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New equations for heat and mass transfer in turbulent pipe and channel flow

Experimental data for large Reynolds numbers and high Prandtl numbers are used to develop an equation which incorporates both the transition region and the region of fully developed flow in pipes and channels.

1. Introduction

In the revision of the "VDI-Wärmeatlas" [1] it was desirable to test whether the equations in this reference work represent the present state of research. For calculation of heat transfer coefficients in turbulent flow through pipes and channels, this heat atlas has hitherto contained an equation by H. Hausen [2], the final modified form of which (1959) reads

$$Nu = 0.037\,(Re^{0.75} - 180)\,Pr^{0.42}\,[1 + (d/L)^{2/3}]\,(\eta_F/\eta_W)^{0.14} \quad (1)$$

Here $Nu = \alpha d/\lambda$ is the Nusselt number, $Re = wd/\nu$ is the Reynolds number, $Pr = \nu/a$ is the Prandtl number, d is the equivalent diameter, L the length and w the velocity of the fluid in the pipe or channel. The thermal conductivity λ, the kinematic viscosity ν, the thermal diffusivity a and the average dynamic viscosity η_F of the flowing medium are to be inserted at the average temperature $\vartheta_m = (\vartheta_i + \vartheta_0)/2$, where ϑ_i and ϑ_0 are the temperatures of the fluid at the inlet and outlet of the tube. In addition, η_W denotes the dynamic viscosity of the fluid at the wall temperature ϑ_W.

The average heat transfer coefficient α for a pipe is defined in terms of the heat flux density q by the equation

$$\dot{q} = \alpha\,\Delta\vartheta_{ln} \quad (2)$$

where $\Delta\vartheta_{ln}$ is the logarithmic-mean temperature difference:

$$\Delta\vartheta_{ln} = \frac{\vartheta_A - \vartheta_E}{\ln\left(\frac{\vartheta_W - \vartheta_E}{\vartheta_W - \vartheta_A}\right)} \quad (3)$$

Hausen developed Equation (1) from the experimental values collected by E. N. Sieder and G. E. Tate [3] for heat transfer in the turbulent pipe flow of liquids. These values are plotted against Equation (1) [2] in Figure 1.

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The range of validity of Hausen's equation is given in the "VDI Heat Atlas" [1] as

$$0 < d/L < 1\,,\ 0.6 < Pr < 10^3,\ 2300 < Re < 10^6$$

On examining the more recent literature two papers were found which provided a further opportunity to test the validity of Equation (1).

W. Hufschmidt et al. [9] reported in 1966 that their experimental values for high Reynolds numbers deviated considerably from the heat transfer coefficients calculated by the Hausen equation. Figure 2 compares their experimental values for water flowing through electrically heated pipes with Equation (1).

In 1969 H. Reinicke [10] published experimental values for heat transfer in the flow of viscous liquids having different Prandtl numbers through short pipes for small temperature differences. The average Nusselt numbers measured by Reinicke for short tubes and water at Pr = 9 are plotted in Figure 3.

It is seen that the experimental values for short pipes, and therefore for large values of the ratio d/L and for relatively small values of the Reynolds number, i.e. in the region between laminar and turbulent flow do not follow the Hausen equation at all, but rather the following theoretically derived equation by E. Pohlhausen [11] for heat transfer in laminar flow in the hydrodynamic and thermal inlet region:

$$Nu = 0.664\,\sqrt[3]{Pr}\,\sqrt{Re\,\frac{d}{L}} \quad (4)$$

E. U. Schlünder [12] has also indicated (1970) that it is necessary to limit the range of applicability of the Hausen equation in the transition region. Figure 3 also contains his equation for heat transfer for thermal development in hydrodynamically developed laminar flow

$$Nu = \sqrt[3]{3.66^3 + 1.61^3\,Re\,Pr\,\frac{d}{L}} \quad (5)$$

This equation provides higher values for the Nusselt number than Equation (4), depending on the Prandtl number and the d/L ratio.

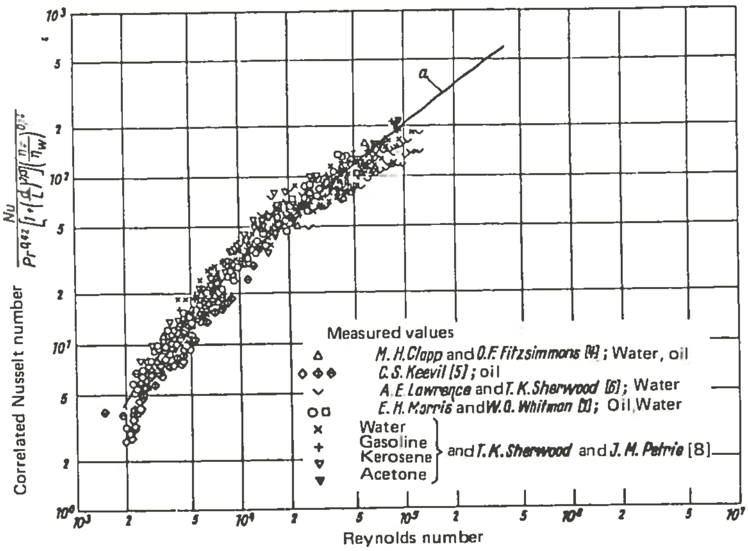


Fig. 1. Nusselt numbers collected by Sieder and Tate [3] compared with Equation (1).

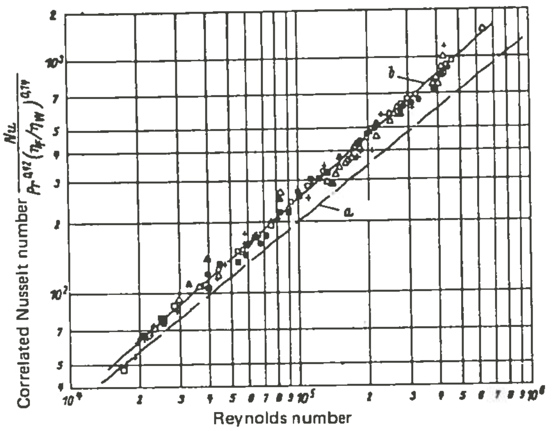


Fig. 2. Nusselt numbers measured by Hufschmidt et al. [9] compared with Equation (1). a) Equation (1) for d/L = 0; b) Smooth curve through the measured points (symbols as in [9]).

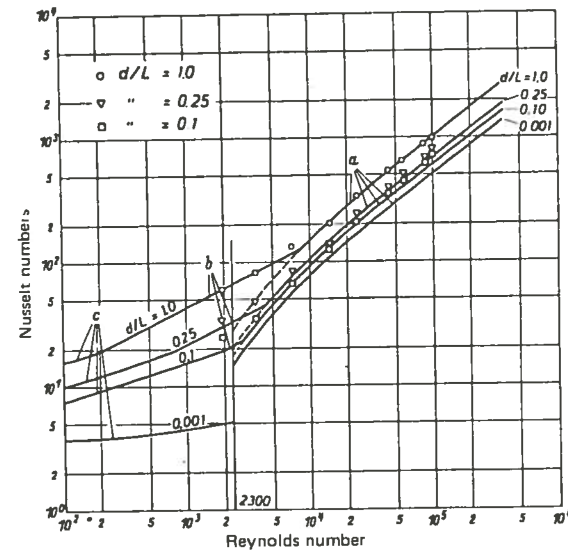


Fig. 3. Nusselt numbers calculated from Equations (1), (4), and (5) for different values of the relative diameter d/L and those measured by Reinicke [10] for Pr = 9. a) Equation (1); b) Equation (4); c) Equation (5); d = pipe diameter; L = pipe length.

2. Evaluation of experimental data

The above findings provided the motive to assemble as many as possible of the recent experimental results for heat transfer in turbulent flow and compare them with each other.

In Figure 4 the experimental values reported by several investigators are plotted as suggested by Hausen [2] together with Equation (1). It can be seen that the isothermal Nusselt numbers deviate appreciably from Equation (1) at high Reynolds numbers. The slope of the smooth line drawn through the experimental values is 0.87. The Nusselt numbers in the region below $Re = 10^4$ differ no more from Equation (1) than do the experimental values of Sieder and Tate [3] plotted in Figure 1. In this region, owing to the effect of inlet effects one must accept a greater scattering of the experimental values. On the other hand, the heat transfer coefficients for gases in turbulent flow, which are plotted in Fig. 5, do not deviate as much at high Reynolds numbers from Equation (1). The slope of the smooth curve drawn through the experimental points for gases is 0.8. This means that at high Reynolds numbers the dependence of the exponent of the Reynolds number on the Prandtl number is very noticeable. It must further be noted that the heat transfer coefficients measured for gases in the transition region $Re \leq 10^4$ do not show the same large decrease as the heat transfer coefficients for liquids plotted in Figure 4. On comparing the experimental results plotted in Figures 4 and 5 it is apparent that to reproduce all

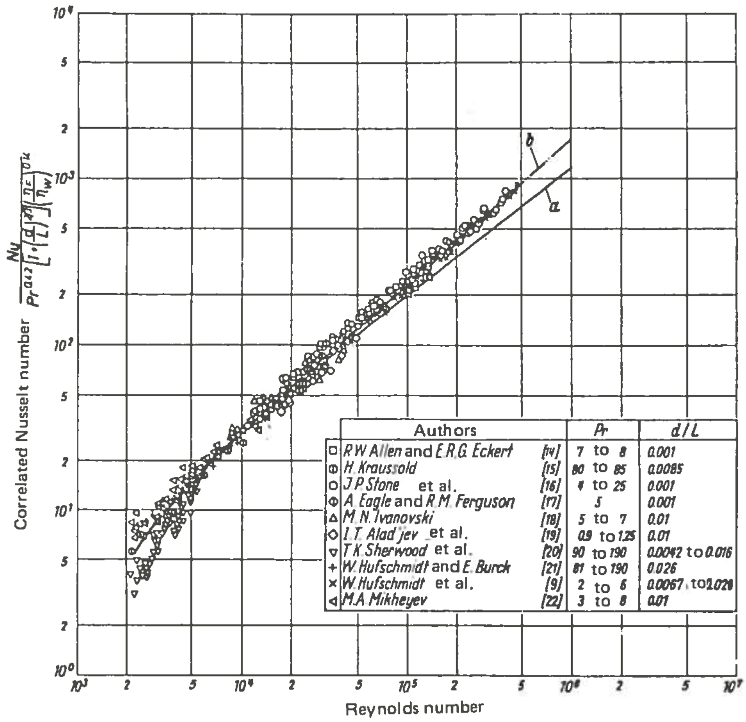


Fig. 4. Heat transfer coefficients for liquids measured by different investigators and those calculated from Equation (1). a) Equation (1); b) Smooth curve through the experimental points. d = pipe diameter, L = pipe length, Pr = Prandtl number.

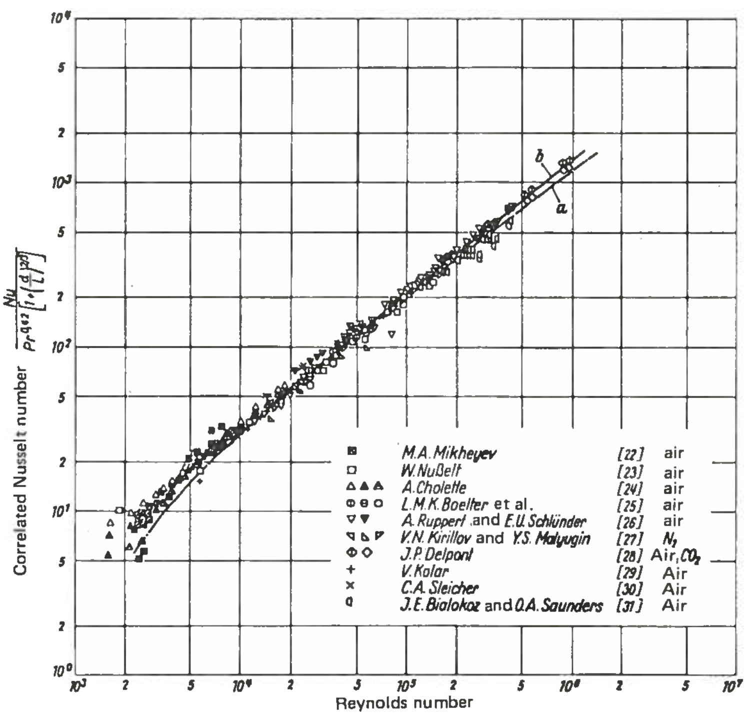


Fig. 5. Heat transfer coefficients measured by various authors for gases and those calculated from Equation (1), (a and b as in Fig. 4).

the experimental values in the literature satisfactorily an equation has to be sought which, like Equation (1) from Hausen [2], correctly describes the decrease in the heat transfer coefficient in the transition region between laminar and turbulent flow in the region Re < 10^4, and which, for large Reynolds numbers takes account of the dependence of the exponent of the Reynolds number on the Prandtl number.

3. Derivation of a new equation

A dependence of the exponent of the Reynolds number on the Prandtl number is produced by the equations for heat transfer which are based on the pressure drop. L. Prandtl [32] presented the first relationship of this type, which reads

$$\frac{\mathrm{Nu}}{\mathrm{Re\,Pr}} = \frac{\xi/8}{1 + 8.7\sqrt{(\xi/8)}\,(\mathrm{Pr}-1)} \quad \cdots \tag{6}$$

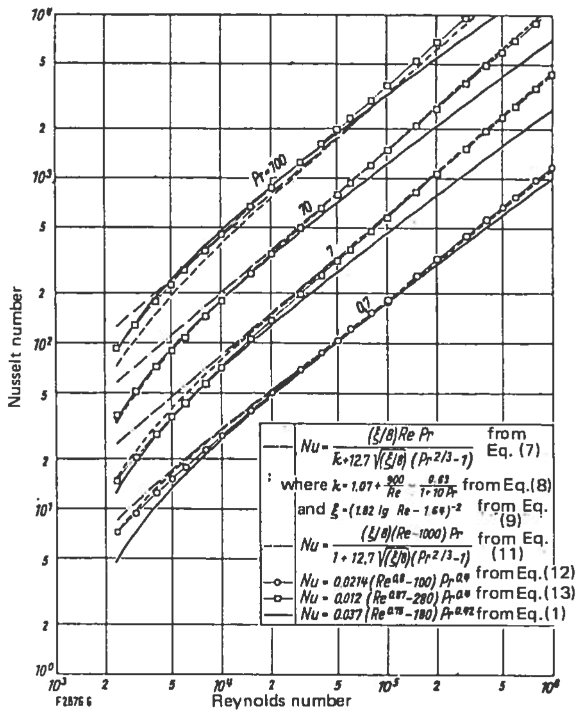


Fig. 6. Nusselt numbers calculated from Equations (7) to (9), (11), (12), (13), and (1) versus the Reynolds numer Re for various Prandtl numbers Pr. ξ = drag coefficient, k = function from Equation (8).

where ξ is the drag coefficient of the tube through which flow is taking place. Since then the Prandtl equation has been further improved.

B. S. Petukhov et al. [33] gave the relationship

$$\mathrm{Nu} = \frac{(\xi/8)\,\mathrm{Re}\,\mathrm{Pr}}{k + 12.7\sqrt{(\xi/8)}\,(\mathrm{Pr}^{2/3} - 1)} \quad \ldots\ldots \tag{7}$$

for heat transfer in fully developed turbulent flow in long pipes, where

$$k = 1.07 + (900/\mathrm{Re}) - [0.63/(1 + 10\,\mathrm{Pr})] \ . \tag{8}$$

The drag coefficient ξ can be calculated from the equation given by G. K. Filonenko [34] for isothermal flows in smooth tubes

$$\xi = (1.82 \lg \mathrm{Re} - 1.64)^{-2} \quad \ldots\ldots\ldots \tag{9}$$

This equation also satisfactorily produces the drag coefficient in the range of applicability of the Blasius equation

$$\xi = 0.3164/\sqrt[4]{\mathrm{Re}} \quad \ldots\ldots\ldots\ldots\ldots \tag{10}$$

W. Hufschmidt et al. [9] used a similar equation by B. S. Petukhov and V. N. Popov [35] to represent their experimental values for Nu.

In Figure 6, Equation (7) is plotted with Equation (1) for various Prandtl numbers. In the region of large Reynolds numbers Equation (7) corresponds exactly to the previously formulated requirement, i.e., irrespective of the Prandtl number Pr, Equation (7) provides higher Nusselt numbers than Equation (1). By modifying Equation (7) correct representation of the experimental values can also be obtained in the range $2300 < \mathrm{Re} < 10^4$. The modified equation

$$\mathrm{Nu} = \frac{(\xi/8)\,(\mathrm{Re} - 1000)\,\mathrm{Pr}}{1 + 12{,}7\sqrt{(\xi/8)}\,(\mathrm{Pr}^{2/3} - 1)} \quad \ldots\ldots \tag{11}$$

is also plotted in Figure 6. For high Reynolds numbers the modified equation (11) is identical with Equation (7); in the transition region $2300 < \mathrm{Re} < 10^4$ Equation (11) also satisfactorily reproduces the decrease in the heat transfer coefficient with decreasing Re, which was represented satisfactorily by Equation (1). It is also to be noted that over the transition region $\mathrm{Re} < 10^4$, Equation (1) does not show the same large decrease for gases ($\mathrm{Pr} = 0.7$) as for liquids ($\mathrm{Pr} = 7$ etc.), as was required earlier.

Using Equation (11), as is also seen, the experimental heat transfer values for turbulent flow in pipes collected so far from the literature can be reproduced over a wide range of Prandtl numbers. However, for approximate calculations, such as the engineer must frequently adopt, Equation (11) is not very convenient. For this purpose the curve from Equation (11) can be reproduced with good accuracy over the technically important range of Prandtl numbers by two equations of the Hausen type. We can use the equation

$$\mathrm{Nu} = 0.0214\,(\mathrm{Re}^{0.8} - 100)\,\mathrm{Pr}^{0.4} \quad \ldots\ldots \tag{12}$$

over the range $0.5 < \mathrm{Pr} < 1.5$ (and thus for gases), and the equation

$$\mathrm{Nu} = 0.012\,(\mathrm{Re}^{0.87} - 280)\,\mathrm{Pr}^{0.4} \quad \ldots\ldots \tag{13}$$

over the range $1.5 < \mathrm{Pr} < 500$ (i.e. for liquids). The behavior of Nu from Equations (12) and (13) is reproduced in Figure 6. Good agreement is found over the cited ranges of validity.

Figure 7 shows the experimental Nusselt numbers collected from the more recent literature for liquids, as compared with the Nusselt numbers calculated from Equation (11). Here Equation (11) was modified further by the correction

$$1 + (d/L)^{2/3}$$

given by Hausen [2] for the effect of pipe length, and by the ratio

$$(\mathrm{Pr}/\mathrm{Pr_w})^{0.11}$$

of the Prandtl number Pr at the average liquid temperature ϑ_m and the Prandtl number $\mathrm{Pr_w}$ at the wall temperature ϑ_w, introduced inter alia by Hufschmidt and Burck [21] and by V. V. Yakovlev [36] to take account of the temperature dependence of the properties in Equation (11). Then Equation (11) reads

$$\mathrm{Nu} = \frac{(\xi/8)(\mathrm{Re} - 1000)\,\mathrm{Pr}}{1 + 12.7\sqrt{(\xi/8)}\,(\mathrm{Pr}^{2/3} - 1)}\left[1 + \left(\frac{d}{L}\right)^{2/3}\right]K \qquad (14)$$

where

$$K = (\mathrm{Pr}/\mathrm{Pr_w})^{0.11} \qquad (15)$$

As was found from the discussion on the experimental results of Reinicke [10] plotted in Figure 3, Equations (4) and (5) give larger Nusselt numbers than Equation (14) for short tubes in the transition region $2300 < \mathrm{Re} < 10^4$. Therefore, the largest value given by Equations (4), (5), and (14) was entered in Figure 7 as the calculated Nusselt number. Nearly 90 pct of the approximately 800 experimental values in Figure 7 differ by less than ±20 pct from the calculated values.

Furthermore, the experimental values of Sieder

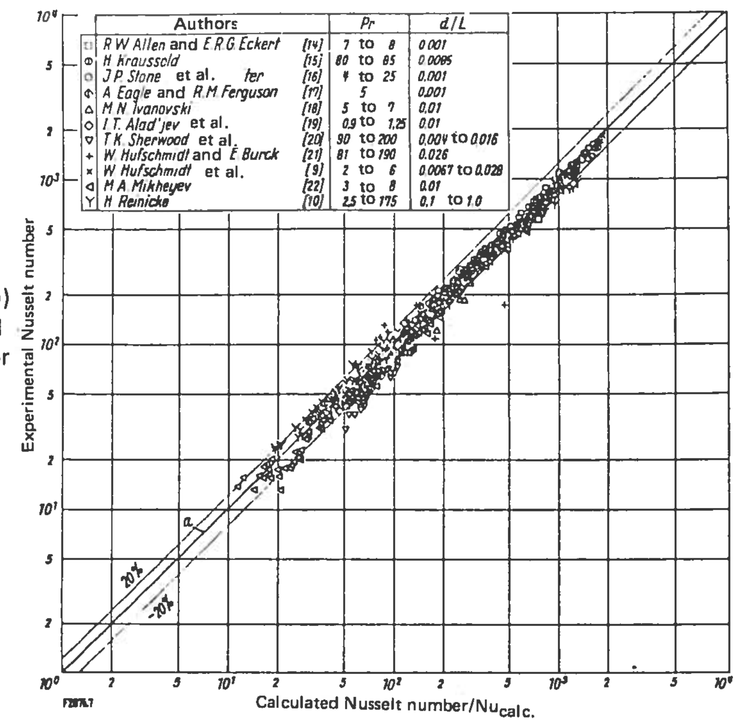
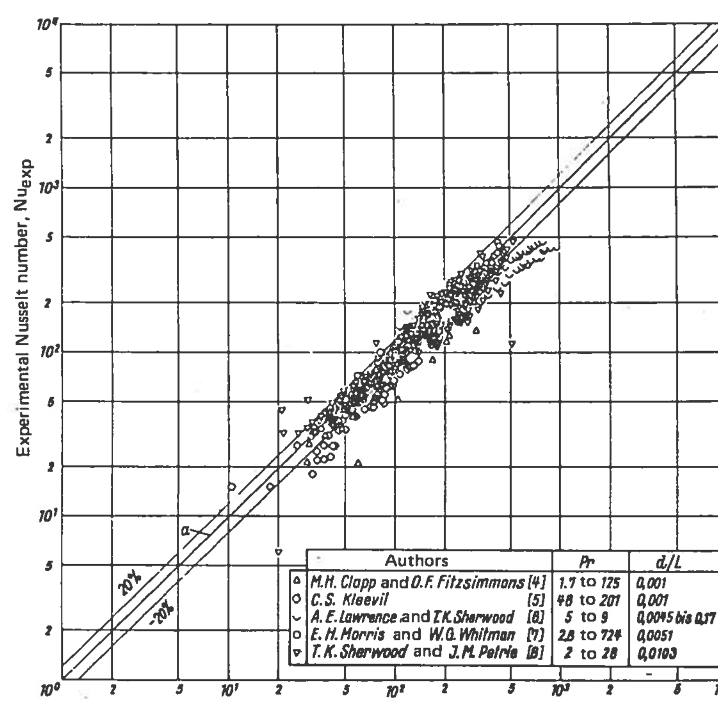
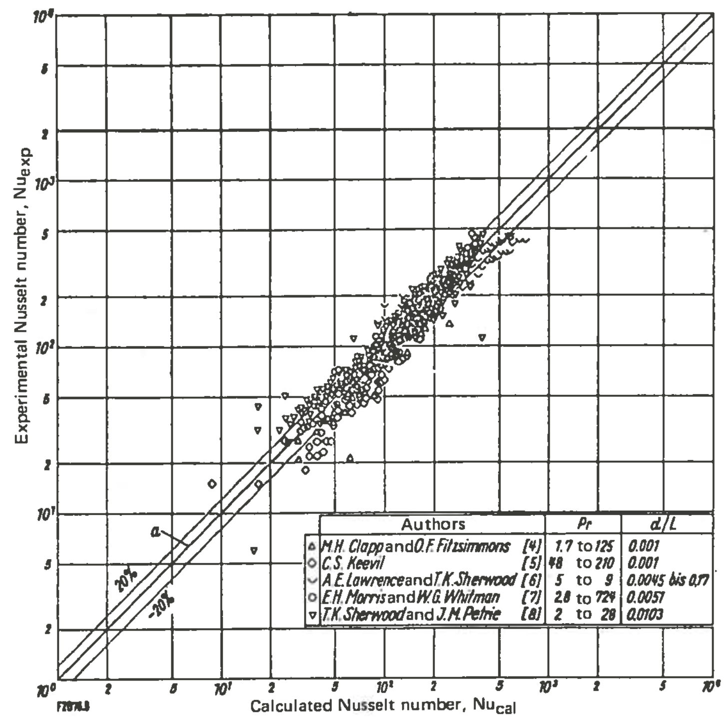


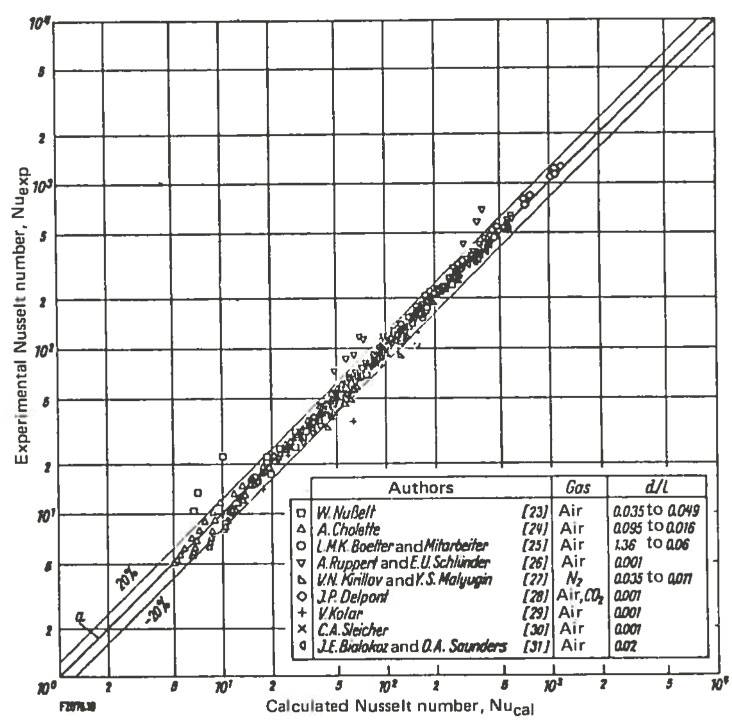
Fig. 7. Comparison of Nusselt numbers calculated from Equation (4), (5) or (14) (each incorporating Equation (15)) with the Nusselt numbers measured for liquids. Pr = Prandtl number, d = pipe tube diameter, L = pipe length, a = line for $\mathrm{Nu_{calc}} = \mathrm{Nu_{exp}}$.

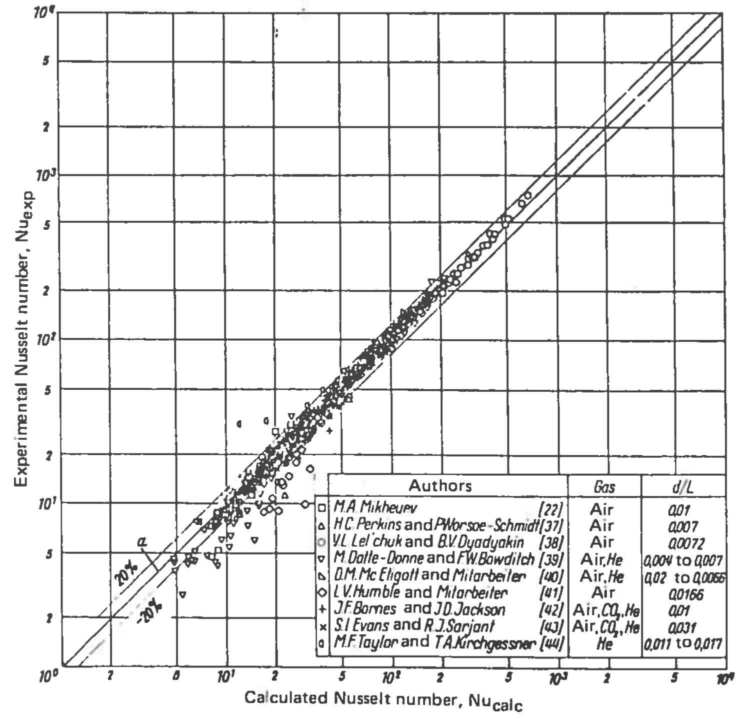


and Tate [3] in Figure 8, which Hausen used to formulate Equation (1), differ no more from the values calculated from Equation (14) than from the values determined from the Equation (1). This is proven by comparing Figures 8 and 9; the Nusselt numbers calculated from Equation (1) were entered in Figure 9. Figure 10 compares the Nusselt numbers measured for gases flowing through pipes with small differences between the average gas temperature and the wall temperature against the Nusselt numbers calculated from Equation (14). Here also good agreement is shown between experiment and calculation.

For heat transfer between liquids and solid walls it is possible to consider the dependence of the properties on temperature by the ratio of the







Prandtl numbers in Equation (15) at the average liquid temperature and the wall temperature, because only the viscosity of the liquid depends very much on temperature, and large temperature differences are not usual for convective heat transfer. This is not true for gases because all properties affecting heat transfer depend greatly on temperature and large temperature differences frequently occur between the gas and the wall. In the literature it has often been proposed that the temperature dependence of the materials properties be taken account of by the ratio of the average absolute temperature T_m of the flowing gas to the average absolute temperature T_W of the pipe wall.

An evaluation of the heat transfer coefficient of gases collected from the literature, which were measured for large temperature differences, showed that these experimental values can also be reproduced by Equation (14) if instead of the correction factor $K = (Pr/Pr_W)^{0.11}$ from Equation (15), we substitute the factor

$$K' = (T_m/T_W)^{0.45} \quad \ldots\ldots\ldots\ldots \quad (16)$$

into Equation (14) for $0.5 < T_m/T_W < 1.5$.

The experimental results of various investigators so evaluated are plotted in Figure 11. Here also good agreement is obtained between the measured and calculated Nusselt numbers. R. Gregorig [45] has constructed a nomogram by means of which account can be taken of the effect of the temperature dependence of the properties of gases on heat transfer

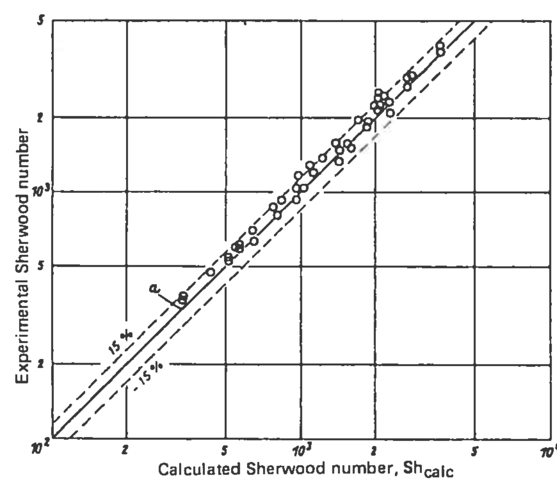


Fig. 12. Comparison of the Sherwood numbers calculated from Equation (14), rewritten for mass transfer (without the correction factor $(Sc/Sc_W)^{0.11}$), and the experimental Sherwood numbers of Harriot and Hamilton [46] a = line for $Sh_{exp} = Sh_{calc}$ where

$$Sh = \frac{(\xi/8)\,(Re - 1000)\,Sc}{1 + 12.7\sqrt{(\xi/8)}\,(Sc^{2/3} - 1)}\left[1 + \left(\frac{d}{L}\right)^{2/3}\right]$$

corresponding to Equation (11) for the Nusselt number (with ξ as the drag coefficient, Re as the Reynolds number, and Sc or Sc_W as the Schmidt number at the average liquid temperature respectively).

both for very large and very small temperature ratios T_m/T_W.

The analogy between heat and mass transfer can be used to calculate the heat transfer coefficients from Equation (14). Here the Sherwood number $Sh = \rho d/\delta$ replaces the Nusselt number Nu in Equation (14) and the Schmidt number $Sc = \nu/\delta$ replaces the Prandtl number Pr (with ρ as the mass transfer coefficient and δ as the diffusion coefficient). P. Harriott and R. M. Hamilton [46] determined the mass transfer between a tube made from benzoic acid and a glycerol solution flowing therein. The measured Sherwood numbers reported by them are plotted in Figure 12 against the Sherwood numbers calculated from Equation (14). Although the Schmidt numbers were varied between 430 and 98,000, the experimental Sherwood numbers do not differ by more than ±15 pct from the Sherwood numbers calculated from Equation (14).

Summary

Using the numerous experimental values from the literature it has been shown that the average heat and mass transfer coefficients for the turbulent flow of gases and liquids in smooth pipes and channels can be calculated by means of Equation (14) (by replacing Nu and Pr by Sh and Sc for mass transfer). Using the correction factor $K = (Pr/Pr_W)^{0.11}$ for liquids $(0.05 < Pr/Pr_W < 20)$ and $K' = (T_m/T_W)^{0.45}$ for gases $(0.5 < T_m/T_W < 1.5)$ the dependence of the properties on temperature is taken into account.

The range of validity of Equation (14), illustrated by comparison with experimental values, covers $2300 < Re < 10^6$ and $0.6 < Pr < 10^5$. Over the range of application $0.6 < Pr < 1.5$ the experimental Nusselt numbers for gases are reproduced with the same accuracy by the relationship

$$Nu = 0.0214\,(Re^{0.8} - 100)\,Pr^{0.4}\left[1 + \left(\frac{d}{L}\right)^{2/3}\right]\left(\frac{T_m}{T_w}\right)^{0.45} \quad . \quad . \quad . \; . \; . \qquad (15)$$

and over the range of application $1.5 < Pr < 500$ the experimental Nusselt numbers for gases are reproduced by the relationship

$$Nu = 0.012\,(Re^{0.87} - 280)\,Pr^{0.4}\left[1 + \left(\frac{d}{L}\right)^{2/3}\right]\left(\frac{Pr}{Pr_w}\right)^{0.11} \quad . \quad . \quad . \quad . \; . \; . \; . \qquad (16)$$

For short pipes and channels the equation for the thermal development for hydrodynamically developed laminar flow (Equation (5)) or the equation for the thermal and hydrogynamic development for laminar flow (Equation (4)) occasionally provides higher Nusselt numbers than Equation (14), (15), or (16) for turbulent flow over the transition range $2300 < Re < 10^4$, depending on the Prandtl number. The higheat value obtained for the Nusselt number is then always valid.

It is essential to recognize that a generally valid heat transfer equation in the form of products from characteristic qualities and powers proposed by Nusselt [23] cannot be formulated. On the other hand, it has been shown that a generally valid equation can be formulated if we commence from the basic form already developed by L. Prandtl [32].

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SOVIET UNION

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The critical heat flux during the boiling of aqueous solutions at low pressures

An experimental study is reported of the critical heat flux during the boiling of aqueous solutions of sodium chloride, oxalic acid, boric acid and glycerol at pressures of 0.1 to 1.0 atm. Non-dimensional equations are presented to summarize the results.

INVESTIGATIONS have been made of the critical heat flux during the boiling of pure liquids on heating surfaces under conditions of natural convection at low pressures [1-5]. These established that there is an increase in the diameter of the vapor bubbles at separation and a decrease in the number of centers of vapor formation as the pressure is reduced, an explosive nature of the bubble growth [1], an effect of the heater dimensions, and a fall in q_{cr} as the pressure is reduced [6]. The effect of vacuum becomes particularly marked at pressures below 0.3–0.4 atm. abs. [7].

The determination of q_{cr} for aqueous solutions at atmospheric pressure showed [8, 9] that q_{cr} is a function of the molar concentration, the boiling point and the boiling point elevation of the solution. There is no information in the literature on the determination or calculation of q_{cr} for aqueous solutions under vacuum.

The purpose of the present work was to study the relationships governing the critical heat flux during the boiling of aqueous solutions of NaCl, oxalic acid, boric acid and glycerol under vacuum.

The experiments were carried out in an apparatus having automatic recording of the boiling crisis with an electronic device. Boiling was studied in an apparatus with a working volume of solution of about 5 liters on the surface of a heating element in the form of a stainless steel wire of diameter 2 mm. placed horizontally.

In order to prevent leakage of the solution and to provide electrical insulation, the copper rods serving as external leads to the heater passed through gaskets and were tightened by means of ebonite and rubber. The heater wire, which formed part of an alternating current circuit produced by two transformers connected in series.

The automatic boiling crisis recorder shut off the heating element, in order to prevent burn-out; ten experiments were carried out on one heating wire.

The volume of solution was maintained constant by use of a reflux condenser. The experiments were carried out at pressures of 0.1, 0.25, 0.5, 0.75 and 1.0 atm. abs.

One of the main parameters determining the state of a boiling salt solution is the temperature factor $\Delta T/T$ [9]. The experimental results were therefore described by using the dimensionless quantities $q_{cr}(\Delta T/T)$ and X, where X is the molar concentration of the solution. ΔT and T are the boiling point elevation and boiling point of the solution.

The experimental data are shown graphically in double logarithmic coordinates in Figures 1 and 2 in terms of the relationships

$$q_{cr}\left(\frac{\Delta T}{T}\right) = f(X) \text{ and } q_{cr}\left(\frac{\Delta T}{T}\right) = f(P)$$

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