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Cognitive Radio: Interference Management and Resource Allocation

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Problem Description

Cognitive radio is a system with the ability to make intelligent decisions about its transmission modes. The goal of cognitive radios are to utilize under-used spectrum, as spectral resources are becoming sparse. There are two primary research areas regarding the theoretic aspect of cognitve radio, spectrum-sensing and interference and resource management.

This project will explore the aspect of interference and resource management in cognitive radio, to see how this can be exploited to enhance the performance of a cognitive radio system. Especially the aspect of simultaneous transmission between cognitive and primary users is of interest, as this could potentially enhance the performance of cognitive systems. Scenarios involving different number of cognitive users and different QoS requirements for the primary users will be reviewed. Review of these problems should shed light on the potential of cognitive radio and, if not solve, at least identify important problems in the area of cognitive radio.

Assignment given: 18. January 2010
Supervisor: Tor Audun Ramstad, IET

Abstract

In this thesis the performance of different cognitive systems are analyzed in different environments and scenarios. The main scenarios are: one cognitive and one primary user, multiple cognitive users and channels and multiple cognitive and primary users. With primary users in the vicinity, cognitive systems are evaluated both when no degradation to primary user QoS is allowed and when some degradation is allowed, measured by an outage probability.

In all scenarios involving one or more primary users, the performance is evaluated over two phases. In Phase 1 the channel is idle, i.e. the primary users are silent, and in Phase 2 the primary users are active on the channel. One of the questions in this thesis is how can cognitive users transmit simultaneously with the primary user in Phase 2. Schemes that show that this is possible is presented and evaluated and performance is compared to a standard cognitive system only transmitting when the channel is idle.

In scenarios with multiple cognitive users and channels, power allocation schemes are reviewed. A novel power allocation algorithm presented in [3], called *modified water filling* in this thesis, is implemented and referenced against other well-known power allocation schemes.

All implementation and simulations were done in MATLAB. It was assumed infinite processing power at all cognitive users, i.e. no processing delay, and perfect spectral sensing at all cognitive users.

The results showed that the performance gain of cognitive system utilizing simultaneous transmission achieves only a slight performance gain over a standard cognitive system, when no degradation to primary user QoS is allowed. However, by allowing only a slight degradation in primary user QoS, the gain is significant and should be included in future work on cognitive radio as it shows a promising way to exploit spectra.

Preface

This thesis ends my five years of study in the Communication Technology Master's program with in-depth study of wireless communications. The program is under the Department of Electronics and Telecommunications at the Norwegian University of Science and Technology.

The topic of the thesis is cognitive radio, a fairly new phenomenon in the world of communication theory as it was first introduced in a PhD thesis by J. Mitola in 1999. Cognitive radios are systems which are able to make intelligent decisions about their transmit modes, with a goal of utilizing underused spectra. The main research in this thesis is with regard to optimization of simultaneous transmission between independent users on the same frequency band.

This work was carried out in the period January 2010 till June 2010, under the supervision of Professor Tor A. Ramstad. I would like to thank Prof. Ramstad for the many discussions and conversations over the last year.

Trondheim, June 2010
Brage Ellingsæter

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List of Abbreviations

ARQ	Automatic Repeat reQuest
BER	Bit Error Rate
DPC	Dirty Paper Coding
FCC	Federal Communications Commission
MIMO	Multiple Input Multiple Output
MFW	Modified Water Filling
OFDM	Orthogonal Frequency Division Multiplexing
OS	Optimal user Selection
PAM	Pulse Amplitude Modulation
PDF	Probability Density Function
PER	Packet Error Rate
QoS	Quality of Service
SINR	Signal to Interference plus Noise Ratio
SNR	Signal to Noise Ratio
THP	Tomlinson-Harashima Precoding
WF	Water Filling

Chapter 1

Introduction

Cognitive radio systems are radios with the ability to exploit their environment to increase spectral efficiency and capacity. As spectral resources become more limited the FCC[1] has recommended that significantly greater spectral efficiency could be realized by deploying wireless devices that can coexist with primary users, generating minimal interference while somehow taking advantage of the available resources [10].

Such devices, known as cognitive radios, would have the ability to sense their communication environment and adapt the parameters of their communication scheme to maximize rate, while minimizing the interference to the primary users. Thus the two most popular research areas when it comes to cognitive radios are spectrum sensing and interference management and resource allocation. Spectrum sensing is the ability to find available frequencies/timeslots to transmit in. The problem is then that the algorithms need to have as little delay as possible so that once channels are available one can transmit immediately. And of course one would want as few false detections and false no-detections as possible.

Research in the area of interference management and resource allocation consists of how to allocate power in channels to maximize capacity while minimizing interference to other users. One way is of course to transmit when no one else is using that frequency/timeslot, but given a scenario where there are multiple cognitive users in the same environment this may not be possible and certainly not the way to maximize capacity.

[1] Federal Communications Commission

When many users transmit at the same frequency, maximizing capacity for one or all users becomes the problem of optimizing power allocation in an interference channel. Even though this problem was considered as early as 1975 ([4]) and certain solutions have been obtained in a few cases, the general solution to the problem has not been found to date.

1.1 Problem Statement

Although extensive research has been done in the area of cognitive radio since the concept first appeared in 1999 [21], one question was put forth in 2007 in [9]: How can we allow a cognitive user to transmit simultaneously with the primary user as long as the level of interference with the primary user remains within an acceptable range? This field of study will be referred to as interference management. And although many studies have been published in this area, such as [16], [8], the research has been very one-dimensional. The research has been done, almost exclusively, with regard to an environment with one primary and one cognitive user and most studies focus only on the theoretical performance limits in this environment.

Power allocation strategies in distributed networks have been studied for a long time. The difference between power allocation strategies in distributed networks, such as sensor networks, and cognitive networks, is that in sensor networks, the nodes either cooperate fully or they do not. In cognitive networks, the cognitive radios have the ability to sense their environment so that even though they do not cooperate, they still can make intelligent decisions to optimize performance.

A practical cognitive system would not only have to consider power allocation among multiple cognitive users, but would also have to consider the possibility of primary users occupying different frequency bands. In this case, schemes to guarantee primary user QoS under simultaneous transmission is of importance. In this thesis models that are more applicable to the real world are considered, both with regard to interference management and power allocation. These models shed light not only on performance limits in such systems, but also on the complexity of such systems and therefore how realistic they are.

1.2 Goal of this Thesis

The goal of this thesis is to investigate different environments and scenarios that are applicable to cognitive radio, review the performance of cognitive systems in these environments and their potential in a real implementation. Some aspects of these environments have been studied before and results obtained in this thesis will be referenced against those, and some new aspects and problems will be put forth in this thesis.

1.3 Thesis Outline

This thesis is outlined as follows:

- Chapter 2 discusses the theory and background information behind this thesis.
- Chapter 3 explains the methods used in the implementation and simulation of the environments and assumptions made about the environment.
- Chapter 4 presents the simulation results and discusses them.
- Chapter 5 concludes the thesis.

Chapter 2

Theory

The key feature that characterizes a cognitive radio is its ability to sense its surrounding spectra, because without this feature it would be nothing but a normal radio. Most research in the area of cognitive radio has also been focused on this part by developing algorithms that detect available spectra. But an important aspect of cognitive radios that need to be studied if cognitive radios are going to go commercial is the case of many cognitive radios in the same environment.

The research in this thesis is done with regard to three different cognitive systems. The first, and most studied in other works, is the setup where the cognitive radio system consists of one primary user, one cognitive user and one channel. In this setup transmission of the cognitive user can be divided into two phases. *Phase 1* is the time period over which the channel is idle, i.e. the primary user is silent. *Phase 2* is the time period over which the channel is used by the primary user. In this scenario, the main research in this thesis is with regard to Phase 2, i.e. how can the cognitive user transmit simultaneously with the primary user without degrading the primary users performance.

The second system consist of n cognitive users and m available channels with no primary users. This means that spectrum sensing has been done by all cognitive radios and they have to "fight" for the available resources. The main research is done with regard to finding an optimal power allocation scheme for the cognitive users.

The third system consists of n cognitive users and 1 or more primary users in the vicinity. The main research is done with regard to optimize the sum rate of all cognitive users in the environment.

Background theory and theoretical performance limits for these systems will be investigated in this Chapter. Notations used in this thesis follow those used in [25] and it is assumed the reader has general knowledge of signal processing, wireless communication and information theory. Throughout this thesis the word channel is used and is defined as the path over which a wireless signal can pass in a given frequency band. If two users are said to use the same channel, it means they use the same frequency band. If m channels are said to be available to a user, it means the user can use m frequency bands to communicate with its receiver.

2.1 Gaussian Interference Channel

With the goal of researching the possibility of simultaneous transmission between two users over the same channel, evaluation of the interference channel must be done. For any two users transmitting over a channel the maximum capacity region is given by Shannon's capacity formula [23]:

$$\begin{aligned} 0 \leq R_1 \leq C &\triangleq \frac{1}{2}\log_2(1+P_1) \\ 0 \leq R_2 \leq C &\triangleq \frac{1}{2}\log_2(1+P_2) \end{aligned} \tag{2.1}$$

If the two users communicate on the same channel and are in such proximity that their signals interfere with each other, this is said be a Gaussian interference channel, which is shown in Figure 2.1. It may seem that generality is lost by having unit gain on desired links, coefficients a and b on interfering links and unit noise power, but [5] showed that one can always apply a scaling transformation to a Gaussian interference channel with arbitrary transmission coefficients and noise powers and reduce it to an equivalent channel. What the capacity over such a channel is, is still an open problem, except in the case of very-strong interference ($a^2 \geq 1+P_1$ and $b^2 \geq 1+P_2$) and strong interference ($a \geq 1$ and $b \geq 1$) and was proven in [4] and [14] respectively.

In the case of very-strong interference, [4] showed that the capacity region of this channel is the same as the capacity region with no interference. The reason for this is that the interfering signals are so strong that the receivers may decode them reliably even if they consider their intended signal as noise.

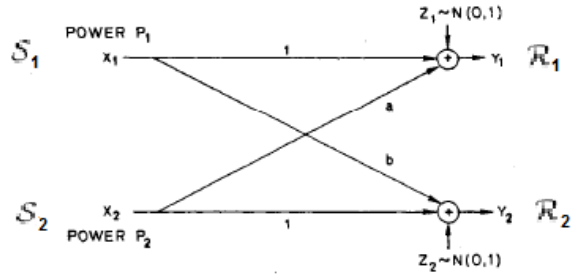


Figure 2.1: Standard Gaussian interference channel [5].

The receivers can then decode the interfering signal, subtract this from the total received signal and then end up with a channel cleared of interference. The capacity region for such a case is thus the full rectangular region given by (2.1).

With two users, the Gaussian interference channel can be viewed as two multiple access channels, one from $\mathcal{S}_1, \mathcal{S}_2 \to \mathcal{R}_1$ and one from $\mathcal{S}_1, \mathcal{S}_2 \to \mathcal{R}_2$. [14] showed that in the case of strong interference, both receivers would be able to decode both messages regardless of the decoding technique used. Thus, the capacity region is the intersection of the capacity regions for the two multiple access channels. Simply put, this is a subset of rate pairs (R_1, R_2) given by (2.1) for which

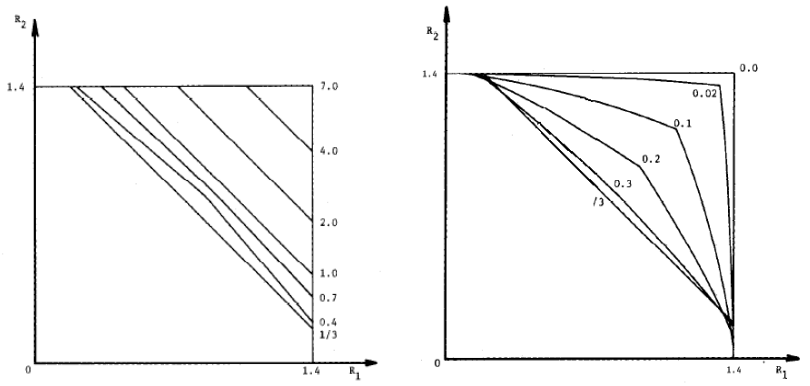
$$R_1 + R_1 \leq \min\left\{\frac{1}{2}\log_2(1 + b^2P_1 + P_2), \frac{1}{2}\log_2(1 + P_1 + a^2P_2)\right\} \tag{2.2}$$

The largest to date known achievable region for arbitrary positive $a, b \in \mathbb{R}$ was proved in [7]. This states that given $\epsilon \geq 0$ and $R_1 \geq C_1 - \epsilon$, then

$$R_2 \leq \frac{1}{2}\log_2(1 + \frac{a^2P_2}{1 + P_1}) + \delta(\epsilon) \tag{2.3}$$

where $\delta(\epsilon) \to 0$ as $\epsilon \to 0$. Achievable rate tuples for different values of $a = b$ is shown in Figure 2.2.

As is seen by the plots in Figure 2.2, even when $\mathcal{S}_1$ is transmitting at rates close to capacity, $\mathcal{S}_2$ can still transmit, though at very low rates. But when $\mathcal{S}_1$ reduces its rate to 2/3 of the capacity, $\mathcal{S}_2$ can increase its rate dramatically. This provides the first basis for investigating simultaneous transmission for a cognitive system.



(a) $1/3 \leq a = b$ (b) $0 \leq a = b \leq 1/3$.

Figure 2.2: Achievable rate regions for different values of $a = b$. $P_1 = P_2 = 6$ [14].

2.2 Genie-aided Cognitive System

As information theory tells us, knowledge can only enhance performance. Thus the theoretical performance limits for a real cognitive system must be upper-bounded by the performance of a system where the cognitive user knows all about the primary user. All means, in this context, that the primary user's message is available to the cognitive user, and this is obtained through an imaginary genie.

The capacity of a system with known interference given by a Gaussian source S, as shown in Figure 2.3, was solved in [6]. With the state of S known, the capacity of the system can be shown to be equal to the Shannon capacity, i.e. the interference, S, does not degrade performance. The coding technique used to achieve capacity is called dirty paper coding. One way to combat the known interference is to use a portion, α, of the power P available to the sender to cancel out S as much as possible and then achieve a rate of $R = \frac{1}{2}\log_2(1+(1-\alpha)P/N+(\sqrt{Q}-\sqrt{\alpha P})^2)$, where Q is the power of the interfering signal. However, dirty paper coding uses codewords in the direction of S. This means that one looks at the space surrounding the vector S and chooses codewords that are compatible with the power constraint and far enough apart to be distinguishable at the channel output. Thus, with dirty paper coding, interference does not degrade the performance of a system if the interference is known. In this Section it is assumed that the interference from the primary user is known at the cognitive user, but the primary user does not know the

interference from the cognitive user. I.e. information about the environment is asymmetric.

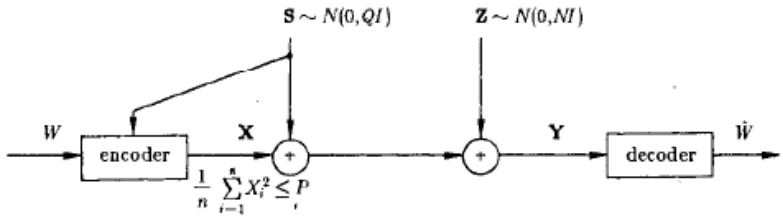


Figure 2.3: System where interference is known at the encoder. W is the intended message, S is the known interference, X is the transmitted symbol, Z is additive, white, Gaussian noise. Y is the receiver symbol and $\hat{W}$ is the reconstructed message [6].

As stated above information is only present at the cognitive user. Thus it is the cognitive user's responsibility to guarantee that the primary user obtains its desired performance. The genie-aided cognitive system is shown in Figure 2.4. Assume n is the length of a codeword in bits. The primary sender, $\mathcal{S}_p$, draws a message m_p from the index set $\{1, 2, \ldots, 2^{nR_p}\}$, resulting in the signal $X_p(m_p)$. Likewise, the cognitive sender has a message m_c drawn from the index set $\{1, 2, \ldots, 2^{nR_c}\}$, but since it also knows m_p the resulting signal is dependent on both m_c and m_p, so $X_c(m_c, m_p)$. R_p and R_c is the rate of the primary and cognitive user, respectively. Both users have average power constraints given as $||X_p||^2 \leq nP_p$ and $||X_c||^2 \leq nP_c$.

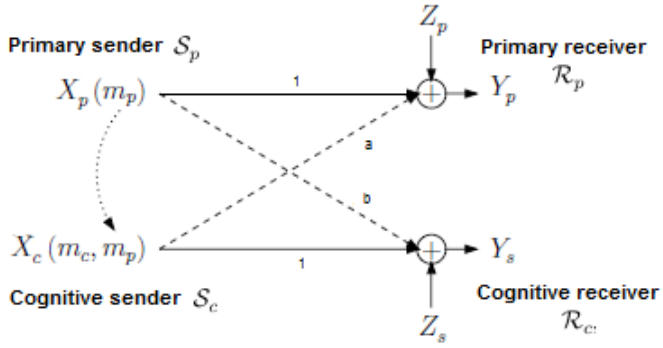


Figure 2.4: Genie-aided cognitive system. One primary user and one cognitive user, with subscripts $_p$ and $_c$ respectively. Cognitive sender is handed the primary senders message by a genie [16].

Below some key definitions of the cognitive system is given.

Definition 2.1. A cognitive sender can achieve a rate R_c in Phase 2 if there exists a code $(2^{nR_c}, n)$ for which:

1. The probability of error $\rightarrow 0$ as $n \rightarrow \infty$
2. A rate $R_p = C_p \triangleq \frac{1}{2}\log_2(1+P_p)$ can be achieved for the primary sender

Since the capacity for the cognitive sender is dependent on the primary senders rate, the capacity for the cognitive sender is defined as

Definition 2.2. The capacity for a cognitive user in Phase 2, C_c, is the largest achievable rate R_c, where R_c is defined in 2.1

Before providing mathematical limits on performance some intuitive boundaries will be presented. Section 2.1 showed the limit for the Gaussian interference channel with no cooperation and independent transmission, this must therefore be the lower bound on performance in any cognitive system. Considering full cooperation, this is equivalent to the 2×2 MIMO channel, thus this is the upper bound on performance of the cognitive system.

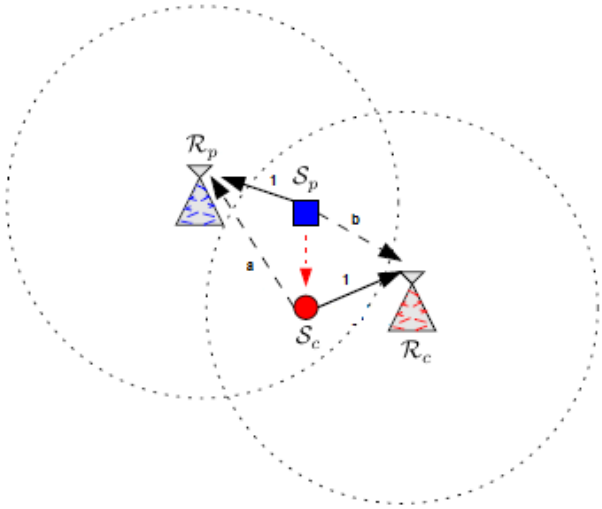


Figure 2.5: An environment where the cognitive sender is closer to the cognitive receiver than the primary receiver [16].

As in the case of the Gaussian interference channel, the capacity of a cognitive user in the one primary user, one cognitive user environment is still an open problem except in a few cases. [16] found the capacity of a cognitive user, for simultaneous transmission (Phase 2), in an environment where the channel gain from the cognitive sender to the primary receiver was less than the channel gain from the primary sender to the primary receiver. Assuming that channel gains are most dependent on path loss given by distance, this models a fairly realistic scenario with a cognitive sender farther from the primary receiver than the primary sender. Such an environment is depicted in Figure 2.5. This means that $a \leq 1$ in Figure 2.3, and this will be assumed throughout this Section.

Given that $a \leq 1$ and $b \in \mathbb{R}$, [16] found the capacity for the cognitive user in Phase 2 as

$$C_c^{(g)} = \frac{1}{2}\log_2(1 + (1-\alpha^*)P_c) \tag{2.4}$$

where $\alpha^* \in [0,1]$ and is defined in (2.7) and the term $^{(g)}$ is used to denote that this is genie-aided capacity. Proof of achievability will be given below as this describes, to some degree, how to achieve this rate. The converse part of the proof is given in [16].

To prove the achievability of (2.4), it has to be proven that there exists two codes $(2^{nR_c}, n)$ and $(2^{nR_p}, n)$ such that $R_p = C_p$ and for both codes the probability of error $\to 0$ as $n \to \infty$. Specifically, the codes are given as:

- Generate a code for the primary user that achieves capacity [25]. Codewords of this code is denoted X_p^n.
- Since the cognitive user knows both the primary user's message and encoding, the cognitive user can perform superposition coding:

$$X_c^n = \hat{X}_c^n + \sqrt{\frac{\alpha P_c}{P_p}} X_p^n \tag{2.5}$$

  where $\alpha \in [0,1]$. The message m_c is encoded into codeword $\hat{X}_c^n$ by dirty paper coding [6], where $(b + \sqrt{\frac{\alpha P_c}{P_p}})X_p^n$ is known interference that will affect the cognitive receiver.

The codeword $\hat{X}_c^n$, generated as described above, will be independent of X_p^n. To satisfy the average power constraint of the cognitive user on the components of X_c^n, the codeword $\hat{X}_c^n$ must satisfy $\frac{1}{n}\sum_{k=1}^{n} \hat{X}_c^2(k) \leq (1-\alpha)P_c$. The decoding

at the primary receiver is done without consideration of the cognitive signal, thus a decoding scheme that achieves capacity, such as the joint-typicality decoder [25], will suffice. The interfering cognitive signal, $\hat{X}_c^n$, is treated as independent Gaussian noise. The decoder at the cognitive receiver, is a Costa decoder [6].

With this scheme the cognitive receiver is not affected by the interfering signal from the primary user, and transmits its desired signal, $\hat{X}_c^n$, at a power $(1-\alpha)P_c$. Thus the probability of error $\to 0$ as $n \to \infty$ for all rates below

$$\frac{1}{2}\log_2(1+(1-\alpha)P_c).$$

The power of the desired signal to the primary receiver from both the primary sender and cognitive sender is $(\sqrt{P_p}+a\sqrt{\alpha P_c})^2$. The extra noise due to interference from the cognitive signal has power $a^2(1-\alpha)P_c$. Therefore the probability of error $\to 0$ as $n \to \infty$ for all rates below

$$\frac{1}{2}\log_2\left(1+\frac{(\sqrt{P_p}+a\sqrt{\alpha P_c})^2}{1+a^2(1-\alpha)P_c}\right).$$

For Definition 2.1 to hold, the maximum achievable rate for the primary user has to equal $\frac{1}{2}\log_2(1+P_p)$. Setting the achievable rates just found above equal to this rate we obtain:

$$\frac{1}{2}\log_2\left(1+\frac{(\sqrt{P_p}+a\sqrt{\alpha P_c})^2}{1+a^2(1-\alpha)P_c}\right)=\frac{1}{2}\log_2(1+P_p)\triangleq C_p. \tag{2.6}$$

As expected, if $a=0$, i.e. no interference from the cognitive user to the primary receiver, any choice of α will suffice. Considering the achievable rate, given by the left hand side of (2.6) as a function of α, $f(\alpha)$, and that $f(\alpha)\in[0,C_p]$, then by the Intermediate Value Theorem there has to be an α for which (2.6) holds. Especially, if $0\le a\le 1$ the equation in (2.6) always has unique solution of $\alpha\in[0,1]$ and can be computed as:

$$\alpha^*=\left(\frac{\sqrt{P_p}(\sqrt{1+a^2P_c(1+P_p)}-1)}{a\sqrt{P_c}(1+P_p)}\right)^2 \tag{2.7}$$

The whole calculation of α is given in Appendix A.

2.3 Causal Cognitive System

Any real cognitive system must obtain its necessary information through some realistic scheme. In the previous Section achievable rates where found given non-causal information about the primary sender. In this Section achievable rates given causal information will be reviewed. Causal information about the primary sender in this context does not only mean information of what has happened, but also that the cognitive user has to use some realistic scheme to obtain this information (i.e. not through a genie).

Achievable rate regions in the causal cognitive system have been studied in a few papers, [8] [22] [20]. But all these papers impose some assumptions about the primary user, such as partial cooperation [8] or Markov block decoding [22], [20]. Since the primary user is suppose to operate in complete obliviousness of the cognitive user, these kinds of assumptions are not considered in this thesis.

That the cognitive sender can be able to obtain the necessary information to perform superposition coding for interference cancellation (as in the previous Section) in a causal manner, may seem impossible. But in theory there are scenarios, where the use of cognitive radios seem suitable, where this can be achieved.

Consider the scenario where a cognitive sender is closer to the primary sender than the primary receiver is to the primary sender. In this case it is very likely that the channel between the primary sender and the cognitive sender has a higher channel gain than that between the primary sender and primary receiver. Mathematically this is written as $G_{PS-PR} \leq G_{PS-CS}$.

Assume that the primary user transmits codewords consisting of N consecutive symbol intervals. During every symbol interval the primary user transmits at a rate of R_p bits per channel use. According to Definition 2.1, $R_p = \frac{1}{2}\log_2(1 + G_{PS-PR}^2 P_p)$. The cognitive user, on the other hand, listens to the primary user until the mutual information between its received signal and primary signal exceeds NR_p. Then it can decode the primary message with arbitrary small probability of error, given a Gaussian code ensemble, and use the signaling scheme presented in the previous Section. This requires, however, that the cognitive user is informed of the state of the received message at the primary receiver. This is possible if the primary user employs some kind of Automatic-Repeat-reQuest (ARQ) protocol. More specifically we have the

following inequalities:

$$\sum_{n=1}^{N'-1} \log\left(1+G_{PS-CS}^2 P_p\right) \leq \sum_{n=1}^{N} \log\left(1+G_{PS-PR}^2 P_p\right) \leq \sum_{n=1}^{N'} \log\left(1+G_{PS-CS}^2 P_p\right). \tag{2.8}$$

Dividing both sides by N and setting $\beta = \lim_{N\to\infty} \frac{N'}{N}$, we can obtain

$$\log(1+G_{PS-PR}^2 P_p) \leq \frac{N'}{N}\log(1+G_{PS-CS}^2 P_p) \tag{2.9}$$

$$\beta \geq \frac{\log(1+G_{PS-PR}^2 P_p)}{\log(1+G_{PS-CS}^2 P_p)} \tag{2.10}$$

$$\beta = \frac{\log(1+G_{PS-PR}^2 P_p)}{\log(1+G_{PS-CS}^2 P_p)} \tag{2.11}$$

This shows that if the channel gain between the primary and cognitive sender is higher than the channel gain between primary sender and receiver, decoding is possible, i.e. $\beta \leq 1$. β is the time the cognitive sender must spend listening and decoding and thus one wants β as small as possible, thus (2.11) follows from this fact [18]. For a more detailed discussion the reader is referred to Section 3.2 in [2].

With the equations derived above, the theoretical capacity for the cognitive user in Phase 2 is:

$$C_c^{(c)} = (1-\beta)C_c^{(g)} = (1-\beta)\frac{1}{2}\log(1+(1-\alpha^*)P_c') \tag{2.12}$$

where $C_c^{(g)}$ is the genie-aided capacity given in (2.4). The term $^{(c)}$ is used to denote that this is causal capacity. Note that since the cognitive user is silent over a period of β, the power constraint is $(1-\beta)P_c' \leq P_c$.

2.4 n-users, m-channels

As stated in the introduction to this Chapter, it is likely that a radio environment will contain multiple cognitive users. How a cognitive radio should allocate resources in such an environment is thus of great importance. In the following a system model consisting of n cognitive users and m channels will be presented, where it is assumed no primary users in the vicinity. Further, different power allocation schemes are described and an algorithm to obtain the maximum sum rate of all cognitive users is presented.

System Model

Figure 2.6 shows the overall system model, in which a set of n point-to-point wireless users share a set of m channels. Different users on the same channel are subject to interference, where the interference experienced by user l on channel k is $\sum_{i=1,i\neq l}^{n} a_{il}^2 \times P_i(k)$. There is no interference between different channels. All users are subject to a power constraint P_l, so that $\sum_{k=1}^{m} P_l(k) = P_l$. Thermal noise at each receiver is assumed to be N and the interference from other users is treated as noise.

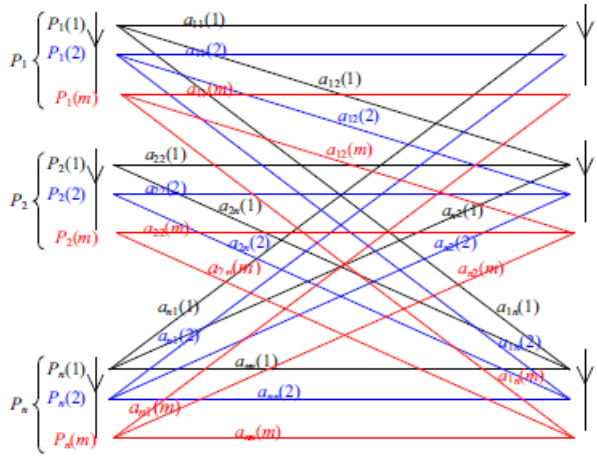


Figure 2.6: System model with n transmitters, n receivers and m channels with interfering links between all transmitters and receivers [3].

The capacity over one channel for a given user l is given by:

$$C_l(k) = \frac{1}{2}\log_2(1 + \frac{P_r(k)}{N + I(k)}) \tag{2.13}$$

where P_r is the received power from the desired user, and I is the interference from other users. The received power from user l on channel k is given by the channel gain $a_{ll}(k)$ and transmit power $P_l(k)$, as can be seen by Figure 2.6, and the interference experienced by user l from the other users is given by $\sum_{j=1,j\neq l}^{n} a_{jl}^2(k)P_j(k)$. The capacity for one user over all channels is then:

$$C_l = \sum_{k=1}^{m} C_l(k) = \frac{1}{2}\sum_{k=1}^{m} \log_2\left(1 + \frac{a_{ll}^2(k)P_l(k)}{N + \sum_{j=1,j\neq l}^{n} a_{jl}^2(k)P_j(k)}\right) \tag{2.14}$$

and the total capacity over all users is given by:

$$C = \sum_{l=1}^{n} C_l = \sum_{l=1}^{n} \frac{1}{2} \sum_{k=1}^{m} \log_2 \left(1 + \frac{a_{ll}^2(k) P_l(k)}{N + \sum_{j=1, j \neq l}^{n} a_{jl}^2(k) P_j(k)} \right) \tag{2.15}$$

where all users are subject to a power constraint defined as

$$\sum_{k=1}^{m} P_l(k) = P_l, l = 1, ..., n, P_i(k) \geq 0, \forall i, k \tag{2.16}$$

Power Allocation

To maximize system performance one needs to know what system feature to maximize. In our case it could either be to maximize capacity for a given user or maximizing the total capacity over all users and all channels. To maximize total capacity we need information about all channel gains and power allocations and even though this is unlikely to be the case of most cognitive radio systems it yields an upper bound on performance.

There are usually three ways to allocate power for a user over multiple channels. The first is to allocate all power to the channel with the best signal to noise ratio, called *channel allocation* in this thesis. The second is to allocate equal power in all channels and the third is water filling. The challenge in this environment is that we have to consider interference in each channel and this may change over time.

Before going further to try to find an optimal scheme to allocate power, a few fundamental properties of the system model considered in this Section is given in Theorem 2.1 and the following corollaries.

Theorem 2.1. *In a system with n users, $m \geq n$ channels and non-zero interfering channel gains,* channel allocation *is the optimal power allocation when $P \to \infty$.*

Proof of Theorem 2.1 is given in Appendix E.

Corollary 2.2. *In a system with n users, $m \geq n$ channels and non-zero interfering channel gains,* channel allocation *is the optimal power allocation when $n \to \infty$.*

Corollary 2.3. *In a system with n users, $m < n$ channels and non-zero interfering channel gains, if all n users transmit at the same time the rate $R \to 0$ as $n \to \infty$, due to the interference.*

The proof of Corollary 2.2 and 2.3 follows exactly the proof of Theorem 2.1, since with the number of users approaching infinity, interference also approaches infinity except in the case of channel allocation. Thus given high power or large number of users channel allocation is the optimal power allocation scheme for this system.

Maximizing C_l is an optimization problem subject to the power constraint. With this type of problem Lagrange multipliers is the obvious method to achieve this optimization. First we will use Lagrange multipliers to optimize the capacity of each user individually. For user l the capacity is given by (2.14) with power constraint given by (2.16). The Lagrange function is then:

$$\begin{aligned} \Lambda(P_l^k, \lambda) &= C_l(P_i^k) + \lambda(\sum_{k=1}^{m} P_l(k) - P_l) \\ \Lambda(P_l^k, \lambda) &= \frac{1}{2}\sum_{k=1}^{m} \log_2(1 + \frac{a_{ll}^2(k)P_l(k)}{N + I(k)}) + \lambda(\sum_{k=1}^{m} P_l(k) - P_l) \end{aligned} \quad (2.17)$$

We then find the partial derivatives of the Lagrange function with respect to each variable and set this equal to zero:

$$\begin{aligned} \frac{\partial \Lambda}{\partial P_l(1)} &= \frac{1}{2}\frac{D_1}{1 + D_1 P_l(1)} + \lambda = 0 \\ &\vdots \\ \frac{\partial \Lambda}{\partial P_l(m)} &= \frac{1}{2}\frac{D_m}{1 + D_m P_l(m)} + \lambda = 0 \\ \frac{\partial \Lambda}{\partial \lambda} &= \sum_{k=1}^{m} P_l(k) - P_l = 0 \end{aligned} \quad (2.18)$$

where $D_i = \frac{a_{ll}^2(i)}{N + I(i)}$. Thus we have $m + 1$ equations with m unknowns. This can be quite cumbersome to solve when m exceeds 3 or 4, but as is shown in [25] this method is equivalent to the power allocation strategy known as water filling.

Water filling is the procedure where one finds the optimal power allocation over a set of channels for a given user. This is done by calculating the SNR with full power at the receiver for each channel and then pouring power into the channels with the best SNRs. Denoting the SNR for channel k as $\gamma_k = a_k^2 P/N$ the percentage of power that should be allocated to channel k is given by:

$$\frac{P_k}{P} = \begin{cases} 1/\gamma_0 - 1/\gamma_k & \gamma_k \geq \gamma_0 \\ 0 & \gamma_k < \gamma_0 \end{cases} \quad (2.19)$$

for some cutoff value γ_0 [12]. Since the total percentage has to be 1, γ_0, which is the point where the noise is so large that no power should be allocated to the channel, can be calculated as:

$$\sum_{k=1}^{m}\left(\frac{1}{\gamma_0}-\frac{1}{\gamma_k}\right)=1\Rightarrow\frac{m}{\gamma_0}=1+\sum_{k=1}^{m}\frac{1}{\gamma_k} \tag{2.20}$$

Considering the interference from other users as noise, the SNR can be altered to consider this term as well. We then have a signal to interference plus noise ratio (SINR) $\gamma_k = a_k^2 P/(N + I(k))$. The problem now is that the term I, which is the interference, depends on all other channel gains and power allocations in the system. Assuming this information is known at all receivers and transmitters, water filling by the procedure just described can be done by an iterative algorithm.

A scheme, which attempts to maximize capacity over all users and channels was presented in [3]. This is done by trying to maximize C in (2.15), subject to the constraint in (2.16). But as was shown above, direct solutions from using Lagrange multipliers are cumbersome at best. The procedure uses Lagrange multipliers to find a stationary point of C:

$$\delta C=\sum_{l=1}^{n}\sum_{k=1}^{m}\frac{\partial C}{\partial P_l(k)}\delta P_l(k)=0 \tag{2.21}$$

$\forall\delta P_l(k)$ such that $\sum_{k=1}^{m}\delta P_l(k)=0,\forall l, P_l(k)\geq 0,\forall l,k$. By reviewing this equation it is clear that it is satisfied given that $\frac{\partial C}{\partial P_l(k)}=c_l,\forall P_l(k)\neq 0$. Further, differentiating C with respect to $P_l(k)$ yields:

$$\frac{\partial C}{\partial P_l(k)}=\frac{a_{ll}^2(k)}{N+a_{ll}^2P_l(k)+\sum_{j=1,j\neq l}^{n}a_{jl}^2(k)P_j(k)}-$$

$$\sum_{i=1,i\neq l}^{n}\frac{a_{li}^2(k)a_{ii}^2(k)P_i(k)}{\begin{pmatrix}N+a_{ii}^2(k)P_i(k)\\+\sum_{j=1,j\neq i}^{n}a_{ji}^2(k)P_j(k)\end{pmatrix}\left(N+\sum_{j=1,j\neq i}^{n}a_{ji}^2(k)P_j(k)\right)}$$

$$=c_l,\forall k\in\left\{k',P_l(k')>0\right\} \tag{2.22}$$

A detailed derivation of $\frac{\partial C}{\partial P_l(k)}$ is given in Appendix C. The sum $\sum_{j=1,j\neq l}^{n}a_{jl}^2(k)P_j(k)$ is the interference from all other users experienced by user l on channel k. By denoting this sum as:

$$\sum_{j=1,j\neq l}^{n}a_{jl}^2(k)P_j(k)=I_l(k) \tag{2.23}$$

and

$$\sum_{i=1,i\neq l}^{n} \frac{a_{li}^2(k)a_{ii}^2(k)P_i(k)}{\begin{pmatrix} N + a_{ii}^2(k)P_i(k) \\ +\sum_{j=1,j\neq i}^{n} a_{jl}^2(k)P_j(k) \end{pmatrix}\left(N + \sum_{j=1,j\neq i}^{n} a_{ji}^2(k)P_j(k)\right)} = B_l(k) \tag{2.24}$$

then (2.22) simplifies to

$$\frac{\partial C}{\partial P_l(k)} = \frac{a_{ll}^2(k)}{N + a_{ll}^2 P_l(k) + I_l(k)} - B_l(k) = c_l \tag{2.25}$$

which can be rewritten as

$$\frac{1}{c_l + B_l(k)} = P_l(k) + \frac{N + I_l(k)}{a_{ll}^2(k)}. \tag{2.26}$$

If the B term is ignored this is equivalent to the water filling procedure obtained through Lagrange multipliers. However, if the goal is to maximize the overall capacity, i.e. sum rate of all users, one should take into account the effect of the power allocation of user l in channel k on the capacity if the other users. The B term does exactly this by calculating the renewed interference the other users will experience due to the power level of user l in channel k. $w_l(k) = \frac{1}{c_l + B_l(k)}$ takes the role as the water level in each channel, but is now no longer constant in each channel. As this is a modified version of water filling, it will be referred to as *modified water filling* [3].

As with the water filling procedure, an iterative algorithm is constructed to find the power allocation of all users in all channels. The algorithm is given in Algorithm 1.

As stated by (2.21) the goal of the algorithm is to find a stationary point of C. Thus if the algorithm converges for all users l over a number of iterations less than the maximum number of iterations, the capacity achieved is at least a stationary point of the global optimization. However it is not guaranteed to be a global maximum of the capacity [3].

(2.24) shows that $B_l(k)$ is dependent on the power allocation of all other users and the channel gains between all users. These are terms not likely known at a cognitive sender in any real implementation of the algorithm. Given that there is a feedback channel from the cognitive receiver to the cognitive sender, it can be assumed that the interference experienced on channel k at the cognitive receiver, $I_l(k)$, is known at the sender. From (2.24) one can

Algorithm 1 Modified Water Filling

Assume an initial power allocation is defined for all channels and all users (e.g. equal power).

1: **for** $it = 1 : IT_{max}$ **do**
2: **for** each user l **do**
3: Calculate $I_l(k)$ and $B_l(k)$ for all channels.
4: Calculate the current water level in each channel $w_l(k) = P_l(k) + \frac{N+I_l(k)}{a_{ll}^2(k)}$.
5: Calculate an estimate of $c_l(k) = \frac{1}{w_l(k)} - B_l(k)$.
6: Calculate the mean of c_l and use this as c_l.
7: The new water level should then be $\frac{1}{w_l(k)} = c_l + B_l(k)$, however this could be negative and lead to negative power or be close to zero and lead excessive power. So instead

$$\frac{1}{w_l(k)} = \max\left(c_l + B_l(k), \frac{1}{P_l + (N + I_l(k))/a_{ll}^2(k)}\right)$$

8: The new power allocation for each channel is

$$P_l(k) = w_l(k) - \frac{N + I_l(k)}{a_{ll}^2(k)}$$

9: **end for**
10: **end for**

see that the denominator consists of the interference experienced by user i on channel k. With increasing number of users, the interference power at any receiver is almost independent of the location of the receiver [13], and thus the interference experienced by user i approaches the interference experienced by user l, thus $\sum_{j=1,j\neq i}^{n} a_{ji}^2(k)P_j(k)$ can be approximated by $I_l(k)$. Noticing that $P_l(k)$ is included in the original expression, the original expression can be more tightly approximated as $I_l(k)\frac{N-1}{N} + a_{li}^2(k)P_l(k)$.

The channel gains in (2.24) are approximated by an average channel gain value, $a_{\text{avg}}^2(k)$. Assuming all users use an initial power allocation, this can be

estimated at the receiver. Then the estimated $B_l(k)$ is computed as:

$$\hat{B}_l(k) = \sum_{i=1}^{n} \frac{I_l(k)/n}{\begin{pmatrix} N + I_l(k)/(na^2_{\text{avg}}(k)) + I_l(k)n/(n-1) \\ +a^2_{\text{avg}}(k)P_l(k) \end{pmatrix} \begin{pmatrix} N + I_l(k)n/(n-1) \\ +a^2_{\text{avg}}(k)P_l(k) \end{pmatrix}} \tag{2.27}$$

Modified water filling using this estimated value of B, is referred to as *cognitive modified water filling* or cognitive MWF.

2.5 Practical Cognitive System

If cognitive radio is to be part of the commercial radio environment, there are a few more aspects of cognitive systems that have to be considered. As stated before, a likely scenario of a complete radio environment with cognitive radios consists of one or more primary users and one or more cognitive users. The primary users will have certain quality of service requirements such as received SNR, bits per second, and the cognitive users wants to maximize their rates subject to the QoS requirements of the primary users.

The environment that is considered in the following consists of m channels and n cognitive users. The cognitive users can transmit on any m channel, but on each channel there is a primary user using the channel for a fraction p_m of the time. The cognitive users can be thought of as using OFDM signaling on m subbands, where each subband is occupied by a primary user for a fraction p_m of the time. Note that in this thesis it is assumed only one primary user in each subband. The setup with 2 primary users communicating with a base station and 4 cognitive users are shown in Figure 2.7.

In Section 2.2 and 2.3 the QoS requirement of the primary user was defined as no degradation of any sort. In this Section the coexistence constraints will be slacked, so that a certain degradation in primary performance is allowed. The question is how to measure this degradation. The FCC first proposed the concept of interference temperature as a way to have unlicensed users share licensed spectra without causing harmful interference. The idea was to regulate the power at unlicensed users on a variable basis to limit the energy at licensed receivers. However, the FCC recently abandoned the idea because it was not a practical concept [11].

[13] proposed a new scheme to measure how cognitive users affect primary users performance. Information outage probability, defined as the probability

that the mutual information of the channel is below the transmitted code rate, was used in [13] to protect the primary users from interference from cognitive users. The outage probability is written as

$$P_{\text{out}}(R) = Prob\{I(x:y) \leq R\} \tag{2.28}$$

where $I(x:y)$ is the mutual information between transmitted signal x and received signal y over the channel and R is the target rate in bits/s/Hz. Reliable communication over the channel can therefore be done when the mutual information over the channel is strong enough to support the target rate R.

With the concept of outage probability as QoS constraint, the coexistence constraint used in this Section is defined as:

Definition 2.3. A cognitive user l can transmit on channel k with a power $P_l(k)$ as long as the information outage probability at a primary user is below a desired threshold q. Mathematically this is written as:

$$P_{\text{out}} = Prob\{C_{\text{pu}} \leq R_{\text{pu}} | R_{\text{pu}}, q\} \leq q \tag{2.29}$$

where the mutual information over the channel is the channel capacity [23].

This Section will, to some degree, use the modified water filling algorithm from Section 2.4 to allocate power and maximize rate. When the channels are idle, i.e. the primary users are silent, this algorithm can be applied directly. When the channels are used by the primary users, the algorithm has to be modified to include the constraint to ensure that the outage probability of the primary user on that channel is below the desired value q.

The optimization problem of Phase 2 can be expressed mathematically similar to the optimization problem in Section 2.4 with an additional constraint:

$$\begin{aligned}
\max_{\{P_1,\ldots,P_N\}} C(P_1,\ldots,P_N) &= \max_{\{P_1,\ldots,P_N\}} \sum_{i=1}^{N}\sum_{k=1}^{M} p_k \frac{1}{2}\log_2(1+SINR_i(k)) \\
&= \max_{\{P_1,\ldots,P_N\}} \sum_{i=1}^{N}\sum_{k=1}^{M} \log_2\left(1+\frac{a_{ii}^2(k)P_i(k)}{\left(N+\sum_{l=1,l\neq i}^{N} a_{li}^2(k)P_l(k) \atop +a_{Pu,i}^2(k)P_{Pu}(k)\right)}\right)
\end{aligned} \tag{2.30}$$

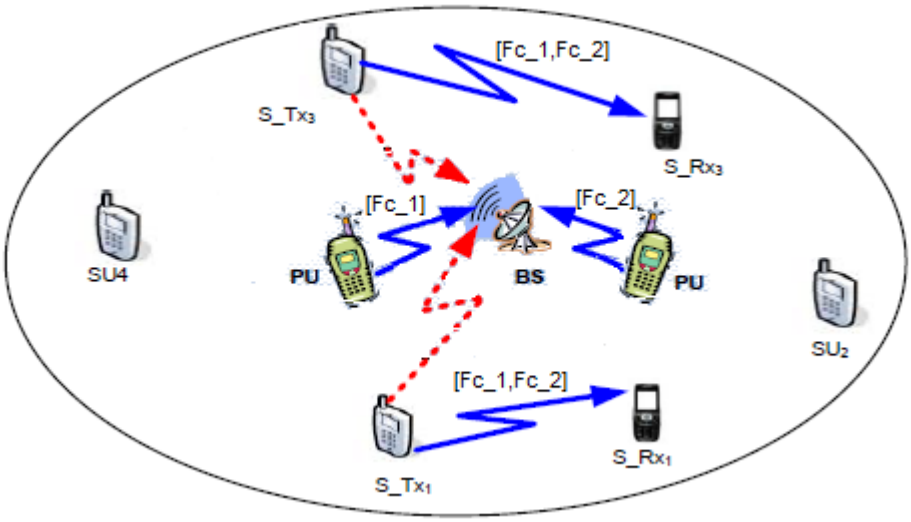


Figure 2.7: Practical cognitive system, with two primary users in the vicinity. The two primary users communicates with a base station on two different carrier frequencies, while the cognitive users can use both carrier frequencies to communicate with their respective links [13].

subject to

$$\sum_{k=1}^{M} P_l(k) \leq P_l \tag{2.31}$$

$$P_{\text{out}} = Prob\{C_{\text{pu}} \leq R_{\text{pu}} | R_{\text{pu}}, q\} \leq q \tag{2.32}$$

where $a_{xy}(k)$ is the channel gain between cognitive sender x and receiver y, $a_{Pu,i}(k)$ is the channel gain between primary sender and cognitive receiver i on channel k and $P_{Pu}(k)$ is the primary user's transmit power on channel k.

It may be tempting to suggest that the solution to this problem lies in simply increasing the transmit-power level of each cognitive sender until either the power constraint or interference constraint is met. However, increasing the transmit-power level of any one sender has the undesirable effect of also increasing the level of interference to which the receivers of all the other senders are subjected. Thus, in reality it is not possible to represent the overall system performance with a single index of performance. Instead performance must be evaluated with different tradeoffs, e.g. is it important to achieve a sense of

fairness in the achievable rates or should one cognitive sender be allowed to be greedy [15].

Given an iterative power allocation algorithm, such as the modified water filling algorithm described in Section 2.4, one can see from (2.32) that if all cognitive users are greedy, i.e. only want to maximize their own rate, the first user in the iterative algorithm would use as much power as possible, until either the power constraint or outage probability is reached. With a high power limit, no other cognitive user would be able to transmit without violating the outage probability.

In this thesis two protocols are presented for signaling in Phase 2. *Protocol 1* tries to allocate resources fairly, so that each cognitive user achieves its desired performance. *Protocol 2* only wants to maximize the sum rate regardless of how it affects each cognitive user. For both protocols, capacity in Phase 1 is given as

$$C^{(1)} = \sum_{l=1}^{n}\sum_{k=1}^{m}(1-p_k)\frac{1}{2}\log_2(1+\frac{a_{ll}^2(k)P_l(k)}{N+\sum_{j=1,j\neq l}^{n}a_{jl}^2(k)P_j(k)}) \tag{2.33}$$

subject to

$$\sum_{k=1}^{m}P_l(k) \leq P_l, l=1,...,n, P_i(k) \geq 0, \forall i,k \tag{2.34}$$

and both protocols optimally allocate power as described in Section 2.4.

In Protocol 1, it is assumed each cognitive user has a desirable rate it wants to achieve. Those users that have achieved their desired rate after Phase 1, will not continue to transmit in Phase 2. In Phase 2 power allocation is initially done with the modified water filling algorithm as in Phase 1 with the added interference from primary users and interference constraint. But those that continue to signal in Phase 2 only allocates enough power to achieve its desired rate, thus increasing the allowed power on the channels for those cognitive users with higher desired rates. The algorithm for Protocol 1 is given in Algorithm 2.

In Protocol 2, the system only wants to maximize the overall capacity. Ideally, the goal would be to find a global solution to this problem. But unfortunately, finding this global solution would require an exhaustive search through the space of all possible power allocations, in which case the computational complexity needed for attaining the global maximum assumes a prohibitively high level.

(2.30) states that we want to maximize the rate over all subbands and users. Given m subbands, if we are able to maximize rate in each of these subbands, the sum rate over these subbands will also be maximized. To maximize rate in a given subband, we simplify the problem by assuming that a cognitive user either transmits with all its power, only constrained by the power constraint, or does not transmit at all. This is known as binary power control and as stated in [13], using binary power control only leads to a negligible capacity loss and is in fact optimal in the low $SINR$ region (where $\log(1 + SINR) \approx SINR$) [17].

The goal of binary power control is to find the maximum number of cognitive users that are allowed to transmit, denoted N^*, for which

$$C(P_1, \ldots, P_{N^*-1}) < C(P_1, \ldots, P_{N^*}) > C(P_1, \ldots, P_{N^*+1}). \tag{2.35}$$

[13] found that user l is allowed to transmit if:

In the low $SINR$ region

$$\begin{aligned} &SINR_l = \frac{a_{ll}^2(k)P_l(k)}{N + \sum_{j=1, j\neq l}^{n} a_{jl}^2(k)P_j(k) + a_{Pu,l}^2(k)P_{Pu}(k)} > 1 \\ &\text{given that} \\ &P_{\text{out}} = Prob\{C_{\text{pu}} \leq R_{\text{pu}} | R_{\text{pu}}, q\} \leq q \end{aligned} \tag{2.36}$$

In the high $SINR$ region

$$\begin{aligned} &SINR_l = \frac{a_{ll}^2(k)P_l(k)}{N + \sum_{j=1, j\neq l}^{n} a_{jl}^2(k)P_j(k) + a_{Pu,l}^2(k)P_{Pu}(k)} > e \\ &\text{given that} \\ &P_{\text{out}} = Prob\{C_{\text{pu}} \leq R_{\text{pu}} | R_{\text{pu}}, q\} \leq q \end{aligned} \tag{2.37}$$

Thus, N^* is the number of users where for $l = N^* + 1$ the criteria in order to transmit is not met. The algorithm for Protocol 2 is shown in Algorithm 3.

Algorithm 2 Protocol 1

Assume an initial power allocation is defined for all channels and all users (e.g. equal power).

for each user l **do**
  Do modified water filling
  Calculate rate
  if rate $<$ desired rate **then**
    increase power until desired rate is achieved or power constraint is met
  end if
  if rate $>$ desired rate **then**
    decrease power intil desired rate is achieved
  end if
  Check outage probability
  if $P\{C_{pu} < R_{pu}\} > q$ **then**
    decrease power until outage constraint is met
  else
    Done
  end if
end for

Algorithm 3 Protocol 2

$P_l = P_{max} \forall l$
for each subband k **do**
  for each user l **do**
    in high SINR regime:
    if $SINR_l < e$ **then**
      $p_l = 0$
    end if
    in low SINR regime:
    if $SINR_l < 1$ **then**
      $p_l = 0$
    end if
    Check outage probability
    if $P\{C_{pu} < R_{pu}\} > q$ **then**
      $p_l = 0$
    else
      Done
    end if
  end for
end for

Chapter 3

Method and Implementation

To investigate the possible performance gains of interference management and power allocation schemes presented in Chapter 2, one of the goals of this thesis is to simulate different environments where these strategies are applicable. This Chapter will explain the implementation of the different systems, as well as important parameters. Since real systems and environments are complex and hard to simulate, simplifications had to be done and these will be explained.

All implementation of code and simulations were done in MATLAB. Built-in MATLAB functions used, that are not trivial, will be noted.

3.1 Dirty Paper Coding

Section 2.2 showed the maximum achievable rate for a cognitive user, given that the performance of the primary user was not degraded. The proof of achievability of (2.4) involved capacity achieving codes for the primary user and Costa's dirty paper coding scheme for the cognitive user. Creating capacity achieving codes is in it self almost impossible and the codes that are closest to achieve capacity today are turbo codes, if Joint-Source Channel Coding is not considered. Turbo codes, or other high performing codes, are very complex and were thus not considered implementing in this thesis.

One way to implement dirty paper coding is a coding technique known as Tomlinson-Harashima Precoding (THP) [26]. This was originally designed to remove the effect of inter-symbol interference, but has recently been investigated for broadcast channels to combat interference [24]. The basic concept of THP is shown in Figure 3.1. The intended signal is denoted U and the

interfering signal is denoted S. Since S is known at the transmitter, in order to convey the intended signal U, the transmitter may send $U' = U - S$ to compensate for the interference of S. However, if $|S|$ is large, the power to transmit U' may violate the power constraint.

Given that U is in a finite interval, the power to transmit U' is constrained by applying the modulo operation to U' and transmitting X, the output of the modulo operation. Thus, setting $X = U' \mod \Delta$, X is uniformly distributed $\in [-\frac{\Delta}{2}, \frac{\Delta}{2}]$, if S is Gaussian with large enough power. As a consequence of the modulo operator all symbols that differ by an integer multiple of Δ are considered to be the same symbol. To reconstruct the originally intended signal U, the same modulo operation is done at the receiver.

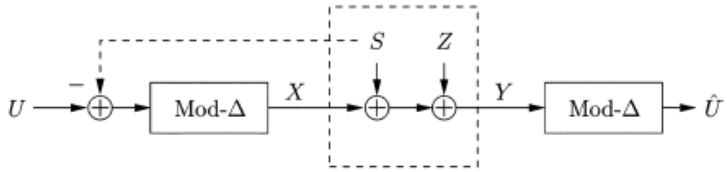


Figure 3.1: Principle of THP. U is the intended message, S is the known interference and X is the symbol transmitted on the channel. Z is additive, white, Gaussian noise, Y is the received symbol and $\hat{U}$ is the reconstructed message [27].

The goal of the original paper [6] and the goal of THP are to minimize the effect of interference to maximize the rate over the channel. However, in this setting the effect of the coding on the performance of another system was not considered. As this is the case in the cognitive system, the precoding has to be modified as described in the proof of (2.4) to minimize interference from the cognitive user to the primary receiver.

To review the effect of using THP, the genie-aided cognitive system was implemented with a cognitive sender transmitting M-PAM signals. For simplicity the primary sender also used M-PAM signaling. Further it is assumed that U, the intended signal at the cognitive sender, is equiprobable. Note that since both the primary message and cognitive message is M-PAM signals, X as described above will not be uniformly distributed between $[-\frac{\Delta}{2}, \frac{\Delta}{2}]$ and hence the average power constraint will not be met. To achieve this distribution a dither variable that has a uniform PDF is introduced without any consequence for performance. This variable has to be known both at the sender and receiver.

As mentioned above, TH precoding has to be modified to limit the effect of interference at the primary receiver, according to Section 2.2. The transmitted signal $X_c = \hat{X}_c + \sqrt{\alpha P_c / P_p} X_p$, where α was given in (2.7). $\hat{X}_c$ is the intended signal from the cognitive sender and is precoded as described above with $(b + \sqrt{\frac{\alpha P_c}{P_p}}) X_p^n$ as the known interference. Given that the average power constraint at the cognitive sender is P_c and remembering that $\hat{X}_c$ and X_p are independent, we get:

$$\begin{aligned} P_c = E[X_c^2] &= E[(\hat{X}_c + \sqrt{\frac{\alpha P_c}{P_p}} X_p)^2] && (3.1) \\ &= E[\hat{X}_c^2] + \frac{\alpha P_c}{P_p} E[X_p^2] && (3.2) \\ &= E[\hat{X}_c^2] + \frac{\alpha P_c}{P_p} P_p && (3.3) \\ &= E[\hat{X}_c^2] + \alpha P_c && (3.4) \\ E[\hat{X}_c^2] &= (1 - \alpha) P_c && (3.5) \end{aligned}$$

The signal output from the TH precoder, $\hat{X}_c$, is uniformly distributed between $[-\frac{\Delta}{2}, \frac{\Delta}{2}]$. The average power is then

$$\begin{aligned} E[\hat{X}_c^2] &= \int_{\frac{\Delta}{2}}^{\frac{\Delta}{2}} \frac{1}{\Delta} x^2 dx \\ &= \frac{1}{\Delta} \left[2 \frac{x^3}{3} \right]_0^{\frac{\Delta}{2}} \\ &= \frac{\Delta^2}{12} \end{aligned} \qquad (3.6)$$

Setting this equal to (3.5), Δ is found to be $\Delta = \sqrt{12(1-\alpha)P_c}$.

As mentioned above, all symbols that differ by an integer multiple of Δ will be regarded as the same symbol. Therefore all intended symbols must be within $[-\frac{\Delta}{2}, \frac{\Delta}{2}]$ to achieve distinguishability. Then, to minimize the effect of noise, the distance between each symbol should be maximized, and is given by Δ/M. The M-PAM constellation is then given by

$[\frac{(-M+1)\Delta}{2M}, \frac{(-M+3)\Delta}{2M}, ..., \frac{(M-3)\Delta}{2M}, \frac{(M-1)\Delta}{2M}]$.

3.2 Maximum Rate Evaluation

The goal of the previous Section was to show that given the genie-aided information, one can in fact precode a signal at the cognitive sender that eliminates interference from the cognitive sender at the primary receiver. However, the goal of this thesis is not to come up with a code that achieves capacity (2.4), but assuming such a code exists does simultaneous transmission provide any performance gain over a system only signaling when the channels are idle.

To investigate the performance gain of using interference management, comparison between the capacity of a cognitive user transmitting over both Phase 1 and Phase 2 versus a cognitive user only transmitting in Phase 1 was done. As given in Section 2.5, Phase 1 lasts for a fraction $(1-p)$ of the time and Phase 2 lasts for a fraction p of the time. Denoting the power of Phase 1 P_1 and power of Phase 2 P_2, the overall power constraint is $(1-p)P_1 + pP_2 \leq P$, where P is the total transmit power. Introducing a new parameter t, the power of each phase can be rewritten as $P_1 = \frac{Pt}{(1-p)}$ and $P_2 = \frac{P(1-t)}{p}$. Thus the power constraint can be rewritten as $tP+(1-t)P \leq P$, and t is thus a parameter that describes how much power is put in the different phases. Then the capacity of cognitive user l over both Phase 1 and Phase 2 is given as:

$$C_l = \max_{t_l \in [0,1]} \left\{ C_l^{(1)}(t_l) + C_l^{(2)}(1-t_l) \right\}. \tag{3.7}$$

With only one cognitive user and one primary user in the environment the capacity of Phase 1 is given by the well known Shannon capacity formula. Capacity of Phase 2 is then given by (2.4) in the genie-aided cognitive system and by (2.12) in the causal cognitive system. With only one cognitive user and one primary user, the optimal t in the genie-aided case is given as:

$$t^{(g)} = \frac{(1-\alpha)P/p + \alpha}{(1-\alpha)P(1/p + 1/(1-p))} \tag{3.8}$$

and the optimal t in the causal case is given as:

$$t^{(c)} = \frac{(1-\alpha)P/((1-\beta)p) + \alpha}{(1-\alpha)P(1/((1-\beta)p) + 1/(1-p))}. \tag{3.9}$$

A detailed calculation of $t^{(g)}$ and $t^{(c)}$ is given in Appendix F.

In scenarios with multiple cognitive users, the performance is evaluated as the total rate achieved by the system, i.e. the sum rate of all users. In the

practical cognitive system, two protocols were presented to evaluate performance. Protocol 1 had as a goal to satisfy as many of the cognitive users as possible, whereas Protocol 2 had as a goal to maximize the total rate. It would seem intuitive that finding the optimal power allocation between the two phases should be done for these two protocols as well. However, with multiple cognitive users, a given cognitive user does not know what rate it can achieve in Phase 2 due to the unknown interference from the other users. Thus for the two protocols in the practical cognitive system $P_1 = P_2$.

3.3 Power Allocation

In the genie-aided cognitive system and the causal cognitive system, there is a problem of power allocation due to the two phases. As described in the previous Section, the problem is to find the correct t, which is the percentage of power to be used in Phase 1 and Phase 2. The optimal t for the genie-aided and causal cognitive system was given in (3.8) and (3.9), respectively.

The different power allocation schemes for the n cognitive users, m channels environment were implemented in MATLAB according to the different algorithms described in Section 2.4. In Section 2.4, four power allocation schemes were presented. The first is for user l to allocate all power to the channel with the best $SINR$. This procedure is referred to as *channel allocation*. When the number of channels is equal to or exceeds the number of users, it is assumed all users use different channels in channel allocation. I.e. there is no interference on the used channels.

The second is for user l to distribute all its power equally across all channels, from now on referred to as *equal power allocation*. The third is water filling according to (2.19) and (2.20), and will from now be referred to as *water filling*. The fourth is the modified water filling calculated from (2.26) and given step by step in Section 2.4. This procedure is referred to as *modified water filling*.

In the implementation of the modified water filling algorithm, a few problems were encountered. This algorithm tries to find the optimum power allocation through iterations and in some channels this can be channel allocation or equal power. If the algorithm finds that channel allocation is the optimum allocation, it is necessary that each user allocates all its power in different channels to avoid interference and maximize rate. This was not guaranteed by Algorithm 1. Therefore a MATLAB function was written to ensure that in the case of channel allocation, the channel was not used by someone else.

A more realistic way of obtaining $B_l(k)$ was given at the end of Section 2.4. This $\hat{B}_l(k)$ depends on an average of channel gains. In the implementation of this estimate, $a_{\text{avg}}(k)$ was computed as the average of the channel gains on the links to cognitive receiver l. The power allocation algorithm using this estimate of B is referred to as *cognitive modified water filling*.

[3] also simulated an algorithm using an estimated value of B. [3] argued that $B_l(k)$ given in (2.24) is most sensitive to the product of $a_{li}(k)$ and $P_i(k)$, since with increasing number of users the sum of interferences becomes constant and independent of which user one computes the interference for. It was further assumed that all channel gains where known to user l, all users use a feedback channel, that channel gains are the same on the feedback channel as on the feed forward channel and that the power allocation on the feedback channel is the same as on the feed forward. Then B can be estimated from the interference received during the return transmission. [3] found that $B_l(k)$ can be estimated by multiplying $I_l(k)$ by $1/36$, based on estimates of the average signal to interference plus noise ratio in each channel at 10 dB SNR. Power allocation using this estimate for $B_l(k)$ is referred to as *cognitive modified water filling from Burr*.

For Protocol 1 in the practical cognitive system, the modified water filling algorithm had to be modified to account for the added probability outage constraint. In Protocol 1 each user first performs the steps of modified water filling algorithm as given in Section 2.4. Since Protocol 1 tries to satisfy as many cognitive users as possible, a cognitive user only transmits with enough power to reach its desired rate. Thus if a user initially used more power than necessary, reducing the rate was done be decreasing the power in each channel equally until the desired rate was met. If, however, the achieved rate is less than the desired rate, power was increased equally in all channels until either the desired rate was achieved, or the power constraint was met or the outage probability constraint was violated.

Protocol 2 in the practical cognitive system tries to maximize the rate of a system using binary power control limited by the outage probability at the primary user. However, the algorithm for Protocol 2 given in Algorithm 3, can in fact be greatly improved. Let us assume that there are 6 cognitive users spread out through the perimeter and that the outage constraint is such that only 3 of them are allowed to transmit. Going trough the steps of Algorithm 3 may lead the first 3 users to transmit and the last 3 to be silent. But if the 3 allowed to transmit are close to each other, the sum rate would be improved if

those 3 farthest from each other were allowed to transmit instead. Thus in the implementation of Protocol 2, implementation of optimal user selection (OS) was done to compare to the regular algorithm of Protocol 2.

3.4 Environments and Channels

The environments and channels implemented for the different cognitive systems in this thesis were chosen for different reasons. When simulating systems, ideally one would want to use models that resemble reality as close as possible, but this is often very difficult. Also, over the years many standard models have been used in different studies so that one are able to compare ones own results to those obtained by others. Therefore, standard environmental models and channels have been used for the different cognitive systems in this thesis so that the results obtained can be compared to the work of others.

Genie-aided and Causal Cognitive System

For the genie-aided cognitive system, with one primary and one cognitive user, the simple Gaussian interference channel shown in Figure 2.4 models the environment. The difference between the genie-aided cognitive system and the causal cognitive system, is that there is a channel between the primary sender and cognitive sender. In this thesis a simplified path loss model is used to obtain the channel gain between the primary sender and cognitive sender, and is given as:

$$G = \frac{\sqrt{K}}{d} = \frac{\lambda}{4\pi}\frac{1}{d} \propto \frac{1}{d}, \tag{3.10}$$

where λ is the wavelength and d is the distance from sender to transmitter [12]. Remembering (2.11), β is given as:

$$\begin{aligned} \beta &= \frac{\log(1+G_{PS-PR}^2 P)}{\log(1+G_{PS-CS}^2 P)} \\ &= \frac{\log(1+\frac{\lambda^2}{(4\pi)^2}\frac{1}{d_1^2}P)}{\log(1+\frac{\lambda^2}{(4\pi)^2}\frac{1}{d_2^2}P)} \end{aligned} \tag{3.11}$$

Since $\lim_{x\to 0} \log(1+x) \approx x$

$$\begin{aligned} \beta &\approx \frac{\frac{\lambda^2}{(4\pi)^2}\frac{1}{d_1^2}P}{\frac{\lambda^2}{(4\pi)^2}\frac{1}{d_2^2}P} \\ &= \frac{d_2^2}{d_1^2} \\ &= \frac{\log(1+P/d_1^2)}{\log(1+P/d_2^2)} \end{aligned} \tag{3.12}$$

thus the channel gain is only given as $G = \frac{1}{d}$. The above approximation holds for $P \leq 20$ dB, with a power above 20 dB, the carrier frequency has to be accounted for.

As mentioned in Section 2.3, it is assumed that the cognitive sender is closer to the primary sender than the primary receiver is to the primary sender. In this thesis the distances are normalized so that the distance from the primary sender to the primary receiver is $d_1 = 1$, and the distance between the primary sender and cognitive sender is $0 \leq d_2 \leq 1$.

n-users, m-channels

For n cognitive users and m channels, three different channel gain matrices was used to evaluate the power allocation algorithms. These channel gain matrices will be referred to as Channel 1, Channel 2 and Channel 3. Channel 1 is defined as follows: for all wanted links the channel gain is unity ($a_{ii}(k) = 1$) and for all interfering links the channel gain is a ($a_{ji}(k) = a, \forall i \neq j$).

Channel 2 is defined as follows:

$$a_{ji}^2(k) = \begin{cases} \text{-7 dB} & \text{if } i \neq j \\ \text{-1 dB} & \text{if } i = j, \text{ but } i \neq k \\ \text{0 dB} & \text{if } i = j = k \end{cases} \tag{3.13}$$

Thus all interfering links have channel gains of -7 dB ($a_{ji}(k) \approx 0.2$) and all users have one link which as a channel gain of unity and all other links have channel gains of -1 dB.

Channel 3 is defined as follows:

$$[a_{ij}^2(k), i, j = 1, \ldots, 4] = \begin{bmatrix} 1 & 0.25 & 0.0625 & 0.0156 \\ 0.25 & 1 & 0.25 & 0.0625 \\ 0.0625 & 0.25 & 1 & 0.25 \\ 0.0156 & 0.0625 & 0.25 & 1 \end{bmatrix} \quad k = 1, \ldots, 4. \tag{3.14}$$

I.e. interfering links are -6, -12 or -18 dB. These channels were chosen in the implementation simply because the different power allocation schemes perform differently in these channels. Also, two of these were also used in [3], thus making it easy to compare performance.

Practical Cognitive System

For the practical cognitive system the environment and channel gains were modeled more realistically. Figure 3.2 show the outline of the environment considered in 1 subband, with a hexagonal cellular system functioning at 1800 MHz and a cell radius of 800 meters. The cognitive users are assumed to have positions around a circle with a distance of 600 meters to the base station.

The channel gains are based on the COST-231 path loss model [1]. More specifically the path loss is given as

$$\begin{aligned} PL(dB) &= 46.3 + 33.9\log_{10}(Fc) - 13.82\log_{10}(h_b) - a(h_m) \\ &+ (44.9 - 6.55\log_{10}(h_b))\log_{10}(d) + C \qquad (3.15) \\ a(h_m) &= (1.1\log_{10}(Fc) - 0.7)h_m - (1.56\log_{10}(Fc) - 0.8) \qquad (3.16) \\ C &= \begin{cases} 0\text{dB} & \text{for medium cities and suburban areas} \\ 3\text{dB} & \text{for metropolitan cities} \end{cases} \qquad (3.17) \end{aligned}$$

where d is the distance from sender to receiver in km, F_c is the carrier frequency in MHz, h_b is the height of the receiver and h_m is the height of the sender in meters. In this thesis h_b is sat to 10 meters and h_m is sat to 2 meters. It is assumed a medium sized city, thus $C = 0$ dB.

For the sake of simplicity it is assumed all cognitive senders and receivers are stationary, thus no fading is considered. But due to the surroundings in a city, shadowing is considered. The shadowing considered is assumed to be log-normal shadowing with standard deviation of 10 dB. Thus the received $SINR$ of any given cognitive user can be written as

$$SINR(dB) = P_t(dB) - PL(dB) - X(dB) - 10\log_{10}(N + I) \tag{3.18}$$

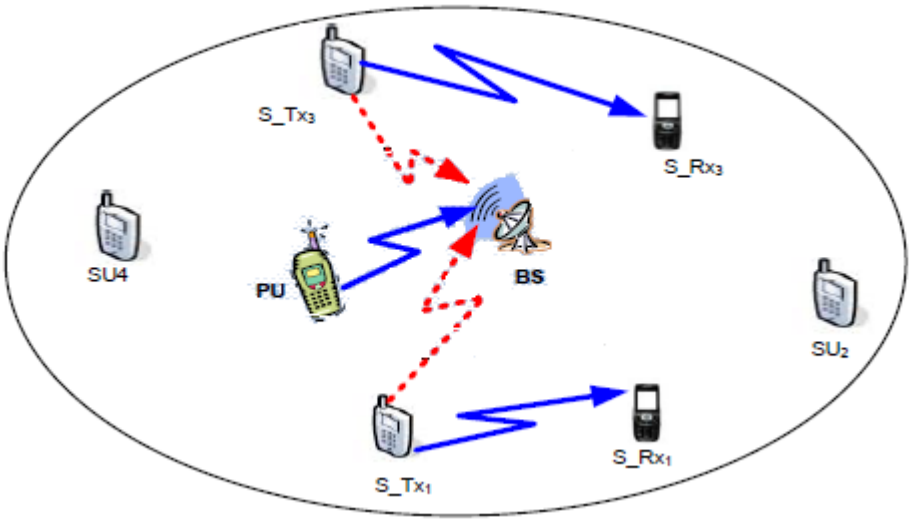


Figure 3.2: Practical cognitive system with 1 subband, with one primary users in the vicinity. The primary user communicates with a base station while the cognitive users tries to communicate with their respective receivers [13].

where P_t is the transmit power, X is shadowing, N is additive white Gaussian noise and I is interference. N is in this thesis given as

$$N = 4k_bTB \tag{3.19}$$

where k_b is Boltzmann's constant, T is the temperature in Kelvin and B is the bandwidth. In this thesis it is assumed $T = 290$ and $B = 1$ MHz.

3.5 Primary user QoS issues

In both the genie-aided and causal cognitive system, primary users QoS requirements are ensured by superposition coding, where a percentage α of the cognitive user's power is used to relay the primary user's message, guaranteeing no degradation in primary user performance. In the practical cognitive system there is no superposition coding, and primary user QoS is guaranteed by means of an outage probability constraint. The outage probability constraint is given in (2.29) and can be written as

$$P_{\text{out}} = Prob\left\{\log_2\left(1 + \frac{a_{pu,pu}^2 P_{pu}}{N + \sum_{l=1}^{N^*} P_l a_{l,pu}^2}\right) \leq R_{pu}\right\} \leq q \tag{3.20}$$

Now we introduce the primary user average channel gain estimate G_{pu} based on the following decomposition:

$$a_{pu,pu} \triangleq G_{pu} a'_{pu,pu} \tag{3.21}$$

where $a'_{pu,pu}$ is the random component of the channel gain and represents the normalized channel impulse response tap. In a dense network the interference experienced by any user is only weakly dependent on the users position [13], thus the interference from the cognitive users at the primary user's base station can be approximated as

$$\sum_{l=1}^{N^*} P_l a_{l,pu}^2 \backsimeq G_{su}^2 \sum_{l=1}^{N^*} P_l \tag{3.22}$$

where N^* is the number of active cognitive users. Now, the outage probability can be estimated as

$$\begin{aligned} P_{\text{out}} &\backsimeq Prob\left\{ \frac{P_{pu} G_{pu}^2 a_{pu,pu}^{'2}}{N + G_{su}^2 \sum_{l=1}^{N^*} P_l} \leq 2^{R_{pu}} - 1 \right\} \leq q \\ &\backsimeq Prob\left\{ a_{pu,pu}^{'2} \leq (2^{R_{pu}} - 1) \left(\frac{N + G_{su}^2 \sum_{l=1}^{N^*} P_l}{G_{pu}^2 P_{pu}} \right) \right\} \leq q. \end{aligned} \tag{3.23}$$

For simplicity of analysis it is assumed in this thesis that the primary user experiences Rayleigh fading, so that the channel gains are i.i.d. Rayleigh distributed. However, this can readily be translated into results for any other channel model by using the desired probability density function. With a Rayleigh fading distribution we get the following integral for the outage probability

$$P_{\text{out}} \backsimeq \int_0^{(2^{R_{pu}}-1)\left(\frac{N + G_{su}^2 \sum_{l=1}^{N^*} P_l}{G_{pu}^2 P_{pu}} \right)} e^{-t} dt \leq q \tag{3.24}$$

and solving this integral yields the following expression for the outage probability

$$P_{\text{out}} \backsimeq 1 - \exp\left[-(2^{R_{pu}} - 1) \left(\frac{N + G_{su}^2 \sum_{l=1}^{N^*} P_l}{G_{pu}^2 P_{pu}} \right) \right] \leq q. \tag{3.25}$$

Given Protocol 2 for the practical cognitive system, the expression for the outage probability can be further simplified since each active cognitive user transmits with a power equal to P_{max}. Thus, (3.25) can be rewritten as

$$P_{\text{out}} \backsimeq 1 - \exp\left[-(2^{R_{pu}} - 1) \left(\frac{N + G_{su}^2 N^* P_{max}}{G_{pu}^2 P_{pu}} \right) \right] \leq q. \tag{3.26}$$

Now the maximum number of active users in Protocol 2 can be found by solving the equation with respect to N^*, given primary users rate R_p and maximum outage probability q:

$$0 \leq N^* \leq \frac{-\ln(1-q)}{(2^{R_{pu}}-1)} \frac{G_{pu}^2 P_{pu}}{G_{su}^2 P_{max}} - \frac{N}{G_{su}^2 P_{max}} \tag{3.27}$$

where the left hand side of (3.27) prevents from obtaining a negative number of active users. In this thesis the distance from the primary sender to the primary receiver is assumed to be 220 meters and G_{pu}^2 is then given by the path loss model (3.15). With the cognitive users spread around a circle, G_{su}^2 is assumed to be the path loss at a distance of 600 meters.

(3.27) gives a new way for the cognitive users to evaluate their impact on primary user performance. For each cognitive user to evaluate (3.25) with sufficient accuracy means that each cognitive user would have to know the transmit power used by all other active cognitive users, in addition to the primary user's desired rate and outage probability. However, with the binary power control, employed in Protocol 2, a cognitive user only has to know the primary user's desired rate and outage probability and sense how many cognitive users are using the subband. Thus this way of ensuring primary user QoS is more suited for a distributed system, which is always desirable when it comes to designing cognitive networks.

Chapter 4

Results and Discussion

All simulations involve to some extent simplifications and it is therefore important to have this in mind when reviewing the results from simulations. This Chapter will present the results from the simulations done in this thesis and discuss them in light of practicality. As mentioned in Chapter 3 all simulations were done in MATLAB. To verify the results, they will be compared to theoretical expectations and results published in other works.

In the following the words channels and Channel are used. Channel, with a capital C, will refer to the different types of channel gain environments used in the simulations. E.g. Channel 2 is the Channel given by (3.13). Number of channels will be used as the number of different frequency bands that are available to the users.

The observant reader may also notice that some graphs have rates in bits per channel use and some graphs have rates in bits/s/Hz. More specifically results from the Tomlinson-Harashima precoding and genie-aided and causal cognitive systems show rate in terms of bits per channel use, whereas the n-users, m-channels and practical cognitive system show rates given as bits/s/Hz. This is done because the related work of others in the respective areas show rates with these units, and using the same units simplifies comparisons. The difference between bits per channel use and bits/s/Hz is:

$$\begin{aligned} C &= \frac{1}{2}\log_2(1+SINR) \text{ bits per channel use} \\ C &= \log_2(1+SINR) \text{ bits/s/Hz} \end{aligned}$$

4.1 Tomlinson-Harashima Precoding

In the simulation of signaling using THP, it is assumed a $BER \leq 10^{-5}$ and thus the rates plotted are those achievable with $BER \leq 10^{-5}$. All rates are plotted against P_t/N_0 (dB) where P_t is the available transmit power at the sender and N_0 is white noise power. Figure 4.1 shows M-PAM rates with and without THP. As expected, using THP results in a power loss since the transmitted signal X can have more power than the intended signal U (Figure 3.1). This power loss at the transmitter is given as P_X/P_U, which on average for a M-PAM constellation is $M^2/(M^2-1)$ [27]. Thus the power loss decreases with increasing constellation size, and for 2-PAM the power loss is $\frac{4}{3} = 1.333$. In Figure 4.1 2-PAM reaches a rate of 1 bit per channel use at $P_t/N_0 = 12.6$ dB which also yield a $SNR = 12.6$ dB. 2-PAM with THP reaches 1 bit per channel use at $P_t/N_0 = 13.85$ dB, yielding a $SNR = 12.6$ dB, which corresponds to a power loss of 1.333 and is exactly as expected by theory.

Figure 4.2 shows again 2-PAM rates with and without THP. In addition the rate of a primary user using 2-PAM and fixed power at 10 dB in a channel with interference from the cognitive user using THP is plotted. The channel is that given in Figure 2.1, with $a = b = 0.5$. With increasing power at the cognitive user it is clear that the performance of the primary user decreases since the interference increases, whereas the cognitive user only suffers from the power loss of using THP.

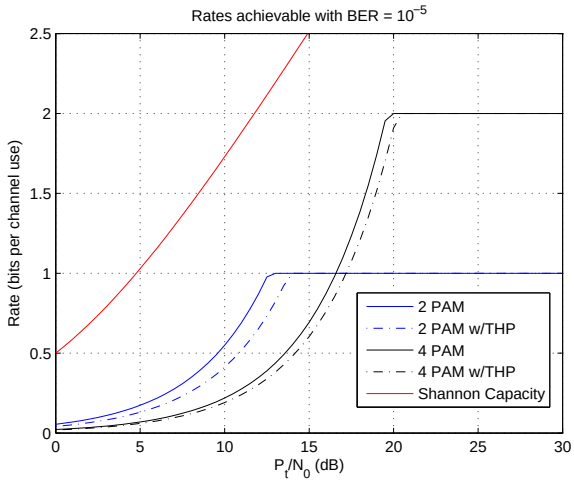


Figure 4.1: Rate of M-PAM signaling with and without THP. Shannon capacity as reference.

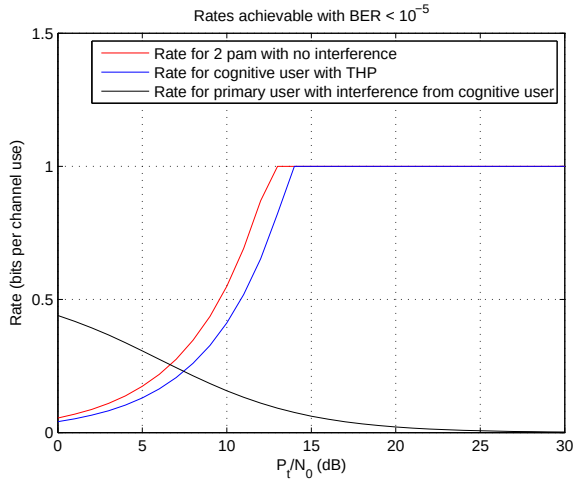


Figure 4.2: Rate of 2 PAM signaling with THP, but no interference cancellation.

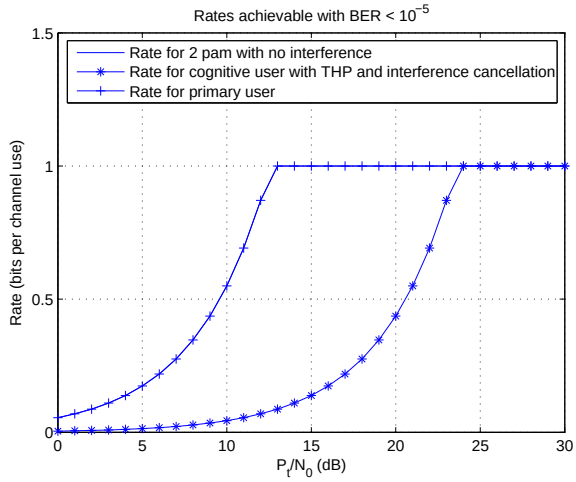


Figure 4.3: Rate of 2 PAM signaling with THP and interference cancellation.

In Figure 4.3 performance in the same scenario as in Figure 4.2 is plotted. The difference is that the cognitive user is now employing THP with interference cancellation so that its signaling does not affect the primary user. As can be seen, the primary user now performs as if the cognitive user was absent all together, whereas the cognitive user suffers from an additional power loss due to the interference cancellation. This power loss depends on α (2.7) which depends on the channel parameters, transmit power at the primary user and

transmit power at the cognitive user. In this plot the channel is that of Figure 2.1 with $a = b = 0.5$ and the primary user transmitted at 12.6 dB which is the necessary power to transmit at $BER = 10^{-5}$ with 2-PAM and no interference.

[27] simulated the same TH precoding for a user using M-PAM constellation and experiencing known interference. The result from Figure 4.1 was compared and verified to that of [27]. However, to the author's knowledge no other work has implemented superposition coding with TH precoding to avoid interference at the primary receiver.

The simulation results show that THP can be used to achieve simultaneous signaling between a primary and cognitive user, although at a severe power penalty at the cognitive sender. In [27] modified trellis codes and convolutional codes have been shown to decrease the power loss compared to THP, which is due to the modulo operation and shaping loss of the M-PAM constellation. But the fact that the cognitive user has to limit its interference on the primary user and thus use a portion α of its power to transmit the primary message is the main cause of the power penalty.

In the simulations, a primary user using only M-PAM signaling was considered. Using the same TH precoding for a primary user using any other modulation scheme, such as FSK or QAM, would in essence be the same, because when the cognitive user uses a portion α of its power (2.5) to transmit the primary signal, it ensures that the primary user exhibits no degradation in performance. However, the TH precoding would have be modified so that the cognitive user does not experience any interference from the primary user. This simplifies if the two uses the same modulation technique.

Therefore, in light of practicality, THP seems suited as an implementation of Dirty Paper Coding since the precoding is independent of the nature of the interference. Thus adding other primary and cognitive users in the environment does not affect performance, only computational complexity and requirements to have sufficient channel knowledge.

4.2 Genie-aided and Causal Cognitive Performance

Below, the simulations of the maximum rate evaluation of the genie-aided and causal cognitive systems are presented. The rates are given as described in Section 3.2, with a channel between the primary user and cognitive user as seen in Figure 2.5. For the causal cognitive system the channel gain between the

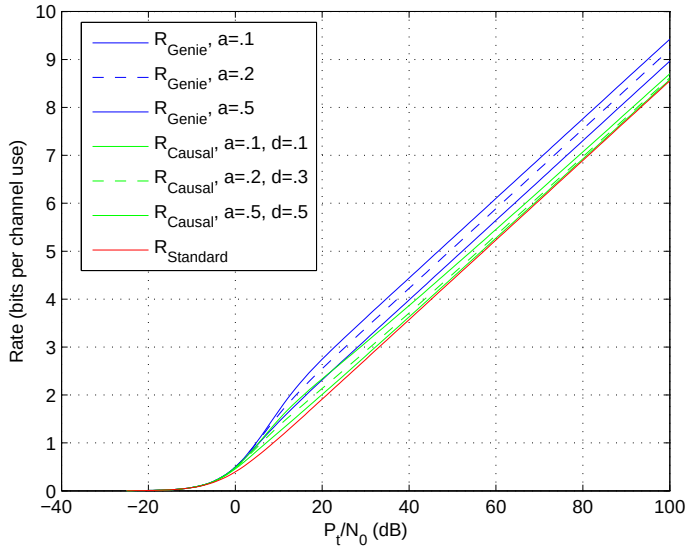


Figure 4.4: Rate vs. P_t/N_0 (dB), where P_t is the total transmit power at the transmitter and N_0 is white noise power, for different cognitive systems. $p = 0.5$.

primary sender and cognitive sender also affects performance. As described in Section 3.4, this depends on the distance between the two. With a normalized gain between the primary sender and receiver equal to unity, this distance is $d \leq 1$, and the channel gain between the primary sender and cognitive receiver is d^{-2}. Note that with the channel parameter $a \leq 1$, performance is independent of the channel parameter b.

Figure 4.4-4.7 shows the rate of genie-aided cognitive radio and causal cognitive radio referenced against a standard cognitive radio. The standard cognitive radio transmits only when the channel is idle, which in Figure 4.4-4.6 is half the time and in Figure 4.7 is 10% of the time. In Figure 4.4 the primary user is set to transmit at the same power as the cognitive user, whereas in Figure 4.5 the primary user has a set power of $P_t/N_0 = 10$ dB. In Figure 4.6 performance of the cognitive systems are referenced against the performance of a MIMO system and performance given by the interference channel.

From these simulations, it is clear that simultaneous transmission between a cognitive and primary user does provide a certain performance gain. The gain does of course depend heavily on the amount of time the primary user is active on the channel. If the primary user is almost absent, i.e. low p, then

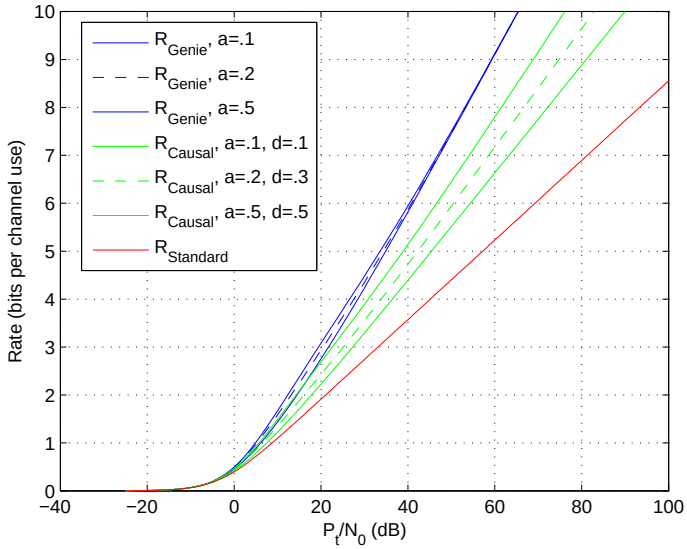


Figure 4.5: Rate vs. P_t/N_0 (dB), where P_t is the total transmit power at the transmitter and N_0 is white noise power, for different cognitive systems. $p = 0.5$ and primary power is sat to 10 dB.

the genie-aided and causal cognitive systems reduce to the standard cognitive system (only transmits when the channel is idle) due to the power constraint. With increasing p (Figure 4.7), the gain provided by the genie-aided and causal cognitive systems increases, as would be expected.

The gain also depends on the channel parameter a, which is the gain between the cognitive sender and primary receiver. Clearly, with a small a, α in (2.7) is also small and thus less power has to be used to transmit the primary signal at the cognitive sender.

In the low transmit power region there is negligible performance difference between the different cognitive systems. In fact, it can be proven that in the low P_t/N_0 regime there is no cognitive transmission scheme that satisfies Definition 2.1 and 2.2 that outperforms the standard cognitive system. Since the primary user is completely oblivious to the presence of the cognitive user, removing it from the environment can only increase the cognitive performance. Assuming still that $p \neq 0$, the standard cognitive system will only transmit during Phase 1 and achieve a rate of $R^S = (1-p)\frac{1}{2}\log_2(1+\frac{P}{1-p})$. The cognitive system that is able to transmit in both phases will now perform power

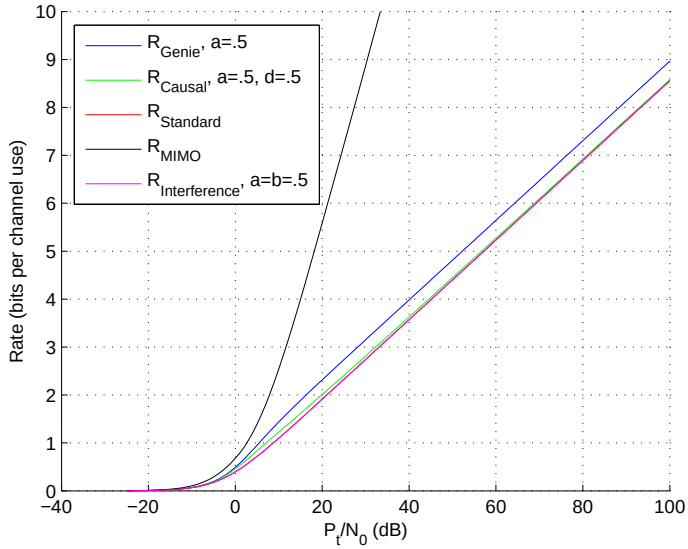


Figure 4.6: Rate vs. P_t/N_0 (dB), where P_t is the total transmit power at the transmitter and N_0 is white noise power, for different cognitive systems. $p = 0.5$

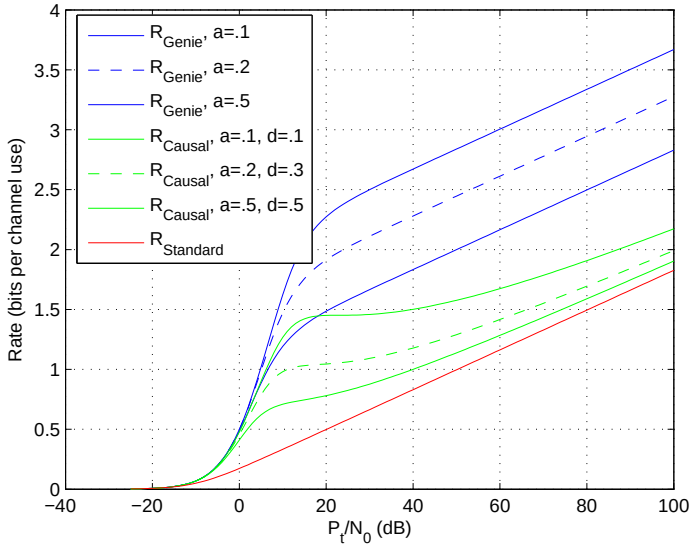


Figure 4.7: Rate vs. P_t/N_0 (dB), where P_t is the total transmit power at the transmitter and N_0 is white noise power, for different cognitive systems. $p = 0.9$

allocation between the two phases, but since the primary user is absent the rate functions are the same, $R = (1-p)\frac{1}{2}\log_2(1+\frac{Pt}{(1-p)}) + p\frac{1}{2}\log_2(1+\frac{P(1-t)}{p})$. As $P \to 0$ we get

$$\begin{aligned}\lim_{P\to 0} R - R^S = \lim_{P\to 0} & \frac{(1-p)}{2}\log_2(1+\frac{Pt}{(1-p)}) + \frac{p}{2}\log_2(1+\frac{P(1-t)}{p}) \\ & - \frac{(1-p)}{2}\log_2(1+\frac{P}{(1-p)})\end{aligned} \tag{4.1}$$

$$= \lim_{P\to 0} \frac{(1-p)}{2}\frac{Pt}{(1-p)} + \frac{p}{2}\frac{P(1-t)}{p} - \frac{(1-p)}{2}\frac{P}{(1-p)} = 0, \tag{4.2}$$

where we have used the fact that $\lim_{x\to 0}\log(1+x) = x$.

When the primary sender's power is high, $P_p \to \infty$, there are two factors that limit performance in the genie-aided and causal cognitive systems compared to the standard cognitive systems. The first is that the fraction of power allocated to the cognitive signal, $1-\alpha$, diminishes as $P_p \to \infty$. The second is that for the causal cognitive system, $\beta \to 1$ as $P_p \to \infty$. When the primary sender's power is constant, β is of course also constant. Also the fraction of power allocated to the cognitive signal at the cognitive sender, $1-\alpha \to 1 - \frac{P_p}{1+P_p}$ as $P_c \to \infty$ when P_p is constant. Proof of these limits are given in Appendix D

Figure 4.4 and 4.5 shows the rate of the cognitive systems when the primary sender's power equals the cognitive power and when the primary sender's power is constant, respectively. In the first of the two figures the causal cognitive gain over the standard cognitive system diminishes with increasing P_t/N_0. In the latter cognitive gain over the standard cognitive system increases with increasing P_t/N_0.

In Section 2.2 it was noted that performance would be lower and upper bounded by performance given by the interference channel and MIMO channel, respectively. This is shown in Figure 4.6. Due to the power allocation between the two phases, the power allocated to transmit in Phase 2 with a rate given by (2.3) is zero and thus reduces to the standard cognitive system. As the MIMO rate is given by a system assuming full cooperation between the two users, this bound was expected to be very loose and as is seen by Figure 4.6 it clearly is.

Many other works have studied the one cognitive, one primary user scenario [8][16][22][20][18], where [16] found the capacity achieving scheme when the

channel gain between the cognitive sender and primary receiver is less than 1. However, only [18] studied the two phase power allocation problem, which is necessary to reference the performance against a standard cognitive user that transmits only when the channel is idle. The results obtained in this thesis coincide with those obtained in [18].

An interesting scenario would be to include multiple cognitive users in this environment and see how it affects performance. With multiple cognitive users, it is shown in Appendix B that superposition coding at each cognitive user and an α found independently for each cognitive user, (2.7), does not degrade performance at the primary user. But to minimize interference at the cognitive receiver, the interference at the cognitive receiver due to the other cognitive users must be known at the cognitive sender. This requires a feedback channel from each cognitive receiver to its sender, which can only be used when the channel is idle as not to affect the primary users performance.

By overall review of the results in Figure 4.4 - 4.7, it is clear that for the causal cognitive system to have any significant performance over the standard cognitive system, the time the primary user is active has to be high and there has to be a significantly better channel between the primary sender and cognitive sender than between the primary sender and primary receiver. If this is not the case, i.e. the channel between the primary sender and cognitive sender is equal to or even worse than the channel between the primary sender and primary receiver, a causal cognitive radio can still exploit packet errors, assuming the primary user employs some form of ARQ. But as given in [19], packet error rate can be approximated as:

$$PER = 1 - (1 - BER)^{N_p} \tag{4.3}$$

where BER is bit error rate and N_p is number of bits in one packet. Thus it is unrealistic to think that exploiting these errors will improve performance of a cognitive system significantly.

In this section it has been assumed that all cognitive users have perfect spectral sensing. I.e. they have always known when the channel is idle and when the primary user is using it. This means that for the causal cognitive system, the users know perfectly what is the signal from the primary user that it needs to decode and what is noise and the knowledge is instantaneous, i.e. no processing time. In reality this is of course not the case. One of the problems of spectral sensing algorithms is that those with high accuracy have high complexity, i.e. high time delay, and those with low complexity

have poor accuracy [28]. These drawbacks would compromise the cognitive user's ability to obtain and decode the primary user's message, and the slight performance gain seen in the results of the causal cognitive system over the standard cognitive system would most likely disappear.

4.3 n users, m channels

Figure 4.9 and Figure 4.10 show the evolution of the modified water filling algorithm in Channel 1 with $a^2 = 0.5$ and Channel 2 respectively. The y-axis is the user number and the x-axis is the channel number. The color mapping is named "HOT" in MATLAB and can be seen in Figure 4.8. In Figure 4.8 "white" is the color at 10, but in general "white" is maximum power. In the simulation results presented here, both the number of users and the number of channels are four.

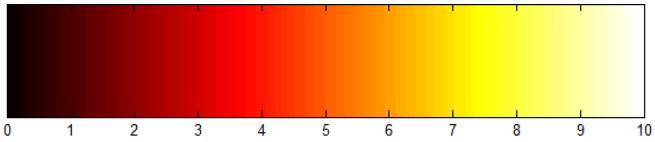


Figure 4.8: Colormap "HOT".

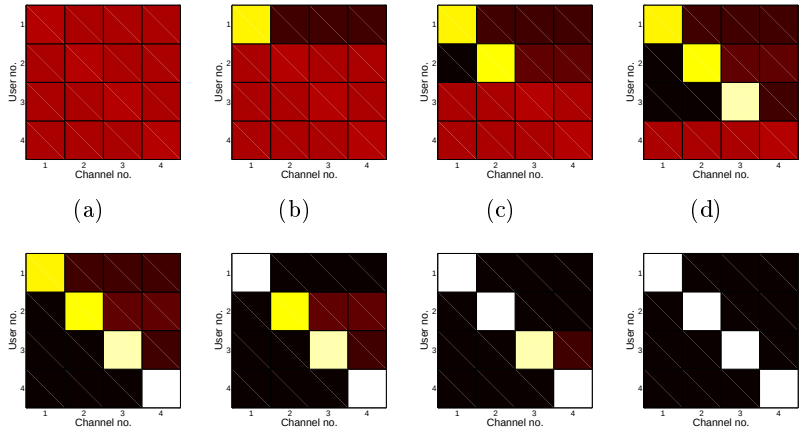


Figure 4.9: Evolution of the modified water filling algorithm over Channel 1 with $a^2 = 0.5$ and trasnmit power at each user 10 dB over the noise floor.

Figure 4.9 shows the evolution of the modified water filling algorithm in Channel 1 with $a^2 = 0.5$ and a transmit power at each user equal to 10 dB. In this scenario channel allocation is the optimum power allocation scheme, as can be seen in Figure 4.13, and the modified water filling algorithm reaches channel allocation after only 2 iterations at each user.

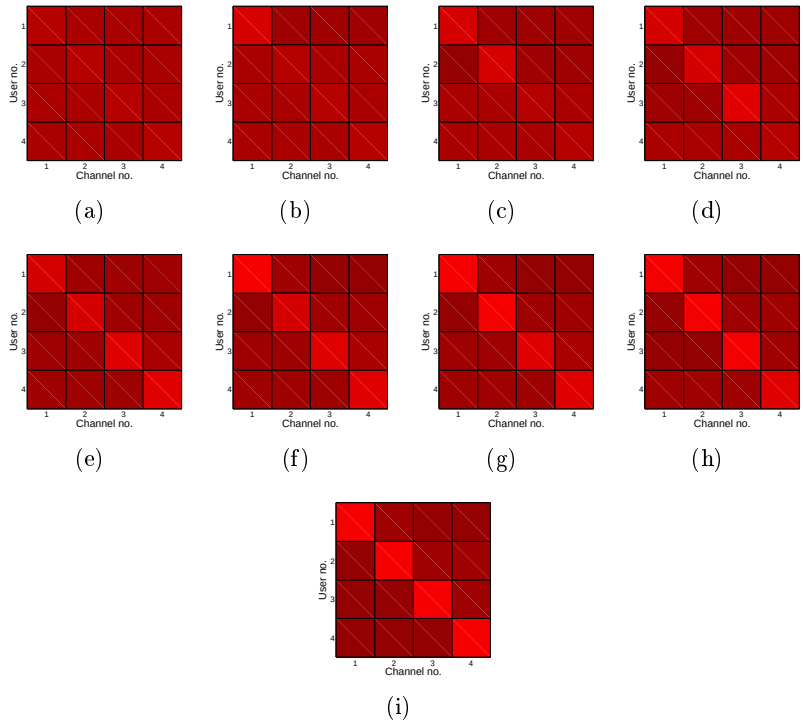


Figure 4.10: Evolution of the modified water filling algorithm over Channel 2 and trasnmit power at each user 10 dB over the noise floor.

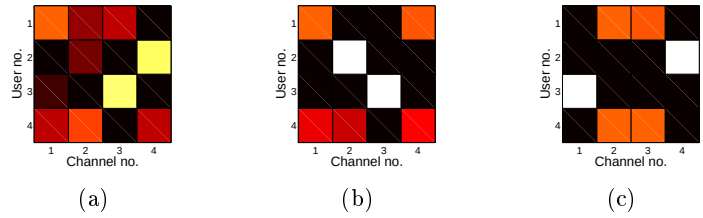


Figure 4.11: Final power allocation in Channel 3, at $P_t/N_0 = 12$, 20 and 30 dB.

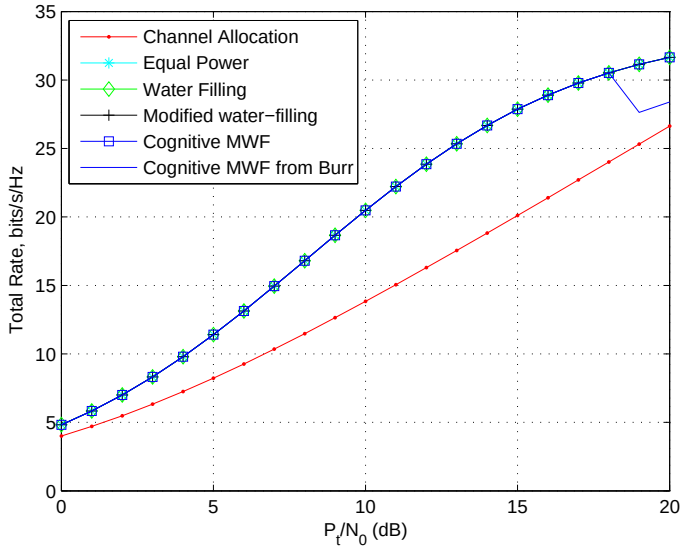


Figure 4.12: Total rate for Channel 1 with $a^2 = 0.1$ against P_t/N_0 in dB. P_t is total transmit power at each cognitive user and N_0 is white noise power.

The same type of evolution can be seen in Figure 4.10 for Channel 2. In this case each user has a preferred channel to transmit on and thus uses more power in this channel number than in the others. But the interference from the other users are so low that all channel numbers are used when the transmit power is at 10 dB. By Figure 4.14 it is evident however that with power at each user above 10 dB, channel allocation becomes the optimal allocation scheme.

Figure 4.12-4.15 show the total rate of four users and four channels in different Channels. Note that the term Total Rate is used on the y-axis, not total capacity. The term rate is used because, as mentioned in Section 2.4, the modified water filling algorithm is not guaranteed to find the global maximum of the capacity. And thus there is a possibility that the rate achieved with the modified water filling algorithm is less than the capacity.

Figure 4.12 and 4.13 show the total rate obtained for different power allocation schemes in Channel 1 with $a^2 = 0.1$ and $a^2 = 0.5$ respectively. With $a^2 = 0.1$ equal power is the optimal allocation scheme for P_t/N_0 below 25 dB, whereas with $a^2 = 0.5$ channel allocation is the optimal allocation scheme for P_t/N_0 above 0 dB. This is intuitive since with $a^2 = 0.1$ the interference is so low that only at high P_t/N_0 is it preferable that all users use different channels.

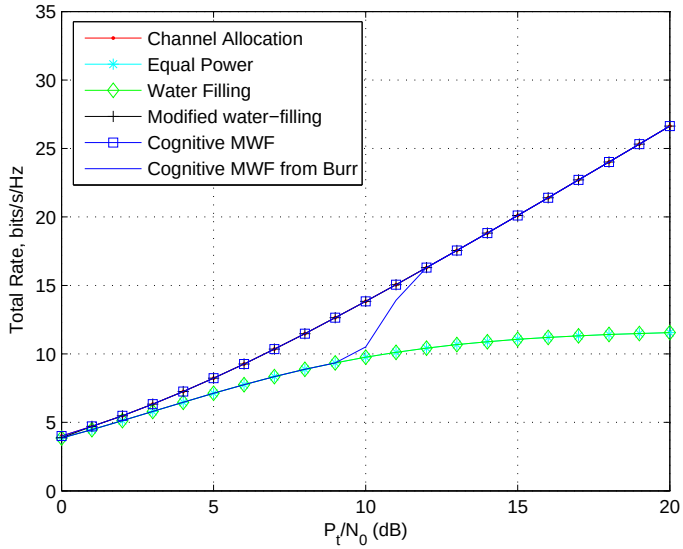


Figure 4.13: Total rate for Channel 1 with $a^2 = 0.5$ against P_t/N_0 in dB. P_t is total transmit power at each cognitive user and N_0 is white noise power.

On the other hand, with $a^2 = 0.5$ the interference is so strong that only at very low P_t/N_0 does equal power outperform channel allocation.

In Channel 2 the optimal power allocation switches from equal power to channel allocation at $P_t/N_0 = 10$ dB. And in Channel 3 channel allocation outperforms equal power at P_t/N_0 above 20 dB. In Channel 3 however, at $10 < P_t/N_0 < 36$ dB neither equal power or channel allocation is the optimum allocation scheme. The reason for this is that this Channel is not symmetric, i.e. the channel gains of interfering links to user l is not necessarily the same as those to user i. As can be seen from Figure 4.11, with increasing P_t/N_0 the modified water filling algorithm finds a middle ground between the two, with channel allocation for 2 of the users and equal power for the two others.

From the results depicted in Figure 4.12-4.15, it is seen that the modified water filling algorithm is able to find the optimum allocation scheme in all the different Channels. In Channel 2 it is able to change from equal power to channel allocation, when this becomes the optimal choice. And in Channel 3 it is able to find some other power allocation that outperforms both channel allocation and equal power. As given in Theorem 2.1, channel allocation *is* the optimal allocation scheme with high interference power, and thus the fact that

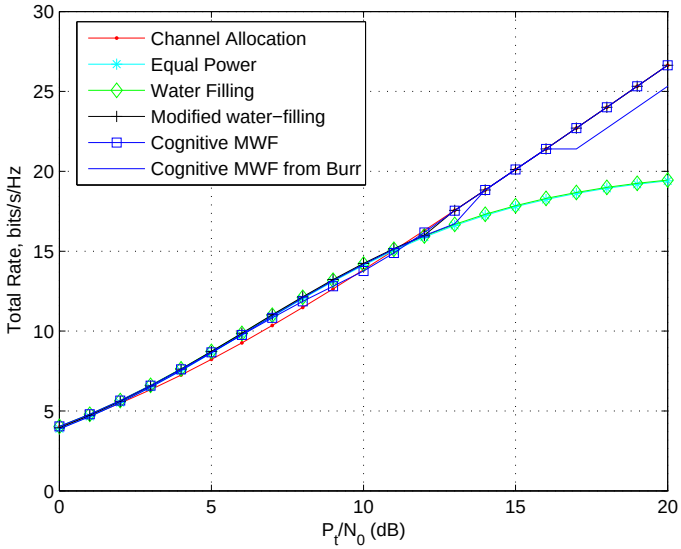


Figure 4.14: Total rate for Channel 2 against P_t/N_0 in dB. P_t is total transmit power at each cognitive user and N_0 is white noise power.

the modified water filling algorithm reaches channel allocation at some point for all the different Channels simulated, is one indicator that this algorithm is able to find the optimal power allocation.

Those familiar with power allocation schemes, especially in distributed networks, may wonder why simple water filling does not perform optimally, since simple water filling is optimal in many other cases such as MIMO channels. The is because simple water filling does not take into account the effect its allocation has on the performance of others. Thus all users are ignorant to the performance of the other users, which is what the B term in the modified water filling actually accounts for. Therefore, with the goal of maximizing the sum rate of all users, simple water filling does not find the optimal power allocation.

The modified water filling algorithm implemented was taken from [3]. [3] also implemented Channel 1 and Channel 3, thus referencing results against those in [3] was straight forward. The only difference was the calculation of the estimate $B_l(k)$, which is part of the cognitive MWF algorithm. As explained in Section 2.4, it is not realistic to assume that all terms included in $B_l(k)$ is known at the cognitive sender, and thus an estimated value computed more realistically was done in the cognitive MWF algorithm. In this thesis an

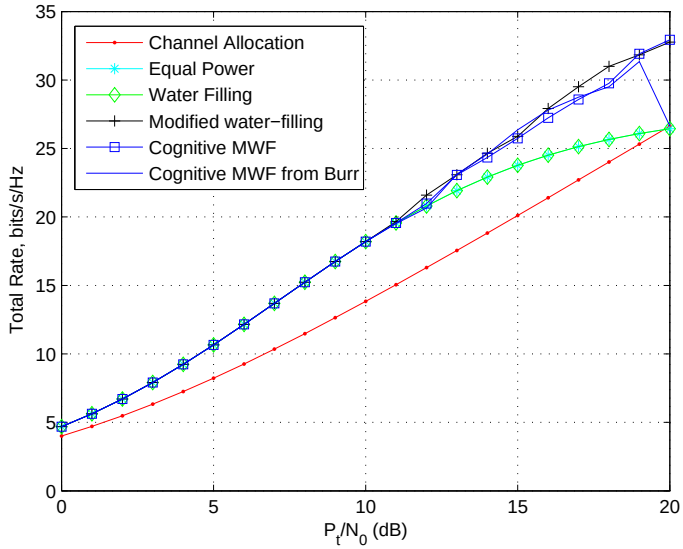


Figure 4.15: Total rate for Channel 3 against P_t/N_0 in dB. P_t is total transmit power at each cognitive user and N_0 is white noise power.

estimate value was found by (2.27) and in [3] $B_l(k)$ was found by multiplying $I_l(k)$ by 1/36 based on estimates of the average signal to interference plus noise ratio in each channel at 10 dB SNR.

For Channel 1, the cognitive MWF finds the optimal allocation scheme for both $a^2 = 0.1$ and $a^2 = 0.5$. The cognitive MWF from Burr ([3]) however, deters from the optimal allocation at $P_t/N_0 > 18$ dB for $a^2 = 0.1$ and only achieves optimal allocation at $P_t/N_0 > 12$ dB for $a^2 = 0.5$. For Channel 2 the cognitive MWF is suboptimal for $6 < P_t/N_0 < 12$ dB. This means that the cognitive MWF reaches channel allocation before it is optimum, because the estimate of $B_l(k)$ is not accurate enough. The cognitive MWF from Burr finds the optimum allocation for $P_t/N_0 < 16$ dB, after which it is suboptimal.

For Channel 3 both cognitive MWF algorithms are suboptimal for $P_t/N_0 > 11$ dB, but they are both performing above equal power. At $P_t/N_0 = 20$ dB, the cognitive MWF from Burr drops to channel allocation and is outperformed by the cognitive MWF for $20 < P_t/N_0 < 36$ dB, where at $P_t/N_0 = 36$ dB channel allocation becomes the optimal allocation. Based on these results, it seems that the cognitive MWF based on (2.27) is more stable than that from [3]. A reason for this is that the estimate from [3] is based on the average signal to

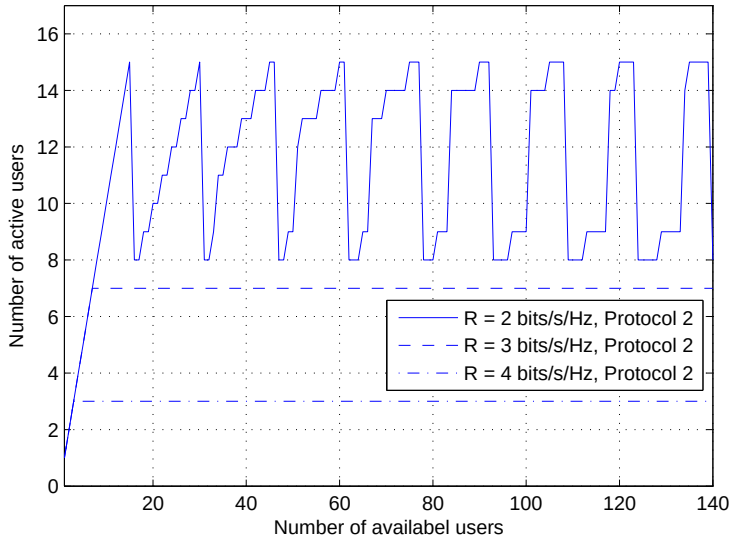


Figure 4.16: Number of available users vs number of active users in Protocol 2. R_p is given by R and the outage probability $q = 1\%$. $P_{max} = P_{pu} = 1$ watt.

interference plus noise ratio in each channel at $P_t/N_0 = 10$ dB. And as can be seen from the results, the cognitive MWF from Burr is quite accurate at P_t/N_0 around 10 dB. The cognitive MWF presented in this thesis however, is not based on a measurement at a certain P_t/N_0, but adapts to the power level on the channels.

4.4 Practical Cognitive System

Figure 4.16 show the number of active users versus the number of available users, given Protocol 2 and a maximum outage probability at 1% . As one can see with a primary user rate at 4 bits/s/Hz, only 3 cognitive users are allowed to transmit. With a primary user rate at 3 bits/s/Hz, 7 cognitive users are allowed to transmit. However, with a primary user rate at 2 bits/s/Hz an interesting characteristic of Protocol 2 is revealed. According to theory, with a primary user rate at 2 bits/s/Hz, 16 cognitive users is the maximum number allowed to transmit. But, since increasing the number of available users also leads each cognitive user to have neighboring cognitive users closer to its receiver, the number of cognitive users satisfying the $SINR$ constraint to transmit varies.

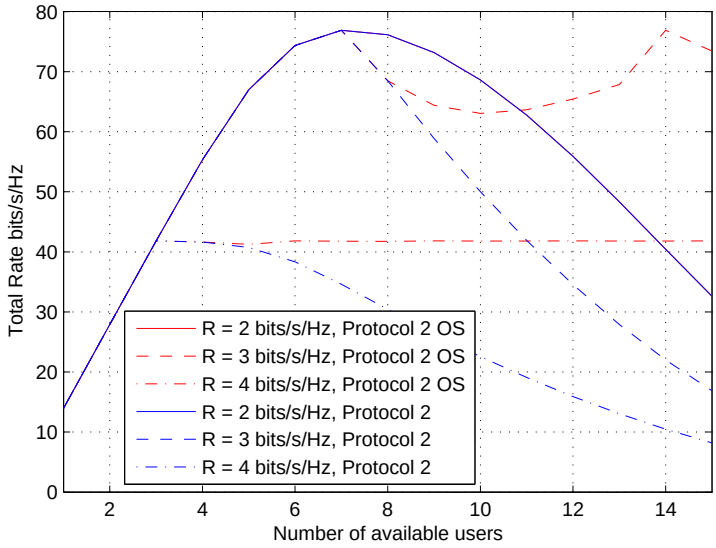


Figure 4.17: Number of available users vs total rate obtained by the system in Protocol 2. R_p is given by R and the outage probability $q = 1\%$. $P_{max} = P_{pu} = 1$ watt.

Figure 4.17 show the total rate of the cognitive system plotted against the number of available users, for Protocol 2 and Protocol 2 using optimal user selection (OS). With $R_p = 2$ bits/s/Hz the number of active users increases linearly along with increasing number of available users up to 15, and thus optimal user selection equals the standard Protocol 2 in this plot. With $R_p = 3$ bits/s/Hz, Protocol 2 and Protocol 2 employing optimal user selection is equal up to 8 available users. This is due to the fact that with $R_p = 3$ bits/s/Hz, 7 users can be active at the same time and with 8 available users only one is shut off, but there is no gain in having cognitive user l turned of instead of cognitive user x. But with 9 or more available users, this gain exists and reaches its maximum at 14 available users where every other cognitive user around the perimeter is turned on.

With $R_p = 4$ bits/s/Hz, only 3 cognitive users can be turned on and therefore for 5 or more available users there is a gain from using optimal user selection compared to standard Protocol 2. In fact, the total rate of the system using optimal user selection is constant with number of available users above 3, due to the low number of users allowed to transmit.

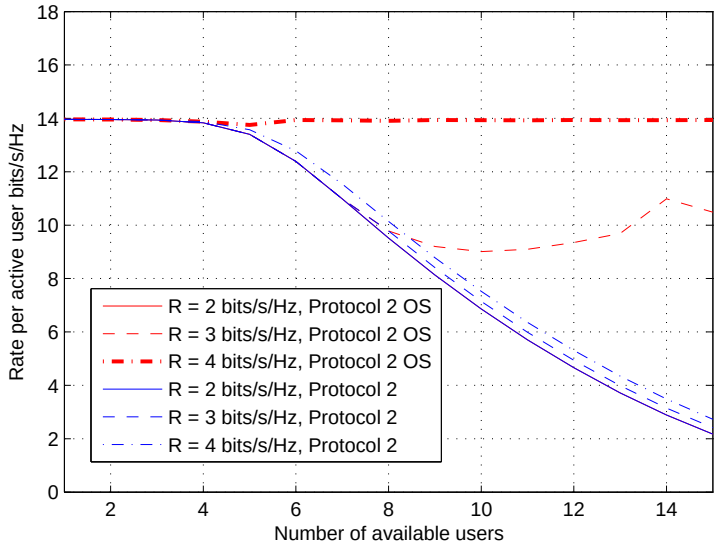


Figure 4.18: Number of available users vs rate per active user in Protocol 2. R_p is given by R and the outage probability $q = 1\%$. $P_{max} = P_{pu} = 1$ watt.

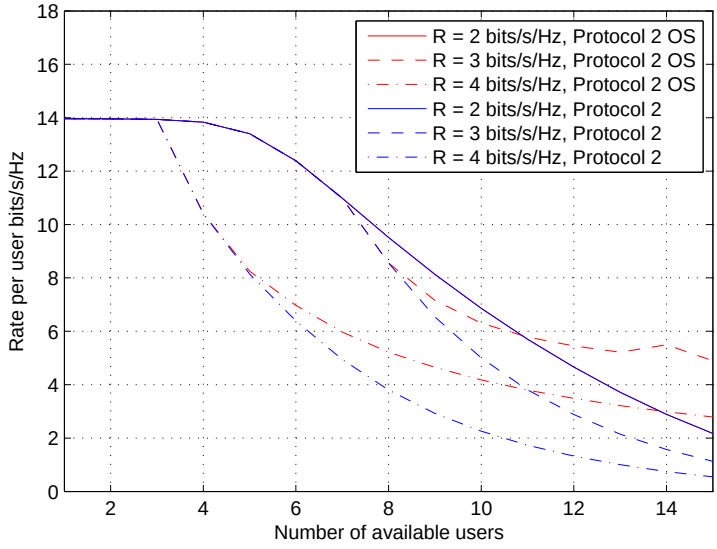


Figure 4.19: Number of available users vs rate per available user in Protocol 2. R_p is given by R and the outage probability $q = 1\%$. $P_{max} = P_{pu} = 1$ watt.

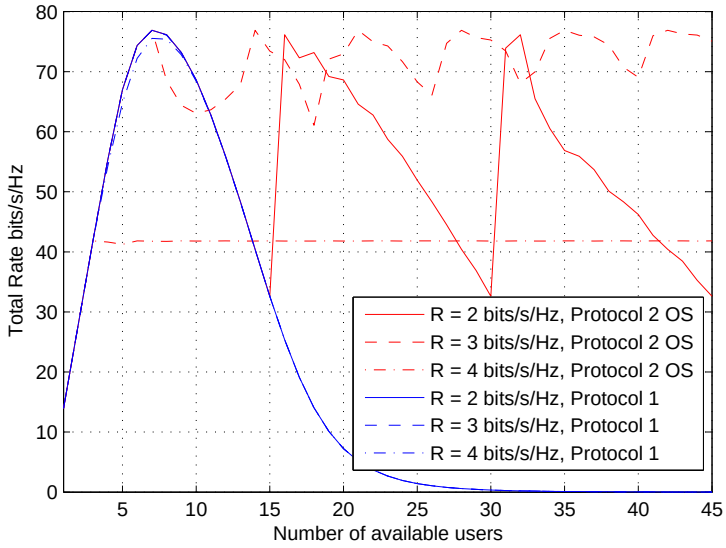


Figure 4.20: Number of available users vs total rate of the system. R_p is given by R and the outage probability $q = 1\%$. $P_{max} = P_{pu} = 1$ watt.

Rate per active user is plotted against number of available users in Figure 4.18. As in Figure 4.16, there is a substantial gain between optimal user selection and standard Protocol 2. With $R_p = 4$ bits/s/Hz the rate of optimal user selection is constant for number of available users above 5 and with $R_p = 3$ bits/s/Hz we see the same convex curve from 8 available users to 14 available users as in Figure 4.17. Even with the rate per available user versus number of available users, as is seen in Figure 4.19, there is a gain from using optimal user selection. However, the gain is not as big as when the rate is divided by the number of active users and will diminish with increasing number of available users.

Figure 4.20 show the rate of Protocol 2 using optimal user selection and Protocol 1 plotted against number of available users. For Protocol 1 it is assumed that all users have a high desired rate, thus all available users transmit with equal power constrained by the outage probability. With $R_p = 4$ bits/s/Hz there is a large performance gain of Protocol 1 over Protocol 2 for number of available users between 3 and 14 after which Protocol 2 with optimal user selection is optimum. With $R_p = 3$ bits/s/Hz there is only a slight gain of Protocol 1 over Protocol 2 when the number available users is between 7 and 11 and with $R_p = 2$ bits/s/Hz Protocol 2 has a performance equal to or above

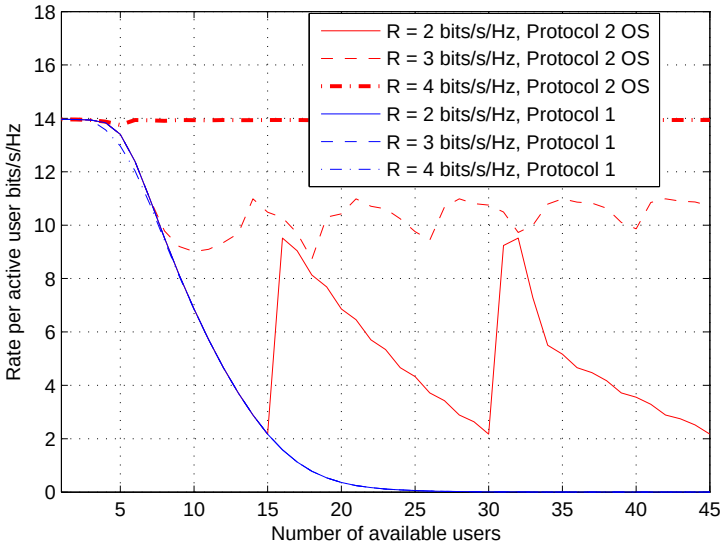


Figure 4.21: Number of available users vs rate per active user. R_p is given by R and the outage probability $q = 1\%$. $P_{max} = P_{pu} = 1$ watt.

Protocol 1 for all numbers of available users.

The reason why Protocol 1 performs so well with a low number of available users is the fact that reducing the power at each user so that each user can transmit and not violate the outage constraint increases the rate in the low interference regime. This can be verified by Figure 4.12 where the optimum power allocation was equal power for all users in all channels. But with increasing number of available users, the interference increases correspondingly and thus the fairness of Protocol 1 leads to a total rate approaching 0.

For Protocol 2 with optimal user selection one can see a periodic pattern in the rate with $R_p = 2$ bits/s/Hz and $R_p = 3$ bits/s/Hz. With $R_p = 3$ bits/s/Hz this is due to the fact that with increasing number of available users, one can optimally choose which users to signal in order to minimize the interference experienced by the signaling users. Peaks in the rate occur whenever the number of available users is a multiple of 7, since 7 is the maximum number of users allowed to transmit. With $R_p = 2$ bits/s/Hz, peaks actually occur when the number of users allowed to transmit changes from 15 to 8 and thus reducing the interference. Therefore, when the number of active users drops from a peak to a minimum in Figure 4.16 there is reversed minimum to peak

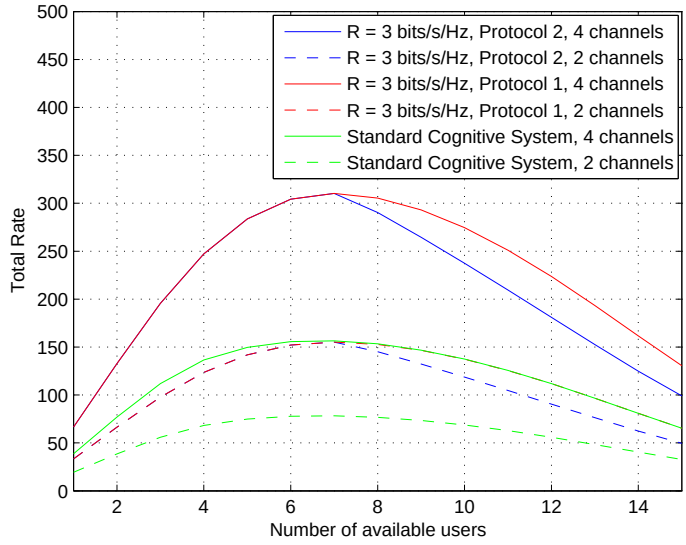


Figure 4.22: Number of available users vs total rate. R_p is given by R and the outage probability $q = 1\%$. $P_{max} = P_{pu} = 1$ watt.

in the rate.

In Figure 4.21 the rate divided by the number of active users of Protocol 2 using optimal user selection and Protocol 1 is plotted against number of available users. As in Figure 4.20 the same periodic patterns can be seen for Protocol 2 with optimal user selection and $R_p = 4$ bits/s/Hz and $R_p = 3$ bits/s/Hz. With $R_p = 2$ bits/s/Hz the rate is constant. Since the rate is divided by the number of active users Protocol 1 has no gain over Protocol 2 and the rate of Protocol 1 diminishes with increasing number of available users.

To see the performance gain of exploiting simultaneous transmission, even when the system is constrained by the outage probability, performance of Protocol 1 and Protocol 2 is referenced against a cognitive system only transmitting when the channels are idle. This is shown in Figure 4.22 and 4.23 where Phase 1, the time the subbands are idle, is 50% of the time. Different numbers of subbands were available in the plots where each user had a peak power constraint of 1 watt in each subband.

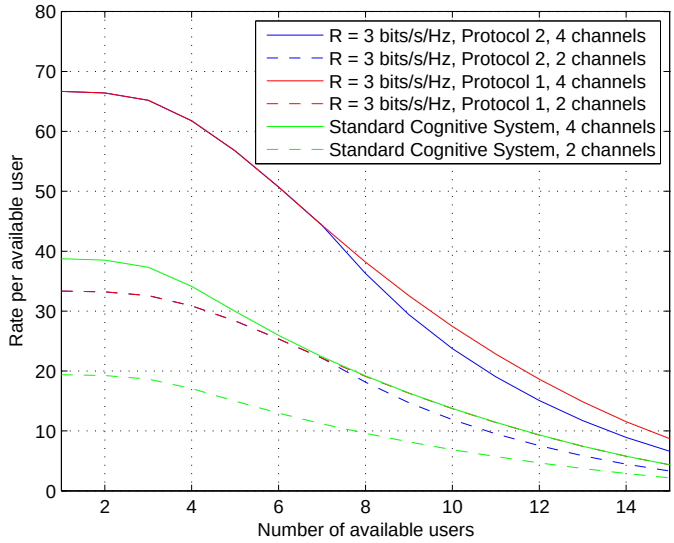


Figure 4.23: Number of available users vs rate per available user. R_p is given by R and the outage probability $q = 1\%$. $P_{max} = P_{pu} = 1$ watt.

The total rate of the systems is plotted against number of available users in Figure 4.22. As one can see there is a substantial gain from exploiting both phases which peaks at 7 available users, which is the maximum allowed number of active users for Protocol 2 with $R_p = 3$ bits/s/Hz. But without optimal user selection being used in Protocol 2, both the rate of Protocol 1 and Protocol 2 reduce to the rate of the standard system as number of available users increases. This is due to the interference from all the other cognitive users and with a limited number of subbands and increasing number of cognitive users signaling, the rate of Phase 1 will also approach 0, as stated in Corollary 2.3.

Figure 4.23 show the rate per available user plotted against the number of available users. Again there is a substantial performance gain from exploiting the two phases when the number of available users is small. But with increasing number of available users the rate of Protocol 1 and Protocol 2 reduces to the rate of the standard system.

These results have shown that in a disperse network both Protocol 1 and Protocol 2 provides a substantial gain over a standard cognitive system. Protocol 1 has the advantage of being able to satisfy as many cognitive users as possible, especially if different cognitive users have different desirable rates.

However, for Protocol 1 to work, i.e. a user only transmits with enough power to reach its desired rate, a cognitive user must know the interference power at its receiver, and thus there is a need for a feedback channel. Also to guarantee that the outage probability is not violated by the transmission of a cognitive user, the cognitive user has to know the current interference power at the primary receiver, i.e. base station. Using the fact that in a dense network the interference power experienced at any one receiver is (almost) independent of the location of the receiver [13] and the proposed scheme for the cognitive modified water filling algorithm at the end in Section 2.4, this can be estimated, but at a cost of complexity and uncertainty in accuracy.

Protocol 2 has the advantage of being fairly simple, because a user either transmits at full power or stay silent. To ensure primary user QoS, knowledge about the current interference power at the primary receiver should also be known. But due to (3.27), calculation of the outage probability at each cognitive user is not necessary. Given knowledge about primary users desired rate and outage probability, as is necessary in any case, each cognitive user knows the number of cognitive users that can be active without violating the outage constraint. Thus Protocol 2 is more suited for a distributed implementation.

The drawback of Protocol 2 is the fact that the performance of the total system is heavily dependent on the manner the active users are chosen. As can be seen from Figure 4.17 - 4.19, choosing the optimal users yields a significant gain, and with increasing number of available users the rate approaches zero without optimal user selection. In this thesis optimal user selection was implemented by an iterative algorithm selecting those users that were farthest from each other to transmit. With the goal of constructing a distributed cognitive system, future work on Protocol 2 would have to include schemes to do, if not optimal, improved user selection in a practical manner.

As explained above, these two protocols both have some advantages and some drawbacks. In essence, Protocol 2 seems suited for data transmission where a cognitive user maximize rate whenever it can. With data transmission, there is no immediate crisis if a cognitive user is not allowed to transmit for a few channel uses, it would only seem like the connection was slow to the user. And taking into account that the user is exploiting unlicensed spectra, the cognitive users should not expect rates as high as if they had a license. Protocol 1 seems suited for communication when the cognitive users have a fairly low desired rate, while at the same time there is QoS requirement involved, such as telephone transmission, e.g. GSM.

All results plotted in this Section for the practical cognitive system, has assumed that an average outage probability q, will satisfy the primary user QoS. Decreasing q will decrease the performance of the cognitive system and increasing q will increase the performance of the cognitive system, since decreasing q makes the primary user less tolerable to interference from the cognitive users while increasing q makes the primary user more tolerable to this interference. Exactly what value q will have for a real primary user depends on the communication standard employed by the primary user. Thus different primary users might have different outage probabilities to satisfy their QoS.

As mentioned above, the outage probability measured is an average. Using this average outage probability is fine for data transmission, but if the primary user is transmitting e.g. real time audio, such as a telephone, this will probably not suffice. Imagine the primary user is experiencing fading on the channel from its sender to its receiver. If the channel is in a deep fade, the primary user can not tolerate the same interference power as it could if the channel was at its average. Thus, if an average outage probability is used and the channel is in a deep fade the primary signal might break down, which is intolerable in the case of real time transmission at the primary user. Therefore, the metric for guaranteeing primary user QoS must also depend on the communication standard employed by the primary user.

Chapter 5

Conclusion and Further Work

In this thesis power allocation optimization and interference management in cognitive radio have been investigated. This has been done for different scenarios that are applicable to cognitive radio. The scenarios that have been reviewed are: 1) one primary and one cognitive user, 2) multiple cognitive users and multiple channels and 3) multiple cognitive users with primary users in the vicinity. In scenarios with primary users, the systems were evaluated over two phases, one where the primary user is active and one where the primary user is silent. The question was then, is it possible for the cognitive user to transmit simultaneously with the primary user and if so, how should power be allocated between the two phases to maximize performance.

In the scenario with 1 primary user and 1 cognitive user, the environment considered was one where the channel gain of the interfering channel from the cognitive sender to the primary receiver was less than the channel gain from the primary sender to the primary receiver. In this case superposition coding, as introduced in [16], is optimal.

With multiple cognitive users and channels, the problem was defined as how each user should allocate power to maximize its performance. In this thesis a novel power allocation algorithm (in this thesis referred to as *modified water filling*) presented in [3] was reviewed and implemented. In addition other standard power allocation strategies were presented and implemented to review the potential of modified water filling.

In the environment with multiple cognitive and primary users, primary user QoS was guaranteed through an outage constraint. One novel scheme to fairly split the resources among the different cognitive users and one scheme to optimize performance taken from [13] was implemented and reviewed.

5.1 Main Findings and Results

This thesis is the first to implement and simulate THP with interference cancellation, to the author's knowledge. Simulation results showed that THP has its advantage in the simplicity of the interference cancellation. The simplicity has its drawback in performance with a modulo loss and power loss, but THP seems a good first step towards a practical implementation of dirty paper coding.

For the genie-aided and causal cognitive systems the capacity as found in [16] was used to evaluate performance. In this thesis it was found that even with increasing numbers of cognitive users, superposition coding given by α in (2.7) does not degrade primary users performance. In the scenario with one primary and one cognitive user it was found both analytically and through simulations that in the low SNR region there is no performance gain of the genie-aided or causal cognitive system over a standard cognitive system. For the causal cognitive system this is also true for the high SNR region. Emphasizing on practicality, there is thus only a slight gain in the middle SNR region in using the two phases compared to only signaling when the channel is idle. This slight gain is dependent on how active the primary user is on the channel.

Simulations showed that the novel power allocation algorithm presented in [3], in this thesis referred to as modified water filling, seems to find the optimal power allocation. As argued before in this thesis, it is not guaranteed to find the globally optimal allocation, but compared to other standard power allocation strategies it performs better or equal. The drawback is the algorithms need for channel state information and simulations showed that results based upon estimates of these, limit the accuracy of the algorithm.

In this thesis a new approach to estimating $B_l(k)$ and thus making the MWF algorithm more applicable to the real world was presented and compared with the approach used in [3]. Simulations showed that this new approach was more stable, since it changes dynamically according to the interference experienced. Using this cognitive configuration, this algorithm is fitted for distributed systems and should be considered in future cognitive systems.

With multiple cognitive users in the environment and primary user QoS guaranteed by an outage probability, a new power allocation scheme based on fairness was presented. This scheme showed good performance in a disperse network, but with increasing number of users and limited number of subbands, the rate quickly approached zero. In contrast, power allocation based on binary power levels showed no degradation in performance with increasing number of users, given that the active users are chosen in an intelligent manner.

In the simulation results, it has been assumed infinite processing power at the cognitive users, i.e. all processing are done instantaneously and without delay. It has also been assumed throughout this thesis that the cognitive users have perfect spectral sensing. In any real cognitive system, this would of course not be the case. Thus in any real cognitive system, the slight gain the causal cognitive system have compared to a standard cognitive system, would be decreased due to these factors. Other papers, [8][16][22][20], conclude that exploiting the spectrum while being used by primary users and not degrading the primary performance in any way, has great potential and is worth considering in any implementation of a cognitive radio. This thesis has found that, in all essence, the added complexity of a causal cognitive system over a standard cognitive system is not justified by the gain it provides.

However, by allowing a slight degradation in primary user QoS, as done in the practical cognitive system, there is actually a significant gain that can be achieved with simultaneous transmission. But the algorithms presented in this thesis are far from complete and must be investigated further before any certainty about their place in the real world can be concluded.

5.2 Future Work

The two most promising aspects considered in this thesis are the modified water filling algorithm and the binary power allocation in Protocol 2. As mentioned above, the drawback of the modified water filling algorithm is the need for accurate channel state information. If this algorithm is going to be part of a real system, the aspect of fading has to be reviewed. Then one would need to look at channel estimation, different kinds of fading (slow fading, fast fading, flat fading, frequency selective fading) as these vary with different rates and environments.

The results presented in the practical cognitive system show the importance of intelligent user selection in Protocol 2. With a goal of making these al-

gorithms suited for a distributed implementation, future work must consider how to make these decisions and, most preferably, how to do it in a distributed manner.

Appendix A

Calcualtion of α

In this appendix a detailed calculation of α is given.

It is clear that to find α, (2.6) reduces to:

$$1 + \frac{(\sqrt{P_p} + a\sqrt{\alpha P_c})^2}{1 + a^2(1-\alpha)P_c} = 1 + P_p \quad \text{(A.1)}$$

$$(\sqrt{P_p} + a\sqrt{\alpha P_c})^2 = P_p(1 + a^2(1-\alpha)P_c \quad \text{(A.2)}$$

$$P_p + 2a\sqrt{\alpha P_c P_c} + a^2\alpha P_c = P_p + a^2(1-\alpha)P_c P_p \quad \text{(A.3)}$$

$$\alpha a P_c(1+P_p) + \sqrt{\alpha}2\sqrt{P_c P_p} - aP_c P_p = 0. \quad \text{(A.4)}$$

By substituting $\sqrt{\alpha} = y$ we get:

$$y^2 a P_c(1+P_p) + y2\sqrt{P_c P_p} - aP_c P_p = 0. \quad \text{(A.5)}$$

Thus we have a quadratic equation with a solution found by:

$$y = \frac{-2\sqrt{P_c P_p} \pm \sqrt{4P_c P_p + 4a^2 P_c^2 P_p(1+P_p)}}{2aP_c(1+P_p)} \quad \text{(A.6)}$$

$$y = \frac{-2\sqrt{P_c P_p} \pm 2\sqrt{P_c P_p}\sqrt{1 + a^2 P_c(1+P_p)}}{2aP_c(1+P_p)} \quad \text{(A.7)}$$

$$y = \frac{\sqrt{P_p}(\pm\sqrt{1 + a^2 P_c(1+P_p)} - 1)}{a\sqrt{P_c}(1+P_p)} \quad \text{(A.8)}$$

Given that $a \in [0,1]$ the positive solution of (A.8) yields a $y \in [0,1)$ and thus an $\alpha = y^2 \in [0,1)$.

Appendix B

Superpostion coding with multiple users

In this appendix it is shown that increasing number of cognitive users, where all does superposition coding according to (2.5) and where all users finds α independent of the other users according to (2.7), does not degrade the performance of the primary user.

Since the rate of the primary user is given as

$$R_P = \frac{1}{2}\log_2(1 + \frac{(\sqrt{P_p} + \sum_{i=1}^{N} a_i\sqrt{\alpha_i P_{c_i}})^2}{1 + \sum_{i=1}^{N} a_i^2(1-\alpha)P_{c_i}}) \tag{B.1}$$

and should equal $\frac{1}{2}\log_2(1+P_p)$, we need to prove that

$$\frac{(\sqrt{P_p} + \sum_{i=1}^{N} a_i\sqrt{\alpha_i P_{c_i}})^2}{1 + \sum_{i=1}^{N} a_i^2(1-\alpha)P_{c_i}} \geq P_p. \tag{B.2}$$

Inserting α from (2.7) for α_i, the numerator can be written as:

$$\begin{aligned} &(\sqrt{P_p} + \sum_{i=1}^{N} a_i\sqrt{P_p P_{c_i}}\frac{(\sqrt{1 + a_i^2 P_{c_i}(1+P_p)} - 1)}{a_i\sqrt{P_{c_i}}(1+P_p)})^2 \\ = \quad & P_p\frac{((1+P_p) + \sum_{i=1}^{N}(\sqrt{1 + a_i^2 P_{c_i}(1+P_p)} - 1))^2}{(1+P_p)^2} \end{aligned} \tag{B.3}$$

Inserting α in the denominator as done in the numerator, the denominator can be written as:

$$\begin{aligned} & 1+\sum_{i=1}^{N} a_i^2 P_{c_i}(1-P_p\frac{(\sqrt{1+a_i^2 P_{c_i}(1+P_p)}-1)^2}{a_i^2 P_{c_i}(1+P_p)^2} \\ = \quad & 1+\sum_{i=1}^{N} a_i^2 P_{c_i} - P_p\frac{(\sqrt{1+a_i^2 P_{c_i}(1+P_p)}-1)^2}{(1+P_p)^2} \end{aligned} \tag{B.4}$$

With P_p factored out in the numerator, we now have to prove that

$$\frac{\frac{((1+P_p)+\sum_{i=1}^{N}(\sqrt{1+a_i^2 P_{c_i}(1+P_p)}-1))^2}{(1+P_p)^2}}{1+\sum_{i=1}^{N} a_i^2 P_{c_i} - P_p\frac{(\sqrt{1+a_i^2 P_{c_i}(1+P_p)}-1)^2}{(1+P_p)^2}} \geq 1. \tag{B.5}$$

I.e. that

$$\begin{aligned} & \frac{((1+P_p)+\sum_{i=1}^{N}(\sqrt{1+a_i^2 P_{c_i}(1+P_p)}-1))^2}{(1+P_p)^2} \geq \\ & 1+\sum_{i=1}^{N} a_i^2 P_{c_i} - P_p\frac{(\sqrt{1+a_i^2 P_{c_i}(1+P_p)}-1)^2}{(1+P_p)^2} \end{aligned} \tag{B.6}$$

This can again be rewritten as:

$$\begin{aligned} & 1+\sum_{i=1}^{N} 2\frac{(\sqrt{1+a_i^2 P_{c_i}(1+P_p)}-1)}{(1+P_p)} + \sum_{i=1}^{N}\frac{(\sqrt{1+a_i^2 P_{c_i}(1+P_p)}-1)^2}{(1+P_p)^2} \\ & + \sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N} 2\frac{(\sqrt{1+a_i^2 P_{c_i}(1+P_p)}-1)(\sqrt{1+a_j^2 P_{c_j}(1+P_p)}-1)}{(1+P_p)^2} \geq \\ & 1+\sum_{i=1}^{N} a_i^2 P_{c_i} - P_p\frac{(\sqrt{1+a_i^2 P_{c_i}(1+P_p)}-1)^2}{(1+P_p)^2} \end{aligned} \tag{B.7}$$

Removing the ones and moving the last part of sum on the right hand side to the left side, we get:

$$\sum_{i=1}^{N} 2\frac{(\sqrt{1+a_i^2 P_{c_i}(1+P_p)}-1)}{(1+P_p)} + \sum_{i=1}^{N} \frac{(\sqrt{1+a_i^2 P_{c_i}(1+P_p)}-1)^2}{(1+P_p)} + \sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N} 2\frac{(\sqrt{1+a_i^2 P_{c_i}(1+P_p)}-1)(\sqrt{1+a_j^2 P_{c_j}(1+P_p)}-1)}{(1+P_p)^2} \geq \sum_{i=1}^{N} a_i^2 P_{c_i} \tag{B.8}$$

Solving the term in the second sum on the left hand side of the inequality we obtain:

$$\sum_{i=1}^{N} 2\frac{(\sqrt{1+a_i^2 P_{c_i}(1+P_p)}-1)}{(1+P_p)} + \sum_{i=1}^{N} 2\frac{(1-\sqrt{1+a_i^2 P_{c_i}(1+P_p)})}{(1+P_p)} + \sum_{i=1}^{N} a_i^2 P_{c_i} + \sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N} 2\frac{(\sqrt{1+a_i^2 P_{c_i}(1+P_p)}-1)(\sqrt{1+a_j^2 P_{c_j}(1+P_p)}-1)}{(1+P_p)^2} \geq \sum_{i=1}^{N} a_i^2 P_{c_i} \tag{B.9}$$

And we are left with

$$\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N} 2\frac{(\sqrt{1+a_i^2 P_{c_i}(1+P_p)}-1)(\sqrt{1+a_j^2 P_{c_j}(1+P_p)}-1)}{(1+P_p)^2} \geq 0 \tag{B.10}$$

which proves the statement.

Appendix C

Detailed derivation of $\frac{\partial C}{\partial P_l(k)}$

In this appendix a detailed calculation of $\frac{\partial C}{\partial P_l(k)}$ is given.

C is given by (2.15). It is clear that the term $P_l(k)$ is part of all segments involving channel k, since $P_l(k)$ is interference at the other users. All segments containing $P_l(k)$ in C can be written as:

$$\begin{aligned}
C(\text{with } P_l(k)) &= \frac{1}{2}\log_2(1+\frac{a_{ll}^2(k)P_l(k)}{N+\sum_{j=1,j\neq l}^{n}a_{jl}^2(k)P_j(k)}) \\
&\quad + \sum_{i=1,i\neq l}^{n}\frac{1}{2}\log_2(1+\frac{a_{ii}^2(k)P_i(k)}{N+\sum_{j=1,j\neq i}^{n}a_{ji}^2(k)P_j(k)}) \\
&= \frac{1}{2}\log_2(1+\frac{a_{ll}^2(k)P_l(k)}{N+\sum_{j=1,j\neq l}^{n}a_{jl}^2(k)P_j(k)}) \\
&\quad + \sum_{i=1,i\neq l}^{n}\frac{1}{2}\log_2(\frac{N+\sum_{j=1,j\neq i}^{n}a_{ji}^2(k)P_j(k)+a_{ii}^2(k)P_i(k)}{N+\sum_{j=1,j\neq i}^{n}a_{ji}^2(k)P_j(k)}) \\
&= \frac{1}{2}\log_2(1+\frac{a_{ll}^2(k)P_l(k)}{N+\sum_{j=1,j\neq l}^{n}a_{jl}^2(k)P_j(k)}) \\
&\quad + \sum_{i=1,i\neq l}^{n}\Big(\frac{1}{2}\log_2(N+\sum_{j=1,j\neq i}^{n}a_{ji}^2(k)P_j(k)+a_{ii}^2(k)P_i(k)) \\
&\quad - \frac{1}{2}\log_2(N+\sum_{j=1,j\neq i}^{n}a_{ji}^2(k)P_j(k))\Big)
\end{aligned} \tag{C.1}$$

Then by partial differentiation $\frac{\partial C}{\partial P_l(k)}$ is obtained:

$$\begin{aligned}
\frac{\partial C}{\partial P_l(k)} &= \frac{1}{2}\frac{1}{1+\frac{a_{ll}^2(k)P_l(k)}{N+\sum_{j=1,j\neq l}^{n} a_{jl}^2(k)P_j(k)}}\frac{a_{ll}^2(k)}{N+\sum_{j=1,j\neq l}^{n} a_{jl}^2(k)P_j(k)} \\
&\quad \frac{1}{2}\sum_{i=1,i\neq l}^{n}\Big(\frac{a_{li}^2(k)}{N+\sum_{j=1,j\neq i}^{n} a_{ji}^2(k)P_j(k)+a_{ii}^2(k)P_i(k)} \\
&\quad -\frac{a_{li}^2(k)}{(N+\sum_{j=1,j\neq i}^{n} a_{ji}^2(k)P_j(k)}\Big) \\
&= \frac{1}{2}\frac{a_{ll}^2(k)}{N+a_{ll}^2(k)+\sum_{j=1,j\neq l}^{n} a_{jl}^2(k)P_j(k)} \\
&\quad -\frac{1}{2}\sum_{i=1,i\neq l}^{n}\frac{a_{li}^2(k)a_{ii}^2(k)P_i(k)}{\left(\begin{matrix}N+a_{ii}^2(k)P_i(k)\\ +\sum_{j=1,j\neq i}^{n} a_{ji}^2(k)P_j(k)\end{matrix}\right)\left(N+\sum_{j=1,j\neq i}^{n} a_{ji}^2(k)P_j(k)\right)}
\end{aligned} \tag{C.2}$$

Since this is suppose to be equal to a constant c_l, the term $\frac{1}{2}$ can be neglected and this is equal to (2.22)

Appendix D

Properties of α and β

In this appendix some properties of α and β are shown.

$$\begin{aligned}
\lim_{P_p\to\infty}\beta &= \lim_{P_p\to\infty}\frac{\log(1+h_1^2P_p)}{\log(1+h_2^2P_p)} = \frac{\frac{h_1^2}{1+h_1^2P_p}}{\frac{h_2^2}{1+h_2^2P_p}} \\
&= \lim_{P_p\to\infty}\frac{(1+h_2^2P_p)h_1^2}{(1+h_1^2P_p)h_2^2} = 1. \qquad \text{(D.1)} \\
\lim_{P_c\to\infty}\alpha &= \lim_{P_c\to\infty}\left(\frac{\sqrt{P_p}(\sqrt{1+a^2P_c(1+P_p)}-1)}{a\sqrt{P_c}(1+P_p)}\right)^2 \\
&= \lim_{P_c\to\infty}\frac{P_p(1+a^2P_c(1+P_p))}{a^2P_c(1+P_p)^2} = \frac{P_p}{1+P_p}. \qquad \text{(D.2)} \\
\lim_{P_p\to\infty}\alpha &= \lim_{P_p\to\infty}\left(\frac{\sqrt{P_p}(\sqrt{1+a^2P_c(1+P_p)}-1)}{a\sqrt{P_c}(1+P_p)}\right)^2 \\
&= \lim_{P_p\to\infty}\frac{P_p(2+a^2P_c(1+P_p)-2\sqrt{1+a^2P_c(1+P_p)})}{a^2P_c(1+P_p)} \\
&= \lim_{P_p\to\infty}\frac{2P_p}{a^2P_c(1+P_p)^2}+\frac{P_pa^2P_c(1+P_p)}{a^2P_c(1+P_p)^2}-\frac{2P_p\sqrt{1+a^2P_c(1+P_p)}}{a^2P_c(1+P_p)^2} \\
&= \lim_{P_p\to\infty}\frac{P_p}{1+P_p} = 1. \qquad \text{(D.3)}
\end{aligned}$$

Appendix E

Proof of Theorem 2.1

In this appendix proof of Theorem 2.1 is given.

With out loss of generality it is assumed a system with 2 users and 2 channels. Each user has power P. Next assume that each user has allocated a portion $t_l \neq 0, 1, l = 1, 2$ of its power to channel 1 and $(1 - t_l)$ to channel 2. The total rate of the system then becomes:

$$\begin{aligned} R = &\frac{1}{2}\log(1 + \frac{Pt_1}{1 + a_{21}(1)^2 Pt_2}) + \frac{1}{2}\log(1 + \frac{P(1-t_1)}{1 + a_{21}(2)^2 P(1-t_2)}) \\ &+ \frac{1}{2}\log(1 + \frac{Pt_2}{1 + a_{12}(1)^2 Pt_1}) + \frac{1}{2}\log(1 + \frac{P(1-t_1)}{1 + a_{12}(2)^2 P(1-t_1)}) \end{aligned} \tag{E.1}$$

with $P \to \infty$, R reduces to

$$\begin{aligned} \lim_{P\to\infty} R = &\frac{1}{2}\log(1 + \frac{t_1}{a_{21}(1)^2 t_2}) + \frac{1}{2}\log(1 + \frac{(1-t_1)}{a_{21}(2)^2 (1-t_2)}) \\ &+ \frac{1}{2}\log(1 + \frac{t_2}{a_{12}(1)^2 t_1}) + \frac{1}{2}\log(1 + \frac{(1-t_1)}{a_{12}(2)^2 (1-t_1)}) < \infty. \end{aligned} \tag{E.2}$$

Assuming channel allocation

$$\lim_{P\to\infty} R = \lim_{P\to\infty} \frac{1}{2}\log(1 + P) + \frac{1}{2}\log(1 + P) = \infty. \tag{E.3}$$

Appendix F

Calculation of Optimal t

To find the optimal t, the capacity over the two phases, given in (3.7), is differentiated with respect to t. In the genie-aided case, the capacity is given as:

$$C = \frac{(1-p)}{2}\log_2(1+\frac{Pt}{(1-p)}) + \frac{p}{2}\log_2(1+(1-\alpha)\frac{P(1-t)}{p}). \tag{F.1}$$

Differentiating C with respect to t yields

$$\frac{\partial C}{\partial t} = \frac{(1-p)}{2}\frac{1}{1+Pt/(1-p)}\frac{P}{(1-p)} + \frac{p}{2}\frac{1}{1+(1-\alpha)P(1-t)/p}\frac{(-1)(1-\alpha)P}{p}. \tag{F.2}$$

Setting this equal to zero, the optimal t is found as

$$\frac{1}{1+Pt/(1-p)} - \frac{(1-\alpha)}{1+(1-\alpha)P(1-t)/p} = 0 \tag{F.3}$$

$$\frac{1}{1+Pt/(1-p)} = \frac{(1-\alpha)}{1+(1-\alpha)P(1-t)/p} \tag{F.4}$$

$$1+\frac{(1-\alpha)P(1-t)}{p} = (1-\alpha)(1+\frac{Pt}{(1-p)}) \tag{F.5}$$

$$t^{(g)} = \frac{(1-\alpha)P/p+\alpha}{(1-\alpha)P(1/p+1/(1-p))} \tag{F.6}$$

In the causal case C is given as

$$C = \frac{(1-p)}{2}\log_2(1+\frac{Pt}{(1-p)}) + \frac{p}{2}(1-\beta)\log_2(1+(1-\alpha)\frac{P(1-t)}{p(1-\beta)}). \tag{F.7}$$

Differentiating C with respect to t yields

$$\begin{aligned}\frac{\partial C}{\partial t} = &\frac{(1-p)}{2}\frac{1}{1+Pt/(1-p)}\frac{P}{(1-p)} \\ &+\frac{p(1-\beta)}{2}\frac{1}{1+(1-\alpha)P(1-t)/((1-\beta)p)}\frac{(-1)(1-\alpha)P}{p(1-\beta)}.\end{aligned} \tag{F.8}$$

Setting this equal to zero, the optimal t is found as

$$\frac{1}{1+Pt/(1-p)} - \frac{(1-\alpha)}{1+(1-\alpha)P(1-t)/((1\beta)p)} = 0 \tag{F.9}$$

$$\frac{1}{1+Pt/(1-p)} = \frac{(1-\alpha)}{1+(1-\alpha)P(1-t)/((1-\beta)p)} \tag{F.10}$$

$$1+\frac{(1-\alpha)P(1-t)}{(1-\beta)p} = (1-\alpha)(1+\frac{Pt}{(1-p)}) \tag{F.11}$$

$$t^{(c)} = \frac{(1-\alpha)P/((1-\beta)p)+\alpha}{(1-\alpha)P(1/((1-\beta)p)+1/(1-p))}. \tag{F.12}$$

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