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Ideal thermal efficiency for geothermal binary plants

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Abstract

The Carnot cycle is reviewed as to its appropriateness to serve as the ideal model for geothermal binary power plants. It is shown that the Carnot cycle sets an unrealistically high upper limit on the thermal efficiency of these plants. A more useful model is the triangular (or trilateral) cycle because binary plants operating on geothermal hot water use a non-isothermal heat source. The triangular cycle imposes a lower upper bound on the thermal efficiency and serves as a more meaningful ideal cycle against which to measure the performance of real binary cycles. Carnot and triangular cycle efficiencies are contrasted and the thermal efficiencies of several actual binary cycles are weighed against those of the ideal triangular cycle to determine their relative efficiencies. It is found that actual binary plants can achieve relative efficiencies as high as 85%. The paper briefly discusses cycles using two-phase expanders that in principle come close to the ideal triangular cycle.

Keywords: Geothermal power plant; Binary power plant; Carnot efficiency; Triangular cycle; Trilateral cycle; Thermal efficiency

1. Introduction

This paper presents a basis for comparing geothermal power plants of the binary type with the ideal thermodynamic cycle appropriate for such plants. It is common to assume that the ideal cycle for a binary plant is the Carnot cycle. There are, however, particular aspects of geothermal binary plants that render that assumption inappropriate and which can lead to misleading conclusions. In this paper the common energy-based thermal efficiency has been used as the measure of performance, leaving aside the exergy-based utilization efficiency, which is at least equally useful (Milora and Tester, 1976; DiPippo, 2004).

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Nomenclature

h	specific enthalpy
$\dot{m}$	brine mass flow rate
$\dot{Q}_{IN}$	heat transfer rate or thermal power added to the cycle
$\dot{Q}_{OUT}$	heat transfer rate or thermal power discharged from the cycle
Q^{rev}	reversible heat transfer
S	entropy
T	absolute temperature
T_C	absolute temperature of the brine leaving the plant
T_H	absolute temperature of the heat source
T_L	absolute temperature of the heat sink
T_0	absolute dead-state temperature
$\dot{W}_N$	net work transfer rate or net power

Greek letters

η_{rel}	relative efficiency, Eq. (7)
η_s	expander isentropic efficiency, Eq. (8)
η_{th}	thermal efficiency, Eq. (2)
$\eta_{th,\,act}$	actual thermal efficiency, Eq. (7)
η_{th}^{C}	Carnot cycle thermal efficiency, Eq. (3)
$\eta_{th,mx}^{C}$	maximum Carnot cycle thermal efficiency, Eq. (4)
η_{th}^{TRI}	triangular cycle thermal efficiency, Eq. (5)
$\eta_{th,mx}^{TRI}$	maximum triangular cycle thermal efficiency, Eq. (6)

2. Ideal Carnot cycle

From elementary thermodynamics, it is known that the ideal closed power cycle in terms of thermal efficiency is the Carnot cycle, which was devised by Sadi Carnot in 1824 (Bejan, 1997; Moran and Shapiro, 2004). Carnot's ideal cycle produces the highest thermal efficiency of any cycle operating between a heat source at a temperature T_H and a sink at a temperature T_L. It is important to remember, however, that the Carnot cycle is stipulated to operate between two constant-temperature heat reservoirs, i.e. a heat source and a sink that are thermally so large that they can deliver or accept any amount of thermal energy without changing their own temperatures.

Furthermore, Carnot's remarkable finding applies only to a cycle composed of reversible processes (i.e. no thermodynamic irreversibilities). Carnot's conclusion stems from what is usually called Carnot's theorem, namely, that the efficiency of the ideal thermodynamic cycle can depend only on the temperature of the heat source, T_H, and the temperature of the heat sink, T_L, measured on the absolute thermodynamic temperature scale (Kestin, 1979).

The stipulation of reversible processes means that all heat transfer and work processes must be thermodynamically perfect. For heat transfer, this means that there must be zero temperature difference between the heating (or cooling) medium and the cycle working fluid at every point in the heat exchanger. For work processes, there must be no heat loss and no increase in the entropy

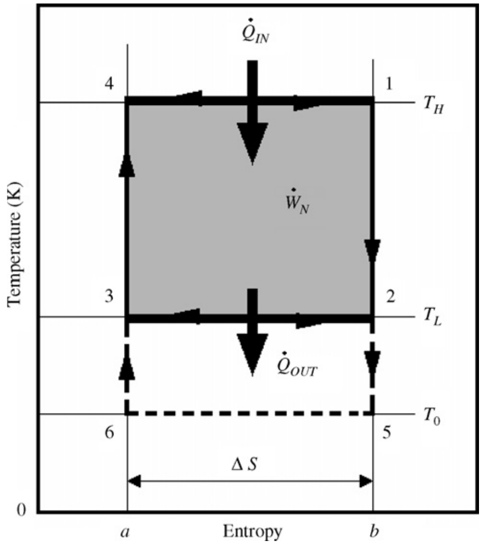

Fig. 1. Ideal Carnot cycle. See text for explanation of symbols.

of the working fluid that would arise from friction or any other dissipative mechanisms within the system. It is impossible, of course, for any real, practical cycle to conform to these constraints.

The Carnot cycle consists of the four reversible processes shown in Fig. 1, a so-called *T–S* diagram having the temperature, *T*, on the ordinate and the entropy, *S*, on the abscissa.

The processes are:

- $1 \to 2$: Isentropic expansion during which work is produced by the cycle working fluid
- $2 \to 3$: Isothermal heat rejection from the working fluid to a cooling medium
- $3 \to 4$: Isentropic compression during which work is performed on the cycle working fluid
- $4 \to 1$: Isothermal heat addition to the working fluid from a heating medium.

As can be seen from the figure, the temperatures of the heating medium and of the cooling medium are identical to that of the working fluid during the processes $4 \to 1$ and $2 \to 3$, respectively. While the working fluid changes from states 4 to 1, the heating medium changes from states 1 to 4, and while the working fluid changes from states 2 to 3, the cooling medium changes from states 3 to 2. This is a practical impossibility for real cycles since there must be a finite temperature difference to drive the heat transfer from one system to the other. Furthermore, the two isentropic work processes ($1 \to 2$ and $3 \to 4$) are unrealistic; irreversibilities, such as generated from friction, can never be completely eliminated and will cause increases in the entropy, even if the systems are perfectly insulated (another practical impossibility). Thus, real cycles have lower efficiencies than the ideal Carnot cycle.

The temperature labeled T_0 is the lowest available temperature of the surroundings, also known as the "dead-state" temperature. It constitutes the lowest temperature that could be used for the heat rejection process, i.e. the lowest sink temperature.

The shaded area 12341 in Fig. 1 represents the net useful work that can be generated by the cycle. The rectangular area 1ba41 lying beneath the process $4 \to 1$ down to absolute zero represents the heat input to the cycle. This area property of the *T–S* diagram is one of the basic consequences of the second law of thermodynamics and follows directly from

$$dS = \frac{\delta Q^{\text{rev}}}{T} \tag{1}$$

where the change in the entropy (a mathematical potential) is given by the amount of reversible heat transfer, Q^{rev}, divided by the absolute thermodynamic temperature associated with each incremental amount of heat transferred. Thus, the area beneath any reversible process in a T–S diagram is equal to the amount of heat transferred during the process (see, e.g. Van Wylen, 1967).

Clearly, as the heat rejection temperature, T_L, is made lower, the net work increases for the same heat input. Since the thermal efficiency, η_{th}, is given in general by

$$\eta_{th} = \frac{\dot{W}_N}{\dot{Q}_{IN}} = 1 - \frac{\dot{Q}_{OUT}}{\dot{Q}_{IN}} \tag{2}$$

η_{th} also increases as T_L is reduced.

Note that Eq. (2) has been written in terms of the rates of work and heat transfer, namely, the mechanical and thermal power, respectively, as indicated by the over-dots. This represents a leap of faith, commonly taken, because in principle no power can be generated from an ideal reversible Carnot cycle because no heat can be transferred in a finite time across a zero temperature difference (Curzon and Ahlborn, 1975; Callen, 1985). The extension from energy to power is tacitly justified because the cycle is an ideal one whose main purpose is to represent the performance limit of real energy conversion systems. As a further note, Curzon and Ahlborn in their paper and Callen in his book discuss the effect of imposing finite temperature differences between the Carnot cycle working fluid and the hot and cold heat reservoirs, an aspect which has not been treated in this paper. They go on to optimize these temperature differences such that the cycle produces the maximum work output for a given set of reservoir temperatures.

It is easy to show (e.g. Moran and Shapiro, 2004) that the efficiency of the ideal Carnot cycle, η_{th}^C, can be expressed in terms of T_H and T_L as follows:

$$\eta_{th}^C = 1 - \frac{T_L}{T_H} = \frac{T_H - T_L}{T_H} \tag{3}$$

It might be noted from Eq. (3) that $\eta_{th}^C = f(T_H, T_L)$ in accordance with the Carnot's theorem.

Since the lowest conceivable heat rejection temperature is T_0, it follows that the maximum ideal Carnot thermal efficiency, $\eta_{th,mx}^C$, is found from:

$$\eta_{th,mx}^C = 1 - \frac{T_0}{T_H} = \frac{T_H - T_0}{T_H} \tag{4}$$

for a given heat source temperature, T_H.

Everything discussed in this section is based on the thermodynamics of the processes that constitute the ideal cycle and are independent of the working fluid used in the cycle. Indeed, this is another of the fundamental principles of the Carnot cycle, namely, that the efficiency of a reversible Carnot cycle does not depend on the nature of the cycle working fluid (Kestin, 1979).

3. Ideal triangular cycle

When geothermal binary plants are considered, it is recognized that the heating medium is not an isothermal source, but rather a fluid (e.g. a hot brine) that cools as it transfers heat to the cycle

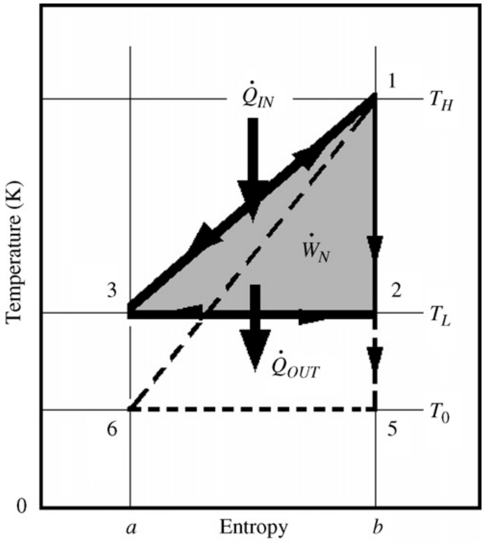

Fig. 2. Ideal triangular cycle. See text for explanation of symbols.

working fluid. The ideal cycle appropriate for such a case is a modification of the rectangular cycle shown in Fig. 1, namely, one of a triangular shape as shown in Fig. 2. It is obvious that the triangular cycle 1231 must have a lower thermal efficiency than the ideal Carnot cycle for a given pair of temperatures, T_H and T_L.

In Fig. 2, the cycle 1231, sometimes called the "trilateral" cycle (Smith, 1993; Brown and Mines, 1998), is composed of three processes, 1 → 2, 2 → 3 and 3 → 1, the first two of which are the same as in the ideal Carnot cycle. Process 3 → 1 represents the heating process for the cycle working fluid and, in the opposite direction (1 → 3), the cooling process for the heating medium. This is true because in the ideal case of reversible heat transfer, as mentioned earlier, there can be no temperature difference between the heating medium and the cycle working fluid at any point along this process. The discharge temperature of the brine, T_3, can be no lower than the condensing temperature of the working fluid, T_L. In the ideal case, these two temperatures are equal.

It is easy to show (DiPippo, 1989) that the thermal efficiency of the ideal triangular cycle 1231 is given by

$$\eta_{th}^{TRI} = \frac{T_H - T_L}{T_H + T_L} \tag{5}$$

It is obvious that the optimum energy conversion efficiency of the triangular cycle would occur if the brine could be cooled down to the dead-state temperature, T_0, thus allowing all of the waste heat to be discharged at the lowest possible temperature. The cycle 1561 represents the maximum-efficiency triangular cycle, given the temperature of the heat source and the prevailing dead-state temperature. The thermal efficiency for this cycle is:

$$\eta_{th,mx}^{TRI} = \frac{T_H - T_0}{T_H + T_0} \tag{6}$$

The units of the temperatures appearing in Eqs. (3)–(6) must be absolute, i.e. Kelvins, K or degrees Rankine, °R.

At the cold end of the cycle, we have assumed an ideal cycle-heat sink interaction, namely, that the heat rejected from the cycle is absorbed by the surroundings (i.e. a thermodynamic heat sink) that have an infinite heat capacity, allowing it to do so without increasing its own temperature. While this cannot be precisely true in practice, it can be approximated if a sufficiently large flow of coolant is available. Its use here is justified since we are considering the ideal limit case for the triangular cycle.

4. Triangular and Carnot cycle comparison

The Carnot and triangular cycle thermal efficiencies are compared in Fig. 3 for specified values of the condensing and dead-state temperatures.

Obviously, the upper limit on the cycle thermal efficiency is given by the maximum Carnot efficiency curve, labeled "Carnot max". The curve labeled "Carnot" constitutes a lower upper limit since the condensing (heat rejection) temperature remains above the dead-state temperature. However, for geothermal binary plants in which the heat source is non-isothermal, the true upper limit is given by the maximum triangular cycle efficiency, the curve labeled "Triangular max". It is this value that should be taken as the standard efficiency against which to measure the performance of actual binary cycles. The lowest curve in Fig. 3, labeled "Triangular", is for an ideal triangular cycle for which the condensing (heat rejection) temperature is higher than the dead-state temperature. As the condensing temperature is reduced, the lowest curve rises; it approaches the maximum triangular curve as the condensing temperature approaches the dead-state temperature.

From Fig. 3, it can be seen, for example, that for a 150 °C resource (for the condensing and dead-state temperatures chosen), the maximum Carnot efficiency is 30.7% and the Carnot efficiency is 23.6%, whereas the maximum triangular cycle efficiency is 18.2% and the triangular cycle efficiency is 13.4%.

5. Actual thermal efficiencies relative to the triangular cycle limit

Binary plants or those designed to follow the trilateral cycle as closely as possible may be compared against the maximum triangular limit case in terms of a relative efficiency, η_{rel}, which

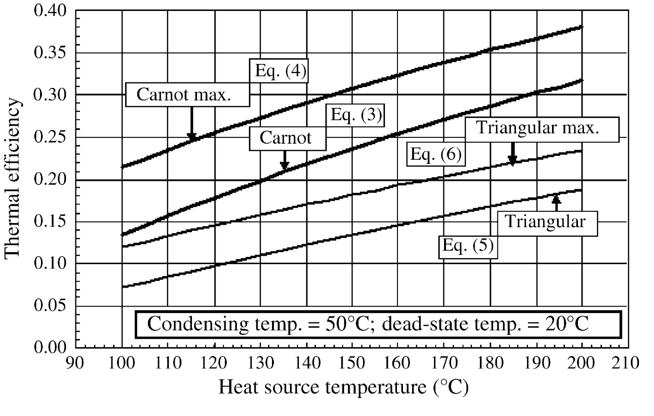

Fig. 3. Thermal efficiencies of Carnot and triangular cycles compared for fixed condensing and dead-state temperatures.

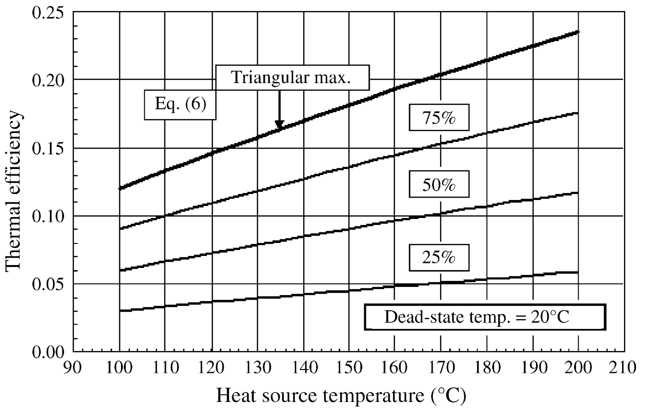

Fig. 4. Thermal efficiency for the optimum triangular cycle and for three values of relative efficiency.

is defined as:

$$\eta_{\mathrm{rel}} = \frac{\eta_{\mathrm{th,act}}}{\eta_{\mathrm{th,mx}}^{\mathrm{TRI}}} \tag{7}$$

Fig. 4 shows several curves for representative values of relative efficiency for a dead-state at 20 °C. Thus, a geothermal binary plant having a brine inlet temperature of 150 °C and a demonstrated thermal efficiency of 13.5% would be operating at a 75% relative efficiency.

The thermal efficiencies of several geothermal binary plants have been published elsewhere (DiPippo, 2004; Moya and DiPippo, 2007), and the relative efficiencies for some of these are shown in Table 1, along with the Steamboat 2 and 3 plants. The average relative efficiency for these examples is about 55%. The dead-state temperatures used correspond to the local conditions appropriate for each plant site.

Fig. 5 illustrates the relationship between the relative efficiency and the resource temperature, where the lower (most recent) value of efficiency has been used for Miravalles Unit 5. It should be noted that the highest (Miravalles) and the lowest (Steamboat) have significantly different parasitic power requirements. Miravalles uses the gravity-fed outflow of waste brine from a set of flash plants, whereas the brine at Steamboat must be pumped from the reservoir and returned to it after the brine leaves the plant. Also, Miravalles has a water-cooling tower, whereas each of Steamboat's two units uses a 120-cell air-cooled condenser, each cell's fan being driven by an 11 kW motor.

Table 1
Relative efficiencies for several geothermal binary plants

Plant and location	Cooling method	T_{H} (°C)	T_0 (°C)	η_{th} (%)	$\eta_{\mathrm{th,mx}}^{\mathrm{TRI}}$ (%)	η_{rel} (%)
Brady Bottom Cycle, NV, USA	ACC	108	16.8	8.0	13.6	59
Heber SIGC, CA, USA	WCT	165	15	13.0	24.8	52
Husavik Kalina, Iceland	WOT	122	5	10.6	17.4	61
Miravalles Unit 5, Costa Rica	WCT	165	23.9	12.8–16.3	19.2	67–85
Nigorikawa, Hokkaido, Japan	WCT	140	13	9.8	18.2	54
Steamboat, NV, USA[a]	ACC	152	23	7.9	14.4	44

ACC, air-cooled condenser; WCT, water cooling tower; WOT, water once-through.
[a] Data from R. Campbell (personal communication), 28 September 2005.

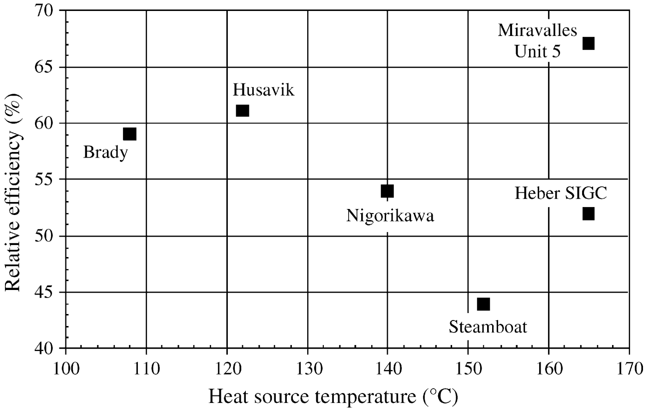

Fig. 5. Relative efficiency for selected binary plants as a function of resource temperature.

6. Practical approaches to implement the triangular cycle

There have been some recent attempts to build machines that would approach the ideal triangular cycle (EISG, 2005; Smith et al., 2005). The *T–S* diagram for such a machine is shown in Fig. 6, along with the liquid–vapor saturation curve for a typical working fluid. The ideal cycle 1231 is unattainable in actuality; a more realistic cycle is 12a31, in which the expansion process, $1 \rightarrow 2a$, is irreversible. The pump work needed to boost the working fluid pressure from that in the condenser to that in the heater is assumed small relative to the work of the expander, and for this ideal cycle is ignored.

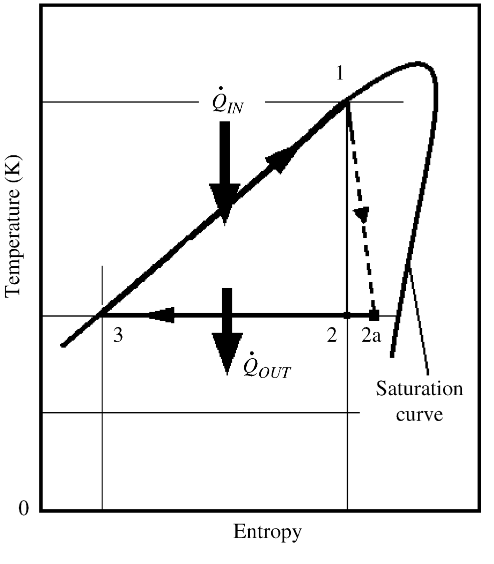

Fig. 6. A triangular cycle using a working fluid having a retrograde saturated vapor line. See text for explanation of symbols.

The process $1 \rightarrow 2a$ is characterized by the isentropic efficiency, η_s, defined as:

$$\eta_s = \frac{h_1 - h_{2a}}{h_1 - h_2} \tag{8}$$

The numerator represents the actual work output of the expander and the denominator represents the ideal work that could be produced if the process were reversible and adiabatic, i.e. isentropic. Steam turbines in state-of-the-art fossil-fueled power plants are capable of achieving isentropic efficiencies as high as 88–90%. Turbines used in organic Rankine binary cycles and in geothermal flash-steam plants typically have efficiencies in the range of 77–82%. However, the process $1 \rightarrow 2a$ is fundamentally different from these in that the working fluid originates as a saturated liquid, not a saturated or superheated vapor.

It is obvious that a cycle as shown in Fig. 6 based on the direct expansion of a saturated liquid has a far better chance of approaching the ideal triangular cycle than does a Rankine-type cycle, owing mainly to the presence of the "pinch point" in the evaporator of the latter (DiPippo, 2005). The main difficulty with direct-expansion systems is the lack of a practical two-phase expander capable of receiving a saturated liquid and efficiently expanding it through the two-phase liquid–vapor region. Positive-displacement expanders, such as a helical-screw expander, suffer from extremely large volume ratios across the machine since the specific volume of the high-vapor-content two-phase fluid at state 2a is much larger than that of the saturated liquid at the inlet state 1 (DiPippo, 1982).

Two-phase expanders have been the subject of much research and development for over 30 years with varying degrees of success (e.g., Hays and Elliott, 1975; Cerini and Hays, 1980). Two-phase expansion machines with at least 75% isentropic efficiency would be widely welcomed in a number of power generation applications including the emerging field of enhanced (or engineered) geothermal systems (EGS) (Sanyal and Butler, 2005; Tester et al., 2006).

7. Conclusions

The Carnot cycle is the standard textbook ideal for power generation. The simplicity of the formula for the maximum thermal efficiency entices many to apply this in certain instances where an alternative is more appropriate. Such a case occurs with geothermal binary plants. There is in this case a lower upper bound on the thermal efficiency; that limit is found from the triangular cycle.

Estimates are sometimes made using Carnot efficiencies to define binary cycle performance for potential applications when it would be better to base such estimates on the maximum triangular cycle efficiency. No matter what ideal cycle is used for finding the ideal efficiency, it is always necessary to apply a relative efficiency to convert from the ideal to the practical. Real binary plants have demonstrated relative efficiencies (based on the maximum triangular cycle) of about $55 \pm 10\%$ over a wide range of resource temperatures. The relatively wide variation is caused by differences in the parasitic power requirements, which are dependent on site conditions.

For the selected case studies with resource temperatures between 100 and 140 °C, the relative efficiency is roughly $58 \pm 4\%$. Thus, one may estimate the efficiency of a binary plant in this range using the approximate formula:

$$\eta_{th} \cong 0.58 \frac{T_H - T_0}{T_H + T_0} \tag{9}$$

The approximate net power may be found from

$$\dot{W}_{\text{net}} \cong 2.47\dot{m}\left(\frac{T_{\text{H}} - T_0}{T_{\text{H}} + T_0}\right)(T_{\text{H}} - T_{\text{C}}) \tag{10}$$

where an average brine specific heat of 4.25 kJ/(kg K) has been adopted, and T_{C} is the brine temperature leaving the cold side of the heat exchangers. Of course, Eqs. (9) and (10) are intended for quick estimates of efficiency and power output and cannot replace detailed heat balance analyses needed for plant design.

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