NTNU – Trondheim
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Optimization of an Upstream Supply Chain

Developing the Optimal Supply Chain for
Exploration Drilling Operations on the NCS

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Background
Exploration drilling is performed to prove the existence of the petrochemical resources. This operation involves extensive logistics operations in terms of mobilizing the equipment and resources required, and the operators are therefore dependent on safe and reliable supply services. The supply chain for the upstream supply services represents a significant cost for offshore operations. If these costs can be reduced through efficient logistics, it can contribute to the reduction in the overall costs of offshore operations.

Primary Objective
The aim of this thesis is to utilize optimization to determine the most cost-effective supply operation for exploration operations. The problem will address the tactical and operational aspect of the traditional supply chain, and a mathematical model will determine the optimal supply alternatives for both planned (deterministic) and unplanned (stochastic) demands by the use of a two-stage recourse model. The results and findings from the study will be utilized to initiate a discussion on the current and alternative transportation alternatives.

The overall objective of the thesis is to develop a model that can support the analysis of logistic strategies for supply to exploration drilling operations on the NCS.

Scope of work
The candidate will cover the following main points:
1) Describe the real problem
2) Present relevant literature
3) Develop a mathematical model that describes a simplified version of the real problem
4) Collect the data necessary to run the model
5) Implement and solve the mathematical data in Xpress IVE
6) Utilize the results and findings to initiate a discussion on the supply chain and possible alternative supply chains

Modus operandi
Professor Stein Ove Erikstad will be the main supervisor from NTNU. The work shall follow the guidelines made by NTNU for project work. The workload shall correspond to 30.0 credits.

Stein Ove Erikstad

Professor/Main Supervisor

SUMMARY

The oil and gas industry is currently the largest industry in Norway, and it is expected to remain so in years to come. Only 44 % of the projected total recoverable resources have been extracted, and new discoveries are still being made. In order for this industry to maintain its position, new resources must be proven, and this requires exploration drilling. During these operations, the operators require an extensive supply of equipment and services, and are therefor highly dependent on safe and reliable supply services. Furthermore, there is an increasing focus on cost-effectiveness for offshore activities, and it is believed that an optimal upstream supply logistic will provide a significant reduction in these cost levels.

This report presents an academic study of supply logistic strategies for exploration drilling operations on the Norwegian Continental Shelf (NCS). The aim of this study is to utilize optimization to determine the most cost-effective supply operation for exploration drilling for both planned and unplanned demands, and use this to initiate a discussion on the supply logistics.

Offshore supply operations are complex, and good logistics and planning are therefore essential elements in achieving excellence. Traditionally, experienced logistic personnel perform the scheduling and route planning manually. But due to the significant amount of variables that must be considered, manual planning may fail to investigate all possible solutions. Therefore, optimization can be used as a decision support tool, and aid the planners in their work.

The problem presented in this thesis is a planning problem in which the operator is responsible for the inventory management, and the routing and scheduling of the deliveries. These problems are classified as Inventory Routing Problem (IRP). This methodology enables the planners to evaluate both the optimal inventory levels and the routing and scheduling decisions, which provides a supply chain that optimize both aspects. The thesis addresses the tactical and operational aspect of the traditional supply chain, and a mathematical model will determine the optimal supply alternatives for both planned (deterministic) and unplanned (stochastic) demands. The mathematical model that is developed for this study is a mixed integer, two-stage recourse model.

The model is implemented in the commercial software FICO™ Xpress Optimization Suit, and tested on a case study in which four offshore facilities require supply services. The solution from the case study yields that the four offshore installations can be serviced by two PSVs during a time horizon of five days, and all the installations should have two visits each. To address the issue with unexpected demands that makes the estimated deterministic stock levels insufficient, late deliveries are performed. The preferred alternative for the late deliveries is to use helicopters. However, as the amount of the unexpected demands increases, an alternative that combines an additional storage at the onshore base and the spot chartering of an additional PSV, becomes the preferable solution. The cost of the estimated planned deliveries is 88,707 $, and the estimated cost of the late deliveries is 17,514 $.

The cost saving of using late deliveries compared to the risk of downtime, is estimated to be approximately 20 %, this is therefor the preferred solution. Still, the cost of the late deliveries might get extensive. If information about the real demands can be revealed during the planning of the initial schedules, these demands can be incorporated in the schedules, which has a potential cost saving of 16 %.

SAMMENDRAG

Olje- og gassindustrien er i dag den største industrien i Norge, og det er forventet at den vil holde sin posisjon i flere år. I dag er kun 44 % av de prosjekterte gjenvinnbare reservene på norsk kontinentalsokkel hentet ut, og det gjøres fremdeles nye funn. For at denne industrien skal kunne holde sin produksjonen og aktivitet ved like, må nye reserver bevises, og dette krever leteboring. Under disse operasjonene, krever operatørene en omfattende mengde forsyninger og andre tjenester, og de er derfor avhengige av trygge og pålitelige forsyningstjenester. Det er et stadig økende fokus på kostnadseffektivitet, og det hevdes at optimale forsyningskjeder kan føre til en signifikant reduksjon av kostnadsnivået for offshore operasjoner.

Denne rapporten representerer et akademisk studie av strategier for forsyningskjeder for leteboring på norsk kontinentalsokkel. Formålet med oppgaven er å benytte optimering til å bestemme den mest kostnadseffektive forsyningskjeden for boreoperasjoner. Dette vil bli undersøkt for estimerte etterspørsler som kan planlegges på et tidlig stadium, og etterspørsler som ikke kan bli forusett, og derfor anses som uventede.

Offshore forsyningsoperasjoner er komplekse, og gode logistikkløsninger og planlegging er derfor essensielle elementer for å oppnå de mest effektive operasjonene. I tradisjonell planlegging, er det erfarne planleggere som utformer logistikken og seilmønstre manuelt. Men disse operasjonene er svært sammensatte, og det er derfor lett å overse mulige løsninger. Optimering kan derfor benyttes som et redskap for beslutningsstøtte, og hjelpe planleggerne med å oppnå de beste løsningene.

Denne masteroppgaven ser på et problem der operatøren er ansvarlig for både lagrene på offshore installasjonen, og leveringen av inventaret. Denne klassen problemer kan defineres som inventar og rute problemer (IRP). Metoden muliggjør at planleggeren kan gjøre beslutninger for både inventar, samt rute- og tidsplanleggingen, noe som kan gi en optimal forsyningskjede i alle ledd. Oppgaven ser på de taktiske og operasjonelle aspektene ved en tradisjonell forsyningskjede, og en matematisk modell er utviklet for å bestemme de optimale forsyningsalternativene for både planlagte og uventede forespørsler. Den matematiske modellen er en blandet to stegs modell med korrigering.

Modellen er implementert i den kommersielle softwaren FICO^{TM} Xpress Optimization Suit, og testet på et tenkt problem med fire offshore installasjoner som krever forsyningstjenester. Løsningen fra studiet viser at det kreves to forsyningsfartøy for å betjene de fire installasjonene, og alle installasjonene besøkes to ganger hver. De uventede forespørslene kan bli betjent ved hjelp av senere leveringer, og det foretrukne alternativet er å benytte helikoptre til dette formålet. Det viser seg imidlertid at dersom etterspørslene øker i omfang, er det foretrukne alternativet å ha nødlager på den landbaserte forsyningsbasen og benytte spot chartrede forsyningsfartøy til å gjøre disse leveringene. Kostnaden ved å utføre de planlagte forsyningene er beregnet til 88 707 $, og estimatet for kostnaden ved sene leveringer er satt til 17 514 $.

Kostnadsbesparelsen ved å ha sene leveringer når alternativet er å risikere nedetid, er estimert til 20 %, sene leveringer er derfor den foretrukne løsningen. Kostnadsnivået for de sene leveringene kan imidlertid bli svært høye, og øker med økende etterspørsel. Hvis informasjonen om de uventede forespørslene kan bli avdekket på et tidligere tidspunkt, vil de kunne bli inkludert i tidsplanen for de planlagte forespørslene. Dette har en estimert kostnadsbesparelse på 16 %.

PREFACE

This master thesis has been written during the spring 2015 at the Department of Marine Technology, Norwegian University of Science and Technology (NTNU). The objective and scope of this thesis was developed in cooperation with my supervisor, and the workload for the thesis is equivalent to 30.0 credits.

This work is a continuation of my project thesis from the fall semester of 2014, which discussed alternative supply logistics for wireline operations in the Barents Sea. Therefore, the information on supply chain logistics, the optimization process and the literature study of inter-modal optimization, build up on the information that was obtained during the work on the project thesis.

Working with this thesis has been rewarding. I have become aware of how interesting and rewarding it can be to get the opportunity to really dig into a problem, and get a thorough understanding of it.

I would like to thank my academic supervisor at NTNU, Professor Stein Ove Erikstad at the Department of Marine Technology for guidance, advice and support during the work with this thesis. I would also like to thank PhD Øyvind Selnes Patricksson for all his help with the Mosel Xpress programming. Finally, I would like to thank Per Killingmo for valuable input and help with the understanding of offshore operations on the NCS.

Trondheim, June 10th, 2015

Hanne Dreyer Engh

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

List of Abbreviations

DWT	Dead Weight Ton
EVPI	Estimated Value of Perfect Information
FCNF	Fixed Charge Network Flow
IRP	Inventory Routing Problem
ISRP	Inventory Ship Routing Problem
LNG	Liquefied Natural Gas
LP	Linear Programming
MDP	Markov Decision Problem
MIP	Mixed Integer Problem
MIRP	Maritime Inventory Routing Problems
MODU	Mobile Offshore Drilling Unit
NCS	Norwegian Continental Shelf
NPD	Norwegian Petroleum Directorate
PDP	Pick-up-and-Delivery Problem
PSV	Platform Supply Vessel
SCM	Supply Chain Management
SIRP	Stochastic Inventory Routing Problems
TSP	Travelling Salesman Problem
VRP	Vehicle Routing Problem
VSS	Value of Stochastic Solution
WOW	Waiting on Weather

CHAPTER 1

INTRODUCTION

The Norwegian petroleum industry came into being in 1969 with the discovery of Ekofisk, and is today the largest industry in Norway. It has been of high importance to the Norwegian economy and has been the main contributor to the development of the welfare state. Furthermore, the industry employs a substantial amount of people, both directly in oil or oil service companies, but also through a ripple effect. Thus, the petrochemical industry is without question of considerable importance to Norway. Some figures that illustrates this are provided in Figure 1.(NPD, 2014)

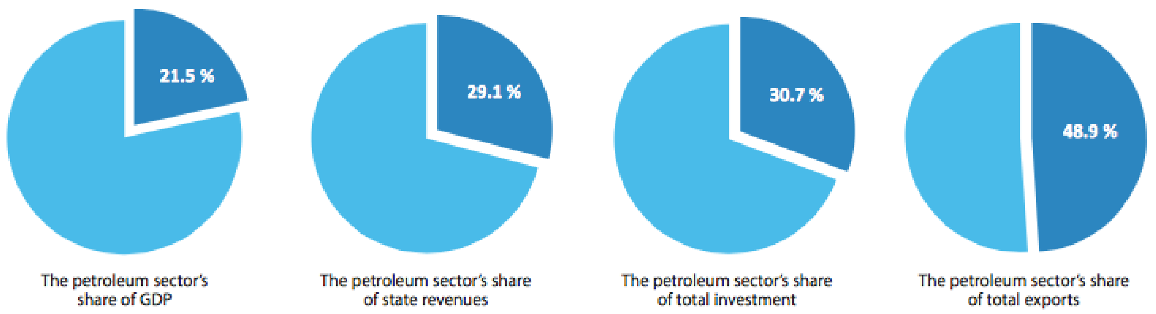

FIGURE 1 MACRO-ECONOMIC INDICATORS FOR THE PETROLEUM SECTOR 2013 SOURCE: (NPD, 2014)

The oil and gas industry is in growth and will continue to be a significant contributor to the Norwegian economy in years to come. Only 44 % of the projected total recoverable resources have been extracted since the production started in 1971, and new discoveries are still being made on the Norwegian continental shelf (NCS). This can be emphasized by the major discoveries of Johan Sverdrup and Johan Castberg that has been done in recent years.(NPD, 2014) A recent review by the Norwegian Petroleum Directorate (NPD) shows that the current recoverable volume of oil in fields and discoveries, are larger than the estimated figures from 2005, and it is assumed that there are more undiscovered resources.(Veggeland, 2015) NPD's production forecast for oil and gas is illustrated in Figure 2, and clearly illustrates that the production will remain high in years to come.

The cost level for the operations at the NCS is a major challenge for the development of the industry, and must therefore be addressed. In recent years the cost level has increased, and in combination with falling oil prices, this has led to postponements or cancellations of several projects.(DN, 2015) Drilling wells is the most important operation to maintain and increase production, but there are drilled fewer wells than what is planned, and this may jeopardize the utilization of the resources.(Osmundsen, 2015)

The operators of offshore oil and gas fields are dependent on safe and reliable supply services. The supply chain for the upstream supply services represents one significant cost for offshore operations, and if these costs can be reduced through efficient supply logistics, it can contribute to the reduction of the overall costs, and thus increased exploration activities. Ensuring a cost-effective supply service is therefore crucial for the industry. Furthermore, a large share of the undeveloped resources is located in frontier areas, such as the Arctic. These areas often have sparse and underdeveloped infrastructure, and place higher requirements on the supply services. The lead times in these areas are significant and consequently also the costs, which means that ensuring a cost-effective supply chain might become even more significant in the future with operations at new frontiers.

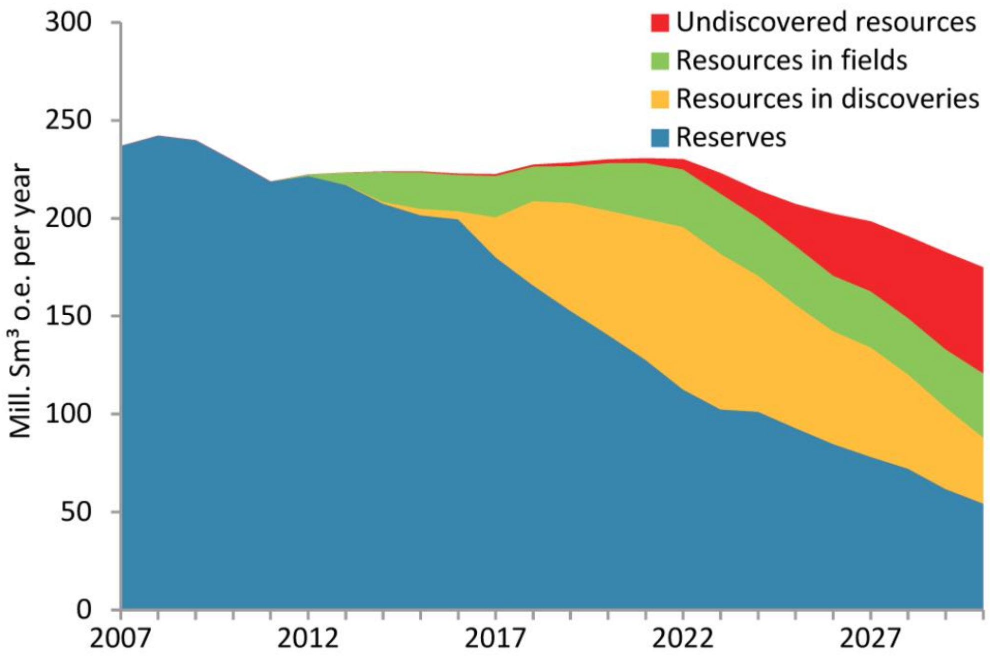

FIGURE 2 PRODUCTION FORECAST FOR OIL AND GAS. SOURCE: (NPD, 2014)

The aim of this thesis is to utilize optimization to determine the most cost-effective supply operation for the supply services in exploration operations. The problem will address the tactical and operational aspect of the traditional supply chain, and a mathematical model will determine the optimal supply alternatives for both planned (deterministic) and unplanned (stochastic) demands by the use of a two-stage recourse model. The results and findings from the study will be utilized to initiate a discussion on the current and alternative supply chains.

The thesis is structured as follows. Firstly, an introduction to the problem is provided in the background description in Chapter 2. It will discuss the exploration services, the offshore supply chain and integrated logistics. The methodology used to solve this problem and the reasoning for choosing this methodology will be described in Chapter 3. Chapter 4 will present relevant literature and research done on similar problems, with a special focus on Inventory Routing Problems. The problem formulation, which is a simplification of the real problem, will be given in Chapter 5. Chapter 6 will present the mathematical model developed for this problem. The implementation and the collection of the data necessary to run the model in the commercial software FICO™ Xpress will be described in Chapter 7. A discussion of the results and model will follow in Chapter 8. Lastly, the concluding remarks and ideas for further work will be provided in Chapter 9.

CHAPTER 2

BACKGROUND

This chapter will provide the background information that is considered relevant for this thesis and the development of the mathematical model. The chapter will begin with a brief introduction to the importance of exploration drilling in section 2.1. A traditional supply chain for offshore operations, and the main characteristics for the individual stages will be described in section 2.2. Finally, section 2.3 will describe the challenges faced by the supply chain that may prevent it from being optimal, and the concept of integrated logistics will be presented as a solution to these challenges.

2.1 EXPLORATION DRILLING

The petrochemical value chain consists of four phases, which are illustrated in Figure 3.

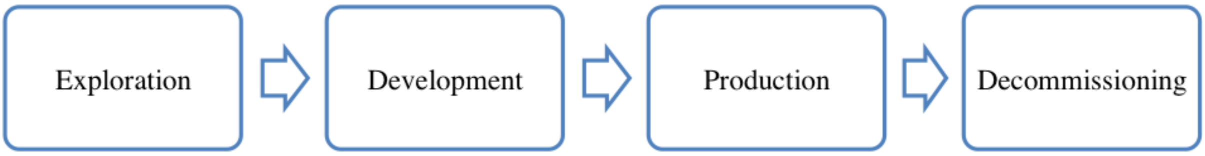

FIGURE 3 PETROCHEMICAL VALUE CHAIN. SOURCE: (YEO, 2013)

In order for the petrochemical resources to be extracted, they must be proven, which is done in the exploration phase. Consequently, the decision of whether or not an area is suited for production is formed by the result of the exploration. Exploration can be described as a sequence of activities; first the geoscientists must gain information and analyse the potential wells through the collection of electrical data, physical sampling profiles and logs. If the analysis provides promising results, an exploration well is drilled, which is the only way to confirm the presence of hydrocarbons.(Mohn & Osmundsen, 2008)

This thesis will focus on the exploration drilling stage, which can last from weeks to a few months, there can also be more than one well at a site.(Kaiser, 2010) The drilling operations can be performed in areas with a potential for new fields, or in an existing field in order to increase its recoverable resources. Exploration drilling often distinguishes between exploration wells and appraisal wells. The exploration wells test a given volume or area for hydrocarbons, and there are usually one or two exploration wells in a prospect. If the exploration well is a discovery, the operators often drill appraisal wells in order to define the extension of the reservoir.(Amado, 2013)

Exploration drilling are costly and risky operations. If the operations are successful, and discoveries are made, the profit may be significant. However, there is a considerable risk that the wells are dry and the operations are considered unsuccessful, in which case they will be costly without any prospect of profits. Still, the potential rewards are so substantial that the companies are willing to take the risk. The risk of performing appraisal drilling is often less significant than the exploratory drilling, but the rewards are also reduced.(Mohn & Osmundsen, 2008) The costs related to exploration drilling are provided in the pie chart in Figure 4, and the cost of oil service, logistics and equipment and materials, make up a great part of the costs related to these operations.(Osmundsen, Roll, & Tveterås, 2010)

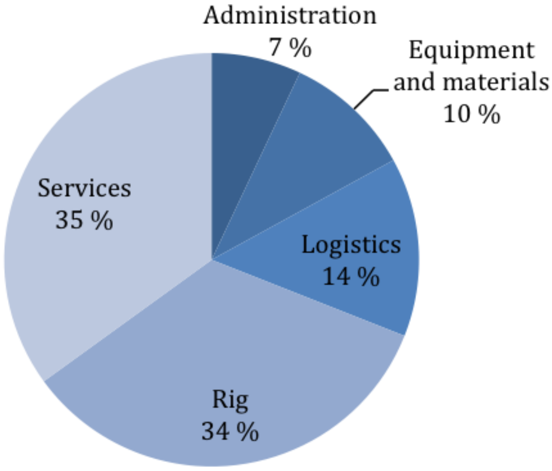

FIGURE 4 COMPOSITIONS OF DRILLING COSTS. SOURCE: (OSMUNDSEN ET AL., 2010)

Companies operating at the NCS are currently in a changeover process, and the focus on reducing costs is increasing. In combination with low oil prices, this has led to a decrease in the activities on the shelf, and it is a concern that there are not being drilled enough wells. (Osmundsen, 2015) One of the reasons why this is considered a problem is that the mature areas have an extensive infrastructure, which should be fully utilized. In these areas, it is likely that new discoveries can be made, but it is not likely that these are major. It is therefore important to recover the resources in these areas before they are shut down. Minor discoveries might not justify the establishment of new infrastructure, which means that these resources may be lost.(NPD, 2014) Reducing the costs of the supply services may be an important step in the process of increasing the exploration activity, which reduces the potential losses, which again might lead to increasing production.

2.2 THE UPSTREAM SUPPLY CHAIN

Chima (2011) defines Supply Chain Management (SCM) as *"the configuration, coordination and continuous improvement of a sequentially organized set of operations"*. Furthermore, if seaborne transportation constitute one vital link, the supply chain is a maritime supply chain.(Christiansen et al., 2007) Hence, this thesis will deal with the supply chain management of a maritime supply chain problem.

The goal of a supply chain is to provide optimal customer service at the lowest price possible, and the company must maximize profits and minimize costs along the chain. According to Chima (2011) there are few industries that can benefit more from maximizing supply chain efficiency than the oil and gas industry. A discussion on the potential for improvements will be provided in section 2.3.

The upstream supply chain for exploration operations is complex, and made up by several links. A traditional supply chain constitute the registration of demand, delivery from the supplier's facility, land-based transportation from the supplier to the onshore base, sea transportation from the onshore base to the offshore facility, and the transportation of backloads from the offshore facility to the onshore base.(Logistikkportalen, 2014) The various segments of the chain and how they are linked are illustrated in Figure 5.

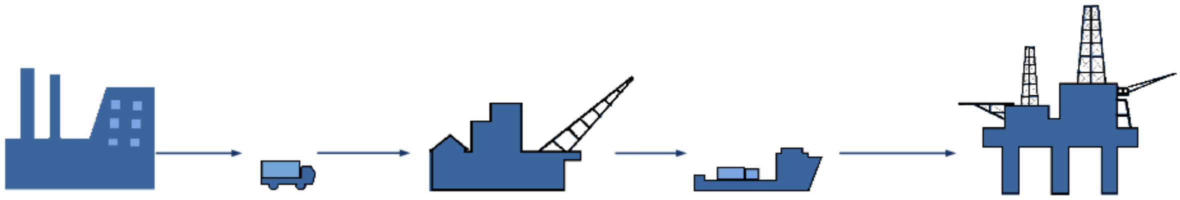

FIGURE 5 TRADITIONAL SUPPLY CHAIN

In the supply chain, all the contributors are highly dependent on each other, thus the entire chain is highly sensitive to delays and unforeseen events that will propagate through the remaining chain. Offshore operations are complex, the investments large and the margin for error is small; therefore, it is of outmost importance that the supply chain is robust in order to avoid operational failure due to late or wrong deliveries. An example of how the elements may be distributed and linked geographically is provided in Figure 6, and a description of the contributors will be provided in the sections below.

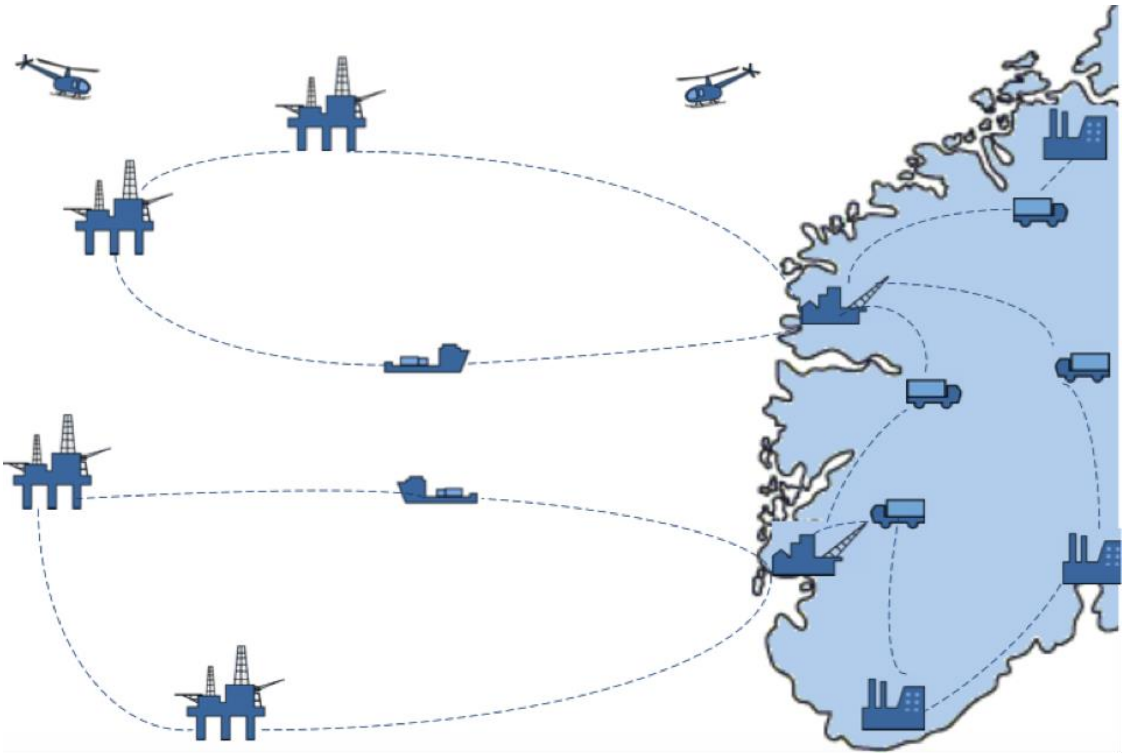

FIGURE 6 SUPPLY NETWORK

SERVICE PROVIDERS

The service providers are the oilfield service companies. These deliver services for the entire value chain from exploration through production, and eventually decommissioning. When the offshore installations place an order for equipment, commodities or services, the service providers must ensure that these requirements are met in time.

When the offshore operators place an order with the service provider, this order is registered and the demands are prepared for transportation. This preparation includes a control for defects, packing, securing and labelling of the commodities and equipment. The labelling is crucial in order to prevent misplaced deliveries and mishandling. The supplier must also provide a bill of lading that must accompany the delivery from the supplier site to the final offshore destination. Finally, the demands are loaded to trucks or semi-trailers, and transported to the onshore supply base.(Logistikkportalen, 2014) If the offshore facility requires service personnel, these are transported with helicopters.

The inventory costs may be significant for the demands. The contractors charge a standby charge for a given service or piece of equipment, and this applies for a predetermined period of time, which typically is minimum 30 days. The standby charge may also be prolonged if the service is provided in remote regions for which the lead times are longer. Additionally, there will be an operational charge that is based on the equipment usage. There are different procedures amongst the contractors for when the fee is charged.(Nardone, 2009b) Some contractors apply the charges the moment the equipment departs for the offshore facility. Furthermore, some apply the charges when service personnel arrive at the platform, whereas some apply the charges when the operations are completed. It is a common practice that the standby charges are stopped when the operational charges are applied.(Nardone, 2009b)

Equipment for drilling operations is expensive, and due to the high costs of the mobilization charge, short lead times and timely deliveries are desirable for the offshore operators. Some equipment is limited or custom-built, and must be obtained from an overseas or foreign location. This equipment is therefore not widely available, which is something that must be considered as equipment can fail and may need to be replaced. Thus, the operators must make a trade off between the risk of downtime due to missing equipment or increased hiring costs due to redundant equipment.

LAND-BASED TRANSPORTATION

The equipment and commodities are loaded to a land-based transportation mode, often trucks or semi-trailers, which deliver the demands to the onshore supply base. The location of the suppliers’ facilities and onshore supply base will be the deciding factors for the lead time for this transportation.

In Norway, there are laws and regulations that require the driver to take regular breaks, and this place an upper bound on the number of consecutive hours a driver is allowed to drive. This is something that must be evaluated for long distances, as it might require additional time for a change of driver or the driver to rest.

ONSHORE SUPPORT BASE

Exploration activities often take place in remote areas, thus the companies must have support bases to support the operations. These bases are often placed at the nearest port to the operations.(Nardone, 2009a) The offshore facilities are assigned an area at the base for storage of equipment and spare parts. Hence, the required equipment and commodities can be retrieved from the storage or from the supplier. Onshore support bases often have opening hours, which is something that must be considered in the scheduling of the loading and unloading of the vessels, as these operations require available service personnel.

The demands that are retrieved from the supplier are upon the arrival at the supply base checked for defects and the accompanying documentation is controlled. If the goods are delivered from a foreign supplier, they may be subject to duties and must therefore be cleared before they can be shipped to the offshore facility.

The sailing schedule is prepared at the onshore supply base. The schedule describes which facilities that will be visited and at which facility the equipment will be delivered. The onshore base must develop an activity plan for the unloading of the vessel; this plan includes the handling of the loads and the necessary lifting capacity. The planning of load is complex and must consider how to best utilize the deck area, the weight and size of the load, in addition to the physical characteristics that might affect the loading and unloading at the various offshore facilities. If these plans are optimized the lay time at the facilities and the number of crane lifts can be reduced, which will reduce the delivery time.

The onshore base must also be prepared to receive backorders. Before a vessel arrives, the base must be informed about the backloads and the services required.(Logistikkportalen, 2014)

PLATFORM SUPPLY VESSELS

The platform supply vessels (PSV) represent one of the most significant costs in the offshore supply chain. PSVs are purposely built vessels that are powerful and designed to maximize cargo capacity, and they can carry everything that is required for offshore operations. The vessels are designed with large, open flat decks that are easily accessible for cranes. The vessels transport essential supplies from an onshore location to the offshore facility.

The offshore facilities may require supplies of both liquid cargo and equipment. The load and type of order must be carefully considered in the scheduling process. The limiting factor is often the deck space for deck cargo. Equipment is placed in offshore containers or baskets on deck, and the equipment that is too large to be fit in these require specially manufactured protective frames with lifting points that enable them to be lifted like the containers. The liquefied cargo, such a drilling fluids, fresh water and fuels, is placed in containers below deck.(Nardone, 2009a)

The number of vessels and the deck space on each vessel is limited, therefore it is necessary to prioritize the equipment load out, and this require coordination between the rig logistics, support base logistics, contractor logistics and the well test engineers. The cargo type on the preceding shipment might also affect which types of cargo the vessel can transport on the next voyage; such cargo types are often chemical. (Nardone, 2009b)

The supply vessels used for offshore operations are usually not owned by the oil and gas companies, but hired from shipowners or brokers. Vessels can be hired on a long-term basis, which is referred to as a time charter, and the charter party for time charters are often valid for one to three years. Vessels can also be hired for individual operations, which is referred to as a spot charter, and the rates of chartering a vessel on spot is significantly higher than the time charter rates. Thus, the oil and gas companies strive to utilize their fleet of time chartered vessels as efficiently as possible. If the fleet of time chartered vessels is not sufficient, spot charters are often used to cover the excess demands. The fleet of time chartered vessels change dynamically over the course of a the year, and makes the foundation for the weekly sailing schedules.(Maisiuk & Gribkovskaia, 2014)

OFFSHORE FACILITY

The exploration is performed at the offshore site, and this is done from a Mobile Offshore Drilling Unit (MODU). There are three types of MODUs; jack-ups which are standing on the sea bed, semi-submersibles which are floaters, and drill ships, but there is only one drill ship on the NCS.(Osmundsen, 2015) The exploration installations are often open 24/7, thus the opening hours is not an issue for the loading and unloading operations.

The demand from the offshore site may vary from week to week, but also within a week, depending on the operational conditions. Due to the high equipment costs and limited storage on the facilities, the redundant equipment and waste must be returned to the onshore base. These loads are the backloads, which are loaded to the vessel after it has delivered the required cargo at the facility, and then returned to the onshore base. Approximately 75% of the loads delivered to the offshore facilities are backloads, and upon the arrival at the onshore base they are sent for reparations, stored at the base, returned to the supplier or disposed of.(Logistikkportalen, 2014)

Before the PSV arrives at the facility, the deck space at the facility must be cleared in order to unload the equipment from the PSV and the required crew must be present. The loading and unloading of the demands and backloads require lifting, which is the most critical and hazardous operation. Therefore, good communication and trained personnel is of outmost importance. The loading and unloading operations are time consuming, but good logistics may reduce lay times while maintaining safe and reliable operations. Weather conditions must always be considered during these operations, and special care must be taken in case of rough weather.

HELICOPTER

The common practice on the NCS is to transport personnel by helicopters. The helicopters also transports the travellers’ luggage, and mail and newspapers. Helicopters are also to a certain extent used for emergency deliveries, but the application is limited due the capacity.

This is an efficient, safe and reliable mode of transportation, and numerous safety regulations apply to the operation. During night, the helicopters can only perform at manned installations, the only exception are technical emergencies, and there is a limit to the distance helicopters kan fly without refuelling. There are also several other regulations that apply to helicopter operations in connection to the weather conditions, some of which will be discussed in section 2.3. (Norwegian Oil and Gas Association, 2011)

2.3 The Supply Chain Planning and Integrated Logistics

Offshore supply operations are complex, and therefore logistics and planning are essential elements in achieving excellence in the operations. The traditional planning levels are defined as strategic, tactical and operational. The strategic level concerns the planning of long-term decisions. Fleet size and mix, and location and size of warehouses, onshore bases and offshore facilities, are typical strategic planning issues. Strategic planning relies on forecasts and assumptions, and may often make decisions based on highly fluctuating and uncertain variables.(Christiansen et al., 2007)

Strategic planning forms the foundation for the tactical and operational planning. Tactical planning deals with decisions that are applicable for short periods, for instance during an exploration phase. Tactical decisions include routing and scheduling, and adjusting the fleet size and mix. Whether to charter in a vessel from the spot market is another tactical decision. Studies on the supply chain aspect in tactical planning are very limited. Operational decisions deal with short-term decisions, like the selection of cruising speed, scheduling and ship loading. (Christiansen et al., 2007) As this thesis will investigate routing and scheduling, and inventory decisions, it will focus on tactical and operational planning.

2.3.1 Challenges in Supply Chain Planning

The offshore logistics are subject to several challenges, and these can be addressed with robust and efficient planning. The most important challenges will be discussed in this section.

Geographical Consideration and Weather Conditions

Favilla et al. (2012) state that some of the more pressing challenges in traditional offshore logistics are the large number of platforms that are located at increasing distances from shore. There are several reasons why this is such a significant challenge, and it is becoming more prominent as offshore operations are moving to new frontiers, such as deep water operations in Brazil and operations in the Arctic. Longer distances mean longer lead times, which increases the time the inventory must be hired and thus the inventory cost, additionally the transportation costs increase with longer distances.

The metocean conditions are also factors that may have a significant impact on the lead times and even more so when the distances increase.(Aas et al., 2009) Metocean is a term that is derived from “meteorological” and “oceanic”, and refers to the conditions at sea.(Nardone, 2009a) Offshore operations are vulnerable to wind, sea and weather conditions, and there are several safety regulations that restrict operation in severe weather conditions. Moreover, the captain and operational crew may also determine if the operation is unsafe due to prevailing conditions.

Wave height is critical, if waves exceed 4.5 meters the safety regulations will not allow for offshore loading or unloading, this means that the weather can delay operations at the offshore site significantly. Wave heights less than 4.5 meters can also increase the time of operation and sailing times, as the sailing speed must be reduced.(Halvorsen-Weare & Fagerholt, 2011) If the conditions are considered unsafe for the vessels to operate in, the vessels are required to Wait on Weather (WOW), which means that they wait for better conditions. If these conditions are long lasting and the deliveries to the facilities are hindered for a long period of time, these conditions can add significantly to the lead times, which can lead to downtime.(Nardone, 2009a)

The land-based modes of transportation can also be severely affected by the weather conditions, although it is often stated that this mode is less affected then the seaborne transportation.(Christiansen et al., 2013) The driving conditions may require a reduction in the travelling speed and more cautious driving that may cause delays. Accidents can be caused by the weather conditions, such as icy and slippery roads, and poor visibility. Furthermore, accidents can also be caused by unrelated weather conditions such as poor driving skills, influence of narcotics and drugs, speed and the driver's level of awareness, which are the main reasons that cause accidents.

The weather conditions are also an uncertainty factor for helicopter transportation, and severe conditions may delay or cancel the flight. During the operation, the helicopter operator shall be informed of the relevant weather conditions. Normal flying can be performed at wind speeds up to 60 knots. There should however be a continuous evaluation of the risk of landing at the helideck. In case of lighting, thunder or cumulonimbus activity, the operator must consider alternative flight patterns or cancellations.(Norwegian Oil and Gas Association, 2011)

Norway is characterized by its geography with its many fjords and mountains, and these conditions do have a significant impact on the land-based transportation. The great number of fjord on the western coast often make ferries a necessary supplement to road transportation. Several of these ferry services lack sufficient capacity, and it is common that vehicles have to wait for the next ferry, as the initial ferry is fully loaded. Some roads are cross-mountains, and mountain pass may be closed due to harsh weather or it necessitates bumper-to-bumper driving, both of which may cause a rapid increase in travelling times.

POOR INFORMATION AND LACK OF INTEGRATION

As discussed in chapter 2.2 there are several stages in the upstream supply chain, and each stage often consists of numerous elements, which adds to the complexity. Furthermore, the supply chain is divided, and there is very little synchronization between the elements, and the information between the various actors is poor. This provides a narrow focus for which various operators are focused on single tasks, thus making it difficult to get a comprehensive understanding of the entire operation. Therefore, the solutions in each stage are often optimal, but that does not mean that the supply chain as a whole is optimal. When the interested parties often have conflicting interests, this poses another challenge for the planners, as these operations must be synchronized.(Favilla et al., 2012)

Asset tracking is also limited, and this hinders the identification of bottlenecks and possible delays. This often leaves the operator with limited and out-dated information that they cannot act upon, and the operators are not left with the right foundation for the decision making. If they could foresee potential pitfalls, they could have made alternative arrangements that could avoid downtime as a result of delays and unexpected events. Traditionally problems are solved as they occur. However, with better information, the planners can detect the problems before they occur and come up with mitigating actions to avoid them from happening.(Favilla et al., 2012)

VARYING DEMANDS

Demand management can also be a challenge. The equipment that must be delivered is varied, and the operations must comply with strict health, working and safety regulations. During drilling operations, the demands are unpredictable.(Favilla et al., 2012) The operators might experience that the demands vary due to the operational conditions, such as unexpected bedrock conditions, equipment may also fail during operations or there can be an emergency situation.(Ose, 2013)

The planners develop detailed schedules, but these may change during the operations due to the variable demands. Possible problems may be control cables that snag during installation, downhole tools that fails to operate after repeated operations, and valves and pipes that develop leaks after prolonged exposure to the harsh production conditions.(Nardone, 2009a) If such emergencies or unexpected demands occur simultaneously for several operations, there may be a variety of conflicting requirements, and one may experience that some are prioritized on the account of others.(Favilla et al., 2012)

If the equipment or personnel is missing, the operators may risks expensive downtime. As discussed above, metocean conditions can result in highly stochastic lead times, and this may jeopardize deliveries that have little time slack. On the other hand, the inventory costs increase with the hiring time. Thus, there is a trade-off between the risk of not possessing the right equipment at the right time, and the cost of having redundant equipment. To predict and determine the supply of these resources is therefore a significant challenge for the planners.

2.3.2 INTEGRATED LOGISTICS

In resent years, there has been an increasing focus on integrated logistics, which is believed to provide better offshore supply logistics. According to Asbjørnslett (2008) integrated logistics is attracting greater attention by the operators as it can provide a demand management that *"integrate planning, balance demands, requirements and supply resources"*. The result of integrated logistics may provide a more customer-service oriented and cost-effective supply operation.(Asbjørnslett, 2008)

Favilla et al. (2012) propose a solution approach for integrated logistics. This system will provide an inventory planning that determine the optimal requirements for parts and the target service level, and the optimal re-stocking point or policy. In the suggested approach, the demands are forecasted and the forecast are continuously reviewed and adjusted if new information occurs. They suggest that backward recursion can be used to find the latest possible start time of the operations, and then a forward simulation will find the earliest start possible. Thus, the planning of the demands will match the demand forecast, part availability and capacities. The integrated logistics guarantee that all involved parts, which could be onshore transportation, warehouse logistics, vessels and helicopters, will operate in synchronization. In this process, they may detect shortcomings and other unforeseen events, such that mitigating actions can be taken.

Integrated logistics are complex and must address many variables – thus decision support tools may prove to be useful aids. This thesis concerns the planning and scheduling of the upstream supply chains in offshore exploration operations. Traditionally, experienced logistic personnel perform the logistics and route planning manually. Due to the complexity and high amount of variables that must be considered, manual planning may fail to investigate possible solutions. With a decision support tool, it is easier to detect the best solution for the problem as a whole, and consider alternatives that easily could be excluded by a planner as it provides a poor solution in one stage of the supply chain. Thus, optimization might encourage the planner to evaluate a wider range of possible solutions, and find a solution that is optimal for the problem as a whole and not just the suboptimal solutions.

Furthermore, a mathematical model describes a problem more concisely than the verbal formulation. Thus the overall structure may seem more comprehensive, and it may be easier to reveal the cause-and-effect relationships, which may be useful if the planners are trying to uncover where improvements can be made.(Hillier & Lieberman, 2010) The models can also be used to simulate and test different scenarios, which can provide even more insight into the problem and the decisive characteristics.

It is important to keep in mind that optimization does not solve the actual problem, but the mathematical formulation which is a simplified version of the problem. This means that the solutions are only optimal for the modelled problem.

CHAPTER 3

METHODOLOGY

The problem described in Chapter 2, will be solved with optimization. Optimization is applied mathematics, and the methodology is used to gain insight into a system and find possible solutions to the problem. Optimization can be viewed as a part of Operational Research, which also comprises research areas such as statistics, simulation, control theory, queuing theory and production economics.(Lundgren, Rönnqvist, & Värbrand, 2010)

Section 3.1 will provide some basic information on optimization as a methodology, and why this is considered a useful methodology for the problem presented in this thesis. The optimization methodology that is used to solve this problem will be discussed in section 3.2.

3.1 OPTIMIZATION AS METHODOLOGY

According to Lundgren et al. (2010) optimization is the *"science of making the best decision or making the best possible decision"*. In this context, "best" indicates that there is an objective function, whereas "possible" is related to the presence of restrictions. Thus, there is a given problem description for which there is a need to make the best possible solution subject to the prevailing restrictions.

Optimization was first utilized as a decision support tool half a century ago, and some of the first applications were military planning. Lundgren et al. (2010) state that many consider World War II the starting point of optimization. In recent years, optimization has been applied in several economic and technical areas such as production planning, transport and logistics, telecommunication, structure design, and scheduling of staff or resources, to name a few.(Lundgren et al., 2010)

Optimization can be said to be a multidisciplinary field, and in order to solve these problems efficiently, one must master both the art of mathematical and computer science, in addition to having technical and economical competence. The optimization models are often complex with large input files, thus the need for computer capacity can be a restriction in the solution process. However, there has been a rapid development in computer performance, which the development of algorithms also has contributed to.(Lundgren et al., 2010)

3.1.1 THE OPTIMIZATION PROCESS

The optimization process constitutes four phases: *identify*, *formulate*, *solve* and *evaluate*. These phases are often performed in parallel, and the time required to complete the process is determined by the model's problem size, structure and complexity.

The process of solving a real life problem with optimization is illustrated in Figure 7.

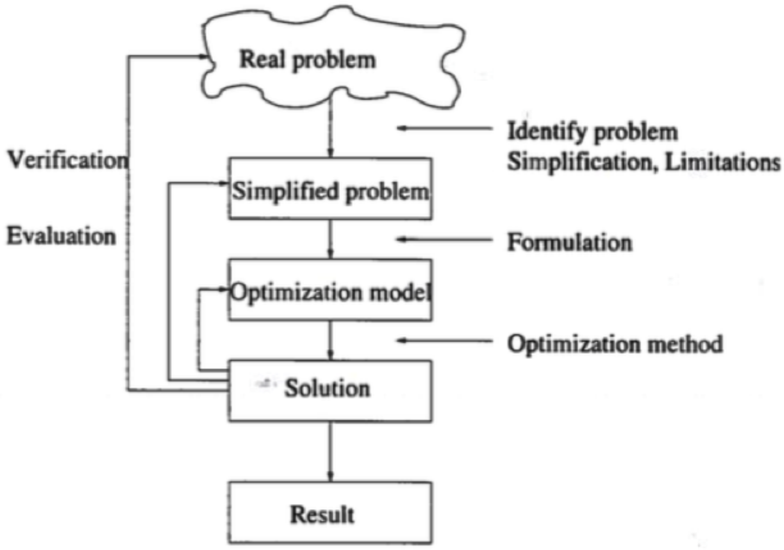

FIGURE 7 OPTIMIZATION PROCESS. SOURCE: (LUNDGREN ET AL., 2010)

The real problem is the actual problem that needs to be solved, like the problem described in Chapter 2. These are complex and there are too many details to model, thus it is necessary to analyse the problem and *identify* which elements that are important and must be evaluated, and which that are irrelevant and can be excluded from the remaining solution approach. Another crucial aspect is to evaluate whether the problem is appropriate for an optimization model, or if there are more suitable solution methods. When this assessment is completed, the real problem can be converted into the simplified problem, the simplified problem in this thesis is provided in Chapter 5.

When the simplified problem is *constructed* it is converted into a mathematical formulation, which describes the essence of the problem and this is the optimization model. The model consists of decision variables, an objective function and constraints. The complexity, size and structure of the model is a decisive factor for how difficult it is to solve, and it may be necessary to perform more simplifications in this phase. The amount of available data must also be considered.

The problem must then be *solved*. This is often done using a solution algorithm or the implementation of the problem in a commercial software such as FICO™ Xpress. There are also some problems that require software adapted for the specific problem. In order for the model to be solved, the relevant data must also be gathered. This is a critical step, and it can be a challenge to collect the correct data to model the real problem. Often these data are approximations of the real data. There is a trade-off between exact and more solvable solutions.

The final step is to *evaluate* and verify the solution, which is necessary to achieve the result. This process is the post optimality analysis, and this phase is important as the solution provided only is the solution for the mathematical model and not necessarily the real problem. Sensitivity analysis is a useful tool in this evaluation as it determines the most critical parameters of the solution. The critical parameters are the parameters that will change the value of the objective function if they are changed.(Hillier & Lieberman, 2010) When the solution is evaluated, it can be utilized in the decision making, which makes the mathematical model a decision support tool.

3.1.2 Optimization Methods and Modelling

There are two types of optimization methods; exact and heuristic. There is also a method that is called metaheurisitcs, which is a combination of the two methods, but this will not be discussed any further in this thesis. The exact method provides the optimal solutions and can verify that it has been found. The heuristic method provides solutions that are close to the optimal solutions. However, these cannot provide estimates for the deviation from the optimal solution, and the optimal solution is not guaranteed.(Lundgren et al., 2010) Heuristic solutions are often used on complex problems, and due to the fact that optimization primarily is used as a decision support tool, these solutions are often considered good enough. The complexity of the model is a decisive factor for which of the methods that is most convenient.

The mathematical model is expressed in mathematical symbols and expressions, and consists of an objective function, decision variables and constraints.(Hillier & Lieberman, 2010) The objective function is the element that the model optimizes. A general optimization problem can be formulated as:

$$\begin{aligned} &min \quad && f(\boldsymbol{x}) \\ &s.t. \quad && x \in X \end{aligned}$$

The objective function $f(\boldsymbol{x})$ can be minimized or maximized, depending on the goal of the decision making. In transportation and logistics problems, the objective functions primarily make decisions regards economic considerations such as cost or profit, or time considerations like transportation time.(Hillier & Lieberman, 2010) Some objectives are multi-objective and the goal is to find the best trade-off between two or more objectives.

The objective function depends on the decisions variables $\boldsymbol{x}$, which are the variable aspect of the problem that must be decided on. Real life situations are often subject to restrictions, which are modelled as constraints. The objective function and constraints consist of decision variables and constants (right-hand side values and coefficients) that are referred to as parameters. Thus, the objective and constraints must be quantified for a real life situation to be modelled mathematically.(Hillier & Lieberman, 2010)

There are several problem classes in optimization, and which class the problem belongs to depend on how the functions are specified and the feasible values for the variables. The main problem cases are Linear Programming problems (LP problems), nonlinear problems, integer programming problems and network problems. In LP problems, all the functions are linear, and the decision variables are continuous. If the LP problems have one or more functions that are nonlinear, the problem is a nonlinear problem. These problems are often difficult to solve with commercial software, and the nonlinear functions should be linearized if possible.(Hillier & Lieberman, 2010)

If a problem has a subset of variables that are integers, thus not continuous, it is an integer programming problem. These variables can either be integers or binary, thus only taking the value 0 or 1. Mixed integer programming problems, are problems with both integer and non-integer variables. Many problems can be described with arcs and nodes, and these are often referred to as network problems. They have an underlying network structure that can be exploited in the solution algorithms. The network problems can be classified as LP problems or integer programming models.(Lundgren et al., 2010)

3.2 Mathematical Modelling for This Thesis

As previously described, this problem will be modelled as an Inventory Routing Problem (IRP). The reason for choosing this methodology is that it enables the planners to evaluate both the optimal inventory levels and the routing and scheduling decisions, and this provide a supply chain that optimize both aspects.

This section will provide some general ideas for the techniques that are used in the mathematical model that is described in Chapter 6. They will not be discussed specifically for this problem, but are meant as an additional aid for the reader to understand the mathematical model. The reasoning for choosing these techniques will also be discussed.

3.2.1 Deterministic Stage

The routing aspect of the inventory routing problems is modelled based on the classical Vehicle Routing Problem (VRP) formulation, with the addition of necessary extensions. The model in this thesis will utilize a visiting system, this has previously been done in the work of Agra et al., (2013), Al-Khayyal and Hwang (2007) and Agra et al., (2015).

The problem in this thesis is a highly dynamic problem; hence a visiting system is considered a better approach than using multiple-periods. Dynamic programming is a solution strategy that can be viewed as sequential decision-making in which one must make decisions in each stage. Thus, the state of a given stage will affect the state of the consecutive stage. In order for a problem to be solved with dynamic programming, it must have a dynamic structure that allows it to be divided into the sequential stages.(Lundgren et al., 2010) To model this problem with multiple periods, in which the days are the sequential stages, will require constraints that connected the sailing of the vessel and the inventory levels for each period. Furthermore, to model the inventory levels with the desired accuracy, this approach will require an additional variable that considers hours per day, which will make the model more complex. The visiting system only requires an update of the stock levels and location of the vessel for each visit, which constitute the sequential stage. Furthermore, the time horizon does not have to be divided into periods, which makes the problem easier to model.

The visiting system enumerates each visit at a node in the network. To illustrate how the visiting system works, an instance with two vessels and four offshore facilities is illustrated in Figure 8. The vessel's movement in the network is described by a pair (i, m), in which i provides the location the vessel visits and m provides which number in the sequence of visits this visit makes.

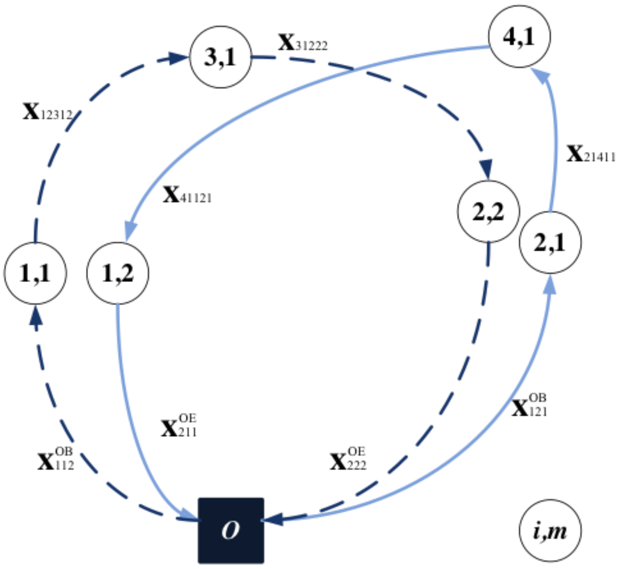

FIGURE 8 VISITING SYSTEM

The corresponding sequences of the sailing pattern for the two vessels are illustrated in Figure 9. For this instance, the two vessels depart from the depot simultaneously. Vessel 2, which is represented with the dotted line, sails to offshore facility 1 for the facility's first visit, and continues to offshore facility 3 for the first visit at this facility. The last facility the vessel sails to is facility 2, but the first visit to this installation is already made by vessel 1, thus vessel 2 makes the second visit, then the vessel returns to the depot. Vessel 1 is represented with the solid line, and visits installation 2, 4 and 1, respectively. The vessel will make the first visits at facilities 2 and 4, and the second visit at facility 1.

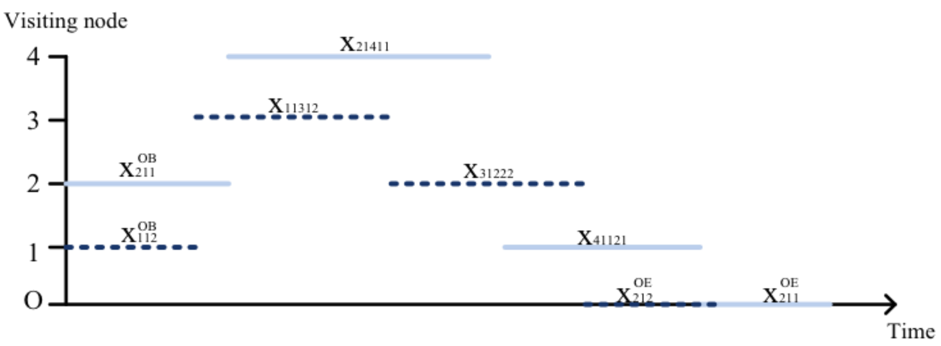

FIGURE 9 VISITING SCHEDULE

The inventory is used to determine the optimal amount of commodities to be delivered to the offshore facilities for each visit. The demand at the facilities is determined by the current inventory level, which is defined by the inventory level from the previous period and potential deliveries.(Lundgren et al., 2010) The inventory levels are illustrated in Figure 10.

Forward recursion can be used, and this provides information about the best way to get the required inventory level at the end of the planning horizon.(Lundgren et al., 2010) The inventory levels are reduced with a fixed rate per unit time, and can only be increased if there is a delivery. Often, these problems have an upper and lower bound on the inventory, which are illustrated by the horizontal dotted lines. This illustration is a typical representation of how inventory levels with fixed rates, are modelled in IRPs.

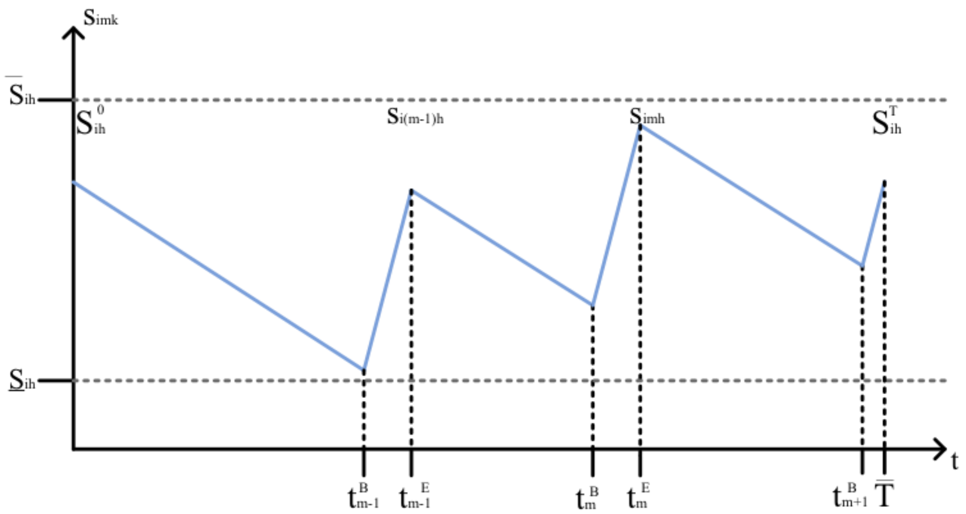

FIGURE 10 DEVELOPMENT IN THE INVENTORY LEVEL OVER THE PLANNING HORIZON

3.2.2 TWO-STAGE RECOURSE FORMULATION

The model in this problem will have a two-stage recourse formulation, in which the deterministic solution is made in the first stage, and the stochastic elements are considered in the second stage. A recourse model provides the opportunity to adjust or adapt the solution after the uncertainty data is revealed. This is a useful modelling approach when some decisions must be made before the actual data are known. In such problems, the variables in the second stage are allowed to vary with the scenarios, and may be defined as “recourse variables”.(Higle, 2005)

According to Higle (2005) the recourse problem can be characterized by a scenario tree, scenario problems and non-anticipatively constraints. A scenario is the realization of the stochastic elements, in this case that would be the unexpected demands. The scenario tree is a structured distribution of the stochastic elements and how they may evolve over the time horizon. Such a scenario tree is illustrated in Figure 11. The tree consists of a root node, which is the initial stage where no information about the stochastic elements is revealed, and there is a deterministic decision. The leaf nodes provide information about the decisions after the stochastic elements are revealed.

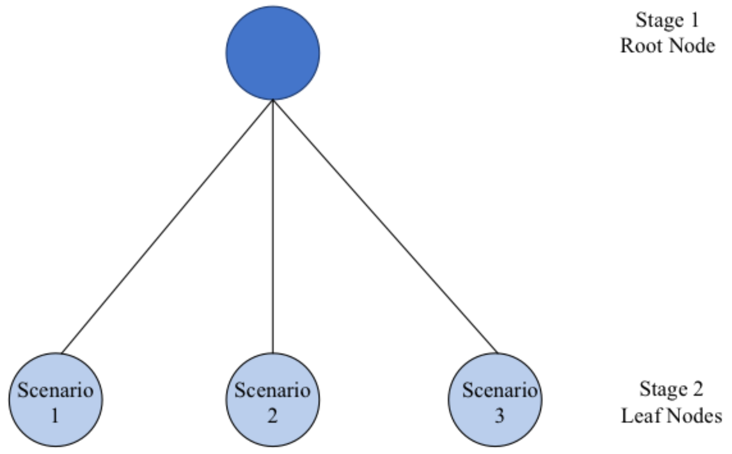

FIGURE 11 SCENARIO TREE

The scenario problem is the problem associated with a given scenario, and can be considered a deterministic problem. The non-anticipatively constraints ensure that the scenarios that share the same history of information also must make the same decisions. The decisions that are made at a specific point in time, is only dependent on the information available at that time and cannot be based on estimates.

The general formulation of a two-stage recourse model is presented by Higle (2005), and the model consist of both the deterministic part and the stochastic part:

$$Min \quad cx + E[h(x, \tilde{\omega})]$$

$$s.t. \; Ax \geq b$$

$$x \geq 0$$

$$where \quad h(x, \omega) = \quad Min \;\; g_\omega y$$

$$s.t. \; W_\omega y \geq r_\omega - T_\omega x$$

$$y \geq 0$$

The first stage, which is the deterministic solution, is provided by x. The x-variable does not respond to the uncertainty data ω. The stochastic elements of the second stage is represented by $h(x, \omega)$, and the estimated value of this element is added to the deterministic model. Thus y is the second stage variable, and will be determined after the uncertainty is revealed.

The objective with the deterministic solution is to provide the solution that is best positioned to respond to all instances of the uncertainty, $\tilde{\omega}$.(Higle, 2005) The stochastic formulation will be added to the deterministic model, by the introduction of scenarios and the recourse decisions.

There are methods that investigate the value of the stochastic solutions, and these can also be utilized to evaluate the stochastic solutions. The value of the stochastic solution (VSS) is the difference between the objective value of the stochastic solution and the expected value of the decision based on the expected value of the deterministic solution. The stochastic solution is given as the here-and-now solution (RP). The value of the stochastic solution is the difference between the stochastic solution and the expected result of using the optimal value of the deterministic solution (EEV). If the difference is zero, the parameters are equal, which means that the deterministic solution is equally as good as the stochastic.

$$VSS = EEV - RP \tag{3.1}$$

Expected value of perfect information (EVPI) provides the difference between the expected value of the solution based on perfect information and the objective function of the stochastic solution. Thus, it measures the amount the decision makers are willing to pay for complete information about the actual conditions.(Birge & Louveaux, 2011; Hillier & Lieberman, 2010) The value of perfect information is defined as the difference between the wait-and-see (WS) and here-and-now solutions (RP). The here-and-now-solution is the value of the recourse problem, whereas the wait-and-see solution is the solution of the perfect information. This can be calculated by formula (3.2)

$$EVPI = RP - WS \tag{3.2}$$

Testing a stochastic model can be challenging compared to a deterministic model. A deterministic model is expected to provide an exact result, and as long as the input is not altered, neither will the model's output. The stochastic model will have input data with a level of randomness, which consequently also provides outputs with a level of randomness, thus it can be a challenge to determine whether the stochastic solution is optimal or if there is an error. Therefore it has become a common approach to develop a deterministic model in addition to the stochastic, and compare the two models' results.(Powell & Topaloglu, 2003)

CHAPTER 4

LITERATURE REVIEW

This chapter will present some contributions within the field of optimization that are considered relevant for the problem in this thesis, the focus will primarily be maritime optimization problems. The chapter will discuss some papers on supply vessel planning problems, and problems that address intermodal problem that illustrates some of the complexity of maritime supply chains. The model in this thesis is a Maritime Inventory Routing Problems (MIRP), thus the characteristics of these problems and some relevant literature on both deterministic and stochastic models will be addressed. The objective of this chapter is not to give the reader a comprehensive overview of related literature, but to provide an overview of articles that are considered relevant for this thesis.

This chapter is organized as follows; the first section 4.1 will present some optimization models developed for offshore supply vessel problems; the optimization models studied for intermodal transportation problems are discussed in section 4.2. Section 4.3 discusses some MIRPs that are considered relevant for this paper.

4. 1 SUPPLY VESSEL PLANNING

The cost of chartering PSVs is significant, and consequently it has in recent years been an increasing focus on reducing these costs by determining the optimal fleet size and mix, and the routing and scheduling of the available fleet. These problems are called the Supply Vessel Planning Problems, and it is a problem faced by many oil and gas companies operating offshore.(Christiansen et al., 2013) The main characteristic of these problems are a set of offshore installations that need services from a common onshore depot. The vessels load at the depot, and discharge the cargo at supply depots along the coast, thus each of the supply depots have a separate planning problem. The schedules that are valid until there is a large change in demands or operational conditions. Such changes could be a result of the occurrence of new installations, or a change from a drilling to a producing phase. The routing and scheduling is subject to constraints due to opening hours, limited capacity at the installations and vessels, and maximum durations of the voyage. The objective of these problems is to minimize the operating costs.

Fagerholt and Lindstad (2000) study a supply vessel problem for supply services in the Norwegian Sea. The study considers a real problem, and was performed on the request of Statoil, who wanted to investigate the effect of closing some of the offshore installations for service during night. The objective of the study is to determine the optimal fleet, and its schedules, which are restricted by the opening hours. The problem is characterized as a multitrip VRP, and the suggested solution algorithm consists of two steps. The feasible candidate schedules for each vessel are generated in the first step, in which a dynamic

Travelling Salesman Problem (TSP) is solved to ensure that the schedules have an optimal visiting sequence. In the second step, an integer programming model is solved to find the vessels to be used and their weekly schedules. The study shows a potential cost saving of 7 million dollars, and the policy was implemented by Statoil.

Halvorsen-Weare et al. (2012) study another supply vessel planning problem. The objective of the study is to determine the optimal fleet composition and the weekly routes and schedules for the supply vessels, in order to minimize the charter and sailing costs. Similar to the study by Fagerholt and Lindstad (2000), the authors propose a two-stage solution approach. The candidate voyages are generated in the first stage, and the optimization model is solved in the second. The supply vessel planners at Statoil utilized the model as a decision support tool, and it played an essential part in the reduction of the supply vessel fleet by one vessel, which has an estimated annual cost saving of 3 million dollars.

Sopot and Gribkovskaia (2014) study routing of supply vessels with delivery and pick-ups of multiple commodities. This is a common problem in the offshore upstream logistics. The authors emphasize that to their knowledge, there are several papers that study similar problems with single commodities, but there are no other papers that study the multi-commodity problems. In their study, a single vessel departs from a depot, which is the onshore base, and service four offshore installations that has both pick-up and delivery demands. The delivered goods are materials, fuel, food and equipment, and the pickup demands are the backhauls such as wastes and empty containers. The vessel can visit the installations one or two times on a voyage, and it will attempt to do both pickup and deliveries on the first visit if this violates the capacity of the vessel, it will do the deliveries on the first visit and pickups on the second. Thus, the solution may be a double path and not a full Hamiltonian solution as is common for these problems. The problem is solved with a metaheuristic algorithm, in which a tabu search algorithm is used to generate a lasso solution for the TSP.

It is a common assumption that the input data is known in advance in mathematical programming. Therefore, the problems are modelled deterministically, and there is no randomness in the input data. However, due to stochastic sailing times in addition to stochastic demands, real life supply vessel problems are often highly stochastic and dynamical. These uncertainties can be modelled with stochastic optimization, but also deterministic models that allow for more robustness. Christiansen et al. (2007) emphasize that uncertainty can be addressed by the combination of simulation and optimization, or with deterministic models that incorporate penalties or slack.

A combination of optimization and simulation is used by Halvorsen-Weare and Fagerholt (2011) in a study that is the continuation of the deterministic study by Halvorsen-Weare et al. (2012) that is presented above. With the implementation of the deterministic solution, Statoil experienced that the impact of the weather conditions made the generated schedules infeasible. This often resulted in rescheduling of the routes, which is an expensive operation and revealed the need for more robust models. To address this problem, Halvorsen-Weare and Fagerholt (2011) added robustness to the deterministic model by the introduction of slack, which can be defined as the vessel's idle time after its completion of a route and the preparation for a new.

The extended model has a three-step solution approach, in which the candidate voyages are generated first step. The voyages are simulated and assigned a robustness measure based on the statistical weather data in the second step, and the model is solved in the third step. Thus, the first and third steps are similar to the first and second step in the deterministic model. The paper describes several solution approaches that were tested on real life data from Statoil, and the authors estimate a cost saving of 3% from the original deterministic model. Furthermore, the authors found that the effect is even higher with increased problem sizes.

Penalties have been studied for several maritime problems, some of which are studied in Christiansen and Fagerholt (2002) and Fagerholt (2001). Researchers consider the deterministic methods "good enough", but it is emphasized that stochastic modes can provide solutions that are significantly better.(Powell & Topaloglu, 2003) Albareda-Sambola et al. (2007) state that the deterministic models often fail to model the practical limitations caused by the high level of uncertainties. As a consequence, a deterministic model may fail to provide a correct solution for the real problem and this lack of robustness in the solutions might make the solutions invalid. Hence, there is an increasing attention towards stochastic modelling.

A supply vessel problem with stochastic sailing and service times is studied by Maisiuk and Gribkovskaia (2014). They state that the economic effect of determining the optimal fleet size a year ahead is significant, thus the objective is to find the annual fleet size of supply vessels operating from an onshore base. The problem is highly stochastic due to the uncertainty in the weather conditions and the volatility of future spot rates. They utilize a discrete event-simulation methodology, and argue that the problem cannot be described and modelled analytically due to the stochastic phenomena. The authors introduce a two-stage solution method. In the first stage, the installations are clustered for each of the vessels in the fleet, and a multi-period TSP with multiple time-windows is solved to find the voyages with the shortest durations. The TSP must consider several operational restrictions, such as opening hours and maximum amount of installations to be serviced. In the second stage the voyages with durations of two or three days are used as input to a set-covering model. The model assigns vessels to start days, and the objective is to minimize charter costs and fuel costs with respect to the requirements from the installations. An annual schedule is generated from a set of consecutive weekly schedules. The model was tested and validated on real data, and the result showed that the company had an optimal fleet size for their operations. The model provides an exact solution for relatively small instances, whereas larger instances can be solved using a large neighbourhood search algorithm.

A common solution approach in supply vessel planning problem is to pre-generated routes. This approach is considered a good solution for these problem as they often are tightly constrained and the size is smaller than for land-based transportation, and this limits the number of feasible routes. The advantage with this approach is that it reduces the complexity of the mathematical model severely as there is one variable per route, instead of one per edge. Additionally, set partitioning problems have a nice structure that is easier to solve than the direct formulations. There is also much flexibility in the way the routes are generated. On the other hand, the drawbacks with this approach is that it will increase exponentially with problem size, and it requires that the optimal route is generated in order to provide the optimal solution.(Fagerholt, 2014) Several of the characteristics of the supply vessel planning problem is equal to the ones presented in this thesis, such as a common depot, limited capacity on vessels and installations, and maximum duration of the voyage. It is difficult to achieve the desired flexibility in the inventory levels with this method, and it is therefore considered necessary to look to other maritime problems to find a better representation of the problem faced in this thesis.

4.2 INTERMODAL TRANSPORTATION AND MARITIME SUPPLY CHAINS

Intermodal transportation is defined as *"transportation [...] by a sequence of at least two transportation modes, the transfer from one mode to the next being performed at an intermodal terminal"*.(Crainic & Kim, 2007) As this thesis will investigate transportation with both land-based and seaborne transportation and the transfer is being performed at the onshore base, it has several similarities with the problems faced in intermodal transportation. During the research in relation to the work with this thesis, there was not found any papers on the application of intermodal transportation models for the supply vessel planning problem. Still, in order to provide the reader with an understanding of the complexity these problems may hold, a few relevant papers that discuss intermodal transportation in maritime applications will briefly be discussed in this section.

One of the more studied supply chain problems in relation to maritime transportation is the one arising in container port terminals. The main function of the terminal is to provide transfer facilities between the vessels and land-based mode of transportation. Container operations represent a complex problem that depends on numerous pieces of equipment, operations and container handling. Crainic and Kim (2007) emphasize that intermodal transportation is a relatively new area of research and widely accepted models are lacking in many areas. In such problems efficient resource management and allocation strategies are crucial in order to maximize the utilization of available resources.

Mehrez et al. (1995) introduce a model for solving an industrial shipping problem for bulk shipping to customers located overseas. The problem involves the shipping of bulk from a source port, to the transhipment ports and the distribution from the transhipment ports to the customers on land. The model includes decisions for ships (size and number) and land transportation routes, and for warehouse storage at the ports. The model addresses the trade-off between having storage in the warehouse with large deliveries in low cost seasons and smaller shipments with "just-in-time" deliveries. It also describe which ports to use, including the warehouse in that port. The problem in formulated as a mixed integer linear program (MIP), and follows the movement of the product from factory to end-customer.

Tsiakis et al. (2001) formulate a multiproduct, multi-echelon supply chain network under demand uncertainties. The model aims to determine the number, location and capacity of warehouses and distribution centres, the transportation links in the network, and the flow and production rates of the material. The manufacturing sites are located at fixed locations, whereas the warehouses and distribution centres will be selected from a set of potential locations. The problem is modelled as a MIP optimization problem, and the objective is to minimize the annual costs for the network.

4.3 INVENTORY ROUTING PROBLEMS

The problem presented in this thesis is a planning problem in which the operator is responsible for the inventory management, and the vessel routing and scheduling of the deliveries. These problems are known as Inventory Routing Problems (IRP) and dates 30 years back.(Coelho et al., 2013) Andersson et al. (2010) emphasize that coordination of inventory management and routing can have beneficial effects in terms of economy, flexibility and improved robustness. Furthermore, they argue that integration may be particularly beneficial if the product is critical for the supply chain, and large costs may be induced if the product is unavailable. This is the case of offshore operations, which face downtime if the required equipment and commodities are missing.

According to Coelho et al. (2013), the original IRPs were primarily variations of the VRP and heuristics developed to include the element of inventory costs in the decisions. The VRP determines the optimal routing from a central depot to a set of geographically distributed customers with pick-up and delivery demands, and it is considered a success story in operational research. The VRP is subject to a variety of constraints such as capacity, time-windows and durations, to name a few.(Laporte, 2007) However, there is often a need for more powerful and versatile routing models than what the classical VRP can provide. This has led to the development of more complex models based on the VRPs, such as the IRP.(Hoff, 2010)

Before the introduction of the IRP there were several papers on inventory problems and on distribution, but the combination of the problems was difficult to handle due to limited computing power and the limitations of the available algorithms. Furthermore, Coelho et al. (2013) emphasize that there are several versions of the IRPs, but there does not exist a standard version. Thus they refer to the basic version that is classified according to seven criteria: time horizon, structure, routing, inventory policy, inventory decisions, fleet composition and fleet size. There are also several extensions of this basic model. Andersson et al. (2010) have developed classification criteria for the inventory routing problem, these are provided in Table 1. A more supplementary description will be provided in the paragraphs below.

TABLE 1 CLASSIFICATION CRITERIA FOR IRPS. SOURCE: (ANDERSSON ET AL., 2010)

Characteristics	Alternatives			
Time	Instant	Finite	Infinite	
Demand	Deterministic	Stochastic		
Topology	One-to-one	One-to-many	Many-to-many	
Routing	Direct	Multiple	Continuous	
Inventory	Fixed	Stock-out	Lost sale	Back-order
Fleet Composition	Homogeneous	Heterogeneous		
Fleet size	Single	Multiple	Unconstrained	

TIME

A short planning period in which there is only time for one visit per customer, is defined as *instant*. The main focus is the balancing between inventory and transportation costs, in addition to the risk of stock-outs. The planning horizon may consist of more than one planning period; if this is the case, earlier visits or postponing visits to the next period should be considered.

If more than one visit is needed and there is a natural end to planning period, the time period is *fixed*. Furthermore, if there is no connection between the time before and after the horizon, long-term effects do not have to be examined. If this is necessary, a rolling horizon algorithm can be utilized.

Some problems deal with distribution strategies rather than schedules, and the cost minimization is considered for the long-run average cost. These problems are *infinite*. The dominant approach is to reduce the average daily costs; another approach is to use a discounting factor that makes costs and revenues from previous periods less important. These problems often have repeatable replenishment plans.

DEMAND

Model formulations, in which all the demand parameters are considered know, are *deterministic* formulations. Models in which there is uncertainty in the demand parameters are considered *stochastic*.

TOPOLOGY

The *one-to-one* topology refers to direct routing between a producer and a customer. This topology also includes the many-to-one case, but there are few studies on this, and it can easily be transformed to a *one-to-many* case. The one-to-many topology is the dominant case in road-based applications. It includes a central facility, which is the depot, and the fleet of vessels departs from and return to this depot. The *Many-to-many* topology is dominant in maritime applications, where there are several ports, in which the vessels can perform both loading and unloading.

Routing

Routing can be seen as either a VRP or a Pick-up-and-Delivery problem (PDP). In the VRP the depot serves as a point in which all the routes start and terminate, and a central warehouse from which the goods are distributed. In a PDP there are no central warehouses, and the pick-up and deliveries can be done at several locations. The VRP approach is common in road-base applications, whereas the PDP is more common in maritime applications.

Moreover, routing can be described as three cases. In *direct* routing, the commodities are picked up at a central depot, and distributed to a central customer, before the vehicle returns to the depot. Direct routing, in which more than one customer is serviced, is referred to as *multiple visits*. In both direct and multiple routing, the vehicle begins and terminates the route at a central depot. Whereas in the PDPs there are no start or end, thus this is referred to as *continuous* routing.

Inventory

Inventory levels that are not allowed to be negative are considered *fixed*. The fixed value can be zero or a level that is based on a *safety stock*. If the demands are not satisfied and inventory falls below the limit, there is a stock-out. A stock-out is usually followed by an emergency delivery or lost sales. Demands that are postponed are referred to as *back-orders*.

Vehicle Fleet

In a *homogenous* fleet, all the vehicle parameters such as speed, capacity and costs, are the same, whereas a *heterogeneous* fleet have vehicle with different parameters. The number of vehicles it consists of determines the size of the fleet. In a single fleet, there is only one vehicle, and a multiple fleet consists of several vehicles. A fleet can also be unconstrained, which means that extra capacity can be purchased, and the planners will always have enough capacity.

4.3.1 Deterministic MIRP

In maritime applications Christiansen et al. (2013) defines IRP as a Maritime Inventory Routing Problem (MIRP). Surveys on MIRPs can be found in the papers by Andersson et al. (2010) and Papageorgiou et al. (2014).

A basic version of this problem is the Inventory Ship Routing Problem (ISRP). In the ISRP there is a single product, and the number of calls at a given port and the quantity to be loaded and unloaded, is not predetermined. It is important that the inventory level does not violate the upper and lower storage capacities at the production or consumption site. It is assumed that the consumption and production rates are provided at each site. The ship can both load and unload at multiple successive ports. The planning problem is to design routes and schedules that minimize the transportation cost, without compromising the level of production or consumption. Inventory costs are not included, as the shipper owns both the production sources and consuming destinations. The fleet is heterogeneous, and partial loading and unloading is allowed. (Christiansen & Fagerholt, 2009)

Most real maritime problems have a more complex structure than what is modelled in the basic ISRP. Thus, there are several extensions that can model the more complex problems. In their survey, Christiansen and Fagerholt (2009) evaluate seven extensions, and these extensions can be combined in order to make the ISRP a better representation of the problem in this thesis. In the problem of this thesis there is a single loading port at the onshore base, whereas in the ISRP there are several loading ports. Furthermore, this problem contains a fleet of homogenous ships, whereas in the ISRP there is only a fleet of one ship. There are only inventory constraints at the offshore facilities, while these also exist in the production port in the ISRP.

One extension relevant for this thesis is the introduction of a single supplier or consumer. Problems with one central supply node are common in road-based IRPs, but rare in maritime transportation problems.(Christiansen & Fagerholt, 2009) This extension is, however, often used in optimization of Liquefied Natural Gas (LNG) transportation, which is an area of research that has received much attention in recent years. In these problems, the LNG is distributed from a single producer to multiple customers. One paper that address this problem is developed by Halvorsen-Weare et al. (2013). They propose a formulation for a real-life tactical LNG ship routing and scheduling problem, in which the producer is responsible for the transportation from the production site to the customers. In their proposed formulation, they also consider the uncertainty in sailing times and production rates, and develop robust schedules that consider this uncertainty.

Inventory routing can be modelled with continuous or discrete time, which is a decision depending on the nature of the production and consumption rates. The common approach is to use continuous time to model rates that are fixed and constant over the planning period, whereas variable rates for production and consumption can be modelled by extending the model to a discrete time model.(Christiansen et al., 2013) Agra et al. (2013) study a short sea inventory routing problem for which they formulate both a continuous and discrete time model. They model arc-load-flow formulations, and emphasize that the main difference between the two formulations is the network structure. Ships may visit the same port several times during the planning horizon, which is complicated to model. To overcome this, the authors developed a discrete time model in which the time horizon is divided into a discrete number of periods. This model utilizes the combined discrete and continuous consumption rates, thus both constant and varying consumption. The other formulation is a continuous time formulation, and use continuous rates only. In this formulation an index is created that indicates the visit number of the ports, thus creating a system for the order of the visits where the network node represents an event, this system is described in section 3.2. The first formulation can only have cycles within the same time period, whereas the other has cycles that can include many time periods.

The basic ISRP only consider a single product, there are however several problems that have multiple products and the ISRP can be extended to a multiple product model.(Christiansen & Fagerholt, 2009) There exist several versions and levels of complexity of this extension. Ronen (2002) presents a multi-product problem faced by producers of large volumes of liquid bulk. The objective of the problem is to minimize the total shipping costs, without violating the storage capacities or the safety stock. This is done by determining the route and schedules of the vessels, and the type and amount of cargo to transport on each voyage. The mathematical model is a MIP model, which is solved in two stages. The inventory decisions are made in the first stage, and the scheduling of the vessels in the next.

In real life problems, vessels that transport multiple products have separate compartments, and there are restrictions to which products that can be assigned to a compartment based the previous product. Al-Khayyal and Hwang (2007) study another problem with multiple products, this is a problem that is commonly experienced by maritime chemical transportation companies. In their study there is a fleet of heterogeneous vessels with dedicated compartments for each product type, and they formulate a mixed-integer nonlinear program to solve the problem.

Some models extend the problem with multiple products even further, and address the issue of allocation of the products in the various ship compartments. However, as the aforementioned models, the majority of the models do not consider this issue. It is often considered to be solved as a separate planning problem, or by the people responsible for the stowing.(Christiansen et al., 2013)

Another extension to the ISRP is to combine inventory routing and cargo routing. Ship scheduling problems are often cargo routing problems, and these problems are often tightly constrained and a significantly more restricted problem than the ISRP. In cargo routing problems, the cargo is given a specific loading and unloading port, and the quantity at each port is provided. In addition, there are often time windows for deliveries. In the case of demands that cannot be planned in advance due to unforeseen events, cargo routing is often a better representation of the problem.

Christiansen and Fagerholt (2009) emphasize that inventory routing problems can be combined with other planning aspects. This is often necessary, as the ISRPs only constitute a small part of the supply chain.

4.3.2 Stochastic MIRP

The oil and gas, and maritime industry often have problems with a high level of uncertainty. According to Christiansen et al. (2013) there are few studies that address the uncertainty in the parameters, and a common approach is to formulate deterministic models in which robustness is added by the introduction of safety stocks and penalty costs on sailing times.(Ronen, 2002)

In order to provide a better representation of the real life problems, stochastic IRPs (SIRP) are introduced. The demands can be modelled as probabilistic parameters, thus the SIRP can provide solutions that are better representation of the real problem than the deterministic equivalents.(Coelho et al., 2013) In these problems, the probability distribution of demand is assumed known, which is not the case in real life applications. Coelho et al (2012) argue that stochastic information can generate better solutions at the expense of more computer time. In recent years, there has been an increase in papers that deal with dynamic and stochastic IRPs.

Kleywegt et al. (2002) study a stochastic inventory routing problem in which the stochastic demands are formulated as a Markov Decision Problem (MDP). They model a problem where one supplier serves several customers, the fleet is fixed and homogenous, and there is an infinite planning horizon. The formulation only allows direct deliveries between the supplier and customers, and stock-outs are allowed, but backordering is not possible. The objective is to decide on a distribution policy that maximizes the discounted value. The problem is solved heuristically with approximate dynamic programming. The formulation is also extended by Kleywegt et al. (2004), and they formulate the same problem only with multiple routing, in which each vehicle is allowed to service up to three customers. In the solution approach the problem is decomposed into sub-problems that that are easier to solve, and an optimization model combines the solutions of the sub problems to obtain an approximate optimal value.

Bertazzi et al. (2013) introduce a dynamic problem formulation for a SIRP. The problem consists of one supplier and multiple customers, the demands are stochastic and the problem is modelled over a fixed time horizon. The objective is to minimize the routing, penalty and inventory costs. Inventory costs are generated when the inventory levels are positive, whereas penalty costs will be generated if the levels are negative. Thus stock-outs may occur, but excess demands are not backlogged. The demands are stochastic and gradually revealed over time, and the customers have a maximum inventory level. The problem is formulated as a MIP formulation, and solved with a hybrid rollout algorithm, which use the average of historical data to estimate the unknown demands. Then the problem is solved as a deterministic instance.

In their paper Coelho et al. (2012) solve the same problem. However, their approach is different from that of Bertazzi et al. (2013) as they develop and compare four policies to solve the problem instead of one. They utilize demand forecasts based on historical data to set the anticipated demand as approximations of the future demands. The static problem is optimized each time new information becomes available, and the initial inventory levels are set equal to the last known levels. The problem is modelled both reactive and proactive, both with and without lateral transhipments between the customers after the actual demands are revealed. The reactive policies observe the inventory state of the system and trigger replenishment when the inventory level reaches the reorder point. The proactive policies use forecasts. When the actual demands are revealed, new forecasts are computed and the process is repeated. The problem formulation only has one vehicle in the fleet, and it can service one customer per time period, but the model also allows subcontracting of carriers to perform a direct delivery. Higher stock levels reduce the costs, as the customers are better covered against variations in stock levels.

Agra et al. (2015) study a scenario-based stochastic programming model, which objective is to design cost effiective routing and scheduling, and inventory decisions for a short sea shipping problem. The model use a heterogeneous fleet of ships that transports several products from two suppliers to multiple customers. In their paper, they state that this is the first model to solve maritime inventory routing problems with stochastic sailing times and port waiting times. Due to the size and complexity of the model, the problem is modelled as a two-stage model in which the routing, loading and unloading decisions are made in the first stage, and scheduling and inventory decisions are made in the second stage.

To summarize this chapter, the aforementioned MIRPs will be categorized according to the categorizations described by Andersson et al. (2010) and summarized in Table 2.

TABLE 2 CATEGORIZATION OF MIRPS

Author	Time	Demand	Topology	Routing	Inventory	Fleet comp. and size
This paper (2015)	Finite	Det./ Stoch.	One-to-many	Multiple	Fixed	Homogenous, multiple
Agra et al. (2013)	Finite	Det.	Many-to-many	Multiple	Variable	Heterogeneous, multiple
Agra et al. (2015)	Finite	Stoch.	Many-to-Many	Multiple	Fixed	Heterogeneous, multiple
Al-Khayyal (2007)	Finite	Det.	Many-to-Many	Multiple	Fixed	Heterogeneous, multiple
Bertazzi et al.(2013)	Finite	Stoch.	One-to-many	Multiple	Stock-out	Homogenous, single
Coelho et al.(2012)	Finite	Stoch.	One-to-many	Multiple	Stock-out	Homogenous, single
Kleywegt et al.(2002)	Infinite	Stoch.	One-to-many	Direct	Lost sale	Homogenous, multiple
Kleywegt et al.(2004)	Infinite	Stoch.	One-to-many	Multiple	Lost sale	Homogenous, multiple
Ronen (2002)	Finite	Det.	One-to-many	Direct	Fixed	Heterogeneous,

CHAPTER 5

PROBLEM DESCRIPTION

This chapter will describe the simplified problem of this thesis, and it is based on the information given in Chapter 2. This problem description is necessary in order for the reader to understand the mathematical model of the problem, and it is an important step in the process of developing the model. The simplifications and assumptions that have been made in the making of the mathematical model will be addressed.

Exploration activities require significant supply of services and commodities. Traditionally several different suppliers provide these services, and the level of co-operation amongst the companies is limited and the information between the actors scarce. For the purpose of this thesis, it is assumed that all services are integrated and there is one planner responsible for the entire supply chain. The overall objective is to ensure a safe and reliable supply chain that delivers the right service, to the right location, and at the right time, while keeping costs as low as possible. The costs to be minimized are the chartering costs of the various modes of transportation, the travelling costs, and potential downtime and storage costs. The stand-by and operational costs for the equipment, are referred to as the inventory costs, which are assumed to be relatively small compared to the other cost elements, and are therefore excluded from the model. The problem is a tactical and operational planning problem with a fixed time horizon, and the planning horizon is broken down to hours.

The problem has a fixed number of supplier facilities and offshore facilities, and one onshore base. The demands are determined by the inventory levels at the offshore facility, which are reduced at a fixed consumption rate. When a demand is placed, it must be obtained from a facility that holds these commodities, and there are three modes of transportation that can transport the demands; PSVs, helicopters and semi-trucks, all of which are assumed to be homogenous and unlimited. This problem will consider multiple commodities, thus it is a multi-commodity problem. The various commodity types are geographically dispersed at various suppliers. This thesis will model the commodities, however in terms of equipment it is assumed that this also can be modelled with inventory levels. It is assumed that the expected operational time of the equipment can be used as the consumption rate. An upper inventory level that only allows a maximum number of equipment at a time, can ensure that the equipment do not exceed the inventory bounds. If the equipment fail the entire “inventory level” will be emptied, and new equipment must be provided right away unless there is a spare.

The demands can be defined as planned or unplanned, and they are defined in cubic meters. The planned demands are the ones that the planners are considering at the beginning of the time horizon, when the supply operations are scheduled. These are therefore based on estimates of the expected demands. The unplanned operations are revealed during the planning horizon, in this thesis that will be upon a vessel’s visit at an installation, and decisions concerning these must therefore be made as they occur. Both of the demand scenarios will be addressed in this thesis, but they require different solutions. Therefore, the problem with planned demands will be described first, followed by the description of the problem with unplanned demands.

The planned supply operation is a two-echelon problem, and it has a network structure similar to the one illustrated in Figure 12. In a two-echelon problem, there are two transportation links. The first echelon is transportation with a semi-trailer (TT), in which supplies are picked up at the suppliers' facilities (SF) and transported to the onshore base (OB) where it is loaded to the PSV. The second echelon deals with the ship transportation (TS), in which the equipment is loaded to the PSV at the onshore base, and then transported to the offshore facility (OF).

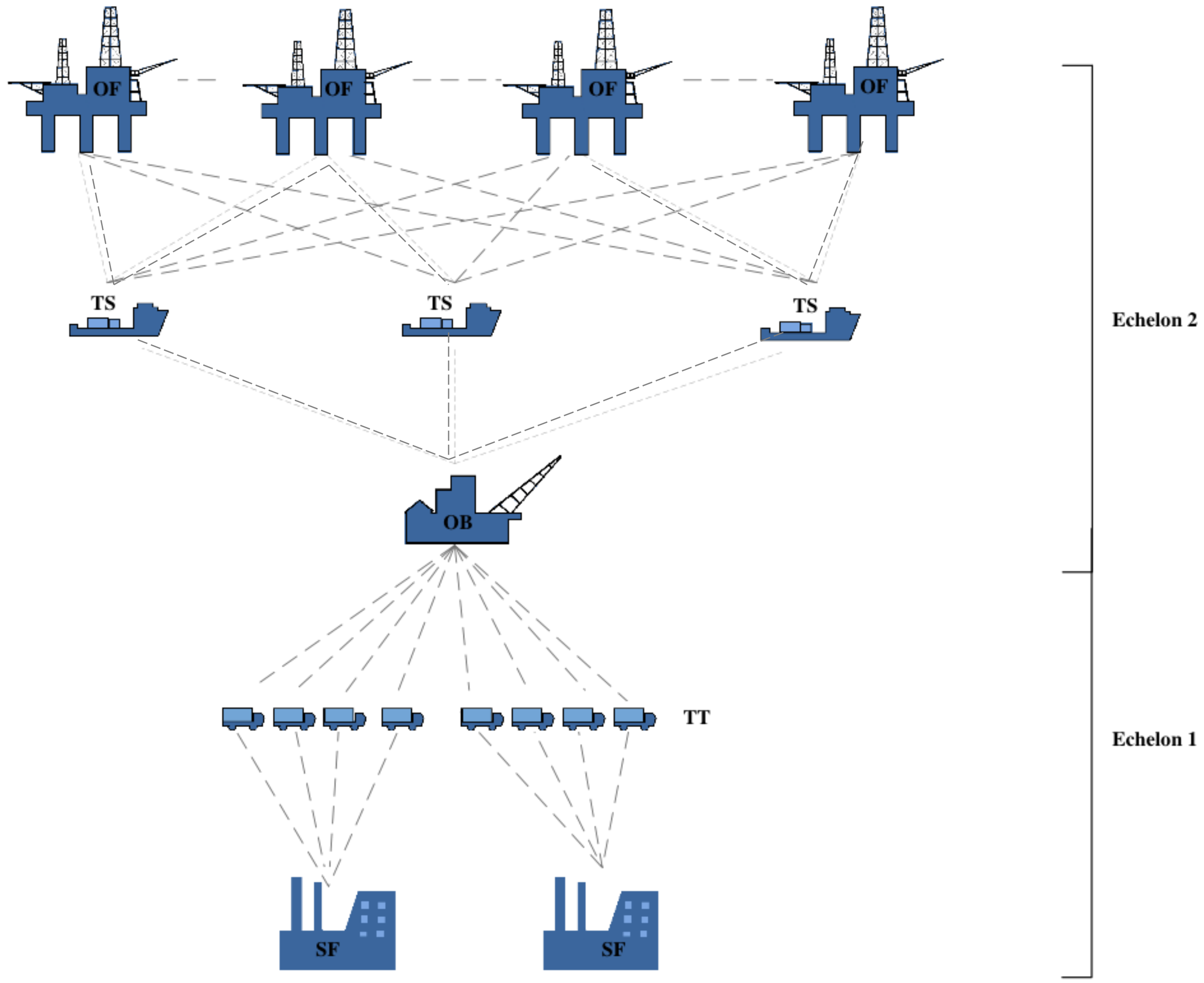

FIGURE 12 SUPPLY NETWORK

The supply vessels and semi-trailers are chartered on fixed time charter, and it is assumed that a voyage cost is generated each time one of these are utilized. There is also a cost for the travelling times, which are linearly dependent on the duration of the route. The transportation costs are assumed to be independent of the loading of commodities.

The semi-trailers will travel directly from the supplier's facility to the onshore base, and the only decisions that must be made for these are the scheduling of the departure and the number of vehicles necessary to deliver the required amount of commodities. The semi-trailers may be scheduled to depart before the planning horizon begins, and this is assumed to be a valid assumption since the schedule must be prepared before the horizon is initiated, and it is assumed that this is done with sufficient time before the vehicles must depart. The time to prepare and load the commodities before transportation is also excluded.

The supply vessel can only sail once the semi-trailers have arrived at the onshore base with the commodities for the specific order. It is assumed that the running costs of hiring equipment for a longer period of time makes storage uneconomical for the planned demands, thus the model follows the just-in-time principle, consequently the onshore base will not be utilized for storage for the planned demands. It is assumed that the unloading of the semi-trailer and the loading of the PSV is direct, and it is therefore assumed a common time for the unloading and loading operation. There is no upper berth capacity at the onshore base; hence there is no limit to the amount of PSVs that can be serviced simultaneously. Furthermore, it is not added any extra time between the unloading of each semi-trailer, therefore it is only the amount of commodities that determine the unloading and loading times.

The PSV can sail various routes depending on the offshore facilities' locations and demands, therefore the routing and scheduling of the PSVs must be decided. In this thesis, a route is the combination of one or more voyages that start and end at the onshore supply depot, and a vessel may visit one or more offshore installations on a route. A facility can only be visited by one vessel at a time, thus there is only one vessel that can perform an individual service demand. The vessels are also required to make a supply service in order to visit an offshore facility. There is not given any minimum time between two succeeding service operations at an offshore site, thus one vessel can immediately succeed another. Furthermore, the vessels are given an upper limit on the duration of the voyage, and they are also required to return to the onshore base before the end of the time horizon. The duration of a route is decided by the vessel's routing and which offshore facilities that are visited, and the loading and unloading times. The vessels cannot wait for an operation, and there is no slack in the sailing times.

The capacity is limited, both for the semi-trailers and PSVs, and the offshore facilities. This is an important aspect to include in the model, as it is decisive for the amount of commodities that can be transported. The semi-trailers cannot carry demands that exceed their capacity; and there may therefore be several trucks that depart from the supplier's facility to service each vessel. The offshore facilities have limited storage space, thus the amount of commodities cannot exceed this capacity, and this is considered the upper bound on the inventory level. As the inventory level must be positive in order to avoid downtime, the deterministic model will not allow stock-outs and the lower bound is zero. In order to ensure that the stock level at the beginning of the next period is of a certain size, there is a minimum stock level at the end of the time period. There will be no requirement to the minimum amount delivered for each visit.

Unplanned deliveries are typically caused by uncertainty in demand due to the operational conditions, or equipment failure. Consequently, these can cause a severe risk of downtime, and require significantly shorter lead times than the planned demands. In order to ensure fast deliveries of these requirements, it is assumed that there are two possible supply scenarios. Ad-hoc helicopters can be chartered, or the onshore base can have additional storage with commodities that can be transported directly to the offshore facility by a spot chartered vessel.

This thesis will only consider uncertainty in demand for demand scenarios that are higher than expected. The reason for this is that the aim is to investigate the optimal supply strategies for unexpected demands. If the demands are lower than expected such deliveries are not necessary. It might even become an issue with inventory that exceeds the upper bounds on the inventory level, which is backhaul issue, and these are not considered in this thesis. Complete equipment failure is also excluded from the modelling, as this require replacement of the entire storage. Due to the high downtime costs compared to the cost of chartering the emergency deliveries, this would always require the fastest delivery possible due to capacity constraints.

The two alternative supply scenarios both constitute one transportation link only, and are therefore one-echelon problems. The first scenario is illustrated in Figure 13. It is a direct shuttle by helicopter (TH) that will transport the commodities directly from the onshore supplier (SF) to the offshore facility (OF). It is assumed that both supplier facilities and offshore facilities have the required space and depots for the helicopters to land and take-off. Furthermore, it is assumed that the time to load and unload is included in the transportation times, and that helicopters and supplies are available at all times. Furthermore, there is not added any upper constraint on the helicopter's travelling time.

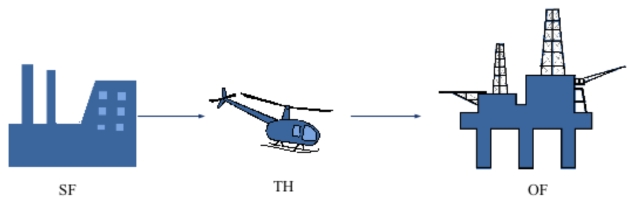

FIGURE 13 ALTERNATIVE SUPPLY CHAIN 1

The other supply alternative is to transport equipment from the onshore base (OB), where spares are stored, to the offshore facility (OF) by a vessel (TS) on spot charter. This supply scenario is illustrated in Figure 14, and is equivalent to the second echelon in the supply network. As with the helicopter alternative, it is assumed that loading and unloading times are included in the transportation times.

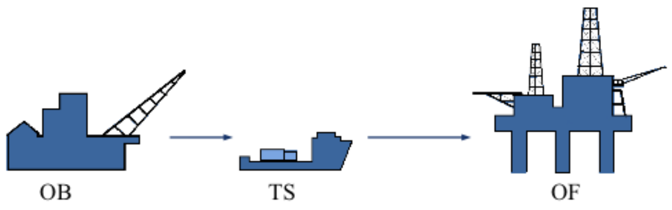

FIGURE 14 ALTERNATIVE SUPPLY CHAIN 2

The helicopter and spot chartered PSVs will only have a single possible route, as they will travel directly to the offshore facility from the supplier's facility or onshore base. Therefore, it is sufficient to determine the scheduling, the number of chartered vehicles, and the loading.

The costs related to these supply alternatives, are the chartering and travelling costs of the helicopters and vessels. The PSV is a spot charter, thus the charter rate is higher than for the time chartered vessels. Furthermore, if supply alternative two is chosen, a storage costs is generated at the onshore base. If no operation for the late deliveries is performed, operations at the offshore facility are delayed due to missing or commodities, and the facility will have downtime. Downtime can be expensive, and will generate a significant downtime cost, which also is included in the model. For simplicity, it is assumed that the excess demands only can be delivered with late deliveries. If this is not done, a downtime cost will be generated for the amount of time it will be necessary to stop the operation due to lack of commodities. Additionally, it is assumed that no downtime will be generated if the operators decide to have late deliveries. It is however assumed that there are some costs related to these operations such as stress or lost reputation, that cannot be quantified, therefore a penalty cost will be added for the number of hours the vehicles travels, which is an element that favour short lead times.

This is a complex problem, and it is necessary to make several additional assumptions and simplifications. It is assumed that there is no queuing for pick-up and delivery of the commodities. It is also assumed that there is no shortage of equipment at the suppliers' facilities, thus no waiting for equipment. For the purpose of this thesis, it is assumed that the bases and facilities are open at all times, and it is not necessary to consider opening hours. Unavailable vehicles due to maintenance and docking will not be included in the model, and it is assumed that these are available throughout the operational period.

This thesis will only address costs that vary as a result of the decisions made in terms of the fleet, routing and scheduling, and inventory decision. In a real life problem, there are several other cost elements that must be considered such as the amount of products transported, which often will be a determining factor for the transportation costs.

In a real life problem, it is necessary to transport the backloads from the offshore facility. The reason for this is the limited capacity at the offshore facility, and the hiring cost of the equipment that will continue to run until the equipment is delivered to the facility. For this thesis, it is assumed that backloads require less space than the demands, and can be loaded to the visiting vessels and transported back to the depot.

Service demands often require the right personnel, for this thesis it is assumed that assembling the right crew for the operations is a less time demanding operations than any of the supply alternatives. The reason for this is that personnel are transported with helicopters for operations on the NCS, thus it is not considered as a problem in this thesis.

CHAPTER 6

MATHEMATICAL FORMULATION

This chapter will elaborate on the deterministic and stochastic stage in the model developed to solve the problem described in the previous chapter. The model solves a mixed integer problem (MIP). The model is a two-stage recourse model, as described in chapter 3.2.2, and is made up of a deterministic and a stochastic stage. The deterministic stage will treat the uncertainty in the parameters as known, thus it only considers the planned demands. Whereas the stochastic stage will treat the uncertainty in demand, thus this model is extended to include the alternative methods for delivery, which are the recourse decisions.

This chapter is structured as follows: the approach and modelling of the deterministic stage is described in section 6.1, and the stochastic stage is described in section 6.2. The complete and compressed mathematical models can be found in appendices A and B, respectively

6.1 FIRST STAGE - DETERMINISTIC MODEL

The problem has an underlying network structure, and can be defined as a network problem in which the suppliers' facilities, onshore bases and the offshore facilities are the nodes and the distances travelled between them are the arcs. In classic VRPs the network of offshore facilities are defined on a graph $G = (V, E)$ where $V = \{1, \dots, \mathcal{N}\}$ is the vertex set and $E = \{[i,j] : i,j \in V, i \neq j\}$ is the arc set, and it is common to denote the depot as vertex 0, and include it as one of the nodes in the network. In this model, the depot will be modelled as a single start node, and not a part of the network of nodes, $\mathcal{N}$.

The set of constraints will be divided into the following parts: routing, loading and unloading, inventory and time.

6.1.1 ROUTING CONSTRAINTS

In this mathematical model it is only the seaborne transportation (echelon 2) that requires routing. It is assumed that the land-based transportation (echelon 1) will have direct routing from the suppliers' facilities to the onshore depot, thus the routing aspect of this echelon will not be considered in the routing constraints. The routing formulations are similar to the one defined by Agra et al. (2013)

The available fleet of vessels is denoted by $\mathcal{K}$ and indexed k. The route is created by the combination of arcs the vessels sail, and the set of offshore facilities is denoted $\mathcal{N}$. The offshore facilities can be visited several times by different vessels. Each visit at an offshore facility is enumerated by m, and the set of visits $\mathcal{M}_i$ at a facility i is denoted $\mathcal{S}$.

For the routing aspect of the model, the following binary variables are constructed: the binary variable x^{OB}_{imk} is 1 if a vessel k departs from the onshore base to make the mth visit at offshore facility i, and 0 otherwise. The corresponding binary variable for the vessel's return to the depot is x^{OE}_{imk}. The binary variable x_{imjnk} determines whether a vessel k sails from (i, m) to (j, n), it takes the value 1 if the vessel sails, and 0 otherwise. The binary variable z_{im}will take the value 1 if facility i is visited the mth time, and 0 otherwise, and w_{imk} will take the value 1 if the visit (i, m) is made by vessel k, and 0 otherwise.

Thus the routing constraints are as follows:

$$\sum_{(i,m)\in\mathcal{S}} x^{OB}_{imk} \leq 1 \qquad for\ k \in \mathcal{K} \qquad (6.1)$$

$$w_{imk} = x^{OB}_{imk} + \sum_{(j,n)\in\mathcal{S}} x_{jnimk} \qquad for\ (i, m) \in \mathcal{S}, k \in \mathcal{K} \qquad (6.2)$$

$$w_{imk} = x^{OE}_{imk} + \sum_{(j,n)\in\mathcal{S}} x_{imjnk} \qquad for\ (i, m) \in \mathcal{S}, k \in \mathcal{K} \qquad (6.3)$$

$$\sum_{k\in\mathcal{K}} w_{imk} = z_{im} \qquad for\ (i, m) \in \mathcal{S} \qquad (6.4)$$

$$z_{i(m-1)} - z_{im} \geq 0 \qquad for\ (i, m) \in \mathcal{S}|m > 1 \qquad (6.5)$$

$$x^{OB}_{imk}, x^{OE}_{imk}, w_{imk} \in \{0,1\} \qquad for\ (i, m) \in \mathcal{S}, k \in \mathcal{K} \qquad (6.6)$$

$$x_{imjnk} \in \{0,1\} \qquad for\ (i, m, j, n) \in \mathcal{S}, k \in \mathcal{K} \qquad (6.7)$$

$$z_{im} \in \{0,1\} \qquad for\ (i, m) \in \mathcal{S} \qquad (6.8)$$

The constraints (6.1)-(6.3) describe the flow on the route. Constraints (6.1) ensure that a vessel leaves the depot maximum one time, this also allows for the vessels to remain at the base. Route continuity is imposed by (6.2) and (6.3), which are the flow conservation constraints. These ensure that all subsequent visits between offshore facilities have equal ingoing and outgoing flow, or starts or ends at the depot. Constraints (6.2) state that the vessel either arrives at a facility from the depot or another facility, and constraints (6.3) state that the vessel after a visit either continuous to another facility or returns to the base. Constraint (6.4) and (6.5) ensure that the numbering of the visits are correct. The visit (i, m) can only be made by one vessel, and this is modelled in (6.4). The relation in constraints (6.5) ensure that higher visiting numbers are not used unless the preceding is used, thus the model will only use the smallest subsequent numbering. Constraints (6.6), (6.7) and (6.8) define the variables as binary.

6.1.2 LOADING AND UNLOADING

This problem is a multi-commodity problem, thus this element must be included. A set of products, $\mathcal{H}$, must be distributed from the suppliers to the offshore facilities. In this model the various producers deliver a single type of commodity, thus the set of producers are assigned the same set as the product types. Hence product h is provided by supplier h. It is assumed a homogenous fleet of vehicles and a homogenous fleet of vessels, which means that there are no restrictions to which vehicle or vessel in the fleet that can carry the different types of products.

If there is a demand for a product at an offshore facility, a vehicle is generated to transport the product type to the onshore base, from which the products will be distributed to the offshore facilities by a vessel. This mathematical model will follow the just-in-time principle, thus the products will be delivered directly from the vehicles to the vessels, without any intermediate storage at the onshore base. The integer variable y_{kh} determines the number of vehicles that travels from the producer to the onshore base for each vessel k that departs from the depot. The fleet of vehicles will only perform direct deliveries, thus the carrying capacity of a vehicle that departs from supplier h is denoted Q_h^V.

The amount of product h loaded to vessel k is expressed by the variable r_{kh}^O. The variable d_{imkh} determines the amount of product h that is delivered at visit (i,m) at by vessel k.

The vessel's capacity is a significant constraint to consider. A vessel will service the demand, and the total demands serviced by one vessel cannot exceed its capacity, it is assumed that the vessels have designated compartments for each product type h, thus the capacity of each compartment in the vessel is denoted by Q_h^K. This model will not include any lower or upper bound for the amount that is unloaded from the vessels.

Thus, the constraints for the loading and unloading of the vessel are:

$$r_{kh}^O = \sum_{(i,m)\in\mathcal{S}} d_{imkh} \qquad for\ k \in \mathcal{K}, h \in \mathcal{H} \qquad (6.9)$$

$$r_{kh}^O \leq Q_h^V y_{kh} \qquad for\ k \in \mathcal{K}, h \in \mathcal{H} \qquad (6.10)$$

$$r_{kh}^O \leq Q_h^K \sum_{(i,m)\in\mathcal{S}} x_{imk}^{OB} \qquad for\ k \in \mathcal{K}, h \in \mathcal{H} \qquad (6.11)$$

$$d_{imkh} \leq Q_h^K w_{imk} \qquad for\ (i,m) \in \mathcal{S}, k \in \mathcal{K}, h \in \mathcal{H} \qquad (6.12)$$

$$\sum_{h\in\mathcal{H}} d_{imkh} \geq w_{imk} \qquad for\ (i,m) \in \mathcal{S}, k \in \mathcal{K} \qquad (6.13)$$

$$y_{kh} \in \mathbb{Z}^+ \qquad for\ k \in \mathcal{K}, h \in \mathcal{H} \qquad (6.14)$$

$$r^{O}_{kh}, d_{imkh} \geq 0 \qquad for\ (i,m) \in \mathcal{S}, k \in \mathcal{K}, h \in \mathcal{H} \qquad (6.15)$$

The amount of cargo loaded at the depot is the sum of the products delivered on the voyage, this initial load is provided in (6.9). The constraints (6.10) ensure that the quantity of a commodities loaded to the vessel at the onshore depot, is transported from the producers. It also ensures that the necessary amount of vehicles is generated, and that the vehicle's capacity is not exceeded. Constraints (6.11) ensure that the products loaded to a vessel do not exceed the vessel's capacity. Constraints (6.12) state that if a vessel unloads at a facility, it must also visit the facility. Constraints (6.13) ensure that a vessel that visits a facility must make a delivery. This will also ensure that the vessel returns to the onshore depot when it is unloaded. The non-negativity constraints for the integer variables are ensured in (6.14), and the continuous variables in (6.15).

6.1.3 INVENTORY CONSTRAINTS

The deck space and storage space at the offshore facility is limited therefore an upper stock level S^{MAX}_{ih} is defined for each product and this limit cannot be exceeded. In this problem, stock-outs are not allowed, thus the stock levels cannot be negative. The minimum stock level is not included due to the fact that it is zero, and this therefore can be assured by the integer stock variables which cannot be negative. The initial stock level is denoted S^{0}_{ih}, and the stock level at the end of the time horizon is $S^{\overline{T}}_{ih}$.

The consumption rate of product h at facility i is given as W_{ih}, and the consumption rates determine the demands.

The amount to be delivered is determined by d_{imkh}. The stock level before a visit (i,m) is decided by an non-integer variable s^{B}_{imh}, and the stock level after this visit is decided by an non-integer variable, s^{E}_{imh}. It would have been sufficient to have one variable that determined the stock level at the end of an operation, but this model will also have the one that provides the stock level before an operation to ease the reading.

The inventory constraints are therefore:

$$S^{0}_{ih} - W_{ih} t^{B}_{im} = s^{B}_{imh} \qquad for\ (i,m) \in \mathcal{S} | m = 1\ h \in \mathcal{H} \qquad (6.16)$$

$$s^{B}_{imh} - W_{ih}\left(t^{E}_{im} - t^{B}_{\mathrm{im}}\right) + \sum_{k \in \mathcal{K}} d_{imkh} = s^{E}_{imh} \qquad for\ (i,m) \in \mathcal{S}, h \in \mathcal{H} \qquad (6.17)$$

$$s^{E}_{i(m-1)h} - W_{ih}\left(t^{B}_{im} - t^{E}_{i(m-1)}\right) = s^{B}_{imh} \qquad for\ (i,m) \in \mathcal{S} | m > 1, h \in \mathcal{H} \qquad (6.18)$$

$$S^{\overline{T}}_{ih} \leq s^{E}_{imh} - W_{ih}\left(\overline{T} - t^{E}_{im}\right) \qquad for\ (i,m) \in \mathcal{S} | \mathrm{m} = \mathcal{M}_i, h \in \mathcal{H} \qquad (6.19)$$

$$s_{imh}^{B}, s_{imh}^{E} \leq \overline{S}_{ih} \qquad for\ (i,m) \in \mathcal{S}, h \in \mathcal{H} \qquad (6.20)$$

$$s_{imh}^{B}, s_{imh}^{E} \leq 0 \qquad for (i,m) \in \mathcal{S}, h \in \mathcal{H} \qquad (6.21)$$

The stock level cannot fall below the lower limit, this is ensured by the stock level which cannot be negative, and generate the start time of the beginning of the visit, thus the inventory level at the beginning of the first visit is defined in (6.16). The set of constraints (6.17) provides the connection between the start and end stock of an operation. The transitioning function (6.18) ensures correlation between the inventory levels between two sequential visits. The inventory level at the end of a period must be above a minimum end stock level; this is ensured by (6.19). The set of constraints (6.20) ensure that stock levels are below the upper limits. The non-negativity is ensured in constraints (6.21).

6.1.4 TIME CONSTRAINTS

Time constraints are necessary to keep track of the inventory levels, and ensure that the vessels return to the depot within the end of the time horizon.

The sailing time to and from the depot, and to the offshore facilities is denoted by T_i^O. The time to sail from node i to j, is provided by T_{ij}. The loading time at the depot for one unit of product h, is T_h^L, whereas the unloading time at offshore facility i of one product of h is T_{ih}^U. The vessel must complete the route during the time period $\overline{T}$, so that it can be ready for a voyage in the consecutive period.

The consumption rates are constant, whereas the time variables t_{it}^B and t_{it}^E introduced for start and end time, respectively are integers. As with the stock variables, it is not necessary to have time variables for both the beginning and end of the operations, but they are included to make the reading of the results easier.

Thus, the model formulation is:

$$t_{im}^{B} - t_{i(m-1)}^{E} \geq 0 \qquad for\ (i,m) \in \mathcal{S} | m > 1 \qquad (6.22)$$

$$t_{\mathrm{im}}^{E} = t_{im}^{B} + \sum_{k \in \mathcal{K}} \sum_{h \in \mathcal{H}} T_{ih}^{U} d_{imkh} \qquad for\ (i,m) \in \mathcal{S} \qquad (6.23)$$

$$\sum_{k \in \mathcal{K}} x_{imjnk} \left(t_{im}^{E} + T_{ij} - \mathrm{t}_{jn}^{B} \right) = 0 \qquad for\ (i,m,j,n) \in \mathcal{S} \qquad (6.24)$$

$$\sum_{k \in \mathcal{K}} \left(T_i^O \mathrm{x}_{imk}^{OB} + \sum_{h \in H} T_h^L r_{kh}^{\mathrm{O}} \right) \leq t_{im}^{B} \qquad for\ (i,m) \in \mathcal{S} \qquad (6.25)$$

$$t_{im}^{E} + T_{i}^{O} \leq \overline{\mathrm{T}} \qquad for\ (i,m) \in \mathcal{S} \qquad (6.26)$$

$$t_{im}^{B}, t_{im}^{E} \in \mathbb{Z}^{+} \qquad for\ (i,m) \in \mathcal{S} \qquad (6.27)$$

Two operations cannot be performed simultaneously at the facility, thus constraints (6.22) ensure that the consecutive operation at the facility is initiated after the previous operation is completed. Constraints (6.23) link the start and end time of an operation. Constraints (6.24) relate the end time of operation (i, m) and start time of operation (j, n) for a vessel that sails between two facilities. The operation (j, n) cannot be initiated before the sailing from the preceding facility has been completed. This is a nonlinear constraint and must be linearized for the implementation. Constraints (6.25) ensure that no operations at the facilities are initiated before a vessel will have the time to be loaded and sail the distance from the onshore base to the offshore facility. All the operations, including the sailing back to the depot, must be completed before the end time of the period; this is defined by constraints (6.26). The integer of the time variables is ensured by (6.27).

6.1.5 Objective function

The objective is to minimize the costs. The planned demands have hard time-windows so that all the demands must be delivered in time, therefore there will not be generated a downtime cost in this phase.

The cost of sailing edge (i, j) is determined by the time for the leg to be completed, and the cost is provided by the constant C_{ij}^{K}. The cost for the vessel to sail to and from the depot to an installation i is provided by C_{i}^{OK}. The transportation cost of the vehicles, which only travels directly from the suppliers to the depot, is included in the vehicle's charter cost.

The cost of using a vehicle or vessel is fixed. A cost is generated for each voyage a vehicle or vessels perform, as in a voyage charter. Introduce a charter cost $C^{TC,K}$ for each vessel that is used. The cost of chartering a vehicle for a voyage is set to $C^{TC,V}$. These costs are linearly dependent on the number of vessels and vehicles, respectively, that are used.

Thus, the objective function is:

$$\min f = \sum_{(i,mj,n)\in\mathcal{S}} \sum_{k\in\mathcal{K}} \mathrm{C}_{ij}^{K} x_{imjnk} \tag{6.28a}$$

$$+ \sum_{(i,m)\in\mathcal{S}} \sum_{k\in\mathcal{K}} C_{i}^{OK} \left(\mathrm{x}_{imk}^{OB} + \mathrm{x}_{imk}^{OE}\right) \tag{6.28b}$$

$$+ \sum_{(i,m)\in\mathcal{S}} \sum_{k\in\mathcal{K}} \mathrm{C}^{TC,K} \mathrm{x}_{imk}^{OB} \tag{6.28c}$$

$$+ \sum_{k\in\mathcal{K}} \sum_{h\in\mathcal{H}} \mathrm{C}_{h}^{TC,V} y_{kh} \tag{6.28d}$$

The first part (28a) defines the sailing costs, the second (28b) defines the cost of sailing to and from the depot; the charter cost of the vessels is provided by (28c) and the charter cost of the vehicles is provided by (28d).

6.2 Second Stage - Stochastic Model

The stochastic formulation of the problem will be presented in this section, and the stochastic elements are the uncertainty in demand. It is assumed that the actual demands are revealed after each visit, which means that upon the visit, the amount of commodities that are lacking will be revealed. The actual demands depend on the scenario, Ω, of which each scenario is indexed c. Thus, the actual consumption rates are provided by W_{ihc}^{R}.

Upon the arrival and realization of the actual stock levels, the operators are provided three alternatives, these are the recourse decisions and these will be based on the time of the visits that are decided in the first stage. The first alternative is to stop the production for the amount of commodities they are lacking for, which will generate downtime. The two other alternatives involve late deliveries. In this model two alternatives for late deliveries are suggested: transportation from the storage at the supply base or to send a helicopter directly from the suppliers' facilities. It is assumed that the offshore facilities have an emergency storage that can be utilized for the missing demand until the late deliveries are performed. Thus, the stock levels determined in stage 1 will not be altered.

6.2.1 Alternative 1

If no last deliveries are ordered, the offshore facility will have downtime. The downtime that is generated will be equal to the number of hours, t_{imhc}^{R}, it would take for the amount of planned demands to be utilized, this connection is provided by the constraints (6.29) and (6.30). The binary variable δ_{imc}^{1} is set to 1 if there is downtime, and zero otherwise.

$$\left(W_{ihc}^{R} - W_{ih}\right)t_{im}^{B} = W_{ih}t_{imhc}^{R} \qquad for\ (i,m) \in \mathcal{S} | m = 1, h \in \mathcal{H}, c \in \Omega \qquad (6.29)$$

$$\left(W_{ihc}^{R} - W_{ih}\right)\left(t_{im}^{B} - \mathrm{t}_{i(m-1)}^{E}\right) = W_{ih}t_{imhc}^{R} \qquad for\ (i,m) \in \mathcal{S} | m > 1, h \in \mathcal{H}, c \in \Omega \qquad (6.30)$$

$$\sum_{h \in \mathcal{H}} t_{imhc}^{R} \leq \overline{T}\delta_{imc}^{1} \qquad for\ (i,m) \in \mathcal{S}, c \in \Omega \qquad (6.31)$$

$$t_{imhc}^{R} \in \mathbb{Z}^{+} \qquad for\ (i,m) \in \mathcal{S}, h \in \mathcal{H}, c \in \Omega \qquad (6.32)$$

$$\delta_{imc}^{1} \in \{0,1\} \qquad for\ (i,m) \in \mathcal{S}, c \in \Omega \qquad (6.33)$$

Constraint (6.29) and (6.30) set the value for the downtime for the first and consecutive visits, respectively. Constraints (6.31) ensure that the binary variable is set to one if there is downtime. The downtime is an integer variable, this is ensured in (6.32) and the binary variable is ensured in (6.33).

The downtime cost will be linearly dependent on the number of hours delay, and if no late deliveries are performed, this will be generated. Thus, the estimated value of (6.34) must be added to the objective function from step 1.

$$\sum_{(i,m) \in \mathcal{S}} \sum_{h \in \mathcal{H}} \sum_{c \in \Omega} C^{DT} t_{imhc}^{R} \delta_{imc}^{1} \qquad (6.34)$$

6.2.2 ALTERNATIVE 2

Late deliveries can be performed with a supply vessel that is spot chartered. The vessel will depart from the onshore supply base where there is storage for spare commodities. The amount of late deliveries required by an offshore facility i for a visit m for a given scenario c, is provided by d_{imhc}^{R}.

If this alternative is used for one visit in the scenario, a storage cost will be generated for the entire planning horizon of this scenario. This cost is independent of the storage levels and the amount used, the level of commodities in storage are assumed to be unlimited. The delivery must be shipped directly to the offshore facility. Furthermore, the demands are assumed to be sufficiently small and the planners want to utilize the capacity efficiently, so that the capacity of the vessel will not be constrained by the individual commodity types, but the total load.

If this alternative is chosen for visit (i,m) in scenario c, the binary variable δ_{imc}^{2} is generated. Furthermore, if this alternative is chosen for any of the visits in any of the scenarios, the binary variable δ^{2} is set to 1.

Thus, the constraints for this alternative are:

$$(W_{ihc} - W_{ih})t_{im}^{B} = d_{imhc}^{R} \qquad for\ (i,m) \in \mathcal{S}|m = 1, c \in \Omega \qquad (6.35)$$

$$(W_{ihc} - W_{ih})\left(t_{im}^{B} - t_{i(m-1)}^{E}\right) = d_{imhc}^{R} \qquad for\ (i,m) \in \mathcal{S}|m > 1, c \in \Omega \qquad (6.36)$$

$$\sum_{h \in \mathcal{H}} Q_{h}^{K}\, \delta_{imc}^{2} \geq \sum_{h \in \mathcal{H}} d_{imhc}^{R} \qquad for\ (i,m) \in \mathcal{S}, c \in \Omega \qquad (6.37)$$

$$\sum_{(i,m) \in \mathcal{S}} \delta_{imc}^{2} \leq \sum_{h \in \mathcal{H}} Q_{h}^{K}\, \delta_{c}^{2} \qquad for\ c \in \Omega \qquad (6.38)$$

$$d_{imhc}^{R} \in \mathbb{Z}^{+} \qquad for\ (i,m) \in \mathcal{S}, c \in \Omega \qquad (6.39)$$

$$\delta_{imc}^{2} \in \{0,1\} \qquad for\ (i,m) \in \mathcal{S}, c \in \Omega \qquad (6.40)$$

$$\delta_{c}^{2} \in \{0,1\} \qquad for\ c \in \Omega \qquad (6.41)$$

The constraints (6.35) and (6.36) set the necessary amount of late demands that must be delivered for a visit (i,m). Constraints (6.37) ensure that the binary variable is given the value 1 if this alternative is chosen for a visit. Whereas constraints (6.38) ensure that if this alternative is chosen for one single visit in a scenario, the binary variable for the storage cost is generated. The integer is ensured by (6.39), and the binary variables are ensured in (6.40) and (6.41).

Thus, the estimated value of the stochastic objective function (6.42) must be added to the objective function in step 1. Where $C^{SPOT,K}$ is the spot charter rate and sailing cost per chartered vessel, and $C^{P}$is the penalty cost per hour of sailing time. The vessels will sail directly from the onshore depot to the offshore site, therefore the sailing costs are constant and can be added to in the spot rate. C^{S} is the storage cost.

$$\sum_{c \in \Omega} \left(\sum_{(i,m) \in \mathcal{S}} \delta_{imc}^{2} \left(C^{SPOT,K} + C^{P} T_{i}^{O} \right) + C^{S} \delta_{c}^{2} \right) \qquad (6.42)$$

6.2.3 Alternative 3

The third alternative is to utilize helicopters to perform late deliveries. The helicopters have limited capacity and this must be considered. Furthermore, the helicopters depart from various suppliers, and it is assumed that the helicopters only do direct deliveries. The number of chartered helicopters for each visit in each scenario is denoted, u_{imhc}.

If this alternative is chosen for visit (i,m) in scenario c, the binary variable δ^3_{imhc} is generated.

Thus the mathematical formulation is provided by (6.43) to (6.48).

$$(W_{ihc} - W_{ih})t^B_{im} = d^R_{imhc} \qquad for\ (i,m) \in \mathcal{S}|m = 1, c \in \Omega \qquad (6.43)$$

$$(W_{ihc} - W_{ih})\big(t^B_{im} - t^E_{i(m-1)}\big) = d^R_{imhc} \qquad for\ (i,m) \in \mathcal{S}|m > 1, c \in \Omega \qquad (6.44)$$

$$Q^H_h u_{imhc} \geq d^R_{imhc} \qquad for\ (i,m) \in \mathcal{S}, h \in \mathcal{H}, c \in \Omega \qquad (6.45)$$

$$\sum_{h\in\mathcal{H}} u_{imhc} \leq \sum_{h\in\mathcal{H}} Q^H_h\, \delta^3_{imc} \qquad for\ (i,m) \in \mathcal{S}, c \in \Omega \qquad (6.46)$$

$$u_{imhc} \in \mathbb{Z}^+ \qquad for\ (i,m) \in \mathcal{S}, h \in \mathcal{H}, c \in \Omega \qquad (6.47)$$

$$\delta^3_{imc} \in \{0,1\} \qquad for\ (i,m) \in \mathcal{S}, c \in \Omega \qquad (6.48)$$

As with alternative 2, the constraints (6.43) and (6.44) set the necessary amount of late demands that must be delivered for a visit (i,m). Constraints (6.45) ensure that the binary variable is given the value 1 if supplies are delivered with helicopters, and it generates the required amount of helicopters to transport the load. Whereas constraints (6.46) ensure that the same decision is made for all the commodities in one visit. The integral variable is ensured in (6.47), and the binary variable is ensured in (6.48).

Thus, the estimated value of (6.49) must be added to the objective function. Where $C^{SPOT,H}$ is the spot rate per chartered helicopter, and $C^P$is the penalty cost per hour of sailing time.

$$\sum_{(i,m)\in\mathcal{S}} \sum_{c\in\Omega} \left(\sum_{h\in\mathcal{H}} \mathrm{C}^{SPOT,H} \mathrm{u}_{imhc} + C^P T^H_i \delta^3_{imc} \right) \qquad (6.49)$$

6.2.4 Combined

In order to compress the stochastic stage, a set for the three alternatives is introduced. The set is denoted $\mathcal{A}$ and indexed a.

It is assumed that the operators only can chose one of the alternatives for each visit (i, m), thus constraint (6.53) must be added, these are the recourse variables. Constraints (6.31) becomes redundant due to constraint (6.53).

$$\left(W_{ihc}^{R} - W_{ih}\right)t_{im}^{B} = d_{imhc}^{\mathrm{R}} \qquad for\ (i,m) \in \mathcal{S}|m = 1, h \in \mathcal{H}, c \in \Omega \tag{6.50}$$

$$\left(W_{ihc}^{R} - W_{ih}\right)\left(t_{im}^{B} - t_{i(m-1)}^{E}\right) = d_{imhc}^{R} \qquad for\ (i,m) \in \mathcal{S}|m > 1, h \in \mathcal{H}, c \in \Omega \tag{6.51}$$

$$W_{ih}t_{imhc}^{R} = d_{imhc}^{R} - \sum_{a \in \mathcal{A}/\{1\}} d_{imhca}^{R} \qquad for\ (i,m) \in \mathcal{S}, h \in \mathcal{H}, c \in \Omega \tag{6.52}$$

$$\sum_{a \in \mathcal{A}} \delta_{imca} = 1 \qquad for\ (i,m) \in \mathcal{S}, c \in \Omega \tag{6.53}$$

$$d_{imhc}^{R} = \sum_{a \in \mathcal{A}/\{1\}} d_{imhca}^{R} \qquad for\ (i,m) \in \mathcal{S}, h \in \mathcal{H}, c \in \Omega \tag{6.54}$$

$$\sum_{h \in \mathcal{H}} Q_h^K\, \delta_{imca} \geq \sum_{h \in \mathcal{H}} d_{imhca}^{R} \qquad for\ (i,m) \in \mathcal{S}, c \in \Omega\,, a \in \mathcal{A}|a = 2 \tag{6.55}$$

$$\sum_{(\mathrm{i},m) \in \mathcal{S}} \delta_{imca} \leq \sum_{h \in \mathcal{H}} Q_h^K\, \delta_{ca} \qquad for\ c \in \Omega, a \in \mathcal{A}|a = 2 \tag{6.56}$$

$$Q_h^H u_{imhc} \geq d_{imhca}^{R} \qquad for\ (i,m) \in \mathcal{S}, c \in \Omega, a \in \mathcal{A}|a = 3 \tag{6.57}$$

$$\sum_{h \in \mathcal{H}} u_{imhc} \leq \sum_{h \in \mathcal{H}} Q_h^H\, \delta_{imca} \qquad for\ (i,m) \in \mathcal{S}, c \in \Omega, a \in \mathcal{A}|a = 3 \tag{6.58}$$

$$t_{imhc}^{R}, d_{imhc}^{R}, u_{imhc} \in \mathbb{Z}^{+} \qquad for\ (i,m) \in \mathcal{S}, h \in \mathcal{H}, c \in \Omega \tag{6.59}$$

$$d_{imhca}^{R} \in \mathbb{Z}^{+} \qquad for\ (i,m) \in \mathcal{S}, h \in \mathcal{H}, c \in \Omega, a \in \mathcal{A} \tag{6.60}$$

$$\delta_{imca} \in \{0,1\} \qquad for\ (i,m) \in \mathcal{S}, c \in \Omega, a \in \mathcal{A} \tag{6.61}$$

$$\delta_{ca} \in \{0,1\} \qquad c \in \Omega, a \in \mathcal{A} \tag{6.62}$$

The objective function for the stochastic problem is provided by (6.63a-e).

$$\min g_c = C^S \delta_{c2}$$

$$+ \sum_{(i,m)\in \mathcal{S}} \left(C_i^{SC,K} + C^P T_i^O\right) \delta_{imc2} \quad (6.63a)$$

$$+ \sum_{(i,m)\in \mathcal{S}} \sum_{h\in H} C^{DT} t_{imhc}^R \, \delta_{imc1} \quad (6.63b)$$

$$+ \sum_{(i,m)\in \mathcal{S}} \sum_{h\in H} \left(C_{ih}^{SC,H} \quad (6.63c)\right.$$

$$\left. + C^P T_{ih}^O\right) u_{imhc} \quad (6.63d)$$

$$(6.63e)$$

Thus, the total recourse model is provided by the sum of the objective functions (6.28a-d) and the estimated value of (6.63a-e), and can be expressed as:

$$\min f + \sum_{c\in\Omega} \{\min g_c\}$$

The constraints are provided by the constraints in (6.1) to (6.27), and (6.50) to (6.62). Thus, the time for the visits, stock levels, and routing and scheduling of planned demands, are decided in stage 1. The decisions regarding the time of the visits are implemented in stage 2, and used to optimize the unexpected demands.

CHAPTER 7

COMPUTATIONAL STUDY

The mathematical model described in Chapter 6 is tested with a numerical study. For this purpose, the model is written in the modelling language Mosel version 2.2.3, and implemented in Xpress-IVE version 1.22.04. The optimization is solved with an Intel(R) Xeon(R) 3.33GHz processor, with a 32 GB memory.

Several of the input parameters are matrixes that are computed before the implementation, this is primarily done with MATLAB or Microsoft Excel. The calculations are exported to a text file that serves as the input file in Xpress-IVE. The model is solved in Xpress-IVE, and the results are written to text files. This workflow is provided in Figure 15.

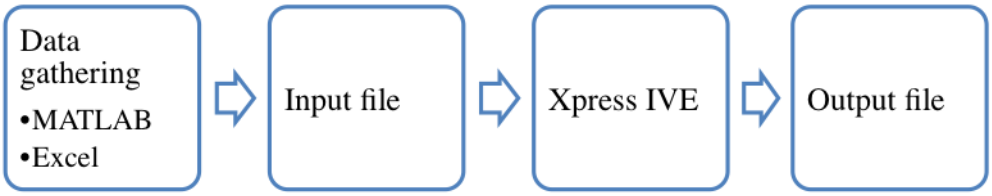

FIGURE 15 FLOW CHART OF THE WORK FLOW

The remainder of this chapter is structured as follows: some adjustments must be made before the implementation of the model, and these will be described in section 7.1. Test case and input data for the computational study will be provided in section 7.2, and the results of the study are provided in section 7.3. The source codes for the stochastic and deterministic stages are given in appendices D and E, respectively. Whereas the corresponding input files are provided in appendices I and J. The MATLAB code can be found in Appendix F.

7.1 MODEL ADJUSTMENT

There are several adjustments that must be made to the model before the implementation in Xpress IVE, all of which will be described in the following sections.

7.1.1 LINEARIZATION

Xpress Optimizer can only solve instances that are linear. Therefore, the time constraints in (6.24) which are not linear must be linearized by the use of the Big M method. The maximum value for the time variables is the time horizon $\overline{T}$, thus this will be used as the big M. Furthermore, these constraints are equality constraints and require therefore two linearizing constraints.

This provides the linearization in (7.1) and (7.2).

$$t_{im}^{E} + T_{ij} - t_{jn}^{B} + \overline{T} \sum_{k \in \mathcal{K}} x_{imjnk} \leq \overline{T} \qquad for\ (i,m,j,n) \in \mathcal{S} \tag{7.1}$$

$$t_{im}^{E} + T_{ij} - t_{jn}^{B} - \overline{T} \sum_{k \in \mathcal{K}} x_{imjnk} \geq -\overline{T} \qquad for\ (i,m,j,n) \in \mathcal{S} \tag{7.2}$$

7.1.2 Simplifications

The time for the loading and unloading of commodities is originally given as a fixed rate for units commodity per unit time, in which one unit time equals one hour. Hence, the parameters are linearly dependent on the amount loaded. In the modelling this is simplified by the elimination of the product index h from the parameter T_h^L and T_{ih}^U, and the parameters are changed to T^L and T_i^U, respectively. Thus the time to load and unload commodities will not vary with the amount or type of product loaded or unloaded. .

The formulation of constraints (6.23) and (6.25) in the implementation is therefore changed to (7.3) and (7.4):

$$t_{\mathrm{im}}^{E} = t_{im}^{B} + T_i^U \qquad for\ (i,m) \in \mathcal{S} \tag{7.3}$$

$$\sum_{k \in \mathcal{K}} \left(T_i^O \mathrm{x}_{imk}^{OB} + \mathrm{T}^L w_{imk} \right) \leq t_{im}^{B} \qquad for\ (i,m) \in \mathcal{S} \tag{7.4}$$

Furthermore, the inventory constraints in (6.18) can also be simplified, and the formulation in (7.5) is implemented in Xpress IVE.

$$s_{imh}^{B} - W_{ih} T_i^U + \sum_{k \in \mathcal{K}} d_{imkh} = s_{imh}^{E} \qquad for\ (i,m) \in \mathcal{S}, h \in \mathcal{H} \tag{7.5}$$

The mathematical model requires the set of visits to be generated dynamically. However, during the work with this thesis, there was not found any solutions for how this dynamic set can be generated in Mosel Xpress. As a result of this, the model cannot consider the last visit $\mathcal{M}_i$ as constraints (6.19) require. Thus, this constraint must be replaced with constraint (7.6) in the implementation.

$$S_{ih}^{0} - \overline{T} W_{ih} + \sum_{k \in \mathcal{K}} d_{imkh} \geq S_{ih}^{\overline{T}} \qquad for\ (i,m) \in \mathcal{S}, h \in \mathcal{H} \tag{7.6}$$

Constraints (7.6) summarize the inventory level at an offshore facility for the entire planning horizon. Therefore, it ensures that the necessary amount of demands are serviced within the time horizon, so that the inventory level at the end of the time horizon is equal or larger than the required end stock.

In order to ensure that a visit is made for the deliveries, constraints (7.7) are also added in the implementation. This formulation will force the binary visit variable to be one if the sum of the stock level at the beginning of a visit, the delivered products and the consumed inventory are larger than zero. Furthermore, this ensures that the upper stock levels are not exceeded, and this makes both constraints (6.20) and (6.12) redundant.

$$\overline{S}_{ih} z_{im} \geq s_{im}^{B} + \sum_{k \in \mathcal{K}} d_{imkh} - W_{ih} T_{i}^{U} \qquad for\ (i,m) \in \mathcal{S}, h \in \mathcal{H} \qquad (7.7)$$

The initial running of the model showed that the vessels have a tendency to travel the same distance back and forth, and service an installation several times. This resulted in a vessel that sailed the entire duration of the time period. This is not assumed to be a realistic representation of the real life problem. Therefore, an upper limit on the operating time, T^{MAX}, for the vessels is added by constraint (7.8), which summarize the vessels' total operational time.

$$\sum_{(i,m) \in \mathcal{S}} \left(\mathrm{T}^{L} \mathrm{x}_{imk}^{OB} + T_{i}^{O} \left(\mathrm{x}_{imk}^{OB} + \mathrm{x}_{imk}^{OE} \right) + \mathrm{T}_{i}^{U} w_{imk} \right) + \sum_{(i,m,j,n) \in \mathcal{S}} T_{ij} x_{imjn\mathrm{k}} \leq T^{MAX} \qquad for\ k \in \mathcal{K} \qquad (7.8)$$

All of the simplifications above only apply to the deterministic stage, but several changes are also made for the implementation of the stochastic stage. The stochastic variables are simplified by the elimination of the index for the products h, thus the model does not distinguish between the various types of commodities, but rather the total demand requirements for a visit. This is a relatively simplified version of the problem. In this adjusted formulation the various product types might be mixed, and the capacity of a specific product type do no longer apply. This is however considered necessary due to the relatively small amounts of each product type that is required for late deliveries. If the model was to consider each individual product type and these could not be mixed for one voyage, it would generate a significant amount of vehicles that would transport an insignificant amount of each product type, which is not a likely scenario as the planners always will attempt to fully utilize the resources.

Due to the fact that the stochastic stage does not distinguish between the product types, the helicopter routing is limited to one single distributor. Hence, all the commodities, regardless of type, can be picked up at one location, and then transported directly to the offshore facility that requires a service.

These simplifications provide a significant change to the formulations presented in Chapter 6, therefore the stochastic stage that is implemented in Xpress is provided in Appendix C.

In order for the implementation of the stochastic stage to run, Xpress IVE requires the demands to be integer. The demands for the late deliveries in each scenario are determined by the difference between the estimated and real consumption rate, multiplied with the time of the deliveries. The difference in demands between the scenarios per hour is significantly smaller than an integer and setting these values to integer would provide unrealistically high or low demands for late deliveries. However, as the time of the deliveries are determined in the deterministic model, and therefore can be defined as a parameter in the stochastic stage, the late demands in each scenario can be considered a parameter. The constraints (6.50) and (6.51) can therefore be included as parameters instead of constraints in the implementation. Thus, the total late demands are calculated, and the round-function rounds the value to the nearest integer, which provides an implementation that runs in Xpress-IVE.

7.2.2 VARIABLE REDUCTION

With increased problem size, the solution time also increases significantly. This problem might get extensive due to the high number of potential variables that can be generated by the indexes.

If this model is to be used as a decision support tool, it is required that the solution times are short, as decisions often must be made in reasonable time. The elimination of variables is one procedure that can limit the problem size. In this thesis the number of variables x_{imjnk} is reduced as this variable only is created for the combinations in which $i \neq j$. The elimination of the product index in the stochastic stage does also provide a significant reduction of variables.

Several of the variables in the deterministic step are derived from other variables by summation. These are primarily included to ease the reading of the models and the results, but do not contribute to the solution. Therefore, these could be eliminated to reduce the solution time. A more thorough discussion on this will follow in Chapter 8.

7.2 TEST CASE

The case study is created for this thesis, and does not represent a real-life problem. Therefore, several of the input data provided in this case are based on assumptions. The assumptions are to the extent possible based on real data in order to provide a relatively realistic case. The data and assumptions, and the reasoning for these, are described in this section.

7.2.1 LOCATIONS

The locations of the exploration wells are determined based on the current exploration operations performed in the North Sea as of May 20th, 2015. The locations and the name of the wells are provided in Table 3 and illustrated in Figure 16 Exploration sites. Source: (NPD, 2015) Each of the wells are assigned a site number, which will be used for the remainder of the case study.

TABLE 3 EXPLORATION WELLS IN THE NORTH SEA

Wellbore	Site	Main Area	Purpose	Entered date	Degrees	
16/1-22 S	1	North Sea	Appraisal	April 26th 2015	58° 54’ 23.10” N	2° 09’ 43.20” E
15/6-13 A	2	North Sea	Appraisal	May 17th 2015	58° 36’ 55.51” N	1° 45’ 40.49” E
15/12-24 S	3	North Sea	Wildcat	April 15th 2015	58° 07’ 26.79” N	1° 55’ 22.64” E
2/4-23 S	4	North Sea	Wildcat	March 13th 2015	56° 41’ 26.85” N	3° 06’ 07.21” E

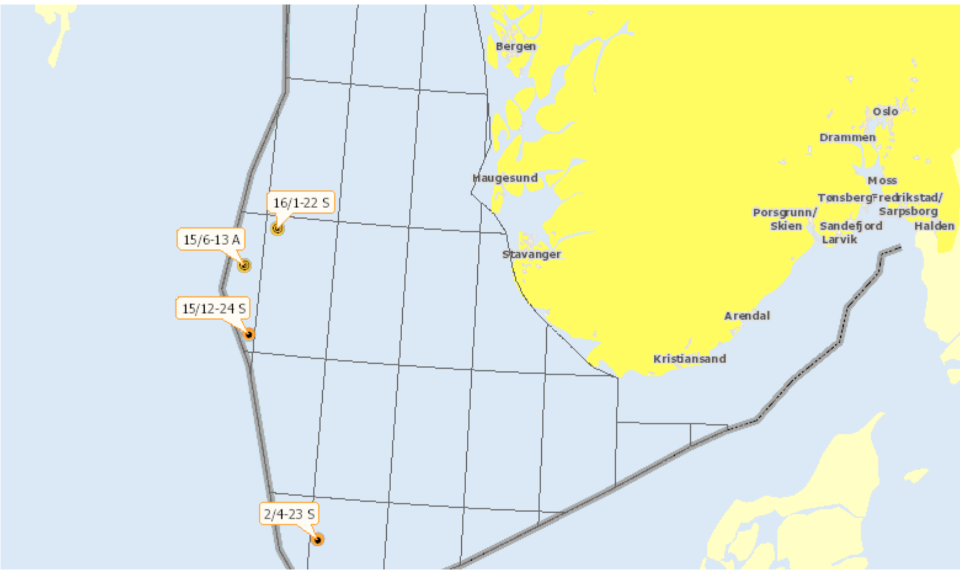

FIGURE 16 EXPLORATION SITES. SOURCE: (NPD, 2015)

Based on the location of the exploration wells, Tananger is assumed to be the supply base that will provide the exploration facilities with the shortest response times. Therefore, Tananger will be used as the supply base in this case study. The location of the base is provided in Table 4.

TABLE 4 LOCATION OF ONSHORE SUPPLY BASE

Location	Type of facility	Degree	
Tananger	Onshore Supply Base	56° 55’ 59.00” N	05° 36’ 43.20” E

It is assumed that the products are distributed at various onshore bases along the coast of Norway, which means that these bases will correspond to what in this thesis has previously been referred to as the supplier locations. The supplies are distributed between the onshore bases by semi-trucks, which is a common practice for the operating companies at the NCS.(Bring, 2015)

As for this case study, water and fuel is assumed to be present at the onshore base in Tananger, and there is no need to transport these supplies with trucks. As for the dry bulk, brine and mud, these are assumed to be located at the onshore bases in Kristiansund, Haugesund and Mongstad, respectively. Equipment will not be considered in this case study, due to lack of data. The coordinates of the onshore bases are provided in Table 5.

TABLE 5 LOCATION OF SUPPLIES

Product	Base	Degrees	
Water	Tananger	56° 55’ 59” N	05° 36’ 43” E
Dry Bulk	Kristiansund	63° 06′ 37″ N	07° 43′ 40″ E
Mud	Mongstad	60° 47′ 39″ N	05° 04′ 01″ E
Brine	Haugesund	58° 56′ 10″ N	05° 34′ 26″ E
Fuel	Tananger	56° 55’ 59” N	05° 36’ 43” E

The distances are used as the basis for the calculations of travelling times. The distance from the onshore base in Tananger to the offshore facilities is determined by the use of MATLAB. It is assumed that the sailing speeds of the vessels are 12 knots, and this speed provides the sailing duration. These numbers are rounded to the nearest integer solution for the implementation in Xpress IVE.

As the helicopters are assumed to travel in a straight line between the facilities, the same MATLAB code was used to determine the distance between the onshore bases (supplier facilities) and the offshore installations. It is assumed that the helicopters have a travelling speed of 162 knots. Furthermore, for the implementation in Xpress it is assumed that the all the commodities can be retrieved from the supplier in Kristiansund. Therefore, it is only the travelling distances from this facility that is included in the computational study.

The onshore distances between the supply bases with the various commodities, and the onshore base in Tananger is determined by NAF’s route planner (NAF, 2015). The western coast of Norway has several fjords and islands, which means that trucks may have to travel by ferries, additionally the speed limits may vary significantly and there may be queues on the road due to harsh weather or accidents. All of these factors are to a certain extent included in NAF’s route planner. Thus, it is assumed that this tool provides the most accurate transportation times and distances, these data are provided in Table 6.

TABLE 6 THE TRAVELLING TIMES AND DISTANCES FOR THE SEMI-TRAILERS

Product	Site	Base	Distance [km]	Hours [h]
Water	Base	Tananger	0	0
Dry bulk	Kristiansund	Kristiansund	765.00	14
Mud	Mongstad	Mongstad	272.00	6
Brine	Haugesund	Haugesund	69.00	2
Fuel	Base	Tananger	0	0

All of the data for the travelling times and distances for the PSVs and helicopters, are provided in the input files in Appendix I and J.

7.2.2 TIME

The common approach in optimization of offshore supply services planning is to have a planning period of a week, for reference see Fagerholt and Lindstad (2000), Halvorsen-Weare et al. (2012) and Maisiuk and Gribkovskaia (2014). These studies divide the time horizon into days, whereas the study in this thesis will break the time horizon down to hours. Due to the quantities of the demand, setting a time horizon equal to seven days provided several demands equal or close to zero, which would not make for an interesting case. Therefore, five days provided better data, and is chosen for this implementation, thus this case study will study operation from Monday to Friday. It is assumed that the operation is equal on all days. Furthermore, as the model defines the time as hours, all the parameters such as consumption and costs that are given per hour.

7.2.3 DEMANDS

The demands are based on the data provided in Appendix G. It is assumed that the offshore sites are at various stages in the exploration operations, and therefore require different amounts of supply. The model will use volume, as the measure for demands. Thus, the demand data are converted from tons to cubic meters. The densities used for the calculations are also provided in Appendix G.

It is assumed that the demands are equally distributed over the week. The demands are therefore divided on the number of hours in five days, and the result yields the consumption rates for each commodity at the offshore sites. The deterministic demands at each offshore facility for a week and the corresponding hourly consumption rates are provided in Table 7.

TABLE 7 WEEKLY DEMANDS AND CONSUMPTION RATES OF COMMODITIES AT OFFSHORE SITE

	Demand	Dry bulk [m^3]	Mud [m^3]	Brine [m^3]	Fuel [m^3]	Water [m^3]
Site 1	Weekly	62.00	167.00	185.00	184.00	304.00
	Per hour	0.51	1.39	1.54	1.54	2.53
Site 2	Weekly	34.00	55.00	47.00	138.00	154.00
	Per hour	0.28	0.46	0.39	1.15	1.28
Site 3	Weekly	62.00	39.00	32.00	109.00	249.00
	Per hour	0.52	0.33	0.26	0.91	2.08
Site 4	Weekly	19.00	12.00	157.00	88.00	99.00
	Per hour	0.15	0.10	1.31	0.73	0.83

The stochastic demands are modelled as scenarios, and for simplicity, it is assumed that the operating conditions that cause a change in demand applies to all the offshore sites. Thus, the same scenarios apply to all the installations throughout the time horizon.

It is assumed that the uncertainty in demand may increase up to 5 %. It is however likely to assume that the uncertainty in demand also may decrease with 5 %, which could result in storage levels at the offshore facilities that exceeds the tank capacity. The demands that are lower than expected will not require late deliveries, but it may be a backhaul issue, which is not addressed in this thesis. Therefore, it is only the demands that exceed the predicted demands that will be considered as uncertain.

Four scenarios are modelled in this case study. The first scenario has demands that are equal to the ones modelled in the deterministic stage. The remaining three scenarios represent an increase in the demand of 1%, 3% and 5%, respectively. The development of these demands over a time horizon is provided in Figure 17. The probability of the first scenario is set to 85%, the remaining three scenarios with the increased demand have a probability of 5%.

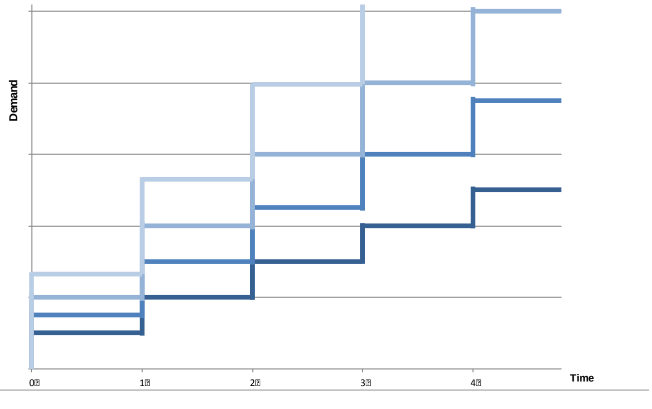

FIGURE 17 DEVELOPMENT OF DEMANDS FOR VARIOUS SCENARIOS

7.2.3 MODES OF TRANSPORTATION: COST AND CAPACITY

The modes of transportation that are modelled in this thesis are PSVs, helicopters and semi-trucks. The costs of utilizing these are significant, and due to limited information on real life data, these are primarily based in assumptions. All of the data used is summarized in Table 8.

The fleet of vessels that are modelled in this case study are PSVs, and the fleet is homogenous. The "World Pearl", which is a PSV of 3300 DWT that operates in the North Sea, is used as the reference vessel. The necessary data for this vessel is provided in appendix H.

The charter rates for PSVs depend on the deck capacity, which often provides the limiting constraint for the amount of cargo the vessel can carry. The spot charter rate for the PSVs is based on the rates provided by Westshore (2015). Furthermore, it is assumed that the spot charter of the vessel is 1.5 times more expensive than the time charter rate. The vessels' travelling costs are obtained from transportation cost models developed by Grønland (2011). These provide the cost per hour of travelling for a PSV of 3000 DWT, and are considered a reasonable estimate for this case study. The cost of loading and unloading the vessel is not included in the rates.

The capacity of the PSV is obtained from capacity data for the reference vessel, which are rounded to the nearest ten. These data were provided in cubic meters, and for the individual commodities, thus it is not necessary to investigate whether the volume capacity violates the weight capacity.

Semi-trailers travel between the bases to service the onshore bases.(Bring, 2015) The travelling cost is often linear with the quantity of product transported, which equals the amount of cargo that leaves the depot. Due to lack of data and the increased complexity of the mathematical model, this element is not included in the model. As for the travelling costs of the semi-trailers, these are also obtained from transportation cost models developed by Grønland (2011). It is assumed that the relation between the travelling costs of semi-trailers and PSVs is a valid representation for the ratio between the chartering costs of the two modes of transportation, thus the charter cost of a semi-trailer is found by this ratio and the time charter rate for the PSVs. In the implementation, the costs for the semi-trucks are constant from each supplier facility as they only have direct routing. Therefore, the total cost of utilizing these can be calculated as the sum of the charter cost and the travel cost multiplied with the hours the route requires.

The vehicles are given an upper limit for total weight and space requirements, thus constraints for both volume and weight must be investigate in order to avoid violation of constraints. The dimension of a semi-truck's carrying space is 82.96 m^3, in which the length is 13.60 m, the width 2.44 m and the height 2.50 m. The total loading capacity is 30.00 ton.(Bring, 2014) If the truck is loaded with 83.00 m^3 with the commodities, this will exceed the weight capacity of all the commodities. Hence, the weight sets the upper limit for the carrying capacity and the vehicles can transport the volume of each commodity that equals 30 ton of the product.

Helicopters are used to transport personnel and emergency equipment to offshore installations at the NCS. The helicopter type Sikorsky S-92 is a large and robust helicopter, and is commonly used for offshore operations, therefore this will be used as a basis for the data provided for the helicopters. These helicopters have a range of 542.00 nm, thus they can travel from all the suppliers to the offshore facilities without refuelling in this case study. The charter costs and travelling costs are assumed to be the double of the sailing and PSV's spot charter costs.

Similar to the semi-trucks, the helicopters do also have capacity constraints on both the weight and volume it can carry. The commodities can be transported in buckets, which can have a volume up to 5.00 m^3. The weight capacity of the Sikorsky helicopters is approximately 4.00 ton. There are however helicopters that can carry up to 20.00 ton (TU, 2015), it is assumed that helicopters used for late deliveries will be purposely built for the task, thus the upper capacity on the weight is set to 10.00 ton. Calculations show that a helicopter loaded with 10.00 ton of any of the commodities will violate the dimensions of the buckets. Thus, the limiting constraint is the volume, of 5.00 m^3.

The cost related to the spot charter of both the vessels and helicopter, includes the chartering and the travelling costs as both alternatives have direct transportation. Thus, if a vessel or helicopter is chartered on a spot charter, the travelling costs to a specific installation will be constant. It is assumed that the spot charters are released from the contract after the mission is performed; consequently the return travel is not included in the price.

TABLE 8 COSTS AND CAPACITY OF MODES OF TRANSPORTATION

	Capacity [m^3]					**Charter cost [$/charter]**		**Travelling [$/h]**
	Dry Bulk	Mud	Brine	Fuel	Water	Fixed	Spot	
PSV	260.00	870.00	870.00	450.00	800.00	$2,100	$3,150	$710
Vehicle	15.00	17.00	24.00	34.00	30.00	$195	NA	$66
Helicopter	5.00	5.00	5.00	5.00	5.00	NA	$6,230	$1,420

7.2.4 STORAGE AND LOADING

It is assumed that the maximum demands during a week are the maximum demands for each commodity throughout the entire operation, thus the upper stock level is decided based on the highest demand for each commodity. As the consumption rates in the real life problem vary from week to week, it is assumed that all the offshore facilities have the same storage capacity. The data is rounded to the nearest 50, and the storage for each commodity is provided in Table 9.

TABLE 9 CAPACITY OF INVENTORY AT THE OFFSHORE FACILITIES

	Dry bulk [m^3]	Mud [m^3]	Brine [m^3]	Fuel [m^3]	Water [m^3]
Site 1	100.00	200.00	200.00	200.00	300.00
Site 2	50.00	100.00	50.00	150.00	150.00
Site 3	100.00	50.00	50.00	100.00	250.00
Site 4	50.00	50.00	200.00	100.00	100.00
Capacity	100.00	200.00	200.00	200.00	300.00

The initial and end inventory levels are assumed to be half of the weekly demands, and these are provided in Table 10.

TABLE 10 INITIAL AND END INVENTORY LEVELS AT THE OFFSHORE FACILITIES

	Dry bulk [m^3]	Mud [m^3]	Brine [m^3]	Fuel [m^3]	Water [m^3]
Site 1	31.00	84.00	92.00	92.00	152.00
Site 2	17.00	27.00	24.00	69.00	77.00
Site 3	31.00	20.00	16.00	54.00	125.00
Site 4	9.00	6.00	78.00	44.00	50.00

Loading and unloading times are calculated based on the data in appendix G. The table provides data for the total loading and unloading time for the various installations. These times are divided on the number of visits that are performed at each facility, which provides the average loading time per visit, and the numbers are rounded to the nearest integer. The loading time at the depot is set equal to the longest loading time. These data are provided in Table 11.

TABLE 11 LOADING AND UNLOADING TIMES

	Visits per week	Loading [h]	Time per visit [h/visit]
Site 1	4.30	29.00	7.00
Site 2	3.80	24.00	6.00
Site 3	3.60	21.00	6.00
Site4	2.70	14.00	5.00
Depot			7.00

There are no costs related to the inventory, except for the potential storage cost at the onshore depot. The storage cost is also found in Grønland (2011) cost models, and is assumed to be 0.05 $ per ton commodity. It is assumed that the storage tanks at the onshore base are equal to the size of the largest demand of the various commodities of all the offshore facilities, hence the cost for storage is 7,507 $ per week.

7.2.5 Downtime and penalty cost

The downtime cost is based on the day rate of Songa Trym, which is the drilling facility that performs the drilling operations at site 15/6-13A. The daily rig rate is 365,000$ per day.(NPD, 2015) The pie chart provided in the background chapter, shows the distribution of costs in for drilling operations. According to this chart, the rig rate constitutes 34 % of the total drilling cost. Thus, this percentage was used to find the total daily cost for the drilling operations. The downtime is thus assumed to be the total cost per hour, divided on the number of products that must be supplied, which yields a downtime cost of 8,946 $ per hour per product. The penalty cost is assumed to be 10 % of the downtime cost, thus 895 $ per hour per product.

7.3 Results of computational study

The data from the section above is implemented as input parameters in Xpress-IVE. The results obtained from the implementation will be provided in this section.

The fleet of vessels is assumed to be unconstrained, but as the solution time increases with the number of variables, the number of vessels was set equal to three. This assumption does not alter the optimal solution as it was solved to optimality, and the model aims to minimize the number of vessels chartered.

7.3.1 Stage 1

The running of the deterministic problem yields the optimal scheduling and routing of the supply vessels, and the optimal time and amount for each delivery. The optimal solution was obtained in 4 hours and 36 minutes, and the estimated cost is 88,707 $ per week. Two vessels are generated, and both will service all the facilities. The sailing pattern and the number of the visits are illustrated in Figure 18.

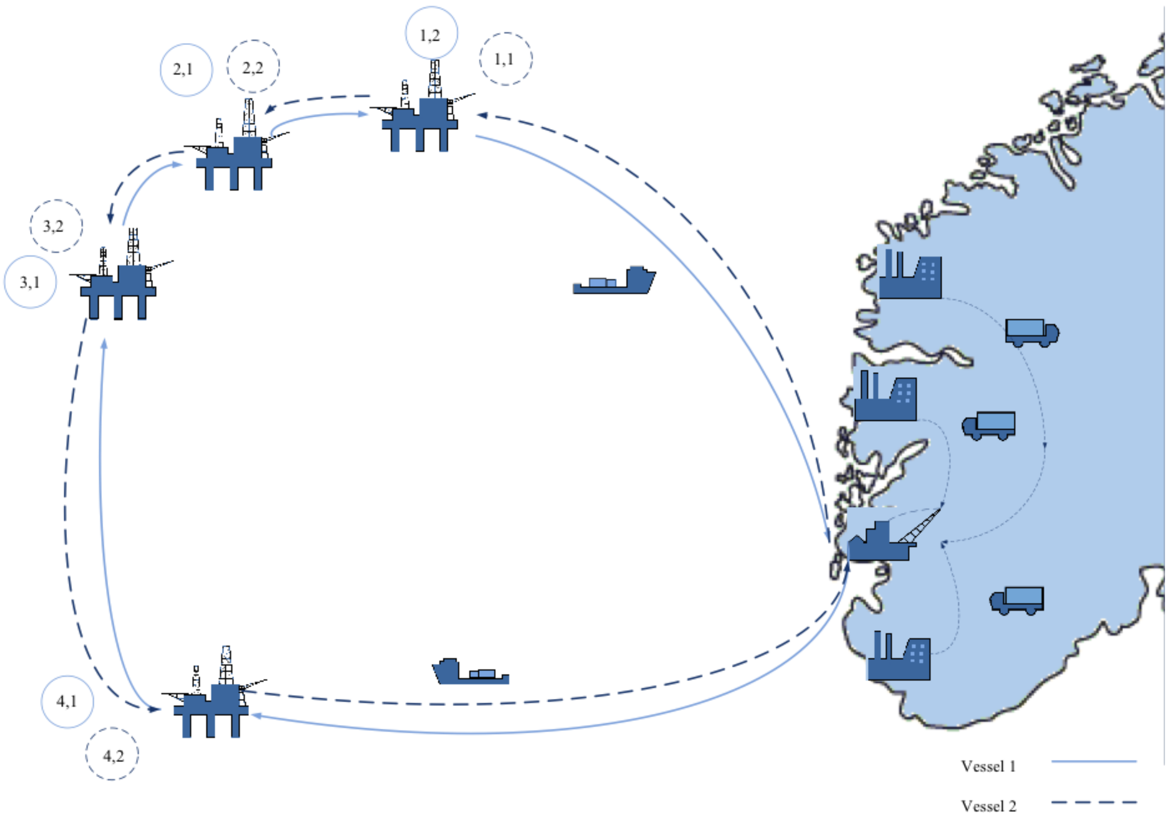

FIGURE 18 SAILING PATTERN AND VISING SEQUENCE FOR THE VESSELS

The corresponding schedules of the second vessel is provided in Table 12, and the schedule for the first vessel is provided in Table 13.

TABLE 12 SCHEDULE FOR VESSEL 2

Day	Hour	Action
Monday-Tuesday	23:00-06:00	Loads the vessel at the depot
Tuesday	06:00-20:00	Depart from depot and sail to site 1
Tuesday-Wednesday	20:00-03:00	Unloads at site 1
Wednesday	03:00-05:00	Departs from site 1 and sail to site 2
Wednesday	05:00-11:00	Unloads at site 2
Wednesday	11:00-14:00	Departs from site 2 and sail to site 3
Wednesday	14:00-20:00	Unloads at site 3
Wednesday-Thursday	20:00-04:00	Departs from site 3 and sail to site 4
Thursday	04:00-09:00	Unloads at site 4
Thursday	09:00-16:00	Departs from site 4 and sail to depot

TABLE 13 SCHEDULE FOR VESSEL 1

Day	Hour	Action
Sunday-Monday	03:00-11:00	Vehicles are loaded and depart from Kristiansund
Monday	05:00-11:00	Vehicles are loaded and depart from Mongstad
Monday	09:00:11:00	Vehicles are loaded and depart from Haugesund
Monday	11:00-18:00	Loads the vessel at the depot
Monday-Tuesday	18:00-01:00	Depart from depot and sail to site 4
Tuesday	01:00-06:00	Unloads at site 4
Tuesday	06:00-14:00	Departs from site 4 and sail to site 3
Tuesday	14:00-20:00	Unloads at site 3
Tuesday	20:00-23:00	Departs from site 3 and sail to site 2
Tuesday-Wednesday	23:00-05:00	Unloads at site 2
Wednesday	05:00-07:00	Departs from site 2 and sail to site 1
Wednesday	07:00-14:00	Unloads at site 1
Wednesday-Thursday	14:00-04:00	Departs from site 1 and sail to depot

The vessels are loaded with various amounts of commodities. The loading of the first vessel is provided in Table 14. The total load is 1162.74 m^3, and the vessel is not fully loaded with any of the commodities. It requires 48 vehicles to carry these commodities from the suppliers' facilities to the onshore base.

TABLE 14 LOAD AT VESSEL 1

Product type	Load [m^3]	Vehicle
Dry bulk	175.10	12
Mud	273.60	17
Brine	420.00	19
Fuel	196.80	0
Water	474.24	0

The loading of the second vessel is provided in Table 15. The total load is 673.68 m^3, and these loads are not constrained by the capacity of the vessel. The vessel is only loaded with fuel and water, which can be obtained from the onshore base, therefore no vehicles are required to supply the vessel.

TABLE 15 LOAD AT VESSEL 2

Product type	Load [m^3]	Vehicle
Dry bulk	0	0
Mud	0	0
Brine	0	0
Fuel	324.54	0
Water	349.14	0

The inventory level at the offshore facilities will vary linearly with time, and the deliveries. The inventory level at the first offshore facility (16/1-22 S) throughout the planning horizon is illustrated in Figure 19. The facility is first visited at time 44, which is Tuesday 08:00, by vessel 2. At this point the vessel delivers 186.40 m^3 fuel, and the delivery is constrained by the upper bound of the inventory level. The second visit is made by vessel 1 at time 55, which is Wednesday 07:00. At this point, all inventory level of the commodities that were not services in the first visit, are below 7.00 m^3 and will be emptied within 5 hours. During this visit, the vessel delivers 61.20 m^3 dry bulk, 166.80 m^3 mud, 184.80 m^3 brine and 303.60 m^3 of water. The water is filled to 300.00 m^3, and is therefore restricted by the upper bound of the inventory level.

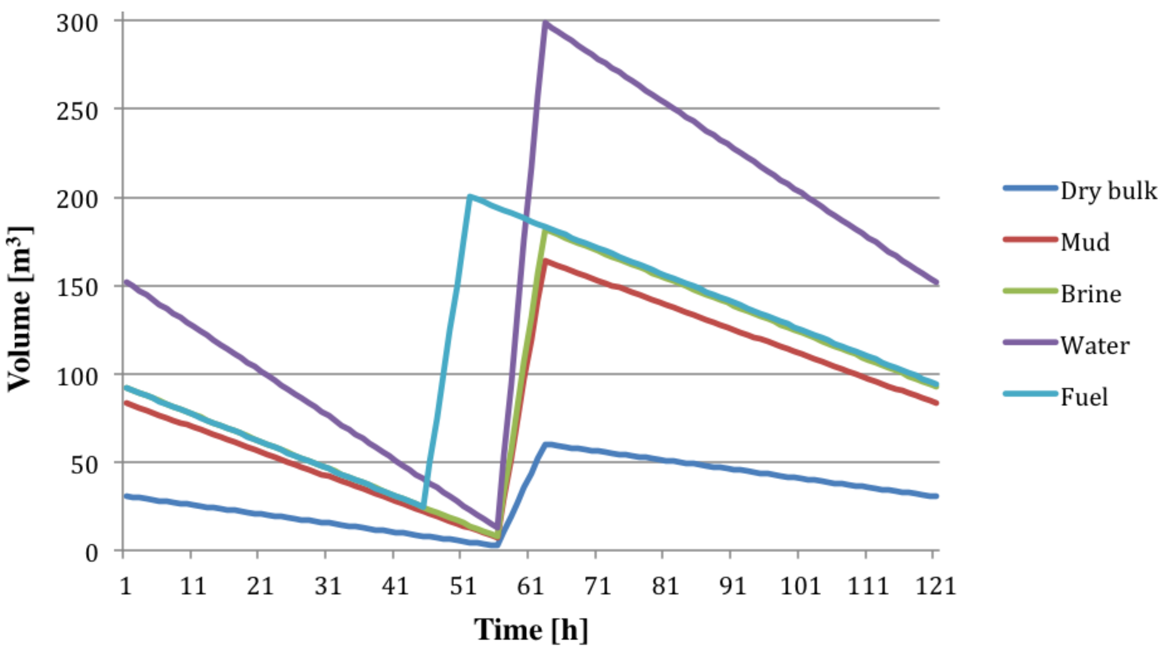

FIGURE 19 INVENTORY LEVEL AT OFFSHORE FACILITY 1

The inventory level at the second offshore facility (15/6-13A) throughout the planning horizon is illustrated in Figure 20. The facility is first visited by vessel 1 at time 47, which is Tuesday 23:00. At this point, none of the commodities are at a critical stock level, and have 13 hours before they are emptied. During the visit the vessel delivers 33.60 m^3 dry bulk, 55.20 m^3 mud, 46.80 m^3 brine and 153.60 m^3 of water. The inventory levels are not filled to the upper bound. The second visit is made at time 53, which is Wednesday 05:00, by vessel 2. By this point, the inventory level of fuel will only have 7 more hours of operation before the inventory level reach zero. During this visit, the vessel delivers 138.00 m^3 fuel, which is the required amount for the stock level to be above the required limit at the end of the planning horizon.

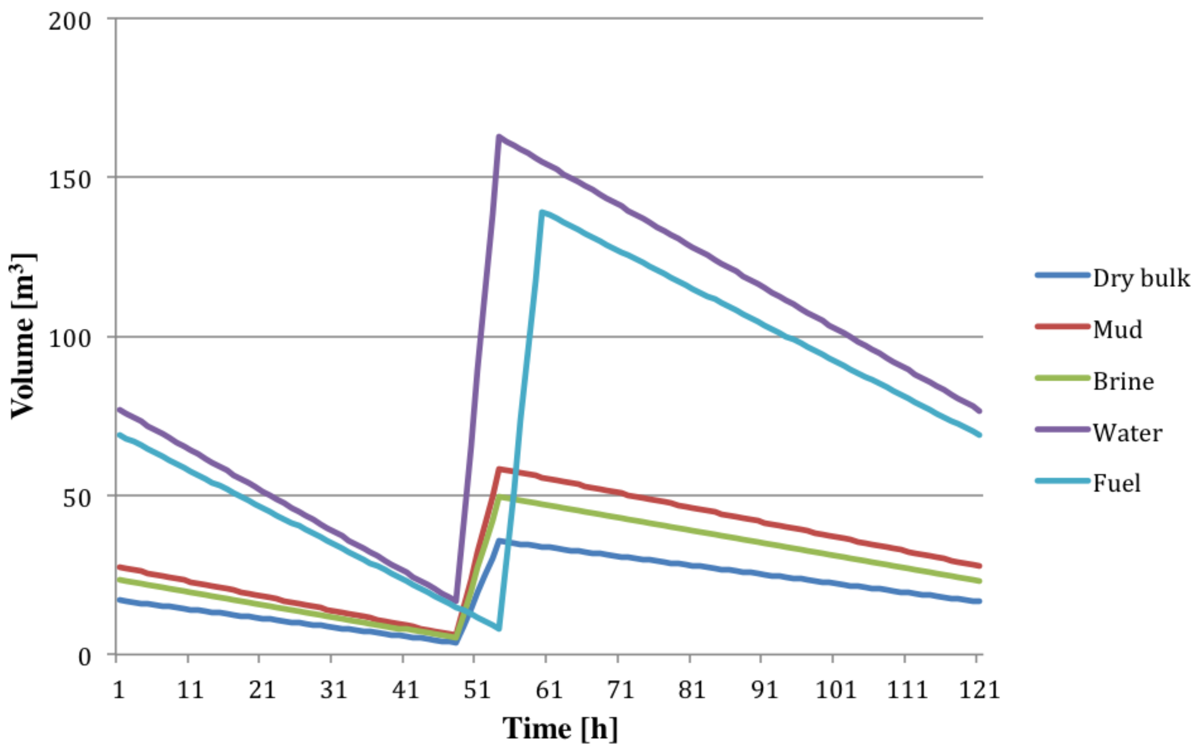

FIGURE 20 INVENTORY LEVEL AT OFFSHORE FACILITY 2

The inventory level at the third offshore facility (15/12-24 S) throughout the planning horizon is illustrated in Figure 21. The facility is first visited at time 38, which is Tuesday 14:00, by vessel 1. At this point, the inventory levels will not be empty for another 22 hours. Upon the visit, the vessel delivers 62.40 m^3 dry bulk, 39.60 m^3 mud, 31.30 m^3 brine, 109.20 m^3 fuel and 3.96 m^3 of water. The second visit is made at time 62, which is Wednesday 14:00, by vessel 2. Upon this visit the inventory level of water is emptied, and must be refilled, thus the vessel delivers 249.50 m^3 water.

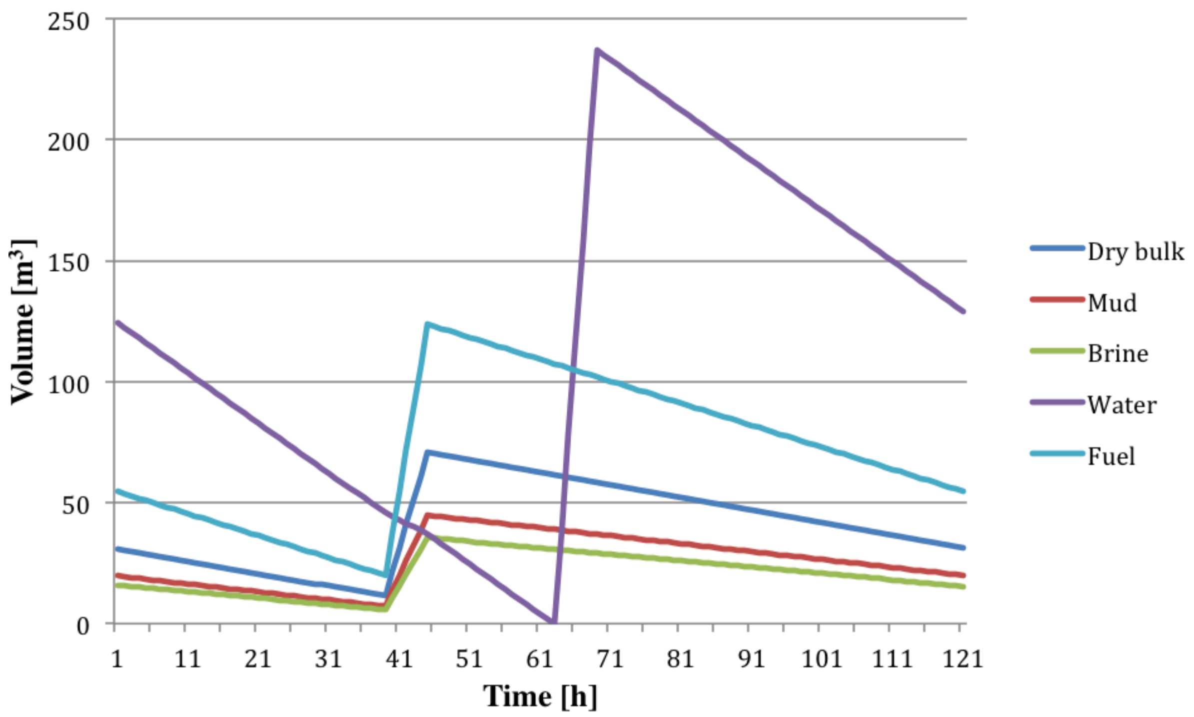

FIGURE 21 INVENTORY LEVEL AT OFFSHORE FACILITY 3

Finally, the inventory level at the second offshore facility (2/4-23S) throughout the planning horizon is illustrated in Figure 22. The consumption of mud and dry bulk is close to none. The facility is first visited at time 25, which is Tuesday 01:00, by vessel 1. Upon the visit, the inventory levels have sufficient stock to last for 35 more operating hours. During this visit the vessel delivers 18.00 m^3 dry bulk, 12.00 m^3 mud, 157.20 m^3 brine 87.60 m^3 fuel and 13.08 m^3 of water. As for the first installation, the consumption of brine is relatively high, and the inventory level is filled to the upper bound. The second visit is made by vessel 2 at time 76, which is Thursday 04:00. Upon this point, the inventory level of water is emptied and must be refilled, thus this is the only commodity that requires two services. During this visit, the vessel delivers 99.60 m^3 water.

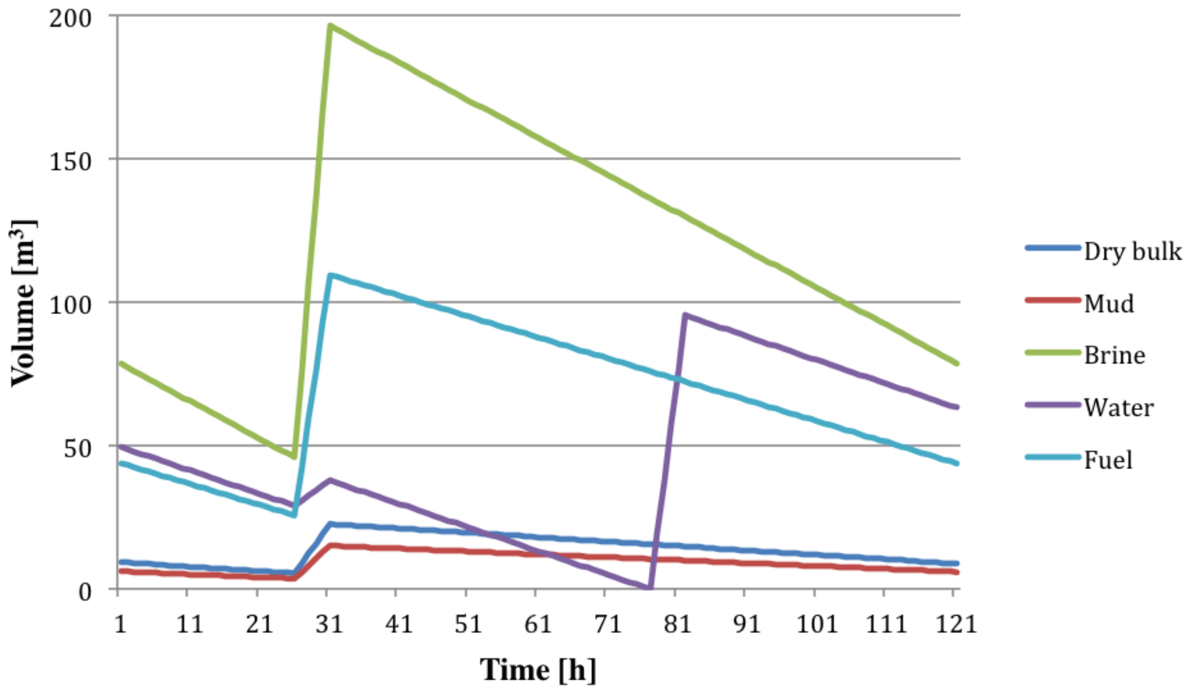

FIGURE 22 INVENTORY LEVEL AT OFFSHORE FACILITY 4

At the end of the planning horizon, all installations are visited two times and all the inventory levels are above the required levels.

7.3.2 STAGE 2

The late deliveries are calculated in the second stage. The model can chose between three alternatives; to perform no late deliveries at the cost of downtime (alternative 1), to have storage and send a spot charter PSV (alternative 2) or send helicopters (alternative 3). The solutions will be provided in this section.

In the first scenario, the demands are equal to the forecasted demands, thus there will be no excess demands, and the cost of this alternative will be equal to the cost from the deterministic calculations, thus 88,707 $.

The second scenario has an increase in demand of 1%. The optimal solution for this scenario is provided in Table 16.

TABLE 16 STOCHASTIC SOLUTION FOR SCENARIO 2

	Demand [m³]		Alt		Helicopter		Penalty [$]		Charter [$]		Cost [$]	
Visit	1	2	1	2	1	2	1	2	1	2	1	2
Site 1	4.00	2.00	3	3	1	1	1,780	1,780	9,070	9,070	10,850	10,850
Site 2	2.00	0.00	3	1	1	0	1,780	0	9,070	0	10,850	0
Site 3	1.00	1.00	3	3	1	1	1,780	1,780	9,070	9,070	10,850	10,850
Site 4	1.00	1.00	3	3	1	1	2,670	2,670	10,490	10,490	13,160	13,160

The optimal solution for scenario 1 includes helicopter transportation of the late demands, for all the visits at all the installations. The only exception is visit 2 at installation 2, for which there is no excess demand. It is sufficient to charter one helicopter after each visit, and the additional cost is 80,570 $, which gives a total cost of 169,277 $ for this scenario.

The third scenario has an increase in demand of 3 %, and the optimal solution for this scenario is provided in Table 17.

TABLE 17 STOCHASTIC SOLUTION FOR SCENARIO 3

	Demand [m³]		Alt		Helicopter		Penalty [$]		Charter [$]		Cost [$]	
Visit	1	2	1	2	1	2	1	2	1	2	1	2
Site 1	10.00	4.00	3	3	2	1	3,560	1,780	18,140	9,070	21,700	10,850
Site 2	6.00	1.00	3	3	2	1	3,560	1,780	18,140	9,070	21,700	10,850
Site 3	4.00	4.00	3	3	1	1	1,780	1,780	9,070	9,070	10,850	10,850
Site 4	5.00	5.00	3	3	1	1	2,670	2,670	10,490	10,490	13,160	13,160

The optimal solution for scenario 2 includes helicopter transportation of the late demands, for all the visits at all the installations. It is sufficient to charter one helicopter after each visit, except for the first visits to installation 1 and 2, which has excess demands require two helicopters. The additional cost is of late deliveries is 113,120 $, which gives a total cost of 201,809 $ for this scenario.

The fourth scenario has an increased demand of 5 %, and the optimal solution for this scenario is provided in Table 18.

TABLE 18 STOCHASTIC SOLUTION FOR SCENARIO 4

	Demand [m³]		Alt		Helicopter		Penalty [$]		Charter [$]		Cost [$]	
Visit	1	2	1	2	1	2	1	2	1	2	1	2
Site 1	17.00	7.00	2	3	0	2	12,460	3,560	12,735	18,140	25,195	21,700
Site 2	9.00	2.00	3	3	2	1	3,560	1,780	18,140	9,070	21,700	10,850
Site 3	8.00	7.00	3	3	2	2	3,560	3,560	18,140	18,140	21,700	21,700
Site 4	4.00	4.00	3	3	1	1	2,670	2,670	10,490	10,490	13,160	13,160
									+ Storage		7,057	

The optimal solution for scenario 3 is a combination of alternative 2 and 3. Helicopter transportation of the late demands is the optimal solution for all the visits, except for the first visit at the first installation. This visit will require four helicopters, and therefore it is more cost-effective to choose supply alternative 2, with the extra storage at the onshore base and the spot chartering of a PSV. The demands that must be serviced by helicopters require one or two helicopters, depending on whether or not the demands exceed the capacity of 5.00 m^3. The additional cost for late deliveries is 156,240 $, which gives a total cost of 244,947 $ for this scenario

The expected value of the stochastic solution is thus 17,514 $, and the preferred and most cost-effective solution is primarily helicopter transportation.

It is assumed that downtime costs will be generated if there are no late deliveries and the real demands exceeds the estimated demands. These downtime costs are calculated manually by the additional demands for the installations that are provided in the second columns of Table 16 to 18. The downtime is found by equation (6.50), and the consumption rates are assumed to be the average of each consumption rate for each commodity at the offshore installations.

The reduced cost of having late deliveries compared to downtime for the different scenarios are provided in Table 19, and the estimated cost is 43,524 $.

TABLE 19 DOWNTIME COST WITHOUT LATE DELIVERIES

	Downtime costs [$]	Cost of late deliveries [$]	Cost saving [$]	Cost saving [%]
Scenario 1	0	0	0	0
Scenario 2	116,289	80,570	35,719	31
Scenario 3	313,110	113,120	199,990	64
Scenario 4	545,706	156,240	389,466	71

The evaluation of the stochastic solutions can be done by equation (3.1) and (3.2) from Chapter 3. The stochastic solution (RP) is 106,221 $, whereas the EEV can be calculated from the cost of the downtime in Table 19. This is done, as it is assumed that without the late deliveries, these will be generated and added to the deterministic solution. The estimated cost of the deterministic solution and downtime equals 132,231 $. This gives a VSS of negative 26,010 $, which is a cost reduction of 20 %. This is the potential reduced cost of having late deliveries, compared to downtime.

If the late deliveries were revealed at the beginning of the planning horizon, and included in the initial schedule, the late deliveries would not be necessary. The deterministic solution with perfect information (WS) is 88,835 $. Compared to the total cost with the stochastic solution, which is 106,221 $, the potential cost saving and EVPI is 17,386 $, which equals 16%.

CHAPTER 8

DISCUSSION

This chapter will discuss the results presented in Chapter 7 in relation to the supply logistics. It will also discuss the elements of the mathematical model and its application as a decision support tool. It must be emphasized that the uncertainty in the data provided is a significant source of error. However, as the main objective of this thesis is to develop a model that can initiate a discussion on the supply logistics, the results from the computational study are assumed to provide adequate accuracy to make a realistic discussion of the results.

Optimization models are often used as decision support tools, and this place some practical requirements in terms of solution times, which is an aspect that will be discussed in section 8.1. The results from the case study will be used to initiate a discussion on the modelled supply chain, and possible improvements and extensions that can make the model a better representation of the real problem. This discussion is provided in section 8.2.

8.1 APPLICATION OF THE MODEL

The solution time is an important aspect for a model that is to be used as a decision support tool. The planners must often make the decisions relatively fast, and must therefore have a model that can provide an optimal or near optimal solution within a short period of time. The deterministic stage has a relatively long solution time of 4 hours and 36 minutes for the optimal solution. It is however observed that the optimal solution is found after 22 seconds only, but at this point the entire solution space is not investigated. A near-optimal solution is often considered to provide a sufficiently good result for the planners, and obtaining the optimal solution is therefore not considered necessary. It might be that the excessive time the model uses to solve the problem to optimality is more expensive than the slightly increased cost of utilizing the heuristic solution. Thus, a good heuristic solution might be more valuable than the optimal.

It was also observed that the solution time was significantly reduced if the demands are modelled as integers instead of non-integer. The problem should, however, still be modelled with non-integer demands as this increases the solution space and provides more flexibility, in addition to providing a more accurate solution.

The deterministic stage of the model contains several variables, some of which are included to ease the reading, but they will not have any impact on the solution, and can therefore be excluded. As described in chapter 7.2.2, the number of variables has a significant impact on the solution time. Therefore, in order to reduce the solution time, the model should ideally be revised and some of these variables eliminated.

Two of the variables that are derived from other variables, are the ones that determine stock and times. The model has one of each for both the beginning and end of an operation. These variables are defined with equality constraints, and can therefore be replaced by the parameter and variables that are defined in this equality, in this case that is the unloading time and the demand. Furthermore, this approach will also reduce the number of constraints, as the constraints that link the start and end time, and start and end inventory levels, no longer are necessary. Thus, the reduction of these variables will lead to a reduction of two variables, and the two sets of constraints (6.17) and (6.23). This will however also require some remodelling of the constraints in which these variables are used. Moreover, the variable that determines the loading of the vessel can also be eliminated and replaced by the sum of the demand variables. Thus, the set of constraints (6.9) can be removed and inserted in the remaining constraints instead of the loading variable. As the first constraint (6.1) only allows each vessel to sail one route per time horizon, the visiting variable can also be eliminated, and replaced with the variable that describes which visits that are made by which vessel. The relation is modelled in constraints (6.4), which can be eliminated. To illustrate these changes, a revised deterministic stage is included in Appendix K.

These changes are not done in this thesis as the solution time proved to be sufficiently short for the application. However, if the problem size is to be increased which easily can be done with an increasing number of installations, or an increased time horizon, which must allow for a bigger fleet of vessels and number of visits, these changes can provide reduced solution times.

The stochastic solution is provided immediately, thus the question of whether or not this should be solved to optimality or a near optimal solution is not necessary. However, the model is a very simplified version of the real problem. This is sufficient for this problem, as the main objective is to investigate the best suited alternative transportation for late delivery, and not explicitly the cost. If this model is to be used as a decision support tool, it should be revised, and this will be discussed in the succeeding sections, some of these changes will also alter the optimal solution and consequently the cost level.

8.2 Using the Models as Decision Support Tools

The following section will discuss the results from the case study, the logistics strategies and how the model can be improved to provide a better representation of the real-life problem. As previously stated the model does only represent a simplified version of the problem, and some of the simplifications are likely to have a significant impact on the result.

The deterministic solution yields an optimal solution in which two vessels are chartered, and both vessels are scheduled to visit all the offshore facilities. This solution coincide with the solution of other supply vessel problems, such as the one studied by Halvorsen-Weare et al. (2012) in which the optimal solution is to utilize two vessels to service seven installations over a planning horizon of one week, and the vessels perform two routes each. Due to the long distance from the depot to the installations, and the relatively short distances between the offshore installations, it seems reasonable that both vessels visits all the installations on one route. Moreover, if the offshore installations have longer distances to shore or larger distances between the installations, the current solution of visiting all the installations on one route would violate the maximum sailing time of each vessel. This illustrates the importance of the distances. There is little slack in the sailing times and this reduces the flexibility in the model. If the vessels were allowed to wait for a service, the delivery times might have been more optimal in terms of when the deliveries are made, and perhaps it would be sufficient with one visit at each offshore facility. Ultimately this can reduce the fleet, given that the capacity constraints are not violated.

The vessels have a given capacity for each commodity. Previously in this thesis, it has been argued that the load may provide a significant restriction to the routing and scheduling. That is not the case for the solution provided in this case study as none of the capacity constraints are exceeded for any of the commodities. The reason for this might be that the case study does not consider deck loads, which often are the loads that exceed the capacities. Based on this finding, it can be assumed that for this problem, smaller vessels can be chartered and these are likely to have a cheaper charter cost, which would reduce the total costs. It is important to emphasize that these vessels must have a capacity that can service all the demands, so that it is not necessary to charter in an additional vessel that would increase the total costs. In order to investigate the effect of constrained loads, the case study can also be solved for vessels with smaller capacities. The result is likely to yield a solution in which more vessels are chartered, and fewer vehicles that are generated per vessel.

In this case study, it is only the commodities that are modelled, and these can be adjusted, and distributed so that the total load or volume can be split. This is not necessarily the case for equipment, and this means that some of the modes of transportation are infeasible for certain equipment types. Single equipment types may also be too heavy or large to be transported with a mode of transportation if it exceeds the capacity. Thus, modelling demands with equipment might add more complexity to the problem. In offshore supply logistics, it is likely that some demands are considered more important than others. This is not included in this model, and all the demands are considered equally important. This aspect can easily be included by a weighted objective function. In this approach, the various demands are given a weight according to their importance, this weight can be included as an element in the objective function, and be dependent on waiting time for the operating to be performed or another variable. This extension can have a significant impact on the schedules, and change the optimal solutions so that they better correspond to the prioritisations.

Backloads are often an issue for the planners, and the model can be extended to include this element. If backloads are included in the modelling, the vessels can perform both delivery and pick-up operations at the offshore installations. This will provide a problem that is more similar to the one studied by Agra et al. (2013) which also allows pick-ups. If the amount of backloads is extensive and the capacity of the vessel is limited, the backloads can provide a significant restriction to the routing and scheduling of the vessels, as they limit the amount of loads the vessel can carry. Thus, including this element can provide a different solution from the one presented in this thesis.

If the amount of generated semi-trailers is high, like in this thesis, it can be interesting to investigate whether the capacity of these vehicles is poorly utilized. In order increase the probability of good utilization of the semi-trailers, the first echelon of the model can be extended to a VRP in which the vehicles can visit several producers and transport mixed commodities. However, based on the results from the case study, most of the vehicles are fully or almost fully loaded and this approach will not have made a significant difference. The cost saving of using one truck to pick-up at two locations is not likely to amount to major cost savings compared to the direct routing. Furthermore, the difference in travelling costs per unit time which is generated between the facilities, and the cost of chartering a vehicle for direct routing, will only make this a profitable solution if the suppliers are less than two hours apart.

Due to the high number of semi-trailers generated to supply the first vessel, other alternative supply scenarios should be considered. It can be interesting to investigate whether it is a more effective solution to utilize a PSV to transport these commodities from the supplier to the onshore base. It must be emphasized that for this alternative to be valid, the suppliers must be located at the coast and have the necessary equipment to load and unload the commodities to the vessel. For this case, the relatively low demand and the low cost of the vehicles does not favour this solution. However, if the amount of commodities is larger and the relative cost between the PSVs and vehicles is smaller, this conclusion can be altered. Another alternative for the onshore transportation is to utilize goods trains. Trains have a significantly larger capacity than the semi-trucks, and can therefore be used to pick-up larger quantitates. This alternative does require that rails are present, which is not the case for the supplier sites that are discussed in this case study. Furthermore, the rail system is limited along the Norwegian coast, thus it is likely that these systems will have to be developed in order for this to be a feasible alternative for operations at the NCS. A third alternative is to allow the vessels to stop by the onshore bases to pick-up the commodities on their way to the offshore facilities; like a pick-up-and-delivery problem. This will also require the supplies' facilities to be located at the coast. It is likely that upper bounds on the sailing time might make this solution unpractical, as the vessel might use a significant part of the sailing time on the pick-up of commodities. However, if the vessel can refuel and the crew can be changed at the offshore suppliers, the sailing time might be increased, which would make this a more flexible alternative.

The vehicles that depart from Kristiansund are scheduled to leave on Sunday, which is a day before the planning period starts. As previously stated this is not considered an issue as the schedules are developed before the actual planning period is initiated. The model can, however add a restriction that forces the vehicles to leave within the planning periods. This restriction, or restrictions related to when the vehicles can depart from the supplier and arrive at the onshore base, will have a significant impact on the solution. In this solution, all the facilities are assumed open at all times, and therefore there are no limitations in terms of these considerations. A natural extension can be to include opening hours for the supplier's facilities, and only allow loading of the vehicles during the opening hours. This will also make the problem more tightly constrained, and it is likely that the solution provided in the previous chapter infeasible, as the both vessels must be served at the onshore depot during night.

Another aspect that can be interesting to consider in terms of the scheduling, is the impact of loading and unloading times that varies with the amount loaded, as it is modelled in the deterministic model. In the implementation of the case study, this is simplified and the problem is solved with constant loading and unloading times. If the time to load and unload varies with the amount, the operational times will vary with the loading, and this must be considered in the scheduling. This will have a significant impact on the scheduling of the semi-trailers, and the vessels, and the current solution will not be feasible.

The various installations require two visits, and one or more commodities will be refilled for each visit. The inventory levels are increased each time a vessel visits the facilities, and the initial and end levels are equal to half of the inventory levels. It is important to emphasize that if the inventory level at the beginning of the planning horizon is different, or the required level at the end of the planning horizon are different the problem is likely to have another solution. The reason for this is that the inventory levels will be emptied at different points in time, which will make for a more interesting problem compared to the one modelled in this case study where all the inventory levels at a given facility will be emptied at the same time without supply services. In the current solution, the vessels deliver the exact amount of commodities that ensure that the inventory levels are compatible with the requirements to the inventory level at the end of the planning horizon. This means that the inventory levels have little robustness. This lack of robustness will be discussed in relation to the stochastic demands in the following paragraphs.

The stochastic solution investigates the best supply alternative for each demand that is revealed upon the vessel's visit and that exceed the planned demand. The preferred supply scenario is to transport the late deliveries with helicopters. The alternative of not having late demands is also included, but this is only chosen if there is no excess demand, which means that no downtime cost is generated. Furthermore, all the scenarios require one or two helicopters to supply the late deliveries, with one exception. The first visit at the first installation in the last scenario, in which the actual demand is 5 % higher than the estimated demands, will require four helicopters to provide the excess demand. Thus, the optimal solution is the second alternative, which requires the storage at the onshore base and a spot chartered vessel. It can therefore be assumed that if the late demands require more than two helicopters, the optimal solution is to use the storage at the onshore base. If late deliveries not are an option, the installations will have downtime, and as the potential cost saving of 20 % compared to the cost generated by down time, there is little doubt that late deliveries provide the best solutions.

As the stochastic solution in this thesis only depends on time, and not the stock levels, more robustness in the inventory levels will not alter and reduce the stochastic solutions. This means that late deliveries are performed for all the visits that have excess demand, regardless of the stock level. However, as the stock levels for many of the visits not are critical in terms of downtime upon the visit, some of the late deliveries do not have to be provided right away in order to avoid downtime. Therefore, the model should be extended to include the aspect of the size of the stock levels, as this will give a more realistic representation of the real problem. Furthermore, it will provide a more flexible and realistic solution space.

A model that includes the stock levels must investigation how fast these will be emptied, and use this as the foundation in the decision making. In the current stochastic model, the late deliveries can only be performed directly, and routing is not considered as an alternative. If the element of stock levels and routing is added to the model, the spot chartered vessel might be allowed to wait and see if there are more demands at other installations that can be serviced. If one vessel can service two installations instead of one, this alternative becomes significantly more cost-effective than helicopter transportation. The routing aspect is not considered applicable to the helicopters, due to their limited capacity. Thus, it is assumed that these always will utilize direct routing. Another solution approach is to investigate the effect of lateral shipments. This approach was used in the study by Coelho et al. (2012), in which lateral shipments are allowed for the late deliveries. This solution assumes that a vessel is available at all times, and that one of the other installations have excess stock of the commodities, and do not risk downtime if parts of the stock is transported to another installation. The amount taken from the offshore facilities must be resupplied, but given that the stock levels are relatively high, it is likely that downtime can be avoided. Thus, adding stock levels in the stochastic solution will provide many additional alternatives. The planners can also investigate whether these demands can be fitted to a scheduled voyage, hence, there will be no need to generate any additional vessels. If this can be used as a solution, it is important to also consider the time for the land-based travelling, and the sailing, and loading and unloading time for the vessel. To model this with optimization will be difficult due to that many dynamic parameters, but simulation can also be used to test the feasibility of the solution.

If the stock levels are included in the decision-making, this will also favour more robust stock levels in the deterministic stage, as this will reduce the probability of stock-outs. The model can be made more robust in several ways. One method is to set the minimum stock level to a value so that the inventory levels are less likely to be affected by the uncertainty in demand. If this is implemented, an interesting extension could be to include the inventory cost, in order to see the trade off between the inventory costs, excess stocks in order to avoid lacking inventory, and the cost of late deliveries. Robustness can also be added to the inventory levels by the introduction of penalties for lower levels. A safety stock level can be introduced, and penalty costs added for each hour the stock level is below the safety level. This approach will encourage more deliveries. Furthermore, in classical IRPs it is a common approach to introduce an order-up-to-level. This means that for each delivery, the vessels are required to fill the entire stock level, which would also add robustness. However, this approach is likely to provide stock levels that are higher than what is required, especially for the offshore facilities that have inventory levels that are significantly higher than the demands.

A solution that is believed to provide the best and most robust supply chain is to combine the deterministic and the stochastic stage, and model the entire problem stochastically in one stage. This is not done in the work with this thesis due to limited time. If however this is done, the model will provide a solution that considers the uncertainty in the stock and the stock levels. It is assumed that this provides a supply chain with earlier and potentially more visits, and the stock levels will be prevented from getting too low before the deliveries are provided. The model will also allow for the late deliveries to be scheduled if this proves to be the most economical solution. Another procedure is to solve the problem deterministically multiple times, by rerunning the deterministic stage after each operation with the actual conditions that are revealed upon the visit. The newly obtained information about the stock levels and positions of the vessels must be implemented in the model, then it can be rerun, and a new schedule will be provided. For this approach to work, some remodelling must be done, as there are no elements in the current deterministic model that allows the planner to set to position of each vessel.

This model only considers uncertainty in demand, but can also be extended to include uncertainty in weather conditions. Weather conditions are one of the uncertainties that can increase the lead times significantly. If the conditions are harsh, it might hinder both the vessels, semi-trailers and helicopters from travelling, or they will have significantly increased travelling times. In their study Halvorsen-Weare et al. (2011) state that the supply vessel planning problem is highly affected by the weather conditions, especially in the North Sea and Norwegian Sea during the winter season. The delays caused by weather conditions can be a significant risk to the stock levels. To address this issue time slack can be added to the operations so that this can cover potential delays. This may however induce unnecessary costs, as the modes of transportation might be chartered for a longer period than necessary to perform the operation.

It is not likely that an operator experiences the same uncertainties in all operations, or that the planners do not change the schedules when the same uncertainties are experienced several times. Therefore, the scenarios presented in this thesis are a very simplified representation of the real problem. Furthermore, if the planners are provided with information about the conditions and they can see a pattern, it might be possible to reschedule, so that the same uncertainties are not experienced for the entire planning horizon. This is where the information aspect of integrated operations becomes important, as extensive exchange of information can increase the probability of detecting unforeseen events. These are aspects that the mathematical model will not consider, and is also the reason why the model must be used solely as a decision support tool, and not a decision tool.

To summarize this chapter, there are several improvements and extensions that can be made to the model and input parameters in order to investigate alternative supply scenarios. The model is a very simplified version of the real problem, and there are many elements that are left out that can alter the solutions significantly if they are included in the model.

CHAPTER 8

CONCLUSION

Exploration drilling is performed to prove the existence of the petrochemical resources, and is therefore crucial for the continued development of offshore operations on the NCS. This operation involves extensive logistics in terms of mobilizing the required equipment and resources, and the operators are dependent on safe and reliable supply services. The cost level is high, and the operators are constantly looking to reduce this level. The supply chain for the upstream supply services represents one significant cost for offshore operations, and if these costs can be reduced through efficient supply logistics, it can contribute to the reduction of the overall costs of exploration drilling.

The offshore logistics are often complex and subject to several variables, thus optimization might prove to be a useful decision support system for the planners. The problem can be studied as an IRP, which will consider both routing and scheduling, and inventory levels. For this thesis, a two-stage recourse model is developed, which considers logistics strategies for both planned and unplanned demands.

The solution from the case study yields that the four offshore installations can be serviced by two PSVs during a time horizon of five days, and all the installations should have two visits each. Furthermore, the stock levels are in accordance with all the requirements for the deterministic solution, but the robustness is poor. To address the issue with poor robustness in relation to unexpected demands that makes the deterministic stock levels insufficient, late deliveries are performed. The preferred alternative for the late deliveries is to use helicopters for direct transportation to the offshore facilities. However, as the amount of the unexpected demands increases, the alternative with additional storage at the onshore base and the chartering of an additional PSV becomes the preferable solution. The total cost of the planned deliveries is 88,707 $, and the estimated cost of the late deliveries is 17,514 $. The cost of the late deliveries increases significantly with increased unplanned demands.

The value of having the stochastic solutions is also assessed in order to investigate the cost saving of having this option. The cost saving of using late deliveries is estimated to be approximately 20 %, and is therefore significant. Thus, this is the preferred solution compared to the risk of downtime. Still, the cost of the late deliveries is significant. If information about the real demands are revealed at the point in time when the planning of the initial schedules is done, these demands could be incorporated in the schedules and this have a potential cost saving of 16 %. This only emphasize the importance of robust stock levels. It must be emphasized that the stochastic solution is highly dependent on the probabilities of each scenario, thus changing these would also change the stochastic solution. Furthermore, the generation of downtime costs is exaggerated compared to the real-case, as no consideration is made to the stock levels, thus it is assumed that downtime is caused for all the unexpected demands with demands that exceeds the planned demands.

The problem studied in this thesis is simplified, and if the model is to be used to solve real-life situations, the model must be improved and extended. The stochastic solution has a significant potential for improvement, and should be extended to include the stock levels, and let the stock levels be a determining factor for the best alternative supply scenarios. The models developed for this thesis only consider alternative supply scenarios for the unplanned demands, but this could also be evaluated for the planned deliveries. Transportation by rails and supply vessels could be evaluated as alternatives to transportation with semi-trucks. Furthermore, instead of direct routing, the potential of multiple visits should be investigated, especially for late deliveries with the spot chartered vessels, which will have a very poor utilization of the capacity for one single late delivery. For an optimal solution, the two models should be combined and solved stochastically. Furthermore, for this model to be used as a decision support tool, there should be a significant reduction in variables and, to the extent possible, the constraints.

However, the study shows that planners that can evaluate both routing, scheduling and inventory considerations are better fit to find the overall best solutions. Therefore, this is considered to be a good solution. The developed model serves well for the solution of these problem instances, and provides a sufficient foundation for the investigation of several important aspects with the offshore supply logistics.

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APPENDIX A: DETERMINISTIC MODEL

A.1 DEFINITIONS

Sets

$\mathcal{N}$	Set of facilities, indexed by i, j
$\mathcal{M}_i$	Set of visit numbers at facility i, indexed by m, n
$\mathcal{S}$	Set of visits (i, m) where $i \in \mathcal{N}$ and $m \in \mathcal{M}_i$, indexed by (i, m)
$\mathcal{K}$	Set of vessels in the fleet, indexed by k
$\mathcal{H}$	Set of product types, indexed by h

Parameters

C_{ij}^{K}	Operating cost for sailing from i to j per unit time
C_i^{OK}	Operating cost for sailing from depot to i per unit time
$C^{TC,K}$	Fixed charter cost for a vessel
$C_h^{TC,V}$	Fixed charter cost for a vehicle from supplier facility h
Q_h^V	Capacity of product h for the vehicles
Q_h^K	Capacity of product h for the vessels
W_{ih}	Consumption rate at facility i for product h per unit time
S_{ih}^0	Initial stock level at offshore facility i for product h
$\overline{S}_{ih}$	Upper stock level at offshore facility i for product h
$S_{ih}^{\overline{T}}$	Stock level of product h at offshore facility i the end of the time horizon
T_h^L	Time to load product type h at the onshore depot
T_i^O	Time to sail from onshore base to offshore facility i
T_{ij}	Time to sail from facility i to j

T_{ih}^{U}	Time to unload at offshore facility i for product h
$\overline{T}$	Time horizon
T^{MAX}	Maximum time a vessel can travel

Decision variables

x_{imjnk}	1 if vessel k sails from visit (i, m) to (j, n), 0 otherwise
x_{imk}^{OB}	1 if vessel k sails from depot to visit (i, m), 0 otherwise
x_{imk}^{OE}	1 if visit (i, m) is the last visit made by vessel k, 0 otherwise
z_{im}	1 if visit (i, m) is made, 0 otherwise
w_{imk}	1 if visit (i, m) is made by vessel k, 0 otherwise
r_{kh}^{O}	Amount of units of commodity of type h loaded to vessel k
d_{imkh}	Amount of units of commodity of type h delivered by vessel k at visit (i, m)
y_{kh}	Number of vehicles that leaves facility h to supply vessel k
s_{imh}^{B}	Stock level of commodity h before visit (i, m)
s_{imh}^{E}	Stock level of commodity h after visit (i, m)
t_{im}^{B}	Start time of visit (i, m)
t_{im}^{E}	End time of visit (i, m)

A.2 Mathematical Model

Objective function

$$\min z = \sum_{(i,mj,n)\in\mathcal{S}} \sum_{k\in\mathcal{K}} \mathrm{C}_{ij}^{K} x_{imjnk} \tag{A.1a}$$

$$+ \sum_{(i,m)\in\mathcal{S}} \sum_{k\in\mathcal{K}} C_{i}^{OK} (\mathrm{x}_{imk}^{OB} + \mathrm{x}_{imk}^{OE}) \tag{A.1b}$$

$$+ \sum_{(i,m)\in\mathcal{S}} \sum_{k\in\mathcal{K}} \mathrm{C}^{TC,K} \mathrm{x}_{imk}^{OB} \tag{A.1c}$$

$$+ \sum_{k\in\mathcal{K}} \sum_{h\in\mathcal{H}} \mathrm{C}_{h}^{TC,V} y_{kh} \tag{A.1d}$$

Routing constraints

$$\sum_{(i,m)\in\mathcal{S}} x_{imk}^{OB} \leq 1 \qquad for\ k \in \mathcal{K} \tag{A.2}$$

$$w_{imk} = x_{imk}^{OB} + \sum_{(j,n)\in\mathcal{S}} x_{jnimk} \qquad for\ (i,m) \in \mathcal{S}, k \in \mathcal{K} \tag{A.3}$$

$$w_{imk} = x_{imk}^{OE} + \sum_{(j,n)\in\mathcal{S}} x_{imjnk} \qquad for\ (i,m) \in \mathcal{S}, k \in \mathcal{K} \tag{A.4}$$

$$\sum_{k\in\mathcal{K}} w_{imk} = z_{im} \qquad for\ (i,m) \in \mathcal{S} \tag{A.5}$$

$$z_{i(m-1)} - z_{im} \geq 0 \qquad for\ (i,m) \in \mathcal{S} | m > 1 \tag{A.6}$$

Loading constraints

$$r_{kh}^{O} = \sum_{(i,m)\in\mathcal{S}} d_{imkh} \qquad for\ k \in \mathcal{K}, h \in \mathcal{H} \tag{A.7}$$

$$r_{kh}^{O} \leq Q_{h}^{V} y_{kh} \qquad for\ k \in \mathcal{K}, h \in \mathcal{H} \tag{A.8}$$

$$r_{kh}^{O} \leq Q_{h}^{K} \sum_{(i,m)\in\mathcal{S}} x_{imk}^{OB} \qquad for\ k \in \mathcal{K}, h \in \mathcal{H} \tag{A.9}$$

$$d_{imkh} \leq Q_{h}^{K} w_{imk} \qquad for\ (i,m) \in \mathcal{S}, k \in \mathcal{K}, h \in \mathcal{H} \tag{A.10}$$

$$\sum_{h\in\mathcal{H}} d_{imkh} \geq w_{imk} \qquad for\ (i,m) \in \mathcal{S}, k \in \mathcal{K} \qquad \text{(A.11)}$$

Inventory constraints

$$S_{ih}^{0} - W_{ih} t_{im}^{B} = s_{imh}^{B} \qquad for\ (i,m) \in \mathcal{S} | m = 1\ h \in \mathcal{H} \qquad \text{(A.12)}$$

$$s_{imh}^{B} - W_{ih}\left(t_{im}^{E} - t_{\mathrm{im}}^{B}\right) + \sum_{k\in\mathcal{K}} d_{imkh} = s_{imh}^{E} \qquad for\ (i,m) \in \mathcal{S}, h \in \mathcal{H} | m = 1 \qquad \text{(A.13)}$$

$$s_{i(m-1)h}^{E} - W_{ih}\left(t_{im}^{B} - t_{i(m-1)}^{E}\right) = s_{imh}^{B} \qquad for\ (i,m) \in \mathcal{S} | m > 1, h \in \mathcal{H} \qquad \text{(A.14)}$$

$$S_{ih}^{\overline{T}} \leq s_{imh}^{E} - W_{ih}\left(\overline{T} - t_{im}^{E}\right) \qquad for\ (i,m) \in \mathcal{S} | \mathrm{m} = \mathcal{M}_i,\ h \in \mathcal{H} \qquad \text{(A.15)}$$

$$s_{imh}^{B}, s_{imh}^{E} \leq \overline{S}_{ih} \qquad for\ (i,m) \in \mathcal{S}, h \in \mathcal{H} \qquad \text{(A.16)}$$

Time constraints

$$t_{im}^{B} - t_{i(m-1)}^{E} \geq 0 \qquad for\ (i,m) \in \mathcal{S} | m > 1 \qquad \text{(A.17)}$$

$$t_{\mathrm{im}}^{E} = t_{im}^{B} + \sum_{k\in\mathcal{K}} \sum_{h\in\mathcal{H}} T_{ih}^{U} d_{imkh} \qquad for\ (i,m) \in \mathcal{S} \qquad \text{(A.18)}$$

$$\sum_{k\in\mathcal{K}} x_{imjnk}\left(t_{im}^{E} + T_{ij} - \mathrm{t}_{jn}^{B}\right) = 0 \qquad for\ (i,m,j,n) \in \mathcal{S} \qquad \text{(A.19)}$$

$$\sum_{k\in\mathcal{K}} \left(T_i^{O} \mathrm{x}_{imk}^{OB} + \sum_{h\in\mathcal{H}} T_h^{L} r_{kh}^{0} \right) \leq t_{im}^{B} \qquad for\ (i,m) \in \mathcal{S} \qquad \text{(A.20)}$$

$$t_{im}^{E} + T_i^{O} \leq \overline{\mathrm{T}} \qquad for\ (i,m) \in \mathcal{S} \qquad \text{(A.21)}$$

Variable constraints

$$x_{imk}^{OB}, x_{imk}^{OE}, w_{imk} \in \{0,1\} \qquad for\ (i,m) \in \mathcal{S}, k \in \mathcal{K} \qquad \text{(A.22)}$$

$$x_{imjnk} \in \{0,1\} \qquad for\ (i,m,j,n) \in \mathcal{S}, k \in \mathcal{K} \qquad \text{(A.23)}$$

$$z_{im} \in \{0,1\} \qquad for\ (i,m) \in \mathcal{S} \qquad \text{(A.24)}$$

$$y_{kh} \in \mathbb{Z}^+ \qquad for\ k \in \mathcal{K}, h \in \mathcal{H} \qquad \text{(A.25)}$$

$$\mathrm{r}_{kh}^{O} \geq 0 \qquad for\ k \in \mathcal{K}, h \in \mathcal{H} \qquad \text{(A.26)}$$

$$d_{imkh} \geq 0 \qquad for\ (i,m) \in \mathcal{S}, k \in \mathcal{K}, h \in \mathcal{H} \qquad \text{(A.27)}$$

$$\mathrm{s}_{imh}^{B}, s_{imh}^{E} \geq 0 \qquad for(i,m) \in \mathcal{S}, h \in \mathcal{H} \qquad \text{(A.28)}$$

$$t_{im}^{B}, t_{im}^{E} \in \mathbb{Z}^+ \qquad for\ (i,m) \in \mathcal{S} \qquad \text{(A.29)}$$

APPENDIX B: STOCHASTIC MODEL

B.1 DEFINITIONS

Sets

$\mathcal{N}$	Set of facilities, indexed by i, j
$\mathcal{M}_i$	Set of visit numbers at facility i, indexed by m, n
$\mathcal{S}$	Set of visits (i, m) where $i \in \mathcal{N}$ and $m \in \mathcal{M}_i$, indexed by (i, m)
$\mathcal{H}$	Set of product types, indexed by h
Ω	Set of scenarios, indexed by c
$\mathcal{A}$	Set of alternative supply operations, indexed by a

Parameters

$C_i^{SC,K}$	Spot charter and travelling cost for a vessel to offshore facility i
$C_{ih}^{SC,H}$	Spot charter cost for a helicopter travelling from facility h to offshore facility i
C^S	Storage cost
C^P	Penalty cost per unit time
C^{DT}	Downtime cost per unit time
Q_h^K	Capacity of product h for the vessels
Q_h^H	Capacity of product h for the helicopters
W_{ih}	Consumption rate at facility i for product h per unit time
W_{ihc}^R	Consumption rate at facility i for product h per unit time in scenario c
T_i^O	Hours to sail from onshore base to offshore facility i
T_{ih}^{OH}	Hours to sail from onshore base to offshore facility i

π_c	Probability of scenario c

Decision variables

d^R_{imhc}	Amount of commodity of type h required at visit (i, m) in scenario c
d^R_{imhca}	Amount of commodity of type h required at visit (i, m) in scenario c alternative a
t^B_{im}	Start time of visit (i, m)
t^E_{im}	End time of visit (i, m)
t^R_{imhc}	Downtime for visit (i, m) for product h in scenario c
u_{imhc}	Amount of hired helicopters for visit (i, m) for product h in scenario c
δ_{imca}	1 if alternative a is chosen for visit (i, m) in scenario c, 0 otherwise
δ_{ca}	1 if alternative $a = 2$ is chosen in scenario c, 0 otherwise

B.2 Mathematic Formulation

Objective function

$$\min g = \sum_{c\in\Omega}\Bigg\{C^S\delta_{c2} \qquad \text{(B.1a)}$$

$$+ \sum_{(i,m)\in\mathcal{S}}\left(\mathrm{C}_i^{SC,K} + C^P T_i^O\right)\delta_{imc2} \qquad \text{(B.1b)}$$

$$+ \sum_{(i,m)\in\mathcal{S}}\sum_{h\in H} C^{DT} t_{imhc}^R\,\delta_{imc1} \qquad \text{(B.1c)}$$

$$+ \sum_{(i,m)\in\mathcal{S}}\sum_{h\in H}\left(\mathrm{C}_{ih}^{SC,H} + C^P T_{ih}^O\right)u_{imhc}\Bigg\} \qquad \text{(B.1d)}$$

Stochastic constraints

$$\left(W_{ihc}^R - W_{ih}\right)t_{im}^B = d_{imhc}^R \qquad for\ (i,m)\in\mathcal{S}|m=1, h\in\mathcal{H}, c\in\Omega \qquad \text{(B.2)}$$

$$\left(W_{ihc}^R - W_{ih}\right)\left(t_{im}^B - t_{i(m-1)}^E\right) = d_{imhc}^R \qquad for\ (i,m)\in\mathcal{S}|m>1, h\in\mathcal{H}, c\in\Omega \qquad \text{(B.3)}$$

$$W_{ih}t_{imhc}^R = d_{imhc}^R - \sum_{a\in\mathcal{A}/\{1\}} d_{imhca}^R \qquad for\ (i,m)\in\mathcal{S}, h\in\mathcal{H}, c\in\Omega \qquad \text{(B.4)}$$

$$\sum_{a\in\mathcal{A}}\delta_{imca} = 1 \qquad for\ (i,m)\in\mathcal{S}, c\in\Omega \qquad \text{(B.5)}$$

$$d_{imhc}^R = \sum_{a\in\mathcal{A}/\{1\}} d_{imhca}^R \qquad for\ (i,m)\in\mathcal{S}, h\in H, c\in\Omega \qquad \text{(B.6)}$$

$$\sum_{h\in H} Q_h^K\,\delta_{imca} \geq \sum_{h\in H} d_{imhca}^R \qquad for\ (i,m)\in\mathcal{S}, c\in\Omega\,, a\in\mathcal{A}|a=2 \qquad \text{(B.7)}$$

$$\sum_{(i,m)\in\mathcal{S}}\delta_{imca} \leq \sum_{h\in H} Q_h^K\,\delta_{ca} \qquad for\ c\in\Omega, a\in\mathcal{A}|a=2 \qquad \text{(B.8)}$$

$$Q_h^H u_{imhc} \geq d_{imhca}^R \qquad for\ (i,m)\in\mathcal{S}, c\in\Omega, a\in\mathcal{A}|a=3 \qquad \text{(B.9)}$$

$$\sum_{h\in H} u_{imhc} \leq \sum_{h\in H} Q_h^H\,\delta_{imca} \qquad for\ (i,m)\in\mathcal{S}, c\in\Omega, a\in\mathcal{A}|a=3 \qquad \text{(B.10)}$$

Variable constraints

$$t_{imhc}^{R}, d_{imhc}^{R}, u_{imhc} \in \mathbb{Z}^{+} \qquad for\ (i,m) \in \mathcal{S}, h \in \mathcal{H}, c \in \Omega \tag{B.11}$$

$$d_{imhca}^{R} \in \mathbb{Z}^{+} \qquad for\ (i,m) \in \mathcal{S}, h \in \mathcal{H}, c \in \Omega, a \in \mathcal{A} \tag{B.12}$$

$$\delta_{imca} \in \{0,1\} \qquad for\ (i,m) \in \mathcal{S}, c \in \Omega, a \in \mathcal{A} \tag{B.13}$$

$$\delta_{ca} \in \{0,1\} \qquad c \in \Omega, a \in \mathcal{A} \tag{B.14}$$

APPENDIX C: STOCHASTIC MODEL XPRESS

C.1 DEFINITIONS

Sets

$\mathcal{N}$	Set of facilities, indexed by i, j
$\mathcal{M}_i$	Set of visit numbers at facility i, indexed by m, n
$\mathcal{S}$	Set of visits (i, m) where $i \in \mathcal{N}$ and $m \in \mathcal{M}_i$, indexed by (i, m)
$\mathcal{H}$	Set of product types, indexed by h
Ω	Set of scenarios, indexed by c
$\mathcal{A}$	Set of alternative supply operations, indexed by a

Parameters

$C_i^{SC,K}$	Spot charter and travelling cost for a vessel to offshore facility i
$C_i^{SC,H}$	Spot charter cost for a helicopter travelling from facility h to offshore facility i
C^S	Storage cost
C^P	Penalty cost per unit time
C^{DT}	Downtime cost per unit time
Q_h^K	Capacity of product h for the vessels
Q^H	Capacity of product h for the helicopters
W_{ih}	Consumption rate at facility i for product h per unit time
W_{ic}^R	Consumption rate at facility i for product h per unit time in scenario c
T_i^O	Hours to sail from onshore base to offshore facility i
T_i^{OH}	Hours to sail from onshore base to offshore facility i

π_c Probability of scenario c

Decision variables from deterministic solution

t_{im}^{B} Start time of visit (i, m)

t_{im}^{E} End time of visit (i, m)

Decision variables

d_{imc}^{R} Amount of commodity required at visit (i, m) in scenario c

d_{imca}^{R} Amount of commodity required at visit (i, m) in scenario c alternative a

t_{im}^{B} Start time of visit (i, m)

u_{imc} Amount of hired helicopters for visit (i, m) in scenario c

δ_{imca} 1 if alternative a is chosen for visit (i, m) in scenario c, 0 otherwise

δ_{ca} 1 if alternative $a = 2$ is chosen in scenario c, 0 otherwise

C.2 MATHEMATICAL FORMULATION

Objective function

$$\min g \quad = \sum_{c\in\Omega}\Bigg\{C^S\delta_{c2} \tag{C.1a}$$

$$+ \sum_{(i,m)\in\mathcal{S}} \left(C_i^{SC,K} + C^P T_i^O\right)\delta_{imc2} \tag{C.1b}$$

$$+ \sum_{(i,m)\in\mathcal{S}} C^{DT} t_{imc}^R \tag{C.1c}$$

$$+ \sum_{(i,m)\in\mathcal{S}} \left(C_i^{SC,H} + C^P T_i^{OH}\right)u_{imc}\Bigg\} \tag{C.1d}$$

Stochastic constraints

$$\left(W_{ic}^R - \sum_{h\in H} W_{ih}\right)t_{im}^B = d_{imc}^R \qquad for\ (i,m)\in\mathcal{S}|m=1, c\in\Omega \tag{C.2}$$

$$\left(W_{ic}^R - \sum_{h\in H} W_{ih}\right)\left(t_{im}^B - t_{i(m-1)}^E\right) = d_{imc}^R \qquad for\ (i,m)\in\mathcal{S}|m>1, c\in\Omega \tag{C.3}$$

$$\sum_{h\in H} W_{ih}\, t_{imc}^R = d_{imc}^R - \sum_{a\in\mathcal{A}/\{1\}} d_{imca}^R \qquad for\ (i,m)\in\mathcal{S}, c\in\Omega \tag{C.4}$$

$$\sum_{a\in\mathcal{A}} \delta_{imca} = 1 \qquad for\ (i,m)\in\mathcal{S}, c\in\Omega \tag{C.5}$$

$$d_{imc}^R = \sum_{a\in\mathcal{A}/\{1\}} d_{imca}^R \qquad for\ (i,m)\in\mathcal{S}, c\in\Omega \tag{C.6}$$

$$\sum_{h\in H} Q_h^K\, \delta_{imca} \geq d_{imca}^R \qquad for\ (i,m)\in\mathcal{S}|, c\in\Omega\, , a\in\mathcal{A}|a=2 \tag{C.7}$$

$$\sum_{(i,m)\in\mathcal{S}} \delta_{imca} \leq \sum_{h\in H} Q_h^K\, \delta_{ca} \qquad for\ c\in\Omega, a\in\mathcal{A}|a=2 \tag{C.8}$$

$$Q^H u_{imc} \geq d_{imca}^R \qquad for\ (i,m)\in\mathcal{S}, c\in\Omega, a\in\mathcal{A}|a=3 \tag{C.9}$$

$$u_{imhc} \leq Q^H \delta_{imca} \qquad for\ (i,m)\in\mathcal{S}, c\in\Omega, a\in\mathcal{A}|a=3 \tag{C.10}$$

Variable constraints

$$t_{imc}^{R}, d_{imc}^{R}, u_{imc} \in \mathbb{Z}^{+} \qquad for\ (i,m) \in \mathcal{S}, c \in \Omega \tag{C.11}$$

$$d_{imca}^{R} \in \mathbb{Z}^{+} \qquad for\ (i,m) \in \mathcal{S}, c \in \Omega, a \in \mathcal{A} \tag{C.12}$$

$$\delta_{imca} \in \{0,1\} \qquad for\ (i,m) \in \mathcal{S}, c \in \Omega, a \in \mathcal{A} \tag{C.13}$$

$$\delta_{ca} \in \{0,1\} \qquad c \in \Omega, a \in \mathcal{A} \tag{C.14}$$

APPENDIX D: DETERMINISTIC SOURCE CODE

```
! This file is the mathematical mode of the deterministic problem
! Created by Hanne Dreyer Engh, 2015-05-27
! Norwegian University of Science and Technology
! ------------------------------------------------------------------------------------------

! ------------------------------------------------------------------------------------------
!                          Gain access to the Xpress-Optimizer solver
! ------------------------------------------------------------------------------------------
model DeterministicProblem
uses  "mmxprs";                        ! Library including optimizer, MIP
options explterm                         ! All commands must end with a ;
options noimplicit                       ! Everything must be declared before it is used

! -----------------------------------------------------------------------------------------
!                          Importing data file
! -----------------------------------------------------------------------------------------
parameters
   DataFile ='Input_Deterministic_v.2.dat';
   RUNTIME = 500000;
end-parameters

setparam("XPRS_maxtime", RUNTIME);

! ----------------------------------------------------------------------------------------
!                          Declaration of indices
! ----------------------------------------------------------------------------------------
declarations
    Site:           set of integer;
    VisitM:         dynamic array(integer) of set of integer;
    Vessel:         set of integer;
    Product:        set of integer;
end-declarations


! -----------------------------------------------------------------------------------------
!                          Declaration of amount of indices as integer
! -----------------------------------------------------------------------------------------
declarations
    nSite:          integer;
    nMaxVisit:      integer;
    nVisitM:        dynamic array(integer) of integer;
    nVessel:        integer;
    nProduct:       integer;
end-declarations

! ----------------------------------------------------------------------------------------
!                          Retrieve indices from file
! ----------------------------------------------------------------------------------------
initializations from DataFile
    nSite;
    nMaxVisit;
    nVessel;
    nProduct;
end-initializations

! ----------------------------------------------------------------------------------------
!                          Definition of indices
! ----------------------------------------------------------------------------------------
Site                :=  1 .. nSite;
Vessel              :=  1 .. nVessel;
Product             :=  1 .. nProduct;

forall(ii in Site, mm in 1..nMaxVisit) do
    VisitM(ii) += {mm};
end-do
!Assigning the visitnumber to each installation

! ----------------------------------------------------------------------------------------
!                          Finalizing the indices
! ----------------------------------------------------------------------------------------
finalize(Site);
finalize(Vessel);
finalize(Product);

! ----------------------------------------------------------------------------------------
!                          Declaration of Parameters
! ----------------------------------------------------------------------------------------
declarations
    Cost:           array(Site, Site)                               of integer;
    CostO:          array(Site)                                     of integer;
    CostTCK:        integer;
    CostTCV:        array(Product)                                  of integer;
    CapV:           array(Product)                                  of integer;
    CapK:           array(Product)                                  of integer;
```

```
    Consume:        array(Site, Product)                                of real;
    StockIni:       array(Site, Product)                                of integer;
    StockLow:       array(Site, Product)                                of integer;
    StockUpper:     array(Site, Product)                                of integer;
    StockEnd:       array(Site, Product)                                of integer;
    TimeLoad:       integer;
    TimeDepot:      array(Site)                                         of integer;
    TimeSail:       array(Site, Site)                                   of integer;
    TimeUnload:     array(Site)                                         of integer;
    TimePeriod:     integer;
    TotalSail:      integer;
end-declarations

! -----------------------------------------------------------------------------------------
!                            Retrieve parameters from file
! -----------------------------------------------------------------------------------------
initializations from DataFile
    Cost;
    Cost0;
    CostTCK;
    CostTCV;
    CapV;
    CapK;
    Consume;
    StockIni;
    StockLow;
    StockUpper;
    StockEnd;
    TimeLoad;
    TimeDepot;
    TimeSail;
    TimeUnload;
    TimePeriod;
    TotalSail;
end-initializations

! -----------------------------------------------------------------------------------------
!                            Declaration of Decision Variables
! -----------------------------------------------------------------------------------------
declarations !Variable derfor dynamic
    flow:           dynamic array(Site, integer, Site, integer, Vessel)            of mpvar;
    startflow:      dynamic array(Site, integer, Vessel)                           of mpvar;
    endflow:        dynamic array(Site, integer, Vessel)                           of mpvar;
    visit:          dynamic array(Site, integer)                                   of mpvar;
    visitvessel:    dynamic array(Site, integer, Vessel)                           of mpvar;

    loading:        dynamic array(Vessel, Product)                                 of mpvar;
    demand:         dynamic array(Site, integer, Vessel, Product)                  of mpvar;
    vehicle:        dynamic array(Vessel, Product)                                 of mpvar;

    startstock:     dynamic array(Site, integer, Product)                          of mpvar;
    stock:          dynamic array(Site, integer, Product)                          of mpvar;

    starttime:      dynamic array(Site, integer)                                   of mpvar;
    endtime:        dynamic array(Site, integer)                                   of mpvar;
end-declarations

! -----------------------------------------------------------------------------------------
!                            Create Decision Variables
! -----------------------------------------------------------------------------------------
forall (ii in Site, mm in VisitM(ii), jj in Site, nn in VisitM(jj), kk in Vessel| (ii)<>(jj)) do
    create(flow(ii, mm, jj, nn, kk));
        flow(ii, mm, jj, nn, kk) is_binary;
end-do

forall(ii in Site, mm in VisitM(ii), kk in Vessel) do
    create(startflow(ii, mm, kk));
        startflow(ii, mm, kk) is_binary;
end-do

forall(ii in Site, mm in VisitM(ii), kk in Vessel) do
    create(endflow(ii, mm, kk));
        endflow(ii, mm, kk) is_binary;
end-do

forall(ii in Site, mm in VisitM(ii)) do
    create(visit(ii, mm));
        visit(ii, mm) is_binary;
end-do

forall(ii in Site, mm in VisitM(ii), kk in Vessel) do
    create(visitvessel(ii, mm, kk));
        visitvessel(ii, mm, kk) is_binary;
end-do
```

```
forall(kk in Vessel, hh in Product) do
    create(loading(kk, hh));

end-do

forall(ii in Site, mm in VisitM(ii), kk in Vessel, hh in Product) do
    create(demand(ii, mm, kk, hh));

end-do

forall(kk in Vessel, hh in Product) do
    create(vehicle(kk, hh));
        vehicle(kk, hh) is_integer;
end-do

forall (ii in Site, mm in VisitM(ii), hh in Product) do
    create(startstock(ii, mm, hh));
end-do

forall (ii in Site, mm in VisitM(ii), hh in Product) do
    create(stock(ii, mm, hh));
end-do

forall (ii in Site, mm in VisitM(ii)) do
    create(starttime(ii, mm));
        starttime(ii, mm) is_integer;
end-do

forall(ii in Site, mm in VisitM(ii)) do
        create(endtime(ii, mm));
        endtime(ii, mm) is_integer;
end-do

! -------------------------------------------------------------------------------------------
!                     Declaration of objective function and constraints
! -------------------------------------------------------------------------------------------
declarations
    TotalCost:                                                                linctr;
    Con1:       array(Vessel)                                                 of linctr;
    Con2:       array(Site, integer, Vessel)                                  of linctr;
    Con3:       array(Site, integer, Vessel)                                  of linctr;
    Con4:       array(Site, integer)                                          of linctr;
    Con5:       array(Site, integer)                                          of linctr;
    Con6:       array(Vessel, Product)                                        of linctr;
    Con7:       array(Vessel, Product)                                        of linctr;
    Con8:       array(Vessel, Product)                                        of linctr;
    Con9:       array(Site, integer, Vessel, Product)                         of linctr;
    Con10:      array(Site, integer, Product)                                 of linctr;
    Con11:      array(Site, integer, Product)                                 of linctr;
    Con12:      array(Site, integer, Product)                                 of linctr;
    Con13:      array(Site, integer, Product)                                 of linctr;
    Con14:      array(Site, Product)                                          of linctr;
    Con15:      array(Site, integer)                                          of linctr;
    Con16:      array(Site, integer)                                          of linctr;
    Con17a:     array(Site, integer, Site, integer, Vessel)                   of linctr;
    Con17b:     array(Site, integer, Site, integer, Vessel)                   of linctr;
    Con18:      array(Site, integer)                                          of linctr;
    Con19:      array(Site, integer)                                          of linctr;
    Con20:      array(Vessel)                                                 of linctr;
end-declarations

! -------------------------------------------------------------------------------------------
!                     Formulations objective function and constraints
! -------------------------------------------------------------------------------------------
TotalCost :=
(sum(ii in Site, mm in VisitM(ii), jj in Site, nn in VisitM(jj), kk in Vessel) Cost(ii, jj)*flow(ii
+ (sum(ii in Site, mm in VisitM(ii), kk in Vessel) Cost0(ii)*(startflow(ii, mm, kk)+endflow(ii, mm,
+ (sum(ii in Site, mm in VisitM(ii), kk in Vessel) CostTCK*startflow(ii, mm, kk))
+ (sum(kk in Vessel, hh in Product) CostTCV(hh)*vehicle(kk, hh));

! -------------------------------------------------------------------------------------------
!                     Formulations constraints
! -------------------------------------------------------------------------------------------

! A.X refers to the number of the constraint in appendix A
!(6.X) and (7.X) refer to where in the text the constraints can be found

!Constraint A.2 (6.1)
forall(kk in Vessel) do
    Con1(kk) :=
        (sum(ii in Site, mm in VisitM(ii)) startflow(ii, mm, kk))
        <= 1;
end-do
```

```
!Constraint A.3 (6.2)
forall(ii in Site, mm in VisitM(ii), kk in Vessel) do
    Con2(ii,mm,kk) :=
        visitvessel(ii, mm, kk)
        = startflow(ii, mm, kk)
        + (sum(jj in Site|(ii)<>(jj), nn in VisitM(jj)) flow(jj, nn, ii, mm, kk));
end-do

!Constraint A.4 (6.3)
forall(ii in Site , mm in VisitM(ii), kk in Vessel) do
    Con3(ii, mm, kk) :=
        visitvessel(ii, mm, kk)
        = endflow(ii, mm, kk)
        + (sum(jj in Site |(ii)<>(jj), nn in VisitM(jj)) flow(ii, mm, jj, nn, kk));
end-do

!Constraint A.5 (6.4)
forall(ii in Site, mm in VisitM(ii)) do
    Con4(ii, mm) :=
        (sum(kk in Vessel) visitvessel(ii, mm, kk))
        = visit(ii, mm);
end-do

!Constraint A.6 (6.5)
forall(ii in Site, mm in VisitM(ii)|(mm) > 1) do
    Con5(ii, mm) :=
        visit(ii,mm-1) - visit(ii, mm)
        >= 0;
end-do


!Constraint A.7 (6.9)
forall(kk in Vessel, hh in Product) do
    Con6(kk, hh) :=
        loading(kk, hh)
        = (sum(ii in Site, mm in VisitM(ii)) demand(ii, mm, kk, hh));
end-do

!Constraint A.8 (6.10)
forall(kk in  Vessel, hh in Product) do
    Con7(kk,hh) :=
        loading(kk, hh) <= CapV(hh)*vehicle(kk, hh);
end-do

!Constraint A.9 (6.11)
forall(kk in Vessel, hh in Product) do
    Con8(kk, hh) :=
        loading(kk, hh)
        <= CapK(hh)*(sum(ii in Site, mm in VisitM(ii)) startflow(ii, mm, kk));
end-do

!Constraint A.10 (6.12)
forall(ii in Site, mm in VisitM(ii),kk in Vessel, hh in Product) do
    Con9 (ii, mm, kk, hh) :=
        demand(ii, mm, kk, hh)
        <= CapK(hh)*visitvessel(ii, mm, kk);
end-do


!Constraint A.12 (6.16)
forall(ii in Site, mm in VisitM(ii), hh in Product|(mm)=1) do
    Con10(ii,mm, hh) :=
        StockIni(ii, hh) - Consume(ii, hh)*starttime(ii, mm)
        = startstock(ii, mm, hh);
end-do


!Constraint A.13 (7.5)
forall(ii in Site, mm in VisitM(ii), hh in Product|(mm)=1) do
    Con11(ii, mm, hh) :=
        startstock(ii, mm, hh)
        + sum(kk in Vessel) demand(ii, mm, kk, hh)
        - Consume(ii, hh)*TimeUnload(ii)
        = stock(ii, mm, hh);
end-do

!Constraint (7.7)
forall(ii in Site, mm in VisitM(ii), hh in Product) do
    Con12(ii, mm, hh) :=
        StockUpper(ii,hh)*visit(ii, mm)
        >= startstock(ii, mm, hh)+ sum(kk in Vessel) demand(ii, mm, kk, hh)
        - Consume(ii, hh)*TimeUnload(ii);
end-do
```

```
!Constraint A.14  (6.18)
forall(ii in Site, mm in VisitM(ii), hh in Product|(mm)>1) do
    Con13(ii, mm, hh) :=
        stock(ii, (mm)-1, hh)
        - Consume(ii, hh)*(starttime(ii, mm)-endtime(ii, (mm)-1))
        = startstock(ii, mm, hh);
end-do

!Constraint A.15 (7.6)
forall(ii in Site, hh in Product) do
    Con14(ii, hh) :=
        StockIni(ii, hh)-TimePeriod*Consume(ii, hh)
        + sum(mm in VisitM(ii), kk in Vessel) demand(ii, mm, kk, hh)
        >= StockEnd(ii,hh);
end-do

!Constraint A.17 (6.21)
forall(ii in Site, mm in VisitM(ii)|(mm)>1) do
    Con15(ii, mm) :=
        starttime(ii, mm) - endtime(ii, (mm)-1)
        >= 0;
end-do

!Constraint A.18 (7.3)
forall(ii in Site, mm in VisitM(ii)) do
    Con16 (ii, mm) :=
        endtime(ii, mm)
        = starttime(ii, mm)
        + TimeUnload(ii);
end-do


!Constraint A.19 (7.1)
forall(ii in Site, mm in VisitM(ii), jj in Site, nn in VisitM(jj),kk in Vessel| (ii)<>(jj)) do
    Con17a(ii, mm, jj, nn, kk) :=
        endtime(ii, mm) + TimeSail(ii, jj) - starttime(jj, nn)
        + TimePeriod*flow(ii, mm, jj, nn, kk)
        <= TimePeriod;
end-do

!Constraint A.19 (7.2)
forall(ii in Site, mm in VisitM(ii), jj in Site, nn in VisitM(jj),kk in Vessel| (ii)<>(jj)) do
    Con17b(ii, mm, jj, nn, kk) :=
        endtime(ii, mm) + TimeSail(ii, jj) - starttime(jj, nn)
        - TimePeriod*flow(ii, mm, jj, nn, kk)
        >= - TimePeriod;
end-do


!Constraint A.20 (7.4)
forall(ii in Site, mm in VisitM(ii)|(mm)=1) do
    Con18(ii, mm) :=
        (sum(kk in Vessel) ((TimeDepot(ii)*startflow(ii, mm, kk))
        + TimeLoad*visitvessel(ii, mm, kk)))
        <= starttime(ii, mm);
end-do

!Constraint A.21 (6.26)
forall(ii in Site, mm in VisitM(ii)) do
    Con19(ii, mm) :=
        endtime(ii, mm) + TimeDepot(ii)
        <= TimePeriod;
end-do

!Constraint (7.8)
forall(kk in Vessel) do
Con20(kk) :=
    (sum(ii in Site, mm in VisitM(ii), jj in Site, nn in VisitM(jj))
    TimeSail(ii, jj)*flow(ii, mm, jj, nn, kk))
    +(sum(ii in Site, mm in VisitM(ii)) TimeUnload(ii)*visitvessel(ii, mm, kk))
    +(sum(ii in Site, mm in VisitM(ii)) TimeDepot(ii)*(startflow(ii, mm, kk)+endflow(ii, mm, kk)))
    +(sum(ii in Site, mm in VisitM(ii)) TimeLoad*startflow(ii, mm, kk))
    <= TotalSail;
end-do


! ------------------------------------------------------------------------------------------
!                          Investigate parameters
! ------------------------------------------------------------------------------------------

setparam("XPRS VERBOSE", TRUE);
```

```
declarations
                status:      string;
end-declarations

case getprobstat of
                XPRS_OPT: status := "Optimal";
                XPRS_UNF: status := "Unfinished";
                XPRS_INF: status := "Infeasible";
                XPRS_UNB: status := "Unbounded";
                XPRS_OTH: status := "Failed";
                else      status := "Unknown";
end-case

! ------------------------------------------------------------------------------------------
!                           Minimize objective function
! ------------------------------------------------------------------------------------------
minimize(TotalCost);
```

APPENDIX E: STOCHASTIC SOURCE CODE

```
! This file is the mathematical mode of the stochastic problem
! Created by Hanne Dreyer Engh, 2015-05-27
! Norwegian University of Science and Technology
! -------------------------------------------------------------------------------------------

! -------------------------------------------------------------------------------------------
!                          Gain access to the Xpress-Optimizer solver
! -------------------------------------------------------------------------------------------
model StochasticProblem
uses  "mmxprs";                         ! Library including optimizer, MIP
options explterm                          ! All commands must end with a ;
options noimplicit                        ! Everything must be declared before it is used

! -------------------------------------------------------------------------------------------
!                          Importing data file
! -------------------------------------------------------------------------------------------
parameters
   DataFile ='Input_Stochasticv2.dat';
   RUNTIME = 10000;
end-parameters

setparam("XPRS_maxtime", RUNTIME);

! -------------------------------------------------------------------------------------------
!                          Declaration of indices
! -------------------------------------------------------------------------------------------
declarations
    Site:           set of integer;
    VisitM:         set of integer;
    Product:        set of integer;
    Scenario:       set of integer;
    Alt:            set of integer;
end-declarations


! -------------------------------------------------------------------------------------------
!                          Declaration of amount of indices as integer
! -------------------------------------------------------------------------------------------
declarations
    nSite:          integer;
    nMaxVisit:      integer;
    nProduct:       integer;
    nScenario:      integer;
    nAlt:           integer;
end-declarations

! -------------------------------------------------------------------------------------------
!                          Retrieve indices from file
! -------------------------------------------------------------------------------------------
initializations from DataFile
    nSite;
    nMaxVisit;
    nProduct;
    nScenario;
    nAlt;
end-initializations

! -------------------------------------------------------------------------------------------
!                          Definition of indices
! -------------------------------------------------------------------------------------------
Site                := 1 .. nSite;
Product             := 1 .. nProduct;
VisitM              := 1 .. nMaxVisit;
Scenario            := 1 .. nScenario;
Alt                 := 1 .. nAlt;

! -------------------------------------------------------------------------------------------
!                          Finalizing the indices
! -------------------------------------------------------------------------------------------
finalize(Site);
finalize(VisitM);
finalize(Product);
finalize(Scenario);
finalize(Alt);

! -------------------------------------------------------------------------------------------
!                          Declaration of Parameters
! -------------------------------------------------------------------------------------------
declarations
    CostSCK:        array(Site)                                        of integer;
    CostSCH:        array(Site)                                        of integer;
    CostStor:       integer;
    CostPen:        integer;
    CostDT:         integer;
```

```
    CapK:           array(Product)                                          of integer;
    CapH:           integer;
    Consume:        array(Site, Product)                                    of real;
    ConsumeS:       array(Site, Scenario)                                   of real;
    TimeDepot:      array(Site)                                             of integer;
    TimeHeli:       array(Site)                                             of integer;
    Prob:           array(Scenario)                                         of real;
    starttime:      array(Site, VisitM)                                     of integer;
    endtime:        array(Site, VisitM)                                     of integer;
    demandreal:     array(Site, VisitM, Scenario)                           of integer;
end-declarations

! ------------------------------------------------------------------------------------------
!                            Retrieve parameters from file
! ------------------------------------------------------------------------------------------
initializations from DataFile
    CostSCK;
    CostSCH;
    CostStor;
    CostPen;
    CostDT;
    CapK;
    CapH;
    Consume;
    ConsumeS;
    TimeDepot;
    TimeHeli;
    Prob;
    starttime;
    endtime;
end-initializations

! ------------------------------------------------------------------------------------------
!                            Create parameters from deterministic model
! ------------------------------------------------------------------------------------------
!Constraint C.2 (6.50)
forall(ii in Site, mm in VisitM, cc in Scenario|(mm)=1) do
    demandreal(ii, mm, cc) :=
    integer(round(getsol((ConsumeS(ii,cc)-(sum(hh in Product) Consume(ii, hh)))*starttime(ii, mm)))
end-do

!Constraint C.3 (6.51)
forall(ii in Site, mm in VisitM, cc in Scenario|(mm)>1) do
    demandreal(ii, mm, cc) :=
    integer(round(getsol((ConsumeS(ii,cc)-(sum(hh in Product) Consume(ii, hh)))*(starttime(ii, mm)-
end-do


! ------------------------------------------------------------------------------------------
!                            Declaration of Decision Variables
! ------------------------------------------------------------------------------------------
declarations
    demandalt:      dynamic array(Site, VisitM, Scenario, Alt)                    of mpvar;
    down:           dynamic array(Site, VisitM,  Scenario)                        of mpvar;
    helicopter:     dynamic array(Site, VisitM, Scenario)                         of mpvar;
    delta:          dynamic array(Site, VisitM, Scenario, Alt)                    of mpvar;
    alt2:           dynamic array(Scenario, Alt)                                  of mpvar;
end-declarations

! ------------------------------------------------------------------------------------------
!                            Create Decision Variables
! ------------------------------------------------------------------------------------------
forall(ii in Site, mm in VisitM, cc in Scenario, aa in Alt) do
    create(demandalt(ii, mm, cc, aa));
end-do

forall(ii in Site, mm in VisitM, cc in Scenario) do
    create(down(ii, mm, cc));
end-do

forall(ii in Site, mm in VisitM, cc in Scenario) do
    create(helicopter(ii, mm, cc));
        helicopter(ii, mm, cc) is_integer;
end-do

forall(ii in Site, mm in VisitM, cc in Scenario, aa in Alt) do
    create(delta(ii, mm, cc, aa));
        delta(ii, mm, cc, aa) is_binary;
end-do

forall(cc in Scenario, aa in Alt|(aa)=2) do
    create(alt2(cc, aa));
        alt2(cc, aa) is_binary;
end-do
```

```
! -------------------------------------------------------------------------------------
!                         Declaration of objective function and constraints
! -------------------------------------------------------------------------------------
declarations
    TotalCost:                                                                  linctr;
    Con25:          array(Site, VisitM, Scenario)                               of linctr;
    Con26:          array(Site, VisitM, Scenario)                               of linctr;
    Con27:          array(Site, VisitM, Scenario)                               of linctr;
    Con28:          array(Site, VisitM, Scenario, Alt)                          of linctr;
    Con29:          array(Scenario, Alt)                                        of linctr;
    Con30:          array(Site, VisitM, Scenario, Alt)                          of linctr;
    Con31:          array(Site, VisitM, Scenario, Alt)                          of linctr;
end-declarations

! -------------------------------------------------------------------------------------
!                         Formulations objective function and constraints
! -------------------------------------------------------------------------------------
TotalCost :=
(sum(cc in Scenario) Prob(cc)*(
    CostStor*alt2(cc, 2)                                                 !C.1a
    +(sum(ii in Site, mm in VisitM)                                      !C.1b
    ((CostSCK(ii)+CostPen*TimeDepot(ii))*delta(ii, mm, cc, 2)))
    +(sum(ii in Site, mm in VisitM) (CostDT*down(ii, mm, cc)))           !C.1c
    +(sum(ii in Site, mm in VisitM)                                      !C.1d
    ((CostSCH(ii)+CostPen*TimeHeli(ii))*helicopter(ii, mm, cc)))));
! -------------------------------------------------------------------------------------
!                         Formulations constraints
! -------------------------------------------------------------------------------------

!C.x refers to the number of the constraint in appendix C
!(6.X) refers to where in the text the constraint can be found

!Constraint C.4 (6.52)
forall(ii in Site, mm in VisitM, cc in Scenario) do
Con25 (ii, mm, cc) :=
    (sum(hh in Product) Consume(ii,hh))*down(ii, mm, cc)
    =demandreal(ii, mm, cc)
    -(sum(aa in Alt|(aa)<>1) demandalt(ii, mm, cc, aa));
end-do

!Constraint C.5 (6.53)
forall(ii in Site, mm in VisitM, cc in Scenario) do
Con26 (ii, mm, cc) :=
    sum(aa in Alt) delta(ii, mm, cc, aa)
    =1;
end-do

!Constraint C.6 (6.54)
forall(ii in Site, mm in VisitM, cc in Scenario) do
Con27(ii, mm, cc) :=
    demandreal(ii, mm, cc)
    =(sum(aa in Alt|(aa)<>1) demandalt(ii, mm, cc, aa));
end-do


!Constraint C.7 (6.55)
forall(ii in Site, mm in VisitM, cc in Scenario, aa in Alt| (aa)=2) do
Con28 (ii, mm, cc, aa) :=
    (sum(hh in Product) CapK(hh)*delta(ii, mm, cc, aa))
    >= demandalt(ii, mm, cc, aa);
end-do

!Constraint C.8 (6.56)
forall(cc in Scenario, aa in Alt| (aa)=2) do
Con29 (cc, aa) :=
    (sum (ii in Site, mm in VisitM) delta(ii, mm, cc, aa))
    <= (sum(hh in Product) CapK(hh)*alt2(cc, aa));
end-do

!Constraint C.9 (6.57)
forall(ii in Site, mm in VisitM, cc in Scenario, aa in Alt| (aa)=3) do
Con30 (ii, mm, cc, aa) :=
    CapH*helicopter(ii, mm, cc)
    >= demandalt(ii, mm, cc, aa);
end-do

!Constraint C.10 (6.58)
forall(ii in Site, mm in VisitM, cc in Scenario, aa in Alt| (aa)=3) do
Con31 (ii, mm, cc, aa) :=
    helicopter(ii, mm, cc)
    <= CapH*delta(ii, mm, cc, aa);
end-do
```

```
! -------------------------------------------------------------------------------------------
!                             Investigate parameters
! -------------------------------------------------------------------------------------------

setparam("XPRS_VERBOSE", TRUE);

declarations
                status:      string;
end-declarations

case getprobstat of
                XPRS_OPT: status := "Optimal";
                XPRS_UNF: status := "Unfinished";
                XPRS_INF: status := "Infeasible";
                XPRS_UNB: status := "Unbounded";
                XPRS_OTH: status := "Failed";
                else      status := "Unknown";
end-case

! -----------------------------------------------------------------------------------------
!                             Minimize objective function
! -----------------------------------------------------------------------------------------
minimize(TotalCost);
```

APPENDIX F: MATLAB CODE


```
%Planning the shortest path
%Define coordinates
base = [56.9331, 5.6119; 56.9331, 5.6119; 56.9331, 5.6119; 56.9331, 5.6119]; %Tananger
site = [58.9064, 2.1619; 58.6156, 1.7611; 58.1242, 1.9231; 56.6908, 3.1019]; %Offshore↙
site
supplier = [56.9311, 5.6119; 63.1104, 7.7279; 60.7944, 5.0670; 58.9362, 5.5741]; %↙
Supplier
%Location 1, 2, 3, 4, respectively


%Calculate the distance from Tananger to Facilities, C0(i)
startDist = zeros(length(base), length(site));
for i=1:length(base)
    for j=1:length(site)
        startDist(i,j)= round(deg2nm(distance('gc',[base(i,1),base(i,2)],...
            [site(j,1),site(j,2)])));
    end
end

%Calculate the distance between the Offshore Facilties C(i,j)
siteDist = zeros(length(site), length(site));
for i = 1:length(site)
    for j= 1:length(site)
            siteDist(i,j) = round(deg2nm(distance('gc',[site(i,1),site(i,2)],...
                [site(j,1),site(j,2)])));
     end
end

%Calculate the distance between the Offshore Facilties C(i,j)
heliDist = zeros(length(supplier), length(site));
for i = 1:length(supplier)
    for j= 1:length(site)
            heliDist(i,j) = round(deg2nm(distance('gc',[supplier(i,1),supplier(i,↙
2)],...
                [site(j,1),site(j,2)])));
     end
end



%Calculate the distance between x and y:
%dgc=distance('gc',x, y);
%distgcnm=deg2nm(dgc);                        %Nautical miles along great circle
```

APPENDIX G: DEMAND DATA FOR INSTALLATIONS

The data provided in the table below are average data from some installations. The data was obtained from the Nordbø (2013), who was given the data by Professor Bjørn Egil Asbjørnslett.

Activity	Date	No. Months	Average visits per week	Total loading and unloading time	Dry bulk	Mud	Brine	Fuel	Mineral oil	Water	Sum tons	Square meters
Production	Jan. - May	5	2.3	6	0	0	0	16	0	65	82	118
Production	Jan. - May	5	3.8	18	82	77	238	27	0	133	558	571
Production	Jan. - April	4	3.2	10	40	6	12	60	0	167	285	200
Exploration drilling	Jan. - May	5	4.3	29	123	289	227	164	0	304	1106	671
Exploration drilling	Jan, April, May	3	3.8	24	37	21	193	78	0	99	428	251
Exploration drilling	Jan. - May	5	3.6	21	68	95	58	123	0	154	498	405
Exploration drilling	Jan. - May	5	2.7	14	124	68	39	97	0	249	577	321
Sum per week			3.4	124	474	555	768	565	0	1171	3533	2 536

The data are converted into volume by the following densities [ton/m^3]:

Dry bulk	2.00
Mud	1.73
Brine	1.23
Fuel	0.89
Water	1.00

APPENDIX H: REFERENCE VESSEL

The information about the reference vessel is obtained from (Westshore, 2015)

WORLD PEARL

LAST 5 FIXTURES

DATE	CHARTERER	SCOPE OF WORK	RATE
18.05	Enquest	Supply duties	GBP 2650
14.04	Enquest	Supply duties	GBP 2750
19.02	Enquest	C/R + Option	GBP 5000
04.02	CNR	Supply duties, D/D 10	RNR
11.12	CNR	Supply duties, D/D 8	GBP 5000

GENERAL INFORMATION

Name:	WORLD PEARL
BHP:	6598
YearBuilt:	2013
Flag:	Norway
Owner:	World Wide Supply Shipping AS

View More...

SPECIFICATIONS

Tonnage & Dimensions | Class/Services | Capacities

Tonnage & Dimensions

Length:	80.03 m
Deck Area:	728 m2
Draft:	6.15
DWT:	3300
Breadth MLD:	16.2 m

Class/Services

Design:	Damen 3300 CD
Accommodation:	22
FiFi:	N/A
Oil Rec.:	No
Standby:	No

Capacities

Base Oil:	869
Brine:	869 m3
Dry Bulk:	259 m3
Fresh Water:	817 m3
Fuel Oil:	454 m3
Liquid Mud:	869 m3
Methanol:	N/A

•

APPENDIX I: DETERMINISTIC INPUT FILE

```
! This file is the mathematical mode of the stochastic problem
! Created by Hanne Dreyer Engh, 2015-05-24
! Norwegian University of Science and Technology
! ---------------------------------------------------------------------
!Input data to stage 1  - Deterministic Model
! ---------------------------------------------------------------------

! ---------------------------------------------------------------------
!                   Define sets
! ---------------------------------------------------------------------
nSite           :   4           !16/1-22S, 15/6-13A, 15/12-24A, 2/4-23S
nMaxVisit       :   3           !Each site must be visited 3 times
nVessel         :   3           !The available vessel fleet
nProduct        :   5           !Water, Dry Bulk, Mud, Brine, Fuel
                                !Tananger, Kristiansund, Mongstad,
                                !Haugesund, Tananger

! ---------------------------------------------------------------------
!                   Data - Stage 1
! ---------------------------------------------------------------------
!Travelling cost for the vessel
Cost        :   [
    -1      1243    2840    8047
    1243    -1      1775    7278
    2840    1775    -1      5562
    8047    7278    5562    -1
    ]

!Travelling Cost from depot
CostO       :   [
    9585    9408    8224    9408
    ]

!Charter cost of Vessel
CostTCK     :   2100


!Charter and travelling cost of vehicle
CostTCV     :   [
    1119    591     327     0       0
    ]

!Capacity of Vessel
CapK        : [
    260     870     870     450     800
    ]

!Capacity of Vehicle
CapV        : [
    15      17      24      34      30
    ]

!Consumption rate per hour
Consume     : [
    0.51    1.39    1.54    1.54    2.53
    0.28    0.46    0.39    1.15    1.28
    0.52    0.33    0.26    0.91    2.08
    0.15    0.10    1.31    0.73    0.83
    ]

!Initial Stock at beginning of time horizon
StockIni    : [
    31      84      92      92      152
    17      27      24      69      77
    31      20      16      54      125
    9       6       78      44      50
    ]

!Lower limit for stock level
StockLow    : [
    1       1       1       1       1
    1       1       1       1       1
    1       1       1       1       1
    1       1       1       1       1
    ]

!Upper limit for stock level
StockUpper  : [
    100     200     200     200     300
    100     200     200     200     300
    100     200     200     200     300
    100     200     200     200     300
    ]
```

```
!Lower stock level at the end of the time horizon
StockEnd     : [
    31      84      92      92      152
    17      27      24      69      77
    31      20      16      54      125
    9       6       78      44      50
    ]

! Time to load
TimeLoad     :   7

! Time to sail from depot to facility i
TimeDepot        :    [
    14      13      12      7
    ]

! Time to sail between facilities
TimeSail     : [
    -1      2       4       11
    2       -1      3       10
    4       3       -1      8
    11      10      8       -1
    ]

!Time to unload
TimeUnload   : [
    7       6       6       5
    ]

! Time horizon
TimePeriod   : 120

!
TotalSail    : 72
```

APPENDIX J: STOCHASTIC INPUT FILE

```
! This file is the mathematical mode of the stochastic problem
! Created by Hanne Dreyer Engh, 2015-05-24
! Norwegian University of Science and Technology
! ---------------------------------------------------------------------
!Input data to stage 1 and 2 - Stochastic Model
! ---------------------------------------------------------------------

! ---------------------------------------------------------------------
!                  Define sets
! ---------------------------------------------------------------------
nSite           :   4          !16/1-22S, 15/6-13A, 15/12-24A, 2/4-23S
nMaxVisit       :   2          !Each site must be visited 3 times
nProduct        :   5          !Water, Dry Bulk, Mud, Brine, Fuel
nScenario       :   4          !Tananger, Kristiansund, Mongstad,
nAlt            :   3          !Haugesund, Tananger

! ---------------------------------------------------------------------
!                  Data - Stage
! ---------------------------------------------------------------------

!Capacity of Vessel
CapK        : [
    260     870     870     450     800
    ]

!Consumption rate per hour
Consume     : [
    0.51    1.39    1.54    1.54    2.53
    0.28    0.46    0.39    1.15    1.28
    0.52    0.33    0.26    0.91    2.08
    0.15    0.10    1.31    0.73    0.83
    ]

! Time to sail from depot to facility i
TimeDepot       :   [
    14      13      12      7
    ]

! ---------------------------------------------------------------------
!                  Data - Stage 2
! ---------------------------------------------------------------------

!Starttimes are obtained from the deterministic solution
starttime       : [
    44      55
    47      53
    38      62
    25      76
    ]

!Endtimes are obtained from the deterministic solution
endtime         : [
    36      81
    44      72
    25      63
    49      59
    ]

!Town from site for helicopter

TimeHeli    : [
    2       2       2       3
    ]

!Downtime cost per hour
CostDT      :   8946

!Penalty cost per hour
CostPen     :   895

!Spot charter rate and travel cost for vessel per installation
CostSCK     :   [
    12735       12558       11374       12558
    ]

!Spot charter and travelling costs for helicopters
CostSCH     : [
    9070        9070        9070        10490
```

Page 1 of 2

```
    ]

!Storage cost
CostStor     :   7057


!Capacity of helicopter
CapH         : 5

!Demand scenario: expected, +1%, 3%, 5%
ConsumeS     : [
    7.51    7.59    7.74    7.89
    3.56    3.60    3.68    3.75
    4.10    4.13    4.21    4.30
    3.12    3.15    3.21    3.27
    ]

Prob         :[
    0.85        0.05        0.05    0.05
    ]
```

APPENDIX K: REVISED DETERMINISTIC MODEL

K.1 DEFINITIONS

Sets

$\mathcal{N}$	Set of facilities, indexed by i, j
$\mathcal{M}_i$	Set of visit numbers at facility i, indexed by m, n
$\mathcal{S}$	Set of visits (i, m) where $i \in \mathcal{N}$ and $m \in \mathcal{M}_i$, indexed by (i, m)
$\mathcal{K}$	Set of vessels in the fleet, indexed by k
$\mathcal{H}$	Set of product types, indexed by h

Parameters

C_{ij}^K	Operating cost for sailing from i to j per unit time
C_i^{OK}	Operating cost for sailing from depot to i per unit time
$C^{TC,K}$	Fixed charter cost for a vessel
$C_h^{TC,V}$	Fixed charter cost for a vehicle from supplier facility h
Q_h^V	Capacity of product h for the vehicles
Q_h^K	Capacity of product h for the vessels
W_{ih}	Consumption rate at facility i for product h per unit time
S_{ih}^0	Initial stock level at offshore facility i for product h
$\underline{S}_{ih}$	Lower stock level at offshore facility i for product h
$\overline{S}_{ih}$	Upper stock level at offshore facility i for product h
$S_{ih}^{\overline{T}}$	Stock level of product h at offshore facility i the end of the time horizon
T_h^L	Time to load product type h at the onshore depot

T_i^O	Time to sail from onshore base to offshore facility i
T_{ij}	Time to sail from facility i to j
T_{ih}^U	Time to unload at offshore facility i for product h
$\overline{T}$	Time horizon
T^{MAX}	Maximum time a vessel can travel

Decision variables

x_{imjnk}	1 if vessel k sails from visit (i,m) to (j,n), 0 otherwise
x_{imk}^{OB}	1 if vessel k sails from depot to visit (i,m), 0 otherwise
x_{imk}^{OE}	1 if visit (i,m) is the last visit made by vessel k, 0 otherwise
w_{imk}	1 if visit (i,m) is made by vessel k, 0 otherwise
d_{imkh}	Amount of units of commodity of type h delivered by vessel k at visit (i,m)
y_{kh}	Number of vehicles that leaves facility h to supply vessel k
s_{imh}^B	Stock level of commodity h before visit (i,m)
t_{im}^B	Start time of visit (i,m)

K.2 MODEL FORMULATION

Objective function

$$\min z = \sum_{(i,mj,n)\in\mathcal{S}} \sum_{k\in\mathcal{K}} \mathrm{C}_{ij}^{K} x_{imjnk} \tag{K.1a}$$

$$+ \sum_{(i,m)\in\mathcal{S}} \sum_{k\in\mathcal{K}} C_{i}^{OK} \left(\mathrm{x}_{imk}^{OB} + \mathrm{x}_{imk}^{OE}\right) \tag{K.1b}$$

$$+ \sum_{(i,m)\in\mathcal{S}} \sum_{k\in\mathcal{K}} \mathrm{C}^{TC,K} \mathrm{x}_{imk}^{OB} \tag{K.1c}$$

$$+ \sum_{k\in\mathcal{K}} \sum_{h\in\mathcal{H}} \mathrm{C}_{h}^{TC,V} y_{kh} \tag{K.1d}$$

Routing constraints

$$\sum_{(i,m)\in\mathcal{S}} x_{imk}^{OB} \leq 1 \qquad for\ k \in \mathcal{K} \tag{K.2}$$

$$w_{imk} = x_{imk}^{OB} + \sum_{(j,n)\in\mathcal{S}} x_{jnimk} \qquad for\ (i,m) \in \mathcal{S}, k \in \mathcal{K} \tag{K.3}$$

$$w_{imk} = x_{imk}^{OE} + \sum_{(j,n)\in\mathcal{S}} x_{imjnk} \qquad for\ (i,m) \in \mathcal{S}, k \in \mathcal{K} \tag{K.4}$$

$$\sum_{k\in\mathcal{K}} w_{imk} = z_{im} \qquad for\ (i,m) \in \mathcal{S} \tag{K.5}$$

$$\sum_{k\in\mathcal{K}} w_{i(m-1)k} - w_{imk} \geq 0 \qquad for\ (i,m) \in \mathcal{S} | m > 1 \tag{K.6}$$

Loading constraints

$$\sum_{(i,m)\in\mathcal{S}} d_{imkh} \leq Q_{h}^{V} y_{kh} \qquad for\ k \in \mathcal{K}, h \in \mathcal{H} \tag{K.7}$$

$$\sum_{(i,m)\in\mathcal{S}} d_{imkh} \leq Q_{h}^{K} \sum_{(i,m)\in\mathcal{S}} x_{imk}^{OB} \qquad for\ k \in \mathcal{K}, h \in \mathcal{H} \tag{K.8}$$

$$d_{imkh} \leq Q_{h}^{K} w_{imk} \qquad for\ (i,m) \in \mathcal{S}, k \in \mathcal{K}, h \in \mathcal{H} \tag{K.9}$$

$$\sum_{h\in\mathcal{H}} d_{imkh} \geq w_{imk} \qquad for\ (i,m)\in\mathcal{S}, k\in\mathcal{K} \tag{K.10}$$

Inventory constraints

$$S_{ih}^{0} - W_{ih}t_{im}^{B} = s_{imh}^{B} \qquad for\ (i,m)\in\mathcal{S}|m=1\ h\in\mathcal{H} \tag{K.11}$$

$$s_{imh}^{B} - W_{ih}\left(t_{\mathrm{im}}^{B} + \sum_{k\in\mathcal{K}} T_{ih}^{U}d_{imkh}\right) + \sum_{k\in\mathcal{K}} d_{imkh} \geq \underline{S}_{ih} \qquad for\ (i,m)\in\mathcal{S}, h\in\mathcal{H}|m=1 \tag{K.12}$$

$$s_{imh}^{B} = s_{i(m-1)h}^{B} + \sum_{k\in\mathcal{K}} d_{imkh} - W_{ih}\left(t_{im}^{B} - \sum_{k\in\mathcal{K}} T_{ih}^{U}d_{imkh} - t_{i(m-1)}^{B}\right) \qquad for\ (i,m)\in\mathcal{S}|m>1, h\in\mathcal{H} \tag{K.13}$$

$$S_{ih}^{\overline{T}} \leq s_{imh}^{E} - W_{ih}\left(\overline{T} - t_{im}^{E}\right) \qquad for\ (i,m)\in\mathcal{S}|\mathrm{m}=\mathcal{M}_i, h\in\mathcal{H} \tag{K.14}$$

Time constraints

$$t_{im}^{B} - t_{i(m-1)}^{B} \geq \sum_{h\in\mathcal{H}} T_{h}^{L}d_{imkh} \qquad for\ (i,m)\in\mathcal{S}|m>1 \tag{K.15}$$

$$\sum_{k\in\mathcal{K}} x_{imjnk}\left(t_{im}^{B} + \sum_{k\in\mathcal{K}}\sum_{h\in\mathcal{H}} T_{h}^{L}d_{imkh} + T_{ij} - \mathrm{t}_{jn}^{B}\right) = 0 \qquad for\ (i,m,j,n)\in\mathcal{S} \tag{K.16}$$

$$\sum_{k\in\mathcal{K}}\left(T_{i}^{O}\mathrm{x}_{imk}^{OB} + \sum_{(i,m)\in\mathcal{S}}\sum_{h\in\mathcal{H}} T_{h}^{L}d_{imkh}\right) \leq t_{im}^{B} \qquad for\ (i,m)\in\mathcal{S} \tag{K.17}$$

$$t_{im}^{B} + \sum_{k\in\mathcal{K}}\sum_{h\in\mathcal{H}} T_{h}^{L}d_{imkh} + T_{i}^{O} \leq \overline{\mathrm{T}} \qquad for\ (i,m)\in\mathcal{S} \tag{K.18}$$

Variable constraints

$$x_{imk}^{OB}, x_{imk}^{OE}, w_{imk} \in \{0,1\} \qquad for\ (i,m)\in\mathcal{S}, k\in\mathcal{K} \tag{K.19}$$

$$x_{imjnk} \in \{0,1\} \qquad for\ (i,m,j,n)\in\mathcal{S}, k\in\mathcal{K} \tag{K.20}$$

$$y_{kh} \in \mathbb{Z}^+ \qquad for\ k \in \mathcal{K}, h \in \mathcal{H} \qquad \text{(K.21)}$$

$$d_{imkh} \geq 0 \qquad for\ (i, m) \in \mathcal{S}, k \in \mathcal{K}, h \in \mathcal{H} \qquad \text{(K.22)}$$

$$\mathrm{s}_{imh}^{B} \geq 0 \qquad for(i, m) \in \mathcal{S}, h \in \mathcal{H} \qquad \text{(K.23)}$$

$$t_{im}^{B} \in \mathbb{Z}^+ \qquad for\ (i, m) \in \mathcal{S} \qquad \text{(K.24)}$$