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Transmission and Power Generation Investment under Uncertainty

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Problem description

Regulated transmission system operators, (TSOs), need to anticipate generation capacity additions and accommodate future growth when building transmission lines.

We study a TSO that maximises social welfare and a power company that maximises profit, and analyse how the two influence each others' investment decisions. The TSO holds an option to invest in a new transmission line, while the power company has installed some generation capacity but holds an option to expand. We use a real options approach and account for game theoretic interactions to find the optimal timing and size of the two investment decisions.

Preface

This master's thesis has been written during the Spring of 2015 as the concluding part of our five year master's degree programme. We are both enrolled in the Industrial Economics and Technology Management programme at the Norwegian University of Science and Technology (NTNU). We have specialised in Production and Quality Engineering as well as Financial Engineering. The thesis has been a unique opportunity to combine our financial background with experience from several mathematics and programming courses. The result of our work is a scientific paper that will be submitted to a peer-reviewed journal. We aim for one of the following journals: European Journal of Operational Research or Journal of Economic Dynamics and Control.

We would like to thank our supervisor, Associate Professor Verena Hagspiel, for providing us with great knowledge, valuable discussions and further insight into the fields of real options and game theory. She has been a great inspirational source. We also thank participants of the 13th Viennese Workshop on Optimal Control and Dynamic games for valuable comments after Hagspiel presented our work in Vienna in May 2015. Furthermore, we appreciate the opportunity to present our work at the 17th British-French-German Conference on Optimization in London this summer.

We also want to express our gratitude to Dr. Afzal Siddiqui at University College London, Professor Peter Kort at Tilburg University and Professor Stein-Erik Fleten at NTNU for their helpful comments.

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Executive Summary

Regulated transmission system operators, (TSOs), need to anticipate generation capacity additions and accommodate future growth when building transmission lines. Given the last decades' deregulation of the European power sectors, EU policymakers are facing the challenge of achieving targets to mitigate climate change since they have relinquished control of the power sector. This challenge has contributed to an increased need for understanding how market participants will respond to TSOs' investment decisions, and how TSOs can accommodate generation expansion and increase adoption of renewable energy technologies among power companies.

We study a market consisting of a welfare-maximising TSO and a profit-maximising power company. The TSO holds an option to invest in a new transmission line, while the power company has installed some generation capacity but holds an option to expand. The proposed model captures both the investment strategies of the TSO and the power company and accounts for the conflicting objectives and game-theoretic interactions of the distinct agents. Taking a real options approach allows us to study the effect of uncertainty on the investment strategies and take into account timing as well as sizing flexibility.

We provide insight to TSOs by studying 1) how much welfare a TSO will forgo by disregarding a power company's optimal investment decision, 2) the effect of uncertainty on optimal transmission and generation investment strategies, and 3) the value of managerial flexibility.

We find that disregarding the power company's optimal investment decision can have a large negative impact on social welfare for a TSO. This is because, in most cases, the TSO will want both agents to invest in a larger capacity than what is optimal for the power company. This implies that the TSO faces a risk of investing in transmission capacity that will be left unused by the power company if it does not consider the power company's optimal capacity decision. The only time we find that the optimal capacity of the TSO is less than that of the power company is if the TSO does not have timing flexibility and is forced to invest at a low demand level. Then, for low uncertainties, the optimal capacity of the TSO is dominating. Furthermore, we find that if the TSO considers only the power company's sizing flexibility and not the flexibility in timing, then it risks investing in a too small capacity. This is because the power company would optimally want to delay investment, and invest in a larger capacity than the TSO anticipates it to install if it assumes that the power company invests at the same time as itself. We furthermore conclude that not only

does a subsidy of the power company's investment cost increase the optimal capacity, but it also triggers earlier investment by the power company. Therefore, a subsidy can be used as a tool to increase social welfare.

We find that increased demand uncertainty leads to an increase in optimal capacity and a delay in investment because of the increased value of waiting. Also the welfare loss from not taking the power company's optimal investment decision into account increases in uncertainty.

This paper extends the theoretical real options literature by considering a two-firm setting with different objectives tackling both timing and capacity choice of the agents. It provides insight into how the conflicting objectives affect the optimal investment strategy of the TSO. Therefore, it is a step in the direction of providing better policies to increase power companies' adoption of renewable energy technologies.

Sammendrag

Når systemansvarlig for kraftnettet tar investeringsbeslutninger knyttet til utbygging av kraftlinjer må den ta hensyn til forventet økning i kraftselskapenes produksjon og legge til rette for vekst. På bakgrunn av de siste tiårenes deregulering av det europeiske kraftmarkedet står EU ovenfor en utfordring knyttet til å nå klimamålene organisasjonen har satt seg ettersom de ikke lenger kontrollerer kraftproduksjonen. Denne utfordringen har bidratt til et økt behov for å forstå hvordan kraftselskapene reagerer på systemansvarlig sine investeringsbeslutninger, og hvordan systemansvarlig kan bidra til å øke kraftproduksjonen samt andelen kraft fra fornybare energikilder.

I denne artikkelen studerer vi et marked som består av en systemansvarlig med mål om å maksimere velferd og et profittmaksimeriende kraftselskap. Systemansvarlig har en opsjon på å bygge ut en kraftlinje, mens kraftselskapet har noe eksisterende produksjonskapasitet i tillegg til en opsjon på å øke kapasiteten. Modellen vi har utviklet inkluderer både systemansvarlig og kraftselskapets investeringsbeslutning. Videre tar den hensyn til at aktørene har motstridende mål og de spillteoretiske aspektene som oppstår på bakgrunn av dette. En realopsjonstilnærming er valgt, noe som gjør det mulig å studere effekten av usikkerhet samtidig som vi tar hensyn til at aktørene har fleksibilitet til å bestemme både tidspunkt for og størrelse på investeringen sin.

Vi gir systemansvarlig bedre innsikt ved å undersøke 1) hvor mye velferd som vil gå tapt dersom systemansvarlig ser bort fra kraftselskapets frihet til å ta sin egen investeringsbeslutning både med tanke på tid og kapasitet, 2) hvordan usikkerhet påvirker hver av de to akørenes optimale investeringsstrategier, og 3) verdien av fleksibilitet knyttet til investeringsbeslutningen for begge aktører.

Vi finner at dersom systemansvarlig ignorerer kraftselskapets frihet til å velge tidspunkt og størrelse på investeringen sin vil det ha en betydelig negativ innvirkning på velferd. Dette fordi systemansvarlig som regel vil at kraftselskapet skal investere i en høyere kapasitet enn det som er optimalt for kraftselskapet med tanke på profitt. Dermed risikerer systemansvarlig å investere i kraftlinjekapasitet som ikke vil bli benyttet av kraftselskapet hvis den ser bort fra kraftselskapets frihet til å bestemme sin egen kapasitet. Unntaket er når systemansvarlig ikke kan velge tidspunkt for sin egen investering, men må investere ved et lavt etterspørselsnivå. For lav usikkerhet vil da systemansvarligs optimale kapasitet være lavere enn den optimale kapasiteten til kraftselskapet. På den annen side finner vi at hvis systemansvarlig kun tar hensyn til at kraftselskapet har mulighet til å bestemme størrelsen

på kapasitetsutvidelsen, men ikke at det har mulighet til å utsette investeringen, risikerer den å bygge en kraftlinje med for lav kapasitet. Det er fordi kraftselskapet optimalt vil utsette investeringen og investere i en høyere kapasitet enn systemansvarlig forutser hvis den antar at kraftselskapet investerer på samme tidspunkt som seg selv. Videre konkluderer vi med at et subsidie av kraftselskapets investeringskostand ikke bare vil føre til at det er optimalt for kraftselskapet å investere i en høyere kapasitet, men også gjøre det optimalt å investere på et tidligere tidspunkt. Derfor kan et subsidie være et verktøy for å øke velferd.

Vi finner at økt usikkerhet i etterspørsel fører til en økning i optimal kapasitet og utsatt investering på grunn av økt verdi av å vente. Også velferdstapet fra å ikke ta hensyn til kraftselskapets optimale investeringsbesluting øker med usikkerhet.

Denne artikkelen utvider den teoretiske realopsjonslitteraturen ved å presentre en modell med to aktører med ulike målsetninger. Den bidrar til innsikt knyttet til hvordan de motstridende målsetningene påvirker systemansvarligs optimale investeringsstrategi. Arbeidet er et skritt på veien mot å kunne utvikle reguleringer som vil bidra til økning av fornybar energiproduksjon.

Contents

Nomenclature

α	Drift rate
β_1	Positive root of the quadratic function $\frac{1}{2}\sigma^2\beta(\beta-1)+\alpha\beta-\rho=0$
δ	Marginal investment cost of the power company
η	Constant > 0 in inverse demand function
γ	Marginal investment cost of the TSO
ρ	Exogenous discount rate
σ	Volatility parameter
τ_P	First time the stochastic variable θ hits the trigger level, θ_P, i.e., the investment timing of the power company
τ_T	First time the stochastic variable θ hits the trigger level, θ_T, i.e., the investment timing of the TSO
θ_P	Investment trigger of the power company
θ_T	Investment trigger of the TSO
K_0	Existing capacity of the power company
K_P	Capacity expansion of the power company
K_T	Capacity of the transmission line exceeding K_0

1 Introduction

Regulated transmission system operators, (TSOs), need to anticipate generation capacity additions and accommodate future growth when building transmission lines. In practice, they are responsible for keeping a balance between generation and consumption of power at every point in time to ensure system stability (Viljanen et al., 2011). A few decades ago, all European electricity industries were regulated, i.e., vertically integrated monopolies, which controlled generation, retail, transmission, and distribution functions. However, deregulation of most industries started early in the 1990s. The goal was to create more efficient markets by introducing competition (Hyman, 2010). By 2002, 80% of the European electricity market was opened to competition, while Denmark, Finland, Germany, Norway, Sweden, and the UK had close to fully deregulated markets (Nord Pool Spot, 2015; Strauss-Kahn and Traca, 2004). However in most countries, transmission is still regulated[1].

At the same time, the European Union (EU) has been facing an increasing pressure of meeting its targets on greenhouse gas emissions, renewable energy, and energy efficiency. In 2007, the EU's strategic Energy Technology Plan (SET-plan) was adopted as a response, to promote research and development (R&D) of renewable energy technologies and to increase their adoption by the market (Huemer, 2012). But, given the deregulation of energy sectors in most EU member states, power companies will only adopt renewable energy technologies if they contribute to their profit-maximising incentives. Thus, when trying to achieve its targets to mitigate climate changes, the EU policymakers face the challenge of having committed to achieve certain environmental standards, while at the same time having relinquished control of the power sector. Furthermore, since renewable technologies like wind, hydro, and solar power are typically geographically dispersed, power companies will invest in such technologies only if there is transmission line capacity available in their geographical area (Kassakian et al., 2011). Transmission investment decisions, however, are typically made by regulated welfare-maximising TSOs. These challenges have contributed to an increased need for understanding how market participants will respond to TSOs' investment decisions and governmental policies, as the EU can no longer influence the power companies' investment decisions directly. As of today, most policy-enabling models of the EU energy

[1]Transmission may be characterized as a natural monopoly (Nelson and Primeaux, 1988). Natural monopolies often arise when fixed costs represent a fundamental proportion of total costs, and they as a consequence are more efficient if operated by a unique player rather than having competitive systems (Rudnick et al., 1995).

system overlook that TSOs and power companies have different objectives, which may lead to flawed market designs[2].

We study a market consisting of one TSO and one power company. The TSO holds an option to invest in a new transmission line, while the power company has installed some generation capacity but holds an option to expand. The power company is dependent on the TSO to invest in grid lines so that it can transmit its power, while the TSO is dependent on the power company as the amount of power transmitted to the market is equal to the production of the power plant. We take into account that the two agents have conflicting objectives as the TSO maximises social welfare while the power company maximises profit. Furthermore, we introduce uncertainty into the model by assuming that future demand is stochastic and consider both optimal timing and size of the two investments. The problem has similarities to a Stackelberg game with the TSO as the leader and the power company as the follower. However in our case, the agents are dependent on each other instead of competing on market share. We solve the problem by taking a real options approach that accounts for game-theoretic interactions. Therewith, we contribute to the theoretical real options literature as, to our best knowledge, we are the first to consider a two-firm setting with different objectives tackling both timing and capacity choice of the agents.

Besides this theoretical contribution, the main goal of the thesis is to provide insights into optimal transmission investment decisions by studying 1) how much welfare a TSO will forgo by disregarding a power company's optimal investment decision, 2) the effect of uncertainty on optimal transmission and generation investment strategies, and 3) the value of managerial flexibility.

We find that if the TSO disregards the power company's optimal investment decision it can have a large negative impact on social welfare. In most cases, the TSO will want both agents to invest in a larger capacity than what is optimal for the power company. Therefore, the TSO faces a risk of investing in transmission capacity that will be left unused by the power company if it does not consider the power company's optimal capacity decision. On the contrary, we find that if the TSO considers only the power company's sizing flexibility and not the flexibility in timing, then it risks investing in a too small capacity. This is because the power company would optimally want to delay investment, and invest in a

[2]E.g., the MARKAL model does not take into account this aspect (Loulou et al., 2004). As of 2011, this model was adapted for use in many countries, including the UK, the US, the Netherlands, Sweden, Australia and Germany (Department of Energy & Climate Change UK, 2011).

larger capacity than the TSO anticipates it to install if it assumes that the power company invests at the same time as itself. Furthermore, we conclude that not only does a subsidy of the power company's investment cost increase the optimal capacity, but it also triggers earlier investment by the power company. Therefore, a subsidy can be used as a tool to increase social welfare.

Moreover, we find that increased demand uncertainty leads to an increase in optimal capacity and a delay in investment because of the increased value of waiting. Also the welfare loss from not taking the PC's optimal investment decision into account increases in uncertainty.

The study contributes to an improved understanding of how conflicting objectives affect the optimal investment strategy of both agents, and the social welfare loss that might occur if TSOs do not anticipate the response of power companies to their investment decisions. The study is a step in the direction of providing better policies to increase power companies' adoption of renewable energy technologies.

We proceed in Section 2 by discussing related literature. In Section 3, we first introduce the assumptions and notation and then formulate the model. In Section 4, the solution approach used to solve the model is described. In Section 5, the analytical expressions for the investment thresholds, corresponding optimal capacities, and the resulting value functions given the possible outcomes of the game are derived. In Section 6, we present numerical results and highlight the economic insight the model provides. Section 7 concludes the paper and offers directions for future research. Detailed derivations, proofs of propositions, as well as analytical solutions to sub-problems that will be introduced in the model section can be found in the Appendix.

2 Related Literature

A number of existing research papers have been motivated by the deregulation of power markets and analyse how deregulation affects the market participants' optimal investment decisions. Most of the existing papers use either a real options or an optimization approach.

Few real options papers consider both transmission and generation investments simultaneously. However, several have studied either transmission planning or generation expansion planning. Siddiqui and Gupta (2007) analyse the effects of deregulating the transmission sector by modelling the investment decision of a private investor holding a perpetual option to construct a transmission line. Using the real options approach, they find optimal investment timing and line capacity under uncertain congestion rents. They compare the cases of limited liability, i.e., the private investor does not incur losses from operating the transmission line, and unlimited liability. Based on their analysis, they conclude that limited liability might be an effective policy tool that induces private investors to invest in transmission lines. Saphores et al. (2004) study a firm that must undergo a costly and time-consuming regulatory process before investing in a transmission line. They consider optimal timing of the stepwise investment, and find that the optimal start of the regulatory review and the project construction depend on the project benefits and the duration of the regulatory review. Boyle et al. (2006) set out a simple analytical framework for incorporating real options in transmission investment decisions but do not address directly the issue of coordination between generation investment and transmission investment. They treat new generation assets as exogenous, i.e., current and projected transmission investments do not influence generation investments. Botterud et al. (2005) present a stochastic dynamic investment model for investments in power generation under both centralised social welfare and decentralised profit objectives. However, they analyse generation without taking transmission capacity into account. Additional to these papers, we consider the interrelation between transmission line and generation investments.

Several papers based on optimisation models including game-theoretic aspects conclude that the interrelation between generation and transmission investments should be considered. Sauma and Oren (2006) include game-theoretic aspects and evaluate the welfare implications of transmission investments based on equilibrium models. They take into account the competitive interaction among generation firms whose decisions in generation capacity investments and production are affected by both transmission investments and the congestion management protocols of the TSO. Their analysis shows that both the size of the welfare gains associated with transmission investments and the location of the best transmission investments might change when the generation expansion response is taken into account.

Sauma and Oren (2007) formulate transmission planning as an optimisation problem using a multistage game-theoretic framework. They consider alternative conflicting objectives and investigate the policy implications of divergent expansion plans resulting from the planner's level of anticipation of strategic responses. They assume that there are several competing power companies maximising profit, and a TSO whose objective is to maximise social welfare while satisfying transmission constraints. Their study shows that optimal transmission expansion plans may be very sensitive to supply and demand parameters. Based on this observation, they also conclude that interrelation between generation and transmission investments should be taken into account when evaluating transmission investment projects.

Other optimisation papers have also considered both transmission and generation planning. One example is Maurovich-Horvat et al. (2015). They compare two markets designs using two bi-level optimisation models. One model with a welfare-maximising TSO and another one with a profit-maximising merchant investor (MI) making transmission investment decisions, while generation investments are determined by wind power companies. They find that social welfare is always higher when the TSO decides transmission investments because the MI has incentives to boost congestion rents by restricting capacities of the transmission lines, which also limits investment in wind power by producers.

Baringo and Conejo (2012) also use a bi-level optimisation model and consider both transmission and generation investments. They however consider a welfare-maximising TSO at the upper level making *both* transmission and wind investment decisions. In addition to them, we take into account that in a deregulated market these investment decisions are typically made by different agents with conflicting objectives.

Compared to Sauma and Oren (2006), Sauma and Oren (2007), Maurovich-Horvat et al. (2015), and Baringo and Conejo (2012), we consider a continuous-time framework and introduce uncertainty into our model. This allows us to consider the timing of investment in addition to capacity. However, in our model, we consider only one power company and not several competing power companies like done in these papers.

In addition to the energy-specific papers mentioned, a range of other real options papers are relevant for our work. The theory of real options determines the optimal time to invest in a given capacity and find that uncertainty generates a value of waiting. Recent contributions in addition determine the optimal size of the investment (Dangl, 1999; Bøckman et al., 2008; Hagspiel et al., 2010; Sarkar, 2011; Chronopoulos et al., 2015). A general result obtained is that when uncertainty increases, firms invest later in a larger capacity. As an example, Dangl (1999) discusses an investment problem in which a profit-maximising firm has to determine both optimal investment timing and optimal capacity choice. He finds that uncertainty in future demand leads to an increase in optimal installed capacity and causes investment to be delayed. Furthermore, our problem is similar to sequential investment problems like Kort et al. (2010) and Chronopoulos et al. (2015) but unlike them we consider that the two investments undertaken at distinct points in time are made by two different agents with conflicting objectives.

Due to the strategic aspect arising in our problem, game-theoretic papers are also relevant for our work (Huberts et al., 2015; Kamoto and Okawa, 2014; Huisman and Kort, 2014). Huisman and Kort (2014) extend the literature that considers both timing and capacity by including competition. They consider both a monopoly and a duopoly case and analyse timing and capacity decisions simultaneously. They find that the capacity level of a social planner is twice the level of the monopolist and that both agents invest in a larger capacity when uncertainty increases. As in Huisman and Kort (2014), we consider two different agents. However, compared to Huisman and Kort (2014), the two agents are not competing on market share. Rather, they are dependent on each other's investment decision both with regards to timing and sizing. Still, one can argue that competition arises in the sense that they have conflicting objectives.

Sinha et al. (2013) consider a similar problem as ours, including a regulating authority and a mining company. Their objectives are conflicting as the regulating authority strives to maximise social welfare through higher taxes and pollution reduction, while the mining company is profit-maximising. This leads to a Stackelberg competition with the regulating authority as the leader. The leader has a first-mover advantage as it can set a tax structure that directly affects the follower's profit and, thus, its investment decision. The problem is solved as a bi-level optimisation problem. Because of extensions they introduce to the model, they do not handle the problem using an analytical approach but rather a bilevel evolutionary algorithm. Our problem differs from Sinha et al.'s (2013) as the TSO's investment decision does not directly affect the power company's profit or investment cost and, thus, neither the investment decision. The TSO only sets lower and upper bounds on the power company's timing and capacity choice, respectively. In addition to Sinha et al. (2013), we introduce uncertainty into the model and derive analytical solutions for the optimal investment strategies.

Review of existing literature reveals a gap in the literature within the field of real options. To the best of our knowledge, we are the first to consider a two-firm setting with different objectives solving for optimal investment strategy for both the leader and the follower with respect to timing and size of investment.

3 Model Assumptions and Problem Formulation

We consider two decision makers, a TSO and a power company (PC), that serve a market characterised by uncertain demand. The TSO holds an option to invest in a new transmission line to connect the capacity of the PC to the main grid and has the flexibility to choose both size and timing of the investment. The PC currently has installed a generation capacity of size K_0 and holds an option to expand. However, the PC receives no profit before the TSO undertakes its investment as we assume that there is no transmission capacity available before the new transmission line is installed[3]. Assuming existing capacity is reasonable as renewable energy sources often are located in remote places were the existing grid is limited and must be replaced in order to meet higher levels of demand. Also, the PC has timing and sizing flexibility with regards to the possible future expansion. Both investments are considered to be irreversible as transmission lines and power plants tend to have low residual values. The problem is similar to a sequential investment problem but with two different agents investing at each step. Moreover, the two agents have different objectives. The TSO strives to maximise social welfare, while the PC maximises profit. We assume perfect information implying that the TSO can anticipate the investment decision of the PC. This adds strategic aspects to the problem, which will influence the TSO's investment strategy since we assume that the TSO makes its investment decision before the PC. Therefore, the problem is similar to a Stackelberg game with the TSO as the leader and the PC as the follower. However, as transmission capacity complements generation capacity rather than substituting it (Chao and Wilson, 2012), the considered problem does not have the same competitive aspects as the traditional Stackelberg model where companies compete on market share. Instead, each agent's value is dependent on the other agent's investment decision. The PC is dependent on the TSO to invest in grid lines so that it can transmit its power, while the TSO is dependent on the PC as the amount of power transmitted to the market is equal to the production of the PC. Still, a competitive aspect arises in the sense that they have conflicting objectives they both want to achieve.

The power plant and the transmission line serve stochastic demand given by the following

[3] In reality there would most likely be an old transmission line available to transmit the initial capacity of the PC before the new transmission line is installed. However, we assume this is not the case as we want to focus on the investment decision of the TSO. Relaxing this assumption would complicate our already complex problem.

inverse demand function[4]:

$$P(\theta_t, K) = \theta_t(1 - \eta K), \tag{1}$$

where θ_t is a stochastic demand shift parameter, $\eta > 0$ is a constant, and K is the capacity of the power plant. We consider a continuous-time framework where the stochastic demand shift parameter is assumed to undergo multiplicative geometric Brownian Motion (GBM) shocks, i.e., $\{\theta_t, t \geq 0\}$ follows a stochastic process of the form:

$$d\theta_t = \alpha\theta_t dt + \sigma\theta_t dW_t, \tag{2}$$

where $\alpha \in \Re$ is the trend parameter or drift, $\sigma \in \Re_+$ is the volatility parameter, and dW_t is an increment of a Wiener process. The current value of the demand parameter is known to the agents, but future values are uncertain and assumed to be log-normally distributed. The stochastic demand shift parameter introduces uncertainty into the investment problem. Furthermore, ρ is the exogenously given discount rate. We assume that $\rho > \alpha$ as otherwise it would never be optimal to invest. Then, both agents would prefer to wait forever. Furthermore, we assume that the PC produces up to capacity[5], K. As we want to focus on deciding optimal timing and size of the investments, and already consider a very complex problem, we refer to Dangl (1999), Sarkar (2009), and Chicu (2012) for further insight into volume flexibility. For ease of notation, production costs are assumed to be implicitly included in the investment cost as the results do not qualitatively depend on them. Also, we assume that the TSO charges the PC no tariffs for transmitting power. Thus, the continuous profit flow of the PC is equal to:

$$\pi(\theta_t, K) = P(\theta_t, K)K. \tag{3}$$

The TSO has to invest in a capacity greater than or equal to K_0. K_T denotes the transmission capacity *exceeding* K_0. Using this notation, the total capacity of the transmission line is equal to $K_0 + K_T$ after the transmission investment is undertaken. Similarly, we

[4] A multiplicative demand function is chosen, i.e., current and future prices depend on the capacity of the PC and, therefore, on the investment decision. This implies an upper bound on quantity, being independent of θ_t, in order to guarantee a positive output price (Boonman and Hagspiel, 2013). See Boonman and Hagspiel (2013) for a broader discussion on demand functions.

[5] This assumption is often called the 'market clearance assumption', and is widely used in the literature, e.g., in Chod and Rudi (2005), Anand and Girotra (2007), Goyal and Netessine (2007), and Boonman et al. (2015).

define K_P as the size of the PC's capacity expansion. Then, the total capacity of the PC is K_0 before expansion, and $K_0 + K_P$ after expansion. As we do not allow for disinvestment, i.e., reduction of the generation and transmission capacity below K_0, we require both K_P and K_T to be positive.

Similar to Sauma and Oren (2007), Huisman and Kort (2014) or Boonman et al. (2015), we assume investment costs for both agents to be linear in capacity. Moreover, we assume that the TSO and the PC face different marginal investment costs. The total investment cost, including operating costs for the TSO, is assumed to be $\gamma(K_0 + K_T)$, while the PC faces an investment cost, including production costs, of δK_P. Note that the PC has already installed a capacity of K_0.

As the TSO's objective is to maximise social welfare, we need to define total surplus. Like Sauma and Oren (2006) and Maurovich-Horvat et al. (2015), we define total surplus as the sum of the consumer and producer surplus net of investment costs for both agents.

Regarding investment timing, we distinguish two possible cases. In case 1, the PC invests later than the TSO, i.e., the investment threshold of the PC, θ_P, is higher than that of the TSO, θ_T. In case 2 the PC invests at the same time as the TSO. The first case will result in three regions, see Figure 1. In the first region, both companies are waiting to invest. In the second region, only the TSO has invested, and an amount of power equal to K_0 is being generated and transmitted. In the last region, the PC has increased its generation capacity by K_P, which contributes to increased total surplus of the TSO and additional profit for the PC, respectively. In case 2 we distinguish only two regions, where both agents have invested in the second region and an amount of power equal to $K_0 + K_P$ is being generated and transmitted, see Figure 2.

Therewith we can formulate the investment problems of the TSO and the PC. First, the

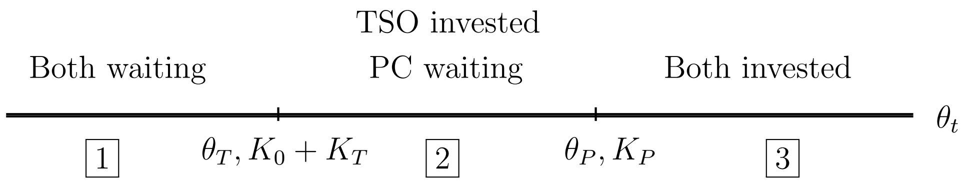


Figure 1: In Region 1, both agents are waiting to invest. In Region 2, the TSO has installed a transmission line with capacity $K_0 + K_T$, however, K_0 is the amount of power being transmitted. In Region 3, the PC has expanded its capacity by K_P and generates and transmits an amount of power equal to $K_0 + K_P$.

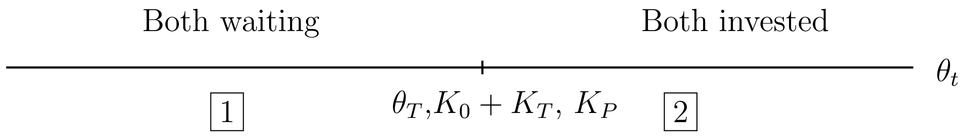


Figure 2: In Region 1, both agents are waiting to invest. In Region 2, both the TSO and the PC have invested and an amount of power equal to $K_0 + K_P$ is being generated and transmitted.

TSO's investment problem at time zero is equal to the following optimal stopping problem:

$$\sup_{\tau_T}\left[\max_{K_T}\mathbb{E}\Bigg[\int_{s=\tau_T}^{\infty} e^{-\rho s} ts(\theta_s, K_0)ds - e^{-\rho\tau_T}\gamma(K_0 + K_T) + \int_{s=\tau_P}^{\infty} e^{-\rho s}[ts(\theta_s, K_0 + K_T) - ts(\theta_s, K_0)]ds - e^{-\rho\tau_P}\delta K_T\Bigg|\theta_0\Bigg]\right], \quad (4)$$

where *ts(.)* denotes the continuous part of total surplus, which is equal to the sum of the continuous parts of the consumer and producer surplus. The first part of Equation (4) is the present value of the total surplus if the PC produces at capacity K_0 forever. The second part denotes the present value of the additional total surplus if the PC expands its capacity to $K_P = K_T$ at time τ_P. The inner optimisation problem states that the TSO at the moment of investment will choose the capacity that maximises the present value of total surplus given that it can choose the size of the PC's capacity expansion, i.e., $K_P = K_T$. In reality, the PC has flexibility to decide its optimal capacity but to solve for the optimal capacity of the TSO, one needs to solve the problem as if the PC will install the TSO's optimal capacity. The outer optimisation problem corresponds to the flexibility of choosing the optimal time to invest in the transmission line.

The solution to the optimal stopping problem is defined by a threshold, θ_T^*. For θ_t levels greater than θ_T^*, we are in the stopping region where it is optimal for the TSO to invest immediately. For $\theta_t < \theta_T^*$, demand is too low to undertake the investment, and it is optimal for the TSO to wait. The TSO invests at the moment θ_t hits the optimal investment level, θ_T^*, the first time. Thus, the optimal investment time, τ_T^*, is equal to the first time the stochastic variable θ hits the optimal level, θ_T^*; $\tau_T^* \equiv \min\{t : \theta_t \geq \theta_T^*\}$. The corresponding optimal capacity is denoted by $K_T^*(\theta_T^*)$.

To find the expression for the total surplus, we start by deriving an expression for the consumer surplus, see Huisman and Kort (2014) for similar derivations. The instantaneous

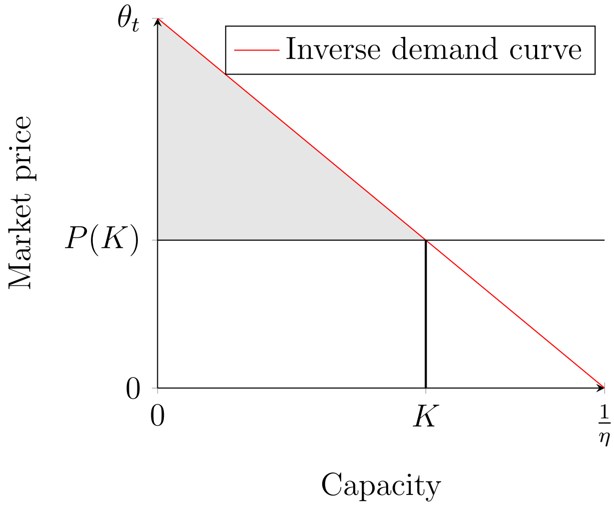


Figure 3: The consumer surplus is equal to the grey area when the inverse demand curve is given by $P(\theta_t, K) = \theta_t(1 - \eta K)$ and the total generation capacity of the PC is equal to K.

consumer surplus is given by $\int_{P(K)}^{\theta_t} K(P)dP$, which is illustrated by the grey area in Figure 3. K is the total generation capacity of the PC and, therefore, the amount of power being generated and transmitted to the consumers. Since $P(\theta_t, K) = \theta_t(1 - \eta K)$, it holds that $K(P) = \frac{1}{\eta}(1 - \frac{P}{\theta_t})$. This leads to the following expression for the instantaneous consumer surplus:

$$cs(\theta_t, K) = \int_{\theta_t(1-\eta K)}^{\theta_t} \frac{1}{\eta}\left(1 - \frac{P}{\theta_t}\right) dP = \frac{1}{2}\theta_t K^2 \eta. \tag{5}$$

Taking into account the three regions where either both are waiting, only the TSO has invested or both have invested, see Figure 1, the instantaneous consumer surplus (cs) at time t is given by:

$$cs(\theta_t, K) = \begin{cases} 0 & \text{if } t \leq \tau_T, \\ \frac{1}{2}\theta_t K_0^2 \eta & \text{if } \tau_T \leq t \leq \tau_P, \\ \frac{1}{2}\theta_t (K_0 + K_T)^2 \eta & \text{if } \tau_P \leq t. \end{cases} \tag{6}$$

The instantaneous part of the producer surplus, on the other hand, is equal to the profit flow of the PC given in Equation (3). At time t it is equal to:

$$ps(\theta_t, K) = \begin{cases} 0 & \text{if } t \leq \tau_T, \\ \theta_t(1 - \eta K_0)K_0 & \text{if } \tau_T \leq t \leq \tau_P, \\ \theta_t(1 - \eta(K_0 + K_T))(K_0 + K_T) & \text{if } \tau_P \leq t. \end{cases} \tag{7}$$

Therefore, the instantaneous part of total surplus is equal to:

$$ts(\theta_t, K) = \begin{cases} 0 & \text{if } t \leq \tau_T, \\ \frac{1}{2}\theta_t K_0^2 \eta + \theta_t(1 - \eta K_0)K_0 & \text{if } \tau_T \leq t \leq \tau_P, \\ \frac{1}{2}\theta_t (K_0 + K_T)^2 \eta + \theta_t(1 - \eta(K_0 + K_T))(K_0 + K_T) & \text{if } \tau_P \leq t. \end{cases} \quad (8)$$

Given the TSO's investment decision, the PC's investment problem at time zero is equal to:

$$\sup_{\tau_P \geq \tau_T} \left[\max_{K_P \leq K_T} \mathbb{E}\left[\int_{s=\tau_T}^{\infty} e^{-\rho s}\pi(\theta_s, K_0)ds \right.\right. \\ \left.\left. + \int_{s=\tau_P}^{\infty} e^{-\rho s}[\pi(\theta_s, K_0 + K_P) - \pi(\theta_s, K_0)]ds - e^{-\rho\tau_P}\delta K_P \middle| \theta_0 \right]\right], \quad (9)$$

where τ_T is the moment in time when the TSO undertakes its investment. The first part of Equation (9) is the present value of the PC if it produces at capacity K_0 forever. The second part denotes the present value of the net additional value of the PC if it expands its capacity by K_P at time τ_P.

The inner optimisation problem corresponds to the flexibility to choose the size of the capacity expansion that maximises the present value of the PC. It will never be optimal for the PC to invest in a larger capacity than that of the transmission line, as it will not be able to transmit the quantity exceeding the transmission line's capacity to the consumers. Hence, we have $K_0 + K_P \leq K_0 + K_T$, which gives $K_P \leq K_T$. The outer optimisation problem corresponds to the flexibility to choose the optimal timing of the investment. The investment timing of the PC is restricted by the TSO's. As we assume that there is no capacity available to transmit power, it does not make sense for the PC to expand capacity before the TSO has installed the new transmission line. Consequently, τ_P is assumed to be greater than or equal to τ_T. The solution to the optimal stopping problem is defined by the threshold, θ_P^*. The optimal investment time of the PC, τ_P^*, is equal to the first time the stochastic variable θ_t hits the optimal level, θ_P^*, $\tau_P^* \equiv \min\{t : \theta_t \geq \theta_P^*\}$. The corresponding optimal capacity expansion of the PC is denoted by $K_P^*(\theta_P^*)$.

In the next section, we describe the solution approach before we present the solution to the problem in Section 5. To gain additional insight into the investment decision of a TSO, we solve several sub-problems in addition to the full problem where we reduce the agents' level of flexibility. An overview of each sub-problem's level of managerial flexibility is given in Table 1. The following sub-problems have been studied:

- *Sub-problem 1:* Both agents face a now-or-never investment decision but have flexibility to choose the capacity size of investment, K_T^* and K_P^*.
- *Sub-problem 2:* Both agents have flexibility to choose the timing of their own investment given by the trigger levels θ_T^* and θ_P^*, but the TSO decides the size of both investments, $K_P = K_T^*$.
- *Sub-problem 3:* Both agents have flexibility to choose the timing of their own investment, but the PC decides the size of both investments, $K_T = K_P^*$.
- *Sub-problem 4:* Both companies have sizing flexibility, K_T^* and K_P^*. The TSO has to invest at time zero, while the PC can choose optimal timing, given by θ_P^*.
- *Sub-problem 5:* Both agents have flexibility to choose sizing, K_T^* and K_P^*, respectively, while the TSO decides timing for both, i.e., $\theta_P = \theta_T^*$

The analytical solutions to the sub-problems are derived in Appendix C.

	Timing		**Sizing**	
	PC	TSO	PC	TSO
Sub-problem 1	θ_0	θ_0	K_P^*	K_T^*
Sub-problem 2	θ_P^*	θ_T^*	$K_P = K_T^*$	K_T^*
Sub-problem 3	θ_P^*	θ_T^*	K_P^*	$K_T = K_P^*$
Sub-problem 4	θ_P^*	θ_0	K_P^*	K_T^*
Sub-problem 5	$\theta_P = \theta_T^*$	θ_T^*	K_P^*	K_T^*
Full problem	θ_P^*	θ_T^*	K_P^*	K_T^*

Table 1: Overview of the sub-problems

4 Solution Approach

Due to the assumption of perfect information, the problem is similar to a two-stage game[6]. The TSO makes its investment decision first, whereas the PC can invest at the same time or later than the TSO. Although the agents choose to invest at the same time, it is assumed

[6]A game is defined as any situation in which players make strategic decisions, i.e., decisions that take into account each other's actions and responses (Pindyck and Rubinfeld, 2009).

that the TSO is the one that decides first, i.e., the leader, and the PC is the follower. The game is similar to a Stackelberg game since it is a sequential game with a leader and a follower. However, it is not a traditional Stackelberg game as the agents do not compete on quantity.

Given that the TSO has invested, the PC cannot influence the investment strategy of the TSO but optimises its own timing and capacity based on the observed investment decision of the TSO. This means that the PC's investment decision includes no strategic aspects. However, when the TSO determines its optimal investment strategy, it takes the PC's reaction into account. Therefore, the problem is solved backwards. First, we find the optimal investment decision of the PC given K_T^* and θ_T^*. Next, we find the optimal strategy of the TSO while taking into account the expected response of the PC.

We derive the following remark:

Remark 1 *The TSO can only affect the PC's investment decision in two ways. 1) Setting a lower bound on the PC's investment timing, θ_T^*, which will force the PC to delay expansion of its generation capacity if θ_P^* is less than θ_T^*. 2) Setting an upper bound on capacity, which will affect the PC's decision if K_T^* is lower than the optimal capacity expansion of the power plant, K_P^*.*

Neither the PC's profit nor its investment cost directly depend on the TSO's investment strategy, i.e., K_T^* or θ_T^*. So, the only way that the TSO can affect the PC is by restricting the size or the timing of the PC's investment decision through the size or timing of the transmission line investment[7]. Given that the TSO installs K_T^* when θ_t hits θ_T^*, the PC will find its corresponding optimal investment decision[8]. The PC can choose among one of the four decisions illustrated in Table 2. Only if decision 1 is optimal will the PC not be bounded by either capacity or timing since the TSO will optimally invest in a larger capacity at an earlier point in time than itself. If one of the other three decisions are optimal, then the TSO's choice of θ_T^* and K_T^* will restrict the optimal decision of the PC.

Due to perfect information, it is the investment strategy of the TSO that eventually

[7] Compared to Sinha et al. (2013), we face a challenge when solving our problem. In their model, the profit of the mining company depends directly on the leader's decision, whereas in our model the PC's value is only affected by the TSO's decision through constraints, i.e., lower and upper bounds.

[8] Note that the investment thresholds and capacities will from now on be written without a star when one of the agents is bounded by the optimal investment decision of the other agent.

Decision 1	Decision 2	Decision 3	Decision 4
$\theta_P^* > \theta_T^*$ $K_P^* < K_T^*$	$\theta_P = \theta_T^*$ $K_P^* < K_T^*$	$\theta_P^* > \theta_T^*$ $K_P = K_T^*$	$\theta_P = \theta_T^*$ $K_P = K_T^*$

Table 2: Overview of possible decisions for the PC given a θ_T^* and a K_T^*

determines which of the decisions the PC will choose, i.e., it can manipulate the investment decision of the PC. Consequently, the TSO will never choose a timing and capacity that will make it optimal for the PC to choose decision 1 or 2 where the TSO ends up having overinvested. To install additional capacity, $K_T^* > K_P^*$, will be reasonable only if the TSO has other ways of utilising the extra capacity, which we assume not to be the case. Thus, it holds that upon investment K_P and K_T will be equal and determined by the lower of the two optimal capacities.

Since the PC's investment decision does not depend on the TSO's decision in a direct way, only through constraints, we choose to solve the problem in two steps. First, we present the analytical solution to each agent's investment problem when solved without taking into account the constraints on capacity. We only consider timing. The reason for including timing, and not capacity, is that the investment problem of the TSO, given in Equation (4), depends directly on the investment timing of the PC, θ_P. Therefore, we can take into account the optimal timing of the PC when solving for the optimal investment strategy of the TSO. Second, we compare the two agents' initial optimal capacities to decide which agent has the power to decide capacity. This decision is based on which one of them has the lower optimal capacity, i.e., the dominating capacity. Next, we update the other agent's investment trigger given that it has to invest in a lower capacity than it initially found optimal. Either it holds that the PC will delay investment beyond the TSO, or it is optimal for the PC to invest as soon as the TSO has invested. The possible outcomes of the game are summarised in Table 3. In outcomes 1 and 2, the TSO's optimal capacity choice is dominating, whereas in outcomes 3 and 4, the PC has the power to decide capacity.

TSO decides capacity		PC decides capacity	
Outcome 1	Outcome 2	Outcome 3	Outcome 4
$\theta_P^* > \theta_T^*$ $K_P = K_T^*$	$\theta_P = \theta_T^*$ $K_P = K_T^*$	$\theta_P^* > \theta_T^*$ $K_T = K_P^*$	$\theta_P = \theta_T^*$ $K_T = K_P^*$

Table 3: Overview of possible outcomes taking all constraints into account

5 Optimal Investment Strategies

In this section, we derive the optimal solution to the investment problems of the TSO and the PC, respectively. As described in Section 4, we start by presenting the analytical solutions of the two investment problems disregarding the capacity constraints. Then, we compare the two optimal capacities to decide which one will be dominating to find the optimal investment strategies of the two agents.

5.1 Optimal investment strategies disregarding capacity constraints

5.1.1 PC's investment decision

When we do not take into account constraints on capacity, the PC's investment problem at time zero is as follows:

$$\sup_{\tau_P \geq \tau_T} \left[\max_{K_P} \mathbb{E}\left[\int_{s=\tau_T}^{\infty} e^{-\rho s}\pi(\theta_s, K_0)ds + \int_{s=\tau_P}^{\infty} e^{-\rho s}[\pi(\theta_s, K_0+K_P) - \pi(\theta_s, K_0)]ds - e^{-\rho\tau_P}\delta K_P \middle| \theta_0 \right]\right]. \tag{10}$$

First, we derive the now-or-never optimal capacity expansion for the PC, denoted by K_P^*, for a fixed t, i.e. the capacity that maximises the additional present value of the PC at time t:

$$\max_{K_P} \mathbb{E}\left[\int_{s=t}^{\infty} e^{-\rho s}[\pi(\theta_s, K_0 + K_P) - \pi(\theta_s, K_0)]ds - e^{-\rho\tau_P}\delta K_P \middle| \theta_t \right], \tag{11}$$

which is equal to:

$$\max_{K_P} \left[\int_{s=t}^{\infty} e^{-\rho s}\mathbb{E}[\pi(\theta_s, K_0 + K_P) - \pi(\theta_s, K_0)\big|\theta_t]ds - \delta K_P \right]$$

$$= \max_{K_P} \left[\int_{s=t}^{\infty} e^{-\rho s} \theta_t e^{\alpha s} [(1 - \eta(K_0 + K_P))(K_0 + K_P) - (1 - \eta K_0) K_0] ds - \delta K_P \right]$$

$$= \max_{K_P} \left[\frac{\theta_t [1 - \eta(K_0 + K_P)](K_0 + K_P)}{(\rho - \alpha)} - \frac{\theta_t (1 - \eta K_0) K_0}{(\rho - \alpha)} - \delta K_P \right].$$

We find the derivative of the expression with respect to K_P and set it equal to zero. Therewith, the optimal capacity, $\hat{K}_P^*$, is given by:

$$\hat{K}_P^*(\theta_t) = \frac{1}{2\eta} \left[1 - \frac{\delta(\rho - \alpha)}{\theta_t} \right] - K_0. \tag{12}$$

As the PC cannot decrease the total capacity level since we do not account for disinvestment, K_P^* is restricted by the lower bound zero and, therefore, equal to:

$$K_P^*(\theta_t) = \max \left(\frac{1}{2\eta} \left[1 - \frac{\delta(\rho - \alpha)}{\theta_t} \right] - K_0, 0 \right). \tag{13}$$

After having solved for $K_P^*(\theta_t)$, the PC's investment problem at time zero reduces to:

$$\sup_{\tau_P \geq \tau_T} \mathbb{E} \left[\int_{s=\tau_T}^{\infty} e^{-\rho s} \pi(\theta_s, K_0) ds + \int_{s=\tau_P}^{\infty} e^{-\rho s} [\pi(\theta_s, K_0 + K_P^*) - \pi(\theta_s, K_0)] ds - e^{-\rho \tau_P} \delta K_P^* \middle| \theta_0 \right], \tag{14}$$

where τ_P is the time at which the investment in additional generation capacity is undertaken. Next, we solve the outer optimisation problem following a dynamic programming approach inspired by Dixit and Pindyck (1994). The solution to the optimal stopping problem is defined by a threshold, θ_P^*. For θ_t levels greater than θ_P^*, we are in the stopping region where it is optimal for the PC to invest immediately. For $\theta_t < \theta_P^*$, we are in the continuation region where demand is too low to undertake the investment, and it is optimal to wait for the PC. The PC invests at the moment θ_t hits the optimal investment level, θ_P^*, the first time. Thus, the optimal investment time, τ_P^*, is equal to the first time the stochastic variable θ_t hits the optimal level; $\tau_P^* \equiv \min\{t : \theta_t \geq \theta_P^*\}$.

Given that the TSO has already invested in a transmission line, the value of the PC can be described by:

$$F(\theta_t, K_P^*(\theta_t)) = \begin{cases} A_1\theta_t^{\beta_1} + V_1(\theta_t, K_0) & \text{if } \theta_T \leq \theta_t \leq \theta_P, \\ V_2(\theta_t, K_0 + K_P^*(\theta_t)) & \text{if } \theta_P \leq \theta_t, \end{cases} \tag{15}$$

where $\beta_1 > 1$ is the positive root of $\frac{1}{2}\sigma^2\beta(\beta - 1) + \alpha\beta - \rho = 0$. The endogenous constant, A_1, and the investment threshold, θ_P^*, are obtained by employing the boundary condition stated below as well as the value-matching and smooth-pasting conditions between the two branches of the value function stated in Equation (15). The value in the continuation region is derived by finding the solution to the ordinary differential equation (ODE) that stews from the Bellman equation:

$$\rho F dt = \mathbb{E}[dF] + \pi(\theta_t, K_0)dt. \tag{16}$$

After expanding the equation applying Itô's Lemma, we get the following ODE:

$$\frac{1}{2}\sigma^2\theta_t^2\frac{d^2F}{d\theta_t^2} + \alpha\theta_t\frac{dF}{d\theta_t} - \rho F + \pi(\theta_t, K_0) = 0. \tag{17}$$

We guess a solution to the ODE of the following form:

$$F(\theta_t) = A_1\theta_t^{\beta_1} + A_2\theta_t^{\beta_2} + V_1(\theta_t, K_0). \tag{18}$$

The boundary condition says that the value of the option to invest goes to zero if θ_t goes to zero. When θ_t goes to zero, it will stay at zero given its stochastic process, see Equation (2). Applying the boundary condition, we get that A_2 is equal to zero:

$$\lim_{\theta_t \to 0} F(\theta_t) = 0 \to A_2 = 0. \tag{19}$$

Subsequently, we are left with:

$$F(\theta_t) = A_1\theta_t^{\beta_1} + V_1(\theta_t, K_0). \tag{20}$$

The first term in this expression is the value of the option to expand capacity at time t, while the second term, $V_1(\theta_t, K_0)$, denotes the present value at time t of generating and selling an amount of power equal to the initial capacity of K_0 forever;

$$V_1(\theta_t, K_0) = \mathbb{E}\left[\int_{s=t}^{\infty} e^{-\rho(s-t)}\pi(\theta_s, K_0)ds\bigg|\theta_t\right] = \frac{\theta_t(1-\eta K_0)K_0}{\rho-\alpha}. \tag{21}$$

The value in the stopping region is the net value of the PC at time t if it invests in additional capacity:

$$\begin{aligned} V_2(\theta_t, K_0 + K_P^*) &= \mathbb{E}\left[\int_{s=t}^{\infty} e^{-\rho(s-t)}\pi(\theta_s, K_0 + K_P^*(\theta_t))ds - \delta K_P^*(\theta_t)\bigg|\theta_t\right] \\ &= \frac{\theta_t(1-\eta(K_0+K_P^*(\theta_t)))(K_0+K_P^*(\theta_t))}{\rho-\alpha} - \delta K_P^*(\theta_t). \end{aligned} \tag{22}$$

To determine the optimal investment threshold, θ_P^*, and the value of the endogenous constant A_1, we employ the value-matching and smooth-pasting conditions. The detailed derivations are in Appendix A.1, where we state specifically the value-matching and smooth-pasting conditions. They give the value of the endogenous constant A_1;

$$A_1 = \frac{K_P^*(\theta_P^*)(1-\eta(2K_0+K_P^*(\theta_P^*)))}{\rho-\alpha}\frac{1}{\beta_1}\hat{\theta}_P^{*1-\beta_1}, \tag{23}$$

and the optimal investment threshold $\hat{\theta}_P^*$, which is given by the solution to the following implicit equation:

$$\hat{\theta}_P^* = \frac{\beta_1}{\beta_1-1}(\rho-\alpha)\frac{\delta}{1-\eta(2K_0+K_P^*(\hat{\theta}_P^*))}. \tag{24}$$

Taking the timing constraint into account, we get the optimal investment timing of the PC given that the TSO invests at θ_T:

$$\theta_P^* = \begin{cases} \hat{\theta}_P^* & \text{if } \theta_T < \hat{\theta}_P^*, \\ \theta_T & \text{if } \theta_T \geq \hat{\theta}_P^*. \end{cases} \tag{25}$$

In the case where K_P^* is equal to zero, generation capacity will never be added, i.e., $\theta_P^*(0)=\infty$. The optimal investment decision of the PC is summarised in the following Proposition:

Proposition 1 *The optimal investment decision of the PC, without taking into account capacity restrictions caused by the TSO's investment strategy, is to expand generation capacity by K_P^* equal to:*

$$K_P^*(\theta_P^*) = \max\left(\frac{1}{2\eta}\left[1 - \frac{\delta(\rho-\alpha)}{\theta_P^*}\right] - K_0, 0\right), \tag{26}$$

at the moment in time when θ_t first hits θ_P^*, equal to:

$$\theta_P^* = \begin{cases} \hat{\theta}_P^* & \text{if } \theta_T^* < \hat{\theta}_P^*, \\ \theta_T & \text{if } \theta_T^* \geq \hat{\theta}_P^*, \end{cases} \tag{27}$$

where $\hat{\theta}_P^*$ is given by the solution to the following implicit equation:

$$\hat{\theta}_P^* = \frac{\beta_1}{\beta_1 - 1}(\rho - \alpha)\frac{\delta}{1 - \eta(2K_0 + K_P^*(\hat{\theta}_P^*))}, \tag{28}$$

and θ_T for the PC is exogenously given by the TSO. In the case where K_P^* is equal to zero, generation capacity will never be added, i.e., $\theta_P^*(0)=\infty$.

5.1.2 TSO's investment strategy

Next, we derive analytical expressions for the optimal investment trigger and capacity of the TSO. As discussed in Section 3, we assume that the PC will expand its generation capacity by $K_P = K_T^*$ when solving the TSO's investment problem.

The TSO's investment problem at time zero is equal to:

$$\sup_{\tau_T}\left[\max_{K_T}\mathbb{E}\left[\int_{s=\tau_T}^{\infty} e^{-\rho s} ts(\theta_s, K_0)ds - e^{-\rho\tau_T}\gamma(K_0 + K_T) + \int_{s=\tau_P}^{\infty} e^{-\rho s}[ts(\theta_s, K_0 + K_T) - ts(\theta_s, K_0)]ds - e^{-\rho\tau_P}\delta K_T \middle| \theta_0\right]\right], \tag{29}$$

where *ts(.)* denotes the continuous part of total surplus, see Equation (8).

By introducing the stochastic discount factor[9], the investment problem can also be writ-

[9] $\left(\frac{\theta_0}{\theta_T}\right)^{\beta_1}$ is the stochastic discount factor from θ_T to θ_0. It holds that:

$$\mathbb{E}\left[e^{-\rho\tau_T}\right] = \left(\frac{\theta_0}{\theta_T}\right)^{\beta_1}, \tag{30}$$

where τ_T is the expected first passage time of reaching θ_T. This expression for the stochastic discount factor is derived in, e.g., Dixit and Pindyck (1994).

ten as[10]:

$$\max_{\theta_T}\left[\max_{K_T}\mathbb{E}\left[\left(\frac{\theta_0}{\theta_T}\right)^{\beta_1} TS(\theta_T,\theta_P(K_T),K_T)\Big|\theta_0\right]\right], \tag{31}$$

where TS(.) is equal to the present value of the sum of total consumer surplus and producer surplus at time τ_T; $TS(\theta_T,\theta_P(K_T),K_T) = CS(\theta_T,\theta_P(K_T),K_T) + PS(\theta_T,\theta_P(K_T),K_T)$.

First, we derive the total expected consumer surplus (CS) at time t:

$$\mathbb{E}\left[\int_{s=t}^{\infty} e^{-\rho(s-t)}cs(\theta_s,K_0)ds + \int_{s=\tau_P}^{\infty} e^{-\rho(s-t)}[cs(\theta_s,K_0+K_T) - cs(\theta_s,K_0)]ds\Big|\theta_t\right] \tag{32}$$

$$= \mathbb{E}\left[\int_{s=t}^{\infty} e^{-\rho(s-t)}\frac{1}{2}\theta_s e^{\alpha s}K_0^2\eta ds + \int_{s=\tau_P}^{\infty} e^{-\rho(s-t)}[\frac{1}{2}\theta_s e^{\alpha s}(K_0+K_T)^2\eta - \frac{1}{2}\theta_s e^{\alpha s}K_0^2\eta]ds\Big|\theta_t\right] \tag{33}$$

$$= \frac{1}{2}\frac{\eta K_0^2}{\rho-\alpha}\theta_t + \frac{1}{2}\frac{\eta[(K_0+K_T)^2 - K_0^2]}{\rho-\alpha}\theta_P(K_T)\mathbb{E}\left[e^{-\rho(\tau_P - t)}\right], \tag{34}$$

where $\theta_P(K_T) = \theta_{\tau_P}(K_T)$. Note that τ_P is a decision variable of the PC and depends on the size of the PC's capacity expansion. Substituting for the stochastic discount factor we get that the total expected consumer surplus, which we will denote by $CS(.,.,.)$ in the following, is equal to:

$$CS(\theta_t,\theta_P(K_T),K_T) = \frac{1}{2}\frac{\eta K_0^2}{\rho-\alpha}\theta_t + \frac{1}{2}\frac{\eta[(K_0+K_T)^2 - K_0^2]}{\rho-\alpha}\theta_P(K_T)\left(\frac{\theta_t}{\theta_P(K_T)}\right)^{\beta_1}. \tag{35}$$

As discussed beforehand, we assume that the total expected producer surplus (PS) is equal to the present value of the PC's future income minus the PC's and the TSO's investment cost. The total expected producer surplus at time t is given by:

$$\mathbb{E}\Bigg[\int_{s=t}^{\infty} e^{-\rho(s-t)}ps(\theta_s,K_0)ds - \gamma(K_0+K_T) + \int_{s=\tau_P}^{\infty} e^{-\rho(s-t)}[ps(\theta_s,K_0+K_T) - ps(\theta_s,K_0)]ds - e^{-\rho\tau_P}\delta K_T\Big|\theta_t\Bigg]. \tag{36}$$

where *ps(.)* is the continuous part of the producer surplus, i.e., the profit flow of the PC. Therefore, the total expected producer surplus, which we will denote by $PS(.,.,.)$ in the

[10] From solving the PC's investment problem we know that its investment trigger depends on the capacity it has to install, therefore we write $\theta_P(K_T)$

following, at time t is equal to:

$$PS(\theta_t, \theta_P(K_T), K_T) = \frac{(1-\eta K_0)K_0}{\rho - \alpha}\theta_t - \gamma(K_0 + K_T)$$
$$+ \frac{(1-\eta(K_0+K_T))(K_0+K_T) - (1-\eta K_0)K_0}{\rho-\alpha}\theta_P(K_T)\left(\frac{\theta_t}{\theta_P(K_T)}\right)^{\beta_1} - \delta K_T\left(\frac{\theta_t}{\theta_P(K_T)}\right)^{\beta_1}. \tag{37}$$

Taking the sum of the expressions for the total consumer surplus (CS) and the producer surplus (PS), we find the total expected surplus or social welfare at time t:

$$TS(\theta_t, \theta_P(K_T), K_T) = \left[\frac{1}{2}\eta K_0^2 + (1-\eta K_0)K_0\right]\frac{\theta_t}{\rho-\alpha} - \gamma(K_0+K_T)$$
$$+ \left[\frac{1}{2}\eta[(K_0+K_T)^2 - K_0^2] + (1-\eta(K_0+K_T))(K_0+K_T) - (1-\eta K_0)K_0\right]\frac{\theta_P(K_T)}{\rho-\alpha}\left(\frac{\theta_t}{\theta_P(K_T)}\right)^{\beta_1}$$
$$- \delta K_T\left(\frac{\theta_t}{\theta_P(K_T)}\right)^{\beta_1}. \tag{38}$$

The first two terms on the right-hand side of Equation (38) can be interpreted as the total surplus given that the PC does not expand its generation capacity, while the last two terms can be interpreted as the additional total surplus if the PC increases its capacity by $K_P = K_T$ at time τ_P. From this expression, we obtain insight into the benefits and costs for the TSO of investing before the PC expands its generation capacity. First, if the existing capacity of the power plant K_0 is low, then the TSO achieves little social welfare from investing before the PC expands. Second, low levels of demand, θ_t, gives little benefit from investing before the PC. Third, if the TSO's marginal investment cost is high, then a higher demand level and K_0 is needed for the investment to be undertaken before the PC. On the other hand, if γ is close to zero, then the TSO will invest immediately as long as the price is positive.

If θ_P increases, then the total surplus less the investment cost is higher at time τ_P. However, when the PC delays investment, the present value of the net additional surplus also diminishes as it is discounted more.

After finding the expression for the total surplus, the TSO's investment problem at time zero can be rewritten. Note that we from now on consider the PC's optimal investment timing, $\theta_P^*(K_T)$. Then the investment problem is equal to:

$$\max_{\theta_T}\left[\max_{K_T}\left(\frac{\theta_0}{\theta_T}\right)^{\beta_1}\left[\left[\frac{1}{2}\eta K_0^2+(1-\eta K_0)K_0\right]\frac{\theta_T}{\rho-\alpha}-\gamma(K_0+K_T)\right.\right.$$
$$+\left[\frac{1}{2}\eta[(K_0+K_T)^2-K_0^2]+(1-\eta(K_0+K_T))(K_0+K_T)-(1-\eta K_0)K_0\right]\frac{\theta_P^*(K_T)}{\rho-\alpha}\left(\frac{\theta_T}{\theta_P^*(K_T)}\right)^{\beta_1}$$
$$\left.\left.-\delta K_T\left(\frac{\theta_T}{\theta_P^*(K_T)}\right)^{\beta_1}\right]\right]. \quad (39)$$

We continue by solving the inner investment problem and maximise the expression for the total surplus with respect to K_T to find the now-or-never optimal K_T^* for a given time t. When doing this, we need to anticipate the PC's investment timing response if it has to invest in a generation capacity of size K_T^*. We take into account that the PC maximises profit and that its optimal investment time depends on the size of its capacity expansion. As the optimal timing of investment for the PC, see Equation (28), is increasing in capacity;

$$\frac{\partial\theta_P^*(K_T)}{\partial K_T}=\frac{\beta_1}{\beta_1-1}(\rho-\alpha)\frac{\delta\eta K_T^2}{[(1-\eta(K_0+K_T))(K_0+K_T)-(1-\eta K_0)K_0]^2}>0, \quad (40)$$

the larger capacity the TSO forces the PC to invest in, the longer it will wait with undertaking the capacity expansion. In other words, the TSO will have to consider the trade-off between a large capacity and the PC delaying investment.

Next, we solve the inner extremum by finding the derivative of Equation (38) with respect to K_T and setting it equal to zero. Note that before finding the derivative, we substitute for $\theta_P^*(K_T)$ into Equation (38). Then, we get the following implicit equation for $\hat{K}_T^*$:

$$-\gamma+\left[\frac{\beta\delta[2\eta\hat{K}_T^*(\eta K_0(\beta-2)-\beta+1)+(2\eta K_0-1)(2\eta K_0(\beta-2)-\beta+1])}{2(\beta-1)(\eta\hat{K}_T^*+2\eta K_0-1)^2}+\frac{\delta}{\beta-1}-\frac{\beta\delta}{2}\right]*$$
$$\left[\frac{(\beta-1)\theta_t(\eta\hat{K}_T^*+2\eta K_0-1)}{\beta\delta(\alpha-\rho)}\right]=0. \quad (41)$$

As we require the TSO to at least be able to distribute an amount of power equal to the current capacity of the PC, K_0, the optimal K_T^* is equal to:

$$K_T^*(\theta_t)=\max\left(\hat{K}_T^*(\theta_t),0\right). \quad (42)$$

After having solved for K_T^*, the outer extremum of the TSO's investment problem is equal to:

$$\max_{\theta_T} \left(\frac{\theta_0}{\theta_T}\right)^{\beta_1} \Bigg[\left[\frac{1}{2}\eta K_0^2 + (1-\eta K_0)K_0\right] \frac{\theta_T}{\rho-\alpha} - \gamma(K_0 + K_T^*(\theta_T))$$
$$+ \left[\frac{1}{2}\eta[(K_0+K_T^*(\theta_T))^2 - K_0^2] + (1-\eta(K_0+K_T^*(\theta_T)))(K_0+K_T^*(\theta_T)) - (1-\eta K_0)K_0\right] *$$
$$\frac{\theta_P^*(K_T^*(\theta_T))}{\rho-\alpha}\left(\frac{\theta_T}{\theta_P^*(K_T^*(\theta_T))}\right)^{\beta_1} - \delta K_T^*(\theta_T)\left(\frac{\theta_T}{\theta_P^*(K_T^*(\theta_T))}\right)^{\beta_1}\Bigg]. \quad (43)$$

We proceed by following a dynamic programming approach to solve the optimal stopping problem and find θ_T^*. The value for the TSO, F, at time t is equal to:

$$F(\theta_t, K_T^*(\theta_t), \theta_P^*(K_T^*(\theta_t))) = \begin{cases} B_1\theta_t^{\beta_1} & \text{if } \theta_t \leq \theta_T, \\ TS(\theta_t, K_T^*(\theta_t), \theta_P^*(K_T^*(\theta_t))) & \text{if } \theta_T \leq \theta_t. \end{cases} \quad (44)$$

The value in the continuation region is equal the value of the option to invest in the transmission line, while the value in the stopping region is equal to the value of the total surplus given that investment has occurred. The value in the continuation region is derived by finding the solution to the ordinary differential equation (ODE) that stews from the Bellman equation:

$$\rho F dt = \mathbb{E}[dF]. \quad (45)$$

After expanding the equation applying Itô's Lemma, we get the following ODE:

$$\frac{1}{2}\sigma^2\theta_t^2\frac{d^2F}{d\theta_t^2} + \alpha\theta_t\frac{dF}{d\theta_t} - \rho F = 0. \quad (46)$$

We guess a solution of the following form:

$$F(\theta_t) = B_1\theta_t^{\beta_1} + B_2\theta_t^{\beta_2}, \quad (47)$$

and apply the boundary condition to find that B_2 is equal to zero:

$$\lim_{\theta_t\to 0} F(\theta_t) = 0 \to B_2 = 0. \quad (48)$$

Given that the PC invests after the TSO, the net value of the total surplus at time t is given by Equation (38).

To determine the optimal investment threshold, θ_T^*, and the value of the endogenous constant B_1, we employ the value-matching and smooth-pasting conditions. The detailed derivations are in Appendix A.2, where we state specifically the value-matching and smooth-pasting conditions. We find that:

$$B_1 = \left[K_0 - \frac{1}{2}\eta K_0^2\right]\frac{\theta_T^{*1-\beta_1}}{\beta_1(\rho-\alpha)} + \left[K_T^*(\theta_T^*)(1-\eta(K_0 - \frac{1}{2}K_T^*(\theta_T^*)))\right]\frac{\theta_P^*(K_T^*)^{*1-\beta_1}}{\rho-\alpha} \\ - \delta K_T^*(\theta_T^*)\theta_P^*(K_T^*)^{*-\beta_1}, \quad (49)$$

and that the optimal investment threshold of the TSO is given by the solution to the following implicit equation:

$$\theta_T^* = \frac{\beta_1}{\beta_1 - 1}(\rho-\alpha)\frac{\gamma(K_0 + K_T^*(\theta_T^*))}{K_0(1-\frac{1}{2}\eta K_0)}. \quad (50)$$

After finding the optimal investment strategy of the TSO, the optimal investment trigger of the PC, $\theta_P^*(K_T^*)$, is equal to Equation (28) but with K_T^* instead of K_P^*:

$$\theta_P^*(K_T^*) = \frac{\beta_1}{\beta_1 - 1}(\rho-\alpha)\frac{\delta K_T^*}{1-\eta(2K_0 + K_T^*)}. \quad (51)$$

In the case where K_T^* is equal to zero, generation capacity will never be added, i.e., $\theta_P^*(0)=\infty$.

When finding K_T^*, the TSO takes into account the optimal investment threshold of the PC given that it has to invest in K_T^*. $\theta_P^*(K_T^*)$ needs to be larger than $\theta_T^*(K_T^*)$ for the analytical solutions above to be valid. If this is not the case, then we assume the corner solution $\theta_P = \theta_T^*$ to be optimal, see Figure 2. However, then we need to derive the TSO's optimal investment problem given that it decides capacity and timing for both.

Corner solution: $\theta_P = \theta_T^*$

In this case, the PC does not hold an option to decide either capacity or timing and will have to follow the TSO's investment strategy. Therefore, we need to solve only the TSO's investment problem, given that they both invest at the same time, which at time zero is equal to::

$$\sup_{\tau_T}\left[\max_{K_T}\mathbb{E}\left[\int_{s=\tau_T}^{\infty} e^{-\rho s} ts(\theta_s, K_0 + K_T)ds - e^{-\rho\tau_T}\gamma(K_0+K_T) - e^{-\rho\tau_T}\delta(K_T)\middle|\theta_0\right]\right]. \quad (52)$$

The total expected surplus at time t is given by:

$$TS(\theta_t, K_T) = \left[\frac{1}{2}\eta(K_0+K_T)^2+(1-\eta(K_0+K_T))(K_0+K_T)\right]\frac{\theta_t}{\rho-\alpha}-\gamma(K_0+K_T)-\delta K_T. \quad (53)$$

Taking the derivative of Equation (53) with respect to K_T and setting it equal to zero, we find the now-or-never optimal capacity, K_T^*, given that we do not allow for disinvestment:

$$K_T^*(\theta_t) = \max\left(\frac{1}{\eta}\left[1-\frac{(\delta+\gamma)(\rho-\alpha)}{\theta_t}\right]-K_0, 0\right). \quad (54)$$

We proceed by following a dynamic programming approach to solve the optimal stopping problem. The value for the TSO, F, at time t is equal to:

$$F(\theta_t, K_T^*(\theta_t)) = \begin{cases} B_1\theta_t^{\beta_1} & \text{if } \theta_t \leq \theta_T, \\ TS(\theta_t, K_T^*(\theta_t)) & \text{if } \theta_T \leq \theta_t, \end{cases} \quad (55)$$

where the value in the continuation region is derived by finding the solution to the ordinary differential equation (ODE) that stews from the Bellman equation:

$$\rho F dt = \mathbb{E}[dF]. \quad (56)$$

Standard calculations similar to those in the two preceding cases are performed, which lead to the value function stated in Equation (55).

To determine the optimal investment threshold, θ_T^*, and the value of the endogenous constant B_1, we employ the value-matching and smooth-pasting conditions. The detailed derivations are in Appendix A.3, where we specifically state the value-matching and smooth-pasting conditions. We find that:

$$B_1 = \left[(K_0+K_T^*(\theta_T^*))(1-\frac{1}{2}\eta(K_0+K_T^*(\theta_T^*)))\right]\frac{\theta_T^{*1-\beta_1}}{\beta_1(\rho-\alpha)}, \quad (57)$$

and θ_T^* is given by the solution to the following implicit equation:

$$\theta_T^* = \frac{\beta_1}{\beta_1-1}(\rho-\alpha)\frac{\gamma(K_0+K_T^*(\theta_T^*))+\delta K_T^*(\theta_T^*)}{(K_0+K_T^*(\theta_T^*))(1-\frac{1}{2}\eta(K_0+K_T^*(\theta_T^*)))}. \quad (58)$$

The optimal investment strategy of the TSO is summarised in the following Proposition:

Proposition 2 *The optimal investment strategy of the TSO, assuming that the PC has to expand capacity by $K_P = K_T^*$, is to invest in a transmission capacity of $K_0 + K_T^*$. K_T^* is equal to:*

$$K_T^*(\theta_T^*) = \max\left(\hat{K}_T^*(\theta_T^*), 0\right), \tag{59}$$

where $\hat{K}_T^*(\theta_T^*)$ is given by the following implicit equation:

$$-\gamma + \left[\frac{\beta\delta[2\eta\hat{K}_T^*(\eta K_0(\beta-2)-\beta+1)+(2\eta K_0-1)(2\eta K_0(\beta-2)-\beta+1])}{2(\beta-1)(\eta\hat{K}_T^*+2\eta K_0-1)^2} + \frac{\delta}{\beta-1} - \frac{\beta\delta}{2}\right] * \left[\frac{(\beta-1)\theta_T^*(\eta\hat{K}_T^*+2\eta K_0-1)}{\beta\delta(\alpha-\rho)}\right] = 0. \tag{60}$$

The optimal investment time is the moment in time when θ_t reaches θ_T^*, which is given by the solution to the following implicit equation:

$$\theta_T^* = \frac{\beta_1}{\beta_1-1}(\rho-\alpha)\frac{\gamma(K_0+K_T^*(\theta_T^*))}{K_0(1-\frac{1}{2}\eta K_0)}. \tag{61}$$

The PC would then expand capacity when θ_t reaches $\theta_P^*(K_T^*)$ given that it had no flexibility to choose the size of its investment:

$$\theta_P^*(K_T^*) = \frac{\beta_1}{\beta_1-1}(\rho-\alpha)\frac{\delta K_T^*}{1-\eta(2K_0+K_T^*)}. \tag{62}$$

In the case where K_T^* is equal to zero, generation capacity will never be added, i.e., $\theta_P^*(0)=\infty$.

However, if this solution is not valid, i.e., $\theta_P^*(K_T^*) < \theta_T^*(K_T^*)$, then we assume the corner solution $\theta_P = \theta_T^*$ to be optimal. The optimal investment strategy of the TSO, given that both agents invest at the same time, is to invest in a capacity of:

$$K_T^*(\theta_T^*) = \max\left(\frac{1}{\eta}\left[1 - \frac{(\delta+\gamma)(\rho-\alpha)}{\theta_T^*}\right] - K_0, 0\right), \tag{63}$$

at the moment in time when θ_t hits θ_T^*, which is given by the solution to the following implicit equation:

$$\theta_T^* = \frac{\beta_1}{\beta_1-1}(\rho-\alpha)\frac{\gamma(K_0+K_T^*(\theta_T^*))+\delta K_T^*(\theta_T^*)}{(K_0+K_T^*(\theta_T^*))(1-\frac{1}{2}\eta(K_0+K_T^*(\theta_T^*)))}. \tag{64}$$

5.2 Optimal investment strategies when considering constraints on capacities

We continue by taking into account the capacity constraints. First, we present optimal strategies for both agents if the TSO's capacity is dominating, i.e., $K_T^* \leq K_P^*$, and continue with the outcomes where the PC's optimal capacity is dominating, i.e., $K_P^* < K_T^*$, see Table 3. We also compare the total value for the PC and the TSO corresponding to each possible outcome. We take into account that we cannot compare the two agents' values at their investment thresholds, because we would compare situations at different moments in time as their investment thresholds can be different. Therefore, we compare all values at the current level, θ_0, considered sufficiently small so that the optimal investment triggers will be larger than θ_0.

5.2.1 Optimal investment strategies when the capacity of the TSO is dominating

If $K_T^* \leq K_P^*$ is the solution to the investment problems in Section 5.1, then the TSO's capacity, K_T^*, will be the dominating capacity. The TSO will invest in K_T^* at θ_T^*, and it can therewith force the PC to invest in a smaller capacity than it found optimal, i.e., $K_P = K_T^*$. Therefore, we find a new optimal trigger θ_P^* for the PC. In the case where $\theta_P^*(K_T^*) > \theta_T^*(K_T^*)$, this is the same $\theta_P^*(K_T^*)$ as the TSO expected for the PC in Section 5.1.2. However when we get that $\theta_P^*(K_T^*) < \theta_T^*(K_T^*)$, we assume the corner solution $\theta_P = \theta_T^*$ to be optimal, i.e., it is optimal for the PC to invest as soon as the TSO has invested. Note that the two possible outcomes when K_T^* is the dominating capacity corresponds to outcomes 1 and 2 in Table 3. We get the following Proposition:

Proposition 3 *The optimal investment strategies of the two agents, given that the TSO's capacity is dominating, are given in Table 4. Two possible outcomes might occur. Either they invest at the same time or the PC delays investment beyond the investment time of the TSO.*

Optimal investment strategies given that the TSO's optimal capacity is dominating $K_T^* \leq K_P^*$

$K_P = K_T^*$	Outcome 1	$\theta_P^* > \theta_T^*$	**TSO's optimal strategy:** $K_T^* = \max\left(\hat{K}_T^*(\theta_T^*), 0\right)$, where $\hat{K}_T^*(\theta_T^*)$ is given by the following implicit equation: $-\gamma + \left[\frac{\beta\delta[2\eta\hat{K}_T^*(\eta K_0(\beta-2)-\beta+1)+(2\eta K_0-1)(2\eta K_0(\beta-2)-\beta+1])}{2(\beta-1)(\eta\hat{K}_T^*+2\eta K_0-1)^2} + \frac{\delta}{\beta-1} - \frac{\beta\delta}{2}\right]$ $* \left[\frac{(\beta-1)\theta_T^*(\eta\hat{K}_T^*+2\eta K_0-1)}{\beta\delta(\alpha-\rho)}\right] = 0$ $\theta_T^* = \frac{\beta_1}{\beta_1-1}(\rho-\alpha)\frac{\gamma(K_0+K_T^*(\theta_T^*))}{K_0(1-\frac{1}{2}\eta K_0)}$
			PC's optimal decision: $K_P = K_T^*$ $\theta_P^* = \frac{\beta_1}{\beta_1-1}(\rho-\alpha)\frac{\delta}{1-\eta(2K_0+K_T^*)}$
	Outcome 2	$\theta_P = \theta_T^*$	**TSO's optimal strategy:** $K_T^* = \max\left(\frac{1}{\eta}\left[1 - \frac{(\delta+\gamma)(\rho-\alpha)}{\theta_T^*}\right] - K_0, 0\right)$ $\theta_T^* = \frac{\beta_1}{\beta_1-1}(\rho-\alpha)\frac{\gamma(K_0+K_T^*(\theta_T^*))+\delta K_T^*(\theta_T^*)}{(K_0+K_T^*(\theta_T^*))(1-\frac{1}{2}\eta(K_0+K_T^*(\theta_T^*)))}$
			PC's optimal decision: $K_P = K_T^*$ $\theta_P = \theta_T^*$

Table 4: Overview of optimal investment strategies if $K_T^* \leq K_P^*$ corresponding to outcomes 1 and 2

If the PC invests after the TSO, then the resulting value at time zero for each agent is equal to[11]:

$$V_{TSO}(\theta_T^*, \theta_P^*, K_T^*) = \left(\frac{\theta_0}{\theta_T^*}\right)^{\beta_1}\left[\left[\frac{1}{2}\eta K_0^2 + (1-\eta K_0)K_0\right]\frac{\theta_T^*}{\rho-\alpha} - \gamma(K_0+K_T^*)\right.$$
$$+\left[\frac{1}{2}\eta[(K_0+K_T^*)^2 - K_0^2] + (1-\eta(K_0+K_T^*))(K_0+K_T^*) - (1-\eta K_0)K_0\right]\frac{\theta_P^*(K_T^*)}{\rho-\alpha}\left(\frac{\theta_T^*}{\theta_P^*(K_T^*)}\right)^{\beta_1}$$
$$\left. - \delta K_T^*\left(\frac{\theta_T^*}{\theta_P^*(K_T^*)}\right)^{\beta_1}\right], \quad (65)$$

[11]Note, in all value functions in this and the following section we have refrained from writing the capacity's dependency on the optimal investment trigger for readability.

$$V_{PC}(\theta_T^*, \theta_P^*, K_T^*) = \left(\frac{\theta_0}{\theta_T^*}\right)^{\beta_1} \Bigg[\Big[(1-\eta K_0)K_0\Big] \frac{\theta_T^*}{\rho - \alpha}$$

$$+ \Big[(1-\eta(K_0+K_T^*))(K_0+K_T^*) - (1-\eta K_0)K_0\Big] \frac{\theta_P^*(K_T^*)}{\rho - \alpha} \left(\frac{\theta_T^*}{\theta_P^*(K_T^*)}\right)^{\beta_1} - \delta K_T^* \left(\frac{\theta_T^*}{\theta_P^*(K_T^*)}\right)^{\beta_1} \Bigg]. \tag{66}$$

Note that when stating the value functions throughout this paper, the first term within the dependency bracket corresponds to the investment threshold of the TSO. The second corresponds to the investment threshold for the PC, while the last term corresponds to the capacity the agents install.

If it is optimal for the PC to invest at the same time as the TSO, i.e., $\theta_P(K_T^*) = \theta_T^*(K_T^*)$, then the value functions simplify to:

$$V_{TSO}(\theta_T^*, \theta_T^*, K_T^*) = \left(\frac{\theta_0}{\theta_T^*}\right)^{\beta_1} \Bigg[\Big[\frac{1}{2}\eta(K_0+K_T^*)^2 + (1-\eta(K_0+K_T^*))(K_0+K_T^*)\Big] \frac{\theta_T^*}{\rho - \alpha} - \gamma(K_0+K_T^*) - \delta K_T^* \Bigg], \tag{67}$$

$$V_{PC}(\theta_T^*, \theta_T^*, K_T^*) = \left(\frac{\theta_0}{\theta_T^*}\right)^{\beta_1} \Bigg[\Big[(1-\eta(K_0+K_T^*))(K_0+K_T^*)\Big] \frac{\theta_T^*}{\rho - \alpha} - \delta K_T \Bigg]. \tag{68}$$

In the case where K_T^* is equal to zero, generation capacity will never be added, i.e., $\theta_P^*(0)=\infty$. The value for the TSO and the PC are equal to:

$$V_{TSO}(\theta_T^*, \infty, 0) = \left(\frac{\theta_0}{\theta_T^*}\right)^{\beta_1} \Bigg[\Big[\frac{1}{2}\eta K_0^2 + (1-\eta K_0)K_0\Big] \frac{\theta_T^*}{\rho - \alpha} - \gamma K_0 \Bigg], \tag{69}$$

and:

$$V_{PC}(\theta_T^*, \infty, 0) = \left(\frac{\theta_0}{\theta_T^*}\right)^{\beta_1} \Bigg[\Big[(1-\eta K_0)K_0\Big] \frac{\theta_T^*}{\rho - \alpha} \Bigg], \tag{70}$$

respectively.

5.2.2 Optimal investment strategies when the capacity of the PC is dominating

On the other hand, if $K_P^* < K_T^*$ is the solution to the investment problems in Section 5.1, K_P^* will be the dominating capacity. The TSO will invest in K_P^* and find a new optimal investment trigger θ_T^* based on K_P^*, which is equal to:

$$\theta_T^*(K_P^*(\theta_P^*)) = \frac{\beta_1}{\beta_1 - 1}(\rho - \alpha)\frac{\gamma(K_0 + K_P^*(\theta_P^*))}{K_0(1 - \frac{1}{2}\eta K_0)}. \tag{71}$$

If $\theta_T^*(K_P^*)$ is lower than $\theta_P^*(K_P^*)$, then this will be the optimal investment strategy.

On the contrary, if $\theta_P^*(K_P^*) < \theta_T^*(K_P^*)$, then we assume the corner solution $\theta_P = \theta_T^*$ to be optimal. Subsequently, the TSO will have the power to decide timing and the PC the power to decide capacity. In this case, we need to solve the PC's investment problem given that it only has sizing flexibility and the TSO's optimal stopping problem given that both agents invest at the same time in K_P^*. The investment problem of the PC at time zero is then equal to:

$$\max_{K_P} \mathbb{E}\left[\int_{s=\tau_T}^{\infty} e^{-\rho s}\pi(\theta_s, K_0 + K_P)ds - e^{-\rho\tau_P}\delta K_P \Big| \theta_0\right], \tag{72}$$

while the investment problem of the TSO at time zero is equal to:

$$\sup_{\tau_T} \mathbb{E}\left[\int_{s=\tau_T}^{\infty} e^{-\rho s}ts(\theta_s, K_0 + K_P)ds - e^{-\rho\tau_T}\gamma(K_0 + K_P) - e^{-\rho\tau_T}\delta K_P \Big| \theta_0\right]. \tag{73}$$

See Appendix B.3 for the derivation of the solution to these investment problems. The optimal investment strategy of the TSO given that they both invest at the same time, is to invest in a capacity of $K_T = K_P^*$, where K_P^* is equal to:

$$K_P^*(\theta_T^*) = \max\left(\frac{1}{2\eta}\left[1 - \frac{\delta(\rho-\alpha)}{\theta_T^*}\right] - K_0, 0\right). \tag{74}$$

Furthermore, the optimal investment threshold of the TSO, θ_T^*, is given by the solution to the following implicit equation:

$$\begin{aligned}&\frac{\beta_1 - 1}{\beta_1}\frac{(K_0 + K_P^*(\theta_T^*)(1 - \frac{1}{2}\eta(K_0 + K_P^*(\theta_T^*)))}{\rho - \alpha}\theta_T^{*2} \\ &- \left[\gamma(K_0 + K_P^*(\theta_T^*)) + \delta K_P^*(\theta_T^*) + \frac{\delta(1 - \eta(K_0 + K_P^*(\theta_T^*))}{2\eta\beta_1}\right]\theta_T^* + \frac{\delta(\gamma+\delta)(\rho-\alpha)}{2\eta\beta_1} = 0.\end{aligned} \tag{75}$$

We get the following Proposition:

Proposition 4 *The optimal investment strategies of the two agents, given that the PC's capacity is dominating, are given in Table 5. Two possible outcomes might occur. Either they invest at the same time or the PC delays investment beyond the investment time of the TSO.*

Optimal investment strategies given that the PC's optimal capacity is dominating $K_P^* < K_T^*$			
$K_T = K_P^*$	Outcome 3	$\theta_P^* > \theta_T^*$	**TSO's optimal strategy:** $K_T = K_P^*$ $\theta_T^* = \frac{\beta_1}{\beta_1 - 1}(\rho - \alpha)\frac{\gamma(K_0 + K_P^*(\theta_P^*))}{K_0(1-\frac{1}{2}\eta K_0)}$
			PC's optimal decision: $K_P^* = \max\left(\frac{1}{2\eta}\left[1 - \frac{\delta(\rho-\alpha)}{\theta_P^*}\right] - K_0, 0\right)$ $\theta_P^* = \frac{\beta_1}{\beta_1 - 1}(\rho - \alpha)\frac{\delta}{1-\eta(2K_0 + K_P^*(\theta_P^*))}$
	Outcome 4	$\theta_P = \theta_T^*$	**TSO's optimal strategy:** $K_T = K_P^*$ θ_T^* is given by the following implicit equation: $\frac{\beta_1 - 1}{\beta_1}\frac{(K_0 + K_P^*(\theta_T^*)(1-\frac{1}{2}\eta(K_0 + K_P^*(\theta_T^*)))}{\rho-\alpha}\theta_T^{*2}$ $-\left[\gamma(K_0 + K_P^*(\theta_T^*)) + \delta K_P^*(\theta_T^*) + \frac{\delta(1-\eta(K_0+K_P^*(\theta_T^*))}{2\eta\beta_1}\right]\theta_T^*$ $+\frac{\delta(\gamma+\delta)(\rho-\alpha)}{2\eta\beta_1} = 0$
			PC's optimal decision: $K_P^* = \max\left(\frac{1}{2\eta}\left[1 - \frac{\delta(\rho-\alpha)}{\theta_T^*}\right] - K_0, 0\right)$ $\theta_P = \theta_T^*$

Table 5: Overview of optimal investment strategies if $K_P^* < K_T^*$ corresponding to outcomes 3 and 4

Note that the two possible outcomes when K_P^* is the dominating capacity corresponds to outcomes 3 and 4 in Table 3.

If the PC invests after the TSO, then the resulting value at time zero for each agent is equal to:

$$V_{TSO}(\theta_T^*, \theta_P^*, K_P^*) = \left(\frac{\theta_0}{\theta_T^*}\right)^{\beta_1}\left[\left[\frac{1}{2}\eta K_0^2 + (1-\eta K_0)K_0\right]\frac{\theta_T^*}{\rho - \alpha} - \gamma(K_0 + K_P^*)\right.$$
$$+\left[\frac{1}{2}\eta[(K_0 + K_P^*)^2 - K_0^2] + (1-\eta(K_0+K_P^*))(K_0+K_P^*) - (1-\eta K_0)K_0\right]\frac{\theta_P^*(K_P^*)}{\rho-\alpha}\left(\frac{\theta_T^*}{\theta_P^*(K_P^*)}\right)^{\beta_1}$$
$$\left. - \delta K_P^*\left(\frac{\theta_T^*}{\theta_P^*(K_P^*)}\right)^{\beta_1}\right], \quad (76)$$

$$V_{PC}(\theta_T^*, \theta_P^*, K_P^*) = \left(\frac{\theta_0}{\theta_T^*}\right)^{\beta_1} \Bigg[\Big[(1-\eta K_0)K_0 \Big] \frac{\theta_T^*}{\rho - \alpha}$$
$$+ \Big[(1-\eta(K_0+K_P^*))(K_0+K_P^*) - (1-\eta K_0)K_0 \Big] \frac{\theta_P^*(K_P^*)}{\rho-\alpha} \left(\frac{\theta_T^*}{\theta_P^*(K_P^*)}\right)^{\beta_1} - \delta K_P^* \left(\frac{\theta_T^*}{\theta_P^*(K_P^*)}\right)^{\beta_1} \Bigg]. \tag{77}$$

If it is optimal for the PC to invest at the same time as the TSO, i.e., $\theta_P(K_P^*) = \theta_T^*(K_P^*)$, then the value functions simplify to:

$$V_{TSO}(\theta_T^*, \theta_T^*, K_P^*) = \left(\frac{\theta_0}{\theta_T^*}\right)^{\beta_1} \Bigg[\Big[\frac{1}{2}\eta(K_0+K_P^*)^2 + (1-\eta(K_0+K_P^*))(K_0+K_P^*) \Big] \frac{\theta_T^*}{\rho-\alpha}$$
$$- \gamma(K_0+K_P^*) - \delta K_P^* \Bigg], \tag{78}$$

$$V_{PC}(\theta_T^*, \theta_T^*, K_P^*) = \left(\frac{\theta_0}{\theta_T^*}\right)^{\beta_1} \Bigg[\Big[(1-\eta(K_0+K_P^*))(K_0+K_P^*) \Big] \frac{\theta_T^*}{\rho-\alpha} - \delta K_P \Bigg]. \tag{79}$$

In the case where K_P^* is equal to zero and generation capacity will never be added, i.e., $\theta_P^*(0){=}\infty$. The value for the TSO and the PC are equal to:

$$V_{TSO}(\theta_T^*, \infty, 0) = \left(\frac{\theta_0}{\theta_T^*}\right)^{\beta_1} \Bigg[\Big[\frac{1}{2}\eta K_0^2 + (1-\eta K_0)K_0 \Big] \frac{\theta_T^*}{\rho-\alpha} - \gamma K_0 \Bigg], \tag{80}$$

and:

$$V_{PC}(\theta_T^*, \infty, 0) = \left(\frac{\theta_0}{\theta_T^*}\right)^{\beta_1} \Bigg[\Big[(1-\eta K_0)K_0 \Big] \frac{\theta_T^*}{\rho-\alpha} \Bigg], \tag{81}$$

respectively.

6 Results

In this section, we will present a numerical analysis in order to illustrate the analytical results and to gain additional insights[12]. The following parameter values are used as a base case: $\rho = 0.03$, $\alpha = 0.015$, $\eta = 0.01$, $\sigma = 0.15$, $K_0 = 100$, $\delta = 100$, and $\gamma = 40$. The marginal investment cost of the TSO is chosen to be lower than that of the PC as for example Baringo and Conejo (2012) argue that the marginal investment cost of transmission facilities

[12] All numerical results are obtained using the software program MATLAB R2014a.

is comparatively much lower than the marginal investment cost of wind power plants[13]. In the end of the analysis, we will vary δ, the marginal investment cost of the PC, to analyse how the decision of the PC is affected by its marginal investment cost.

When building renewable power plants in remote areas, often a small plant is already installed while it has to undergo a *large* expansion to be connected to the main grid. For this reason, K_0 is chosen to be relatively low compared to the values of K_T^* and K_P^*. When comparing value functions, the value of each agent at investment has to be discounted back to the current demand level[14], which is set equal to $\theta_0 = 1$.

We start by presenting economic results from the sub-problems before we proceed with numerical analysis of the full model. We are not studying a specific case example, therefore the model parameters are not calibrated on a real data set. The goal is to provide more general economic insight into the optimal investment decision of a TSO. Throughout the analysis, the focus is on providing insights with respect to 1) how much welfare a TSO will forgo by disregarding the PC's optimal investment decision, 2) the effect of uncertainty on optimal transmission and generation investment strategies, and 3) the value of managerial flexibility. In addition, we perform a sensitivity analysis with respect to the PC's marginal investment cost, δ, in order to gain insight into how a subsidy of the PC's investment cost can affect its optimal capacity choice, K_P^*, as well as its optimal investment threshold, θ_P^*, and hence social welfare.

6.1 Welfare loss from not having power to decide the PC's capacity

In Norway, we see that the TSO, Statnett, tries to influence PCs' capacity choice by setting a minimum capacity that the PCs have commit to install for the TSO to build specific transmission lines[15]. The motivation behind the following analysis is to reveal how much

[13] The marginal investment cost of the TSO depends on several factors like the voltage, thickness and length of the power lines, while the marginal investment cost of the PC, among other things, depends on the type of power plant. Therefore the difference between the two marginal investment costs will vary from project to project. However, when consulting the Norwegian TSO, Statnett, their general impression is that marginal transmission investment costs are considerably lower than marginal generation investment costs.

[14] As the optimal investment timing of the two agents might differ, the value functions need to be discounted to the same point in time in order to be comparable.

[15] Currently, Statnett considers building transmission lines from Trollheim to Snillfjord and Namsos to Storheia but it requires a group of PCs to commit to install a total generation capacity of 1000 MW

generation capacity the TSO will want the PC to commit to install, and the welfare loss from not being able to do so. In this section, we first study the welfare loss when the agents do not have timing flexibility, i.e., they have to make now-or-never investment decisions. Next, we extend the analysis by providing both agents with timing flexibility.

6.1.1 Without timing flexibility

First, we investigate how the optimal capacities of a TSO and a PC differ due to their different objectives when both agents face a now-or-never investment decision, i.e., sub-problem 1. The solution to the sub-problem and the corresponding derivations can be found in Appendix C.1.

If the PC can choose the optimal size of its generation expansion, it will be equal to:

$$K_P^*(\theta_0) = \max\left(\frac{1}{2\eta}\left[1 - \frac{\delta(\rho-\alpha)}{\theta_0}\right] - K_0, 0\right). \tag{82}$$

And, if the TSO can decide its own capacity exceeding K_0 and the generation capacity of the PC, they will be equal to:

$$K_T^*(\theta_0) = K_P(\theta_0) = \max\left(\frac{1}{\eta}\left[1 - \frac{(\delta+\gamma)(\rho-\alpha)}{\theta_0}\right] - K_0, 0\right). \tag{83}$$

Comparing these results, we find that the TSO's total optimal capacity is twice the total optimal capacity of the PC minus a correction term and equal to:

$$K_0 + K_T^* = 2(K_0 + K_P^*) - \frac{1}{\eta}\frac{\gamma(\rho-\alpha)}{\theta_0}. \tag{84}$$

The following Proposition states the condition for when the capacity of the TSO will be larger than that of the PC if they both invest at time zero.

Proposition 5 *If both agents invest immediately at time zero, then the total optimal capacity of the TSO will be larger than the total optimal capacity of the PC if the following condition holds:*

$$\frac{1}{\eta}\frac{\gamma(\rho-\alpha)}{\theta_0} < K_0 + K_P^*, \tag{85}$$

before it will start building. See http://www.norwea.no/nyhetsarkiv/visning-nyheter/ett-av-prosjektene-i-snillfjord-vraket.aspx?PID=1145Action=1,http://www.tu.no/kraft/2015/03/09/nve-statnett-krav-i-strid-med-energiloven *which is equivalent to:*

$$\frac{\theta_0}{\rho - \alpha} > 2\gamma + \delta. \tag{86}$$

In this case, the flexibility of sizing will be of no value for the TSO.

The result is similar to the result of Huisman and Kort (2014) who consider one firm that can undertake an investment to enter a market where there is no firm active. They compare the optimal capacity level of the firm given a social welfare and a profit objective and find that the optimal capacity level given a welfare objective is twice the level of a monopolist. The difference of Huisman and Kort (2014) to this paper is that they compare the optimal capacity for a single firm optimising either social welfare or profit, while we consider two firms in the same model. We find that the optimal capacity for the TSO is twice the optimal capacity of the PC minus a correction term. The reason is that when both agents operate in the same market, we need to include both investment costs when calculating total surplus. If we considered only one agent, we would get the same result as in Huisman and Kort (2014).

If we use the base case parameters, then we get the numerical results shown in Figure 4. When the demand level, θ_0, is very low, then the size of the optimal capacity expansion of the PC is higher than that of the TSO. The TSO will not want to invest at this demand

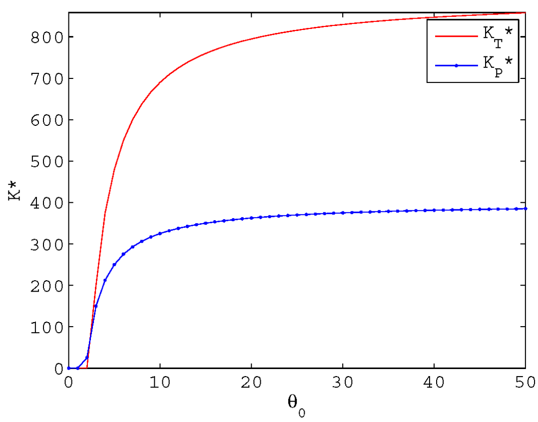


Figure 4: Optimal capacities as functions of θ_0 for the base case parameters in the case of now-or-never investments.

level, i.e., $K_T^*(\theta_0) = 0$. For higher demand levels, the TSO's optimal capacity exceeding K_0 is more than twice the optimal capacity expansion of the PC. Thus, the TSO will want the PC to commit to install a capacity that is considerably larger than the one the PC finds optimal.

The producer surplus is both positively and negatively affected by a capacity increase. The amount of power generated increases, but the price per unit decreases and the total investment cost increases. On the contrary, the consumer surplus is strictly increasing in capacity. Thus, the total surplus is growing at a higher rate than the producer surplus when capacity is increasing. The objective of the PC only takes into account a part of the total producer surplus, which is the continuous profit flow and the investment cost of the PC. The objective of the TSO, on the other hand, includes the consumer surplus in addition to the total producer surplus. Therefore, as the PC does not take into account the consumer surplus when finding its optimal capacity, it is expected that the TSO has a larger optimal capacity.

If the TSO has to accept the PC's optimal capacity, the social welfare achieved will be considerably lower than if it could make the PC to commit to install a capacity of $K_P = K_T^*$. The welfare loss for the TSO from not having the power to decide the size of the PC's capacity expansion, expressed in percentage, is equal to:

$$\text{Welfare loss} = \frac{V_{TSO}(\theta_0, \theta_0, K_T^*) - V_{TSO}(\theta_0, \theta_0, min(K_T^*, K_P^*))}{V_{TSO}(\theta_0, \theta_0, K_T^*)}. \tag{87}$$

Figure 5 illustrates that the percentage welfare loss is increasing in the demand level θ_0. This is because the difference between the two agents' optimal capacities, $K_T^* - K_P^*$, is increasing. The percentage welfare loss increases strongly for low values of θ_0 while more moderately for higher values. For demand levels above $\theta_0 = 10$, the welfare loss from not being able to make the PC commit to install K_T^* is higher than 20%. The result can explain why in practice TSOs try to force PCs to commit to install a minimum capacity before building a transmission line. In the next section, we will extend this analysis by including timing flexibility for both agents.

6.1.2 With timing flexibility

Here, we extend the model from Section 6.1.1 by providing both agents with timing flexibility. We compare sub-problem 2 and 3 where in sub-problem 2 the PC decides capacity for

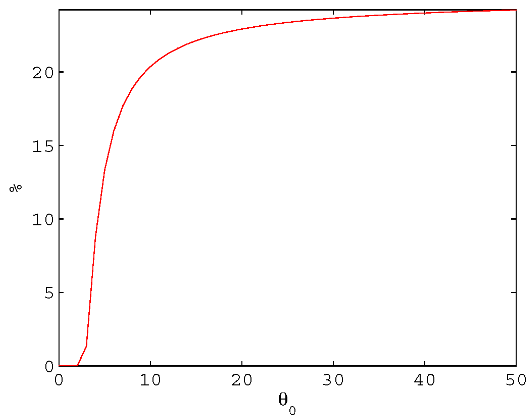


Figure 5: Percentage welfare loss from not having the power to force the PC to invest in K_T^* in the case of now-or-never investments for the base case parameters.

both agents, while in sub-problem 3 the TSO decides. Note that the derivations and optimal investment strategies corresponding to these sub-problems can be found in Appendix C.2 and C.3.

With the parameters from the base case, we get the numerical results shown in Figure 6. The TSO will want the PC to commit to install a capacity that is significantly larger than the capacity the PC will want to install, i.e., $K_T^* > K_P^*$. Given that $\theta_P^*(K) > \theta_T^*(K)$, which is the case here, both investment thresholds are strictly increasing in K, see Proposition 6.

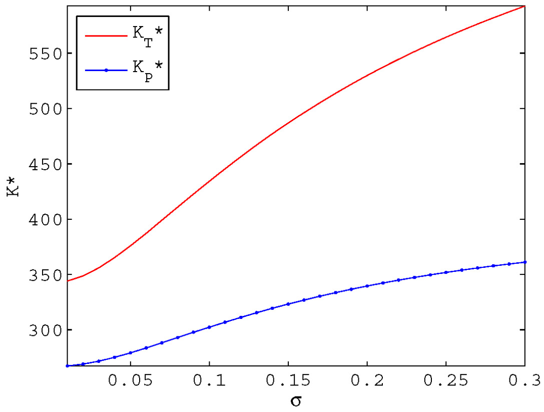
Proposition 6 *Given that $\theta_P^*(K) > \theta_T^*(K)$, the investment threshold of the PC and the TSO, respectively, are strictly increasing in K as:*

$$\frac{\partial \theta_P^*(K)}{\partial K} = \frac{\beta_1}{\beta_1 - 1}(\rho - \alpha)\frac{\delta\eta}{(1 - \eta(2K_0 + K))^2} > 0, \tag{88}$$

and:

$$\frac{\partial \theta_T^*(K)}{\partial K} = \frac{\beta_1}{\beta_1 - 1}(\rho - \alpha)\frac{\gamma}{\frac{1}{2}\eta K_0^2 + (1 - \eta K_0)K_0} > 0. \tag{89}$$

Therefore, the investment triggers θ_T^* and θ_P^* will be higher if both agents have to invest in a capacity of size K_T^*, compared to if both have to invest in K_P^*. Thus, it is optimal for the TSO to make the PC commit to install a larger capacity at a later point in time than what is optimal for the PC.



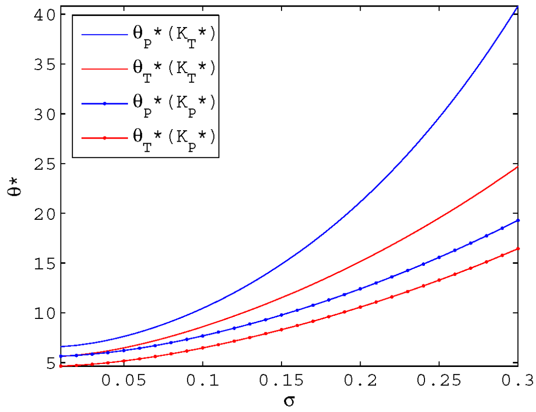


Figure 6: Optimal capacities (left) and optimal investment thresholds (right) for the TSO and the PC as functions of σ for the base case parameters.

Furthermore, we find that the PC will invest later than the TSO in both cases, see Figure 6. The condition for when the PC's investment trigger will be higher than that of the TSO, given that they have to invest in the same capacity, is given in Proposition 7.

Proposition 7 *If both agents have to invest in the same capacity, then the PC will delay investment beyond the investment threshold of the TSO as long as:*

$$\frac{\delta}{\gamma} > \frac{(K_0 + K)(1 - \eta(2K_0 + K))}{K_0(1 - \frac{1}{2}\eta K_0)}. \tag{90}$$

If this condition is satisfied, the flexibility to choose timing, i.e., the ability to wait for more information, *is* of value for the PC and it will choose to delay investment beyond the moment in time the when the TSO invests.

Furthermore, Figure 6 shows that the optimal capacities and the investment triggers are increasing in uncertainty. This can be shown analytically for the investment triggers and the optimal capacity of the PC given that $\theta_P^*(K) > \theta_T^*(K)$, see Proposition 8. However, the implicit expression for K_T^* forces us to bend towards numerical analysis to show the effect of uncertainty on the TSO's optimal investment trigger. Extensive numerical analyses lead to the result that the TSO's optimal capacity is also increasing in uncertainty. That both optimal investment triggers and capacities are increasing in uncertainty is consistent with existing real options literature. However, to our best knowledge, we are the first ones to confirm that this also holds for a model that includes two firms with different objectives.

Proposition 8 *Given that $\theta_P^*(K) > \theta_T^*(K)$ the following holds:*

$$\frac{d\theta_T^*}{d\sigma} > 0, \tag{91}$$

$$\frac{d\theta_P^*}{d\sigma} > 0, \tag{92}$$

$$\frac{dK_P^*}{d\sigma} > 0. \tag{93}$$

The percentage welfare loss if the TSO cannot make the PC commit to install a capacity of $K_P = K_T^*$ when both agents have timing flexibility and K_P^* is the dominating capacity is equal to:

$$\text{Welfare loss} = \frac{V_{TSO}(\theta_T^*(K_T^*), \theta_P^*(K_T^*), K_T^*) - V_{TSO}(\theta_T^*(K_P^*), \theta_P^*(K_P^*), K_P^*)}{V_{TSO}(\theta_T^*(K_T^*), \theta_P^*(K_T^*), K_T^*)}. \tag{94}$$

The numerical result is shown in Figure 7. When uncertainty is very low, the welfare loss is only around $2-3\%$. However, when uncertainty increases the loss increases to more than 10%. Therefore when uncertainty in future demand is high, it is even more important for TSOs to make PCs commit to install a certain capacity than when uncertainty is low.

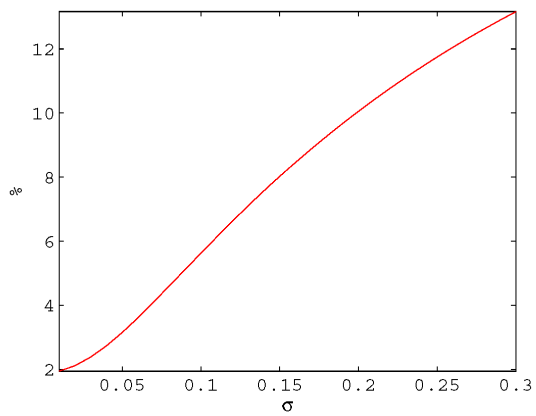


Figure 7: Percentage welfare loss from not being able to decide the PC's capacity as a function of σ for the base case parameters.

6.2 Welfare loss from disregarding the PC's flexibility to choose capacity

Based on the large differences between the two agents' optimal capacities revealed in the last section, the TSO will face a welfare loss if it disregards the PC's flexibility to choose capacity, and rather assumes that the PC will install a generation capacity equal to the capacity of the transmission line. In this section, we study the welfare loss when both agents do not have timing flexibility, i.e., face a now-or-never investment decision, and when both agents have timing flexibility.

6.2.1 Without timing flexibility

The numerical results shown in Figure 4 show that if both agents invest at time zero and the TSO disregards that the PC has flexibility to decide its own capacity, then the TSO will overinvest by choosing to invest in $K_0 + K_T^*$ in case $K_T^* > K_P^*$. Then a transmission capacity of $K_T^* - K_P^*$ will be left unused. The TSO will pay for a transmission capacity of $K_0 + K_T^*$, but the rest of the producer surplus as well as the consumer surplus will depend on the generation capacity of the PC, i.e., the amount of power that is being generated and transmitted. Therefore, if the TSO disregards the investment decision of the PC, the present value of the total surplus is equal to the following expression[16]:

$$\begin{aligned} &V_{TSO}(\theta_0, \theta_0, K) = \\ &\Bigg[\Big[\frac{1}{2}\eta(K_0 + min(K_P^*, K_T^*))^2 + (1 - \eta(K_0 + min(K_P^*, K_T^*)))(K_0 + min(K_P^*, K_T^*))\Big]\frac{\theta_0}{\rho - \alpha} \\ &\qquad - \gamma(K_0 + K_T^*) - \delta min(K_P^*, K_T^*)\Bigg]. \end{aligned} \tag{95}$$

This will yield a lower social welfare than if the TSO instead anticipated the PC's optimal investment decision and invested in $K_T = K_P^*$. The welfare obtained from anticipating the PC's decision and investing in the lower of the two optimal capacities is equal to $V_{TSO}(\theta_0, \theta_0, min(K_T^*, K_P^*))$.

Therewith, the welfare loss from not taking the PC's optimal investment decision into account expressed in percentage is equal to:

$$\text{Welfare loss} = \frac{V_{TSO}(\theta_0, \theta_0, min(K_T^*, K_P^*)) - V_{TSO}(\theta_0, \theta_0, K)}{V_{TSO}(\theta_0, \theta_0, min(K_T^*, K_P^*))}. \tag{96}$$

[16] K is used in the dependency bracket as the two agents install different capacities.

As shown in Figure 8, the absolute value of the welfare loss is strictly increasing in θ_0 when $K_P^* < K_T^*$. However, the loss expressed in percentage is decreasing in θ_0. This is because the loss from having overinvested, $\gamma(K_T^* - K_P^*)$, constitutes a diminishing part of the welfare the TSO could have achieved if it instead anticipated the PC's decision, i.e., the denominator in Equation (96), when θ_0 increases.

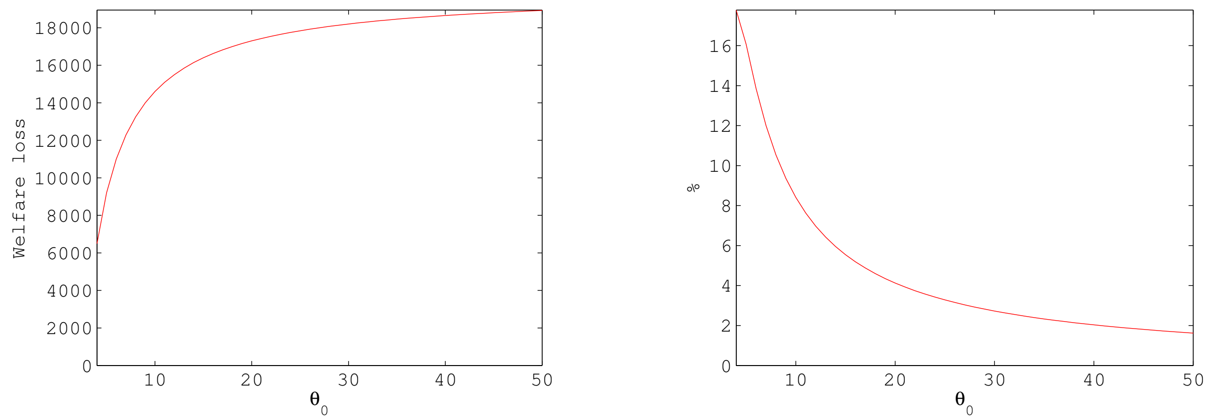


Figure 8: Absolute welfare loss (left) and percentage welfare loss (right) if the TSO does not take into account that the PC has flexibility to decide capacity as a function of θ_0 for the base case parameters in the case of now-or-never investments.

6.2.2 With timing flexibility

Here, we extend the model from Section 6.2.1 by providing both agents with timing flexibility. As in Section 6.1.2, we compare sub-problems 2 and 3 where in sub-problem 2 the PC decides capacity for both agents, while in sub-problem 3 the TSO decides.

Figure 6 shows that it is optimal for the TSO that both agents invest in a larger capacity at a later point in time than what is optimal for the PC. Consequently, if the TSO wrongly assumes that the PC will install a generation capacity equal to the optimal capacity of the transmission line exceeding K_0, i.e., $K_P = K_T^*$, the TSO will install a capacity of K_T^* as shown in Figure 6, and invest at $\theta_T^*(K_T^*)$. The TSO will then expect the PC to wait until $\theta_P^*(K_T^*)$ and invest in $K_T^*(\theta_T^*)$. However, the PC will obtain a higher value by investing immediately after the TSO, i.e., $\theta_P = \theta_T^*$, in a capacity of size $K_P^*(\theta_T^*)$ as shown in Figure 9.

Therefore, if the TSO disregards that the PC can choose its own optimal capacity size ,

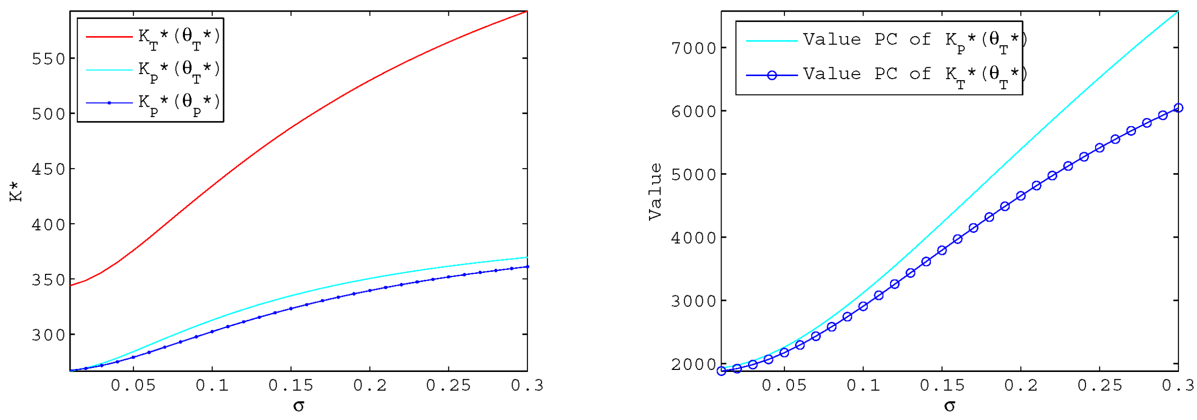


Figure 9: Optimal capacities (left) and value of the PC (right) as functions of σ for the base case parameters when including timing flexibility for both agents.

the present value of the total surplus is equal to the following expression[17]:

$$V_{TSO}(\theta_T^*, \theta_T^*, K) = \left(\frac{\theta_0}{\theta_T^*}\right)^{\beta_1} \Bigg[\Big[\frac{1}{2}\eta(K_0 + min(K_T^*, K_P^*))^2$$

$$+(1-\eta(K_0+min(K_T^*, K_P^*)))(K_0+min(K_T^*, K_P^*))\Big]\frac{\theta_T^*}{\rho - \alpha} - \gamma(K_0+K_T^*) - \delta min(K_T^*, K_P^*)\Bigg]. \tag{97}$$

However, if it anticipates that the PC will invest in a lower capacity, the resulting total surplus will be equal to $V_{TSO}(\theta_T^*, \theta_P^*, \min(K_P^*, K_T^*))$. Therewith, the welfare loss for the TSO from not anticipating the investment decision of the PC, expressed in percentage, is equal to:

$$\text{Welfare loss} = \frac{V_{TSO}(\theta_T^*, \theta_P^*, \min(K_P^*, K_T^*)) - V_{TSO}(\theta_T^*, \theta_T^*, K)}{V_{TSO}(\theta_T^*, \theta_P^*, \min(K_P^*, K_T^*))}. \tag{98}$$

As shown in Figure 10, the absolute welfare loss for the TSO from not taking into account that the PC can choose its optimal capacity is increasing in uncertainty. Furthermore, the percentage welfare loss is between 3 and 5% for the levels of uncertainty considered. When there is no uncertainty the TSO will invest when θ_t hits $\theta_T^* = 6$, which can be seen from Figure 6. Then the welfare loss from not anticipating the PC's investment decision is around 5%. Comparing this number to the case where both agents face a now-or-never investment decision, we see that the welfare loss from not anticipating the PC's investment decision

[17] K is used in the dependency bracket as the two agents install different capacities.

is higher, and around 11%, for an initial demand level of $\theta_0 = 6$. This suggests that the welfare loss from not anticipating the PC's decision is lower when both agents have timing flexibility. This is because the difference in the two agents' optimal capacities is larger when they both have to invest at time zero, than when the PC can delay its investment beyond the investment time of the TSO. The reason for this is that the PC's optimal capacity is increasing in the demand level, see Proposition 9. Therefore, when it delays investment, it will invest in a larger capacity closer to the optimal capacity of the TSO, compared to if it had to invest at time zero.

Proposition 9 *The optimal capacity of the PC is strictly increasing in the demand level upon investment:*

$$\frac{dK_P^*}{d\theta_t} = \frac{1}{2\eta}\frac{\delta(\rho - \alpha)}{\theta_t^2} > 0. \tag{99}$$

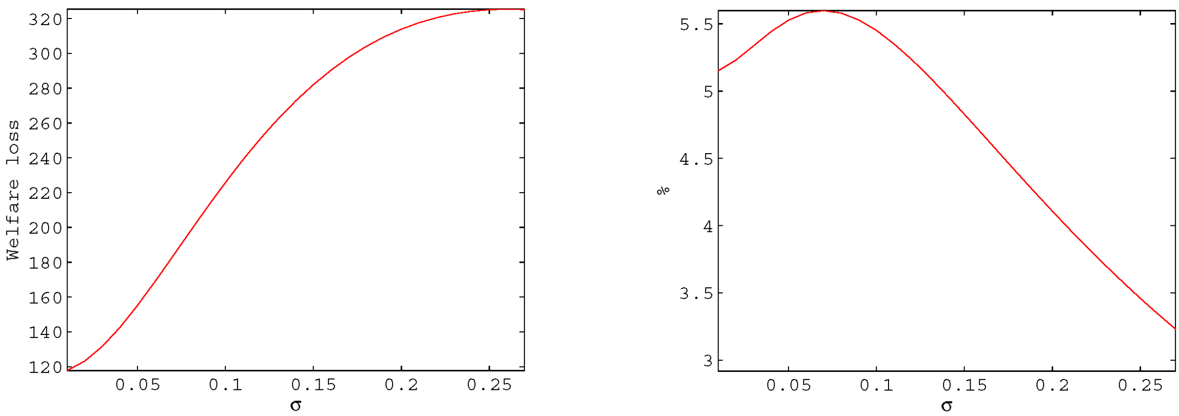
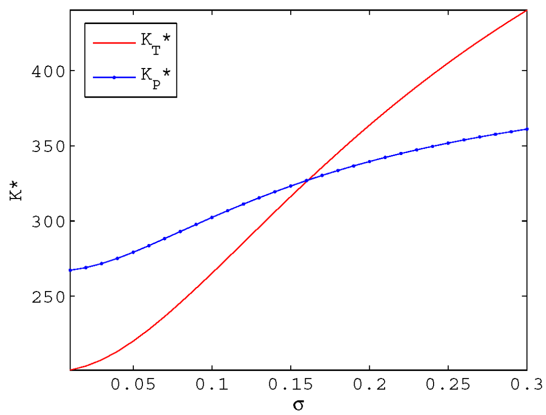


Figure 10: Absolute welfare loss (left) and percentage welfare loss (right) if the TSO does not take into account that the PC has flexibility to decide capacity as a function of σ for the base case parameters with timing flexibility for both agents.

6.3 Welfare loss from the TSO not having timing flexibility

Flexibility in timing is valuable for the TSO due to two reasons. 1) The option value from being able to choose its optimal investment timing rather than having to make a now-or-never investment decision, and 2) the strategic value from being able to postpone the investment timing of the PC through its own investment timing. Since K_P^* is increasing in θ_P, see Proposition 9, the PC will invest in a larger capacity if it is forced to delay investment. However, when the TSO has to make a now-or-never decision, it can only affect the PC's investment decision through its own capacity choice. The optimal triggers and capacities for sub-problem 4, which is described here, are given in Appendix C.4.

Using the base case parameters, we get the numerical results shown in Figure 11 for a current demand level of $\theta_0 = 3$. This demand level is chosen because at $\theta_0 < 3$ $K_T^* = 0$, i.e., it is not optimal for the TSO to invest in a larger capacity than K_0, and consequently, the PC will never expand its generation capacity. Figure 11 shows that the TSO will be able to affect the PC's investment decision through its capacity choice when uncertainty is relatively low, i.e., the PC will be forced to invest in a lower capacity than it finds optimal, $K_P = K_T^*$, and will therefore invest at an earlier point in time than it would find optimal if it could transmit an amount of power equal to K_P^*. On the other hand, K_P^* will be dominating when



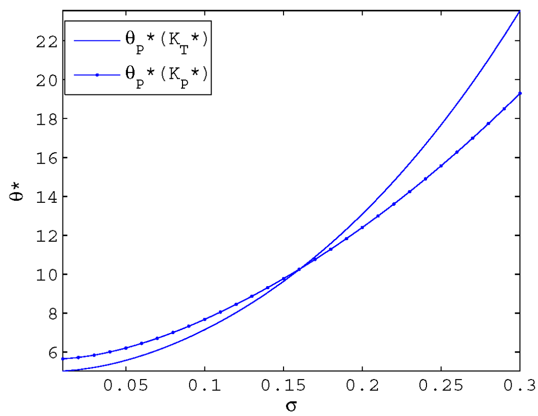


Figure 11: Optimal capacities (left) and optimal investment thresholds for the PC (right) when the TSO does not have timing flexibility as functions of σ for the base case parameters and $\theta_0 = 3$.

uncertainty is high, and then the TSO will not be able to affect the PC's decision through its capacity choice. For all uncertainties, the PC will choose to delay investment compared to the TSO that has to invest immediately at $\theta_0 = 3$. This is because the current demand level is too low for it to be optimal for the PC to expand generation capacity. Moreover, the value of waiting to get more information increases for the PC when uncertainty increases.

The welfare loss due to the inability for the TSO to choose its own investment time is defined as[18]:

$$\text{Welfare loss} = \frac{V_{TSO}(\theta_T^*, \theta_P^*, min(K_T^*, K_P^*)) - V_{TSO}(\theta_0, \theta_P^*, min(K_T^*, K_P^*)}{V_{TSO}(\theta_T^*, \theta_P^*, min(K_T^*, K_P^*))}, \tag{100}$$

which is equal to the percentage difference between the welfare from solving the full problem, where both agents have timing and sizing flexibility, and the welfare when the TSO cannot decide timing and has to make a now-or-never investment decision.

By varying the value of θ_0, we reveal how the welfare loss changes depending on the current demand level. The results are shown in Figure 12. We find that the welfare loss

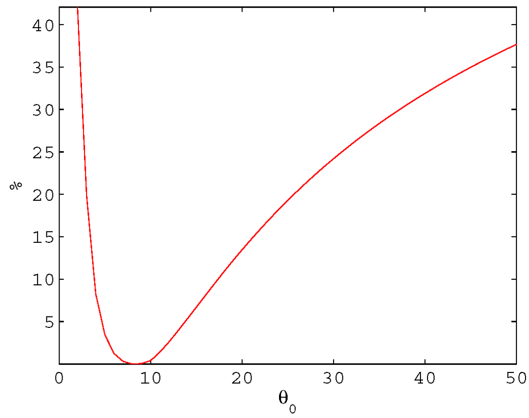


Figure 12: Percentage welfare loss when the TSO does not have timing flexibility as a function of the current demand level, θ_0, for the base case parameters.

with regards to θ_0 is largest if the TSO is forced to invest when the value of θ_0 is very low. For $\theta_0 < 3$, $K_T^* = 0$ and the welfare loss is very high as the PC will never expand its generation capacity, i.e. $\theta_P^*(0) = \infty$. Furthermore, for low demand levels above $\theta_0 = 3$, the TSO can only justify to install a small capacity. For $\theta_0 = 8.3$ the welfare loss is equal to zero as it would be optimal for the TSO to invest immediately at this demand level if it had timing flexibility. The welfare loss first decreases steeply for low values of θ_0, while it increases for high values at a more moderate rate. This is because for low demand levels, the TSO loses both the real option value of postponing investment and the strategic value of being able to affect the PC's investment decision thorough its timing choice. For higher

[18] Here, we have refrained from writing the triggers' dependency on $min(K_T^*, K_P^*)$ for readability.

demand levels the TSO is forced to invest at a sup-optimal point in time, since it optimally would invest earlier. However, the TSO will be able to get the strategic value from making the PC delay investment and hence invest in a larger capacity. The strategic effect will mitigate the welfare loss from having to invest at a sub-optimal time.

The results show that when the TSO does not have timing flexibility, it can lead to a significant loss in social welfare compared to when it does have this flexibility.

6.4 Welfare loss from disregarding the PC's flexibility to choose timing

In Appendix C.5, the optimal investment strategies for sub-problem 5 where both agents have sizing flexibility, but only the TSO has timing flexibility is presented. In this sub-problem, it is assumed that the PC will invest at the same time as the TSO. Following the optimal investment strategy from solving this problem and ignoring that the PC has timing flexibility, will only be optimal for the TSO if the optimal investment trigger of the PC, θ_P^*, is not larger than that of the TSO, θ_T^*. For parameter sets where the inequality in Proposition 7 holds, the PC will delay its investment beyond the investment time of the TSO if the expansion has to be of size K, i.e., $\theta_P^*(K) > \theta_T^*(K)$. Consequently, in these cases the TSO will suffer a welfare loss from not taking into account that the PC can choose to delay investment.

We define the percentage welfare loss from not considering the PC's timing flexibility as[19]:

$$\text{Welfare loss} = \frac{V_{TSO}(\theta_T^*, \theta_P^*, min(K_T^*, K_P^*)) - V_{TSO}(\theta_T^*, \theta_T^*, min(K_T^*, K_P^*)}{V_{TSO}(\theta_T^*, \theta_P^*, min(K_T^*, K_P^*))}, \tag{101}$$

which is the difference between the welfare achieved following the optimal strategy from the full model and the welfare achieved when the TSO follows its optimal investment strategy from sub-problem 5 while the PC actually chooses to delay investment and invest at $\theta_P^* > \theta_T^*$.

For the parameters in the base case, the inequality in Proposition 7 does hold, i.e., the PC will want to delay investment compared to the TSO. Hence there will be a welfare loss from assuming that the PC invests at the same time as the TSO. The numerical results are shown in Figure 13 and 14[20].

[19]Also here we have refrained from writing the triggers' dependency on $min(K_T^*, K_P^*)$ for readability.

[20]K_T^* is not included in the graphs because it in both cases will be higher than K_P^*, i.e., K_P^* is the dominating capacity.

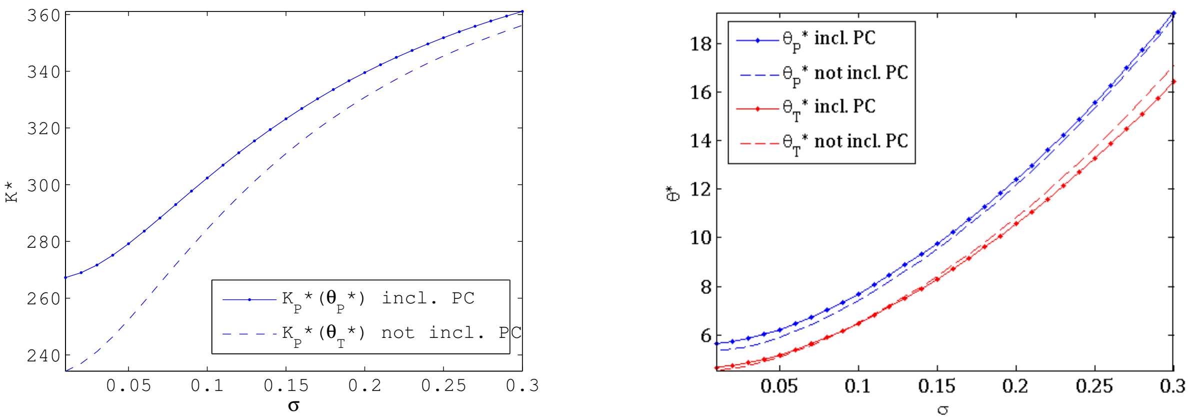


Figure 13: Comparison of optimal capacities for the PC (left) and the optimal investment thresholds for the PC and the TSO (right) as functions of σ for the base case parameters when the TSO considers the PC's timing flexibility and when it does not.

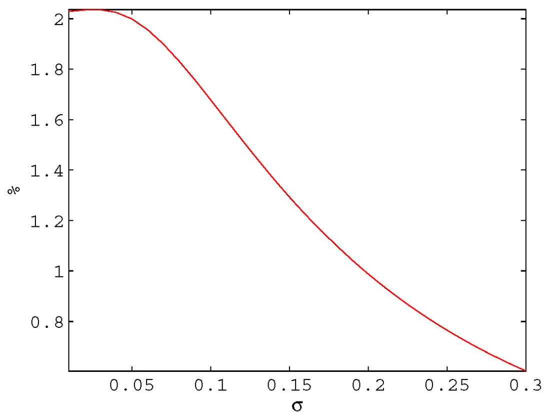


Figure 14: Percentage welfare loss from assuming that the PC invests at the same time as the TSO while it actually delays investment as a function of σ for the base case parameters.

The numerical results show that the TSO will underinvest if it does not take into account that the PC can delay investment. Since the capacity is increasing in the demand level, see Proposition 9, the PC will want to invest in a larger capacity given that it delays investment compared to if it has to invest at the same time as the TSO. When the TSO invests in $K_P^*(\theta_T^*)$, it sets an upper bound for the PC and prevents it from investing in its own optimal capacity, $K_P^*(\theta_P^*)$, which is higher. This leads to a loss in social welfare compared to if it anticipates that the PC will want to delay investment and invest in a larger capacity. From Figure 14, we see that the loss is decreasing in uncertainty. This is because the difference in the two optimal capacities is decreasing in uncertainty, as illustrated in Figure 13. When the difference between the capacity the TSO assumes the PC will install and the capacity the PC will optimally install decreases, the welfare loss from wrongly assuming that the PC will invest at the same time also decreases. Note that the welfare loss for the TSO from disregarding the PC's flexibility to choose its own timing is never above 2 % and hence is lower than the welfare loss from disregarding that the PC can choose its own capacity, which is between 3 and 5%.

6.5 Results of the full model

The numerical results from solving the full model where both agents have timing and sizing flexibility are shown in Figure 15 for the base case parameters. We see that the TSO, if it has the power to decide, will want to invest in a larger capacity at a later point in time than the PC. But since the PC's capacity will be dominating, the TSO anticipates this capacity choice and invests in the same capacity. Furthermore, the TSO will find its

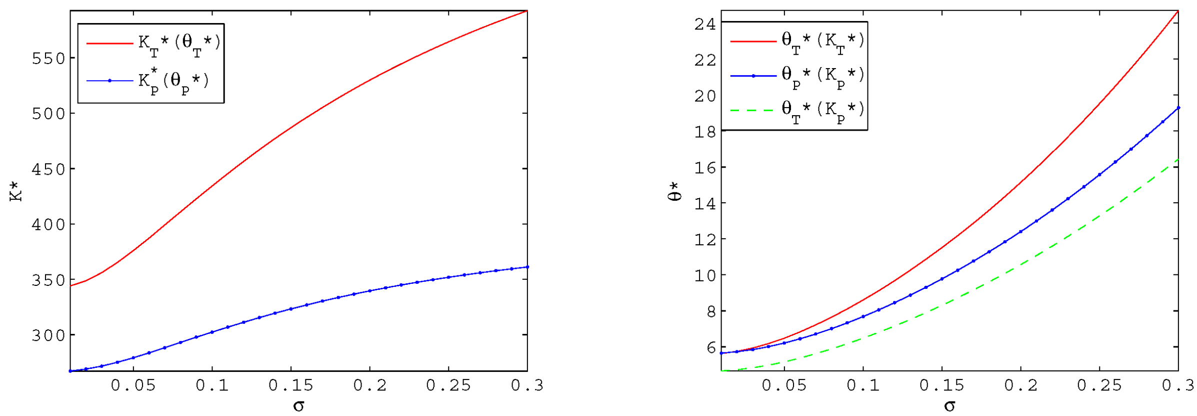


Figure 15: Optimal capacities (left) and the optimal investment threshold of the PC and the initial and final optimal investment thresholds for the TSO (right) when both agents have timing and sizing flexibility as functions of σ for the base case parameters.

optimal investment timing based on the PC's capacity choice, K_P^*. Since $\theta_T^*(K_P^*) < \theta_P^*(K_P^*)$, the TSO will choose to invest earlier than the PC as shown in Figure 15. When investing before the PC, the TSO gains welfare from the existing generation capacity, K_0, before the PC undertakes its expansion.

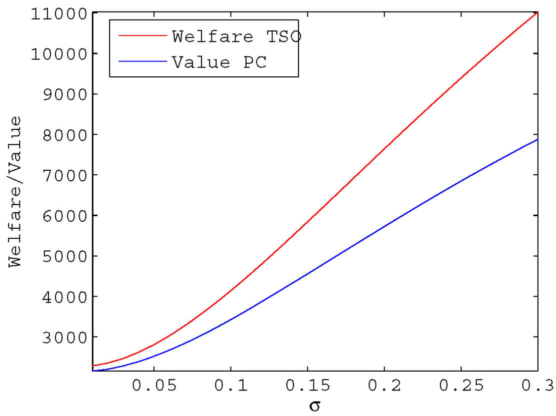


Figure 16: Social welfare and the value of the PC as functions of σ for the base case parameters.

As Dixit (1993), Dangl (1999) and Chronopoulos et al. (2015), we find that both the size and the investment thresholds are increasing in uncertainty as can be seen from the numerical results and Proposition 8. High uncertainty leads to a high value of waiting. This delays investment with the implication that at the moment of investment the market has grown enough to invest in a larger capacity. Figure 16 shows the welfare for the TSO and the profit of the PC as functions of uncertainty. Both are increasing in uncertainty. It is not possible to show analytically that the values are always increasing in uncertainty as it depends on the parameter values and optimal values of the variables.

The numerical analysis of the full model with the base case parameters, shows that the PC will not be bounded by the TSO on timing or capacity, see Outcome 3 in Table 3. Rather the TSO must adapt to the PC's decision, which is sub-optimal for the TSO. Hence, it will be beneficial for the TSO if it can make the PC commit to install $K_P = K_T^*$ to avoid that the PC invests in a capacity that is lower than what is optimal from a social welfare perspective. In the next section, we analyse how the results from the full model changes when we introduce a subsidy of the PC's investment cost.

6.6 Welfare gain from a subsidy of the PC's investment cost

The numerical analysis of the full model suggests that given the base case parameters, the PC has the dominating capacity and hence the power to decide the size of the TSO's investment. The TSO can choose to delay investment to make the PC invest later in a larger capacity but, given the base case parameters, this is sub-optimal for the TSO. Therefore, the TSO will seek to influence the PC to make it invest in a larger capacity to increase social welfare. One way to give the PC an economic incentive to invest in a larger capacity is through a subsidy of some of the PC's investment costs. In the left panel of Figure 17, we compare the optimal capacity of the PC in the case of no subsidy with the case where 50 % of the PC's investment cost is subsidised by an external party, i.e., $\delta = 50$.

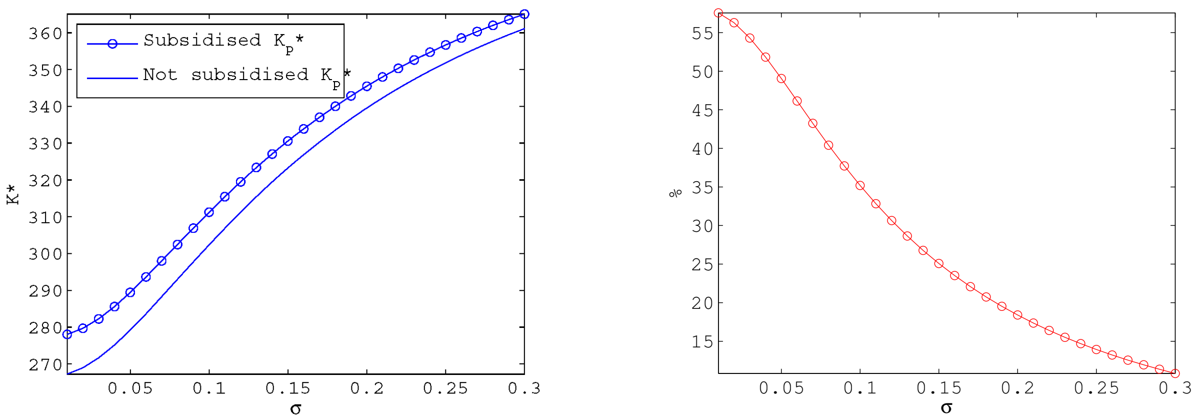


Figure 17: Optimal capacities in the case of no subsidy, $\delta = 100$, and a 50 % subsidy, $\delta = 50$ (left) and the percentage gain in welfare from a 50 % subsidy (right) as functions of σ.

We find that when δ is reduced by 50%, the PC invests in a larger capacity. This will result in a higher social welfare. Since we assume that the subsidy is paid by an external party, the gain in total surplus from providing a subsidy, expressed in percentage, is defined as[21]:

$$\text{Welfare gain} = \frac{V_{TSO,\text{with subsidy}} - V_{TSO,\text{without subsidy}}}{V_{TSO,\text{without subsidy}}}. \tag{102}$$

The percentage gain from providing a subsidy is shown in the right panel of Figure 17. The welfare gain is decreasing in uncertainty. This is because the difference between the

[21]If the subsidy was not paid by an external party, then the cost of the subsidy should have been included when calculating total surplus to find whether the subsidy provided a net increase in social welfare or not.

capacity the PC will install with and without the subsidy is decreasing in uncertainty. This can be seen from the left panel of Figure 17. The analysis shows that a subsidy of the PC's investment cost increases the optimal capacity of the PC and hence social welfare.

In Figure 18, we extend the analysis by studying how the optimal capacities and thresholds change with different subsidy levels[22]. We find that an increasing subsidy level not

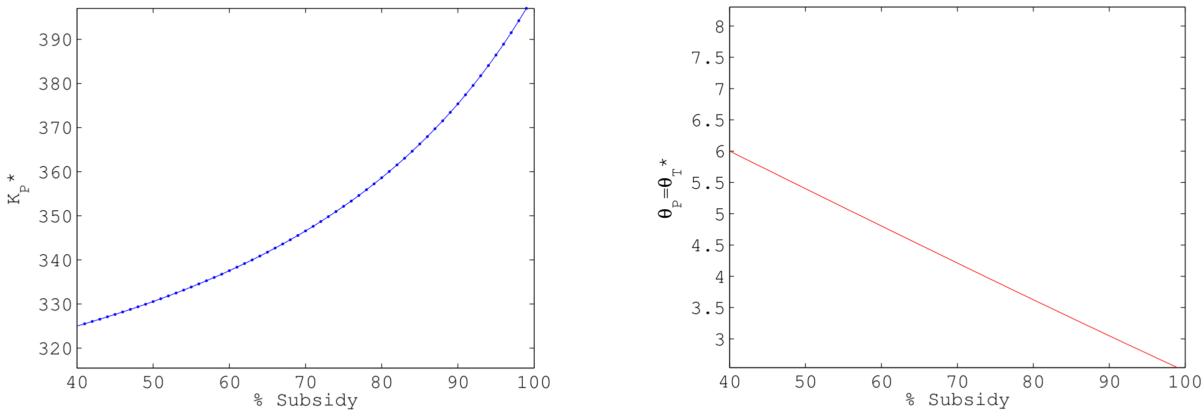


Figure 18: Optimal capacity (left) and the optimal investment threshold (right) for a subsidy between 40 and 100% for the base case parameters.

only increases the optimal capacity expansion of the PC, it also triggers earlier investment. When more than 40% of the PC's investment cost is subsidised by an external party, it becomes optimal for the PC to invest as soon as the TSO has invested instead of waiting. Therefore, the investment timing of the PC is equal the TSO's as shown in the right panel of Figure 18. Also, the TSOs investment timing is decreasing with increasing subsidy level as the investment cost of the PC, which the TSO takes into account when finding the investment trigger that maximise social welfare, decreases. Hence, not only does a subsidy of more than 40% of the PC's investment cost increase the optimal capacity, it also triggers earlier investment by the PC than in the case of no subsidy.

Last, we evaluate how the percentage gain in welfare from a subsidy varies with the size of the subsidy as shown in Figure 19. The gain in welfare is increasing when the subsidy increases. We conclude that a subsidy might be a tool to make the PC invest in a larger capacity at an earlier point in time to increase total surplus.

[22]We find that K_P^* is the dominating capacity for all levels of subsidy. Therefore, K_T^* is not included in the graph.

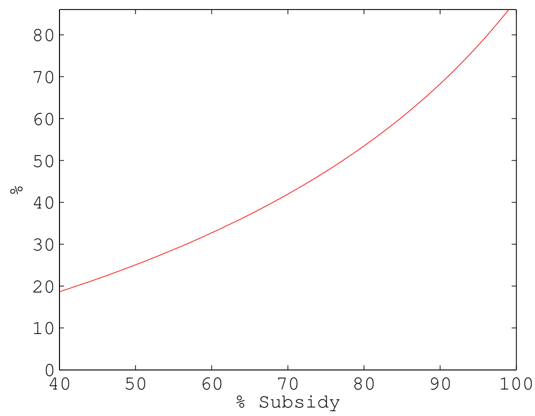


Figure 19: The percentage gain in welfare from a subsidy between 40 and 100% for the base case parameters.

7 Conclusion

This paper extends the theoretical real options literature by considering a two-firm setting with different objectives. In particular, we determine the optimal timing and sizing strategy of a welfare-maximising TSO taking into account the optimal timing and sizing decision of a profit-maximising PC.

We find that disregarding the PC's optimal investment decision can have a large negative impact on social welfare for a TSO. This is because, in most cases, the TSO will want both agents to invest in a larger capacity than what is optimal for the PC. This implies that the TSO faces a risk of investing in transmission capacity that will be left unused by the PC if it does not consider the PCs' optimal capacity decision. The only time we find that the optimal capacity of the TSO is less than that of the PC is if the TSO does not have timing flexibility and is forced to invest at a low demand level. Then, for low uncertainties, the optimal capacity of the TSO is dominating. Furthermore, we find that if the TSO considers only the PC's sizing flexibility and not the flexibility in timing, then it risks investing in a too small capacity. This is because the PC would optimally want to delay investment, and, therefore, invest in a larger capacity than the TSO anticipates it to install if it assumes that the PC invests at the same time as itself.

We find that increased demand uncertainty leads to an increase in optimal capacity and a delay in investment because of the increased value of waiting. This is similar to what has been shown in previous real options literature with respect to timing and sizing. Also the welfare loss from not taking the PC's optimal investment decision into account increases in uncertainty.

We find that not only does a subsidy of the PC's investment cost increase the optimal capacity, but it also triggers earlier investment by the PC. Therefore, a subsidy can be used as a tool to increase social welfare. The analysis can be a starting point for a more comprehensive study to find the optimal subsidy level given that the cost of the subsidy is taken into account in the calculation of the total surplus. Furthermore, one could analyse the effect of providing a subsidy only above a certain capacity level. A similar approach to Boomsma et al. (2012) could also be incorporated into the model to study how different support schemes affect the optimal investment decision of the PC, and thereby the optimal strategy of the TSO.

The model could be extended by introducing volume flexibility, i.e., relax the assumption that the PC produces up to capacity. It would be interesting to study as it might cause the PC to invest in a larger capacity and hence shift the current power structure from the PC having the dominating capacity, in most cases, to the TSO. However, this extension would likewise be at the expense of being able to obtain an analytical solution.

Finally, it would be interesting to apply the model in a case study. Then, it would be necessary to do an empirical analysis to find more realistic parameters for both agents' investment cost, variable and fixed production costs, drift and discount rates. One could expect the two companies to have different discount rates. Also the price process for the considered market should be evaluated to reveal if it really follows a GBM or if it is mean-reverting.

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Appendix

A Detailed derivations

A.1 Derivation of and solution to the VM and SP conditions in Section 5.1.1

To determine the investment threshold θ_P^* and the endogenous constant A_1, we employ the value-matching and smooth-pasting conditions:

$$A_1\theta_P^{*\beta_1} + V_1(\theta_P^*, K_0) = V_2(\theta_P^*, K_0 + K_P^*(\theta_P^*)), \tag{A.1}$$

$$A_1\beta_1\theta_P^{*\beta_1-1} + \left.\frac{dV_1}{d\theta_t}\right|_{\theta_t=\theta_P^*} = \left.\frac{\partial V_2}{\partial\theta_t}\right|_{\theta_t=\theta_P^*} + \left.\frac{\partial V_2}{\partial K_P}\frac{\partial K_P^*}{\partial\theta_t}\right|_{\theta_t=\theta_P^*}. \tag{A.2}$$

Note that K_P^* depends on θ_P^*. However, after maximising the present value of the PC after investment, $V_2(\theta_P^*, K_0 + K_P^*(\theta_P^*))$, with respect to K_P, for a given value of the demand shift parameter at the time of investment, θ_t, by the envelope theorem we have that $\frac{\partial V_2}{\partial K_P^*} = 0$. The smooth-pasting condition reduces to:

$$A_1\beta_1\theta_P^{*\beta_1-1} + \left.\frac{dV_1}{d\theta_t}\right|_{\theta_t=\theta_P^*} = \left.\frac{\partial V_2}{\partial\theta_t}\right|_{\theta_t=\theta_P^*}. \tag{A.3}$$

The smooth-pasting condition gives the value of the endogenous constant A_1:

$$A_1 = \frac{K_P^*(\theta_P^*)(1-\eta(2K_0 + K_P^*(\theta_P^*)))}{\rho-\alpha}\frac{1}{\beta_1}\theta_P^{*1-\beta_1}. \tag{A.4}$$

Substituting the expression for A_1 into the value-matching condition and solving for θ_P^*, we get that the optimal investment threshold is given by the solution to the following implicit equation:

$$\hat{\theta}_P^* = \frac{\beta_1}{\beta_1 - 1}(\rho - \alpha)\frac{\delta}{1 - \eta(2K_0 + K_P^*(\hat{\theta}_P^*))}. \tag{A.5}$$

A.2 Derivation of and solution to the VM and SP conditions in Section 5.1.2

To determine the optimal investment threshold θ_T^* and the value of the endogenous constant B_1, we employ the value-matching and smooth-pasting conditions. First, the value-matching condition is given by:

$$B_1 \theta_T^{*\beta_1} = TS(\theta_T^*, K_T^*(\theta_T^*), \theta_P^*(K_T^*(\theta_T^*)). \tag{A.6}$$

If we substitute for $\theta_P^*(K_T^*)$ into the expression for the total surplus, then we can write the value-matching condition as:

$$B_1 \theta_T^{*\beta_1} = TS(\theta_T^*, K_T^*(\theta_T^*)). \tag{A.7}$$

When deriving the smooth-pasting condition, one has to take into account that K_T^* depends on θ_T. Therefore, when deriving the smooth-pasting condition we get:

$$B_1 \beta_1 \theta_T^{*\beta_1 - 1} = \left.\frac{\partial TS}{\partial \theta_t}\right|_{\theta_t = \theta_T^*} + \left.\frac{\partial TS}{\partial K_T}\frac{\partial K_T^*}{\partial \theta_t}\right|_{\theta_t = \theta_T^*}. \tag{A.8}$$

However, after maximising the total surplus (TS), where we have substituted the optimal $\theta_P^*(K_T)$, with respect to K_T, by the envelope theorem we have that $\frac{\partial TS}{\partial K_T} = 0$, see Equation (41). Therefore, the smooth-pasting condition reduces to:

$$B_1 \beta_1 \theta_T^{*\beta_1 - 1} = \left.\frac{\partial TS}{\partial \theta_t}\right|_{\theta_t = \theta_T^*}. \tag{A.9}$$

The smooth-pasting condition gives:

$$B_1 = \left[K_0 - \frac{1}{2}\eta K_0^2\right]\frac{\theta_T^{*1-\beta_1}}{\beta_1(\rho - \alpha)} + \left[K_T^*(\theta_T^*)(1 - \eta(K_0 - \frac{1}{2}K_T^*(\theta_T^*)))\right]\frac{\theta_P(K_T^*)^{*1-\beta_1}}{\rho - \alpha} - \delta K_T^*(\theta_T^*)\theta_P(K_T^*)^{*-\beta_1}. \tag{A.10}$$

Substituting the expression for B_1 into the value-matching condition and solving for θ_T^*, we get that the optimal investment threshold is given by the solution to the following implicit equation:

$$\theta_T^* = \frac{\beta_1}{\beta_1 - 1}(\rho - \alpha)\frac{\gamma(K_0 + K_T^*(\theta_T^*))}{K_0(1 - \frac{1}{2}\eta K_0)}. \tag{A.11}$$

A.3 Derivation of and solution to the VM and SP conditions corresponding to the corner solution in Section 5.1.2

To determine the indifference level θ_T^* and the value of the endogenous constant B_1, we employ the value-matching and smooth-pasting conditions. First, the value-matching condition is given by:

$$B_1 \theta_T^{*\beta_1} = TS(\theta_T^*, K_T^*(\theta_T^*)). \tag{A.12}$$

where the total expected surplus is equal to:

$$TS(\theta_t, K_T, \theta_P^*) = \left[\frac{1}{2}\eta(K_0 + K_T)^2 + (1 - \eta(K_0 + K_T))(K_0 + K_T)\right]\frac{\theta_t}{\rho - \alpha} - \gamma(K_0 + K_T) - \delta K_T. \tag{A.13}$$

Next, when deriving the smooth-pasting condition, one has to take into account that K_T^* depends on θ_T. We get:

$$B_1 \beta_1 \theta_T^{*\beta_1 - 1} = \left.\frac{\partial TS}{\partial \theta_t}\right|_{\theta_t = \theta_T^*} + \left.\frac{\partial TS}{\partial K_T}\frac{\partial K_T^*}{\partial \theta_t}\right|_{\theta_t = \theta_T^*}. \tag{A.14}$$

However, after maximising the total surplus (TS) with respect to K_T we have that $\frac{\partial TS}{\partial K_T} = 0$ and the smooth-pasting condition reduces to:

$$B_1 \beta_1 \theta_T^{*\beta_1 - 1} = \left.\frac{\partial TS}{\partial \theta_t}\right|_{\theta_t = \theta_T^*}. \tag{A.15}$$

By solving the value-matching and smooth pasting conditions we find the following expression for B_1:

$$B_1 = \left[(K_0 + K_T^*(\theta_T^*))(1 - \frac{1}{2}\eta(K_0 + K_T^*(\theta_T^*)))\right]\frac{\theta_T^{*1-\beta_1}}{\beta_1(\rho - \alpha)}, \tag{A.16}$$

The optimal investment threshold is given by the solution to the following implicit equation:

$$\theta_T^* = \frac{\beta_1}{\beta_1 - 1}(\rho - \alpha)\frac{\gamma(K_0 + K_T^*(\theta_T^*)) + \delta K_T^*(\theta_T^*)}{(K_0 + K_T^*(\theta_T^*))(1 - \frac{1}{2}\eta(K_0 + K_T^*(\theta_T^*)))}. \tag{A.17}$$

B Proofs of Propositions

B.1 Proof of Proposition 1

The proof is given in Section 5.1.1.

B.2 Proof of Proposition 2

The proof is given in Section 5.1.2.

B.3 Proof of Proposition 3

The proof is given in Section 5.1.2.

B.4 Proof of Proposition 4

The proof of the optimal investment strategies in the region where $\theta_T^*(K_P^*) < \theta_P^*(K_P^*)$ is given in Section 5.1.1 and 5.2.2. However here we derive the proof of the optimal investment strategies when we assume that the corner solution $\theta_P(K_P^*) = \theta_T^*(K_P^*)$ is optimal.

In this case, the PC holds only the option to decide capacity, not timing, and will have to follow the TSO's investment timing strategy. The investment problem of the PC at time zero is equal to:

$$\max_{K_P} \mathbb{E}\left[\int_{s=\tau_T}^{\infty} e^{-\rho s}\pi(\theta_s, K_0 + K_P)ds - e^{-\rho\tau_P}\delta K_P \Big| \theta_0\right]. \tag{B.1}$$

The optimal K_P^* is given by the same expression as in Section 5.1.1, except that it depends on θ_T^* in this case, therefore, K_P^* is given by:

$$K_P^*(\theta_T^*) = \max\left(\frac{1}{2\eta}\left[1 - \frac{\delta(\rho - \alpha)}{\theta_T^*}\right] - K_0, 0\right). \tag{B.2}$$

To find the TSO's optimal investment time, we need to solve its optimal stopping problem.

At time zero it is equal to:

$$\sup_{\tau_T} \mathbb{E}\Big[\int_{s=\tau_T}^{\infty} e^{-\rho s} ts(\theta_s; K_0 + K_P^*)ds - e^{-\rho\tau_T}\gamma(K_0 + K_P^*) - e^{-\rho\tau_T}\delta(K_P^*)\Big|\theta_0\Big]. \tag{B.3}$$

We follow a dynamic programming approach to solve the optimal stopping problem. The value for the TSO, F, at time t is equal to:

$$F(\theta_t, K_T^*(\theta_t)) = \begin{cases} B_1\theta_t^{\beta_1} & \text{if } \theta_t \leq \theta_T, \\ TS(\theta_t, K_P^*(\theta_t)) & \text{if } \theta_T \leq \theta_t, \end{cases} \tag{B.4}$$

where the value in the continuation region is derived by finding the solution to the ordinary differential equation (ODE) that stews from the Bellman equation:

$$\rho F dt = \mathbb{E}[dF]. \tag{B.5}$$

Standard calculations similar to those in Section 5.1.1 and 5.1.2 are performed, which lead to the value function stated in Equation (B.4).

In order to find the TSO's optimal investment time, we need to determine the optimal investment threshold, θ_T^*, and the value of the endogenous constant B_1 by employing the value-matching and smooth-pasting conditions. First, the value-matching condition is given by:

$$B_1\theta_T^{*\beta_1} = TS(\theta_T^*, K_P^*(\theta_T^*)), \tag{B.6}$$

where the total expected surplus at time τ_T^*, given that both invest at the same time, is equal to:

$$\begin{aligned} TS(\theta_T^*, K_P^*(\theta_T^*)) = \Big[\frac{1}{2}\eta(K_0 + K_P^*(\theta_T^*))^2 + (1 - \eta(K_0 + K_P^*(\theta_T^*)))(K_0 + K_P^*(\theta_T^*))\Big]\frac{\theta_T^*}{\rho - \alpha} \\ - \gamma(K_0 + K_P^*(\theta_T^*)) - \delta K_P^*(\theta_T^*). \end{aligned} \tag{B.7}$$

Next, when deriving the smooth-pasting condition, one has to take into account that K_P^* depends on θ_T. We get:

$$B_1\beta_1\theta_T^{*\beta_1-1} = \frac{\partial TS}{\partial\theta_t}\Big|_{\theta_t=\theta_T^*} + \frac{\partial TS}{\partial K_P}\frac{\partial K_P}{\partial\theta_t}\Big|_{\theta_t=\theta_T^*}. \tag{B.8}$$

As K_P^* is chosen to maximise profit not total surplus, we do not have that $\frac{dTS}{dK_P} = 0$ as in the previous cases, but that $\frac{d\pi}{dK_P} = 0$, thus the smooth-pasting condition does not reduce in this case. The smooth pasting condition gives:

$$B_1 = \frac{1}{\beta_1}\Bigg[\frac{(K_0 + K_P^*(\theta_T^*)(1 - \frac{1}{2}\eta(K_0 + K_P^*(\theta_T^*)))}{\rho - \alpha} + \Big[\frac{1 - \eta(K_0 + K_P^*(\theta_T^*))}{\rho - \alpha}\theta_T^* - \gamma - \delta\Big] * \\ \frac{1}{2\eta}\frac{\delta(\rho - \alpha)}{\theta_T^{*2}}\Bigg]\theta_T^{*1-\beta_1}. \quad \text{(B.9)}$$

Substituting the expression for B_1 into the value-matching condition, we get the following implicit equation for the optimal investment level, θ_T^*:

$$\frac{\beta_1 - 1}{\beta_1}\frac{(K_0 + K_P^*(\theta_T^*)(1 - \frac{1}{2}\eta(K_0 + K_P^*(\theta_T^*)))}{\rho - \alpha}\theta_T^{*2} \\ - \Bigg[\gamma(K_0 + K_P^*(\theta_T^*)) + \delta K_P^*(\theta_T^*) + \frac{\delta(1 - \eta(K_0 + K_P^*(\theta_T^*))}{2\eta\beta_1}\Bigg]\theta_T^* + \frac{\delta(\gamma + \delta)(\rho - \alpha)}{2\eta\beta_1} = 0. \quad \text{(B.10)}$$

B.5 Proof of Proposition 5

We find that if both agents invest at time zero, the total optimal capacity of the TSO will be larger than the total optimal capacity of the PC if:

$$\frac{1}{\eta}\frac{\gamma(\rho - \alpha)}{\theta_0} < K_0 + K_P^*. \quad \text{(B.11)}$$

By substituting K_P^*, given in Equation (82), into the equation above, we find that in order for K_T^* to be larger than K_P^* Equation (86) must hold.

B.6 Proof of Proposition 6

Given that $\theta_P^*(K) > \theta_T^*(K)$ the following holds:

$$\frac{\partial\theta_P^*(K)}{\partial K} = \frac{\beta_1}{\beta_1 - 1}(\rho - \alpha)\frac{\delta\eta}{(1 - \eta(2K_0 + K))^2} > 0, \quad \text{(B.12)}$$

and:

$$\frac{\partial\theta_T^*(K)}{\partial K} = \frac{\beta_1}{\beta_1 - 1}(\rho - \alpha)\frac{\gamma}{\frac{1}{2}\eta K_0^2 + (1 - \eta K_0)K_0} > 0. \quad \text{(B.13)}$$

Note that $(1 - \eta K_0) > 0$ has to hold due to the upper bound on capacity given by the inverse demand function, see Equation(1).

B.7 Proof of Proposition 7

In the case where the two agents invest in the same capacity, K, and the PC invests after the TSO we have the following investment triggers for the two agents:

$$\theta_T^*(K) = \frac{\beta_1}{\beta_1 - 1}(\rho - \alpha)\frac{\gamma(K_0 + K)}{K_0(1 - \frac{1}{2}\eta K_0)}, \tag{B.14}$$

$$\theta_P^*(K) = \frac{\beta_1}{\beta_1 - 1}(\rho - \alpha)\frac{\delta}{1 - \eta(2K_0 + K)}. \tag{B.15}$$

By solving the inequality $\theta_P^*(K) > \theta_T^*(K)$, we can derive an expression for when the PC will delay investment beyond the investment threshold of the TSO. The relation is equal to:

$$\frac{\delta}{\gamma} > \frac{(K_0 + K)(1 - \eta(2K_0 + K))}{K_0(1 - \frac{1}{2}\eta K_0)}. \tag{B.16}$$

.

B.8 Proof of Proposition 8

Given that the PC invests after the TSO we have the following investment triggers for the two agents:

$$\theta_T^*(K) = \frac{\beta_1}{\beta_1 - 1}(\rho - \alpha)\frac{\gamma(K_0 + K)}{K_0(1 - \frac{1}{2}\eta K_0)} \tag{B.17}$$

$$\theta_P^*(K) = \frac{\beta_1}{\beta_1 - 1}(\rho - \alpha)\frac{\delta}{1 - \eta(2K_0 + K)} \tag{B.18}$$

First, $\theta_T^*(K)$ is evaluated. The total derivative of θ_T with respect to σ is equal to:

$$\frac{d\theta_T^*}{d\sigma} = \frac{\partial\theta_T^*}{\partial\sigma} + \frac{\partial\theta_T^*}{\partial\beta}\frac{\partial\beta}{\partial\sigma}. \tag{B.19}$$

From Equation (B.17), we have that $\frac{\partial\theta_T^*}{\partial\sigma} = 0$. Furthermore;

$$\frac{\partial\theta_T^*}{\partial\beta} = \frac{-1}{(\beta_1 - 1)^2}(\rho - \alpha)\frac{\gamma(K_0 + K)}{K_0(1 - \frac{1}{2}\eta K_0)} < 0. \tag{B.20}$$

Also $\frac{\partial\beta}{\partial\sigma} < 0$. The proof can be found in Dixit and Pindyck (1994). Therewith, we have that:

$$\frac{d\theta_T^*}{d\sigma} > 0. \tag{B.21}$$

Next, $\theta_P^*(K)$ is evaluated. The total derivative of θ_P with respect to σ is equal to:

$$\frac{d\theta_P^*}{d\sigma} = \frac{\partial\theta_P^*}{\partial\sigma} + \frac{\partial\theta_P^*}{\partial\beta}\frac{\partial\beta}{\partial\sigma} > 0, \tag{B.22}$$

From Equation (B.18), we have that $\frac{\partial\theta_P^*}{\partial\sigma} = 0$. Furthermore;

$$\frac{\partial\theta_P^*}{\partial\beta} = \frac{-1}{(\beta_1 - 1)^2}(\rho - \alpha)\frac{\delta}{1 - \eta(2K_0 + K)} < 0. \tag{B.23}$$

Therewith, we have that:

$$\frac{d\theta_P^*}{d\sigma} > 0. \tag{B.24}$$

It is also possible to show that the optimal capacity of the PC is increasing in uncertainty. This holds independent of whether it has to invest at its own or the TSO's optimal investment threshold.

$$K_P^*(\theta_t) = \max\left(\frac{1}{2\eta}\left[1 - \frac{\delta(\rho - \alpha)}{\theta_t}\right] - K_0, 0\right). \tag{B.25}$$

First we have that:

$$\frac{dK_P^*(\theta_P^*)}{d\sigma} = \frac{\partial K_P^*}{\partial\sigma} + \frac{\partial K_P^*}{\partial\theta_P^*}\frac{\partial\theta_P^*}{\partial\sigma}. \tag{B.26}$$

From Equation (B.25), we see that $\frac{\partial K_P^*}{\partial\sigma} = 0$. Furthermore, from Equation (B.24) we have that $\frac{\partial\theta_P^*}{\partial\sigma} > 0$. Also:

$$\frac{\partial K_P^*}{\partial\theta_P} = \frac{1}{2\eta}\frac{\delta(\rho - \alpha)}{\theta_P^2} > 0. \tag{B.27}$$

Therewith, we have that:

$$\frac{dK_P^*(\theta_P^*)}{d\sigma} > 0, \tag{B.28}$$

Similarly we have that:

$$\frac{dK_P^*(\theta_T^*)}{d\sigma} = \frac{\partial K_P^*}{\partial\sigma} + \frac{\partial K_P^*}{\partial\theta_T^*}\frac{\partial\theta_P^*}{\partial\sigma} > 0, \tag{B.29}$$

since $\frac{d\theta_T^*}{d\sigma} > 0$.

B.9 Proof of Proposition 9

The proof is given in Section 6.3.

C Analytical Solutions to the Sub-Problems

C.1 Sub-problem 1: Optimal capacity for each agent when they both have to invest at time zero

C.1.1 PC's investment problem

When the PC only has sizing flexibility, and has to invest at time zero, its now-or-never investment problem is equal to:

$$\max_{K_P} \mathbb{E}\left[\int_{s=0}^{\infty} e^{-\rho s}\pi(\theta_s, K_0 + K_P)ds - \delta K_P \middle| \theta_0\right]. \tag{C.1}$$

The solution to this problem is equal to Equation (13) but with θ_0 instead of θ_t:

$$K_P^*(\theta_0) = \max\left(\frac{1}{2\eta}\left[1 - \frac{\delta(\rho-\alpha)}{\theta_0}\right] - K_0, 0\right). \tag{C.2}$$

C.1.2 TSO's investment problem

When also the PC has to invest at the zero, the TSO's now-or-never investment problem is equal to:

$$\max_{K_T} \mathbb{E}\left[\int_{s=0}^{\infty} e^{-\rho s} ts(\theta_s; K_0 + K_T)ds - \gamma(K_0 + K_T) - \delta K_T \middle| \theta_0\right], \tag{C.3}$$

or equivalently:

$$\max_{K_T} \mathbb{E}\left[TS(\theta_0; K_0 + K_T) \middle| \theta_0\right], \tag{C.4}$$

where total surplus is defined as the sum of the consumer surplus and the producer surplus.

Building on the theory in Section 3, the present value of the total expected consumer surplus (CS) at time zero is equal to:

$$CS(\theta_0; K_0 + K_T) = \mathbb{E}\left[\int_{s=0}^{\infty} e^{-\rho s}\frac{1}{2}\theta_s(K_0 + K_T)^2\eta ds \middle| \theta_0\right] = \frac{\theta_0(K_0 + K_T)^2\eta}{2(\rho-\alpha)}. \tag{C.5}$$

As discussed under assumptions, the total expected producer surplus (PS) is equal to the expected present value of the PC's future income minus the PC's and the TSO's investment cost. At time zero we have:

$$PS(\theta_0; K_0 + K_T) = \frac{\theta_0(1 - \eta(K_0 + K_T))(K_0 + K_T)}{\rho - \alpha} - \delta K_T - \gamma(K_0 + K_T). \tag{C.6}$$

The total expected surplus at time zero, $TS(\theta_0; K_0 + K_T)$, is then equal to:

$$\begin{aligned} TS(\theta_0; K_0 + K_T) &= CS(\theta_0, K_0 + K_T) + PS(\theta_0; K_0 + K_T) \\ &= \frac{\theta_0(K_0 + K_T)(2 - \eta(K_0 + K_T))}{2(\rho - \alpha)} - \delta K_T - \gamma(K_0 + K_T). \quad \text{(C.7)} \end{aligned}$$

Next, we solve the TSO's investment problem to obtain K_T^*:

$$\max_{K_T} \Big[TS(\theta_0; K_0 + K_T) \Big] \quad \text{(C.8)}$$

$$\frac{d}{dK_T} \left[\frac{\theta_0(K_0 + K_T)(2 - \eta(K_0 + K_T)}{2(\rho - \alpha)} - \delta K_T - \gamma(K_0 + K_T) \right] = 0 \quad \text{(C.9)}$$

$$\frac{\theta_0}{2(\rho - \alpha)} \big[- \eta K_0 + 2 - \eta K_0 - 2\eta K_T \big] - \delta - \gamma = 0 \quad \text{(C.10)}$$

Then the optimal $\hat{K}_T^*$ is equal to:

$$\hat{K}_T^*(\theta_0) = \frac{1}{\eta} \left[1 - \frac{(\delta + \gamma)(\rho - \alpha)}{\theta_0} \right] - K_0. \quad \text{(C.11)}$$

However, we require the TSO to at least be able to distribute an amount of power equal to the current capacity of the PC, K_0. Therefore, K_T^* is given by:

$$K_T^*(\theta_0) = \max \left(\frac{1}{\eta} \left[1 - \frac{(\delta + \gamma)(\rho - \alpha)}{\theta_0} \right] - K_0, 0 \right). \quad \text{(C.12)}$$

C.1.3 Optimal investment strategies

The optimal investment strategies of the two agents depend on the lower of the two agents' optimal capacities. If both agents invest in the dominating capacity, i.e., the lower optimal capacity, the resulting social welfare for the TSO will be:

$$\begin{aligned} &V_{TSO}(\theta_0, \theta_0, \min(K_T^*, K_P^*)) = \\ &\left[\frac{1}{2}\eta(K_0 + \min(K_T^*, K_P^*))^2 + (1 - \eta(K_0 + \min(K_T^*, K_P^*)))(K_0 + \min(K_T^*, K_P^*)) \right] \frac{\theta_0}{\rho - \alpha} \\ &\qquad - \gamma(K_0 + \min(K_T^*, K_P^*)) - \delta \min(K_T^*, K_P^*). \quad \text{(C.13)} \end{aligned}$$

And the value of the PC:

$$V_{PC}(\theta_0, \theta_0, \min(K_T^*, K_P^*)) =$$
$$\Big[(1 - \eta(K_0 + \min(K_T^*, K_P^*)))(K_0 + \min(K_T^*, K_P^*))\Big] \frac{\theta_0}{\rho - \alpha} - \delta \min(K_T^*, K_P^*). \quad \text{(C.14)}$$

C.2 Sub-problem 2: Both decide timing, while the TSO decides capacity, $K_P = K_T^*$

C.2.1 PC's investment problem

When the PC only has timing flexibility, its investment problem at time zero is equal to:

$$\sup_{\tau_P \geq \tau_T} \mathbb{E}\Bigg[\int_{s=\tau_T}^{\infty} e^{-\rho s}\pi(\theta_s, K_0)ds + \int_{s=\tau_P}^{\infty} e^{-\rho s}[\pi(\theta_s, K_0 + K_T^*) - \pi(\theta_s, K_0)]ds - e^{-\rho\tau_P}\delta K_T^* \Big| \theta_0\Bigg], \quad \text{(C.15)}$$

where K_T^* is the optimal capacity of the TSO. The solution to this optimal stopping problem is equal to the solution derived in Section 5.1.1, i.e., Equation (23) and (24), but with K_T^* instead of K_P^*:

$$A_1 = \frac{K_T^*(1 - \eta(2K_0 + K_T^*))}{\rho - \alpha} \frac{1}{\beta_1} \hat{\theta}_P^{*1-\beta_1}, \quad \text{(C.16)}$$

and:

$$\hat{\theta}_P^* = \frac{\beta_1}{\beta_1 - 1}(\rho - \alpha)\frac{\delta}{1 - \eta(2K_0 + K_T^*)}. \quad \text{(C.17)}$$

Taking the timing constraint into account, we get that the optimal investment timing of the PC given the TSO's investment time is equal to:

$$\theta_P^* = \begin{cases} \hat{\theta}_P^* & \text{if } \theta_T < \hat{\theta}_P^*, \\ \theta_T & \text{if } \theta_T \geq \hat{\theta}_P^*. \end{cases} \quad \text{(C.18)}$$

In the case where K_T^* is equal to zero, generation capacity will never be added, i.e., $\theta_P^*(0)=\infty$.

C.2.2 TSO's investment problem

The TSO's investment problem at time zero, given that it has both sizing and timing flexibility, is equal to:

$$\sup_{\tau_T}\left[\max_{K_T}\mathbb{E}\left[\int_{s=\tau_T}^{\infty}e^{-\rho s}ts(\theta_s;K_0)ds-e^{-\rho\tau_T}\gamma(K_0+K_T)+\right.\right.$$
$$\left.\left.\int_{s=\tau_P}^{\infty}e^{-\rho s}[ts(\theta_s;K_0+K_T)-ts(\theta_s;K_0)]ds-e^{-\rho\tau_P}\delta K_T\middle|\theta_0\right]\right]. \quad \text{(C.19)}$$

The solution to this problem is equal to the solution derived in Section 5.1.2. The optimal capacity, K_T^*, after having substituted for $\theta_P^*(K_T^*)$ is equal to:

$$K_T^*(\theta_t)=\max\left(\hat{K}_T^*(\theta_t),0\right), \quad \text{(C.20)}$$

where $\hat{K}_T^*(\theta_t)$ is given by the solution to the following implicit equation:

$$-\gamma+\left[\frac{\beta\delta[2\eta\hat{K}_T^*(\eta K_0(\beta-2)-\beta+1)+(2\eta K_0-1)(2\eta K_0(\beta-2)-\beta+1])}{2(\beta-1)(\eta\hat{K}_T^*+2\eta K_0-1)^2}+\frac{\delta}{\beta-1}-\frac{\beta\delta}{2}\right]*$$
$$\left[\frac{(\beta-1)\theta_t(\eta\hat{K}_T^*+2\eta K_0-1)}{\beta\delta(\alpha-\rho)}\right]=0. \quad \text{(C.21)}$$

The value of the endogenous constant B_1 is equal to:

$$B_1=\left[K_0-\frac{1}{2}\eta K_0^2\right]\frac{\theta_T^{*1-\beta_1}}{\beta_1(\rho-\alpha)}+\left[K_T^*(\theta_T^*)(1-\eta(K_0-\frac{1}{2}K_T^*(\theta_T^*)))\right]\frac{\theta_P^*(K_T^*)^{*1-\beta_1}}{\rho-\alpha}$$
$$-\delta K_T^*(\theta_T^*)\theta_P^*(K_T^*)^{*-\beta_1}, \quad \text{(C.22)}$$

while the optimal investment threshold θ_T^* is given by the solution to the following implicit equation:

$$\theta_T^*=\frac{\beta_1}{\beta_1-1}(\rho-\alpha)\frac{\gamma(K_0+K_T^*(\theta_T^*))}{K_0(1-\frac{1}{2}\eta K_0)}. \quad \text{(C.23)}$$

The PC will then expand capacity when θ_t hits $\theta_P^*(K_T^*)$ given that it has no flexibility to choose the size of its investment:

$$\theta_P^*(K_T^*)=\frac{\beta_1}{\beta_1-1}(\rho-\alpha)\frac{\delta K_T^*}{1-\eta(2K_0+K_T^*)}. \quad \text{(C.24)}$$

However, if this solution is not valid, i.e., $\theta_P(K_T^*) < \theta_T^*(K_T^*)$, we assume the corner solution $\theta_P = \theta_T^*$ to be optimal. The optimal investment strategy of the TSO given that they both invest at the same time, is to invest in a capacity of:

$$K_T^*(\theta_T^*) = \max\left(\frac{1}{\eta}\left[1 - \frac{(\delta+\gamma)(\rho-\alpha)}{\theta_T^*}\right] - K_0, 0\right), \tag{C.25}$$

at the moment in time when θ_t hits θ_T^* given by the solution to the following implicit equation:

$$\theta_T^* = \frac{\beta_1}{\beta_1 - 1}(\rho-\alpha)\frac{\gamma(K_0 + K_T^*(\theta_T^*)) + \delta K_T^*(\theta_T^*)}{(K_0 + K_T^*(\theta_T^*))(1 - \frac{1}{2}\eta(K_0 + K_T^*(\theta_T^*)))}. \tag{C.26}$$

The value of the endogenous constant B_1 is equal to:

$$B_1 = \left[K_0 - \frac{1}{2}\eta K_0^2\right]\frac{\theta_T^{*1-\beta_1}}{\beta_1(\rho-\alpha)} + \left[K_T^*(\theta_T^*)(1 - \eta(K_0 - \frac{1}{2}K_T^*(\theta_T^*)))\right]\frac{\theta_P^*(K_T^*)^{*1-\beta_1}}{\rho-\alpha}$$
$$- \delta K_T^*(\theta_T^*)\theta_P^*(K_T^*)^{*-\beta_1}. \tag{C.27}$$

C.2.3 Optimal investment strategies

The optimal investment strategies for Sub-problem 2 are summarised in Table C.1.

If the PC invests after the TSO, the resulting value at time zero for each agent is equal to:

$$V_{TSO}(\theta_T^*, \theta_P^*, K_T^*) = \left(\frac{\theta_0}{\theta_T^*}\right)^{\beta_1}\left[\left[\frac{1}{2}\eta K_0^2 + (1-\eta K_0)K_0\right]\frac{\theta_T^*}{\rho-\alpha} - \gamma(K_0 + K_T^*)\right.$$
$$+ \left[\frac{1}{2}\eta[(K_0+K_T^*)^2 - K_0^2] + (1-\eta(K_0+K_T^*))(K_0+K_T^*) - (1-\eta K_0)K_0\right]\frac{\theta_P^*(K_T^*)}{\rho-\alpha}\left(\frac{\theta_T^*}{\theta_P^*(K_T^*)}\right)^{\beta_1}$$
$$\left. - \delta K_T^*\left(\frac{\theta_T^*}{\theta_P^*(K_T^*)}\right)^{\beta_1}\right], \tag{C.28}$$

$$V_{PC}(\theta_T^*, \theta_P^*, K_T^*) = \left(\frac{\theta_0}{\theta_T^*}\right)^{\beta_1}\left[\left[(1-\eta K_0)K_0\right]\frac{\theta_T^*}{\rho-\alpha}\right.$$
$$\left. + \left[(1-\eta(K_0+K_T^*))(K_0+K_T^*) - (1-\eta K_0)K_0\right]\frac{\theta_P^*(K_T^*)}{\rho-\alpha}\left(\frac{\theta_T^*}{\theta_P^*(K_T^*)}\right)^{\beta_1} - \delta K_T^*\left(\frac{\theta_T^*}{\theta_P^*(K_T^*)}\right)^{\beta_1}\right]. \tag{C.29}$$

Optimal investment strategies Sub-problem 2		
K_T^*	$\theta_P^* > \theta_T^*$	**TSO's optimal strategy:** $K_T^* = \max\left(\hat{K}_T^*(\theta_T^*), 0\right)$, where $\hat{K}_T^*(\theta_T^*)$ is given by the following implicit equation: $-\gamma + \left[\frac{\beta\delta[2\eta\hat{K}_T^*(\eta K_0(\beta-2)-\beta+1)+(2\eta K_0-1)(2\eta K_0(\beta-2)-\beta+1])}{2(\beta-1)(\eta\hat{K}_T^*+2\eta K_0-1)^2} + \frac{\delta}{\beta-1} - \frac{\beta\delta}{2}\right] *$ $\left[\frac{(\beta-1)\theta_T^*(\eta\hat{K}_T^*+2\eta K_0-1)}{\beta\delta(\alpha-\rho)}\right] = 0$ $\theta_T^* = \frac{\beta_1}{\beta_1-1}(\rho-\alpha)\frac{\gamma(K_0+K_T^*(\theta_T^*))}{K_0(1-\frac{1}{2}\eta K_0)}$
		PC's optimal decision: $K_P = K_T^*$ $\theta_P^* = \frac{\beta_1}{\beta_1-1}(\rho-\alpha)\frac{\delta}{1-\eta(2K_0+K_T^*)}$
	$\theta_P = \theta_T^*$	**TSO's optimal strategy:** $K_T^* = \max\left(\frac{1}{\eta}\left[1-\frac{(\delta+\gamma)(\rho-\alpha)}{\theta_T^*}\right] - K_0, 0\right)$ $\theta_T^* = \frac{\beta_1}{\beta_1-1}(\rho-\alpha)\frac{\gamma(K_0+K_T^*(\theta_T^*))+\delta K_T^*(\theta_T^*)}{(K_0+K_T^*(\theta_T^*))(1-\frac{1}{2}\eta(K_0+K_T^*(\theta_T^*)))}$
		PC's optimal decision: $K_P = K_T^*$ $\theta_P = \theta_T^*$

Table C.1: Overview of optimal investment strategies for Sub-problem 2

If it is optimal for the PC to invest at the same time as the TSO, i.e., $\theta_P(K_T^*) = \theta_T^*(K_T^*)$, the value functions simplify to:

$$V_{TSO}(\theta_T^*, \theta_T^*, K_T^*) = \left(\frac{\theta_0}{\theta_T^*}\right)^{\beta_1}\left[\left[\frac{1}{2}\eta(K_0+K_T^*)^2 + (1-\eta(K_0+K_T^*))(K_0+K_T^*)\right]\frac{\theta_T^*}{\rho-\alpha} - \gamma(K_0+K_T^*) - \delta K_T^*\right], \quad (C.30)$$

$$V_{PC}(\theta_T^*, \theta_T^*, K_T^*) = \left(\frac{\theta_0}{\theta_T^*}\right)^{\beta_1}\left[\left[(1-\eta(K_0+K_T^*))(K_0+K_T^*)\right]\frac{\theta_T^*}{\rho-\alpha} - \delta K_T\right]. \quad (C.31)$$

In the case where K_T^* is equal to zero and generation capacity will never be added, i.e.,

$\theta_P^*(0)$=∞, the value for the TSO and the PC are equal to:

$$V_{TSO}(\theta_T^*, \infty, 0) = \left(\frac{\theta_0}{\theta_T^*}\right)^{\beta_1} \left[\left[\frac{1}{2}\eta K_0^2 + (1-\eta K_0)K_0\right]\frac{\theta_T^*}{\rho-\alpha} - \gamma K_0\right], \tag{C.32}$$

and:

$$V_{PC}(\theta_T^*, \infty, 0) = \left(\frac{\theta_0}{\theta_T^*}\right)^{\beta_1} \left[\left[(1-\eta K_0)K_0\right]\frac{\theta_T^*}{\rho-\alpha}\right], \tag{C.33}$$

respectively.

C.3 Sub-problem 3: Both decide timing, while the PC decides capacity, $K_T = K_P^*$

C.3.1 PC's investment problem

When the PC has flexibility to decide both timing and capacity, its investment problem at time zero can be summarised as below:

$$\sup_{\tau_P \geq \tau_T} \left[\max_{K_P} \mathbb{E}\left[\int_{s=\tau_T}^{\infty} e^{-\rho s}\pi(\theta_s, K_0)ds \right.\right.$$
$$\left.\left. + \int_{s=\tau_P}^{\infty} e^{-\rho s}[\pi(\theta_s, K_0+K_P) - \pi(\theta_s, K_0)]ds - e^{-\rho\tau_P}\delta K_P \middle| \theta_0\right]\right]. \tag{C.34}$$

The solution to this problem is equal to the one derived in Section 5.1.1. The optimal decision is to expand generation capacity with K_P^* equal to:

$$K_P^*(\theta_P^*) = \max\left(\frac{1}{2\eta}\left[1 - \frac{\delta(\rho-\alpha)}{\theta_P^*}\right] - K_0, 0\right), \tag{C.35}$$

at the moment in time when θ_t first hits θ_P^*, equal to:

$$\theta_P^* = \begin{cases} \hat{\theta}_P^* & \text{if } \theta_T^{<}\hat{\theta}_P^*, \\ \theta_T & \text{if } \theta_T^* \geq \hat{\theta}_P^*, \end{cases} \tag{C.36}$$

where θ_T is the moment in time when the TSO invests in the transmission line and θ_P^* is given by the solution to the following implicit equation:

$$\hat{\theta}_P^* = \frac{\beta_1}{\beta_1 - 1}(\rho-\alpha)\frac{\delta}{1-\eta(2K_0 + K_P^*(\theta_P^*))}. \tag{C.37}$$

In the case where K_P^* is equal to zero, generation capacity will never be added, i.e., $\theta_P^*(0)=\infty$.

The value of the endogenous constant A_1 is equal to:

$$A_1 = \frac{K_P^*(\theta_P^*)(1-\eta(2K_0 + K_P^*(\theta_P^*)))}{\rho-\alpha}\frac{1}{\beta_1}\hat{\theta}_P^{*1-\beta_1}. \tag{C.38}$$

C.3.2 TSO's investment problem

The TSO's investment problem at time zero, when it holds only the option to decide the timing of investment, is equal to:

$$\begin{aligned}\sup_{\tau_T} \mathbb{E}\Bigg[\int_{s=\tau_T}^{\infty} & e^{-\rho s} ts(\theta_s; K_0)ds - e^{-\rho\tau_T}\gamma(K_0+K_P^*) \\ &+ \int_{s=\tau_P}^{\infty} e^{-\rho s}[ts(\theta_s; K_0+K_P^*) - ts(\theta_s;K_0)]ds - e^{-\rho\tau_P}\delta K_P^* \Big| \theta_0\Bigg]. \end{aligned} \tag{C.39}$$

The solution to this problem is equal to Equations (49) and (50) derived in Section 5.1.2, but with K_P^* instead of K_T^*:

$$\begin{aligned}B_1 = \left[K_0 - \frac{1}{2}\eta K_0^2\right]\frac{\theta_T^{*1-\beta_1}}{\beta_1(\rho-\alpha)} + \left[K_P^*(\theta_P^*)(1-\eta(K_0 - \frac{1}{2}K_P^*(\theta_P^*)))\right]\frac{\theta_P^*(K_P^*)^{*1-\beta_1}}{\rho-\alpha} \\ - \delta K_P^*(\theta_P^*)\theta_P^*(K_P^*)^{*-\beta_1}, \end{aligned} \tag{C.40}$$

while θ_T^* is given by the solution to the following implicit equation:

$$\theta_T^* = \frac{\beta_1}{\beta_1-1}(\rho-\alpha)\frac{\gamma(K_0+K_P^*(\theta_P^*))}{K_0(1-\frac{1}{2}\eta K_0)}. \tag{C.41}$$

The only way the TSO can affect the investment decision of the PC, within this set-up, is by investing later than what is optimal for the PC, and force it to delay its investment. If the PC would like to invest earlier than the TSO, it will have to wait and the TSOs optimal investment time will be dominating. In this case, the PC only holds the option to decide capacity not timing and will have to follow the TSO's investment timing strategy.

Then the optimal K_P^* will depend on θ_T^* and be given by:

$$K_P^*(\theta_T^*) = \max\left(\frac{1}{2\eta}\left[1-\frac{\delta(\rho-\alpha)}{\theta_T^*}\right] - K_0, 0\right). \tag{C.42}$$

To find the TSO's optimal investment time given that they invest at the same time, we need to solve the following optimal stopping problem:

$$\sup_{\tau_T} \mathbb{E}\left[\int_{s=\tau_T}^{\infty} e^{-\rho s} ts(\theta_s; K_0 + K_P^*) ds - e^{-\rho \tau_T} \gamma(K_0 + K_P^*) - e^{-\rho \tau_T} \delta(K_P^*) \Big| \theta_0 \right]. \tag{C.43}$$

This solution to this problem is derived in Section B.4 where we get:

$$B_1 = \frac{1}{\beta_1} \Bigg[\frac{(K_0 + K_P^*(\theta_T^*)(1 - \frac{1}{2}\eta(K_0 + K_P^*(\theta_T^*)))}{\rho - \alpha} + \Big[\frac{1 - \eta(K_0 + K_P^*(\theta_T^*))}{\rho - \alpha} \theta_T^* - \gamma - \delta \Big] \frac{1}{2\eta} \frac{\delta(\rho - \alpha)}{\theta_T^{*2}} \Bigg] \theta_T^{*1-\beta_1}. \tag{C.44}$$

Substituting the expression for B_1 into the value-matching condition, we get the following implicit equation for the optimal investment level, θ_T^*:

$$\frac{\beta_1 - 1}{\beta_1} \frac{(K_0 + K_P^*(\theta_T^*)(1 - \frac{1}{2}\eta(K_0 + K_P^*(\theta_T^*)))}{\rho - \alpha} \theta_T^{*2} - \left[\gamma(K_0 + K_P^*(\theta_T^*)) + \delta K_P^*(\theta_T^*) + \frac{\delta(1 - \eta(K_0 + K_P^*(\theta_T^*))}{2\eta\beta_1} \right] \theta_T^* + \frac{\delta(\gamma + \delta)(\rho - \alpha)}{2\eta\beta_1} = 0. \tag{C.45}$$

C.3.3 Optimal investment strategies

The optimal investment strategies for Sub-problem 3 are summarised in Table C.2.

If the PC invests after the TSO, the resulting value at time zero for each agent is equal to:

$$V_{TSO}(\theta_T^*, \theta_P^*, K_P^*) = \left(\frac{\theta_0}{\theta_T^*} \right)^{\beta_1} \Bigg[\Big[\frac{1}{2}\eta K_0^2 + (1 - \eta K_0) K_0 \Big] \frac{\theta_T^*}{\rho - \alpha} - \gamma(K_0 + K_P^*) + \left[\frac{1}{2}\eta[(K_0 + K_P^*)^2 - K_0^2] + (1 - \eta(K_0 + K_P^*))(K_0 + K_P^*) - (1 - \eta K_0) K_0 \right] \frac{\theta_P^*(K_P^*)}{\rho - \alpha} \left(\frac{\theta_T^*}{\theta_P^*(K_P^*)} \right)^{\beta_1} - \delta K_P^* \left(\frac{\theta_T^*}{\theta_P^*(K_P^*)} \right)^{\beta_1} \Bigg], \tag{C.46}$$

$$V_{PC}(\theta_T^*, \theta_P^*, K_P^*) = \left(\frac{\theta_0}{\theta_T^*} \right)^{\beta_1} \Bigg[\Big[(1 - \eta K_0) K_0 \Big] \frac{\theta_T^*}{\rho - \alpha} + \Big[(1 - \eta(K_0 + K_P^*))(K_0 + K_P^*) - (1 - \eta K_0) K_0 \Big] \frac{\theta_P^*(K_P^*)}{\rho - \alpha} \left(\frac{\theta_T^*}{\theta_P^*(K_P^*)} \right)^{\beta_1} - \delta K_P^* \left(\frac{\theta_T^*}{\theta_P^*(K_P^*)} \right)^{\beta_1} \Bigg]. \tag{C.47}$$

Optimal investment strategies Sub-problem 3

<table>
<tr><td rowspan="4">K_P^*</td><td rowspan="2">$\theta_P^* > \theta_T^*$</td><td>TSO's optimal strategy:
$K_T = K_P^*$
$\theta_T^* = \frac{\beta_1}{\beta_1 - 1}(\rho - \alpha)\frac{\gamma(K_0 + K_P^*(\theta_P^*))}{K_0(1 - \frac{1}{2}\eta K_0)}$</td></tr>
<tr><td>PC's optimal decision:
$K_P^* = \max\left(\frac{1}{2\eta}\left[1 - \frac{\delta(\rho-\alpha)}{\theta_P^*}\right] - K_0, 0\right)$
$\theta_P^* = \frac{\beta_1}{\beta_1 - 1}(\rho - \alpha)\frac{\delta}{1 - \eta(2K_0 + K_P^*(\theta_P^*))}$</td></tr>
<tr><td rowspan="2">$\theta_P = \theta_T^*$</td><td>TSO's optimal strategy:
$K_T = K_P^*$
θ_T^* is given by the following implicit equation:
$\frac{\beta_1 - 1}{\beta_1}\frac{(K_0 + K_P^*(\theta_T^*)(1 - \frac{1}{2}\eta(K_0 + K_P^*(\theta_T^*)))}{\rho - \alpha}\theta_T^{*2}$
$-\left[\gamma(K_0 + K_P^*(\theta_T^*)) + \delta K_P^*(\theta_T^*) + \frac{\delta(1 - \eta(K_0 + K_P^*(\theta_T^*))}{2\eta\beta_1}\right]\theta_T^*$
$+\frac{\delta(\gamma+\delta)(\rho-\alpha)}{2\eta\beta_1} = 0$</td></tr>
<tr><td>PC's optimal decision:
$K_P^* = \max\left(\frac{1}{2\eta}\left[1 - \frac{\delta(\rho-\alpha)}{\theta_T^*}\right] - K_0, 0\right)$
$\theta_P = \theta_T^*$</td></tr>
</table>

Table C.2: Overview of optimal investment strategies for Sub-problem 3

If, however, it is optimal for the PC to invest at the same time as the TSO, i.e., $\theta_P(K_P^*) = \theta_T^*(K_P^*)$, the value functions simplify to:

$$V_{TSO}(\theta_T^*, \theta_T^*, K_P^*) = \left(\frac{\theta_0}{\theta_T^*}\right)^{\beta_1}\left[\left[\frac{1}{2}\eta(K_0 + K_P^*)^2 + (1 - \eta(K_0 + K_P^*))(K_0 + K_P^*)\right]\frac{\theta_T^*}{\rho - \alpha} - \gamma(K_0 + K_P^*) - \delta K_P^*\right], \quad (C.48)$$

$$V_{PC}(\theta_T^*, \theta_T^*, K_P^*) = \left(\frac{\theta_0}{\theta_T^*}\right)^{\beta_1}\left[\left[(1 - \eta(K_0 + K_P^*))(K_0 + K_P^*)\right]\frac{\theta_T^*}{\rho - \alpha} - \delta K_P\right]. \quad (C.49)$$

In the case where K_P^* is equal to zero and generation capacity will never be added, i.e., $\theta_P^*(0)=\infty$, the value for the TSO and the PC are equal to:

$$V_{TSO}(\theta_T^*, \infty, 0) = \left(\frac{\theta_0}{\theta_T^*}\right)^{\beta_1}\left[\left[\frac{1}{2}\eta K_0^2 + (1 - \eta K_0)K_0\right]\frac{\theta_T^*}{\rho - \alpha} - \gamma K_0\right], \quad (C.50)$$

and:

$$V_{PC}(\theta_T^*, \infty, 0) = \left(\frac{\theta_0}{\theta_T^*}\right)^{\beta_1} \left[\Big[(1-\eta K_0)K_0\Big]\frac{\theta_T^*}{\rho-\alpha}\right], \tag{C.51}$$

respectively.

C.4 Sub-problem 4: Both decide capacity and PC decides timing while TSO has to invest at time zero

C.4.1 PC's investment problem

When the PC has timing and sizing flexibility while the TSO has to invest at time zero, the investment problem of the PC is equal to:

$$\sup_{\tau_P \geq 0} \left[\max_{K_P} \mathbb{E}\left[\int_{s=0}^{\infty} e^{-\rho s}\pi(\theta_s, K_0)ds \right.\right.$$
$$\left.\left. + \int_{s=\tau_P}^{\infty} e^{-\rho s}[\pi(\theta_s, K_0+K_P) - \pi(\theta_s, K_0)]ds - e^{-\rho\tau_P}\delta K_P \middle| \theta_0\right]\right]. \tag{C.52}$$

As in Sub-problem 3, the solution to this problem is equal to the one derived in Section 5.1.1. The optimal decision is to expand generation capacity with K_P^* equal to:

$$K_P^*(\theta_P^*) = \max\left(\frac{1}{2\eta}\left[1 - \frac{\delta(\rho-\alpha)}{\theta_P^*}\right] - K_0, 0\right), \tag{C.53}$$

at the moment in time when θ_t first hits θ_P^*, equal to:

$$\theta_P^* = \begin{cases} \hat{\theta}_P^* & \text{if } \theta_0 < \hat{\theta}_P^*, \\ \theta_0 & \text{if } \theta_0 \geq \hat{\theta}_P^*, \end{cases} \tag{C.54}$$

where $\hat{\theta}_P^*$ is given by the solution to the following implicit equation:

$$\hat{\theta}_P^* = \frac{\beta_1}{\beta_1 - 1}(\rho-\alpha)\frac{\delta}{1-\eta(2K_0 + K_P^*(\theta_P^*))}. \tag{C.55}$$

In the case where K_P^* is equal to zero, generation capacity will never be added, i.e., $\theta_P^*(0)=\infty$.

The optimal value of the endogenous constant A_1 is equal to:

$$A_1 = \frac{K_P^*(\theta_P^*)(1-\eta(2K_0+K_P^*(\theta_P^*)))}{\rho-\alpha}\frac{1}{\beta_1}\hat{\theta}_P^{*1-\beta_1}. \tag{C.56}$$

C.4.2 TSO's investment problem

The TSO's investment problem now only includes the flexibility to decide on capacity. At time zero the investment problem is equal to:

$$\max_{K_T} \mathbb{E}\Bigg[\int_{s=0}^{\infty} e^{-\rho s} ts(\theta_s; K_0) ds - \gamma(K_0 + K_T) \\ + \int_{s=\tau_P}^{\infty} e^{-\rho s}[ts(\theta_s; K_0 + K_P) - ts(\theta_s; K_0)] ds - e^{-\rho \tau_P} \delta K_T \Big| \theta_0 \Bigg]. \quad \text{(C.57)}$$

The only way the TSO can influence the PC's investment decision in this case is by investing in a capacity which is smaller than the PC's optimal capacity. Making it invest in a smaller capacity than it finds optimal, will not only affect the size of the PC's investment, but also force it to invest earlier as its investment threshold is decreasing in capacity. We assume that the TSO has the power to affect the PC's investment decision. Then we need to substitute the expression for $\theta_P^*(K_T)$ into the total surplus expression, before we maximise it with respect to K_T. We get the same expression for K_T^*, Equation (59), as in Section 5.1.2. K_T^* is equal to:

$$K_T^*(\theta_0) = \max\left(\hat{K}_T^*(\theta_0), 0\right), \quad \text{(C.58)}$$

where $\hat{K}_T^*(\theta_0)$ is given by the following implicit equation:

$$-\gamma + \left[\frac{\beta\delta[2\eta\hat{K}_T^*(\eta K_0(\beta-2) - \beta + 1) + (2\eta K_0 - 1)(2\eta K_0(\beta - 2) - \beta + 1])}{2(\beta-1)(\eta\hat{K}_T^* + 2\eta K_0 - 1)^2} + \frac{\delta}{\beta - 1} - \frac{\beta\delta}{2} \right] * \\ \left[\frac{(\beta-1)\theta_0(\eta\hat{K}_T^* + 2\eta K_0 - 1)}{\beta\delta(\alpha - \rho)} \right] = 0. \quad \text{(C.59)}$$

C.4.3 Optimal investment strategies

To find the optimal investment strategies after taking capacity constraints in to account, we need to compare K_T^* and K_P^* to determine which one of them will be dominating. If $K_T^* > K_P^*$, the PC will expand its capacity with K_P^* when θ_t reaches $\theta_P^*(K_P^*)$, and the TSO will have to accept the PC's optimal investment strategy and that it has no power to affect it. However, if $K_T^* < K_P^*$ the PC will have to accept the optimal capacity of the TSO, and choose it's optimal investment time based on K_T^*, $\theta_P^*(K_T^*)$. This optimal investment time will be the same as the TSO took into account when it found its optimal K_T^*. The optimal investment strategies when K_T^* is the dominating capacity is summarised in Table C.3 while the optimal investment strategies when K_P^* is the dominating capacity is summarised in Table C.4.

<table>
<tr><th colspan="3">Optimal investment strategies Sub-problem 4
$K_T^* < K_P^*$</th></tr>
<tr><td rowspan="4">$K_P = K_T^*$</td><td rowspan="2">$\theta_P^* > \theta_0$</td><td>TSO's optimal strategy:
$K_T^* = \max\left(\hat{K}_T^*(\theta_0), 0\right)$, where $\hat{K}_T^*(\theta_0)$ is given by the following implicit equation:
$-\gamma + \left[\frac{\beta\delta[2\eta\hat{K}_T^*(\eta K_0(\beta-2)-\beta+1)+(2\eta K_0-1)(2\eta K_0(\beta-2)-\beta+1])}{2(\beta-1)(\eta\hat{K}_T^*+2\eta K_0-1)^2} + \frac{\delta}{\beta-1} - \frac{\beta\delta}{2}\right] *$
$\left[\frac{(\beta-1)\theta_0(\eta\hat{K}_T^*+2\eta K_0-1)}{\beta\delta(\alpha-\rho)}\right] = 0$
$\theta_T = \theta_0$</td></tr>
<tr><td>PC's optimal decision:
$K_P = K_T^*$
$\theta_P^* = \frac{\beta_1}{\beta_1-1}(\rho-\alpha)\frac{\delta}{1-\eta(2K_0+K_T^*)}$</td></tr>
<tr><td rowspan="2">$\theta_P = \theta_0$</td><td>TSO's optimal strategy:
$K_T^* = \max\left(\frac{1}{\eta}\left[1 - \frac{(\delta+\gamma)(\rho-\alpha)}{\theta_0}\right] - K_0, 0\right)$
$\theta_T = \theta_0$</td></tr>
<tr><td>PC's optimal decision:
$K_P = K_T^*$
$\theta_P = \theta_0$</td></tr>
</table>

Table C.3: Overview of optimal investment strategies for Sub-problem 4 if K_T^* is the dominating capacity

Given the dominating capacity, $\min(K_T^*, K_P^*)$, and the resulting investment threshold for the PC, the social welfare at time zero will be equal to:

$$V_{TSO}(\theta_0, \theta_P^*, \min(K_T^*, K_P^*)) = \left[\frac{1}{2}\eta K_0^2 + (1-\eta K_0)K_0\right]\frac{\theta_0}{\rho-\alpha} - \gamma(K_0 + \min(K_T^*, K_P^*)) +$$
$$\left[\frac{1}{2}\eta[(K_0+\min(K_T^*, K_P^*))^2 - K_0^2] + (1-\eta(K_0+\min(K_T^*, K_P^*)))(K_0+\min(K_T^*, K_P^*)) - (1-\eta K_0)K_0\right] *$$
$$\frac{\theta_P^*(\min(K_T^*, K_P^*))}{\rho-\alpha}\left(\frac{\theta_0}{\theta_P^*(\min(K_T^*, K_P^*))}\right)^{\beta_1} - \delta\min(K_T^*, K_P)^*\left(\frac{\theta_0}{\theta_P^*(\min(K_T^*, K_P^*))}\right)^{\beta_1}, \tag{C.60}$$

Optimal investment strategies Sub-problem 4 $K_P^* \leq K_T^*$		
$K_T = K_P^*$	$\theta_P^* > \theta_0$	**TSO's optimal strategy:** $K_T = K_P^*$ $\theta_T = \theta_0$
		PC's optimal decision: $K_P^* = \max\left(\frac{1}{2\eta}\left[1 - \frac{\delta(\rho-\alpha)}{\theta_P^*}\right] - K_0, 0\right)$ $\theta_P^* = \frac{\beta_1}{\beta_1 - 1}(\rho-\alpha)\frac{\delta}{1-\eta(2K_0+K_P^*(\theta_P^*))}$
	$\theta_P = \theta_0$	**TSO's optimal strategy:** $K_T = K_P^*$ $\theta_T = \theta_0$
		PC's optimal decision: $K_P^* = \max\left(\frac{1}{2\eta}\left[1 - \frac{\delta(\rho-\alpha)}{\theta_0}\right] - K_0, 0\right)$ $\theta_P = \theta_0$

Table C.4: Overview of optimal investment strategies for Sub-problem 4 if K_P^* is the dominating capacity

and the value for the PC at time zero will be equal to:

$$V_{PC}(\theta_0, \theta_P^*, \min(K_T^*, K_P^*)) = \Big[(1-\eta K_0)K_0\Big]\frac{\theta_0}{\rho-\alpha} +$$
$$\Big[(1-\eta(K_0+\min(K_T^*,K_P^*)))(K_0+\min(K_T^*,K_P^*)) - (1-\eta K_0)K_0\Big]\frac{\theta_P^*(\min(K_T^*,K_P^*))}{\rho-\alpha} *$$
$$\left(\frac{\theta_0}{\theta_P^*(\min(K_T^*,K_P^*))}\right)^{\beta_1} - \delta\min(K_T^*,K_P^*)\left(\frac{\theta_0}{\theta_P^*(\min(K_T^*,K_P^*))}\right)^{\beta_1}. \quad \text{(C.61)}$$

In the special case where K_T^* or K_P^* is equal to zero, the PC will never expand its capacity above K_0, the present value for the TSO and the PC, given that the TSO invests in K_0 at time θ_T^*, is equal to:

$$V_{TSO}(\theta_0, \infty, 0) = \Big[\frac{1}{2}\eta K_0^2 + (1-\eta K_0)K_0\Big]\frac{\theta_0}{\rho-\alpha} - \gamma K_0, \quad \text{(C.62)}$$

$$V_{PC}(\theta_0, \infty, 0) = \Big[(1-\eta K_0)K_0\Big]\frac{\theta_0}{\rho-\alpha}. \quad \text{(C.63)}$$

C.5 Sub-problem 5: Both have sizing flexibility, while the TSO decides timing

C.5.1 PC's investment problem

When the PC only has sizing flexibility, and has to follow the TSO's timing decision, the investment problem of the PC at time zero is equal to:

$$\max_{K_P} \mathbb{E}\left[\int_{s=\tau_T}^{\infty} e^{-\rho s}\pi(\theta_s, K_0 + K_P)ds - e^{-\rho\tau_P}\delta K_P \middle| \theta_0 \right]. \tag{C.64}$$

The solution to this problem is equal to Equation (13) but with θ_T^* instead of θ_t:

$$K_P^*(\theta_T^*) = \max\left(\frac{1}{2\eta}\left[1 - \frac{\delta(\rho-\alpha)}{\theta_T^*}\right] - K_0, 0 \right). \tag{C.65}$$

C.5.2 TSO's investment problem

When the PC has to invest at the same time as the TSO, the TSO's investment problem at time zero is equal to:

$$\sup_{\tau_T} \left[\max_{K_T} \mathbb{E}\left[\int_{s=\tau_T}^{\infty} e^{-\rho s} ts(\theta_s; K_0 + K_T)ds - e^{-\rho\tau_T}\gamma(K_0 + K_T) - e^{-\rho\tau_T}\delta K_T \middle| \theta_0 \right]\right]. \tag{C.66}$$

If both invest at θ_t, the total expected surplus at time t is given by:

$$TS(\theta_t, K_T) = \left[\frac{1}{2}\eta(K_0+K_T)^2 + (1-\eta(K_0+K_T))(K_0+K_T)\right]\frac{\theta_t}{\rho-\alpha} - \gamma(K_0+K_T) - \delta K_T. \tag{C.67}$$

The solution to the inner maximisation problem is equal to the one in Sub-problem 1, see Appendix C.1 but with but with θ_t instead of θ_0:

$$K_T^*(\theta_t) = \max\left(\frac{1}{\eta}\left[1 - \frac{(\delta+\gamma)(\rho-\alpha)}{\theta_t}\right] - K_0, 0 \right). \tag{C.68}$$

After having solved for K_T^*, the TSO's investment problem reduces to:

$$\sup_{\tau_T} \mathbb{E}\left[\int_{s=\tau_T}^{\infty} e^{-\rho s} ts(\theta_s; K_0 + K_T^*)ds - \gamma(K_0 + K_T^*) - \delta K_T^* \middle| \theta_0 \right]. \tag{C.69}$$

We proceed by following a dynamic programming approach to solve the optimal stopping problem and find θ_T^*. The value for the TSO at time t is equal to:

$$F(\theta_t, K_T^*(\theta_t)) = \begin{cases} B_1\theta_t^{\beta_1} & \text{if } \theta_t \leq \theta_T, \\ TS(\theta_t, K_T^*(\theta_t)) & \text{if } \theta_T \leq \theta_t. \end{cases} \tag{C.70}$$

The value in the continuation region is equal to the value of the option to invest in the transmission line, while the value in the stopping region is equal to the value of the total surplus given that they both have invested. The value in the continuation region is derived by finding the solution to the ordinary differential equation (ODE) that stews from the Bellman equation:

$$\rho F dt = \mathbb{E}[dF]. \tag{C.71}$$

Standard calculations similar to those in Section 5.1.1 and 5.1.2 are performed, which lead to the value function stated in Equation (C.70).
The second branch is equal to:

$$\begin{aligned} TS(\theta_t, K_T^*(\theta_t)) = \left[\frac{1}{2}\eta(K_0 + K_T^*(\theta_t))^2 + (1 - \eta(K_0 + K_T^*(\theta_t)))(K_0 + K_T^*(\theta_t))\right]\frac{\theta_t}{\rho - \alpha} \\ - \gamma(K_0 + K_T^*(\theta_t)) - \delta K_T^*(\theta_t). \end{aligned} \tag{C.72}$$

To determine the investment threshold θ_T^* and the value of the endogenous constant B_1, we employ the value-matching and smooth-pasting conditions. First, the value-matching condition is given by:

$$B_1\theta_T^{*\beta_1} = TS(\theta_T^*, K_T(\theta_T^*)). \tag{C.73}$$

When deriving the smooth-pasting condition, we have to take into account that K_T^* depends on θ_T. We get:

$$B_1\beta_1\theta_T^{*\beta_1 - 1} = \left.\frac{\partial TS}{\partial \theta_t}\right|_{\theta_t = \theta_T^*} + \left.\frac{\partial TS}{\partial K_T}\frac{\partial K_T^*}{\partial \theta_t}\right|_{\theta_t = \theta_T^*}. \tag{C.74}$$

However after maximising the total surplus (TS) with respect to K_T, we have by the envelope theorem that $\frac{\partial TS}{\partial K_T} = 0$. Therefore the smooth-pasting condition reduces to:

$$B_1\beta_1\theta_T^{*\beta_1 - 1} = \left.\frac{\partial TS}{\partial \theta_t}\right|_{\theta_t = \theta_T^*}. \tag{C.75}$$

The smooth-pasting condition gives:

$$B_1 = \left[(K_0 + K_T^*(\theta_T^*)) \left[1 - \frac{1}{2}\eta(K_0 + K_T^*(\theta_T^*)) \right] \right] \frac{\theta_T^{*1-\beta_1}}{\beta_1(\rho - \alpha)}. \tag{C.76}$$

Substituting the expression for B_1 into the value-matching condition and solving for θ_T^*, we get that θ_T is given by the solution to the following implicit equation:

$$\theta_T^* = \frac{\beta_1}{\beta_1 - 1}(\rho - \alpha) \frac{\gamma(K_0 + K_T^*(\theta_T^*)) + \delta K_T^*(\theta_T^*)}{(K_0 + K_T^*(\theta_T^*))[1 - \frac{1}{2}\eta(K_0 + K_T^*(\theta_T^*))]} \tag{C.77}$$

C.5.3 Optimal investment strategies

The final optimal investment strategy depends on which agent that has the lowest optimal capacity. If K_P^* is larger than K_T^*, the TSO will decide the timing and the size of both investments. However, if the opposite holds, that K_T^* is larger than K_P^*, the PC decides the size of both investments, while the TSO decides the timing. The optimal investment strategies when K_T^* is the dominating capacity is summarised in Table C.5 while the optimal investment strategies when K_P^* is the dominating capacity is summarised in Table C.6.

<table>
<tr><th colspan="3">Optimal investment strategies Sub-problem 5
$K_T^* < K_P^*$</th></tr>
<tr><td rowspan="2">$K_P = K_T^*$</td><td rowspan="2">$\theta_P = \theta_T^*$</td><td>TSO's optimal strategy:
$K_T^* = \max\left(\frac{1}{\eta}\left[1 - \frac{(\delta+\gamma)(\rho-\alpha)}{\theta_T^*}\right] - K_0, 0\right)$
$\theta_T^* = \frac{\beta_1}{\beta_1-1}(\rho - \alpha)\frac{\gamma(K_0+K_T^*(\theta_T^*))+\delta K_T^*(\theta_T^*)}{(K_0+K_T^*(\theta_T^*))[1-\frac{1}{2}\eta(K_0+K_T^*(\theta_T^*))]}$</td></tr>
<tr><td>PC's optimal decision:
$K_P = K_T^*$
$\theta_P = \theta_T^*$</td></tr>
</table>

Table C.5: Overview of optimal investment strategies for Sub-problem 5 if K_P^* is the dominating capacity

<table>
<tr><th colspan="3">Optimal investment strategies Sub-problem 5
$K_P^* \leq K_T^*$</th></tr>
<tr><td rowspan="2">$K_T = K_P^*$</td><td rowspan="2">$\theta_P = \theta_T^*$</td><td>TSO's optimal strategy:
$K_T = K_P^*$
$\theta_T^* = \frac{\beta_1}{\beta_1 - 1}(\rho - \alpha)\frac{\gamma(K_0 + K_P^*(\theta_T^*)) + \delta K_P^*(\theta_T^*)}{(K_0 + K_P^*(\theta_T^*))[1 - \frac{1}{2}\eta(K_0 + K_P^*(\theta_T^*))]}$</td></tr>
<tr><td>PC's optimal decision:
$K_P^* = \max\left(\frac{1}{2\eta}\left[1 - \frac{\delta(\rho - \alpha)}{\theta_T^*}\right] - K_0, 0\right)$
$\theta_P = \theta_T^*$</td></tr>
</table>

Table C.6: Overview of optimal investment strategies for Sub-problem 5 if K_T^* is the dominating capacity

The resulting social welfare for the TSO will be:

$$\begin{aligned} V_{TSO}(\theta_T^*, \theta_T^*, \min(K_T^*, K_P^*)) = \left(\frac{\theta_0}{\theta_T^*(\min(K_T^*, K_P^*))}\right)^{\beta_1} \Bigg[\Big[\frac{1}{2}\eta(K_0 + \min(K_T^*, K_P^*))^2 \\ + (1 - \eta(K_0 + \min(K_T^*, K_P^*)))(K_0 + \min(K_T^*, K_P^*)) \Big] \frac{\theta_T^*(\min(K_T^*, K_P^*))}{\rho - \alpha} \\ - \gamma(K_0 + \min(K_T^*, K_P^*)) - \delta \min(K_T^*, K_P^*) \Bigg]. \quad \text{(C.78)} \end{aligned}$$

And the value of the PC:

$$\begin{aligned} V_{PC}(\theta_T^*, \theta_T^*, \min(K_T^*, K_P^*)) = \left(\frac{\theta_0}{\theta_T^*(\min(K_T^*, K_P^*))}\right)^{\beta_1} * \\ \Bigg[\Big[(1 - \eta(K_0 + \min(K_T^*, K_P^*)))(K_0 + \min(K_T^*, K_P^*)) \Big] \frac{\theta_T^*(\min(K_T^*, K_P^*))}{\rho - \alpha} - \delta \min(K_T^*, K_P^*) \Bigg]. \\ \text{(C.79)} \end{aligned}$$

If K_T^* or K_P^* is equal to zero, the PC will never expand its capacity above K_0, then the present value for the TSO and the PC, given that the TSO invests in K_0 at time θ_T^*, is equal to:

$$V_{TSO}(\theta_T^*, \infty, 0) = \left(\frac{\theta_0}{\theta_T^*}\right)^{\beta_1} \left[\left[\frac{1}{2}\eta K_0^2 + (1 - \eta K_0)K_0 \right] \frac{\theta_T^*}{\rho - \alpha} - \gamma K_0 \right], \quad \text{(C.80)}$$

$$V_{PC}(\theta_T^*, \infty, 0) = \left(\frac{\theta_0}{\theta_T^*}\right)^{\beta_1} \left[\left[(1 - \eta K_0)K_0 \right] \frac{\theta_T^*}{\rho - \alpha} \right]. \quad \text{(C.81)}$$