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Portfolio Optimization of Wind Power Projects

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Problem Statement

In this thesis I will look into the energy production situation in Central Norway by investigating the wind power projects that are currently being evaluated. I will value and compare different investment opportunities in wind power for the purpose of optimizing TrønderEnergi’s project portfolio. As opposed to traditional discounted cash flow methods for project valuation, which can be inaccurate in the presence of high uncertainty and managerial flexibility, the thesis will focus on finding more advanced and nontraditional approaches to portfolio optimization.

Preface

This master thesis has been prepared at the Norwegian University of Science and Technology, Department of Industrial Economics and Technology Management in the spring of 2015. The thesis accounts for 30 ECTS credits and is part of the specialization TIØ4900 Financial Engineering.

First of all I would like to thank my teaching supervisor, Associate Professor Verena Hagspiel, for guidance and helpful comments and feedback. The report is a case study on TrønderEnergi's investment opportunities in wind power, and I wish to thank them for providing relevant data, helpful discussions and industry insight. I would also like to thank Arne Fredrik Lånke at Rambøll for contributing with his knowledge about the wind power projects considered in this thesis.

Trondheim, June 11, 2015

Executive Summary

The use of traditional discounted cash flow approaches for valuing and comparing investment opportunities are prevailing in most industries, and the same is the case for renewable energy investments. As the wind power industry is subject to high uncertainty in electricity prices, the traditional approach might fail to establish the desired overview for making the correct investment decisions because it does not incorporate the value of managerial flexibility.

In this thesis a real options approach to capital budgeting decisions in wind power is analyzed, and the method's suitability is evaluated in relation to a traditional discounted cash flow approach. The hypothesis is that the real options approach will improve the quality of the information in the decision making process, and optimize the project selection for wind power portfolios.

The model developed in this thesis is applied to TrønderEnergi's investment portfolio. The projects considered are located in Central Norway, an area where several companies are currently evaluation investments in wind power. The companies currently evaluating investments are facing a deadline due to the tradable green certificate market. In addition, many of the wind parks in Central Norway depend on the upcoming decision about the upgrade in the central grid, thus resulting in a complex situation for the decision makers. The upgrade in the central grid is triggered if enough capacity is developed in the Fosen area. Companies with concessions to invest in this area created the joint company Fosen Vind to coordinate investments of enough capacity to trigger the grid investment. TrønderEnergi currently owns 14.5 percent of this company, but has the option to reduce its interest. This thesis will therefore consider several investment strategies for TrønderEnergi, depending on the ownership interest they hold in Fosen Vind.

To investigate the investment opportunities, the practical conditions affecting the investment decision are discussed. First, some general information about investment in wind power and an overview of the current wind power development in Central Norway is presented, followed by a detailed presentation of TrønderEnergi's investment situation. Further the relevant literature on real options and capital budgeting under uncertainty is briefly presented followed by a detailed introduction to the relevant theory used in the thesis. The theory presents all relevant equations needed to perform the calculations, however, it is assumed that the reader has some basic knowledge in calculus, stochastic processes and integer programming. References are made to relevant literature if the derivation of the equations used are omitted. The theory on real options analysis and integer programming is used

to develop a dynamic scenario based optimization model. The model is applied to find the optimal project selection for TrønderEnergi's wind power portfolio and the result is compared to the result from optimizing the portfolio based on the traditional discounted cash flow method.

To solve the dynamic scenarios based optimization model, a set of scenarios for the possible development in the projects' value in the future is required. In a real options framework, the value of the option to invest into a project is based on the expected future value of the revenue excluding the investment cost, where the investment cost is considered to be the strike price for the option. All projects are assumed to be exposed to the same sources of uncertainty and the development in the future expected revenue for the projects are considered to follow the same stochastic process as the electricity price. It is further assumed that the development in the electricity price can be described by a mean reverting model, due to its tendency to revert back to its long term mean. The parameters for the stochastic process have been estimated using historical data for the electricity price available at Nord Pool Spot. The binomial option pricing model is used to generate a subset of scenarios for the value of investments. The stochastic process is discretised and the value of the option to invest in each scenario is determined. The critical threshold for investment for each period is calculated to make sure investments do not take place when the option value of waiting is higher than the net value of investing. A value function, representing the value of investing in all the scenarios, is defined. The value function returns zero when investment is not optimal in a given scenario and returns the net expected value of investing when investing is optimal. The dynamic optimization model maximizes the total expected value of the portfolio based on the set of scenarios defined by the value function.

The results demonstrate roughly three times higher expected portfolio value when the dynamic model is applied, compared to the traditional capital budgeting model. It also results in significantly different project selection for several of the scenarios. The dynamic real option based model is capable of including the value of waiting to invest in projects that might have higher net expected value in the future, even though they are currently not considered the most profitable. The limitations of the traditional techniques and the potential of real options theory lead to the conclusion that a real option based capital budgeting approach should be considered when dealing with investments in wind power.

In addition to the conclusion and recommendations, a program for the valuation and scenario generation and two optimization models for the selection of projects are created. The program for the valuation and scenario generation is written in *Visual Basics for Application* and is included in Appendix A, while the optimization models are written in Mosel for the optimization software *FICO Xpress Optimization Suite* and is included in Appendix B and C. The program was created for TrønderEnergi's portfolio, but takes all parameters as inputs, is flexible in number of projects and time to maturity for the investments and can therefore be applied to other wind power portfolios with similar risk and flexibility.

Sammendrag

Bruken av tradisjonelle kontantstrømsanalyser for verdsettelse og sammenligning av investeringsmuligheter er rådende i de fleste industrier, og det samme gjelder for selskaper innen fornybar energi. Vindkraftindustrien er utsatt for stor usikkerhet i kraftprisen, og ved å ignorere verdien av beslutningstakernes fleksibilitet kan den tradisjonelle tilnærmingen gi et manglende bilde for å gjøre de riktige investeringsbeslutningene.

I denne masteroppgaven vil en realopsjonstilnærming til kapital budsjetteringsbeslutninger i vindkraft analyseres, og metoden vurderes i forhold til en tradisjonell diskontert kontantstrømtilnærming. Hypotesen er at realopsjonstilnærmingen vil forbedre kvaliteten på informasjonen i beslutningsprosessen, og optimalisere prosjektvalget i vindkraftporteføljer.

Modellen utviklet i denne masteroppgaven er anvendt for TrønderEnergis investeringsportefølje. Prosjektene som vurderes er lokalisert i Midt-Norge, et område der flere aktører vurderer investeringer i vindkraft. Selskapene som vurderer investering står ovenfor en frist på grunn av sertifikatmarkedet. I tillegg er mange av vindparkene som vurderes i Midt-Norge avhengig av den kommende avgjørelsen om oppgradering i sentralnettet, noe som resulterer i en kompleks situasjon for beslutningstakerne. Oppgraderingen i sentralnettet utløses ved utbygging av tilstrekkelig kapasitet i Fosen-området. Selskaper med konsesjoner i dette området gikk sammen og skapte et felles selskap, Fosen Vind, for å koordinere beslutninger og utvikle tilstrekkelig kapasitet til å utløse nettinvesteringen. TrønderEnergi eier i dag 14.5 prosent av dette selskapet, men har muligheten til å redusere denne eierinteressen. Denne masteroppgaven vil derfor vurdere flere investeringsstrategier for TrønderEnergi, avhengig av hvor stor eierinteresse de har i Fosen Vind.

For å undersøke investeringsmulighetene, vil de praktiske forholdene som påvirker investeringsbeslutningen bli diskutert. Først vil generell informasjon om investeringer i vindkraft og en oversikt over dagens vindkraftutbygging i Midt-Norge bli presentert, etterfulgt av en detaljert presentasjon av TrønderEnergis investeringssituasjon. Videre vil relevant litteratur innen realopsjoner og kapitalbudsjettering under usikkerhet bli presentert, etterfulgt av en detaljert gjennomgang av relevant teori som benyttes i oppgaven. Teorien presenterer alle relevante ligninger nødvendig for å utføre beregningene, men det er antatt at leseren har en viss kunnskap i matematisk analyse, stokastiske prosesser og heltallsprogrammering. Hvis utledninger av likninger er utelatt, vil det bli gitt referanser til relevant litteratur. Realopsjonsteori og heltallsprogrammering benyttes for å utvikle en dynamisk, scenario-basert mo-

dell. Modellen benyttes for å finne det optimale prosjektvalget for Trønderenergis portefølje. Resultatet sammenlignes med resultatet fra optimalisering av porteføljen basert på den tradisjonelle diskontert kontantstrømtilnærmingen.

For å løse den dynamiske, scenario-baserte modellen, kreves et sett med scenarier for utviklingen i prosjektenes verdi over tid. I rammeverket for realopsjoner er verdien av opsjonen til å investere i et prosjekt basert på forventet fremtidig verdi av inntektene eksklusiv investeringskostnaden. Investeringskostnaden regnes som *strike*-prisen for opsjonen. Alle prosjektene antas å være utsatt for samme usikkerhet og utviklingen i forventet inntekt for prosjektene antas å følge samme stokastiske prosess som kraftprisen. Videre er det antatt at utviklingen i kraftprisen kan beskrives av en *mean-reversion* modell, på grunn av tendensen kraftprisen har til å revertere tilbake til den langsiktige middelverdien. Parameterene for den stokastiske prosessen ble estimert fra historiske data for kraftprisen i det Nordiske markedet, tilgjengelig på Nord Pool Spot. Den binomiske opsjonsprisingsmodellen er benyttet for å generere en undergruppe av scenarier for verdien av investeringene. Den stokastiske prosessen er diskretisert og verdien av opsjonen til å investere i hvert senario bestemmes. Den kritiske terskelen for investering beregnes fra opsjonsverdiene, for å være sikker på at investering ikke skjer når opsjonsverdien av å vente er høyere enn nettoverdien av å investere. En verdifunksjon, som representerer nettoverdien av å investere i hvert scenario, er definert. Verdifunksjonen returnerer null når investering ikke er optimalt i et gitt scenario, og returnerer netto forventede verdi av investeringen når det er optimalt å investere. Den dynamisk optimaliseringsmodellen maksimerer total forventet verdi av porteføljen basert på settet med scenarier definert av verdifunksjonen.

Resultatene viser omtrent tre ganger høyere porteføljeverdi ved bruk av den dynamiske modellen, sammenlignet med den tradisjonelle kapital budsjetteringsmodellen. Det er også en vesentlig forskjell i prosjektvalget for flere av scenariene. Den dynamiske realopsjonsmodellen er i stand til å inkludere verdien av å vente for å investere i prosjekter som kan ha høyere netto forventet verdi i fremtiden, selv om prosjektet for øyeblikket ikke er ansett som det mest lønnsomme. Begrensningene i tradisjonelle evalueringsteknikker og potensialet i realopsjonsteorien fører til konklusjonen om at en realopsjonsbasert kapital budsjetteringtilnærming bør vurderes for investeringer i vindkraft.

I tillegg til konklusjonen og anbefalingene, er et program for vurdering og scenariogenerering, og to optimaliseringsmodeller for valg av prosjekter, utviklet. Programmet for scenariogenerering er skrevet i *Visual Basics for Application* og er inkludert i vedlegg A, mens optimaliseringsmodellene er skrevet i Mosel for optimaliseringsprogramvaren *FICO Xpress Optimization Suite*, og er inkludert i vedlegg B og C. Programmene er utviklet for Trønderenergis portefølje, men tar alle parametere som input, er fleksibel i antall prosjekter og tid til forfall i investeringene og kan derfor benyttes på andre vindkraftporteføljer med tilsvarende risiko og fleksibilitet.

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1 Introduction

1.1 Background

Norway's electricity production is dominated by hydro power plants, accounting for 99 percent of the production (Statkraft, 2015). This production covers the base load demand in Norway and is supplemented for peak period demand in the winter by import from nuclear power plants in Sweden. The region of Central Norway has limited hydro power resources, relative to the region's consumption, compared to the west and north of Norway. All three counties in Central Norway are deficit areas. Central Norway, including the counties Møre og Romsdal, Sør-Trøndelag and Nord-Trøndelag, has a power deficit varying between 7 TWh in a normal year to 12 TWh in a dry year (TrønderEnergi, 2014). This results in a constrained power situation in the winter and therefore periods of high power prices in the spot market. Today the deficit is covered by importing power through a 300 kV connection from the surplus region in the North and a 420 kV connection from Sweden in the East (Statnett, 2013). It is expected that the consumption in this area will increase, mainly because of the new gas facility at Nyhamna and increased industry consumption. Without new production the power deficit in Sør-Trøndelag is expected to increase from 570 Gwh in 2014 to 1 915 Gwh by 2034 (TrønderEnergi, 2014). Because Sør-Trøndelag is surrounded by other areas with a power deficit, the required power must either be imported from other regions or new production capacity must be established in Central Norway. As part of the plan to solve the power situation in Central Norway Statnett is planning several upgrades in the central grid in this area. The most essential for assuring delivery of power in the future is the new 420 kV interconnector between Ørskog and Fardal. This interconnector will ensure supply in the region when it is completed in 2016, and will enable increase in consumption and facilitate new power production in Sogn og Fjordane and Sunnmøre. There are also several plans for building new wind power facilities in Sør-Trøndelag, which can potentially lead to 2 675 GWh of surplus by 2034 (TrønderEnergi, 2014). This surplus must be transported out of the region, making upgrades in the central grid necessary either way.

The upcoming decisions from grid operators and power production companies in Central Norway are important to meet the expected increase in consumption and assure sufficient delivery of power in the future, but there are several other potential advantages of the development. Firstly, the development of new power production can lead to the region being a power surplus area instead of a deficit area. This will to some extent remove bottlenecks in the grid into the region and possibly reduce power prices in peak load hours. Secondly, increased power production can facilitate development of new power intensive industry by making it more attractive to invest in the area. One example is the continuously growing server industry. Streaming services for data, movies and pictures alone are expected to double the global demand for server halls, requiring 500 TWh of power by 2025 (Bloomberg, 2014). This industry is already established in Sweden and Finland, where Google and Facebook have started operating huge data centers. Other companies in need of data storage capacity are also looking to the Nordic countries because of the power surplus, stable grid connections and cold climate to cool the equipment. The Nordic region in general has plans to develop more nuclear and wind power and the surplus is expected to reach 50 TWh by 2025 according to Markedskraft AS(Bloomberg, 2014). This need for data storage capacity demonstrate possible benefits of taking advantage of the wind potential in Central Norway, especially considering the decreasing number of good hydro power sites in the country.

1.2 Motivation

This expected future increase in power consumption in Norway lead to more import of electricity in peak load periods if new production is not established. Because of Norway's large resource potential for renewable energy it is considered attractive to develop more power production in the country to meet the increase in demand. Hydro power has been the dominant production technology in Norway, but this also means that most of the good production sites are already developed. Because hydro power is a mature technology its cost is not expected to decrease further. Therefore it is becoming more and more attractive to invest in wind power, an industry with large potential for cost reduction and access to good production sites. Not only is it becoming economically profitable to develop wind power but the large future potential of this industry makes it attractive to build competence in the field. However, because the industry is not as mature there is higher uncertainty related to wind power investments, and the process of valuing projects becomes more complicated. An investment in wind power requires a relatively high irreversible up front cost, while the future rewards are subject to uncertainty in prices and other factors. Irreversible investments and high uncertainty adds value to possible decision flexibilities, and traditional theory on valuation of projects might fail to capture the implications of this flexibility, and of handling uncertainty correctly (Dixit and Pindyck, 1994). Because using traditional valuation approaches does not reflect the true value of a project it might also fail when used in capital budgeting decisions where several projects are considered and compared.

The companies with investment opportunities in wind power in Central Norway are facing a complex investment situation. Decisions to invest must be coordinated with upgrades in the grid to make sure the grid capacity is sufficient, and there are substantial uncertainties for the different players. TrønderEnergi AS is one of the companies with investment opportunities in Central Norway and this thesis will investigate possible investment strategies and approaches to optimize their project portfolio. The optimization of project selection will consider capital budgeting under uncertainty as an alternative to traditional capital budgeting approaches. The goal is to investigate the effect of uncertainty on the investment decision by considering a real options approach. The real options approach will be compared to the traditional capital budgeting approach based on the discounted cash flow (DCF) method, in order to get insight in how the investment decision depends on the uncertainty. The thesis will also consider how business strategy is important when selecting investments by also considering how the investments fit into the company's long term strategy. This is done by proposing different strategies for TrønderEnergi's upcoming decisions and comparing the results.

1.3 Overview of Remaining Sections

Section 2 provides general information about investment in wind power and an overview of the current wind power development in Central Norway. Section 3 presents TrønderEnergi and their current investment opportunities to provide necessary information about the case study considered. Section 4 presents previous literature on investment in wind power and capital budgeting under uncertainty and Section 5 explains the general basic concepts of capital budgeting and real options theory used in this thesis. Two main approaches for capital budgeting are considered. The first is a traditional net present value approach and is presented in Section 6, the second is a real option based optimization approach and is presented in Section 7. The results are presented in Section 8 and the results and assumptions and discussed in Section 9. Section 10 concludes.

2 Wind Power

The most attractive locations for wind power production are areas with a high average wind speed and even wind conditions throughout the year. This is the case for large parts of Norway, which is considered to have some of the best wind resources in Europe and a physical wind potential far above what is realistic to develop. At the end of 2013 an installed capacity of 800 MW wind power was in operation, equivalent to a normal production of 2 TWh per year or 1.5 percent of the total power production in Norway (NVE, 2015). Projects that would account for a total of 7.5 GW of new capacity has received concession from the Norwegian Water Resources and Energy Directorate (NVE) to develop wind park facilities in Norway. Almost 3 GW of this capacity is located in Central Norway (NVE, 2009). In the following all relevant background information about wind power will be presented.

2.1 Investing in Wind Power

There are a lot of factors to consider when investing in wind power. In particular it is important to find a site with good wind conditions. A good wind site however is not sufficient. One has to consider distance to and capacity of nearby transmission grid, environmental aspect and access to land. Another factor that has to be considered is which turbine technology that is best suited for the given site and wind conditions. To invest in wind power access to capital is necessary. It is important to understand the economics of wind power, both the expected costs and the dynamics of the power market and the tradable green certificates (TGC). Below some of the most important key factors to consider when investing in wind power will be discussed further.

2.1.1 Cost of Energy

Hydro power and wind power are seen as the most attractive forms of large scale power production in Norway, because of low costs, large resource potential and limited impact on the environment. However, as opposed to hydro power the cost of wind power is still decreasing. This makes it more and more attractive for power companies to invest in wind power in Norway. In a report by NVE, about the cost of energy for several production technologies in Norway (NVE, 2015), the current cost of energy for wind power was estimated. The following discussion is to a large extent based on the results from this report.

The cost for a wind power park can be divided into investment cost, operation and maintenance (O&M) cost and cost of capital. Approximately 75 percent of the total costs are related to the up front investment cost of the turbine and the balance of plant (BoP) cost (Krohn et al., 2009). The BoP cost includes all the infrastructure and facilities, with the exception of the turbine itself, and mainly comprises of costs associated with foundation, connection to grid, transformers, roads and cranes (ERSU, 2012). In general both the investment cost and the O&M cost depend on factors such as site, size of the wind park, number of turbines and type of technology.

The average distribution of the investment cost, based on a sample of five wind parks that started operation in Norway between 2011 and 2013, are presented in Figure 1. The five projects had an average investment cost of approximately 12 000 NOK/kW (NVE, 2015). The cost distribution is not necessarily representative for the current cost of energy for wind power, because the data is based on only five projects that had significantly different cost structures. These projects were the most mature at the moment but not necessarily the most cost effective. In the report by NVE it is estimated that the investment cost for projects with concession in 2014 is about 20 percent lower than the average investment cost between 2011 and 2013. This cost reduction is mainly because of lower turbine prices as well as lower construction and project management costs (NVE, 2015). After the financial crisis in 2008 the demand for wind turbines decreased heavily in many markets, while at the same time the production of turbines increased and the costs for material and work decreased. This resulted in a significant reduction in the cost of wind turbines after 2008.

The O&M cost for wind power is relatively low compared to other forms of power production. This is because the wind is given by nature and under normal circumstances frequent maintenance of the equipment is not required. Usually the wind park owners enter into long term service contracts with a turbine supplier for operation and maintenance, stating a guarantee for a maximum down time. In addition to maintenance of the turbines, other contributors to total O&M cost for wind power are feed tariffs, balancing costs, operations personnel, maintenance of roads and grid, insurance and lease of land (NVE, 2015). These costs are often project specific and data about the cost is hard to collect. NVE estimates that an O&M cost of approximately 15 øre/kWh is reasonable (NVE, 2015).

A method for comparing different production technologies and to investigate the development of the cost over time is calculation of the levelized cost of electricity

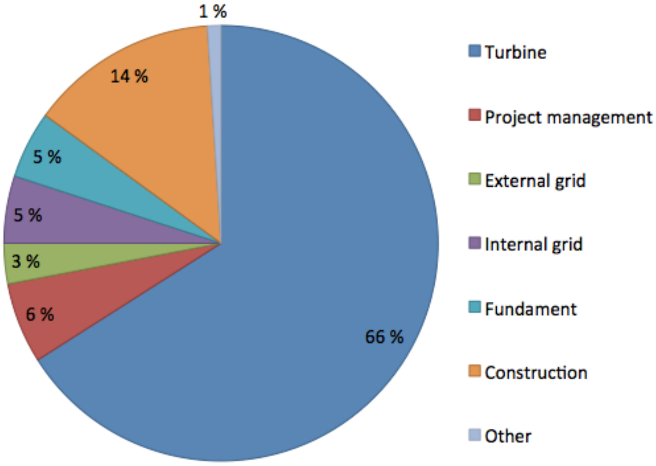


Figure 1: Average distribution of investment costs for wind power projects. Source: NVE (2015).

(LCOE) (NVE, 2015). The LCOE represents the present value of the total cost per produced kWh over the entire operation period, including investment cost, O&M cost and financing cost, and is calculated as;

$$LCOE = \frac{\sum\limits_{t=0}^{T} \frac{I_t + M_t}{(1+r)^t}}{\sum\limits_{t=0}^{T} \frac{P_t}{(1+r)^t}}, \tag{1}$$

where T is the lifetime of the project, I_t is the investment cost in year t, M_t is the maintenance cost in year t, P_t is the annual production quantity and r is the discount rate. NVE found that the cost of energy for several wind power projects with concession ranged between 35-46 øre/kWh. In the report the cost of energy for a representative wind power project in 2014 was estimated to be 40 øre/kWh considering a discount rate of 4 percent and 44 øre/kWh with a discount rate of 6 percent. The discount rate that should be used varies between technologies, depending on the risk of investing in different types of power production (NVE, 2015). In the report NVE has used the same discount rate for all power production technologies, for comparing reasons. These discount rates are not representative for the cost of capital usually required in wind power investments, as these are riskier investments compared to mature power production technologies, and therefore require a higher expected return.

The cost distribution and cost of energy for the representative wind park considered by NVE is presented in Table 1. It can be seen that the cost of the turbines represent the largest cost. The cost of energy was shown to be most sensitive to the production (NVE, 2015). A high number of full load hours, or a large yearly production quantity compared to the performance of the production facility, contributes to a significant reduction in the cost of energy. This is related to the fact that the fixed investment cost is significantly higher than the variable cost of operation. When production increases the cost of operation does not increase at the same rate and therefore the total cost of energy decrease. Second to the full load hours, the cost of energy was sensitive to variations in the investment cost and to the lifetime of the project.

Cost distribution	NOK/kW	Percentage
Turbines	7 000	68.3
Foundation	600	5.6
Construction	1 200	11.7
Internal grid	500	4.9
External grid	500	4.9
Projects management	300	2.9
Other	150	1.5
Total	10 250	100
Construction period interest	410	
Cost of energy	øre/kWh	
O&M costs	15	
FCOE (discount rate: 4 %)	40	
FCOE (discount rate: 6 %)	44	

Table 1: Distribution of investment cost and cost of energy for a representative wind park in 2014 with 100 MW capacity and 3200 full load hours. Source: NVE (2015).

The wind power industry has experienced a significant reduction in cost of energy during the last decades due to a reduction in capital costs and an increase in production efficiency. The potential for future decrease in costs mainly lies in improving the wind turbine technology in order to increase the number of full load hours.

2.1.2 Technology

When developing a wind park there are usually several choices in wind turbines available from different suppliers. Two of the main specifications of a wind turbine are the capacity, or the amount of power it produces, and the turbine class. Which turbine class is optimal for a site depends on the average wind speed at the site, the degree of turbulence and the maximum wind speed that might occur in a 50 year period (Renewablesfirst, 2015). The main difference between the turbine classes is the wind speed that results in optimal production. Table 2 presents the specifications of the different turbine classes.

	Class 1	Class 2	Class 3
Annual average speed (max)	10.0 m/s	8.5 m/s	7.5 m/s
50 year return gust	70.0 m/s	59.5 m/s	52.5 m/s

Table 2: Specifications for IEC wind turbine classes. Source: Renewablesfirst (2015).

In addition, many other parameters vary between turbine models. Rotor diameter and hub height are the most obvious ones. Another clear distinction between turbines is whether they have a gear box or a direct drive system. The overall efficiency of a wind turbine depends on how well all the components in the system are optimized; including blades, electro mechanic parts and software. Regarding the choice between a gear and a direct drive turbine, one may expect the latter to be the more expensive technology. The gear based technology may imply higher O&M cost. Thus, deciding which turbine technology to use is often a trade-off between energy production, CAPEX and OPEX. The efficiency and energy output of a wind park also depends on the positioning of the turbines as well as the balance of plant design (BoP).

Wind power technology has developed quickly over the last decades, with several technological leaps. The main factors resulting in improved efficiency and reduced costs are the height of the turbine and the size of the rotor blades. Larger blades results in a larger area to exploit the available wind energy. According to an estimate by NVE (NVE, 2015), the average full load hours for wind parks can be expected to increase with 1.5 percent each year towards 2020. This increase is due to improvements in the rotor design, advanced control systems (that can adjust for variable wind conditions) and micrositing (advanced data technology and techniques for wind measurements and optimal placement of the turbines).

2.1.3 Certificate Market and Rules for Taxation

If new production capacity is to be established in Central Norway, the companies with investment opportunities in wind power has to make the decision to invest soon. There is an implicit deadline for investment in renewable energy because of the tradable green certificate market in Norway. In 2012 Sweden and Norway entered into a common market for green certificates with the goal of assuring development of 26.4 TWh of renewable production by 2020 (Thema, May, 2014). The certificate marked is a market based support scheme where the certificates are an instrument to facilitate new production of renewable energy. The development of renewable energy projects are then partly financed through certificates, received for each MWh produced, which can be sold in the market. To assure demand for the certificates the electricity supply companies are obligated to buy certificates to cover a consumer quota, which is divided equally between Sweden and Norway. The quota is equal to a certain share of the power consumption each year and this share changes according to a predetermined curve. If the power consumption subject to quotas turns out to be different from what was predicted the curve is to be reevaluated. These adjustments are meant to assure that the total demand for certificates over the entire period is 396 TWh (Thema, Jan., 2014). The certificate market will last until 2035 and the production facilities have to be in operation by 2020 to receive the certificates. Because most of the projects currently depend on the revenue from the certificate market to be profitable this results in an implicit deadline for the investment decisions in renewable energy (Thema, May, 2014).

Which projects that are eventually realized in the certificate market depends on many factors, among which are the rules for taxation in the two countries. Because wind power projects in Norway are taxed at a higher rate than in Sweden, projects that are socioeconomically more expensive can end up being developed before those that are cheaper. The differences in taxation can therefore have large impact on the development of certificate power towards 2020. In a report by Thema consulting group, about the certificate market and rules of taxation (Thema, May, 2014), they suggest that the current tax system will result in approximately 9 TWh of certificate power in Norway, compared to 17 TWh in Sweden. However, the government of Norway is currently considering changing the rules for taxation for renewable energy, so that it becomes more attractive to invest in Norwegian wind power. Thema (Thema, May, 2014) has estimated that green depreciations can increase development in Norway by 2 TWh, while implementation of identical rules for taxation in Norway and Sweden can increase development in Norway by up to 3 TWh. They conclude that the current rules for taxation lead to underdevelopment in Norway, and thereby increased costs and socioeconomic loss because cheap projects in Norway might be displaced by more expensive projects in Sweden. Table 3 presents the current tax depreciation relevant for wind power in Norway using the declining balance method. The income tax rate is 27 and the property tax is 0.7 percent of the appraised value (Altinn, 2015). If the new depreciation rules for renewable investments that are currently awaiting approval with the Norwegian government are implemented, all assets except for construction related assets can be depreciated using straight-line depreciation over 5 years.

D	Machine equipment	20 %
G	Electronic equipment	5 %
H	Constructions	4 %

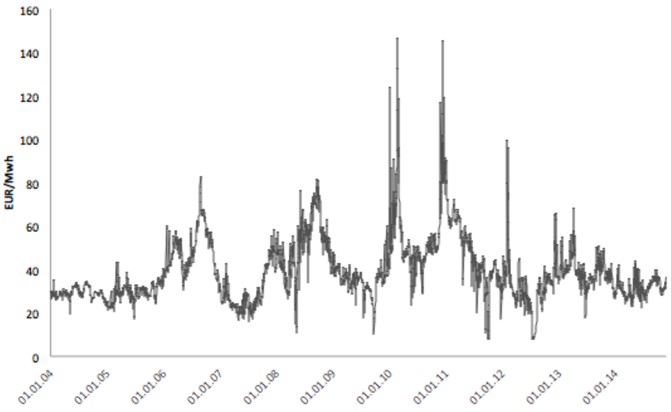
Table 3: Tax depreciation for wind power in Norway. Source: Altinn (2015).

2.2 Risk Factors in Wind Power Development

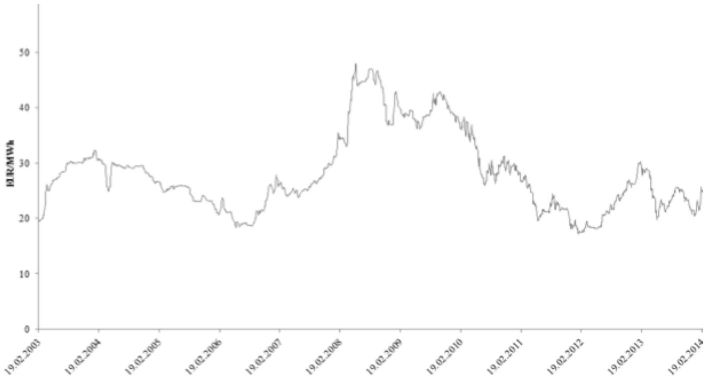
The investment situation in wind power in Central Norway is very complicated because there are a lot of uncertainties related to the profitability of the projects. Therefore, the investment analyses becomes difficult. The uncertainties in cost and revenue are related to the price of electricity and tradable green certificates, investment cost, maintenance cost and sharing of grid cost. In addition, wind power projects are subject to uncertainties in the production, both with regards to how much capacity is eventually installed and how good the wind conditions are in the operating period. There is also uncertainty about the application process for the concession that is required to develop a wind park. In the following some important risk factors for wind power investments are discussed further.

2.2.1 Price of Electricity

The revenues for a wind park strongly depend on the electricity price. Since electricity prices are very volatile they contribute significantly to the uncertainty in valuing a wind power project. The electricity spot price in the Nordic power market is determined through trading at Nord Pool Spot AS, and the forces of supply and demand establish the price. The volatility of the spot price in the Nordic power market can be observed from the historical daily electricity prices from Nord Pool Spot between 2004 and 2014, displayed in Figure 2a.



(a) Electricity prices from Nord Pool Spot between 2004 and 2014



(b) TGC price between 2003 and 2014

Figure 2: Daily historical spot prices for the Nordic power market. Source: Nord Pool Spot (2015).

2.2.2 Price of Tradable Green Certificates

The price of the tradable green certificates are also established in the market through supply and demand, and are therefore subject to uncertainty. As mentioned in Section 2.1.3 the supply of certificates depend on the production of certificate power, which to some extent is determined by the amount of investments in renewable energy. The demand for certificates are established through quotas. The uncertainty in supply is related to the amount of capacity that will actually be developed by 2020. But even if investments are made to reach the goal of 26.4 TWh there is also uncertainty related to the production quantity each year. If the investments are not sufficient or the production is low this will result in low supply of certificates and therefore the price will increase. If investments are sufficient or production is high the supply will be high resulting in a lower price for certificates. In theory the price of certificates will reflect the markets perception of the cost of building the most expensive facility to cover demand for certificates and the expected revenue from the electricity market. The difference between these two will reflect the certificate price that ensures sufficient investments in certificate power. In other words, the revenues from the certificate market are equal to the subsidy level that is necessary to make the marginal project profitable (Thema, Jan., 2014). This dynamic will however, also in theory, only hold in the investment period, because after 2020 the supply is to some degree given by the amount of investments. After 2020 the certificate price is determined by the balance in the market. The price level is controlled by expectations about the future and prospects of certificate deficit in the next years. If a given year does not produce enough certificates to cover the demand, the price will be high. The electricity suppliers must pay a tax levy of 150 percent of the certificate price if they do not have the right amount to cover the quota. To avoid this tax levy they will therefore be willing to pay more for the certificates in periods of deficit. The suppliers have to deliver the certificates once a year. How high the price becomes in deficit years depends on how early the market becomes aware of the deficit.

Currently the main uncertainty in the certificate price is related to the uncertainty in the amount of investments during the next few years, and the possibility that more than 26.4 TWh can be developed. Another important uncertainty is the technical adjustments to the quota curve that determines demand. Some projects depend on high certificate prices in the period after 2020, and this period is currently very hard to analyze with respect to these prices. This is a challenge for companies that are considering investing in wind power, because many projects depend on high certificate prices to be profitable. Historical volatility in the certificate price can be observed from Figure 2b which present the daily prices for certificates between 2003 and 2014. Historical prices are however not representative in terms of specifying the volatility and the development in the certificate price in the future, as the certificate market has only existed for a short period. The historical data does not necessarily reflect the price development after 2020, therefore it is not considered descriptive for the future.

2.2.3 Costs

As mentioned in Section 2.1.1 the key costs of wind power investments are turbine cost, BoP cost, O&M costs and cost of capital. Because approximately 75 percent of the costs are related to the investment cost, the uncertainty in this cost represents the largest risk factor.

The uncertainty in the investment cost is related to market uncertainty for turbines and civil work. High demand for turbines and civil work will increase the prices. The wind turbines are the largest part of the investment cost. This makes investment costs very sensitive to fluctuations in the turbine price (NVE, 2015). If a contract is entered into for the purchase of the turbines the uncertainty in the investment cost is low. However, without a contract this cost is subject to uncertainty due to market fluctuations.

Part of the investment cost are grid investments, which includes building all the grid that is required to transport the produced electricity to the existing grid. Sometimes this cost can be shared between several investors that benefit from building the grid. Before the developer of a wind park enter into any potential contracts for the grid development there is uncertainty in the size of this cost.

The level of future O&M costs will to a large degree depend on the future development in international prices of service agreements. The uncertainty in the O&M cost for the wind park developer also depends on who has the responsibility for the maintenance of the wind turbines. Usually the manufacturer of the turbines is responsible for the maintenance during the first years of operation, and therefore also holds the risk of the cost. After a few years the developer of the wind park might take over and do the maintenance themselves. In general, the risk is lower for the developer when the manufacturer has the responsibility of maintenance. However, the developer can choose to take on the risk for the O&M cost if they expect to do the maintenance at a lower cost compared to what the manufacturer offer. The O&M cost will vary between projects, depending on the negotiations between the developer and the turbine supplier. Estimating the risk for the O&M cost is a complex matter as there are several options for contracts with the manufacturer. The risk depends on which strategy is chosen, where alternatives for wind park developers are long term fixed price with the manufacturer or taking over O&M early in the operation period. The uncertainty also depends on how the maintenance is exercised. Preventive maintenance may be less risky than corrective maintenance but normally requires extra costs. The complexity of the terrain and the wind conditions at the site also affect the uncertainty. Typically O&M costs will increase with high production and high turbulence, but the required maintenance will also be less predictable in these situations and therefore the uncertainty is higher. Finally, the uncertainty in the O&M cost is related to the choice of turbine technology as less experience regarding the required maintenance of new technology increase the uncertainty.

2.2.4 Concession

In most cases a concession from the government is required if a company wants to develop an energy facility in Norway. A concession is an approval or license to build. For wind power in Norway this concession is required if the voltage in one or more components are higher than 1 000 volts (NVE, 2009). The developer has to send an application to NVE, which includes analyses of the projects potential, environmental impact and financial aspects. After receiving approval from NVE the concession can be appealed and final concession is received from the Ministry of Petroleum and Energy (OED). A wind power project might be denied in the application process due to various reasons, such as environmental regards, complaints from local authorities or low expected production.

2.2.5 Production

The revenues for a wind power project depends on the amount of power produced and the production depends heavily on the speed, distribution and quality of wind. The amount of wind on the site in the future is off course uncertain but estimates can be made using historical data. The degree of uncertainty in the wind measurements are therefore related to the quality of the measurements done in the area. The number of measurement masts at a site, and their placement relative to where the turbines will be placed, are important factors for the quality of the measurements. Another factor of uncertainty related to the production is the actual capacity that is developed. Sometimes it is optimal to develop less than the capacity approved in the concession, or use a smaller area, because the wind measurements show worse potential than expected in certain areas. In other situations it might be possible to extend the capacity or area to make the project more profitable. This often requires applying for a change in the concession, which might be denied in the application process.

2.3 Wind Power Development in Central Norway

To achieve the goal to increase renewable energy production in Norway to 67.5 percent of the total power production by 2020 the government is facilitating development of wind power in Norway. In addition to the green certificate subsidy scheme, the government in Norway decided to bolster Statkraft's equity by 5 billion NOK in 2014, and the Ministry of Trade, Industry and Fisheries will reduce its overall dividend from Statkraft by 5 billion NOK towards 2018 (The Norwegian Government, 2011). This total equity increase of 10 billion NOK is supposed to help Statkraft make major investments in renewable energy for a total of approximately 60 billion NOK (Statkraft, 2014). Some of Statkraft's investments in wind parks are located in Central Norway, where a lot of companies hold investment opportunities that are currently being considered.

In Fosen and Snillfjord 8 wind power projects with a total capacity of 1 300 MW, and total estimated investment cost of 20 billion NOK, received final concession from OED in August 2013. Currently a total of 1400 MW wind power has concession to build in the two areas. These investments can provide 3,7 TWh of renewable electricity per year, supply 180 000 households and cover half of the power deficit in Central Norway. At the same time OED gave Statnett concession to build a 420 kV connection from Namsos to Trollheim, with transformer stations at Roan, Storheia and Snillfjord. The grid in the area is currently not equipped to receive the large amount of power generated from the planned wind power projects. The realization of the projects will trigger the grid development. The decision to invest in the wind power projects has not yet been made. Statnett is coordinating development of the grid with development of the wind parks (Statnett, 2013). The wind parks have to be in operation by 2020 to receive the green certificates, and the prerequisite for the central grid is that at least 1 000 MW of wind power will be developed in the area. Statnett has plans to develop the grid in three stages; first the grid from Namsos to Roan will be built if at least 600 MW of capacity is developed at Fosen. Next, the grid from Snillfjord to Trollheim will be developed if at least 400 MW of capacity is developed at Snillfjord. Finally, an interconnector across Trondheimsfjorden will be built when the need for transmission between the two regions is sufficient (Statnett, Aug.,, 2013). The concessions for the wind parks are allocated among different companies; SAE Vind (Statkraft and Agder Energi) and Sarepta Energi (NTE and TrønderEnergi) at Fosen and SAE Vind and Zephyr AS (Østfold Energi, Energiselskapet Buskerud, Vardar og Dong) at Snillfjord (Windcluster, 2014).

To simplify the coordination of the development and invest before the deadline, the companies behind the concessions at Fosen decided to create a joint venture company with sufficient capacity to satisfy the demand to build the grid. Nord-Trøndelag Elektrisitetsverk (NTE), TrønderEnergi, Agder Energi and Statkraft created Fosen Vind AS to develop the 3 wind parks Storheia, Roan and Kvenndalsfjellet with a total capacity of 600 MW. The ownership structure and projects in Fosen Vind AS are illustrated in Figure 3. The three projects are currently being evaluated and a decision is expected by summer 2015. Statkraft will be the operator of the joint venture company in the development phase. The three projects have a total investment cost of approximately NOK 7 billion (Windcluster, 2014). In addition, Sarepta has concession to build Harbaksfjellet and Sørmarksfjellet in the Fosen area.

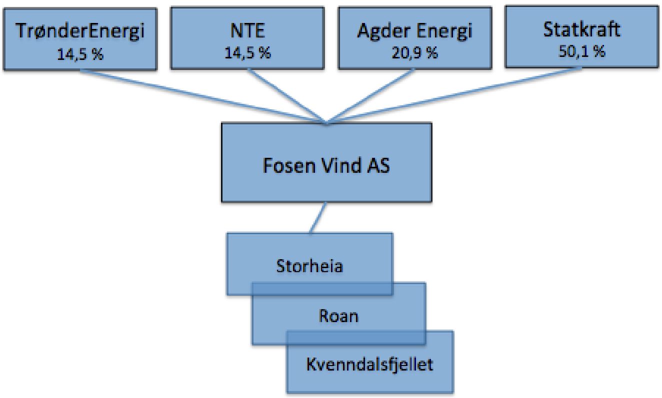


Figure 3: Illustration of ownership structure and projects in Fosen Vind AS.

Measurements demonstrate good wind conditions in the inland and mountain regions in Trøndelag. Projects in this area with a total potential capacity of 1 600 MW are currently registered with NVE. 400 MW of this capacity has applied for concession and none of the projects has yet received concession (Statnett, 2013). These projects are not as mature as those at Fosen and Snillfjord, but can potentially be developed to further take advantage of the wind resources in the area. New wind power facilities are also under construction at Ytre Vikna and there are plans to develop projects at Hitra, Frøya, Nordmøre and Sunnmøre. Realization of these projects will demand further upgrades to the existing grid internally and out of the region. Figure 4 presents all the projects with concession in Sør- and Nord-Trøndelag.

Figure 4: Overview of wind power projects with concession to build in Central Norway as of 2014.

3 Investment Opportunities for TrønderEnergi

This section will briefly introduce TrønderEnergi AS and present information about the considered investment opportunities. First, the company will be presented with an overview of the current investment situation and its available investment capital, followed by a detailed description of the different investments, with required investment cost and the risk and profitability respectively, for the projects considered.

3.1 TrønderEnergi AS

TrønderEnergi is based in Sør-Trøndelag in Norway and is organized under the group TrønderEnergi AS with a number of subsidiaries. The group generates annual sales of approximately NOK 1,7 billion allocated among three business areas; Energy, Networks and Markets. The business areas are responsible for the generation, distribution and sale of 2,1 TWh of power each year (TrønderEnergi, 2015). TrønderEnergi strives to create value by producing and distributing environmentally friendly hydro power and wind power to customers in the local region. As of 2014 200 GWh of their production resulted from wind power.

3.1.1 Investment Opportunities

TrønderEnergi is currently evaluating several investment opportunities in new wind power. This requires extensive analysis and leads to complicated decision making processes. Their upcoming investment decisions are restricted by access to capital and will require the need for strict prioritizing among the various opportunities. Figure 5 illustrates TrønderEnergi's investment opportunities in wind power. They currently has a concession to build the wind park Stokkfjellet and has applied for concession for a second wind park; Brungfjellet. The company owns 50 percent of Sarepta AS, together with NTE, where they are evaluating four wind parks. They also hold 14.5 percent ownership in Fosen Vind AS where the wind parks Storheia, Roan and Kvenndalsfjellet are being evaluated. The company also has plans for investments in grid companies and to reinvest in their own existing grid.

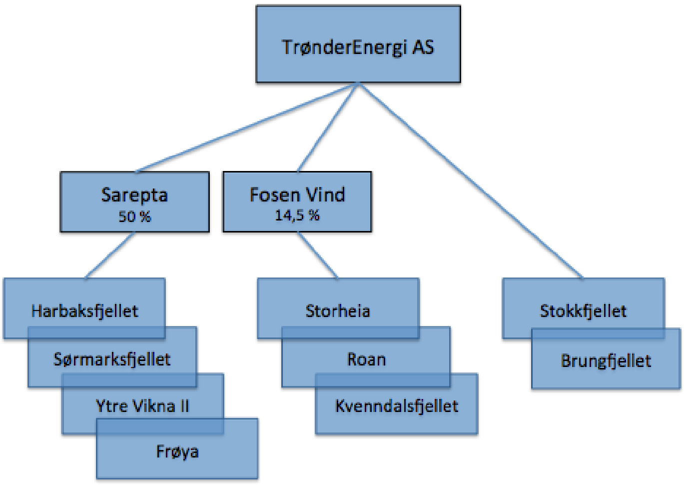


Figure 5: Illustration of investments and projects in TrønderEnergi AS.

3.1.2 Investment Capital

Full development of TrønderEnergi's wind power portfolio will require an extensive amount of capital and is considered unrealistic from the company's perspective. TrønderEnergi has therefore actively pursued the establishment of the joint ventures referred to above. The Board of Directors has additionally approved an increase of the equity in the parent company, which could result in as much as NOK 1 billion available for use in wind power investments. The current investment capital is estimated based on the annual report from 2013, which according to TrønderEnergi is a good estimate for the current situation. TrønderEnergi's liquidity reserve was NOK 1276 million as of 2013. The company's financing strategy requires that the reserve should always amount for 20 percent of the turnover, which in 2013 was NOK 1536 million, resulting in NOK 969 million of cash equivalents available for investments. The surplus from operation in the next few years can also be used for future investments. Approximately 50 percent of the profit is payed in dividend to the company's shareholders. The annual profit was NOK 245.6 million in 2013 and NOK 320.4 million in 2012 and an average profit of NOK 250 million is expected for the next four years. TrønderEnergi can partly finance their investments using debt. The debt coverage can, however, not exceed 70 percent for the grid investments and 60 percent for the wind power investments. The investments therefore require 30 and 40 percent equity, respectively. Table 4 presents the calculation of the current equity capital available for investments.

Liquidity reserve	1 276
Turnover	1 536
Required liquid reserves	20 % of turnover
Available cash equivalents	969
New equity issue	1 000
Average annual result	250
Generated investment capital	500
Total equity capital	2 469

Table 4: Available equity capital for TrønderEnergi in NOK million.

TrønderEnergi currently faces a complex decision regarding which wind parks to invest in. Even though the Board approved issue of new equity and the company has the option to invite more co-owners to Sarepta, there are still significant restrictions on capital and several investment strategies to consider. The next sections will present each of TrønderEnergi's wind investment opportunities and potential investment strategies.

3.2 Fosen Vind AS

The joint venture Fosen Vind AS has plans to develop the three wind parks Storheia, Kvenndalsfjellet and Roan in the Fosen area, provided that the investments in the projects are meeting the company's required rate of return. If the company decides to invest in the three wind parks, the total investment cost will be approximately NOK 7 billion, divided among its shareholders.

TrønderEnergi only holds a financial role in Fosen Vind, owning 14.5 percent of the company. If TrønderEnergi maintains this ownership position they have to contribute approximately NOK 1 billion of capital for the investment. In addition, they also hold the option to decrease their interest in Fosen Vind before the investment decision is made. Alternatives being considered are either to keep their current position, decrease it to 10 percent or not to invest in Fosen Vind. If they choose to decrease their interest in Fosen Vind, they would be able to sell their part of the company in the market. It is, however, uncertain how much such a sale would profit them. Figure 6 shows the average value from transactions of wind power projects depending on the stage in development. The average value is based on data collected by Deloitte (2011) and TrønderEnergi. As can be seen in the figure the average profit from selling wind power projects in Norway is quite low prior to operation, compared to Europe and USA. It is assumed that the same apply for selling shares in Fosen Vind. The differences in value of selling projects in Europe and USA and Norway can partly be explained by the risk preferences of investors in Norway.

Fosen Vind will not be financed by debt and the company requires its shareholders to contribute with capital free of debt. There are no official numbers for the expected return for the Fosen projects, but they are considered to be good projects and economies of scale for purchase of turbines and O&M costs suggest a higher return than the other projects considered by TrønderEnergi.

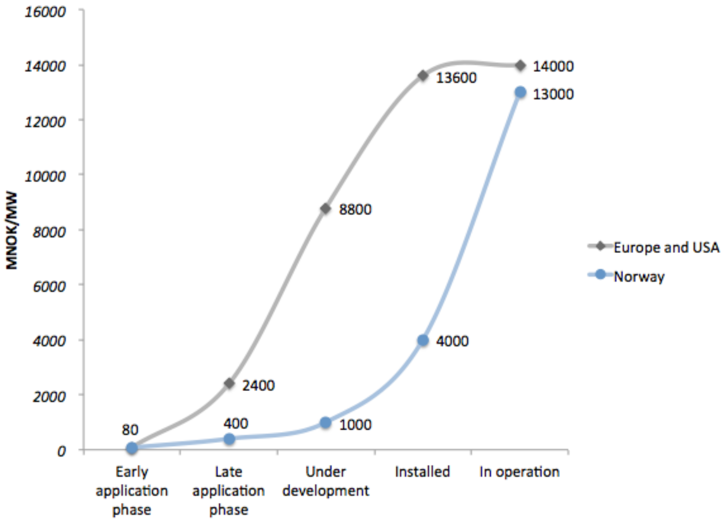


Figure 6: Transactions of wind power projects depending on stage in development. Source: The data for Europe and USA is collected by Deloitte (2011) and the data for Norway was collected by TrønderEnergi.

3.3 Grid Investments

TrønderEnergi's business area *Networks* has plans to make investments in grid companies in Central Norway using a total of approximately NOK 1500 million towards 2020. They also plan to upgrade existing grid facilities for approximately NOK 250 million towards 2020. It is possible to finance the investments with 70 percent debt, resulting in a total required equity of NOK 525 million.

3.4 TrønderEnergi's Wind Parks

TrønderEnergi currently holds a concession to build the wind park Stokkfjellet, has applied for a concession for Brungfjellet and is planning development of Skomakerfjellet wind park. There are a lot of uncertainties regarding the profitability of Brungfjellet and there is a high possibility the wind park will not receive a concession. In addition, Brungfjellet is not considered to be very profitable, compared to the other projects. Therefore we decide not to considered this investment opportunity. TrønderEnergi has recently made the decision to invest in the wind park Skomakerfjellet which will require NOK 124 million of investment capital in the coming year. The location of the project being considered for investment and key information is presented in Figure 7.

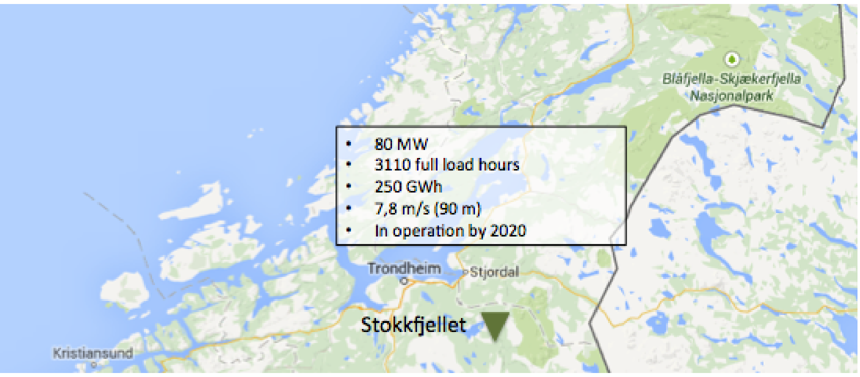


Figure 7: Location and key information of TrønderEnergi's wind power project Stokkfjellet.

3.4.1 Stokkfjellet

Stokkfjellet wind park is located in Selbu municipality in Sør-Trøndelag. The project received concession from NVE in 2014 and is waiting for the final approval from OED. TrønderEnergi applied to develop a total capacity of 90 MW in an area of 5.8 km^2. Due to capacity constraints in the grid no more than 80 MW can be developed and as the grid required is expensive the full capacity must be developed for the project to be profitable. The wind park is expected to have 3110 full load hours, or an average annual production of 250 GW, but there is some uncertainty to these numbers related to the quality of wind measurements. The site is complex, with wind turbulence and possibility of icing on the turbines. Both class 1 and class 2 turbines are considered to be possible technology choices for this wind power project. The project requires an expected investment cost of approximately 11.2 MNOK/MW. There is little uncertainty about the grid cost because there is no possibility of sharing this cost with anyone. TrønderEnergi will have to pay for all the grid development required themselves. The project will require two years of development and the investment decision has to be made before 2018. The distribution of the investment cost is presented in Table 5.

Cost distribution	Stokkfjellet
Turbines	680 200
Foundation	35 800
Construction	89 500
Internal grid	8 950
External grid	53 700
Projects management	8 950
Other	17 900
Total	895 000
Total per MW	11 188

Table 5: Cost distribution for Stokkfjellet in 1000 NOK.

3.5 Sarepta Energi's Wind Parks

Sarepta Energi was established in 2005 and is jointly owned and operated by TrønderEnergi and NTE. Sarepta has one wind park in operation at Ytre Vikna which produces 120 GWh each year and generated operating revenue of NOK 30.8 million in 2013 (Sarepta, 2014). Sarepta also has concession to build and operate four wind parks at Ytre Vikna, Sørmarksfjellet, Harbaksfjellet and Frøya, which will demand investments of NOK 6 billion towards 2020. The different projects will be organized as separate entities in special purpose vehicles (SPV) that will be financed separately with debt and equity. The parent company Sarepta will be financed by fees for management services from each project. If the projects are developed in the joint venture, TrønderEnergi will own 50 percent of each project and is required to add capital to Sarepta equivalent to half of the investment cost. The projects can, however, also be sold separately and another possibility is that TrønderEnergi can buy the 50 percent they do not currently own.

This section will present all the relevant information about each of the Sarepta projects. All data about the projects are taken from publicly available information in the concession applications (NVE, 2009) and from the home page of the company (Sarepta, 2015). The location of the four projects being considered and key information about each project are presented in Figure 8.

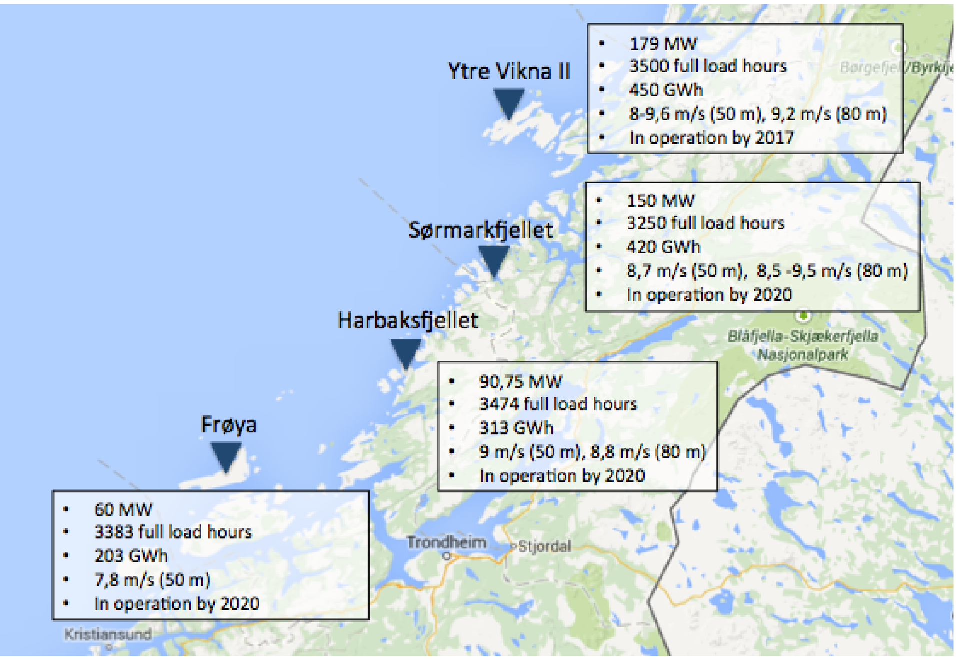


Figure 8: Location and key information of Sarepta Energi's wind power projects.

3.5.1 Ytre Vikna II

Ytre Vikna II is the second stage of a wind park development in Vikna municipality in Nord-Trøndelag. The first stage started operation in October 2012 and has a capacity of 39 MW and an annual production of 120 GWh. The second stage of the wind park development has concession to build in an area of 9.7 km^2 and can have a total capacity of 179 MW. However, Sarepta expect to develop 128 MW as this is considered the best alternative for the site. Good and representative wind measurements have been done in the area, demonstrating low turbulence and an expected number of full load hours of 3500, or a average annual production of 450 GWh. Because of the wind conditions, class 2 turbines would be used. Both stages of the wind park received final concession from OED in 2006, but changes to the concession have been approved since then. The latest changes gave an extension to the deadline for developing the wind park until 2017. It is very likely that it is possible to extend this to 2020 if necessary. However, there are possible strategic incentives to make the investment decision early. Due to the fear of excess capacity of renewable energy being developed, which will cause overflow of certificates in the market, a signal of commitment might have the advantage of scaring off competitors. The project requires two years to be developed. Therefore, the investment decision has to be made by 2017 at the latest. The turbines at Ytre Vikna I are 2.3 MW, but the second stage will most likely use 3 MW turbines. The total investment cost is approximately 10 MNOK/MW. Ytre Vikna has high production and low investment cost but the project will require a higher O&M cost than the other projects due to the long grid connections required. This connection is expensive to maintain and leads to high transmission losses.

3.5.2 Sørmarksfjellet

Sørmarksfjellet wind park is located in the municipality of Osen and Flatanger in Nord-Trøndelag. The wind park received the final concession from OED in 2013 for 150 MW in an area of 9.3 km^2, but developing 144 MW is considered to be the best alternative according to TrønderEnergi. The wind measurements in the area are less representative, therefore more uncertain. Additionally there are some wind turbulence in the area. It is possible to use both class 1 and class 2 turbines at the site. The expected number of full load hours is 3250, or an average annual production of 468 GWh. According to TrønderEnergi the wind park depends on the development of the new central grid and an investment decision can not be made before it is certain that the grid will be developed. The decision whether to develop the grid is expected in June 2015. The project requires two years of development and the investment decision has to be made no later than 2017. There has not yet been made any agreements for sharing the grid cost. The project requires an extra transformer, which will be expensive and increases the risk in the grid cost. The investment cost is estimated to be approximately 11.7 MNOK/MW if 48 turbines with 3 MW capacity each will be used.

3.5.3 Harbaksfjellet

Harbaksfjellet wind park is located in Åfjord municipality in Sør-Trøndelag. The final concession for 90.75 MW in an area of 10.1 km^2 was received in 2005. The wind measurements in the area are representative and demonstrate good wind conditions. The estimated number of full load hours are 3474, equivalent to an annual production of 313 GWh. It is possible to use both class 1 and class 2 turbines. The investment cost for 30 turbines with a capacity of 3 MW each is estimated to be 11.76 MNOK/MW. According to TrønderEnergi the wind park depends on the development of the central grid and an investment decision can not be made before it is certain that the grid will be developed. The project is relatively small and requires a shorter development period than Ytre Vikna and Sørmarksfjellet. The investment decision can therefore be delayed until 2018 at the latest.

3.5.4 Frøya

Frøya wind park is located in Frøya municipality in Sør-Trøndelag. The wind park received final concession from OED in 2013 for 60 MW in an area of 6.6 km^2. There is limited flexibility in the capacity to be developed because the area is quite small. The wind measurements demonstrate good wind conditions and low wind turbulence in the area, but the measurements are less representative and therefore uncertain. Class 2 turbines seems to be the most reasonable choice. The estimated number of full load hours are 3383, equivalent to an annual production of 203 GWh. The wind park depends on the development of the central grid, as well as grid connections from Hitra which will be developed if Hitra wind park is built. Hitra, however, is according to TrønderEnergi likely to be developed if the central grid is built. The project will result in high grid costs, but other infrastructure costs will be low. Estimated investment costs for the project are NOK 695 mill, or 11.6 MNOK/MW, where NOK 32.8 mill relates to grid investments. As the project is relatively small it requires a short development period and the investment decision can therefore be delayed until 2018.

3.5.5 Investment Cost

The distribution of the investment cost for Sarepta's projects are presented in Table 6. As seen from the table the most expensive project is Sørmarksfjellet, which is related to the large size of this project. Frøya is the smallest project and also requires the lowest investment cost.

Cost distribution	Ytre Vikna	Frøya	Sørmarksfjellet	Harbaksfjellet
Turbines	844 880	492 676	1 115 009	697 891
Foundation	64 000	37 324	130 925	72 536
Construction	179 200	80 000	244 880	151 643
Internal grid	64 000	52 200	80 971	65 776
External grid	38 400	32 800	56 491	23 256
Projects management	76 800	-	41 600	32 891
Other	12 800	-	15 452	14 533
Total	1 280 000	695 000	1 685 328	1 058 526
Total per MW	10 000	11 583	11 704	11 761

Table 6: Cost distribution for the Sarepta projects in 1000 NOK.

3.6 Qualitative Analysis

3.6.1 Risk

In the following the projects are compared in relation to different risk factors for wind parks. The main risk factors identified are the price of electricity and certificates, production from the wind park (Prod), investment cost (IC), operation and maintenance cost (O&M), grid cost (GC) and concession (Con). The risk associated with the real production cost, which includes grid loss and feed tariffs, is related to the production and is therefore not considered as a separate risk factor. To compare the risk for the projects the relative risk is ranked as high, medium, low or zero and is presented in Table 7.

The revenues of the projects depend on the future electricity price. The project risk related to the uncertainty of future electricity prices is considered high and the projects are equally exposed to this uncertainty.

The revenue of the projects are also highly dependent on the production of the wind park. The uncertainty in the production is related to the actual capacity that is developed and the wind in the operation period. Ytre Vikna is not very flexible in the capacity that will be installed, and will most likely develop the full 60 MW, and the wind measurements are very representative resulting in less production risk for this project. The other projects either have more uncertainty in the installed capacity or less representative wind measurements.

The profitability of the projects depends on the investment cost. The uncertainty in the investment cost is considered low for Ytre Vikna as they already have contracts in place, and is relatively low for Frøya as Sarepta has received offers from turbine suppliers. The profitability for the rest of the projects are exposed to the market risk in the price of turbines.

The uncertainty in the O&M cost is related to the complexity of the site and wind conditions. Sørmarksfjellet and Stokkfjellet are complex sites with high turbulence and has higher O&M cost risk. Ytre Vikna and Harbaksfjellet are high load sites which also results in relatively high risk. Frøya, however, is a low complexity site with less wind and has lower O&M cost risk.

None of the projects have contracts for sharing of grid cost and are therefore relatively exposed to uncertainty in this cost. Sørmarksfjellet potentially has to pay for an extra transformer in the Roan substation which increases this risk slightly. The possibility of sharing the grid cost required for Stokkfjellet is considered unrealistic and it is quite certain they have to develop the grid themselves. Thereby the cost for the required grid is quite certain compared to the other projects.

Four of the projects have received final concession from OED, resulting in no uncertainty in the application process. Stokkfjellet, however, is waiting for the final approval from OED and is subject to some risk of not receiving final concession.

Project	Price	Prod	IC	O&M	GC	Con
Ytre Vikna II	H	L	L	L/M	M	0
Frøya	H	L/M	L/M	L	M	0
Sørmarksfjellet	H	M/H	M	M	M/H	0
Harbaksfjellet	H	M	M	L/M	M	0
Stokkfjellet	H	M	M	M	L	M

Table 7: Qualitative risk assessment for the projects. Risk classified as high (H), medium (M), low (L) or zero (0) risk.

3.6.2 Profitability

The expected profitability of the projects are evaluated compared to a benchmark for the cost of energy for a representative project from a report by NVE (NVE, 2015). The cost of energy depends on production quantity, investment cost, O&M cost, lifetime and financial costs. If we assume that the lifetime and financial costs are the same for all the projects we can evaluate them by comparing the production and investment costs. It was shown in the report by NVE that the cost of energy had highest sensitivity to these two factors, so the assumption above is considered reasonable. The production, or number of full load hours, were shown to be the most important factor when calculating cost of energy, and will be emphasized.

The full load hours and investment costs for the different projects are presented in Table 8. The different projects are compared to the data for the benchmark project stated in Table 1. Figure 9 shows the profitability and uncertainty for the five projects. The vertical line represents the average profitability of the representative benchmark project. The relative profitability of the projects, evaluated using the cost of energy, are presented as horizontal lines accounting for different discount rates. The relative uncertainty is based on the information in Table 7 presenting the relative risk associated with each project.

Ytre Vikna and Harbaksfjellet have roughly the same expected full load hours. But Ytre Vikna has lower investment cost and is therefore expected to be more profitable. Frøya is very similar to the benchmark project in terms of the full load hours and investment cost. The project is potentially more expensive but has better production potential. Sørmarksfjellet and Stokkfjellet are the least attractive projects it terms of the profitability, because they have either lower production potential or higher investment costs compared to the benchmark project.

Project	Full load hours	Investment cost
Benchmark	3200	10.25
Ytre Vikna II	3500	10.00
Frøya	3383	11.60
Sørmarksfjellet	3250	11.70
Harbaksfjellet	3474	11.76
Stokkfjellet	3110	11.20

Table 8: Full load hours and investment cost (MNOK/MW) for each project.

3.6.3 Flexibility

The managerial flexibilities related to the uncertain decisions are evaluated for each project. The possible managerial flexibilities considered for the projects are timing of the initial investment decision, adaption of the size of the project and the choice of technology. Adaption of size is related to the possibility of extending the planning area and choose size without regards for grid capacity. It is also related to the possibility of reducing the installed capacity to reduce wake loss. Flexibility in choice of technology is related to the possible technologies and sizes of turbines that can be considered and whether one can choose freely among these.

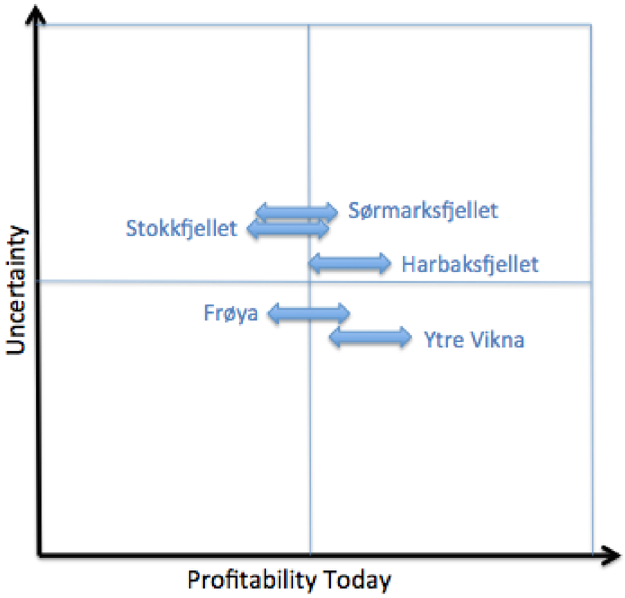


Figure 9: Qualitative assessment of profitability and uncertainty for each project.

All projects need to be in operation before 2020. As Ytre Vikna and Sørmarksfjellet require longer development processes the investment decision has to be made by 2017. The other projects also require relatively long development processes and the investment decision is required by 2018. None of the projects are mature enough to reach an investment decision before 2016 at the earliest, resulting in relatively little flexibility in terms of the timing of the initial investment decision.

Frøya has an upper limit of 60 MW due to the grid capacity and the limited planning area. Ytre Vikna has more flexibility in capacity to be installed as anything up to 179 MW can be developed. Because Ytre Vikna and Frøya are class 2 turbine sites, there is less flexibility in the technology choice for these projects. Also, Ytre Vikna has already signed contracts for the turbines and Frøya has longer time to decide on the turbine technology, making Frøya more flexible than Ytre Vikna in the technology choice. Sørmarksfjellet and Harbaksfjellet can potentially use both class 1 and class 2 turbines and are therefore very flexible in the technology choice. The managerial flexibilities for each project are summarized in Table 9.

Project	Investment decision	Optimal size adaption	Technology choice
Ytre Vikna	2017	M/H	L
Frøya	2018	L	M
Sørmarksfjellet	2017	M	H
Harbaksfjellet	2018	M	H
Stokkfjellet	2018	L	M

Table 9: Qualitative assessment of the flexibility in each project.

3.6.4 Synergies

As synergies between projects can lead to reduced costs the potential synergies for the projects considered are investigated. Harbaksfjellet is close to the two Fosen Vind projects Storheia and Kvenndalsfjellet. It can be expected that this projects can take advantage of synergies in the development phase and for the O&M cost if the same turbine technology is chosen. It is also expected that Sørmarksfjellet and Harbaksfjellet can be realized with the same turbine technology and to some degree take advantage of synergies in the O&M cost. Sørmarksfjellet is however quite isolated and does not necessarily benefit from any synergies in the development phase.

3.7 Investment Strategy

When evaluating investment opportunities the underlying strategy of the company considering the investments are important. The investments should be in accordance with the company's long term goal to sustain competitive advantage. An important strategic choice for TrønderEnergi when choosing investments is whether they want to focus on the industrial role of operating the wind parks or if they want to focus on financial investment opportunities.

TrønderEnergi is currently an important industrial participant in power production in Central Norway and has knowledge and skills that can be used to develop and operate these wind parks. If TrønderEnergi chooses to invest in projects that they will be able to develop and operate themselves, they can benefit from the additional knowledge and experience. These intangible resources might be strategically important for an industrial company in the long run.

An important strategic goal is to maximize the return on all future investments. Possible strategic advantages for future investments as well as current synergies for operating multiple wind parks must be weighed against maximizing current return. An investment in Fosen Vind would strictly be a financial investment for TrønderEnergi and the potential high returns must be weighed against the potential operational benefits of other investments.

Another consideration that relates to the business strategy is to what degree TrønderEnergi will operate the wind parks. As mentioned previously operation and maintenance of wind turbines can either be the responsibility of the wind park developer or the turbine manufacturer. This choice is of strategic relevance because to take responsibility of operation early in the project one might benefit from learning and improving the company's competence in the area. This might reduce the cost of operating future wind park investments. This consideration might also be relevant if choosing to invest in the Sarepta projects. The joint responsibility for operation between two industrial companies might reduce the degree of operational learning for each company compared to operating the project alone. The operation of the projects will be handled by the parent company Sarepta and it is uncertain if this will result in the same learning effect for TrønderEnergi compared to operating projects themselves. In the following different investment scenarios for TrønderEnergi will be presented depending on the strategies pursued.

In the following it will be assumed that a decision to invest in the Fosen Vind projects is made and that the central grid will be built. TrønderEnergi's investment decision, given this assumption, results in a very interesting and complicated situation which this thesis aims to find an optimal solution for. It is also considered very likely that the Fosen projects will be developed and this situation is therefore the most interesting to investigate further. Given this assumption the remaining decision for TrønderEnergi is the equity interest they wish to hold in Fosen Vind and which of the remaining wind power investments they should pursue. The investment opportunities with the required investment capital for different ownership in the investments are summarized in Table 10.

Depending on TrønderEnergi's long term strategy, of having either a more financial or a more industrial focus, five different investment strategies are suggested. The two extremes are the cases considered to be the most financial or the most industrial. On the one hand TrønderEnergi could maintain its position in Fosen Vind and optimize its selection of wind power projects by considering to jointly develop projects in Sarepta. On the other hand TrønderEnergi does not invest in Fosen and prioritizes developing projects that they own and control themselves to achieve full industrial responsibility and benefits of their own project portfolio. For this investment strategy it is assumed that TrønderEnergi has the option to buy the remaining 50 percent in the Sarepta projects, thereby owning 100 percent of these projects. The five investment strategies are presented in Figure 10 together with the resulting investment capital available for investment in the relevant wind power projects. For each investment strategy the figure presents the percentage interest TrønderEnergi holds in Fosen Vind and whether they consider owning 50 percent or 100 percent of the Sarepta projects. The available investment capital for investment in the remaining wind power projects is calculated by subtracting the equity required for the Fosen Vind investment, grid investments and Skomakerfjellet wind park and adding the debt coverage.

Sarepta Projects	100%	50%
Ytre Vikna	1 280	640
Frøya	695	348
Sørmarksfjellet	1 685	843
Harbaksfjellet	1 058	529
TrønderEnergi's Project		
Stokkfjellet	895	
Financial Investments	10%	14.5%
Fosen Vind	700	1 015
Grid Investments		
Grid companies	1 500	
Re-investments in existing grid	250	

Table 10: TrønderEnergi's share of investments with required investment cost (MNOK).

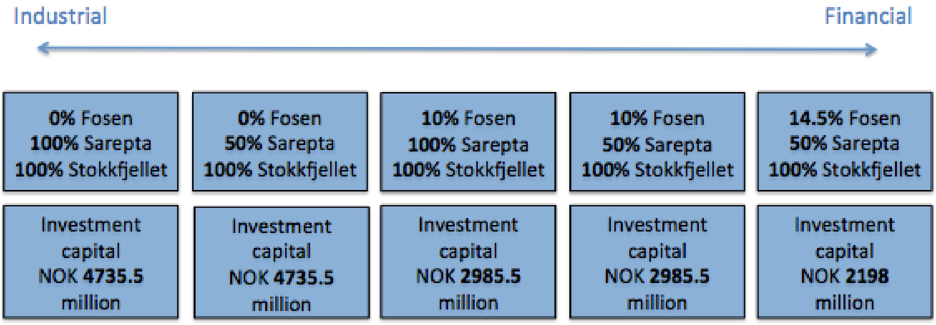


Figure 10: Investment strategies for TrønderEnergi.

4 Literature Review

The idea of viewing investments in real assets analogous to financial options was first proposed by Myers in 1977. The idea gradually developed due to the growing concern towards using the traditional approaches for valuing investment opportunities in uncertain environments. Indeed, Dixit and Pindyck (1994) showed that when future cash flows are uncertain and the decisions are partially irreversible, the simple net present value method does not properly reflect the true value of an investment project. These characteristics can therefore profoundly affect the investment decision.

The work of Black and Scholes (1973) and Merton (1973) led to an explosion of research in pricing of derivative products, which could in turn also be utilized to evaluate investments in real assets. The main approaches used to find the real options value of projects can be identified as either discrete or continuous time. Cox et. al. (1979) developed a discrete binomial options approach using risk neutral valuation and Rendleman and Bartter (1979) developed a binomial lattice valuation process. The continuous time approaches are based on closed form equations. Black and Scholes (1973), Margrabe (1978), Geske (1979) and Carr (1988) developed closed form equations for option valuation in a continuous time context, given different sets of assumptions. To derive the closed form equations, stochastic differential equations must be solved. For most real life applications, the equations are too complicated to be solved analytically and you have to refrain to numerical methods or Monte Carlo simulations. Wilmott (1998), Hull (2000) and Neftci (2000) present introductions to stochastic differential equations, finite-difference methods and Monte Carlo simulation.

Real options application papers have addressed several areas of industry to investigate the applicability, including biotechnology, manufacturing and inventory, natural resources, research and development and technology (Miller and Park, 2002). Lint and Pennings (2001) recognize that projects fall into one of four categories, depending on the expected payoff and volatility, and suggest that projects with high volatility in the cash flow would benefit from real options analysis (ROA). They further suggest that the method should be used as a complement to DCF tools. In order to perform a ROA the DCF approach is often needed to calculate inputs for the option value. The DCF analysis should be performed first, followed by the more labor intensive ROA if necessary. Park and Herath (2000) divided investments according to varying degree of uncertainty, and found that the higher the uncertainty the more ROA will impact the decision. For application of real options valuation in renewable energy investments, Deng and Oren (1995) suggests that the DCF approach is not the best choice because it does not model high volatility and fails to include managerial flexibility. Lee and Shih (2010) shows, through analytical results, that the traditional approach underestimates the policy value of wind power, and that real options analysis more accurately reflects the actual value. Lee (2011) values investment in wind power from the real options perspective and found that the value of the investment increases with underlying price, volatility, time to maturity and risk free rate and decreases with exercise price (investment cost). Munos et al. (2009) present a decision making tool based on real options for investment in wind energy, that considers both the volatility in market prices and wind regimes. Fleten and Ringen (2006) present a real option model to calculate the trigger level for investment in renewable electricity production in Norway, by considering the electricity price and certificate price as stochastic factors.

The stochastic behavior of a project is central in valuing the real option. Dixit and Pindyck (1994) found that the trigger level to initiate investment depends heavily on the stochastic model chosen. Miller and Park (2002) points out that the volatility is the most important variable when valuing uncertain projects using ROA and argue that projects based on commodities can utilize the price patterns in the underlying commodity to estimate the volatility in the project. Fleten et al. (2007) and Bøckman et al.(2008) applies the assumption that projects that produce electricity have the same volatility as the electricity price for real options valuation of renewable energy investments.

As mentioned above, the choice of the stochastic model is important in a real option investment analysis. In the literature commodity prices are often described by a mean reverting process, because of the tendency of the price to revert back to its long term mean. Schwartz (1997) presents a one factor model for mean reversion in the price of commodities where the logarithm of the price follows a mean reverting process, and demonstrates that this model appropriately describes the development in the price for several commodities. Lucia and Schwartz (2002) presents a mean reverting model with seasonal variations, based on historical spot data in the Nordic power market. Davison et al. (2002) developed a hybrid, mean reverting switching model and Barlow (2002) presents a non-linear Ornstein-Uhlenbeck model for the spot price. Jablonska et al. (2011) presents a jump diffusion process, that is superimposed on a mean reverting Ornstein-Uhlenbeck model, calibrated with hourly prices from the Nord Pool spot market.

Real options analysis has proven to be a good valuation tool for projects in uncertain environments, leading to the issue of how to use the real option analysis to address capital budgeting situations where several projects are compared. Several optimization models for capital budgeting have been proposed to deal with the uncertainty in the cash flows of projects. Korhonen (2001) discusses the Dynamic Scenario Optimization (DYSCO), which derives a scenario tree from the combination of all possible uncertain outcomes. A new way of approaching capital budgeting, that includes the possibility of postponing the investment decision, was proposed by Meier et al. (2001). Up to our best knowledge they are the first to propose an optimization model, beyond a conceptual level, for capital budgeting decisions that includes the flexibilities in investments by means of option based project valuation. Meier et al. (2001) developed two optimization models for projects that can be deferred indefinitely and whose values develop stochastically over time. The first model is an option value maximization approach, that simply maximizes the value of the portfolio based on the real option value for each project. The model does not assure that the projects chosen by the model are invested in during a specified budgeting period, and should only be applied when the budgeting period is indefinite. The second model is a dynamic approach based on a number of scenarios for the projects' value. The model suggests scenario dependent portfolios that maximizes the overall value of projects that have the potential of being commenced within a certain budgeting period. Meier et al. (2001) apply the optimization models for projects when the development in the projects' value are assumed to follow Geometric Brownian motion.

Literature related to the use of real option analysis when selecting among several projects in capital budgeting decisions in wind power is insubstantial. Kumbaroğlu et al.(2008) introduce a study that integrates learning curves for renewable power generation into a dynamic programming formulation using real options. The model evaluates a set of investment alternatives on a year-by-year basis and thereby takes into account the flexibility to delay an irreversible investment. The capabilities of the model allow for the planning of future investment decisions. However, the focus of the study is that the flexibility to delay an irreversible investment expenditure can affect the diffusion prospects of renewable power generation technologies. The objective is to guide policy planning in the electricity supply sector. The model does not account for cost differences and scale effects and should therefore not be directly used as a guide for investment planning decisions. Other researchers have proposed models for project selection in wind power, without the use of real options approaches. Roques et al. (2010) demonstrate how the Mean-Variance Portfolio theory can be used to identify optimal cross-country portfolios that minimize the total variance of wind production. Geographic diversification smooth out the fluctuations in generation and reduce system balancing and reliability costs. Lee et al. (2009) develops critical success criteria for selection of wind parks. Factors that affect the success of wind parks (benefits, opportunities, costs and risks) are analyzed by considering experts' opinions and a multi-criteria decision-making model is developed.

In this thesis the previous work done in real options in wind power will be extended by considering how the real options analysis can be used to optimize project selection in a wind power portfolio. Even though real options represents a good tool for valuing investment opportunities in wind power, there is not much literature on how dealing with several projects affect the investment decision. The purpose of this thesis is to compare the real options value for several projects by incorporating the real options approach into a capital budgeting decision model. In reality the projects can include different real options. However, in this thesis only the option to delay the investments will be considered. The work will extend the dynamic capital budgeting model developed by Meier et al. (2001) to account for the mean reverting properties of the underlying commodity in wind power projects.

5 Theory Review

5.1 Capital Budgeting

Capital budgeting is the allocation of resources among investment projects on a long term basis (Trigeorgis, 1996). The financial goal of a company is to maximize the wealth of its owners by maximizing the value of the company. It is therefore the goal of capital budgeting to allocate resources to maximize the value of the company's projects. Traditional capital budgeting rely on the DCF method to select the projects that maximize the portfolio value under the restrictions of a total budget. The DCF method was developed by Fisher (1907, 1930) and has since then been adopted as a valuation tool for financial and real assets and used as the basis for investment decisions. The method is based on the net present value (NPV) and requires relatively simple calculations. Another widely used tool for selecting investment projects is the performance measure return on investment (ROI), which indicates the benefit of an investment compared to the required investment cost.

5.1.1 Static Net Present Value

The NPV is calculated using Equation 2, where I_0 is the initial investment cost, T is the number of operating years, FCF_t is the free cash flow in year t and r is the required rate of return.

$$NPV = I_0 + \sum_{t=1}^{T} \frac{FCF_t}{(1+r)^t}. \tag{2}$$

Usually the investment cost and the time horizon are typically known and the cash flows and discount rate must be forecasted and estimated. The free cash flow for each time period can be calculated as; (earnings before interest and tax) × (1- tax rate) + (depreciation and amortization) - (change in working capital) - (capital expenditure). The discount rate used should reflect the investors required return and is called the cost of capital (Koller et al., 2010). It is the expected rate of return an investor can earn by undertaking other investments with the same risk. It is common to use the weighted average cost of capital (WACC) for discounting the cash flows of a project. This parameter differ among companies and projects. As the company themselves has the best knowledge about the capital structure and investor's risk preferences, the parameter should be estimated internally.

The simple NPV rule states that one should invest in all project with a positive

NPV and discard those with a negative NPV. In addition, a project should not be undertaken if it prevents the undertaking of other projects with higher NPV. This results in the importance of capital budgeting when selecting projects. Since companies are subject to resource constraints they can not invest in every project with a positive NPV, as undertaking one project might require resources that could be better used for more valuable projects.

The NPV approach to capital budgeting is widely adopted by practitioners for evaluating investment opportunities because of its relatively simple and intuitive calculations, that are understandable for managers and decision makers. The method is widely accepted, economically rational, considers the time value of money and gives clear, consistent decision criteria for projects selection (Mun, 2006). There are however, several issues concerning the NPV calculations, of which the most important is related to the expected stream of profits. In the NPV approach the cash flows are deterministic and need to be estimated, leading to the issue of how to correctly estimate these values. In reality the cash flows are stochastic and the NPV approach can not reflect managerial flexibility in investment decisions and underestimates the opportunity and actual value of the investment by not considering the rapidly changing investment climate(Trigeorgis, 1996). Other issues are how to treat inflation and what discount rate to use (Dixit and Pindyck, 1994).

5.1.2 Return On Investment

The return on investment (ROI) evaluates the efficiency of an investment and can be used to compare the efficiency of several investment projects. The measure considers the profit of an investment in relation to the capital required (Investopedia, 2015);

$$ROI = \frac{earnings \quad from \quad investment - cost \quad of \quad investment}{cost \quad of \quad investment}. \tag{3}$$

The ROI can be an additional tool in project selection for an investment portfolio by prioritizing the projects with the best ROI.

5.1.3 Portfolio Optimization

The traditional capital budgeting approach, of selecting the projects that maximizes the net present value of the portfolio under the restriction of a budget, was first formulated as a optimization model by Weingartner (1974). Weingartners model is stated in Equations 4 to 6. The objective function to be maximized is given in Equation 4, where $a_{j,t}$ is the cash flow for project j in year t and r is the discount rate. The budget restriction is given in Equation 5, where $b_{j,t}$ is the investment cost in year t and M_t is the budget in year t.

$$max \sum_{j=1}^{J} \sum_{t=0}^{T} \frac{a_{j,t}}{(1+r)^t} x_j \tag{4}$$

$$s.t \quad \sum_{j=1}^{J} b_{j,t} x_j \leq M_t \quad t = 0, ..., T \tag{5}$$

$$s.t \quad x_j \leq 0 \quad j = 1, ..., J \tag{6}$$

The optimization model is a so called knapsack problem. Many solution methods and algorithms for such problems have been developed, Martello and Toth (1990) present several of them.

5.2 Real Option Theory

5.2.1 Investment under uncertainty

According to Dixit and Pindyck (Dixit and Pindyck, 1994) most investment decisions are subject to three characteristics of varying degree; the decision is partially or completely irreversible, there is uncertainty about the future rewards of the investment and there might be some flexibility in the timing of the investment. Traditional theory does not capture the implications of this irreversibility, uncertainty and choice of timing. If we assume the project value follows a stochastic process, with volatility greater than zero, and that the decision is at least partially irreversible, it can be proven that the simple NPV rule underestimates the value of projects. Dixit and Pindyck (1994) presents this proof for a project that follows Geometric Brownian motion, but the validity of the result is general.

An investment decision can be viewed as an option, analogous to a financial option, because the firm has the right but not the obligation to buy the asset at some time in the future. When the firm decides to invest this exercises the option and they loose the possibility of waiting for more information that might affect the desirability of the project. The value of the real options approach is related to the uncertainty and the managerial flexibility, i.e. the decision maker's ability to affect the future uncertain cash flows of a project in a way that increases the expected return or reduces the expected loss (Miller and Park, 2002). From a real options perspective Lee (2011) found that the value of investments in wind power increase with volatility and time to maturity.

In traditional theory the investment rule is to invest whenever the present value of the revenue flow is equal to or exceeds the costs. According to McDonald and Siegel (McDonald and Siegel, 1986) this rule is incorrect because it ignores the opportunity cost of making the investment now and giving up the option to wait for new information. When the future value of the project, V, is uncertain, this opportunity cost should be included in total costs of investing, resulting in an investment rule where the value of the project that leads to optimal investment is at least some V^* greater than the investment cost. To account for this opportunity cost the value of investing today must be compared with the value of investing at all possible times in the future. If the present value of investing in the future is higher than the present value of investing today this means the investment decision should be deferred (McDonald and Siegel, 1986). In these situations the value of a project is higher than the net present value because the option to wait adds additional value. The value of investing in the future can be accounted for by solving the problem using dynamic programming.

Dynamic programming

Dynamic programming is a systematic method to compare values of immediate decision and decisions in the future by splitting the decision into the immediate choice and the continuation value of all the remaining decisions (Dixit and Pindyck, 1994). The procedure is to work backward from the optimal decision at the last decision point and optimize all the preceding decisions with backward induction. An important idea in dynamic programming is Bellman's principle of optimality, which states that the optimal value of all the remaining choices are subsumed in the continuation value and only the immediate decision must be optimized. This principle results in the Bellman equation which can be used to evaluate the optimal immediate decision of investing now or waiting (Dixit and Pindyck, 1994).

Bellman equation

The Bellman equation is a decomposition of choices into the immediate choice and the continuation value, which subsumes all remaining choices. In the sense of the investment opportunities discussed in this report this is equivalent to the immediate choice of investing or waiting, when the value of the decisions to invest at all possible times in the future are summarized in the continuation value.

$$\rho F(x,t) = max_u \left(\Omega(x,u,t) + \frac{1}{dt}\mathcal{E}(dF) \right). \tag{7}$$

In economic terms Equation 7 demonstrates the return equilibrium condition. The left side is the return required for holding asset F with discount rate ρ, Ω is the immediate payout or dividend and the last term is the expected rate of capital gain (Dixit and Pindyck, 1994). In the case of a simple invest or wait decision there is no profit if we wait and there is no capital gain if we invest. It is assumed that if we invest we immediately receive the value of the underlying project and we loose the possibility that the project might increase in value. The Bellman equation for a case like this becomes:

$$F(V,t) = max \left(V - I, \frac{1}{1+\rho dt}\mathcal{E}(F(V+dV, t+dt) \mid V) \right), \tag{8}$$

where F is the value of the option to invest, V is the value of the project, I is the investment cost and the last term is the continuation value.

5.2.2 Stochastic processes

In the Bellman equation in the previous section the value of the project, V, is considered uncertain and is often assumed to evolve stochastically through time. To solve the investment problem using dynamic programming we need to know the process for the possible development of the value of the project. This value depends on a number of project specific parameters and some uncertain factors of the project. Some of the uncertainties relevant for wind power investments are presented in Section 2.2. Some of these uncertain factors of the value can also be assumed to follow stochastic processes. In this section the theory on stochastic processes relevant for this thesis is reviewed.

Process for asset prices

Asset prices are often assumed to follow Ito processes, which are stochastic processes that evolve in continuous time and can be generalized by the following equation:

$$dX = a(X,t)dt + b(X,t)dZ, \tag{9}$$

where the drift term a and the volatility term b can be functions of X, and dZ is an increment of a Wiener process (McDonald, 2014). The functions a and b in Equation 9 can be adjusted to fit the properties of different kinds of assets. Common modifications are Arithmetic Brownian motion, with constant drift and volatility, Geometric Brownian motion, with drift and volatility proportional to the mean and standard deviation, and the Ornstein-Uhlenbeck process, which incorporates mean reversion by modifying the drift term; the price will have negative drift when X rises above the mean and positive drift when X falls below the mean.

Process for commodity prices

There are strong arguments that commodity prices should follow a mean reverting process (Schwartz, 1997). An increase in the spot price is typically followed by an increase in supply and therefore the price decrease back to the commodity's long-term marginal cost of production. Similarly a reduction in the price is an incentive to stop production when it becomes unprofitable resulting in decreased supply and an increase in the price. Prices that deviate from the long-term average cost will have a higher probability of moving towards the long-term average than away from it. The most common mean reversion models are mean reversion to the long term mean of the price and mean reversion to the long term logarithm of the price (Skorodumov, 2008). Volatility of prices tend to be higher when the price is high and lower when the price is low. A simple way to account for this non-constant volatility is to model the logarithm of the price. The logarithm of the price with constant volatility gives some modeling advantages, but results in a price that has higher volatility for high prices (Guthrie, 2009).

Schwartz (Schwartz, 1997) represents a one factor model of mean reversion for the price S_t by the equation:

$$dS_t = \alpha S_t(\mu - \ln S_t)dt + \sigma S_t dZ_t, \tag{10}$$

where α is the mean reversion rate, μ is the mean reversion level, σ is the standard deviation of the process and dZ is the increment to a standard Brownian motion. If we apply Itos lemma for $X_t = lnS_t$ on this equation we get the arithmetic Ornstein-Uhlenbeck process for the logarithm of the price:

$$dX_t = \alpha(\mu^\star - X_t)dt + \sigma dZ_t, \tag{11}$$

where $\mu^\star = \mu - \sigma^2/2\alpha$.

Process for electricity prices

In the literature it is common to differentiate between short-term and long-term development in the electricity price. The factors that influence the price in the short-term are for instance the weather, the short run availability of production capacity and transmission bottlenecks (Schwartz and Smith, 2000). The long-term factors include issues such as demand growth uncertainty, regulatory uncertainty, and fuel and substitute energy prices. In the short term, the electricity price might experience seasonal variations and spikes. Lucia and Schwartz (2002) and Jablonska et al. (2011) presents models that describe such characteristics for the Nordic power market. Even though these more advanced models have a better fit to the data, it is not always better to use them to describe the electricity price for real options analysis. When considering long term investment projects the expected value of the short term variations are zero. For the analysis in this thesis the electricity price will be assumes to follow a one factor model of mean reversion to the long term logarithm of the price.

5.2.3 Solving the investment problem

Option pricing methods require that there exists a marketed security or a portfolio of securities that replicate the payoffs of the asset. For real asset projects such a replicating portfolio does not exist (Brandao and Dyer, 2005). Copeland and Antikarov (Copeland and Antikarov, 2003) suggest that the present value of the projects revenue without options is the best unbiased estimator of the market value of the project. This assumption is called the Marketed Asset Disclaimer (MAD), and assumes that the project itself to be the underlying asset of the replicating portfolio. As a result the option can be valued using traditional option pricing methods. To solve the real option problem, using dynamic programming, the estimated values of the underlying project at all times in the future are required. The development in the underlying project value can be approximated by stochastic processes. Miller and Park (Miller and Park, 2002) argue that commodity based projects can utilize the development in the underlying commodity to estimate the volatility of the project. The stochastic development in the expected revenue of a project can therefore be assumed to follow the same process as the price. In this thesis the values of the projects will be assumed to follow the same process as the price, where the log price is described by a mean reverting model.

Guthrie (2009) presents a discrete modeling approach for solving simple timing options where a constant investment cost gives some future reward that depends on a stochastic process. The method is based on the assumptions that arbitrage opportunities do not exist, which allows us to work as if all investors are risk neutral and would attach the same value to a risk free cash flow and a risky cash flow with the same expected value. The implication of the assumption is that the market value of the project can be found by discounting at the risk free rate using the risk neutral pricing formula. In this approach the future is considered as discrete time steps and a binomial tree for the possible evolution of the future value of a project is created. Following the approach, and the fact that the log price follows a mean reverting model, for each subsequent period in the tree the log price can either increase or decrease by $\sigma\sqrt{\triangle t}$, depending on whether an up or down move occurs. σ is the volatility in the mean reverting process and $\triangle t$ is the time step in the binomial tree. Taking exponentials result in the constant size of up and down movements in the price throughout the tree;

$$U = e^{\sigma\sqrt{\triangle t}} \quad \text{and} \quad D = e^{-\sigma\sqrt{\triangle t}}. \tag{12}$$

Next, probabilities for the up and down movements in each node of the tree is required. The probabilities are calculated in such a way that the expected value of the change in the log price over the next $\triangle t$ is equal to the value that is implied by the stochastic process. Proof and derivation of the equations for the probabilities are found in the book by Guthrie (2009) and the resulting equations, representing the probability of an up move in node (i,n), are presented below;

$$\pi_{i,n} = \theta_{i,n} - \frac{(E(R_m) - R_f)\beta}{U - D}, \tag{13}$$

with

$$\theta_{i,n} = \begin{cases} 0 & \text{if } \ \frac{1}{2} + \frac{(1-e^{-\alpha\triangle t})(\mu^\star - P_{i,n})}{2\sigma\sqrt{\triangle t}} \leq 0 \\ \frac{1}{2} + \frac{(1-e^{-\alpha\triangle t})(\mu^\star - P_{i,n})}{2\sigma\sqrt{\triangle t}} & \text{if } \ 0 > \frac{1}{2} + \frac{(1-e^{-\alpha\triangle t})(\mu^\star - P_{i,n})}{2\sigma\sqrt{\triangle t}} > 1 \\ 1 & \text{if } \ \frac{1}{2} + \frac{(1-e^{-\alpha\triangle t})(\mu^\star - P_{i,n})}{2\sigma\sqrt{\triangle t}} \geq 1 \end{cases} \tag{14}$$

where σ, α and $\mu^\star$ are parameters of the Ornstein - Uhlenbeck process for the log price described by Equation (11), $E(R_m)$ is the expected return of the market, R_f is the risk free rate, β is the project's beta and $P_{i,n}$ is the electricity price in node i at time step n.

5.3 Capital Budgeting under Uncertainty

Based on the net present value of projects, the capital budgeting decision is to select the projects that maximizes the current present value of the portfolio and at the same time does not exceed the company's budget. However, the forecasting of cash flows for the net present value calculations are subject to error. Typical ways of handling this uncertainty is to conduct sensitivity analyses to determine the greatest source of uncertainty, scenario analysis to create the best and worst case scenario and a probabilistic analysis to assess the likelihood of each potential outcome. But even with a thorough analysis of the downside of the uncertainty it is not possible to hedge this uncertainty. The method does not capture the flexibility in the decision making process. When the fact that the actual cash flows of a project will differ from the originally expected cash flows is included, as well as managerial flexibility to react to these changes in the underlying assumptions by applying dynamic programming, the traditional optimization model must be adjusted. A new way of approaching capital budgeting, that includes the possibility of postponing the investment decision, was proposed by Meier et al. (1999). The investment decision now includes choosing the portfolio of options that results in the highest value given the budget restrictions and to choose which projects to invest in immediately and which projects to postpone. The model is a dynamic approach that, based on a number of scenarios, suggest scenario dependent portfolios that maximizes the overall value of projects that have the potential of being commenced within a certain budgeting period.

5.3.1 Dynamic Capital Budgeting Model

The dynamic capital budgeting model is based on generating several scenarios for the project values and finding the optimal portfolio for each scenario. To define this scenario based optimization model a set of scenarios that represent a realistic subset of all scenarios is required. The binomial option pricing (BOP) model, mentioned in Section 5.2.3, is often used to approximate values that depend on stochastic processes. In this optimization model the BOP model is used to generate the scenarios for the value of the investments. By using dynamic programming, to find the option values for all time steps, the threshold values, V^*, that makes investment optimal for each time period, can be calculated. The option to invest in a project should only be exercised if the value of postponing the investment is less than the value of investing immediately. With this threshold value, a value function defining the value of investing in each state can be defined;

$$g(j,k) = \begin{cases} p_{j,k}(V_{j,k} - I_j) & if \quad V_{j,k} \geq V_j^* \\ 0 & otherwise. \end{cases} \tag{15}$$

where $g(j,k)$ is the value of investing in project j in state k, $p_{j,k}$ is the probability of ending up in this state, $V_{j,k}$ is the value of the project in the state, I_j is the required investment cost and V_j^* is the threshold value that makes investing in the state optimal. This value function assures that investment is not undertaken if it is possible and optimal to wait another period by setting its value to zero for these situations. Otherwise the value of investing is equal to the projects net expected value.

When the scenarios for the project values in the future are generated using the stochastic development, the scenarios for the future value of the investments, generated by the value function $g(j, s_i(t))$, can be used in the capital budgeting model. The value function makes sure that each scenario dependent portfolio only contains projects that are going to be exercised, because the value of investments are zero when it is not optimal to invest in the project in a given state. The optimization model formulated by Maier et al. is presented below.

$$max \sum_{j=1}^{N} \sum_{t=0}^{T} \sum_{i=1}^{S} e^{-t \cdot \Delta t \cdot r} g(j, s_i(t)) x_{j,s_i(t)} \tag{16}$$

$$s.t \quad \sum_{t=0}^{T} \sum_{j=1}^{N} I_j x_{j,s_i(t)} \leq B \quad i = 1, ..., S \tag{17}$$

$$s.t \quad \sum_{t=0}^{T} x_{j,s_i(t)} \leq 1 \quad i = 1, ..., S \quad j = 1, ..., N \tag{18}$$

$$s.t \quad x_{j,s_i(t)} \in \{0, 1\} \quad i = 1, ..., S \quad j = 1, ..., N \quad t = 0, ..., T \tag{19}$$

In the optimization model N is the number of projects, T is the number of periods and S is the number of scenarios considered. The factor $e^{-t \cdot \Delta t \cdot r}$ discounts the value function to maximize the present value of the portfolio, where Δt is the length of one time period and r is the discount rate. The decision variables are binary. The constraints in Equation 18 makes sure a project can only be selected once within a certain scenario and the constraint in Equation 17 guarantees that the capital expenditure remains within the given limit for all scenarios, where B is the total capital available.

5.4 Integer programming

A pure integer programming problem is an optimization problem on the form;

$$max \quad cx$$

$$s.t. \quad Ax < b$$

$$x > 0 \quad integer$$

where c is a row vector, $c = (c_1, ..., c_n)$, A is a $n \times m$ matrix, $A = (a_{i,j})$ and b is a column vector, $b = (b_1, ..., b_m)$. The variables to be optimized are the values in the column vector, $x = (x_1, ..., x_m)$ (Conforti et el., 2014). For the optimization model in this thesis the variables will in addition be constrained only to take on binary values, $x \in \{0, 1\}$, and the model will have an additional dimension in c and x Methods for solving such problems include the branch-and-bound method, the cutting plane method and branch-and-cut method (Conforti et el., 2014).

When an optimization model only requires that one of two constraints are satisfied, the big-M method can be used. The formulation of the restrictions for such a case is presented below, where M equals a very large number and y is a binary variable.

$$Dx \leq 0 \quad or \quad Dx \geq S$$

$$\Downarrow$$

$$Dx \leq 0 + M(1 - y)$$

$$Dx \geq S - My$$

These restrictions make sure that one of the two constraints are satisfied by associating the constraints with a large positive or negative constant, and thereby automatically satisfying the other restriction by adding or subtracting this large number (Griva et al., 2008).

6 Traditional Capital Budgeting Approach

In this section a traditional approach to capital budgeting for TrønderEnergi's portfolio is presented. The traditional approach uses the DCF method to select the projects that maximize the portfolio value, subject to the budget constraint. The results are supplemented by evaluating the return on investment for each project and each portfolio selected.

6.1 Static Optimization Model

The net present value of the projects are used as input in a simple static optimization model presented in Equations 20 to 22. Equation 20 is the objective function to be maximized. Constraint 21 makes sure the portfolio selected satisfies the restriction of the budget and the decision variables, x_j, are binary. The model selects the portfolio of projects that maximizes the net present value of the portfolio given the restriction on investment capital. The budget, B, available for each of the investment strategies considered is presented in Figure 10 and the investment cost for each project j, I_j is given in Table 10 in the same section. The model is solved with the optimization software FICO Xpress Optimization Suite, and the code is given in Appendix B. Because of the simplicity there are no computational issues related to solving this model.

$$max \sum_{j=1}^{J} NPV_j x_j \tag{20}$$

$$s.t \quad \sum_{j=1}^{J} I_j x_j \leq B \tag{21}$$

$$s.t \quad x_j \in \{0,1\} \quad j = 0, ..., J \tag{22}$$

6.2 Net Present Value Calculation

The net present value for each project was calculated using Equation 2. The equation is stated below and the results are presented in Table 11.

$$NPV = I_0 + \sum_{t=1}^{T} \frac{FCF_t}{(1+r)^t}.$$

The free cash flow is based on the investment cost and production for each project, stated in the concession applications, and discrete estimates of the electricity price during the operating years. The average O&M cost is based on the estimate from NVE of 15 øre/kWh, but is corrected for project specific factors in collaboration with TrønderEnergi. The total O&M cost is divided into a cost per kWh, a transition loss and a regulation loss. The cost per kWh is set to 10.8 øre/kWh, the marginal loss to 3 percent and the regulation loss to 3.5 percent. For Ytre Vikna the marginal loss is expected to be higher due to a long transition line, and is set to 10 percent. For Sørmarksfjellet and Harbaksfjellet the cost is expected to be lower due to synergies with other projects in the Fosen area, and is therefore set to 9.5 øre/kWh. The exact costs for the projects are not available as public information, but the values used represent good approximations based on the information in the concession applications. The calculations were done using a required rate of return of 8 percent, which is a standard in renewable energy investments for Enova (Gjølberg and Johnsen, 2007), and the depreciation is based on the straight-line depreciation rules currently awaiting approval with the Norwegian government. The depreciation rules are likely to be approved to facilitate wind power investments in Norway.

	Present value operation	Net present value
Ytre Vikna	1 407 420	127 420
Frøya	739 248	44 248
Sørmarksfjellet	1 775 652	90 324
Harbaksfjellet	1 180 511	121 985
Stokkfjellet	908 496	13 496

Table 11: Net present value and value of the operational phase for each project (1000 NOK).

6.3 Return On Investment

The return on investment for each projects is calculated with Equation 3. The equation is repeated below and the results are presented in Table 12.

$$ROI = \frac{earnings \quad from \quad investment - cost \quad of \quad investment}{cost \quad of \quad investment}.$$

Project	ROI
Ytre Vikna	10.0 %
Frøya	6.4%
Sørmarksfjellet	5.4%
Harbaksfjellet	11.5%
Stokkfjellet	1.5%

Table 12: Return on investment for each project.

7 Real Options Based Capital Budgeting Approach

The objective of this thesis is to find a suitable method to select among projects in a wind power portfolio. The empirical analysis of the method is based on data for TrønderEnergi's investment opportunities and the results will be compared to a traditional capital budgeting approach. Traditional capital budgeting approaches are based on the DCF method, which is the method currently used by TrønderEnergi. There are several advantages with the DCF approach, as mentioned in Section 5.1.1, but the approach is based on some underlying assumptions that in some cases might make the method unsuitable. Most importantly it is based on the assumption that either the investment is reversible, and the expenditures can be recovered if the market conditions turn out to be worse than expected, or that the investment is a now or never proposition, and the investment can not be undertaken in the future (Dixit and Pindyck, 1994). For wind power projects the investments are usually neither reversible nor without significant uncertainty and managerial flexibility. On the contrary, most of the investment cost is a sunk cost and the income from operation is highly uncertain due to the commodity risk associated with wind power projects and the uncertainty in the TGC market. Dixit and Pindyck (1994) and Kulatilaka and Marcus (1987), among others, show that in the presence of real options the conventional DCF techniques for project valuation is poorly suited. The flexibility in timing gives managers the ability to reduce the effect of unfavorable outcomes by waiting to receive additional information about the investment environment. This possibility is not considered in traditional DCF methods. Because the DCF method is not considered to properly reflect the project value in uncertain environments it is also assumed that using the DCF method as the basis for capital budgeting might result in selecting the wrong projects for investment. The effect of the real option value on capital budgeting decisions will be investigated by suggesting an optimization model for project selection that accounts for the high uncertainty in wind power. In this section a quantitative dynamic optimization approach is presented and applied to TrønderEnergi's investment opportunities. The approach is based on the BOP model that is used to calculate the value of options to invest in projects when the development in the values of the projects follow a stochastic process. The development of the future project value is discretized to represent possible scenarios which are used as input to the scenario based capital budgeting model.

7.1 Real Options Approach to Project Value

The optimization model used is based on scenarios for the value of the projects determined by the stochastic development in the project value. The project value is the discounted expected value of the operational phase, or in other words the net expected revenue of the project excluding the investment cost. This section presents the real options approach used to evaluate the investment possibilities and generate scenarios for the potential future values of the different investments. First the assumptions are presented followed by the BOP model that is used to approximate the development in the value of the projects at different discrete time steps.

7.1.1 Assumptions

The assumptions for the real options approach are presented below together with argumentation for the different assumptions.

A. 1: *The price of electricity follows a process that reverts to the long term mean.*

The choice of price description is important in an investment analysis. The literature presents many arguments for modeling commodity prices with mean reversion, discussed in detail in Section 5.2.2. As mentioned in Section 5.2.2 there are several more advanced methods for modeling the electricity price. However, when assessing long term power generation projects the short term fluctuations from seasonal variations and spikes are not relevant as the expected value of these fluctuations over time are zero. The use of a simpler mean reverting process was considered sufficient as it should lead to small errors because the value of the investments are not influenced by short term fluctuations such as sudden spikes in the price.

A. 2: *The development in the expected value of the project's revenues follow the same process as the price of electricity.*

The value of the wind power projects will be assumed to have the same volatility and follow the same stochastic process as the commodity that it produces. The projects might have slightly different volatility than the electricity price, but the assumption is widely accepted for real option analysis for projects that produce commodities. The reason is that commodity uncertainty is a major factor to the uncertainty of the value of the underlying project (Miller and Park, 2002). For the projects considered the electricity price is considered to be the largest uncertainty factor for the value of the investments and it seems reasonable that the projects exhibits the same risk as the price. The same assumption has been applied for several applications in the literature (Fleten et al., 2007) (Bøckman et al., 2008).

A. 3: *All costs are constant.*

Assumption 3 is a necessary simplification. The turbine cost is hard to model, due to macro effects and selective pricing to different customers depending on their willingness to pay, and it is beyond the scope of this thesis to account for the uncertainties in costs. In general, companies with investment opportunities in wind power have enough knowledge about the costs related to projects when they are mature. At this point they can request offers from suppliers for the turbines and O&M costs, thereby reducing the uncertainty in these costs.

A. 4: *Assets are priced so that arbitrage opportunities do not exist.*

Assumption 4 enables us to estimate the market value of a cash flow by a replicating portfolio, because the price of any two portfolios that generate the same cash flow must be equal. This assumption allows us to perform our calculations as if all investors are risk neutral, which means they would attach the same value to a risk free cash flow and a higher risk cash flow with the same expected value. The important implication of this assumption is that we can find the market value of the project by discounting at the risk free rate using the risk neutral pricing formula (Guthrie, 2009).

A. 5: *Once invested the present expected value of the project's revenues without flexibility is the best estimate for the market value of the project.*

In practice it is impossible to create a replicating portfolio for the risk neutral pricing formula with the exact same risk profile as the underlying project. However, Assumption 5 provides a method for estimating the market value of the project by using the traditional present value of the project's revenues as a twin security (Copeland and Antikarov, 2003).

7.1.2 Development in Project Value

When we assume that the development in the net expected value of the project's revenues follows the same process as the electricity price, we can use the stochastic model that describes the price to calculate the development in the project value. Because of the modeling advantages, described in Section 5.2.2, it is assumed that the price follows a stochastic model which mean reverts to the long term price logarithm, given by Equation 10.

Estimating Normalized Parameters of the Price Process

The logarithm of the price can be modeled by the O-U process in Equation 11. This equation can be discretized to the first order auto-regressive process, AR(1):

$$X_{t+1} = \beta_0 + \beta_1 X_t + \varepsilon_t, \tag{23}$$

where $\beta_0 = \alpha\mu^\star \triangle t$ and $\beta_1 = 1 - \alpha \triangle t$. Linear regression is performed on monthly electricity data and fitted to this equation by considering the observations of prices through time as a linear relationship of X_{t+1} and X_t and some noise ε_t. The linear parameters are used to estimate the parameters for the O-U process so that they are applicable to situations with arbitrary frequency.

Creating the Binomial Tree for the Development in Project Value

When the parameters for the process of the electricity price are calculated they can be used to create a binomial tree for the development in the value of a project. The method used to create the tree was proposed by Guthrie (Guthrie, 2009), and is presented in Section 5.2.3. The discrete modeling approach for solving simple timing options relies on calculating the size and risk neutral probabilities for up and down movements, based on the parameters for the price process, to create a binomial tree approximating the development. Later these values can be discounted at the risk free rate. The size of the up and down moves in the tree are given by:

$$U = e^{\sigma\sqrt{\triangle t}} \quad \text{and} \quad D = e^{-\sigma\sqrt{\triangle t}}, \tag{24}$$

where the risk neutral probability of an up move from node i in time step n is:

$$\pi_{i,n} = \theta_{i,n} - \frac{(E(R_m) - R_f)\beta}{U - D}, \tag{25}$$

with

$$\theta_{i,n} = \begin{cases} 0 & \text{if} \quad \frac{1}{2} + \frac{(1-e^{-\alpha\triangle t})(\mu^\star - P_{i,n})}{2\sigma\sqrt{\triangle t}} \leq 0 \\ \frac{1}{2} + \frac{(1-e^{-\alpha\triangle t})(\mu^\star - P_{i,n})}{2\sigma\sqrt{\triangle t}} & \text{if} \quad 0 > \frac{1}{2} + \frac{(1-e^{-\alpha\triangle t})(\mu^\star - P_{i,n})}{2\sigma\sqrt{\triangle t}} > 1 \\ 1 & \text{if} \quad \frac{1}{2} + \frac{(1-e^{-\alpha\triangle t})(\mu^\star - P_{i,n})}{2\sigma\sqrt{\triangle t}} \geq 1 \end{cases} \tag{26}$$

where σ, α and $\mu^\star$ are parameters of the Ornstein - Uhlenbeck process for the log price described by Equation (11), $E(R_m)$ is the expected return of the market, R_f is the risk free rate, β is the project's beta and $P_{i,n}$ is the electricity price in node i at time step n.

The development in the project value is calculated by starting with the current net present value of the project and multiplying with the appropriate amount of up and down movements;

$$V_{i,n} = V_{0,0} \cdot U^{n-i} \cdot D^{i}, \tag{27}$$

and the probabilities of ending up in the node are given by:

$$p_{i,n} = p_{i,n-1} \cdot \pi_{i,n-1} + p_{i-1,n-1} \cdot (1 - \pi_{i-1,n-1}) \tag{28}$$

where the probability of ending up in the initial node is certain and equal to 1. The resulting binomial tree for the development is illustrated below.

	n		
i	$V_{0,0}$	$V_{0,1}$	$V_{0,2}$
		$V_{1,1}$	$V_{1,2}$
			$V_{2,2}$

The choice of $\triangle t$ in the calculations above depend on the number of scenarios to be generated each year. For the case study monthly time steps were chosen for the development of the value. Monthly time steps result in 13 different scenarios for the value after one year. The calculations are repeated for each of these scenarios to get 13 new scenarios for the next year.

7.1.3 Finding the Investment Threshold and the Threshold Value

When we know the value of the project at different nodes in a binomial tree we can find the optimal investment policy by option theory and dynamic programming. To apply the Bellman equation to this investment problem we need an expression for the continuation value. The simple timing option is solved with a binomial lattice approach. To use this approach we need the discrete version of the Bellman equation:

$$F_t = max\left(V_t - I, \frac{1}{R_f}\mathcal{E}(F_{t+1})\right). \tag{29}$$

Using this approach the continuation value $\mathcal{E}(F_{t+1})$ is equal to the expected value of F in the next nodes;

$$\pi_{i,n}F_{i,n+1} + (1 - \pi_{i,n})F_{i+1,n+1}, \tag{30}$$

where $\pi_{i,n}$ is the risk neutral probability of an up move in the project value. This continuation value results in the dynamic programming equation for decision node (i,n) for the simple timing option:

$$F_{i,n} = max\left(V_{i,n} - I, \frac{\pi_{i,n}F_{i,n+1} + (1 - \pi_{i,n})F_{i+1,n+1}}{R_f}\right) \tag{31}$$

For the case study investments are considered once a year, and the remaining nodes are non-decision nodes. If the node is not a decision node there is no possibility to invest in the node and the value of the option to invest is simply the continuation value. As explained in Section 5.2.1, the problem can be solved by defining the last decision point and working backwards from this decision by optimizing all the preceding decisions. In wind power investments the option to invest is usually lost at some point in the future resulting in a termination value. At this time the option to wait is lost and the value of waiting becomes zero;

$$F_{i,n} = max\left(V_{i,n} - I, 0\right). \tag{32}$$

The problem is now reduced to the simple now or never investment opportunity, and the optimal decision is given by the traditional rule of investing when $V > I$. After applying Equation 32 to the last nodes we can use Equation 31 and backward induction to find the optimal decision rule. Table 13 demonstrates the calculations for a two period binomial tree.

$F_{i,n}$	0	1	2
0	$max\Big(V_{0,0} - I, \mathcal{E}_{0,0}(F)\Big)$	$max\Big(V_{0,1} - I, \mathcal{E}_{0,1}(F)\Big)$	$V_{0,2} - I$
1		$max\Big(V_{1,1} - I, \mathcal{E}_{1,1}(F)\Big)$	$V_{1,2} - I$
2			$V_{2,2} - I$

Table 13: Illustration of the binomial lattice approach for two periods.

When the binomial tree for the underlying project is created using Equation 27 the optimal investment rule for a decision node is defined by Equation 31 above. The equation below shows that for each time period investment is triggered at the threshold value, V^*, when the value of investing is equal to the value of waiting one more period.

$$V_n^* - I = \frac{\pi_{i,n}F_{i,n+1} + (1-\pi_{i,n})F_{i+1,n+1}}{R_f}.$$

7.2 Scenario Generation

The development of the project value calculated by Equation 27 is the basis for the scenario generation. The different values for the projects after one year and the probabilities of ending up in each node result in the different scenarios considered. To make sure the optimization model only considers investing in projects when investing is optimal, a value function is defined. The function sets the value of investing to zero where the value of waiting to invest is higher than the net present value of investing. The threshold value, V_j^*, for project j, denotes the threshold for investment. When investment is optimal the value function is equal to the net expected value of the investment calculated by subtracting the investment cost for the project from the value of the project and multiplying with the probability for the given scenarios. The value function is defined by Equation 33 and the resulting scenario tree for the values of investing in each year is illustrated in Figure 11. For illustration purposes the figure represent a case where two scenarios are generated each year.

$$G_j(k,t) = \begin{cases} p(k,t)(V_j(k,t) - I_j) & if \quad V_j(k,t) \geq V_j^*(t) \\ 0 & otherwise. \end{cases} \tag{33}$$

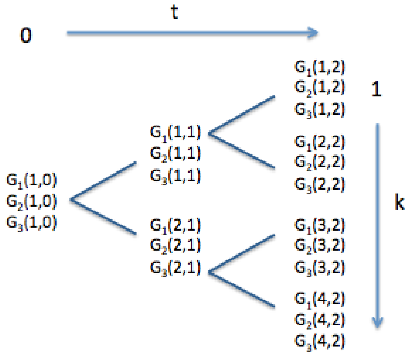


Figure 11: Illustration of the scenario generation for the investment value $G_j(k,t)$. $k \in \{1,4\}$, $t \in \{0,2\}$, $j \in \{1,3\}$.

7.3 Scenario Based Optimization Model

The tree generated by the value function $G_j(k,t)$, that represents the different scenarios, is used as the input in the optimization model. The goal is to maximize the expected net present value of the portfolio for all the scenarios subject to the budget constraint and the fact that if investment is exercised the option is lost and can not be exercised again in the same scenario. The optimization model by Meier et al.(2001) presented in Equations 16 to 19 is applied to find the optimal projects and optimal time to invest for each scenario. The model is dynamic and gives scenario dependent solutions, where the opportunity to wait and invest at a later time is included. As a consequence the result reflects the option value of the flexibility of waiting for more information about which projects to invest in.

7.3.1 Solving the Optimization Model

To solve the dynamic model for a large number of decision variables, the advanced optimization software FICO Xpress Optimization Suite is used. In order to solve the model using this software the different scenarios have to be identified. This is done by creating vectors for each scenario by duplicating the value $G_j(k,t)$ in each node in the tree for each scenario the node belongs to. The result is a matrix with the dimensions $j \times t \times s$, where each row represents one scenario, while the columns represent each time step the investment is considered. An illustration of this matrix is presented in Figure 12.

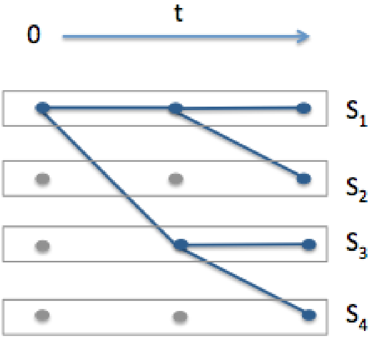


Figure 12: Illustration of the $t \times s$ matrix for the project value for one project.

With the new matrix additional constraints are required to make sure the decision variable for all the duplicated values are equal to the original node in the scenarios tree. These restrictions use the big-M method to make sure the sum of all duplicated decision variables are equal to either 0 or the number of duplicated nodes. The number of duplicated nodes for each position in the scenarios tree are given by the vector $N(p)$ where p is the position. Reference is made to Section 5.4 for detailed explanation of the big-M method. It generally applies in cases where only one of two constraints has to be satisfied. The position in the tree is stored as a separate index in the $j \times t \times s$ matrix. The index does not represent an additional dimension, but gives information about the position in the tree. The matrix with the additional index, p, is illustrated in Figure 13 for a two period tree with two scenarios each year. The vector $N(p)$ for the example is $[\ 4\ 2\ 2\ 1\ 1\ 1\ 1\ ]$. The modified model used to solve the optimization problem is given in Equations 34 to 39.

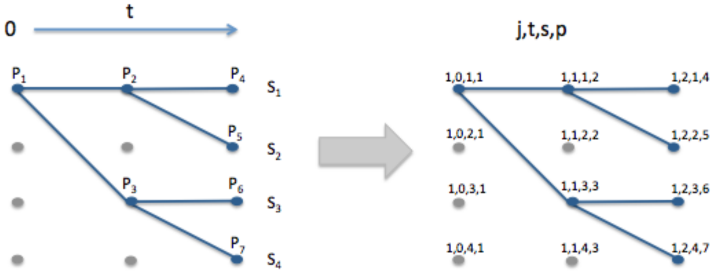


Figure 13: Illustration of the $t \times s$ matrix for the project value for one project with the additional position index p.

$$max \sum_{j=1}^{J} \sum_{t=0}^{T} \sum_{s=1}^{S} \sum_{p=1}^{P} e^{-t \cdot \Delta t \cdot r} x_{j,t,s,p} \frac{G_{j,t,s,p}}{N(p)} \tag{34}$$

$$s.t \quad \sum_{t=0}^{T} \sum_{j=1}^{J} \sum_{p=1}^{P} I_j x_{j,t,s,p} \leq B \quad s = 1, ..., S \tag{35}$$

$$s.t \quad \sum_{t=0}^{T} \sum_{p=1}^{P} x_{j,t,s,p} \leq 1 \quad s = 1, ..., S \quad j = 1, ..., J \tag{36}$$

$$s.t \quad \sum_{s=1}^{S} x_{j,t,s,p} \leq 0 + M(1 - y_{j,p}) \quad t = 1, ..., T \quad j = 1, ..., J \quad p = 1, ..., P \tag{37}$$

$$s.t \quad \sum_{s=1}^{S} x_{j,t,s,p} \geq N(p) - My_{j,p} \quad t = 1, ..., T \quad j = 1, ..., J \quad p = 1, ..., P \tag{38}$$

$$s.t \quad x_{j,t,s,p} \in \{0, 1\} \quad s = 1, ..., S \quad j = 1, ..., J \quad t = 0, ..., T \quad p = 1, ..., P \tag{39}$$

Equation 34 is the objective function to be maximized, which is the sum of the present value of investing in the nodes in the matrix. $G_{j,t,s,p}$ is divided by the number of duplicated value, $N(p)$, so that the expected value of the investing does not include one particular scenarios multiple times. Constraint 35 is the budget constraint and makes sure that the capital used for investment in each scenario does not exceed the available capital B. Constraint 36 makes sure that when the option to invest in one project is only exercised once in each scenario. Constraint 37 and 38 make sure the decision variable for all the duplicated values are equal to the original node in the scenarios tree, and Constraint 39 specifies that all decision variable are binary.

7.3.2 Computational Issues

The number of decision variables, $x_{j,t,s,p}$, is equal to the number of nodes in the matrix $j \times t \times s$. For real applications, the number of projects and the years the investments are considered are usually given by the investment situation. The number of scenarios generated for each year however, can be adjusted when solving the optimization model. Clearly, by considering several possible scenarios each year the result is more realistic, as the value of a project is not likely to take on one of two possible values after one year. However, if one chooses to consider more scenarios for each year, the number of scenarios increases drastically for each year that investment is considered. For example, to demonstrate the increasing number of decision variables, by considering four scenarios each year compared to two, the number of scenarios at year 3 is 64 instead of 8. This results in 192 decision variables, instead of 24, for each project considered. The number of scenarios should depend on the number of projects and the number of periods for which the investment is considered. With a high number of projects and many periods to consider the investments, the number of scenarios for each period should be low. For j projects, t years until the final investment decision and S scenarios generated each year, the number of decision variables are $j \times (t + 1) \times S^t$.

The optimization model was solved on a computer running Windows 7 with an Intel i7-3770 3.40 GHz CPU and 16 GB of RAM. The model was written in Mosel and implemented in Xpress-IVE 1.24.04 with Xpress-Mosel 3.6.0 and solves with Xpress Optimizer 21.01.04. When the investments are only considered for a period of two years, as for the case study in this thesis, the choice of number of scenarios to generate is very flexible. The model can handle both monthly and weekly time steps in the value development, resulting in 13 and 53 scenarios after one year, 169 and 2 809 scenarios in total and 2 535 and 42 135 decision variables. Because of the simplicity in the optimization model, and the fact that many decision variables are duplicated values and should be equal, the model runs efficiently in less than a second for both monthly and weekly time steps. If we however consider the investments for three years, it would not be reasonable to consider weekly time steps in the development in the value. If this is done, the number of scenarios becomes 148 877 and the total number of decision variables for five projects comes to 2 977 540, which leads to computational issues for the model. This demonstrates how quickly computational issues might arise when the investments are considered for additional periods.

7.4 Applying the Model to the Case Study

The procedure for generating the scenarios and solving the optimization problem can be summarized in the following five steps;

1. Finding the parameters for the price process by regression on historical log price data.
2. Use the regression parameters to model the stochastic development in the project values and the probabilities for each value.
3. Find the investment threshold for each project using real option theory and dynamic programming.
4. Generate the scenario tree for the project values based on the stochastic development and the investment thresholds.
5. Optimize project selection with the scenario based optimization model stated in Equations 34 to 39.

In the first step linear regression is done on monthly electricity data from Nord Pool Spot, between January 2001 and January 2015, resulting in the normalized estimates of the parameters presented in Table 14.

Rate of mean reversion, α	0.48
Long term mean $\mu^\star$	3.53
Standard deviation σ	0.18

Table 14: Normalized estimates of the price process parameters.

In the second step, the stochastic process in the project values are discretized to create a binomial tree for the development each year. The equations in Section 7.1.2 are applied using the parameters for the price process, the initial value of each project and the initial electricity price. The value and probability in each node in the tree is calculated using Equations 27 and 28. For the case study investment is considered in the beginning of each year, so that development of the project can start the following summer. The accuracy of the optimization model can be improved by considering more scenarios for each year. Considering a large amount of scenarios can, however, lead to computational issues in the optimization model. The number of scenarios generated for each year should therefore be restricted depending on the number of projects and the number of years the investments are considered. When the model is applied to the case study, monthly time steps are used for the development in the project value. If monthly time steps are used the value of a project will take one of 13 possible values in one year, and for each of these scenarios there will be another 13 possible values the next year. The total number of scenarios after two years is equal to $13^2 = 169$. As neither of the investments are considered for more than two years, both monthly and weekly time steps are computationally possible choices. The accuracy with monthly time steps is sufficient and additional scenarios would complicate the presentation of the results. If, however, the investments were considered for a longer period a larger time step should be considered. With monthly time steps for a period of one year the final nodes in the binomial tree represents the 13 scenarios for the value of the projects one year from now. Using these values as the initial value the calculations are repeated to create additional binomial trees to get the scenarios two years from now.

In the third step the threshold that triggers investment for each project in each year is calculated using the equations in Section 7.1.3 and the development in the value as well as the investment cost. Investment is triggered when the value of investing is larger than the option value of waiting. Equation 31, repeated below, is used to calculate the option value in node in the binomial tree.

$$F_{i,n} = max\left(V_{i,n} - I, \frac{\pi_{i,n}F_{i,n+1} + (1-\pi_{i,n})F_{i+1,n+1}}{R_f}\right)$$

Since the time steps are monthly but investment can only be undertaken in the beginning of a year we have to distinguish between decision nodes and non-decision nodes. In a decision node the option value is equal to the maximum of the value of investing and the continuation value, given by the equation above. While in the non-decision nodes the option value is equal to the continuation value;

$$F_{i,n} = \frac{\pi_{i,n}F_{i,n+1} + (1-\pi_{i,n})F_{i+1,n+1}}{R_f}.$$

In the last decision nodes there are no opportunity to wait, and the option value is therefore equal to the maximum of the net present value and 0;

$$F_{i,n} = max\left(V_{i,n} - I, 0\right).$$

The option value for all the nodes in the binomial tree are calculated by starting at the last decision nodes and working recursively backwards, by using the equations above. For the case study the time until the final investment decision for the projects differ. This is accounted for by using the appropriate number of years for each project in the calculations. The threshold that triggers investment in each year is the lowest value that makes investment optimal, or in other words when the value of investment is equal to the option value of waiting;

$$V_n^* - I = \frac{\pi_{i,n}F_{i,n+1} + (1-\pi_{i,n})F_{i+1,n+1}}{R_f}.$$

In step four the scenarios to consider in the optimization model are calculated using the value function in Equation 33;

$$G_j(k,t) = \begin{cases} p(k,t)(V_j(k,t) - I_j) & if \quad V_j(k,t) \geq V_j^*(t) \\ 0 & otherwise. \end{cases}$$

where $V_j(k,t)$ and $p(k,t)$ are, respectively, the scenarios for the value of the projects and the probabilities for each scenario in year t, representing the values in the last time step from the calculations in step two. $V_j^*(t)$ is the threshold value for investments in year t as calculated in step three. The code for computing the development in the value, the threshold values and applying the value function is written in Visual Basics for Applications (VBA) and is included in Appendix A. With monthly time steps, resulting in a total of 13^2 different scenarios, and the different investment horizons for each project, given in Table 16 below, the value function results in $13^2 \times (3 \times 3 + 2 \times 2) = 2197$ values and thereby 2 197 related decision variables.

In step five, the optimization model is applied to the values generated by the value function, accounting for the constraints on the capital available for investments and solved with the software FICO Xpress Optimization Suite. The VBA code writes the data to a file which is taken as input in the optimization model. The code for the optimization model is included in Appendix C.

7.4.1 Parametrization of the case study

The parameters required for the model include the risk free rate, the market risk premium, company beta and the initial electricity price and are presented in Table 15. The risk free rate is assumed equal to the annual average of the return on 10 year treasury notes which is 2.52 percent (Norges Bank, 2015). The market risk premium is based on a report by PwC on the Norwegian market (PwC, 2013) and assumed equal to 5.6 percent. The Beta is estimated considering the relative risk between the business areas at TrønderEnergi and the required rate of return and is assumed to be 0.6. The initial price is a monthly average from Nord Pool for March 2015 which is 25.34 Euro/MWh.

Risk free rate	2.52 %
Market risk premium	5.60 %
Beta	0.6
Initial electricity price	25.34 Euro/MWh

Table 15: Parametrization of the dynamic model.

In addition, the required project specific inputs are the investment cost, the expected present value of the revenues for each potential project and the number of years the investments are considered, which are summarized in Table 16. The capital available for investments in the wind power projects for each investment strategy to be considered, are stated in Figure 10.

Project	Investment Cost	Present Value	Years of flexibility
Ytre Vikna	1 280 000	1 407 420	1
Frøya	695 000	739 248	2
Sørmarksfjellet	1 685 328	1 775 652	1
Harbaksfjellet	1 058 526	1 180 511	2
Stokkfjellet	895 000	908 496	2

Table 16: Project specific inputs for the dynamic model (1000 NOK).

8 Results

8.1 Static Optimization Model

The static optimization model was solved for the five investment strategies considered. The optimal project selection for each strategy is presented in Table 17, together with the net present value, required capital and ROI for each portfolio.

Investment strategy	Projects	NPV	Required capital	ROI
1: Fosen 0% Sarepta 100%	Ytre Vikna Frøya Sørmarksfjellet Harbaksfjellet	383 977	4 718 854	8.1%
2: Fosen 0% Sarepta 50%	Stokkfjellet Ytre Vikna Frøya Sørmarksfjellet Harbaksfjellet	205 485	3 254 427	6.3%
3: Fosen 10% Sarepta 100%	Ytre Vikna Harbaksfjellet	249 405	2 338 526	10.7%
4: Fosen 10% Sarepta 50%	Ytre Vikna Frøya Sørmarksfjellet Harbaksfjellet	191 989	2 359 427	8.1%
5: Fosen 14.5% Sarepta 50%	Ytre Vikna Sørmarksfjellet Harbaksfjellet	169 865	2 011 927	8.4%

Table 17: Results from the simple optimization model for each investment strategy, with net present value, required capital and return on investment for each portfolio. Amounts in 1000 NOK.

The results from the simple static model illustrate that the prioritization among projects normally follows the simple rule of choosing the projects with the highest NPV. In most cases it is optimal to choose the projects with the highest NPV, unless choosing one of these projects leads to abandoning two projects that jointly have a higher NPV, while still satisfying the restrictions of the budget.

Based on the NPV the project prioritization would be Ytre Vikna > Harbaksfjellet > Sørmarksfjellet > Frøya > Stokkfjellet. The selection of projects in a situation with limited capital would be based on this prioritization. For the investment strategies with limited capital resources only the two or three best projects are chosen for investment. Ytre Vikna and Harbaksfjellet are the best projects, based on the NPV, and are chosen for all investment strategies.

Based on the ROI the project prioritization would be Harbaksfjellet > Ytre Vikna > Frøya > Sørmarksfjellet > Stokkfjellet. This measure is useful when evaluating the attractiveness of the projects, and can be used in addition to the NPV when making the final decision. Based on the ROI the same projects would be selected for every investment strategy except for strategy 5. For strategy 5 Frøya would be selected instead of Sørmarksfjellet, because even though the net present value of Frøya is lower compared to Sørmarksfjellet, the relative investment cost is significantly lower. If Frøya was chosen instead of Sørmarksfjellet the ROI for the portfolio would be 9.6 percent instead of 8.4 percent.

8.1.1 Sensitivity Analyses

To illustrate the sensitivity for the model with respect to the parameters a sensitivity analysis for the investment cost and the present value for the static optimization model was performed for investment strategy 3 and 4. It was not considered necessary to present a sensitivity analysis for all the investment strategies. The two strategies chosen represent both the strategy were TrønderEnergi owns 50 percent and were they own 100 percent in the Sarepta projects. The results are presented in Tables 18 and 19. In addition, a sensitivity analysis for the present value calculations with respect to the O&M cost, production, required rate of return and the electricity price were performed. The present value calculation for Ytre Vikna was chosen to illustrate the sensitivity and Table 20 presents the results.

The static model was most sensitive to the investment cost. The model was also quite sensitive to the present value which in turn was most sensitive to production. This result is in accordance with the result in the report by NVE (2015), where the cost of energy for wind power was most sensitive to the same two parameters. The present value was also quite sensitive to the electricity price. The implication is that these parameters have to be estimated with accuracy. By the time of the investment decision the investment cost is certain, but the production and price, however, are still subject to significant uncertainty and are difficult to estimate.

Investment cost					
Deviation	-20%	-10%	-	10%	20%
Portfolio value (1000 NOK)	994 058	597 006	249 405	16 132	0
Relative change	299%	139%	-	-94%	-100%
Project selection	2,3,4	2,3,5	2,5	5	0
Present Value					
Deviation	-20%	-10%	-	10%	20%
Portfolio value (1000 NOK)	0	3 934	249 405	536 051	854 358
Relative change	-100%	-98%	-	115%	243%
Project selection	0	5	2,5	2,4	2,4

Table 18: Sensitivity analysis on the static optimization model for investment strategy 3; 10% Fosen and 100% Sarepta.

Investment cost					
Deviation	-20%	-10%	-	10%	20%
Portfolio value (1000 NOK)	856 370	530 927	191 989	8 066	0
Relative change	346%	177%	-	-96%	-100%
Project selection	All	All	2,3,4,5	5	0
Present value					
Deviation	-20%	-10%	-	10%	20%
Portfolio value (1000 NOK)	0	1 967	191 989	492 389	801 418
Relative change	-100%	-99%	-	156%	317%
Project selection	0	5	2,3,4,5	1,2,4,5	1,2,4,5

Table 19: Sensitivity analysis on the static optimization model for investment strategy 4; 10% Fosen and 50% Sarepta.

O&M cost					
Deviation	-20%	-10%	-	10%	20%
Present value	1 514 244	1 461 110	1 407 420	1 353 099	1 298 085
Relative change	8%	4%	-	-4%	-8%
Production					
Deviation	-20%	-10%	-	10%	20%
Present value	1 133 691	1 275 782	1 407 420	1 535 349	1 661 359
Relative change	-19%	-9%	-	9%	18%
Required rate of return					
Deviation	-20%	-10%	-	10%	20%
Present value	1 591 746	1 495 171	1 407 420	1 327 519	1 254 616
Relative change	13%	6%	-	-6%	-11%
Electricity price					
Deviation	-20%	-10%	-	10%	20%
Present value	1 152 896	1 285 974	1 407 420	1 525 728	1 642 022
Relative change	-18%	-9%	-	8%	-17%

Table 20: Sensitivity analysis on the present value calculation (1000 NOK) for Ytre Vikna.

8.2 Dynamic Optimization Model

The dynamic optimization model was solved for the five investment strategies. The data sets generated by the VBA code were used as input. The present expected portfolio value for each of the investment strategies are presented in Table 21. The expected value is based on the probabilities for each scenario where investments are undertaken. The optimal project selection for each strategy is presented in Figures 14 to 18 where the projects are represented by the numbers given in the list below.

Stokkfjellet → 1

Ytre Vikna → 2

Frøya → 3

Sørmarksfjellet → 4

Harbaksfjellet → 5

Figures 14 to 18 illustrate the scenario trees for the two time periods, for the strategies 1 to 5, respectively. Ytre Vikna and Sørmarksfjellet can only be delayed by one year and has to be chosen at $t = 1$ at the latest, while the investment decision for the other projects can be delayed until $t = 2$. At $t = 1$ there are 13 scenarios for the value of the projects and the figures illustrate which projects are selected for investment in each scenario, starting with scenario 1 at the top, down to scenarios 13 at the bottom. At $t = 2$ there are another 13 scenarios for each of the scenarios in $t = 1$. Because this results in a total of 169 scenarios at $t = 2$, the illustration is simplified to show only the scenarios where investments should be undertaken. When the same projects are selected for investment in multiple scenarios, they are only presented once together with the number of scenarios where the project selection applies.

Investment strategy	Expected value
1: Fosen 0% Sarepta 100%	1 053 070
2: Fosen 0% Sarepta 50%	691 541
3: Fosen 10% Sarepta 100%	689 676
4: Fosen 10% Sarepta 50%	617 345
5: Fosen 14.5% Sarepta 50%	475 682

Table 21: Expected value of the portfolio for each investment strategy using the dynamic model (1000 NOK).

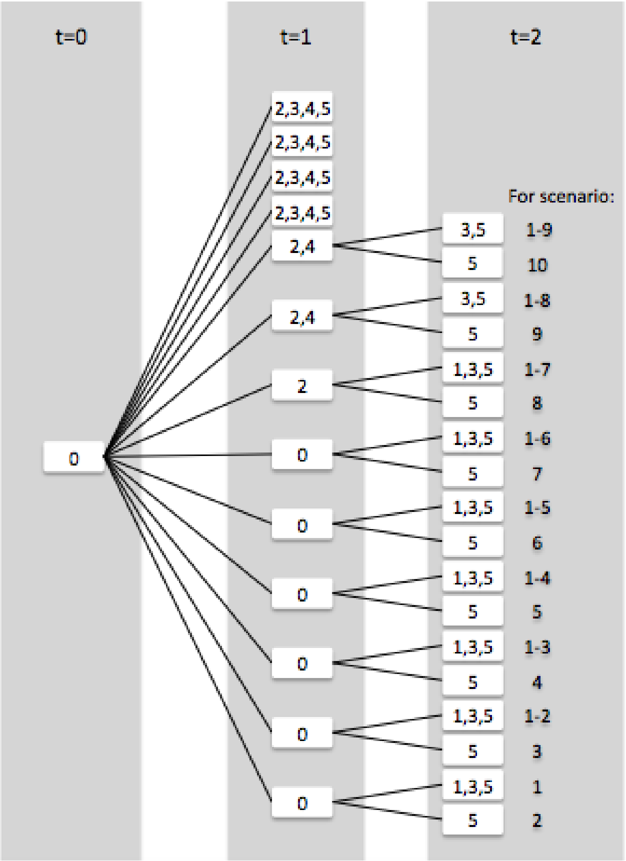


Figure 14: Project selection for investment strategy 1; Fosen 0% and Sarepta 100%. Each column represents investments now, after one year and after two years. At t=1 there are 13 scenarios for the value of the projects. The numbers represent the projects that should be invested in, for each scenario. At t=2 there is a total of 169 different scenarios. For simplicity only the scenarios where investment should be undertaken are presented and when the result is the same for several scenarios the result is presented once with the relevant scenarios next to the result.

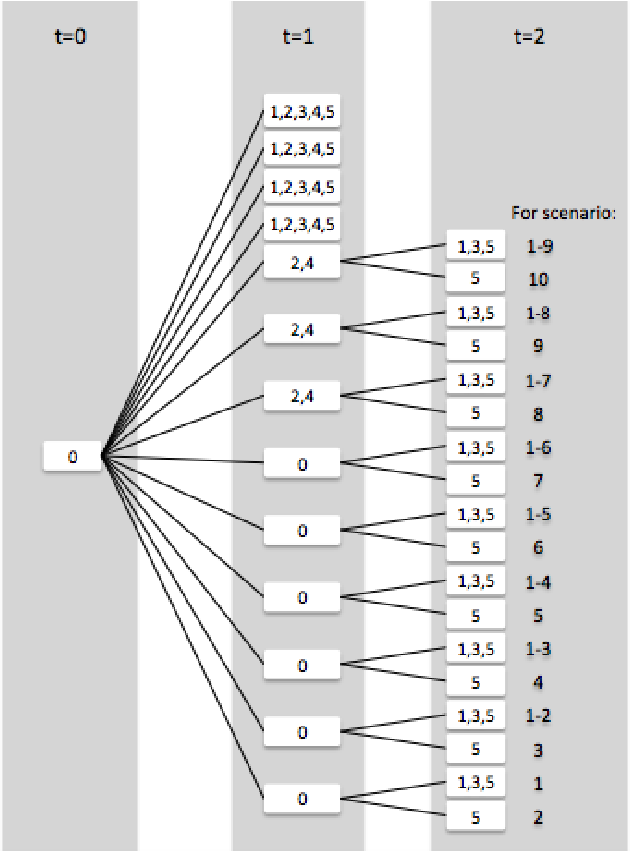


Figure 15: Project selection for investment strategy 2; Fosen 0% and Sarepta 50%. Each column represents investments now, after one year and after two years. At t=1 there are 13 scenarios for the value of the projects. The numbers represent the projects that should be invested in, for each scenario. At t=2 there is a total of 169 different scenarios. For simplicity only the scenarios where investment should be undertaken are presented and when the result is the same for several scenarios the result is presented once with the relevant scenarios next to the result.

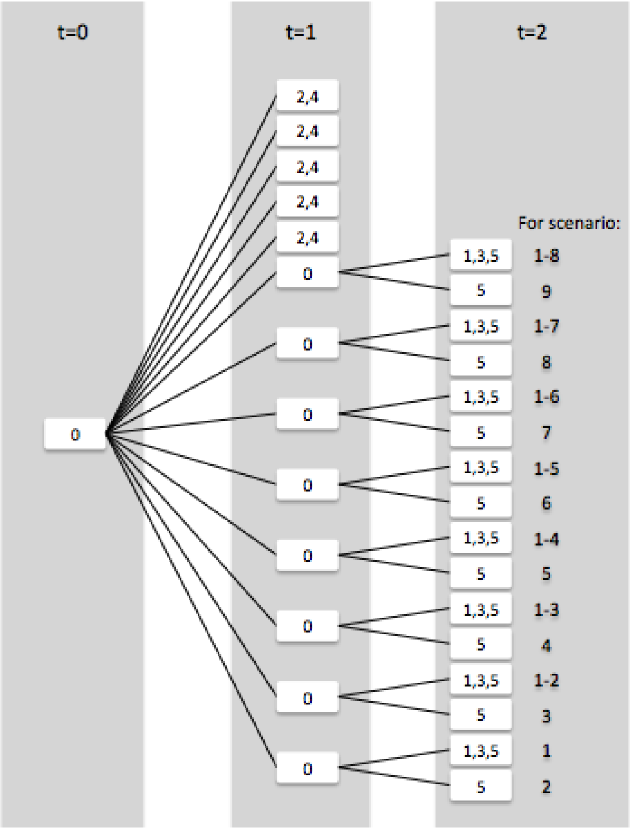


Figure 16: Project selection for investment strategy 3; Fosen 10% and Sarepta 100%. Each column represents investments now, after one year and after two years. At t=1 there are 13 scenarios for the value of the projects. The numbers represent the projects that should be invested in, for each scenario. At t=2 there is a total of 169 different scenarios. For simplicity only the scenarios where investment should be undertaken are presented and when the result is the same for several scenarios the result is presented once with the relevant scenarios next to the result.

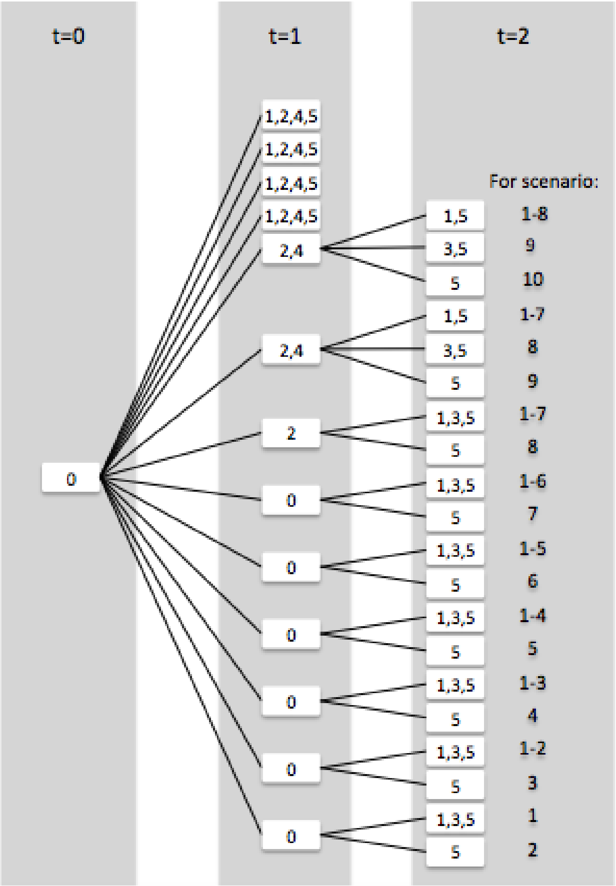


Figure 17: Project selection for investment strategy 4; Fosen 10% and Sarepta 50%. Each column represents investments now, after one year and after two years. At t=1 there are 13 scenarios for the value of the projects. The numbers represent the projects that should be invested in, for each scenario. At t=2 there is a total of 169 different scenarios. For simplicity only the scenarios where investment should be undertaken are presented and when the result is the same for several scenarios the result is presented once with the relevant scenarios next to the result.

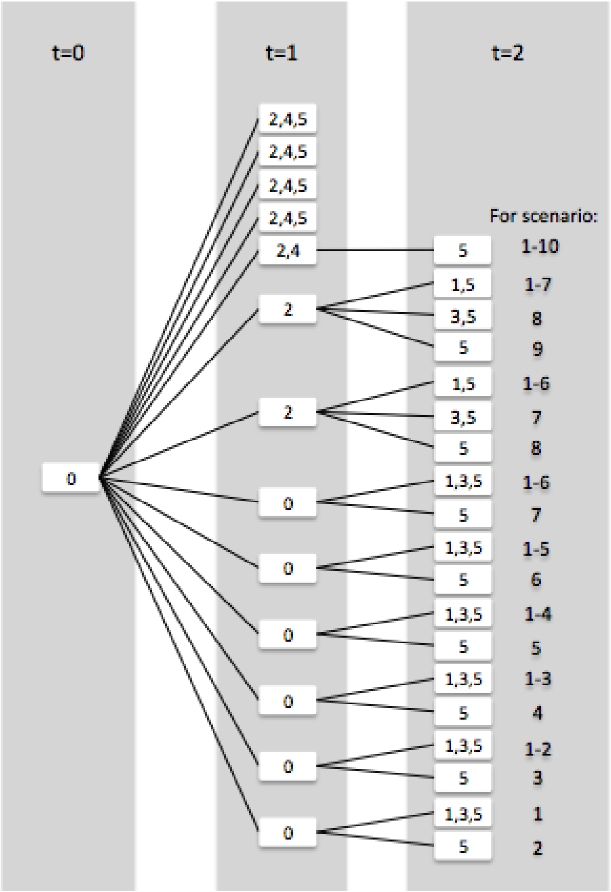


Figure 18: Project selection for investment strategy 5; Fosen 14.5% and Sarepta 50%. Each column represents investments now, after one year and after two years. At t=1 there are 13 scenarios for the value of the projects. The numbers represent the projects that should be invested in, for each scenario. At t=2 there is a total of 169 different scenarios. For simplicity only the scenarios where investment should be undertaken are presented and when the result is the same for several scenarios the result is presented once with the relevant scenarios next to the result.

According to the results of the dynamic model it is not optimal to invest right away in any of the projects. This is because the real option value of waiting to consider investments in the next couple of years are higher than the net present value of investing right away for all of the projects. For the next periods where investments are considered the optimal project selection differ for each of the investment strategies. In general, investments are never made in the scenarios where the projects' value has decreased, except for Harbaksfjellet for one scenarios. Decreasing the projects' value by the size of the down movement, D, makes all projects except for Harbaksfjellet unprofitable. For Harbaksfjellet the project is still profitable for the scenario where the value of the project decreases slightly, and the project is selected for investment even when the value decrease. This demonstrates the sensitivity of the profitability of the projects to decreases in the electricity price, thus the downside of investing right away in projects that might turn out to be unprofitable. Investment in Ytre Vikna and Sørmarksfjellet are easier triggered in $t = 1$ compared to the other projects. This is the last opportunity for investment in these projects and there are no opportunity cost related to the possibility of deferring these investments. Investment decisions for the other projects, however, can be delayed by one additional year. Therefore, investments in these projects would require a higher value to undertake investments now rather than delaying them. Table 22 presents the threshold project values that trigger investment in each time period for both the strategy where TrønderEnergi owns 50 percent and 100 percent of the Sarepta projects.

	t=1		t=2	
	100%	50%	100%	50%
Stokkfjellet	1 233 861		908 496	
Ytre Vikna	1 407 420	703 710		
Frøya	1 003 999	502 000	739 248	369 624
Sørmarksfjellet	1 775 652	887 826		
Harbaksfjellet	1 603 294	801 647	1 065 996	532 998

Table 22: Threshold project value that triggers investments. Values in 1000 NOK.

In the scenarios where the value of the projects have increased significantly it seems that Ytre Vikna and Sørmarksfjellet are prioritized, as demonstrated by Figure 16. Ytre Vikna and Sørmarksfjellet are always selected in the scenarios where the value has increased significantly. However, for some of the investment strategies, when the value has not increased significantly these projects are rejected to allow for the possibility of investing in the remaining projects in $t = 2$. This is because the expected value of these projects in the next year are higher than the current value of investing in Ytre Vikna or Sørmarksfjellet in $t = 1$.

8.2.1 Sensitivity Analyses

Sensitivity analyses with respect to the investment cost, present value, volatility and risk free rate were performed for investment strategy 3 and 4. Tables 23 and 24 present the sensitivity of the portfolio value to these parameters. The dynamic model was very sensitive to the investment cost and the present value of the projects. As should be expected, the portfolio value increases significantly when investment cost decreases or the present value increases. Changes in the volatility of the projects and the risk free rate, however, did not result in substantial changes in the portfolio value.

Investment cost					
Deviation	-20%	-10%	-	10%	20%
Portfolio value (1000 NOK)	1 452 920	1 054 810	689 676	451 010	264 142
Relative change	111%	53%	-	-35%	-62%
Present value					
Deviation	-20%	-10%	-	10%	20%
Portfolio value (1000 NOK)	182 500	401 240	689 676	1 004 370	1 331 340
Relative change	-74%	-42%	-	46%	93%
Volatility					
Deviation	-20%	-10%	-	10%	20%
Portfolio value (1000 NOK)	644 009	666 365	689 676	714 344	740 858
Relative change	-6%	-4%	-	4%	7%
Risk free rate					
Deviation	-20%	-10%	-	10%	20%
Portfolio value (1000 NOK)	694 882	692 274	689 676	687 088	684 512
Relative change	0.8%	0.4%	-	-0.4%	-0.7%

Table 23: Sensitivity analysis on the dynamic optimization model for investment strategy 3; 10% Fosen and 100% Sarepta.

Investment cost					
Deviation	-20%	-10%	-	10%	20%
Portfolio value (1000 NOK)	1 287 570	979 831	617 345	367 938	224 602
Relative change	109%	59%	-	-40%	-64%
Present value					
Deviation	-20%	-10%	-	10%	20%
Portfolio value (1000 NOK)	140 380	341 672	617 345	930 727	1 259 270
Relative change	-77%	-45%	-	51%	104%
Volatility					
Deviation	-20%	-10%	-	10%	20%
Portfolio value (1000 NOK)	581 189	598 366	617 345	638 028	660 360
Relative change	-6%	-3%	-	3%	7%
Risk free rate					
Deviation	-20%	-10%	-	10%	20%
Portfolio value (1000 NOK)	621 871	619 604	617 345	615 096	612 856
Relative change	0.7%	0.4%	-	-0.4%	-0.7%

Table 24: Sensitivity analysis on the dynamic optimization model for investment strategy 4; 10% Fosen and 50% Sarepta.

The project selection was very sensitive to both changes in the investment cost and present value of the projects. To illustrate how changing these parameters might affect the project selection the sensitivity with respect to the investment cost is presented. Figures 19 and 20 show the project selection for investment strategy 3, when the investment cost is 10 percent higher and 10 percent lower than the original estimates.

As should be expected a lower investment cost increases the incentive for early investment, and investment is optimal for additional scenarios. In contrast, a higher investment cost decreases the incentive for investment and leads to more conservative investment choices. With a higher investment cost the results suggest that investments should be undertaken in fewer scenarios and that it is often more optimal to wait and potentially invest in $t = 2$.

Similarly, analysis of the sensitivity of the present value also resulted in quite different projects selected. Changes in volatility and the risk free rate, however, did not change the project selection significantly. There were minor changes in the selection of Harbaksfjellet in $t = 2$, when the volatility for the projects were increased by 20 percent. This seems reasonable because higher volatility can also lead to lower project value for a down movement in the binomial tree. Harbaksfjellet becomes unprofitable when the value decreases and is not selected for this scenario. All other projects were already unprofitable in the scenario where a down movement in the binomial tree occurred.

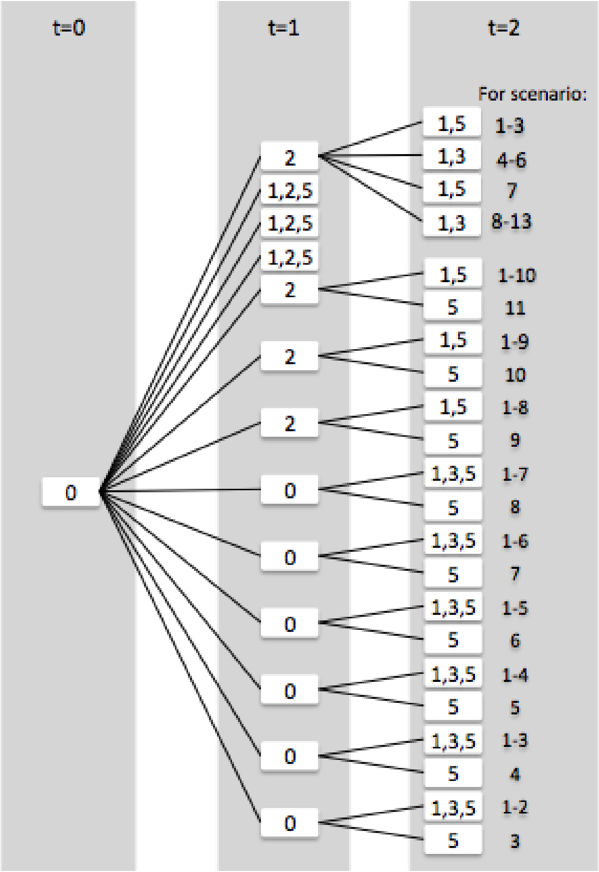


Figure 19: Project selection for investment strategy 3; 10% Fosen and 100% Sarepta, with 10 percent lower investment cost for the projects. Each column represents investments now, after one year and after two years. At t=1 there are 13 scenarios for the value of the projects. The numbers represent the projects that should be invested in, for each scenario. At t=2 there is a total of 169 different scenarios. For simplicity only the scenarios where investment should be undertaken are presented and when the result is the same for several scenarios the result is presented once with the relevant scenarios next to the result.

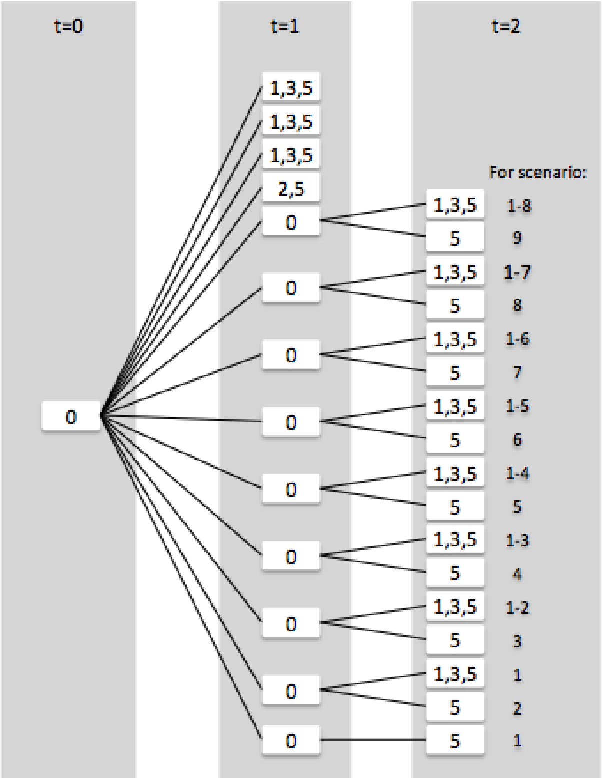


Figure 20: Project selection for investment strategy 3; 10% Fosen and 100% Sarepta, with 10 percent higher investment cost for the projects. Each column represents investments now, after one year and after two years. At t=1 there are 13 scenarios for the value of the projects. The numbers represent the projects that should be invested in, for each scenario. At t=2 there is a total of 169 different scenarios. For simplicity only the scenarios where investment should be undertaken are presented and when the result is the same for several scenarios the result is presented once with the relevant scenarios next to the result.

For the investments considered in this thesis the uncertainty in the investment cost is assumed to be quite low. Additionally, it is assumed that by the time the investment decisions can be made, requests for price quotas can be made to reduce this uncertainty for all the projects. Regarding the uncertainty in the present value there are a lot of uncertain factors going into this calculation. The most important being the production, O&M costs and the required rate of return. The sensitivity of these parameters shows that the present value of the projects are most sensitive to production. A thorough analysis of the production for the wind park should therefore be performed if this optimization model is used.

In addition, a sensitivity analysis with respect to the number of scenarios generated for each year was performed. The results for the value of the portfolio in the case of 24 and 52 time steps each year are presented in Table 25. The relative change in portfolio value for both cases were less than 0.15 percent. The results do not change significantly when additional scenarios are considered, supporting the conclusion that it is sufficient to use monthly time steps in the value development.

Investment strategy	Portfolio value (1000 NOK)		
	n=12	n=24	n=52
3: Fosen 10% Sarepta 100%	689 676	689 820	690 146
4: Fosen 10% Sarepta 50%	617 345	618 460	618 297

Table 25: Sensitivity analysis on the dynamic optimization model for additional number of scenarios.

9 Discussion

This thesis has applied two different optimization models for selection of wind power projects in TrønderEnergi's portfolio. The first model is based on the DCF method. The second is a dynamic scenario based optimization model that considers the uncertainty in the cash flows by including the flexibility in optimal timing of investment decisions. This section contains a discussion about the results and compare the different approaches and investment strategies. In addition, the assumptions and the relevant uncertainties and flexibilities presented in the qualitative analyses are discussed.

9.1 Comparison of Investment Strategies

The value of the portfolio of wind power projects vary significantly depending on which investment strategy is chosen. If TrønderEnergi does not invest in Fosen Vind they can use more capital to invest in additional wind power projects, and the return on these projects will increase the value of the project portfolio. In contrast, if they continue to hold 14.5 percent interest in Fosen Vind, they will have to cut back on their investment in wind power projects, resulting in a lower total value of the project portfolio. For the investment strategy where TrønderEnergi does not invest in Fosen, and they continue to own 50 percent of the Sarepta projects, both optimization models select all projects for investment. This strategy gives the highest available capital for project investments and as all projects can be selected the budget is clearly not a restriction.

The strategy where TrønderEnergi does not invest in Fosen Vind and can own 100 percent of the Sarepta projects results in the highest portfolio value for both of the optimization approaches. As should be expected, the lowest portfolio value is for the strategy where they hold 14.5 percent shares in Fosen Vind.

For the cases where the available capital is the same, the strategies where TrønderEnergi invests 100 percent in the Sarepta projects give the highest portfolio value and highest ROI. This result appears reasonable as they can invest more of the capital in higher quality projects instead of owning 50 percent in more projects, where the additional projects are financially less attractive. The decision to choose among the different investment strategies is, however, not that straight forward. Owning 100 percent in the Sarepta projects also means that they have to buy the additional 50 percent interest in the projects that they do not currently own. First of all, NTE might not be willing to sell their interest in the projects and secondly, the cost of buying the projects might exceed the additional benefits of owning 100 percent compared to remaining co-owners in the projects.

Several factors must be considered when deciding which strategy is the best alternative for TrønderEnergi. First of all, to evaluate the strategies more accurate information regarding the financial return for the Fosen investment is required. It is hard to directly compare the strategies because the public information about the return on the Fosen investment is unknown or at least very uncertain. However, the Fosen projects are considered to be good projects with potentially lower costs than the projects considered in this optimization model. It seems reasonable that TrønderEnergi should invest in Fosen Vind if they believe that the potential additional returns are more important than the potential long term benefits derived from operating their own additional wind parks. Thus the second factor to consider when selecting investment strategy is the long term operating strategy of the company.

If we assume a return on the Fosen projects, we can compare the results for the static model in terms of total portfolio value. Table 26 shows the value and ROI of each portfolio in the static model with a return of 7 and 8 percent on the Fosen investment. The portfolios require approximately the same amount of capital, but have different returns. If the Fosen projects have a return of 7 percent it is more profitable to reduce the investment in Fosen Vind to 10 percent. If, however, the return on Fosen Vind is higher it is more profitable to continue to hold the 14.5 percent ownership in this company. In either case holding some interest in Fosen Vind is more profitable than disposing of the investment, because the total return of the portfolio is always higher than for the portfolios where they do not invest in Fosen Vind. The ROI for the strategies where they do not invest in Fosen Vind, and own either 50 percent or 100 percent in the Sarepta projects, are 6.3 percent and 8.1 percent, respectively. Because the Fosen projects are considered to be good projects, it seems reasonable that TrønderEnergi should continue to hold some interest in the company. In doing so they can disregard the less profitable projects, and receive a high return on Fosen Vind instead, and still benefit from operating some projects themselves. If they pursue this suggestion, of choose to continue to hold a position in Fosen Vind, they still have to decide on which interest to hold. As can be seen in Table 26, this should depend on how good the return on the Fosen investment is. If the expected return is high they should maintain their position of 14.5 percent ownership. If they choose to maintain this position compared to decreasing their interest to 10 percent, the results from the static model suggests that they have to abandon the Frøya project. The results from the model still suggests they should invest in three projects. As Frøya is a small project, the added value of holding 4.5 percent additional interest in Fosen Vind will most likely exceed the potential financial and operational benefits of owning 50 percent of Frøya.

Investment strategy		7 %			8 %	
	Value	Inv. Cost	ROI	Value	Inv. Cost	ROI
5: Fosen 14,5 % Sarepta 50 %	274 305	3 026 927	9,1 %	434 225	3 026 927	14,3 %
4: Fosen 10 % Sarepta 50 %	264 016	3 059 427	8,6 %	374 305	3 059 427	12,2 %
3: Fosen 10 % Sarepta 100 %	321 432	3 038 526	10,6 %	431 722	3 038 526	14,2 %

Table 26: Total portfolio value, investment cost and ROI for the static model in the case of 7 and 8 percent return on the Fosen investment. Amounts in 1000 NOK.

9.2 Comparison of Static and Dynamic Approach

The expected value of the optimal investment portfolio was very different for the two approaches. In general, the dynamic approach gives roughly three times higher portfolio value for each of the investment strategies. This increased value reflects the real option value of the portfolio or the added value of including some of the uncertainty and allow for more informed decisions.

For several scenarios the results for the dynamic model selects different projects compared to the the static model. What first draws our attention is the prioritization of Sørmarksfjellet instead of Harbaksfjellet for investment strategy 3, and Stokkfjellet instead of Frøya for investment strategy 4, in the scenarios where the project's value increases significantly. In $t = 0$ Harbaksfjellet and Frøya are better projects, with higher NPV, than Sørmarksfjellet and Stokkfjellet, respectively, and are therefore selected in the static model. However, for the scenarios where the value of the operational phase of the projects increases, Sørmarksfjellet and Frøya are selected. When generating the scenarios the project values increase by multiplying with the factor U, calculated in Equation 24, which represents the size of the movements in the price process. In reality, the value of the operational phase depends heavily on the electricity price and an increase in price will increase the value of the projects that produce more electricity at a faster rate. The investment cost, however, does not directly depend on the movements in the electricity price, and is assumed constant in this thesis. When the investment cost is constant and the value of the operational phase increases by a constant factor U, the projects with higher initial value of the operational phase will increase at at faster rate. Eventually the NPV of these projects can exceed projects that initially had higher NPV. As a consequence, the prioritization of projects change depending on the scenario. The changes in the prioritization as price increases, and thereby the value of the operational phase of the projects increases, are illustrated in Figure 21. In the scenario where the value of the projects are constant, the prioritization is the same as for the static model. As price increase and thereby the value of the the projects increase, the prioritization, based on the NPV, change according to what is presented in the Figure 21. The changes in prioritization is different for the strategy where TrønderEnergi will own 100 percent in the Sarepta projects compared to if they own 50 percent. This is because the current value of half the Sarepta projects are less than the current value of Stokkfjellet. Stokkfjellet is currently the least profitable project, with a high investment cost compared to the value resulting in a low NPV. However, as the price increases, the value of Stokkfjellet increases at a faster rate compared to 50 percent of the Sarepta projects. Therefore Stokkfjellet becomes more profitable when the price increases significantly. Ytre Vikna is currently the most profitable project, and is among the best projects also for the scenarios when the value increases. Its high prioritization and low investment cost leads to it being selected for investment in several of the scenarios.

The prioritization is however not a strict rule to which projects are selected because of the budget restrictions and the fact that the investment horizon differs for the projects. In some scenarios the projects with the highest NPV at $t = 1$ are not necessarily chosen because the expected value of other projects are higher in $t = 2$. In scenario 6 and 7 at $t = 1$, Ytre Vikna, Harbaksfjellet and Sørmarksfjellet are the three best projects. However, in some cases Ytre Vikna and Sørmarksfjellet are not chosen in $t = 1$, even though this is the last opportunity for investing in these projects. The reason is that the expected value in $t = 2$ of other projects are higher than the current value of Ytre Vikna or Sørmarksfjellet in $t = 1$. The difference in the results for the static and dynamic models can be further investigated by looking at the scenario in the dynamic model where the value remains constant for each time period. In this scenario the prioritization between the projects are the same for the static and dynamic approach, but the resulting projects selected are different for all investment strategies except strategy 2, where the budget is not a restriction

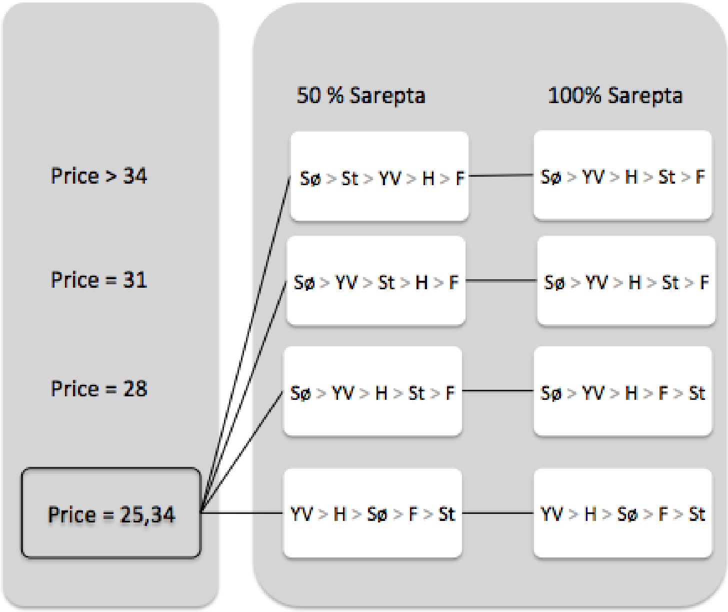


Figure 21: Changes in prioritizing of projects when the price of electricity increase for investments of both 50 percent and 100 percent in the Sarepta projects.

and all projects are selected. The project selection for the static model and for the dynamic model in the scenario where the value is constant is summarized in Table 27. The main difference in the results is the fact that Sørmarksfjellet is never selected in the dynamic model, except for strategy 2. As mentioned above this is because the final investment opportunity for Sørmarksfjellet is in $t = 1$ and the expected value of other projects in $t = 2$ is higher. This demonstrates an important difference between the two approaches; the dynamic model includes the possibility of waiting for a potential increase in project value. The result of this could be to reject projects that are currently more valuable for the opportunity to invest in projects that are currently less valuable but have high expected value for the future. This expected future value is related to the real option value that comes from the additional flexibility in timing. The project selection in the results for the dynamic model demonstrates that this model considers the real option value of the projects as the projects that are the most profitable are not necessarily the ones selected.

Investment strategy		Static model	Dynamic model (s=7)	
			t=1	t=2
1:	Fosen 0%	Ytre Vikna	Ytre Vikna	
	Sarepta 100%	Frøya		Frøya
		Sørmarksfjellet		Stokkfjellet
		Harbaksfjellet		Harbaksfjellet
2:	Fosen 0%	Stokkfjellet		Stokkfjellet
	Sarepta 50%	Ytre Vikna	Ytre Vikna	
		Frøya		Frøya
		Sørmarksfjellet	Sørmarksfjellet	
		Harbaksfjellet		Harbaksfjellet
3:	Fosen 10%	Ytre Vikna		Stokkfjellet
	Sarepta 100%	Harbaksfjellet		Harbaksfjellet
				Frøya
4:	Fosen 10%	Ytre Vikna	Ytre Vikna	
	Sarepta 50%	Frøya		Frøya
		Sørmarksfjellet		Stokkfjellet
		Harbaksfjellet		Harbaksfjellet
5:	Fosen 14.5%	Ytre Vikna	Ytre Vikna	
	Sarepta 50%	Sørmarksfjellet		Frøya
		Harbaksfjellet		Harbaksfjellet

Table 27: Summary of project selection for the static model and for the dynamic model in the scenario where the project value is constant.

Usually there are advantages and disadvantages with all valuation methods when dealing with investments in uncertain projects and one method by itself might not be sufficient. Real option analyses are often used as supplements in decision making processes and a thorough DCF analyses is usually necessary either way. The simplicity and rationality of the DCF method are good arguments for choosing this approach. The benefit of alternative approaches, which often requires extensive calculations and understanding of advanced theory, must make up for the additional work required.

In a previous discussion in this thesis it was suggested that the traditional DCF method is not always sufficient for project valuation of wind power projects. The same argument should also apply for models to be used in capital budgeting decisions related to wind power. The literature suggests that a real option approach for the valuation of wind power projects is appropriate, because of the commodity risk associated with the projects. The comparison of the two portfolio optimization approaches above demonstrates the validity of such an hypothesis related to capital budgeting decisions. The traditional portfolio optimization model largely underestimates the portfolio value compared to the real options approach, and in several scenarios the project selection differs significantly.

The important difference between the approaches is that the real options model incorporates the project's uncertainty. When a volatility of zero was used as input in the dynamic model it yielded the same values and the same project selection as the static optimization model (and selected all projects in $t = 0$). Without uncertainty there is no value in the flexibility in decisions and zero strategic option value. The assumption for the DCF method, that the cash flows are known with certainty and the volatility is zero, is not realistic. In reality the world and market conditions are stochastic and this justifies the need for other methods. The increased portfolio value when the uncertainty is included leads to the suggestion, in accordance with previous literature, that a real options approach should be used in capital budgeting decisions for projects with high uncertainty.

As seen from the sensitivity analysis for the project selection with respect to the investment cost and the present value, an increased initial value of the projects increase the incentive for early investment. The higher the initial value of the investments are the sooner the projects will be selected and the more the results will resemble the results for the static model. It is therefore also suggested that the real option based approach should be utilized for situations when projects have low or possibly negative initial investment value.

Even though the real options approach adds value to the portfolio it is not always worth while to perform the time consuming analysis. In accordance with Lint and Pennings (2001), it is suggested that the DCF approach should be performed first, followed by the more labor intensive real options approach if necessary. As suggested above the capital budgeting decision will benefit more from the real options approach in situations with high uncertainty and low or possibly negative initial project value.

The dynamic model presented in this thesis reflects the dynamic reality of decision making, and is therefore considered to be a better approach. The fact that the dynamic capital budgeting model results in higher portfolio value demonstrates the added value of choosing a real options approach to capital budgeting decisions. However, the model is still a simplification of the complex reality and several assumptions are made. The next sections will discuss these assumptions and present possible improvements to the current dynamic model.

9.3 Assumptions

The electricity price was modeled as an Ornstein-Uhlenbech (O-U) process, which is a so-called mean reverting process. This process captures the tendency of the electricity price to revert back to its long term average, and is considered realistic for modeling electricity prices. However, there are drawbacks associated with the model. Mean reversion does not capture all the properties of the movements in electricity prices, like for example the sudden spikes that are often seen in the price. In addition there might be seasonal variations or non constant volatility. When choosing how to model the electricity price, as well as several other assumptions in the model, it is essential to consider the trade off between accuracy and applicability. In order to establish a model that is useful for practitioners, it should be relatively straightforward and easy to understand, while at the same time reflecting reality as accurately as possible. The mean reverting process is considered sufficient when considering long term investments. The model captures the most important tendencies of the electricity price while at the same time allowing for easy calculations in creating the binomial tree for the price development.

Historical data for the value of the operational phase of a project is difficult to obtain because projects are not normally traded in the market. This makes it difficult to estimate the volatility of the projects and to include the uncertainty in the analysis. For commodity based products it is argued that the volatility of a project is equal to the volatility of the price of the commodity on which it is based. In this thesis it is assumed that the projects' value follow the same process as the electricity price. This assumption will clearly lead to small errors in estimating the projects' value because the value is not strictly a function of the price. First of all, the operational income depends on the TGC price as well as the electricity price, and the TGC price does not exhibit the same tendencies as the electricity price. Secondly, the expenditures of a project do not vary along with the income as a stochastic process. In reality these factors follow their own appropriate dynamics. An alternative way to treat the volatility could be to separate the operational income from the expenditures and perform a thorough analysis of the development in these additional factors and find the appropriate process for each variable. However, the dynamic model was not significantly sensitive to changes in the volatility and the assumption seems reasonable.

For the calculations of the current value of each project it is assumed that the production from the wind parks, as well as all costs, are constant, and the TGC price is assumed to be deterministic with values estimated by TrønderEnergi. In reality both wind and TGC price are stochastic. The O&M cost can be either stochastic or fixed depending on what agreements are made with the turbine manufacturer at the time of investment. The investment cost is also to some degree uncertain, but given the relatively short period for which the investments are considered it is expected to be relatively constant.

The factors mentioned above are treated as deterministic or constant values in the present value calculations. The present value is in turn multiplied with a factor determined by the movements in the price. In reality some of these factors are uncertain and follow separate processes. The production depends on the wind, which thereby directly influences the income of a project. In reality the wind is stochastic and is often assumed to follow the Weibull distribution. As the sensitivity analysis for the present value calculations show, the present value is most sensitive to production. Therefore, treating the production of a wind park as constant might lead to errors in the optimization model. The TGC price also follows a different stochastic process. The certificates are traded in a market with different characteristics than the spot market for the electricity price. First of all, there are no marginal cost of production for TGCs, and therefore no tendency for the price to revert back to the mean. Secondly, the demand for certificates is created artificially through quotas defined by the government and political actions that will affect the volatility of the TGC price are likely. Due to these characteristics and the fact that the TGC market is new and lacks historical data, there is a lot of uncertainty regarding both the TGC price itself and how to best model the price. One possibility is to model the TGC price with Geometric Brownian motion, but the lack of historical data to represent the market is likely to make the parameter estimation inaccurate. One can only speculate on how the market will evolve in the future compared to the past. Despite the arguments in this section, that the cost, production and TGC price should follow separate stochastic processes, this was not done in this thesis due to the complexity of such an analysis. However, the value of the operational phase is still very much dependent on the electricity price and it is considered reasonable to utilize the assumption above to help simplify the calculations.

9.4 Uncertainty and Flexibility

In the dynamic optimization model in this thesis the only uncertainty considered is in terms of the price. In addition to the electricity price, the projects are subject to TGC price risk, production risk, investment cost risk, O&M cost risk, grid cost risk and risk of not receiving concession. The managerial flexibility in decision making is related to other flexibilities than timing, such as flexibility in the size of the projects and in the choice of technology. The following discussion will evaluate the potential impact of the risk factors and flexibilities that are not incorporated in the quantitative analysis.

In the real option literature the value of uncertainty and flexibility is related to the fact that the additional information about the uncertain factors received by waiting, might potentially result in better decision making. More information about a project's cash flows reduces the possible downside of investing in projects where the cash flows turn out to be less than expected. By deferring an investment in wind power the decision maker receives more information about the development of the electricity price in the spot market and can choose to invest if the price increases and disregard the project if the price decreases. The price uncertainty therefore increases the option value of the project. It has been shown that the real option value for wind power projects increase with volatility and time to maturity, because more uncertainty means higher value of the flexibility (Lee, 2011). From this we can conclude that projects with high uncertainty and high flexibility have higher real option value. However, in the dynamic model used in this thesis, the projects are subject to the same risk related to the price, even thought the projects have varying degree of uncertainty related to other risk factors. An interesting question is whether these risk factors should be included when making the investment decision.

The risk preferences of investors differ, but in general additional risk is not preferable unless it gives the potential of a higher expected investment value to commensurate the higher risk. High uncertainty in production and costs for wind power projects can result in both higher and lower value than expected. The advantage of the real options approach is that the downside of this uncertainty to some extent can be removed. However, the risk in production and cost for wind power projects are largely dependent on the maturity of the projects and this risk decrease as the projects matures. The final decision to invest in wind power in Norway is not made until the project is mature and the developer has collected all the necessary information about the wind conditions and obtained quotas for the turbines and O&M cost. Therefore a real options perspective on the downside of these risk factors are already indirectly included in the decision process. The production and cost risk and flexibilities associated with TrønderEnergi's projects are related to the difference in maturity of the projects and the company will wait until they have the necessary information about the production and costs before they make the investment decision. As mentioned in Section 2.2.5, the uncertainty in production is related to the quality of the wind measurements done in the area. After a certain amount of measurements, additional measurements will be expensive and the cost is not justified compared to the additional information it will bring. The remaining aspects of the uncertainty in production and costs are usually revealed subsequent to making the investments, when production has started. Deferring the investments in mature projects will therefore normally not reveal any more useful information about these factors, since the costs of obtaining this information is considered too high. In addition, the flexibility in technology choice and size adaption is minimal for mature projects. However, the projects considered in the optimization model have varying degree of maturity, which rises the question of how to deal with these differences when comparing projects in the model. One solution would be to only consider projects that are mature. If this is not the case, the assumption that the projects are subject to the same uncertainty might lead to large errors in the model. In this thesis investments are considered from January 2016, and it is assumed that all projects are mature by that time.

Even when only considering mature projects, the projects might still be subject to different risk in the production. As the results were very sensitive to production this can have an affect on the investment decision. The fact that high production risk should correspond with higher expected return is not accounted for in the model in this thesis.

9.5 Possible Extensions

From the sensitivity analyses the investment cost and present value represent the parameters to which the results are most sensitive. It is therefore important that these parameters are estimated with high accuracy.

Regarding the present value, the stochastic behavior of several of the factors going in to the calculation makes it hard to get exact estimates. In order to obtain a more accurate model it is suggested that these uncertain factors can be modeled as separate stochastic processes. Most importantly, the accuracy can be significantly improved by modeling the seasonal patterns of the wind input or the TGC price as stochastic variables rather than deterministic values. By doing this the volatility in the model will more accurately reflect the actual uncertainty in the projects.

The capital cost is the most significant cost for wind power projects, and represents an uncertain factor in wind power valuation. For this case study the investments are only considered for two years and it is assumed that the projects have offers for the turbine cost by the time the decision to invest can be made. It was therefore concluded that the uncertainty in the investment cost is minimal. Investment cost has decreased in the recent years and is expected to decrease during the next decade. In this thesis the opportunity value of decreasing capital cost is not captured, but it is suggested that for portfolios with a longer investment horizon, the potential of decreasing capital costs should be incorporated.

In addition, the historical data for the electricity price demonstrate varying volatility and spikes. It can be argued that the price process should also incorporate regime switching volatility and jump diffusion. However, these characteristics are not expected to have a large impact on the result for long term investments.

It is suggested that future work building on this model should consider the extensions mentioned in this section. However, when developing a model that is intended to be used in practice, it is essential to consider the trade off between accuracy and applicability. The objective of this thesis is to find a suitable method for practitioners to select among projects in a wind power portfolio, and while extending the model might be interesting from an academic viewpoint it is not necessarily practical for real applications.

10 Conclusion

The goal of this thesis has been to find a suitable method to select among projects in a wind power portfolio. The method was applied to TrønderEnergi's investment opportunities. According to the literature in the area of valuating and choosing among investments in uncertain environments, the traditional DCF approach underestimates the project value and can potentially lead to incorrect investment decisions. The weaknesses of the DCF approach to value investment opportunities calls for alternative approaches in capital budgeting for wind power portfolios. The literature suggests that real option analysis is appropriate for valuating investments in wind power and that flexibility generally adds a significant amount of value to the investment opportunities. It is believed that a real options approach for the capital budgeting decision is a more appropriate method, which will improve the quality of the investment decisions by including the high uncertainty related to wind power investments.

The portfolio optimization was performed for five different investment strategies, depending on TrønderEnergi's interest in Fosen Vind and the resulting capital available for investment in other wind power projects. For each investment strategy the portfolio was optimized using two different capital budgeting approaches. The first was based on the traditional DCF method and the second was a dynamic real options approach. Both approaches require the net present value calculations for the revenues of each project as an input. The dynamic approach generates scenarios for the future value of the projects by assuming that the development can be described by a mean reverting stochastic model. The DCF approach maximizes the value of the portfolio based on the current values of the projects, while the real options approach includes the flexibility of delaying the investments by optimizing the expected value of the portfolio given a set of scenarios for the projects' value in the future.

The results demonstrate a significant increase in expected portfolio value when the dynamic model was applied. For every investment strategy the expected portfolio value was approximately three times higher when the dynamic model was applied, compared to the traditional DCF approach. The dynamic model results in different project selection for several of the scenarios. The most obvious explanation why different projects are selected in some of the scenarios is related to the fact that the projects with high production will benefit more if the electricity price goes up, and the value will increase at a faster rate. When the investment cost, or exercise price, for the projects are constant, but the value of the projects revenue increase, the prioritization of projects will change. The dynamic real option based model is capable of including the value of deferring investments in projects that might have higher net expected value in the future even though they are currently not the most profitable.

In previous literature real options theory has been applied to several sectors in the energy industry, from generation to evaluation of policies. In accordance with previous literature, this thesis revealed how the traditional approach might lead to selecting the wrong projects by ignoring the opportunity cost of making an investment now and giving up the option to wait for new information about which projects are the most profitable. The limitations of the traditional techniques and the potential of real options theory leads to the conclusion that a real option based capital budgeting approach should be considered when dealing with investments in the energy industry.

In addition to the above findings the different investment strategies for TrønderEnergi are reviewed. Obviously the investment strategy chosen must be related to the company's long term strategy and depends to some extent on the expected return for the Fosen Vind investment. The investment is believed to be at least more profitable than the least profitable projects considered by TrønderEnergi and it is suggested that investing 10 percent in Fosen Vind might be a good solution. After contributing with the capital required to invest 10 percent in the Fosen Vind projects, TrønderEnergi can still invest in several additional projects and thereby develop the company by building competence and experience in the wind power industry to support future investments.

There are a number of ways in which the model discussed above could be improved. Possible extensions could include the uncertainty of factors that are considered deterministically, such as the wind, certificate price or investment cost. If these are treated separately, the development in the value of a project is a function of all uncertain factors and it is possible to also include the development of the price in the operational phase. For such a case another extension could include a more complex process for the electricity price that includes spikes or regime-switching volatilities. However, it can be argued that the complexity of such analyses might exceed the benefits and while extending the model might be interesting, it is not necessarily practical for real applications.

This thesis has proposed an alternative real options based approach to capital budgeting as opposed to optimization models based on the DCF method. Also, the thesis has demonstrated why this approach is better for project selection in a wind power portfolio by applying the model to TrønderEnergi's investment opportunities. Hopefully the results and discussions in this thesis will be useful for decision makers and further the interest of applying the concept of real options in capital budgeting in wind power investments.

References

Altinn (2015). Skatt for aksjeselskap/upersonlige skatteytere. *https://www.altinn.no/no/Starte-og-drive-bedrift/Drive/Skatt-og-avgift/Foretakets-skatt/Skatt-for-aksjeselskapupersonlige-skatteytere/*

Black, F. and Scholes, M. (1973) *The Pricing of Options and Corporate Liabilities* Journal of Political Economy, Volume 81, 1973, pp. 637-659

Bloomberg (2014). Google's Nordic Server Set to Get Asian Imitators. *http://www.bloomberg.com/news/articles/2014-06-27/google-s-nordic-server-to-get-asia-imitators-seeking-cheap-power*

Brandao, L. and Dyer, J.S. (2005) *Decision Analysis and Real Options: A Discrete Time Approach to Real Option Valuation* Annals of Operations Research, Volume 135, 2005, pp. 21–39

Bøckman, T. et al. (2008) *Investment timing and optimal capacity choice for small hydropower projects* European Journal of Operational Research, Volume 190, No. 1, 2008, pp. 255-267

Carr, P. (1988) *The Valuation of Sequential Exchange Opportunities* Journal of Finance, Volume 43, No. 5, 1988, pp. 1235-1256

Cartea, A. and Figueroa, M. G. (2005) *Pricing in Electricity Markets: A Mean Reverting Jump Diffusion Model with Seasonality* Applied Mathematical Finance, Volume 12, Issue 4, 2005

Conforti, M., Cornuejols, G., Zambelli, G. (2014) *Integer Programming* Springer Interntional Publishing Switzerland, 2014

Copeland, T. and Antikarov, V. (2003) *Real Options: A Practitioner's Guide* Cengage Learning, New York, NY10010, US

Cox, J., Ross, S, and Rubinstein, M. (1979) *Option Pricing: A Simplified approach* Journal of Financial Economics, Volume 7, 1979, pp. 229-263

Davison, M., Anderson, C.L., Marcus, B., Anderson, K. (2002) *Development of a hybrid model for electrical power spot prices* Power Systems, IEEE Transactions on , Volume 17, No.2, 2002, pp.257-264

Deloitte (2011) *Valuing wind and solar farm developers* Januar 2011, 2nd edition

Deng, S.-J. and Oren, S. (1995) *Incorporating oporational characteristics and start-up costs in option based valuation of power generation capacity* Probability in the Engineering and Informational Sciences, Volume 17, No. 2, 2003, pp. 155-181

Dixit, A.K. and Pindyck, R.S. (1994) *Investment under uncertainty.* Princeton University Press, Princeton, New Jersey, 1994

ESRU (2012). Balance of plant for wind projects. *http://www.esru.strath.ac.uk/EandE/Web.sites/11-12/Wind.BOP/bop.html*

Fleten, S.E., Maribu, K. and Wangensteen, I. (2007) *Optimal Investment strategies in decentralized renewable power generation under uncertainty* Energy, Volume 32, No. 5, 2007, pp. 803-815

Fleten, S.E. and Ringen, G. (2006) *New renewable electricity capacity under uncertainty: The potential in Norway* Probabilistic Methods Applied to Power Systems, 2006, pp. 1-8

Geske, R. (1979) *The Valuation of Compound Options* Journal of Finance, Volume 7, No. 1, 1979, pp. 63-81

Gjølberg, O. and Johnsen, T. (2007) *Investeringer i produksjon av fornybar energi: Hvilket avkastningskrav bør Enova SF legge til grunn?* http://www2.enova.no/minas27/publicationdetails.aspx?publicationID=282

Griva, I., Nash, S. G. and Sofer, A. (2008) *Linear and Nonlinear Optimization* Society for Industrial and Applied Mathematics, Second Edition, 2008

Guthrie, G. (2009) *REAL OPTIONS in theory and practice.* Oxford University Press, Inc, New York, 2009

Hull, J. (2000) *Options, Futures and Other Derivatives* 4th edition, Prentice-Hall, Upper Saddle River, New Jersey, 2000

Investopedia (2015). Return on investment. *http://www.investopedia.com/terms/r/returnoninvestment.asp*

Jablonska, M. et. al. (2011) *The multiple-mean-reversion jump-diffusion model for Nordic electricity spot spices* The Journal of Energy Markets, Volume 4, No. 2, 2011

Koller, T. et al. (2010) *Valuation: Measureing and managing the value of companies* Fifth Edition, John Wiley & Sons, Inc., Hoboken, New Jersey, 2010

Korhonen, A. (2001) *Strategic financial management in a multinational financial conglomerate: A multiple goal stochastic programming approach* European Journal of Operations Research, Volume 128, 2001, pp. 418–434

Krohn et al. (2009) *The Economics of wind energy* European Wind Energy Association

Kulatilaka, N. and Marcus, A. J. (1987) *Project valuation under uncertainty: When does DCF fail?* Continental Bank J. Appl. Finance, Volume 5, No. 3, 1987, pp. 92-100

Kumbaroğlu, G., Madlener, R., Demirel, M. (2006) *A real options evaluation model for the diffusion prospects of new renewable power generation technologies* Energy Economics, No. 30, 2008, pp. 1882–1908

Lee, S.-C. (2011) *Using real option analysis for highly uncertain technology investments: The case of wind energy technology* Renewable and Sustainable Energy Reviews, Volume 15, No. 9, 2011, pp. 4443-4450

Lee, A. H. I, Chen, H. H, Kang, H.-Y. (2009) *Multi-criteria decision making on strategic selection of wind farms* Renewable Energy, Volume 34, No. 1, 2009, pp. 120–126

Lee, S.-C. and Shih, L. (2010) *Renewable energy policy evaluation using real option model, case of Taiwan* Energy economics, Volume 32, No. 1, 2011, pp. 67-78

Lint, O. and Pennings, E. (2001) *An Option Approach to the New Product Development Process: A case study at Phillips Electronics* R&D Management, Volume 31, No. 2, 2001, pp. 163-172

Lucia, J. and Schwartz, E. (2002) *Electricity prices and power derivatives. Evidence from the Nordic power exchange* Review of Derivatives Research, Volume 5, No. 1, 2002, pp.5–50

Margrabe, W. (1978) *The Value of an Option to Exchange One Asset for Another* The Journal of Finance, Volume 33, No. 1, 1978, pp. 177-186

Martello, S. and Toth, P. (1990) *Knapsack Problems - Algorithms and Computer Implementations* John Wiley and Sons, New York, USA, 1990

McDonald, R.L and Siegel, D. (1986) *The value of waiting to invest.* The Quarterly Journal of Economics, Volume 101, No. 4, pp. 707-728

McDonald, R.L. (2014) *Derivatives Markets.* Pearson Education Limited, Essex, 2014

Meier et al. (2001) *Capital Budgeting Under Uncertainty - An Integrated Approach Using Contingent Claims Analysis and Integer programming* Operations Research, Volume 49, No. 2, 2001, pp. 196-206

Merton, R. (1973) *Theory of Rational Option Pricing* Bell Journal of Economics and Management Science, Volume 4, 1973, pp.141-183

Miller, L. T. and Park, C. S (2002) *Decision making under uncertainty-Real options to the rescue?* The Engineering Economist, Volume 47, No. 2, 2002, pp. 105-150

Mun, J. (2006) *Real Options Analysis: Tools and Techniques for Valuing Strategic Investments and Decisions* John Wiley & Sons, Inc., Hoboken, New Jersey, 2006

Munoz, J.I., Contreras, J., Caamano, J., Correia, P.F.(2009) *Risk assessment of wind power generation project investments based on real options* PowerTech, 2009 IEEE Bucharest, 2009, pp.1,8

Neftci, S. (2000) *An Introduction to the Mathematics of Financial Derivatives* 2nd edition, Academic Press, New York, 2000

Nord Pool Spot (2015)
http://www.nordpoolspot.com/historical-market-data/

Norges Bank (2015). Statsobligasjoner.
http://www.norges-bank.no/Statistikk/Rentestatistikk/Statsobligasjoner-Rente-Arsgjennomsnitt-av-daglige-noteringer/

NVE (2009). Konsesjoner.
http://www.nve.no/no/Konsesjoner/

NVE (2015). Kostnader i energisektoren.
http://publikasjoner.nve.no/rapport/2015/rapport2015.021.pdf

Park, C.S. and Herath, H.S.B (2000) *Exploiting Uncertainty - Investment opportunities as real options: A new Way of Thinking in Engineering Economics* The Engineering Economist, Volume 45, No. 1, 2000, pp. 1-36

PwC (2013). The Norwegian Market Risk Premium 2013 and 2014.
http://www.pwc.no/no/publikasjoner/deals/risikopremieundersokelse-2013-en.pdf

Rendleman, R. and Bartter, B. (1979) *Two-State Option Pricing* The Journal of Finance, Volume 34, No. 5, 1979, pp. 1093-1110

Renewablesfirst (2015). What is the ‘wind class’ of a wind turbine? *http://www.renewablesfirst.co.uk/wind-learning-centre/what-is-the-wind-class-of-a-wind-turbine/.*

Roques, F., Hiroux, C., Saguan, M. (2010) *Optimal wind power deployment in Europe — A portfolio approach* Energy Policy, Volume 38, No. 7, 2010, pp. 3245–3256

Sarepta Vind AS (2015).
http://sareptavind.no

Sarepta Vind AS (2014). Annual report.
http://sareptavind.no/investor/arsrapporter/

Schwatrz, E.S. (1997) *The Stochastic Behavior of Commodity Prices: Implications for Valuation and Hedging.* The Journal of Finance, Volume 52, No. 3, pp. 923-973

Schwartz, E.S and Smith, J.E. (2000) *Short-Term Variations and Long-Term Dynamics in Commodity Prices.* Management Science, Volume 46, No. 7, 2000, pp. 893-911

Skorodumov, B. (2008) *Estimation of mean reversion in Oil and Gas markets.* Technical Report, MITSUI/2008-10-14

Statkraft (2015). Hydropower.
http://www.statkraft.com/energy-sources/hydropower/

Statkraft (2014). Billions for renewable energy.
http://www.statkraft.com/media/news/News-archive/2014/Billions-for-renewable-energy/

Statnett (2013). Nettutviklingsplan.
http://www.statnett.no/Global/Dokumenter/Prosjekter/Nettutviklingsplan%202013/Nettutviklingsplan%202013.pdf

Statnett (2013). Konsesjon for tilrettelegging av vindkraft i Midt-Norge.
http://www.statnett.no/Nettutvikling/Namsos—Roan—Storheia/Nyheter/Konsesjon-for-tilrettelegging-av-vindkraft-i-Midt-Norge/

Thema (May 2014). *Sertifikatkraft og skatt - oppdatering.*

Thema (Jan 2014). *Usikkerhet i Elsertifikatmarkedet.*

The Norwegian Government (2011). Fornybardirektivet en del av EØS-avtalen. *https://www.regjeringen.no/nb/aktuelt/fornybardirektivet-en-del-av-eos-avtalen/id667482/*

Trigeorgis, L. (1996) *Real Options - Managerial Flexibility and Strategy in Resource Allocation* The MIT Press, Cambridge Massachusetts, 1996

TrønderEnergi Nett (2014). Regional Kraftsystemutredning For Sør-Trøndelag 2014–2034 Hovedrapport.
http://tronderenerginett.no/media/pdf/2014-2/rksu2014-hovedrapport.pdf

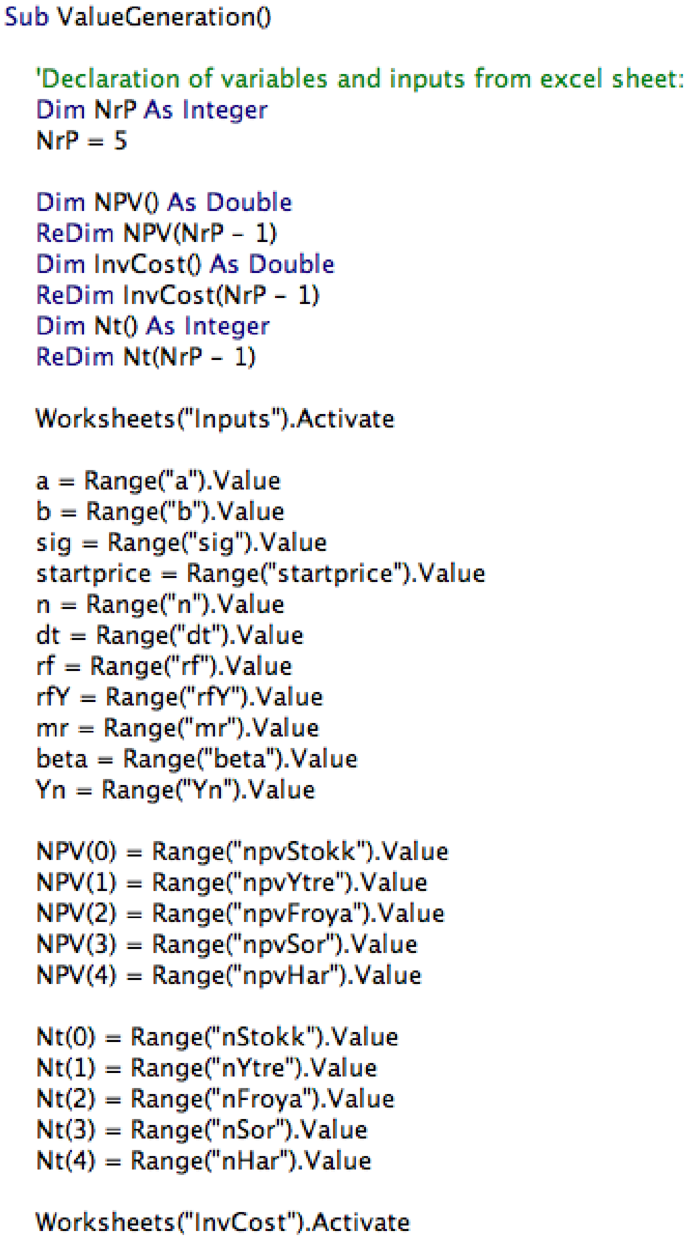
TrønderEnergi (2015).
https://tronderenergi.no

Wilmott, P. (1998) *Derivatives: The Theory and Practice of Financial Engineering* Jon Wiley & Sons, New York, 1998

Windcluster (2014). Vindkraft på Fosen og i Snillfjord. *http://windcluster.no/files/2014/12/Kristian-Aa-Presentasjon-Fosen-Vind-for-WCN-des-2014.pdf*

Appendices

A VBA Code for Scenarios Generation



```
Sub ValueGeneration()

    'Declaration of variables and inputs from excel sheet:
    Dim NrP As Integer
    NrP = 5

    Dim NPV() As Double
    ReDim NPV(NrP - 1)
    Dim InvCost() As Double
    ReDim InvCost(NrP - 1)
    Dim Nt() As Integer
    ReDim Nt(NrP - 1)

    Worksheets("Inputs").Activate

    a = Range("a").Value
    b = Range("b").Value
    sig = Range("sig").Value
    startprice = Range("startprice").Value
    n = Range("n").Value
    dt = Range("dt").Value
    rf = Range("rf").Value
    rfY = Range("rfY").Value
    mr = Range("mr").Value
    beta = Range("beta").Value
    Yn = Range("Yn").Value

    NPV(0) = Range("npvStokk").Value
    NPV(1) = Range("npvYtre").Value
    NPV(2) = Range("npvFroya").Value
    NPV(3) = Range("npvSor").Value
    NPV(4) = Range("npvHar").Value

    Nt(0) = Range("nStokk").Value
    Nt(1) = Range("nYtre").Value
    Nt(2) = Range("nFroya").Value
    Nt(3) = Range("nSor").Value
    Nt(4) = Range("nHar").Value

    Worksheets("InvCost").Activate
```

```
Budget = Range("budget").Value

InvCost(0) = Range("InvcostStokk").Value
InvCost(1) = Range("InvcostYtre").Value
InvCost(2) = Range("InvcostFroya").Value
InvCost(3) = Range("InvcostSor").Value
InvCost(4) = Range("InvcostHar").Value

Dim ValueP(), ValueF(), ValueG(), tvalue(), pvalue(), Critical() As Double
Dim aOption(), aNpv(), aPrice(), aOptionNew(), aNpvNew() As Double
Dim PriceNew(), aProb(), aProbNew() As Double

Dim j As Integer
Dim t As Integer
Dim s As Integer
Dim k As Integer
Dim Count As Integer
Dim Count2 As Integer
Dim i As Integer
Dim l As Integer
Dim Proba As Double
Dim Periods As Integer

ReDim Critical(NrP - 1, n)
ReDim aOption(Yn * n)
ReDim aNpv(Yn * n)
ReDim aPrice(Yn * n)
ReDim aOptionNew(Yn * n)
ReDim aNpvNew(Yn * n)
ReDim aPriceNew(Yn * n)
ReDim aProb(Yn)
ReDim aProbNew(Yn)
ReDim tvalue(n, (Yn + 1) ^ n - 1)
ReDim pvalue(n, (Yn + 1) ^ n - 1)
ReDim ValueP(NrP - 1, n, (Yn + 1) ^ n - 1)
ReDim ValueF(NrP - 1, n, (Yn + 1) ^ n - 1)
ReDim ValueG(NrP - 1, n, (Yn + 1) ^ n - 1)
```

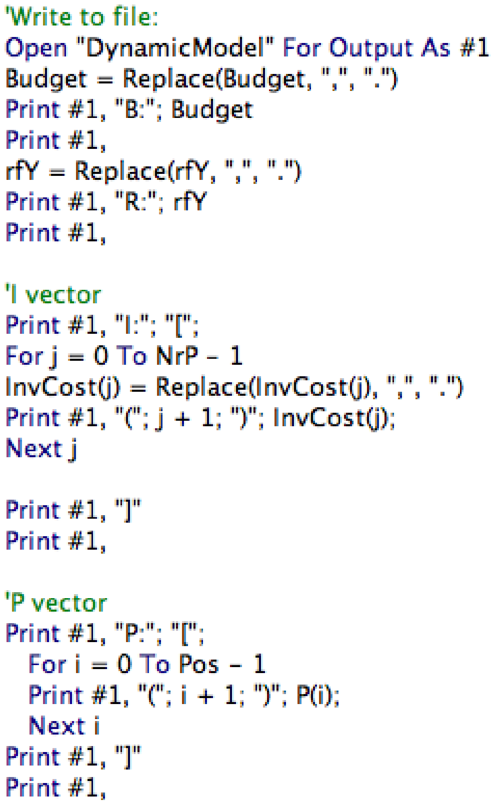
```
'Defines up and down moves:

up = Exp(sig * Sqr(dt))
down = 1 / up


'Calculate critical values for each project in each year:

For j = 0 To NrP - 1
 Critical(j, Nt(j)) = 90000000
 Periods = Nt(j) * Yn

'last column
For i = 0 To Periods

    aNpv(i) = NPV(j) * Exp((Periods - 2 * i) * sig * Sqr(dt))
    aOption(i) = WorksheetFunction.Max(aNpv(i) - InvCost(j), 0)
    aPrice(i) = startprice * Exp((Periods - 2 * i) * sig * Sqr(dt))

    If aNpv(i) - InvCost(j) >= 0 Then
      If Critical(j, Nt(j)) > aNpv(i) Then
      Critical(j, Nt(j)) = aNpv(i)
      End If
    End If

Next i

'rest of the values
For t = (Periods - 1) To 0 Step -1

   If t = 12 Or t = 0 Then ' decision node
   Critical(j, t / Yn) = 90000000

   For i = 0 To t
   aNpvNew(i) = aNpv(i) / up
   aPriceNew(i) = aPrice(i) / up
   Proba = theta_up(a, b, dt, aPriceNew(i), sig, up, down, beta, mr)
   aOptionNew(i) = WorksheetFunction.Max(aNpvNew(i) - InvCost(j), _
   (Proba * aOption(i) + (1 - Proba) * aOption(i + 1)) / (1 + rf))
```

```
  If aNpvNew(i) - InvCost(j) >= (Proba * aOption(i) + (1 - Proba) * _
  aOption(i + 1)) / (1 + rf) Then
      If Critical(j, t / Yn) > aNpvNew(i) Then
      Critical(j, t / Yn) = aNpvNew(i)
      End If
  End If

  Next i

   For i = 0 To t
   aNpv(i) = aNpvNew(i)
   aPrice(i) = aPriceNew(i)
   aOption(i) = aOptionNew(i)
   Next i

   Else ' none decision node

   For i = 0 To t
           aNpvNew(i) = aNpv(i) / up
           aPriceNew(i) = aPrice(i) / up
           Proba = theta_up(a, b, dt, aPriceNew(i), sig, up, down, beta, mr)
           aOptionNew(i) = (Proba * aOption(i) + (1 - Proba) * aOption(i + 1)) / (1 + rf)
   Next i

   For i = 0 To t
   aNpv(i) = aNpvNew(i)
   aPrice(i) = aPriceNew(i)
   aOption(i) = aOptionNew(i)
   Next i

   End If

Next t
Next j
```

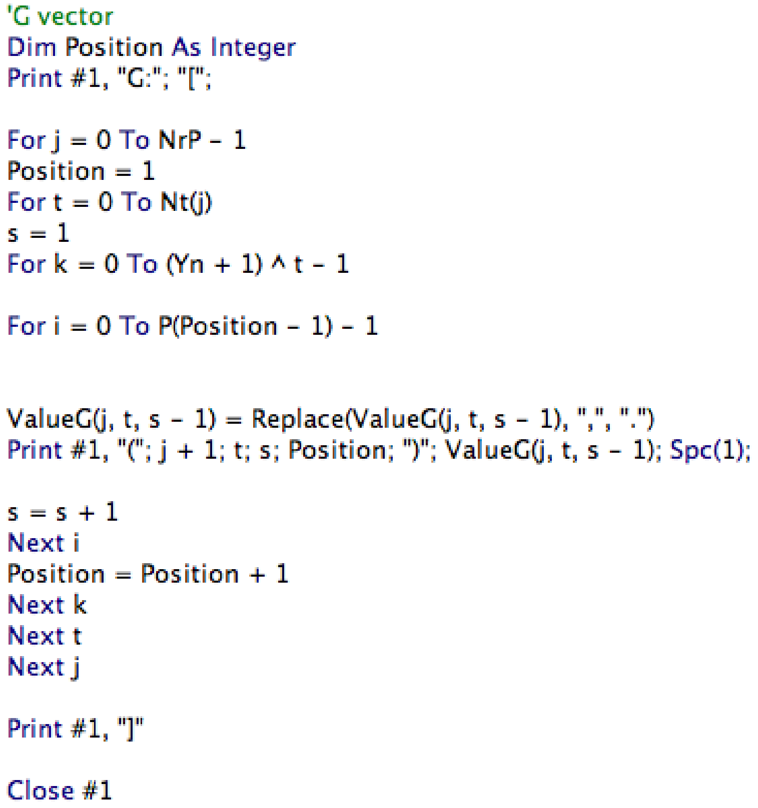
```
' Calculate probabilities

pvalue(0, 0) = startprice
tvalue(0, 0) = 1

For t = 0 To n - 1
Count = 0

For k = 0 To t * Yn
aPrice(0) = pvalue(t, k)
aProb(0) = tvalue(t, k)

For l = 1 To Yn
For i = 0 To l

aPriceNew(i) = pvalue(t, k) * Exp((l - 2 * i) * sig * Sqr(dt))

If i = 0 Then
aProbNew(i) = aProb(i) * theta_up(a, b, dt, aPrice(i), sig, up, down, beta, mr)
Else

If i = l Then
aProbNew(i) = aProb(i - 1) * (1 - theta_up(a, b, dt, aPrice(i - 1), sig, up, down, beta, mr))
Else

aProbNew(i) = aProb(i) * theta_up(a, b, dt, aPrice(i), sig, up, down, beta, mr) + _
aProb(i - 1) * (1 - theta_up(a, b, dt, aPrice(i - 1), sig, up, down, beta, mr))

End If
End If

Next i

For i = 0 To l
aPrice(i) = aPriceNew(i)
aProb(i) = aProbNew(i)
Next i

Next l
```

```
 For i = 0 To Yn
 pvalue(t + 1, Count) = aPrice(i)
 tvalue(t + 1, Count) = aProb(i)
 Count = Count + 1
 Next i

 Next k
 Next t

'Get scenarios from development in value and value function:

 For j = 0 To NrP - 1
 ValueP(j, 0, 0) = NPV(j)

 If ValueP(j, 0, 0) + 1 > Critical(j, 0) Then
 ValueF(j, 0, 0) = tvalue(0, 0) * (ValueP(j, 0, 0) - InvCost(j))
 Else
 ValueF(j, 0, 0) = 0
 End If

 For t = 0 To Nt(j) - 1
 Count = 0
 For k = 0 To t * Yn

 For i = 0 To Yn

    ValueP(j, t + 1, Count) = ValueP(j, t, k) * up ^ (Yn - 2 * i)

    If ValueP(j, t + 1, Count) + 1 > Critical(j, t + 1) Then
    ValueF(j, t + 1, Count) = tvalue(t + 1, Count) * (ValueP(j, t + 1, Count) - InvCost(j))
    Else
    ValueF(j, t + 1, Count) = 0
    End If

    Count = Count + 1
 Next i

 Next k
 Next t
 Next j
```

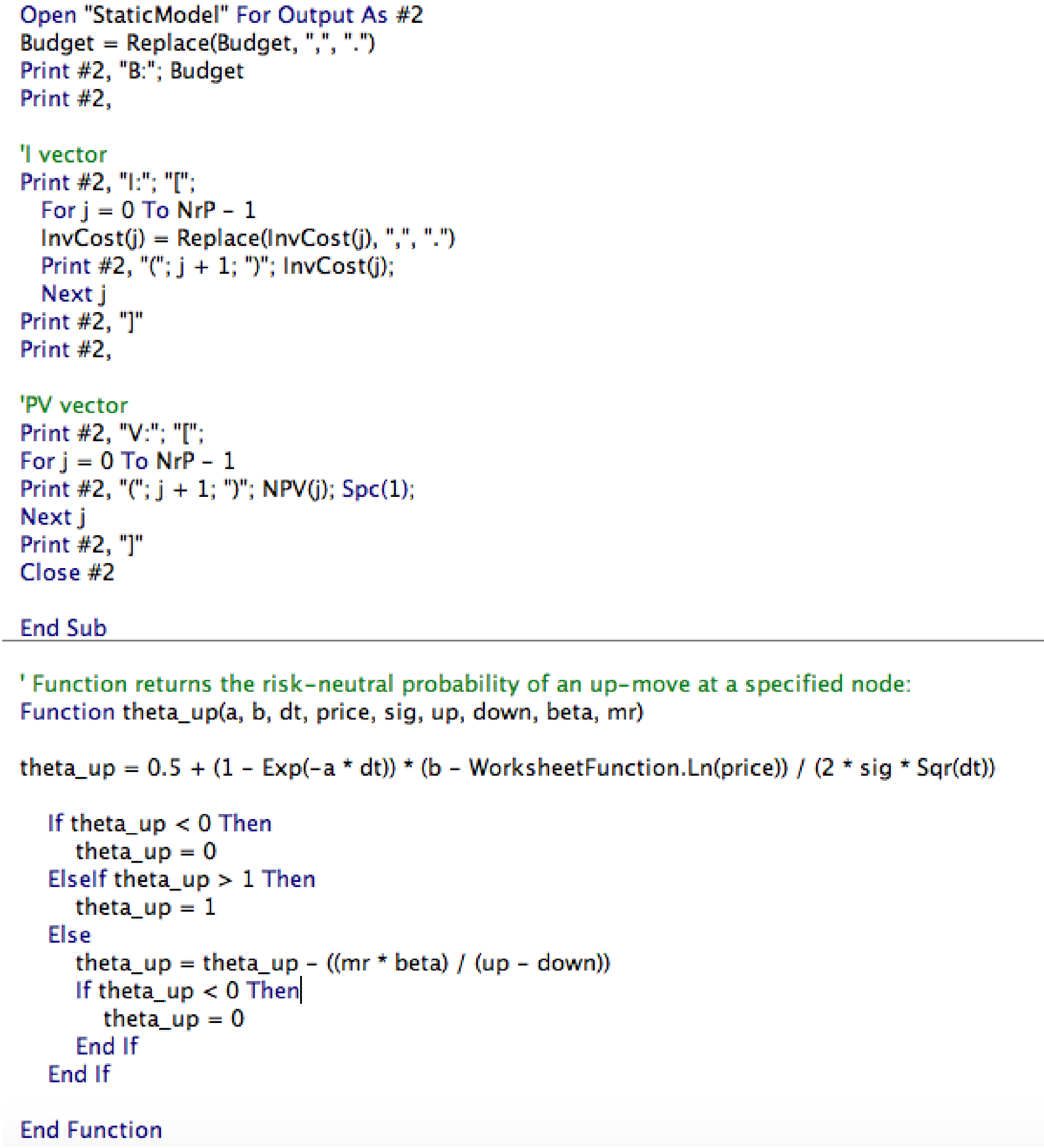
```
 'Duplicate values to get j x t x s matrix

 For j = 0 To NrP - 1
 For s = 0 To (Yn + 1) ^ n - 1
 ValueG(j, 0, s) = ValueF(j, 0, 0)
 Next s

 For t = 1 To Nt(j)
 Count = 0
 Count2 = (Yn + 1) ^ n / (Yn + 1) ^ t

 For k = 0 To (Yn + 1) ^ t - 1

 For i = 1 To Count2
 ValueG(j, t, Count) = ValueF(j, t, k)
 Count = Count + 1
 Next i

 Next k
 Next t
 Next j

'Create position vector:

Dim Pos As Integer
Pos = 0
For t = 0 To n
Pos = Pos + (Yn + 1) ^ t
Next t
Dim P() As Integer
ReDim P(Pos - 1)
Dim CountPos As Integer

CountPos = 0
For t = 0 To n
For k = 0 To (Yn + 1) ^ t - 1
P(CountPos) = (Yn + 1) ^ n / (Yn + 1) ^ t
CountPos = CountPos + 1
Next k
Next t
```



```
'Write to file:
Open "DynamicModel" For Output As #1
Budget = Replace(Budget, ",", ".")
Print #1, "B:"; Budget
Print #1,
rfY = Replace(rfY, ",", ".")
Print #1, "R:"; rfY
Print #1,

'I vector
Print #1, "I:"; "[";
For j = 0 To NrP - 1
InvCost(j) = Replace(InvCost(j), ",", ".")
Print #1, "("; j + 1; ")"; InvCost(j);
Next j

Print #1, "]"
Print #1,

'P vector
Print #1, "P:"; "[";
   For i = 0 To Pos - 1
   Print #1, "("; i + 1; ")"; P(i);
   Next i
Print #1, "]"
Print #1,
```

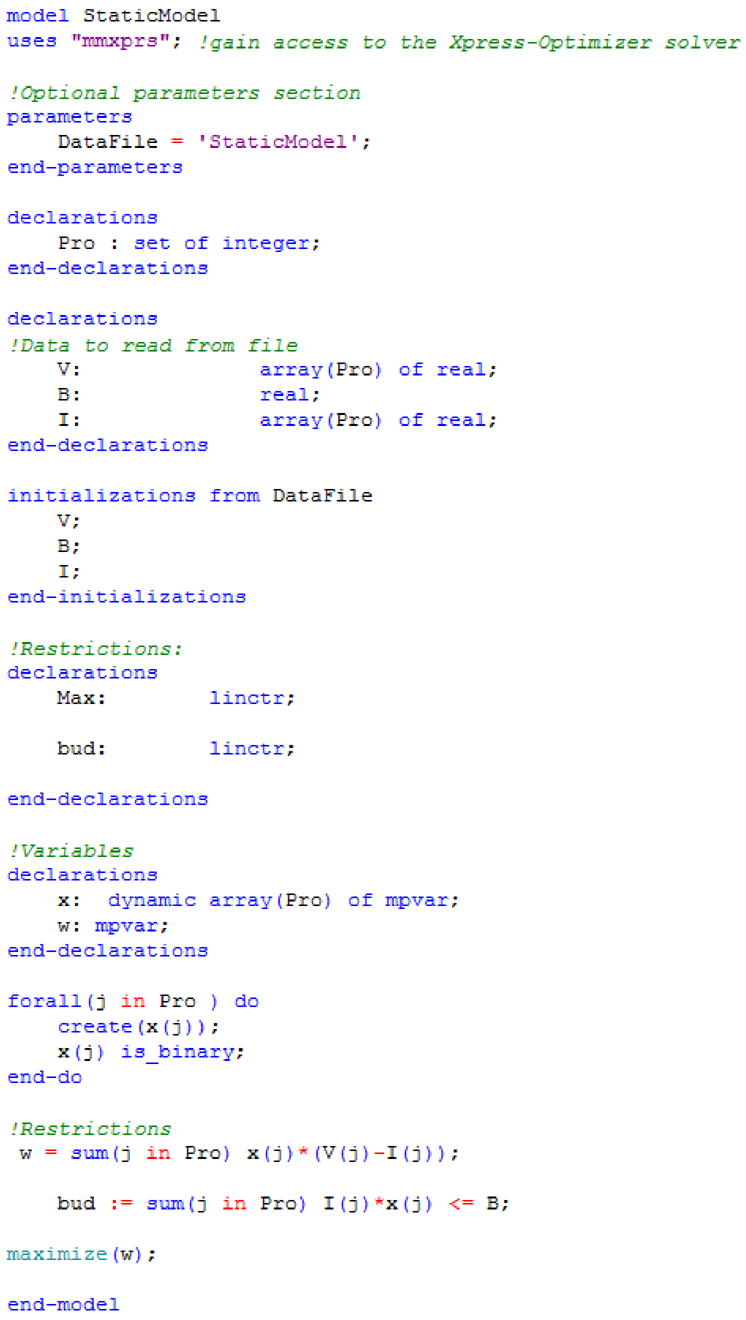


```
'G vector
Dim Position As Integer
Print #1, "G:"; "[";

For j = 0 To NrP - 1
Position = 1
For t = 0 To Nt(j)
s = 1
For k = 0 To (Yn + 1) ^ t - 1

For i = 0 To P(Position - 1) - 1


ValueG(j, t, s - 1) = Replace(ValueG(j, t, s - 1), ",", ".")
Print #1, "("; j + 1; t; s; Position; ")"; ValueG(j, t, s - 1); Spc(1);

s = s + 1
Next i
Position = Position + 1
Next k
Next t
Next j

Print #1, "]"

Close #1
```



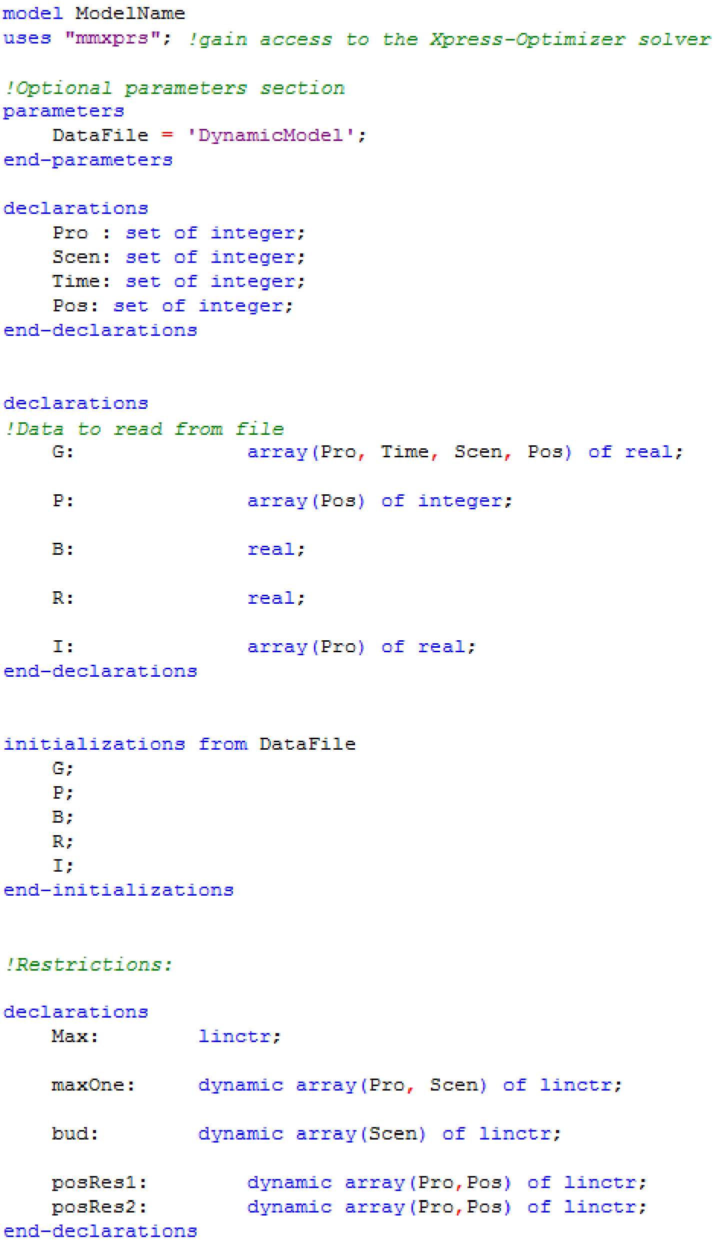
```
Open "StaticModel" For Output As #2
Budget = Replace(Budget, ",", ".")
Print #2, "B:"; Budget
Print #2,

'I vector
Print #2, "I:"; "[";
   For j = 0 To NrP - 1
   InvCost(j) = Replace(InvCost(j), ",", ".")
   Print #2, "("; j + 1; ")"; InvCost(j);
   Next j
Print #2, "]"
Print #2,

'PV vector
Print #2, "V:"; "[";
For j = 0 To NrP - 1
Print #2, "("; j + 1; ")"; NPV(j); Spc(1);
Next j
Print #2, "]"
Close #2

End Sub

' Function returns the risk-neutral probability of an up-move at a specified node:
Function theta_up(a, b, dt, price, sig, up, down, beta, mr)

theta_up = 0.5 + (1 - Exp(-a * dt)) * (b - WorksheetFunction.Ln(price)) / (2 * sig * Sqr(dt))

    If theta_up < 0 Then
        theta_up = 0
    ElseIf theta_up > 1 Then
        theta_up = 1
    Else
        theta_up = theta_up - ((mr * beta) / (up - down))
        If theta_up < 0 Then
            theta_up = 0
        End If
    End If

End Function
```

B Mosel Code for Static Optimization Model

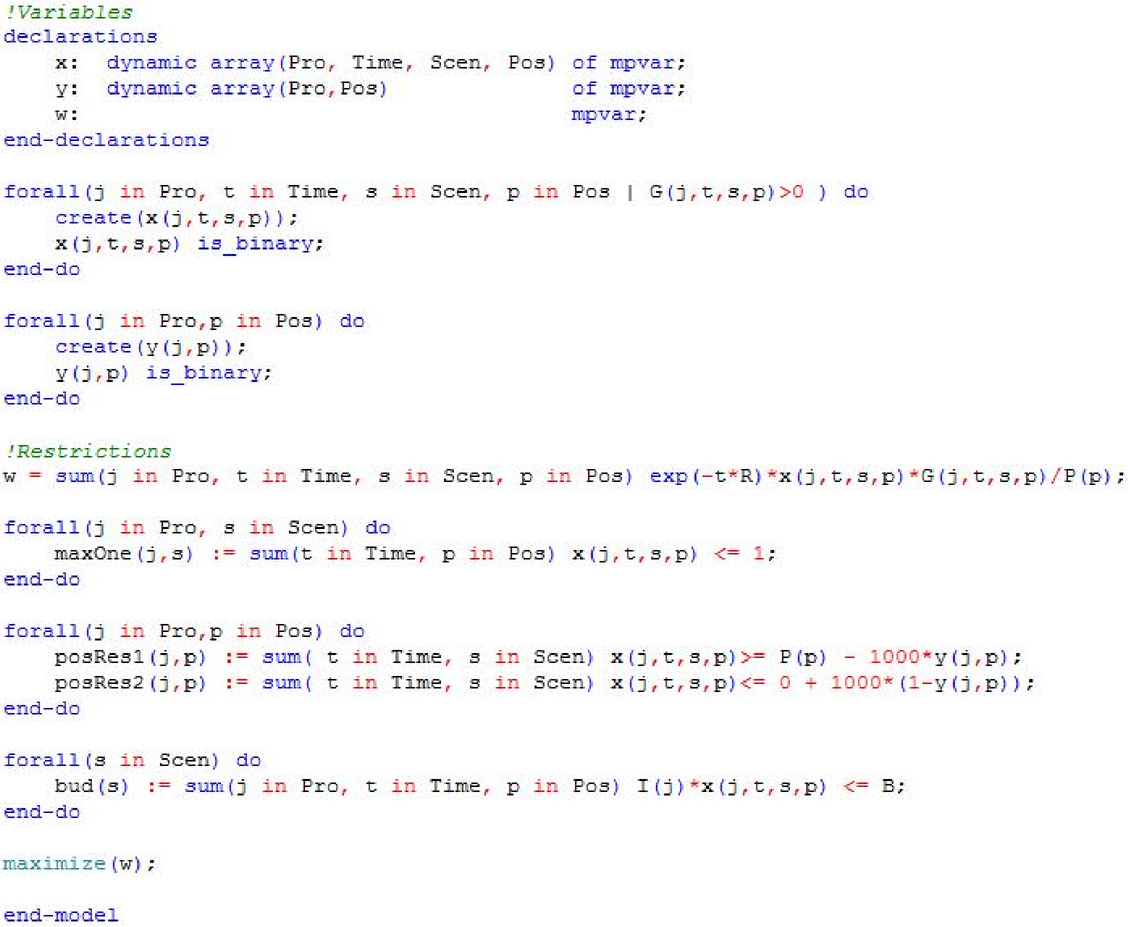


```
model StaticModel
uses "mmxprs"; !gain access to the Xpress-Optimizer solver

!Optional parameters section
parameters
    DataFile = 'StaticModel';
end-parameters

declarations
    Pro : set of integer;
end-declarations

declarations
!Data to read from file
    V:              array(Pro) of real;
    B:              real;
    I:              array(Pro) of real;
end-declarations

initializations from DataFile
    V;
    B;
    I;
end-initializations

!Restrictions:
declarations
    Max:        linctr;

    bud:        linctr;

end-declarations

!Variables
declarations
    x:  dynamic array(Pro) of mpvar;
    w: mpvar;
end-declarations

forall(j in Pro ) do
    create(x(j));
    x(j) is_binary;
end-do

!Restrictions
 w = sum(j in Pro) x(j)*(V(j)-I(j));

    bud := sum(j in Pro) I(j)*x(j) <= B;

maximize(w);

end-model
```

C Mosel Code for Dynamic Optimization Model



```
model ModelName
uses "mmxprs"; !gain access to the Xpress-Optimizer solver

!Optional parameters section
parameters
    DataFile = 'DynamicModel';
end-parameters

declarations
    Pro : set of integer;
    Scen: set of integer;
    Time: set of integer;
    Pos: set of integer;
end-declarations


declarations
!Data to read from file
    G:              array(Pro, Time, Scen, Pos) of real;

    P:              array(Pos) of integer;

    B:              real;

    R:              real;

    I:              array(Pro) of real;
end-declarations


initializations from DataFile
    G;
    P;
    B;
    R;
    I;
end-initializations


!Restrictions:

declarations
    Max:        linctr;

    maxOne:     dynamic array(Pro, Scen) of linctr;

    bud:        dynamic array(Scen) of linctr;

    posRes1:        dynamic array(Pro,Pos) of linctr;
    posRes2:        dynamic array(Pro,Pos) of linctr;
end-declarations
```



```
!Variables
declarations
    x:  dynamic array(Pro, Time, Scen, Pos) of mpvar;
    y:  dynamic array(Pro,Pos)              of mpvar;
    w:                                      mpvar;
end-declarations

forall(j in Pro, t in Time, s in Scen, p in Pos | G(j,t,s,p)>0 ) do
    create(x(j,t,s,p));
    x(j,t,s,p) is_binary;
end-do

forall(j in Pro,p in Pos) do
    create(y(j,p));
    y(j,p) is_binary;
end-do

!Restrictions
w = sum(j in Pro, t in Time, s in Scen, p in Pos) exp(-t*R)*x(j,t,s,p)*G(j,t,s,p)/P(p);

forall(j in Pro, s in Scen) do
    maxOne(j,s) := sum(t in Time, p in Pos) x(j,t,s,p) <= 1;
end-do

forall(j in Pro,p in Pos) do
    posRes1(j,p) := sum( t in Time, s in Scen) x(j,t,s,p)>= P(p) - 1000*y(j,p);
    posRes2(j,p) := sum( t in Time, s in Scen) x(j,t,s,p)<= 0 + 1000*(1-y(j,p));
end-do

forall(s in Scen) do
    bud(s) := sum(j in Pro, t in Time, p in Pos) I(j)*x(j,t,s,p) <= B;
end-do

maximize(w);

end-model
```