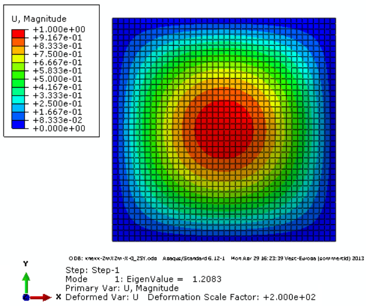
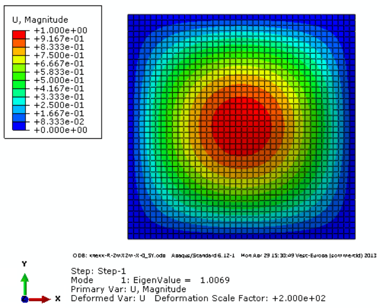
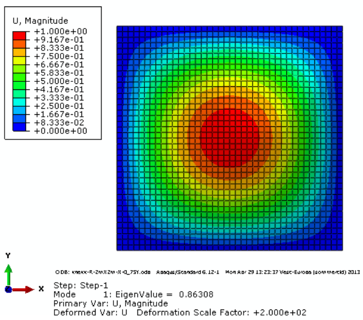
A. Buckling modes and strength curves of biaxially loaded plates

A.1 Buckling modes: Biaxially loaded plates

Table A- 1 Buckling modes of biaxially loaded plate-2000X2000, t=20

No.	Loading [N/mm]	Buckling mode -1	Sigma_cr_abaqus [N/mm^2]
1	X=1000 Y=250		λ = 1,2083 σ _cr,x = λ*(1000 N/mm)/t σ_ cr,y = λ*(250 N/mm)/t σ_cr.x=60,415 σ-cr,y=15,1
2	X=1000 Y=500		λ = 1,0069 σ _cr,x = λ*(1000 N/mm)/t σ_ cr,y = λ*(500 N/mm)/t σ_cr,x=50,3 σ_cr,y=25,2
3	X=1000 Y=750		λ = 0,86308 σ _cr,x = λ*(1000 N/mm)/t σ_ cr,y = λ*(750 N/mm)/t σ_cr,x=43,2 σ_cr,y=32,4

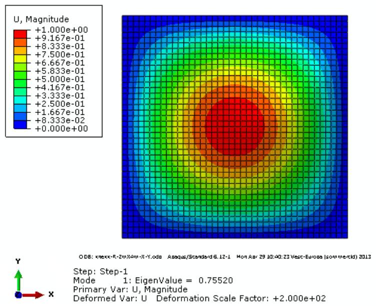
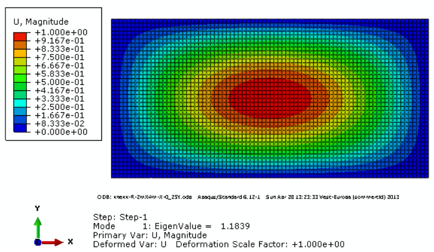
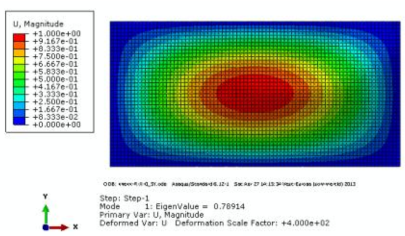
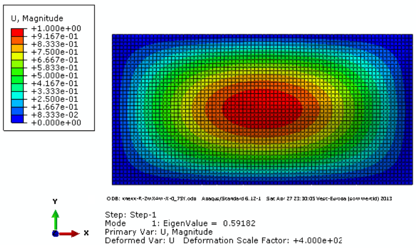
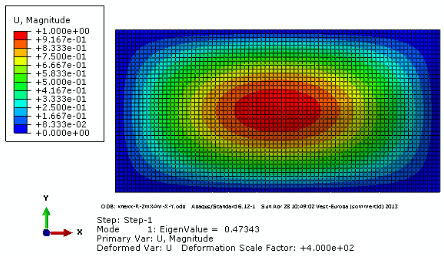
4	X=1000 Y=1000		λ = 0,7552 σ _cr,x = λ*(1000 N/mm)/t σ_ cr,y = λ*(1000 N/mm)/t t= thickness σ_cr,x=37,8 σ_cr,y=37,8

Table A- 2 Buckling modes of biaxially loaded plate-2000X4000, t=20

No.	Loading [N/mm]	Buckling mode -1	Sigma_cr [N/mm^2]
1	X= 1000 Y=250		λ = 1,1839 σ _cr,x = λ*(1000 N/mm)/t σ_ cr,y = λ*(1000 N/mm)/t t= thickness σ_cr,x=59,2 σ_cr,y=14,8
2	X=1000 Y=500		λ =0,78914 σ _cr,x = λ*(1000 N/mm)/t σ_ cr,y = λ*(1000 N/mm)/t σ_cr,x = 39,5 σ_cr,y = 19,7

3	X=1000 Y=750		λ =0,59182 σ _cr,x = λ*(1000 N/mm)/t σ_ cr,y = λ*(1000 N/mm)/t σ_cr,x = 29,6 σ_cr,y = 22,2
4	X= 1000 Y=1000		λ =0,47343 σ _cr,x = λ*(1000 N/mm)/t σ_ cr,y = λ*(1000 N/mm)/t σ_cr,x =23,7 σ_cr,y=23,7

A.2 Strength curves: Biaxially loaded plates

The strength of the plate is extracted as LPF (Load proportionality factor). This is then transformed into stresses as follow:

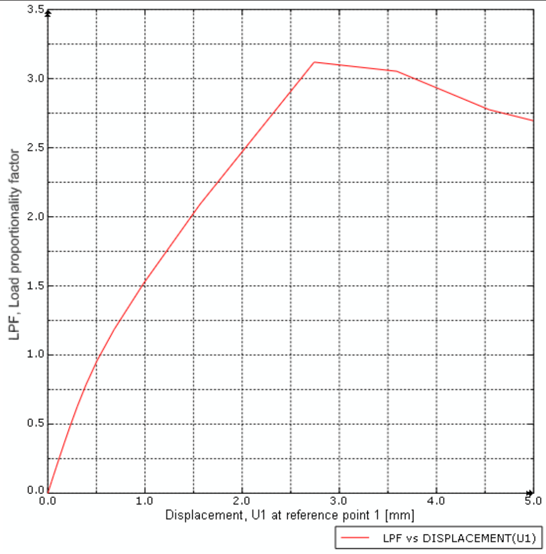
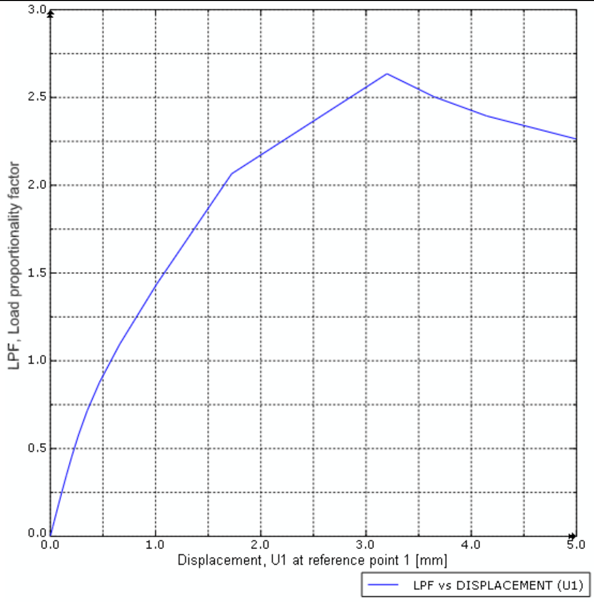
σ_x,Rd= LPF*(Loading_x-direction)/t

σ_y,Rd = LPF*(Loading_y-direction)/t

Imperfection = buckling mode-1 is used for all simulations.

Imperfection amplitude = b/500 = 4 mm for all simulations

Table A- 3 Strength curve for biaxially loaded plate 2000X2000, t=20mm

No.	Loading [N/mm]	Strength curve: LPF vs. Displacement (U1)	Strength
1	X=1000 Y=250		Max LPF =3,11899 U1 at max. LPF = 2,741 mm σ_x,Rd= 156 N/mm^2 σ_y,Rd=39 N/mm^2
2	X=1000 Y=500		Max LPF =2,63529 U1 at max. LPF= 3,199 mm σ_x,Rd = 131,8 N/mm^2 σ_y,Rd =65,9 N/mm^2

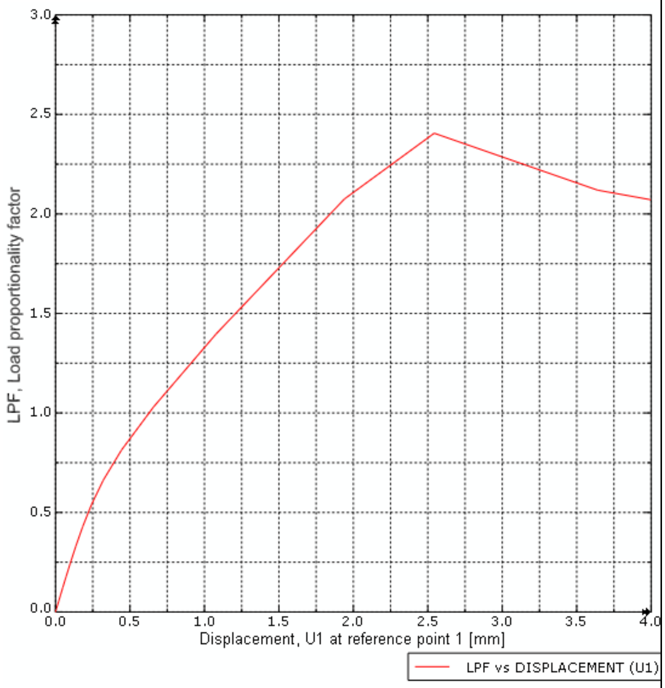
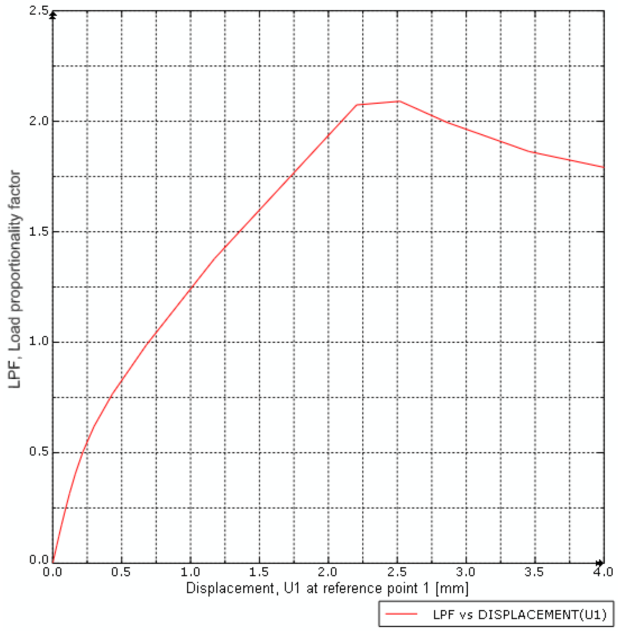
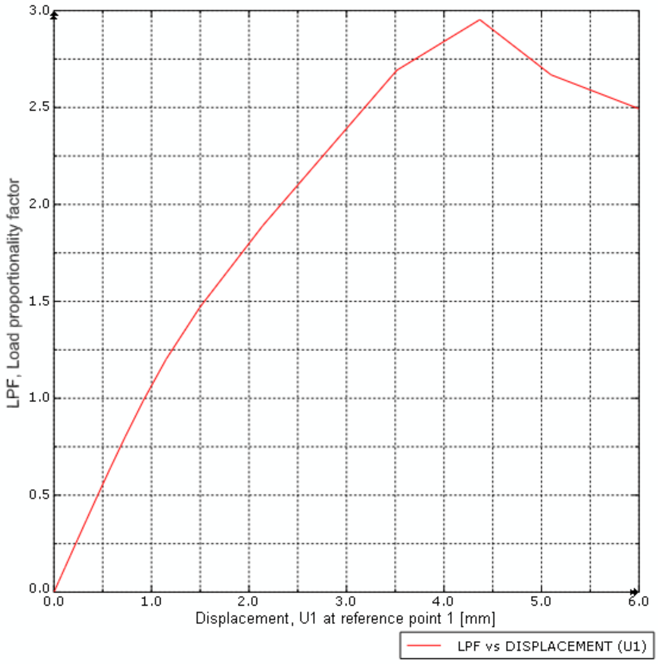
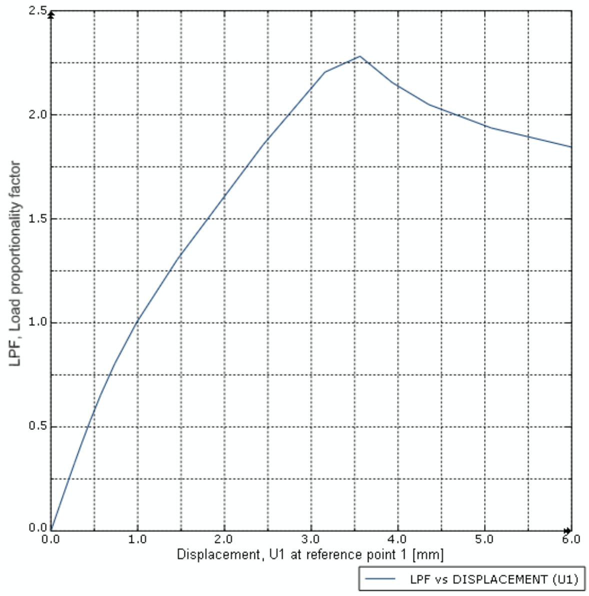
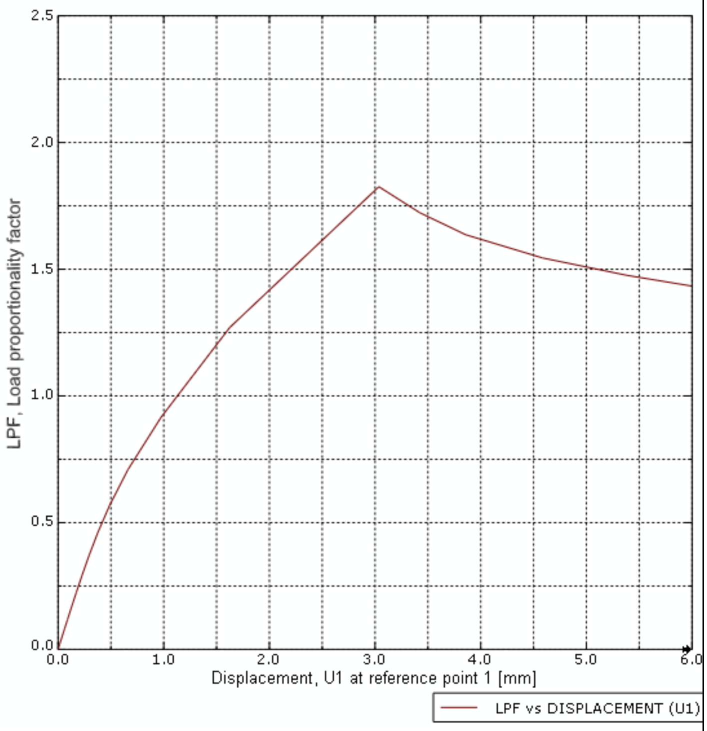
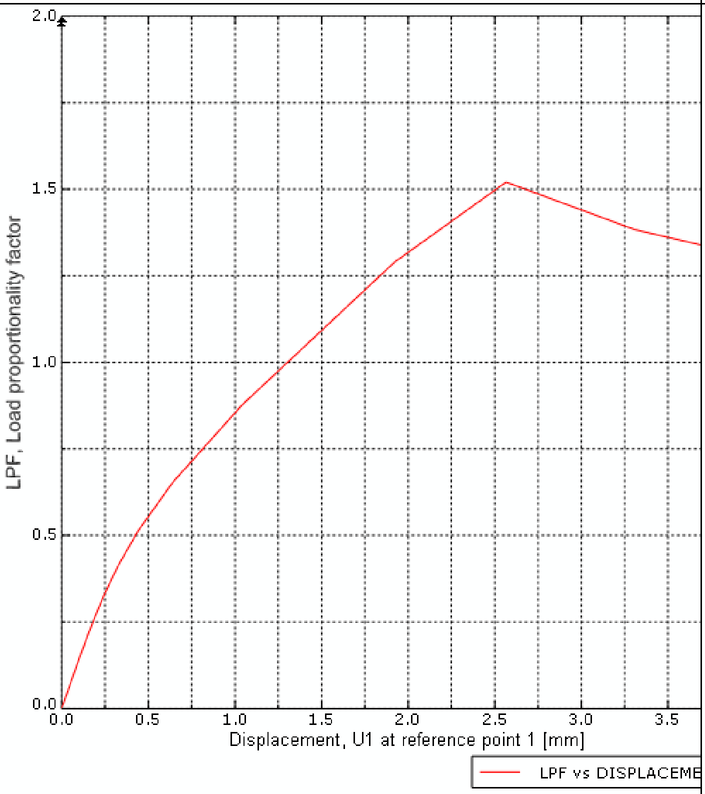
3	**X=1000** **Y=750**		Max. LPF =2,40541 U1 at max. LPF= 2,54301 mm σ_x,Rd = 120,3 N/mm^2 σ_y,Rd = 90,2 N/mm^2
4	**X= 1000** **Y=1000**		Max. LPF = 2,09055 U1 at max. LPF= 2,519 mm σ_x,Rd = 104,5 N/mm^2 σ_y,Rd = 104,5 N/mm^2

Table A- 4 Strength of biaxially loaded plate 2000X4000, t=20mm

No	Loading [N/mm]	Strength curve : LPF vs. Displacement (U1)	Strength
1	X =1000 Y = 250		Max. LPF =2,95329 U1 at max.LPF = 4,37 mm $\sigma_{x,Rd}$ =147,7 N/mm^2 $\sigma_{y,Rd}$ = 36,9 N/mm^2
2	X=1000 Y=500		Max. LPF =2,28143 U1 at max. LPF= 3,56 mm $\sigma_{x,Rd}$ = 114,1 N/mm^2 $\sigma_{y,Rd}$ =57 N/mm^2

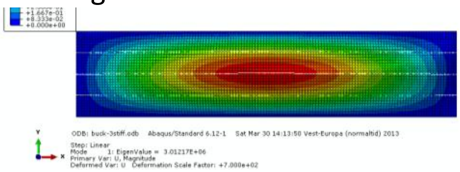
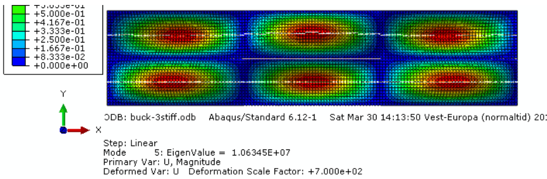
3	**X=1000** **Y=750**		Max. LPF = 1,82541 U1 at max.LPF = 3,04 mm σ_x,Rd =91,2 N/mm^2 σ_y,Rd = 68,5 N/mm^2
4	**X=1000** **Y=1000**		Max. LPF =1,52051 U1 at max. LPF =2,57 mm σ_x,Rd = 76 N/mm^2 σ_y,Rd =76 N/mm^2

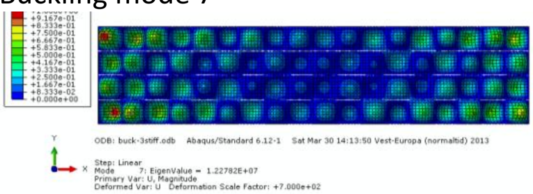
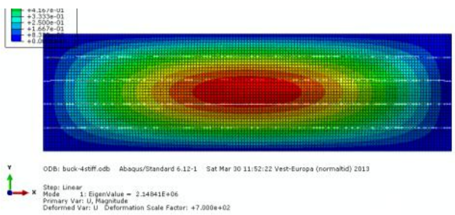
B. Plates with increasing no. of stiffeners : Buckling modes & strength curves

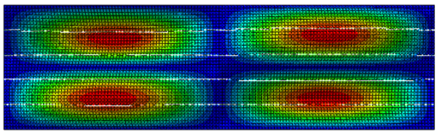
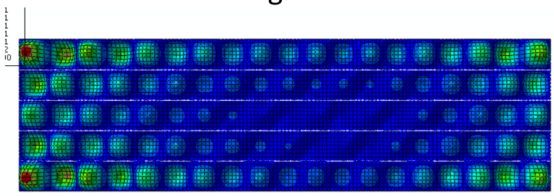
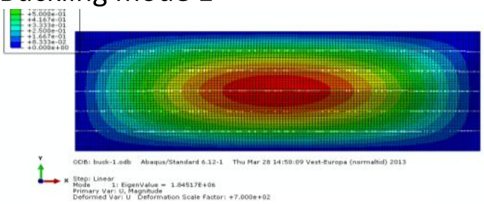
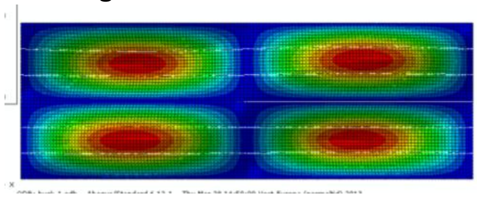
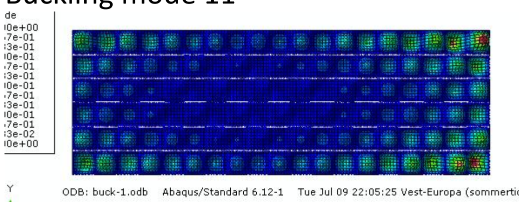
Buckling analysis of stiffened plates (all plates have a thickness of 10 mm).

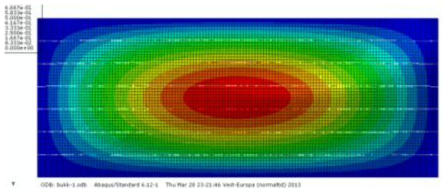
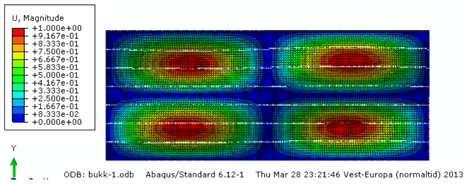
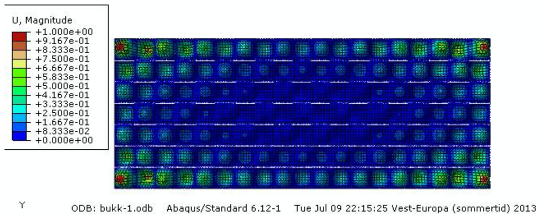
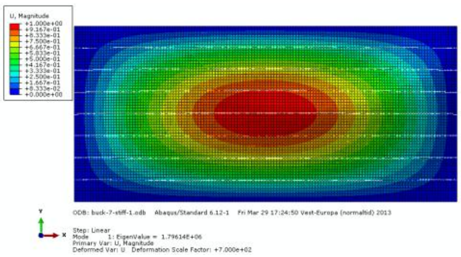
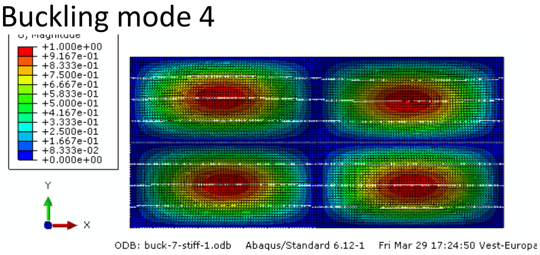
A point load is applied at the reference point.

Table B- 1 Lowest buckling modes: Plates with increasing no. of stiffeners

<table>
<tr><th>No.</th><th>Plate size(mm)</th><th>No. of stiffeners</th><th>Buckling mode</th><th>N_cr,abaqus</th></tr>
<tr><td rowspan="3">1</td><td rowspan="3">1600X7000</td><td rowspan="3">3</td><td>Buckling mode 1
</td><td>N_cr = 3012,2 kN</td></tr>
<tr><td>Buckling mode 5


Lowest mode with two half-sine waves in the transverse direction</td><td>N_cr=10634,5 kN</td></tr>
<tr><td>Buckling mode 7

Lowest local buckling mode</td><td>N_cr =12227,8 kN</td></tr>
<tr><td>2</td><td>2000X7000</td><td>4</td><td>Buckling mode 1
</td><td>N_cr = 2148,4 kN</td></tr>
</table>

			Buckling mode 4  Lowest mode with two half-sine waves in the transverse direction	N_cr=8585,9 kN
			Buckling mode 9  Lowest local buckling mode	N_cr = 15714,7 kN
3	**2400X7000**	**5**	Buckling mode 1 	N_cr = 1845,2kN
			Buckling mode 3  Lowest mode with two half sine waves in the transverse direction	N_cr = 7364,5 kN
			Buckling mode 11  Lowest local buckling mode	N_cr = 19131,5 kN

4	**2800X7000**	**6**	Buckling mode 1 	N_cr= 1767,8 kN
			Buckling mode 3  Lowest mode with two half-sine waves in the transverse direction	N_cr = 7049,69 kN
			Buckling mode 12  Lowest local buckling mode	N_cr = 22537,4 kN
5	**3200X7000**	**7**	Buckling mode 1 	N_cr = 1796,1 kN
			Buckling mode 4  Lowest mode with two half-sine waves in the transverse direction	N_cr = 7159 kN

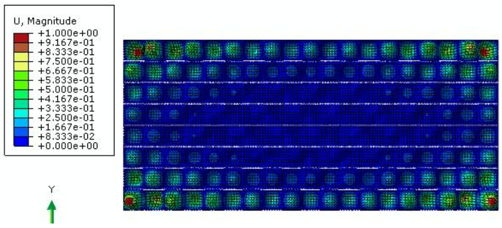
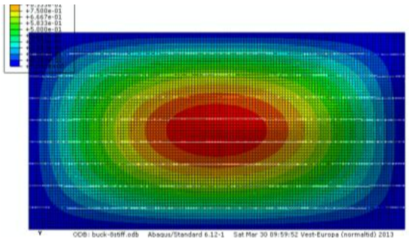
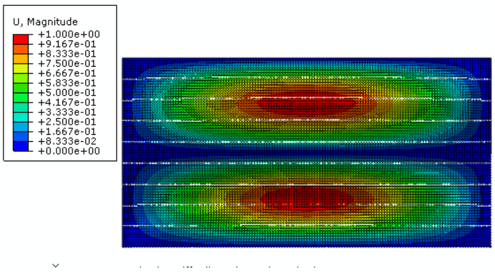
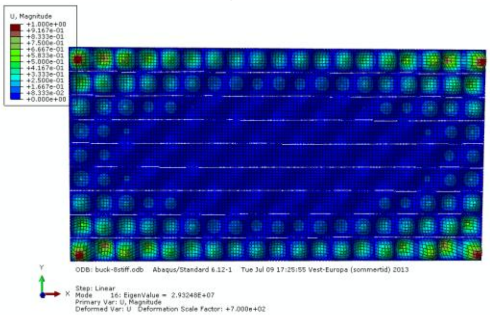
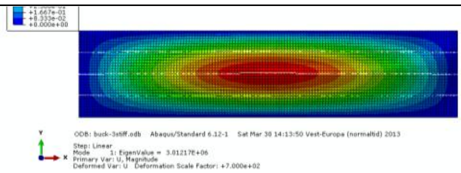
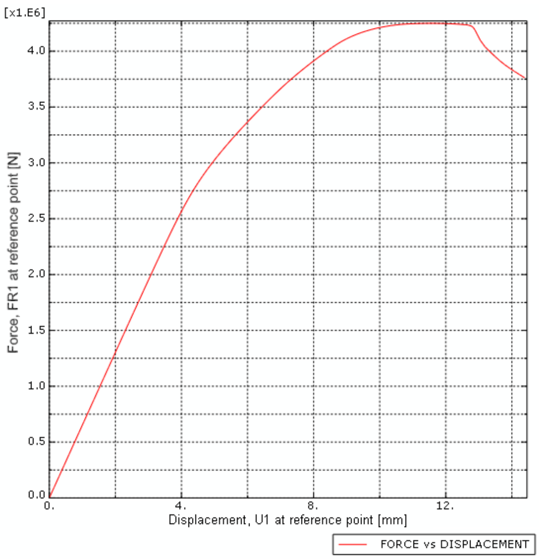
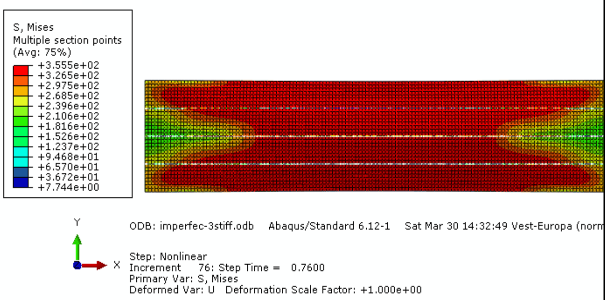
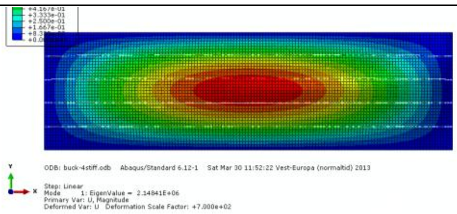
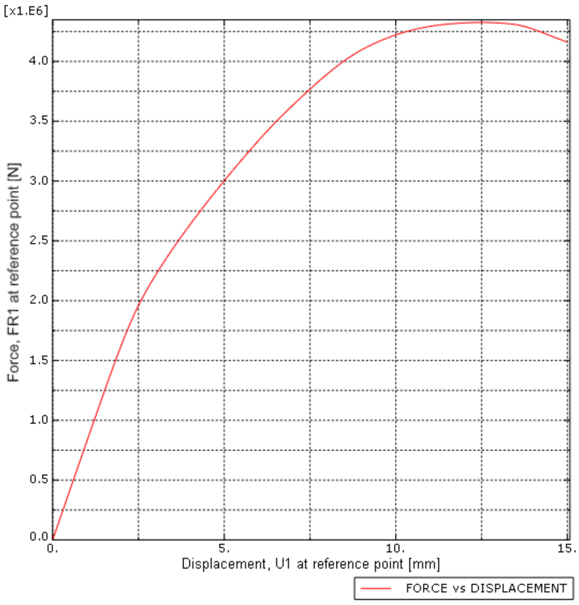
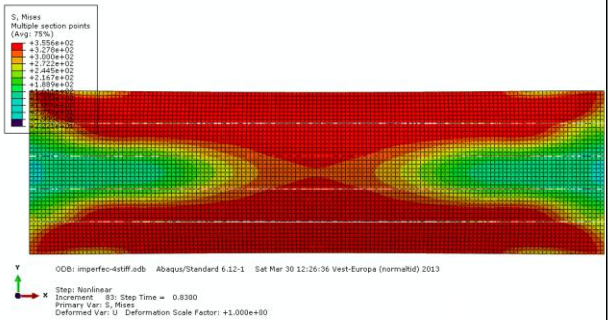
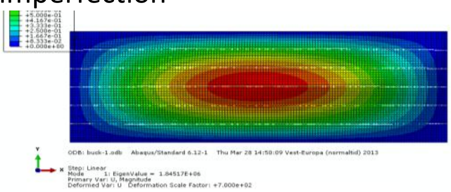
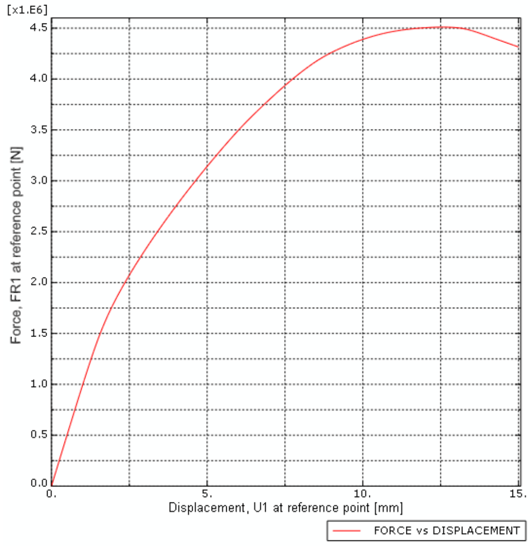
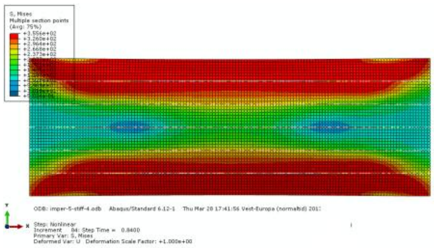
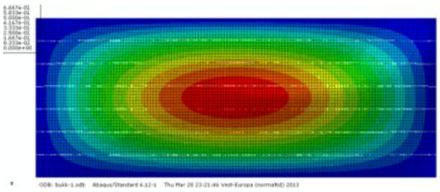
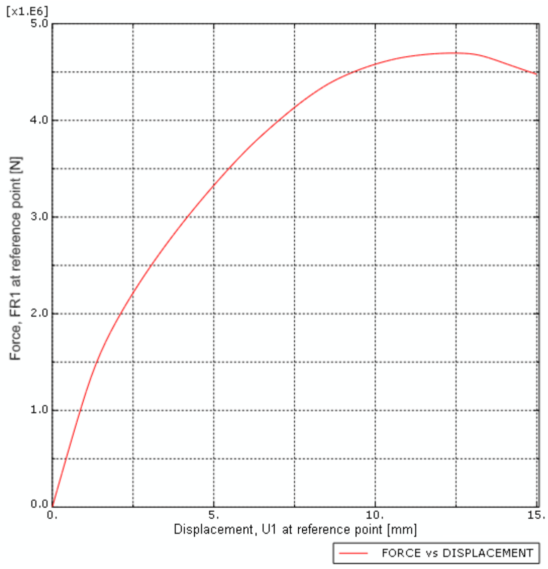
			Buckling mode 15  Lowest local buckling mode	N_cr = 25934 kN
6	**3600X7000**	**8**	Buckling mode 1 	N_cr =1879 kN
			Buckling mode 2 	N_cr = 5173,8 kN
			Buckling mode 16 	N_cr = 29324,8 kN

Table B- 2 Strength curve: plates with increasing no. of stiffeners

No.	Plate size (mm)	No. of stiffeners	Plot (buckling mode used to introduce imperfection and the strength curve)	Strength [kN] (Abaqus)
1	1600X7000	3	 Buckling mode 1 is use to introduce imperfection. 	N_Rd =4249,6 kN U1 at max. load =11,7 mm Imperfection amplitude =3,2 mm
				Stress pattern at maximum loading for the plate with 3 stiffener

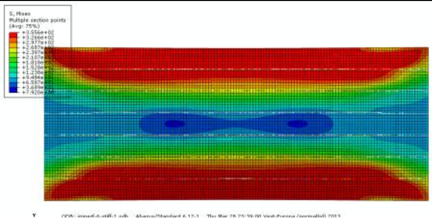
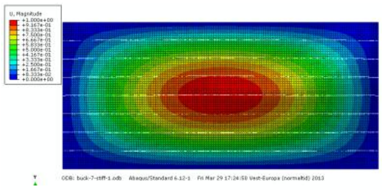
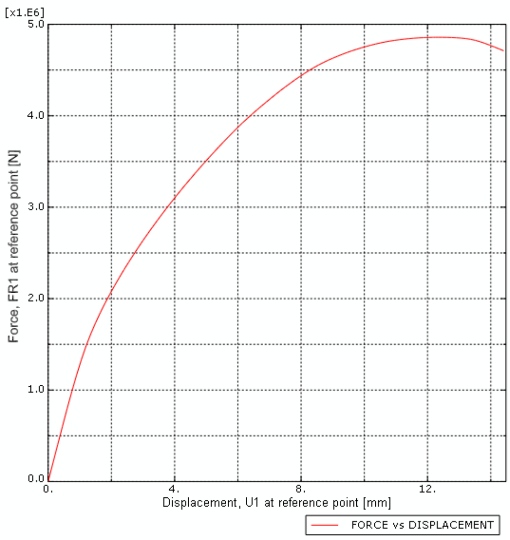
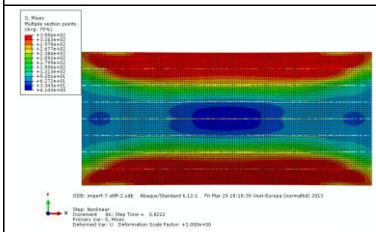
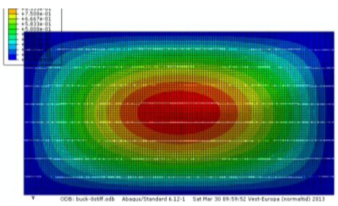
2	**2000X7000**	**4**	 Buckling mode 1 is used to introduce imperfection 	N,Rd = 4326 kN U1 at max load = 12,45 mm Imperfection amplitude = 4 mm
				Stress pattern at Maximum load for plate with 4-stiffeners.
3	**2400X7000**	**5**	Buckling mode1 is used to introduce imperfection 	N_Rd =4510,4 kN Imperfection amplitude = 4,8 mm U1 at max.

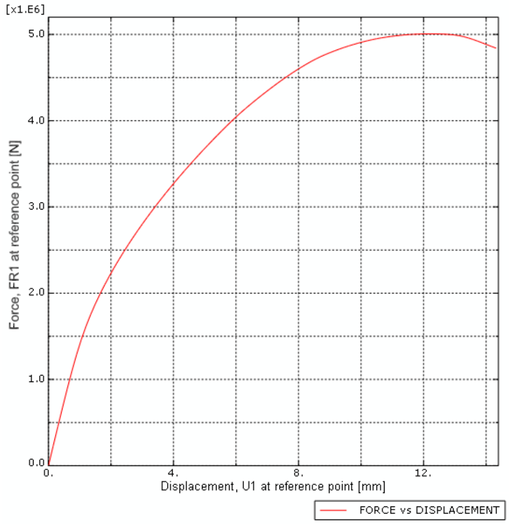
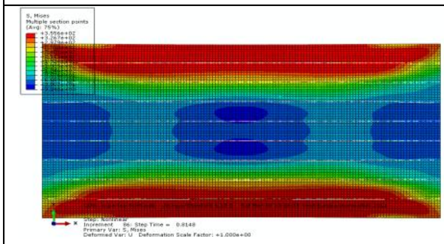
<table>
<tr><td></td><td></td><td></td><td>
</td><td>loading = 12,6 mm</td></tr>
<tr><td></td><td></td><td></td><td></td><td>Stress pattern at maximum loading.

This is stress pattern belongs to plate with 5-stiffeners.</td></tr>
<tr><td>4</td><td>2800X7000</td><td>6</td><td>Buckling mode 1 is used to introduce imperfection


</td><td>N_Rd = 4695,8 kN

U1 at max. Loading = 12,45 mm

Imperfection amplitude = 5,6 mm</td></tr>
</table>

				Stress pattern at max. loading for the plate with 6 stiffener
5	**3200**	**7**	Buckling mode 1is used to introduce imperfection  	N_Rd = 4858,5 kN U1 at max. loading = 12,3 mm Imperfection amplitude = 5,6 mm
				Stress pattern at max. loading for the plate with 7-stiffeners.
6	**3600X7000**	**8**	Buckling mode 1 used to introduce imperfection 	N_Rd = 5006,8 kN U1 at max. loading = 12,2 mm Imperfection amplitude =7,2 mm

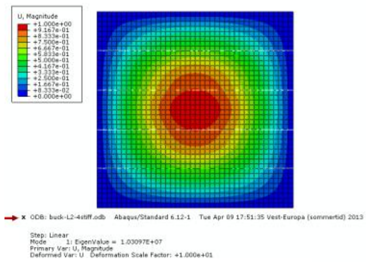
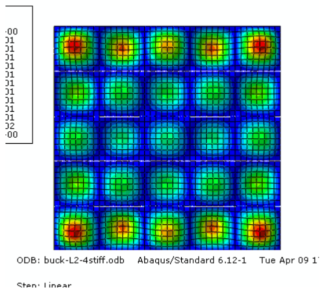
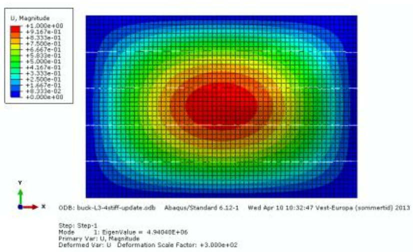
				
				Stress pattern at max. loading for the plate with 8-stiffeners.

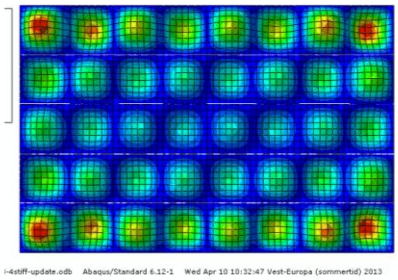
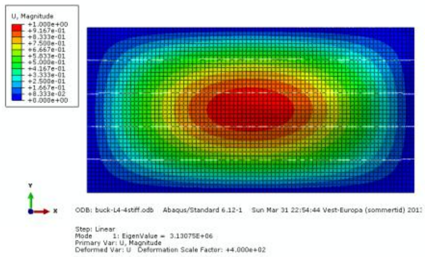
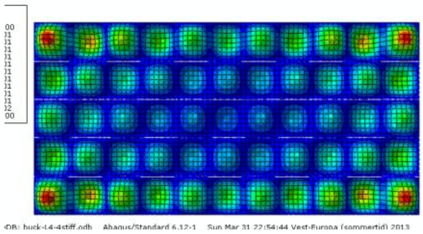
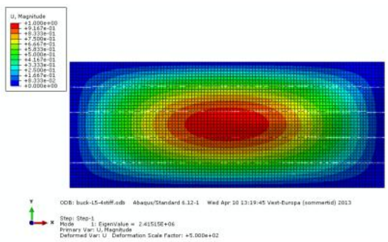
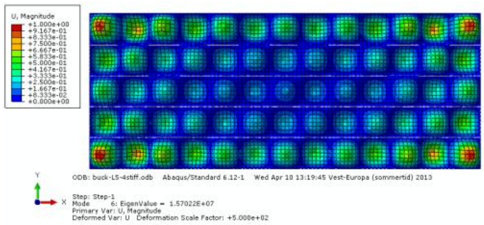
C. Stiffened plate with increasing length : Buckling modes & strength curves

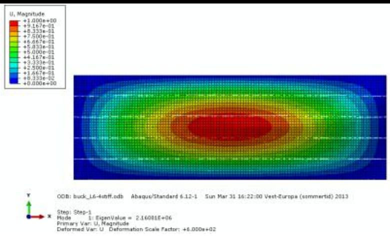
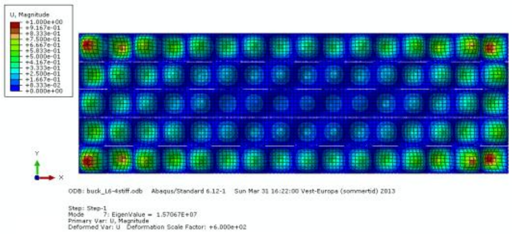
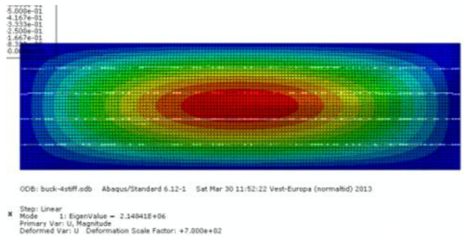
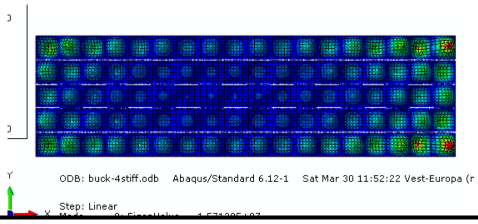
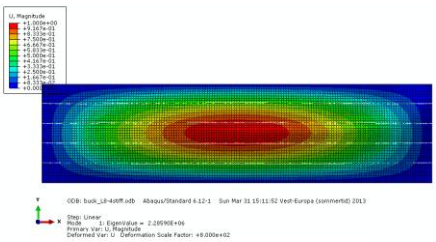
Buckling analysis for stiffened plates (all plates have a thickness of 10mm)

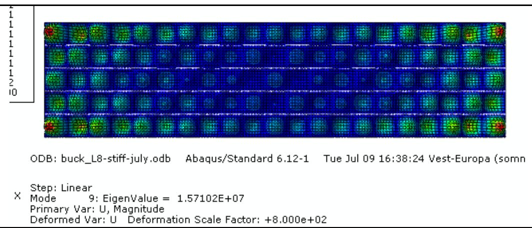
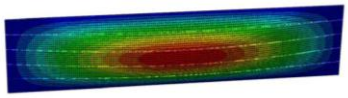
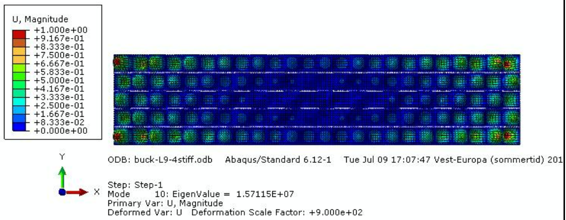
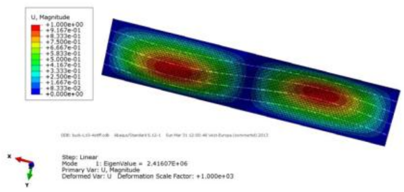
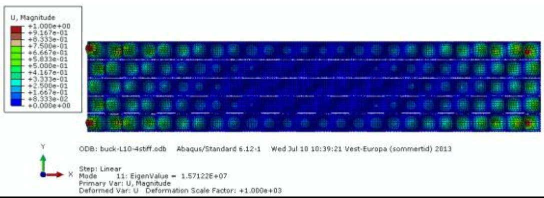
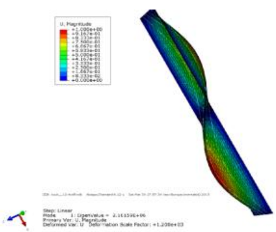
A point load is applied at the reference point

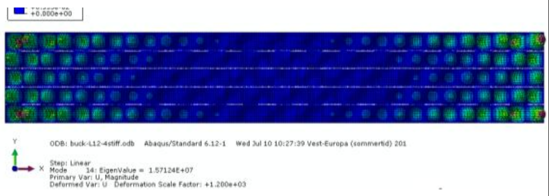
Table C- 1 Buckling plots of plates with increasing number of stiffeners

No	Plate size [mm]	No. of stiffeners	Buckling mode No.	N_cr_abaqus [kN]
1	2000X200	4	Mode 1 	N_cr = 10309,7 kN
			Mode 3 	N_cr = 15671,8 kN
2	2000X3000	4	Mode 1 	N_cr =49040 kN
			Mode 3	N_cr =15677 kN

				
3	2000X4000	4	**Mode 1** 	**N_cr =3130,8 kN**
			Mode 5 	**N_cr = 15711,7 kN**
4	2000X5000	4	**Mode 1** 	**N_cr = 2415,2 kN**
			Mode 6 	**N_cr = 15702,2kN**
5	2000X6000	4	**Mode 1**	

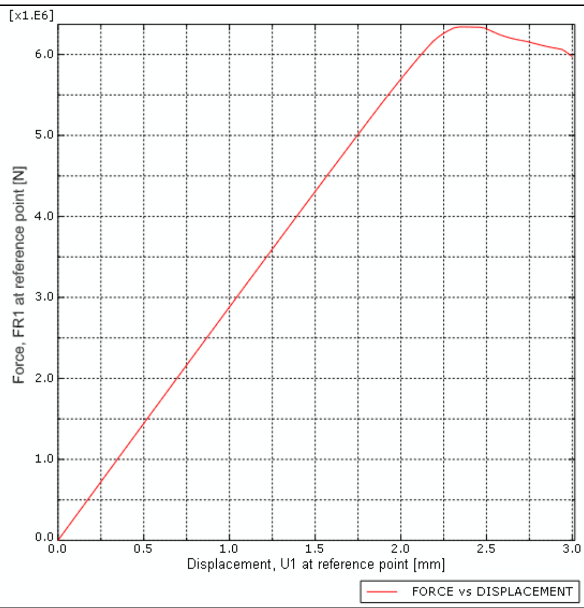
				**N_cr = 2160,8 kN**
			Mode 7	**N_cr = 15706,7 kN**
				
6	**2000X7000**	**4**	**Mode 1** 	**N_cr = 2148,4 kN**
			Mode 8 	**N_cr =15712,8 kN**
7	**2000X8000**	**4**	**Mode 1** 	**N_cr = 2285,9 kN**
			Mode 9	**N_cr =15710,2 kN**

<table>
<tr><td></td><td></td><td></td><td>
</td><td></td></tr>
<tr><td rowspan="2">8</td><td rowspan="2">2000X9000</td><td rowspan="2">4</td><td>Mode 1
</td><td>N_cr =2529,2 kN</td></tr>
<tr><td>Mode 10

</td><td>N_cr = 15711,5 kN</td></tr>
<tr><td rowspan="2">9</td><td rowspan="2">2000 X10000</td><td rowspan="2">4</td><td>Mode 1
</td><td>N_cr =2416,1 kN</td></tr>
<tr><td>Mode 11

</td><td>N_cr = 15712,2 kN</td></tr>
<tr><td>10</td><td>2000 X12000</td><td>4</td><td>Mode 1
</td><td>N_cr = 2161,6kN</td></tr>
</table>

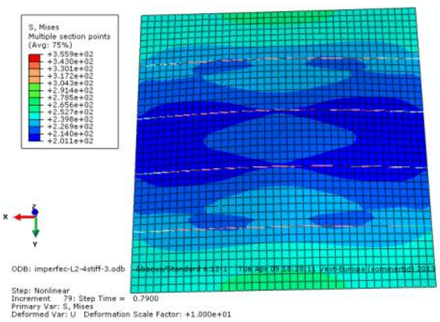
<table>
<tr><td></td><td></td><td></td><td>Mode 14
</td><td>N_cr = 15712,2 kN</td></tr>
</table>

Strength curves of stiffened plates dimensions with increasing length are given on Table C- 2. Buckling mode 1 is used to introduce imperfection.

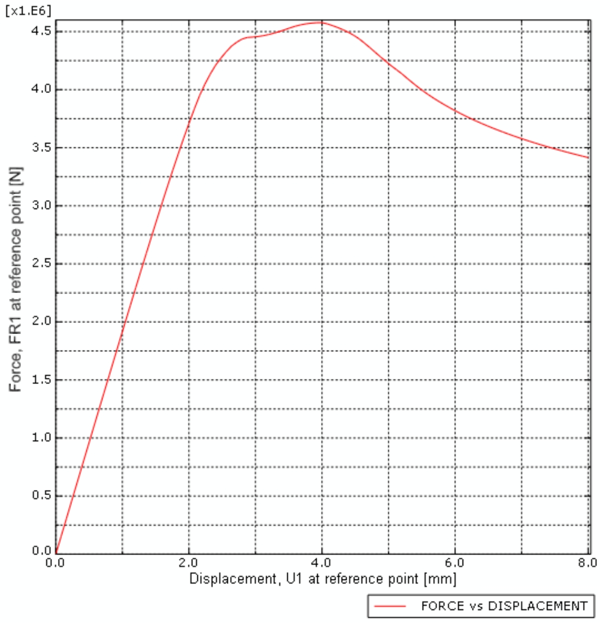
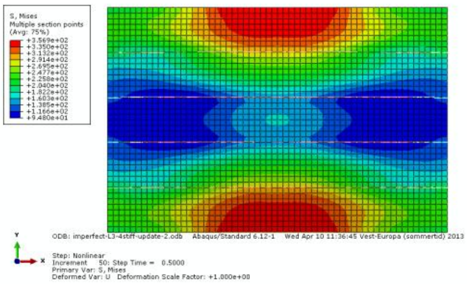
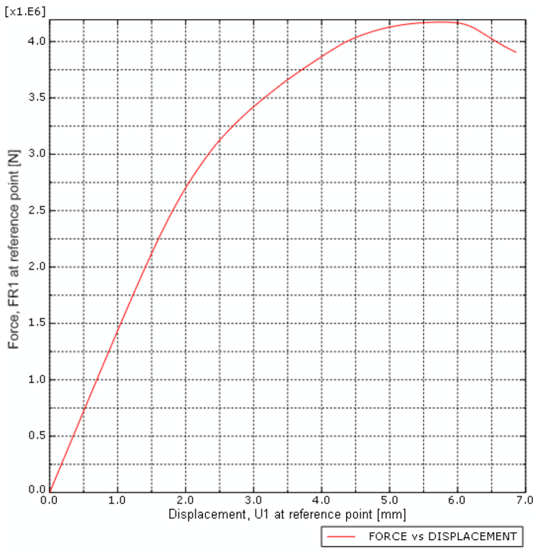
Table C- 2 Strength curves: stiffened plates with increasing length

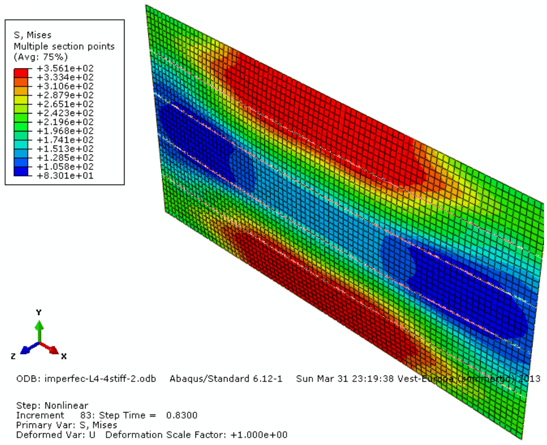
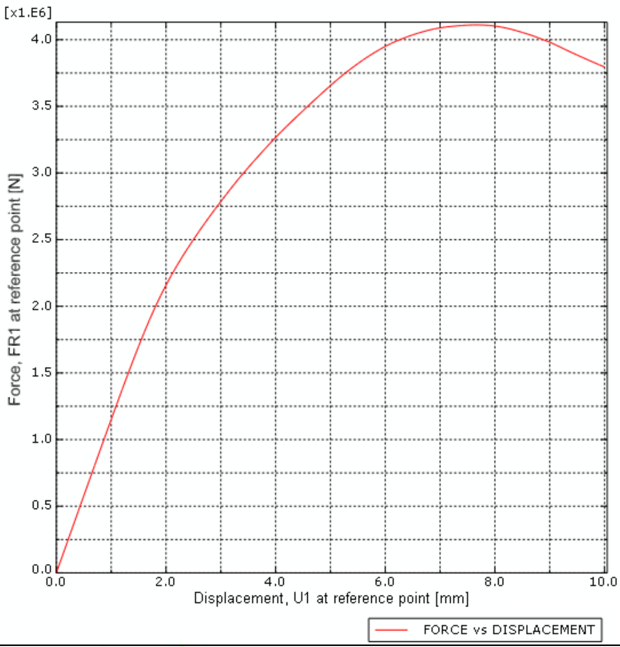
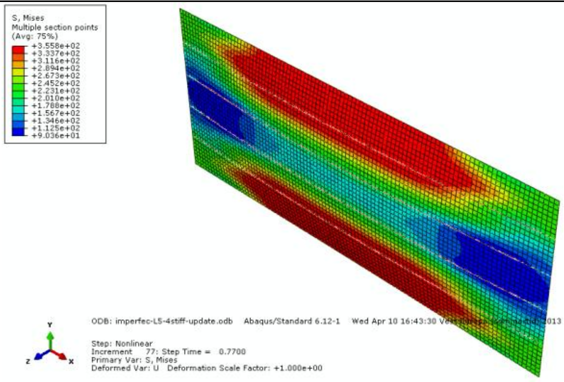
<table>
<tr><th>No.</th><th>Plate size (mm)</th><th>Figure</th><th>strength</th></tr>
<tr><td rowspan="2">1</td><td rowspan="2">2000X2000</td><td>
</td><td>N_Rd =6339,8 kN

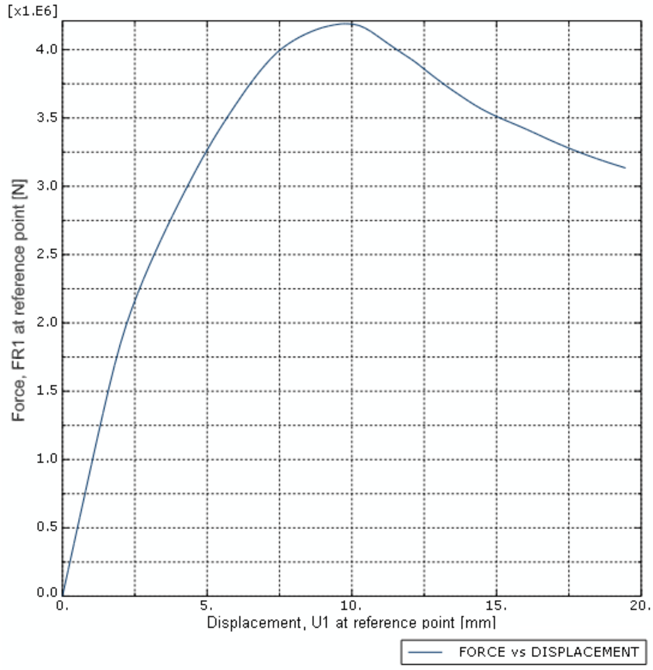
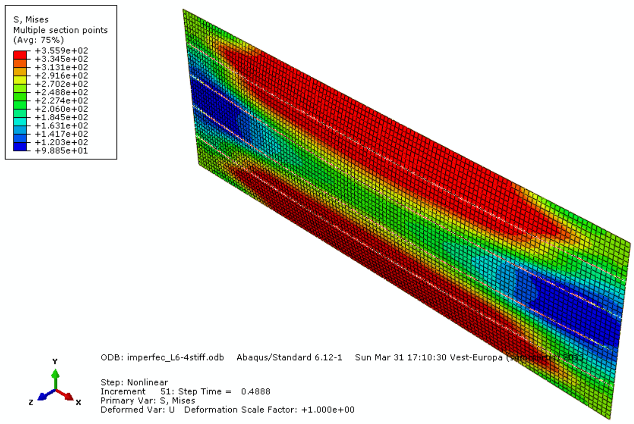
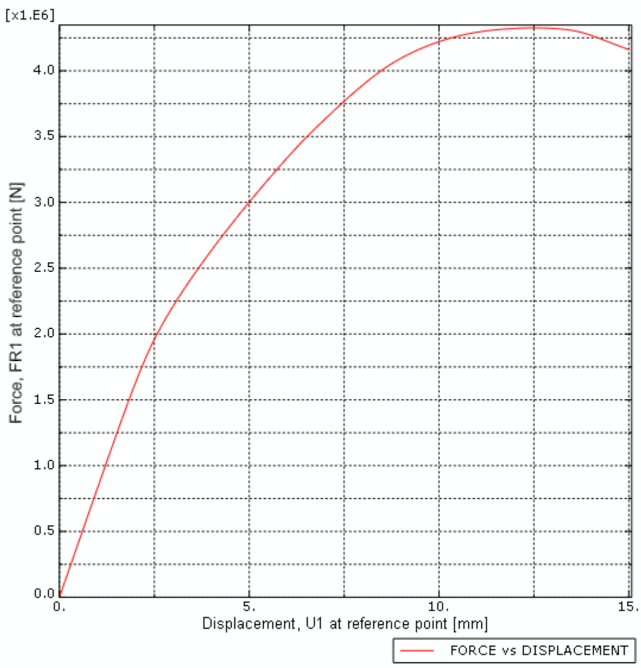
U1 at max load =2,37

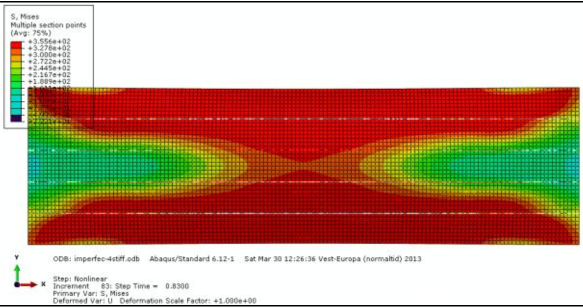
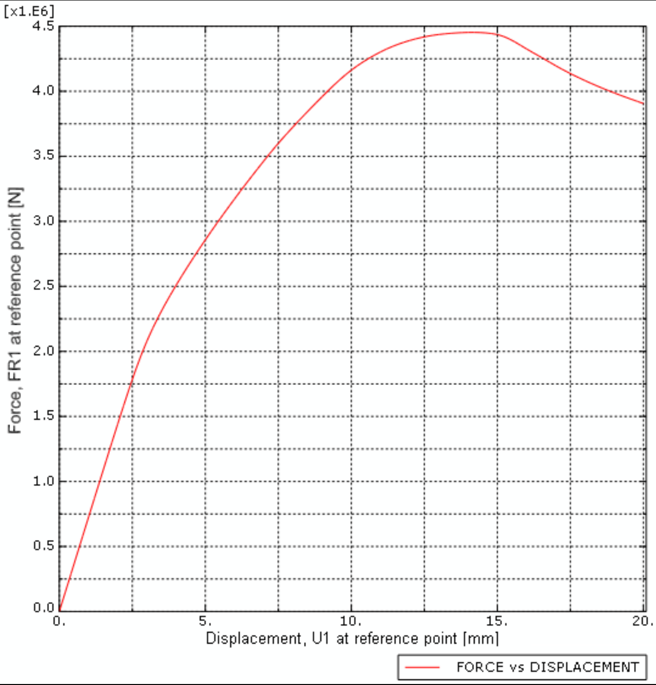
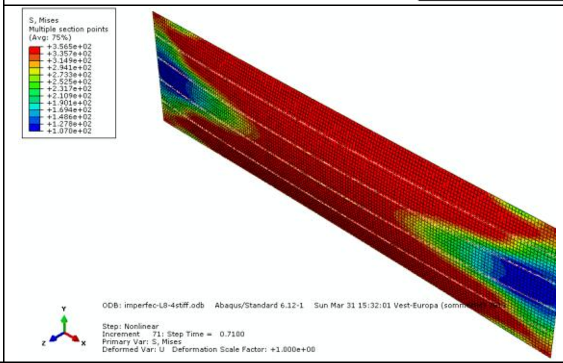
Both the loading and the displacement are taken at the RP (reference point)</td></tr>
<tr><td>
</td><td>Stress pattern at maximum loading.

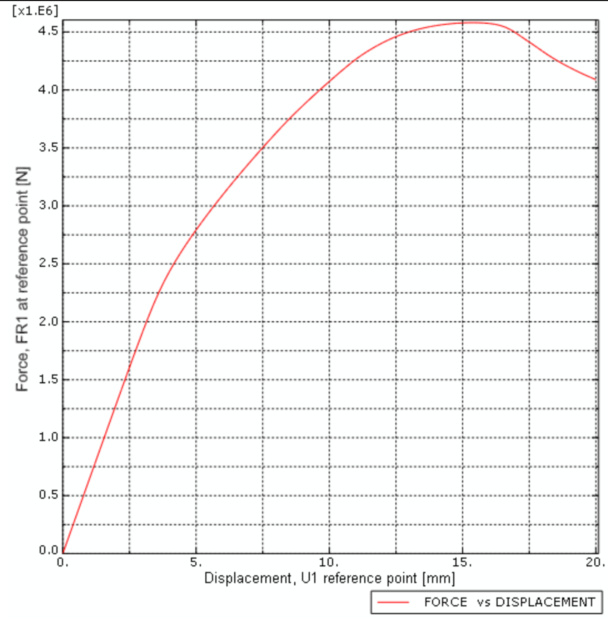
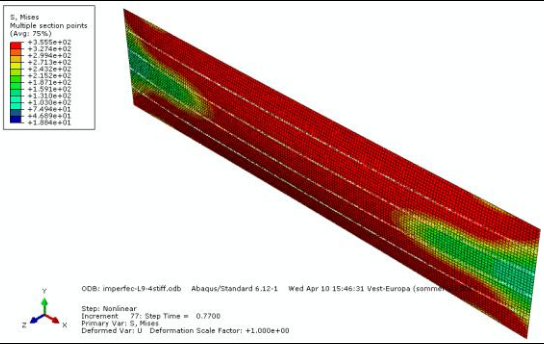
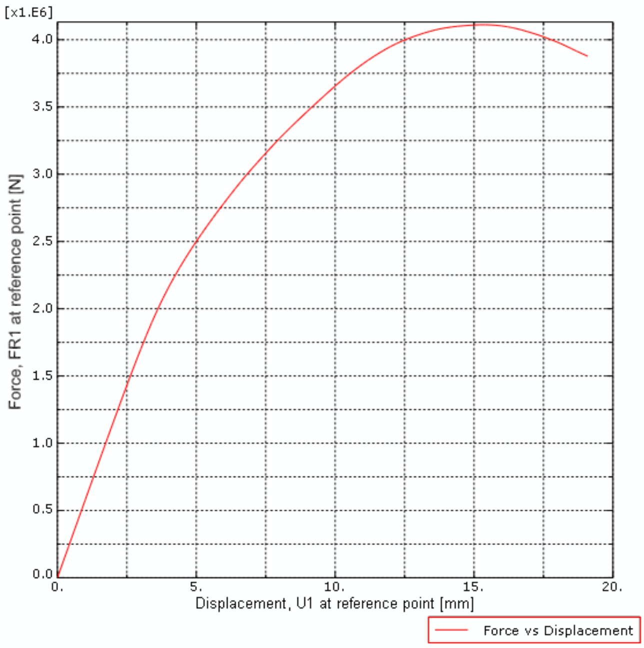
Failure occurred at axial stress of: 229 N/mm^2</td></tr>
</table>

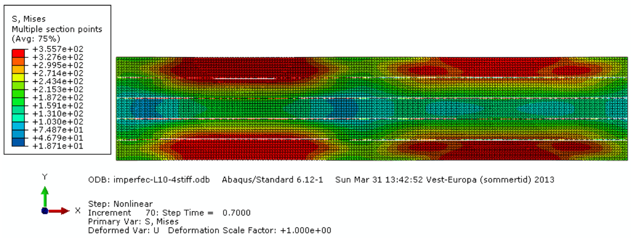
2	2000X3000		N_Rd = 4576,9 kN U1 at max. load = 4 mm Both the loading and the displacement are taken at the RP (reference point)
			Stress pattern at maximum loading. Most of stresses are concentrated Along the longitudinal edges as expected.
3	2000X4000		N_Rd = 4172,4 kN U1 at max. load =5,8 mm Both the loading and the displacement are taken at the RP (reference point)

			Stress pattern at maximum loading. Most of stresses are concentrated Along the longitudinal edges as expected.
4	2000X5000		N_Rd = 4110,6 kN U1 at maximum loading = 7,7 mm Both the loading and the displacement are taken at the RP (reference point).
			Stress pattern at maximum loading

5	2000X6000		N_Rd =4189,6 kN U1 at maximum loading = 9,8 mm Both the loading and the displacement are taken at the RP (reference point)
			Stress pattern at maximum loading
6	2000X7000		N_Rd = 4326 kN U1 at maximum loading = 12,45 mm Both the loading and the displacement are taken at the RP (reference point)

			Stress pattern at maximum loading
7	2000X8000		N_Rd = 4452,6 kN U1 at max loading = 14,2 mm Both the loading and the displacement are taken at the RP (reference point)
			Stress pattern at maximum loading.

8	2000X9000		N_Rd = 4580,7 kN U1 at maximum loading = 15,4 mm Both the loading and the displacement are taken at the RP (reference point)
			Stress pattern at maximum loading.
9	2000X10000		N_Rd = 4111 kN U1 at max. loading = 15,4 mm Both the loading and the displacement are taken at the RP (reference point)

<table>
<tr>
<td></td>
<td></td>
<td>
</td>
<td>Stress pattern at maximum loading</td>
</tr>
<tr>
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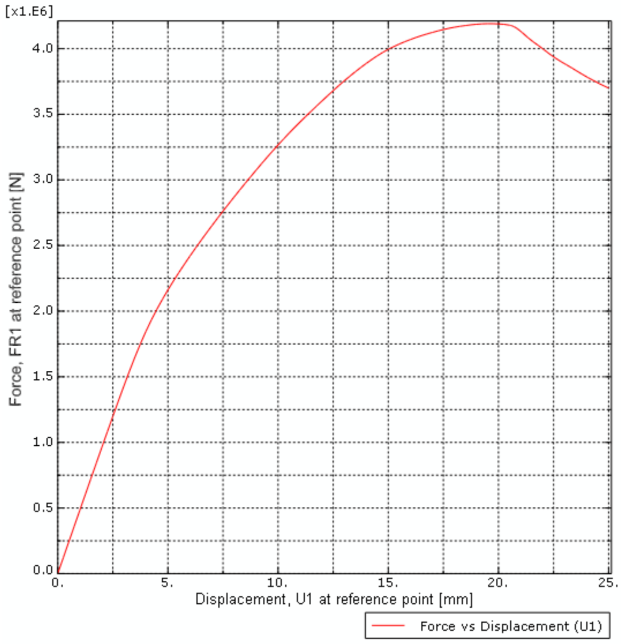
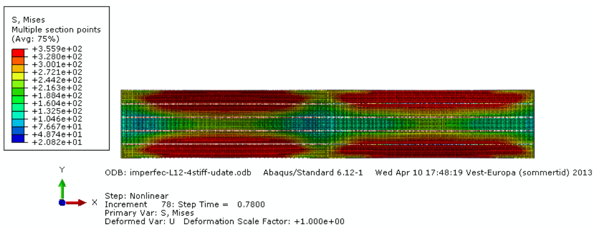
<td rowspan="2">2000X12000</td>
<td>
</td>
<td>N_Rd = 4190 kN

U1 at maximum loading = 19,5 mm

Both the loading and the displacement are taken at the RP (reference point)</td>
</tr>
<tr>
<td>
</td>
<td>Stress pattern at maximum loading.</td>
</tr>
</table>

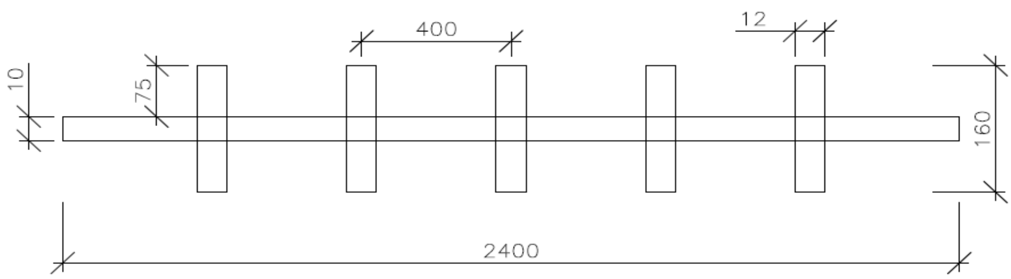
D. Calculation work sheet for longitudinally stiffened plate

Both Effective width method and Reduced stress method are shown.

A typical calculation work sheet showing how the calculations are performed is given below.

The work sheet shows design procedures based on both Effective width method and the Reduced stress method.

Plate with five equally spaced stiffeners



Geometry

Plate:

$a := 7000mm$

$b := 2400mm$

$b_{sup} := 400mm$

$b_{end} := 400mm$

$t_p := 10mm$

Stiffener:

(all flat type stiffeners)

$h_{sl} := 160mm$

$t_{sl} := 12mm$

$n := 5$

(n = no. of stiffeners)

Material date:

$E := 2.1\cdot 10^5 MPa$

$f_y := 355MPa$

$\nu := 0.3$

$$\varepsilon := \sqrt{\frac{235MPa}{f_y}} = 0.814$$

*symbol description:

a = plate length
b = plate width
b_{sup}= plate panel between stiffeners, distance taken c/c stiffener
b_{end} = plate panel between edge of the plate and nearest stiffener

$$A_c := n\cdot b_{sup} t_p + n\cdot h_{sl}\cdot t_{sl} = 2.96\times 10^4 mm^2$$

Cross sectional check

Three types of subpanels to be checked:

1: subpanels at the ends: b_{end} x t_p

2: subpanels between stiffeners: b_{sup} x t_p

3: stiffeners as an outstand plate: h_{sl} x t

Class 3 requirement:

$\frac{C}{t} \leq 42 \cdot \varepsilon$ $\quad 42 \cdot \varepsilon = 34.172$ for internal element with uniform axial compression

$\frac{C}{t} \leq 14 \cdot \varepsilon$ $\quad 14 \cdot \varepsilon = 11.391$ for outstand element with uniform axial compression

For subpanel no.1, i.e. subpanel between the outermost stiffener and the edge of the plate

$$\frac{b_{end} - \left(\frac{t_{sl}}{2}\right)}{t_p} = 39.4$$

It is a class 4.

For subpanel no.2 = it is the subpanel between two internal stiffeners

$$\frac{b_{sup} - t_{sl}}{t_p} = 38.8$$

It is a class 4.

For subpanel no.3 = the stiffener as an outstand plate

$$\frac{\left(\frac{h_{sl} - t_p}{2}\right)}{t_{sl}} = 6.25$$

Ok, class 3 or better

S**lenderness values for the subpanels:**

Both the subpanels at the ends of the plate and those between stiffeners have equal width. We need a single slenderness value.

Aspect ratio α:

$$\alpha_{loc} := \frac{a}{b_{end}} = 17.5$$

Very large aspect ratio:

$$k_{\sigma} := 4$$

$$\lambda_{p.loc} := \frac{\frac{b_{end}}{t_p}}{28.4 \cdot \varepsilon \cdot \sqrt{k_{\sigma}}} = 0.865549$$

Local reduction factor,ρ_{loc}:

$$\psi := 1$$

uniform compression

$$\rho_{loc} := \begin{cases} \frac{\lambda_{p.loc} - 0.055(3+\psi)}{\lambda_{p.loc}^2} & \text{if } \lambda_{p.loc} > 0.673 \\ 1 & \text{otherwise} \end{cases} = 0.861679$$

$$b_{end.eff} := \rho_{loc} \cdot b_{end} = 344.672 mm$$

$$b_{sup.eff} := b_{end.ef}$$

$$A_{c.eff.loc} := n \cdot \left(b_{end.eff} \cdot t_p + h_{sl} \cdot t_{sl}\right) = 26833.589 mm^2$$

NS-EN-1993-1-5:fig.4.4

$$A_c = 29600 mm^2$$

Global plate buckling:

Plate buckling will be calculated by considering an equivalent orthotropic plate, according NS-EN-1993-1-5:A.1

Parameters like I_{sl}, I_p, γ , δ ,α are needed to calculate k_{σ} :

$$I_{sl} := \frac{b \cdot t_p^3}{12} + n \cdot \left(\frac{t_{sl} \cdot h_{sl}^3}{12}\right) = 2.068 \times 10^7 mm^4$$

I_{sl}= second areal moment of the plate

n= no. of stiffeners

$$I_p := \frac{b \cdot t_p^{\,3}}{12 \cdot \left(1 - \nu^2\right)} = 2.198 \times 10^5\,mm^4$$

$$A_{sl} := n \cdot \left(h_{sl} \cdot t_{sl}\right) = 9.6 \times 10^3\,mm^2$$

A_{sl} = sum of area of the individual stiffener

$$A_p := b \cdot t_p = 2.4 \times 10^4\,mm^2$$

A_p = area of the plate part only

Aspect ratio for the whole plate, α_p:

$\psi = 1$

$$\alpha_p := \max\left(\frac{a}{b}, 0.5\right) = 2.917$$

See Annex A.1: NS-EN-1993-1-5

Areal ratio,δ:

$$\delta_p := \frac{A_{sl}}{A_p} = 0.4$$

Inertia ratio, γ_p

$$\gamma_p := \frac{I_{sl}}{I_p} = 94.094 \qquad \sqrt[4]{\gamma_p} = 3.114515$$

$$k_{\sigma.p} := \begin{cases} 2 \cdot \dfrac{\left[\left(1 + \alpha_p^{\,2}\right)^2 + \gamma_p - 1\right]}{\alpha_p^{\,2} \cdot (\psi + 1) \cdot \left(1 + \delta_p\right)} & \text{if } \alpha_p \le \sqrt[4]{\gamma_p} \\ \dfrac{4 \cdot \left(1 + \sqrt{\gamma_p}\right)}{(\psi + 1) \cdot \left(1 + \delta_p\right)} & \text{if } \alpha_p > \sqrt[4]{\gamma_p} \end{cases}$$

Ns-EN-1993-equation (A.2)

$$k_{\sigma.p} = 15.406$$

$$\sigma_E := \frac{\pi^2 \cdot E}{12 \cdot \left(1 - \nu^2\right)} \cdot \left(\frac{t_p}{b}\right)^2 = 3.295 \times 10^6\,Pa$$

σ_E = reference buckling stress

$$\sigma_{cr.p} := k_{\sigma.p} \cdot \sigma_E = 5.076 \times 10^1 \cdot MPa$$

Global plate slenderness, $\lambda_{p.glob}$:

$$\lambda_{p.glob} := \sqrt{\beta_{A.c} \cdot \frac{f_y}{\sigma_{cr.p}}} = 2.5178621$$

$$\beta_{A.c} := \frac{A_{c.eff.loc}}{A_c} = 0.907$$

$$\rho_p := \frac{\lambda_{p.glob} - 0.055(3 + \psi)}{\lambda_{p.glob}^2} = 0.36246$$

Column like buckling:

Need $I_{sl.1}$ and $A_{sl.1}$: for the stiffener nearest to the edge and the plate part adjacent to it, according NS-EN-1993-1-5:Fig(A1):

$$A_{sl.1} := \frac{(b_{end} + b_{sup})}{2} \cdot t_p + h_{sl} \cdot t_{sl} = 5920\,mm^2$$

NS-EN-1993-1-5:Fig.A.1

$$A_{sl.1.eff} := \frac{(b_{end.eff} + b_{sup.eff})}{2} \cdot t_p + h_{sl} \cdot t_{sl} = 5.366718 \times 10^3\,mm^2$$

$$I_{sl.1} := \left[\frac{\frac{(b_{end} + b_{sup})}{2} \cdot t_p^3}{12} + \frac{t_{sl} \cdot h_{sl}^3}{12} \right]$$

$$I_{sl.1} = 4.129333 \times 10^6\,mm^4$$

$$\sigma_{cr.sl} := \frac{\pi^2 E \cdot I_{sl.1}}{A_{sl.1} \cdot a^2} = 2.950402 \times 10^7\,Pa$$

NS-EN-1993-1-5:equation(4.9)

$$\sigma_{cr.c} := \sigma_{cr.s}$$

Relative column slenderness, λ_c:

$$\beta_{A.c.sl} := \frac{A_{sl.1.eff}}{A_{sl.1}} = 0.90654$$

$$\lambda_c := \sqrt{\beta_{A.c.sl} \cdot \frac{f_y}{\sigma_{cr.c}}} = 3.302685$$

NS-EN-1993-1-5:equation(4.11)

$$e := \frac{h_{sl}}{4}$$

NS-EN-1993-1-5:4.5.3(5)

$$i := \sqrt{\frac{I_{sl.1}}{A_{sl.1}}} = 26.411\,mm$$

$\alpha := 0.49$ open section stiffener

$$\alpha_e := \alpha + \frac{0.09}{\left(\frac{i}{e}\right)} = 0.62631$$

$$\Phi_c := \frac{1}{2} \cdot \left[1 + \alpha_e \cdot \left(\lambda_c - 0.2\right) + \lambda_c^{\,2}\right] = 6.925482$$

NS-EN-1993-1-1:6.3.1.2

$$\chi_c := \frac{1}{\Phi_c + \sqrt{\Phi_c^{\,2} - \lambda_c^{\,2}}} = 0.076848$$

Interpolation between plate- like and column-like buckling:

$$\xi := \begin{cases} \left(\frac{\sigma_{cr.p}}{\sigma_{cr.c}} - 1\right) & \text{if } 0 \le \frac{\sigma_{cr.p}}{\sigma_{cr.c}} - 1 \le 1 \\ 1 & \text{if } \frac{\sigma_{cr.p}}{\sigma_{cr.c}} - 1 > 1 \\ 0 & \text{otherwise} \end{cases}$$

$$\xi = 0.721$$

$$\rho_c := \left(\rho_p - \chi_c\right) \cdot \xi \cdot (2 - \xi) + \chi_c = 0.34016$$

$$b_{edge.eff} := \rho_{loc} \cdot \frac{b_{end}}{2} = 172.336\,mm$$

$$\rho_{loc} = 0.862$$

$$A_{c.eff} := \rho_c \cdot A_{c.eff.loc} + 2 \cdot \left(b_{edge.eff}\right) \cdot t_p = 1.2574373 \times 10^4\,mm^2$$

$\gamma_{m.1} := 1$

$$N_{c.Rd} := A_{c.eff} \cdot \frac{f_y}{\gamma_{m.1}} = 4463.902 kN$$

$N_{c.Rd}$ = is a design resistance - it is a value determined by interpolating between plate -like and column- like buckling behavior.

For the sake of curiosity, let us calculate strength neglecting the effect of column- like buckling (i.e. considering only plate behavior) :

$$N_{c.Rd\_plate} := \left(\rho_p \cdot A_{c.eff.loc} + 2b_{edge.eff} \cdot t_p\right) \cdot \frac{f_y}{\gamma_{m.1}} = 4676.351 kN$$

CALCULATION BASED ON REDUCED STRESS METHOD

Loading situation:

$\sigma_{x.Ed} := 116.6422146758 MPa$ uniaxial stress condition

Note: this value of $\sigma_{x,Ed}$ was found by iteration.

Determination of α_{cr} :

There are two possibilities to determine, α_{cr}

1) Hand calculation using NS-EN-1993-1-5, equation (10.6)

2) Using soft ware like EBplate, Abaqus.

1) Using hand calculation to determine, α_{cr} :

I: calculating α_{cr}:

$\sigma_{cr.p} := 50.7635 MPa$ This value of $\sigma_{cr.p}$ was previously calculated for Effective width method. See the calculation done for Effective width method above.

$\sigma_{cr.x} := \sigma_{cr.p}$ For uniaxial stress condition, $\sigma_{cr.x} = \sigma_{cr.p}$ See NS-EN-1993-1-5:10(6)

$\alpha_{cr.x} := \frac{\sigma_{cr.x}}{\sigma_{x.Ed}}$ $\alpha_{cr.\tau} := 0$ $\alpha_{cr.z} := 0$ No transverse & shear loading

NS-EN-1993-1-5:eq. (10.6):

$$\frac{1}{\alpha_{cr}} := \frac{1+\psi_x}{4\cdot\alpha_{cr.x}} + \frac{1+\psi_z}{4\cdot\alpha_{cr.z}} + \left[\left(\frac{1+\psi_x}{4\cdot\alpha_{cr.x}} + \frac{1+\psi_z}{4\cdot\alpha_{cr.z}}\right)^2 + \frac{1-\psi_x}{2\cdot\alpha_{cr.x}^2} + \frac{1-\psi_z}{2\cdot\alpha_{cr.z}^2} + \frac{1}{\alpha_{cr.\tau}^2}\right]^{\frac{1}{2}}$$

$\alpha_{cr} := \alpha_{cr.x} = 0.435$

II: Determination of $\alpha_{ult,k}$:

$$\frac{1}{\alpha_{ult.k}} := \left(\frac{\sigma_{x.Ed}}{f_y}\right)^2 + \left(\frac{\sigma_{z.Ed}}{f_y}\right)^2 - \left(\frac{\sigma_{x.Ed}}{f_y}\right)\cdot\left(\frac{\sigma_{z.Ed}}{f_y}\right) + 3\cdot\left(\frac{\tau_{Ed}}{f_y}\right)^2$$

NS-EN-1993-1-5: eq. (10.3)

Only $\sigma_{x.Ed}$ is different from zero

$\alpha_{ult.k} := \frac{f_y}{\sigma_{x.Ed}} = 3.043$

III: Determination of plate slenderness, λ_p

$\lambda_p := \sqrt{\frac{\alpha_{ult.k}}{\alpha_{cr}}} = 2.644$ NS-EN-1993-1-5: eq. (10.2)

IV: Determination of reduction factor, ρ_p

NS-EN-1993-1-5 allows determination of reduction factor into two ways:

A: Using different buckling curves according the clause 10(5a). Here equation (4.2), (4.3) and clause 5.2(1) could be used if necessary.

B: Using a single generalized curve based on equation (B.1)

In the following calculation, alternative A (i.e. different buckling curves) will be used).

A generalized buckling curve method is usually used when the plate is non regular or when the loading situation is very complex.

$\psi_x := 1$

$$\rho_{p\_r} := \begin{cases} \dfrac{\lambda_p - 0.055(3 + \psi_x)}{\lambda_p^2} & \text{if } \lambda_p > 0.673 \wedge (3 + \psi_x) \geq 0 \\ 1 & \text{otherwise} \end{cases}$$

NS-EN-1993-1-5:eq(4.2)

$\rho_{p\_r} = 0.347$

V: Column like buckling:

According NS-EN-1993-1-5:10(5a), a column-like buckling of the plate should be considered. All relevant equations in section 4.5.3 will be used. The only exception is instead of λ_c (relative column slenderness), the plate slenderness, λ_p which is calculated above will be used.

$$A_{sl.1.} := \frac{b_{sup} + b_{end}}{2} \cdot t_p + h_{sl} \cdot t_{sl} = 5.92 \times 10^3 \, mm^2$$

NS-EN-1993-1-5:4.5.3(3)

$$I_{sl.1.} := \frac{b_{end} \cdot t_p^3}{12} + \frac{t_{sl} \cdot h_{sl}^3}{12} = 4.129 \times 10^6 \, mm^4$$

$$\sigma_{cr.c\_r} := \frac{\pi^2 \cdot E \cdot I_{sl.1}}{A_{sl.1} \cdot a^2} = 2.950402 \times 10^7 \, Pa$$

$\sigma_{cr.c\_r}$ = the subscript "r" indicate Reduced stress method

$$e_{.} := \frac{h_{sl}}{4} = 40mm$$

(open section stiffener)

$\alpha := 0.49$

$$i := \sqrt{\frac{I_{sl.1}}{A_{sl.1}}} = 26.411mm$$

$$\alpha_{e.} := \alpha + \frac{0.09}{\left(\frac{i}{e_{.}}\right)} = 0.6263086$$

$$\varphi_c := 0.5 \cdot \left[1 + \alpha_{e.} \cdot (\lambda_p - 0.2) + \lambda_p^2\right] = 4.7621$$

NS-EN-1993-1-1:6.3.1.2(1)

$$\chi_{c\_r} := \frac{1}{\varphi_c + \sqrt{\varphi_c^2 - \lambda_p^2}} = 0.115$$

χ_{c_r} = the subscript "r" indicate Reduced stress method

VI: Interpolation between plate and column buckling:

$$\xi\_r := \begin{cases} \frac{\sigma_{cr.p}}{\sigma_{cr.c\_r}} - 1 & \text{if } 0 \le \frac{\sigma_{cr.p}}{\sigma_{cr.c\_r}} - 1 \le 1 \\ 1 & \text{if } \frac{\sigma_{cr.p}}{\sigma_{cr.c\_r}} - 1 > 1 \\ 0 & \text{otherwise} \end{cases}$$

$\xi\_r = 0.721$

$\rho_{c\_r} := (\rho_{p\_r} - \chi_{c\_r}) \cdot \xi\_r \cdot (2 - \xi\_r) + \chi_{c\_r} = 0.329$

VII: Determination of resistance:

If there are different loading types, NS-EN-1993-1-5:eq. (10.5) is most appropriate to check for resistance of the plate. When there is only uniaxial loading situation, either eq.(10.5) or eq.(10.1) could be used.

$\gamma_{m1} := 1$ Since results will be compared with that of Abaqus, influence of material factor is avoided

$$\frac{\sigma_{x.Ed}}{\left(\frac{\rho_{c\_r} \cdot f_y}{\gamma_{m1}}\right)} = 1.00000000000028$$ NS-EN-1993-1-5: eq.(10.5)

OK!

$\sigma_{c.Rd\_r} := \sigma_{x.Ed} = 116.642\,MPa$

$\sigma_{cr.p} = 50.7635\,MPa$

OR

$$\frac{\rho_{c\_r} \cdot \alpha_{ult.k}}{\gamma_{m1}} = 0.99999999999972$$ NS-EN-1993-1-5: eq.(10.1)

OK!

2: Using soft ware to determine α $_{cr}$:

By using EBplate we get the critical buckling stress, $\sigma_{cr\_EBplate}$= 54.137 Mpa

$\alpha_{.cr}$ will be calculated based on $\sigma_{cr\_EBplate}$:

$\sigma_{cr\_EBplate} := 54.137\,MPa$

$\sigma_{x.Ed\_2} := 120.0041095605\,MPa$ $\sigma_{x.Ed\_2}$ is determined by

$$\alpha_{cr.EBplate} := \frac{\sigma_{cr\_EBplate}}{\sigma_{x.Ed\_2}} = 0.451$$

$$\alpha_{ult.k\_2} := \frac{f_y}{\sigma_{x.Ed\_2}} = 2.958$$

$$\rho_{p\_2} := \frac{\lambda_{p\_2} - 0.055\left(3 + \psi_x\right)}{\lambda_{p\_2}^{\;2}} = 0.357$$

$$\lambda_{p\_2} := \sqrt{\frac{\alpha_{ult.k\_2}}{\alpha_{cr.EBplate}}} = 2.561$$

Interpolation equation:

$\chi_{c\_2} := \chi_{c\_r} = 0.115$

$\xi_2 := \xi_r = 0.721$

$$\rho_{c\_2} := \left(\rho_{p\_2} - \chi_{c\_2}\right)\cdot\xi_2\cdot\left(2 - \xi_2\right) + \chi_{c\_2} = 0.338$$

Then the resistance check will be:

$$\frac{\sigma_{x.Ed\_2}}{\left(\frac{\rho_{c\_2}\cdot f_y}{\gamma_{m1}}\right)} = 1.0000000000002$$

NS-EN-1993-1-5: eq.(10.5)

$$\frac{\rho_{c\_2}\cdot\alpha_{ult.k\_2}}{\gamma_{m1}} = 0.99999999999978$$

NS-EN-1993-1-5: eq.(10.1)

Strength based on $\alpha_{crEBplate}$:

$\sigma_{c.Rd\_2} := \sigma_{x.Ed\_2} = 120.00411\,MPa$

critical buckling stress

$\sigma_{cr.p\_2} := \alpha_{cr.EBplate}\cdot\sigma_{x.Ed\_2} = 54.137\,MPa$

An example showing how iterations are carried out by MathCAD

All parameters are same as those used in the last calculation above. Symbols used are defined on last two pages above.

$\chi_{c\_2} := 0.114646592210588$ $\psi_x := 1$

$\xi_2 := 0.72056214092270$ $\gamma_{m1} := 1$

$$\alpha_{ult.k\_2}(\sigma_{x.Ed\_2}) := \frac{355MPa}{\sigma_{x.Ed\_2}}$$

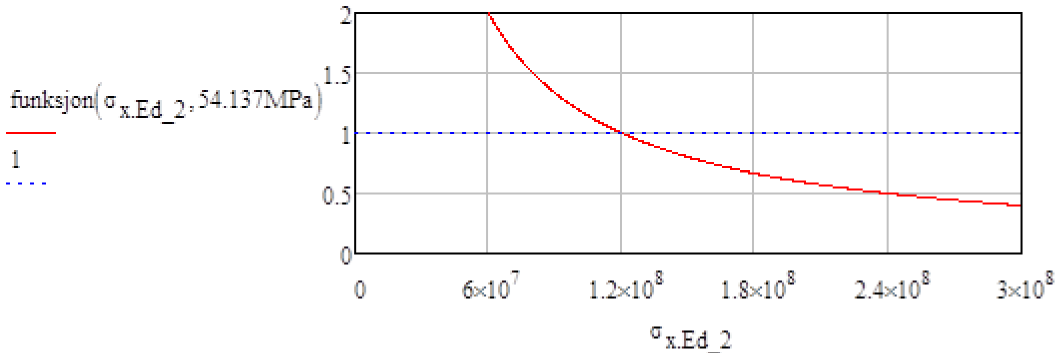
$$\alpha_{cr.EBplate}(\sigma_{x.Ed\_2}, \sigma_{cr\_EBplate}) := \frac{\sigma_{cr\_EBplate}}{\sigma_{x.Ed\_2}}$$

$$\lambda_{p\_2}(\sigma_{x.Ed\_2}, \sigma_{cr\_EBplate}) := \sqrt{\frac{\alpha_{ult.k\_2}(\sigma_{x.Ed\_2})}{\alpha_{cr.EBplate}(\sigma_{x.Ed\_2}, \sigma_{cr\_EBplate})}}$$

$$\rho_{p\_2}(\sigma_{x.Ed\_2}, \sigma_{cr\_EBplate}) := \frac{\lambda_{p\_2}(\sigma_{x.Ed\_2}, \sigma_{cr\_EBplate}) - 0.055(3 + \psi_x)}{\lambda_{p\_2}(\sigma_{x.Ed\_2}, \sigma_{cr\_EBplate})^2}$$

$$\rho_{c\_2}(\sigma_{x.Ed\_2}, \sigma_{cr\_EBplate}) := (\rho_{p\_2}(\sigma_{x.Ed\_2}, \sigma_{cr\_EBplate}) - \chi_{c\_2}) \cdot \xi_2 \cdot (2 - \xi_2) + \chi_{c\_2}$$

$$\text{funksjon}(\sigma_{x.Ed\_2}, \sigma_{cr\_EBplate}) := \frac{\rho_{c\_2}(\sigma_{x.Ed\_2}, \sigma_{cr\_EBplate}) \cdot \alpha_{ult.k\_2}(\sigma_{x.Ed\_2})}{\gamma_{m1}}$$



$\sigma_{x.Ed\_2.trial} := 140MPa$

$\text{root}(\text{funksjon}(\sigma_{x.Ed\_2.trial}, 54.137MPa) - 1, \sigma_{x.Ed\_2.trial}) = 120.0041095605\,MPa$

CONTROL AGAINST TORSIONAL BUCKLING OF STIFFENERS

Flat stiffeners on axially loaded plates are prone to torsional buckling. NS-EN 1993-1-5:9.2.1(8) states that, properties of a longitudinal flat stiffener should satisfy the following criteria to prevent torsional buckling.

$$\frac{I_T}{I_p} \geq 5{,}3\frac{f_y}{E}$$ NS-EN 1993-1-5: eq. (9.3)

Geometry of stiffeners used:

$h_{st} = 160\,\text{mm} \Rightarrow$ height of double sided flat stiffener
$t_s = 12\,\text{mm} \Rightarrow$ thickness of the stiffener

$$I_T = \frac{1}{3}h_{st}t_{st}^3\left(1 - 0{,}63\frac{t_{st}}{h_{st}}\right)$$

$$I_{x,st} = \frac{t_{st}h_{st}^3}{12} = \frac{12 \cdot 160^3}{12} = 4{,}096 \cdot 10^6\,\text{mm}^4$$

$$I_{y,st} = \frac{h_{st}t_{st}^3}{12} = \frac{160 \cdot 12^3}{12} = 23040\,\text{mm}^4$$

$$I_p = I_{x,st} + I_{y,st} = 4119040\,\text{mm}^4$$

$$\frac{I_T}{I_p} = \frac{87805{,}44\,\text{mm}^4}{4119040\,\text{mm}^4} = 0{,}0213$$

$$5{,}3\frac{f_y}{E} = 5{,}3\frac{355\,\text{N/mm}^2}{2{,}1 \cdot 10^5\,\text{N/mm}^2} = 0{,}009$$

$$\Rightarrow \frac{I_T}{I_p} > 5{,}3\frac{f_y}{E}$$ OK!