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The Effect of Gain Saturation in a Gain Compensated Perfect Lens

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Problem Description

Explore the feasibility of using gain to compensate for the large losses experienced in realisations of near-optical metamaterials with a negative refractive index.

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Preface

This report was written as my 5th year masters thesis from the Norwegian University of Science and Technology (NTNU). My degree is in electronics, with a specialisation in photonics and microelectronics. The work was done at the University Graduate Center (UniK), Kjeller.

I would like to thank my supervisor prof. Johannes Skaar for his guidance, and for his work with the article.

Summary

Perfect lenses operating in the near visible spectrum has only recently been introduced, and these kind of metamaterials seem to have a large potential. One problem encountered with these perfect lenses are exceedingly large intrinsic losses, making them impractical for use in applications. This project has explored some of the limitations in using gain to compensate for these losses, specifically the effect of gain saturation has been considered.

Gain saturation has been proven to limit the maximum parallel spatial frequency that can be reproduced by the lens. Even though, it has been shown that amplification has the potential to increase the resolution limit by a measurable factor. In the case of several waves traversing the lens simultaneously, the critical factor is how much of the total wave amplitudes lie in spatial frequencies close to the resolution limit. Waves with relatively small parallel spatial frequencies requires small amplifications, and those with high parallel spatial frequencies will get attenuated or reflected almost immediately, meaning both these types contribute little to gain saturation.

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Chapter 1

Introduction

Minituarization is a very important concept in technology. Smaller devices can be faster, more energy efficient and packed closer. Together with the level of mass production seen in todays electronic industry, this means cheaper and better equipment for the end user. However, if the rate of minituarization is to be maintained, new methods for research and production must be developed.

When imagening this progress, the transistor is the common choice of example. Starting out as a clearly visible blob several cm in size, it is now residing on thin silicon wafers in densities of over 200 millions pr. cm^2 [1]. As technology has reached ever lower dimensions, there has also been a need to develop methods to visualize and probe these structures.

One limitation when looking at a very small scale, is the resolution limit of the optical components. For quality components designed for this purpose, this is most often limited by diffraction, that is $\Delta_{min} \approx \lambda$. In other words, the smallest discernable features in a traditional optical system are in the order of a single wavelength of the incident light. X-ray-diffraction, Scanning Electron Microscope and Atomic Force Microscopy are a few examples of techniques that have been developed to overcome this resolution limit and image the world beyond. Though there are certain positive aspects with these methods compared to optical microscopy, they can be both complicated and invasive.

It was recently reported that a laser operating in the X-ray spectrum had been successfully tested, reaching a new high for laser frequencies [2]. These kind of discoveries helps drive the field onwards, and will hopefully prove to be usable in industry applications as well as a general tool in further research. One drawback with using such high frequency light is the innate complexity compared to lasers and LEDs in the visible spectrum. Visible light is both easy to produce and detect, and it interacts with many materials in desired ways, be it to initiate chemical reactions in photolithography or to carefully probe the structure of electrical components. By raising the resolution limit to subwavelength features, the

idea of perfect lenses is an improvement which may prove to bring such methods to a new level.

The name "perfect lens" has been given to a simple, rectangular slab made out of a new kind of material. First described by Veselago in the late sixties [3], and unrealised until 2000 [4], these materials have simultaneously negative permeability and permittivity, leading to unconventional results. Veselago also noted that the index of refraction, relative to vacuum, in such a material would be negative. These metamaterials are in reality designed structures, with dimensions much smaller than the wavelength of the incident light. As such they are not easy to create, and only in the last years have there been progress in the design of lenses operating in the near IR-spectrum [5–8].

The waves carrying this subwavelength information are not propagating waves, instead they decay exponentially away from the source, and are called evanescent. This is why they are usually ignored when considering the Fourier expansion of the source fields; in classical optics they will not be transmitted through the lens. In a perfect lens though, they will actually be amplified so they can be reproduced at the far side of the lens. If the lens has a refractive index perfectly equal to minus one, all EM-fields in the source plane will theoretically be perfectly recreated in the image plane. Though here another limitation of the lens becomes clear, due to the fact that there can be no discontinuity of the fields in the image plane, the fields between the lens and the image will not have any correspondence to the original fields behind the source.

A second problem is intrinsic losses. Even when they are small, they put a large limitation on the resolution, and these losses seem to increase when approaching the visual spectrum [5–8]. To overcome the problems with large intrinsic losses, it has been suggested to introduce gain [9–11]. In currently realised designs, the intrinsic losses are very large, denoted by a large Figure of Merit; $FOM = \text{Im}\{n\}/\text{Re}\{n\}$, but it has been theoretically shown that they can be made much smaller [12].

When a wave enters a negative index media (NIM) from a vacuum, the wave suddenly travels backwards. Now the direction of the rays, and the energy flow, are opposite that of the wavevector, which points backwards. When looking at evanescent waves, this means that instead of decaying, they will be amplified inside the NIM. By exploiting this natural amplification of evanescent waves it is possible to recreate the near-fields of the object in the image plane. This will allow us to image sub-wavelength features in an object. Due to the inherent diffraction [13, 14], this is only possible to achieve at a single wavelength, and nonzero losses will put a finite limit on the resolution limit.

Higher evanescent waves decay exponentially faster, and due to noise, it is difficult to restore them after distances higher than a few wavelengths. Because of this, the main use of perfect lenses will most probably be in places where one have small distances from the object to the source. It may be possible to use them for lithography in semiconductor production, or as improved optical microscopes for research and development purposes.

This thesis have looked at the feasibility of using gain to improve perfect lenses, specifically concerning gain saturation. The first sections will present the theories behind perfect lenses, and show how they ideally should behave. In the subsequent sections the main problems behind the calculations will be discussed. The methodology, results and conclusion are shown in the article in Appendix A.



Chapter 2

Background theory

This section will explain the physics behind evanescent waves and metamaterials with simultaneously negative electric permittivity and magnetic permeability. It will then present the theories of using these materials to create perfect lenses which may exploit the evanescent waves to achieve subwavelength resolution.

2.1 Evanescent waves

Evanescent waves can be seen as waves that are not supported by the surrounding medium. Instead of propagating, they decay exponentially with distance [15, ch. 23-5]. As such they are only found relatively close to their source, and are usually referred to as “near-fields”. It is these near fields that carry information about the source on the subwavelength scale, and hence they are the key to unlocking higher resolutions. When considering an infinitesimal dipole in front of a lens, with a steady state electrical field strictly in the x/z -plane, this can be seen from the Fourier-expansion [16]:

$$\mathcal{E}(x, z) = \int_{-\infty}^{\infty} \mathcal{E}_0(k_x) e^{i(k_x x + k_y y + k_z z - \omega t)} dk_x, \quad (2.1)$$

where k_x and k_z are the transversal and longitudinal wavenumber, respectively. The latter can be found from the first in conjunction with the frequency of the wave, ω , and the refractive index, n , of the surrounding medium:

$$k_z^2 = k_0^2 - k_x^2 = n^2 \left(\frac{\omega}{c}\right)^2 - k_x^2, \quad (2.2)$$

where c is the speed of light in a vacuum. In a normal media (n positive) one must choose the positive squareroot for k_z . Assuming the parallel spatial frequency is a real value, if it should exceed that of the total wave, the left side in the equation will

become negative. This means the transverse spatial frequency must be a complex value. These are the evanescent waves, and they will decay rapidly. Consider the field of a wave with $k_z = i|k_z|$:

$$\mathcal{E}_{k_x^2 > k_0^2}(x, z) = \mathcal{E}_0(k_x) e^{-|k_z|z} e^{i(k_x x - \omega t)}. \quad (2.3)$$

The electrical field now decay exponential in the \hat{z} -direction proportional with the longitudinal wavenumber, and would not be recreated in the image by a conventional lens. This is why it is normal in Fourier-optics to ignore the evanescent fields in (2.1) by limiting the integral to $\pm k_0$. As will be seen later, this is not the case when you have negative refraction, as the evanescent fields to a certain degree will be amplified by the lens.

One last thing about evanescent fields worth to mention is the time averaged Poynting vector. For waves where $k_x^2 > k_0^2$ this looks like:

$$\langle \vec{S}_z \rangle = -i \frac{1}{2} \frac{|k_z|}{\omega \mu} e^{-2|k_z|z} |\mathcal{E}_0(k_x)|^2, \quad (2.4)$$

which is purely imaginary. Given that the powerflow is defined as $\text{Re}\{\langle \vec{S}_z \rangle\}$, this is zero for all evanescent waves. Since powerflow is often associated with a flux of photons, it would also be appropriate to say that evanescent fields have no photon flux. This is important to remember when discussing the amplification of evanescent waves in perfect lenses.

2.2 Negative refractive materials

There are several naturally occurring materials that have negative parameters for certain frequencies, with a negative ϵ being the most common. These are both known and used, mostly in microwave engineering. What has not been found, are materials where both the permittivity and the permeability are negative simultaneously [16]. Such a material was first made artificially in 2000 [4], over 30 years after their existence was proposed by Veselago [3].

The refractive index is defined as:

$$n^2 \equiv \epsilon \mu, \quad (2.5)$$

where ϵ is the relative electric permittivity, and μ is the relative magnetic permeability. When solving for n , one would normally take the positive squareroot. It has been shown that when the permittivity and the permeability are simultaneously negative, causality forces one to choose the negative squareroot instead, and the refractive index is negative compared to vacuum [3]:

$$n = -\sqrt{\epsilon \mu}, \quad \text{Re}\{\epsilon\} < 0, \quad \text{Re}\{\mu\} < 0. \quad (2.6)$$

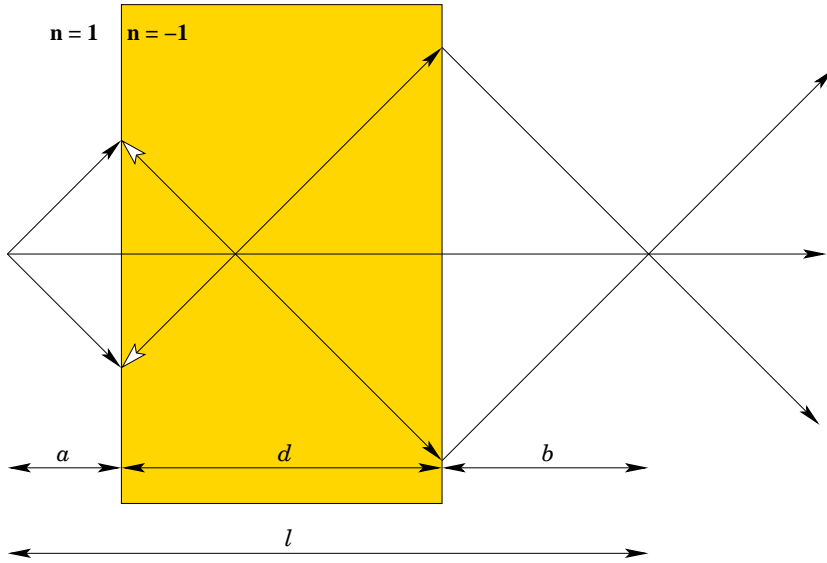


Figure 2.1: Optical path of rays through a negative refractive index material situated in a vacuum. Outside the slab (white), the directions of the rays and the wavevectors are equal. Inside the slab (yellow), the directions of the rays (black arrows) and the wavevectors (white arrows) are opposite.

This has many implications, and though it was not embraced in the beginning [17], it is now an accepted theory [18]. Since the first realisation, there have been several successful experiments confirming this behaviour [19–23]. Some unique properties present in negative refractive materials are backward Cerenkov radiation, inverse Doppler effect and a negative Goos-Hänchen shift [16].

When looking at a normal ray diagram, as seen in figure 2.1, it is not necessary to take any special precautions. Snell's law for the refraction between two materials (denoted 0 and 1) is given as [15, ch. 6.2]:

$$n_0 \sin \theta_i = n_1 \sin \theta_r. \quad (2.7)$$

Where θ_i is the incident angle, and θ_r is the refracted angle. For transitions between materials with non-negative and negative parameters, respectively, this equation still holds true, since:

$$\frac{\sin \theta_i}{\sin \theta_r} = \frac{-|\mathbf{k}_1|}{|\mathbf{k}_0|} \equiv \frac{n_1}{n_0} < 0. \quad (2.8)$$

This means that in a material with index of refraction $n = -1$, one only needs a simple slab to focus the rays from the source into an image. One limitation with this design is that it will not act as a normal lens in any other way. The slab does not have a focal length, and it will therefore not focus parallel incoming rays.

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Neither is it capable of creating parallel rays from a focused image. What it will do, is create a perfectly focused image of a source placed a distance a in front of the lens at a distance b behind the lens. This distance can be found by using Snell's Law, and is found to be governed by the simple formula [24]:

$$b = d - a, \quad (2.9)$$

where d is the thickness of the lens. Of course, this means that it will only be able to image objects that are a maximum distance of $a < d$ in front of the lens.

Another way to look at NIMs are as “negative space” [18], as illustrated in figure 2.2. It has been shown that if a ray travels through two optical systems, where the second one is a complementary of the first, the optical path length will be zero. One example of this is the system of a slab with $n = -1$ in vacuum. If the light travels the same distance in vacuum as inside the lens, the systems are exact opposites of each other, the optical path length experienced by the light will be zero.

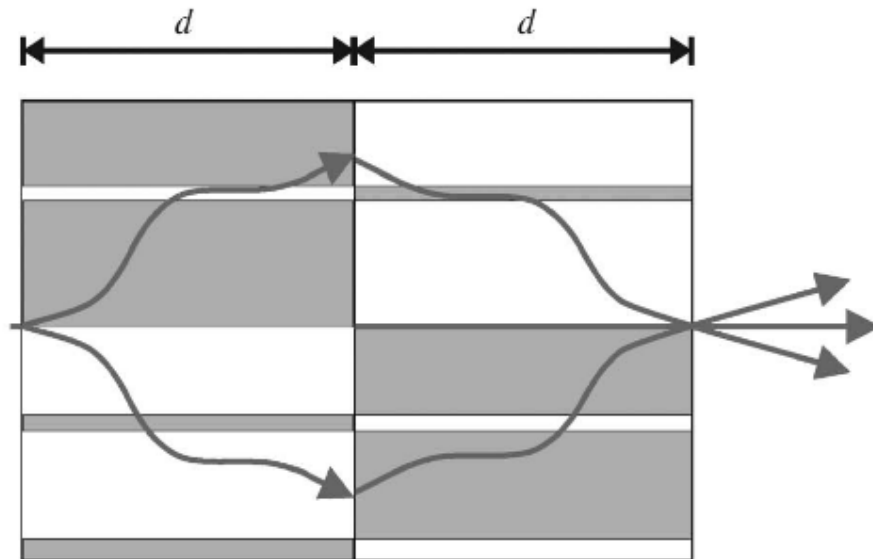


Figure 2.2: As long as two systems are complementary of each other, the optical path length of the system will be zero. Here: Gray and white are complementary media, thus the two halves are complementary systems. Figure from “Negative Refraction”, Pendry [18].

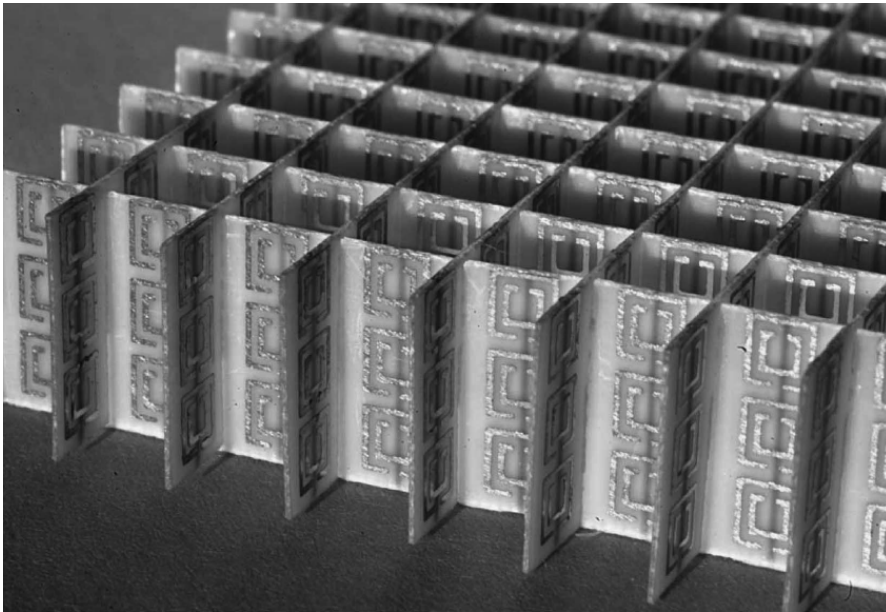


Figure 2.3: Metamaterial with split-ring structure and copper wires which exhibits frequency bands with negative ϵ and μ . Structure made at UCSD by David Smith [18].

A common definition of metamaterials is “materials that cannot be found in nature” [16]. They are artificially composite structures, designed to interact both magnetically and electrically with incident EM-fields. Furthermore, they are usually divided into two categories; photonic crystals and effective media. As the name suggests, the latter obtain their effect by tailoring the effective parameters seen by the waves. This means the structure of the media has to be much smaller than the wavelength of the light. The former relies on its periodicity to achieve effects, and works when the scale of the structure is comparable to the wavelength. A basic example is the layered two film structure, consisting of media with alternating high and low parameters. Depending on the incident light, this could fit within both definitions. When considering high frequency light, it could be seen as a photonic crystal, and when considering light with frequencies below a certain limit, it could be seen as an effective media. This limit is often, confusingly, called the metamaterial limit [25]. When discussing metamaterials and perfect lenses, it is normally implied that they are effective media.

One example of a metamaterial with negative refraction for certain frequencies is shown in figure 2.3. This is most often referred to as the split-ring resonator structure, and consists of two distinct parts [18]. The first is the metallic 3D grid, which simulates a low density plasma. The permittivity will then be negative

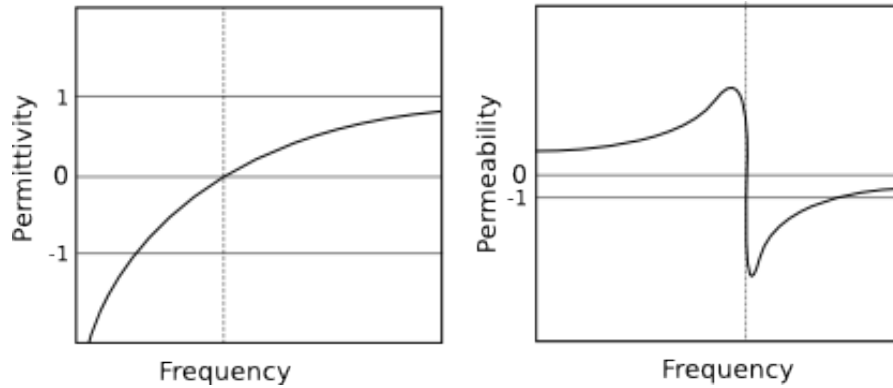


Figure 2.4: Approximate frequency respons of the metamaterial shown in figure 2.3. The plasma frequency, ω_p , is defined as where the permittivity passes through zero, and the resonance frequency for the permeability is defined the same way. Left; Permittivity. Right; Permeability

for frequencies below the effective plasma frequency, ω_p , as seen in figure 2.4. By choosing the thickness and spacing of the grid, the plasma frequency can be adjusted as desired. The important concept when reducing ω_p is the extremely low density of metal, and thus electrons, compared to the bulk case. This means ω_p can be raised to leves previously unattainable.

To achieve a magnetical respons, the split-ring resonator is introduced. Here it is important that large cross sections of the structure are covered, to capture as much of the incident magnetic flux as possible. This leads to a resonance in the frequency response of the permeability, see figure 2.4. When reaching near-optical frequencies there has been some problems in finding suitable materials which are magnetically active [26].

2.3 Perfect lens

The ray approach to the ideal perfect lens was touched upon in the last section. Here, the limitations will be seen more clearly when wave theory is applied, and the lens no longer is considered to be ideal. As stated, an ideal “perfect lens” would be a slab of a metamaterial with permittivity and permeability perfectly equal to -1 . However, all materials with negative refraction must also have large dispersion and losses [27], though the losses can be made arbitrary small [28]. This means that a realisable perfect lens would still only operate at a single frequency, and it would not have unlimited resolution. Most operate in frequency ranges below the metamaterial limit, where they can be characterized with a negative effective index of refraction, though some lie just above this limit, depending on their periodicity

to achieve the desired respons [29, 30].

In a normal lens, even though loss due to absorption is assumed to be non-existent, one will still loose effect due to reflections at the interfaces. By using the equations for the complex wave coefficients, it will be shown that this is not the case for a perfect lens. The general formulas are given as [28]:

$$\mathcal{R} = \frac{(\mu^2 k_z^2 - k_z'^2)[1 - e^{i2k_z' d}]e^{i2k_z a}}{(\mu k_z + k_z')^2 - (\mu k_z - k_z')^2 e^{i2k_z' d}}, \quad (2.10a)$$

$$\mathcal{T} = \frac{4\mu k_z k_z' e^{ik_z' d} e^{ik_z a}}{(\mu k_z + k_z')^2 - (\mu k_z - k_z')^2 e^{i2k_z' d}}, \quad (2.10b)$$

$$\mathcal{S}^+ = \frac{2\mu k_z (\mu k_z + k_z') e^{ik_z a}}{(\mu k_z + k_z')^2 - (\mu k_z - k_z')^2 e^{i2k_z' d}}, \quad (2.10c)$$

$$\mathcal{S}^- = \frac{2\mu k_z (\mu k_z - k_z') e^{ik_z a}}{(\mu k_z - k_z')^2 - (\mu k_z + k_z')^2 e^{-i2k_z' d}}, \quad (2.10d)$$

where \mathcal{R} is the reflection coefficient at the source plane, and \mathcal{T} is the transmission coefficient at the image plane. \mathcal{S}^+ and \mathcal{S}^- are the relative amplitudes of the forward and backward propagating wave inside the lens, respectively. The wavevectors are given as:

$$k_z^2 = \frac{\omega^2}{c^2} - k_x^2, \quad (2.11a)$$

$$k_z'^2 = \mu\epsilon \frac{\omega^2}{c^2} - k_x^2. \quad (2.11b)$$

In vacuum the longitudinal wavevector is given as k_z , where one must choose the positive squareroot in (2.11a). The sign of the longitudinal wavevector inside the lens, k_z' does not matter. In the case of an ideal perfect lens, both μ and ϵ are considered to be -1 , and the complex wave amplitudes simplify to:

$$\begin{aligned} \mathcal{T} &= 1, & \mathcal{R} &= 0, \\ \mathcal{S}^- &= 0, & \mathcal{S}^+ &= e^{jk_z a}, \end{aligned} \quad (2.12)$$

This means that there are no reflections from the lens, neither from the first or second interface. It also shows that the transmitted wave in the image plane is exactly equal to the field at the source, denoted by $T = 1$. The amplitude of the wave just inside the lens, given as \mathcal{S}^+ , is just the amplitude of the wave at the source, but it has propagated a distance a , leading to a possible phase difference. The total wave inside the lens is given as:

$$\mathcal{E}_1 = \mathcal{S}^+ e^{jk_z' z} + \mathcal{S}^- e^{-jk_z' z}, \quad (2.13)$$

$$\mathcal{E}_1 = e^{jk_z a} e^{-jk_z z}, \quad z \in [0, d]. \quad (2.14)$$

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This means that the field just before the wave exits the lens is given as:

$$\mathcal{E}_1(z = d) = e^{jk_z(a-d)} = e^{-jk_z b}, \quad (2.15)$$

that is, exactly as if the original wave had traveled a distance of $-b$. Since the distance from the lens to the image plane is b , the field will be:

$$\mathcal{E}_{image} = e^{-jk_z b} e^{jk_z z}, \quad (2.16)$$

$$\mathcal{E}_{image}(z = b) = e^{jk_z(-b+b)} = 1. \quad (2.17)$$

Which is exactly the same as the wave at the source. Note that this is true for all k_z , even when the wave is evanescent. If so, the results will be the same, but the wave can now be written as:

$$\mathcal{S}^+ = e^{-|k_z|a}, \quad (2.18a)$$

$$\mathcal{E}_1 = e^{-|k_z|a} e^{|k_z|z}, \quad (2.18b)$$

$$\mathcal{E}_1(z = d) = e^{|k_z|(-a+d)} = e^{|k_z|b}, \quad (2.18c)$$

$$\mathcal{E}_{image} = e^{|k_z|b} e^{-|k_z|z}, \quad (2.18d)$$

$$\mathcal{E}_{image}(z = b) = \mathcal{T} = e^{|k_z|(b-b)} = 1 \quad (2.18e)$$

As can be seen, instead of the phase being turned back, as in the propagating case, the amplification of the field inside the lens perfectly compensates for the losses outside the lens. This would not have been the case if the distance from the source to the lens plus from the lens to the image plane was different than d . Also note that the evanescent field as it is *before* it reaches the image plane is *not* a representation of the field before the source. Since there is no physical source at the image side, there can be no discontinuities in the fields, and so the lens will not be able to recreate the fields from the far side of the source. Given that the evanescent fields at the image side will be strictly decreasing, there is no way to see where the image plane is, the position of both the source and the lens must be known to calculate its location.

When constructing a perfect lens it is unrealistic to assume that all losses are negligible. Considering lossy materials, $\text{Im}\{\epsilon\} > 0$ and/or $\text{Im}\{\mu\} > 0$, one can see from equations (2.10) that reflections will start to arise, from both interfaces, and the transmission would no longer be perfect. Though, since the evanescent waves, including the reflected ones, are amplified inside the lens, one could have high transmission *and* reflection simultaneously, exceeding the total amplitude of the incident wave. This is possible, because, as stated at the end of chapter 2.1, the evanescent waves has no energy flow, thus no energy is needed to amplify them. Of course, this only applies to the steady state situation assumed here.

Including these unavoidable losses, the smallest discernable feature, defined as where $\mathcal{T} = 0.5$, can be found to be given as [31]:

$$\Delta_{min} = -2\pi \frac{d}{\ln(|\epsilon + 1|/2)}, \quad (2.19)$$

for TM-polarized waves (assuming the losses in μ are smaller than those in ϵ). This means an exponential decrease in loss is required to increase the resolution linearly. Note that since ϵ is frequency dependent, and these metamaterials are highly dispersive, the resolution will for most cases decrease rapidly when moving away from a center frequency.

Chapter 3

Methodology

As described in the article, there were several assumptions and approximations that had to be made in order to reach an accepted method of calculation. This section will not go into details of every step, but present and discuss the major difficulties. As such, it is assumed that the reader has already read the article presented in Appendix A.

3.1 Quantum theory

In optical amplification, the normal approach is to use quantum theory [15]. The incoming optical fields are considered to be photons, and the optically active material consists of individual atoms. Atoms can experience spontaneous emissions, and incident photons can be absorbed or give rise to stimulated emission. The latter of which is the origin of amplification. By describing the system with probability functions and photon flux, the macroscopic amplification can be found.

When discussing steady state evanescent waves, this method reaches an impasse. It is impractical to talk about photons, since there is zero photon flux. Instead one has to utilize the semi-classical quantum theory of atom-field interaction [32]. In this theory, the atoms are still represented as quantum mechanical systems, but the incident fields are treated classically.

Using this method, and ignoring the common step up to representation by photon flux, the following equation for the susceptibility of the active material was found:

$$\chi_a(\omega) = -\frac{\lambda_0}{2\pi} \vartheta^2 \frac{N_0}{\epsilon_0 \hbar} \frac{\gamma}{\gamma^2 + (\omega_c - \omega)^2} \left(\frac{(\omega_c - \omega)}{\gamma} + i \right) \frac{1}{1 + \frac{|\mathcal{E}|^2}{\mathcal{E}_s^2}}, \quad (3.1a)$$

$$\mathcal{E}_s^2(\omega) = \gamma_a \gamma_b \hbar^2 \frac{\gamma^2 + (\omega_c - \omega)^2}{\gamma \vartheta^2}. \quad (3.1b)$$

Except for the frequency, ω , the free space wavelength, λ_0 , the population difference, N_0 , and the electrical field strength, \mathcal{E} , these are all material constants. They have been ignored in the article, since their specific values are not important for the discussion of amplification. The atomic transition frequency is given as ω_c , and the incident fields are all considered to be at this frequency. γ is the mean decay rate of the given states, where a and b denotes, respectively, the lower and higher energy state of the atoms. \mathcal{E}_s is the saturation constant, and ϑ is the dipole moment. All the parameters are real values.

In this discussion, the important aspect of equation 3.1 is not its specific value but rather its structure. An amplification coefficient, $A(\omega)$, is defined such that when $\omega = \omega_c$:

$$\chi_a(\omega_0) = i\chi_a''(\omega_0) = -\frac{A(\omega_0)}{1 + \frac{|\mathcal{E}|^2}{\mathcal{E}_s^2(\omega_0)}}i. \quad (3.2)$$

As can be seen, only the imaginary value of the susceptibility is disturbed because of the active material. The total electrical permittivity of the medium will then be:

$$\epsilon = 1 + \chi = 1 + \chi_p' + i(\chi_p'' + \chi_a''), \quad (3.3)$$

where subscript p denotes the passive structure.

3.2 Spatial dependency

With the equation for the permittivity at hand, one can see another problem for calculating the transmission of electrical fields inside the lens. The amplification at a given place is fully dependent on the local electrical field. This is always a limitation in optical amplification, but can often be ignored if the incident and emitted fields are much lower than the saturation constant. Given the nature of this exploration, such assumptions can not be done here.

This circular dependency makes it futile to approach the problem in a purely analytical manner. Once the general transfer matrix for the lens has been found, the problem becomes numerical. By choosing a set of appropriate initial conditions for the electrical fields, an approximation for the refractive index can be found:

$$n_{approx}^2 = 1 - i\chi_p'' \left(1 - \frac{1 - \Delta\chi_g}{1 + \frac{|\mathcal{E}_{init}|^2}{\mathcal{E}_s^2}} \right), \quad (3.4)$$

where the permeability has been considered to be $\mu = -1$, and the real part of the permittivity $\text{Re}\{\epsilon\} = -1$. This approximation then forms the basis for calculating a new set of electrical fields inside the lens by using the transfer matrix. Note that the lens is still considered a single entity, which poses a problem when the refractive index is spatially dependent.

To solve this, consider dividing the lens into an infinite number of slices in the x/y -plane, each infinitesimally thin. If one now should calculate the transmission and reflection of the incident field between the vacuum and the first of these slices, one would obtain the local electric fields just inside the lens. By repeating the same method, one would eventually reach the imageplane, though infinite time is required. To make practical calculations, the slices will need to increase in thickness.

The important factor when deciding the thickness of the slices, is how fast the electrical fields are changing. If the fields can be considered invariant within each slice, this would also apply to the resulting permittivity. Thus the lens can be viewed as a layered thin film structure.

3.3 Matlab

The simulations were all done in Matlab. A function for calculating the converged permittivity was written, and a wrapper script was used to generate the results and plots. To ensure that the model was correct, it was tested with a variety of input parameters. These included parameters that would simulate known conditions, as an ideal lens and the non-compensated case, to which the answers were readily available. There was not found any inconsistencies in the model during these simulations.

An overview of the function used is given below.

```
1 function [ ...
2   Transmission,... transmission coefficients: M x 1 x It-matrix
3   Reflection,... reflection coefficients: M x 1 x It-matrix
4   Efields,... calculated fields: M x N x It-matrix
5   Permittivity,... calculated permittivity: M x N x (It+1)-matrix
6   Sp,... calculated forwards going waves: M x N x It-matrix
7   Sm... calculated backwards going waves: M x N x It-matrix
8   ] = ...
9 calculate_permittivity(...
10  Incidentkx,... list of incident wavenumbers (k_x) (size:M)
11  IncidentExtra,... list of simultaneous wavenumbers (k_x) (size:P)
12  PermEq,... equation for calculating eps(E)
13  my,... set value for the permeability
14  InitialValues,... initial values to use in the iteration
15  Wmean,... weighted mean used in the calculations
16  MaxIt,... maximum number of iterations to run
```

3.3. MATLAB

```
17 Dim,... physical dimensions: [d a b]
18 IFamp,... incident field amplitude at source
19 N,... number of slices
20 genphi... the general transfer matrix
21 )
22
23 %% Check incoming parameters %%
24 %% Set constants and create storage matrices %%
25
26 %% Start the iteration %%
27 for it=1:MaxIt
28     % Calculate the wavevectors in vacuum
29     k0list(:,1) = sqrt(1 - Incidentkx.^2);
30     k0list(:,2:P+1) = ones(N,1) * sqrt(1 - IncidentExtra.^2);
31
32     % Step through all the incident k_x
33     for u=1:M
34         % Calculate the wavevectors inside the lens
35         kllist(1) = -sqrt(eps(u,:,it)*my - Incidentkx.^2);
36         kllist(2:P+1) = -sqrt(eps(u,:,it)*my - IncidentExtra.^2);
37         % Step through all simultaneous waves
38         for s=1:P+1
39
40             % Set the wavevectors
41             k0 = k0list(u,s);
42             kl = kllist(s);
43             % Calculate the transfer matrix
44             varphi = zeros(2,2,q);
45             % The first interface
46             varphi(:, :, 1) = genphi(k0,kl(1),1,eps(1),a);
47             % The interfaces inside the lens
48             for t=1:1:r-1
49                 varphi(:, :, t+1) = ...
50                     genphi(kl(t),kl(t+1),eps(t),eps(t+1),(d/q))...
51                     * varphi(:, :, t);
52             end
53             % Calculate the total transfer matrix
54             phi = [exp(j*k0*b),0;0,exp(-j*k0*b)] * ...
55                 genphi(kl(q),k0,eps(q),1,(d/q))*varphi(:, :, end);
56             % Find reflected and transmitted field amplitudes
57             Reflected(u,it,s) = - IFamp * phi(2,1)/phi(2,2);
58             Transmitted(u,it,s) = ...
59                 phi(1,1)*IFamp + phi(1,2)*Reflected(u,runde,s);
60             % Find the reflection and transmission coefficients
61             Reflection(u,runde,s) = ...
62                 abs(Reflected(u,runde,s)) / IFamp;
63             Transmission(u,runde,s) = ...
64                 abs(Transmitted(u,runde,s)) / IFamp;
65
```

```

66         % Find the fields inside the lens
67         for sl=1:N
68             Sp(u,sl,it,s) =...
69                 ( varphi(1,1,sl)*IFamp +...
70                 varphi(1,2,sl)*Reflected(u,it,s) )...
71                 * exp(j*k1(sl)*(0.5*d/N));
72             Sm(u,sl,it,s) =...
73                 ( varphi(2,1,sl)*IFamp + ...
74                 varphi(2,2,sl)*Reflected(u,runde,s) )...
75                 * exp(-j*k1(sl)*(0.5*d/q));
76             Efields(u,sl,it,s) =...
77                 Sp(u,sl,it,s) + Sm(u,sl,it,s);
78         end
79     end
80 end
81 %% Find the new approximation for the permittivity %%
82 % Initial values are located in eps(:, :, 1)
83 for sl=1:N
84     eps(:,sl,it+1) =...
85         (1-Wmean)*PermEq(sum(Efields(:,sl,it,:),4)) +...
86         Wmean*eps(:,sl,it);
87 end
88
89 %% Check for convergens using RMSD %%
90 % If the permittivity has converged, break out of the for-loop
91 end
92
93 end
94 % End function

```

Appendix A

Article

In this appendix the article written in conjunction with this master thesis is presented. It has not yet been released, as it lacks input from one of its co-authors (not credited in the current version). Hopefully it will be presented for publishing in the online journal “Optics Express” during the summer of 2009.

The effect of gain saturation in a gain compensated perfect lens

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Abstract: The transmission of evanescent waves in a gain-compensated perfect lens have been discussed. In particular, the impact of gain saturation have been included in the analysis, and a method for calculating the fields of such nonlinear systems have been developed. Gain compensation clearly improves the resolution; however, a number of nonideal effects arises as a result of gain saturation. The resolution associated with the lens is strongly dependent on the saturation constant of the active medium.

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OCIS codes: (260.2110) Electromagnetic theory; (160.4670) Optical materials; (160.3918) Metamaterials

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I. Introduction

Metamaterials have large potential in electromagnetics and optics due to their possibility of tailoring the permittivity and permeability. This enables construction of, for example, media with negative refractive index n [1], perfect lenses [2], invisibility cloaks [3, 4], and other exciting components transforming the electromagnetic field [5]. Unfortunately, the performance of such devices is strongly limited by loss. Although causality and passivity do not prohibit negative index materials with arbitrary low losses [6], in practice it is difficult to fabricate materials with high figure of merit $FOM = -\text{Re}n/\text{Im}n$, especially at optical frequencies [7, 8, 9, 10]. For perfect lenses, losses limit the amplification of evanescent waves associated with large spatial frequencies. This means that the resolution of a perfect lens is strongly limited by losses [11, 12]. It has therefore been suggested to introduce gain into the metamaterials [7, 13, 14, 15]. This could be a promising approach provided the intrinsic losses can be made relatively small so that compensation by a realistic amount of gain is possible.

Both permittivity ϵ and permeability μ may involve losses; thus in general, gain may be needed to reduce *both* $\text{Im}\epsilon$ and $\text{Im}\mu$. For a perfect lens it is generally not sufficient e.g. to reduce $\text{Im}\epsilon$ below zero such that the refractive index $n = \sqrt{\epsilon\mu}$ becomes real. Thus, in general, gain compensation might not be straightforward for optical frequencies, where gain media capable of reducing $\text{Im}\mu$ may be difficult to achieve. However, as long as the object to be imaged is one-dimensional, only one polarization (TE or TM) of the electromagnetic field is required. Then, provided the lens is sufficiently thin, only one of the parameters ϵ and μ is relevant for the transmission of evanescent waves [2, 12]. Choosing TM polarization, only ϵ matters, enabling gain compensation with dielectric, active media.

Introducing the necessary activatable material into a metamaterial leads to a change not only in the imaginary part but also the real part of the permittivity, and should therefore be kept in mind while designing the metamaterial structure. Other critical considerations include matching of the negative refractive index frequency band to that of the gain lineshape function, the level of loss possible to overcome in the absence of saturation, and the saturation constant of the active medium.

There has been several attempts to create a perfect lens in the near IR-spectrum the last years [7, 8, 9, 16]. The FOM currently reported is of the order of 3 for the frequency where $\text{Re}n \approx -1$ [8, 9]. With these values, traditional optical amplifiers such as Erbium-doped silica or gas laser amplifiers will not be able to reduce the intrinsic losses significantly. Theoretical studies have shown that it may be possible to raise the FOM at near IR-frequencies to as much as 20, while keeping $\text{Re}n \approx -1$ [17]. It has also been reported that laser dyes, or dye-Ag aggregate mixtures, may reach amplifications of up to $\text{Im}n \approx -0.06$ at near IR-frequencies [14, 18]. Taking into account these reports, this article will not speculate further on the choice and design of the metamaterial, but merely assume an appropriate material is physically feasible.

The main purpose of our work is to consider the transmission of evanescent waves in a practical, gain-compensated perfect lens. Clearly, gain saturation is highly relevant in this context, and we demonstrate how this effect leads to limited amplification of evanescent fields, and therefore limited resolution. We calculate the resolution as a function of the saturation constant of the active medium, and also the detailed field profile and reflections from the lens.

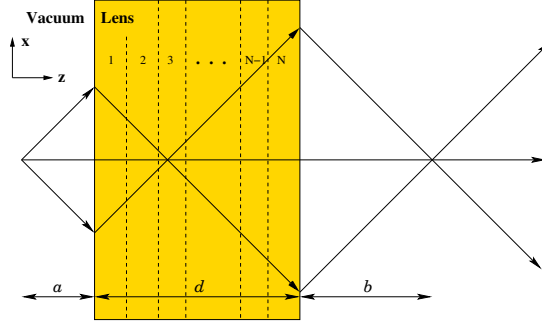


Fig. 1: Perfect lens in vacuum. The parameter d is the thickness of the lens, a and b are the distances from the source to the lens, and from the lens to the image plane, respectively, and are governed by the equation $d = a + b$. The numbers 1 through N indicate the different slices. The lens is considered to be infinite in the xy -plane.

II. Nonlinear gain saturation and field calculations

The relative permittivity of the active metamaterial is given by:

$$\varepsilon(\omega) = 1 + \chi_p(\omega) + \chi_a(\omega), \quad (1)$$

where $\chi_p(\omega)$ denotes the susceptibility of the passive structure, and $\chi_a(\omega)$ the contribution from the active part. This splitting into a passive and an active part is chosen such that $\chi_a(\omega) = 0$ when the pump is turned off.

When discussing evanescent fields in active media, the gain saturation must be formulated directly in terms of the electric field, not the photon flux. With the help of the semi-classical approach in quantum optics [19], we derive the following expression for the active susceptibility:

$$\chi_a(\omega) = \frac{A(\omega) \left(\frac{\omega - \omega_0}{\gamma} - i \right)}{1 + \frac{|\mathcal{E}|^2}{\mathcal{E}_s(\omega)^2}}. \quad (2)$$

Here ω_0 is the center frequency of the lineshape function, ω the frequency of the incident light, γ the inverse lifetime of the electron excitations, \mathcal{E} the complex electric field, and $\mathcal{E}_s(\omega)$ the saturation constant of the active medium. The saturation constant depends on the selected gain material and pumping level. For dye amplifiers a normal value is $\mathcal{E}_s(\omega_0) \sim 10^7$ V/m [18, 20], corresponding to energies in the kW/cm² regime for propagating waves. The numerator in Eq. (2) describes the susceptibility at zero field (small-signal) conditions, i.e. $\mathcal{E} \approx 0$ everywhere. The numerator contains the line shape function and several material parameters; irrelevant factors are absorbed into the function $A(\omega)$. For $\omega = \omega_0$ and $\mathcal{E} \approx 0$, $A(\omega)$ is simply $-\text{Im} \chi_a(\omega)$. Both functions $A(\omega)$ and $\mathcal{E}_s(\omega)$ are real-valued.

Throughout this paper, we will consider a single frequency $\omega = \omega_0$. Eq. (2) now reduces to:

$$\chi_a(\omega_0) = \frac{-iA(\omega_0)}{1 + \frac{|\mathcal{E}|^2}{\mathcal{E}_s(\omega_0)^2}}. \quad (3)$$

This means that the real part of the total permittivity will be independent of the pumping and local field distributions.

We consider a perfect lens slab which extends to infinity in the xy -plane, and has thickness d in the z -direction, see Fig. 1. The source is located a distance a (with $a < d$) from the input end of the lens. The incident field from the source will be taken to be a superposition of plane TM-waves, with the magnetic field in the y -direction. Provided $\omega d/c \ll 1$ and $|\mu| \sim 1$, the specific value of μ is not critical for the operation of the lens for evanescent waves [2, 12]. The permittivity is given by (1), (3), and $\text{Re } \chi_p(\omega_0) = -1$. The remaining losses after gain compensation (in the absence of saturation) is described by the parameter:

$$\Delta\chi = \frac{\text{Im } \chi_p(\omega_0) - A(\omega_0)}{\text{Im } \chi_p(\omega_0)}. \quad (4)$$

To find the steady state solution to Maxwell's equations for our nonlinear medium, an iterative approach can be used. In the zeroth iteration, the field is simply set to zero everywhere. (Alternatively, the initial field could be set to infinity. This does not give any significant difference in performance, in terms of the required number of iterations.) In the next iteration, Eqs. (1) and (3) are used to find an approximation of the permittivity of the lens. Taking the incident electric field to be unity (normalized), we can now compute the field everywhere. Now we may repeat the iteration; calculate a new approximation of the permittivity from the field, computing the resulting field from this new structure, etc. The iteration procedure has an inherent stability, as growing fields leads to less gain in the medium, and vice versa.

Nevertheless, inaccuracies and even divergence may arise if the number of slices is too low, so that the field no longer can be treated as constant in each slice. In the case $\Delta\chi = 0$ the convergence seems to be extremely sensitive to the number of slices. An alternative to increasing the number of slices to a very high number, is to regularize the iterative approach as follows: Rather than setting the permittivity to that resulting from the field in the previous iteration, it can be set to a weighted mean of the permittivities as resulting from the last two iterations. In our computations, the permittivity in iteration i (for $i \geq 2$) was set to 0.3 times the permittivity calculated by the field from iteration $i - 1$, plus 0.7 times that resulting from iteration $i - 2$. For $i = 1$ the permittivity was calculated using the field from iteration 0. This resulted in convergence after $\sim 10 - 20$ iterations.

If the root mean square deviation of three successive iterations were within a specified limit (10^{-7} for the relative permittivity in our computations), and strictly decreasing, the results were deemed converged. Note that when the fields of subsequent iterations coincide, we have a valid solution to Maxwell's equations with constitute relation as implied by (1) and (3). Thus, provided convergence, the choice of initial field (in iteration 0) does not have any impact on the results.

In general, the field in one iteration, and therefore the permittivity in the next iteration, will be dependent on z . Thus the computation of the field in the next iteration requires the solution to Maxwell's equations in an inhomogeneous structure. For this calculation, we employ a transfer matrix technique, considering the different plane waves separately. The lens is divided into N slices in the xy -plane, as seen in Fig. 1. These slices must be sufficiently thin, such that the permittivity inside each slice is approximately uniform. For this condition to be valid for the next iteration as well, the resulting field from the present iteration must also be approximately constant. This means that $k_x d/N \lesssim 1$ for all transverse wavenumbers k_x to be considered.

Solving Maxwell's equations in each slice, and connecting the fields of adjacent slices with the boundary conditions, we find the following transfer relation between slice j and $j + 1$:

$$\begin{bmatrix} \mathcal{S}_{j+1}^+ \\ \mathcal{S}_{j+1}^- \end{bmatrix} = \frac{1}{2} \begin{bmatrix} e^{ik_j d_j \left(\frac{k_j}{k_{j+1}} + \frac{\epsilon_j}{\epsilon_{j+1}} \right)} & e^{-ik_j d_j \left(\frac{k_j}{k_{j+1}} - \frac{\epsilon_j}{\epsilon_{j+1}} \right)} \\ e^{ik_j d_j \left(\frac{k_j}{k_{j+1}} - \frac{\epsilon_j}{\epsilon_{j+1}} \right)} & e^{-ik_j d_j \left(\frac{k_j}{k_{j+1}} + \frac{\epsilon_j}{\epsilon_{j+1}} \right)} \end{bmatrix} \begin{bmatrix} \mathcal{S}_j^+ \\ \mathcal{S}_j^- \end{bmatrix}. \quad (5)$$

Here the slices of the lens are denoted by subscript j , where $1 \leq j \leq N$. Eq. (5) is also valid for the vacuum section between the source and the lens, denoted by $j = 0$. In Eq. (5) the transfer matrix will be called \mathcal{M}_j ; \mathcal{S}_j^+ is the forward travelling electric field wave, while \mathcal{S}_j^- is the backward travelling wave, as evaluated at the left-hand end of slice j . The longitudinal wavenumbers k_j , and the thicknesses d_j , are given by:

$$k_0^2 = k_{N+1}^2 = \frac{\omega^2}{c^2} - k_x^2, \quad k_j^2 = \epsilon_j \mu_j \frac{\omega^2}{c^2} - k_x^2 \quad \text{for } 1 \leq j \leq N, \quad (6a)$$

$$d_0 = a, \quad d_j = \frac{d}{N} \quad \text{for } 1 \leq j \leq N, \quad (6b)$$

where k_x denotes the transversal wavenumber, and c is the vacuum velocity of light. The wavenumbers k_0 and k_{N+1} must be chosen nonnegative, and the sign of k_j does not matter. Given the amplitudes of the forward and backward propagating waves in slice j , the amplitudes in slice $j+1$ can be found. By the successive application of (5), we find:

$$\begin{bmatrix} \mathcal{T} \\ 0 \end{bmatrix} = \begin{bmatrix} e^{ik_0 b} & 0 \\ 0 & e^{-ik_0 b} \end{bmatrix} \prod_{j=N}^0 \mathcal{M}_j \begin{bmatrix} 1 \\ \mathcal{R} \end{bmatrix}. \quad (7)$$

where 1 is the incident field amplitude at the source, \mathcal{R} is the reflection coefficient at the source plane, and \mathcal{T} is the transmission coefficient at the image plane. The matrix written out explicitly in Eq. (7) propagates the wave from the back end of the lens to the image plane (see Fig. 1). Once the total matrix in Eq. (7) has been found, it is straightforward to calculate the unknowns \mathcal{T} and \mathcal{R} , and therefore the field amplitudes in all slices.

III. Numerical results

For simplicity and generality the frequency ω_0 is normalized such that $\omega_0/c = 1$. The thickness of the lens was chosen such that $\omega_0 d/c = 2\pi/10$, and we take $N = 20$. The permeability was set to $\mu = -1$ (although the specific value does not matter significantly for evanescent TM waves).

The transmission coefficient is shown in Fig. 2. It is easy to see significant improvements as a result of gain compensation, when the the saturation constant \mathcal{E}_s is larger than the electric field at the source. Nevertheless, for a fixed amplitude of the incident field, Fig. 3 indicates that an exponential increase in the saturation constant is needed to increase the resolution linearly.

The reflection coefficient is plotted in Fig. 4. We note that significant reflections arise even for the spatial frequencies where the transmission is relatively large.

As can be seen in Fig. 5, the field distributions of the two evanescent components in the lens increase roughly exponentially with $+z$ or $-z$, respectively. For small spatial frequencies, where the lens is essentially perfect, the one increasing in the $+z$ direction dominates. For higher spatial frequencies the two components have a similar amplitude, such that the total field and therefore the imaginary part of the permittivity start to look like a lopsided U-shaped valley.

In general, different plane wave components of the source will couple to each other through Eq. (3). To simulate the gain-compensated lens under more real-world conditions, it was therefore tested with several waves traversing the lens simultaneously. The transmission of one wave as a function of k_x , in the presence of another wave at a fixed k_x , is shown in Fig. 2. The amplitudes of both waves were set to 1/2 to keep the total field at the source equal to the case with a single wave. From a number of simulations with several waves, it was discovered that, as a worst-case estimate, one can judge whether the lens operates as required by assuming that the mode with largest k_x has amplitude equal to the sum of the amplitudes at the source.

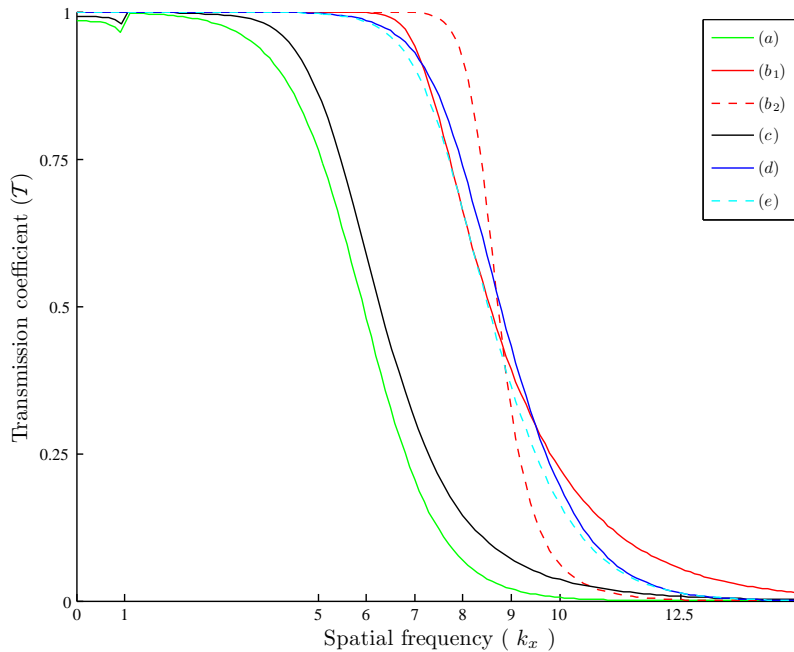


Fig. 2: Transmission coefficient, $|\mathcal{T}|$, when $\omega_0/c = 1$ (normalized), $\omega_0 d/c = 2\pi/10$, $\text{Im}\chi_p(\omega_0) = 0.05$, and $N = 20$: (a) Non-compensated lens; (b₁) $\mathcal{E}_s = 10$, $\Delta\chi = 0$; (b₂) same as b₁ but after only 3 iterations; (c) $\mathcal{E}_s = 1$, $\Delta\chi = 0$; (d) $\mathcal{E}_s = 10$, $\Delta\chi = 0.3$; (e) $\mathcal{E}_s = 10$, $\Delta\chi = 0$, with an extra wave, $k_x = 8.5$, both have amplitude $1/2$. All results except (b₂) after convergence (< 20 iterations).

IV. Conclusion

We have developed a method for calculating the transmission, reflection, and detailed field profile of a gain-compensated perfect lens, taking into account gain saturation. The gain compensation clearly improves the resolution limit of perfect lenses. However, due to gain saturation, a number of non-ideal effects arise, included limited resolution and reflections. The nonideal effects depend heavily on the saturation constant and/or the field strength of the source.

If there are different waves traversing the lens at the same time, they will interact through the material. Waves with a spatial frequency close to the resolution limit will have the greatest impact. As a rule of thumb, it is enough to know the collected amplitudes of the waves at the source, and then assume the mode with the largest spatial frequency has this amplitude. If this single wave is transmitted, then so will any superpositions of waves with less spatial frequencies and the same collected amplitudes.

The calculations in this work was performed for TM polarization and a one-dimensional source. For a two-dimensional source with both polarizations, both dielectric and magnetic losses should be compensated, that is, $\text{Im}\epsilon$ and $\text{Im}\mu$ must be reduced. Although the theory in this paper can trivially be extended to this situation, there may be serious practical problems associated with the fabrication of such active media for optical frequencies.

For a noncompensated lens, the maximum spatial frequency resolved by the lens is approximately $-\frac{1}{d} \ln \frac{|1+\epsilon|}{2}$ [11, 12]. Thus, for a fixed d , an exponential decrease in the losses is necessary to increase the resolution linearly. From our numerical results, a similar relation is approx-

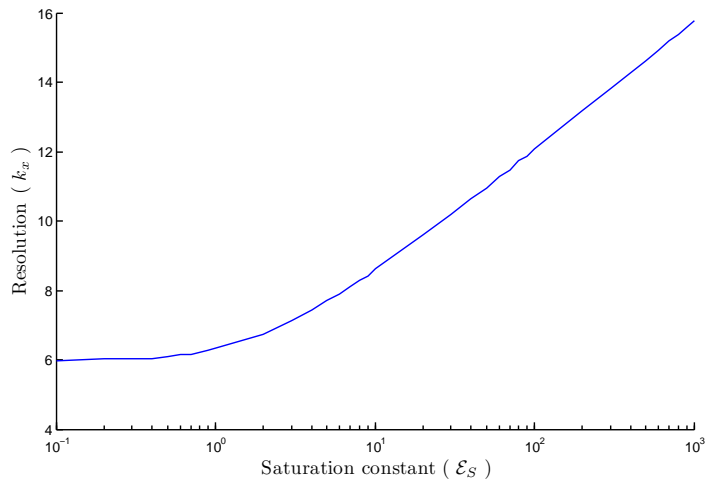


Fig. 3: The resolution of the lens as a function of the saturation constant. The resolution is defined as the k_x value where the transmission is $|\mathcal{T}| = 1/2$. Parameters: $\omega_0/c = 1$, $\omega_0 d/c = 2\pi/10$, $\text{Im} \chi_p(\omega_0) = 0.05$, $\Delta\chi = 0$, $a = b = d/2$, and $N = 20$.

imately valid for the saturation constant; to achieve a linear improvement in the resolution, the saturation constant must increase exponentially. This clearly shows the difficulties of achieving very high resolution.

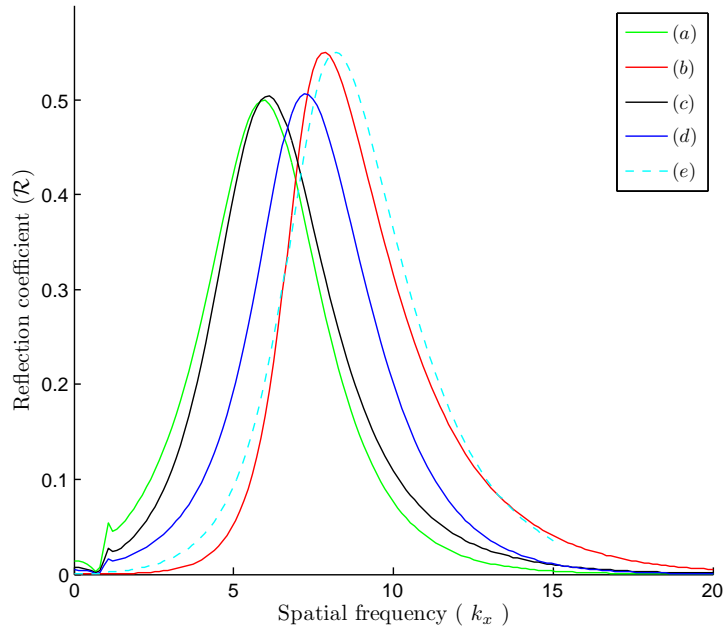


Fig. 4: Reflection coefficient, $|\mathcal{R}|$, after convergence for the same cases as those in Fig. 2.

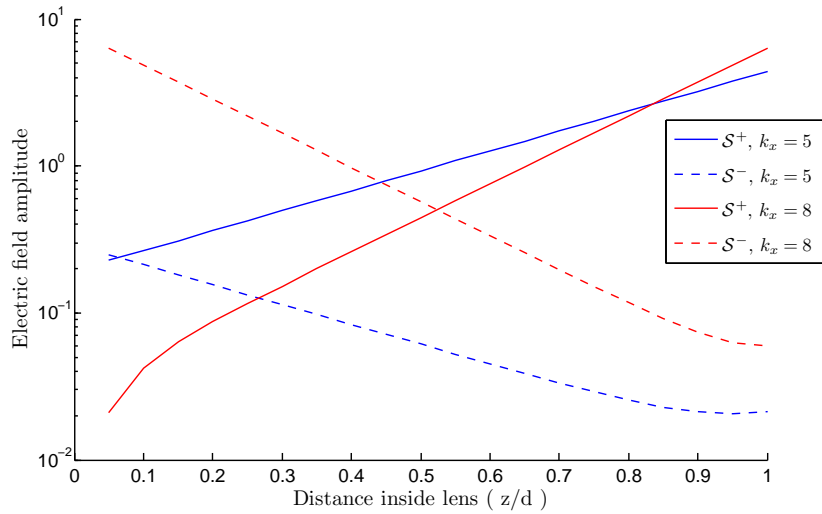


Fig. 5: The distribution of the two components of the evanescent field in the lens, after convergence. Parameters: $\omega_0/c = 1$, $\omega_0 d/c = 2\pi/10$, $\text{Im}\chi_p(\omega_0) = 0.05$, $\Delta\chi = 0$, $\mathcal{E}_s^\ell = 10$, $a = b = d/2$, and $N = 20$.

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