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Analyse av startbetingelser og vannverdier for en stokastisk korttidsmodell

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Abstract

A new model for short term hydro power scheduling (the SHARM model) has been developed and is being tested in an ongoing research project at SINTEF Energi. The SHARM-model accounts for uncertainty in market price and reservoir inflow, and will through this give a better basis for decisions and more robust plans when multiple possible strategies should be considered. With some conditions a stochastic model will give more valuable results. When there are low reservoirs and low inflow the risk of committing to more production than what can be delivered is great. Different forms of water value expressions will also influence this. It is interesting to study how a stochastic model would solve this versus a deterministic model.

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Sammendrag

Denne oppgaven har forsøkt, ved hjelp av Sintefs SHARM modell, å finne betingelser hvor en stokastisk fremgangsmåte for produksjonsplanlegging av vannkraft vil gi målbare fordeler fremfor en deterministisk fremgangsmåte. En deterministisk modell tar ett sett med inndata og gir deg det beste planen, gitt at nøyaktig det du fortalte modellen skjer. Dette er slik dagens kommersielle modeller fungerer og når inndataen er værmeldinger og markedsforustigelser, sier det seg selv at resultatene ikke kan brukes uten å modereres noe. Konsekvensen av detter er at produksjonsplanlegging i dag krever mye skjønn.

Inndataen i en stokastisk modell er en sannsynlighetsfordeling med mange mulige utfall for pris og tilsig. Modellen veier ulike scenarioer mot hverandre og gir et resultat som i en hvis grad tar høyde for usikkerheten. Målet er at denne dataen skal gi et bedre grunnlag til å fatte beslutninger i produksjonsplanleggingen enn det de deterministiske modellene gir. Fokuset til optimaliseringene i denne oppgaven har vært bruken av forskjellige vannverdiformer, og effekten av startmagasinnivået. Optimaliseringene har blitt gjort med både stokastisk og deterministisk inndata og resultatene av dette sammenliknet.

En vannverdisensitivitetsstudie er blitt gjort for å finne ut effekten av to forskjellige måter å uttrykke vannverdier, uavhengige vannverdier og uavhengige vannverdifunksjoner. Den første er en statisk vannverdi, den andre er avhengig av magasinnivået. Vannverdiene ble hevet og senket og testet mot to forskjellige prisprofiler. Resultatene viste at effekten av de mer ekstreme vannverdiene ble dempet av uavhengige vannverdifunksjoner og av en prisprofil med større variasjon. De stokastiske og de deterministiske fremgangsmåtene reagerte veldig likt på inndataen.

Den andre analysen som ble gjort var av startmagasinnivået. Her ble startmagasinene senket til de var nesten tomme. Den stokastiske modellen viste seg å være mer forsiktig enn den deterministiske og ga de beste resultatene når tilsiget var lavt. Forskjellen ble ikke spesielt signifikant før startmagasinnivået var ekstremt lavt.

Analysis of Initial Values and Water Values for a Stochastic Short Term Model

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Summary
This thesis has sought to find conditions where a stochastic approach to production planning of hydro power will give measureable benefit compared to a deterministic approach, using Sintefs SHARM model.
A deterministic model takes one set of inputs and tells you the best course of action if that exact course of events transpires. This is how the commercial models used today works and when the input data is forecasts for meteorological and market data it goes without saying that the output from the deterministic model cannot be used without some moderation. As a consequence production planning today require a great degree of human touch.

A stochastic model uses a range of possible outcomes as input, weighs them by probability and gives solutions that take some of the uncertainty into account. The goal is that this data will aid in the production planning process more that the deterministic.
The focus in the optimizations done has been the use of different water value expressions, and the effect of the initial reservoir level. Optimizations have been done with both stochastic and deterministic input, and the results compared.

A water value sensitivity study tried to determine the effect of two different water value expression form, independent water values and independent water value functions. The first is a static water value, the second dependent of reservoir level. Water values were raised and lowered and tested against two different price profiles. The results showed that the effect of extreme water values was dampened by independent water value functions and a price profile with large variations. Stochastic and deterministic approached reacted very similarly to the input data.

The second analysis was of initial reservoir levels, where the starting reservoirs were lowered to almost empty. Here the stochastic approach proved to be more cautious, giving the best results when inflow was low. The differences only really stood out when initial reservoir levels were extremely low.

Contents

Theory

Power markets

The Nordic power exchange is where producers may sell their power. It is divided into two parts, a financial market called NASDAQ OMX and a physical market run by Nord Pool Spot. In the financial market one may trade power as far as six years ahead in time. To manage their risk producers will sell some production ahead of time. There are also several financial instruments that can be traded on NASDAQ OMX.

The physical market consists of a day-ahead market (Elspot) and an intraday market (Elbas). Power sold in the day-ahead market still accounts for a substantial part of the total produced volume in the Nordic system.

The day-ahead market is cleared by 14:00 every day. Producers have until 12:00 to deliver their bids in the form of a bid matrix. In this bid matrix each hour has a set of prices and a corresponding production. Each participant in the market sends one bid matrix with their aggregated production or consumption. The system price is then calculated by finding the intersection between the aggregated sale and buy curves. The system price is calculated with an assumption of unlimited transmission capacity in the system (Wangensteen, 2011). If the power flow on a line between price areas then exceeds its capacity the price on each side of that line will be changed to facilitate the flow. In the deficit area the price is increased to bring up production, and in the surplus area the price is decreased.

After the market clearing the producers have a commitment to deliver the power they have sold. The sold amount is found by interpolation between the two closest price points. The day-ahead market uses a marginal price, meaning every bid gets the same clearing price. The time between the decision on what to bid to the time of delivery can be as much as 36 hours. Many things may change in this time. From a producer standpoint the most notable is the weather. More rain than expected may drastically increase inflow to smaller reservoirs, which in turn may restrict planned production upstream, or force unplanned production downstream. Failure of equipment can also pose a large problem. These unforeseen circumstances may make it impossible to fulfil the commitment made in the day-ahead market.

The intraday market allows producers to trade with each other up to one hour before delivery. The capacities that created price difference in the day-ahead market still applies in the intraday market. This means that you can sell power to connected areas in a deficit area, but not buy. Trading within your area has no restrictions. The intraday market is a continuous auction where a buyer will get the price of the cheapest seller until their order is filled or their price not met.

If a producer cannot meet the commitment from the day-ahead market and not trade their way to balance in the intraday market they can plan with an unbalance. In practice this means they are committing their unbalance to the regulating power market. The regulating market is part of the balance market and is also called tertiary reserve. This is what the transmission system operator (TSO) use to balance the power system when there are faults on lines, large generators or pumps do not come on line or unexpectedly fall out or when demand differs from the forecast. Participants in the regulating power market submits a bid matrix of available regulation at a price for every hour. These are both up and down regulation and equates to buy and sell bids. The TSO will activate each bid as they see fit and the price for all activated bids will be the price of the highest, in the case of up regulation, or the lowest, in case of down regulation, of the activated bids at that time. Every hour will have a regulating power price for each area which is the average price for regulating power that hour. This is the price you get for your unbalance when planning with an unbalance.

Water values

The resource of hydro power is of course water. Over the course of a year a producer may only use as much water as enters the system in that time, lest they end the year with depleted reservoirs, and have that much less water to spend the following year. As a result of this a producer has a limited amount of production over a year, and to maximize profits must only sell that power in the hours with the highest prices.

Given a single reservoir and generator with a degree of regulation of one, meaning the reservoir storage capacity is equal to its yearly inflow (no risk of spillage), and a usage time of 1000, meaning 1000 hours of production will spend the yearly inflow, the optimal operation would be to only sell power in the 1000 highest priced hours of the year. The price of the lowest of those 1000 hours is the minimum price at which production should be sold. Knowing this one can say that whenever a better price than that can be attained one should sell. This price is what is called the water value.

Each reservoir will have its own water value, and the exact value is decided by a great many factors. Simply, as stated in the previous paragraph the water value is the best price one can expect to get for the water. If a reservoir is nearly full one cannot wait for the ideal price and the water value will have to be lowered to a level that gives enough production to prevent overflow; a river power plant that has to produce all the water that enters during the summer will have a low water value, while a large reservoir plant can save the water for the higher priced winter months and subsequently will have a high water value. From this it is also evident that inflow has an effect on the water values.

To find the optimal operation of a hydro system the profit from production over the optimization period plus the value of stored water remaining is maximized. From this one can see that the water value needs to represent, not the value today, but the value at the end of the optimization period. Because of this the water value depends on the operation of the system. In more complex systems each water value is also dependent on the reservoir levels of the other reservoirs in the system. In cascaded systems one reservoirs water value will depend on the reservoir above and vice versa.

The main objective of the water value is to represent the future after the optimization period. They are made to reflect the results from the long term models, and through that the long term strategy. Calculating water values is a large operation and not something that can be done every day. It is common to calculate new water values once a week. If the forecasts are mistaken the reservoir levels after a few days may differ greatly from the expected development at the time of water value calculation, and recourse may be needed that simple water value descriptions will not be able to reflect. Because of this the water values needs to be robust and able to give good results, even when inflow and prices deviate from the prognosis from which the water values were calculated. The more information held in the water values, the better basis the optimization has to decide whether the water is best spent now or later. There are three ways of expressing water values, each one more detailed than the last.

Independent water values

The simplest way of expressing water values is the independent water value. This can be viewed as one dimensional and contains only one water value per reservoir that is unchanged throughout the optimization period. This water value is found by setting a target for the reservoir level at the end of the optimization period based on price forecasts for the year and the long term strategies. The water value is the value that gives the desired production during that time based on the expected price in the period. This is a large simplification, and while it is effective, may not give optimal results. The independent water values are not robust, and will not trigger additional production when inflow is higher or prices lower than the original prognosis and the reservoir levels rise. Another weakness is that it attributes a too large value to a full reservoir in times of low prices. Especially on reservoirs with a low degree of regulation where the optimization will end the period with brimful reservoirs. Independent water values does not reflect risk of spillage.

Independent water value function

The second option is the two dimensional independent water value function. These water values are dependent on the reservoir level. They are commonly expressed as a straight line from a water value of zero at full reservoir level and increasing with lower reservoir levels. The independent water value function will to a much larger degree keep the reservoirs on the level where the long term strategies wants them. Allowing reservoirs to rise will bring the water value down and incite production, while draining them will stop production unless the prices are high enough to warrant the production. The independent water value function is more robust than independent water values and is able keep reservoirs within the desired limits even when price and inflow forecasts miss their mark.

Dependent water values

Dependent water values of cut files is the last and by far most complicated of the water value descriptions. The dependent water values are not only dependent on the reservoir level of the reservoir in question, but also on the reservoir level of all the other reservoirs in the hydro system.

For a single reservoir a graph of expected income as a function of end-reservoir level can be found. This is the red graph in figure 1. At different reservoir levels the marginal value can be found, expressed by u1, u2 and u3 in the figure. These lines can be added to the optimization model as restrictions and will act as water values. For one reservoir the dependent water values will be non-linear and two dimensional. For two reservoir it will be three dimensional as the right graph in figure 1 shows. For three reservoirs it will be four dimensional and so on.

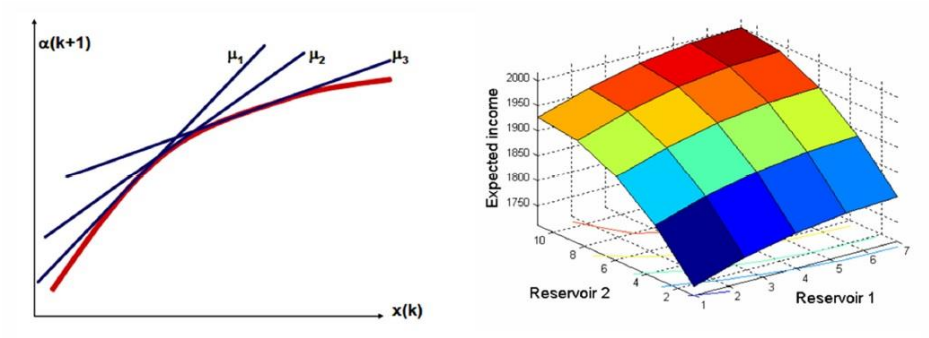

Figure 1: Expected income for one reservoir expressed by three cuts, and for two reservoirs expressed by 16 cuts. (Doorman/Fosso, 2013)

From a production planner's point of view the use of cut files mask a lot of the inner workings of the optimization model, as it is difficult to know which cuts have been used and to know just why the model will trigger production from a given reservoir at a specific time. The other methods are more transparent, as they make it much more predictable when production should take place.

Hydro scheduling

The starting point of planning hydro production is knowing how much production one has available. Production is measured in power, a term that means energy per second, and can be found through the formula for potential energy E=mgh where m is mass, g is the gravitational constant and h is the height. This formula could yield the total stored energy of a reservoir, but to find the power one needs to find the energy of the amount of water that passes through the power plant in a single second. The mass of the water flowing in a second is found by p*Q where p is the density of water (1000 kg/m^3) and Q is the volume-flow (m^3/s) dictated by the tunnel cross section. Because of losses in the tunnel, in the turbine and electrical losses in the generator all of the energy cannot be utilized. This loss is expressed by an overall efficiency n. The end result is the formula

$$P=npQgh$$

The height is the difference between the inlet and outlet water surface and varies with the water level of the reservoir, as restricted by the upper and lower limits of regulation for the reservoir, known as HRV and LRV respectively. These are bounds set by the NVE during the concession process with environmental concerns in mind. A common expression in this regard is head. Head is the energy per unit mass of water and is related to the velocity of moving water (or proportional to the height in case of static head). (Doorman, 2013. When placing a reservoir there is often a tradeoff between catchment area and head. A reservoir built high in the mountains will have a large head, but a small catchment area and subsequently a small inflow. Placing the reservoir lower along the watercourse will increase inflow but decrease the head.

The goal of hydro scheduling is to find the operation of the power system that yields the biggest profit. To find this optimal operation short term earnings, meaning the immediate production at the price it could be sold, and the value of stored water, the remaining energy stored sold at the expected future prices, are maximized. The mathematical formulation below is a simplified version of this to give an idea of the method.

$$Max\left[\sum_{t=0}^{T_k}\left(p_t \cdot \left(q_{s,t} - q_{p,t}\right) - c_{start,t} - c_{penalty,t}\right) + R_T\right]$$

T_k	= Total amount of time steps
p_t	= Price at time step t
$q_{s,t}$	= Quantity sold at time step t
$q_{p,t}$	= Quantity purchased at time step t
$c_{start,t}$	= Cost of start-up at time step t, 0 if there is no change in a generator running/ not running
$c_{penalty,t}$	= Cost of penalty function at time step t, penalties are a set cost for breaking a specific boundary like
R_T	= Value of end reservoir, the value attributed to water remaining decided by water value and reservoir level

When looking at this it becomes apparent that the water value, as part of R_T, plays an important role, and that an accurate water value is imperative to the correct management of resources. Finding this water value requires looking years ahead in the future and taking into account a great many factors. To achieve this, models that simulate inflow, production and demand over the next several years are used.

The detail needed to make a finished production schedule or bid matrix, does not easily scale up to international size over several years. For this reason the process is divided into the hierarchy depicted in figure 2.

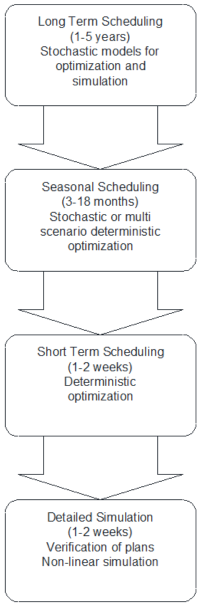

Figure 2

Long term model

The long term model is a stochastic optimization model that analyses fluctuations in inflow and price over a long time span to find the optimal use of production resources. The system boundaries are chosen as large as possible. This could be the Nordic system with connections to a generalized Europe. To make this large system manageable all reservoirs and generation in one area is aggregated into a single reservoir and plant. The timescale can also be aggregated to use a weekly resolution. Inputs for the long term model are statistical meteorological and hydrological data as well as forecasts of demand, outages and new production capacity. Outputs from the long term model are aggregated water values, target reservoirs and price series.

Seasonal model

The seasonal model acts as a link between the long term and short term models. While aggregated reservoirs are used in the long term model, the short term model needs information about each reservoir. It is the seasonal models objective to interpret the output from the long term model and output information that the short term model can use. The coupling between the long term and seasonal model is based on target reservoir levels when they can be considered known, like at the start of the spring flood, where reservoirs should be depleted, or at the beginning of the draining season, where reservoirs should be full. Multiple scenarios are run by deterministic optimization to give the cuts for each reservoir used in the short term model.

Short term model

The final step of the scheduling is the short term. Because the short term model will be used to make operative decisions, the output should be an implementable production schedule. To facilitate this the short term model is more detailed and has complex system descriptions, taking into account the smaller details like head, water course delay and efficiency. The short term model needs to be run many times a day for the production planners to get a good basis to make their decisions. Hence, the model must have little computation time and due to the complexity of the model it needs to be as small as possible. It only accounts for the reservoirs and plants that needs schedules, while any other upstream production only is represented by an expected inflow. Everything else should be part of the water values and strategy from the longer term models.

Risk management

The most profitable operation is not always the ideal as a change in inflow from the prognosis may lead to empty reservoirs and you being unable to fulfill your commitments, or full reservoirs and loss of water, and thereby loss of future profit. The same applies to uncertainty in price. A lower price than expected may lead to too little production and loss of water while a higher price than expected may lead to a larger commitment than you can deliver.

To mitigate this risk a production planner will try to create a bidding matrix and production schedules that are robust at the expense of profit.

SHARM

Tree generation

The first step is to generate the input to the stochastic model. The input is what is called a scenario tree. The stochastic short term model optimizes profit with respect to the distribution for future values of the uncertain variables. The distribution for these variables are given as a scenario tree. Each node in the tree holds one possible realized value for each variable and the branching structure reflects the information flow of the problem. To create this trees a program called Scentreegen has been developed.

Several approaches can be taken to generate a scenario tree. The Scentreegen program used in the SHARM model implements algorithms for scenario reduction and scenario tree generation. The methods are based on probability metrics which are measures of the distance between the reduced and full trees. The implemented algorithms are heuristic algorithms for obtaining a reduced tree that minimizes the distance between the full and reduced trees among all reduces trees of a given size, or for a given degree of reduction (Follestad, 2014). The mean values are not necessarily preserved.

First a fan tree is generated. This is a tree with only one branching point. Each branch is one combination of an inflow series and a price series. For an input of ten inflow series and five price series, 50 scenarios are created.

The tree generation takes the fan tree and creates a tree with several branching points. The method is based on successively reducing sub-trees of the original tree. Next the tree is reduced to limit the number of end nodes and make the optimization less resource demanding. Tree reduction refers to the task of creating a tree that consists of a subset of the scenarios in the original tree. The general idea is to successively delete or select single scenarios untill either a prescibed number of scenarios are selected or deleted, or a prescibed degree of reduction is achieved. The scenarios in the reduced tree is selected such that the reduced tree is as close to the original tree as possible.

Optimization

The optimization will move through the scenario tree node by node, gradually "revealing" the outcomes in the tree. For instance in the tree in figure 3 it starts in the root and sees the information up to node A as deterministic and creates a schedule for the period up to node A accounting for all the probabilities in the tree beyond node A. Next it created two schedules, one with deterministic input from node A to B and stochastic farther down and one with deterministic from node A to E. Lastly two schedules, from B to C and B to D, are created. The end result is three different schedules and reservoir developments, one for each root to end node path; there will be one unique result per end node in the tree.

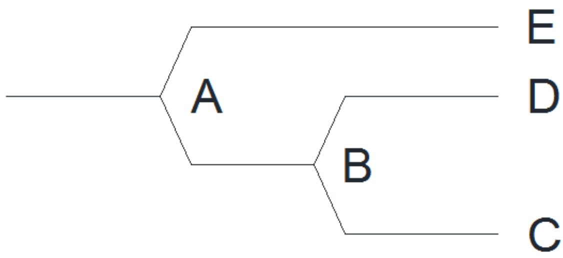

Figure 3

Case study

Input data

Independent water value function calculation

In the model provided from E-CO Energi the water values were given as independent water values of Flævatn 250 NOK/MWh, Vavatn 250 NOK/MWh, Flatsjø 200 NOK/MWh and Eikrabekkdammen 200 NOK/MWh. To do optimizations with independent water value functions they have to be calculated and to get consistent data the independent water values are used as a starting point.

The initial assumption is that the independent water value function will have the same water value as the independent at the starting reservoir level. The independent water value function is a straight line from zero water value at maximum reservoir level going through that point. This line is expressed by the formula

$$VW(V) = WVindependent / (Vmax - Vstart) * (Vmax - V)$$

This is the independent water value function expresses in NOK/MWh. SHARM requires the independent water value functions to be given in NOK/Mm3. To find this the following formula is applied.

$$NOK/Mm3 = NOK/MWh * MWh/Mm3$$

The value of MWh/MM3 is found by roh*h*Q*g*n where Q is the amount of water that would need to flow to expend one MM3 in one hour. Q=10^6/3600 m^3/s, g is gravity with the value of 9,81 m/s^2 and roh is the weight of one m^3 of water = 1000 kg. n is the efficiency and h is the head found by the difference between reservoir level and height of the outflow of the power plant. The resulting equation becomes

$$10^6*10^3*9,81/10^6*3600 * h * n$$

This can be contracted into

$$2,725*h*n$$

The efficiency is in the range of 0,8 to 0,9, but due to the complexity calculating an exact number it is omitted in the first tests. The efficiency is a constant modifier and will be accounted for when adjustments are made to the water values later to ensure consistency. The value of h dependent on the reservoir level in meters above mean sea level (mamsl) while the water value is dependent on reservoir level in Mm3. To convert one into the other the Mm3 value is referred to the reservoir curve, a table showing the relationship between the two based on measurements that are unique to each reservoir. These curves are found in the model file supplied by E-CO.

If the water values are consistent, i.e. represents the same value, the reservoir levels of the long term reservoirs should be equal at the end of the optimization period for optimizations with the different water value expressions given the same price and inflow. The smaller reservoirs varies between full and empty multiple times a day hence the end reservoir levels will not be greatly affected by the water value. They also face forced production meaning the production cannot be used to assess consistency either.

As seen in table xx the optimization with the initial independent water value functions gives too much production and the end reservoir levels are too low. The water values need to be increased to incentivize spending less water. From this point trial and error is used as the water value for all reservoirs are gradually increased until the end reservoirs match the results from the independent water value optimization. At an increase of 40% the end reservoir in Vavatn is slightly under and Flævatn is slightly over. At this point the error is 0,518% for Vavatn and 0,226% for Flævatn. Because of differences in inflow and reservoir curves the independent water value functions of the two reservoirs may differ slightly. The water values are changed independently until the desired outcome is found. Because the short term reservoirs cannot be assessed in the same way they are approximated by increasing them by the average of the increase of the other two.

The end result is found to be an increase of 35% for Vavatn, 43% for Flævatn and the average of the two, 39%, for Flatsjø and Eikrabekkdammen. This gives a fault of 0,056% for Vavatn and 0,022% for Flævatn. The changes in the results when trying to minimize the fault further are very slight, and because of the time consuming nature of the trial and error this is deemed close enough.

In later cases the starting reservoir levels need to be adjusted. The independent water values will not be affected by this, but the independent water value functions will need to be adjusted to compensate for the changed reservoir levels. When finding these water value the starting reservoir level used in the calculation of the independent water value function in the formula above is adjusted to a percentage of the original reservoir level for the two long term reservoirs. The two smaller reservoirs are kept unchanged due to their low degree of regulation. With these adjusted water values one cannot guarantee that the independent water value functions and the independent water values are still consistent. In these cases the optimizations with different water values are not directly compared, rather they are assessed by looking at the differences between deterministic and stochastic optimization with the same water values.

Hydro system description

The hydro system used is the Hemsil system, belonging to E-CO Energi AS, found in Hemsedalen. The topology of the system is shown in figure 4. It consists of four plants and four reservoirs. The largest reservoir called Flævatn (205 Mm^3) is connected to the plant Hemsil 1 (2x35 MW) and runs into Eikrabekkdammen (0,7 Mm^3). The plant Gjuva (8 MW) gets its water from Vavatn (34 Mm^3) and runs into the very small reservoir Flatsjø (0,12 Mm^3) which in its turn produces through Brekkefoss (2 MW). Production water from Brekkefoss ends up in Eikrabekkdammen and finally everything is produced in Hemsil 2 (2x50 MW).

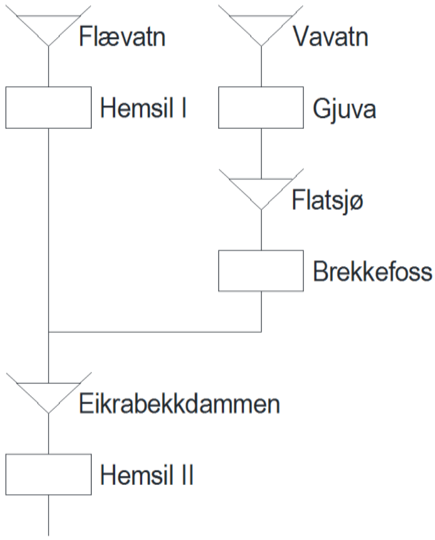

Figure 4: Hemsil hydro system

During the spring flood and whenever there is heavy rainfall both Eikrabekkdammen and Flatsjø overflows. In the late summer there are often prices high enough to warrant production from Hemsil 1 and Gjuva, but uncertainty about inflow can make the risk of spillage downstream too high. This mostly applies to Hemsil 1 as its water throughput at maximum is more than 5 times that of Gjuva, and hence of more consequence to Eikrabekkdammen, but there is a profit to be made from an ideal operation of Gjuva and Brekkefoss as well. There is a hope that a stochastic approach will aid in the making of this decision more than today's deterministic models does.

Inflow

The inflow used in the simulations are based on an ensemble forecast. The ensemble forecast consists of 50 precipitation and temperature series and each set of precipitation and temperature is run through the hbv model (Hydrologiska Byråns Vattenbalansavdeling model) for the hydro system resulting in 50 inflow series. The hbv model accounts for the catchment area and things like snow and soil moisture, and gives a quite accurate estimate of the expected inflow. These series are total inflow to the system and a scaling based on average yearly inflow is used to divide the inflow between each reservoir.

In the water value sensitivity analysis the full ensemble forecast is used with all 50 inflow series. The tree generation is set to make branching points every 24 hours to reflect the planning horizon for production planners who will be using the tool. There are 50 end nodes, meaning that there is one path through the tree, one scenario, for each inflow series.

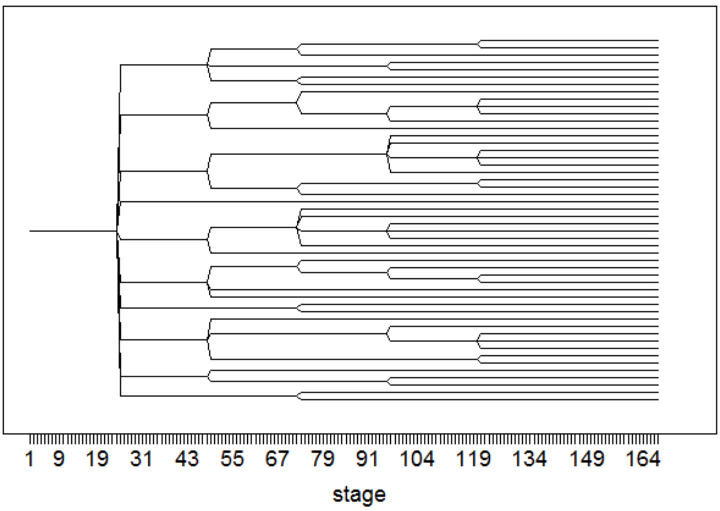

Figure 5

In the initial reservoir analysis only the lowest 10 inflow series are used. This part only seeks to observe the effects of a low inflow on already low reservoirs and the results from the higher inflow series will not yield relevant results, while the presence of high inflow probability could affect the stochastic optimizations to disregard the low inflow that . The inflow tree for this analysis is shown in Figure 6 with indications of which end branches that represent which inflow series. The inflow series are sorted by sum inflow from i1, being the lowest at 8,53 Mm^3, to i10 at the maximum of 11,48 Mm^3.

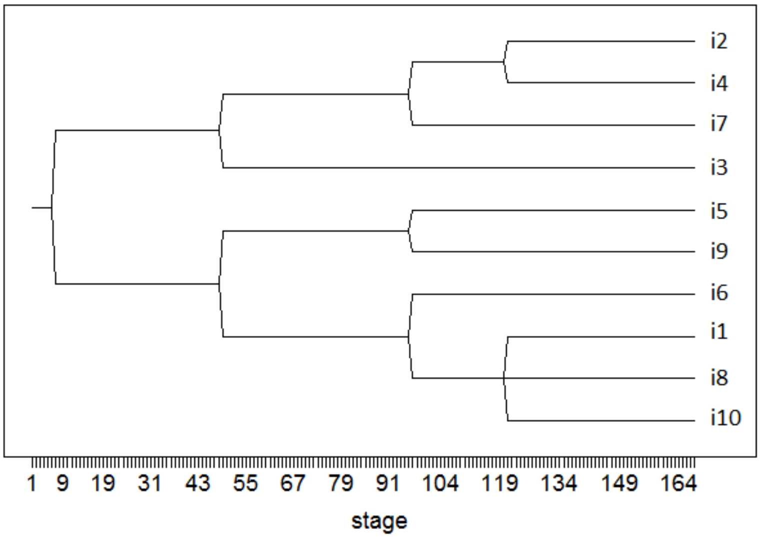

Figure 6

The inflow ensemble forecast is run through the tree generation to create a branching tree.

Price

The prices used in this paper are observed prices from the same time period as the inflow forecast. In the water value sensitivity analysis both German and Norwegian prices are used. The Norwegian prices are from price are NO5, the same area that the Hemsil water course is in. Both prices are used to observe the effect of different water values with different price profiles. In the initial reservoir analysis only the NO5 prices are used.

Figure 7 show a common price development with German prices having large fluctuations between day and night, while the Norwegian stay much more stable. The German day prices are normally higher that Norwegian and the nights lower. There is also often a midday dip in German prices. The changes in German prices is due to the composition of their production which is unfit to deal with changing demand. When power consumption drops off at midday and during the night, thermal plants are unwilling to shut down production due to the large start-up costs, while wind production has no reason to shut down while there is wind.

The Norwegian production consisting almost entirely of hydro power is much better at handling the changing demand. The start-up cost for hydro power is very small and all reservoir power is able to stop when prices drops below the water values, saving the water for later. Smaller reservoirs with large inflow and river plants will have to keep going through low prices because, as with wind power, stopping production will simply mean lost income.

The Norwegian prices are pretty stable around 250 kr/MWh while the German prices are more volatile, but varies around the same level as well. The mean values of the two are 245,91 kr/MWh and 228,34 kr/MWh respectively.

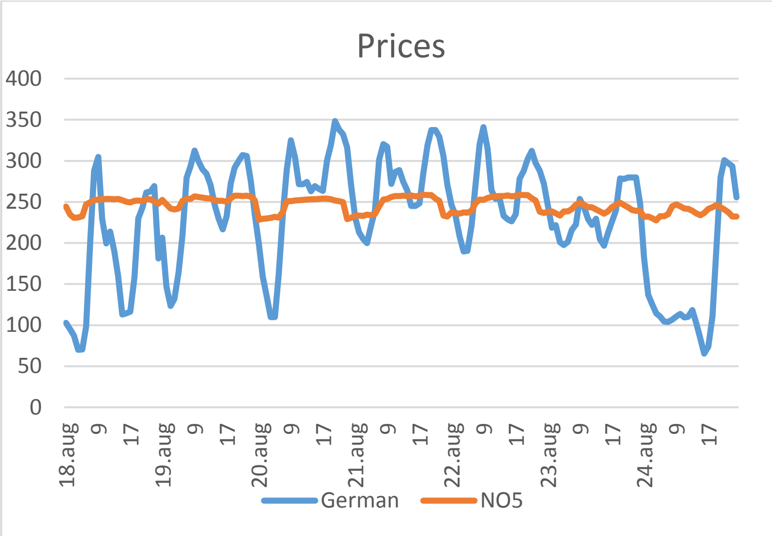

Figure 7

Method

Water value sensitivity

To determine how the model reacts to the different water values and any changes in these a sensitivity study is performed. Here different independent water values and independent water value functions will be tested against two sets of prices, Norwegian and German observed prices from the same time period, with both deterministic and stochastic optimization.

The water values will be increased 25 % and 50 % and decreased 25 % and 50 % from their original values. Adjusting the water values are done by multiplying the original independent water values by 0,5, 0,75, 1,25 and 1,5, giving an independent water value for Vavatn of 125 kr/MWh, 187,5 kr/MWh, 312,5 kr/MWh and 375 kr/MWh. The independent water value function is calculated the same as before, but with the adjusted independent water value as a starting point.

The stochastic optimizations are done with the scenario tree from figure 7, the full ensemble forecast.

While the stochastic model weighs all the possible outcomes, the deterministic only looks at the one. Hence there is only need to optimize a representative selection of scenarios deterministically. With two different price series, two different water value descriptions, five different water values and two different optimization methods the total number of optimizations gets quite large. To limit the time spent doing these optimizations three scenarios have been selected for deterministic optimization, the maximum, minimum and the mean total inflow scenarios. These are shown in figure 8.

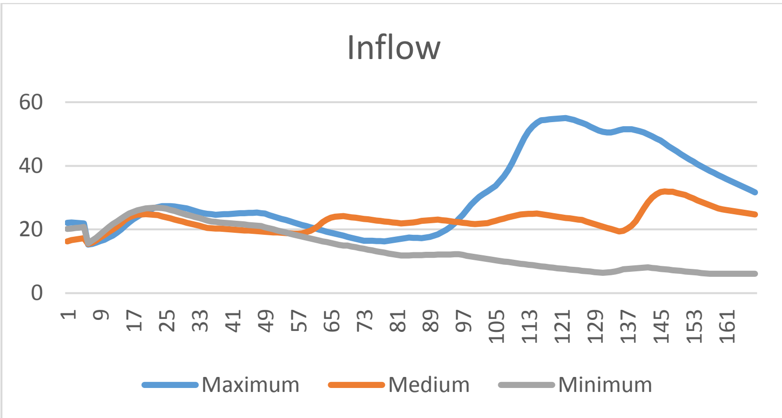

Figure 8

From the results from the complete stochastic optimization the scenarios that represents these inflow series can be found and compared to the deterministic results.

Initial reservoir analysis

It is theorized that the stochastic approach will have the largest advantage in situations where reservoir levels are low and inflow is low. In this situation there is a great risk of scheduling more production than you will be able to deliver. The stochastic approach should mitigate this risk by accounting for the lower inflow scenarios when deciding a schedule. To test this optimizations are done with different initial reservoirs and a reduced scenario tree consisting of only the ten lowest inflow series from the ensemble forecast. A mean inflow scenario is chosen and a deterministic optimization is done. A stochastic optimization is then done for the whole tree, and the results from the mean scenario is extracted.

This leaves two schedules, both made based on the same inflow, but one accounting for the possibilities of inflow deviations. The schedule found from the mean inflow scenario from both optimizations is then tested on all scenarios. To do this the schedule is set as a plan for all plants and simulated 10 times, one for each inflow. If the production commitment cannot be fulfilled water has to be drained from upstream to fulfill it, losing value on the way, and if it cannot do that it will incur penalties for breaking the production boundaries. Either way, this will be reflected in the objective function value for each scenario.

The optimizations are done with both independent water values and independent water value functions to see how the water value expression form affects the results.

The total inflow of the 10 different inflow series can be seen in table xx. The mean inflow is 10256706 m^3 hence i5, being the closest to this, is chosen as the operational scenario, the scenario that the schedule will be found for.

Scenario	Total inflow
i1	8530452 m^3
i2	9464040 m^3
i3	9516708 m^3
i4	9921672 m^3
i5	10150092 m^3
i6	10521648 m^3
i7	10623816 m^3
i8	10923876 m^3
i9	11435220 m^3
i10	11479536 m^3

A very important aspect in the stochastic optimizations is how the scenarios are grouped when the branching scenario tree is made. As seen in figure xx i5 is grouped together with i9 as well as i6, i1, i8 and i10. This means that the stochastic optimization will pay very little attention to the possibility of the lower inflow series i2, i3 and i4. Because of this the optimizations are also done with i4 as the operational scenario to study the impact of the first branching point.

The initial reservoirs are reduced to 5 %, 10 %, 15 % and 20 % of the original initial reservoirs, which are observed values. This is to maintain the relation between them in an attempt to emulate the actual strategies employed by the operators of the system.

Results

Water value sensitivity

The easiest place to see the effects of the water values is the production in Gjuva and Hemsil I. These plants draw from the long term reservoirs and will have production only when prices are higher than the water value. The other two plants are much more dependent on inflow, facing forced production when inflow is high, and limited production when inflow is low. The inflow dependence of the lower reservoirs also affect the upstream plants in that Hemsil II and Brekkefoss will be running at capacity when inflow is high, thereby not being able to accept water from upstream production; high inflow will block production at Hemsil I and Gjuva. This is also apparent when studying the results. The higher inflow a scenario has the more total production in the system, but the less production from the long term reservoirs.

With the stable Norwegian prices the production is influenced heavily by changes in water value. When using the independent water values, the first step up, 125 %, gives full stop of production from the long term reservoirs. The results are exactly the same at the 150 % step, indicating that the highest water value that will trigger production lies somewhere between 125 % and 100 %. The independent water value for Flævatn at this step is 312,5 kr/MWh and the highest price in the NO5 price series is 258,67 kr/MWh. However, the water value for Eikrabekkdammen and Flatsjø is 250 kr/MWh, a price that is exceeded daily, but when increasing this to 300 kr/MWh production stays the same. The amount of production needed to throughput the inflow is greater than the amount of hours with prices above the water value at 125 %.

Going in the other direction, 75 %, production from the long term reservoirs is at maximum production, all hours. Scenario 32 has reduced production because of blocking caused by the higher inflow. The water value at 75 % is 187,5 kr/MWh, lower than any price encountered, and further lowering the water values has very little impact on the deterministic results. The stochastic optimizations decide to run Gjuva to an overflowing Flatsjø, when water values are at their lowest.

The independent water value function moderates the response somewhat. When there is little production, reservoir levels rise, lowering the water value and inciting production. The 125 % water value that stopped all production with independent water values now keeps a lot of production. This is highly dependent on inflow as the results from the 125 % Hemsil I clearly illustrates. The results from Gjuva seems to indicate the opposite, but again this is due to the blocking effect; the inflow to Eikrabekkdammen is high enough that both plants cannot run at maximum and the model prioritizes Hemsil I.

The response is also lessened for a decrease in water value. As reservoirs are drained, water value increases and slows production. The influence of inflow on reservoir level and water value that was prevalent when increasing water values are much less visible here. The blocking effect from the large amount of production water entering Eikrabekkdammen counteracts this.

The German prices vary much more than the Norwegian. The highest prices go up to 348 kr/MWh while the lowest are as low as 65 kr/MWh. This entails that when water values are lowered to 75 % of their original value there are still many hours where prices are lower than the water value, and when raised to 125 % there are hours when prices are high enough to warrant production. This leads to a ramping of production across the spectrum. The highest water values still give zero production from the larger reservoirs, but due to the very low night prices in the German price series the lowest water values does not give the full production that was seen in the NO5 price scenarios.

Using independent water value functions the spread across the different water values are even greater. The 75% and 50 % have some reduced production, and the 125 % has a significant increase. For the first time even the 150 % water values has production from the long term reservoirs, akin to the 125 % with independent water values, and still very inflow dependent as discussed in earlier paragraphs.

Initial reservoir analysis

A challenge when reviewing the results from the initial reservoir analysis is that the changed reservoir levels and the inflow of the different scenarios all change the total value in the system, and thereby change the objective function value. To account for this the difference between the objective function value for the operational scenario and the others are studied. This value is quite constant across reservoir fillings as the added value from more water influences all the scenarios equally. It is only when production is limited by there being too little water to fulfill the commitment that this value starts to change. Table xx shows this value across different initial reservoir levels. With the independent water value function this value is not as stable as with independent water values, but the first being so dependent on reservoir level some variance must be expected.

At the 4 % initial reservoir level the lowest inflow scenario faces empty an empty reservoir at Vavatn. Both the stochastic and the deterministic schedules encounter this for independent water value function. At 3 % multiple scenarios end the period with empty Vavatn for both water value expressions, the deterministic in scenarios 1, 2 and 3, the stochastic still only in 1. At the lowest setting, 1 %, all reservoir end up empty in the dryer scenarios.

This is where trouble arises. The SHARM model does not calculate the production when set to a plant schedule. This allows it to have production without having water in the reservoir, and without incurring any penalties. The objective function values from the simulations were unchanged despite to empty reservoirs, and no useful information was possible to be gotten from them. When making the schedules these penalties are accounted for by the model, but they are not transferred when testing the schedules on all scenarios.

To compensate for this an algorithm was made in Excel VBA that used the data from the result files from the SHARM model. By using the same values as the model for plant discharge, inflow, gate flow and initial reservoirs the reservoir development from SHARM could be matched exactly. A condition was added that when the plant discharge for day d exceeded the reservoir volume from day d-1 plus day d inflow, upstream plant discharge and upstream gate flow, the production for day d was reduced to the production that the day's total water balance would allow. The missing production multiplied by day d prices plus a penalty value is then added to the scenario's objective function value.

The condition for Hemsil II is printed below.

```
If (PlantDischargeHemsilII(d) > (VEikrabekkdammen(d - 1) +
InflowEikrabekkdammen(d) + PlantdischargeHemsilI(d) +
PlantdischargeBrekkefoss(d) - GateEB(d) + GateFvE(d) + GateFsE(d)) Then

        PmissingHemsilII(d) = PHemsilII(d) * (1 - ((VEikrabekkdammen(d - 1) +
        InflowEikrabekkdammen(l) + PlantdischargeHemsilI(d)+
        PlantdischargeBrekkefoss(d) / VEikrabekkdammen(d)))

        VEikrabekkdammen(d)=0

Else

        VEikrabekkdammen(d) = VEikrabekkdammen(d - 1)
        (VEikrabekkdammen(d - 1) + InflowEikrabekkdammen(d) +
        PlantdischargeHemsilI(d) + PlantdischargeBrekkefoss(d) -
        PlantDischargeHemsilII(d) - GateEB(d) + GateFvE(d) + GateFsE(d)

End If
```

It was decided that the missing production from Brekkefoss should not be added to the penalty as it is a very small reservoir and if how it behaves under these conditions is not indicative of any pros or cons to the modelling approach.

The penalty for missing production was set to 500 kr/MWh. With the adjusted objective function values the results follow the patterns that one should expect. The table of results can be found in the appendix. The values in the table are scenario i – scenario 4, a low value reflects a solution that is good for multiple scenarios. The difference line is deterministic – stochastic, and a higher value means a better stochastic solution comparatively. The value at i4 is the objective function value for the operational scenario.

The stochastic approach has a better solution for the lower inflow scenarios than the deterministic, but worse for the high inflow scenarios. In the independent water value results it can also be seen that the stochastic schedule is a better solution for scenario 7. This scenario is closer to the operational scenario than any of the other higher inflow scenarios in the stochastic tree. When the initial reservoir get lower the stochastic solution does even better than the deterministic for low inflow, and also improves for the higher inflow.

Conclusion

Water value sensitivity

The water value is maybe the most important input parameter. As discussed in the hydro scheduling chapter, the value given to the remaining water in the system is a large part of the optimization LP, but how it intersects the price can be just as significant. With stable prices a small deviation in water value will have a very large impact. Full production all the time is rarely the most profitable management of the system, as is stopping all production from unpressured reservoirs. With the Norwegian price profile an accurate water value is essential.

Water values that adapt to changing reservoir levels such as the independent water value function and cut-files will to some degree correct themselves. This behavior makes them more robust to changing conditions in the power markets and in the meteorological situation.

More volatile prices make the response in the system more binary. Either prices are high and maximum production is warranted, or prices are low and everything should be stopped. Operating in this kind of market lowers the need for precise water value. The results from the sensitivity analysis show that variations in the water values still have a large impact, but the changes in water values are severe. Small deviations in water value in a volatile market has small consequences.

Applying water values that depend on reservoir levels to the fluctuating price profile, yields production spread out across the whole spectrum; there is almost as much production between the 100 % and the 125 % steps at NO5 with independent water values as there is between 75% and 150% with German prices and an independent water value function.

Initial reservoir analysis

The deterministic solution will always be better for the operational scenario than the stochastic. The stochastic tries to find a solution that works for more than one scenario, and sacrifices profit in the process. The scenarios closest in the stochastic tree are the ones that are prioritized by the stochastic optimization. With a sufficient initial reservoir and independent water values the stochastic optimization has the best solution for scenario 1, 2, 3, 5 and 7. Scenario 7 is close to the operational scenario in the scenario tree and the lower inflow scenarios are the main concern for the stochastic model.

With independent water value functions the stochastic model only has the best solution for scenarios 1 and 2. The solution for scenario 1 is much better than the deterministic however. The can be explained by the stochastic model seeing the extremely good value that can be found for the lowest scenario and this has shifted the focus of the stochastic model to optimizing for low inflows.

When decreasing the initial reservoirs down to the critical point where reservoir go empty the pattern is still the same. With the stochastic schedule the independent water values has the best results for all low inflow, while the independent water value function only has really a good solution for the lowest inflow.

The initial reservoirs has to be lowered to extremes before the benefit of the stochastic approach really shines through. The 3 % case is quite similar to the 20 % case, and it is really only the 1 % case that sees a large upswing in benefit from the stochastic approach. The penalty value of missing production will certainly play a big part in this. This is a value that is difficult to set a price on in a practical setting. The actual losses are dependent of the reserve power price of the day and hour and will vary a lot. There is even the possibility of make a profit from it. From a production planner's point of view it is important that you make a schedule that you know you can keep. Replanning the schedule entails extra work and late hours. When reservoirs run empty before schedule the generators have to be taken out of the automatic control and new set points has to be set every hour or more often. And of course the transmission system operators want schedules they can trust. There are a lot of quality of life concerns that is hard to set a finite penalty per MWh for.

All these things considered, a stochastic approach is beneficial when reservoirs are in danger of trespassing on their bounds. There will still be a need for people to use judgement in these cases, but a stochastic approach will give a better basis for decisions.

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Appendix

					Independent water value						
	i1	i2	i3	i4	i5	i6	i7	i8	i9	i10	
	586879	316909	452522	4733291	240913	-170693	-172466	-347667	-513008	-486265	Deterministic
1 %	532894	235268	494972	4718997	129762	-151306	-190097	-342165	-468120	-439920	Stochastic
	53986	81641	-42449		111152	-19387	17631	-5502	-44888	-46345	Difference
	568460	235447	275468	5738923	86217	-185475	-167701	-350049	-514926	-490971	Deterministic
2 %	534818	210371	261329	5740119	49975	-142526	-192310	-330975	-467726	-439327	Stochastic
	33642	25075	14139		36243	-42949	24608	-19074	-47200	-51645	Difference
	570645	234245	263805	6763915	77019	-184491	-168021	-350881	-515519	-491768	Deterministic
3 %	534082	206923	248327	6765960	57680	-138791	-192079	-329640	-468250	-441469	Stochastic
	36562	27322	15478		19339	-45701	24058	-21242	-47270	-50298	Difference
	540544	223526	254906	7777168	76164	-180474	-169679	-351434	-515758	-490566	Deterministic
4 %	508441	206676	244870	7779356	54382	-141664	-192450	-330831	-468845	-439706	Stochastic
	32103	16850	10036		21783	-38810	22771	-20603	-46912	-50859	Difference
	540581	223649	254973	8787018	76132	-180248	-170833	-351487	-515796	-490672	Deterministic
5 %	509215	206599	244179	8787658	53727	-139205	-192682	-329878	-469090	-439570	Stochastic
	31367	17050	10794		22404	-41043	21849	-21609	-46707	-51101	Difference
					Independent water value function						
	i1	i2	i3	i4	i5	i6	i7	i8	i9	i10	
	740058	234506	235923	4992822	60137	-20492	-141400	-234319	-366743	-335869	Deterministic
1 %	542796	164531	196899	4931346	36981	-102317	-138875	-236059	-360382	-331507	Stochastic
	197262	69975	39024		23156	81825	-2526	1739	-6361	-4363	Difference
	647828	240332	230271	6010674	41712	-102553	-147121	-244478	-373251	-344905	Deterministic
2 %	532758	168441	184439	5958990	23934	-102709	-139604	-236624	-361431	-332374	Stochastic
	115070	71890	45832		17778	156	-7517	-7854	-11820	-12530	Difference
	631120	233715	252549	7073666	36033	-112140	-146047	-244969	-373843	-347227	Deterministic
3 %	524575	163758	208798	7016953	24107	-105359	-140073	-240505	-363895	-335138	Stochastic
	106545	69957	43751		11926	-6781	-5974	-4464	-9948	-12089	Difference
	612382	217630	248943	8120211	37299	-111232	-146574	-246918	-374695	-348145	Deterministic
4 %	522786	168306	220520	8077232	25708	-104817	-137060	-240050	-362066	-335485	Stochastic
	89597	49324	28423		11591	-6415	-9515	-6868	-12630	-12659	Difference
	651852	216847	244143	9178211	36894	-82634	-147593	-240365	-371196	-341599	Deterministic
5 %	508719	168480	201351	9132108	25510	-105017	-137883	-239995	-362752	-335594	Stochastic
	143133	48367	42792		11384	22383	-9710	-370	-8444	-6005	Difference

	540581.2618	223648.844	254972.7096	8787017.553	76131.83672	-180247.7567	-170833.2462	-351487.0165	-515796.4878	-490671.5537	Deterministic
5 %	509214.6959	206598.615	244178.5959	8787657.945	53727.49505	-139205.1971	-192681.8457	-329878.1255	-469089.7662	-439570.1296	Stochastic
	31366.56591	17050.22899	10794.11369		22404.34167	-41042.55952	21848.59944	-21608.89101	-46706.72166	-51101.42412	Difference
	543362.5969	224321.5745	256357.39	13851328.13	76685.08336	-175650.3782	-168550.3495	-351676.2248	-516410.7169	-491427.6588	Deterministic
10 %	510658.8241	206417.2113	245730.3647	13849998.76	54686.87809	-141291.7819	-192927.9245	-331325.2332	-470227.8898	-441029.6901	Stochastic
	32703.77281	17904.36321	10627.02526		21998.20527	-34358.59624	24377.575	-20350.99154	-46182.82712	-50397.96868	Difference
	544511.2522	224781.6946	256905.3527	18933177.38	76832.37828	-175761.5904	-169592.6407	-352055.1163	-517046.4285	-492101.3259	Deterministic
15 %	510850.4525	207142.5347	246972.4479	18930191.81	55507.43528	-141122.3045	-193193.7656	-330341.2029	-470557.8511	-441282.7592	Stochastic
	33660.79965	17639.1599	9932.904811		21324.94301	-34639.28593	23601.12488	-21713.91344	-46488.57745	-50818.56666	Difference
	545226.0047	225040.6866	257389.8104	24024187.53	77104.55634	-175752.9653	-169931.7474	-352342.3682	-517491.6521	-492503.4919	Deterministic
20 %	512104.8234	207402.7914	247195.8862	24021261.41	55471.89099	-141412.743	-193401.0646	-331217.7272	-470905.9443	-441525.1535	Stochastic
	33121.18125	17637.89522	10193.92417		21632.66535	-34340.22236	23469.31721	-21124.64103	-46585.70786	-50978.33835	Difference
					Independent water value function						
	651851.6089	216847.0783	244143.246	9178211.274	36893.57081	-82633.73296	-147592.9182	-240364.7814	-371195.9273	-341598.8519	Deterministic
5 %	508718.9466	168479.9988	201351.3718	9132108.468	25509.82614	-105017.1184	-137882.9213	-239994.5154	-362751.53	-335594.0398	Stochastic
	143132.6622	48367.07952	42791.87416		11383.74467	22383.38544	-9709.996912	-370.266009	-8444.397283	-6004.812088	Difference
	673234.5631	216868.7138	248898.9056	14628051.02	36659.14346	-71338.37815	-148803.5812	-239267.416	-371692.871	-341148.5399	Deterministic
10 %	510015.5053	168391.5022	215188.7142	14582539.09	24942.9154	-105011.9534	-139113.1116	-241833.9185	-365297.0166	-337566.2953	Stochastic
	163219.0578	48477.21152	33710.19142		11716.22806	33673.57524	-9690.469575	2566.502425	-6395.854406	-3582.244587	Difference
	685170.4906	218255.1484	248147.1891	20383990.49	36761.09798	-64923.19017	-149122.5535	-239346.9429	-371827.1086	-341382.0966	Deterministic
15 %	530165.3424	168113.2088	210831.9137	20337546.26	24701.30989	-103303.1883	-139420.5916	-240057.694	-365370.074	-336286.8697	Stochastic
	155005.1482	50141.9395	37315.27541		12059.78809	38379.99809	-9701.961882	710.751168	-6457.034618	-5095.226882	Difference
	696576.0266	218174.0814	243132.2377	26488859.47	36655.17689	-58764.51928	-149899.7069	-238983.9239	-372827.8612	-342013.8891	Deterministic
20 %	557642.1681	167835.9098	202005.9907	26441017.02	24417.82483	-101527.977	-140106.3998	-239085.1593	-365668.2615	-335841.0149	Stochastic
	138933.8585	50338.17161	41126.24703		12237.35205	42763.45768	-9793.307121	101.235325	-7159.599719	-6172.874248	Difference

	Independent NO5 Stochastic				
32 Total	7404	7404	18424	29152	29627
43 Total	6027	6026	18896	29177	29353
34 Total	5408	5407	19368	28834	28831
32 Gjuva	0	0	979	1194	1533
43 Gjuva	0	0	1480	1617	1678
34 Gjuva	0	0	1535	1678	1678
32 Hemsil I	0	0	5120	11623	11725
43 Hemsil I	0	0	5618	11725	11725
34 Hemsil I	0	0	6200	11725	11725
	Independent Function NO5 Stoch				
32 Total	7605	12172	20973	28635	29521
43 Total	6235	7667	19693	27624	29206
34 Total	5606	6535	17254	27414	28670
32 Gjuva	0	811	906	1628	1534
43 Gjuva	0	491	1114	1482	1678
34 Gjuva	0	471	1335	1654	1678
32 Hemsil I	0	1831	6936	10891	11613
43 Hemsil I	0	313	6696	11034	11725
34 Hemsil I	0	0	5403	11070	11725

	Independent NO5 Deterministic				
32 Total	8955	8955	18241	28317	28431
43 Total	6654	6654	19397	29516	29821
34 Total	4047	4047	20172	28011	28011
32 Gjuva	0	0	851	968	1025
43 Gjuva	0	0	1321	1521	1678
34 Gjuva	0	0	1559	1678	1678
32 Hemsil I	0	0	4214	10631	10637
43 Hemsil I	0	0	5669	11709	11725
34 Hemsil I	0	0	7274	11724	11724
	Independent Function NO5 Det				
32 Total	9144	16093	22749	28005	28419
43 Total	6867	9609	20872	28469	29684
34 Total	4260	6529	15621	26058	27741
32 Gjuva	0	798	876	996	1019
43 Gjuva	0	736	986	1668	1678
34 Gjuva	0	1111	1280	1636	1678
32 Hemsil I	0	3227	7130	10325	10631
43 Hemsil I	0	765	7135	11048	11725
34 Hemsil I	0	0	5193	10863	11724

	Indpendent German Stochastic				
32 Total	7219	8489	16254	23004	26237
43 Total	5857	7446	15728	22948	25860
34 Total	5229	7060	15526	22688	25386
32 Gjuva	0	116	563	981	1318
43 Gjuva	0	131	768	1213	1414
34 Gjuva	0	167	834	1270	1442
32 Hemsil I	0	561	4801	8435	9883
43 Hemsil I	0	682	4995	8826	9975
34 Hemsil I	0	754	5064	8853	10010
	Independent Function German Stoch				
32 Total	8586	12375	17379	22480	25862
43 Total	6933	10179	15795	21421	24828
34 Total	5991	9486	15065	21238	24140
32 Gjuva	169	543	732	1023	1317
43 Gjuva	207	553	818	1189	1414
34 Gjuva	229	456	915	1304	1394
32 Hemsil I	530	2388	5196	7965	9595
43 Hemsil I	299	1853	4933	7875	9595
34 Hemsil I	67	1909	4686	7956	9580

	Independent German Deterministic				
32 Total	8744	10047	17070	23037	25567
43 Total	6481	7988	16143	23243	26384
34 Total	3915	6481	15281	22091	24413
32 Gjuva	0	109	514	838	915
43 Gjuva	0	126	731	1105	1422
34 Gjuva	0	212	885	1283	1493
32 Hemsil I	0	595	4420	7630	9041
43 Hemsil I	0	676	4915	8767	9985
34 Hemsil I	0	855	5326	8891	10015
	Independent Function German Det				
32 Total	10544	15001	19174	23115	25539
43 Total	7516	11297	16155	22401	25521
34 Total	4579	7686	13843	20065	23523
32 Gjuva	138	510	841	939	935
43 Gjuva	186	612	748	1113	1424
34 Gjuva	196	280	1041	1207	1522
32 Hemsil I	817	3068	5239	7490	8927
43 Hemsil I	304	2079	4829	8128	9633
34 Hemsil I	33	1709	4499	7987	9648