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Modelling of Pedestrian Loading on Slender Footbridges

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Modelling of Pedestrian Loading on Slender Footbridges

Modellering av fotgjengerlaster på slanke gangbruer

BY:

Kasper Van de Pontseele

Daniel Seides

SUMMARY:

This thesis concerns itself with the dynamic response of a slender network arch bridge by means of a numerical model. The numerical model is generated using a Python code written by Ph.D. Candidate Anna Ostrycharczyk. The code was modified by the authors to include pedestrian loading. Based on a modal analysis of the bridge, the first ten natural frequencies and corresponding mode shapes were extracted, and attempts at exciting each mode by simulating pedestrian loading were carried out.

The acceleration of the bridge deck was the main variable of study, as this is what pedestrians feel and react to while walking across the bridge. The accelerations achieved through numerical analyses have been compared to the suggested values in design guidelines, both to check if the design guidelines have valid methods for calculating accelerations and to check if the bridge in study fulfils the guideline criteria.

For single pedestrian analyses the dynamic response matched the design guidelines' predicted response fairly well, and were within the presented criteria. Simulating groups of pedestrians yielded extremely high acceleration responses. They were considered worst case, but not very realistic. Other load cases such as running were also simulated.

The specific acceleration values retrieved in this thesis are not useful, because the dimensions of the bridge in study are still subject to change. However, three modes were found critical for the bridge. One mode was concerned with the lateral responses of the arches and the two others with the lateral and vertical responses of the deck. It is fair to assume that similar modes will dominate the response of any slender network arch bridge. These mode shapes should be studied carefully and will be governing in the placement of any external dampers.

The methods for calculating vertical accelerations in the guidelines are often too simple. This should be addressed as more slender footbridges with unique designs are being built every year. The way horizontal accelerations are covered is also deficient. The guidelines propose methods dependent on frequency ranges, disregarding the fact that higher harmonics of the walking frequency might excite the bridge. Horizontal resonance was achieved in this study, despite the fact that most guidelines deemed it unnecessary to check.

The numerical method used in this thesis shows promise, but needs to be developed further, especially for simulating groups of people. The material damping implemented in the numerical model proved the importance of damping, especially at resonance. Through further work a method may be obtained to calculate the dynamic response of a footbridge for a given amount of damping. This way the minimum amount of damping necessary to satisfy the comfort criteria stated in design guidelines can be found.

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TITTEL: **Modellering av fotgjengerlaster på slanke gangbruer**

Modelling of Pedestrian Loading on Slender Footbridges

UTFØRT AV:

Kasper Van de Pontseele

Daniel Seides

SAMMENDRAG:
Denne avhandlingen omhandler dynamisk respons av en slank nettverksbuebro i tre, beregnet ved hjelp av en numerisk modell. Den numeriske modellen er generert ved bruk av en Python kode skrevet av Ph.D. Kandidat Anna Ostrycharczyk. Koden ble omskrevet av forfatterne til å inkludere fotgjengerlaster. Gjennom en modal analyse av brua ble de første ti egenfrekvensene og tilhørende egenmodene funnet, og forsøk på å trigge modene ved å simulere fotgjengerlaster ble utført.

Akselerasjonen av brudekket var den viktigste variabelen, da dette er hva fotgjengere føler og reagerer på når de går over brua. Akselerasjonene ble funnet gjennom numeriske analyser og sammenlignet med beregnede akselerasjonsgrenser gitt i standarder og håndbøker. Dette ble gjort både for å undersøke om metodene presentert i standardene er gyldige, og for å sjekke om den studerte nettverksbuebrua oppfyller kriteriene satt i standardene.

Analysene med én enkelt fotgjenger ga resultater som samsvarte godt med standardene, og var innenfor kriteriene. Analysene med en gruppe fotgjengere ga imidlertid ekstreme akselerasjoner. Disse akselerasjonsverdiene kan regnes som det verst tenkelige tilfellet, men de er antageligvis ikke realistiske. Andre lasttilfeller med én løpende fotgjenger ble også simulert.

Akselerasjonsverdiene funnet i dette studiet er ikke direkte brukbare fordi bruas dimensjoner ikke er fastsatt. Imidlertid ble tre kritiske vibrasjonsmoder funnet for brua - en sideveis utbøyning av buene, og to for utbøyning av dekket i henholdsvis sideveis og vertikal retning. Det er rimelig å anta at samme moder vil dominere responsen til lignende slanke nettverksbuebruer. Disse vibrasjonsmodene burde vies ekstra oppmerksomhet, og vil være dimensjonerende for plasseringen av eventuelle eksterne dempere.

Metodene for å beregne vertikale akselerasjoner i standarder og håndbøker er ofte for enkle, og bedre metoder bør utvikles da flere slanke bruer med unikt design bygges hvert år. Måten horisontale akselerasjoner blir dekket er også mangelfull. Standardene foreslår metoder avhengig av frekvensområder, uten å ta hensyn til at høyere harmoniske komponenter av gangfrekvensen kan bidra til resonans. Horisontal resonans av brua ble oppnådd i dette studiet, til tross for at det i følge standarder ville vært unødvendig å utføre en kontroll av de horisontale akselerasjonene.

Den numeriske metoden som brukes i denne avhandlingen har potensiale, men må utvikles videre, spesielt for å simulere grupper med fotgjengere. Materialdempingen i den numeriske modellen viste viktigheten av demping, spesielt ved resonans. Gjennom videre arbeid kan en fremgangsmåte utvikles for å vurdere den dynamiske responsen til en bru for en gitt mengde demping. Dermed kan man bestemme den minste dempingsmengden som trengs for å tilfredsstille kriteriene satt i standardene.

FAGLÆRER: Kjell Arne Malo

VEILEDER(E): Kjell Arne Malo

UTFØRT VED: Institutt for konstruksjonsteknikk, NTNU

Preface

The work presented in this master thesis was carried out at the Department of Structural Engineering at the Norwegian University of Science and Technology (NTNU) from January to June, 2015. It was written by Kasper Van de Pontseele and Daniel Seides for the group of timber structures.

We would like to express gratitude to our supervisor Professor Kjell Arne Malo and also to Ph.D. Candidate Anna Ostrycharczyk and thank them for all their help and guidance throughout this project. They have supported us with thoughts and ideas critical to the advancement of this thesis.

Lastly, we would also like to thank our fellow students Eivind Torgunrud Røshol and Nicolay Line Holm for being helpful with kicking ideas back and forth.

Trondheim, June 2015

Kasper Van de Pontseele and Daniel Seides

Contents

1 Introduction

1.1 Background

With technology improving and advancing every year, the construction of bridges has tended towards more daring constructions; lightweight and flexible, slender-looking and innovative designs. The development of building materials that can withstand higher stresses has opened up for smaller cross sections and greater spans, which may be visually pleasing. However, smaller cross sections lead to decreased bending and axial stiffness, which, combined with longer spans decrease the natural frequencies of the bridges. When a bridges natural frequencies coincide with the walking or running frequencies of the pedestrians crossing the bridge, resonance becomes a possibility.

Various incidents regarding out-of-control dynamics on footbridges have been recorded through history. A famous example is the Angers bridge in France, which collapsed in 1850 while a battalion was marching across it, killing 200 men [1]. In modern day the problem is more common, and vibrations can occur at events containing less people and activity than a marching battalion. Two incidents especially spurred a lot of attention towards the subject; both the Millennium bridge in London and Pont-de-Solférino in Paris (depicted in Figure 1.1) experienced excessive lateral vibrations during opening day, and needed to close down for structural renovation thereafter.

Figure 1.1: *Pont de Solférino, which is now known as Passerelle Léopold-Sédar-Senghor.*

1.2 Objective

This project investigates the dynamic responses due to pedestrian loading on a slender timber footbridge by means of a numerical model. The bridge is currently in the design phase at NTNU, and has been subject to a wide array of work over the past few years. Two masters students built a small scale version of the bridge for their master in 2013 (see Figure 2.1). At this time several master students are writing about various aspects of the bridge. Our goal is two-parted; can numerical models be used to simulate pedestrian-induced resonance? How well will a numerical model coincide with the regulations set forth by the design guidelines?

1.3 Method

This thesis can be divided into two main parts, the modelling of the bridge and loads in *Abaqus*, and the comparison of the results to the design guidelines. The former part is the bulk of the thesis.

The modelling is based on a Python script written by Ph.D. Candidate Anna Ostrycharczyk. During the work on this thesis the script has been modified to include pedestrian loads, including any other variables pertinent to this. The model was also used to calculate the natural frequencies of the bridge. The various model parameters such as mesh size, step size and numerical damping were determined through preliminary parameter studies.

Attempts to induce resonance were made to look at the worst case scenario and obtain the corresponding dynamic responses. To limit the work, the first ten modes of vibration were selected to study, which is a fair guess keeping in mind that the higher modes are less likely to be excited. Accelerations and displacements were the chosen variables to extract from each analysis. They were evaluated and converted to the frequency domain through a discrete Fourier transform (DFT) to help determine whether or not resonance had occurred.

By running different analyses the hope was to unveil the critical modes, which is important for the further work on the bridge. Perhaps dampers are required. The script has been written as generally as possible, with the notion in mind that someone may continue developing it.

1.4 Limitation

- At the time of writing, the dimensions of any bridge that might be built are still unknown. Therefore the results presented in this thesis are not very useful themselves, but the method is still valid and tailoring the work to a new set of dimensions is fairly painless and will yield useful information.

- No partial factors were attributed the loads. No combinations of loads were tested (for example wind or snow).
- A limited number of guidelines were compared; Eurocode, ISO, Handbook N400, british standards and a technical guide by Sétra.
- Only the ten first eigenmodes of the bridge were considered.
- All materials were given the same amount of damping to give a uniform damping throughout the entire structure. In reality the damping ratio for wood is a factor of ten greater than that for steel.
- The Fourier coefficients used to calculate the pedestrian loading are given for a specific frequency range and activity. Only the latter has been taken into account in this thesis and coefficients have been used independent of the pedestrians pacing rate.

1.5 Thesis Outline

Chapter 2 describes the history and development of network arch bridges, and aims to present the concept from a structural point of view.

Chapter 3 consists of theory relevant to the work done for this thesis. Structural dynamics and modal dynamics are touched briefly. The bulk of the chapter presents the theory around pedestrian induced loading, and how this is described mathematically. The latter part is concerned with dynamics in finite element analysis, especially numerical damping.

Chapter 4 is a literature review. Here the different guidelines dealing with vibration in structures are presented with their methods for calculating limit accelerations.

Chapter 5 revolves around the numerical model. Here the choices of parameters within the model are justified. An attempt at determining the amount of damping in the model is described, and also the choice of load modelling.

Chapter 6 presents the results from the analyses. The results include accelerations, displacements and discrete Fourier transforms. Each of the ten natural modes are presented chronologically for single-person analysis, followed by analyses covering arch movement, groups of people, running pedestrians and the effects of damping.

Chapter 7 is a discussion chapter, comparing the different results with each other and with the design guidelines.

Chapter 8 presents the conclusions made by the authors, based on the previous chapters.

Chapter 9 proposes further work.

In the **appendices** the following additional information can be found:

- **Appendix A** - *MATLAB* scripts used in the thesis
- **Appendix B** - Explanation of *Python* coding
- **Appendix C** - A complete list of performed *Abaqus* analyses
- **Appendix D** - Hand calculations of accelerations according to design guidelines
- **Appendix E** - A guide to running the *Python* script
- **Appendix F** - Complete *Python* code (digital)

2 Network Arch Bridges

2.1 Case Study

Figure 2.1: *Small scale model of bridge located at NTNU Department of Structural Engineering laboratory.*

The basis for this thesis is the bridge in Figure 2.1, which is a small scale model of a bridge that will hopefully be built in Orkanger some time in the future. The bridge is a network arch bridge in timber, which is a relatively new concept in bridge design.

2.2 Concept

Network arches can be defined as those with inclined hangers with multiple interceptions [2]. The inclined hangers make the network arch bridge act like a truss, with only axial compressive and tensile forces. Bending moments and shear forces are small in network arches. Because the arch and ties are mainly subject to axial forces, their cross sections can be quite small. This has made the design and construction of more slender-looking bridges possible.

Inclined hangers have been in use for almost a century, but network arches were developed in the sixties. It appears that the Swedish engineer Octavius F. Nielsen pioneered the concept by using V-shaped hangers nearly a hundred years ago, while Norwegian engineer Per Tveit developed the concept further in the fifties by inclining the hangers in the transverse direction as well. Inclined hangers dramatically lowers the moment in the arch, and makes the bridge much stiffer. Compared to an arch with vertical hangers where loads on the bridge deck transfer as point loads to the arch, creating moment, the inclined hangers greatly reduce the moment-arm of the force, reducing the moment by transforming it into compressive forces in the arch. By using a network hanger design the moment action in the arch may be reduced by 75% [3].

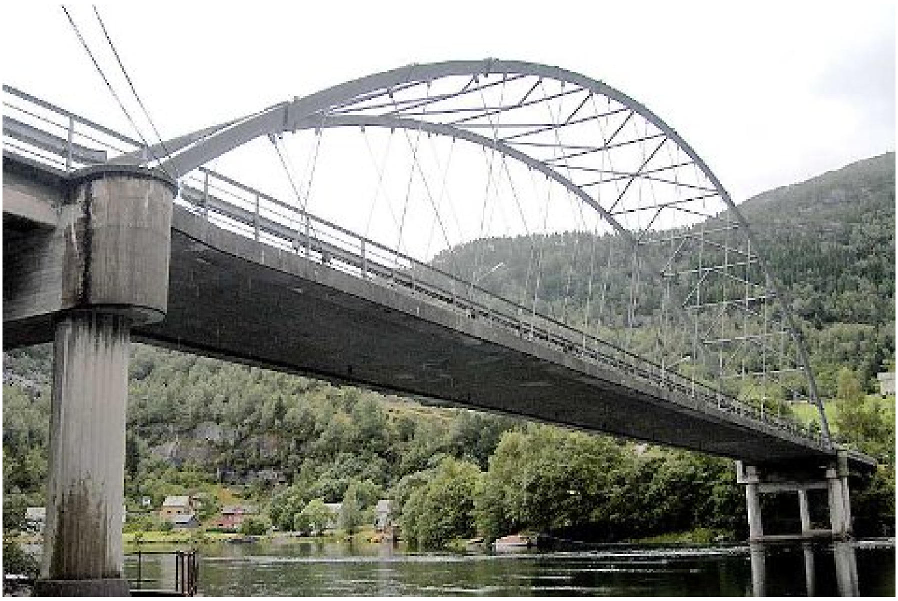


Figure 2.2: *Bolstadstraumen Bru - A network arch bridge in Sognefjord.*

2.3 Stability of Arch

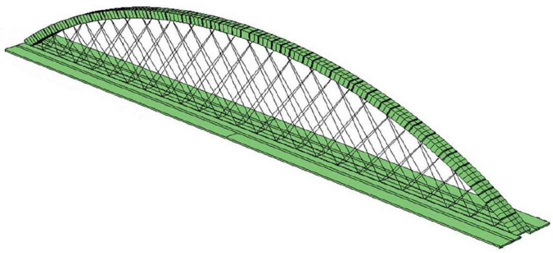


Figure 2.3: *Transversely inclined hangers resemble that of a spoked bicycle wheel.*

For arch bridges in general, if the arches are too slender, some form of sideway support is necessary to increase the stability. One way of doing this is clamping the arches at the supports, but as the span increases, the effect of clamping decreases and other solutions must be employed. Perhaps the most obvious one is a truss system connecting the arches, which can be seen in Figure 2.2. Another option is the use of inclined hangers out of plane.

Consider an arch bridge in which each hanger consists of a pair of parallell hangers. If the paired hangers are fastened to the deck at two separate points in the lateral direction, the hangers will resemble that of a spoked bicycle wheel (see Figure 2.3). This will have a positive effect on the out-of-plane buckling of the arches, which are the primary modes of buckling (see Figure 2.4).

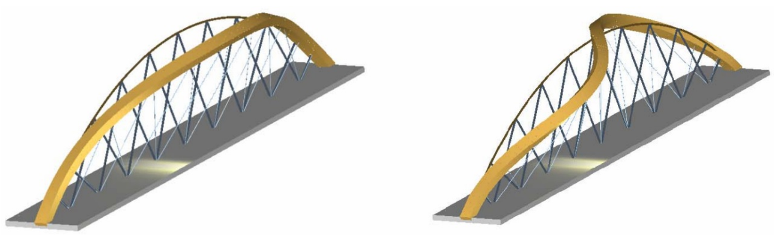


Figure 2.4: *The two primary buckling modes for an arch [3].*

The advantages of parallel hangers are demonstrated in Figure 2.5. By using one vertical hanger, the arch might rotate about the fastening points of the hangers. By using pairs of inclined hangers, any such rotation would imply an elongation of one of the hangers, inducing forces that resist lateral movement of the arch. The strain in the hangers due to any lateral movement is dependent on the curvature of the arch (or specifically, the distance between the arch and the deck, R). By introducing the geometric ratio $r = a/R$ and assuming small angles α, where a, R and α are as shown in Figure 2.5, the strain in the tension hanger becomes:

$$\epsilon = \frac{r}{r^2+1}\alpha \tag{2.1}$$

From this Equation it can be seen that for increasing R, r will decrease and if a < R the strain ϵ will also decrease. The strains are thus higher near the supports, meaning this is where the stability is greatest, and the sideways stability varies along the span[4].

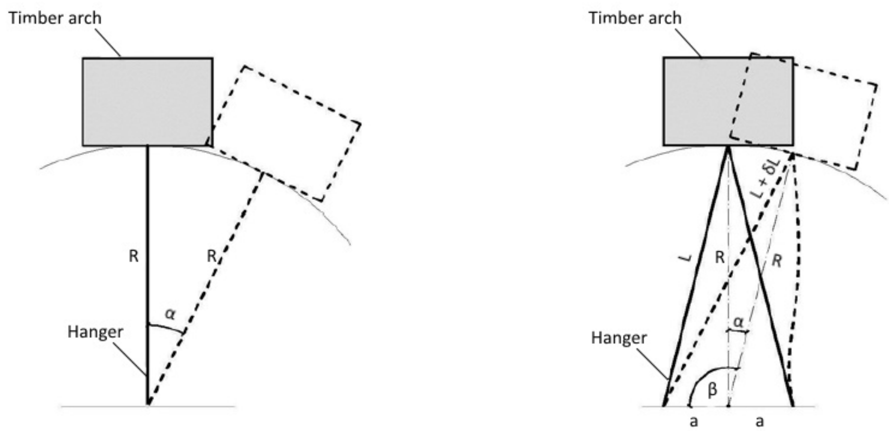


Figure 2.5: *Lateral stiffness from spoked wheel configuration [4].*

3 Theory

This chapter gives a brief summary of the theory and previous research relevant to the work done for this thesis.

3.1 Structural Dynamics

Structural dynamics are usually concerned with the vibration of structural members induced by forces that vary with time, resulting in time varying responses of the structure, such as displacements, velocities and accelerations. All structures have specific natural frequencies, see Subsection 3.1.3 for a thorough explanation. If the structure is subject to a dynamic load with the same frequency as one of the natural frequencies, resonance may occur creating large response amplitudes relative to the static response. In general, resonant responses should be avoided if possible since they can lead to fatigue and structural collapse. If the natural frequencies of the structure are so that resonance is inevitable, the goal should be to ensure small response amplitudes even at resonance. This is especially the case with slender bridges, where the serviceability might be threatened by resonance, even though the structural integrity is not. To consider the inherent dynamic properties of a structure one needs to know the different aspects of vibrations, which will be presented in the next couple of sections.

3.1.1 Single-Degree of Freedom Systems

The simplest model of a vibrating mechanism is that of a single degree of freedom [5].

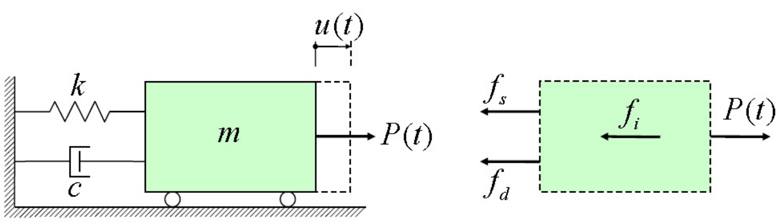


Figure 3.1: *Schematic drawing of a single degree of freedom system, containing a mass, a linear spring and a viscous damper [5].*

The system in Figure 3.1 consists of a mass, *m*, a viscous damper (dashpot) *c*, and a linear spring *k*. The system is subject to an applied force *P(t)*, and the motion of the system is described by a single degree of freedom, $u = u(t)$, being translation in the horizontal direction.

Free oscillations

When there is no externally applied load ($P(t) = 0$), the system will experience free vibration. The forces acting on the body then are:

$$\begin{array}{ll} f_i = m\ddot{u} & \text{Inertia force} \\ f_d = c\dot{u} & \text{Damping force} \\ f_s = ku & \text{Elastic spring force} \end{array} \tag{3.1}$$

The equation of motion for the system is represented by the following 2$^{\text{nd}}$ order differential equation:

$$m\ddot{u} + c\dot{u} + ku = 0 \tag{3.2}$$

The natural circular frequency [rad/s] of the system is given as

$$\omega_n = \sqrt{\frac{k}{m}} = 2\pi f_n \tag{3.3}$$

Where f_n is the natural frequency in Hertz. The damped natural frequency is relevant when the system contains damping ($c \neq 0$):

$$\omega_d = \omega_n\sqrt{1-\zeta^2} \tag{3.4}$$

With damping present, the response will decay exponentially and approach zero. In the equation above ζ is the damping ratio, which is the ratio between the damping, c, and the critical damping, $c_{cr} = 2m\omega_n$ [6]:

$$\zeta = \frac{c}{c_{cr}} \tag{3.5}$$

The obtained response depends highly on the magnitude of the damping ratio as can be seen in Figure 3.2. If $\zeta < 1$ the system is underdamped, and the response becomes oscillatory. With $\zeta = 1$ the system is critically damped, and this gives the fastest exponentially decaying response. The last alternative is when $\zeta > 1$ and the system becomes overdamped, which also leads to an exponential decay in response.

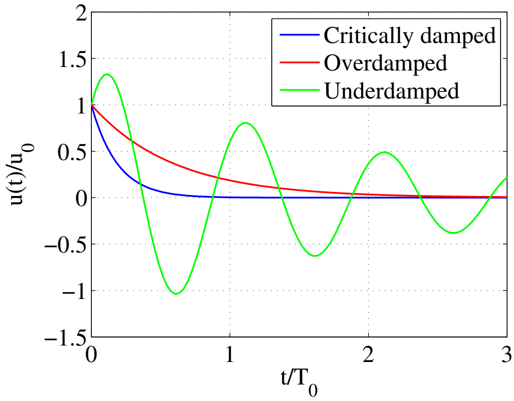


Figure 3.2: *Free vibration of undamped, critically damped and overdamped systems.*

Forced oscillations

If an external force is applied, the equation of motion becomes:

$$m\ddot{u} + c\dot{u} + ku = P(t) \tag{3.6}$$

The frequency ratio is given as the ratio between the frequency of the applied load and the natural frequency of the system:

$$\beta = \frac{\omega}{\omega_n} \tag{3.7}$$

Figure 3.3 shows the Dynamic Magnification Factor (DMF) against the frequency ratio β for a forced single degree of freedom system. DMF is the ratio between the dynamic response amplitude and the corresponding static amplitude that would be obtained with a static load ($\omega = 0$). Large response amplitudes are observed when $\beta \rightarrow 1$, which is when an external load is applied with a frequency equal to the systems natural frequency. This phenomenon is called resonance, and is why the natural frequencies are also referred to as resonant frequencies. Continued forcing at a resonance frequency could lead to unstable excitation, increasing unconditionally if the damping is not sufficient [6].

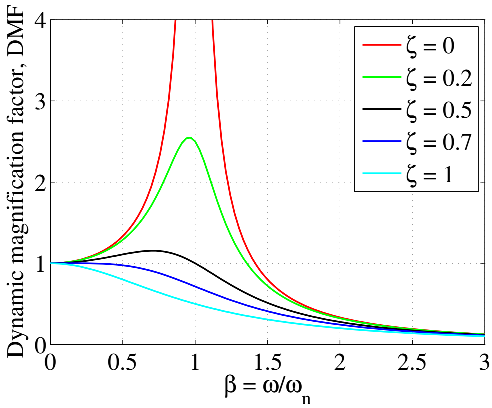


Figure 3.3: *A plot of Dynamic Magnification Factor versus frequency ratio, clearly showing the resonance phenomenon.*

3.1.2 Multiple Degree of Freedom Systems

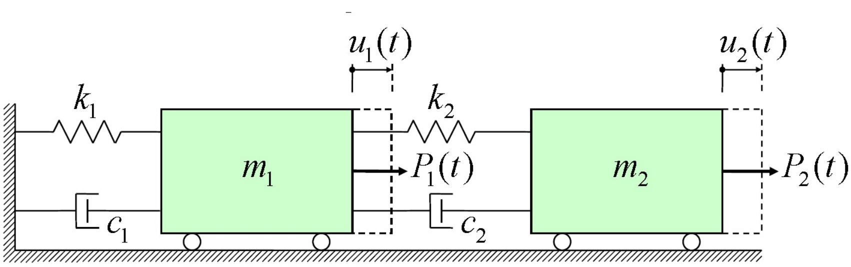


Figure 3.4: *Schematic drawing of a two degree of freedom system [5].*

In reality, structures are rarely simple enough to be modelled as single degree of freedom systems. Instead, one must employ multiple degrees of freedom (MDOF) and the different motions of the system become more complex with n coupled equations of motion, n being the number of degrees of freedom. Through modal techniques, discussed later in this chapter, the equations can be uncoupled and the study of the system can be considered as studying a set of n simple oscillators, each one describing a characteristic vibration of the system [7]. First a closer look into establishing MDOF systems, starting with an example of a two degree of freedom system shown in Figure 3.4. By establishing a free body diagram for the two masses all forces acting upon the two masses can be accounted for:

$$\left.\begin{array}{l} f_{i1} = m_1 \ddot{u}_1 \\ f_{i2} = m_2 \ddot{u}_2 \end{array}\right\} \quad \text{Inertial forces}$$

$$\left.\begin{array}{l} f_{d1} = c_1 \dot{u}_1 \\ f_{d2} = c_2 (\dot{u}_2 - \dot{u}_1) \end{array}\right\} \quad \text{Damping forces} \tag{3.8}$$

$$\left.\begin{array}{l} f_{s1} = k_1 u_1 \\ f_{s2} = k_2 (u_2 - u_1) \end{array}\right\} \quad \text{Spring forces}$$

These equations can be rewritten in matrix form as such:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{Bmatrix} + \begin{bmatrix} (c_1 + c_2) & -c_2 \\ -c_2 & c_2 \end{bmatrix} \begin{Bmatrix} \dot{u}_1 \\ \dot{u}_2 \end{Bmatrix} + \begin{bmatrix} (k_1 + k_1) & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} P_1(t) \\ P_2(t) \end{Bmatrix} \tag{3.9}$$

which often is written more compactly in this fashion:

$$[\mathbf{M}]\{\ddot{\mathbf{u}}\} + [\mathbf{C}]\{\dot{\mathbf{u}}\} + [\mathbf{K}]\{\mathbf{u}\} = \{\mathbf{P}(\mathrm{t})\} \tag{3.10}$$

where **[M]** is the mass matrix, **[C]** is the damping matrix, **[K]** is the stiffness matrix, $\mathbf{P}(t)$ is the vector of forcing functions and **u** is the displacement vector. With larger systems the most common way to solve equations like Equation 3.9 is by use of the finite element method, in which a real structure with infinitely many DOFs is discretized into a finite number of elements which are interconnected at a limited number of points (nodes) and which have a finite number of DOFs at these nodes. Next the mass, damping and stiffness matrices have to be established together with the force vector to get the equation of motion. Having discretized the degrees of freedom in space with a finite amount of elements the equation of motion now has to be solved to approximate how the responses vary with time. Several methods exist for solving this second order differential equation. Modal decomposition and implicit direct integration methods are two of them. They are discussed separately in subsections below, but first, some brief theory about natural frequencies and mode shapes.

3.1.3 Natural Frequencies and Mode Shapes

The natural frequencies of a structure are the frequencies at which the structure tends to vibrate in the absence of externally driving forces or damping forces. The vibration mode shapes are the characteristic deformed shapes of the structure when vibrating at these frequencies. As en example the three first natural frequencies and corresponding mode shapes of a simply supported beam are shown in Figure 3.5.

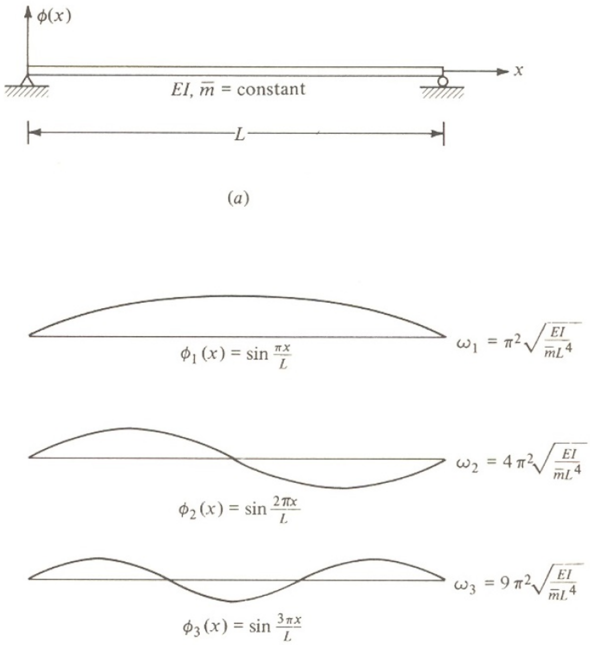


Figure 3.5: *The first three mode shapes of a simply supported beam.*

When studying the dynamic response of a bridge, its modes of vibration are paramount. As mentioned before, if a periodic load is applied with frequency equal to one of the natural frequencies of the bridge, and located at the maximum of the predominant mode shape, resonance can occur unless the structure is heavily damped. The different eigenfrequencies or natural frequencies and corresponding eigenmodes can be found by modal analysis. First the natural frequencies of the system are calculated from the following equation:

$$([\mathbf{K}] - \omega_n^2[\mathbf{M}])[\Phi] = [\mathbf{D}(\omega_n)][\Phi] = 0 \tag{3.11}$$

where $[\mathbf{D}](\omega)$ is the dynamic stiffness matrix and $[\Phi]$ is the eigenvector of the system, containing its eigenmodes. To satisfy this equation the product eigenvector times dynamic stiffness matrix must vanish. Since a zero eigenvector is a trivial solution, and therefore not interesting, the equation is solved by stating that the determinant of the the dynamic stiffness matrix must be zero [8]. There is one solution for each degree of freedom in the system. Next the different eigenmodes are found by calculating the dynamic stiffness matrix with the obtained natural frequencies and solving for the eigenvector. Since $\mathrm{D}(\omega_n)$ is now singular one of the entries in the eigenvector has to be set equal to unity to determine the other entries. The magnitude of the entries in the mode shape vectors are therefore only relative.

3.1.4 Modal Decomposition

One way to solve the equations of motion for multiple degree of freedom systems is by modal decomposition [9]. The mode shapes obtained from a modal analysis are then used to decouple the set of differential equations into a set of linearly independent differential equations of single degree of freedom oscillators. This is done by first acknowledging that since any N-dimensional vector can be expressed as a combination of N orthogonal N-dimensional vectors [10], the total displacement vector $\mathbf{u}(t)$ can be presented as a linear combination of the mode shape vectors, ϕ_n. The coefficients of that combination vary with time and are the generalised displacements $Y_i(t), i = 1, ..., n$:

$$\{\mathbf{u}(t)\} = \phi_1 Y_1(t) + \phi_2 Y_2(t) + ... + \phi_n Y_n(t) = [\Phi]\{\mathbf{Y}(t)\} \tag{3.12}$$

The modal mass, damping and stiffness matrices are obtained by inserting Equation 3.12 into 3.10 and pre-multiplying by the transpose of the mode shape matrix:

$$\begin{gathered}[\Phi]^T[\mathbf{M}][\Phi]\{\ddot{Y}\} + [\Phi]^T[\mathbf{C}][\Phi]\{\dot{Y}\} + [\Phi]^T[\mathbf{K}][\Phi]\{Y\} = [\Phi]^T\{\mathbf{P}(t)\} \\ [\mathbf{M^*}]\{\ddot{Y}\}+[\mathbf{C^*}]\{\dot{Y}\}+[\mathbf{K^*}]\{Y\}=\{\mathbf{P^*}(t)\}\end{gathered} \tag{3.13}$$

Due to the orthogonality of the mode shape matrix with respect to both the mass and stiffness matrices of the system all modal matrices (marked with *) are diagonal [10]. n modal equations are then obtained on the form:

$$\ddot{Y}_n(t) + 2\zeta_n\omega_n\dot{Y}_n(t) + \omega_n^2 Y_n(t) = \frac{P_n^*(t)}{M_n^*} \tag{3.14}$$

Here $\ddot{Y}_n(t)$, $\dot{Y}_n(t)$ and $Y_n(t)$ are modal (or generalised) acceleration, velocity and displacement, ζ_n and ω_n are the damping ratio and natural circular frequency for the n^{th} mode of vibration, while $P_n^*(t)$ and M_n^* are the modal force and mass for the same mode. By knowing the external forces, often modelled as a Fourier series, the modal displacements Y_i(t) can be determined by solving Equation 3.14 as an ordinary second order differential equation. Finally, the physical displacements can be calculated using relation 3.12. The particular solutions for the modal displacements highly depend on the modal forces $\mathbf{P^*}(t)$ as the solutions take the same form. I.e., if the loading is sinusoidal the particular solution will be sinusoidal. Therefore it is important to identify the shape of the forcing, in this case due to pedestrians, and find a proper and easy way to model it. This will be investigated further in Section 3.3.

Quite often with structures like footbridges there is one dominant mode, and in these cases the problem can be simplified and solved fairly accurately by solving only one single degree of freedom system. The procedure is then to use the modal equation for that specific mode (see Equation 3.14) to obtain the modal responses, and inserting into Equation 3.12, then only containing one term on the right side. This is commonly implemented when checking vibration in footbridges. To obtain reliable estimates of the response using modal dynamics it is very important to determine the dynamic properties, such as mass, damping and stiffness accurately on modal form. The stiffness and mass are easily calculated in finite element programs when correct material properties are given. The damping is more difficult to determine both because the amount of damping in the structure is difficult to predict in advance and because damping is mathematically difficult to describe correctly. There are several different damping models, and it's not always obvious which one to use. Nonetheless, damping represents energy dissipation and is very beneficial because it reduces the response, especially to a dynamic force causing resonance, so it is important to model it as accurate as possible [11]. More about damping and how to model it in Section 3.4.

3.1.5 Implicit Direct Integration Methods

The second methods elaborated upon for solving the equation of motion for multiple degree of freedom systems are implicit direct integration methods. These methods subdivide the time interval into n_{step} equal time increments $\Delta t = T/n_{step}$, and the integration scheme considered establishes an approximate solution by satisfying the dynamic equilibrium equation at the discrete times: $t = \Delta t,\ 2\Delta t,\ 3\Delta t,\ ...,\ t_n = n\Delta t,\ t_{n+1} = (n+1)\Delta t,\ ...\ ,\ T$, as illustrated in Figure 3.6. It is assumed that displacements and velocities are known at some time $t = 0$, $\{\mathbf{D}(t=0)\} = \{\mathbf{D}_0\}$ and $\{\dot{\mathbf{D}}(t=0)\} = \{\dot{\mathbf{D}}_0\}$, and that we want to determine the response history from time $t = 0$ to time $t = T$. With implicit direct integration methods the displacement at step $n+1$ is obtained indirectly (implicitly) from the equilibrium conditions at time t_{n+1}. Thus equation solving is required at every time step [12].

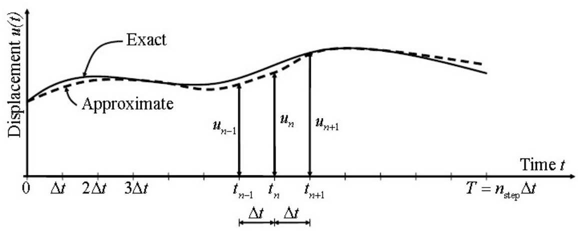
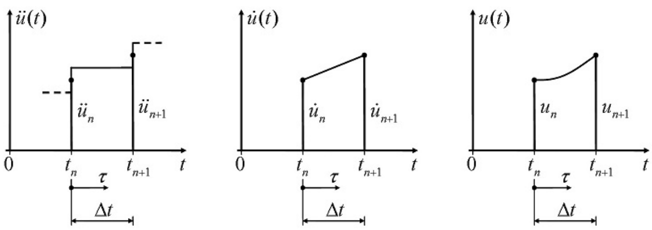
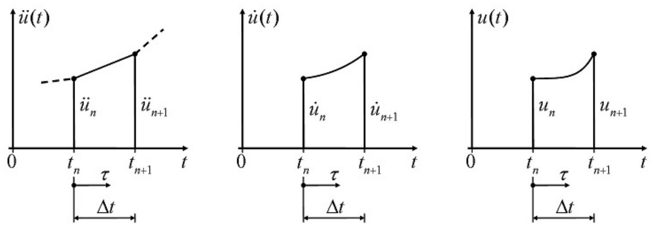


Figure 3.6: *Implicit direct integration methods [12].*

There are many variations of this method, depending on the assumptions used to integrate the accelerations in order to determine the velocities and displacements at the end of each time step. For instance, one can assume constant average acceleration or linear acceleration during one time step, as shown in Figure 3.7:



(a) *Constant Acceleration.*



(b) *Linear Acceleration.*

Figure 3.7: *Shape of displacement-time and velocity-time functions based on assumed variation of acceleration-time function within one time step [12].*

The constant average and linear acceleration methods are both special cases of the Newmark method, which is based on the following approximations:

$$\dot{u}_{n+1} = \dot{u}_n + \Delta t[\gamma \ddot{u}_{n+1} + (1-\gamma)\ddot{u}_n] \tag{3.15}$$

$$u_{n+1} = u_n + \Delta t \dot{u}_n + \frac{\Delta t^2}{2}[2\beta \ddot{u}_{n+1} + (1-2\beta)\ddot{u}_n] \tag{3.16}$$

For which $\gamma = \frac{1}{2}, \beta = \frac{1}{4}$ $\Rightarrow$ Constant average acceleration method

$\gamma = \frac{1}{2}, \beta = \frac{1}{6}$ $\Rightarrow$ Linear acceleration method

The parameters γ and β govern the variation of acceleration over a time step, the stability of the integration scheme, the amount of numerical damping and the accuracy of the method. By solving Equation 3.16 for $\ddot{u}_{n+1}$ and inserting into Equation 3.15 the following expressions are obtained for the acceleration and velocity at time *n+1*:

$$\ddot{u}_{n+1} = \frac{1}{\beta \Delta t^2}(u_{n+1} - u_n - \Delta t \dot{u}_n) - \left(\frac{1}{2\beta} - 1\right)\ddot{u}_n \tag{3.17}$$

$$\dot{u}_{n+1} = \frac{\gamma}{\beta \Delta t}(u_{n+1} - u_n) - \left(\frac{\gamma}{\beta} - 1\right)\dot{u}_n - \Delta t\left(\frac{\gamma}{2\beta} - 1\right)\ddot{u}_n \tag{3.18}$$

The resulting Newmark approximations for acceleration and velocity can be substituted into the equation of motion:

$$m\ddot{u}_{n+1} + c\dot{u}_{n+1} + ku_{n+1} = P_{n+1} \tag{3.19}$$

For which we obtain:

$$\begin{aligned}\left(\frac{m}{\beta\Delta t^2} + \frac{\gamma c}{\beta\Delta t} + k\right)u_{n+1} = P_{n+1} + m\left[\frac{1}{\beta\Delta t^2}u_n + \frac{1}{\beta\Delta t}\dot{u}_n + \left(\frac{1}{2\beta} - 1\right)\ddot{u}_n\right] + \\ c\left[\frac{\gamma}{\beta\Delta t}u_n + \left(\frac{\gamma}{\beta} - 1\right)\dot{u}_n + \Delta t\left(\frac{\gamma}{2\beta} - 1\right)\ddot{u}_n\right]\end{aligned} \tag{3.20}$$

For an MDOF system, with $\{\mathbf{D}\}$ being the vector containing the degrees of freedom, the Newmark relations become:

$$\{\dot{\mathbf{D}}\}_{n+1} = \{\dot{\mathbf{D}}\}_n + \Delta t\left[\gamma\{\ddot{\mathbf{D}}_{n+1} + (1-\gamma)\{\ddot{\mathbf{D}}\}_n\right] \tag{3.21}$$

$$\{\mathbf{D}\}_{n+1} = \{\mathbf{D}\}_n + \Delta t\{\dot{\mathbf{D}}\}_n + \frac{1}{2}\Delta t^2\left[2\beta\{\ddot{\mathbf{D}}_{n+1} + (1-2\beta)\{\ddot{\mathbf{D}}\}_n\right] \tag{3.22}$$

By solving the latter equation for $\{\ddot{\mathbf{D}}\}_{n+1}$, we obtain

$$\{\ddot{\mathbf{D}}\}_{n+1} = \frac{1}{\beta\Delta t^2}\left(\{\mathbf{D}\}_{n+1} - \{\mathbf{D}\}_n - \Delta t\{\dot{\mathbf{D}}\}_n\right) - \left(\frac{1}{2\beta} - 1\right)\{\ddot{\mathbf{D}}\}_n \tag{3.23}$$

$$\{\dot{\mathbf{D}}\}_{n+1} = \frac{\gamma}{\beta\Delta t}\left(\{\mathbf{D}\}_{n+1} - \{\mathbf{D}\}_n\right) - \left(\frac{\gamma}{\beta} - 1\right)\{\dot{\mathbf{D}}\}_n - \Delta t\left(\frac{\gamma}{2\beta} - 1\right)\{\ddot{\mathbf{D}}\}_n \tag{3.24}$$

These equations are substituted into the equation of motion 3.10 written at time step $n+1$, and then solved for $\{\mathbf{D}\}_{n+1}$. The result is

$$\begin{aligned}[\mathbf{K}^{\text{eff}}]\{\mathbf{D}\}_{n+1} = \{\mathbf{R}^{\text{ext}}_{n+1}\} + [\mathbf{M}]\left\{\frac{1}{\beta\Delta t^2}\{\mathbf{D}\}_n + \frac{1}{\beta\Delta t}\{\dot{\mathbf{D}}\}_n + \left(\frac{1}{2\beta} - 1\right)\{\ddot{\mathbf{D}}\}_n\right\} \\ +[\mathbf{C}]\left\{\frac{\gamma}{\beta\Delta t}\{\mathbf{D}\}_n + \left(\frac{\beta}{beta} - 1\right)\{\dot{\mathbf{D}}\}_n + \Delta t\left(\frac{\gamma}{2\beta} - 1\right)\{\ddot{\mathbf{D}}\}_n\right\}\end{aligned} \tag{3.25}$$

Where

$$[\mathbf{K}^{\text{eff}}] = \frac{1}{\beta\Delta t^2}[\mathbf{M}] + \frac{\gamma}{\beta\Delta t}[\mathbf{C}] + [\mathbf{K}] \tag{3.26}$$

Equation 3.26 can then be solved for $\{\mathbf{D}\}_{n+1}$ at every time step. The Newmark family of methods is unconditionally stable in linear problems, meaning the solution never blows up, and is probably the most widely used implicit method [13].

3.2 Pedestrian Induced Vibrations

While walking, humans exert a time-varying dynamic force that has components in three directions; the vertical, horizontal-longitudinal and horizontal-lateral. These forces can cause structures to vibrate and therefore single pedestrian walking has been studied for some time. The vertical component has been studied the most because it has the largest amplitude, but with increasingly slender structures it has been established that also the horizontal components can have a significant impact, which has caused a slight shift in focus over the last decades. The effects of several pedestrians have also been investigated and models to predict the response due to groups of people have been proposed. Other types of human-induced forces like jumping and running may also be important and literature on the matter does exist.

Bachmann and Ammann stated that the shape of the dynamic load-time curve due to a single pedestrian depends on multiple factors such as pacing rate, forward speed, stepping particularities, the persons weight and sex, the type of footwear and the surface conditions. These affect the amplitude, the position of the peaks, the duration of each load cycle and the period between load cycles [14]. One of the most important variables in pedestrian loading is the frequency of the dynamic force, equal to the walking frequency or pacing rate . It is given as number of paces per second. Matsumoto et al. investigated a sample of 505 pedestrians, and concluded that the step frequency followed a normal distribution with a mean of 1.99 pace/s and a standard deviation of 0.173 paces/s, see Figure 3.8 [15][16] .

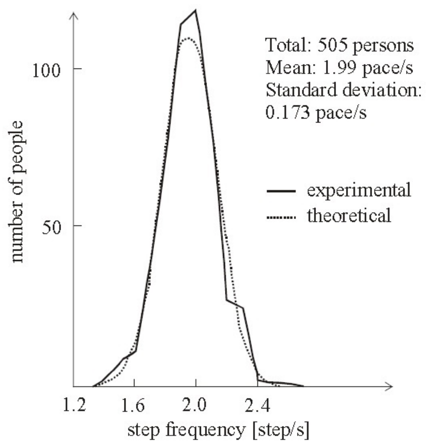


Figure 3.8: *Normal distribution of pacing frequencies for normal walking [15].*

Studies by Kerr and Bishop have produced matching results [17]. Bachmann et al. confirmed this and also provided frequency ranges for several other different human activities; 1.6 - 2.4 Hz for walking, 2.0 - 3.5 Hz for running, 1.8 - 3.4 Hz for jumping, 1.5 - 3.0 Hz for bouncing and 0.4 - 0.7 Hz for horizontal body swaying while staying stationary [18].

The forward speed or velocity (v_s) of a pedestrian is related to the pacing rate through the stride length (l_s). As one would expect people walking with the same velocity can have very different stride lengths and pacing rates. Through experiments average values for pacing rate and forward speed were found and are given in Table 3.1 [14].

	f_s [Hz]	v_s [m/s]	l_s [m]
Slow walk	~1.7	1.1	0.60
Normal walk	~2.0	1.5	0.75
Fast walk	~2.3	2.2	1.00
Slow running (jogging)	~2.5	3.3	1.30
Fast running (sprinting)	>3.2	5.5	1.75

Table 3.1: *Correlation of pacing rate, forward speed and stride length for walking and running according to Bachmann and Amman.*

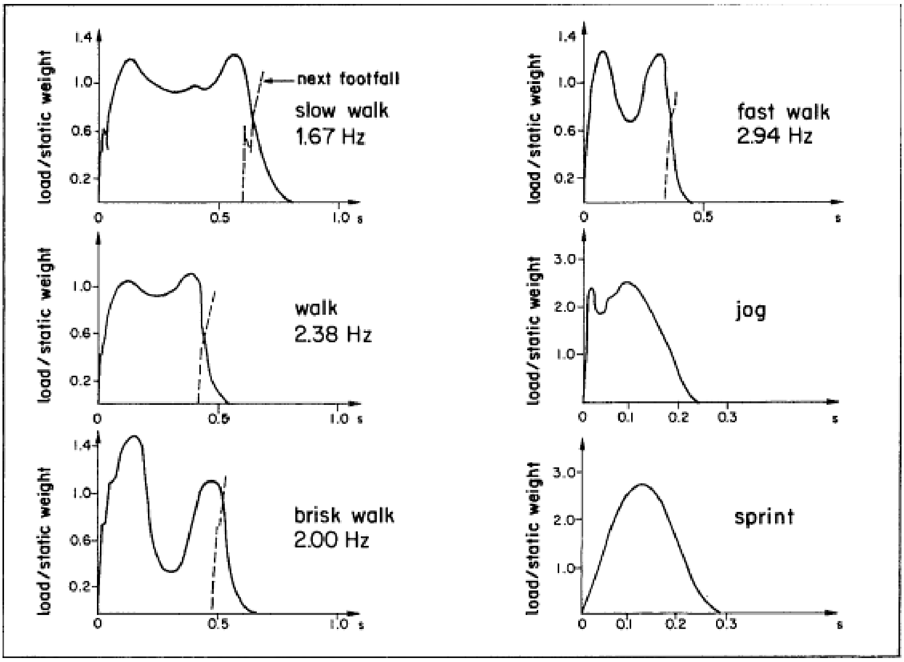


Figure 3.9: *Load-time function with different pacing rates [19].*

Figure 3.9 shows typical load-time functions for different pacing rates and activities. Notice that the shape of the curve for walking looks like a saddle (it is bimodal), which is due to the inverted pendulum movement of the centre of gravity. The centre of gravity will accelerate slightly upwards, which induces a second inertia reaction force. The two peaks are at the "foot-flat" (FF) and the "heel-off" (HO) stage of the walking pace [20] (the terms are explained in Figure 3.10).

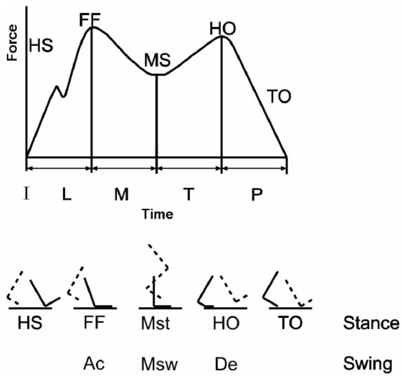


Figure 3.10: *Load-time function with the human gait cycle [20].*

3.2.1 Groups of Pedestrians

When several pedestrians are present on a bridge, a distinction is usually made between "groups" and "crowds" - the difference being that in the former the people involved are more or less synchronized, whereas in the latter there is no synchronization between the individuals. Several studies have been performed to figure out how crowd behaviour can affect a structure. When considering slender and/or lightweight structures, it is important to not only consider the forces exerted by the pedestrians upon the structure, but also the dynamic interaction between human and structure. Firstly, the natural frequencies and damping of a bridge is altered when pedestrians are present, and secondly the frequency of the pedestrian loading usually synchronizes with the bridge's natural frequency to some degree. Another synchronization aspect is the fact that people tend to subconsciously synchronize with each other, not only the bridge's perceived motion. This is partly due to the fact that motion becomes restricted due to the reduction of available space [21].

Figure 3.11: *Millennium bridge in London experienced severe excitations during opening day in 2000.*

Pedestrians tend to synchronize with the bridge's motion when it becomes perceptible. People are most sensitive to lateral motion, and are more prone to synchronize with this motion. This subconscious behaviour is an attempt to maintain body balance. The attempt to synchronize leads to a wider leg stance and a greater motion of the upper torso leading to a greater exerted force. This amplifies the dynamic response, and the only way to stop it is to reduce the number of pedestrians on the bridge, or to disrupt their movement. The frequency of lateral movement is half that of the vertical, i.e. around 1 Hz. Pedestrian-structure synchronization is also known as "lock-in" [22]. This happens after the structure has reached a certain threshold of acceleration [21]. Interestingly, when it comes to vertical vibrations, the pedestrians inability to synchronize their pace with the vertical movement causes the vibration to diminish, meaning they may act as dampers on the system in the vertical direction [22].

After investigating the incidents at the Millenium Bridge (see Figure 3.11) and Solférino bridge, where large groups of people caused severe lateral motion, it was concluded that there exists a transition point where a small increase in the number of people on the bridge leads to a large increase in the bridge's lateral response. With the notion of people acting as negative dampers (i.e. amplifiers), first introduced by Dallard et al., one can calculate a critical number of pedestrians which marks the transition between stability and instability [21]. Unfortunately there is not much research available on this topic and most of what is available antedates the incidents at the Solférino Bridge and the Millenium Bridge. The latest documented critical number is resighted in a Technical guide from the Technical Department of Transport, Roads and Bridges Engineering and Road Safety (Sétra) in France. The formula for critical number of pedestrians based on the Millenium bridge is stated as such:

$$N = \frac{8\pi\zeta m_1 f_1}{K} \tag{3.27}$$

In which f_1 is the first lateral natural frequency, ζ is the modal damping ratio, m_1 is the generalised mass in the mode (modal mass) and K is a proportionality factor (in Ns/m) that must be determined for the structure at hand [7]. The difficulties with such a formula are obvious; not only does K need to be determined for all individual structures, it is established by measuring accelerations under conditions of steady state crowd loading - after the bridge has erected. Additionally, the formula can only be used as a rough estimate to the maximum allowable number of people [23].

Perhaps the simplest way of modelling groups of people crossing a bridge is with Matsumoto et al.'s model [16], although it is derived for bridges vibrating only in the vertical direction. In this model the response from a single pedestrian is simply multiplied with $\sqrt{N}$, where N is the number of pedestrians present on the footbridge at a given time. Assuming that the pedestrians that enter the bridge per second follow a Poisson distribution and that they all walk with the same frequency but with random phases, the model implies that all pedestrian movement is uncorrelated, rendering it useless for any modelling of synchronized behaviour.

3.2.2 Comfort Criteria

Slender footbridges tend to have low natural frequencies in the range of pedestrian walking frequencies, meaning resonance is probable. It is the comfort of the pedestrians on the bridge that poses a problem, and generally not the structural integrity. The loads from a group of pedestrians are simply not large enough to compromise the structure. How much care the designer should pay to the pedestrians comfort needs to be decided in advance. One way to determine this is to classify the bridge or structure into traffic classes, based on how many people are expected to use the bridge. Criteria for comfort are most commonly represented as a limit acceleration for the footbridge [24]. Peak-acceleration is not necessarily representative for the dynamic response of the bridge, so instead root mean square acceleration is preferable. The RMS-acceleration is the square root of the mean value of the square acceleration[25]:

$$RMS - acceleration = \sqrt{\frac{\int_{t1}^{t2} \ddot{x}(t)^2 dt}{t_2 - t_1}} \tag{3.28}$$

Where $\ddot{x}(t)$ is the acceleration time history, and t_1 and t_2 define the beginning and end of the time interval considered. The choice of RMS-accelerations as the vibration perception descriptor was based mainly on the fact that it was easy to measure with both digital and analog methods. Suggested values for the limit acceleration are usually given in standards and design codes, either as tabulated values or as a mathematical formula. Because the perception of motion and tolerance is individual for each person, the limits are usually within a certain bandwidth of values. The appearance of the bridge itself is also important to the pedestrians perception of vibration. If a bridge looks slender and rickety, pedestrians will be mentally prepared for some vibration, which might not be the case for a bridge that appears sturdy. This is demonstrated in the figure below, where the pedestrians perception of vibration of the sturdy Wachtelsteg Footbridge in Pforzheim, Germany was compared to the lighter-looking Kochenhofsteg Footbridge in Stuttgart, also Germany.

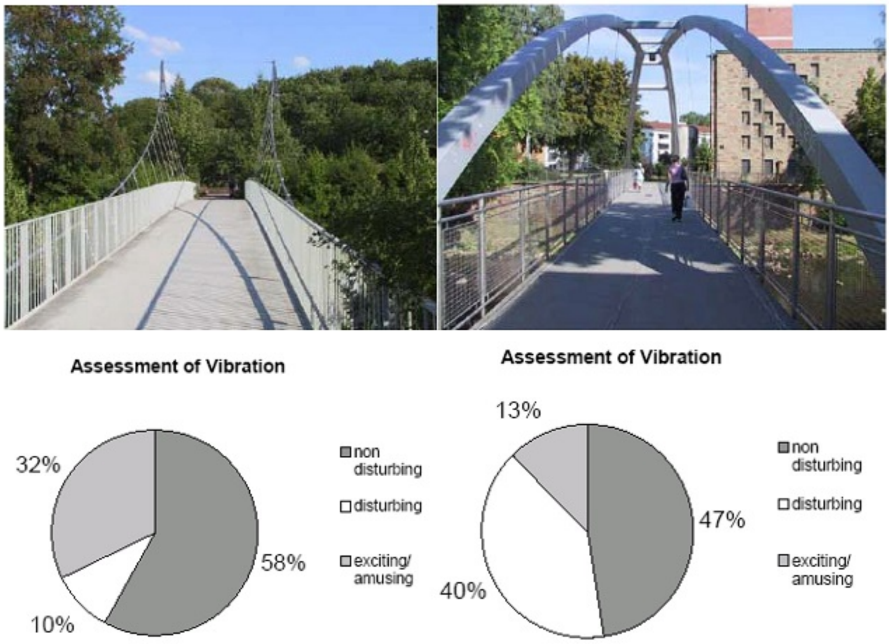


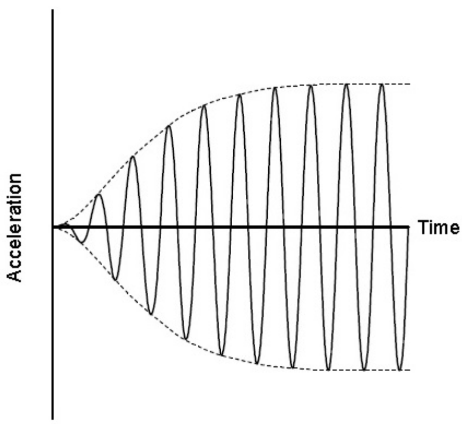
Figure 3.12: *Perception of vibration for a slender-looking bridge (left) versus a sturdy-looking bridge (right).*

The two bridges have very similar dynamic properties, but still the percentage of pedestrians feeling disturbed by the vibrations is 4 times as great on the more sturdy looking bridge. This supports the theory that several aspects, given below, influence the assessment of footbridge vibration and whether or not they are characterized as uncomfortable or not [24]:

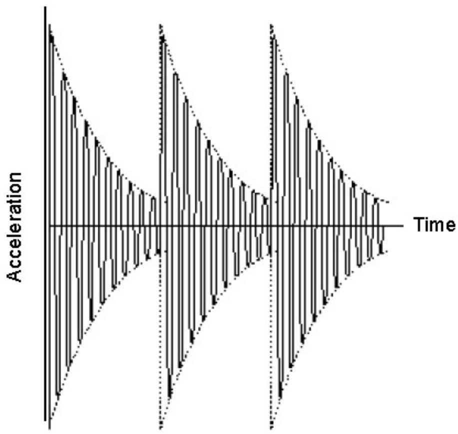
- Number of people walking on bridge
- Frequency of use
- Height above ground
- Position of human body (sitting, standing, walking)
- Harmonic or transient excitation characteristics (vibration frequency)
- Exposure time
- Transparency of the deck pavement and the railing
- Expectancy of vibration due to appearance

3.2.3 Dynamic Response due to Pedestrian Loading

It can be useful to plot the acceleration response due to a pedestrian loading against time to determine whether or not resonance has occurred. The two possible response scenarios are depicted in Figure 3.13. The resonant response (Figure 3.13(a)) occurs when the frequency of the loading, or one of the higher harmonics of the fundamental frequency, is similar to the natural frequency of the structure (more on higher harmonics in section 3.3.1). In this scenario the acceleration of the structure gradually builds up until it reaches a steady state level. In the theoretical case where no damping is present, the build-up would continue infinitely. At steady state the structure is resonating with the excitation, hence the term resonance. Walking activities may induce resonance to some degree, as long as the walking frequency is within the range of natural frequencies of the structure. If the structures natural frequencies don't match the walking frequencies, the response in Figure 3.13(b) is typical. This is known as a transient response, in which case the structure responds to the forcing as if it is a series of impulses, with the vibration caused by one footstep dying away before the next footstep. A combination of the two responses is most likely [26].



(a) *Resonant response.*



(b) *Transient response.*

Figure 3.13: *Response envelopes to dynamic loading [26].*

3.3 Mathematical Modelling of Pedestrian Loading

Establishing an analytical force model due to human movement is a complicated task. There are many variables involved, and even a person walking twice at the same speed will have variations from time to time. For example, increasing walking velocity will lead to increasing step length and peak force magnitude [27]. Additionally, when increasing the walking speed, the variability in vertical and lateral successive steps increases [28]. Both the number of pedestrians and their degree of synchronisation will influence the forcing, making it complicated to generalise. Lastly, research indicates that the forces exerted by pedestrians are dependent on whether or not the motion of the structure is perceptible [11]. All these factors add up and cause great uncertainty and therefore instead of one universal load model, several different ones exist.

There are also differences between walking and running which must be accounted for. While walking there is a period of time when both feet are in contact with the ground, whereas with running there is a period of time when both feet are off the ground, leading to zero force recorded (see Figure 3.14) [11] . However, it is worth mentioning that the loading due to a pedestrian running is often neglected, simply because the time it takes a person to run over the structure at study is too short for the resonance phenomenon to settle, and also so short that it only annoys any other pedestrians for a short time. In special cases, such as a marathon run, more attention should be paid to the loads induced by running masses [7].

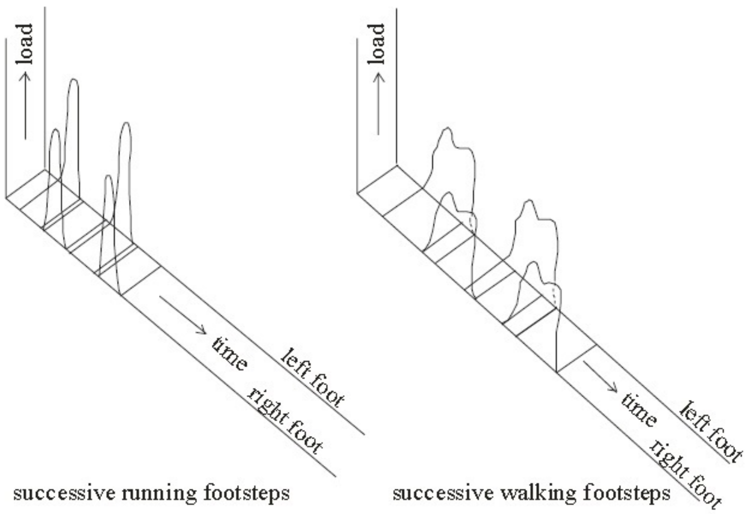


Figure 3.14: *Patterns of walking and running (Galbraith and Barton [11]).*

The two ways to apply dynamic forces to a numerical model are with either time- or frequency domain models. The former is by far the most widely used, and will be the chosen model for this thesis. Furthermore, for dynamic forces due to pedestrian loading the time-domain models are divided into probabilistic and deterministic models. Both models are based on the assumption that both feet produce the same force and that the force is periodic. Only deterministic models will be investigated further and used in the different analyses in this thesis.

The most common load models, and the ones that have been used in the numerical model of the footbridge, are based on a Fourier series with a varying number of terms included. A Fourier series can represent any force as long as it is periodic, and by including a finite number of terms an approximation of the periodic force is obtained. Such series are on the form [18]:

$$F_p(t) = G\alpha_0 + \sum_{i=1}^{n} G\alpha_i \sin\left(2\pi i f_p t - \phi_i\right) \tag{3.29}$$

G	=	Persons weight [N]
α_i	=	Fourier's coefficient of the i^{th} harmonic, i.e. dynamic load factor (DLF)
f_p	=	Pacing rate [Hz]
ϕ_i	=	Phase shift of the i^{th} harmonic
i	=	Order number of the harmonic
n	=	Total number of contributing harmonics

Fourier series are used for all three force components and the Fourier's coefficients (dynamic load factors) are what separate the force components and different load models from each other. The DLFs are determined empirically, and are dependent on both the walking frequency and activity. Table 3.2 gives an overview of DLFs established by different authors. α_0 in Equation 3.29 represents the static component due to gravity and is always equal to 1 for vertical loads and 0 for horizontal loads. It is therefore not mentioned in the table.

Author(s)	DLFs for considered harmonics	Comment	Type of activity and its direction
Blanchard et al. [29]	$\alpha_1 = 0.257$	DLF is lesser for frequencies from 4 to 5 Hz	Walking - vertical
Bachmann & Ammann [14]	$\alpha_1 = 0.4$ - 0.5 $\alpha_2 = \alpha_3 = 0.1$	2.0 Hz - 2.4 Hz At approximately 2.0 Hz	Walking - vertical Walking - vertical
Schulze (after Bachmann & Ammann [14]	$\alpha_1 = 0.37$, $\alpha_2 = 0.10$, $\alpha_3 = 0.12$ $\alpha_4 = 0.04$, $\alpha_5 = 0.08$	At 2.0 Hz	Walking - vertical
	$\alpha_1 = 0.039$, $\alpha_2 = 0.01$, $\alpha_3 = 0.043$ $\alpha_4 = 0.04$, $\alpha_5 = 0.08$	At 2.0 Hz	Walking - lateral
	$\alpha_{1/2} = 0.037$, $\alpha_1 = 0.204$ $\alpha_{3/2} = 0.026$, $\alpha_2 = 0.083$ $\alpha_{5/2} = 0.024$	At 2.0 Hz	Walking - longitudinal
Rainer et al. [30]	α_1, α_2, α_3 and α_4	DLFs are frequency dependent (Figure 10)	Walking, running, jumping - vertical
Bachmann et al. [18]	$\alpha_1 = 0.4/0.5$, $\alpha_2 = \alpha_3 = 0.1/$-	At 2.0/2.4 Hz	Walking - vertical
	$\alpha_1 = \alpha_3 = 0.1$	At 2.0 Hz	Walking - lateral
	$\alpha_{1/2} = 0.1$, $\alpha_1 = 0.2$, $\alpha_2 = 0.1$	At 2.0 Hz	Walking - longitudinal
	$\alpha_1 = 1.6$, $\alpha_2 = 0.7$, $\alpha_3 = 0.2$	At 2.0 - 3.0 Hz	Running - vertical
Kerr [31]	α_1, $\alpha_2 = 0.07$, $\alpha_3 \approx 0.06$	α_1 is frequency dependent (figure 11)	Walking - vertical
Young [32]	$\alpha_1 = 0.37(f - 0.95) \leq 0.5$ $\alpha_2 = 0.054 + 0.0044f$ $\alpha_3 = 0.026 + 0.0050f$ $\alpha_4 = 0.010 + 0.0051f$	These are mean values for DLFs	Walking - vertical
Bachmann et al. [18]	$\alpha_1 = 1.8/1.7$, $\alpha_2 = 1.3/1.1$ $\alpha_3 = 0.7/0.5$	Normal jump at 2.0/3.0 Hz	Jumping - vertical
	$\alpha_1 = 1.9/1.8$, $\alpha_2 = 1.6/1.3$ $\alpha_3 = 1.1/1.8$	High jump at 2.0/3.0 Hz	Jumping - vertical
	$\alpha_1 = 0.17/0.38$, $\alpha_2 = 0.10/0.12$ $\alpha_3 = 0.04/0.02$	At 1.6/2.4 Hz	Bouncing - vertical
	$\alpha_1 = 0.5$	At 0.6 Hz	Body swaying while standing - lateral
Yao et al. [33]	$\alpha_1 = 0.7$, $\alpha_2 = 0.25$	Free bouncing on a flexible platform with natural frequency of 2.0 Hz	Bouncing - vertical

Table 3.2: *DLFs for single person force models by different authors.*

3.3.1 Higher Harmonics

Pedestrians walking at a frequency, f_p, will cause vertical and longitudinal forces consisting of that fundamental frequency and also nth harmonics of that frequency, meaning terms with frequency $2f_p$, $3f_p$, etc. The load models take this into account by including several terms in the Fourier series:

$$F = \alpha_1 \sin\left(2\pi f_p t\right) + \alpha_2 \sin\left(2\pi 2 f_p t\right) + \alpha_3 \sin\left(2\pi 3 f_p t\right) + \dots \tag{3.30}$$

The consequence of this is that a pedestrian can cause resonance at a natural frequency not only by walking at a pacing rate equal to that resonance frequency, but also if one of the nth harmonics of the pacing rate is equal to the resonance frequency. Modes with natural frequencies outside the range of walking frequencies (1.6 - 2.4 Hz) are thus not automatically safe from excitation. This is why many design guidelines suggest that calculation of the vertical accelerations may be omitted only if the natural frequencies are higher than 5 Hz. More about this in Chapter 4.

In contrast to the vertical and longitudinal, the fundamental frequency of the lateral forces due to a pedestrian walking is equal to half the pacing rate due to the left and right foot causing forces in opposite directions [11]. A pedestrian walking at a frequency, f_p, will thus cause lateral forces on the form:

$$F = \alpha_1 \sin\left(2\pi \frac{f_p}{2} t\right) + \alpha_2 \sin\left(2\pi 2 \frac{f_p}{2} t\right) + \alpha_3 \sin\left(2\pi 3 \frac{f_p}{2} t\right) + \dots \tag{3.31}$$

It is important to be aware of this difference in harmonics between vertical and lateral force components when trying to cause resonance in a structure.

3.4 Damping

Damping of vibrating structures happens through energy dissipation. All structures dissipate some amount of energy and therefore have some damping, but the amount is highly dependent on the specific structure, the materials used, etc. The capability to dissipate energy is very beneficial because it reduces the structural response due to dynamic forcing, especially near resonance. Since the resonance response most often governs the serviceability of a footbridge it is very important to model damping as accurately as possible [11].

The total damping is very structure-specific due to the fact that several dissipation mechanisms can contribute, all of which are very hard to assess individually. They are also hard to model mathematically and therefore several damping models exist. The viscous damping model is most widely used due to its simplicity, despite the fact that it does not describe the real behaviour of the structure [34]. The most common way to express this damping model is in terms of the viscous damping ratio ζ_n, defined for each mode n. The best way to get an idea of how much damping there will be is through testing, where the investigated mode is excited near resonance [35].

3.4.1 Pedestrians as Dampers

Pedestrians are interesting in regards to damping, because they can act as both dampers and negative dampers (amplifiers). A pedestrian in itself has certain damping properties that can be divided into two categories. Firstly, energy is dissipated within the body itself, through joints, limbs, muscles and tendons. Secondly energy is dissipated mechanically due to human action. It has been argued that for small accelerations in a system with a pedestrian, the acceleration of the human mass centre will be somewhat smaller [36]. This implies that the human body actively counteracts the inertia forces from the structure, meaning it acts as an active damper system [20]. Zheng & Brownjohn [37] observed that a person standing on a vibrating plank contained a damping ratio of 39%, which is significant. The damping effect from several pedestrians can become quite substantial, depending on the mass ratio between pedestrian and structure [20].

Alternatively, pedestrians may act as negative dampers. If the external force from the pedestrian is in phase with the inertia force from the structure, it may be interpreted as added structural mass, which is the case when the pedestrians synchronize with the movements in the bridge. This tends to happen for larger groups of people. Newland [36] concludes that bridge vibration becomes unstable under pedestrian loading when the mass of people m per unit length is too great a proportion of the bridge mass M per unit length. A permissible ratio m/M depends on the amount of damping present in the modes that are likely to be excited by the pacing rate of pedestrians.

3.4.2 Damping in *Abaqus*

[38] *Abaqus* has four categories of damping sources; material and element damping, global damping, modal damping and damping associated with time integration, known as numerical integration. These damping sources are optional to include and can be combined. All these types of damping can be applied in a combination of two forms:

- Velocity proportional viscous damping
- Displacement proportional structural damping

For mode-based dynamics composite damping is also an option. Viscous damping is implemented using the Rayleigh damping factors α and β. The damping matrix (either element matrix or global matrix based on the category of damping) is then given as a linear combination of the mass and stiffness matrices:

$$[\mathbf{c}] = \alpha[\mathbf{m}] + \beta[\mathbf{k}] \tag{3.32}$$

Structural damping represents damping as complex stiffness. By defining the structural damping factor s you get the structural damping matrix:

$$[\mathbf{k}_s]{=}s[\mathbf{k}] \tag{3.33}$$

When in the time domain, the solution can not include imaginary values, and therefore structural damping in the time-domain is applied as an equivalent viscous damping.

Material and element damping:
Damping can be specified as a part of the material model or as elements such as dashpots and springs. All have the option of both viscous and structural damping coefficients. As indicated by Equation 3.32 you specify an α-factor for mass proportional damping and a β-factor for stiffness proportional damping.

For a given mode n the fraction of critical damping ζ_n is given as

$$\zeta_n = \frac{\alpha}{2\omega_n} + \frac{\beta\omega_n}{2} \tag{3.34}$$

From Equation 3.34 it is apparent that α damps the lower frequencies and β the higher ones. Material damping gives damping proportional to the mass and stiffness matrices of an element [39], so the frequencies in Equation 3.34 are those of each individual element, not the natural frequencies of the entire structure. Only if the same coefficients are used for all relevant materials in the *Abaqus* model can the coefficients be factored out of the global matrices and then give the relation stated in Equation 3.34 for the natural frequencies of the entire structure as well. If different Rayleigh coefficients are implemented in different materials the damping ratio for a given mode will depend on the relative influence of the different materials when deforming into the corresponding mode shape. The consequence being that the damping ratio of each structural mode can not be calculated using Equation 3.34

Global damping and modal damping:
Both viscous Rayleigh damping and structural damping can be applied to the global matrices, thus giving global damping. Unfortunately, this can only be implemented in linear perturbation analyses, all of which do not operate in the time domain. The same is true for modal damping, in which viscous and structural damping factors can be applied to specific modes or frequency ranges.

Damping associated with time integration:
The latter damping category, algorithmic/numerical damping (also numerical *dissipation*), is merely amplitude decay which is generally associated with time integration schemes in which the time increment size is finite. The amount of numerical damping is highly dependent on the integration scheme, the time step and the natural frequencies of the structure. It is implemented in *Abaqus* through the α-method proposed by Hilber, Hughes and Taylor. This method can be regarded as a generalization of the Newmark methods elaborated upon on page 17. It is based on the Newmark difference relations [40], Equation 3.23 and 3.24, and the modified equation of motion becomes:

$$[\mathbf{M}]\{\ddot{\mathbf{D}}\}_{n+1} + (1+\alpha)[\mathbf{C}]\{\dot{\mathbf{D}}\}_{n+1} - \alpha[\mathbf{C}]\{\dot{\mathbf{D}}\}_n + (1+\alpha)[\mathbf{K}]\{\mathbf{D}\}_{n+1} - \alpha[\mathbf{K}]\{\mathbf{D}\}_n = \{\mathbf{R}_\alpha^{\text{ext}}\} \quad (3.35)$$

Where $\{\mathbf{R}_\alpha^{\text{ext}}\}$ is $\{\mathbf{R}^{\text{ext}}\}$ evaluated at time $(1+\alpha)t_{n+1} - \alpha t_n = t_{n+1} + \alpha\Delta t$. If $\alpha = 0$, the equation reduces to the equation used in Newmark methods. Algorithmic damping is introduced by using $\alpha < 0$ [40].

If the direct time integrator operator is only conditionally stable, it can lead to impractically small time steps, and thus increase CPU-time drastically. Unconditional stability (meaning that the solution never blows up) is therefore of great value. To introduce numerical damping yet retain unconditional stability, it is recommended that $-\frac{1}{3} \leq \alpha \leq 0$, with $\gamma \geq \frac{1}{2}(1-2\alpha)$ and $\beta \geq \frac{1}{4}(1-\alpha)^2$. Although both the Newmark method and the α-method damp out higher frequency modes, the α-method introduces less damping in the lower modes, which tends to increase the accuracy. When α is chosen to be $-\frac{1}{3}$, it provides the maximum numerical damping, which gives a damping ratio at about 6% when the time increment is 40% of the time period of oscillation of the mode being studied [13]. The default value for α in *Abaqus* is -0.05 for *transient fidelity*, which is meant to give "slight numerical damping" [13]. Typical dynamic applications fall into three categories in which transient fidelity is one of them. Transient fidelity applications, such as an analysis of satellite systems, require minimal energy dissipation. In these applications small time increments are taken to accurately resolve the vibrational response of the structure, and numerical energy dissipation is kept at a minimum. These stringent requirements tend to degrade convergence behavior for simulations involving contact or nonlinearities [13].

4 Design Guidelines

This chapter presents methods for calculating limit accelerations according to four different guidelines; the first one is Eurocode 5-2, which proposes methods for calculation, but without presenting a maximum limit (although a limit is stated in Eurocode 0, presented in the following section). The second and third are British Standards, and the fourth is a guideline written by the Norwegian Public Roads Administration. In addition to this the sections concerning limit accelerations in ISO-10137 and Sétra are reviewed. Hand calculations based on the guidelines are presented in Appendix D, which are used to compare the results presented in Chapter 6.

4.1 Comfort Criteria in Design Guidelines and Literature

ISO 2631-1, Mechanical Vibration and Shock - Evaluation of Human Exposure to Whole-Body Vibrations has included the following values for likely reactions to various magnitudes of acceleration in Part 1, Appendix C[41]:

Less than 0.315 m/s^2	not uncomfortable
0.315 m/s^2 to 0.63 m/s^2:	a little uncomfortable
0.5 m/s^2 to 1 m/s^2:	fairly uncomfortable
0.8 m/s^2 to 1.6 m/s^2:	uncomfortable
1.25 m/s^2 to 2.5 m/s^2:	very uncomfortable
Greater than 2 m/s^2:	extremely uncomfortable

Obviously such classifications are subjective, which is why limits like that often are given as ranges of accelerations. The same vibrations may be perceived differently from one individual to the next, and even if the perception is the same they might have different thresholds for what they find comfortable. In addition to this, there is a difference between the actual vibrations of the structure and the vibrations perceived by the pedestrian. For instance, the duration for which the pedestrian is exposed to vibration affects what the pedestrian feels [7].

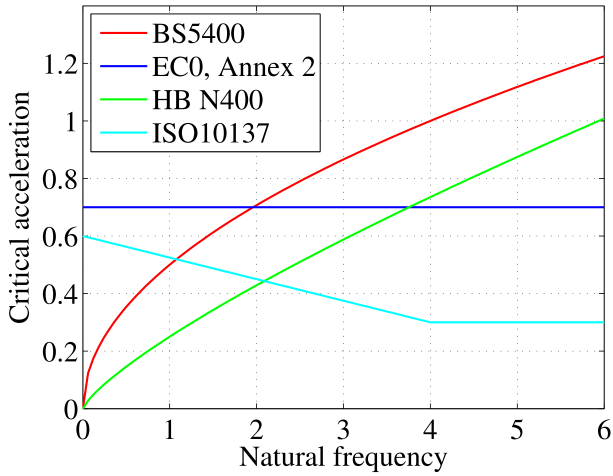


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Figure 4.1: *Vertical critical accelerations as a function of natural frequency according to various guidelines.*

Figure 4.1 shows the vertical limit accelerations given in four of the guidelines mentioned below. Although the methods diverge at the lower and higher frequencies, there seems to be consensus at walking frequency (around 2 Hz), where the limit accerelations are in the range of $0.5 - 0.7$ m/s^2.

4.1.1 Eurocode

Eurocode states that control of peak accelerations needs to be performed if the natural frequency of the bridge is below 5 Hz for vertical vibrations or less than 2.5 Hz for lateral and torsional vibrations, which may be the case for slender footbridges. According to Eurocode, the maximum allowed accelerations in vertical and horizontal direction are as presented in Table 4.1 [42]:

Direction of vibrations	Maximum accelerations [m/s^2]
Vertical	0.7
Horizontal (normal use)	0.2
Horizontal (crowd conditions)	0.4

Table 4.1: *Limit accelerations according to Eurocode 0, Annex A2.4.3.2.*

A method for calculating theoretical accelerations can be found in the Eurocode 5-2, Annex B - Vibrations caused by pedestrians [43]:

For one person crossing the bridge, the vertical acceleration $a_{vert,1}$ in m/s^2 of the bridge should be taken as:

$$a_{vert,1} = \begin{cases} \dfrac{200}{M\zeta} & \text{for } f_{vert} \leq 2,5 \text{ Hz} \\ \\ \dfrac{100}{M\zeta} & \text{for } 2,5 < f_{vert} \leq 5 \text{ Hz} \end{cases} \tag{4.1}$$

$$a_{hor,1} = \frac{50}{M\zeta} \qquad \text{for } 0.5 \text{ Hz} \leq f_{hor} \leq 2.5 \text{ Hz} \tag{4.2}$$

where:

M	is the total mass of the bridge in kg, given $M = m\mathscr{L}$;
$\mathscr{L}$	is the span of the bridge;
m	is the mass per unit length (self-weight) of the bridge in kg/m;
ζ	is the damping ratio;
f_{vert}	is the fundamental natural frequency for vertical deformation of the bridge.
f_{hor}	is the fundamental natural frequency for horisontal deformation of the bridge.

For several persons crossing the bridge, the vertical and horizontal accelerations, $a_{vert,n}$ and $a_{hor,n}$ in m/s^2 of the bridge should be calculated as:

$$a_{vert,n} = 0,23 a_{vert,1} n k_{vert} \tag{4.3}$$

$$a_{hor,n} = 0,18 a_{hor,1} n k_{hor} \tag{4.4}$$

where:

n	is the number of pedestrians;
k_{vert}/k_{hor}	are coefficients dependent on the bridge's natural frequency, according to Figure 4.2;
a_{vert}	is the vertical acceleration for one person crossing the bridge determined

The number of pedestrians, n, should be taken as:

$n = 13$	for a distinct group of pedestrians;
$n = 0.6A$	for a continuous stream of pedestrians

where A is the area of the bridge deck in m^2.

If running persons are taken into account, the vertical acceleration $a_{vert,1}$ in m/s^2 of the bridge caused by one single person running over the bridge should be taken as:

$$a_{vert,1} = \frac{600}{M\zeta} \qquad \text{for } 2.5 \text{ Hz} < f_{vert} \leq 3,5 \text{ Hz} \tag{4.5}$$

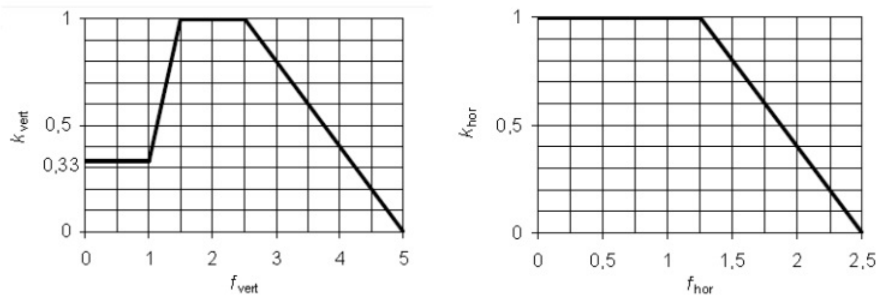


Figure 4.2: *Relationship between the fundamental natural frequencies and the coefficients k_{vert} and k_{hor} according to EC 5-2.*

4.1.2 British Annex to Eurocode

[44] In the British annex to Eurocode 1, referred to as BS-NA in the following chapters, there are proposed methods for calculating the vertical acceleration in the deck of bridge's for single pedestrians and groups of pedestrians. The method is based on modelling the pedestrian loading as a Fourier series and applying the load to the relevant areas of the deck with the direction of the force varied to match the direction of the vertical displacements of the mode for which responses are being calculated. However, any calculated maximum vertical acceleration should be less than the design acceleration limit given by:

$$a_{limit} = 1.0 k_1 k_2 k_3 k_4 \text{ m/s}^2 \tag{4.6}$$

$$\text{and } 0.5 \text{ m/s}^2 \leq a_{limit} \leq 2,0 \text{ m/s}^2$$

Where:

k_1, k_2, k_3 are the response modifiers taken from Table 4.2, 4.3 and 4.4:

k_1 = site usage factor
k_2 = route redundancy factor
k_3 = height of structure factor
k_4 is an exposure factor which is to be taken as 1,0 unless determined otherwise for the individual project.

Table NA.9

Bridge function	k_1
Primary route for hospitals or other high sensitivity routes	0,6
Primary route for school	0,8
Primary routes for sports stadia or other high usage routes	0,8
Major urban centres	1,0
Suburban crossings	1,3
Rural environments	1,6

Table 4.2: *Recommended values for the site usage factor* k_1.

Table NA.10

Route redundancy	k_2
Sole means of access	0,7
Primary route	1,0
Alternative routes readily available	1,3

Table 4.3: *Recommended values for the route redundancy factor* k_2.

Table NA.11

Bridge height	k_3
Greater than 8 m	0,7
4 m to 8 m	1,0
Less than 4 m	1,1

Table 4.4: *Recommended values for the structure height factor* k_3.

Values of k_1, k_2 and k_3 other than those given in tables 4.2 to 4.4 may be determined for the individual project using Figure NA.10 (not included) as guide [44]. k_4 may be assigned a value of between 0,8 and 1,2 to reflect other conditions that may affect the users' perception towards vibration. These may include consideration of parapet design (such as height, solidity or opacity), quality of the walking surface (such as solidity and opacity) and provision of other comfort-enhancing features. The value to be taken should be determined for the individual project.

The avoidance of unstable lateral responses due to crowd loading

Structures should be designed to avoid unintended lateral responses. If there are no significant lateral modes with frequencies below 1.5 Hz it may be assumed that unstable lateral responses will not occur. For all other situations, it should be demonstrated that unstable lateral responses due to crowd loading will not occur, using the following method:

For all deck modes of vibration having a significant lateral horizontal component and a frequency below 1,5 Hz, compare the pedestrian mass damping parameter, D, and the mode frequency with the stability boundary defined in Figure 4.3. If the pedestrian mass damping parameter falls below the indicated boundary divergent lateral responses may be expected. Values above the line should be stable. The pedestrian mass damping parameter *D* is given by:

$$D = \frac{m_{bridge}\zeta}{m_{pedestrian}} \tag{4.7}$$

in which

m_{bridge}	is the mass per unit length of the bridge
$m_{pedestrian}$	is the mass per unit length of pedestrians for the relevant crowd density obtained from **NA.2.44.2** assuming that each pedestrian weighs 70 kg
ζ	is the structural damping when expressed as a damping ratio, $\zeta = \delta/(2\pi)$
δ	logarithmic decrement of decay of vibration between successive peaks

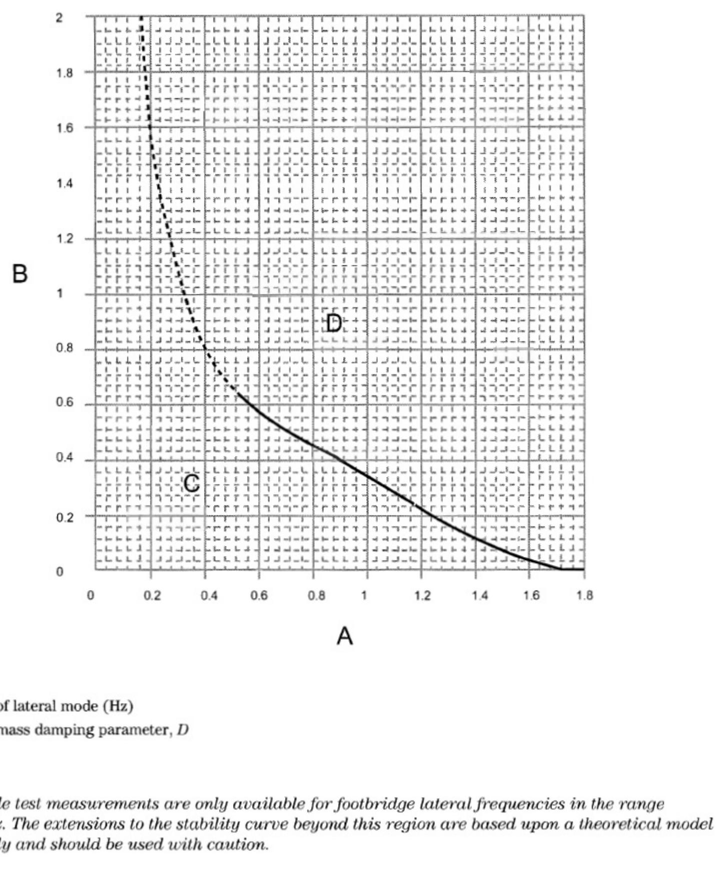


Figure 4.3: *Lateral lock-in stability boundaries.*

4.1.3 BS 5400

[45] BS 5400 is a British Standard code of practice for the design and construction of steel, concrete and composite bridges. Part 2 - Appendix B - Vibration serviceability requirements for foot and cycle track bridges states that the limit acceleration should be taken as

$$a_{lim} = 0.5\sqrt{f_v} \tag{4.8}$$

If the natural frequency of the bridge in vertical direction is less than 5 Hz. For single span, or two-or-three span continuous, symmetric superstructures, of constant cross section and supported on bearings that may be idealised as simple supports, a simplified method is proposed. The maximum vertical acceleration in m/s^2 should then be taken as:

$$a = 4\pi^2 f_v^2 y_s k\psi \tag{4.9}$$

Where

f_v	is the fundamental natural frequency (in Hz)
y_s	is the static deflection (in m)
k	is the configuration factor
ψ	is the dynamic response factor

For values of f_v greater than 4 Hz the calculated maximum acceleration may be reduced by an amount varying linearly from zero reduction at 4 Hz to 70% reduction at 5 Hz.

4.1.4 Handbook N400

[46] The Norwegian Public Roads Administration has issued the *Handbook N400* for bridge design, which is the most commonly used guideline for bridges in Norway. It states that footbridges should be designed so that the reference acceleration will satisfy the following:

$$a_v \leq 0.25 f_v^{0.7782} \tag{4.10}$$

The reference acceleration is calculated as such:

$$a_v = 4\pi^2 f_v^2 W_s K \psi r \tag{4.11}$$

In which

f_v	is the bridge's first natural frequency [Hz] for vertical vibrations
W_s	is the static deflection for a concentrated force of 700 N
K	is a factor dependent on number of spans and span to width ratio
ψ	is a dynamic factor dependent on the span length and damping ratio, ζ, and should be taken from Figure 4.4
r	is a correctional factor for the reference acceleration; a function of f_v [Hz]:

$$r = \begin{cases} 1.0 & ; \quad f_v \leq 4 \\ 3.0 - f_v/2 & ; \quad 4 < f_v < 6 \\ 0.0 & ; \quad f_v \geq 6 \end{cases} \tag{4.12}$$

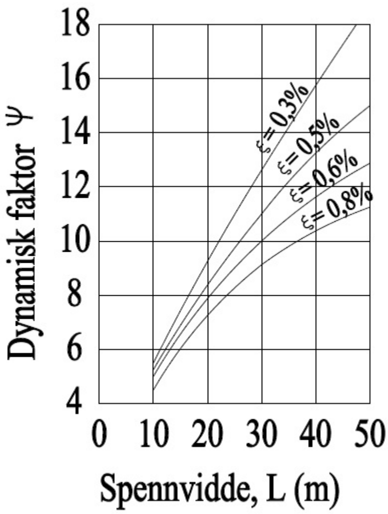


Figure 4.4: *Dynamic factor, ψ, as a function of span length and damping ratio, ζ.*

The handbook states that a control must be performed if the fundamental horizontal frequency of the bridge is in the range 0.5 Hz $\leq f_{hor} \leq$ 1.3 Hz. The control is performed by calculating a critical number of pedestrians on the bridge, which is compared to the expected number of pedestrians on the bridge:

$$N_L = \frac{8\pi\zeta f_{hor} M}{k} \tag{4.13}$$

In which

N_L	is the amount of pedestrians that will cause unacceptable horizontal vibrations
ζ	is the damping ratio
f_{hor}	is the fundamental horizontal frequency of the bridge
M	is the modal mass of bridge
k	is a factor of 300 Ns/m

4.1.5 ISO 10137

[47] ISO 10137 is a guideline for serviceability of buildings and walkways. In annex C.2.1 - Walkways the standard proposes the following scenarios to consider the dynamic response of a footbridge:

- One person walking across the walkway and another (the receiver) standing at mid-span
- An average pedestrian flow based on a daily occurrence rate, e.g. a group size of 8 to 15 people, depending on the length and width of the walkway
- The presence of streams of pedestrians (significantly more than 15 persons)
- Occasional festive or choreographic events (when relevant)

Further, it states that the level of vibrations in the vertical direction for walkways over roads or waterways should not exceed those obtained by a multiplying factor of 60 to the relevant basecurve (Figure 4.5), except where one or more persons standing still on the walkway have to be accounted for (such as in the first scenario), in which case a multiplying factor of 30 should be applicable. With regards to horizontal vibrations, they should not exceed that of 60 times the base curve.

Finally, an average time interval of 1 s is recommended for calculating the RMS-acceleration. This proposed time interval is also stated in ISO-2631-1 [41].

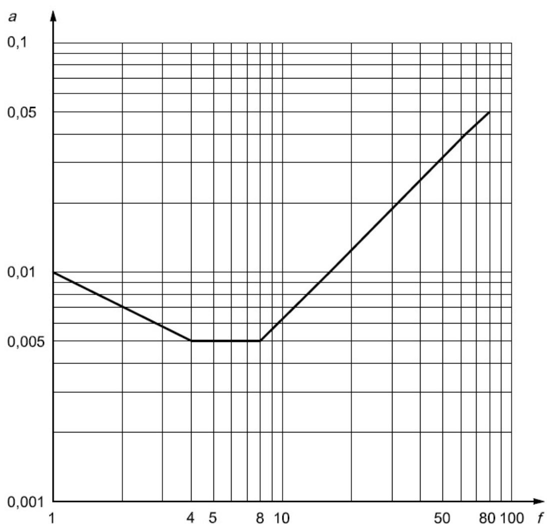


Figure 4.5: *Building vibration base curve for acceleration (foot-to-head vibration direction). RMS-Acceleration along y-axis, frequency along the x-axis.*

4.1.6 Sétra

[7] This technical guide from 2006 with title - Assessment of vibrational behaviour of footbridges under pedestrian loading suggests that footbridge vibrations should be assessed by going through four stages:

Stage 1: Determination of footbridge class
The owner of the bridge needs to determine the class of the bridge based on the level of traffic expected. The classes range from Class I, high pedestrian activity with dense crowds expected, to class IV, seldom used footbridges connecting sparsely populated areas.

Stage 2: Choice of comfort level by the Owner
A comfort level has to be chosen, based on the perceptibility of the accelerations undergone by the structure:
Maximum comfort: The accelerations are practically imperceptible. This comfort level should be chosen in cases where particularly sensitive users such as schoolchildren, elderly or disabled people are expected.
Mean/Average comfort: The accelerations are merely perceptible
Minimum comfort: Under loading configurations that seldom occur, accelerations undergone by the structure are perceived by the users, but do not become intolerable.

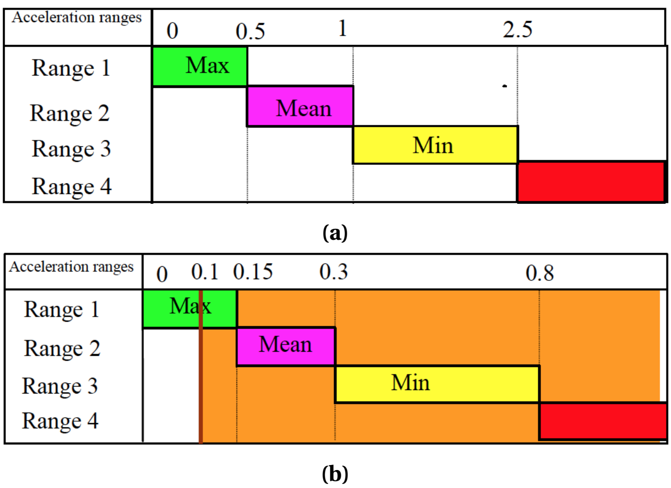


Figure 4.6: *Acceleration ranges* (m/s^2) *for vertical (a) and horizontal (b) vibrations.*

The obtained comfort level is assessed by calculating the accelerations undergone by the structure using different dynamic load cases. Given that the comfort level becomes a subjective measure, Sétra suggests looking at ranges of accelerations instead of thresholds. In Figure 4.6 four suggested ranges for both vertical (a) and horizontal vibrations (b) are shown. The first three ranges correspond to the respective comfort levels described previously while the fourth range corresponds to uncomfortable accelerations that are not acceptable. To avoid "lock-in" effect, which causes severe lateral motions, the horizontal acceleration limit is set to 0.10 m/s^2, indicated by the orange line/area.

Stage 3: Determination of frequencies and of the need to perform dynamic load case calculations or not
For Class I to III bridges the natural vibration frequencies of the structure have to be decided. In cases where the risk of resonance is considered negligible the comfort level can then automatically be considered as sufficient. Class IV structures automatically satisfy the criteria. The vibration frequencies should be decided for all 3 directions (vertical, lateral horizontal and longitudinal horizontal) and for 2 mass assumptions: empty footbridge and loaded footbridge with one 700 N pedestrian per square meter. By finding the ranges where the frequencies are situated the risk of resonance can be assessed.

There are four different frequency ranges in both vertical and horizontal directions. The ranges have a decreasing risk of resonance, with Range 1 being maximum risk of resonance and range 4 negligible risk of resonance. In Figure 4.7 the frequency ranges are shown for vertical and longitudinal vibrations in (a) and for lateral vibrations in (b).

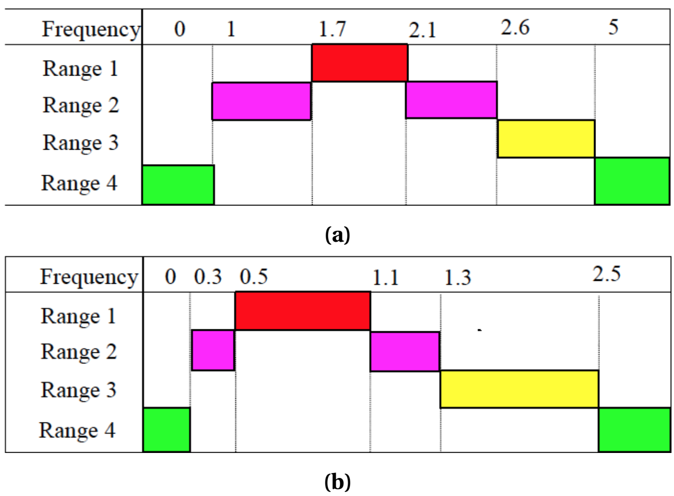


Figure 4.7: *Frequency ranges (Hz) for vertical and longitudinal vibrations (a) and for lateral vibrations (b).*

Based on the footbridge class and the ranges where the natural frequencies are situated calculations for all, or part of, 3 load cases have to be carried out:

Case 1: Sparse and dense crowd
Case 2: Very dense crowd
Case 3: Complement for an evenly distributed crowds (2nd harmonic effect)

Figure 4.8 defines the necessary calculations to be performed based on class and frequency range.

		Load cases to select for acceleration checks		
		Natural frequency range		
Traffic	Class	1	2	3
Sparse	III	Case1	Nil	Nil
Dense	II		Case 1	Case 3
Very dense	I	Case 2	Case 2	Case 3

Figure 4.8: *Load cases based on class and frequency range.*

Stage 4: If necessary: Calculation with dynamic load cases
If the conclusion after the three previous stages is that calculations are needed the load case from stage 3 has to be used to calculate the response of the structure. The response should be checked against both the comfort level criteria from stage 2 and traditional Service Limit State and Ultimate Limit State checks.

Based on the load case and the class, Setra defines the load per m^2 in all three directions. The loads are to be applied to the whole deck of the footbridge, and the sign of the vibration amplitude must, at any point, be selected to produce the maximum effect. This implies that the direction of application of the load must be the same as the direction of the mode shape and must be inverted every time the mode shape direction changes. The forcing depends on the density of the crowd which depends on the load case, the walking frequency, the area of the deck, the damping ratio ζ, and a minus factor ψ between 0 and 1, taking into account the structure´s natural frequency and the fact that risk of resonance becomes less likely when getting further away from the walking frequency.

Comment on Case 3: It takes into account the effect of the 2^{nd} harmonic of the crowd, meaning the stresses caused by pedestrian walking, located at double the frequency of the 1^{st} harmonic.

Stage 5: Modification of the project or of the footbridge If the calculations in stage 4 do not satisfy either the comfort level or the SLS or ULS checks, the project has to be re-started if it's a new footbridge or steps have to be taken if it concerns an already existing footbridge. Steps that can be taken are for example adding mass to reduce the accelerations or adding dampers.

4.2 Damping According to Guidelines

Table 4.5 presents the damping ratio for steel and timber given in different guidelines. Note that the damping ratio specified in Eurocode 5 (National Annex, part 7.3.1) is 1.0% if no mechanical joints are present and 1.5% if mechanical joints are present [43]. Sétra mentions that it is particularly important not to overestimate the damping, in order to avoid under-dimensioning. JRC (Joint Research Centre) published a report during the development of Eurocode 3 in 2009 [48] with the damping ratios shown in the second column. While CEB information bulletin No. 209 presented critical damping ratios for timber and steel, which are also restated here. CEB (Euro-International Committee for Concrete) has since merged with FIP (International Federation for Prestressing) to form FIB (The International Federation for Structural Concrete).

	JRC		**CEB/FIB**		**EC**	**ISO**	**Sétra**
Material	Min. ζ	Avg. ζ	Min. ζ	Avg. ζ	ζ	ζ	ζ
Steel	0.2%	0.4%	0.2%	0.4%	0.5%	0.5%	0.4%
Timber	1.0%	1.5%	1.5%	3.0%	1.0% / 1.5%	-	1%

Table 4.5: *Damping ratio for different materials for serviceability conditions according to Joint Research Centre [48], the International Federation for Structural Concrete, European Commission, ISO 10137 [47], Eurocode 8, part 2 [49] and Sétra [7].*

5 Numerical Model in *Abaqus*

The goal of this project was to find a consistent method to determine the dynamic response of slender foobridges due to pedestrian loading. To do this, a basis model was necessary. The basis model is based on a planned network arch bridge that will be built in Orkanger. The dimensions of the model are based on the tentative dimensions of this bridge. While the outer dimensions of the bridge are fairly certain the dimensions of the different profiles are not final. The dimensions used in further analyses are given in Table 5.1. This chapter describes the basis model that can be used with *Abaqus* to run several analyses with different pedestrian load configurations.

5.1 Basis Model

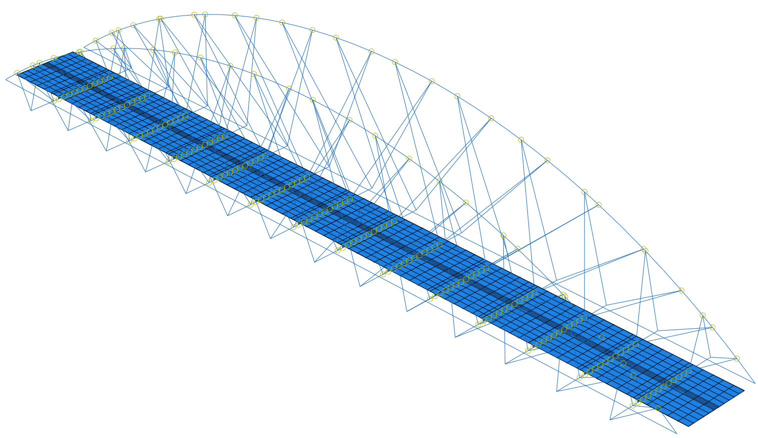


Figure 5.1: *Default bridge generated by Python script.*

A numerical model that can be used to perform simulations within a reasonable time interval and still exhibit good accuracy was desired. Ph.D. Candidate Anna Ostrycharczyk has written a *Python* script compatible with *Abaqus 6.13* that requests 20 input variables through a parameter selector (See Original Input Variables in Table 5.2), and generates a bridge in *Abaqus* with the wanted dimensions. An example of such a bridge can be seen in Figure 5.1. The default values in Table 5.2 are based on the tentative dimensions of the bridge stated in Table 5.1.

Length	60.0 m
Height	9.0 m
Width	5.0 m
Number of hangers	14
Hanger angle	35^o
Deck thickness	500 mm
Asphalt thickness	70 mm
Arc profile (rectangular)	800 mm × 500 mm
Transverse beam profile (hollow circular)	r = 300 mm, t = 50 mm
Hangers (circular)	r = 20 mm
Tie profile	r = 100 mm

Table 5.1: *Tentative dimensions of bridge.*

A preliminary parameter study was conducted to establish a basis model from Anna's original script. During this study different mesh sizes and time increments were tried out to optimize accuracy vs. total run time. The study resulted in the default values for mesh sizes and time increment given in Table 5.2. Quite a lot of effort also went into determining which damping model to use to get a fair amount of damping represented in the numerical model. More about this in Section 5.3. Firstly, an overview of the different aspects of the basis model , created by running the script with default values.

Parts, Properties and Meshing

The deck and pavements are modelled as shells while the crossbeams, hangers, ties and arches are all wires. The seed sizes used to mesh the different parts depend on the 'mesh size' input variables shown in Table 5.2. Four-noded rectangular shell elements with reduced integration (S4R) are assigned to both the deck and the pavements while quadratic 3D beam elements (B32) are assigned to hangers, and linear 3D beam elements (B31) to the crossbeams, arches and ties. The crossbeams, hangers and ties are made out of elastic steel, while the arches, pavements and deck are timber. They too are modelled elastically, but with different Young's and shear moduli in different principal directions due to the anisotropic properties of wood. An extra deck with asphalt properties is also created the same way as the original deck. All material properties are shown in Table 5.3 and 5.4. To introduce damping in the model, material damping was implemented as viscous damping using Rayleigh coefficients. These coefficients are only given in Table 5.4, even though they are assigned to all materials.

Original Input Variables	Default Value
Model name	StudentBridge
Number of dim	3
Variant	1
Length [mm]	60 000
Elevation [mm]	9 000
Width (deck) [mm]	5 000
Pavement width [mm]	0
Hangers 3D spacing (one side) [mm]	400.00
Hangers angle (arc plane) [deg.]	35.
Number of hangers	14
Tie level	0
Deck mesh size	100
Hanger mesh size	100 000
Beam mesh size	100
Tie mesh size	100 000
Arc mesh size	100
Number of bolt on deck	10
Number of bolts on pavement	1
Number of eigenvalues	10
Prestress S11	0
Additional Input Variables	
Rows of people	1
Position of row	1
Number of people per row	1
Walking across bridge or in place	0
Position of pedestrians walking in place	2
Walking frequency [Hz]	2.0
Load model	1
Stride length [mm]	750.0
Step width [mm]	200.0
Time increment	0.01
Modal	0

Table 5.2: *Input variables in Python script.*

Name	Density [t/mm^3]	Young's modulus [N/mm^2]	Poisson's ratio
Asphalt [50], [51]	2.24E-009	2000	0.3
Deck2dMaterial	8.8E-010	30 000	0.3
HangerSteel	7.8E-009	209 000	0.3
Steel	7.8E-009	209 000	0.3
VirtualSteel	7.8E-009	209	0.3
WeakSteel	7.8E-009	150000	0.3

Table 5.3: *Material properties used in Abaqus model.*

Material	Wood
Density	4.90E-10 t/mm^3
Damping	α=0.092091, β=0.000187
E_1	13000 N/mm^2
E_2	410 N/mm^2
E_3	410 N/mm^2
ν_{12}	0.5
ν_{13}	0.6
ν_{23}	0.6
G_{23}	760 N/mm^2
G_{13}	760 N/mm^2
G_{23}	76 N/mm^2

Table 5.4: *Material properties for wood in the model.*

Steps and Interactions

The final script generates two steps in addition to the always present initial step. The former is a static step used to prescribe the gravity load on the structure, while the latter step is an implicit dynamic step where the pedestrian loading is defined. If prestress is to be added in the cables, this is modelled during an extra static step inserted at the beginning of each analysis. Also, if input variable 'Modal' is set to be 1, the implicit dynamic step is replaced by a real and a complex frequency step that will calculate the numerical natural frequencies, mode shapes and damping ratio of each mode. Since the goal of the analysis is to resemble pedestrians crossing a footbridge, the total time of the analysis is quite large, of magnitude 40 - 80 s. An explicit analysis would not be a good option because it is limited by the critical time step which depends on the density, stiffness and size of the smallest element and might therefore be very small. Implicit dynamic analyses are unconditionally stable when linear and are therefore better suited for simulations like these. Nonlinear geometry is included in all steps, which implies that the geometric stiffness matrix is generated and included.

To get accurate results and avoid numerical damping of the lower modes (more about this in Section 5.3) the time step size Δt used during the dynamic implicit analysis step should be in the range [12]:

$$\frac{T_{co}}{30} \leq \Delta t \leq \frac{T_{co}}{10} \tag{5.1}$$

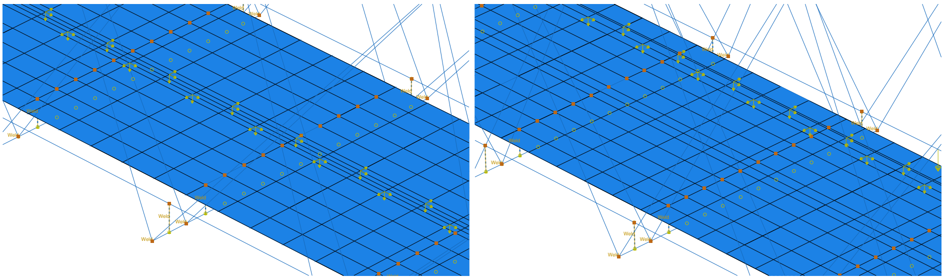
where T_{co} is the period of the highest mode the user of the script wants accurately represented. Based on the frequencies and mode shapes of the bridge (more about this in Section 5.2) the 10 first modes have been assumed to be the maximum number of modes necessary to accurately describe the response of the bridge. The cut off period is thus the period of mode 10: $T_{co} = 0.1514$ s $\Rightarrow 0.0051$ s $\leq \Delta$t ≤ 0.0151 s. Time step size 0.01 s is therefore chosen as default value for the dynamic step. The same is also chosen for the static gravity step, although automatic incrementation could be used instead.

To model the welds that will connect the different parts of the bridge, connectors have been defined in *Abaqus*. The connector type used is weld, which constrains all relative components of motion. These connectors are used to connect hangers to arches, hangers to beams, beams to decks, beams to ties and ties to arches. The only parts that were not connected using connectors were the original deck and the asphalt deck, which were tied together using constraints instead, allowing no relative motion between them.

Loads and Boundary Conditions

By default Anna's script adds boundary conditions by constraining the ends of the original deck and the arches against all translations as well as rotations around the longitudinal and vertical axis. It also adds a gravity load. To model one or more pedestrians walking/running across the bridge, forces simulating footsteps are applied along the deck. This was written into the script in a way such that also the loading is generalized and can be applied to a bridge with any dimensions. By adding more input variables to the script the user can also change the number of pedestrians, their position on the deck, the pacing rate, stride length, step width, which load model to be used, etc. All the additional input variables that have been added to the script to model different pedestrian situations can be seen in Table 5.2.

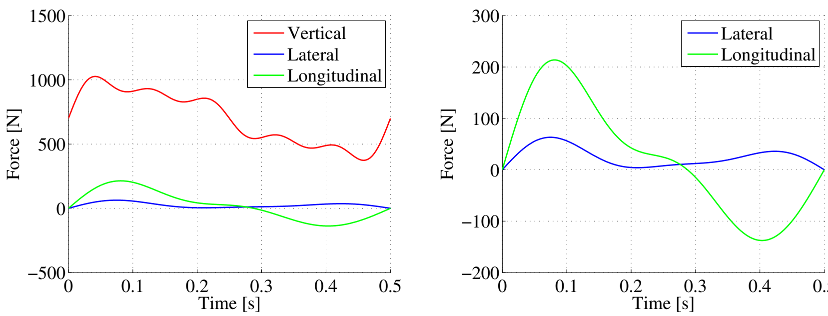
The previously mentioned extra deck with asphalt properties, from now on referred to as the "load deck", is created so that the forces from the pedestrians can be applied there. This was done to ensure that none of the boundary conditions or connector assignments created in the original script were altered. The load deck is partitioned so that sets are defined where the concentrated forces are to be applied. The pattern of the partition depends on the input variables and two examples showing the applied forces along different partition lines can be seen in Figure 5.2(a) and 5.2(b). The examples show pedestrians walking along the middle and the edge of the load deck, respectively.



(a) **(b)**

Figure 5.2: *Concentrated forces with three components applied at specific sets. The position of the walking path can also be altered by changing the partition, as shown in (b).*

The loads are applied as concentrated forces, with three components and specific amplitudes. Each force component due to each footstep has a corresponding amplitude which is equal to zero during most of the analysis and only nonzero when that specific footstep is applied. The shape and magnitude of the corresponding amplitude when it is nonzero is calculated using the Fourier series in Equation 3.29 with DLFs from Table 3.2. The amplitudes are implemented in *Abaqus* using tabulated values. Which DLFs to use can be changed by altering the value for 'Load Model' in the parameter selector. The load model proposed by Schulze is chosen as default and shown for one single footstep with walking frequency 2.0 Hz in Figure 5.3(a) and 5.3(b). This was chosen both because his model is based on Bachmann and Ammanns research [14], and also because the model looks similar to empirical load-time plots, such as the ones in Figure 3.9.

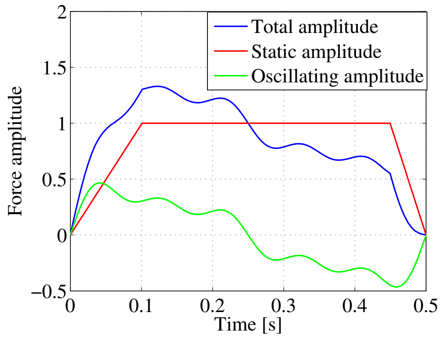


(a) *Schulze's single pedestrian load model* **(b)** *Schulze's single pedestrian load model - only lateral and longitudinal components*

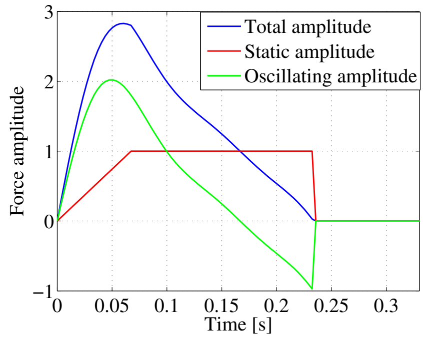
Figure 5.3: *Schulze's single pedestrian load model for all three components.*

While the longitudinal and lateral force components oscillate around 0 N the vertical component oscillates around G = 700 N (see Figure 5.3(a)). The force not starting at zero at the beginning of every footstep caused unphysical accelerations in the numerical results. To get around this a Loading response and a Preswing (L and P in Figure 3.10) were included so that also the vertical forces always start and end up at zero. By linearly increasing ("ramping") the static component of the Fourier series (DLF a_0) from

0 to 1 during a time period longer than the time step Δt the unphysical responses disappeared. After studying the general load-time functions in Figure 3.9 the loading response and preswing were defined equal to: $L = \frac{1}{5} \times T$ and $P = \frac{1}{10} \times T$, T being the total duration of one step equal to $\frac{1}{f_p}$, where f_p is the walking frequency. The result is the load time function seen in Figure 5.4(a).



(a) *Final load model for walking, here shown with pacing rate 2.0 Hz*



(b) *Final load model for running, here shown with pacing rate 3.0 Hz*

Figure 5.4: *Load time functions implemented in Abaqus for walking and running, respectively.*

In reality the forces due to two consecutive footsteps will overlap as long as the pedestrian is walking and not running. For simplicity, this was neglected in this thesis. Thus, the forces due to any right footstep start being applied only after the forces due to the previous left footstep have reached zero, and the following left step starts when the right step has reached zero, and so forth. For running pedestrians (implemented in the numerical model if input variable 'Load Model' is set equal to '3') there is a time between each footstep where both feet are off the ground and therefore no forces are acting on the deck. The duration of the time where the forcing is equal to zero was based on the general load-time functions in Figure 3.9 and was set equal to $0.3 \times \frac{1}{\text{pacing rate}}$, where the latter factor equals the total time between two consecutive footsteps. The resulting load time function for running is shown in Figure 5.4(b). The walking speed of the pedestrians depend on the pedestrian configuration on the footbridge [11]. This has been neglected and the model used for people walking in small groups is the only model implemented in the script. This way the walking speed of all pedestrians is equal to v_s and is given by the walking frequency f_p and stride length l_s as:

$$v_s = f_p l_s \tag{5.2}$$

NB! For a more thorough explanation of how the analysis steps, boundary conditions and loads have been created using Python, the reader can refer to Appendix B and Appendix F.

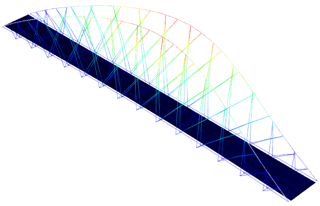
5.2 Natural Frequencies and Mode Shapes

The first 20 natural frequencies of the numerical model of the bridge and the accompanying mode shapes were found by running a modal analysis in *Abaqus*. In accordance with Sétra (See Subsection 4.1.6), the modal analysis was performed both without pedestrians and with a uniform pressure of 700 N/mm^2 equal to 1 person per square meter. As seen in Table 5.6, the differences between the natural frequencies from the two analyses are negligible. Hence, the values in column 2 were used in proceeding analyses.

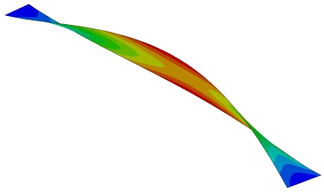
Mode	Natural frequency with gravity [Hz]	Natural frequency with additional pressure load [Hz]
1	1.8838	1.8832
2	2.1405	2.1399
3	3.3217	3.3194
4	3.4370	3.4345
5	3.6216	3.6215
6	3.7528	3.7529
7	5.5485	5.5457
8	5.7278	5.7247
9	6.0945	6.0945
10	6.6034	6.6030
11	7.6070	7.6124
12	7.6414	7.6467
13	8.4343	8.4342
14	8.4352	8.4344
15	8.8650	8.8629
16	8.9017	8.8982
17	10.121	10.121
18	10.476	10.476
19	10.882	10.886
20	10.959	10.962

Table 5.5: *Natural frequencies of Abaqus model.*

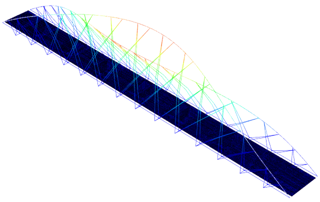
In addition an analysis with refined deck and arc mesh (Seed 50 mm and 10 mm respectively) was performed. Also here, only negligible differences in the natural frequencies were observed, thereby confirming the accuracy of the model with 100 mm mesh. The 10 first mode shapes corresponding to the 10 first natural frequencies in column 2 in Table 5.6 are shown in Figure 5.5 and 5.6. Note that there are no longitudinal natural frequencies. This is because the deck was constrained against movement in the longitudinal direction at both ends. Based on the natural frequencies and the according mode shapes one can anticipate where and at which frequencies the pedestrians will have to walk to have the greatest chance of causing resonance.



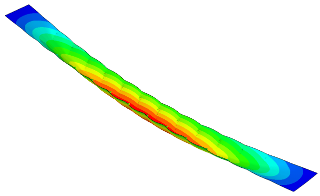
(a) *Mode 1: Lateral movement of arches, 1 sag*



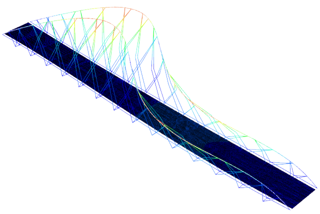
(b) *Mode 1, deck only*



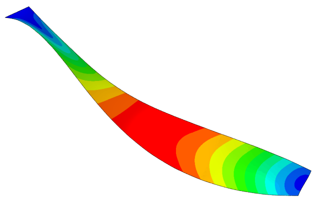
(c) *Mode 2: Lateral movement of arches, 1 sag*



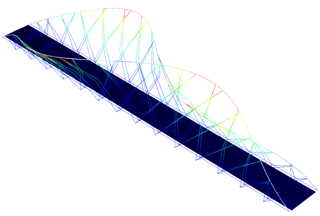
(d) *Mode 2, deck only*



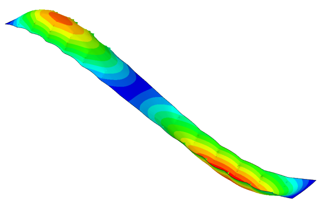
(e) *Mode 3: Lateral movement of arches, 2 sags*



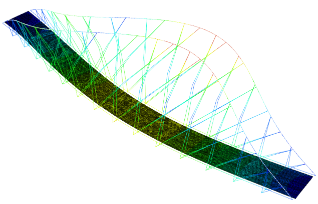
(f) *Mode 3, deck only*



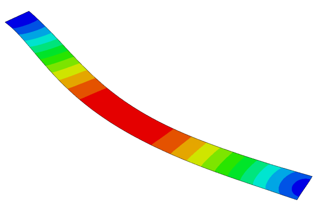
(g) *Mode 4: Lateral movement of arches, 2 sags*



(h) *Mode 4, deck only*

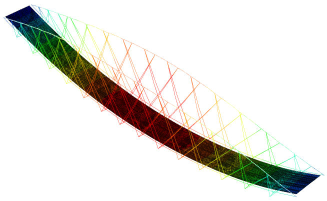


(i) *Mode 5: First lateral mode, 1 sag*

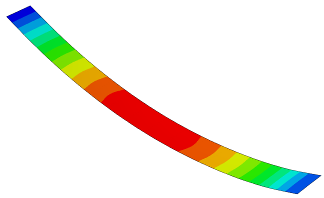


(j) *Mode 5, deck only*

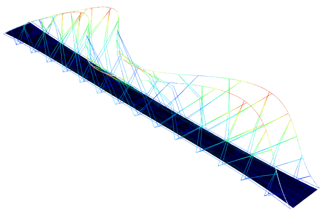
Figure 5.5: *First five mode shapes of the numerical model.*



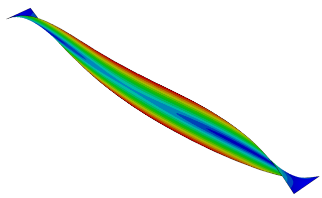
(a) *Mode 6: First vertical mode, 1 sag*



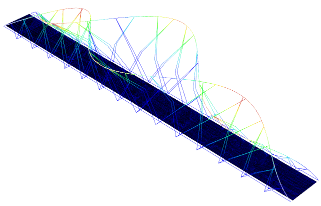
(b) *Mode 6, deck only*



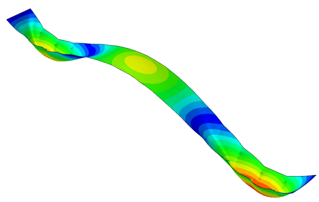
(c) *Mode 7: Lateral movement of arches, 3 sags*



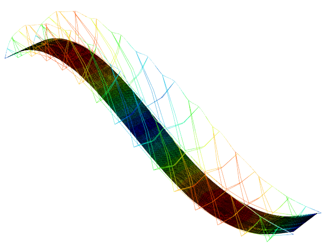
(d) *Mode 7, deck only*



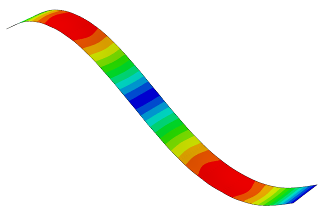
(e) *Mode 8: Lateral movement of arches, 3 sags*



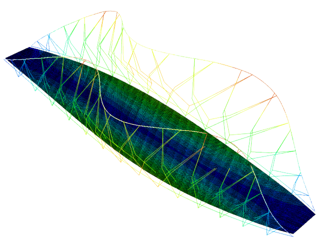
(f) *Mode 8, deck only*



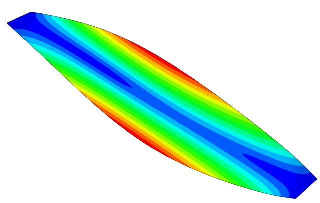
(g) *Mode 9: Second vertical mode, 2 sags*



(h) *Mode 9, deck only*



(i) *Mode 10, first torsional mode*



(j) *Mode 10, deck only*

Figure 5.6: *Last five mode shapes of the numerical model.*

5.3 Damping in *Abaqus* Model

5.3.1 Numerical Damping

Knowing that *Abaqus* by default adds some numerical damping by implementing the α-HHT method (see Section 3.4.2), it was imperative that the amount of damping present be worked out for the results to be useful. $\alpha = 0$ was originally tried out, but this yielded unphysical results because high frequency noise was not damped out.

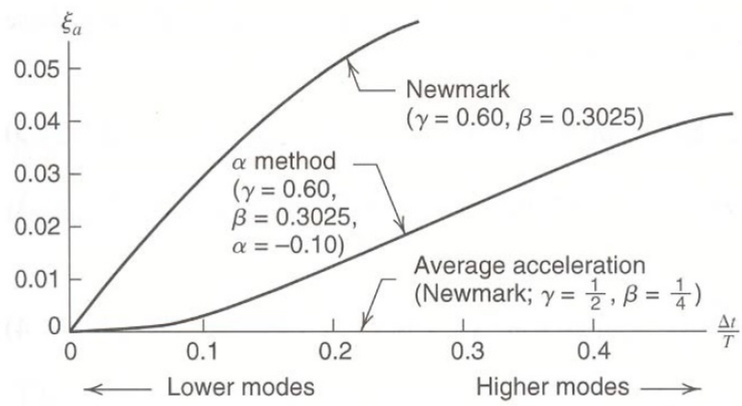
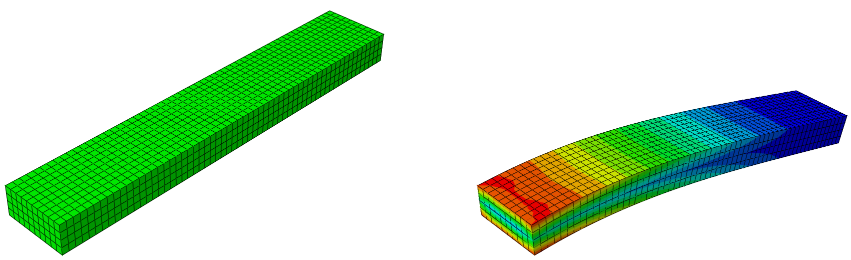


Figure 5.7: *Algorithmic damping ratios ζ_a provided by implicit integration methods, where T is the period of the mode for which ζ_a is depicted.*

As seen in figure 5.7, the amount of damping for a given mode is only dependent on the time step size, Δt, and T, the period of the mode in study. To get accurate results and negligible numerical damping for the lower modes, a small value of $\frac{\Delta t}{T}$ was desired. Executing a simplified analysis helped determine the necessary size of the time step for the implicit integration scheme.



(a) *Cantilever beam* **(b)** *Deformed cantilever beam*

Figure 5.8: *Model of cantilever beam used to determine the amount of numerical damping at each mode.*

A simple cantilever beam (see Figure 5.8(a)) was used to determine the inherent numerical damping in *Abaqus* for a given time step. The cantilever was subject to an initial displacement at the free end (see Figure 5.8(b)). At the beginning of the next step this displacement was "released" so that the following oscillatory displacement at the free end could be examined and help quantize the damping of the cantilever. The quantification was done through use of the logarithmic decrement δ, defined as the natural logarithm of the ratio between two peaks of the response at time t_n and $t_n + T_d$, in which T_d is the response period [52]. For free damped vibrations the expression becomes:

$$\delta = ln\left(\frac{u_n}{u_{n+1}}\right) = ln\left(\frac{\rho e^{-\zeta\omega_n t_n}}{\rho e^{-\zeta\omega_n(t_n+T_d)}}\right) = 2\pi\frac{\zeta}{\sqrt{1-\zeta^2}} \approx 2\pi\zeta \tag{5.3}$$

for $\zeta << 1$. By altering the properties of the cantilever, its first natural frequency was set equal to the natural frequency of each of the modes of the numerical model in turn. Next the fraction of critical damping for the investigated mode was calculated using the relation in Equation 5.3. This was done using a *MATLAB* script, which is included in Appendix A.1. Two different approaches, both found in the attachment, were used to estimate the logarithmic decrement: $\bar{\delta}$ was found by calculating the logarithmic decrement between two consecutive peaks, and then averaging over all the peaks. δ_{tot} is the logarithmic decrement from the first to the last peak. The critical damping factor was calculated for both methods and compared to verify the calculations. To tailor the natural frequencies of the cantilever, the Young's modulus was adjusted. It becomes clear how by the following formula for the first natural frequency of a cantilever beam:

$$\omega_1 = 1,875^2\sqrt{\frac{EI}{\rho AL^4}} \tag{5.4}$$

Which, arranged to be solved for E, becomes:

$$E = \frac{\omega_n^2 \rho AL^4}{1,875^4 I} \tag{5.5}$$

In which

ρ	=	2.24E-9 ton/mm^3
I	=	$\frac{bh^3}{12}$
A	=	bh mm^2
L	=	6000 mm
h	=	444,3 mm
ω_n	=	natural frequency of the investigated mode

Table 5.6 presents the different Young's moduli used to get the first natural frequency of the cantilever equal to the ten first natural frequencies of the numerical model in turn.

Mode	Frequency	Young's modulus [MPa]
ω_1	1.8838	2000
ω_2	2.1405	2583
ω_3	3.3217	6220
ω_4	3.4370	6659
ω_5	3.6216	7393
ω_6	3.7528	7939
ω_7	5.5485	17354
ω_8	5.7278	18493
ω_9	6.0945	20937
ω_{10}	6.6034	24580

Table 5.6: *Eigenfrequencies of cantilever beam.*

Shown in Figure 5.9(a) and 5.9(b) are the oscillations of the cantilever with two different natural frequencies, where both analyses were executed with step size 0.01 s. The increase in numerical damping due to increased frequency, thus decreased period T, is evident.

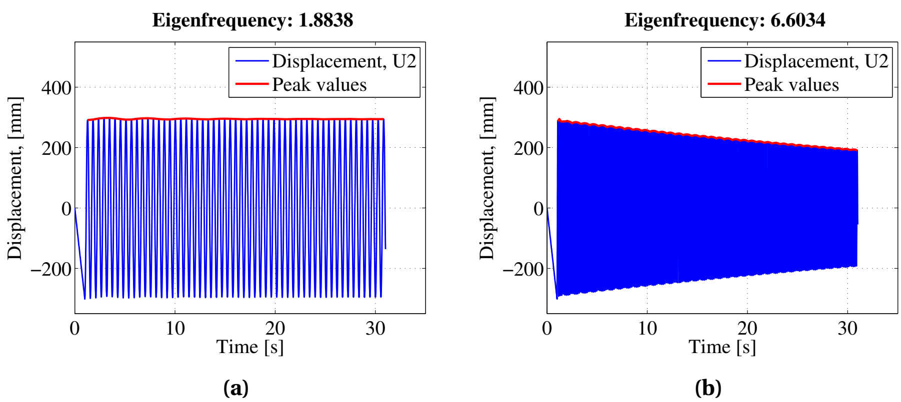


Figure 5.9: *Plots showing the amount of numerical damping at mode 1 and mode 10, respectively.*

Figure 5.10 shows the numerical damping for the first ten modes and also mode 17 and 28, numbered as 11 and 12, for three different time step sizes Δt. The numerical damping increases with increasing step size, as expected from figure 5.7. There is almost no difference in ζ_{avg} and ζ_{tot}, which implies a close to logarithmic decrease in peak amplitudes for the entire time interval. The greatest correlation between the two methods of calculation appears when Δt is the smallest.

In accordance with what was calculated earlier on page 51 the time step size was chosen as 0.01 s for the basis model to avoid significant numerical damping of the first ten

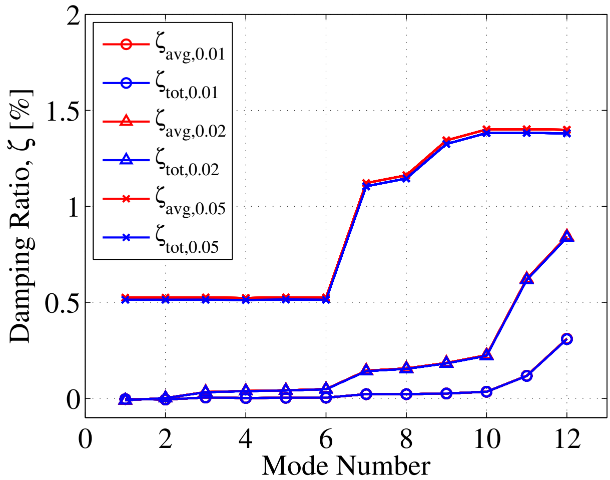


Figure 5.10: *Average and total damping of different modes and step sizes of cantilever beam.*

modes. Instead material damping was added to the system, as it is easier to control and quantify.

5.3.2 Material Damping

As previously mentioned in Subsection 3.4.2 there are a few different ways to implement damping in *Abaqus* [39]. Material damping was chosen for this numerical model and implemented as Rayleigh damping. If the same material damping is added to all materials the fraction of critical damping, ζ_i, for a given structural mode i, can be expressed in terms of the the Rayleigh coefficients α_R and β_R as:

$$\zeta_i = \frac{\alpha_R}{2\omega_i} + \frac{\beta_R \omega_i}{2} \tag{5.6}$$

in which ω_i is the natural frequency of mode i. Based on the inherent damping present in wood alone [53], $\zeta_i \approx 0.005$ was wanted for the first ten modes. In reality the damping ratio for the entire structure will be higher, partly due to friction between different parts, so this is a conservative estimate. The two coefficients α and β were calculated as such:

$$\begin{bmatrix} \zeta_i \\ \zeta_j \end{bmatrix} = \frac{1}{2} \begin{Bmatrix} \frac{1}{\omega_i} & \omega_i \\ \omega_j & \omega_j \end{Bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \tag{5.7}$$

In this calculation two boundary values were chosen for the frequency range, namely the first and tenth natural frequency ($\omega_1 = 2\pi\text{rad}\times 1.8838\text{Hz}$, and $\omega_{10} = 2\pi\text{rad}\times 6.6034\text{Hz}$). This resulted in $\alpha = 0.09209$ and $\beta = 0.000187$. The damping value ζ_i is dependent on each natural frequency, and with this approach any modes above the tenth mode have a higher damping than $\zeta = 0.005$, and any of the modes in between have a lower damping. This is apparent by studying Figure 5.11(a) where the damping ratios should follow the red line. The higher modes are expected to only cause numerical noise, and therefore damping them out will render a more physical result.

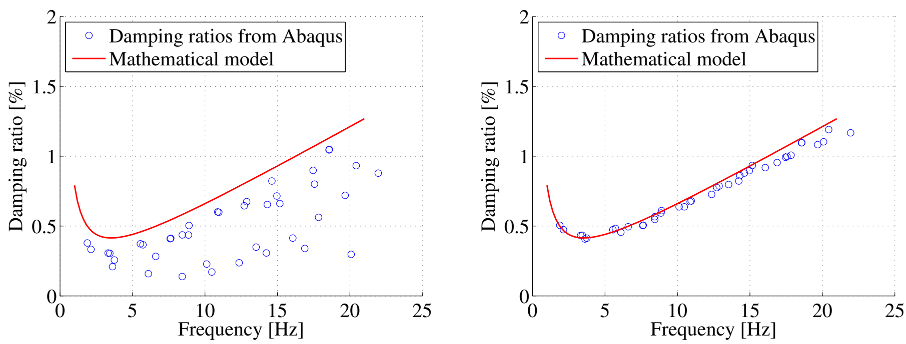
As mentioned the damping coefficients where assigned to all the materials, since this is the only way to ensure that the all the modes of the structure get the expected damping ratio. In reality, steel will have a damping of around 10% of the damping present in wood, but this was neglected. To ensure the right amount of damping had been obtained a complex eigenvalue extraction in *Abaqus* was performed.

Complex eigenvalue analysis in *Abaqus*

In *Abaqus* a frequency step can be followed by a complex frequency step. The *Python* script will automatically generate a numerical model with these to analysis steps if the input variable 'Modal' is set equal to '1'. Running such an analysis will give both the real part, α, and imaginary part, ω, of the eigenvalues of the structure, and calculate the damping ratios of all modes, based on the relation given in Equation 5.8. By doing so it was investigated whether or not the material damping rendered the correct damping for each mode [54].

$$\zeta = -\frac{\alpha}{|\omega|} \tag{5.8}$$

NB: Abaqus calculates the damping ratio by multiplying Equation 5.8 with a factor of 2. This is incorrect and should be noted.



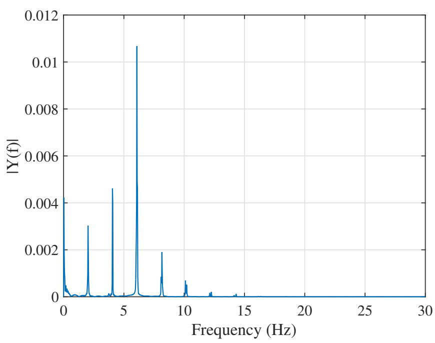
(a) **(b)**

Figure 5.11: *Material damping in the Abaqus model compared to the mathematical model.*

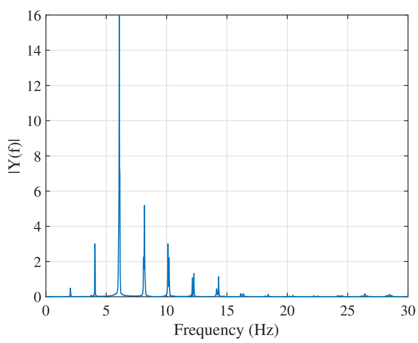
The damping ratios shown in Figure 5.11(b) were retrieved through a complex frequency analysis. The expected damping ratios for each mode according to equation 5.6 are also shown in red and good convergence can be observed. Figure 5.11(a) displays the damping ratios of the structure when material damping is added only to the wood components of the bridge. Most of the modes follow the same trend with a small reduction in damping ratio, but some of the higher modes have a significantly lower damping ratio. This is unwanted due to the numerical noise that can be caused by such higher modes, and therefore the model with damping in all materials was chosen.

5.3.3 Discrete Fourier Transform of Dynamic Response

Chapter 6 presents the results from many different analyses with different pedestrian configurations. With a complex structure it is not always easy to see which frequencies are dominating the response. The relative contribution of each natural frequency and its corresponding mode shape can be obtained by means of a discrete Fourier transform on the response from the analyses. A Fourier transform decomposes a function of time into the frequencies that make it up. The resulting function is a function of frequency, whose absolute value represents the amount of that frequency present in the original function. The FFT-algorithm in *MATLAB* has been used to perform the transform (see Appendix A.2). The physical meaning of the Fourier transform is that it moves the response from the time domain to the frequency domain. Thus, by taking the Fourier transform of an acceleration response the accelerations in the frequency domain are obtained.



(a) *Fourier transform of displacement response*



(b) *Fourier transform of acceleration response*

Figure 5.12: *Fourier transforms of the responses due to pedestrian walking at 2.0319 Hz.*

Figure 5.12(a) and 5.12(b) show the output from Fourier transforms of the displacement and acceleration responses due to a pedestrian walking at 2.0319 Hz. Several frequency peaks are present in both plots, and although the peaks occur at the same frequencies the amplitudes are shifted towards the higher modes in the Fourier transform of the acceleration response. The reason for this is simple: For a harmonic loading the response will also be harmonic and the acceleration, equal to the second derivative of the displacement will be given as shown in Equation 5.9:

$$a = \frac{d^2u}{dt^2} = -\omega^2 u \tag{5.9}$$

When the displacement response consists of several frequencies, the acceleration will consist of the same frequencies, but the amplitudes of the different harmonic terms will have been multiplied by the square of the corresponding frequency, therefore higher frequency harmonics will have higher amplitude in the acceleration response than in the displacement response plots.

When presenting the different results in the following chapter, both Fourier transforms are consistently included to be certain that all frequencies contributing to the response are known.

6 Results from Numerical Simulations

The following chapter presents the results from all analyses performed in *Abaqus*. The aim was to cause resonance for each individual mode so that the responses could be compared to the acceleration limits stated in the guidelines. To limit the amount of work, only the first ten modes were studied. Since they contain the first vertical, lateral and torsional mode of the bridge this range was expected to contain the dominating modes. The first section covers all the single pedestrian analyses and the results are ranged by mode number, starting with mode 1 and proceeding through to mode 10. In each case the acceleration is plotted versus time, and discrete Fourier transforms of the displacement and acceleration data are included. The acceleration responses will be compared to the two possible response scenarios depicted in Figure 3.13 to evaluate if responses are resonant. The Fourier transforms are also an indicator of the degree of resonance achieved. At resonance the natural frequency is dominating the response and the Fourier transform should then only contain one distinct peak. The response of the arches due to single pedestrian loading has also been investigated.

The second part of this chapter includes the other load scenarios that were tested; groups of people, continuous streams of people, a single pedestrian running and lastly a single walking pedestrian with increased amount of material damping in the model. The aim was still to cause resonant responses, and having found the dominating resonant frequencies through single pedestrian analyses only the frequencies thought to give resonance were tested for the groups and streams of people.

6.1 Single Walking Pedestrian

6.1.1 Response of the Deck

The largest response amplitude due to a harmonic load is obtained at resonance [55]. By making sure that either the fundamental frequency or the 2^{nd} or 3^{rd} harmonic of the walking frequency was equal to one of the natural frequencies resonance of the corresponding mode was considered possible. The local mode shapes of the deck were also taken into consideration when trying to excite the different modes, in that the position of the pedestrian was in accordance with the actual mode. For example, a torsional mode is more likely excited when the pedestrian is walking along the edge of the deck, as opposed to the center line.

The final script contains an input variable that determines whether the pedestrians walk across the bridge or just walk in place at a position determined by a second input variable. Although a pedestrian walking across is more likely, a pedestrian stepping in place causes greater responses and was therefore included. In reality a combination of the two might occur. All frequencies have been tested with both scenarios. The following section shows the analysis results from simulations with single pedestrians walking at different pacing rates, with the goal to cause resonance at the ten first natural frequencies of the bridge.

NB! Whenever the pedestrian is walking in place, the response values were extracted at the location where the pedestrian was located since this was where the maximum response amplitude occured.

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Mode 1: f_1 **= 1.8838 Hz**

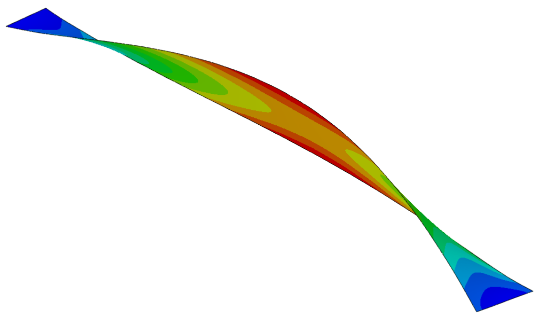
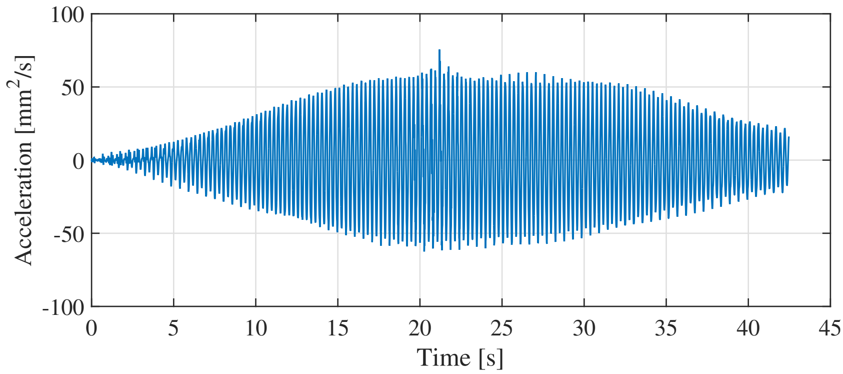


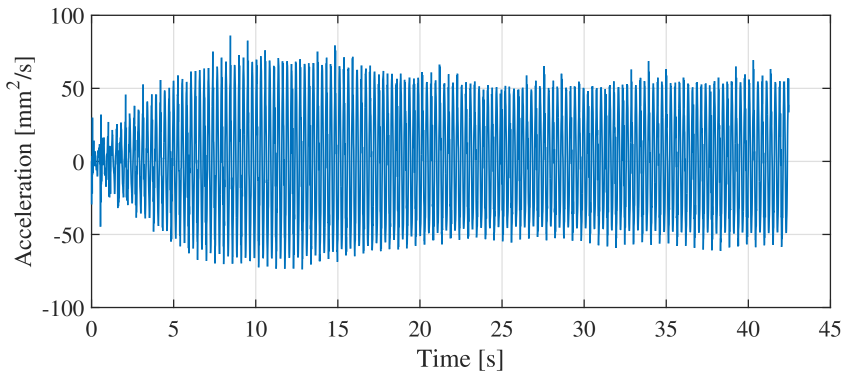
Figure 6.1: *Mode 1, deck only.*

By only presenting the mode shape of the deck and increasing the deformation scale factor in *Abaqus*, it is observed that the deck deforms in a torsional fashion for mode 1, 3, 7 and 10 (see Figure 6.1). Attempts were made to excite all these modes by modelling a pedestrian walking either along or in place at the left side of the bridge. This way the forcing is applied according to the mode shapes, which should cause the largest response amplitude.

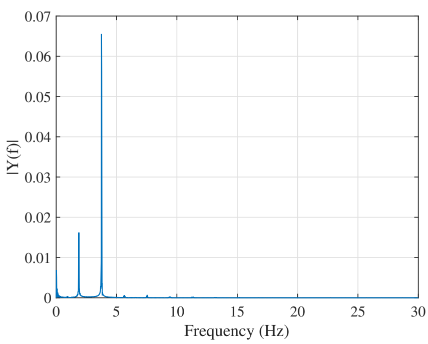
A pedestrian walking at a pacing rate equal to the first natural frequency, 1.8838 Hz, was modelled to try to excite the first mode of the structure. As shown in Figure 6.2(a) and 6.2(b), the peak vertical accelerations reach values of about 60 mm/s^2 and 70 mm/s^2, respectively. The plots of the vertical displacement and acceleration in the frequency domain (see Figure 6.2(d)) show that the response is made up of mainly one frequency. But the peak is in the range of the natural frequency of the sixth mode, not the first. Thus, the 2nd harmonic of the walking frequency (3.7676 Hz) dominates, and it is not the torsional mode of the deck that has been excited. The conclusion is therefore that resonance for the first mode has not occurred. Instead mode six was excited, and by looking at Figure 6.2(b), it appears that larger responses could be obtained if the frequency was tuned to match the natural frequency of the sixth mode. With the gradual increase in response combined with smaller transients due to each footstep (the peaks identified every 0.5 s) the response looks like a combination of the resonant and transient response curves depicted in Figure 3.13. In addition, the decrease in peak accelerations after about 10 seconds indicates that the steady state solution has not reached its maximum. A damped system subject to resonance caused by a harmonic load will have a response like the one shown in Figure 3.13(a). The solution will asymptotically reach the steady state solution, which is not the case here. This implies that the maximum acceleration response due to a harmonic load was not obtained. Interestingly, the arches are resonating at the natural frequency of mode 1 (see Figure 6.23(d)), and this will be addressed later in this chapter.



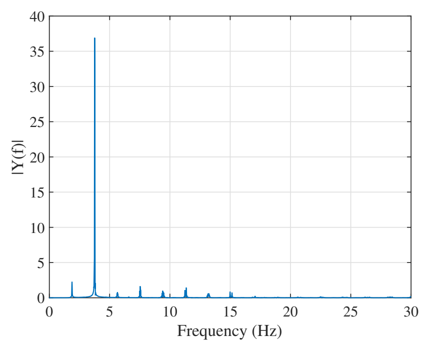
(a) *Vertical acceleration due to pedestrian crossing the bridge along the left edge*



(b) *Vertical acceleration due to pedestrian walking in place at the left edge at the middle of the bridge*



(c) *Vertical displacement in the frequency domain*



(d) *Vertical acceleration in the frequency domain*

Figure 6.2: *Dynamic responses, extracted at the left edge at the middle of the deck, due to a single pedestrian walking at 1.8838 Hz.*

Mode 2: f_2 **= 2.1405 Hz**

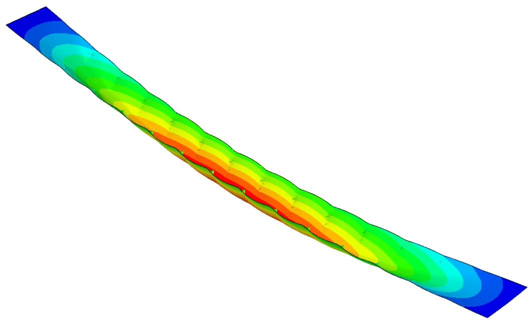
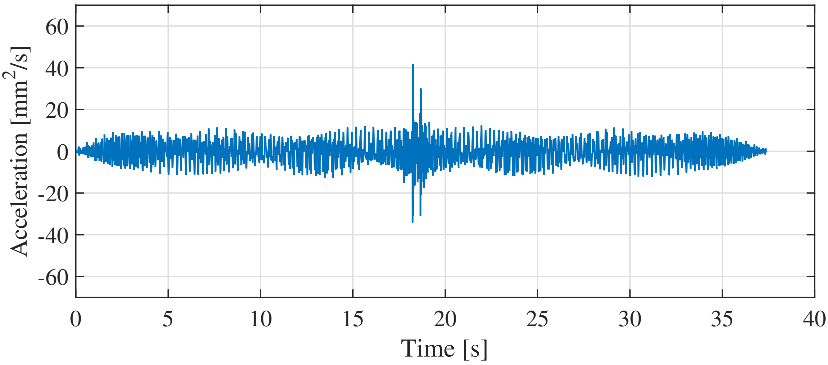
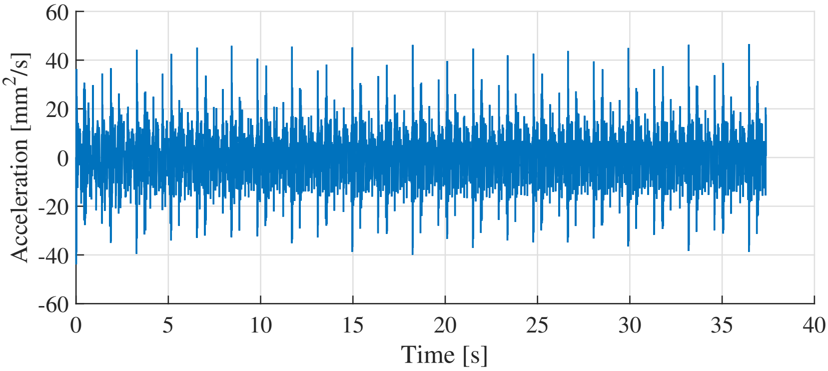


Figure 6.3: *Mode 2, deck only.*

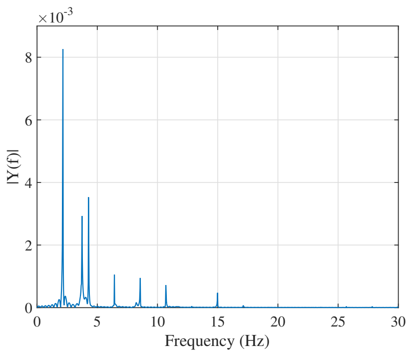
The second mode has a natural frequency of 2.1405 Hz, and is mainly a lateral mode of the arches. Although the deck does have a vertical sag, this deflection is extremely small compared to the arches. The natural frequency is well within the range of walking frequencies, so analyses with one person walking across the center line of the bridge, and with one person walking in place at the middle of the bridge at frequency 2.1405 Hz were conducted. The results in Figure 6.4(a) and 6.4(b) show little of interest as no signs of resonance are apparent in the deck. A transient response is observed with the accelerations due to the impact of each step damping out quickly. Figure 6.4(c) and 6.4(d) reveal that several frequencies are contributing to the response, which is typical for transient responses identified by peaks with decreasing amplitude. All this confirms that the deck will not resonate while pedestrians are walking at frequencies around 2.1405 Hz. Once again the arches seem to approach resonance (see Figure 6.24(d)).



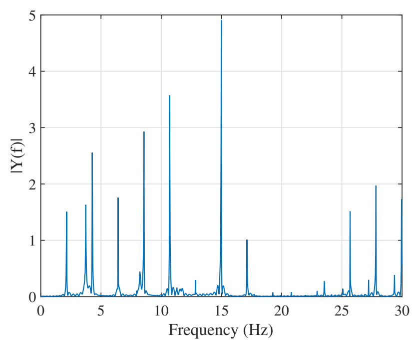
(a) *Vertical acceleration due to pedestrian crossing the bridge along the middle*



(b) *Vertical acceleration due to pedestrian walking in place at the middle of the bridge*



(c) *Vertical displacement in the frequency domain*



(d) *Vertical acceleration in the frequency domain*

Figure 6.4: *Dynamic responses, extracted at the middle of the deck, due to a single pedestrian walking at 2.1405 Hz.*

Mode 3: f_3 **= 3.3217 Hz**

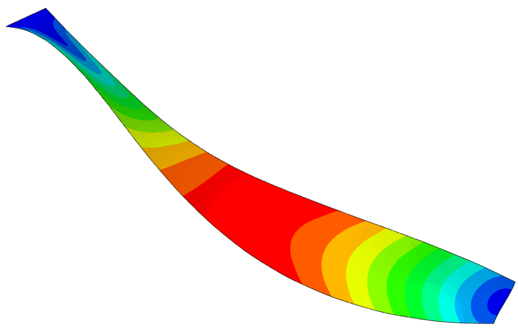
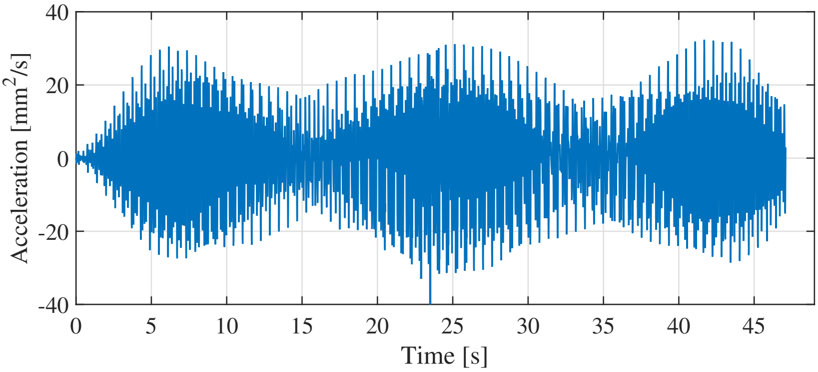
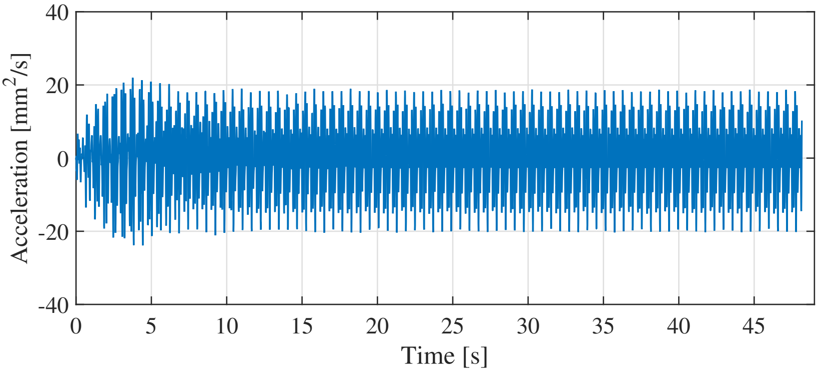


Figure 6.5: *Mode 3, deck only.*

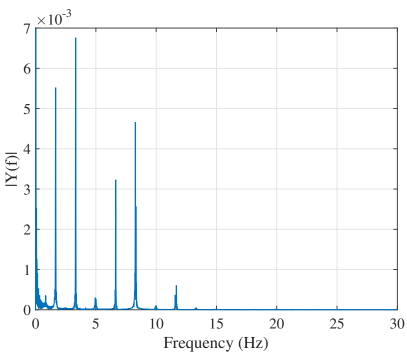
The third natural frequency, 3.3217 Hz, with its corresponding torsional mode shape for the deck, is not within the range of walking frequencies (1.6 - 2.4 Hz). Therefore an attempt to cause resonance with the 2^{nd} harmonic of the pacing rate was made. A pedestrian walking along the left side with frequency 1.6609 Hz and walking in place at the left edge at $\frac{3}{4}$ of the length of the bridge yielded the results shown in Figure 6.6. An envelope response is observed in Figure 6.6(a), which implies that the response consists of several frequencies. This is confirmed by the plot of the displacement and acceleration in the frequency domain (see figure 6.6(c) and 6.6(b)) showing peaks at several n^{th} harmonics in addition to the fundamental frequency. A transient response is also seen in both (a) and (b) confirming that the deck will not resonate with the the mode shape of mode 3 due to a single walking pedestrian.



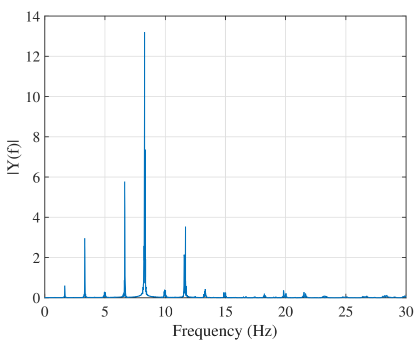
(a) *Vertical acceleration due to pedestrian crossing the bridge along the left edge*



(b) *Vertical acceleration due to pedestrian walking in place at the left edge at 3/4 of the length of the bridge*



(c) *Vertical displacement in the frequency domain*



(d) *Vertical acceleration in the frequency domain*

Figure 6.6: *Dynamic responses, extracted at the left edge at 3/4 of the length of the deck, due to a single pedestrian walking at 1.6609 Hz.*

Mode 4: f_4 **= 3.4370 Hz**

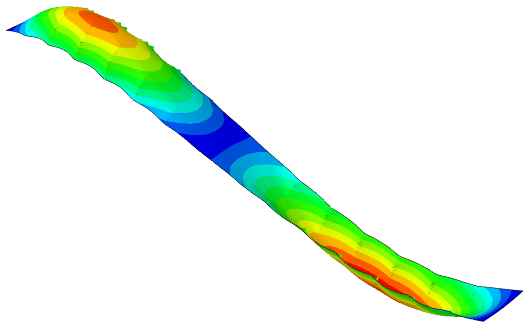
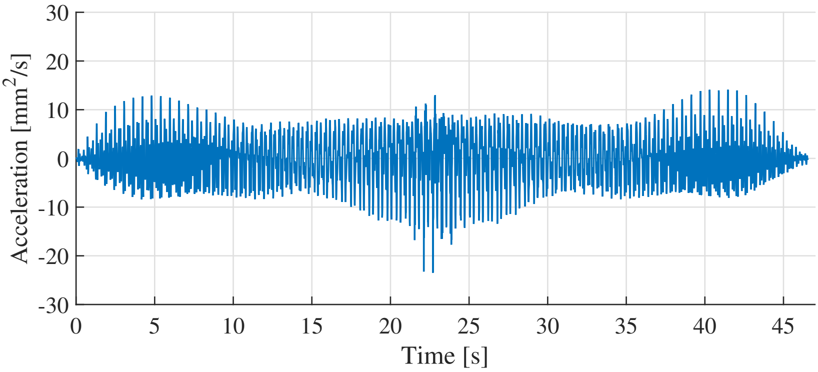


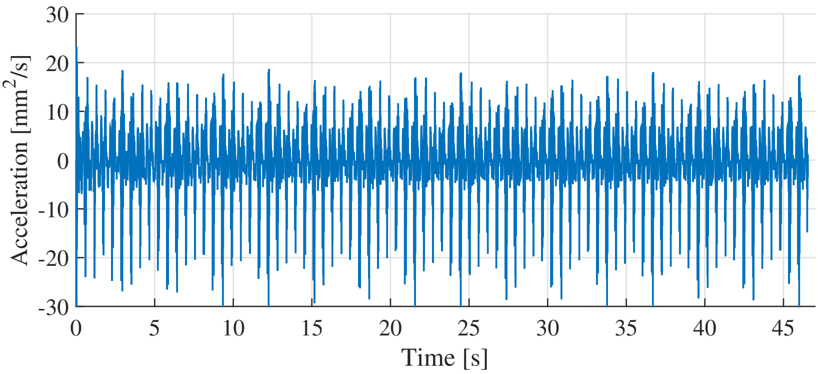
Figure 6.7: *Mode 4, deck only.*

The local mode shape of the fourth mode is a vertical one with two sags and a natural frequency of 3.4370 Hz. Like with the first three modes, the mode shape of the deck is almost negligible compared to the corresponding mode shape of the arches, and the arches will probably be excited long before the deck. Nevertheless, analyses with one single pedestrian with walking frequency 1.7185 Hz were performed to try to cause resonance with the 2^{nd} harmonic (3.4370 Hz). The results are shown in Figure 6.8 and the resemblance to the results for mode 3 (see Figure 6.6 on previous page) is obvious for both the acceleration in the time and frequency domain. The conclusion is therefore the same: A single walking pedestrian will not be able to cause resonant responses in the deck for mode 4.

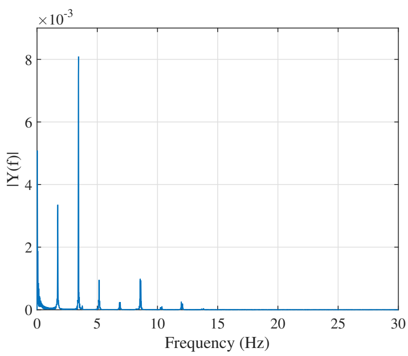
Some extra attention should be given to the distinct acceleration peaks in Figure 6.8(a). These occur exactly when the pedestrian reaches the middle of the bridge and are damped out immediately. Although they may be due to high frequency noise and therefore unphysical, they are unimportant and are smoothed out in the calculated RMS-accelerations when a sufficiently big time interval is used.



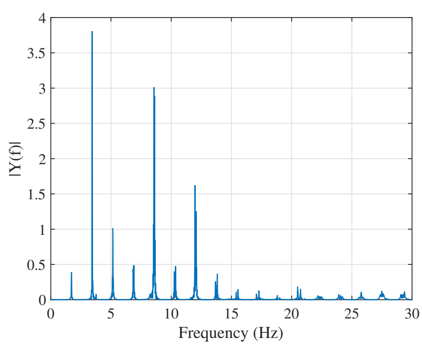
(a) *Vertical acceleration due to pedestrian crossing the bridge along the middle*



(b) *Vertical acceleration due to pedestrian walking in place at the middle at 3/4 of the length of the bridge*



(c) *Vertical displacement in the frequency domain*



(d) *Vertical acceleration in the frequency domain*

Figure 6.8: *Dynamic responses, extracted at the middle at 3/4 of the length of the deck, due to a single pedestrian walking at 1.7185 Hz.*

Mode 5: f_5 = 3.6216 Hz

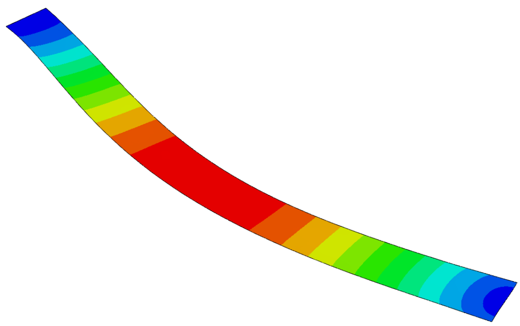
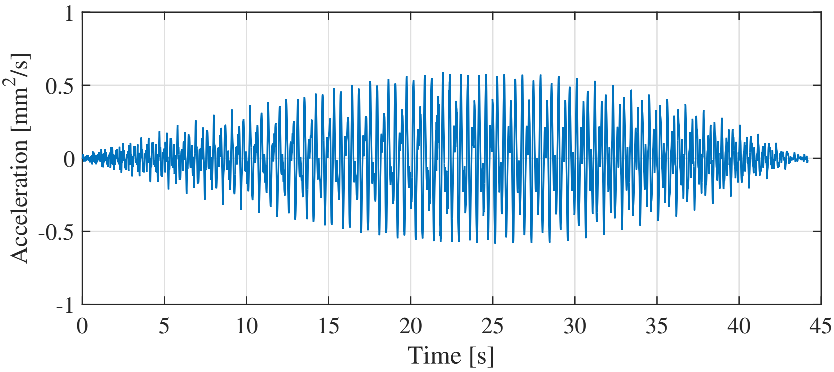


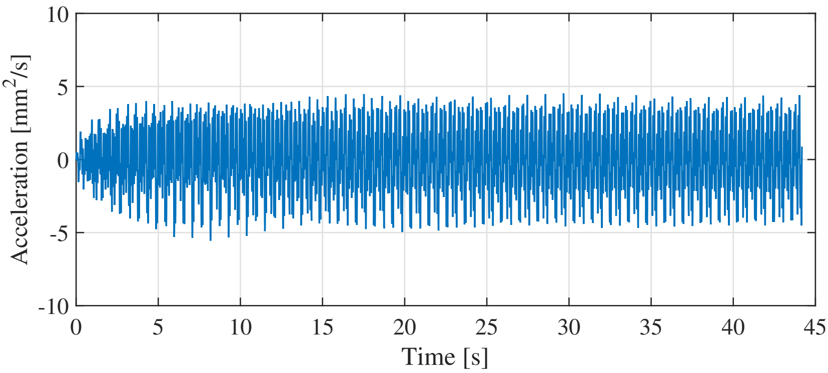
Figure 6.9: *Mode 5, deck only.*

The only lateral mode within the first ten modes of the bridge is mode 5, with a frequency of 3.6216 Hz. As mentioned in Section 3.3.1, the fundamental frequency of the lateral forcing is half that of the vertical and longitudinal forcing because the left and right foot cause forces in opposite directions, laterally. The consequence of this is that resonance can be caused with n^{th} harmonics of two different frequencies within the range of walking frequencies.

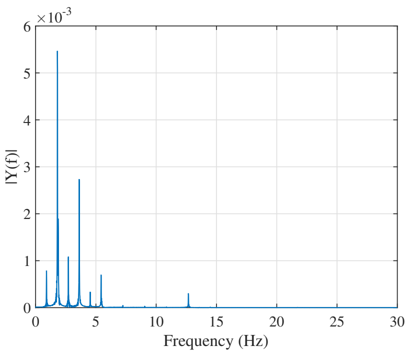
Attempts were made to cause resonance with the 3^{rd} and 4^{th} harmonic of the forcing from a single pedestrian walking at 2.4144 Hz and 1.8108 Hz, respectively. The former results are shown first in Figure 6.10. Notice that the y-axis does not have the same dimensions in Figure (a) and (b). The magnitude of the acceleration is extremely small in both cases and the transient response is dominating. The plots of the Fourier transforms in Figure 6.10(c) 6.10(d) display two distinct peaks at about 2.72 Hz and 4.53 Hz. These frequencies correspond to the third and fifth harmonic of the fundamental lateral frequency, but do not match any of the natural frequencies of the bridge. Thus, a pedestrian walking at 1.8108 Hz will not cause resonant behaviour of the deck.



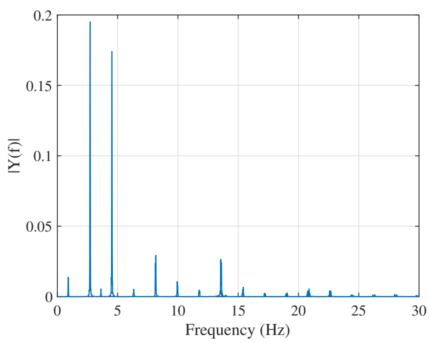
(a) *Lateral acceleration due to pedestrian crossing the bridge along the middle*



(b) *Lateral acceleration due to pedestrian walking in place at the middle of the bridge*



(c) *Lateral displacement in the frequency domain*

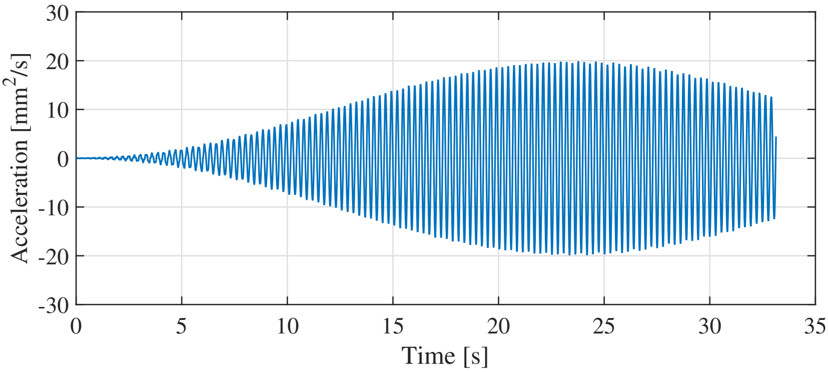


(d) *Lateral acceleration in the frequency domain*

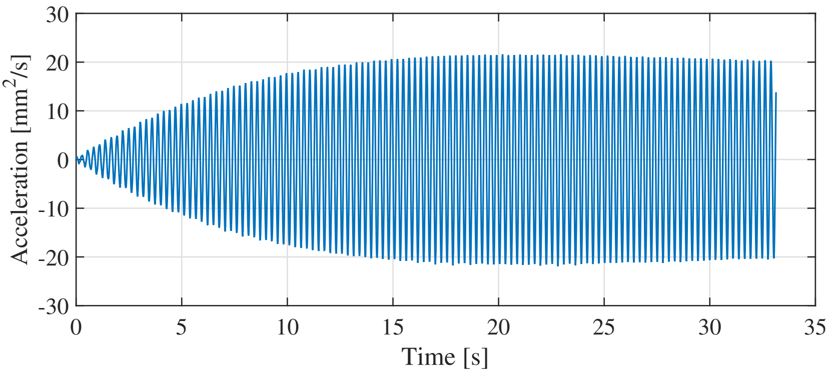
Figure 6.10: *Dynamic responses, extracted at the middle of the deck, due to a single pedestrian walking at 1.8108 Hz.*

The results from the analyses with walking frequency 2.4144 Hz are shown in Figure 6.11 and this time the resonant behaviour is obvious. The discrete Fourier transforms (see Figure 6.11(c) and 6.11(d)) display a single peak at around 3.62 Hz, exactly the fifth natural frequency of the structure. Unlike the previous results there are no other frequencies represented in the Foruier transform, and that is why the curves in Figure 6.11(a) and 6.11(b) are so smooth. The analysis with the pedestrian walking across the bridge (a) almost reaches the same magnitude of acceleration as the analysis with a pedestrian stepping in place (b), but the vibrations start damping out when the pedestrian approaches the end of the bridge. The initial envelope curve that asymptotically reaches the steady state solution is characteristic for damped resonant response [26] and can be observed in Figure 6.11(b).

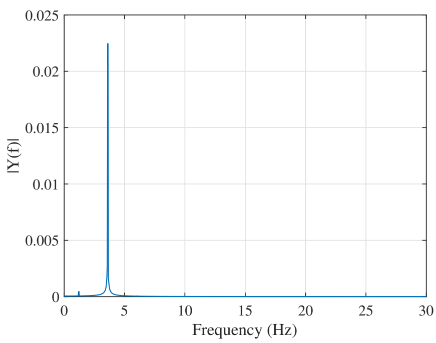
Note that these results are obtained using Schulze's load model (see Figure 5.4(a)). With a different load model different DLFs follow, yielding different results. For example, if the analyses were conducted with Bachmann's load model instead (which has been written into the script and can be implemented in *Abaqus* setting 'Load Model' equal to '2'), the responses would have been greater because the DLFs have greater magnitude which in turn leads to forces with greater magnitude.



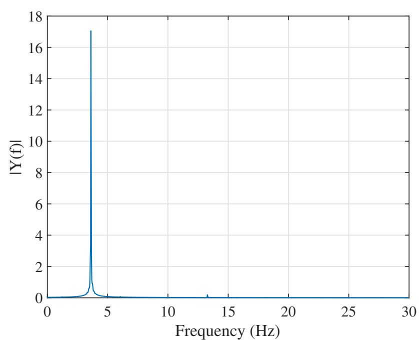
(a) *Lateral acceleration due to pedestrian crossing the bridge along the middle*



(b) *Lateral acceleration due to pedestrian walking in place at the middle of the bridge*



(c) *Lateral displacement in the frequency domain*



(d) *Lateral acceleration in the frequency domain*

Figure 6.11: *Dynamic responses, extracted at the middle of the deck, due to a single pedestrian walking at 2.4144 Hz.*

Mode 6: f_6 **= 3.7528 Hz**

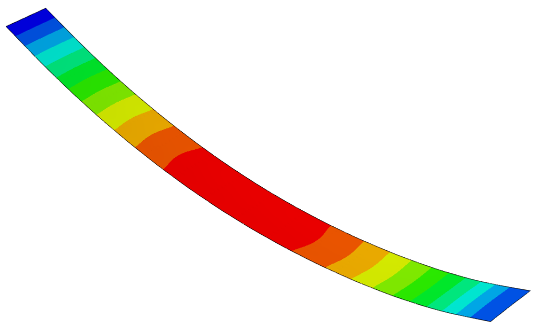
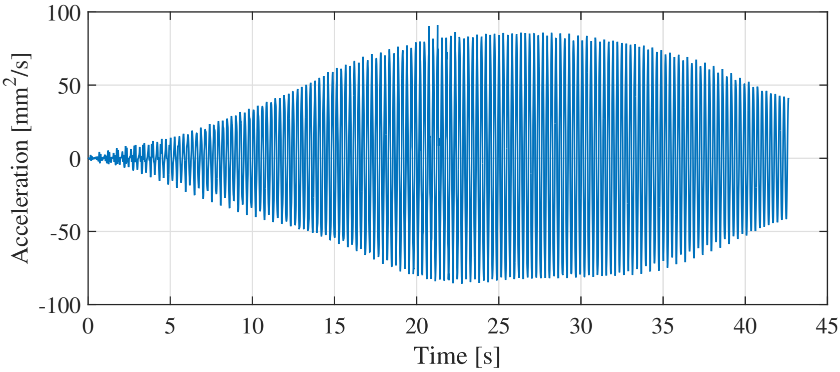


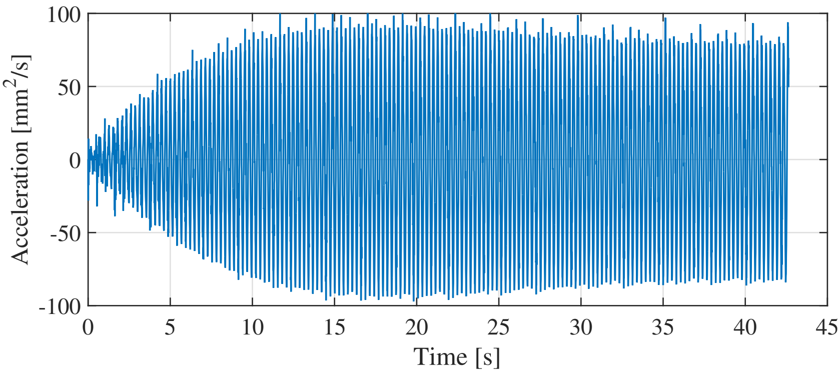
Figure 6.12: *Mode 6, deck only.*

Mode 6, the first vertical mode of the structure, has a natural frequency of 3.7528 Hz, well outside the range of walking frequencies. A runner might run at this frequency, and this will be addressed later. By modelling a pedestrian walking at 1.8764 Hz it has been tested and confirmed for mode 6 that the 2^{nd} harmonic of a walking frequency can cause resonant responses. Figure 6.13(b) shows the accelerations at the middle of the deck due to a pedestrian walking in place. A clear damped resonant response can be observed with an initial, almost linear increase until the solution settles on the steady state solution. Figure 6.13(c) and 6.13(d) show the displacement and acceleration in the frequency domain obtained through a Fourier transform. Although the pedestrian walks at 1.8764 Hz the only frequency represented in the response is the 2^{nd} harmonic (3.7528 Hz) equal to the sixth natural mode of the bridge. This makes it clear that pedestrians will be able to make the bridge resonate.

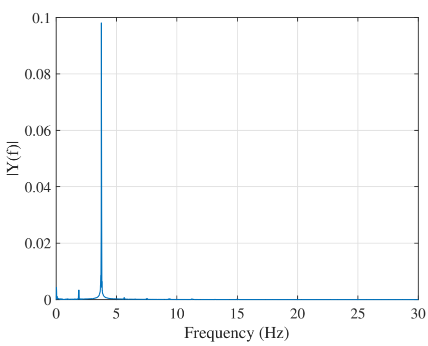
A Small transient response can still be observed in the results, but the differences between the peaks are small compared to the response amplitude. There is also a small decrease in maximum amplitude as the steady state settles in Figure 6.13(b). Compared to the results for mode 1 in Figure 6.2(b), where also the sixth mode was excited, the response amplitudes are higher and the decrease from initial transient phase to steady state is much smaller. This was as expected.



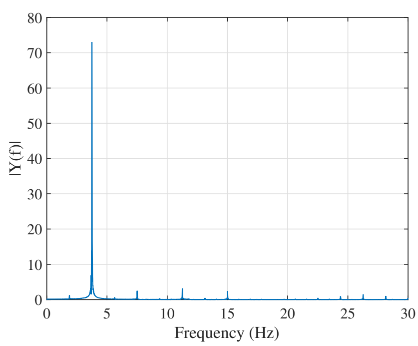
(a) *Vertical acceleration due to pedestrian crossing the bridge along the middle*



(b) *Vertical acceleration due to pedestrian walking in place at the middle of the bridge*



(c) *Vertical displacement in the frequency domain*



(d) *Vertical acceleration in the frequency domain*

Figure 6.13: *Dynamic responses, extracted at the middle of the deck, due to a single pedestrian walking at 1.8764 Hz.*

Mode 7: f_7 **= 5.5485 Hz**

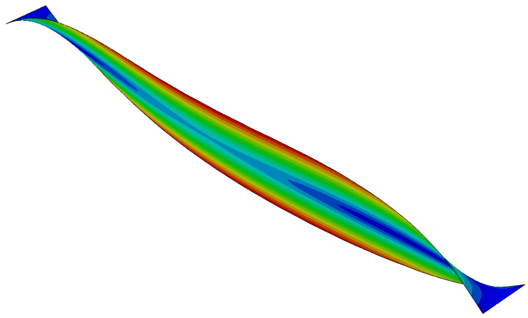
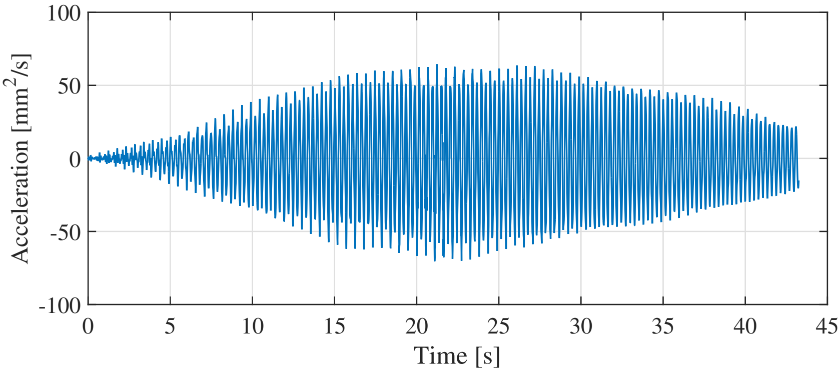
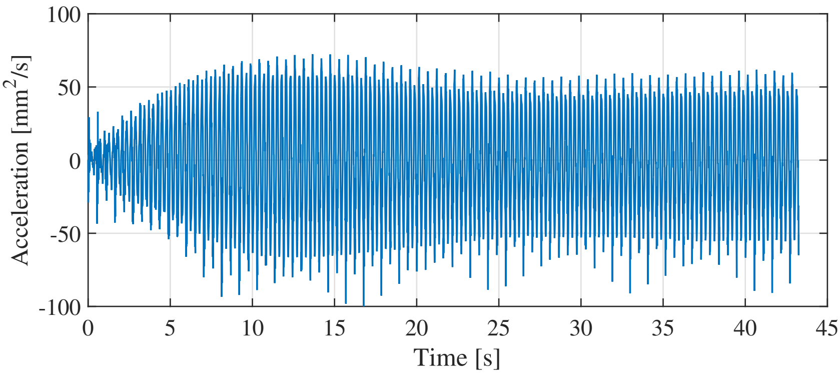


Figure 6.14: *Mode 7, deck only.*

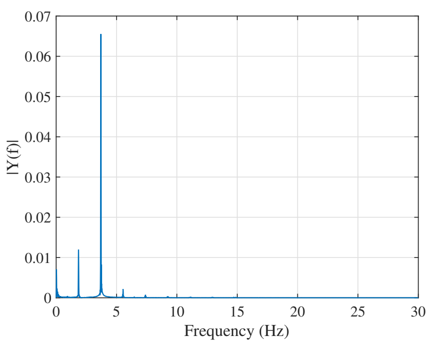
Mode 7 looks quite similar to mode 1 (1.8838 Hz), both of them being torsional modes local to the deck. An attempt was made to excite the seventh mode by means of the 3rd harmonic of the walking frequency (1.8496 Hz) with a person walking along the left edge. The results seen in Figure 6.15 are extremely similar to the results obtained when trying to excite the first mode (see Figure 6.2) and therefore the same conclusion has been drawn; again the sixth mode has been excited instead, through the 2nd harmonic of the walking frequency, and a single pedestrian has not been able to cause resonance of a local torsional deck mode. It is worth noticing that the frequencies used in the attempts to cause resonance at mode 1 (1.8838 Hz) and 7 (1.8496 Hz) are on either side of 1.8764 Hz, the natural frequency of mode 6. They form a range of frequencies all exciting this mode.



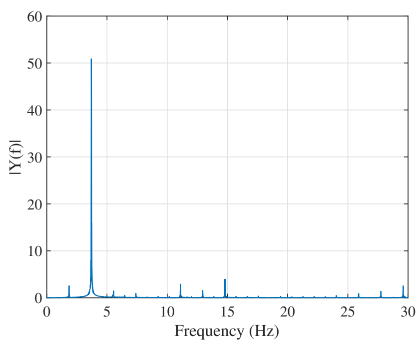
(a) *Vertical acceleration due to pedestrian crossing the bridge along the left edge*



(b) *Vertical acceleration due to pedestrian walking in place at the left edge at the middle of the bridge*



(c) *Vertical displacement in the frequency domain*



(d) *Vertical acceleration in the frequency domain*

Figure 6.15: *Dynamic responses, extracted at the left edge at the middle of the deck, due to single pedestrian walking at 1.8496 Hz.*

Mode 8: f_8 **= 5.7278 Hz**

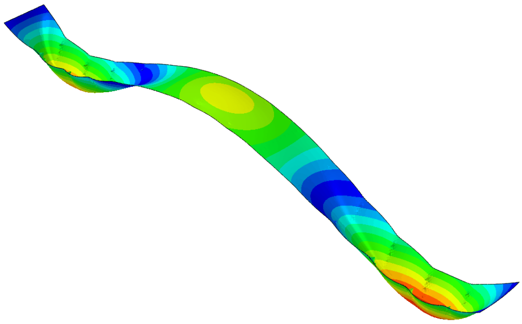
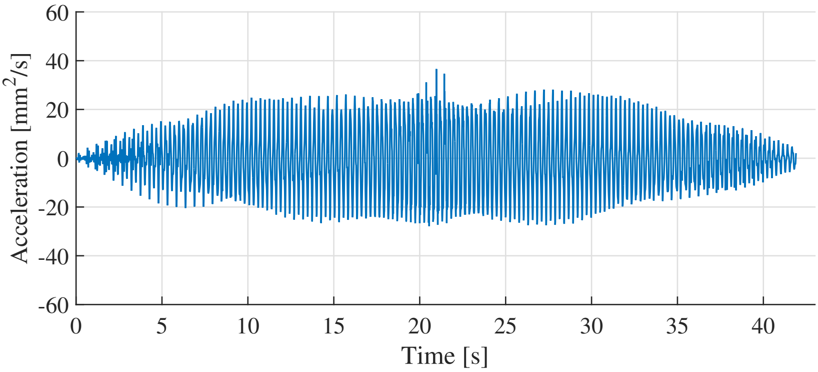
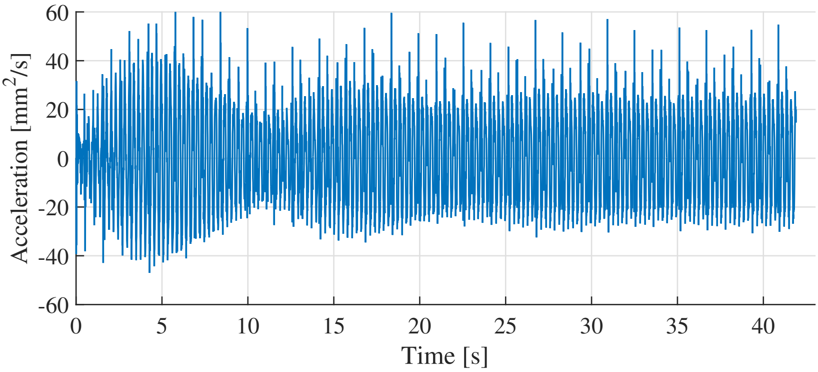


Figure 6.16: *Mode 8, deck only.*

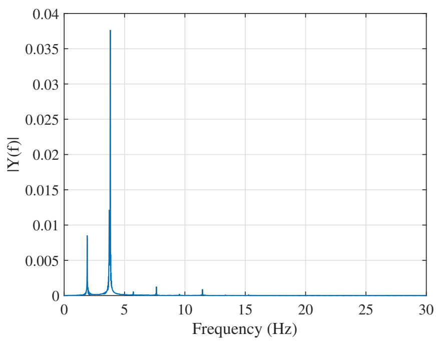
Mode 8, with a natural frequency of 5.7278 Hz, is yet another mode dominated by the arches, and their response has been investigated separately. The local mode shape of the deck is characterized as a vertical mode with three sags, and the pedestrian walking in place was therefore placed at the middle of the deck. The results in Figure 6.17 show modest acceleration values due to the pacing rate of 1.9094 Hz, used in the attempt to excite the structure through the 3rd harmonic. The responses due to each footstep are dying out quickly, shown by the distinct peaks in Figure 6.17(b) and the initial resonant response is small. It can be seen from the Fourier transforms that the 3rd harmonic of the walking frequency is not present in the responses. Instead the 2nd harmonic dominates by exciting, once again, the sixth mode.



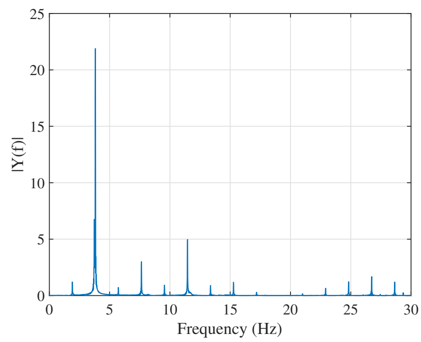
(a) *Vertical acceleration due to pedestrian crossing the bridge along the middle*



(b) *Vertical acceleration due to pedestrian walking in place at the middle of the bridge*



(c) *Vertical displacement in the frequency domain*



(d) *Vertical acceleration in the frequency domain*

Figure 6.17: *Dynamic responses, extracted at the middle of the deck, due to a single pedestrian walking at 1.9094 Hz.*

Mode 9: f_9 **= 6.0945 Hz**

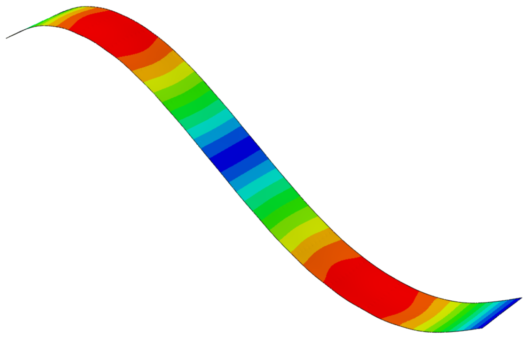
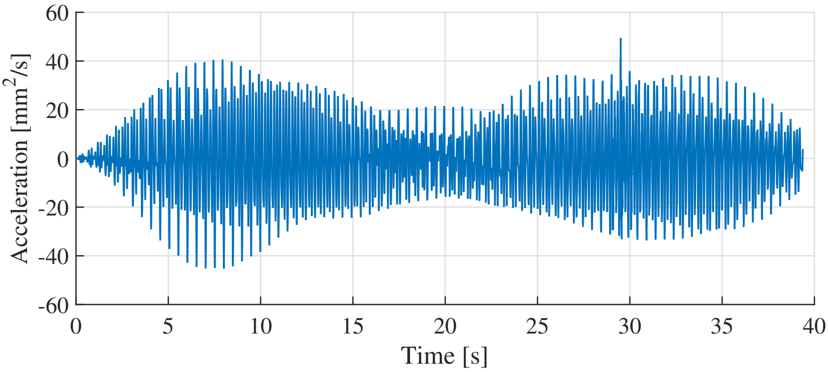


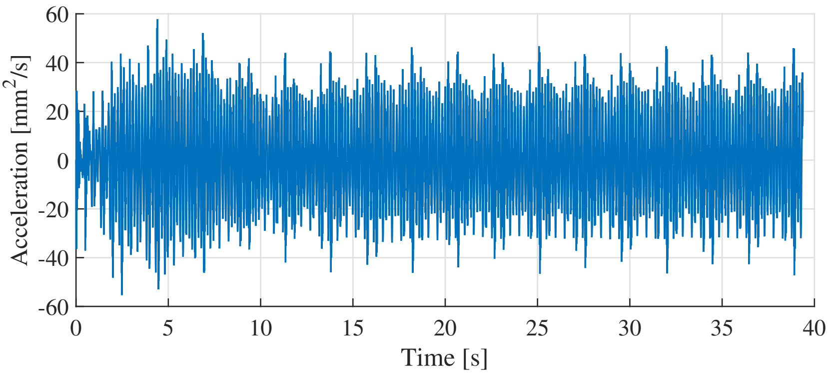
Figure 6.18: *Mode 9, deck only.*

Analyses with a single pedestrian walking at 2.0319 Hz were executed to try to excite the ninth mode by means of the 3rd harmonic of the walking frequency (see Figure 6.19). The mode is the second global vertical mode, consisting of two sags, and therefore the pedestrian walking in place was placed at $\frac{3}{4}$ of the length of the bridge. The accelerations were also extracted there, which is why the distinct acceleration peaks in Figure 6.19(a) are after 30 s, when the pedestrian passes the extraction point.

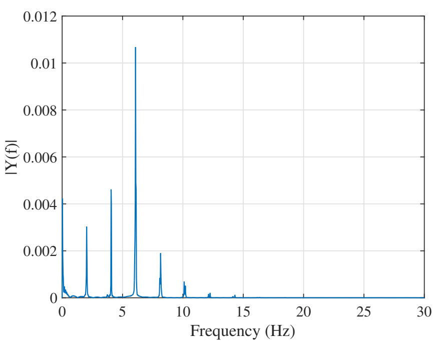
Although the plots of the displacement and acceleration in the frequency domain (Figure 6.19(c) and 6.19(d)) show that the 3rd harmonic is the dominating frequency in the responses, resonance is not apparent. A short initial resonant response can be seen during the first 5 seconds, but there is an equally large transient response due to the impact of each footstep being damped out. The response amplitudes are also small compared to previous results where resonance was obtained.



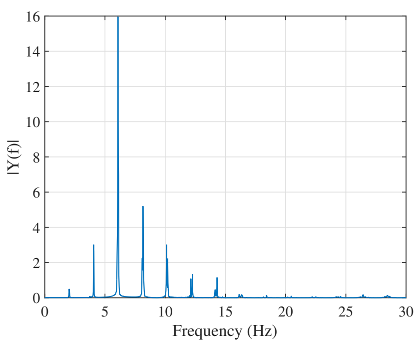
(a) *Vertical acceleration due to pedestrian crossing the bridge along the middle*



(b) *Vertical acceleration due to pedestrian walking in place at 3/4 of the length at the middle of the bridge*



(c) *Vertical displacement in the frequency domain*



(d) *Vertical acceleration in the frequency domain*

Figure 6.19: *Dynamic responses, extracted at the middle at 3/4 of the length of the deck, due to a single pedestrian walking at 2.0319 Hz.*

Mode 10: f_{10} **= 6.6034 Hz**

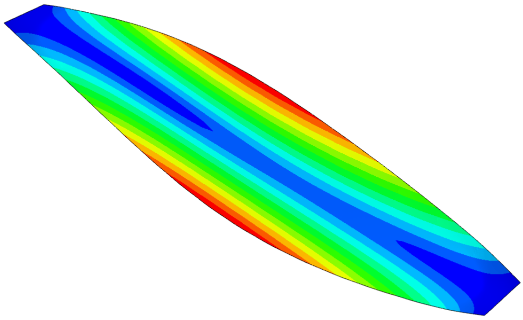
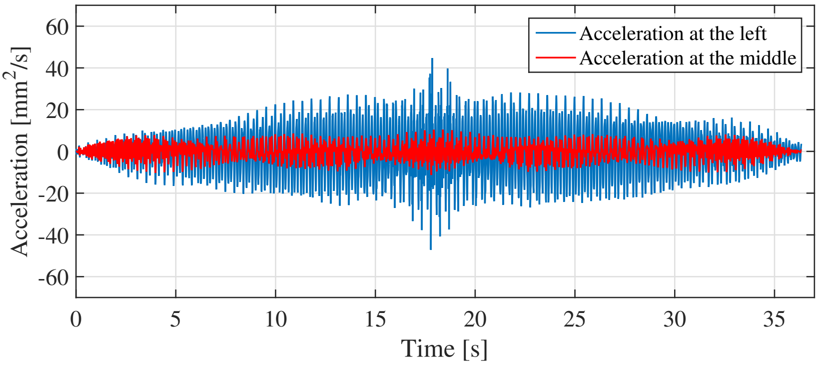


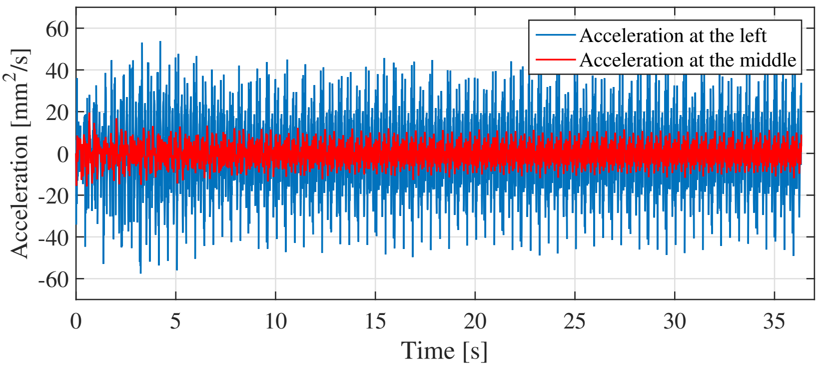
Figure 6.20: *Mode 10, deck only.*

Mode 10, the final investigated mode, displays the first global torsional mode shape. A single pedestrian walking at 2.22012 Hz was simulated to try to excite the structure through the 3rd harmonic. The results are shown in Figure 6.21. For the first time the accelerations at two different locations are shown because they are distinctly different. As expected for torsional motion of the deck, the vertical accelerations at the edges are much greater than the vertical acceleration at the middle, which, for a purely torsional mode, should be zero. This indicates that the deck was excited into a torsional mode shape.

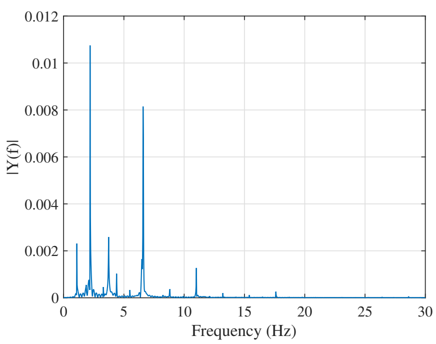
Figure 6.21(c) and 6.21(d), showing the displacement and acceleration at the left edge of the deck in the frequency domain, confirm that the the footbridge has been excited at frequency 6.6034 Hz. However, other frequencies equal to the fundamental frequency, the 2nd, and the 5th harmonic are also present. As with the attempt to excite mode 9 (results in Figure 6.19), an initial resonant response is present but the transient response is dominating. An even better correlation between the third harmonic of the walking frequency and the natural frequency of mode 10 is expected to give somewhat greater responses, but resonance is deemed unlikely.



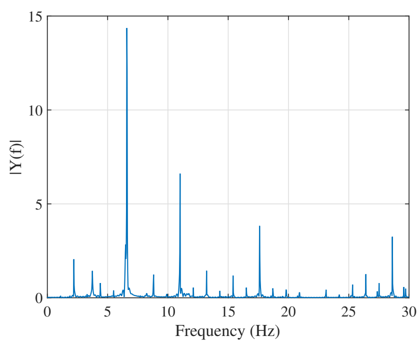
(a) *Vertical accelerations due to pedestrian crossing the bridge along the left edge*



(b) *Vertical accelerations due to pedestrian walking in place at the left edge at the middle of the bridge*



(c) *Vertical displacement at left edge in the frequency domain*

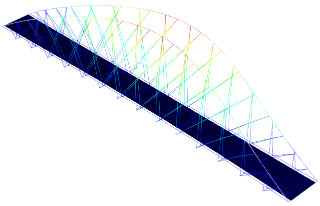


(d) *Vertical acceleration at left edge in the ||frequency domain*

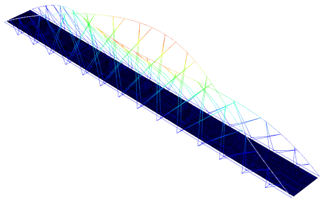
Figure 6.21: *Dynamic responses, extracted at the left edge at the middle of the deck, due to a single pedestrian walking at 2.2012 Hz.*

6.1.2 Response of the Arches

Many of the bridge's mode shapes are mainly deflections of the arches. Because any visible movement of the structure can contribute to the sense of discomfort for present pedestrians, this part of the chapter is dedicated to investigating how the arches will respond to loading on the deck from a single pedestrian. Only the results from the analyses with a single pedestrian walking in place with pacing rate equal to the two first natural frequencies are included here, because these analyses yielded the greatest responses of the arches. Lateral acceleration and displacement values are extracted at the top of the arches, where the responses are assumed to be of greatest magnitude based on the mode shapes of the first and second mode, shown in Figure 6.22.



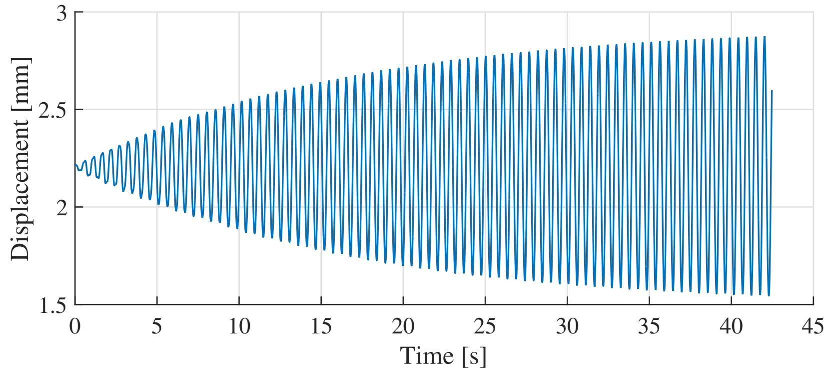
(a) *Mode 1.*



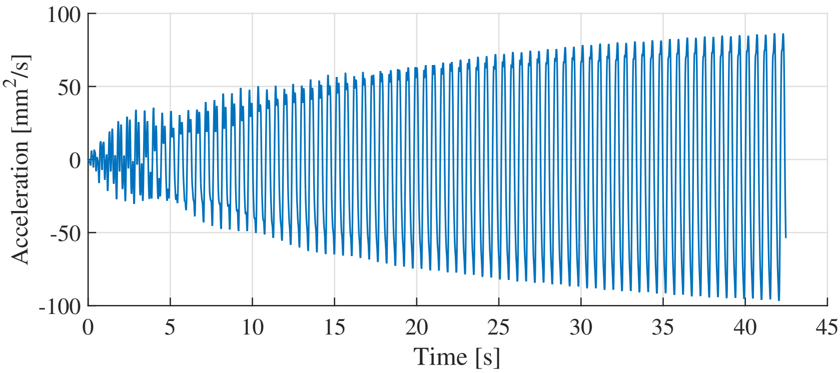
(b) *Mode 2.*

Figure 6.22: *Modes 1 and 2 of the bridge reveal lateral movement of arches.*

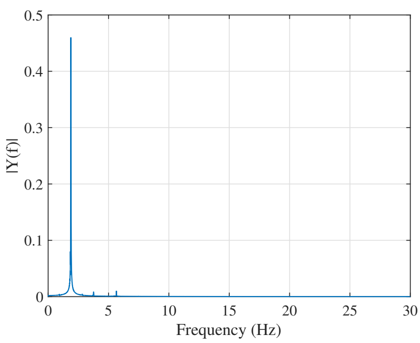
By comparing the results in Figure 6.23 and 6.24 it is clear that resonance has occurred in both cases. This can be seen both by studying the shape of the displacement and acceleration curves, figures (a) and (b) in both cases and also by noticing that the Fourier transforms, figures (c) and (d), each contain only one distinct peak. The difference in response amplitudes is very significant with both the displacement and acceleration response being a factor of about 5 times higher for the analysis exciting the first mode. A small transient response can also be observed in Figure 6.24(a) and 6.24(b), confirming that the the resonant response is stronger in the former analysis. Although not shown here, the similar responses for the other single pedestrian analyses were also much lower than the response obtained with walking frequency 1.8838 Hz. Even though the steady state solution has not quite reached its asymptotic value, the slope of the envelope curve is approaching zero and it is believed that the increase in amplitude response will be negligible.



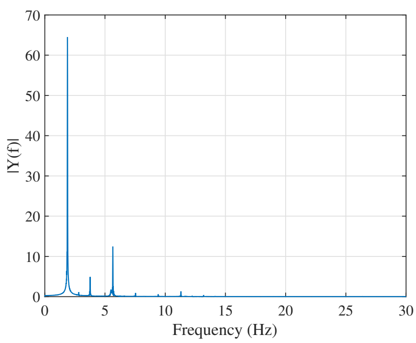
(a) *Lateral displacement*



(b) *Lateral acceleration*

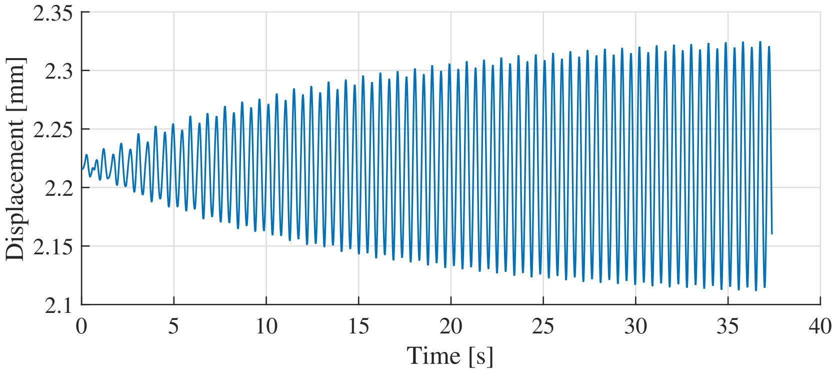


(c) *Lateral displacement in the frequency domain.*

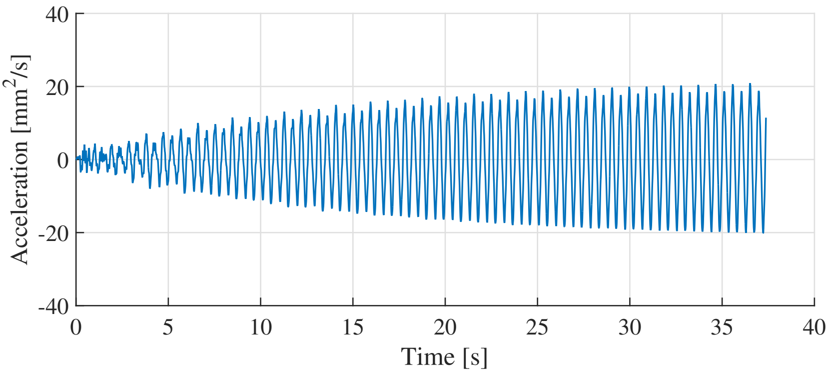


(d) *Lateral acceleration in the frequency domain*

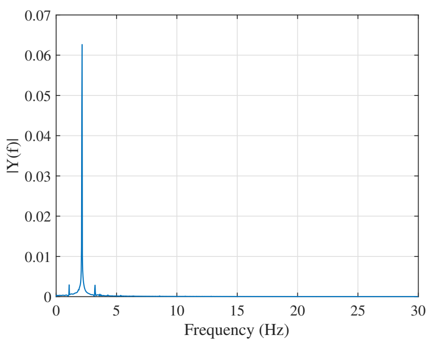
Figure 6.23: *Dynamic responses, extracted at arch top, due to pedestrian walking in place at the left edge at the middle with pacing rate 1.8838 Hz.*



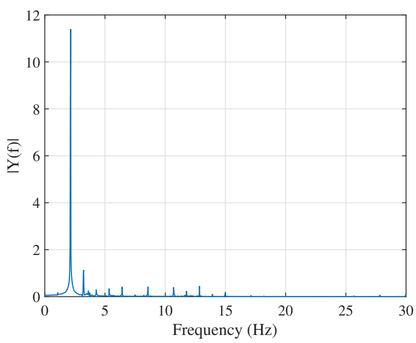
(a) *Lateral displacement*



(b) *Lateral acceleration*



(c) *Lateral displacement in the frequency domain*



(d) *Lateral acceleration in the frequency domain*

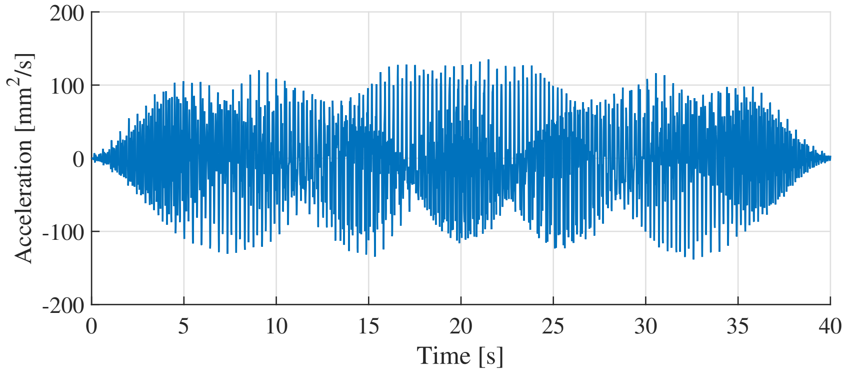
Figure 6.24: *Dynamic responses, extracted at arch top, due top pedestrian walking in place at the middle with pacing rate 2.1405 Hz.*

6.2 Groups of People

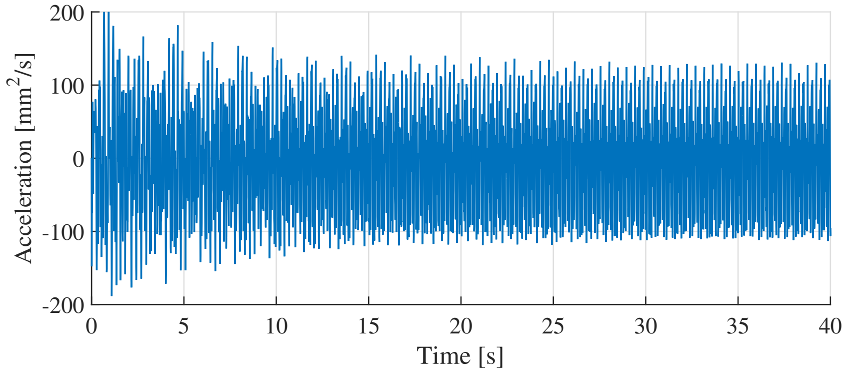
The response of the footbridge due to groups of people walking is also of interest. Based on the results from the analyses with a single pedestrian on the bridge, a couple of modes were characterized as dominating for the response of the deck. These modes are mode 5 (lateral mode) and mode 6 (vertical mode). Attempts were made to excite these modes with groups of people through the 2^{nd} harmonic of the walking frequency. In addition, mode 10 was tried excited with a group of people since it was the only torsional mode that was successfully excited during previous analyses, even though resonance was not observed. Lastly, mode 2 was also tested to confirm that resonance would not occur at that mode for a group of people either.

According to Eurocode 5 a distinct group of pedestrians equals 13 people [43]. Three rows of people where chosen as default by the authors and since the *Python* script is written to apply the same number of people to all rows, 4 pedestrians were modelled to walk in each row, giving a total of 12 pedestrians. They are perfectly synchronized, which may be considered a worst case scenario.

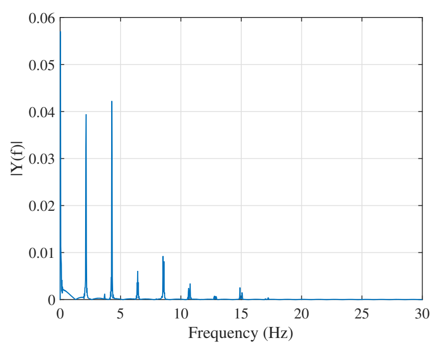
Firstly, analyses with a group walking both across and in place at the middle at 2.1405 Hz, equal to the natural frequency of mode 2, were carried out. The results are shown in Figure 6.25 and back up the results from the same analyses with only one person, showing that resonance in the deck will not occur at this frequency. The peak accelerations are only of magnitude 100 mm/s^2, which is about the same as what was obtained with only one person exciting the sixth mode of the footbridge. The plots of the vertical displacement and acceleration in the frequency domain (see Figure 6.25(c) and 6.25(d)) show several peaks at n^{th} harmonics of the walking frequency. This is typical for non-resonant transient behaviour.



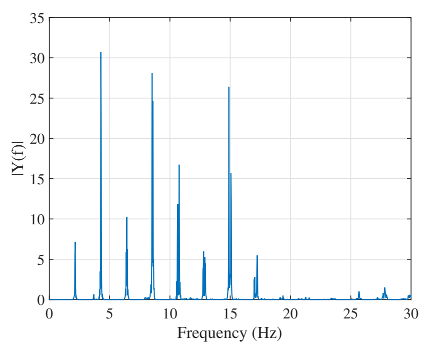
(a) *Vertical acceleration due to a group crossing the bridge along the middle*



(b) *Vertical acceleration due to a group walking in place at the middle of the bridge*



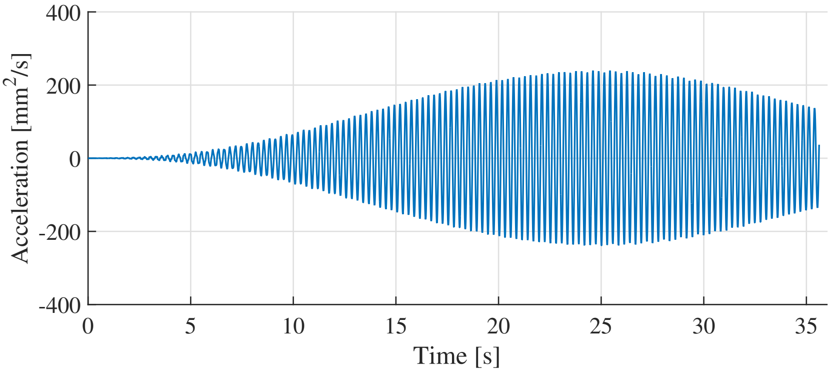
(c) *Vertical displacement in the frequency domain*



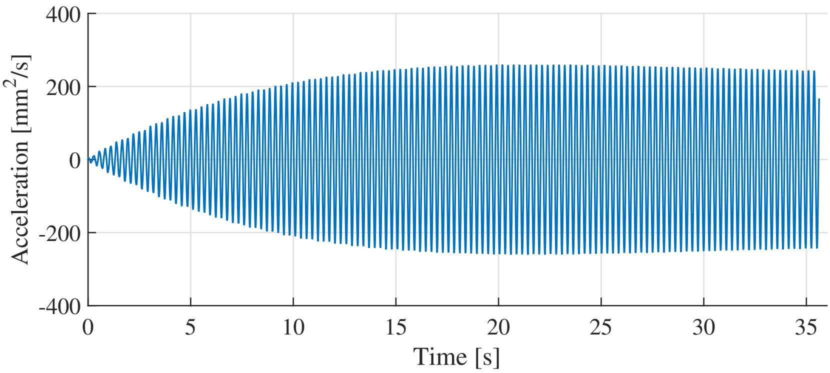
(d) *Vertical acceleration in the frequency domain*

Figure 6.25: *Dynamic responses, extracted at the middle of the deck, due to a group of pedestrians walking at 2.1405 Hz.*

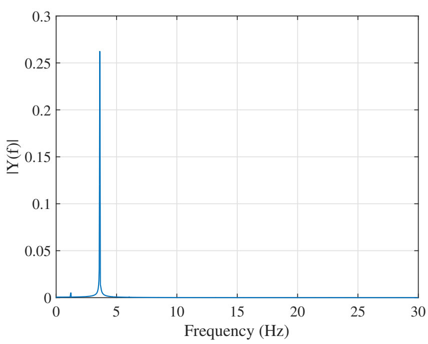
With 12 people walking in phase at 2.4144 Hz the lateral peak accelerations exceed 200 mm/s^2 as shown in Figure 6.26(a) and 6.26(b). By comparing the latter figure to the typical resonant response curve (Figure 3.13(a) it is obvious that the response is resonant, as was the case with a single pedestrian walking at the same frequency. The Fourier transforms in Figure 6.26(c) and 6.26(d) confirm this by showing that only one frequency dominates the response.



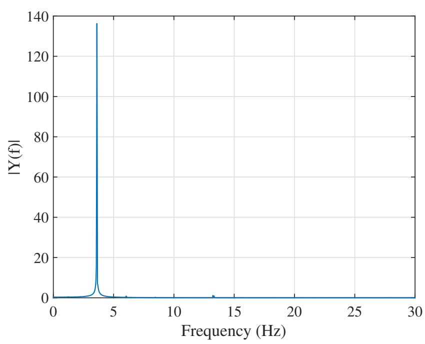
(a) *Lateral acceleration due to a group crossing the bridge along the middle*



(b) *Lateral acceleration due to a group walking in place at the middle of the bridge*



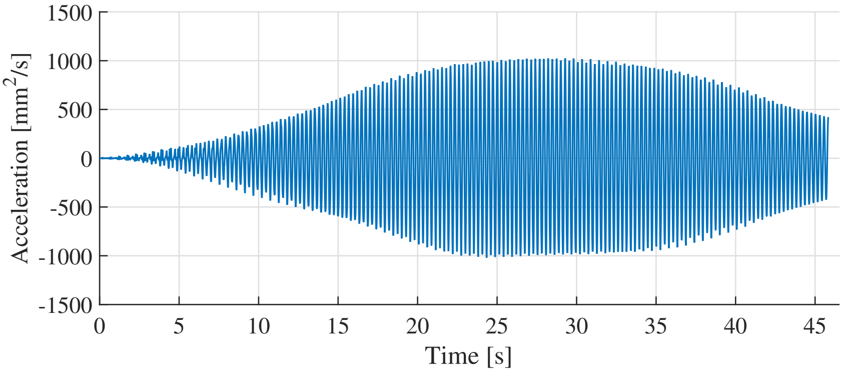
(c) *Lateral displacement in the frequency domain*



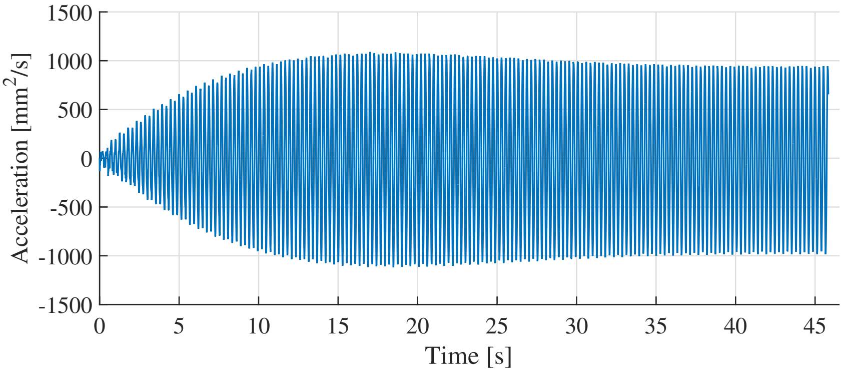
(d) *Lateral acceleration in the frequency domain*

Figure 6.26: *Dynamic responses, extracted at the middle of the deck, due to a group of pedestrians walking at 2.4144 Hz.*

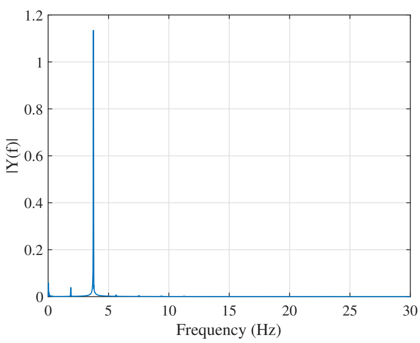
By modelling the same group of people walking at 1.8764 Hz, equal to half of the natural frequency of mode 6, great response amplitudes were obtained. Peak accelerations of 1000 mm/s^2 are observed in Figure 6.27(a) and 6.27(b) and it is clear from this that resonance has occurred. A small decrease in the acceleration peaks can be seen from 6.27(b), which implies that a somewhat higher response would have been obtained if the second harmonic of the walking frequency was tweaked a little to match the resonant frequency even better. This has not been pursued since it is time consuming, and the change in response is expected to be small.



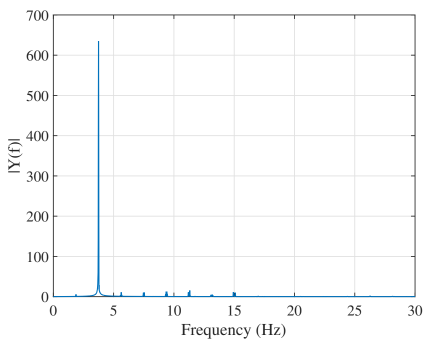
(a) *Vertical acceleration due to a group crossing the bridge aloung the middle*



(b) *Vertical acceleration due to a group walking in place at the middle of the bridge*



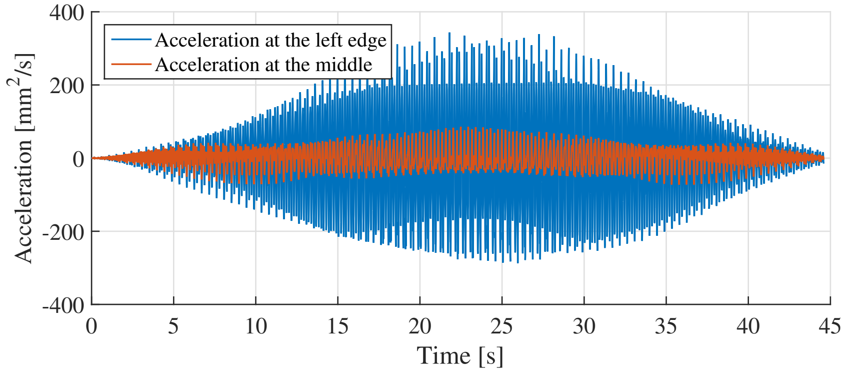
(c) *Vertical displacement in the frequency domain*



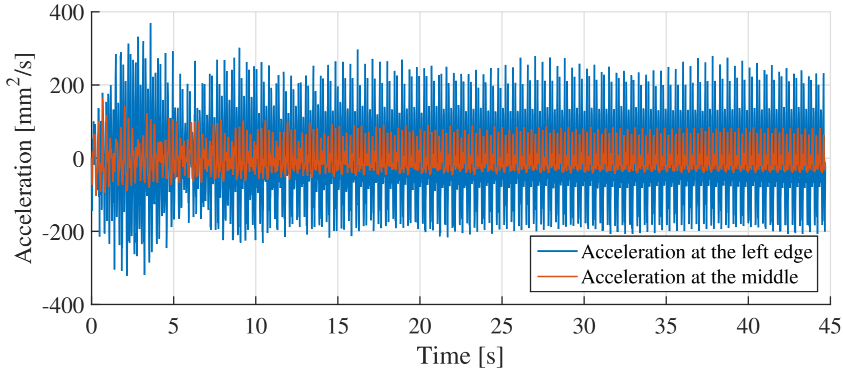
(d) *Vertical acceleration in the frequency domain*

Figure 6.27: *Dynamic responses, extracted at the middle of the deck, due to a group of pedestrians walking at 1.8764 Hz.*

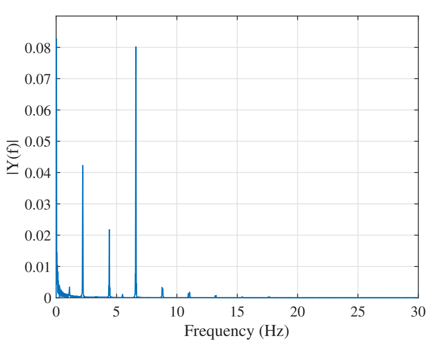
As mentioned at the beginning of this section, attempts were also made to cause resonance by exciting mode 10 through the 3^{rd} harmonic of the walking frequency. A single row of 12 people were therefore modelled to walk along the left edge of the bridge at 2.2012 Hz, and the results from these analyses are shown in Figure 6.28. In figure (a) and (b) both the acceleration at the left edge and at the middle are shown in the same plots. As with the results obtained with only one pedestrian walking at 2.2012 Hz, the vertical accelerations are much greater along the edges than in the middle, which is expected when a torsional mode is successfully excited. The response is fairly large, but resonance is not obtained, which is confirmed by both the large transient responses in Figure 6.28(a) and 6.28(b) and by the presence of many smaller peaks in the Fourier transforms in Figure 6.28(c) and 6.28(d).



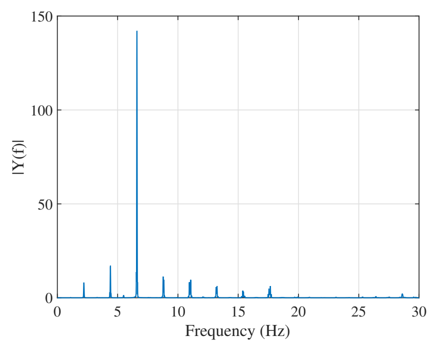
(a) *Vertical accelerations due to a row of pedestrians crossing the bridge along the left edge*



(b) *Vertical accelerations due to a row of pedestrians walking in place at the left edge at the middle of the bridge*



(c) *Vertical displacement in the frequency domain*

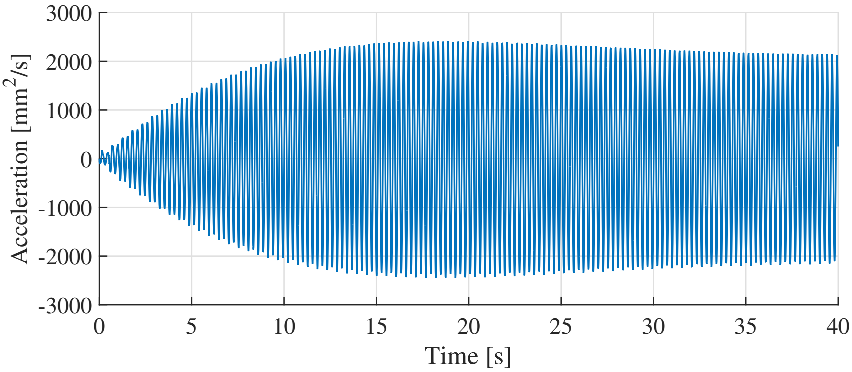


(d) *Vertical acceleration in the frequency domain*

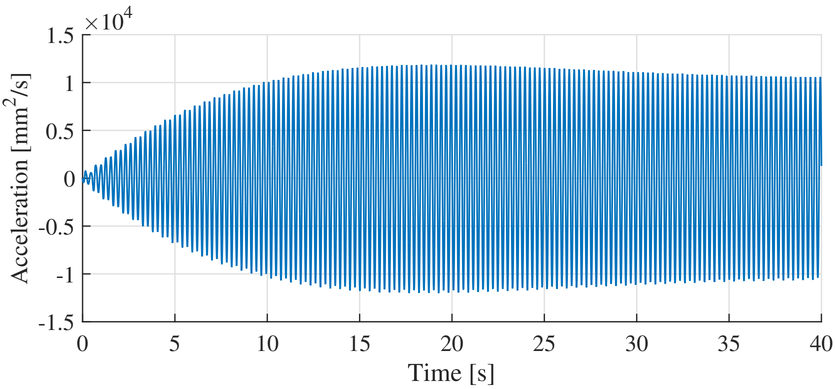
Figure 6.28: *Dynamic responses, extracted at the left edge at the middle of the deck, due to a row of pedestrians walking at 2.2012 Hz.*

Eurocode 5 states that the accelerations due a continuous stream of people walking over the bridge should also be calculated using a given formula shown in Appendix D [43]. To be able to evaluate the calculated acceleration value, two analyses with streams of people were executed. The results in Figure 6.29(a) are from an analysis with one continuous row of people walking across the middle of the bridge at 1.8764 Hz. With a stride length of 750 mm this gives room for 40 people on the bridge at any given time. Figure 6.29(b) shows a similar analysis, but with 5 rows, with 36 pedestrians per row, giving a total of 180 pedestrians on the bridge. This number was chosen to match the number of people in the formula used in EC 5-2 (see Subsection 4.1.1 or Appendix A). Both analyses were modelled as if the people were walking in place, a stride length apart. This is the equivalent of a continuous stream of people crossing the bridge, the only difference being that the people do not walk on to the bridge during the start of the analyses, they are already spread out along the entire length of the bridge when the simulation begins. Simulating it this way ensures that the steady state solution due to the final amount of people is obtained quicker.

The exact same behaviour is observed in both Figure 6.29(a) and 6.29(b), only difference being the magnitude of the accelerations, which is greater in the latter results, as expected. The same results were also obtained with 1 and 12 people instead, see Figure 6.13(a) and 6.27(a), so the same conclusion can be drawn; resonance with the sixth natural frequency occurs when pedestrians are walking at 1.8764 Hz. The results from the analysis with increasing amount of pedestrians could be used to investigate how the response increases with an increasing amount of pedestrians. However, since the analysis with 12, 40 and 180 people are very conservative by assuming that all people walk in phase over the deck, the ratio would not be very realistic and it has therefore not been done.



(a) *Vertical acceleration due to one continuous stream of pedestrians walking along the middle of the bridge*

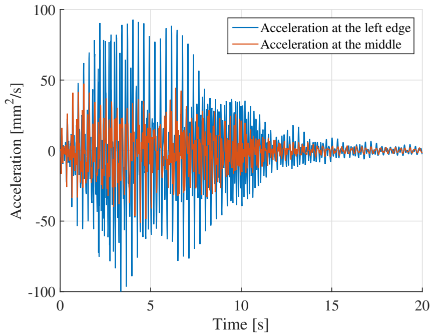


(b) *Vertical acceleration due to five continuous streams of pedestrians walking along the middle of the bridge*

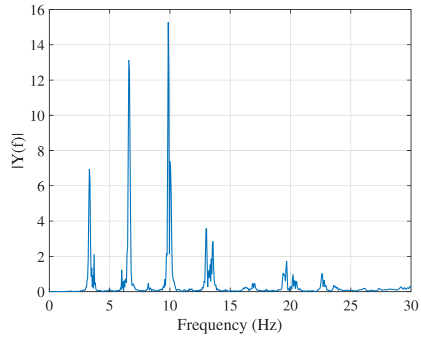
Figure 6.29: *Dynamic responses, extracted at the middle of the deck, due to continuous streams of pedestrians walking at 1.8764 Hz.*

6.3 Running

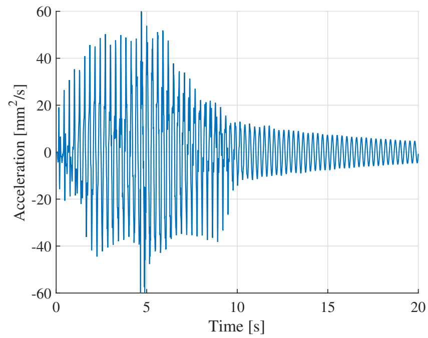
When a pedestrian runs the vertical force he or she exerts increases drastically compared to normal walking, by a factor of more than two. The way runners affect the structural response of slender structures should therefore be considered separately. The frequency range for jogging and running has been proposed to be 2.0 - 3.5 Hz by Bachmann et al. [18]. All natural frequencies within this range should be tested. For the subject footbridge mode 6, the dominating vertical mode, lies just outside this range, but has been tested regardless. The horizontal forces due to running are not mentioned in any found literature, but are assumed similar to those of horizontal walking frequencies, and have therefore been neglected in the analyses with runners. This means that horizontal natural frequencies are of little interest overall when running is considered, so no attempts have been made to excite mode 5, the lateral mode of the bridge. Some of the frequencies in the range of the second running harmonic (4 - 7 Hz) have also been tested to obtain a foundation for comparing different results. It only takes a runner about 10 seconds to cross the bridge, depending on stride length and pacing frequency. The stride lengths chosen for the different analyses were based on Table 3.1 - Pedestrians jogging at frequencies between 2.0 and 3.0 Hz were given a stride length of 1.30 m, while 1.75 m was used for pacing rates higher than 3.0 Hz. Running pedestrians will usually not be disturbed by the acceleration response of the structure because of the short duration the runner is on the bridge. However, other pedestrians on the bridge could experience discomfort. All analyses with running pedestrians lasted for 20 s to investigate the structural response also after the runner was off the bridge.



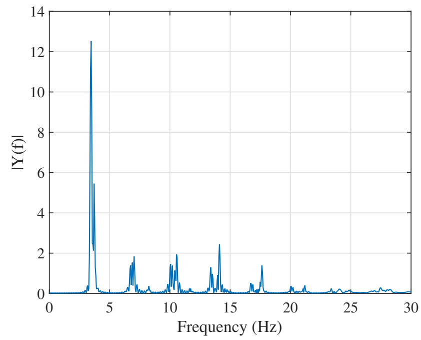
(a) *Vertical accelerations due to a pedestrian running along the left edge at 3.3217 Hz (mode 3)*



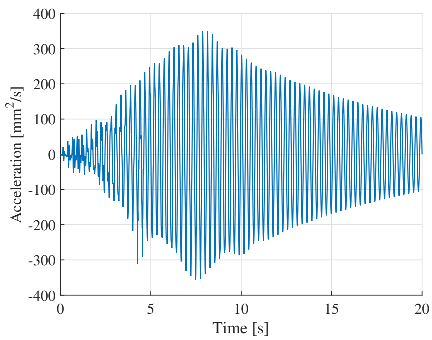
(b) *Vertical acceleration in the frequency domain due to running at 3.3217 Hz*



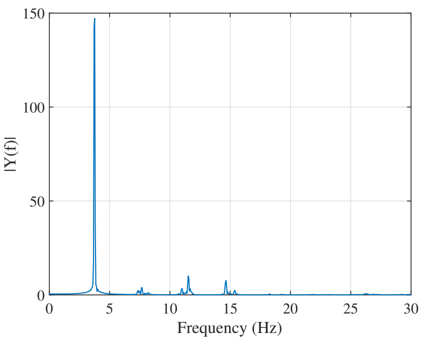
(c) *Vertical acceleration due to a pedestrian running along the middle at 3.4370 Hz (mode 4)*



(d) *Vertical acceleration in the frequency domain due to running at 3.4370 Hz*

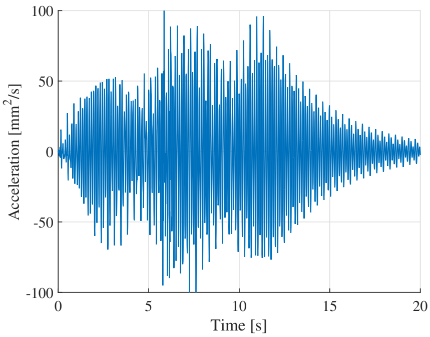


(e) *Vertical acceleration due to a pedestrian running along the middle at 3.7528 Hz (mode 6)*

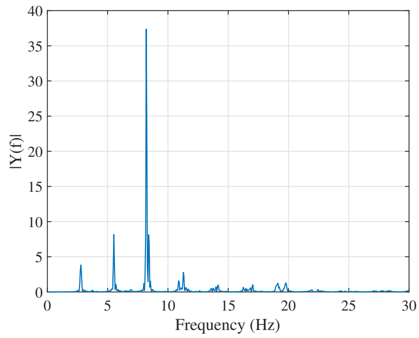


(f) *Vertical acceleration in the frequency domain due to running at 3.7528 Hz*

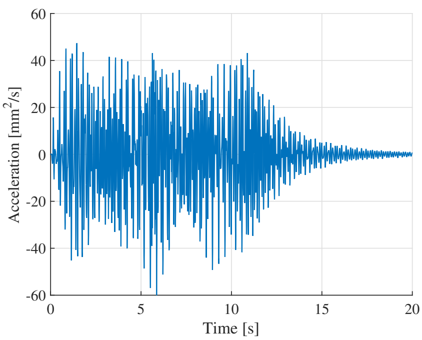
Figure 6.30: *Dynamic responses, extracted at the middle of the deck, due to a running pedestrian attempting to excite mode 3, 4 and 6, respectively.*



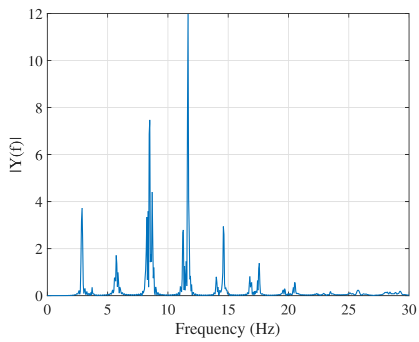
(a) *Vertical acceleration due to a pedestrian running along the left edge at 2.7743 Hz (mode 7)*



(b) *Vertical acceleration in the frequency domain due to running at 2.77423 Hz*

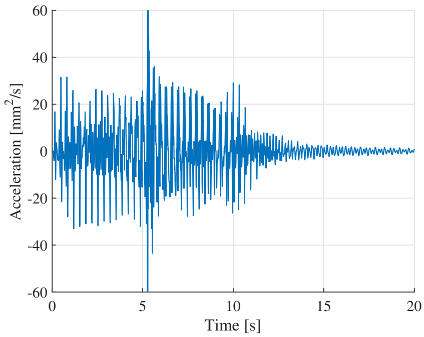


(c) *Vertical acceleration due to a pedestrian running along the middle at 2.8641 Hz (mode 8)*

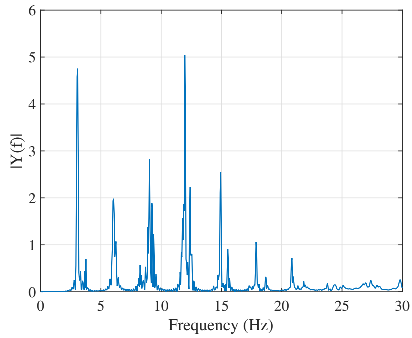


(d) *Vertical acceleration in the frequency domain due to running at 2.8641 Hz*

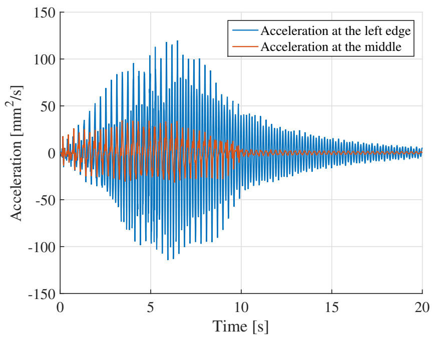
Figure 6.31: *Dynamic responses, extracted at the middle of the deck, due to a running pedestrian attempting to excite mode 7 and 8, respectively.*



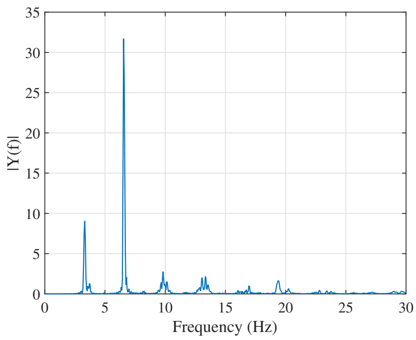
(a) *Vertical acceleration due to a pedestrian running along the middle at 3.0479 Hz (mode 9)*



(b) *Vertical acceleration in the frequency domain due to running at 3.0479 Hz*



(c) *Vertical accelerations due to a pedestrian running along the left edge at 3.3018 Hz (mode 10)*



(d) *Vertical acceleration in the frequency domain due to running at 3.3018 Hz*

Figure 6.32: *Dynamic responses, extracted at the middle of the deck, due to a running pedestrian attempting to excite mode 9 and 10, respectively.*

The results in Figure 6.30, 6.31 and 6.30, showing the vertical accelerations due to pedestrians running at different frequencies, are in accordance with what was expected based on the analysis with walking pedestrians. Mode 6, the vertical mode, is the dominating mode of the structure. As seen in Figure 6.30(e) the acceleration amplitudes caused by exciting the sixth mode are 3.5 - 10 times greater than the acceleration amplitudes from any of the other analysis. It is also the only analysis result where there is almost no transient response. The corresponding Fourier transform shown in 6.30(f) reveals only one peak and hereby confirms that resonance has occurred by modelling a pedestrian running at 3.7528 Hz. This frequency is outside the normal range of running stated by Bachmann et al., but not more than that it could be achieved by a very fast runner, so it was been included.

Other interesting results are shown in Figure 6.32(c), where mode 10 has been excited by means of the 2^{nd} harmonic of the running frequency (3.3018 Hz). Both the difference in acceleration between the left edge and the middle, and the Fourier transform of the acceleration in Figure 6.32(d) confirm that the deck has a torsional motion dominated by a frequency close to the natural frequency of mode 10. However, a transient response, identified by the great decrease in peak amplitude between every footstep, is dominating the motion, and as with the walking analyses resonance with the tenth natural frequency was not obtained. At first glance it looks like mode 3 has also been excited, see Figure 6.30(a). But by taking a closer look at the corresponding Fourier transform in Figure 6.30(b) it can be seen that it is the 2^{nd} harmonic, a frequency in the range of the tenth natural frequency, that is dominating instead. Thus, neither mode 3 nor 7, the two local torsional modes, have been excited by running pedestrians.

6.4 Damping

When the loading is harmonic, which often is the case as all periodic loads can be represented using a Fourier series, the spring and inertia forces for a SDOF are given as shown in Eq. 6.1:

$$
\begin{aligned}
f_i &= m\ddot{u} = -\omega^2 mu = -\beta^2\omega_n^2 mu \\
f_s &= ku = \omega_n^2 mu
\end{aligned}
\tag{6.1}
$$

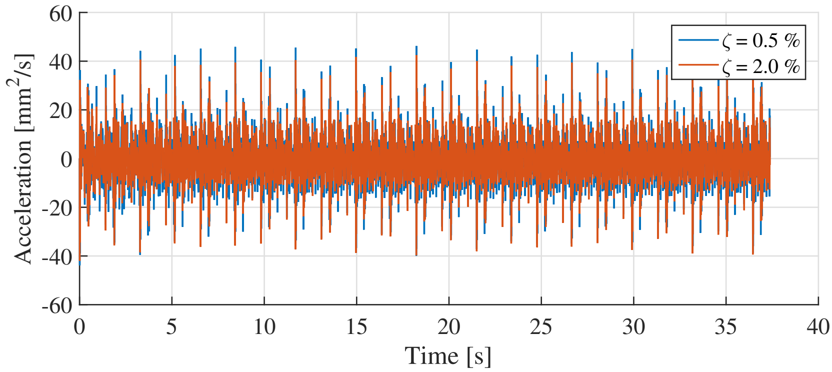
whereβ is the frequency ratio. When resonance occurs $\beta \Rightarrow 1$, the stiffness and inertia forces cancel each other out and the damping forces have to equal the external forces. The amount of damping therefore becomes paramount for the resonant behaviour of a structure. To confirm this, analyses with increased amount of material damping were conducted. According to Eurocode 1-4 [56], a structural damping of 1.9% can be expected. For simplicity, the analyses with increased damping were executed with 2% damping for mode 1 and 10, exactly four times as much as for all previous analyses. This implied multiplying the Rayleigh coefficients by a factor of 4, giving α = 0.368364 and β = 0.000750. Remember that damping for each mode is calculated using Equation 5.6.

Walking in place

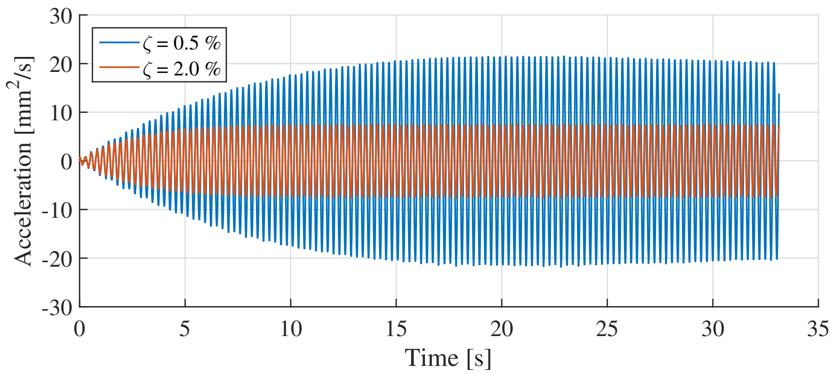
Figure 6.33 shows results from analyses with single pedestrians walking in place at the middle of the deck, trying to excite mode 2, 5 and 6, respectively. The effect of increased damping can be observed by comparing the red and blue curves in each of the figures. As expected the effect is most obvious in Figure 6.33(c) and 6.33(b), the cases were resonance has occurred. With increased damping the steady state solution is obtained after only 5 seconds, whereas this takes up to 35 seconds in the cases with minimal damping.

Walking across

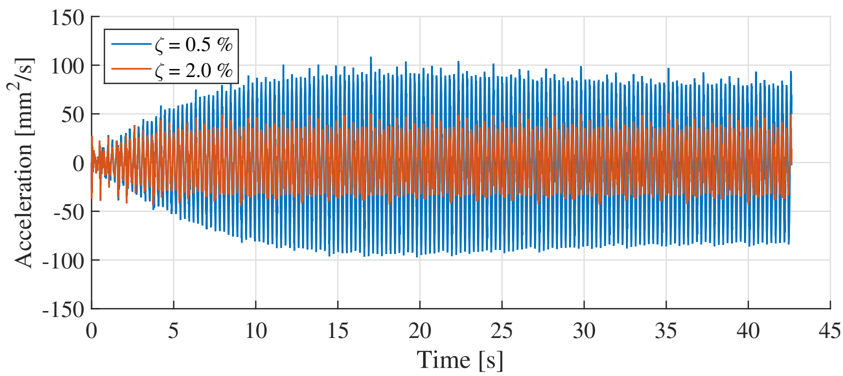
Results from analyses with a single pedestrian walking across the bridge are shown in Figure 6.34. Once again the walking frequency in the different analyses was such that mode 2, 5 and 6 were excited, respectively. As in Figure 6.33 the amount of damping has more impact on the response at resonance, seen by comparing the relative difference between the blue and red curves in Figure 6.34(c) and 6.34(b) against Figure 6.34(a).



(a) *Vertical accelerations due to pedestrian walking in place at the middle of the deck at 2.1405 Hz*

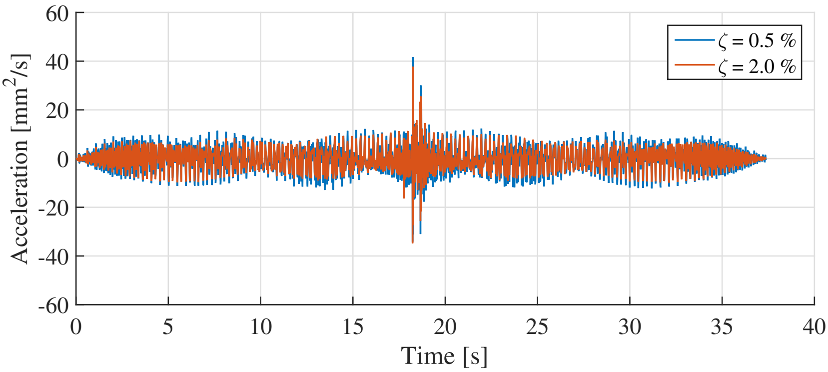


(b) *Lateral accelerations due to pedestrian walking in place at the middle of the deck at 2.4144 Hz*

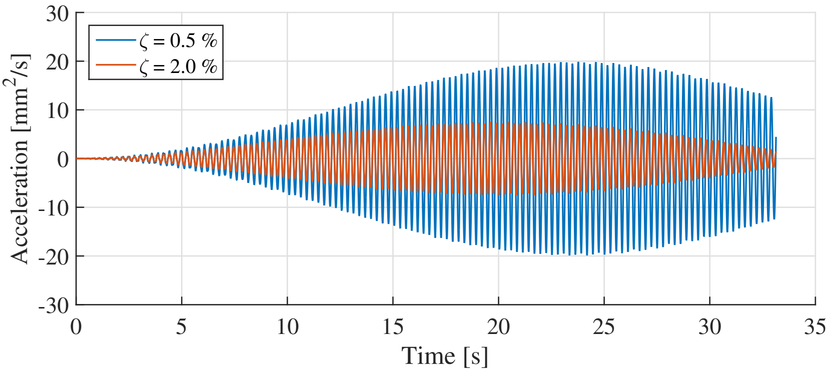


(c) *Vertical accelerations due to pedestrian walking in place at the middle of the deck at 1.8764 Hz*

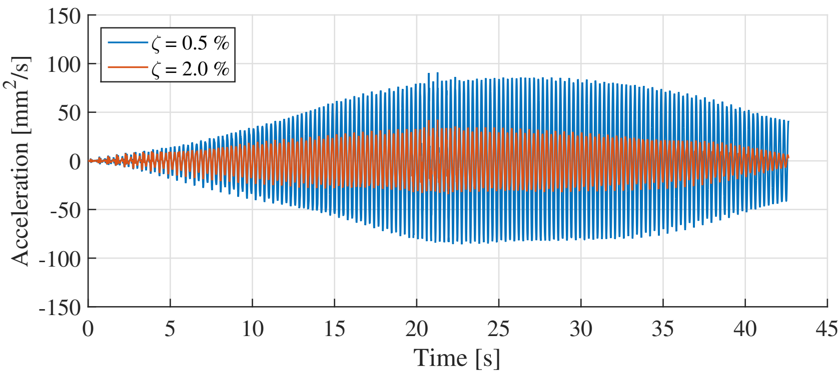
Figure 6.33: *Dynamic responses, extracted at the middle of the deck, due to a single pedestrian, with two different amounts of material damping implemented.*



(a) *Vertical accelerations due to a pedestrian walking across the middle of the bridge at 2.1405 Hz*



(b) *Lateral accelerations due to a pedestrian walking across the middle of the bridge at 2.4144 Hz*



(c) *Vertical accelerations due to a pedestrian walking across the middle of the bridge at 1.8764 Hz*

Figure 6.34: *Dynamic responses, extracted at the middle of the deck, due to a single pedestrian, with two different amounts of material damping implemented.*

7 Discussion of Results

In this chapter the results from Chapter 6 are evaluated and compared, both against each other and against the acceleration values suggested in the different design guidelines mentioned in Chapter 4. The calculations of the suggested acceleration values are given in Appendix D. The peak accelerations given in the various tables hereunder are the maximum absolute value of the accelerations obtained during the different analyses. This implies that the distinct acceleration peaks that might not be physical are still included, and that the peak acceleration values therefore are of less interest. A more sound measure of acceleration is therefore the root mean square acceleration (a_{RMS}), mentioned in Subsection 3.2.2. The time interval over which to integrate the accelerations has been chosen as T = 1 s, which is the recommended value in both ISO 10137 and ISO 2631-1 [47], [41]. The starting point of the time interval, t_1, is set equal to the time when the peak acceleration is obtained. The RMS-algorithm in *MATLAB* was used to calculate the root mean square accelerations, as shown in Appendix A.3.

7.1 Single Walking Pedestrian

7.1.1 Response of the Deck

NB! The acceleration for mode 5 is in lateral direction, whereas all the other accelerations are in vertical direction. This is simply because mode 5 is the only lateral mode. All other modes are vertical and torsional.

Attempts were made to excite the ten first modes of the bridge one at the time, through either the fundamental frequency or the 2^{nd} or 3^{rd} harmonic of the walking frequency, depending on the corresponding natural frequency. The criteria was that the walking frequency had to be in the range of 1.6 - 2.4 Hz, established by Bachmann & Ammann [14]. One small exception was made here by modelling a pedestrian with walking frequency 2.4144 Hz to try to cause resonance with the fifth natural frequency. Peak accelerations and RMS-accelerations for the different analyses are shown in Table 7.1 together with the walking frequency of the pedestrian.

By studying the figures in Section 6.1 and taking into account both the shape of the curves, the magnitude of the accelerations, the actual mode and the Fourier transforms, responses approaching resonance can be observed for mode 1, mode 5 (second attempt), mode 6 and mode 7. For mode 1 and 7 it can be seen from the Fourier transforms, showing the responses in the frequency domain, that the dominating frequency

Mode nr.	f_p [Hz]	Walking across		Walking in place	
		a_{peak} [mm/s^2]	a_{RMS} [mm/s^2]	a_{peak} [mm/s^2]	a_{RMS} [mm/s^2]
1	1.8838	75.66	40.68	86.18	45.27
2	2.1405	41.68	9.54	46.60	11.29
3	1.6609	41.68	16.53	25.41	12.18
4	1.7185	23.48	7.02	32.16	6.12
5	1.8108	0.83	0.37	5.61	2.31
5	2.4144	19.97	14.11	22.89	15.37
6	1.8764	91.02	59.11	108.60	63.09
7	1.8496	70.38	42.09	100.85	44.59
8	1.9094	36.61	17.32	63.48	27.42
9	2.0319	49.47	22.70	57.82	25.98
10	2.2012	47.14	18.22	57.59	24.46

Table 7.1: *Peak and RMS-accelerations from all analyses with a single walking pedestrian.*

is the sixth natural frequency, not the natural frequency of mode 1 and 7. In both cases the 2nd harmonic of the walking frequency has thus dominated the response instead of the 1st and 3rd, respectively. The results for mode 1 and 7 can therefore be directly compared to the results for mode 6 in Table 7.1, where it is seen that the results for mode 6 have greater responses and are therefore the only results discussed any further in this chapter. Note that the RMS-accelerations are almost equal for the analyses with the pedestrian walking across the bridge and the pedestrian walking in place at the middle. This implies that the position of the pedestrian along the deck is not crucial as long as he or she is closer to the middle than to the supports.

The analyses with walking frequency 2.4144 Hz, exciting the fifth mode, show the clearest resonance response with the amplitude of the response reaching its maximum at steady state. The given accelerations are small compared to the results from mode 6 because these are lateral acceleration values. However, lateral accelerations cause discomfort for pedestrians at a much lower magnitude, as illustrated through the different comfort criteria, and they are therefore not directly comparable to the vertical accelerations. Both the peak and RMS-accelerations are almost the same for both analyses and it can be seen from Figure 6.11(a) that the peak is reached after about 24 s, which is long after the pedestrian has passed the middle of the bridge, once again implying that the position of the pedestrian along the length of the deck is not that important.

The final modes of interest are mode 9 and 10. By simulating a single pedestrian walking at 2.0319 Hz and 2.2012 Hz it can be seen from the Fourier transforms of the respective accelerations (Figure 6.19(d) and 6.21(d)) that the 3rd harmonics of the walking frequency, equal to the natural frequencies of the two modes, are the dominating frequencies in the responses. Resonance however, has not occurred, which can be seen by studying Figure 6.19(b) and 6.21(b) and noticing the transient responses and the small response amplitudes. It seems that resonance at the higher frequencies can not occur, even if the correct frequencies are present in the forcing. This backs up the general assumption that footbridge response is dominated by only a few critical modes [11], in this case mode 5 and 6 for the deck. The lateral responses of the arches however are dominated by other modes, and this is investigated in the following section.

7.1.2 Response of the Arches

Although a person walking at 1.8838 Hz (the natural frequency of the first mode) was not able to excite the deck into the corresponding torsional mode shape, the activity did cause resonance in the arches. Large acceleration amplitudes with a peak acceleration of almost 100 mm/s^2 can be seen in Figure 6.23(b). However, with the arches, unlike the deck, it is not the accelerations that will be perceived by the pedestrian, but the displacement. This is because the pedestrian are not in physical contact with the arches, so the acceleration is not felt, but the displacement can be seen. The sight of arches swaying with large displacement amplitudes can cause discomfort and should therefore be avoided. However, the largest displacement amplitude obtained for the arches was about 0.75 mm, which is not perceivable. It should be noted that the results in Figure 6.23(a) and 6.24(a) have not reached steady state yet, and simulations over bigger time periods should have been performed. Still, the slope of the envelope curve is approaching zero, implying that the steady state solution would've been obtained soon after and that any additional increase in amplitude response would be negligible. The conclusion is that the arches won't pose a problem.

7.2 Groups of People

Mode nr.	f_p [Hz]	N	Walking across		Walking in place	
			a_{peak} [mm/s^2]	a_{RMS} [mm/s^2]	a_{peak} [mm/s^2]	a_{RMS} [mm/s^2]
2	2.1405	12	138.41	58.08	211.66	89.31
5	2.4144	12	240.17	169.68	266.07	185.93
6	1.8764	12	1023.99	712.87	1118.92	752.25
10	2.2012	12	343.60	177.46	369.24	152.12
6	1.8764	40	-	-	2448.68	1708.6
6	1.8764	180	-	-	12010.0	8377.10

Table 7.2: *Peak and RMS-accelerations from all group analyses.*

In Table 7.2 characteristic acceleration values for the different group analyses with number of people, N, and walking frequency, f_p are shown together with the mode that was attempted to excite. Based on the results in Figure 6.26(b) and 6.27(b) and the RMS-accelerations, it is apparent that resonance occurred in the analysis with 12 people for mode 5 and 6 and so the maximum dynamic response was obtained. The accelerations for mode 5 are lateral, and were greater in magnitude than the vertical accelerations for mode 2 and 10. This is impressive considering how small the lateral force components are compared to the vertical. From the Fourier transform of the accelerations from the analysis with walking frequency 2.1405 Hz (see Figure 6.25(d)), it is obvious that neither the right mode has been excited nor has resonance occurred. The attempt to excite mode 10 was once again partially successful. The dominating frequency in the Fourier transform seems to be the correct frequency, namely 6.6034 Hz, but a transient response is dominating, indicating that resonance is not the case.

The two latter analyses in Table 7.2 simulate 1 and 5 streams of people crossing the bridge, respectively. They were given a walking frequency equal to half the natural frequency of the sixth mode, the global vertical mode. This was done to compare the obtained accelerations to estimated values of acceleration given in the different guidelines due to groups of people. Analyses with streams of people causing resonance in the lateral direction were not conducted because the horizontal natural frequency of the bridge was outside the ranges given in the guidelines (see Section 7.5).

By comparing the analyses with different amounts of people (1, 12, 40 and 180 people) and pacing rate 1.8764 Hz, an almost linear relation can be observed between number of people and RMS-acceleration. Easily seen by calculating the ratio between a_{RMS} and N, a linear relation would render a constant ratio. This is extremely conservative and suggests that the simulation is less valid for several pedestrians. In reality people will never walk perfectly in phase, as has been modelled here, and accelerations are therefore expected to be much lower in reality.

7.3 Running

Mode nr.	$\mathbf{f_p}$ **[Hz]**	$\mathbf{a_{peak}}$ **[mm/s²]**	$\mathbf{a_{RMS}}$ **[mm/s²]**	$\mathbf{a_{RMS}}$ **10 s after walking off [mm/s²]**
3	3.3217	100.66	49.71	1.84
4	3.4370	64.57	28.30	3.43
6	3.7528	356.85	229.46	77.30
7	2.7743	101.53	50.30	5.18
8	2.8641	67.15	22.74	0.70
9	3.0479	101.84	18.17	0.68
10	3.3018	119.72	65.42	4.03

Table 7.3: *Vertical peak and RMS-accelerations from all analyses with running pedestrians.*

From the analyses with a single running pedestrian, three characteristic acceleration values were extracted. In addition to the peak acceleration the RMS-acceleration was calculated both at the peak and 10 s after the pedestrian had left the bridge. The latter RMS-values were to examine whether the dynamic response induced by the runner could have any effect on pedestrians still on the bridge. As seen in Table 7.3, the greatest accelerations are obtained when mode 6 is excited. Mode 6 also has the only Fourier transform with only one peak, indicating resonance. The accelerations for this mode are quite large, keeping in mind that they are induced by a single person. It should also be noted that the steady state solution did not have time to settle, and that the peak acceleration kept increasing until the runner ran of the bridge at around 10 seconds. The reoccurring round shapes in the acceleration response are due to decrease in acceleration when there is no force on the deck since both feet are off the ground.

Mode 3, 7 and 10 were the torsional modes with natural frequency in the range of either the fundamental running frequency or the 2nd harmonic of the running frequency. Even though none of the torsional mode-plots display the smooth resonance curve, accelerations extracted at the edges and middle of deck reveal that some torsional motion of the deck is present for the analysis attempting to excite mode 3 and 10. This is observable in Figure 6.30(a) and 6.32(c), where the vertical acceleration at the middle of the deck is of noticeably smaller in magnitude than the vertical accelerations retrieved at the edge. The corresponding plots of the accelerations in the frequency domain (See Figure 6.30(b) and 6.32(c)) show that the natural frequency of mode 10 is present in both acceleration responses. The former also contains the fundamental running frequency in addition to the 3rd harmonic, implying that the response is a combination of several mode shapes. The attempt to excite mode 7 was unsuccessful with the accelerations at the middle being equal to the accelerations at the edges. Also, the Fourier transform (see Figure 6.31(b)) was dominated by the third harmonic, implying that a high frequency mode was somehow excited. The conclusion from this is that mode 10, the global torsional mode, is the first torsional mode that was successfully excited. However, as addressed in the previous section about single walking pedestrian analyses, it seems impossible to obtain resonance with frequencies higher than the sixth natural frequency of the structure.

Modes 4 and 8 are local vertical modes, which explain the low accelerations. The deformation of the deck is simply too small, making it impossible to cause the deck to resonate with these mode shapes, because higher harmonics of the forcing function will excite other modes instead.

7.4 Damping

Table 7.4 presents the peak and RMS-accelerations from analyses attempting to excite mode 2, 5 and 6 with normal and high damping (0.5% and 2%). As explained in Section 6.4, the increased damping should have a significantly greater effect on the response in the analyses where resonance occurs, which is the case for mode 5 and 6. The results from mode 2 are included to demonstrate just this. As shown in Table 7.4 the increase in damping leads to a 8.4% decrease in RMS-acceleration for the walking in place scenario when trying to excite mode 2, as opposed to 65.4% and 63.4% for the same analysis exciting mode 5 and 6.

In addition to confirming that resonance really has occurred for the analysis with walking frequency 1.8764 Hz and 2.4144 Hz, these results show the importance of damping

Mode nr.	$\mathbf{f_p}$ **[Hz]**	ζ **[%]**	**Walking across**		**Walking in place**	
			$\mathbf{a_{peak}}$ **[mm/s^2]**	$\mathbf{a_{RMS}}$ **[mm/s^2]**	$\mathbf{a_{peak}}$ **[mm/s^2]**	$\mathbf{a_{RMS}}$ **[mm/s^2]**
2	2.1405	0.5	41.68	9.54	46.60	11.29
2	2.1405	2.0	37.72	8.29	42.47	10.34
Percentage deviance [%]			9.5	13.1	8.9	8.4
5	2.4144	0.5	19.97	14.11	22.89	15.37
5	2.4144	2.0	7.73	4.99	8.26	5.32
Percentage deviance [%]			61.3	64.6	63.9	65.4
6	1.8764	0.5	91.02	59.11	108.60	63.09
6	1.8764	2.0	42.30	21.89	51.02	23.10
Percentage deviance [%]			53.5	63.0	53.0	63.4

Table 7.4: *Peak and RMS-accelerations from all analyses executed with different amounts of damping.*

when resonance is possible due to low natural frequencies. Although not pursued here, analyses with different amounts of damping could be conducted to determine the minimum amount of damping necessary to satisfy the comfort criteria.

7.5 Comparison to Design Guidelines

The last section of this chapter investigates how the numerical results hold up to the regulations stated in the design guidelines. All the calculations for the different guidelines are shown in Appendix D.

Vertical vibrations

Table 7.5 shows the highest vertical RMS-accelerations obtained with the different loading scenarios, and compares these to the acceleration values calculated with the proposed method in each guideline. The bottom row features the maximum limit acceleration of each guideline, which would be the determining criteria if one used the guidelines as basis for a dynamic study. Note that only Eurocode distinguishes between different activities and different numbers of pedestrians on the bridge. HB-N400 and BS5400 propose only one single way of calculating the probable acceleration in a bridge, regardless of the activity or pedestrian density.

Two guidelines, BS-NA and Sétra, propose a range of permitted accelerations rather than specific values. After hand calculations the criteria in BS-NA ranges from 0.5 - 1.747 m/s^2, whereas Sétra states the rather liberal range of 0.5 - 2.5 m/s^2, depending on the bridges comfort class. Both Sétra and BS-NA present a rather arduous method for calculating the accelerations in the deck (see Subsection 4.1.6 and 4.1.2), so they have not been included here.

By comparing the accelerations calculated with the guideline methods with the accelerations in the numerical model it is seen that the correlation varies. Eurocode gives a reasonable value for the acceleration induced by a single pedestrian, whereas the other guidelines have much higher values (which makes sense because their values need to cover all activities). For running Eurocode gives a very conservative estimate, more than twice as high as the numerical result. However, the value in Eurocode is probably based on the assumption that the steady state solution is reached, which was not the case in the running analysis giving the greatest accelerations (Figure 6.30(e)). Had the bridge been longer, giving steady state response time to settle, it is believed that the Eurocode would have given a better correlation. The accelerations for a group of people is much higher than all calculated values, and the stream of people analysis yields accelerations completely out of bounds for all guidelines. This confirms yet again that the analyses with groups and streams of people are very conservative and not very realistic.

Activity	**Numerical model**	**Eurocode 5**	**Handbook N400**	**BS 5400**
Walking	0.063	0.080	0.175	0.175
Running	0.229	0.480	0.175	0.175
Group of people	0.752	0.120	0.175	0.175
Stream of people	8.377	1.656	0.175	0.175
CRITERIA	-	0.700	0.698	0.969

Table 7.5: *Numerical acceleration values compared to guidelines.*

The numerical results are mainly within the limits of the different criteria. The first three activities are within the limits set forth by BS 5400, and are just barely outside the limits for HB-N400 and EC5. The ranges in Sétra and BS-NA are dependent on many factors such as frequency of use and surroundings, so it's hard to say whether or not the numerical RMS-accelerations are all within their ranges. They would be within if the whole range was included.

Horizontal vibrations

Sétra and Eurocode are the only guidelines that provide a limit to horizontal accelerations independent of mode frequencies. The design guidelines with horizontal limits dependent of frequency all state that lateral vibrations will not be a problem due to the relatively high natural frequency of the first lateral mode (3.6216 Hz). Sétra limits the acceleration to 0.10 m/s^2 to avoid lock-in effect, whereas Eurocode limits the horizontal acceleration to 0.2 m/s^2 for normal use and 0.4 m/s^2 for exceptional conditions. As seen in Table 7.6, the numerical model is well within the limit accelerations for a single person. With a group of people the limit acceleration is exceeded according to Sétra. However it's still within the average comfort level defined in Figure 4.6(b) (described as "merely perceptible"), which is in the range 0.15 - 0.3 m/s^2 [7]). According to Eurocode, the numerical model is within limit acceleration. It should also be remembered that the group analyses with the numerical model includes twelve people walking entirely in sync, which is a worst case scenario. The true accelerations in the bridge with twelve people crossing it would almost certainly be lower. Note that the running analyses were performed without the lateral force component because it is assumed equal to that of walking, see 6.3.

Activity	Numerical model	Setra (criteria)	Eurocode 0 (criteria)
Walking	0.015	0.10	0.2
Group of people	0.186	0.10	0.2

Table 7.6: *Numerical horizontal acceleration values compared to Sétra and Eurocode.*

8 Conclusion

Single pedestrian analyses

After studying the results a few important modes stand out, namely mode 1 (first lateral response of the arches), mode 5 (first lateral) and mode 6 (first vertical). Resonant responses of the deck were only achieved by exciting mode 5 and 6, giving lateral and vertical resonance, respectively. None of the torsional modes were excited to resonance. The arches did in fact resonate with the natural frequency of mode 1, but their acceleration will not be felt by the pedestrians. Instead, the sight of them moving might cause discomfort, but their displacement amplitudes were less than 1 mm, which was considered negligible.

The results from the single pedestrian analyses seem reasonable and promising. It is normal for footbridges to have a few modes dominate the response of the structure [11]. The arches are dominated by the first mode, while the deck is dominated by the first global lateral and vertical modes, number 5 and 6, respectively. The response curves due to running are not as smooth as the ones due to walking because the response is damped out a little every time both feet are off the ground. Still, resonance was obtained at one mode, namely mode 6. This shows that with regards to the dynamic behaviour of the structure, there is little difference between running and walking - the dominating mode is excited either way. Being able to simulate resonance like this is an indicator that the method used for applying forces is valid.

Group analyses

The results from analyses with single pedestrian walking and groups of pedestrians (12 people) walking at the same frequency were almost identical, with only the amplitude of the response being greater in the latter analyses. The obtained results from the group analyses are very conservative since all the pedestrians were modelled to walk in phase at the same frequency. This is very unlikely for smaller groups of people and will only happen when lock-in occurs, at which point pedestrians will stop due to discomfort created by the induced vibrations.

Guideline deficiencies

The maximum acceleration response from the single walking pedestrian analyses match the predictions of the acceleration in Eurocode 5-2 well. The other guidelines investigated have poorer predictions, which is expected since they neither take into account the number of pedestrians nor their activity. The RMS-accelerations obtained in the running analyses differ from the values given by Eurocode, the results being lower by a factor of about 2. This is because the steady state response is not reached in the critical running analysis because the time it takes a runner to cross the bridge is too short. The running results match the other guidelines better, but again, these are independent of number of people and activity, so their accuracy is doubted. The group analysis results have no correlation with the predicted acceleration values in the design guidelines and the obtained RMS-accelerations in both lateral and vertical direction exceeded the acceleration limits stated in the Eurocode and Handbook N400.

The formulas stated in the Eurocode are very simplified and don't take into account the length of the bridge at hand. The duration of the pedestrian loading is dependent on the length, and steady state resonance is more likely if the forcing is applied for a longer time. The method implies that a long, slender footbridge would expect the same accelerations as a short, less slender bridge so long as the mass and damping are the same, which is a brute assumption. The way horizontal accelerations are covered in the guidelines is also deficient. The guidelines propose methods within small frequency ranges, meaning that no control check needs to be performed if the relevant horizontal frequency of the bridge is outside that range. This disregards the fact that higher harmonics of a walking frequency might excite the bridge, which was the case here with the lateral mode.

Damping model

The amount of structural damping was proven important for the resonant response of the structure. By increasing the structural damping by a factor of 4, the RMS-accelerations in the single pedestrian analyses experiencing resonance were decreased by an average of 61.0%. There is some uncertainty concerning the validity of the damping model. All materials are given the same Rayleigh coefficients so that the entire structure has the same amount of damping, and this way the damping of each mode is very predictable. In reality it is not that simple. If different coefficients are given to different materials the damping ratio of a mode will depend on the relative influence of the different materials, leading to damping ratios that aren't as easily determined. Some higher modes might then get unfavourably low damping ratios, possibly causing numerical noise. This has not been investigated any further in this thesis. At resonance only the natural frequency is present in the response, which is confirmed by the distinct singular peaks present in the Fourier transform plots. This lead the authors to believe that the resonant responses, which are dimensioning for slender footbridges, would be unaffected by the higher modes. By establishing which modes are critical, it can then be determined how much damping is necessary in these modes to fulfill the comfort criteria.

Final thoughts

The method introduced used in this thesis for calculating the dynamic response of a slender footbridge due to pedestrian loading shows potential. For simulations of single pedestrians the obtained responses are reasonable, and it is reasonable to assume that the mode shapes excited in the numerical model are likely to be excited in reality as well. There is much uncertainty regarding the influence of several pedestrians and how they will interact with each other and the structure. One of the difficulties is that pedestrians can act as both dampers and amplifiers (negative dampers). The way it is done in this thesis is highly conservative, and should be modified by somehow decreasing the degree of synchronization.

The current available methods concerning pedestrian loading seem to be either too simple or too complicated. Some of the design guidelines have very general formulas (Eurocode and Handbook N400), and others, such as Sétra and British Standard have rigorous mathematical approaches similar to the method used in this thesis, which may be too much work for a simple footbridge. The numerical model created for this thesis is believed to have predicted the dynamic response reasonably well. Further development of the script and generalizing it even further (introducing several bridge-structures, various damping models, choices of materials, etc.) may be a valid, new approach for considering the dynamic response of a bridge before it is built. Perhaps a new guideline specifically for slender footbridges needs to be written, and if so, perhaps a numerical study like this can be helpful, either to establish new tabulated limits, or to develop empirical formulas.

9 Further Work

The procedure used in this thesis to calculate the responses due to pedestrian loading shows potential by simulating resonance correctly and also predicting reasonable critical modes. This final chapter gives some pointers on how the procedure might be improved.

- An improved approach for calculating the dynamic response due to groups of people is necessary. One option is to implement the methods stated in Sétra and BS-NA. They both suggest applying a distributed load to the entire deck where the direction of the load is always according to the direction of the vertical displacements of the mode for which responses are being calculated [7], [44].
- Other damping categories should be tested and the alternatives are then to either implement and learn to control structural damping or to perform analyses through modal superposition procedures instead where global and modal damping are possible to implement.
- Modal dynamics is also an interesting alternative approach. With such linear perturbation methods there is no time period and hence the responses are presented in the frequency domain [13]. Modal superposition is only applicable for linear problems, and are in general best suited when the structural response is well captured by a a small number of modes, which is proven to be the case for the footbridge in study.
- An experienced programmer might be able to rewrite parts of the script, making it more compact and efficient.
- A parameter study on the damping coefficients of asphalt should be performed, as it is expected to increase the total damping [57].

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A Appendix A - *MATLAB* Codes

The .txt-files in the different *MATLAB* scripts are obtained by reporting history outputs from *Abaqus*.

A.1 Logarithmic Decay

This script was used to determine the numerical damping of the model based on the cantilever beam. The script reads the time and displacements (reported to 'numdamp<modenumber>Ts<timestep>.txt' in *Abaqus*) from all ten beams and arranges them in two matrices, (tmatrix and umatrix). This is done for all three step sizes; 0.01, 0.02 and 0.05. The plot subsequently calculates the logarithmic decay for each of the ten cases, for each step size, and displays the values in a plot.

```
clear all
clc

set(0,'DefaultAxesFontName','Times New Roman')
set(0,'defaultlinelinewidth',1.3)
set(gca,'fontsize', 18)

%%
format long
D = [1 2 3 4 5 6 7 8 9 10 11 12];
%STEPSIZE 0.01
Input1=textread('numdamp1.txt','','headerlines',3);
Input2=textread('numdamp2.txt','','headerlines',3);
Input3=textread('numdamp3.txt','','headerlines',3);
Input4=textread('numdamp4.txt','','headerlines',3);
Input5=textread('numdamp5.txt','','headerlines',3);
Input6=textread('numdamp6.txt','','headerlines',3);
Input7=textread('numdamp7.txt','','headerlines',3);
Input8=textread('numdamp8.txt','','headerlines',3);
Input9=textread('numdamp9.txt','','headerlines',3);
Input10=textread('numdamp10.txt','','headerlines',3);
Input11=textread('numdamp17.txt','','headerlines',3);
Input12=textread('numdamp28.txt','','headerlines',3);

L1=length(Input1);
L2=length(Input2);
L3=length(Input3);
L4=length(Input4);
L5=length(Input5);
L6=length(Input6);
```

```
L7=length(Input7);
L8=length(Input8);
L9=length(Input9);
L10=length(Input10);
L11=length(Input11);
L12=length(Input12);

Lengths=[L1,L2,L3,L4,L5,L6,L7,L8,L9,L10,L11,L12];
umatrix=zeros(max(Lengths),12);
tmatrix=zeros(max(Lengths),12);

tmatrix(1:L1,1)=Input1(:,1);
tmatrix(1:L2,2)=Input2(:,1);
tmatrix(1:L3,3)=Input3(:,1);
tmatrix(1:L4,4)=Input4(:,1);
tmatrix(1:L5,5)=Input5(:,1);
tmatrix(1:L6,6)=Input6(:,1);
tmatrix(1:L7,7)=Input7(:,1);
tmatrix(1:L8,8)=Input8(:,1);
tmatrix(1:L9,9)=Input9(:,1);
tmatrix(1:L10,10)=Input10(:,1);
tmatrix(1:L11,11)=Input11(:,1);
tmatrix(1:L12,12)=Input12(:,1);

umatrix(1:L1,1)=Input1(:,2);
umatrix(1:L2,2)=Input2(:,2);
umatrix(1:L3,3)=Input3(:,2);
umatrix(1:L4,4)=Input4(:,2);
umatrix(1:L5,5)=Input5(:,2);
umatrix(1:L6,6)=Input6(:,2);
umatrix(1:L7,7)=Input7(:,2);
umatrix(1:L8,8)=Input8(:,2);
umatrix(1:L9,9)=Input9(:,2);
umatrix(1:L10,10)=Input10(:,2);
umatrix(1:L11,11)=Input11(:,2);
umatrix(1:L12,12)=Input12(:,2);

avgdamp = zeros(12,1);
totdamp = zeros(12,1);

%STEPSIZE 0.02
Input12=textread('numdamp1Ts02.txt','','headerlines',3);
Input22=textread('numdamp2Ts02.txt','','headerlines',3);
Input32=textread('numdamp3Ts02.txt','','headerlines',3);
Input42=textread('numdamp4Ts02.txt','','headerlines',3);
Input52=textread('numdamp5Ts02.txt','','headerlines',3);
Input62=textread('numdamp6Ts02.txt','','headerlines',3);
Input72=textread('numdamp7Ts02.txt','','headerlines',3);
Input82=textread('numdamp8Ts02.txt','','headerlines',3);
Input92=textread('numdamp9Ts02.txt','','headerlines',3);
Input102=textread('numdamp10Ts02.txt','','headerlines',3);
Input112=textread('numdamp17Ts02.txt','','headerlines',3);
Input122=textread('numdamp28Ts02.txt','','headerlines',3);

L12=length(Input12);
L22=length(Input22);
L32=length(Input32);
L42=length(Input42);
L52=length(Input52);
```

```
L62=length(Input62);
L72=length(Input72);
L82=length(Input82);
L92=length(Input92);
L102=length(Input102);
L112=length(Input112);
L122=length(Input122);

Lengths2=[L12,L22,L32,L42,L52,L62,L72,L82,L92,L102,L112,L122];
umatrix2=zeros(max(Lengths2),12);
tmatrix2=zeros(max(Lengths2),12);

tmatrix2(1:L12,1)=Input12(:,1);
tmatrix2(1:L22,2)=Input22(:,1);
tmatrix2(1:L32,3)=Input32(:,1);
tmatrix2(1:L42,4)=Input42(:,1);
tmatrix2(1:L52,5)=Input52(:,1);
tmatrix2(1:L62,6)=Input62(:,1);
tmatrix2(1:L72,7)=Input72(:,1);
tmatrix2(1:L82,8)=Input82(:,1);
tmatrix2(1:L92,9)=Input92(:,1);
tmatrix2(1:L102,10)=Input102(:,1);
tmatrix2(1:L112,11)=Input112(:,1);
tmatrix2(1:L122,12)=Input122(:,1);

umatrix2(1:L12,1)=Input12(:,2);
umatrix2(1:L22,2)=Input22(:,2);
umatrix2(1:L32,3)=Input32(:,2);
umatrix2(1:L42,4)=Input42(:,2);
umatrix2(1:L52,5)=Input52(:,2);
umatrix2(1:L62,6)=Input62(:,2);
umatrix2(1:L72,7)=Input72(:,2);
umatrix2(1:L82,8)=Input82(:,2);
umatrix2(1:L92,9)=Input92(:,2);
umatrix2(1:L102,10)=Input102(:,2);
umatrix2(1:L112,11)=Input112(:,2);
umatrix2(1:L122,12)=Input122(:,2);

avgdamp2 = zeros(12,1);
totdamp2 = zeros(12,1);

%STEPSIZE 0.05
Input15=textread('numdamp1Ts05.txt','','headerlines',3);
Input25=textread('numdamp2Ts05.txt','','headerlines',3);
Input35=textread('numdamp3Ts05.txt','','headerlines',3);
Input45=textread('numdamp4Ts05.txt','','headerlines',3);
Input55=textread('numdamp5Ts05.txt','','headerlines',3);
Input65=textread('numdamp6Ts05.txt','','headerlines',3);
Input75=textread('numdamp7Ts05.txt','','headerlines',3);
Input85=textread('numdamp8Ts05.txt','','headerlines',3);
Input95=textread('numdamp9Ts05.txt','','headerlines',3);
Input105=textread('numdamp10Ts05.txt','','headerlines',3);
Input115=textread('numdamp17Ts05.txt','','headerlines',3);
Input125=textread('numdamp28Ts05.txt','','headerlines',3);

L15=length(Input15);
L25=length(Input25);
L35=length(Input35);
L45=length(Input45);
```

```
L55=length(Input55);
L65=length(Input65);
L75=length(Input75);
L85=length(Input85);
L95=length(Input95);
L105=length(Input105);
L115=length(Input115);
L125=length(Input125);

Lengths5=[L15,L25,L35,L45,L55,L65,L75,L85,L95,L105,L115,L125];
umatrix5=zeros(max(Lengths5),12);
tmatrix5=zeros(max(Lengths5),12);

tmatrix5(1:L15,1)=Input15(:,1);
tmatrix5(1:L25,2)=Input25(:,1);
tmatrix5(1:L35,3)=Input35(:,1);
tmatrix5(1:L45,4)=Input45(:,1);
tmatrix5(1:L55,5)=Input55(:,1);
tmatrix5(1:L65,6)=Input65(:,1);
tmatrix5(1:L75,7)=Input75(:,1);
tmatrix5(1:L85,8)=Input85(:,1);
tmatrix5(1:L95,9)=Input95(:,1);
tmatrix5(1:L105,10)=Input105(:,1);
tmatrix5(1:L115,11)=Input115(:,1);
tmatrix5(1:L125,12)=Input125(:,1);

umatrix5(1:L15,1)=Input15(:,2);
umatrix5(1:L25,2)=Input25(:,2);
umatrix5(1:L35,3)=Input35(:,2);
umatrix5(1:L45,4)=Input45(:,2);
umatrix5(1:L55,5)=Input55(:,2);
umatrix5(1:L65,6)=Input65(:,2);
umatrix5(1:L75,7)=Input75(:,2);
umatrix5(1:L85,8)=Input85(:,2);
umatrix5(1:L95,9)=Input95(:,2);
umatrix5(1:L105,10)=Input105(:,2);
umatrix5(1:L115,11)=Input115(:,2);
umatrix5(1:L125,12)=Input125(:,2);

avgdamp5 = zeros(12,1);
totdamp5 = zeros(12,1);

%%
%STEPSIZE 0.01
for k = 1:12
    j=0;
    clear umaks
    clear tmaks
    for i =1:(Lengths(k)-2)
      if umatrix(i+1,k)>0 && umatrix(i+1,k)>umatrix(i,k) && ...
          umatrix(i+1,k)>umatrix(i+2,k)
        j=j+1;
        tmaks(j)=tmatrix(i+1,k);
        umaks(j)=umatrix(i+1,k);
      end
    end

    %umaks
    %j
```

```
    Δ=zeros(1,length(umaks)-1);
    Δt=zeros(1,length(umaks)-1);
    for i=1:length(umaks)-1
        Δ(i)=log(umaks(i)/umaks(i+1));
        Δt(i)=tmaks(i+1)-tmaks(i);
    end

    %Average logarithmic decrement and period
    Δavg=sum(Δ(:))/length(Δ);

    %Total logarithmic decrement calculated using first and last maxima
    Δtot=1/(length(umaks))*log(umaks(1)/umaks(length(umaks)));

    avgdamp(k,1) = 100*Δavg/(2*pi)
    totdamp(k,1) = 100*Δtot/(2*pi)

end

%STEPSIZE 0.02
for k = 1:12
    j=0;
    clear umaks
    clear tmaks
    for i =1:(Lengths2(k)-2)
      if umatrix2(i+1,k)>0 && umatrix2(i+1,k)>umatrix2(i,k) && ...
          umatrix2(i+1,k)>umatrix2(i+2,k)
        j=j+1;
        tmaks2(j)=tmatrix2(i+1,k);
        umaks2(j)=umatrix2(i+1,k);
      end
    end

    %umaks
    %j
    Δ2=zeros(1,length(umaks2)-1);
    Δt2=zeros(1,length(umaks2)-1);
    for i=1:length(umaks2)-1
        Δ2(i)=log(umaks2(i)/umaks2(i+1));
        Δt2(i)=tmaks2(i+1)-tmaks2(i);
    end

    %Average logarithmic decrement and period
    Δavg2=sum(Δ2(:))/length(Δ2);

    %Total logarithmic decrement calculated using first and last maxima
    Δtot2=1/(length(umaks2))*log(umaks2(1)/umaks2(length(umaks2)));

    avgdamp2(k,1) = 100*Δavg2/(2*pi)
    totdamp2(k,1) = 100*Δtot2/(2*pi)

end


%STEPSIZE 0.05
for k = 1:12
    j=0;
    clear umaks
    clear tmaks
    for i =1:(Lengths5(k)-2)
```

```
            if umatrix5(i+1,k)>0 && umatrix5(i+1,k)>umatrix5(i,k) && ...
                umatrix5(i+1,k)>umatrix5(i+2,k)
                j=j+1;
                tmaks5(j)=tmatrix5(i+1,k);
                umaks5(j)=umatrix5(i+1,k);
            end
        end

        %umaks
        %j
        Δ5=zeros(1,length(umaks5)-1);
        Δt5=zeros(1,length(umaks5)-1);
        for i=1:length(umaks5)-1
            Δ5(i)=log(umaks5(i)/umaks5(i+1));
            Δt5(i)=tmaks5(i+1)-tmaks5(i);
        end

        %Average logarithmic decrement and period
        Δavg5=sum(Δ5(:))/length(Δ5);

        %Total logarithmic decrement calculated using first and last maxima
        Δtot5=1/(length(umaks5))*log(umaks5(1)/umaks5(length(umaks5)));

        avgdamp5(k,1) = 100*Δavg5/(2*pi)
        totdamp5(k,1) = 100*Δtot5/(2*pi)

end
%%
%PLOTS
plot(D,avgdamp,'-ro')
hold on
plot(D,totdamp,'-bo')
hold on
plot(D,avgdamp2,'-r^')
hold on
plot(D,totdamp2,'-b^')
hold on
plot(D,avgdamp5,'-rx')
hold on
plot(D,totdamp5,'-bx')
grid on
xlabel('Mode Number')
ylabel('Damping Ratio, \zeta [%]')
l=legend('\zeta_{avg,0.01}','\zeta_{tot,0.01}','\zeta_{avg,0.02}',
'\zeta_{tot,0.02}','\zeta_{avg,0.05}','\zeta_{tot,0.05}','location','northwest');
set(l,'FontSize',14)
axis([0 13 -0.1 2])
```

A.2 Fast Fourier Transform

This script reads the acceleration and time and performs a discrete Fourier transform. The plots are presented throughout Chapter 6. The accelerations are reported in *Abaqus* as "Vol14Hz<frequency>.txt".

```
clear all
close all
clc

set(0,'DefaultAxesFontName','Times New Roman')
set(0,'defaultlinelinewidth',1.3)
set(0,'DefaultAxesFontSize', 18)

Input=textread('Vol14Hz18764.txt','','headerlines',3);
t=Input(:,1);
u=Input(:,2);
Fs=100;
x=t;
y=u;
y = detrend(y,0);
L=length(y)
NFFT = 2^nextpow2(L) % Next power of 2 from length of y
Y = fft(y,NFFT)/L;
f = Fs/2*linspace(0,1,NFFT/2+1);

% Plot single-sided amplitude spectrum.
width=370;

FigHandle = figure('Position', [100, 100, width, 300]);
plot(f,2*abs(Y(1:NFFT/2+1)))
grid on
xlim([0,30])
%title('Single-Sided Amplitude Spectrum of y(t)')
xlabel('Frequency (Hz)')
ylabel('|Y(f)|')
```

A.3 Root Mean Square

This script calculates the root mean square acceleration for the response plots (see 3.2.2).

```
close all
clear all

set(0,'DefaultAxesFontName','Times New Roman')
set(0,'defaultlinelinewidth',1.3)
set(0,'DefaultAxesFontSize', 18)

format long g
Input=textread('Vol11Hz21405.txt','','headerlines',3);
t=Input(:,1)-1; %Minus 1 because the time start at 1 due to the ...
    gravity step, for which we do not have outputvalues for ...
    acceleration since it is a static step
aleft=Input(:,2);
amid=Input(:,3);
aright=Input(:,4);

amax=zeros(3,2);
```

```

[amax(1,1),amax(1,2)]=max(abs(aleft));
[amax(2,1),amax(2,2)]=max(abs(amid));
[amax(3,1),amax(3,2)]=max(abs(aright));

amax

[apeak,i]=max(amax(:,1));

if i==1
    pos=1
    arms=rms(aleft(amax(1,2):amax(1,2)+100)); % +100 because the ...
        timestep
    %is 0.01 s, so 100 entries later equals one second later
    arms2=rms(aleft(length(aleft)-100:length(aleft))); %Only used to
    %calculate rms for the last second for running analyses
elseif i==2
    pos=2
    arms=rms(amid(amax(1,2):amax(1,2)+100));
    arms2=rms(amid(length(amid)-100:length(amid))); %Only used to
    % calculate rms for the last second for running analyses
elseif i==3
    pos=3
    arms=rms(aright(amax(1,2):amax(1,2)+100));
    arms2=rms(aright(length(aright)-100:length(aright))); %Only ...
        used to
    % calculate rms for the last second for running analyses
end
arms
arms2
```

B Appendix B

B.1 Python Coding Explained

This appendix aims to describe the parts of the *Python* code that have been written or rewritten by the authors to perform the analyses discussed in this thesis. Descriptions of what each code section does are included hereunder, while in Appendix F, which contains the entire script (digitally), more in-depth comments are given, explaining how the coding actually works, as a guide for whoever might want to continue developing the script, or just understand it. There are quite a few smaller changes that have been made, such as extra keywords for new parameters, etc. These changes are not included in this part. However, absolutely everything that has been changed is marked in the code by writing ##KVDP!! and an additional comment after the changed line. For both appendices, knowledge of the Python language is recommended.

Part 1 - The load deck

Defining the load deck
Here, the load deck part is defined, with the same dimensions as the deck of the bridge. The load deck was created so that geometric sets could easily be created where the forces were to be applied. In this function four points are defined, which define the perimeter of the load deck.

Segment can be found in line 94-122 in the script.

```
class Loaddeck:
    def __init__(self, length, width):
        self.length = length
        self.width = width
        self.p1 = np.array((-length/float(2), 0), dtype=float)
        self.p2 = np.array((self.p1[0] + length,
                                    self.p1[1] ), dtype=float)
        self.p3 = np.array((self.p1[0] + length,
                                    self.width ), dtype=float)
        self.p4 = np.array((-length/float(2), self.width), dtype=float)
        self.divPoints = []
        self.sketch = None
        self.part = None
        self.instance = None
```

Defining new material and adding material damping

The material that is going to be assigned to the load deck is defined in the shown paragraph, with material damping added. Material damping is added to all materials, but only the asphalt material is seen here.

Segment can be found in line 580-590 in the script.

```
    def createProfiles(self):
        asp = self.bridgeModel.Material(name='Asphalt')
        asp.Elastic(table=((2000, 0.3),), type=ISOTROPIC)
        asp.Density(table=((2.24E-9,),))
        asp.Damping(alpha=0.092091, beta=0.000187)
```

Creating the load deck

Function ceateLoadDeck() creates the load deck with the same dimensions as the bridge deck, and is presented here in parts. This first segment creates the deck and assigns a section and material orientation to it. The load deck has the properties of an asphalt layer, so it serves a physical function. It is later in the script tied to the bridge deck.

Segment can be found in line 1586-1635 in the script.

```
    def createLoadDeck(self):

        self.loaddeck = Loaddeck(self.length, self.width - 2*self.
        hangersZspacing - 2 * self.bufferWidth)
        self.loaddeck.sketch = self.bridgeModel.ConstrainedSketch(
        name='LoadDeck', sheetSize = self.sheetSize)

        self.loaddeck.sketch.Line(self.loaddeck.p1,self.loaddeck.p2)
        self.loaddeck.sketch.Line(self.loaddeck.p2,self.loaddeck.p3)
        self.loaddeck.sketch.Line(self.loaddeck.p3,self.loaddeck.p4)
        self.loaddeck.sketch.Line(self.loaddeck.p4,self.loaddeck.p1)

        self.loaddeck.part = ...
            self.bridgeModel.Part(dimensionality=THREE_D,
        name='LoadDeck', type=DEFORMABLE_BODY)

        self.loaddeck.part.BaseShell(sketch=self.loaddeck.sketch)

        dSet = self.loaddeck.part.Set(name='LoadDeck_Set',
        faces=(self.loaddeck.part.faces))

        self.loaddeck.part.SectionAssignment(region=dSet,
        sectionName='LoadDeck_Section', offset=0.0,
        offsetType=MIDDLE_SURFACE)

        self.loaddeck.part.MaterialOrientation(
        region=self.loaddeck.part.sets['LoadDeck_Set'],
        orientationType=GLOBAL, axis=AXIS_1,
        additionalRotationType=ROTATION_NONE,localCsys=None, ...
            fieldName='')
```

Partitioning the load deck

In the following segment a sketch containing vertical and horizontal lines is created. The sketch will be used to partition the load deck. Later on, sets will be created at the vertices of the partition, and this way forces can be applied where wanted. The partition is dependent on where the loads are selected to be applied in the parameter selector; at the left edge, in the middle or at the right edge, hence the if-sentence at the beginning. Additionally, the partitioning depends on the stride length and step width of the pedestrians. Both can be selected in the parameter selector that appears when running the script. At the end of this segment a instance of the load deck is created and rotated in Assembly so that it becomes horizontal and lines up perfectly with the other instances.

Segment can be found in line 1640-1717 in the script.

```
        skLD = self.bridgeModel.ConstrainedSketch(
        name='LoadDeck_Partition', sheetSize = self.sheetSize)

        if (self.numOfRow==1):
            if (self.rowposition==1):
                a=0.
                b=1.

            elif (self.rowposition==2):
                a=1/2.
                b=0.

            elif (self.rowposition==3):
                a=1.
                b=-1.

            skLD.Line((self.loaddeck.p1[0] ,self.loaddeck.p1[1] +
            a*self.loaddeck.width - self.stepwidth/2. +
            b*(self.stepwidth/2.+ 500.)), (self.loaddeck.p2[0],
            self.loaddeck.p2[1] + a*self.loaddeck.width -
            self.stepwidth/2. + b*(self.stepwidth/2.+ 500.)))

            skLD.Line((self.loaddeck.p1[0] ,self.loaddeck.p1[1] +
            a*self.loaddeck.width + self.stepwidth/2. +
            b*(self.stepwidth/2.+ 500.)), (self.loaddeck.p2[0],
            self.loaddeck.p2[1] + a*self.loaddeck.width +
            self.stepwidth/2. + b*(self.stepwidth/2.+ 500.)))

        else:
            for i in range(1,self.numOfRow + 1):
                skLD.Line((self.loaddeck.p1[0] ,self.loaddeck.p1[1] +
                i*self.loaddeck.width/(float(self.numOfRow +1)) -
                self.stepwidth/2.), (self.loaddeck.p2[0],
                self.loaddeck.p2[1] + i*self.loaddeck.width/(
                float(self.numOfRow +1)) - self.stepwidth/2.))

                skLD.Line((self.loaddeck.p1[0] ,self.loaddeck.p1[1] +
                i*self.loaddeck.width/(float(self.numOfRow +1)) +
                self.stepwidth/2.), (self.loaddeck.p2[0],
                self.loaddeck.p2[1] + i*self.loaddeck.width/(
                float(self.numOfRow +1)) + self.stepwidth/2.))

```

```
        for i in range(1, int(self.length/self.stridelength + 1)):
            skLD.Line((self.loaddeck.p1[0] +i*self.stridelength,
                       self.loaddeck.p1[1]), (self.loaddeck.p4[0] +
                       i*self.stridelength ,self.loaddeck.p4[1]))

        self.loaddeck.part.PartitionFaceBySketch(
        faces=self.loaddeck.part.faces, sketch=skLD)

        self.loaddeck.instance = ...
            self.bridgeModel.rootAssembly.Instance(
        name='LoadDeck', part=self.loaddeck.part)

        self.loaddeck.instance.rotateAboutAxis(axisPoint=(0, 0, 0),
        axisDirection=(1,0,0),angle=90)

        self.loaddeck.instance.translate((0, 0, ...
            self.hangersZspacing +
        self.bufferWidth))
```

Creating sets on the load deck

The segment below creates the sets at which the concentrated forces representing footsteps will be applied. If the pedestrian is walking across the bridge, the right and left footsteps are one stridelength apart in the longitudinal direction, and one stepwidth apart in the lateral direction. If the pedestrian is walking in place, the left and right footsteps are adjacent, one stepwidth apart. The sets are created as 'RightStep{0}_{1}' and 'LeftStep{0}_{1}' in the for-loops, where both {0} and {1} are counters with added value for each cycle. The number replacing 0 represents the row number, counting from the row farthest to the left, while the number replacing 1 represents which footstep it is. 'RightStep{3}_{18}' thus means right footstep number 18 in row number 3. The number of rows depends on how many rows of pedestrians have been specified by the user through the parameter selector.

Segment can be found in line 1725-1772 in the script.

```
    if (self.numOfRow==1):
        for i in range(1, int(self.length/self.stridelength), 2):
                LoadSetLeft = (self.loaddeck.p1[0]+
                (i+1)*self.stridelength , 0, self.loaddeck.p1[1] +
                a*self.loaddeck.width - self.stepwidth/2. +
                b*(self.stepwidth/2.+ 500.) + self.hangersZspacing +
                self.bufferWidth)

                self.bridgeModel.rootAssembly.Set(
                name='LeftStep1_{0}'.format((i+2)/2),vertices=(
                self.loaddeck.instance.vertices.findAt(
                (LoadSetLeft,))))

                if self.walkInPlace == False:
                    LoadSetRight = (self.loaddeck.p1[0] +
                    i*self.stridelength , 0, self.loaddeck.p1[1] +
                    a*self.loaddeck.width + self.stepwidth/2. +
                    b*(self.stepwidth/2.+ 500.) +
                    self.hangersZspacing + self.bufferWidth)

                elif self.walkInPlace == True:
                    LoadSetRight = (self.loaddeck.p1[0] +
                    (i+1)*self.stridelength , 0, self.loaddeck.p1[1]
                    + a*self.loaddeck.width + self.stepwidth/2. +
                    b*(self.stepwidth/2.+ 500.) +
                    self.hangersZspacing + self.bufferWidth)

                self.bridgeModel.rootAssembly.Set(
                name='RightStep1_{0}'.format((i+2)/2),vertices=(
                self.loaddeck.instance.vertices.findAt(
                (LoadSetRight,))))

    else:
        for j in range(1,self.numOfRow + 1):
            for i in range(1, int(self.length/self.stridelength), 2):
                LoadSetLeft = (self.loaddeck.p1[0] +
                (i+1)*self.stridelength , 0, self.loaddeck.p1[1] +
                j*self.loaddeck.width/(float(self.numOfRow +1)) -
                self.stepwidth/2. + self.hangersZspacing +
```

```
self.bufferWidth)

self.bridgeModel.rootAssembly.Set(
name='LeftStep{0}_{1}'.format(j,(i+2)/2), vertices=(
self.loaddeck.instance.vertices.findAt(
(LoadSetLeft,))))

if self.walkInPlace == False:
    LoadSetRight = (self.loaddeck.p1[0] +
    i*self.stridelength , 0, self.loaddeck.p1[1] +
    j*self.loaddeck.width/(float(self.numOfRow +1))
    + self.stepwidth/2. + self.hangersZspacing +
    self.bufferWidth)

elif self.walkInPlace ==True:
    LoadSetRight = (self.loaddeck.p1[0] +
    (i+1)*self.stridelength , 0, self.loaddeck.p1[1]
    + j*self.loaddeck.width/(float(self.numOfRow +1))
    + self.stepwidth/2. + self.hangersZspacing +
    self.bufferWidth)

self.bridgeModel.rootAssembly.Set(
name='RightStep{0}_{1}'.format(j,(i+2)/2),vertices=(
self.loaddeck.instance.vertices.findAt(
(LoadSetRight,))))
```

Meshing the load deck

The function mesh() hereunder seeds and meshes all the different instances (only the part of the function meshing the load deck is included here). The load deck is assigned element type S4R and is given a global seed size equal to variable meshSizeLD. This variable is later in the script set to be equal to input variable meshSizeD.

Segment can be found in line 3025-3060 in the script.

```
    def mesh(self, meshSizeD, meshSizeLD, meshSizeH, meshSizeB, ...
        meshSizeT,
             meshSizeA):
        self.logMesh(meshSizeD, meshSizeLD, meshSizeH, meshSizeB,
        meshSizeT, meshSizeA)

        print "meshing..\n"

        for key in self.bridgeModel.rootAssembly.instances.keys():
            inst = self.bridgeModel.rootAssembly.instances[key]

            if inst.name == 'LoadDeck':
                if self.numOfDim == 3:

                    elemType1 = mesh.ElemType(elemCode=S4R,
                    elemLibrary=STANDARD)
                    f = inst.faces
                    faces = f.getSequenceFromMask(mask=('[#1 ]', ), )
                    pickedRegions =(faces, )

                self.bridgeModel.rootAssembly.seedPartInstance(
                regions=(inst,), size=meshSizeLD)
                self.bridgeModel.rootAssembly.generateMesh(
                regions=(inst,))
```

Part 2 - Defining analysis steps and outputs

Modal and complex frequency analyses

The function defineStepsAndBC() governs the next 7 segments. In addition to creating analysis steps and defining boundary conditions it defines the concentrated forces with their corresponding amplitudes and ties the deck and load deck together. The first segment below defines both a frequency step and a complex frequency step after the gravity step. This is only done if the input variable "Modal" is changed from default value '0' to '1' in the parameter selector. If the frequency steps are included no other analysis steps are defined after that. This option can be used to obtain the natural frequencies of the structure, the corresponding modes and the damping ratios for each mode.

Segment can be found in line 4316-4334 in the script.

```
    def defineStepsAndBC(self):

    #(...)

        if self.modal == True:
            st = self.bridgeModel.FrequencyStep(name='Modal',
            previous='Gravity',normalization=MASS,
            numEigen = self.numEigen)

            name = st.fieldOutputRequestStates.keys()[0]
            st.fieldOutputRequestStates.changeKey(
            fromName=name,toName='FieldOutputStepModal')

            st = self.bridgeModel.ComplexFrequencyStep(
            name='ComplexFrequency', previous='Modal',
            numEigen = self.numEigen)

            name = st.fieldOutputRequestStates.keys()[0]
            st.fieldOutputRequestStates.changeKey(
            fromName=name,toName='FieldOutputComplexFrequency')
```

Creating analysis step for pedestrian loading and defining additional sets for extracting output variables

If the input variable "Modal" is kept unchanged, an implicit dynamic step is created, as seen in the section below. The total time period of the analysis step is dependent on the walking frequency, the length of the bridge and the number of people crossing the bridge.

This section also defines 11 specific sets on the footbridge where acceleration (A1, A2, A3) and displacement values (U1, U2, U3) are extracted by defining history outputs at those points. These 11 sets are at the tops of the two arches, and at the left edge, middle and right edge at 1/4, 1/2 and 3/4 of the length of the bridge. The procedure for defining the sets and extracting the history output is identical for all 11 points, and therefore only one example has been included in the segment below.

Segment can be found in line 4350-4550 in the script.

```
        if self.modal == False:

            self.bridgeModel.ImplicitDynamicsStep(nlgeom=ON,
            initialInc=self.timestep, maxInc=self.timestep,
            minInc=1.E-10, maxNumInc=10000, name='Pedestrians',
            previous='Gravity',
            timePeriod=(self.length/self.stridelength)*1/self.freq +
            (self.numOfPeople-1)*2./self.freq)

            MidPointArch = (self.loaddeck.p1[0] +self.length/2.,
            self.elevation, self.loaddeck.p1[1])
            self.bridgeModel.rootAssembly.Set(
            name='TopArc',vertices=(self.arc.instance.vertices.findAt(
            (MidPointArch,))))

#(...)

            regionTop=self.bridgeModel.rootAssembly.sets['TopArc']
            self.bridgeModel.HistoryOutputRequest(
            name='HistoryOutputArch', createStepName='Pedestrians',
            variables=('A1','A2','A3','U1','U2','U3','V1','V2','V3'),
            frequency=1, region=regionTop, sectionPoints=DEFAULT, ...
                rebar=EXCLUDE)

#(...)
```

Part 3 - Pedestrian Loading

Defining Fourier series

The Fourier series used to describe the three different force components are defined in the following. The dynamic load factors (alpha-values) used in the Fourier series depend on the load model, which is chosen in the parameter selector. At the end of this section there is a small if-sentence, which linearly ramps the static part of the vertical force amplitude up at the beginning and back down at the end, so that the force is applied gradually, as opposed to suddenly.

Segment can be found in line 5426-5616 in the script.

```
        t=np.linspace(0,1./self.freq,100)
        fvert=np.linspace(0,0,100)
        fstatic=np.linspace(1,1,100)
        flat=np.linspace(0,0,100)
        flong=np.linspace(0,0,100)

        if self.loadmodel==1:
            alpha1vert=0.37
            alpha2vert=0.10
            alpha3vert=0.12
            alpha4vert=0.04
            alpha5vert=0.08

            alpha1lat=0.039
            alpha2lat=0.010
            alpha3lat=0.043
            alpha4lat=0.012
            alpha5lat=0.015

            alpha1_2long=0.037
            alpha1long=0.204
            alpha3_2long=0.026
            alpha2long=0.083
            alpha5_2long=0.024

        elif (self.loadmodel==2 and self.freq≤2.2):
            alpha1vert=0.4
            alpha2vert=0.1
            alpha3vert=0.1
            alpha4vert=0.0
            alpha5vert=0.0

            #(...)

        amp=0

        for i in range(0,100):

            fvert[i]=alpha1vert*math.sin(2*math.pi*self.freq*t[i])
            + alpha2vert*math.sin(2*2*math.pi*self.freq*t[i])
            + alpha3vert*math.sin(2*3*math.pi*self.freq*t[i])
            + alpha4vert*math.sin(2*4*math.pi*self.freq*t[i])
```

```
          + alpha5vert*math.sin(2*5*math.pi*self.freq*t[i])

          flat[i]=alpha1lat*math.sin(2*math.pi*self.freq/2.*t[i])
          + alpha2lat*math.sin(2*2*math.pi*self.freq/2.*t[i])
          + alpha3lat*math.sin(2*3*math.pi*self.freq/2.*t[i])
          + alpha4lat*math.sin(2*4*math.pi*self.freq/2.*t[i])
          + alpha5lat*math.sin(2*5*math.pi*self.freq/2.*t[i])

          flong[i]=alpha1_2long*math.sin(2*1/2*math.pi*self.freq*t[i])
          + alpha1long*math.sin(2*1*math.pi*self.freq*t[i])
          + alpha3_2long*math.sin(2*3/2*math.pi*self.freq*t[i])
          + alpha2long*math.sin(2*2*math.pi*self.freq*t[i])
          + alpha5_2long*math.sin(2*5/2*math.pi*self.freq*t[i])

              if i<20:
                  fstatic[i]=amp
                  amp=amp+1/20.

              elif i>89:
                  amp=amp-1/10.
                  fstatic[i]=amp

        elif self.loadmodel ==3:
              if i<20:
                  fstatic[i]=amp
                  amp=amp+1/20.

              elif i≥70:
                  fvert[i]=0
                  fstatic[i]=0
```

Creating load amplitudes

Amplitudes for each force component for every single footstep are created in the following segment. Only the amplitudes for the vertical and lateral components due to the right footsteps are included here because the procedure is identical for all amplitudes, so including all of them here was deemed redundant. The rest of the code can be found in Appendix F.

All amplitudes are defined for the entire time period of the Pedestrian analysis step. An amplitude is equal to 0 as long as the footstep it represents is not supposed to be active, and equal to the Fourier series during the time the footstep is supposed to be applied to the deck. All forces are equal to a constant G times the corresponding amplitude, and this way all forces are present during the entire analysis step, but only nonzero at the right time.

Next two Segment can be found in line 5641-5966 in the script.

```
for i in range(1, int(self.length/self.stridelength + ...
    2*self.numOfPeople - 2), 2):
self.bridgeModel.TabularAmplitude(
name='AmplitudeRightVert_{0}'.format((i+1)/2), timeSpan=STEP,
smooth=SOLVER_DEFAULT, data=(((i-1)*1/self.freq+t[0], fvert[0] + ...
    fstatic[0]), ((i-1)*1/self.freq +t[1] , fvert[1] + ...
    fstatic[1]), ((i-1)*1/self.freq + t[2], fvert[2] + ...
    fstatic[2]), ((i-1)*1/self.freq +t[3] ,fvert[3] + fstatic[3]),
#(...)
((i-1)*1/self.freq +t[97] , fvert[97] + fstatic[97]), ...
    ((i-1)*1/self.freq + t[98], fvert[98] + fstatic[98]), ...
    ((i-1)*1/self.freq +t[99] ,fvert[99] + fstatic[99])))
```

Note that the static load component is excluded from the lateral and longitudinal amplitudes:

```
self.bridgeModel.TabularAmplitude(
name='AmplitudeRightLat_{0}'.format((i+1)/2), timeSpan=STEP,
smooth=SOLVER_DEFAULT, data=(((i-1)*1/self.freq+t[0], flat[0]), ...
    ((i-1)*1/self.freq +t[1] , flat[1]), ((i-1)*1/self.freq + ...
    t[2], flat[2]), ((i-1)*1/self.freq +t[3] ,flat[3]),
#(...)
((i-1)*1/self.freq +t[97] , flat[97]), ((i-1)*1/self.freq + t[98], ...
    flat[98]), ((i-1)*1/self.freq +t[99] ,flat[99])))
```

Defining the concentrated forces

The following two segments define the concentrated forces in all three directions at the correct locations with the corresponding amplitudes defined above. The former segment will create loads along the entire row(s), thereby modelling pedestrians walking across the bridge. The latter section will create loads at a specific number of points equal to the number of pedestrians, and models pedestrians walking in place. Only one of the segments will be executed depending on the value given to input variable 'Walk across bridge or in place'.

The if-loop at the beginning defines the "positionnumber" which is used in the for-loop to place the pedestrians at either 1/4,1/2 or 3/4 of the length of the bridge if they are stepping in place. This is because the different modes have their maximum at different places, and the greatest dynamic response is obtained if the pedestrians excite the different modes at their maximum.

Segment can be found in line 5967-6027 in the script.

```

if self.walkInPlace == True:
    if self.walkInPlacePos == 1:
        positionnumber =1/2.
    elif self.walkInPlacePos == 2:
        positionnumber=1
    elif self.walkInPlacePos ==3:
        positionnumber=3/2.

for i in range(1, int(self.length/self.stridelength +
               2*self.numOfPeople - 2), 2):

#(...)

for k in range(1,self.numOfRow + 1):
    for j in range(1,i+2,2):
         if (self.walkInPlace == False and (abs((i-j)/2 + 1) ≤
            self.numOfPeople) and ...
                j≤int(self.length/self.stridelength)):

                regionRight=self.bridgeModel.rootAssembly.
                sets['RightStep{0}_{1}'.format(k,(j+1)/2)]

                regionLeft=self.bridgeModel.rootAssembly.
                sets['LeftStep{0}_{1}'.format(k,(j+1)/2)]

                self.bridgeModel.ConcentratedForce(
                name='LoadRightStepVert{0}_{1}_{2}'.format(
                k,(i+1)/2,(j+1)/2), createStepName='Pedestrians',
                region=regionRight, cf2=-700, amplitude=
                'AmplitudeRightVert_{0}'.format((i+1)/2),
                distributionType=UNIFORM, field='', localCsys=None)

                self.bridgeModel.ConcentratedForce(
                name='LoadLeftStepVert{0}_{1}_{2}'.format(
                k,(i+1)/2,(j+1)/2),
                createStepName='Pedestrians',
```

```
                    region=regionLeft, cf2=-700,
                    amplitude='AmplitudeLeftVert_{0}'.format(
                    (i+1)/2), distributionType=UNIFORM,
                    field='', localCsys=None)

                    if self.loadmodel < 3:
                        self.bridgeModel.ConcentratedForce(
                        name='LoadRightStepLat{0}_{1}_{2}'.
                        format(k,(i+1)/2,(j+1)/2),
                        createStepName='Pedestrians',
                        region=regionRight, cf3=700,
                        amplitude='AmplitudeRightLat_{0}'.
                        format((i+1)/2),
                        distributionType=UNIFORM, field='',
                        localCsys=None)

                        self.bridgeModel.ConcentratedForce(
                        name='LoadLeftStepLat{0}_{1}_{2}'.
                        format(k,(i+1)/2,(j+1)/2),
                        createStepName='Pedestrians',
                        region=regionLeft, cf3=700,
                        amplitude='AmplitudeLeftLat_{0}'.
                        format((i+1)/2),
                        distributionType=UNIFORM, field='',
                        localCsys=None)

                        self.bridgeModel.ConcentratedForce(
                        name='LoadRightLong{0}_{1}_{2}'.
                        format(k,(i+1)/2,(j+1)/2),
                        createStepName='Pedestrians',
                        region=regionRight, cf3=700,
                        amplitude='AmplitudeRightLong_{0}'.
                        format((i+1)/2),
                        distributionType=UNIFORM, field='',
                        localCsys=None)

                        self.bridgeModel.ConcentratedForce(
                        name='LoadLeftStepLong{0}_{1}_{2}'.
                        format(k,(i+1)/2,(j+1)/2),
                        createStepName='Pedestrians',
                        region=regionLeft, cf3=700,
                        amplitude='AmplitudeLeftLong_{0}'.
                        format((i+1)/2),
                        distributionType=UNIFORM, field='',
                        localCsys=None)
```

Segment can be found in line 6030-6080 in the script.

```

    for j in range(1,int(self.length/self.stridelength +
                    2*self.numOfPeople),2):

        if (self.walkInPlace == True and -self.numOfPeople
            ≤(j-positionnumber*int(self.length/(2*self.stridelength)))
            ≤ (self.numOfPeople-1)):

            regionRight=self.bridgeModel.rootAssembly.
            sets['RightStep{0}_{1}'.format(k,(j+1)/2)]
            regionLeft=self.bridgeModel.rootAssembly.
            sets['LeftStep{0}_{1}'.format(k,(j+1)/2)]

            self.bridgeModel.ConcentratedForce(
            name='LoadRightStepVert{0}_{1}_{2}'.format(
            k,(i+1)/2,(j+1)/2),
            createStepName='Pedestrians',
            region=regionRight, cf2=-700,
            amplitude='AmplitudeRightVert_{0}'.format(
            (i+1)/2), distributionType=UNIFORM,
            field='', localCsys=None)

            self.bridgeModel.ConcentratedForce(
            name='LoadLeftStepVert{0}_{1}_{2}'.format(
            k,(i+1)/2,(j+1)/2),
            createStepName='Pedestrians',
            region=regionLeft, cf2=-700,
            amplitude='AmplitudeLeftVert_{0}'.format(
            (i+1)/2), distributionType=UNIFORM,
            field='', localCsys=None)

            if self.loadmodel < 3:
                self.bridgeModel.ConcentratedForce(
                name='LoadRightStepLat{0}_{1}_{2}'.
                format(k,(i+1)/2,(j+1)/2),
                createStepName='Pedestrians',
                region=regionRight, cf3=700,
                amplitude='AmplitudeRightLat_{0}'.
                format((i+1)/2),
                distributionType=UNIFORM, field='',
                localCsys=None)

                self.bridgeModel.ConcentratedForce(
                name='LoadLeftStepLat{0}_{1}_{2}'.
                format(k,(i+1)/2,(j+1)/2),
                createStepName='Pedestrians',
                region=regionLeft, cf3=700,
                amplitude='AmplitudeLeftLat_{0}'.
                format((i+1)/2),
                distributionType=UNIFORM, field='',
                localCsys=None)

                self.bridgeModel.ConcentratedForce(
                name='LoadRightStepLong{0}_{1}_{2}'.
                format(k,(i+1)/2,(j+1)/2),
                createStepName='Pedestrians',
```

```
                    region=regionRight, cf3=700,
                    amplitude='AmplitudeRightLong_{0}'.
                    format((i+1)/2),
                    distributionType=UNIFORM, field='',
                    localCsys=None)

                    self.bridgeModel.ConcentratedForce(
                    name='LoadLeftStepLong{0}_{1}_{2}'.
                    format(k,(i+1)/2,(j+1)/2),
                    createStepName='Pedestrians',
                    region=regionLeft, cf3=700,
                    amplitude='AmplitudeRightLong_{0}'.
                    format((i+1)/2),
                    distributionType=UNIFORM, field='',
                    localCsys=None)
```

Tieing the deck and load deck

The short paragraph below defines surfaces on both the deck and load deck and ties these two together, allowing no relative motion between them.

Segment can be found in line 6093-6117 in the script.

```
    if self.numOfDim == 3:
        faces = self.deck.instance.sets['Deck_Set'].faces

        regiondeck = self.bridgeModel.rootAssembly.Surface(
        side2Faces=faces, name='Deck_surf_1')

        loadfaces=self.loaddeck.instance.sets['LoadDeck_Set'].faces

        regionloaddeck = self.bridgeModel.rootAssembly.Surface(
        side1Faces=loadfaces, name='LoadDeck_surf_1')

        self.bridgeModel.Tie(adjust=ON, master=regiondeck,
        name='TieLoadDeckToDeck', positionToleranceMethod=COMPUTED,
        slave=regionloaddeck, thickness=ON, tieRotations=ON)
```

Defining input variables

In this final segment the input variables are defined together with their default values that will be shown in the parameter selector.

Segment can be found in line 6823-6968 in the script.

```
class UserIn:
    def __init__(self):
        userInputs=getInputs(
        fields=(('Model name','StudentBridge'),
        ('Number of dim','3'),
        ('Variant:','1'),
        ('Length (mm)', '60000.0'),
        ('Elevation (mm)', '9000.0'),
        ('Width [deck] (mm)', '5000.0'),
        ('Pavement width (mm)', '0'),
        ('Rows of people','1'),
        ('Position of row','2'),
        ('Number of people per row','1'),
        ('Walk across bridge or in place','0'),
        ('Position of pedestrians walking in place','2'),
        ('Walking frequency (Hz)','2.0'),
        ('Stride length (mm)','750'),
        ('Step width','200'),
        ('Hangers 3d spacing [one side] (mm)', '400.0'),
        ('Hangers angle [arc plane] (deg.)', '35.'),
        ('Num. of hangers','14'),
        ('Tie level', '0'),
        ('Deck mesh size','100'),
        ('Hanger mesh size','100000'),
        ('Beam mesh size','100'),
        ('Tie mesh size','100000'),
        ('Arc mesh size','100'),
        ('Timestep','0.01'),
        ('Load Model','1'),
        ('Number of bolts on deck','10'),
        ('Number of bolts on pavement','1'),
        ('Modal','0'),
        ('Number of eig. values','10'),
        ('Prestress s11','0'),
         ),

    dialogTitle='Provide the data',
    label='Please define features for this bridge model\n' +
    '\nVariants:\n' +
    '1 -- equal points on deck\n' +
    '2 -- equal points on arc')

if userInputs[0] == None:
raise MyError('Incorrect input. End of script')

    self.name = userInputs[0]
    self.numOfDim = int(userInputs[1])
    self.variant = int(userInputs[2])
    self.length = float(userInputs[3])
    self.elevation = float(userInputs[4])
```

```
        self.width = float(userInputs[5])
        self.pavementWidth = float(userInputs[6])
        self.numOfRow = int(userInputs[7])
        self.rowposition = int(userInputs[8])
        self.numOfPeople = int(userInputs[9])
        self.walkInPlace = int(userInputs[10])
        self.walkInPlacePos = int(userInputs[11])
        self.freq = float(userInputs[12])
        self.stridelength = float(userInputs[13])
        self.stepwidth = float(userInputs[14])
        self.hangersZspacing = float(userInputs[15])
        self.xyAngle = np.radians(float(userInputs[16]))
        self.numOfHangers = int(userInputs[17])
        self.tieLevel = float(userInputs[18])
        self.meshSizeD = float(userInputs[19])
        self.meshSizeLD = float(userInputs[19])
        self.meshSizeH = float(userInputs[20])
        self.meshSizeB = float(userInputs[21])
        self.meshSizeT = float(userInputs[22])
        self.meshSizeA = float(userInputs[23])
        self.timestep = float(userInputs[24])
        self.loadmodel = int(userInputs[25])
        self.numOfBolts = int(userInputs[26])
        self.numOfBoltsP = int(userInputs[27])
        self.modal = int(userInputs[28])
        self.numEigen = int(userInputs[29])
        self.s11 = int(userInputs[30])
```

C Appendix C

C.1 Complete List of Performed *Abaqus* Analyses

This appendix gives an overview over all the different analyses that were executed in *Abaqus*. They are arranged after which mode they attempted to excite.

Range of walking frequencies: 1.6 - 2.4 Hz
Range of jogging and running frequencies: 2.0 - 3.5 Hz

With two exceptions all pedestrians have been modelled with walking frequencies within the two ranges stated above. To attempt to excite modes with natural frequencies outside these ranges the walking frequencies were decided such that the 2^{nd} or 3^{rd} harmonic of the walking frequency was equal to the natural frequency in focus.

Explanation of terms used in analyses:
Group: 12 pedestrians arranged in three rows with 4 people in each row.
Strip: 12 pedestrians arranged in one row,
Stream1: 40 pedestrians arranged in one row
Stream5: 180 pedestrians evenly arranged over 5 rows (36 per row)
HiDamp: ζ = 2.0 % damping instead of 0.5 % for mode 1 and 10. Implemented as material damping using Rayleigh coefficients

The pedestrians walking in groups, strips and streams are all perfectly synchronized and are one step length apart.

Mode 1 - Torsional - 1 sag

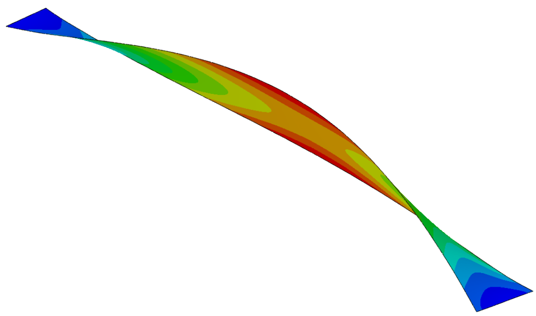


Figure C.1: *Mode 1*

	Frequency [Hz]
	1.8838
Load cases	One pedestrian walking across the bridge along the left edge
	One pedestrian walking in place at the middle of the bridge at the left edge

This natural frequency is below the range of running frequencies and analyses with a running pedestrian have therefore not been performed at this frequency.

Mode 2 - Vertical - 1 sag

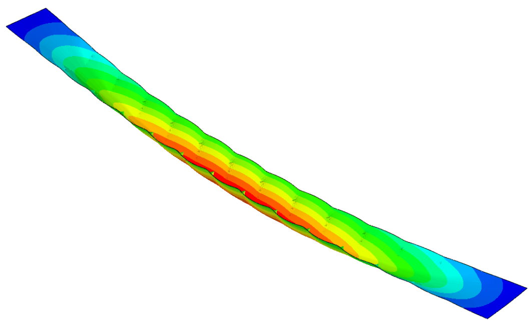


Figure C.2: *Mode 2*

	Frequency [Hz]
	2.1405
Load cases	One pedestrian walking across along the middle
	One pedestrian walking in place at the middle
	Group walking across along the middle
	Group walking in place at the middle
	One pedestrian walking across along the middle (HiDamp)

This natural frequency is below the range of running frequencies and analyses with a running pedestrian have therefore not been performed at this frequency.

Mode 3 - Torsional/Lateral - 2/1 sags

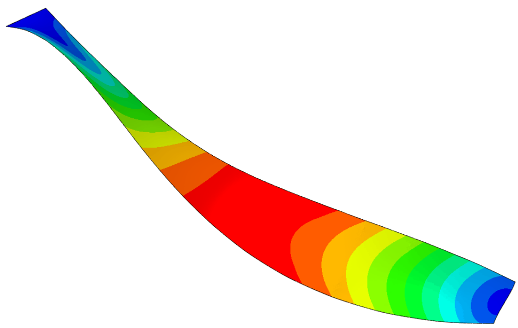


Figure C.3: *Mode 3*

	Frequency [Hz]	
	1.6609	*3.3218*
Load cases	One pedestrian walking across along the left edge	One pedestrian running across along the left edge
	One pedestrian walking in place at $\frac{3}{4}$ of the length at the left edge	

The natural frequency of mode 3 is 3.3218 Hz. Half of that value (1.6609 Hz) is within walking range, and the actual value is within running range.

Mode 4 - Vertical - 2 sags

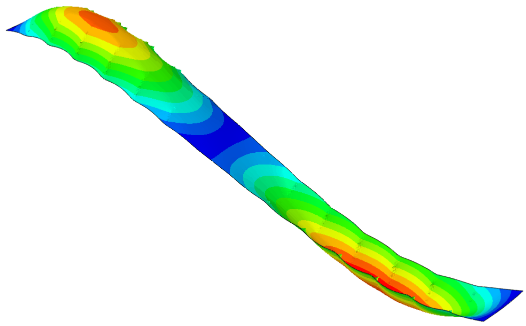


Figure C.4: *Mode 4*

	Frequency [Hz]	
	1.7185	*3.4370*
Load cases	One pedestrian walking across along the middle	One pedestrian running across along the middle
	One pedestrian walking in place at $\frac{3}{4}$ of the length at the middle	

The natural frequency of mode 4 is 3.3470 Hz. Half of that value (1.7185 Hz) is within walking range, and the actual value is within running range.

Mode 5 - Lateral - 1 sag

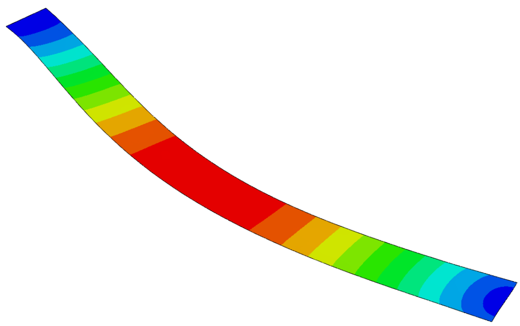


Figure C.5: *Mode 5*

	Frequency [Hz]	
	1.8108	*2.4144*
Load cases	One pedestrian walking across along the middle	One pedestrian walking across along the middle
	One pedestrian walking in place at the middle	One pedestrian walking in place at the middle
		Group walking across along the middle
		Group walking in place at the middle
		One pedestrian walking across along the middle (HiDamp)
		One pedestrian walking in place at the middle (HiDamp)

The natural frequency of mode 5 is 3.6216 Hz. Half of that value (1.8108 Hz) is within walking range. Two thirds of the natural frequency (2.4144 Hz) is just outside the walking range but has been included as well. Two thirds of the natural frequency can be used to cause resonance through the 3rd harmonic since the frequency of the lateral force component is equal to half the walking frequency. The results from the analyses with 1.8108 Hz were so poor that analyses with groups of people were not performed at this frequency. Since this is a lateral mode and the lateral components due to running are not mentioned in any literature lateral vibrations due to running are assumed to be negligible. Therefore running analyses were not performed with any of the frequencies relating to mode 5.

Mode 6 - Vertical - 1 sag

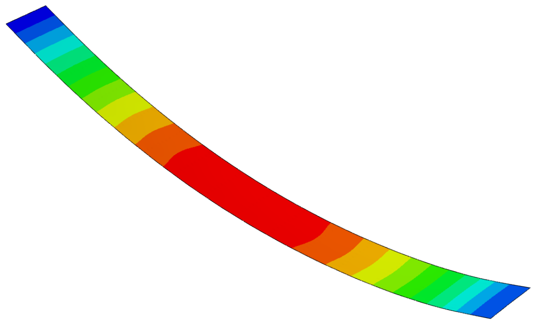


Figure C.6: *Mode 6*

	Frequency [Hz]	
	1.8764	*3.7528*
Load cases	One pedestrian walking across along the middle	One pedestrian running across along the middle
	One pedestrian walking in place at the middle	
	Group walking across along the middle	
	Group walking in place at the middle	
	One pedestrian walking across along the middle (HiDamp)	
	One pedestrian walking in place at the middle (HiDamp)	
	One continuous stream of pedestrians walking across along the middle	
	Five continuous streams of pedestrians walking across along the middle	

The natural frequency of mode 6 is 3.7528 Hz, which is outside the running range. However, a pedestrian running at this frequency is possible and was therefore modelled as well. Half that value (1.8764 Hz) is within walking range. This mode was found to dominate the vertical response of the footbridge. Therefore also streams of people were modelled at this pacing rate to obtain maximum dynamic responses.

Mode 7 - Torsional - 1 sag

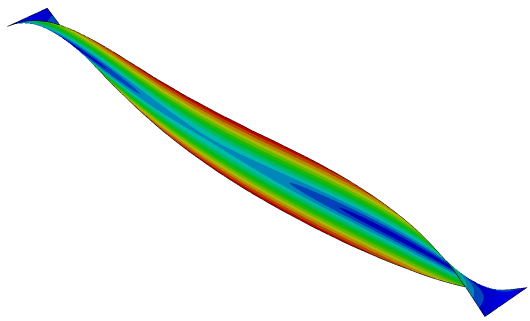


Figure C.7: *Mode 7*

	Frequency [Hz]	
	1.8495	*2.7743*
Load cases	One pedestrian walking across along the left edge	One pedestrian running across along the left edge
	One pedestrian walking in place at the middle of the bridge at the left edge	

The natural frequency of mode 7 is 5.5488 Hz, which is outside the range of both walking and running. A third of that value (1.8495 Hz) is within walking range, and half of the value (2.7744) is within jogging range.

Mode 8 - Vertical - 3 sags

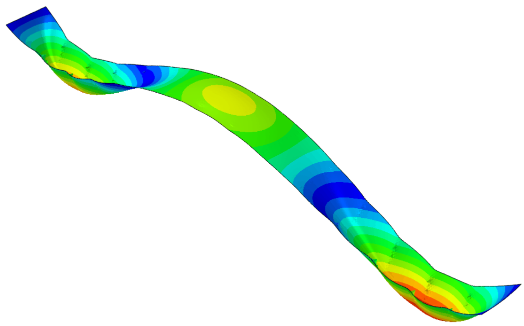


Figure C.8: *Mode 8*

	Frequency [Hz]	
	1.9094	*2.8641*
Load cases	One pedestrian walking across along the middle	One pedestrian running across along the middle
	One pedestrian walking in place at the middle of the bridge	

The natural frequency of mode 8 is 5.7281 Hz, which is outside both walking and running frequency range. A third of that value (1.9094 Hz) is within walking range, and half of that value (2.8641 Hz) is within jogging range.

Mode 9 - Vertical - 2 sags

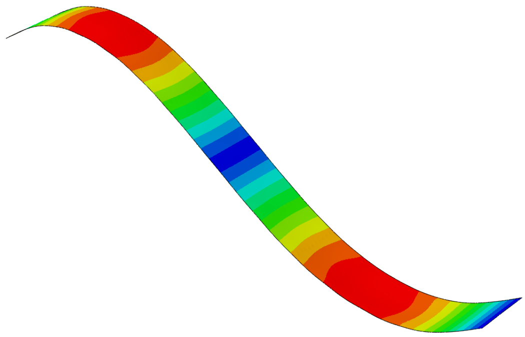


Figure C.9: *Mode 9*

	Frequency [Hz]	
	2.0319	*3.0479*
Load cases	One pedestrian walking across along the middle	One pedestrian running across along the middle
	One pedestrian walking in place at $\frac{3}{4}$ of the length at the middle	

The natural frequency of mode 9 is 6.0957 Hz. A third of that value (2.0319) is within walking range, and half of that value (3.0479 Hz) is within running range.

Mode 10 - Torsional - 1 sag

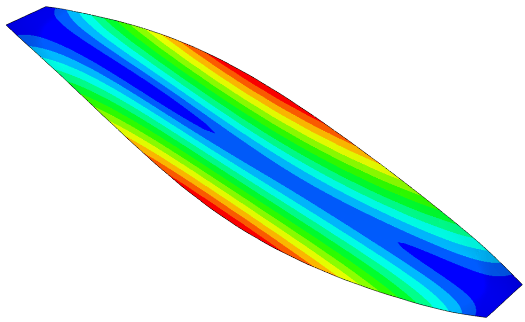


Figure C.10: *Mode 10*

	Frequency [Hz]	
	2.2012	*3.3018*
Load cases	One pedestrian walking across along the left edge	One pedestrian running across along the left edge
	One pedestrian walking in place at the middle of the bridge at the left edge	
	Strip of pedestrians walking across along the left edge	
	Strip of pedestrians walking in place at the middle of the deck along the left edge	

The natural frequency of mode 10 is 6.6035 Hz. A third of this value (2.2012 Hz) is within walking range, and half of that value (3.3018 Hz) is within running range.

Additional analyses

A modal analysis of the bridge with both a frequency and a complex frequency step was performed to obtain the numerical natural frequencies, the corresponding modes and the damping ratios of each mode

In addition an analysis with only one static pedestrian placed at the middle was performed. This was used to find both the mass of the structure through the reaction forces and and the static deflection due to one pedestrian.

D Appendix D - Hand calculations according to design guidelines

The following Appendix shows hand calculations of suggested acceleration values and acceleration limits according to the different design guidelines mentioned in 4. First some characteristic values of the bridge are determined.

Mass of the bridge

The mass of the bridge was calculated by summing all the reaction forces in the vertical direction and dividing by the acceleration of gravity, g. The reaction forces were obtained from *Abaqus* as History Output variables (*RF2*), and the mass was calculated as:

$$M_{bridge} = \frac{\Sigma RF2}{\text{g}} = \frac{2.411 \times 10^6 \text{ N}}{9.81 \text{ m/s}^2} = 245769.6 \text{ kg} = 246 \text{ Tons} \tag{D.1}$$

Damping ratio of the bridge

The chosen value for ζ is 0.5%, which is set to be the damping ratios for mode 1 and 10. All modes in between have a lower damping ratio and all higher modes have a higher damping ratio.

Fundamental frequency of the bridge

The first vertical mode of vibration is mode 6 (Figure C.6).

f_{vert} = 3.7529 Hz

The first horizontal mode of vibration is mode 5 (Figure C.5).

f_{hor} = 3.6216 Hz

D.1 Eurocode

The formulas in this first section are from Annex B in Eurocode 5, part 2 - Vibrations caused by pedestrians.

Vertical acceleration for one pedestrian crossing the bridge

$$a_{vert,1} = \frac{100}{M\zeta} = \frac{100}{245769.6\text{ kg} \times 0.005} = 0.08\text{ m/s}^2 \tag{D.2}$$

Vertical acceleration for a group of pedestrians crossing the bridge (n = 13)

$k_{vert} = 0.5$

$$a_{vert,n} = 0.23 a_{vert,1} \times n \times k_{vert} = 0.23 \times 0.08 \times 13 \times 0.5 = 0.1196\text{ m/s}^2 \tag{D.3}$$

Vertical acceleration for a stream of pedestrians crossing the bridge (n = 0.6A)

$A = 60 \times 5 = 300\text{ m}^2$
$k_{vert} = 0.5$

$$a_{vert,n} = 0.23 a_{vert,1} \times n \times k_{vert} = 0.23 \times 0.08 \times 0.6 \times 300 \times 0.5 = 1.656\text{ m/s}^2 \tag{D.4}$$

Vertical acceleration for a single pedestrian running across the bridge

$$a_{vert,1} = \frac{600}{M\zeta} = \frac{600}{245769.6 \times 0.005} = 0.48\text{ m/s}^2 \tag{D.5}$$

No horizontal accelerations were calculated since the fundamental horizontal frequency is outside the limits stated in EC5 (0.5 Hz $\le f_{hor} \le$ 2.5 Hz).

D.2 British Annex to Eurocode

Maximum vertical acceleration according to UK National Annex to Eurocode 1. Actions on structures. Traffic loads on bridges

$$a_{limit} = 1.0 k_1 k_2 k_3 k_4 \text{ m/s}^2 \tag{D.6}$$

$$\text{and } 0.5 \text{ m/s}^2 \leq a_{limit} \leq 2.0 \text{ m/s}^2$$

Where:

k_1, k_2, k_3 are the response modifiers taken from Tables 4.2, 4.3 and 4.3 in which:

k_1 = site usage factor
k_2 = route redundancy factor
k_3 = height of structure factor
k_4 is an exposure factor which is to be taken as 1,0 unless determined otherwise for the individual project.

The factors need to be carefully considered, so here are the lower and upper bounds presented:

$0.6 \leq$	k_1	≤ 1.6
$0.7 \leq$	k_2	≤ 1.3
	k_3	$= 0.7$
$0.8 \leq$	k_4	≤ 1.2

Which yields the following boundaries:

$$a_{limit,upper} = 1.0 \times 1.6 \times 1.3 \times 0.7 \times 1.2 = 1.747 \text{ m/s}^2 \tag{D.7}$$

$$a_{limit,lower} = 1.0 \times 0.6 \times 0.7 \times 0.7 \times 0.8 = 0.235 \text{ m/s}^2 \tag{D.8}$$

The lower bound is outside the domain stated above, which means that for this particular bridge the following is true according to british standards:

$$\text{and } 0.5 \text{ m/s}^2 \leq a_{limit} \leq 1.747 \text{ m/s}^2$$

Lateral lock-in stability boundaries

As can be seen in Figure 4.3, the bridge is considered stable against lateral responses due to crowd loading if the fundamental horizontal frequency is above 1.8 Hz, which is the case. Therefore no horizontal accelerations were calculated.

D.3 BS 5400

Maximum vertical acceleration

$$a_{vert} \leq 0.5\sqrt{f_{vert}} = 0.5 \times \sqrt{3.7529} = 0.969 \text{ m/s}^2 \tag{D.9}$$

BS 5400 proposes a simplified method for calculating the vertical acceleration in the bridge, which happens to be the same as the one in Håndbok N400. Calculations are therefore superfluous.

D.4 Handbook N400

Pedestrian bridges are to be designed so that the vertical acceleration, a_{vert}, meets the following requirement:

$$a_{vert} \leq 0.25 f_{vert}^{0.7762} = 0.25 \times 3.7529^{0.7762} = 0.698 \text{ m/s}^2 \tag{D.10}$$

The acceleration of the bridge can be calculated as

$$a_{vert} = 4\pi^2 f_{vert}^2 W_s K \psi r \tag{D.11}$$

Where W_s is the static deflection from a concentrated force equal to 700 N, K is a factor dependent on the number of spans and the ratio between the different spans, ψ is a dynamic factor dependent on the span length and damping ratio and r is a correctional factor for the calculated acceleration, a function of the frequency. In this case, W_s was obtained from an analysis performed in *Abaqus* with one static pedestrian standing on the bridge, midspan.

$$\begin{aligned} f_{vert} &= 3.7529 \text{ Hz} \\ W_s &= 0.021 \text{ mm} \\ K &= 1 \text{ (Always 1 for one span)} \\ \psi &= 15 \text{ (see Figure 4.4)} \\ r &= 1 \text{ (see Subsection)} \end{aligned}$$

$$a_{vert} = 4\pi^2 \times 3.7529^2 \times 0.021 \times 10^{-3} \times 1 \times 15 \times 1 = 0.175 \text{ m/s}^2 \tag{D.12}$$

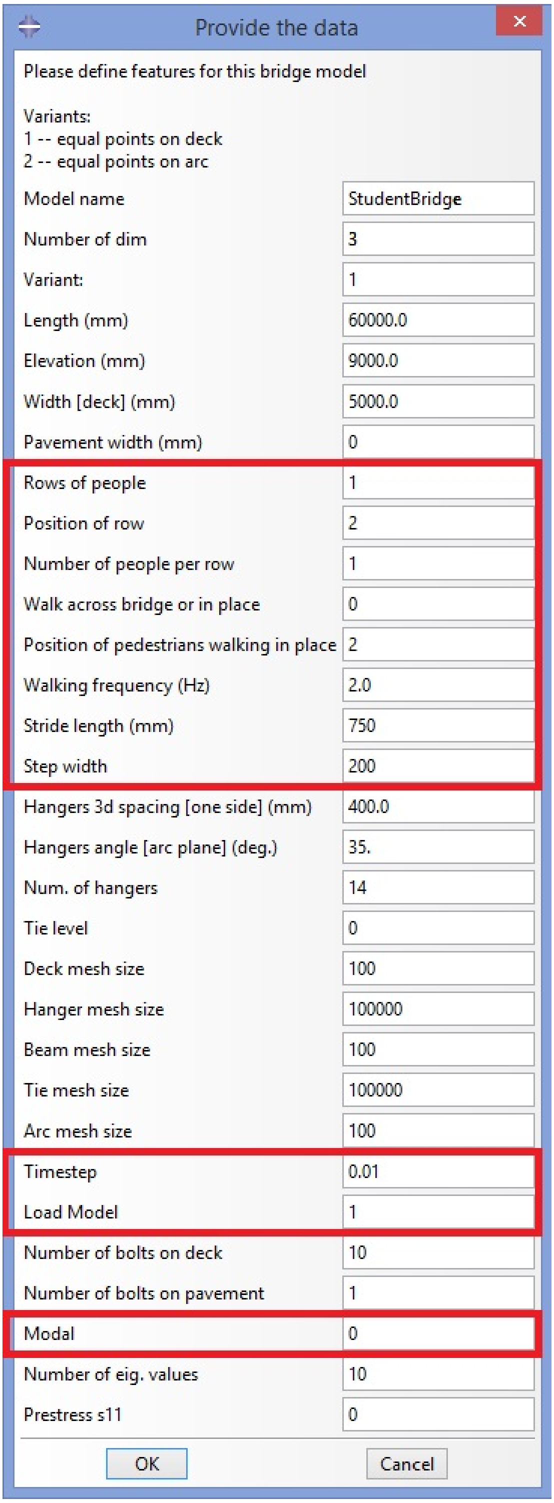
No horizontal accelerations were calculated since the fundamental horizontal frequency is outside the limits stated in HB400 (0.5 Hz $\leq f_{hor} \leq$ 1.3 Hz).

E Appendix E - Running the script

This is a short guide on how to use the attached Python script to generate a *Abaqus* model. Most of this text is written by Anna Ostrycharczyk, and the authors have just added their changes. To run a script in *Abaqus*, click "File", then "Run script...". When running the script, the user is met with the parameter selector shown in Figure E.1. The parameters that have been added to the original script are marked with a red box.

The Python script, StudentBridgeVol14.py, allows the user to create a network arch bridge in *Abaqus*. The aim of the script is to build a bridge model with user-defined geometry and loading. There are a few different bridge layouts (described in the Python script) based on different hanger configurations. All possible layouts are described below.

In addition to creating the geometry, the script also prescribes basic features for physical elements in *Abaqus* model. Those features are defined by script author (considered to be most favourable) and can be changed in the *Abaqus* environment after the geometry of model is generated, or before – in the Python script. Changing and editing the script must be done with care. Incorrect changes in the script might result in mistakes during the generation of the model in the *Abaqus* environment. If user decides to change the script, there are comments that begin with ##PARAM above the parts of the script where the most basic parameters can be found. There are also comments that begin with ##AA which are comments by Anna, or ##KVDP!! which are comments by the authors. The comments are ignored when running the script.


Provide the data

Please define features for this bridge model

Variants:
1 -- equal points on deck
2 -- equal points on arc

Model name	StudentBridge
Number of dim	3
Variant:	1
Length (mm)	60000.0
Elevation (mm)	9000.0
Width [deck] (mm)	5000.0
Pavement width (mm)	0
Rows of people	1
Position of row	2
Number of people per row	1
Walk across bridge or in place	0
Position of pedestrians walking in place	2
Walking frequency (Hz)	2.0
Stride length (mm)	750
Step width	200
Hangers 3d spacing [one side] (mm)	400.0
Hangers angle [arc plane] (deg.)	35.
Num. of hangers	14
Tie level	0
Deck mesh size	100
Hanger mesh size	100000
Beam mesh size	100
Tie mesh size	100000
Arc mesh size	100
Timestep	0.01
Load Model	1
Number of bolts on deck	10
Number of bolts on pavement	1
Modal	0
Number of eig. values	10
Prestress s11	0

OK Cancel

Figure E.1: *Parameters to be selected for running an analysis*

To generate the *Abaqus* model of the bridge, the user needs to run the script and complete the table with various data. The explanation below will helpto generate your own bridge with correct input data.

1. MODEL NAME
– default name is StudentBridge; it is suggested to change it whenever a new model is built to avoid overwriting models due to the same name.

2. NUMBER OF DIM
- number of dimensions that can be used are:

3	⇒	standard model 3D (3D elements in 3D space)
2	⇒	standard model 2D (2D elements in plane)
0	⇒	3D model of one arch with hangers joined to the ground by BC (3D elements in 3D space)

3. VARIANT
- variant of points distribution on the arch/deck; there are two basic layouts of hanger configuration: hangers equally distributed on the deck level, or on the arch

1	⇒	in-plane inclined hangers equally distributed on the deck level
2	⇒	in-plane inclined hangers equally distributed on the arch
111	⇒	in-plane inclined hangers in 'rhombic' configuration; points equally distributed on the deck level; additional configuration (don't need to be use)
11	⇒	vertical hangers equally distributed on the deck level
22	⇒	vertical hangers equally distributed on the arch

4. LENGTH
- length of the bridge (length of the deck; distance between support points of the arch)

5. ELEVATION - arch elevation

6. DECK WIDTH - deck width

7. PAVEMENT WIDTH – pavement is located outside the arches

8. ROWS OF PEOPLE
- number of rows of people. There is no limit to the amount of rows. For this thesis a maximum of 5 rows has been used.

9. POSITION OF ROW
- determines where on the bridge the row of pedestrian(s) is placed. This only works for one row of people. When there is more than one row they are automatically created such that the spacing between the different rows is equal to the spacing between the outer rows and the deck edge.

1	⇒	row is created along the left edge of the deck.
2	⇒	row is created along the middle (default choice).
3	⇒	row is created along the right end. This is useful for exciting torsional modes of vibration.

10 NUMBER OF PEOPLE PER ROW

11 WALK ACROSS BRIDGE OR IN PLACE
- decides whether or not the loads are moving longitudinally through space.

0	⇒	Pedestrians walking across the bridge, loads created such that they move from one end of the bridge to the other (default choice).
1	⇒	Pedestrians walking in place at a location to be decided by the next parameter. The loads are placed at the same location for the entire duration of the analysis and expanding outwards from specified location with a spacing equal to the step width if several pedestrians are to be modelled

This parameter needs to be set equal to 1 if the loads are supposed to model jumping. The difference between stepping and jumping is selected later, in "Load model".

12. POSITION OF PEDESTRIANS WALKING IN PLACE
-determines were the pedestrian(s) are placed in the longitudinal direction of the bridge deck, only valid if they are walking in place.

1	⇒	Loads are applied at $\frac{1}{4}$ of the length of the deck
2	⇒	Loads are applied at $\frac{1}{2}$ of the length of deck (default choice)
3	⇒	Loads are applied at $\frac{3}{4}$ of the length of the deck

In Figure E.2 the three force components due to a pedestrian walking in place can be seen at $\frac{3}{4}$ of the length of the bridge at the left edge.

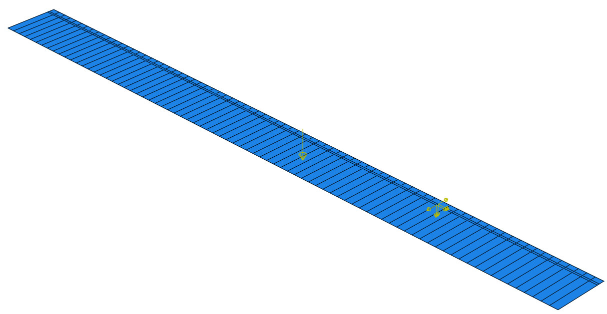


Figure E.2: *The figure illustrates where loads are placed when option 3 is selected. Note that row placement is option 1 (point 9).*

13 WALKING FREQUENCY (Hz) - 2.0 Hz is the default choice

14 STRIDE LENGTH (mm)
-For walking, the stride length is recommended by the authors to be set to 750, which is the default choice
-For running, the stride length is recommended by the authors to be set to 1750

-For jogging, the stride length is recommended by the authors to be set to 1300

15 STEP WIDTH (mm) - Horizontal distance between left and right footstep. 200 is the default choice

16. HANGERS 3D SPACING [one side]
- horizontal distance created by projection of the arch centreline and hanger joining point on the deck level; creation of out-of-plane hanger inclination

17. HANGERS ANGLE
- in variant 1: angle between arch radius and hanger joined on the deck level
- in variant 2: angle between arch radius and hanger joined on the arch level

18. NUMBER OF HANGERS
- in variant 1: define number of points on the deck; model will be built with doubled number of hangers
- in variant 2: define number of points on the arch; model will be built with this number of hangers

19. TIE LEVEL
- tie can be joined to the arch on any level; basic solution is tie joining ends of arch on the deck level (value 0)

20. DECK MESH SIZE – deck mesh size

21. HANGER MESH SIZE – it is desirable to have mesh size bigger than hanger length to obtain one mesh element for each hanger

22. BEAM MESH SIZE – refers to transverse beams

23. TIE MESH SIZE – rule like for hangers apply

24. ARCH MESH SIZE – arch mesh size

25. TIMESTEP - The step time to be used in the dynamic implicit step.

26. LOAD MODEL - determines the shape and amplitude of the force curves.
1 ⇒ Schulze's load model. The default choice
2 ⇒ Bachmann & Amman's load model
3 ⇒ Load model for running (if parameter 11 is set to '0') or jumping (if parameter 11 is set to '1')

27. NUMBER OF BOLTS ON DECK
- number of points, where deck-transverse beam connector is created

28. NUMBER OF BOLTS ON PAVEMENT
- number of points, where pavement-transverse beam connector is created

29. MODAL - a Modal analysis is optional, and can be selected here.

0	⇒	No modal analysis is performed. This is the default choice.
1	⇒	Real and complex frequency analyses are performed instead of a dynamic analysis.

30. NUMBER OF EIGENVALUES
- number of requested eigenvalues in mode analysis

30. PRESTRESS S11
- value of hanger prestress force

Useful information about the script

- Units used in the script: mm, N, t (ton), s, MPa
- Out-of-plane hanger inclination is defined by distance between hangers joint on the deck level and propagation of the centreline of the arch on the deck plane (HANGERS 3D SPACING)
- In-plane hanger inclination is defined by rotation of arch radius around the point on the arch or on the deck
- When out-of-plane inclination is equal zero, script still creates double hangers with the same connecting (to the deck or arch) points; while comparing results for network and spoked arches, the same amount of hangers is required
- Arch is always built on side of the deck; for out-of-plane inclination of hangers, in case when distance between hangers joints on the deck level is wider than arch width projection, this distance determines the position of the arch; in case when out-of-plane inclination of hangers is in projection of arch width, edge of the arch is located on the edge of the deck
- Deck-transverse beam connector length is equal: half of deck thickness + radius of transverse beam; it is created during first build-up process in *Abaqus* and remain constant, even when changes of deck thickness are applied into *Abaqus* model (!)

E.1 Additional tipT in *Abaqus CAE*

Suppressing irrelevant outputs

A useful tip is to suppress the default field and history output created by *Abaqus* as shown below in Figure E.3 and only keep the history outputs created intentionally by the script. This drastically reduces the file size (from about 5-10 GB to 30 MB).

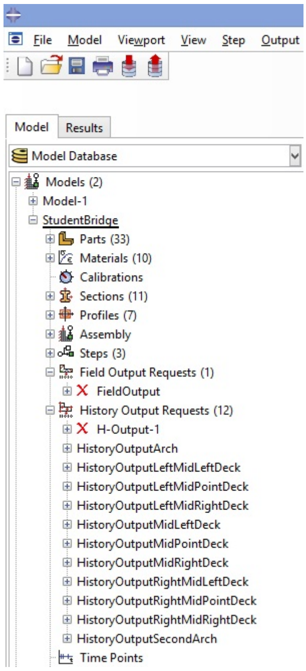


Figure E.3: *Suppresing FieldOutput and H-Output-1 greatly reduces the size of the resulting .odb file*

Reporting data

Figure E.4 shows the possible locations from which to extract output variables. In this case output variable A2 (acceleration in vertical direction) is displayed. There is one set for each arch top, loacated at midspan, and there are three sets across the width of the bridge (MIDLEFTDECK, MIDPOINTDECK, MIDRIGHTDECK), which can be extracted at three different locations along the length of the bridge (RIGHT, MID, LEFT), rendering (RIGHTMIPLEFTDECK, RIGHTMIDPOINTDECK, etc.).

Retrieving the complex eigenfrequencies

When performing a complex modal analysis, the complex eigenfrequencies are found tabulated at the bottom of the .dat file.

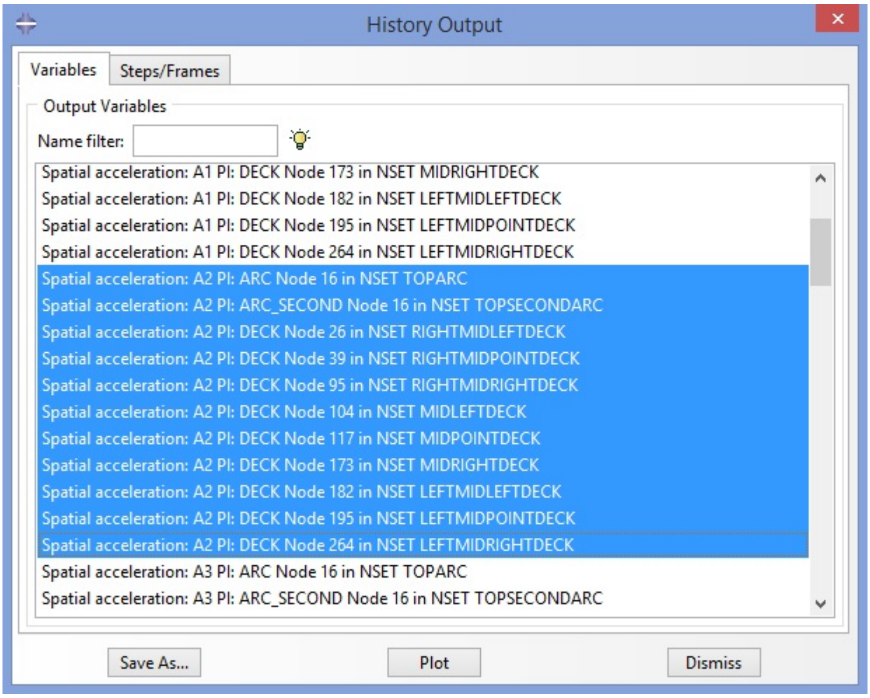


Figure E.4: *All possible locations to extract output variable A2.*

F Appendix F - Digital

This appendix is on digital form on a USB-stick given to supervisor Kjell Arne Malo