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Optimization of Resource Allocation Using Queueing Theory

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Master Thesis In Marine Technology

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for

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Optimization of Resource Allocation Using Queueing Theory

Background

The oil industry is moving north into remote areas such as the Barents Sea and the Arctic. The companies' main warehouses are located in the southwestern part of Norway, creating long supply chains.

Overall aim and focus

The aim of the thesis is to find optimal allocation of equipment going from the main warehouses to the offshore installations using queueing theory. An analysis of the financial viability of a floating offshore depot will also be performed.

Problem description

The thesis will consider a supply chain going from the main warehouses to the offshore installations in Barents Sea and the Arctic. Along the supply chain, a series of intermediate depots will be placed. The thesis will focus on mission critical equipment.

Contents of thesis

- Generation of a mathematical model
- Implement the model in an optimization routine
- Collect necessary data
- Discuss results and the financial viability of a floating offshore depot

Thesis supervisor

Professor Bjørn Egil Asbjørnslett

Deadline: 10.06.2015

Preface

This Master Thesis completes my degree of Master of Science in Marine Technology at the Norwegian University of Science and Technology, NTNU. The thesis was written in the spring semester 2015. This thesis continous the work done in the project thesis which was written in the fall semester 2014.

The objective of the thesis was to use queueing theory to determine the optimal allocation of inventory within a supply chain going to the Barents Sea and the Arctic. A model of the supply chain was therefore created. The rationale for including queueing theory is that the real world is stochastic, therefore stochastic methods should be used. The model was implemented into a mathematical formulation which was then implemented into MATLAB. The MATLAB- scripts are attached as appendix and also attached in the folder.

I would also like to than my supervisor, Professor Bjørn Egil Asbjørnslett for valuable guidance through the process. I would also thank you for your providing me with data from your communication with Statoil ASA.

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Trondheim, 10th of June 2015

Håkon Hellum

Abstract

As the ice in the Arctic region is melting, new areas will open for oil exploration and production. These areas are however remote, with long sailing times from shore. In addition, the main warehouses of the oil companies are located in the southwestern part of Norway, thus increasing the transportation distances further. To operate, the offshore installations need a lot of equipment, some of which is absolutely necessary for the operation, so called mission critical equipment. This type of equipment have been the main focus in this thesis. When this type of equipment breaks down, or it is no longer needed, it is sent to one of the main warehouses for maintenance and recalibration. By storing spares of this equipment closer to the installation, response time when equipment breaks down is reduced.

Three possible supply chain scenarios were created. The first scenario is the current scenario, where equipment is sent by trucks from one of the main warehouses and to the Hammerfest depot, from where it is shipped to the offshore installations by Platform Supply Vessels (PSVs). The two other scenarios utilize a offshore depot. The depot is assumed to be a converted bulk-carrier. In scenario 2 this depot vessel sails from Hammerfest, while in Scenario 3 it sails from one of the main warehouses. For scenario 2, the equipment is transported by truck to Hammerfest. For a continous operation, it was assumed that two vessels are needed.

By setting an operability constraint the supply chain could be optimized with respect to costs. Operability is the percentage of time inventory of the equipment is present at the installation. Then, by combining the Genetic Algorithm in MATLAB with queueing theory, this optimization problem was solved. It was created as a closed queueing network, meaning that a finite population of customers travel inside the network. This was chosen due to the nature of the equipment studied. The steady state probabilities was calculated using Buzen's algorithm.

Three demand cases were studied. The first case was low demand, where demand arises twice a year. In the medium demand case, demand arises every month. The final case, high demand, demand arises twice a month. In queueing theory, demand is modelled as arriving customers. In a closed queueing network where there are no customers arriving from outside the system, this arrival rate is equal to the service rate at the offshore installation. By varying the transportation costs for the system, an offshore depot vessel seemed more viable for the high demand case, thus preferring to allocate the inventory closer to the installation.

Sammendrag

Isen i Arktis smelter, noe som vil åpne nye områder for oljeleting og produksjon. Disse områdene ligger fjernt fra land, noe som gir lange seillingstider. I tillegg ligger hovedlagrene til de store oljeselskapene i det sørvestre hjørnet av Norge, noe som øker lengden på forsyningskjeden ytterligere. Installasjonene trenger mye forskjellig utstyr. Noe av utstyret er absolutt nødvendig for operasjonene, såkalt kritisk utstyr. Dette har vært fokus i oppgaven. Ved å lagre dette utstyret nærmere platformen kan etterspørselen etter utstyr tilfredstilles hurtigere.

Tre ulike forsyningskjede-scenarier ble laget. Det første scenariet er forsyningskjeden slik den er i dag, der utstyr sendes fra et av hovedlagrene med lastebil til Hammerfest der utstyret lastes over på PSVer. De to andre scenariene benytter seg av fremskutte lagre plassert nærmere installasjonen. I scenario 2 seiler skipet fra Hammerfest, mens det i scenario 3 seiler fra et av hovedlagrene. I scenario 2 blir utstyret fraktet med lastebil fra hovedlageret til Hammerfest. For en kontinuerlig operasjon må to lagerskip benyttes.

Ved å sette et oppetidskrav, kunne forsyningskjeden optimeres for å minimere kostnader. Oppetiden er prosentandelen av tiden det finnes utstyr ombord på innstallasjonen. Dette ble løst ved å kombinere den genetiske algoritmen i MATLAB med køteori. Et køteorinettverk ble derfor laget. Det ble valgt å bruke et lukket netverk der kundene ikke forlater systemet, der kundene er det kritiske utstyret. Et lukket netverk ble valgt siden ødelagt utsyr repareres.

Tre etterspørselsprofiler ble studert. I den første profilen, lav etterspørsel, oppstår etterspørsel to ganger i året. I den mellomste profilen oppstår etterspørsel hver måned. Ved høy etterspørsel oppstår etterspørselen to ganger i måneden. Ved å variere transportkostnaden ble det avdekket når et lagerskip ble mer gunstig enn direkte forsyning. I tilfellet med høy etterspørsel inntraff dette tidligere enn i de to andre tilfellene. Dette indikerer at lagerskip er mer nyttig for utstyr med høyere etterspørsel, slik at dette utstyret bør lagres nærmere installasjonene.

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Chapter 1

Introduction

Much of the remaining oil and gas in the world is located in remote areas such as the Barents Sea and the Arctic. This will open up large areas for oil exploration and later production. These areas are however very remote, creating long supply chains. Increased supply chain length combined with pressure for lowering the costs requires cost efficient supply chains. As of today, the large oil companies in Norway have their main warehouses located in the southwestern part of Norway. The distances from these main warehouses to the remote areas in the Barents Sea and the Arctic are enormous. Thus, transportation by times can be several days. Air transportation on parts or on the entire supply chain length will reduce transportation time drastic, but increase the costs.

When equipments onboard a drilling or a production rig breaks down, they must be repaired as quickly as possible. Depending on equipment it can be repaired locally with the workers already on the rig, it can be repaired locally with repair crew transported from shore or it must be transported to a repair facility on shore. If the equipment must be replaced from an onshore warehouse, storing it in a warehouse closer to the installation than one of the main warehouses will reduce response time. There is already a warehouse in Hammerfest, but in the future floating offshore warehouses may be available.

These long supply chains may need new approaches and solution methods. In this initial phase of studies regarding these supply chains, it is important to search widely for the optimal solutions. The optimal setups of these supply chains have been studied before, see Nordbø (2013) and Akselsen (2014). This thesis will implement stochastic behaviour and focus on inventory stock. To maintain a specified service level there must be a certain amount of inventory available to fill the demand within a time period. This inventory however comes at a cost. Solutions to these problems can be either deterministic or stochastic. The deterministic models are easier to solve while the stochastic models are more realistic. They can however both serve a purpose as the deterministic models can be used as a foundation and later be expanded. Queueing networks have proven to be a valuable tool in supply chains over the last decades and will be used in this thesis.

Chapter 2

Problem description

This section outlines the problem in this thesis.

As the oil industry moves north, the supply chains needed to support the exploration and production installations and vessels operating will become stretched. While sailing time in the North Sea is typically hours, the sailing time in the Barents Sea in the Arctic from land to the installations can. To find good solutions to the problems related to these supply chains one must first identify all the challenges that can be met and also search for new approaches.

The installations or vessels operating offshore have demands of everything from food to spare parts for the onboard equipment. Consumables, such as food and other necessities, have a relative constant demand, or in the very least one that is easy to predict. The amount of food needed is proportional to the number of crew. Apart from the basic needs of the installations, type of demand and amount will change and fluctuate. The demand amount and type is of course also very linked to the type of installation or vessel.

2.1 Supply chains

Supply chains have a vital role in any corporation, and they are an absolute necessity for manufacturers of anything from toys to cars. They are also immensely important for oil companies operating around the globe. A well functioning supply chain will satisfy the demand at the minimal cost with a given demand satisfaction. In a supply chain there is a cost of providing service, in the sense that one has to use transportation modes to transport the cargo from supplier to customer. There is however also a cost of not providing service, both related to unmet demand and also goodwill. For a newspaper unmet demand simply means that the customers are not able to buy the newspaper and they could have sold more newspapers. For an oil company unmet demand may grind the production at an oilrig to a halt. The cost of lost sales or unmet demand also leads to lost goodwill among customers. This may cause the oil company to change their suppliers.

The simplest of supply chain methods is the Economic Order Quantity (EOQ). This seeks to calculate the correct amount of inventory to carry, given a deterministic and constant demand(Goetschalckx, 2011, p. 446-452). The optimal amount of inventory is the amount that minimizes annual ordering costs and annual inventory holding costs. Ordering costs consists of all costs related to purchasing the goods, such as overhead and transport. For this thesis the largest ordering cost would be transportation between the warehouses. Inventory costs come from the fact that inventory ties up capital that could be used elsewhere or invested on the stock market. For markets with stochastic demand a very general approach is the newsvendor model. This model also seeks to find the optimal amount of inventory to carry(Goetschalckx, 2011, p. 453-458).

The most basic versions of a supply chain is a situation where one warehouse or production facility supplies one customer. The customer can be a retail point from where demand is satisfied. Cost will arise from transportation and holding inventory. Such a model can then be expanded to serve several retailers. Further, several warehouses can be added to the model. A common assumption is that a retailer is served by only one warehouse since model with split deliveries are difficult to treat(Fagerholt, 2014). Split deliveries means that half of the cargo can be supplied by one warehouse and the rest from an other warehouse. The model can be further expanded by adding echelons upstream of the warehouse node, typically production facilities and raw material supply points. These types of models are typical in production supply chains.

Supply chain planning can be divided into three main categories. These are strategic, tactical and operational(Goetschalckx, 2011, p. 11-14). The strategic planning is the overall strategy for the company which has a horizon of several years. The tactical planning has a horizon of some months to a year. The lowest planning level, the operational, has a horizon of a couple of days to one week. The supply chain decision discussed in this thesis is on a strategic or tactical level. Whether or not to invest in a depot vessel would be on a strategic level, while the allocation of the inventory and number of chartered Platform Supply Vessels (PSVs) typically would be on a tactical level.

There has been much attention to supply chains with repairable items since the Multi-Echelon Technique for Recoverable Item Control (METRIC) was published by Sherbrooke (1968). The US Air Force have had a large interest in such supply chains, as they operate a lot of expensive and highly advanced equipment, such as aircraft engines, see Rappold and Roo (2009) Diaz and Fu (1997) expanded the Metric model and introduced approximations that could handle a large variety of problems. The above works uses stochastic distributions, most notably the Poisson process, Rappold and Roo (2009), or the compund Poisson, Sherbrooke (1968). Some works, such as Park and Lee (2011), relax the (S-1,S) supply policy found in much of the litterature, and found the optimal (S,Q) stocking policy. Thus, the interface between supply chains and queueing theory are of interest and also holds possibilities for wide applications.

2.2 Offshore supply chains

As of today, there are 36 Statoil-operated fields with 34 fixed installations and 15-20 movable rigs on the Norwegian Continental Shelf(NCS) (Logistikkportalen.no, 2015a). These installations generate demand for all types of consumables and goods. To satisfy their demand there are many interacting supply chains which all must perform optimal. The supply chains consists of several parts, both on land, on sea and in the air. The supply chains objective is to deliver the cargo as economical as possible, at the right time and without damages to the goods, installations or environment. Much of the cargo which is sent to the installations is returned to land. In fact, circa 75 % of all cargo being sent to the installations return as backlog (Logistikkportalen.no, 2015b). This may be anything from reusable equipment to dangerous waste which must be processed in the correct manner.

2.3 The main warehouse

The main warehouse in this thesis is located in the south-western part of Norway. Three of Statoil's warehouses can be treated as this main warehouse, namely Dusavikbase outside Stavanger, Ågotnes outside Bergen and Florø. The southernmost of this bases is the warehouse in Dusavik. On the Norwegian Continental Shelf, the authorities sets limitations as to which installations are to be served from which onshore depots (Halvorsen-Weare et al., 2012). Therefore, some of the inventory at these warehouses will be loaded directly onto to PSVs while other will be sent by truck or ship to the depot specified by the authorities. During the current regulations this will be the case for Statoil's operation in the far north. The main warehouse is the initial storing location of the items and will also be the preferred storing location. It is the preferred location since the equipment stored here can quickly be deployed on the installations in the North Sea. The main warehouses have more repair and recalibration tools than the other warehouses.

2.4 Intermediate warehouses

These intermediate warehouses are placed around the coast of Norway. In this thesis the intermediate warehouses are called depots. This makes it easier to distinguish between the two, as the depots are assumed not to have any repair capacities. Kristiansund, Sandnessjøen and Hammerfest all have depots servicing offshore installations. For the development in the Barents Sea the Hammerfest depot is the most interesting, as this is the northernmost depot.

2.5 Fixed offshore depot

Since the long supply chain requires new approaches, a fixed offshore depot may be needed. This offshore depot can have several functions other than being only a depot. As a depot it will receive inventory from PSVs sailing from the Hammerfest depot. These PSVs will most likely sail in shuttle between these depots. From the offshore depot and to the installations an other set of PSVs will be used. This depot serves then two purposes in the supply chain. Firstly, it can alter the composition of the PSV-fleet. If the installations are to be served directly from shore the PSVs may need to be larger, while this offshore depot may relax this requirement. Secondly, if the depot store either spare parts, safety inventory or both the response time can be lowered.

The offshore depot may also serve as a storage for oil-spill or other emergency equipment, thus reducing lead time also for these operations. The range of Helicopters are limited and for long flights an intermediate stop for refuelling may be needed. In these cases the offshore depot can be useful. Also, if the Helicopter experiences mechanical problems the offshore depot can be important.

2.6 Temporary offshore depot

A temporary offshore depot will serve the same purposes as a fixed, but this depot will be a self-propelled vessel which sails from the onshore depot to a suitable offshore position. This depot vessel will preferably be a large ship, for instance a bulk carrier with some modifications.

This depot type has some advantages and some disadvantages in respect to the fixed depot. The number of ship-to-ship transfers will be reduced as there, in a normal situation, will be no PSVs sailing from the onshore depot to the offshore depot. This is a true advantage in a hostile environment such as the Barents Sea and will reduce risk.

The temporary depot also gives the planners a larger degree of freedom as the depot needs not to be located in the same position in each inventory cycle or even within each inventory cycle. This can be an advantage if an installation is planning an operation that will require more inventory and more frequent visits. To shift position during an inventory cycle is however possible also for a fixed depot if this is too is a self-propelled vessel.

If the regulations on which onshore depot each installation must be serviced by, the depot vessel may sail from any onshore depot. If the temporary depot is allowed to sail from one of the main warehouses, the need for depot-to-depot transfer will be reduced. The ship will then position itself at a location from where PSVs will serve the installations. When all the supplies are offloaded the ship will sail back to the onshore warehouse. Nordbø (2013) argues that for this type of requires two ships, for instance converted bulk carriers, to sail between the warehouse and the location offshore. This allows for a service without disruptions when one ship has to return to the shore to load cargo as the ships will operate in opposite phases.

2.7 Installations

In general there are two types of operations, exploration and production. The two different operations have different demand for equipment.

The two different types of installations or vessels will create different challenges for the supply chain. One of the most obvious differences is while a production vessel lies in a fixed position the drilling vessel will move around. For the production vessel it is therefore known where the demand will be for the next 40 years and a supply chain can be built to serve this installation. For a drilling vessel, creating an optimal supply chain will be more difficult as the sailing distances will change from mission to mission. A drilling vessel will also have a very fluctuating demand with strong peaks during drilling and periods of lower demand during transit. The demand can therefore be grouped into two groups, fixed and fluctuating. The fixed demand is the demand can be said to be deterministic, or at least possible to determine precisely using some prevision method. The fluctuating demand will vary over time and may be caused by planned operations such as planned maintenance or drilling operations. It may also be caused by unforeseen events such as a breakdown of equipment. The fluctuating demand will therefore experience some stochastic behaviour with a probability distribution.

2.8 PSV

A fleet of PSVs will be needed to supply cargo from either the onshore, fixed offshore or temporary offshore depot. The PSVs must be able to carry the six types of demand categories outlined by Fagerholt and Lindstad (2000). These categories are: Deck cargoes, dry bulk, mud, brine, diesel and water. The same requirement will of course also apply to the offshore depots.

PSVs are often chartered on a time basis where several factors determine the cost of hire. One of the obvious parameters are their size as this varies typically from 500 to $1100m^2$ (Fagerholt and Lindstad, 2000). The durations of this charters vary from long term contracts to single trip. The chartering cost is also determined by the market situation, and to hire spot ship are usually costly.

The harsh environment of the Barents Sea and the Arctic where ice can be encountered sets limitations on the pool of possible vessels to charter. Ice-class and other appropriate measures must be required, which will drive up the chartering cost.

2.9 Equipment and inventory

Inventory systems
For every business it is important to know the right amount of inventory to buy and maintain. Shortages of inventory may mean lost profits, while excessive amounts means unnecessary amount of capital tide to the inventory.

Figure 2.1 shows how inventory vary over time. When a replenishment arrives, inventory increases. As demand is satisfied, the inventory decreases. In figure 2.1 the demand is satisfied by discrete deliveries, which then can be simplified to the linear function. This is a good example of the inventory process in the depot vessel. When the ship arrives at the depot position, the inventory is full. Then it gradually decreases.

A stochastic extension is the *Newsvendor problem*, also known as the *Newsboy problem*. In this well known problem the demand is uncertain, but the mean value and distribution of the demand is known. The newsvendor must decide how many newspapers to buy based on this information. If he has too many he will not be able to sell all and he will have to dispose

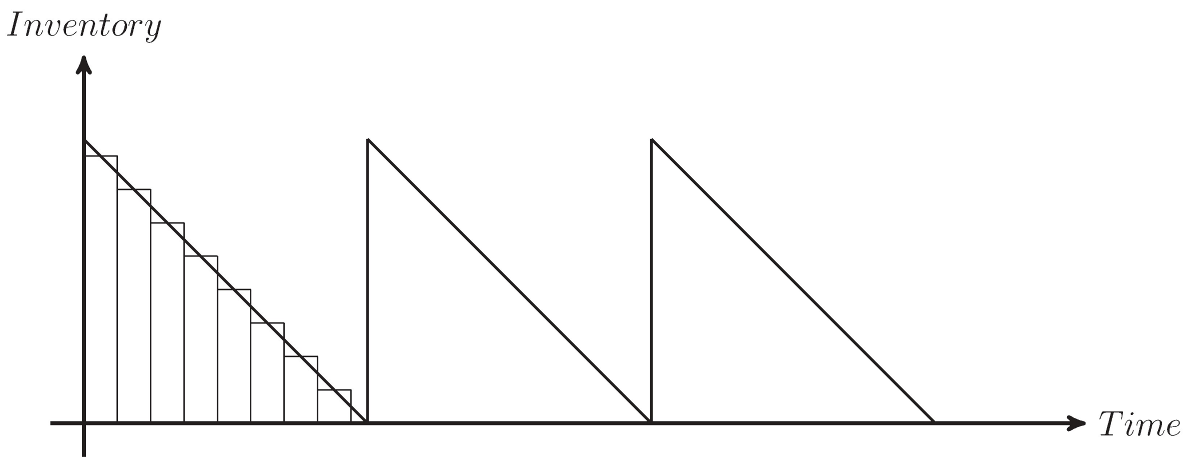

Figure 2.1: Inventory onboard the offshore depot will change over time

them as yesterdays news is of little value. If he has too few he will not be able to satisfy demand, hence loosing profit. The classic newsvendor problem has a horizon of only one cycle and deals with goods that looses its value after the period. However, the newsvendor model can be extended to incorporate multiple periods.

The classical newsboy problem uses overage and underage costs. This poses a problem for this supply chain as the underage costs outweigh everything else. Thus, service levels can be used instead. Service levels are used to determine how well the inventory system performs. Service level can be divided into three types, type 1, 2 or 3 (Goetschalckx, 2011, p.437-438). Type 1 is called *cycle service level* or *in-stock probability* and describes the probability of not stocking out during an inventory cycle. Type 1 is often denoted α or *P1*. Type 2 service is called the *fill rate (fr)*, denoted as β or *P2*. Type 2 describes the amount of demand met by in stock inventory and it also measures the amount of backorders. A backorder occurs when the demand at a retail node cannot be met. The retailer then sends a request for inventory to an upstream warehouse, thus backordering the demand. The third type, type 3, is known as the *Ready Rate(rr)*. It measures the percentage of the demand satisfied immediately(Sabri and Beamon, 2000).

Service policies

In supply chain logistics products are often placed into three categories, A, B and C. The products are divided into three groups depending on their value, or contribution to the total value of the inventory. The distribution typically follows what is known as *Pareto curves*(Goetschalckx, 2011, p. 504). The size of the three classes are typically around 80-15-5, meaning that the goods classified as A sums up to 80 percent of the inventory value, B to 15 percent and C to 5 percent. These classes often reflects the nature of the products, and they are sometimes called *fast*, *medium* and *slow movers* instead of A, B and C. The mission critical equipment has a high cost and can be defined as a class A product.

For the different inventory types, one often chose different ordering policies. Some of the inventory one monitors continuous and orders when the inventory reaches a given level. This inventory value is often set on the background of lead times to delivery to avoid stock-outs. One such policy is the (s,Q) policy. Q is the order amount, while s is the reorder point. The time between replenishments is R, thus this policy is often referred to as (R,Q)(Goetschalckx, 2011, p. 465).

An other type of reordering policty is the (s,S) policy. This is a preiodic review policy When the inventrory is equal to or lower than s, one orders the quantity that will bring the inventory back to S. A special case of this is the (S-1,S) policy. This policy is so that when inventory decreases with one, an order is issued to bring the inventory back to S. This is also known as one-for-one replenishment (Goetschalckx, 2011, p. 477). This policy is typical for high cost, repairable items, as they are usually not batched for repair or resupply(Sherbrooke, 1968).

To operate the installations, various types of cargo and equipment are needed. When this cargo and equipment is in the supply chain it is called on-hand inventory. While this inventory is the very basis of supply chain, holding excess inventory is unfavourable as this binds capital. This capital could otherwise have been invested on the stock-market. The inventory in the depots will vary over time, but will show different behaviour for the different depots. The mission critical equipment is assumed to be sent as deck-load on the PSVs.

Some of the equipment which is needed at the platforms are not needed all the time. This may be the case for equipment being used only for certain operations. Storing this equipment on the platform is then inconvenient as this equipment will use space on the platform. Hence, the equipment is preferred to be stored at an other location than the platform. Storing the equipment at an offshore depot will also use space and capacity that could otherwise have been used for needed cargo, thus reducing the duration of the period at sea, and then increasing the number of trips the depot vessels must sail. However, when this equipment then becomes needed, having it stored in a warehouse in the southern part of Norway may mean it will not arrive in time, and operations must pe postponed. The same is for scenario when equipment brake down. The oil companies tries to avvoid breakdowns at all costs, but they are unfortunately inevitable. If the equipment needs to be repaired on land, sending the equipment from Stavanger may mean shutting down the operations for days. To reduce the lead time of this equipment one may want to store it further north. This will reduce the responce time, but it will require more equipment. An example is equipment which is used for a period and then sent on shore and to Stavanger for maintenance and recalibration. In order to reduce costs, minimizing the number of such equipment is preferable. However, should there be a demand for this equipment, and the only one is being recalibrated, processes must be postponed.

Chapter 3

Method

The objective of this thesis was to compare different supply chain designs using queueing theory. The model in this thesis will be based upon the model made my the project thesis during the autumn semester 2014, see Hellum (2014). Three scenarios were studied using queueing theory and an optimization routine. By comparing the different design scenarios, most notably the differences between design scenario one on one hand, and design scenarios two and three on the other, one can see how allocating inventory along the supply chain affects costs and operability.

One of the major challenges in the model presented in Chapter 4 is the presense of several echelons. The multi-echelon approach makes calculations more difficult as the problems generally become non-linear, see Rappold and Roo (2009) and Kerbache and Smith (2004).

To use queueing theory, the supply chains were divided into nodes. Each node represent either a warehouse or a transportation leg and consists of a queue and one or several servers in parallel. Using a MATLAB-script it was possible to calculate the marginal probabilities for the number of customers at each node. This script utilizes Buzen's-algorithm to calculate the normalizing constant G(N), see Buzen (1973). Buzen's algorithm is well suited for this purpose. It is a recursive algorithm, which makes it suitable for large networks with a large number of customers. This ability to handle larger number of customers makes it suitable for a wide range of networks, also networks with more inventory than in this thesis. Once the marginal distributions are calculated these are used as input into an optimization script.

3.1 Queueing Theory

There are several ways to cope with the uncertainty in the world. Both deterministic and stochastic models can be used, as both have pros and cons. The deterministic models are usually easier to model, and provide good basic knowledge of the problems. Since they are deterministic measures must be taken to cope with the uncertainty. Fagerholt and Lindstad (2000) set the demand to 150% of the average demand to ensure sufficient capacity. By doing so, they made sure to always have enough capacity and also introduced slack to the routes. However, using such estimates might lead to install excessive capacity in a supply chain. One of the most important outputs of a queueing model is server utilization. This measure, ρ, equals the arrival rate of entities to the node, λ divided by the service capacity of the node,μ. By adding several service nodes,c, the utilization will reduce, see Equation (3.1).

$$\rho = \frac{\lambda}{c * \mu} \tag{3.1}$$

This measure should be lower than one, otherwise the queue will forever build up in front of the node, thus hindering the steady state. Also, when the utilization approaches one, the system is more prone to delays. However, by installing excessive capacity, e.g. using more PSVs than needed, the vessels will spend time idling. This is of course unfavourable and should be minimized. In a supply chain with repairable items, several parts of the supply chain can be modelled as servers. For instance, the repair time of equipment will vary depending on its state, but it is not unreasonable to assume the repair times to be fairly similar. Further, the literature suggests the failures of equipment to follow a Poisson distribution, see for instance Rappold and Roo (2009). Poisson processes and the exponential distributions are closely linked (Walpole et al., 2007). Poisson distributions of arrivals is one of the main assumptions in basic queueing theory. This assumption has of course been relaxed, but it does complicate the calculations. Thus, queueing theory can decide needed installed capacity.

In a network where the entities can move from node to node and the nodes have different service times, the arrival rates will naturally change, thus also the utilization. Using a queueing network it is possible to determine these measures. Using the steady-state properties of queueing theory it is possible to calculate the marginal probability of the number of entities present at the different nodes. This gives the connection to service levels. The interface between queuing theory and supply chains are however greater than this, and can be used for other measures. The utilization at the nodes is already mentioned, but also throughput can be used as a constraint in an optimization routine, see Kerbache and Smith (2004).

The queuing network takes a number of inputs, all of which can be used to create an optimal network. The inputs include service times, routing probabilities, number of servers and number of nodes.

Using queueing theory alone will only give technical data of the network from the input values. For queueing theory to become interesting it must be paired with an optimization routine. Gross et al. (2011, p. 212) state that closed, multi class Jackson networks is of "immense importance for modelling a variety of systems of interest in the computer, communication and logistics fields". Further, Rappold and Roo (2009) states that there are still research opportunities in the field of combining queueing theory, multi-echelon invnetory and facility location problems.

One major advantage in queueing theory is that it allows for finite repair time. Queueing networks can be computational challenging, but if they are treated as Jackson networks the steady state joint probabilities can be presented in a product form, making calculations easier. Some algorithms have also been constructed to solve the joint and marginal probabilities.

Queueing networks take several inputs, all which can be changed to find optimal solutions.

The inputs are number of customers, number of nodes, service time at each node and routing probabilities. Some algorithms also allow for multiple servers, state dependent servers or other extensions. In an open network there is an external arrival rate. In a closed network the arrival rate equals the service rates from the nodes feeding into the node in question.

Using Buzens's algorithm the steady state probabilities are found directly. Buzen's algorithm requires the normalizing constant to be calculated, but during this calculations it also extract the data needed to determine the marginal state probabilities. The normalizing constant G(N) can account for other demand distributions than exponential with approximation algorithms.

Although this is not considered here, queueing theory allows for several classes, where the different classes have different attributes. Both this and routing optimality will not be optimized in this thesis, and will be left for later research.

Among the drawbacks of queuing theory is worth mentioning the high number of equations making it computationally challenging. However, the system treated here will be adapted so that they are reasonably small, and due to the nature of the equipment, the number of customers are relatively low. It is also important to keep in mind that queueing theory can only aid in an optimization routine, not completely solve the problem on its own.

3.2 Optimization

The optimization related to queueing networks are complex. The constraints and objective functions are not necessary linear(Kerbache and Smith, 2004), and in some cases not convex (Rappold and Roo, 2009). This makes optimization of these problems more difficult. Additionally, decision variables such as number of customers and number of servers in each node is integer values. Therefore, all this must be accounted for in the optimization routine.

The Genetic Algorithm in MATLAB is suited for optimization of such complex problems (MathWorks, 2015)). The algorithm has several steps to find the optimal solution. First, it creates a population randomly. From this initial population new populations are created. When the algorithm creates the next population it performs several steps. The fitness value of each member of the population is calculated. The fitness value is the objective value. These fitness scores are then scaled and based on their fitness some members are chosen and called *parents*. The best members of the current population are called *elite* and these are passed on to the next population. From the *parents*, points called children are produced, using either a method called *crossover* or a method called *mutation*. The *elite* members are passed on as they are and renamed children. A *crossover* child is created by combining two parents, while a *mutation* child is created by performing random changes to the parent. Then the process continuous using the children as parents for the next population until some stopping condition is met. The stopping criteria can be chosen by the user and can for example be time related, related to the number of generations, closeness to the fitness function, constraint tolerance.

3.3 Literature review

The literature studied is in the interfaces between supply chains, queueing networks and optimization.

Sherbrooke (1968) introduced METRIC, one of the first papers to treat a multi-echelon, multi-item problem. In the paper of Sherbrooke (1968) there was one central depot, but several bases. The failed items could be repaired locally according to a probability, otherwise it was sent to the central depot. This model assumed ample service for repair.

One of many extensions to the METRIC was proposed by Diaz and Fu (1997). Diaz and Fu (1997) created a model with two convolutions, where the first feeds into the second. The authors used negative binomial distributions. The model was expanded with respect to METRIC with limited repair capacity and also a model with several classes. The authors argue that approximations are more effective in comparison to exact methods such as the Buzen's algorithm. However, for the case in this thesis with relatively few entities in the system, the exact method works sufficiently fast. However, for instances with many entities, such as Diaz and Fu (1997), the solution time of Buzens' algorithm increases, and it is then likely that an approximation is preferable. Approximations and two or more convolutions are thus possible future extensions of the model presented in this thesis.

Rappold and Roo (2009) studied a problem from the US Air Force, treated as a two-echelon single item system. Their system design can however be extended to multiple echelons. The main goal of their work was to minimize expected cost of the system. The decision variables of the problem were the base stock levels at the main facilities and the field stocking locations. Repair costs at the main facilities are lower than at the field stocking locations due to economies of scale. Similar to the offshore supply problem treated in this thesis, service constraints are present, and long transportation times from the main repair facility to the field stocking locations could be too long for this restriction to be fulfilled. Rappold and Roo (2009) also considered the number of facilities to open, which repair facilities to supply which field stocking locations, and repair capacity at the repair facilities. The authors solved the problem using a two step method, where the first step solved the location problems. The second step solved the inventory allocation problem. The expected inventory holding cost was only near-convex, so the authors proposed an approximation in order to deal with the non-convexity.

Both the work of Diaz and Fu (1997) and Rappold and Roo (2009) studied the interface between supply chains and queueing theory and networks. Rappold and Roo (2009) created a subnetwork for every repair facility that was opened, each with a $M/M/k$-queue. Thus, it was possible to add capacity to these subnetworks to increase repair capacity. Rappold and Roo (2009) used a Poisson process for the breakdowns, similar to the case in this thesis. Diaz and Fu (1997) represented failure, repair and transportation as queues. Their model shows and extension to the one in this thesis, due to the two convolutions. This indicates that this is a viable path to better decide allocation of inventory between the warehouses.

Park and Lee (2011) used a closed queueing network to analyse a supply chain system. The system consisted of multiple classes and also used a (S,Q) inventory policy for needed spare parts. The main outline of the paper was: If a part in the system fails it is sent to a Part Inventory System where it enters a queue. If a spare part needed for the repair is available the failed item and the spare part are sent to the repair facility. In the case that a spare part is not available the failed item must wait until a batch of spare parts arrives before being sent to the repair facility. Park and Lee (2011) also treats the operational node as a queue, similar to what is done in this thesis. There are R such operating nodes called Bases, all with D servers. Due to the focus on repair policy Park and Lee (2011) concentrate on the inventory node for their analysis. Park and Lee (2011) used a two step approach.

Gross et al. (1983) studied a three node system with "up" and "down" nodes. When the components are working, they are int the "up"-node. When they fail, a certain amount can be repaired locally, while the rest must be sent to a central repair facility. The "up" node has M servers, representing M machines to be working, and y is the number of spares. M and y together represent the total number of customers in the system, N. Gross et al. (1983) used Buzen's algorithm to calculate the steady-state probabilities. The results from the queueing theory was used as a service constraint, which ensured the system to be working a specified percentage of the time. The system was shown to have monotonic behaviour, as increasing the number of customers in the system or the number of servers at the repair nodes all increased availability of the system. The monotonicity allowed the authors to use the Lawler and Bell algorithm.

Madu and Kuei (1996) studied a network similar to Gross et al. (1983). Their study featured only load independent servers. Their case considered four nodes, but resembles Gross et al. (1983) closely, as the goal was to optimize the number of repair persons and spares subject to an operability constraint.

Teimoury et al. (2010) used queueing theory to determine safety stock for a chemical company. This article treated consumable products, as opposed to this thesis' repairable items.

Sahba and Balcıog~lu (2011) compared two supply chain designs, one with a centralized repair shop and one with repair shops at the sites. They used the MVA algorithm as opposed to Buzen's algorithm.

Toktay et al. (2000) used a closed queuing system to find the optimal ordering policy so to minimize the costs of the procurement, holding and lost sales cost. The authors studied supply chain for Kodak single-use cameras. The cameras were reusable, so that after use a portion was sent back to the manufacturer, while others were not returned. This reuse is similar to the equipment needed for drilling operations as also this equipment after use needs to be sent back to base to be recalibrated and maintained. Kodak's supply chain was modelled with six nodes, of which one node was outside the control of Kodak. The five first nodes were Vendor, Shipping, Production, Distribution and Retailer, while the last was named Customer and Lab. To analyse the arrival rate at the nodes the authors used an approach called Fixed Population Mean (FPM). This approach is well suited to solve networks with large populations, infinite number of servers and low service rate. The two last assumptions could be applicable to the supply chain in this thesis, but here the numbers of customers are assumed to be lower, given the difference in costs between equipment for the petroleum industry and single-use cameras.

Kerbache and Smith (2004) claim that closed queueing networks are suitable to link optimal inventory level and the characteristics of the network design. The authors also state that "performance measures such as throughput, cycle time, utilization of resources, can be measured as a function of the target product population of the system"(Kerbache and Smith, 2004). The authors use the MVA- algorithm to calculate the network characteristics. This approach can give the same results as Buzen's algorithm, but the state probabilities are calculated after the other characteristics such as queue lengths(Gross et al., 2011, p. 204). Kerbache and Smith (2004) studied the long supply chain with many stages of a suitcase producer.

Verma et al. (2013) studied a problem related to oil spill response. Oil spill response is important in the Barents Sea and the Arctic as the areas are home to important eco-systems. This response can be assigned to different warehouses, both onshore as well as offshore depots, temporary and fixed. If the offshore depots also work as storages for oil-spill equipment they may prove very useful.

Chapter 4

Model

This chapter shows how the problem is translated into a model that can be used for analysis and implementation into an optimization routine. Three scenarios are outlined as possible supply chain designs.

4.1 Developing the model

In Chapter 2 the different types of depots was outlined. Utilizing these depots in different ways one can create different supply chains, called scenarios. It is possible to design the supply chain in other ways than the scenarios created here, but these three can be seen as extremes relative to each other, thus giving a better understanding of the problems and possibilities. During the development of the model some simplifications were made.

The demand point in the supply chain is at the installation. In the model presented here there is only one demand point. In a real life situation there would typically be several demand points. In queueing theory this demand is equal to the arrival of customers. The customers are entities moving around in the network of queues and servers and they can represent any type of cargo. If the focus is on mission critical equipment breaking down, an arrival of customer is the equivalent of a breakdown. The broken part is then sent to a repair facility, in this model located at the main warehouse. To ensure operability of the installation, a new part must be installed.

In a closed queueing network there is no arrival of external customers. Similar to the examples found in the literature such as, Gross et al. (1983), the service capacity the installation node, $\mu_{Installation}$, will be set equal to the arrival of customers, λ. This is therefore the demand rate. In queueing theory the demand rate is denoted as number of entities per time unit, where the time unit can be chosen, for instance hours.

4.2 Scenario 1

The first scenario is the basis design since it is the most straightforward solution. It is the scenario that requires the least investments as the infrastructure is ready and there are suitable ships available. In this scenario only one intermediate depot is used. In the real world the equipment may be stored in any of the other on-land bases, for instance in Kristiansund or Sandnessjøen, but this is not treated here. Figure 4.1 is a basic representation of what the supply chain will look like. Between the main warehouse and the Hammerfest depot the inventory can be transported in different manners. However, in 2010, 75 % of Statoil's Base-to-Base transfers were done by truck while only 13 % were done by ship (Haram). Thus, land transport was chosen as the preferred solution in this thesis.

Figure 4.1: Scenario 1. The circular figures represent the transportation legs.

Figure 4.2 shows the queuing model of the scenario. Following the course of a broken equipment, the part is initially located at node I which represents the installation. When the part breaks down, it waits at the node PSV_{wait}. This node represents the capacity of the PSVs in the system. Following this node is the PSV_{sail} node representing the sailing delay. This is modelled as an infinite server, similar to Toktay et al. (2000) and Kerbache and Smith (2004). In the routine this is ensured by using a sufficiently large number, chosen so that increasing or decreasing the number of servers do not influence the solution. This was done by trial and error. The node HD represents the Hammerfest depot. The service time at this node is the time it takes to transport the equipment to the main warehouse, repair it and then return it to Hammerfest. The number of servers at this node represents the installed capacity in the system. The inventory then waits in an other PSV_{wait} node before it is shipped via an other PSV_{sail} back to the installation.

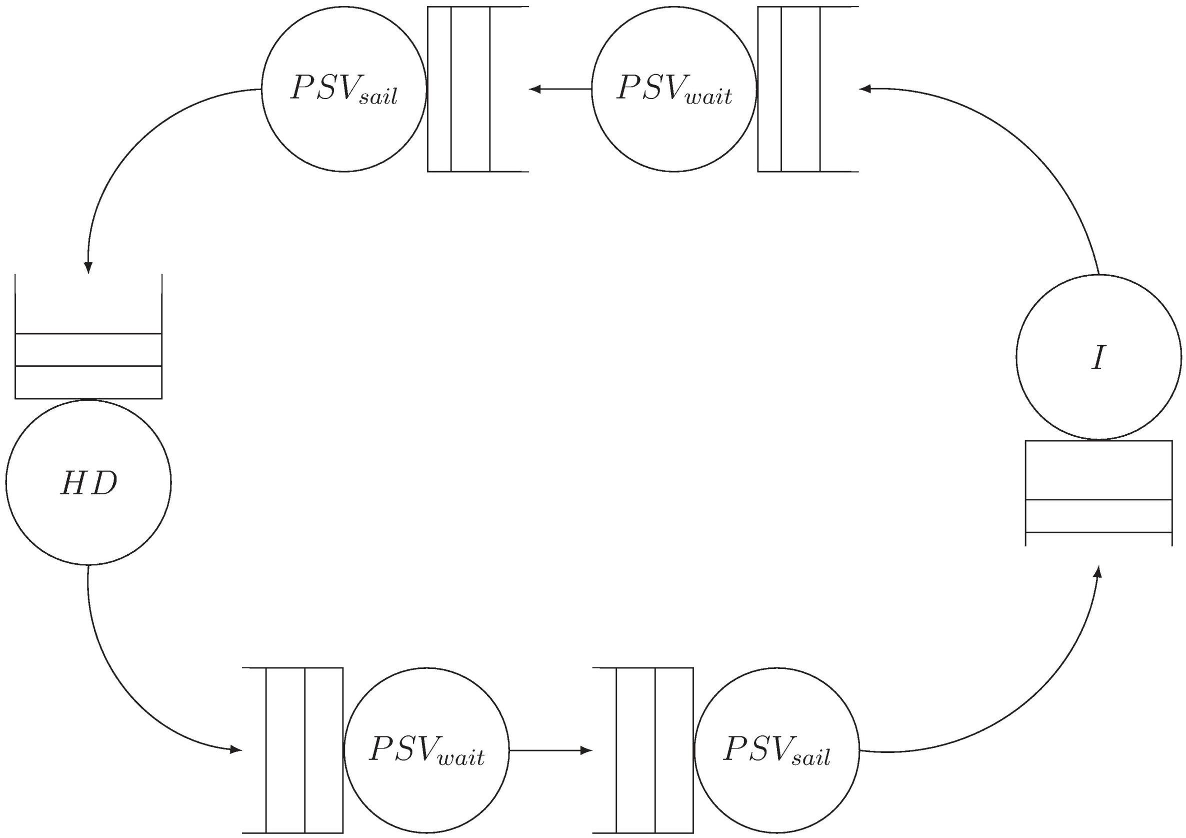

Figure 4.2: Queueing model for Scenario 1.

4.3 Scenario 2

Scenario 2 is identical to scenario 1 from the main warehouse and to Hammerfest, but differ greatly as it involves an offshore floating depot. The floating depot can be a cargo vessel sailing to and from a depot location or a stationary semi submersible platform. Due to operational limitations, it is preferred to minimize the number of ship-to-ship transfers. This can be achieved if this scenario consists of one or several large vessels that sail from the Hammerfest depot and to the offshore depot location. Scenario 2 would then be very similar to scenario 3 in the last stages of the supply chain. Figure 4.3 is a visual representation of the supply chain.

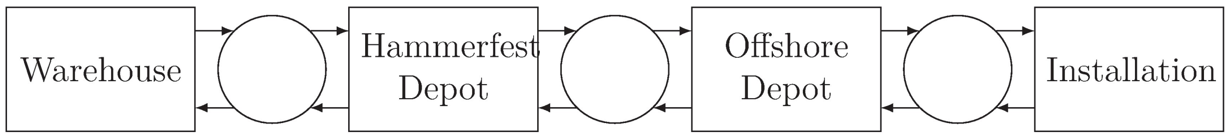

Figure 4.3: Scenario 2. The circular figures represent the transportation legs.

Figure 4.4 shows the queuing model of scenario 2. Similar to the model for scenario 1, when an item breaks down at node I, it is sent to the PSV_{wait} where it waits for a PSV. The node PSV_{sail} is once again an infinite server node representing the transportation time. The node D represents the depot vessel. The service time at this node represents the time the depot vessel stays at sea, as this represents an inventory cycle. The number of servers at this node represents the installed capacity in the system. The inventory then waits in an other PSV_{wait} node before it is shipped via an other PSV_{sail} back to the installation.

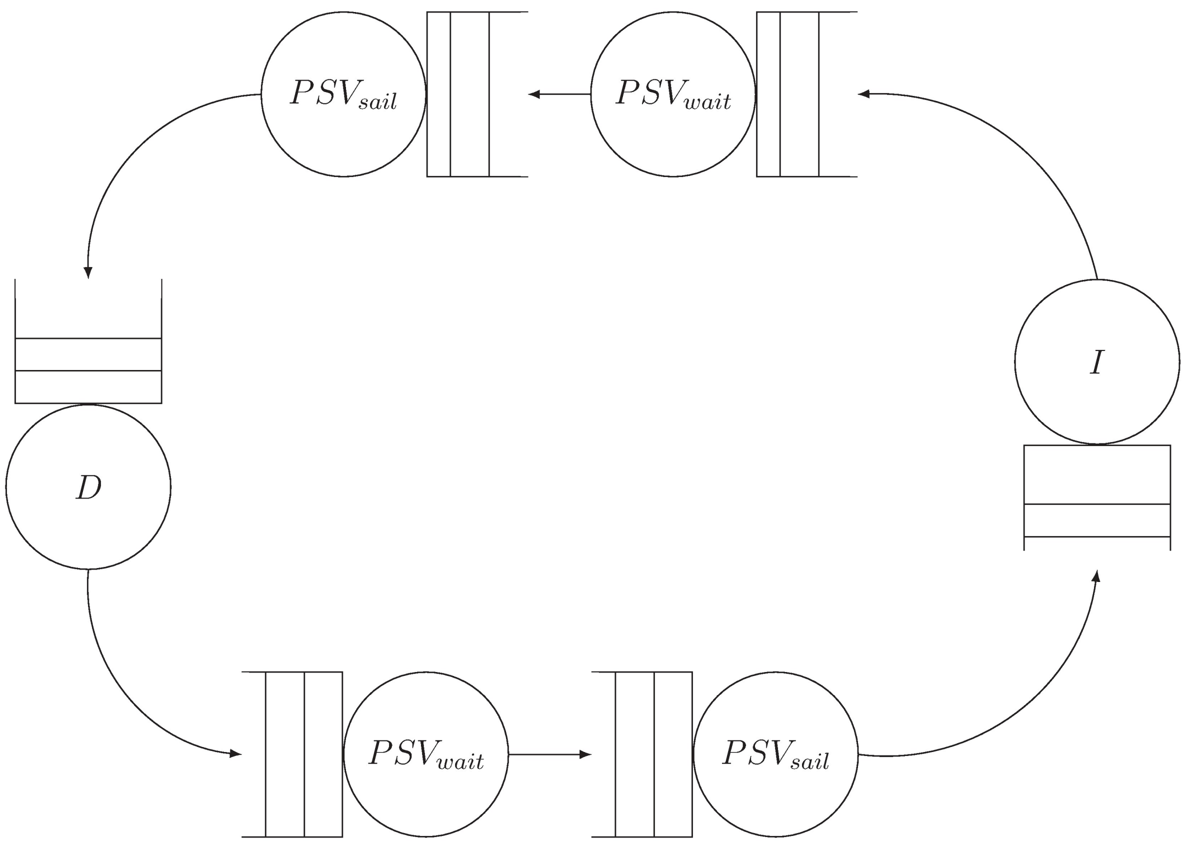

Figure 4.4: Queueing model of scenario 2

4.4 Scenario 3

Scenario 3 is similar to scenario 2 in the sense that they both utilize an offshore depot. In the case of scenario 3, this depot needs to be a self-propelled vessel as it is to sail from the main warehouse and to the depot location, from where it will supply PSVs.

Scenario 3 differs greatly from the others because it does not utilize the Hammerfest depot or any of the other land-based depots. A large vessel is loaded at the main warehouse from where it sails to a given location in the Barents Sea or the Arctic. When it reaches this location it positions itself here for a time period decided by the onboard supplies. Between the cargo vessel and the installations a fleet of PSVs distributes the cargo and equipment. Figure 4.5 is a visual representation of the supply chain, while Figure 4.6 shows the differences between scenario 2 and 3. The depot vessel node in Figure 4.5 is called "Temporary Depot" to emphasize the fact that ii is a moving vessel.

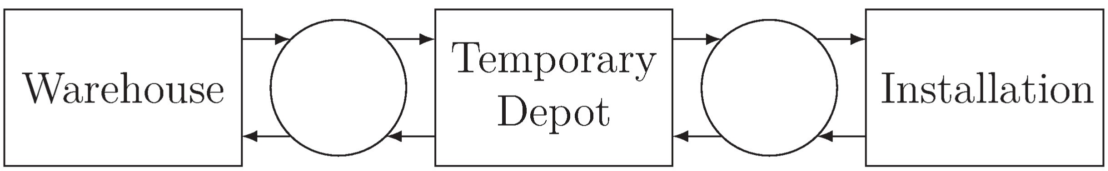

Figure 4.5: Scenario 3. The circular figures represent the transportation legs.

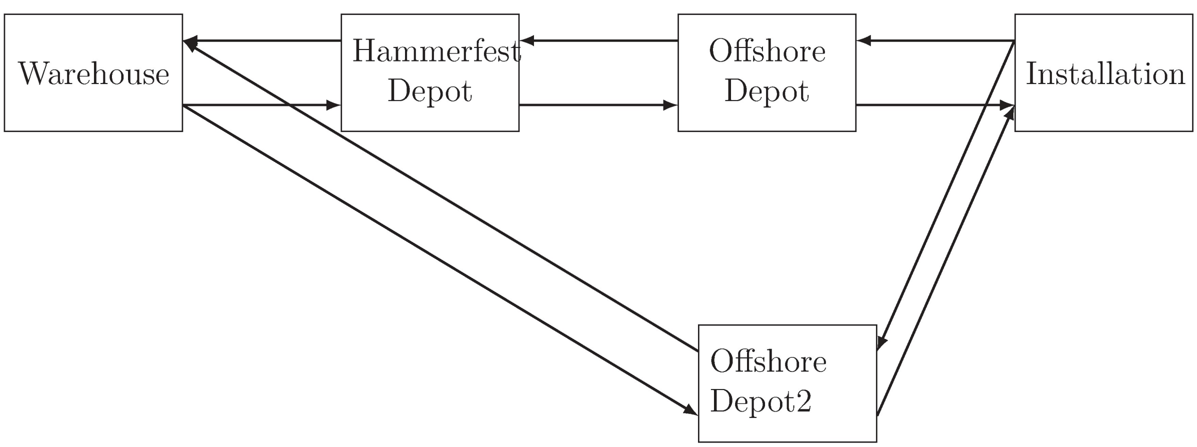

Figure 4.6: Scenario 2 and 3 combined.

Figure 4.7 shows the queuing model of scenario 2. Similar to the model for scenario 1, when an item breaks down at node I, it is sent to the PSV_{wait} where it waits for a PSV. The node PSV_{sail} is once agian an infinite server node representing the transportation time. The node D represents the depot vessel. The service time at this node represents the time the depot vessel stays at sea, as this represents an inventory cycle. The number of servers at this node represents the installed capacity in the system.

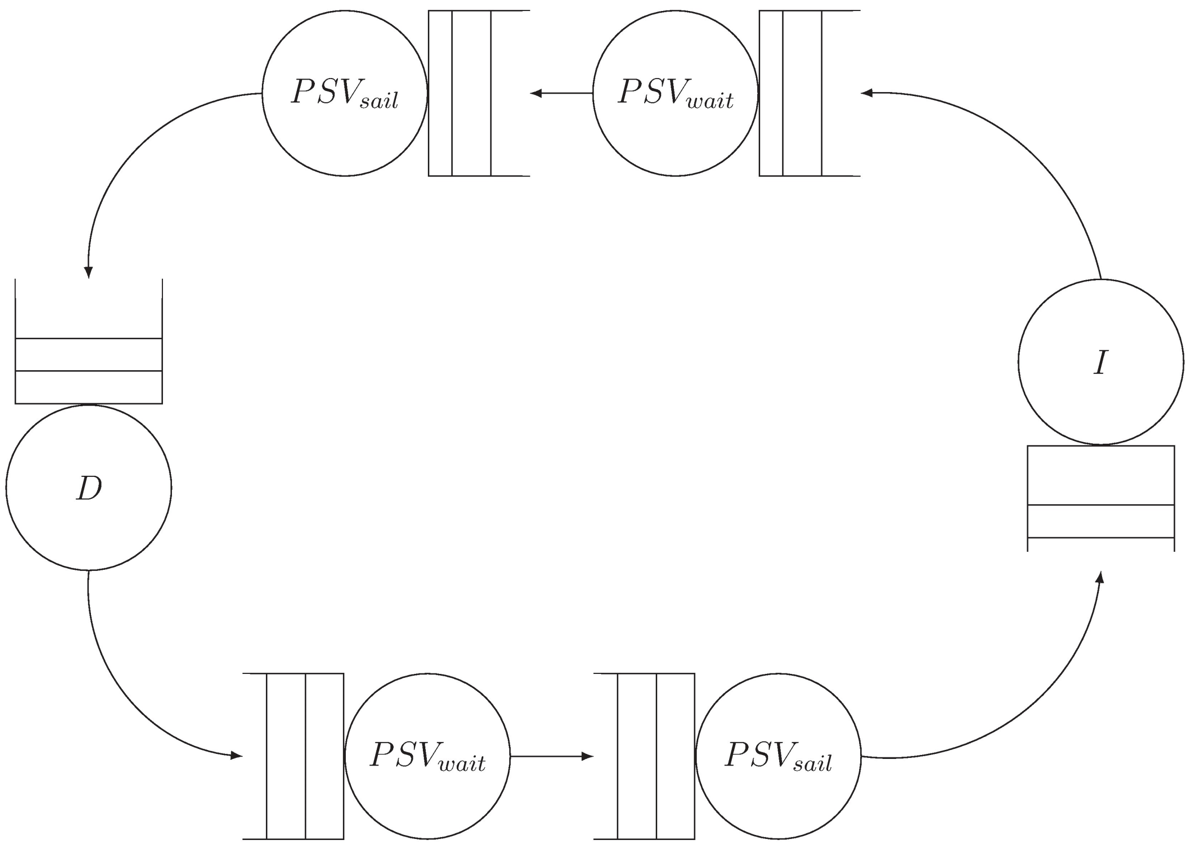

Figure 4.7: Queueing model of scenario 3

Chapter 5

Mathematical formulation

In this chapter the mathematical foundation and formulation is outlined. First, the queueing network mathematics is explained, then the optimization formulation is explained.

5.1 Queueing part

In the optimization routine the queueing part is used as a constraint, but it is presented before the optimization part with its objective function. This is done because it is believed that the optimization part will be easier to understand when the queueing theory is already presented.

The most basic queueing system is a system consisting of one queue and a single server node. Both the external interarrival times and the service time are assumed to be exponential. Over the years the distribution assumption has been relaxed, but queueing theory originally used the exponential distribution. Several servers can be added to this system to increase capacity. The interarrival times, can also be expressed as arriving customers per time unit, denoted λ. Similarly, the service time can also be viewed as how many customers can be served per time unit, denoted μ. The connection between service time and μ is $\mu = \frac{1}{service\ time}$ This is where the connection between the exponential distribution lies. The interarrival times and service times are exponentially distributed, while arrival rates and service rates are Poisson processes (Gross et al., 2011, p. 16).

When several queues and servers are in a chain and the customers leaving one server immediately enters an other queue, many interesting and real-life applications can be modelled. This will however naturally increase the complexity of the system. For instance, the effective arrival rate to each queue depends on how many servers feed into that queue and also these servers' properties. Luckily, these problems are possible to overcome, and with algorithms even larger systems they can be solved easily on computers.

To describe a queue the notation used in this thesis is on the form A/B/X/Y/Z Table 5.1 describes the notations.

Position	Meaning	Symbol	Explanation
A	Interarrival- time	M	Exponential
		D	Deterministic
		E_k	Erlang type k (k=1,2,...)
		H_k	Mixture of k exponentials
		PH	Phase type
		G	General
B	Service- time distribution	M	Exponential
		D	Deterministic
		E_k	Erlang type k (k=1,2,...)
		H_k	Mixture of k exponentials
		PH	Phase type
		G	General
X	Number of parallel servers	1,2,...,∞	
Y	Maximum system capacity	1,2,..„∞	
Z	Queue discipline	FCFS	First Come, First Served
		LCFS	Last come. First Served
		RSS	Random Selection of Service
		PR	Priority
		GD	General Discipline

Table 5.1: Notations in queueing theory (Gross et al., 2011, p. 8)

It is often assumed that there is no limit in the maximum system capacity and that the queueing discipline is First Come, First Served. Then Y=∞ and Z=FCFS and the notation reduces to A/B/X. For instance an M/M/2 queue is a queue with exponential interarrival-time distribution, exponential service- time distribution and 2 servers.

The number of customers present at a node, i.e. a queue and a server, follow a *birth-death pattern*. When a customer arrives, a birth is said to have taken place. Taking advantage of the *Markov-chain properties*, the state changes from *n* customers to *n+1* customers. A Markov process is a memoryless process which depends on the present condition of the system(Gross et al., 2011, p. 24). Similarly, when a customer has finished service, it leaves the node, thus creating a *death*. Over the long term the number of customers leaving and entering the queue will balance and a *steady state* situation is achieved. This property allows one to calculate the *traffic equations*. For an open queueing network the traffic equations takes the form:

$$\lambda_i = \gamma_i + \sum_{j=1}^{k} \lambda_j r_{ij} \tag{5.1}$$

λ_i is the total mean flow into node i, γ_i is the flow of customers from outside the network to node i and λ_j is the flow of customers from the other nodes in the network to node i.

In a closed queueing network there are no external arrivals, i.e., γ_i equals zero. Thus, the traffic equations reduce to

$$\lambda_i = \mu_i \rho_i = \sum_{j=1}^{k} \lambda_j r_{ji} = \sum_{j=1}^{k} \mu_j r_{ji} \rho_j \tag{5.2}$$

ρ_i can be found by solving the eigenvector-like system

$$\mu_i \rho_i = \sum_{j=1}^{k} \mu_j r_{ji} \rho_j \tag{5.3}$$

In the case of the closed network, one of the equation sets obtained from Equation (5.3) is redundant, so one ρ_i can be set equal to one. This is only to initialize the solution method, and the normalizing constant can later be used to find the true utilization rate of the nodes.

In a closed queuing network the state probability at each node is determined from Equation (5.4)

$$p_n = \frac{1}{G(N)} \prod_{i=1}^{k} \frac{\rho_i^{n_i}}{a_i(n_i)} \tag{5.4}$$

The normalizing constant $G(N)$ was found using Buzen's algorithm.

To simplify the notation, $f(n_i)$ is introduced.

$$f_n(n_i) = \frac{\rho_i^{n_i}}{a_i(n_i)} \tag{5.5}$$

$G(N)$ is a constant that sums all p_n to one. Thus,

$$G(N) = \sum_{n_1+n_2+\cdots+n_k=N} \prod_{i=1}^{k} f_i(n_i) \tag{5.6}$$

To calculate the G(N), Buzen created an auxilliary function called $g_m(n)$

$$g_m(n) = \sum_{n_1+n_2+\cdots+n_m=n} \prod_{i=1}^{m} f_i(n_i) \tag{5.7}$$

When $n = N$ and $m = k$ $g_m(n) = G(N)$

$g_m(n)$ can be calculated recursively so that

$$g_m(n) = \sum_{i=0}^{n} f_m(n_i) g_{m-1}(n-i) \tag{5.8}$$

To initalize the algorithm, $g_1(n) = f_1(n)$ and $g_m(0) = 1$

The G(N) can then be found in the bottom right of the matrix created by the algorithm.

Using the equation 5.9 the marginal probabilities at node k can be found. To find the marginal distributions of all nodes one has to do the calculations k times, i.e. corresponding to the number of nodes, so that every node at one point is the $k'th$ node.

$$p_n = \frac{f_k(n)g_{k-1}(N-n)}{G(N)} \tag{5.9}$$

where

$$a_i(n_i) = \begin{cases} n!, & (n_i < c_i) \\ c_i^{n_i - c_i} c_i!, & (n_i \geq c_i) \end{cases} \tag{5.10}$$

$a_i(n_i)$ adjusts for several servers. c_i is the number of servers at each node, while n_i is the number of customers.

Then

$$p_n = \frac{1}{G(N)} \prod_{i=1}^{k} \frac{\rho_i^{n_i}}{a_i(n_i)} \tag{5.11}$$

The inputs in the queueing part are:

- Number of nodes, i.e., number of warehouses and transportation legs
- Number of customers, i.e., number of parts
- Service time
- Number of servers
- Routing

All of these inputs can be varied to model the system in question. The understanding of these values is presented here. The number of nodes decides the layout of the system. This is explained in Chapter 4. The number of customers or entities in the system represents the amount of inventory needed in the system. The service time has different meanings. At the installation node it indicates the arrival rate of demand, caused by failure or other events. At the waiting node the service rate describes time between ship arrivals, thus the capacity of the system. It is intuitive that if several ships are used, the time between their arrivals would decline, thus giving higher capacity. At the sailing nodes the service capacity relies on the transportation time. At the depot node, the service time represents different things for the various scenarios. For scenario 1 the service time represents the time the equipment will use to be shipped to the main depot and back by the trucks and the repair time. For scenario 2 this time is set equal to the duration of the offshore stay for the depot vessel. Thus, it arrives new functioning equipment for example every two weeks. This assumes that there is always equipment ready at the main warehouse. The number of servers also have various meanings.

At the installation node the number of servers represent the number of equipment desired to be operational. At the PSV-waiting nodes the number of servers describes the amount of cargo picked up by each vessel. Since the replenishment policy is one-for-one, this value is set equal to one. The number of servers at the sailing nodes is a high number, effectively making the number of servers equal to infinity. The number of servers at the depot node is a representation of installed capacity. The routing in this network is very straightforward, as each node feeds directly only into the next node. For each combinations of the above inputs the algorithm outputs the steady state probabilities.

5.2 Optimization

The objective of the optimization routine presented here is to minimize the costs for the supply chain designs. By comparing these it is then possible to establish if and when a depot vessel could be financially viable.

The optimization routine will minimize the number of customers in the system, N. N represents the amount of inventory necessary in the system to maintain service. Further, it minimizes the service rate at the $PSV_{waitnode}$ node. Thus, it minimizes the number of PSVs. Last, the objective function minimizes the capacity in the supply chain between the main warehouse and the depot in question.

The objective function has three decision variables, x_1, x_2 and x_3 .

x_1 is the number of customers in the system, x_2 is the service capacity in the $PSV_{waitnode}$ nodes, and x_3 is the capacity in the supply chain.

Parameters:

$C_{inventory}$: Cost of inventory

C_{PSV}: Cost of PSVs

C_D: Cost of Depot Ship

$C_{Transportation}$: Cost of Transportation from warehouse to depot

Decision variables

x_1: Number of entities in the system

x_2: Service rate at nodes 2 and 5

x_3: number of servers at node 4, i.e. the needed transportation capacity

$$min z = C_{inventory} * x_1 + C_{PSV} * x_2 + C_{Transport} * x_3 \tag{5.12}$$

subject to

$$\sum_{c}^{N} p_{n_{installation}} \geq SL \tag{5.13}$$

$$x_1 \geq 1, \quad integer \tag{5.14}$$

$$x_2 > 0 \tag{5.15}$$

$$x_3 \geq 1, \quad integer \tag{5.16}$$

The cost of the depot is added as a fixed cost in scenario 2 and 3. To adjust for the different sailing lengths, a constant is multiplied to the cost of PSVs. This is explained thoroughly in Chapter 6.

Chapter 6

Routine

The optimization routine is written in MATLAB and consists of two main parts which are connected. The scripts are enclosed in the appendices. The main script is the *opti_main.m* which in turn calls on the other functions described in this chapter.

6.1 Queueing part

The queueing function is called *QT.m*. It is a MATLAB-function whose outputs are the steady state probabilities, *pn*, and the effective utilization of the servers, *rho_eff*. The inputs to *QT.m* come from the MATLAB-function *constraintfun.m*. These inputs are the number of nodes, service time at each node, number of customers, routing matrix and number of servers.

The first step in the *QT.m*-function is to calculate *N1*, which is simply the number of customers plus one to account for the fact that there is a possibility that there is zero customers in a node.

As Buzen's algorithm calculates the sate probability for only the *k'th* node the routine must account for this. This is done by creating a third dimension to each of the matrices containing the inputs. This operation is done by the *circshift* command. The next step is to calculate the traffic equations. This is done according to Equation (5.2), and then divided by μ_i. Then the eigenvectorlike traffic equation is solved, and this is done for all the different matrices created by *circshift* command. The element in the eigenvector with the highest value is set to one and the other ρ are scaled accordingly. To account for the fact that some nodes have several servers a_i is calculated.

Then, the special features of Buzen's algorithm are initiated. First, all the $F(i, j, y)$ are calculated, corresponding to Equation (5.5). Then, the $g(n, m, y)$ matrix is created. Also this has a third dimension to allow all nodes to be the *k'th* node at one point. The algorithm is initialized by setting the first row and first column equal to one, maintaining the other values equal to zero. In the next loop operations, firstly the first row in the $g(n, m, y)$ matrices is substituted to containing the corresponding $F(i, j, y)$ values. Then the the rest of the matrix is filled in using the recursive properties of the algorithm. The *GN* value is found in the bottom right of the matrix.

The $g(n, m, y)$, $F(i, j, y)$ and *GN* values are then used to calculate the steady state probabilities. These values are stored temporarily in the matrices*pn_temp(i,j,y)* As only the column to the extreme right in each of the *pn_temp(i,j,y)* matrices, these columns are extracted and put in the matrix *pn*. With all these values other measures can be calculated, such as the length of the queues, *L*, and also the effective utilization of the nodes, *rho_eff*.

The function is checked for correctness by comparing it to the *Qts-Plus* software distributed with the book of Gross et al. (2011) in Appendix E. The related Excel-spreadsheet also worked as a basis for the MATLAB-function.

6.2 Optimization part

The main script, *opti_main.m* is the optimization script as it calls the Genetic Algorithm. The objective function is called from the function *objfun.m* and the constraint function is called from *constraintfun.m*. The main script governs the number of variables, lower and upper bounds and integer constraints. It is also possible to set options for the optimization, such as the size of the elite population. The settings are similar to the settings Veflingstad (2014) used, where they gave good results.

The objective function is found in the *objfun.m* and is described in Section 5.2. The script inputs the vector with the decision variables and outputs the objective value. *objfun.m* contains two sets of objective functions, one for scenario 1 and one for scenario 2 and 3. Scenario 2 and 3 is treated together since the model does not distinguish between them. Within the sets there are ten diffenet functions, one for each transportation cost. Further, a constant is multiplied to the cost of the PSVs to account for the varying sailing times between the scenarios.

The constraint function in *constraintfun.m* is also described in Section 5.2. This too takes the decision variables vector as input. The decision variables and other relevant values are then used as inputs for the queueing function. The outputs of the queueing function, steady-state probabilities and effective utilization is then used as constraints in the inequality constraint matrix, *c*.

Chapter 7

Input data

This chapter describes the input data used in the model. To produce relevant results relevant data must be used as input to the optimization routine.

7.1 Transportation

The transportation in this model consists of several transportation modes, both ships and trucks. Air transport is also a possibility due to the long distances, but this is not taken into consideration here.

Since 75 % of all Statoil's base-to-base transports are performed by truck it is most likely that this transport mode will be chosen between the main warehouse and Hammerfest(Haram). Trucks are also a very flexible transportation mode.

Transportation time

The transportation times will vary between the alternatives. To show off the differences between the scenarios, relatively long sailing times is preferable. From the correspondence with Statoil, typical sailing times in for the depot vessel in the Kara Sea will be three and a half days. Although the Kara Sea is east of the Barents Sea, this is used as sailing time. Thus, a roundtrip will take 7 days. Hence, this is the sailing time chosen, since this sailing time most definitely is for remote fields.

Truck transportation, which will be Statoil's preferred transportation mode also for the future takes 50 hours from the main warehouses and to Hammerfest. This will vary somewhat between the warehouses, but 50 hours is still a good estimate.

Transportation costs

The transportation costs will also vary between the modes. If the offshore depot is to be financially viable, the presence of the depot must lower transportation costs. This will either be through lower need for PSVs or less costs of the first echelons of the supply chain. These input values are changed in the range 100 000 NOK per day to 1 000 000 NOK per day in order to produce results.

PSV costs

The PSVs are as mentioned in Chapter 2 chartered on a time basis. These costs vary greatly, and as of May 2015 the rates on some contracts are as low as 20 000 NOK a day, far less than the operating costs of the vessels(Segrov, 2015). As the oilfields are postponed due to the low oil price this day rate cannot be assumed in the calculations. With an NOK-USD exchange rate of 7,54 20th of May 2015 this equals aprroximately 2650 USD (DN.no, 2015). After Professor Bjørn Egil Asbjørnslett's contact with Statoil, he recommended using a day rate of 200 000 NOK per day. This high price assumption is justified by the fact that the Arctic will not be opened to production when the oil price is low.

7.2 Cost of offshore depot

The offshore depot can be either a self propelled vessel sailing to and from a port, or a moored installation lying at a fixed position. The moored installation can also be a self propelled vessel, but the difference will be the operational profile. In this thesis it is assumed that the depot will consist of a bulk carrier. This is due to the fact that retrofitting them with a crane and with some customization they are suitable for this task (Nordbø, 2013). Using a converted bulk carrier instead of a semi-submersible platform gives the depot higher speed and better, and therefore the possibility of being used both as a fixed and a mobile depot. The bulk carrier chartering market is of course varying, but at the moment it is quite low. For the months March to June 2015 the Baltic Dry Index, indicating the chartering cost of bulk carriers,fluctuated arond 600 (Bloomberg.com, 2015), down from 11 800 before the financial crisis in May 2008 (Evans, 2015). How this will evolve in the future is of course difficult to tell, so some outer estimates should be used. The cost related to the depot will in addition to chartering costs also include fuel cost. What type of charter the oil company should choose, depends on time perspective and availability of ships. It is fair to assume that the oil company, regardless of chartering method, would have to pay a premium for the retrofitting. Nordbø (2013) calculated approximate day rates of 53 000 USD for a converted bulk carrier. Thus, for a two ship operation, the cost is then 106 000 USD, or approxiamtely 800 000 NOK per day. This is a resonable amount and Nordbø (2013) have taken high safety margins, givng confidence that this can be seen as a good approximation also in times with high bulkcarrier dayrates. Nordbø (2013) used a 5 year old ship as starting point, which is a good assumption.

7.3 Operational constraint

Downtime is, although very expensive, not possible to prevent. When the oil price is high it can be financially viable to spend more to prevent downtime, while during periods with low oil prices a stop in production may not be that critical. This also depends on the type of operation. A drilling rig is often time chartered and downtime will in any case be unfavourable. Thus, a service level must be set. The number of servers at the installation node is one, thus there must be one or more entities at the installation node the specified percentage of time. The service level in is set at 99%.

7.4 Inventory cost and demand estimation

Inventory binds capital that could otherwise be used elsewhere in the organization or invested on the stock market. The price used here is only indicative and is set relatively high in order to make sure the model does not prefer increasing the number of parts beyond reasonable amounts.

The demand estimation is also just indicative, and the model is flexible enough to handle different demand quantities. Three demand cases were chosen, one where demand rises twice a year, one where there is demand every month and one where it is demand twice a month.

Chapter 8

Results and Discussion

In this chapter the results from the optimization routine are presented. The results can be used to understand when an offshore depot is viable, thus, when it is preferable to store the equipment close to the installations.

The calculations were done for three demand categories: Low, medium and high. The results for Scenario 2 and 3 are identical because it is assumed that a new depot vessel arrives with inventory every two weeks, regardless from where.

The optimization was performed on an Acer Aspire Switch 10.1" Full HD with an Intel Atom Z3735F / 1.33 GHz processor with 2 GB RAM. Solution times varied from 85 seconds to 215 seconds, with an average of 146 seconds.

Low Demand For the case with low demand, the costs of scenario 1 varies between 457 094 NOK and 1 357 094 NOK, see Table 8.1. For scenario 2 the cost varies between 1 112 022 NOK and 2 012 022 NOK. For all instances in both scenarios the lowest possible capacity is installed in the depot node. This is logical, as low demand needs little capacity. When the transportation cost in scenario 1 is 800 000 NOK and the transportation cost in scenario 2 is 100 000 NOK, scenario 2 is financially favourable. By using simple interpolation, the transition happens approximately midway between 700 000 and 800 000 NOK of transportation costs of scenario 1. However, since the data are so approximate, it is wiser to round up. This difference, approximately 700 000 NOK is lower than the cost of the depot, indicating that the operating costs of scenario 2 and 3 minus the cost of the depot vessel is lower than for scenario 1.

Table 8.1: Results from the low demand case

Scenario 1	x1	x2	x3	T cost	Obj.value
0,0055	3	0,0408	1	100000	457094
0,0055	3	0,0408	1	200000	557094
0,0055	3	0,0408	1	300000	657094
0,0055	3	0,0408	1	400000	757094
0,0055	3	0,0408	1	500000	857094
0,0055	3	0,0408	1	600000	957094
0,0055	3	0,0408	1	700000	1057094
0,0055	3	0,0408	1	800000	1157094
0,0055	3	0,0408	1	900000	1257094
0,0055	3	0,0408	1	1000000	1357094
Scenario 2	x1	x2	x3	T cost	Obj.value
0,0055	2	0,2404	1	100000	1112022
0,0055	2	0,2404	1	200000	1212022
0,0055	2	0,2404	1	300000	1312022
0,0055	2	0,2404	1	400000	1412022
0,0055	2	0,2404	1	500000	1512022
0,0055	2	0,2404	1	600000	1612022
0,0055	2	0,2404	1	700000	1712022
0,0055	2	0,2404	1	800000	1812022
0,0055	2	0,2404	1	900000	1912022
0,0055	2	0,2404	1	1000000	2012022

Medium Demand The results are presented in see Table 8.2. Similar to the case for low demand, only when the transportation cost for scenario 1 is sufficiently high the offshore depot is viable. Also in this case the transition happens approximately midway between 700 000 NOK and 800 000 NOK. When the transportation cost is low in this scenario, the model adds capacity in the depot node rather than inventory.

Table 8.2: Results from the medium demand case

Scenario 1	x1	x2	x3	T cost	Obj.value
0,0329	5	0,1285	1	100000	779958
0,0329	5	0,1286	1	200000	879982
0,0329	5	0,1285	1	300000	979957
0,0329	5	0,1285	1	400000	1079957
0,0329	5	0,1285	1	500000	1179958
0,0329	5	0,1285	1	600000	1279969
0,0329	5	0,1285	1	700000	1379958
0,0329	5	0,1285	1	800000	1479957
0,0329	5	0,1285	1	900000	1579957
0,0329	5	0,1285	1	1000000	1679962
Scenario 2	**x1**	**x2**	**x3**	**T cost**	**Obj.value**
0,0329	4	0,2921	2	100000	1414607
0,0329	6	0,1726	1	200000	1608631
0,0329	6	0,1726	1	300000	1708631
0,0329	6	0,1726	1	400000	1808631
0,0329	6	0,1726	1	500000	1908631
0,0329	6	0,1726	1	600000	2008631
0,0329	6	0,1726	1	700000	2108631
0,0329	6	0,1726	1	800000	2208631
0,0329	6	0,1726	1	900000	2308631
0,0329	6	0,1726	1	1000000	2408631

High Demand The results are presented in see Table 8.2 If the transportation cost for scenario 3 is set to the lowest value the transition for when the offshore depot is viable is lower than for the other cases. The transition in the high demand case happens when the transportation cost of scenario 1 is between 600 000 NOK and 700 000 NOK. This indicates that parts which are in higher demand should be placed on a depot vessel, i.e. allocated closer to the installation. When the transportation cost is low in this scenario, the model adds capacity in the depot node rather than inventory.

Table 8.3: Results from the high demand case

Scenario 1	x1	x2	x3	T cost	Obj.value
0,0658	7	0,19159	1	100000	1068232
0,0658	7	0,1916	1	200000	1168234
0,0658	7	0,1916	1	300000	1268233
0,0658	7	0,1916	1	400000	1368240
0,0658	7	0,1916	1	500000	1468244
0,0658	7	0,1916	1	600000	1568233
0,0658	7	0,1916	1	700000	1668288
0,0658	7	0,1916	1	800000	1768233
0,0658	7	0,1916	1	900000	1868240
0,0658	7	0,1916	1	1000000	1968263
Scenario 2	x1	x2	x3	T cost	Obj.value
0,0658	5	0,5832	3	100000	1629161
0,0658	6	0,6172	2	200000	1830859
0,0658	6	0,6175	2	300000	2030876
0,0658	6	0,6172	2	400000	2230859
0,0658	6	0,6171	2	500000	2430855
0,0658	6	0,6192	2	600000	2630961
0,0658	6	0,6171	2	700000	2830855
0,0658	6	0,6171	2	800000	3030856
0,0658	6	0,6175	2	900000	3230873
0,0658	6	0,6182	2	1000000	3430909

The results give a slight indication that the offshore depot is more suited for the equipment with a higher demand. This is reasonable as higher demand naturally costs more to transport, thus seeking the alternative with low transportation cost sooner. Although this is not encountered for directly in the model, bringing equipment that might not be needed use space that could otherwise be used for in-demand cargo. This will reduce the offshore time for a depot vessel, thus increasing the sailing distance due to an increased number of roundtrips. Also, shorter time at sea means that the ship has less time to sail to posrt, reload and sail out to position again since the other ship in the operation has similary reduced offshore time.

When demand is low, the demand is probably best satisfied using some sort of direct shuttle. The opposite will then be true for high demand cargo. This is illustrated in Figure 8.1. The red line illustrates the optimal transportation mode as a function of demand frequency.

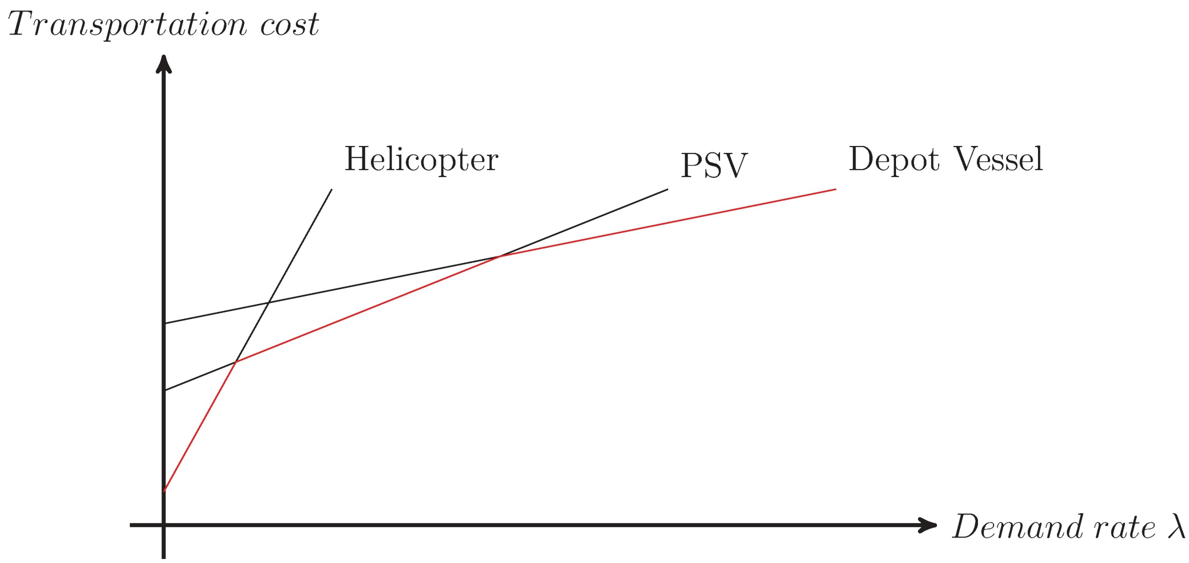

Figure 8.1: Optimal transportation mode as a function of demand

The graph illustrates how utilization of the transport modes is important. If the demand for a type of equipment is low, the utilization of the transportation node is low and thus installed capacity should also be lower. Thus, storing the equipment upstream and utilizing high speed delivery methods is favourable. As the demand for the equipment increases, the installed capacity will increase. Helicopter transport is expensive for large quantities, therefore PSVs in direct shuttle will at a certain demand be more favourable. This transportation method will have lower transportation speed, forcing the equipment to be stored closer to the installations. When the demand is high a depot vessel may be viable, given that the transportation costs upstream is lower. Thus, using queuing theory one can calculate the utilization of the transportation servers. Using the utilization as a measure, it is possible to decide which transportation mode to use. The slopes of the graph are only meant as an illustration, but they show how the high cost of investment for the depot vessel can be prohibitively expensive for small cargo volumes.

The different supply chains will react to unforeseen events differently as they have different flexibility. Scenario 1 and 2 have a relatively steady supply from land, and as long as inventory is available somewhere in the chain it can be transported to the installation.

Creating models for long supply chain proved difficult. One of the main reasons is that if one sees the chain as one, from the warehouse to the installation, the model will hesitate to use the forward storages. This is due to the fact that the total transportation time is the same, regardless of where the parts are stored. Hence, investigating the supply chain with the entire supply chain in one model proved difficult. Extension with several convolutions, similar to Diaz and Fu (1997) is could probably solve this problem. Their model does however not perform too well with small part quantities.

Different methods were used to try to allocate the inventory. One such is the Figure 8.2. This is a model that could be applied if one have repair capacity at for instance the Hammerfest Depot. In this figure the arcs between the nodes represent the routing of the system. The installation node is node seven, denoted μ_7. μ_7 equals the arrival frequency of the demand, either due to breakdown of equipment or due to planned operations. This parameter has an exponential distribution with a known mean. When the equipment has been used, it is shipped to the main warehouse with a shipping delay, μ_8. In this model the parts can follow two paths. One of the decision variables is the percentage of equipment that is sent in each path. It is possible to allow one of the paths to be faster, but more expensive, and consequently, one path which is slower but more economical. Using the service constraint, this model can then determine how much of the equipment must be sent the through the fast path and how much through the slow path. The rest of the nodes can then be chosen as the repair nodes, warehouses and transportation nodes.

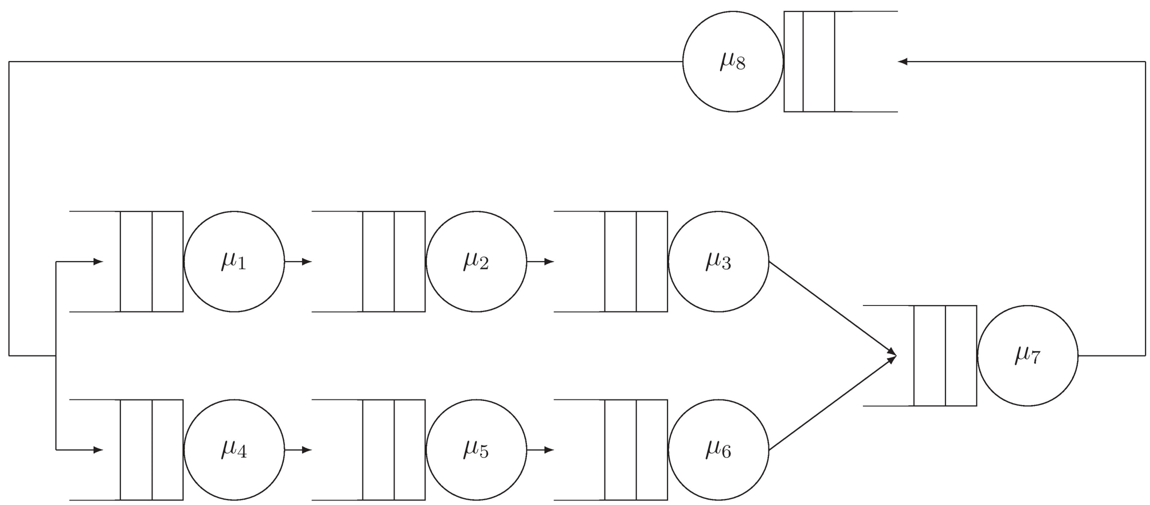

Figure 8.2: Possible network layout

Whether or not an offshore storage is viable depends on several things. Firstly, the transportation cost must be lowered as the depot vessel will incur costs. The depot vessel will, if used as a mobile depot sailing from shore to depot position, reduce the need for PSVs. The costs of bulk carriers are not necessarily corresponding to the oil price, it follows the Baltic Dry Index. However, one should be prepared for worst case scenarios if one is to convert a bulk carrier. Thus, the oil company should do a continuous tradeoff between the value of having the bulk carrier versus not having it depending on costs. As the bulk market at the moment is poor, establishing long term contracts at favourable prices could be an option. Although this is out of scope of this thesis, the oil company should continuously see the value of using the converted ship as a storage or use it as a bulk carrier. This again depends of course on the degree of customization and the value the oil company sees in using the forward storage.

In any supply chain there is a trade off between service and costs. For most supply chains there is a service level requirement which must be fulfilled. Incremental service above this level will of course cost. A supply chain must not necessarily be all about saving costs, it can very well be about maximizing profit. In an environment with high cost equipment, this means operability. In this thesis service requirements are used as benchmarks since using downtime costs will skew the optimization to apply overly amounts of capacity in the chain. However, the two are not independent thus one needs to takes downtime costs into consideration. Increasing the operability will cost, but the cost may be outweighed by the income. One must therefore establish an understanding of what the extra service is worth. This may lead the way for multi-objective optimization, for example with weights, where the weights are determined based on the oil price.

A major issue related to the forward storage is if and when it becomes viable financially. As the distance from shore increases, so does the transportation cost. By using a larger vessel, economics of scale become present as it is often more convenient to ship large batches. However, from the inventory theory, this means that the holding costs will increase due to the larger quantities stored on board. Nordbø (2013) argues that the vessel should be offshore in the depot position for two weeks. Hence, there would be some extra inventory holding costs, but the relative size of this is arguable. The average inventory on board the ship would then equal inventory sufficient for one week of supply. Akselsen (2014) assumed the PSVs to have a sailing speed of 15 knots, while the depot ships had a sailing speed of 12 knots. It is therefore clear that the inventory costs related to the depot vessel would be higher. This too must then be balanced by lower transportation costs.

A natural extension from the queueing model in this thesis is to implement it into a simulation environment, such as SIMEVENTS. SIMEVENTS is an event triggered simulation environment. The failure of a part or the sudden demand for equipment is indeed an event. There is a logical connection between queueing theory and SIMEVENTS as a supply chain design in SIMEVENTS will contain queues and servers. Thus, queueing theory works as one of the foundations for SIMEVENTS. The advantages of such simulation environments is that one can model very complex systems in a manner that gives a good overview. This also gives the possibility to extend the model to implement data from MetOceanData and other sources. This can however also be added in the queuing model presented here, but the main difference is still the transparency of a simulation environment. Which of the environments is most flexible depends on the user, but the simulation environment with its graphic user interface gives a better visual overview and it is possibly easier to follow the logic and movements of the entities in the model. It is also easier to add functions such as batching and blocking of servers. Blocking of servers is possible in queueing theory, but this is a complicating factor for the routine. This kind of works has been done before, see Ulstein (2014), but can be expanded to treat the entire supply chain.

In the offshore supply chain several types of cargo are present. This could be introduced into the queueing model as multiple customer classes. This is a feature in queueing theory, as it allows different customer classes to have different routing probabilities and service times. Also, customers may have different priorities. If queueing theory is used to determine necessary installed capacity for the consumables, the chain should be able to facilitate the rare orders as there should be some slack. Thus, the focus in this thesis has been solely on the repairable parts.

The drawback of a bulk carrier in comparison to a semi submersible installation is motions in heave. This could restrict the operability of the vessel, but when one takes into consideration the reduced need for transfers between vessels the bulk carrier seems as an attractive option.

In the results, scenario 2 and 3 were treated as one as it was assumed that a new depot vessel arrives every two weeks. The choice of two weeks was done to fit the ship proposed by Nordbø (2013), and thus the ship chartering cost. It is not certain that a two week schedule will suffice if the ship is to sail all the way between Stavanger and the Arctic. However, if this would not suffice with two ships, one either must increase ship sizes or number of ships, thus increasing costs. As the cost of the depot vessel already is high, adding an other depot ship would make that solution prohibetively expensive.

In scenario 2 and 3 it is assumed that the large vessel is loaded with some amount of inventory when it leaves port. This amount can either be chosen to last for a given set of weeks, or it can be optimized. Figure 2.1 shows haw the inventory will change over time in the offshore depot. In the figure it is assumed that the vessel takes eight batches of cargo, which for a scenario where the demand is two batches per week would last four weeks.

This demand must be found using some prevision method. If one allows this demand prediction to embrace also the stochastic peaks in demand, the model can be solved using deterministic methods. This method will however always carry too much inventory. Using methods like multiperiod newsvendor can be used to estimate the optimal inventory in scenario 2 and 3. Scenario 3 is slightly less complex than the others given that it has fewer echelons than the other. It is however the scenario with the least flexibility as the depot vessels inventory is filled up with long time intervals, while the other scenarios have a more periodic refillment.

In this thesis a closed queueing network was chosen. The rationale of this choice was that the network handles high value equipment which can be assumed to have a finite population. This network assumed that there are always inventory in the supply chain. This is reasonable for the high cost equipment. For the low cost consumables an open design could have been chosen instead. The closed system has one disadvantage that it is somewhat more computationally challenging, This was solved using Buzen's algorithm, one of several algorithms used to overcome this challenge.

Queueing theory and networks have some strengths and some weaknesses. Among the weaknesses is the fact that the arrival and service time distribution normally is exponentially distributed. For other distributions, algorithms and approximations can be applied. The exponential distribution does however mimic the distribution of breakdowns of equipment quite well. Another feature in a queueing system is that the load on the server, ρ, must be less than one. This is intuitive since a queue would continue to build up in front of the node if there were more arriving customers than the server could handle. In that case, steady-state could never be achieved.

Chapter 9

Conclusions and further work

From the calculation no clear conclusions can be made. However, the model seems to favourize the offshore depot at an earlier stage when the demand is high in contrast to lower demand, thus allocating the resources closer to the installation.

The main contribution of this thesis has been to prove how queueing theory can be applied to this supply chain problem. The model has some shortcomings related to allocation of inventory, but the thesis shows the versatility of queueing theory and its many possible extensions.

Among these extensions is the createtion of several convolutions, similar to Diaz and Fu (1997). This should allow for a more careful distribution of inventory. An other possible extension of the model is allowing for blocking of the servers. This means that when an entity is present at a node, no new entities are allowed to enter that node.

For future research simulation environments should be considered, such as SIMEVENTS. SIMEVENTS does to a large degree consist of queues and server, thus validating the usefulness of queueing theory.

An other interesting and viable path is multi-objective optimization. By using multi-objective optimization it is possible to further investigate the tradeoffs that lies between costs and service. A possible path within multi-objective optimization could be using the weighting method. The weights set by the decision makers could then be changed according to the market situation. Also, Bill-Of-Material (BOM) models can also be an approach to the problem.

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Appendix A

obti_main.m

```
clear all
clc

tic


nvars = 3;

ObjectiveFunction = @objfun;

IntCon=[1,3];


LB = [1 0.001 1];    % Lower bound
UB = [10 1 100];  % Upper bound




ConstraintFunction = @constraintfun;


options = gaoptimset('PlotFcns',
{@gaplotbestf,@gaplotmaxconstr,@gaplotrange},'FitnessScalingFcn',{
@fitscalingrank },'TolCon', 1e-20, ...
'PopInitRange',[LB ; UB],...% Decides the rangeof variables for the initial
population
'StallGenL',100,...
'Generations',100, ... %Sets the number of generations
'PopulationSize',25,...% Sets the populationsize
'EliteCount',2, ... %Size of elite population
'CrossoverFraction',0.4,... % Percentage of the population made of
crossover children
'UseParallel','always',... %Enables parallel computing
'MigrationInterval',10,...
'MigrationFraction',0.1,...
'MigrationDirection','both', ...
'PopulationType','doubleVector');


[x, fval, exitflag, output, population, scores]...
    = ga(ObjectiveFunction,nvars,[],[],[],[],LB,UB, ...
    ConstraintFunction,IntCon,options)
toc
```

Appendix B

objfun.m

```
function obj=objfun(x)

 sail1=7; %Sailing time is 7 days. Uncommentfor scenario 1
%sail2=0.25; %Sailing time is 6 hrs=0.25 days. Uncommentfor scenario 2

 obj=100000*x(1)+(200000*sail1)*x(2)+100000*x(3); %Scenario 1
% obj=100000*x(1)+(200000*sail1)*x(2)+200000*x(3); %Scenario 1
% obj=100000*x(1)+(200000*sail1)*x(2)+300000*x(3); %Scenario 1
% obj=100000*x(1)+(200000*sail1)*x(2)+400000*x(3); %Scenario 1
% obj=100000*x(1)+(200000*sail1)*x(2)+500000*x(3); %Scenario 1
% obj=100000*x(1)+(200000*sail1)*x(2)+600000*x(3); %Scenario 1
% obj=100000*x(1)+(200000*sail1)*x(2)+700000*x(3); %Scenario 1
% obj=100000*x(1)+(200000*sail1)*x(2)+800000*x(3); %Scenario 1
% obj=100000*x(1)+(200000*sail1)*x(2)+900000*x(3); %Scenario 1
% obj=100000*x(1)+(200000*sail1)*x(2)+1000000*x(3);  %Scenario 1

%  obj=100000*x(1)+(200000*sail2)*x(2)+100000*x(3)+800000; %Scenario 2
%  obj=100000*x(1)+(200000*sail2)*x(2)+200000*x(3)+800000; %Scenario 2
%  obj=100000*x(1)+(200000*sail2)*x(2)+300000*x(3)+800000; %Scenario 2
%  obj=100000*x(1)+(200000*sail2)*x(2)+400000*x(3)+800000; %Scenario 2
%  obj=100000*x(1)+(200000*sail2)*x(2)+500000*x(3)+800000; %Scenario 2
%  obj=100000*x(1)+(200000*sail2)*x(2)+600000*x(3)+800000; %Scenario 2
%  obj=100000*x(1)+(200000*sail2)*x(2)+700000*x(3)+800000; %Scenario 2
%  obj=100000*x(1)+(200000*sail2)*x(2)+800000*x(3)+800000; %Scenario 2
% obj=100000*x(1)+(200000*sail2)*x(2)+900000*x(3)+800000; %Scenario 2
%  obj=100000*x(1)+(200000*sail2)*x(2)+1000000*x(3)+800000; %Scenario 2
```

Appendix C

constraintfun.m

```
function [c, ceq] = constraintfun(x)
fHandle=@QT;


    m=6; %Nr. of nodes

mu_j=[0.0055;x(2);0.2857;0.1935;x(2);0.2857]; %Scenario 1,1
% mu_j=[0.0329;x(2);0.2857;0.1935;x(2);0.2857]; %Scenario 1,2
% mu_j=[0.0658;x(2);0.2857;0.1935;x(2);0.2857]; %Scenario 1,3




% mu_j=[0.0055;x(2);4;0.0714;x(2);4]; %Scenario 2,1
% mu_j=[0.0329;x(2);4;0.0714;x(2);4]; %Scenario 2,2
% mu_j=[0.0658;x(2);4;0.0714;x(2);4]; %Scenario 2,3

% mu_j=[0.0055;x(2);4;0.0714;x(2);4]; %Scenario 3,1
% mu_j=[0.0329;x(2);4;0.0714;x(2);4]; %Scenario 3,2
% mu_j=[0.0658;x(2);4;0.0714;x(2);4]; %Scenario 3,3




 N=x(1); % Nr. of customers
SL=0.99; % Service level


R=[0,1,0,0,0,0;...
   0,0,1,0,0,0;...
   0,0,0,1,0,0;...
   0,0,0,0,1,0;...
   0,0,0,0,0,1;...
   1,0,0,0,0,0];       %Routing matrix


c_serv=   [1;        %Nr. of operating equipment
           1;        %PSV Wait node
           1000;     %PSV sail node
           x(3);     %Depot node
           1;        %PSV Wait node
           1000];    %PSV sail node

[pn_num, rho_eff]=fHandle(m,mu_j,N,R,c_serv);


     c=[
       pn_num(1,1)-(1-SL)
     rho_eff(:)-(1-10e-16);
         ];

     ceq = [];
```

Appendix D

QT.m

```
function [pn, rho_eff]=QT(m,mu_j,N,R,c_serv)



N1=N+1; % Nr of customers+1 to account for 0 customers


sizeR=[m m];



RR=zeros(m,m,m);

for y=1:m
        RR(:,:,y)=circshift(R,[y-1 y-1]);
end




for i=1:m
mu_j(i)=mu_j(i)';
end
mu_r=zeros(m,1,m);
for y=1:m
        mu_r(:,:,y)=circshift(mu_j,y-1);
end




mu=zeros(m,m,m);
for y=1:m
    for i =1:m
        mu(i,i,y)=mu_r(i,:,y);
    end
end




for i=1:m
    c(i)=c_serv(i);
end

c=c';


c_2=zeros(N1,m,m);
c_s=zeros(m,1,m);

for y=1:m
        c_s(:,:,y)=circshift(c,y-1);
end
```

```
for y=1:m
    for i=1:m
        c_2(:,i,y)=c_s(i,:,y);
    end
end

c=c_2;



%Traffic equations

rho=zeros(m,m,m);


for y=1:m
    rho(:,:,y)=mu(:,:,y)*RR(:,:,y);
end

for y=1:m
    rho(:,:,y)=rho(:,:,y)';
end




for y=1:m
for i=1:m
    rho(i,:,y)=rho(i,:,y)/mu_r(i,1,y);
end
end

for y=1:m
[V(:,:,y),D(:,:,y)]=eig(rho(:,:,y));

rho_eig(:,:,y)=abs(V(:,1,y));
end


% Make one value 1

for y=1:m
[max_value(:,:,y), index(:,:,y)] = max(rho_eig(:,:,y));
end

rho_1=zeros(length(rho_eig),1,m);

corr=zeros(m,1);

for y=1:m
    corr(y)=1/rho_eig(index(:,:,y),y);
end
```

```
for y=1:m
    for i=1:length(rho_eig)
        rho_1(i,y)=rho_eig(i,y)*corr(y);
    end
end


rho_i=rho_1;



for y=1:m
n(:,:,y)=[0:N];
end
a_i=zeros(N1,m,m);

for y=1:m
    for i=1:N1
        for j=1:m
        if n(1,i,y)<c(i,j,y)
            a_i(i,j,y)=factorial(n(1,i,y));

        elseif n(1,i,y)>=c(i,j,y)
            a_i(i,j,y)=c(i,j,y)^(n(:,i,y)-c(i,j,y))*factorial(c(i,j,y));
        end
        end
    end
end


%%%%%%%%%%%%%%%%%%
%%% Buzen's algorithm


F=zeros(N1,m,m);
for y=1:m
    for i=1:N1

        F(i,:,y)=rho_i(:,:,y);
    end
end

for y=1:m
    for i=1:N1
        for j=1:m
            F(i,j,y)=F(i,j,y)^(i-1);
        end
    end
end


F=F./a_i;

F=permute(F,[2 1 3]);
```

```
g=zeros(m,N1,m);
pn_temp=zeros(N1,m,m);
pn=zeros(N1,m);



for y=1:m



for i=1:N1
        for j=1:m
            if i==1 || j==1
                g(j,i,y)=1;
            else
                g(j,i,y)=0;
            end
        end
end


      for j=1:m
         for k=2:N1
            for l=1:m
                if l==1
                    g(l,k,y)=F(l,k,y);
                else
                    g(l,k,y)=0;
                    for k1=1:k
                       g(l,k,y)=g(l,k,y)+F(l,k1,y)*g(l-1,k-(k1-1),y);
                    end
                end
            end
         end

         GN = g(m,N1,y);
        G1 = g(m,N,y);

         for i=1:N1

             pn_temp(i,j,y)  = F(end,i,y)*g(m-1,N1+1-i,y)/GN;

         end
      end

end



for i=1:m
    pn(:,i,N)=pn_temp(:,end,i);
end
```

```
pn=pn(:,:,end);
        csum=sum(pn);
pn=fliplr(pn);

L=zeros(1,m);
    for i=1:m
        for j=2:N1
            L_temp(j,i)=(j-1)*pn(j,i);
            L(i)=L(i)+L_temp(j,i);
        end
    end
 L=fliplr(L);

 rho_eff=zeros(1,m);
 for i=1:m
     rho_eff(i)=1-pn(1,i);


 end


end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```