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Socio-metabolic analysis of the educational sector in Norway

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Abstract

Currently, industrial ecology frameworks and methods are increasingly used to study the social metabolism and address environmental implications and climate change mitigation. Despite many models in these studies use the population as a driver, demographical dynamics and interactions in the social environment have not been integrated. To continue the development of this integration in Material Flow Analysis (MFA) models, we focus on the Norwegian education sector from a demographic and anthropological life cycle perspective. Using MFA methods, we designed a stock flow model of users and suppliers in the education system to identify the patterns and drivers of shape these stocks and flows, which in turn may have an effect in the magnitude of the supply of other services. The boundaries of the model include the population of Norway and its transformations when it moves from, within, and across the education system. Our results confirm that the supply of teachers by the Norwegian education system was insufficient in the year of study (2013) and we have identified and quantified patterns in the population that cause such insufficiency. Among them: retirement, deaths, and enrollment and graduation rates.

Preface

The purpose of this master thesis is to illustrate and analyze the education system of Norway both as a service supplied and used by the population of the country while new applications of material flow assessment methodologies are explored.

With the integration of MFA and demographics, we aim to give insights about the influence of demographic changes and behaviors in human activities, as we believe that a better understanding of services is key for the development and implementation of strategies to tackle environmental, and social, and economic aspects.

The core of this thesis is a mathematical model of the Norwegian education system with a demographic approach using Material Flow Analysis methods. The result is a model of anthropological stocks and flows of users in the education system and the working force of educators to satisfy educational services in the country. In other words, the units of the model are not conventional mass units, but people. To our best knowledge, this is the first time that this method has been used to model and assess service systems with an anthropological perspective.

A considerable part of the time of the development of this thesis was invested in identifying and understanding the great number of possible flows in the education system. Later on, the relationships between stocks and flows, and parameters were studied to find unknown and hidden flows of the model, most of which are not reported in conventional statistics of education. The best data quality was found to be

that of 2013 and some previous years. For that reason, year 2013 was chosen for this study and the best possible educated estimates were made whenever data was unavailable or fragmented.

Acknowledgements

I thank my family for their love and support even thousands of km away; to my mother, Elizabeth, and my brother, Roy. I thank also my friends at Kringsjåvegen for the home we built together in Trondheim, my professors at NTNU for their inspiring lectures and discussions, and my co-supervisor, Luis Felipe Vásquez, for his patient advice and follow up in the development of this work.

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Abbreviations

SSB	Statistics Norway (*Statistik Sentralbyrå*)
PPU	Practic Pedagogic Education (*Praktisk Pedagogisk Utdanning*)
NOKUT	National Organ for Quality in Education (*Nasjonalt organ for kvalitet i utdanningen*)
UiB	University of Bergen
NSD	Norwegian Social Scientific Data Services
DBH	NDS's Database of Statistics of Higher Education
OECD	Organization for Economic Cooperation and Development
BPIE	Buildings Performance Institute Europe
MFA	Material Flow Analysis
LCA	Life Cycle Analysis
EIOA	Environmental Input Output Analysis

1. Introduction

The satisfaction of human needs in combination with demographic, technological, and cultural changes have shaped our social metabolism for millennia (Grünbühel et al. 2003; Fischer-Kowalski & Haberl 1998; Haberl 2006). We understand this social metabolism as the magnitude, drivers and patterns of the interactions between society and the environment (Fischer-Kowalski & Haberl 1998; Ayres 1998), including the natural, built, and social environments.

Currently, industrial ecology frameworks and methods, like life cycle analysis, (LCA), material flow analysis (MFA), and Environmental Input Output Assessment (EIOA) have been used to model and assess social metabolism and environmental impacts. Particularly, MFA methods have been used to study social metabolism and the built environment due to its environmental implications and potential to reduce energy use and mitigate climate change. In the EU and Norway, for example, dwellings are responsible for 40% of the energy use in these regions (Economidou et al. 2011; Rapf & BPIE 2012; Sartori et al. 2009) and are also expected to achieve considerable energy reduction gains.

Although many of these studies use the population as a driver, demographical dynamics and interactions in the social environment have not been integrated into MFA models. In the Industrial Ecology Master Project[1] *Socio-metabolic analysis of the educational building stock in the Trondheim municipality* (Sigüenza 2014), a first

[1] The Industrial Ecology Master Project is a compulsory work of the MSc in industrial ecology programme at NTNU.
[2] *Forecasting demand and supply of labor by education (Insert reference) and Frmskrivinger av*

attempt was made to explore this gap by the study of two different resources required by pre-school services: floor area, and labor force for education. In this work, demographics were integrated into a MFA model of infrastructure as separate subsystems or layers, shown as users and suppliers in Figure 1.

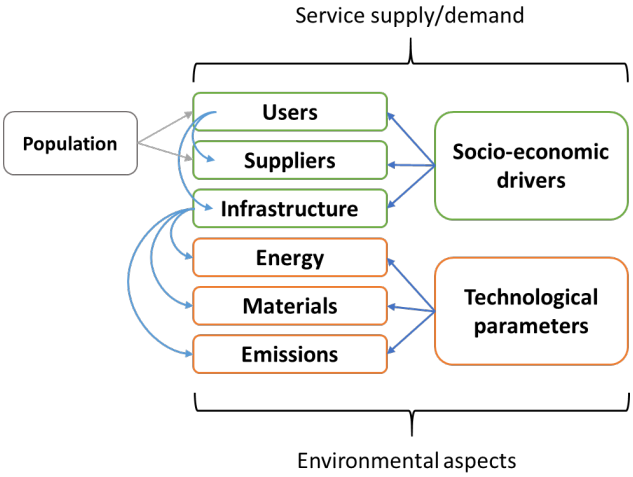


Figure 1. Socio-metabolic concept framework of services **(Sigüenza, 2014)**

To further explore this integration, we continue to study the education sector, this time at a national level. This sector as a service has the peculiarity that for the population to become a supplier, first it needs to become a user. As the population studies, some people may become teachers that eventually re-integrate the education system as workers. This makes the education system of Norway its own factory of human resources for education.

In Norway, the population has increasingly participated more in education and attained more qualifications in the last four decades (OECD 2012), and the requirements to work as a teacher have tightened (Utdanning.no 2015; Roksvaag & Texmon 2012). On the other hand, reports by the SSB suggest that Norway may face a lack of up to 20 000 teachers by 2020 (Gjefsen et al. 2014; Cappelen et al. 2013). However, the social and demographic mechanisms of cause and possible solutions to these scenarios are not addressed or discussed in these reports.

With this thesis, we aim to contribute to a deeper understanding and knowledge of the education sector of Norway and the modeling of services. We will explore and study the population stocks and patterns to identify possible drivers that affect the need and supply for educational services and try to answer the following questions:

- How are the stocks of students in the education system conformed?
- Which behaviors or patterns may affect the size of these stocks?
- How does the education system of Norway supplies teachers?
- Can we confirm a current undersupply of teachers?
- If so, which social or demographic patterns may be causing such imbalance?
- Can we apply MFA methods to answer these questions?

1.1 The Norwegian Education System explained

The education system in Norway consists of different education levels. These are: pre-school, primary and lower secondary education, upper secondary education, folk high schools, tertiary vocational education and higher education.

In general, the educational offer is tiered. This means that the satisfactory completion of each level of education grants the student access to the following level. However, the completion of some education levels such as pre-school, folk high schools, and some strains of secondary education do not qualify students to enroll in other types of education. The main paths in the education system are visualized in Figure 2.

Any person age five or younger can attend pre-school. Since 2007 a statutory right to a place in pre-school for children under the age of 6 was introduced (Haug & Storø 2013; Holmseth 2013). At the age of six, most pupils start compulsory

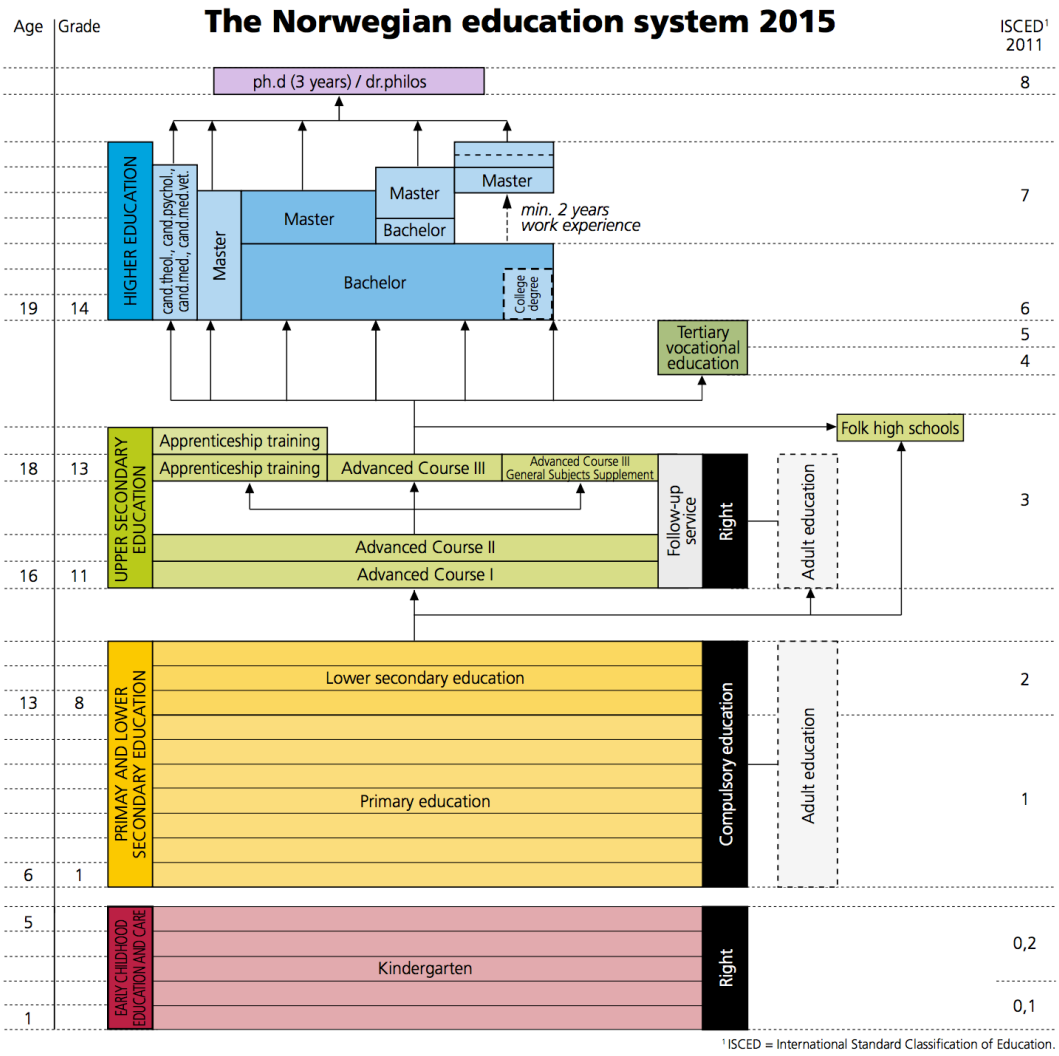


Figure 2. The Norwegian education system 2015. SSB 2015

education in Norway, which has duration of 10 years since the reform of 1997 (Holmseth 2013) and consists of primary and lower secondary education.

After completing compulsory education, normally at age 16 (Nygård 2014), students have the right to take part in upper secondary education. The upper secondary education has two main strains: a vocational strain and an academic strain. The first gives the student professional competence to start working, while the latter gives access to tertiary education. However, students of the vocational strain may take

a complimentary year in upper secondary education to earn access to higher education.

The higher education offer in Norway consists of college, bachelor, master, and doctoral, and professional degrees. The professional degree programmes have duration of 5 or more years and cover fields like medicine, psychology, nursery, veterinary, among others, but in Norway are not categorized as bachelor or master degrees.

Tertiary vocational education can be taken when upper secondary education is completed. While their programmes last between six months and two years and give vocational qualifications, they do not give qualifications to start higher education (insert reference).

Additionally, any student aged 16 or older may enroll, folk high schools. The duration of these programmes can be up to ten months and they have mostly integration purposes (Nygård 2014). These programmes do not give qualifications to enroll to any other educational programme.

1.2 Labor force for education

The labor force of the Norwegian education system consists of persons with different activities and backgrounds, from administration and services to teaching staff. In this thesis, however, we will focus in the latter, to which we will refer to from now on as teachers.

In Norway, there are several types of teachers: pre-school teachers, general teachers, subject teachers, special education teachers, professors, and lecturers.

There exist different study paths to become a teacher in Norway. One path is to study a teacher programme in higher education with an ordinary duration of three, four, or a recent integrated five-year programme. These programmes can give qualifications as pre-school teachers, general teachers, or subject teachers. There does not exist specific programmes to become a special education teacher, but many of these have general teacher qualifications (Nygård 2014; Foreign Credits 2012).

Another way to become a teacher is by completing a one-year complementary programme called P*raktisk Pedagogisk Utdanning (PPU)* or practical pedagogic education in English, which gives teacher qualifications for persons who already have a higher education degree of at least three-year duration or a two-year tertiary vocational education degree or at least two years of relevant vocational experience. For simplicity, we will refer to this programme as *PPU,* for its initials in Norwegian.

Pre-school teachers have the capacity to work as teaching leaders or assistants. Many of them have taken a complimentary course to be able to teach in the first four grades of primary school (Roksvaag & Texmon 2012). General teachers are qualified to teach in primary and lower secondary schools. Depending on their specialization of their higher education, they can teach in grades 1 to 7 or 5 to 10 of compulsory education.

Subject teachers are teachers that can teach a group of subjects or a single subject (Roksvaag & Texmon 2012), and they are entitled to teach in single subjects in primary and lower secondary school, upper secondary schools, and adult and other types of education for youth (Roksvaag & Texmon 2012).

Professors, associate professors and lecturers in higher education are the equivalent as teachers in higher education. To become a teacher in higher education, usually a longer educational and professional career is required. These teachers have at least a doctoral degree, and it is common that they continue with a post-doc or research position before they become lecturers, associate professors or professors (European University Institute 2015)

1.3 Education and labor demand and supply models

We identified three models that are used by the SSB to analyze and forecast labor supply and education. One of them is MOSART, a dynamic micro-simulation model that forecasts the demand and supply of labor force by level of education and educational background for different sectors (Gjefsen 2013). This model uses individual propensities of the population to attain different levels of education based on possible choices starting education, choices of areas of study, completion, and age (Gjefsen 2013).

The other is MODAG. MODAG is a macro-economic model for the Norwegian Economy developed by SSB (Statistics Norway 2015; Cappelen et al. 2013). This model has an inter-industry economic matrix of 28 sectors and calculates the demand of 5 different educational levels for each sector (Cappelen et al. 2013). Projections with this model take into account technological changes in the multi-sectorial part (Cappelen et al. 2013).

The SSB published in 2013 and 2014 reports[2], in which it compares the results and forecasts of the demand of labor force of the model MODAG with the results and forecasts of the supply of labor force by education of the model MOSART (Roksvaag & Texmon 2012; Cappelen et al. 2013). The results include the misbalances between the demand and supply of labor for different sectors. Among their results, excess in the demand of teachers and nurses and an excess supply of engineers and other fields of science were forecasted. Some of these figures are available in Appendix I.

The third model is LÆRERMOD. This model is a more specialized tool than MOSART used to forecast the demand and the supply of the educational labor force (Roksvaag & Texmon 2012). In LÆRERMOD, the educational work force is divided into five categories: pre-school teachers, general teachers, subject teachers, practical pedagogic education and special pedagogues, which are finally allocated as

[2] *Forecasting demand and supply of labor by education (Insert reference) and Frmskrivinger av befolkning og arbeidsstyrke etter utdanning med alternative forutsetninger for innvandring (Insert reference)*

educational personnel in several levels in the education system with one personnel composition for each level.

In addition, in LÆRERMOD, the next factors are part of the supply side of the model: labor force participation, average working time, economic growth, population growth and age (by sub model BEFINN[3]), trends of student admission and completion to relevant pedagogy related programmes, as well and leaves by deaths.

In general, the SSB warns/notes that the time span of these studies is rather long, and many variables that can affect the labor supply and demand forecasts of all of the models explained previously and advices the reader to interpret the results with caution.

[3] BEFINN is a dynamic population model that the SSB uses to forecast population (Aase et al. 2014)

2. Methods

Several alternative system designs were proposed for the study at hand. Some of them are available in Appendix II. In this chapter we present and explain the most optimal model to our educated understanding that adapts to the complexity of the education system, population behavior, and the most complete and recent available data.

The model is a quasi-stationary model that uses conventional MFA methodology. It has a temporal design that describes the natural-life and occupational cycles of the population as it participates in the educational services as students to eventually supply for the same educational services as teachers. The model includes demographic aspects such as births, deaths, and migration as people study, work, and finally retire.

The system is divided in five main components: three main process groups and two single processes:

- Process group 1: Education
- Process group 2: Markets of labor force for education
- Process group 3: Labor force in education
- Process 7: Retirement
- Process 1: Rest of population

The boundaries of the system are drawn around the group processes 1, 2, and 3, and process 1 and 7 because the scope of the study is Norway, and the stocks of these processes and process groups are the total population of Norway.

Each process in the system (including inside process groups) are considered processes because they give the population new characteristics as they conform and leave each stock of each process, similarly to the way in which materials are transformed in industrial processes, and respecting mass balance principles. Or in this case, population balance.

Due to the large number of variables, equations, and parameters used to solve this system, only the most relevant of them are explained in this chapter. Nonetheless, a complete set of variables, equations, and parameters is available in Appendix III, IV and V, respectively.

In the next section we explain the process groups and the processes retirement and rest of the population along with the main assumptions and mathematical approaches that characterize them.

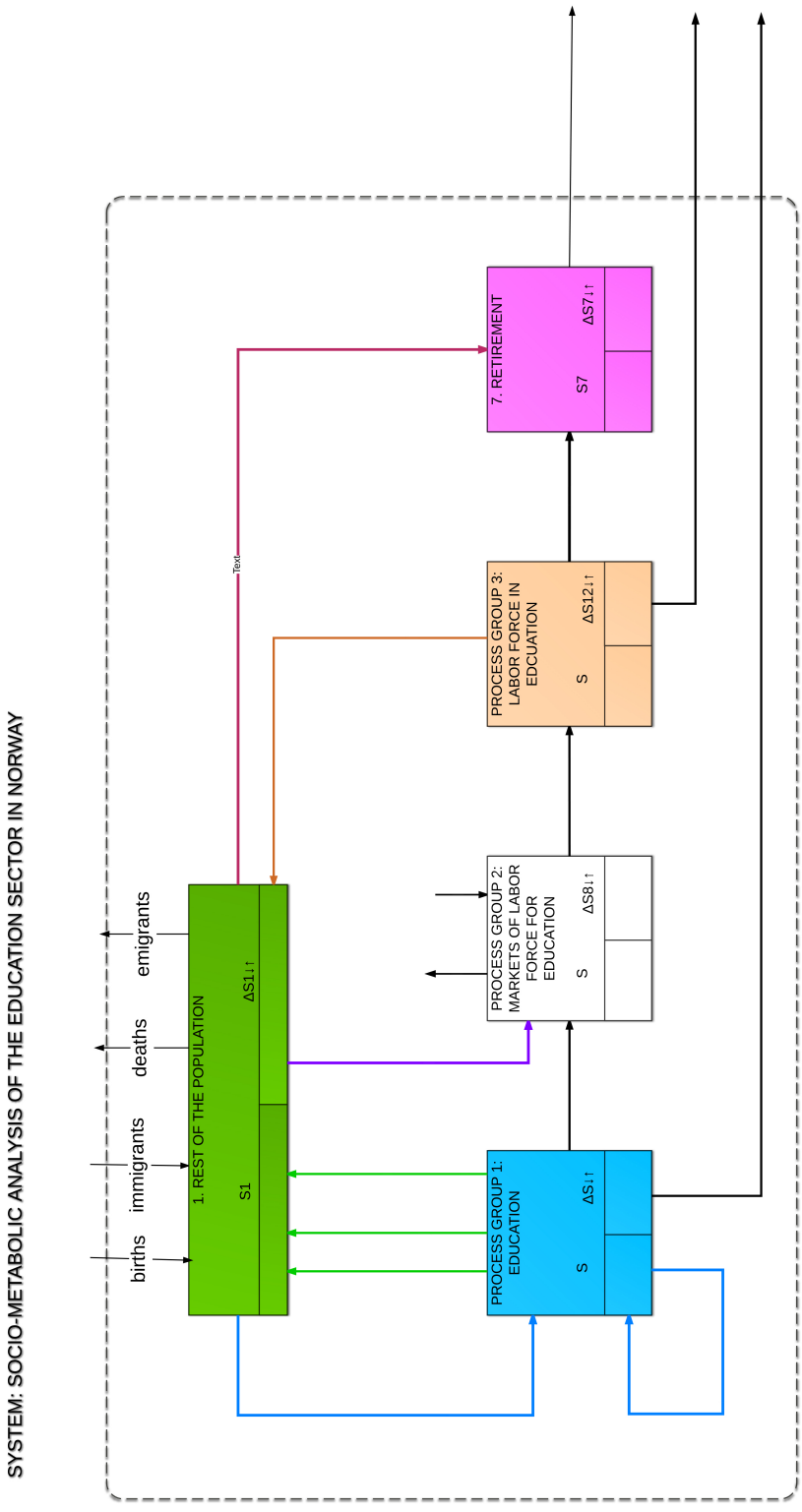


Figure 3. Model by process groups for the socio-metabolic analysis of the education sector in Norway

2.1 Process group 1: Education

The process group Education describes the stocks of students in each formal education level and the flows of students across the different educational levels as they finish or leave each educational level.

Each process in this group represents one of the formal educational levels in Norway:

- Process 2: Pre-school
- Process 3: Primary and lower secondary education
- Process 4: Upper secondary education
- Process 5: Tertiary vocational education
- Process 6: Higher education

Each of these education levels or processes have several and different years of duration. For simplicity, they are represented as single processes. Process 6 (higher education) needs a special mention. In this education level, bachelor, master, PhD, and other professional degrees are offered. Flows between these sublevels are complicated and unclear, as many students change programmes, finish, drop out, or enroll other levels of higher education, at the same time not sufficient data on these flows was available. By aggregating these sub processes into one, it is therefore assumed that all students of higher education conform one stock, regardless of their programme of study.

On the other hand, the output of students with higher education degrees by study area are relevant for the labor force for education and they were differentiated from other types of degrees.

Additionally, process 17 represents the students that formally participate in the in higher education system in Norway, but that are abroad (e.g. exchange students).,

Process 17 is part of process 6 and its stock is included in the system only for visualization purposes.

2.2 Behavior of the population in education

Most levels of education are tiered. However, not all students enroll a "higher" level of education after attaining a "lower" one. Many students, especially during and after upper secondary education, take breaks or leave education permanently. Emphasis was placed in the modeling and assessment of these flows, which are not regularly reported in statistics of education.

In the model, we make a distinction between the flows of students that enroll to each level of education and their origin, those that leave education abruptly, and those that attain one level of education but do not enroll in another one. It was possible to make these distinctions for all the processes in this group with exception of pre-school, where drop-outs are not relevant, and tertiary vocational education, where very limited data was available.

To assess the flows to, from, and between each process, some data on the composition of the student stocks by grade[4] and/or age provided by the SSB were used. These data became then parameters to develop model approach equations to eventually solve the system by algebraic substitution.

[4] One grade is the equivalent to one year of education. For example, primary and secondary education consists of ten years, or grades 1st to 10th.

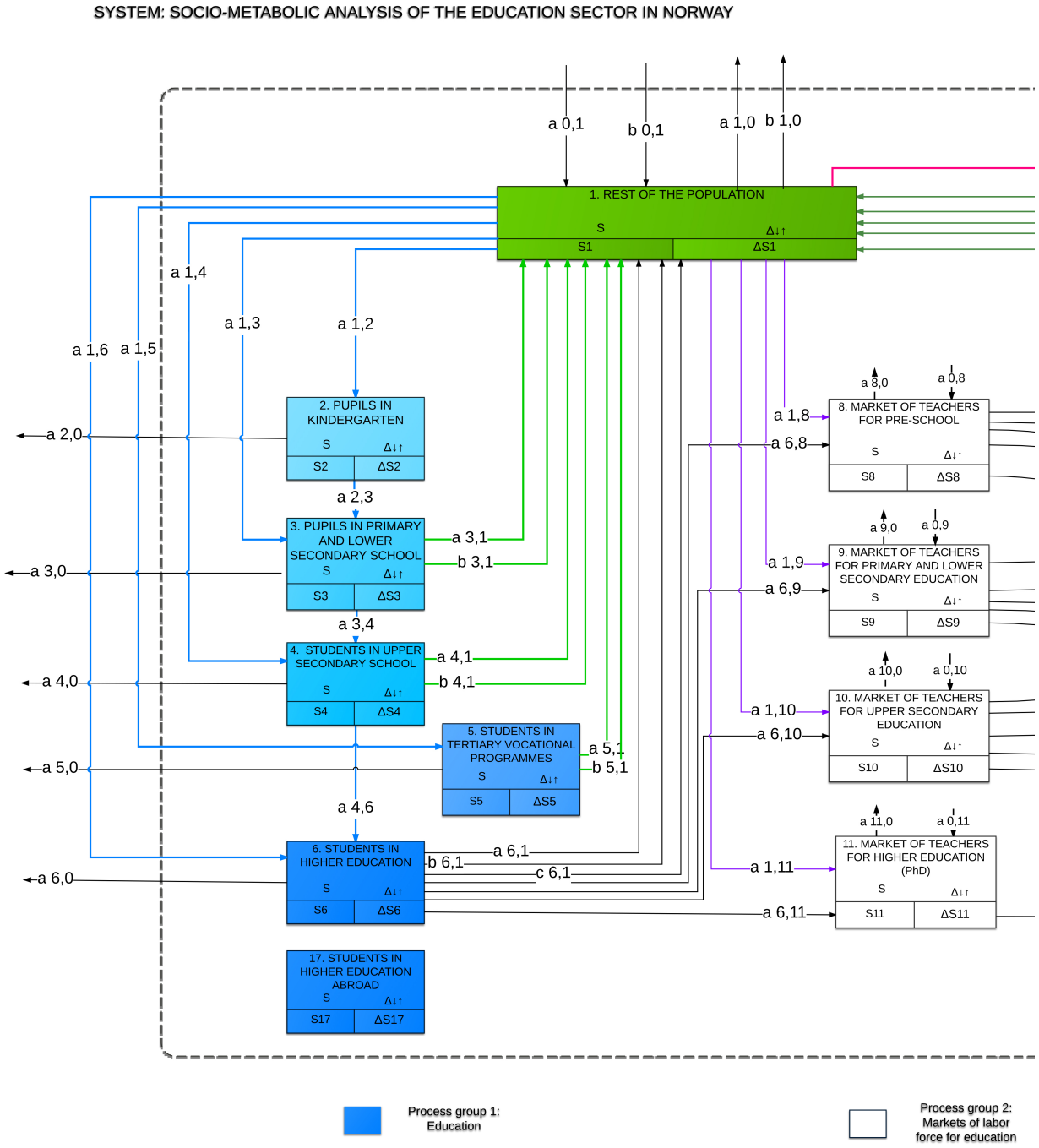
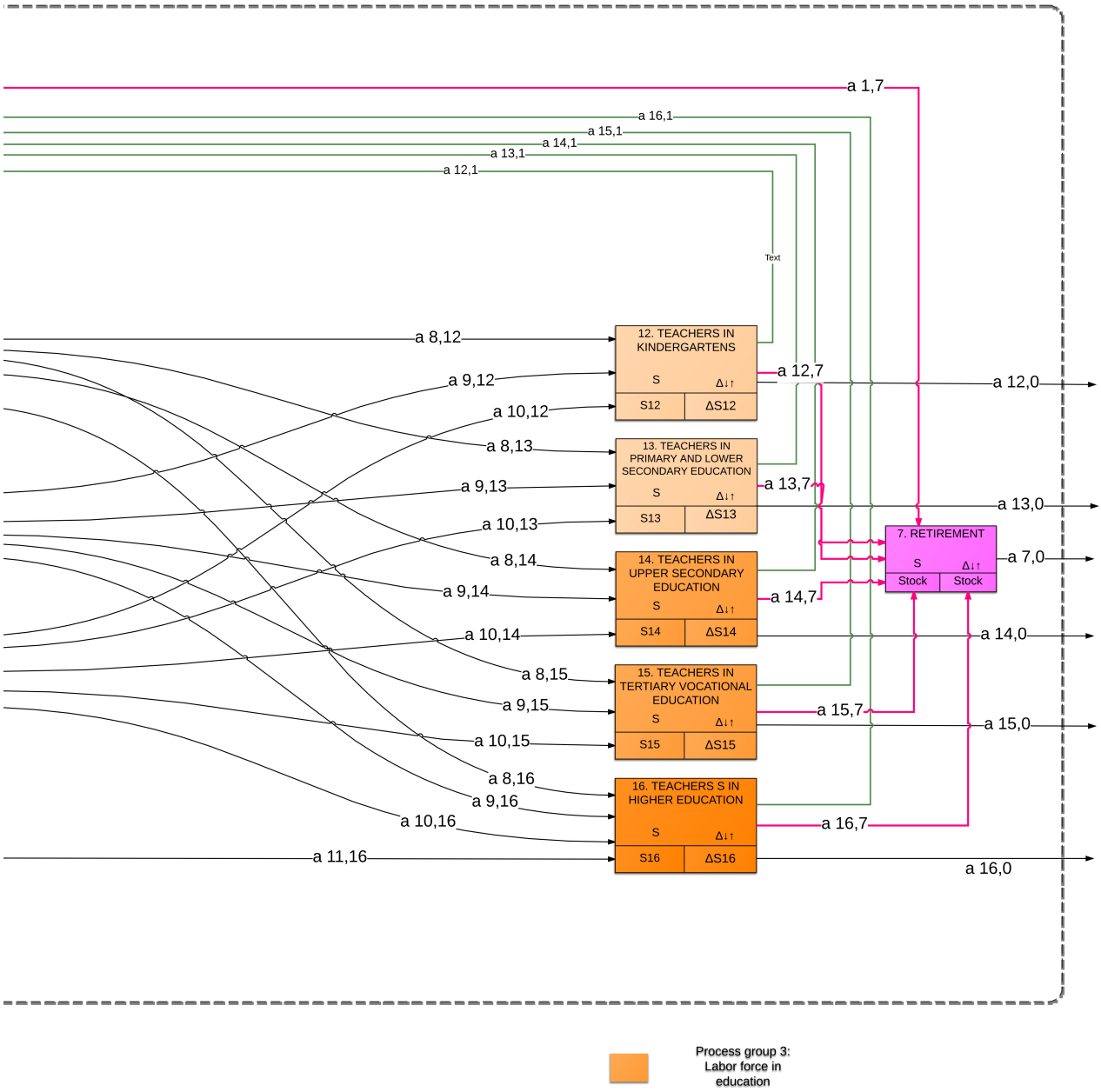


Figure 4. Model by processes of the education sector in Norway


a 1,7
a 16,1
a 15,1
a 14,1
a 13,1
a 12,1
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a 11,16
12. TEACHERS IN KINDERGARTENS
S
Δ↓↑
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13. TEACHERS IN PRIMARY AND LOWER SECONDARY EDUCATION
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14. TEACHERS IN UPPER SECONDARY EDUCATION
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Δ↓↑
S14
ΔS14
15. TEACHERS IN TERTIARY VOCATIONAL EDUCATION
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16. TEACHERS S IN HIGHER EDUCATION
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7. RETIREMENT
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Process group 3: Labor force in education

Examples of this approach are flows $a_{2,3}$ (Equation 1) and $a_{4,6}$ (Equation 2). Flow $a_{2,3}$ is the number of children that leaves pre-school and enrolls primary and lower secondary school. This flow is determined by the stock of pupils in kindergarten of ages 5 and 6 of the previous year ($S2_{5-6_{t-1}}$), since these children become age 6 (and a few turn 7) in 2013 and the normal age of enrollment to compulsory education is age 6.

$$a_{2,3} = S2_{5-6_{t-1}} \tag{1}$$

Equation 2 describes the number of students that finish upper secondary school and enroll higher education. This expression takes into account the age composition of the newly enrolled students in higher education. Since most students in upper secondary are aged under 20 (Statistics Norway 2015), we assumed that the newly enrolled students in higher education 20 or younger ($I6_{0-20}$) did it directly after finishing upper secondary school. To this number of enrollments known from the higher education "side", we need to add the fraction of students of age 21 or older that coursed and passed the last year of the academic strain of upper secondary education and that enrolled in higher education: $\eta\, V_{21}\, V_{ac3}$; where η is the fractional rate of enrollment from upper secondary education to higher education, V_{21} is the share of students 21 or older in upper secondary education, and V_{ac3} is the number of students of the last year of the academic strain of upper secondary education.

$$a_{4,6_t} = I6_{0-20} + \eta\, V_{21}\, V_{ac3} \tag{2}$$

Additionally, the flow $c_{6,1}$ from higher education was calculated by mass balance (Equation 3), and it reflects the number students in higher education that changed programme and those that finished exchange student programmes in Norway, but who did not obtain a higher education degree.

$$c_{6,1} = a_{1,6} + a_{4,6} - a_{6,8} - a_{6,9} - a_{6,10} - a_{6,11} - b_{6,1} - a_{6,0} - \Delta S6 \tag{3}$$

2.2.1 Drop out flows

Data on fractional drop out rates from education reports by the SSB and OECD were used to model and assess drop out flows. At the same time, average residence times of each level of education and total enrollments of previous years were also taken into account for a more accurate assessment.

The equations for the drop out flows of processes 3, 4, and 6 follow the construction of Equation 4.

$$b_{i,1} = \frac{\mu_i}{\sigma_i} \sum_t I(i,t)\, Y(i,t) \tag{4}$$

In Equation 4, $b_{i,1}$ is the flow of students that drop out from each process i and goes to process 1. i equals processes 3, 4, or 6. μ_i represents the correspondent fractional drop out rate of each process i. (i,t) represents the total enrollments of each process by each relevant year t, and $Y(i,t)$ is the weighting factor for each inflow of each process. The total weight of $Y(i,t)$ sums up the average residence times of each process i. Finally, everything is divided by the average residence time of each process σ_i.

This approach was used in order to more accurately assess and not underestimate the flows of students that drop out education. With this approach, the drop-out flows represent the students that dropped out in 2013 taking into account those that enrolled several years before.

Table 1 shows the fractional drop out rates of the students that enroll a level of education, but leave abruptly. The residence times in education are explained and shown in the sub section 0

Residence times in education.

Table 1. Fractional drop-out rates in selected education levels

	Symbol	Value	Source
Fractional drop out rate from primary and lower secondary school	μ_3	0,01	OECD
Fractional drop out rate from upper secondary school	μ_4	0,3	SSB
Fractional drop out rate from higher education	μ_6	0,17	OECD

2.2.2 Outflows of higher education and tertiary vocational programmes

The outflow of students from tertiary vocational education was modeled with a static approach and an average residence time of 1,25 years. This residence time is the average duration of these programmes, which can be from 6 months to two years (Statistics Norway 2014). Equation 5 illustrates the solution for this flow, where $\sigma_5$is the average residence time, $a_{5,1}$ is the estimated outflow of students from tertiary vocational programmes and S5 is the stock of students in the process at hand.

$$a_{5,1} = \frac{S5}{\sigma_5} \tag{5}$$

For the outflows of higher education, data about the number of graduates by different degrees in 2013 was rearranged and grouped to assess the flows of graduates with teaching qualifications. Five outflows (or types) of graduates were distinguished, four of which are relevant for the markets of labor force for education. The outflows that were distinguished are:

- Pre-school teachers
- Teachers for primary and lower secondary education
- Teachers for upper secondary education
- Professors and teachers for higher education (PhDs)
- Rest of graduations

Table 2 shows the criteria for grouping the outflows of graduates and the corresponding flows in the system. These criteria are based on the level of education in which graduates are qualified to teach at, as well as the data available on graduations of higher education by the SSB.

Table 2. Graduations from higher education grouping by type

	Flow	Degrees (graduations)
Pre-school teachers	$a_{6,8}$	Bachelor degree, pre-school-/kindergarten teacher training
Primary and lower secondary school teachers	$a_{6,9}$	Bachelor degree, teacher training and education, not general teacher training programme General teacher training programme Higher degree, teacher education and education in pedagogy Teacher training programme (1 year)
Upper secondary teachers	$a_{6,10}$	Bachelor degree, vocational teacher, foundation programme Subject teacher training, practical arts and subjects
Higher education professors and teachers	$a_{6,11}$	PhD (all)
Rest of graduations	$a_{6,1}$	Rest of the programmes of higher education (non teaching or pedagogy oriented)

2.2.3 Residence times in education

The residence times in the education processes are the result of averaging in some cases the length of the programmes (primary and lower secondary education and tertiary vocational education). In other cases (upper secondary and higher education), the average residence times were calculated from statistics and the share of students that take different times to finish (or not) these educational levels. Table 3 shows the values used in the model. In Appendix VI additional information of the derivation of the residence times of upper secondary and higher education is available.

Table 3. Average residence times of students in the education system by level

	Symbol	Value (years)	Source or method
Average residence time in primary and lower secondary education	σ_3	10	Average duration of programmes (SSB)
Average residence time in upper secondary education	σ_4	4	Shares of students that spent certain years in this level of education (SSB 2014)
Average residence time in tertiary vocational programmes	σ_5	1,25	Average duration of programmes (SSB)
Average residence time in higher education	σ_6	5,64	Shares of students that spent certain years in this level of education (SSB 2014)

2.3 Process group 2: Markets of labor force for education

The processes in this group reflect the balance between the output of teachers of higher education and the labor force for education hired to replace the teachers that leave the labor force in education (process group 3). It is assumed that all the teachers that graduate from higher education ingress to these markets the same year of graduation. The stocks of these markets remain unknown (as no sufficient data was available), and only the stock changes were identified (balance between teachers graduated and actually teachers hired).

The markets of labor force for education are four:

- Process 8: Market of teachers for pre-school
- Process 9: Market of teachers for primary and lower secondary education

- Process 10: Market of teachers for upper secondary education
- Process 11: Market of teachers for higher education (PhDs)

The inflows to these processes are flows $a_{6,8}$, $a_{6,9}$, $a_{6,10}$, and $a_{6,11}$. The outflows of the markets are in fact the inflows of the processes of process group 3 calculated by mass balance, explained in more detail in the next sub section.

$$\Delta S(i) = a_{6,i} - \sum a_{i,j} \tag{6}$$

The balance equations for these stock changes follow the construction of Equation 6, where i is each process in the process group 2, j is each process in process group 3; $a_{6,i}$ is the flow of teachers from higher education to each process i, and $a_{i,j}$ is the flow of teachers hired from each process i by each process j.

2.4 Process group 3: Labor force in education

The stocks in this process group represent the number of teachers working at each level of education in 2013. The processes that conform this group are 5:

- Process 12: Teachers in pre-school
- Process 13: Teachers in primary and lower secondary education
- Process 14: Teachers in upper secondary education
- Process 15: Teachers in tertiary vocational education, and
- Process 16: Teachers in higher education

The outflows of these processes are the teachers that leave the labor force in education. The following aspects for the modeling and assessment of these flows were considered:

- Retirement at age 67
- Premature retirement
- Desertion of the newly hired teaching staff (newly hired teachers that leave before the first year of work)
- Desertion of ”permanent” teaching staff
- Deaths

2.4.1 Retirement flows

Retirement and deaths are sensitive to the age of the workers. Therefore, age composition of each stock in combination with different fractional retirement rates and fractional death rates by age groups were used to estimate these flows.

The data found on the age composition of most of these stocks were quite limited. A survey performed by the Oxford Research and the University of Aarhus provided for age compositions of the stocks of teachers of compulsory and upper secondary education and only an average age of teachers and professors in higher education was found to be 47-48 years old (European University Institute 2015). Therefore, the age composition of teachers in kindergartens was assumed to be the same as that of teachers in compulsory education, and a normal distribution from ages 29 to 67 was used to estimate the age composition of the stock of teachers and professors in higher education. Additional information on the age composition of teachers is available in Appendix VII.

Equation 7 describes the construction of the equations for the flows of teachers that leave work (processes j) and enter retirement (process 7). The term $A_{60}(j)$ is the share of teachers aged 60 or older in each process j. $S(j)$ represents the stock of each process j in process group 3. ϑ is the relative change of the number of early retired people (that retire between ages 25 and 66; see Appendix V for more detail).

Therefore, the term $\frac{A_{60}(j)}{67-60}\ S(j)$ represents the number of teachers that retire at age 67, while $\vartheta\left(S(j)-\frac{A_{60}(j)}{67-60}\right)$ represents the number of teachers that retire early. The net value of the denominator $67-60$, is 7. We have to use this value to estimate the number of people aged 67, since we only know the share of those that are 60 or older in statistics. Hence, it is assumed that all teachers retire at age 67 and that there are no teachers older than 67 in the working force[5].

$$a_{j,7}=\frac{A_{60}(j)}{67-60}\ S(j)+\vartheta\left(S(j)-\frac{A_{60}(j)}{67-60}\right) \tag{7}$$

2.4.2 Desertion flows

The desertion flows were considering two different fractional desertion rates. The first is the fractional desertion rate of the newly hired teachers that desert before the first year of work. The second is the fractional desertion of the rest of the stock of teachers. This splits the desertion flows in two parts: one dependent on the size of the inflow and the other dependent on the size of the stock. Specific fractional desertion rates were not found in literature for each of the types of teachers that work in different levels of education. Instead, it was assumed that these fractional desertion rates in Table 4 are the same for teachers working at all levels.

These flows follow the construction of Equation 8, where *i* is each process of process group 2, *j* is every process in process group 3. $\Omega\ \sum a_{i,j}$ is the teachers that drop out before completing the first year of work, and $\zeta\ S(j)$ is the number of "permanent" teachers that desert. The term Ω is the fractional desertion rate of newly hired teachers, and ζ is the fractional desertion rate of "permanent" teachers.

$$a_{j,1}=\ \Omega\sum a_{i,j}+\zeta\ S(j) \tag{8}$$

[5] In Norway, the common age for retirement is 67 (China 2011).

Table 4. Fractional desertion rates of teachers

Fractional desertion rate	Symbol	Value
Newly hired teachers	Ω	9% (UiB)6 or 6,6% (Utdanningsnytt.no)7
"Permanent" teachers	ζ	2%

2.4.3 Inflows of teachers

These inflows are calculated by mass balance of the processes in the process group 3, and then disaggregated according to the mix of teachers at every education level. A teacher mix for 2010 was available in reports by the SSB (Roksvaag & Texmon 2012). This teacher mix was adapted to meet the criteria used to characterize the output of teachers of higher education and the markets of labor force for education of the system at hand.

Table 5 shows the teacher mix of each level of education. And in Appendix VIII, the original teacher composition suggested by the SSB and its adaptation to the model is available.

[6] 45% of newly graduated teachers hired leave education in 5 years or less (UiB)
[7] One third of newly graduated teachers hired leave work in 5 years (Utdanningsnytt)

Table 5. Teacher mix

	Composition of teachers in pre-school	Composition of teachers in primary and lower secondoary school	Composition of teachers in upper secondary education	Composition of teachers in tertiary vocational education	Composition of teachers in higher education
Pre-school teachers	96,6%	13,9%	2%	2%	1,53%
Primary and lower secondary teachers	1,5%	67,6%	12%	12%	4,67%
Upper secondary teachers	1,9%	18,5%	86%	86%	15,28%
PhDs	-	-	-	-	78,5%
Total	100%	100%	100%	100%	100%

This

	Parameter	Parameter	Parameter	Parameter	Parameter
Pre-school teachers	$l_{8,12}$	$l_{8,13}$	$l_{8,14}$	$l_{8,14}$	$l_{8,16}$
Primary and lower secondary teachers	$l_{9,12}$	$l_{9,13}$	$l_{9,14}$	$l_{9,14}$	$l_{9,16}$
Upper secondary teachers	$l_{10,12}$	$l_{10,13}$	$l_{10,14}$	$l_{10,14}$	$l_{10,16}$
PhDs	-	-	-	-	$l_{11,16}$

2.5 Process 7: Retirement

This process consists of all the people that are retired. The inflows are the people retiring from the labor force in education and the people retiring from the rest of the population. The total number of retired people and age composition in Norway for 2012 and 2013 were taken from statistics of the SSB.

The deaths of the retired people are considered the only outflow of this process. This outflow was modeled according to the age composition of the stock together with fractional death rates by age. This allowed estimating the deaths of retired people by model approach equations and the inflow of people retiring from the rest of the population by mass balance approach.

The way all deaths in the system were modeled is described in the sub section 2.7 Deaths.

2.6 Process 1: Rest of the population

The stock in this process represents all the population of Norway except those that study, work as teachers, are retired, emigrated, and died. It functions as a buffer stock for the rest of the processes and ensures that the population balance is preserved, since it is a limited resource.

The inflows of this process are births, immigrants, teachers that deserted work, as well as the flows of students that interrupted or paused education, and those that finished a degree of higher education but did not go to the markets of teachers.

The outflows of this process are deaths, emigrants, and the flows of people that enroll in education without coming directly from another education level.

2.7 Deaths

Deaths in the process group of education, labor force for education, retirement and rest of the population were taken into account to be consistent with mass balance and the relevance of deaths of teachers. To assess all deaths in the system, fractional death rates by age and age composition of the stocks were used. Equation 9 shows the approach used for these assessments.

$$a_{i,0} = S(i) \sum d(c,i)\ A(c,i) \tag{9}$$

In equation 10, *i* can be process: 3, 4, 5, 6, 7, 12, 13, 14, 15, or 16. $a_{i,0}$ is the deaths that occur in each process *i*, $S(i)$ is the stock of each relevant processes, $d(c,i)$ is the fractional rate of each age group relevant to each process *i*, and $A(c,i)$ is the share of each relevant age group that composes the stock of each process *i*.

The total number of deaths is known (parameter *D*), and the deaths from process 1 were modeled with the following model approach equation:

$$\begin{aligned} a_{1,0} = D - a_{7,0} - a_{2,0} - a_{3,0} - a_{4,0} - a_{5,0} - a_{6,0} - a_{12,0} - a_{13,0} \\ - a_{14,0} - a_{15,0} - a_{16,0} \end{aligned} \tag{10}$$

Detailed age compositions and fractional death rates are available in Appendices IX and X, respectively.

2.8 Limitations of the model

2.8.1 Migration

All processes in the model are prone to have migration flows. In the model, however, migration flows are only addressed to the rest of the population process. This approach was chosen for three reasons. The first is the limited data found to relate migration to all the processes in the system. Only basic data on migration and students with immigrant background were found. The second is how immigrants can become Norwegian citizens[8]; and the third is that no differentiation of immigrants that live temporarily or permanently could be derived from statistics[9].

These factors make the identification and disaggregation of migration flows to every process in all the processes of the system difficult. Therefore, migration flows were allocated only to the rest of the population process. As a result, the outflows of process 1 embed some immigration. These outflows are students that enroll any level of education from the process "Rest of the population", teachers that enter the markets of labor force for education, and people of the rest of the population that retire.

This approach shall not affect the mass balance in the system, but it affects the transparency and detail of the model concerning migration. At the same time, it might underestimate the balance of retired people and the direct imports and exports of teachers to the markets of labor force for education.

[8] For example, an immigrant student can enroll in primary and lower secondary school. After some years, the student may apply for a Norwegian citizenship; therefore this student becomes part of the non-immigrant population and still is part of the sock of students. In addition, students can migrate at any time at any education level.

[9] Note that foreign exchange students, foreign students living permanently in Norway, and foreign students in full length programmes living temporarily in Norway account all as "foreign students" in most statistics and cannot be disaggregated.

2.8.2 Aggregation of processes of higher education

This aggregation was made due to the lack of information about the origin of students that enroll each level of higher education (e.g. bachelor, master, PhD, PPU, professional studies, etc). By this aggregation, it is assumed that all graduates leave the stock of education but some may re-integrate into the stock of students after an unknown residence time in the rest of the population.

Therefore, the flows $a_{6,1}$ and $a_{1,6}$ are gross flows of students enrolling to any higher education programme and those graduating, but we cannot distinguish those that for example, finish a bachelor degree and start a master degree the same year. In this sense, the model is short in detail about the characteristics of the throughput of students in higher education except for those that enroll and graduate.

2.8.3 Behavior of the labor force for education

The process groups 2 and 3 represent the overall behavior of the teaching staff when it comes to enrollment, desertion, and retirement of work. However, the model does not account for all the possible flows of teachers within different levels of education as work. For example, if a teacher leaves work in kindergarten and starts work in primary and lower secondary education. Instead, these interactions have been synthesized as gross flows that leave each of the processes of the group. It is therefore assumed that teachers that leave work do so permanently.

The flows $a_{1,8}$, $a_{1,9}$, $a_{1,10}$, and $a_{1,11}$ are visualized in the system indicating the possible flows of other than newly graduated teachers entering the teacher markets, but remain without assessment for the distinctions mentioned above could not be made.

For this reasons, the stock changes in the markets of teachers are limited to reflect the extra teachers hired other than newly graduated teachers.

3. Results

In this chapter, we present the results and main findings of the work in this thesis. For a visual comprehension of results, please refer Figure 5 and Figure 6, which show the values of all the variables assessed in the system individually and grouped by processes. The complete list of results of the model is also available in Appendix XI.

3.1 Aggregated results

We found that 28,3% of the population of Norway attended formal education in 2013. In contrast, only 3,5% of the population of the country worked as teachers, and 14% of the population was retired. Table 6 describes the classification of population of Norway according to the system in this study.

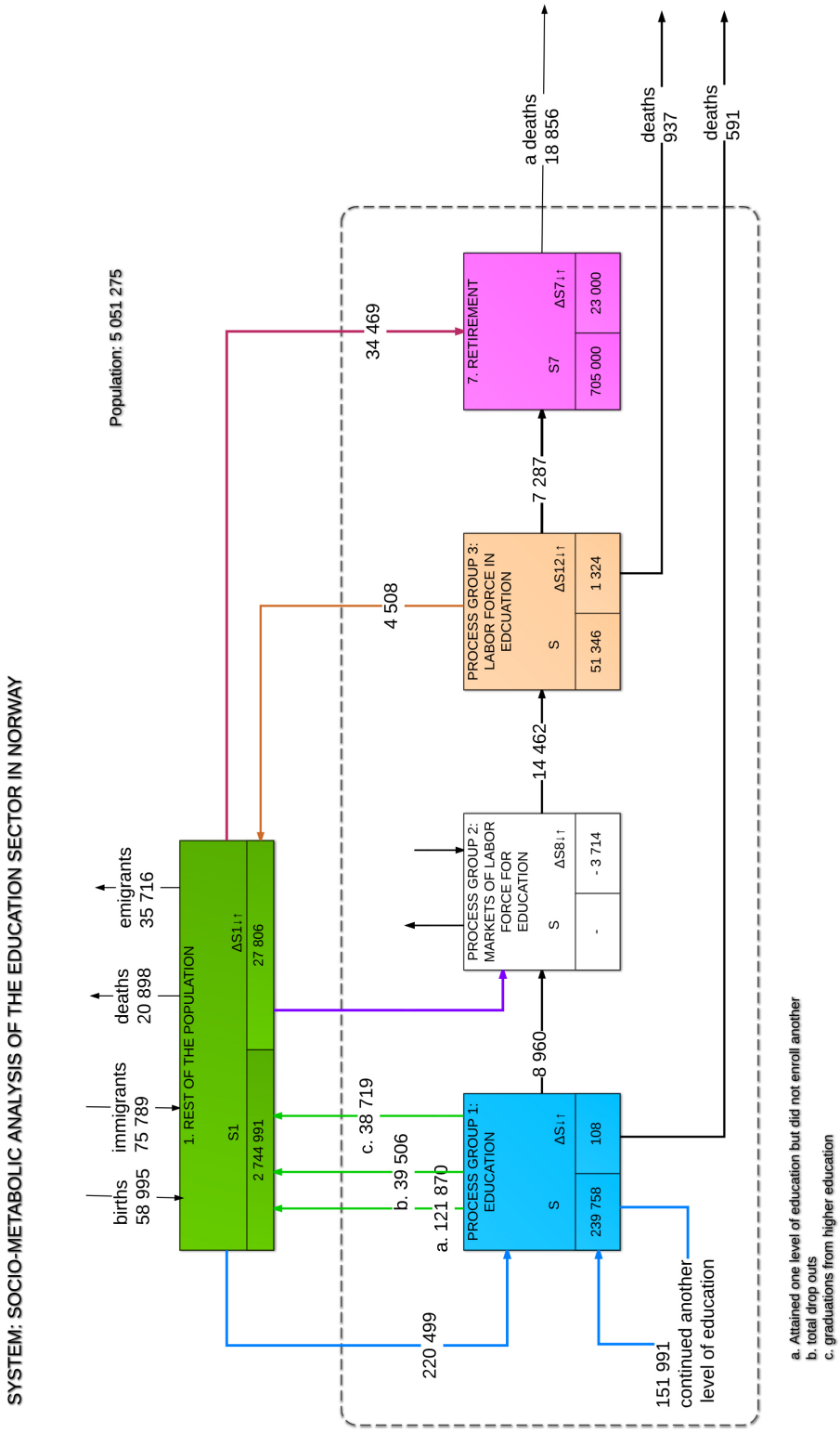

Figure 5. Results: Summary by process groups

Table 6. Results: Composition of the population of Norway

Classification	Population	
Students	1 426 820	28,30%
Teachers	174 464	3,50%
Retirement	705 000	14%
Rest of the population	2 279 245	45,5%
Total	5 035 529	100%

We found that 39 506 students left education abruptly in 2013, of which, 23 201 left upper secondary and 15 706 left higher education. From this education level, there were 48 466 graduations, of which, only 8 960 were related to pedagogy and academia. At the same time, there were 220 499 enrollments in education from the rest of the population.

In the process group 3, 12 833 teachers left work, 14 462 started work, and 8 960 graduated from higher education. This results in a negative stock change of 5 818 teachers in the markets of labor force for education, which are covered from the rest of the population.

At the same time, of the teachers that left work in 2013, 4 508 deserted, 7 287 retired, and 937 died. These numbers represent 2,6%, 4,2%, and 0,5% respectively of the total stock of teachers working.

In retirement, there were 705 000 people in 2013, with an increase of 23 000 from the previous year. The people that retired that year were 41 856, of which 18% were teachers.

3.2 Process group 1: Education

In this process group, we observe that the flows of students that leave education abruptly increased as students scale up in the education system. In 2013, 599 students dropped out from primary and lower secondary, 23 201 dropped out from upper secondary, and 15 706 dropped out from higher education. These flows are insignificant for primary and lower secondary education, but the drop out flows represent 10% and 6% of the stocks of students in upper secondary and higher education, respectively. Table 7 shows the enrollments and drop outs of each education level.

We also observe that the enrollment of students from lower secondary to upper secondary and the enrollment of students of upper secondary to higher education do not occur all in the same year. In fact, we found that only 60% of the students that finished upper secondary education enrolled in higher education. This represents 57% of the new enrollments[10] and 28% of the total enrollments of higher education, which were 52 372 and 104 456 respectively.

In contrast, 94% of pupils that enrolled primary school were in pre-school the same year. Similarly, 99% of the students that finished primary and lower secondary school enrolled upper secondary school. On the other hand, we estimate that almost the same number of students participate, enter, and leave tertiary vocational education as seen in Table 7.

In higher education, we found that 18,3% of the graduations are teacher/pedagogy related, and this ration increases to 21,3% including PhD graduates. Of the total number of graduations, we found that 4,9% are graduates with qualifications to teach in kindergarten, 6,7% to teach in primary and lower secondary schools, 6% to teach in upper secondary schools, while 3,7% are PhDs (See

Table 10).

[10] Students that enrolled higher education for the first time.

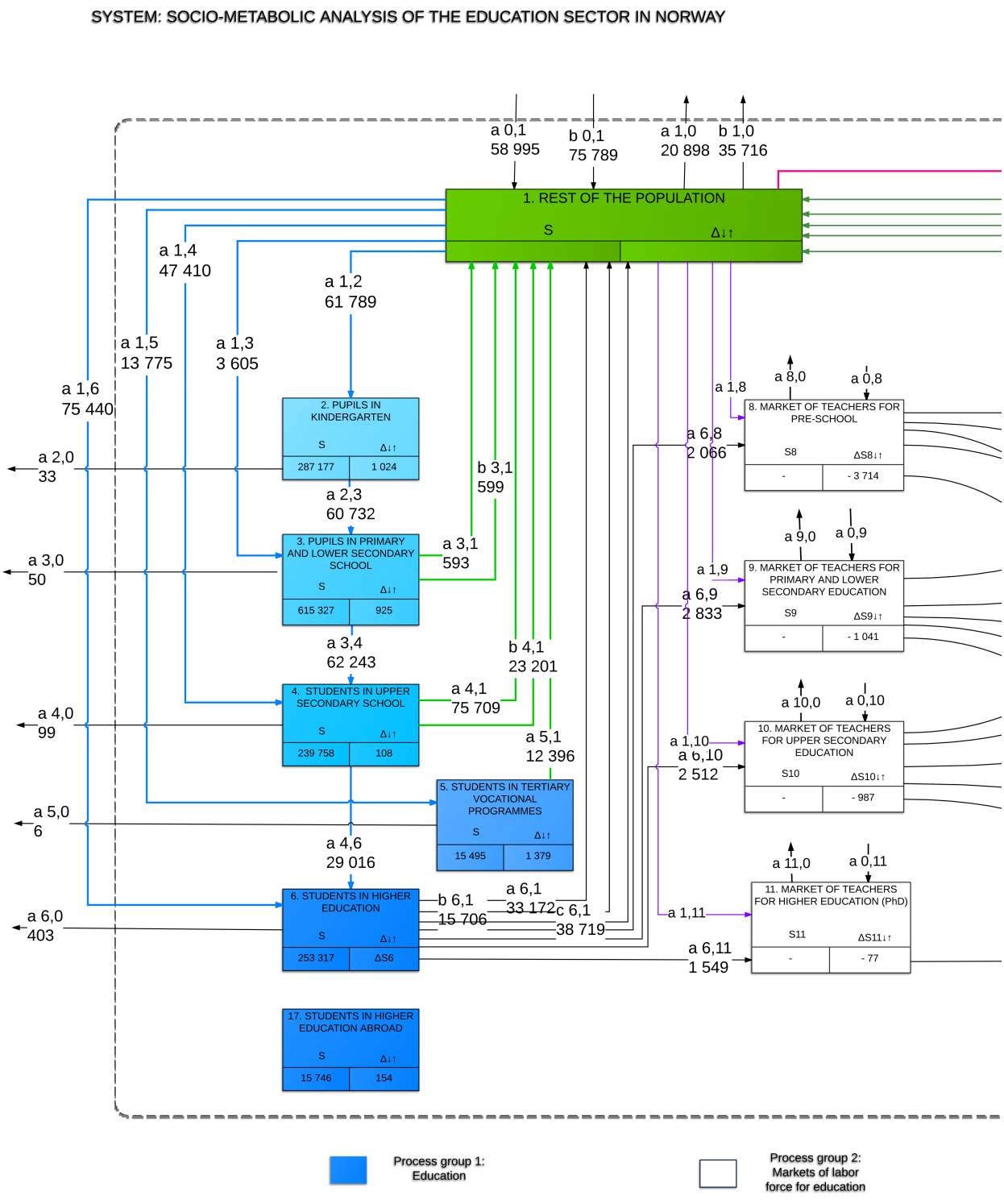

Figure 6. Results by process

H

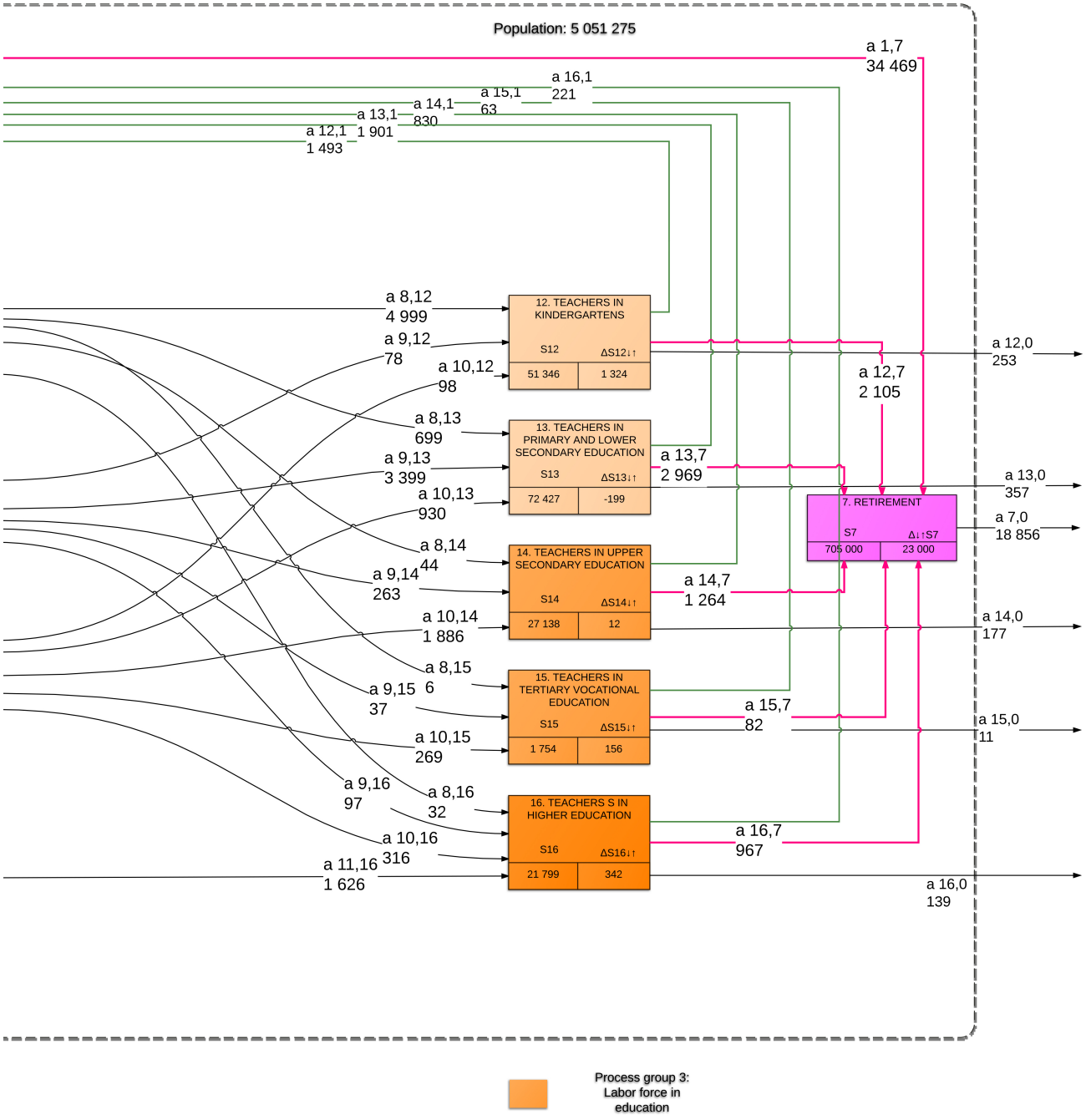
Population: 5 051 275
a 1,7
34 469
a 16,1
221
a 15,1
63
a 14,1
830
a 13,1
1 901
a 12,1
1 493
a 8,12
4 999
a 9,12
78
a 10,12
98
a 8,13
699
a 9,13
3 399
a 10,13
930
a 8,14
44
a 9,14
263
a 10,14
1 886
a 8,15
6
a 9,15
37
a 10,15
269
a 9,16
97
a 8,16
32
a 10,16
316
a 11,16
1 626
12. TEACHERS IN KINDERGARTENS
S12
ΔS12↓↑
51 346
1 324
13. TEACHERS IN PRIMARY AND LOWER SECONDARY EDUCATION
S13
ΔS13↓↑
72 427
-199
14. TEACHERS IN UPPER SECONDARY EDUCATION
S14
ΔS14↓↑
27 138
12
15. TEACHERS IN TERTIARY VOCATIONAL EDUCATION
S15
ΔS15↓↑
1 754
156
16. TEACHERS S IN HIGHER EDUCATION
S16
ΔS16↓↑
21 799
342
a 13,7
2 969
a 14,7
1 264
a 15,7
82
a 16,7
967
a 12,7
2 105
7. RETIREMENT
S7
Δ↓↑S7
705 000
23 000
a 12,0
253
a 13,0
357
a 7,0
18 856
a 14,0
177
a 15,0
11
a 16,0
139
Process group 3: Labor force in education

Table 7. Flows of students across education levels

Education level	Direct enrollments from the rest of the population (process 1)	Enrollments from previous education level	Students that dropped out	Students that finished the level of education at hand and enrolled the next level	Students that finished the level of education at hand and did not enroll the next level	Deaths	Stock change
Pre-school	61 789	n.a.	n.a.	60 732	n.a.	33	1 024
Primary and lower secondary education	3 605	60 732	599	62 243	593	50	925
Upper secondary education	65 890	62 243	23 201	29 016*	75 709**	99	108
Higher education	75 440	29 016	15 706	n.a.	71 891***	6	7 899
Tertiary vocational education	13 775	n.a.	n.a.	n.a.	12 396	403	1 379

** Enrolled in higher education only*

*** Enrollments to tertiary vocational education not considered*

**** 33 172 graduated and 38 719 changed programme or were exchange students*

Also, we found that the number of students that change programme in higher education and those that complete exchange programmes, and who do not obtain a degree adds up to 38 719 students and represents 14% of the total stock of students in this education level.

In addition, we found that in the average residence time of students in higher education, which is of 5,64 years, 40% obtain a degree (see Appendix VI).

Table 8. Results: Enrollment rates from and to selected education levels

Symbol	Value	Description	Notes
θ	99%	Percentage of students that finished primary and lower secondary education and enrolled upper secondary education the same year	Calculated by algebraic substitution of two model approach equations. Reported in statistics but not as a fraction.
η	60%	Percentage of students that finished the academic strain of upper secondary education[11] and enrolled higher education the same year	Calculated by algebraic substitution of three model approach equations. Not reported in statistics.

Table 9. Results: Enrollments in higher education and their origin

Total enrollments	**First-time enrollments**		**Rest of enrolments**
	From upper secondary education	From the rest of the population	From the rest of the population
104 456	29 016	23 356	52 084

Table 10. Results: Graduates from higher education by type

Type of graduates	**Number of graduates**	
Teachers for pre-school	2 066	4,9%
Teachers for primary and lower secondary education	2 833	6,7%
Teachers for upper secondary education	2 512	6,0%
Teachers for higher education (PhDs)	1 549	3,7%
Other graduates	33 172	78,7%
Total	42 132	100%

[11] Only the academic strain of upper secondary education gives qualifications to enroll higher education.

About other patterns of the population as students, we found two fractional enrollment rates that are not reported in statistics of education of Norway. One of them is the fraction of students than finishes primary and lower secondary and enrolls upper secondary. The second is the fraction of students that finishes the academic strain of upper secondary and enrolls in higher education, as seen in Table 8.

3.3 Markets of labor force for education

In the markets of labor force for education we found that all the balances between the output of teachers from the process 6 (higher education) and the teachers hired to replace the teachers that left work are negative. As seen in Table 11, the largest difference was found in the market of teachers for pre-school, with a balance of
-3 714 teachers, followed by the markets of teachers of primary and lower secondary, upper secondary, and higher education.

When we compare the stock changes of the markets of labor force for education and the output of teachers from higher education, we observe that the graduates of higher education supply only for 61% of the teachers needed in the market of labor force for education. Table 12 shows this supply by type of market of labor force for education.

Table 11. Results: Stock changes in the markets of labor force for education

Pre-school teachers	**Primary and lower secondary teachers**	**Upper secondary teachers**	**Higher education teachers**
- 3 714	- 1 041	- 987	- 77

Table 12. Results: Sufficiency of teacher supply by the education system

	Markets			
	Pre-school teachers	Primary and lower secondary teachers	Upper secondary teachers	Higher education teachers
Supply by graduates of higher education	36%	73%	73%	95%

Table 13. Results: Labor force in education: Stocks and outflows of teachers

	Stock	**Desert**	**Retire**	**Die**	**Sum of desertions, retirements and deaths**
Pre-school	51 346	1 493	2 105	253	3 851
Primary and lower secondary	72 427	1 901	2 969	357	5 227
Upper secondary	27 138	830	1 264	177	2 271
Tertiary vocational programmes	1 754	63	82	11	156
Higher education	21 799	221	967	139	1 327
Total	174 464	4 508	7 387	937	12 832
Pre-school	100%	2,9%	4,1%	0,5%	7,5%
Primary and lower secondary	100%	2,6%	4,1%	0,5%	7,2%
Upper secondary	100%	3,1%	4,7%	0,7%	8,4%
Tertiary vocational programmes	100%	3,6%	4,7%	0,7%	8,9%
Higher education	100%	1%	4,4%	0,6%	6,1%
Total	100%	2,6%	4,2%	0,5%	

Table 14. Results: Retirement: Stock changes, inflows, and outflows

Retirement					
	Teachers that retired	**Rest of the population that retired**	**Retired population**	**Deaths**	**Stock change**
	7 387	34 469	705 000	18 856	23 000
Sum	41 856				

3.4 Process group 3: Labor force in education

Of the teachers that left work, those of primary and lower secondary education lead with the highest number (5 227), followed by pre-school teachers (3 851), and upper secondary teachers (2 271), higher education teachers(1 327) and tertiary vocational teachers (156). Overall, the leave of teachers represent between 6,1 and 8,9% of the stocks of teachers at each level. Detailed flows of teachers that leave work are in Table 13.

4. Discussion

In our model, the negative balances in the stocks of markets of labor force in education confirm that the supply of teachers by the education system is insufficient to substitute the teachers that leave work. The largest insufficiency was found to be that of teachers of pre-school followed by teachers for primary and lower secondary school, while the least insufficiency was that of teachers for higher education (even assuming that all PhDs enter the market of labor force for higher education.

In contrast, results in reports of the SSB (Roksvaag & Texmon 2012) show that the largest accumulated undersupply of teachers (for 2015) is that of general teachers, followed by pre-school teachers (See Appendix I). Although the results of both studies cannot be compared directly due to the different scopes and years of reference, we advise to understand the drivers and mechanisms of the systems with caution: if possible, with a holistic approach.

In the decade 2003-2013 the number of teachers in kindergarten has almost doubled. This sunk the pupil/teacher ratio from 8,13 to 5,6 in that time period (See Appendix X). In contrast, the number of teachers in primary and lower secondary decreased slightly from 2010 to 2013 with an average student/teacher ratio of 8,4 in the same period; the same value as 1999 (See Appendix XII). Yet, according to our results, almost as many teachers in pre-school as in primary and lower secondary school started work in 2013.

On the other hand, we might have estimated the age composition of teachers in pre-school to be too old, which increased the flow of teachers to retirement and overestimated the need for newcomers. In spite of this, even if we halve the flows of retirement and desertion of teachers of pre-school, the education system is not capable of providing for new teachers that year; the stock change of the market of pre-school teachers remains larger: -2 596 and -1 024 for the markets of pre-school and primary and lower secondary teachers, respectively.

This suggests that pre-school education is substituting their labor force with teachers from the rest of the population more effectively than compulsory education. Other reasons may be the stricter requirements to work as a teacher in primary and lower secondary school than in pre-school, or even the attractiveness to work as a pre-school teacher is higher.

Although the sufficiency of the supply of teachers in higher education was calculated of 95% in 2013, we did not study the number of PhDs or professors that start work in other sectors, or those that leave the country. The latter flow might be significant, for ca. 35% of PhD students in 2012 were foreign citizens (SSB 2014).

The supply of teachers is clearly dependent on the type of degree students choose to pursue. Increasing the number of enrollments and graduations of teachers is a key factor to secure the supply of teachers in all education levels. At the same time, increasing the number of students in the academic strain of upper secondary education and the total enrollments to higher education can increase the chances of providing for more teachers. In addition, decreasing the average time of students in higher education can lead to a faster supply of all types of graduates including teachers. The higher education system in Norway faces a challenge in this regard, for although 40% of students complete higher education studies in 5,64 years, 30% of students complete their degrees in 8 years or more (see Appendix VI). This decreases the rate at which students attain qualifications, affecting of all types of graduates.

4.1 Suggestions for future work

The study of services used and provided by the population and the resources necessary to supply them may benefit by the implementation of a dynamic approach models. For the education sector, this thesis is a small contribution to the understanding of the dynamics, nature, and behavior of the population in the education system, as suggestions of parameters that drive the size of stocks and flows in the education system at hand.

This work could be continued by two different approaches. One approach could be development of a dynamic model to analyze the supply and demand of the labor force for education in relation with the behavior and choices of students (particularly from upper secondary and higher education) and the behavior of the labor force. In addition to the dynamics of the population related to age (e.g. retirement age), the drivers in this model could be variable and include more detailed desertion and early retirement rates of teachers as well as a more detailed output of graduates of higher education. Systematic comparisons of scenarios and sensitivities could give insights on which changes in the system are more effective or efficient to avoid the undersupply of the working force.

Another approach for continuing this work could focus on the assessment of the stocks of demanders of educational services as drivers for other stocks of infrastructure, resources, and other services. For an infrastructure approach, for instance, a model would require specific data on buildings like units of service and their lifetimes. In this case, insights in this study such as drop out rates, time spent during studies, migration, throughputs of foreign students, and other parameters and behaviors may aid more accurate assessments of these stocks.

For a dynamic approach of the system and population, a model would require more data. Especially data on the behavior and needs of the population as they study, work and age, but these could provide for a better analysis of the production of teaching staff and possibly other types of labor force for future years.

4.2 Conclusions

We conclude that MFA frameworks and methods are helpful to conceptualize system services and to find patterns that affect the social environments. In the education system in Norway we found that age is a strong factor that shapes the demand of teachers in addition to desertion patterns. For the stocks and flows of students, age is a determinant factor for some education levels, but for more advanced levels of education, the real durations of the programmes and the choices of the population are more determining factors.

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Appendix I. Balances in supply and demand of labor force by education

Tabell A.5. Behovet for lærere i antall personer[1], 2015, 2020, 2025, 2030 og 2035

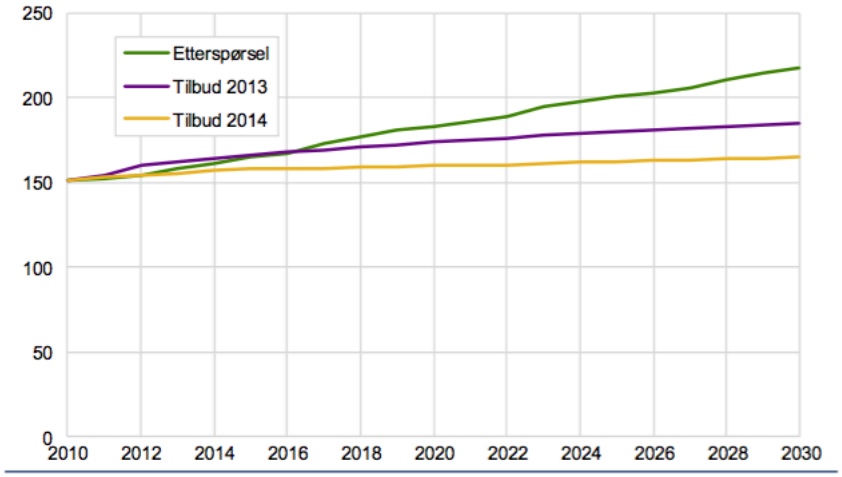
	Balanse i demografialternativet					Balanse i referansealternativet				
	Allmenn-lærere	Førskole-lærere	Faglærere	PPU for universitets- og høgskole-kandidater	PPU for yrkesfag	Allmenn-lærere	Førskole-lærere	Faglærere	PPU for universitets- og høgskole-kandidater	PPU for yrkesfag
Årsverl										
2015	-5 091	-1 333	491	639	1 332	-6 497	-2 546	175	-111	999
2020	-8 964	-950	1 151	955	2 626	-11 139	-2 785	662	-179	2 126
2025	-11 657	-241	1 653	1 130	3 289	-14 972	-2 930	905	-581	2 534
2030	-13 127	2 070	2 029	1 430	3 736	-18 802	-2 249	748	-1 472	2 458
2035	-13 718	4 105	2 135	1 047	3 700	-22 746	-2 474	68	-3 628	1 630
Antall personer										
2015	-5 720	-1 457	525	730	1 522	-7 300	-2 783	187	-127	1 142
2020	-10 072	-1 038	1 231	1 092	3 001	-12 516	-3 044	708	-204	2 430
2025	-13 098	-263	1 768	1 292	3 759	-16 822	-3 202	968	-664	2 896
2030	-14 749	2 262	2 170	1 634	4 270	-21 126	-2 458	800	-1 682	2 809
2035	-15 413	4 486	2 283	1 196	4 229	-25 557	-2 704	73	-4 146	1 863

[1] Behovet for antall personer er beregnet ved å se på gjennomsnittlig avtalte årsverk blant de sysselsatte. For eksempel for allmennlærerne hvor gjennomsnittlig avtalte årsverk er 89 prosent. 1 allmennlærer utfører dermed 0,89 årsverk, eller sagt på en annen måte: For å utføre ett årsverk er det behov for behov for 1,12 allmennlærere. Dette regnestykket tar ikke hensyn til endringer i kjønns- og alderssammensetningen i løpet av perioden.

Figure 7. Balances of the need of different types of teachers. LÆRERMOD results (SSB 2012)

Source: Statistics Norway (2012)

Figur 27. Lærere, kort høyere utdanning. 1000 personer



Kilde: Statistisk sentralbyrå.

Figure 8. Supply and demand for teachers (SSB 2014)

Source: Statistics Norway (2014)

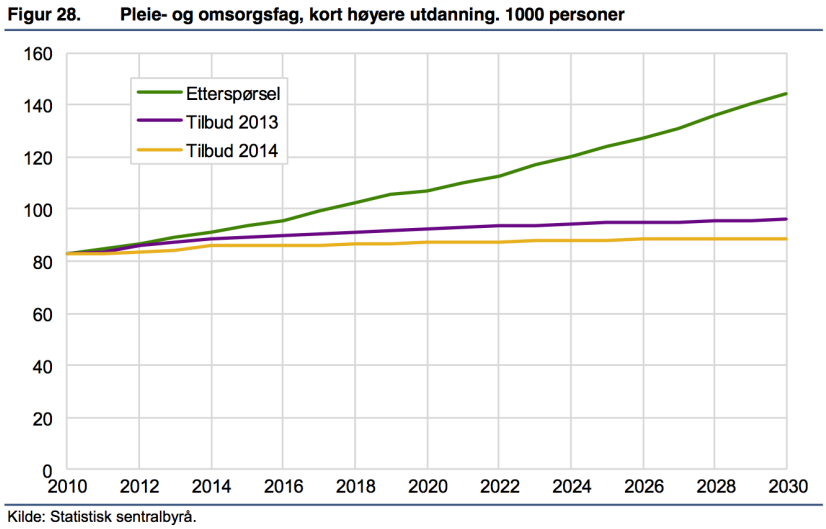


Figure 9. Supply and demand of nurses (SSB, 2014)

Source: Statistics Norway (2014)

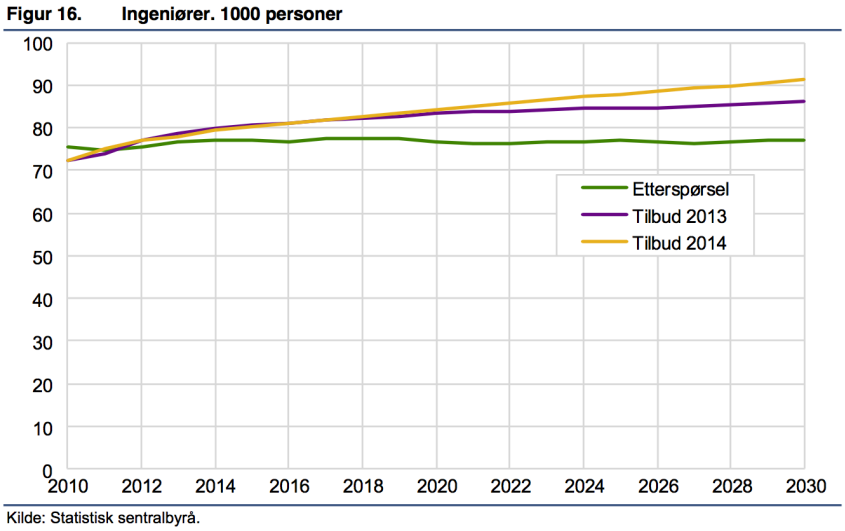


Figure 10. Supply and demand of engineers (SSB 2014)

Source: Statistics Norway (2014)

Appendix II. Conceptual MFA systems of the Norwegian education system

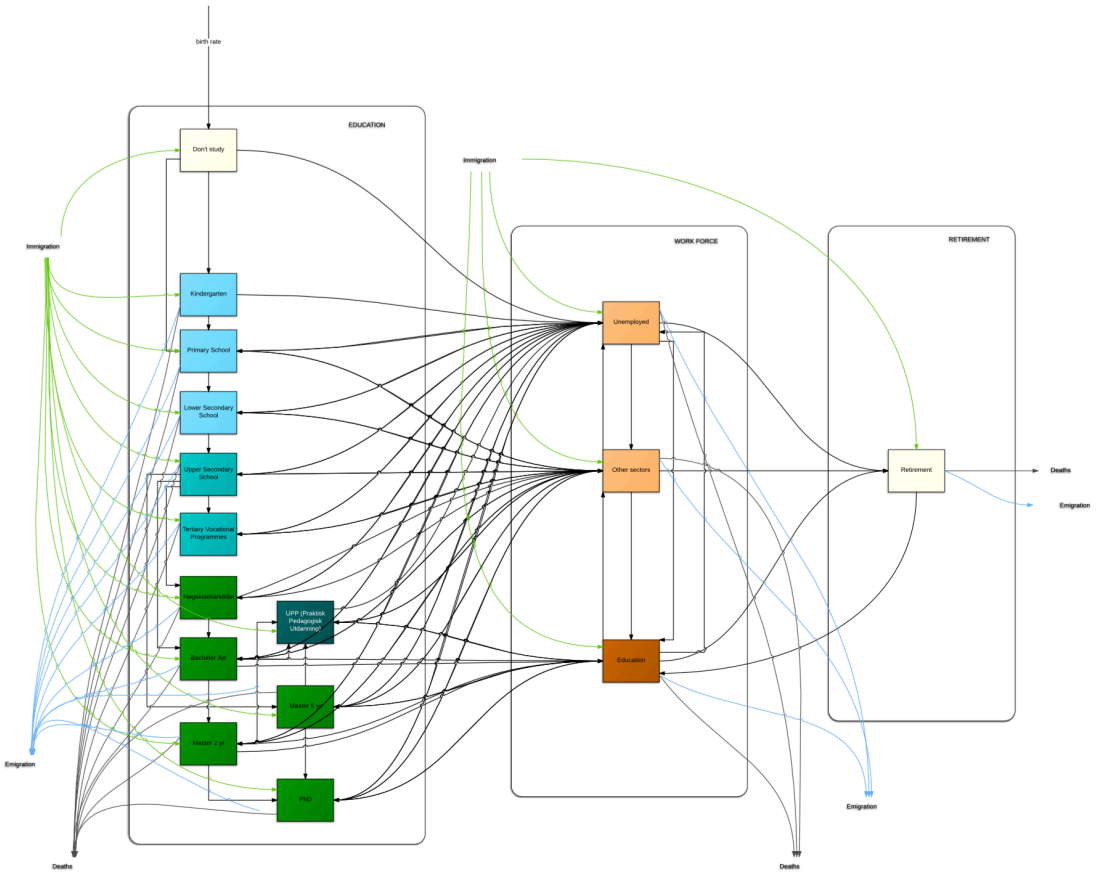


Figure 11. System concept 1.

In this system concept, the blue and green boxes represent the education system and the orange boxes are unemployed, employed in other sectors, and employed in education. The white box on the top left represents the people that do not study, and the white box on the right represents the retired people. The education system is very disaggregated, distinguishing between strains of upper secondary education and the different offer of higher education including PPU.

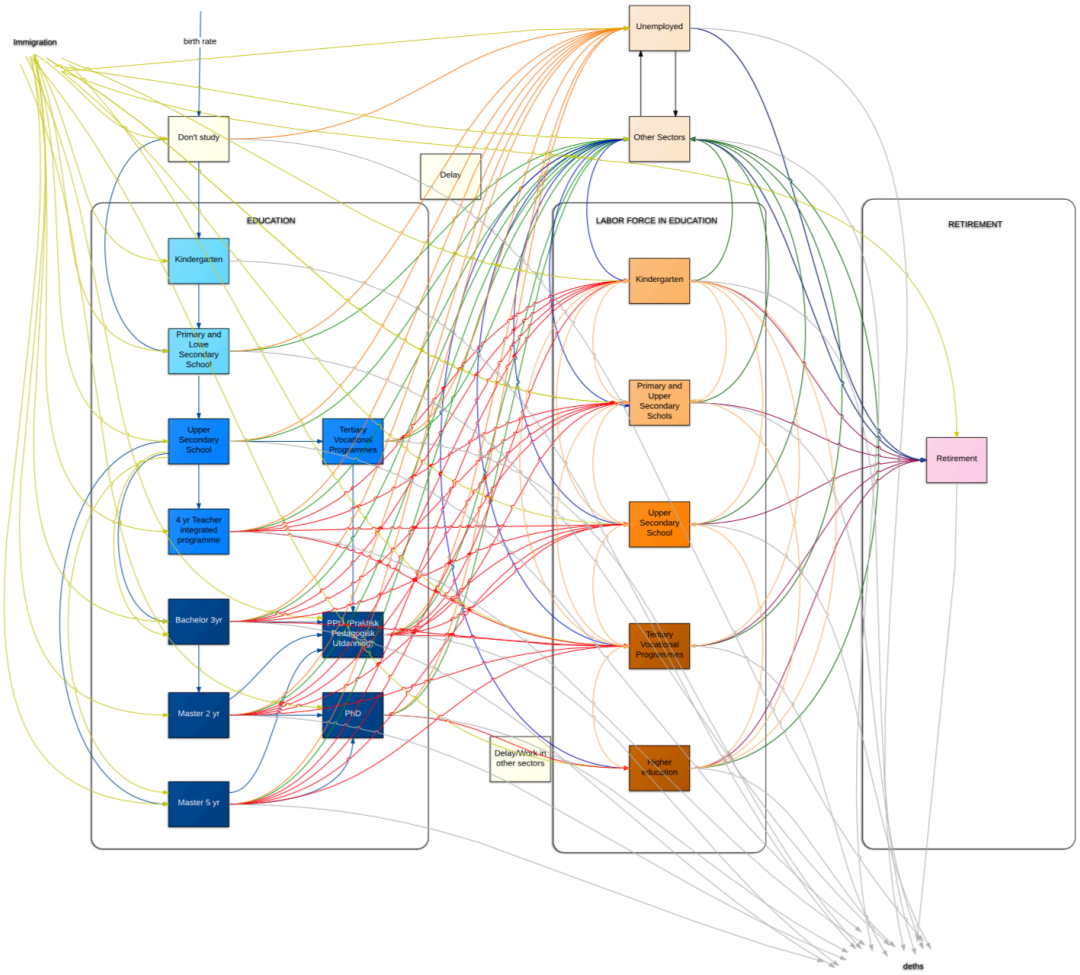


Figure 12. System concept 2.

This system is the closest to the system modeled in this thesis, but including stocks of unemployment and employment in other sectors than education. It also visualizes the flows of migration and deaths from all the processes.

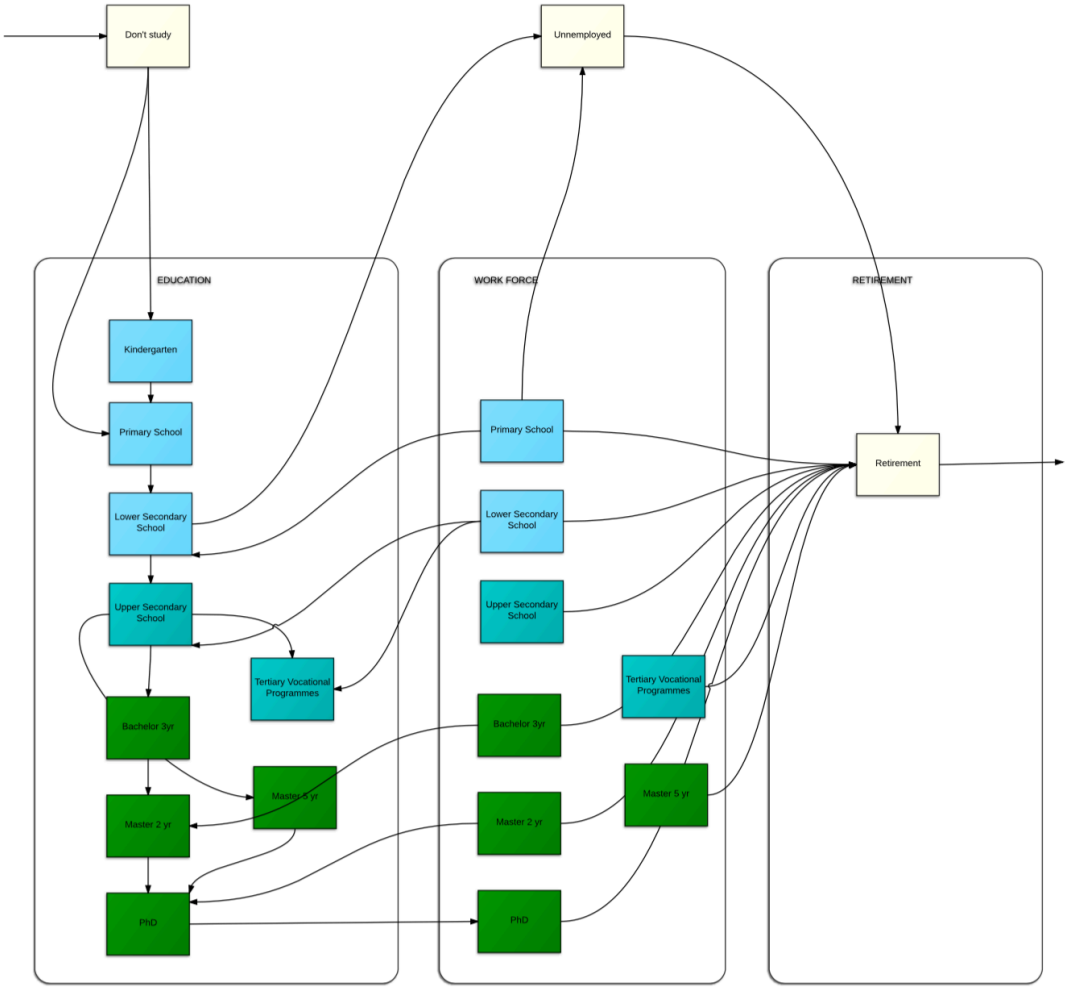


Figure 13. System concept 3.

This system concept is focused only in education (including higher education in different processes) and would model the education system (left), the qualified people as teachers (middle), and their retirement (right).

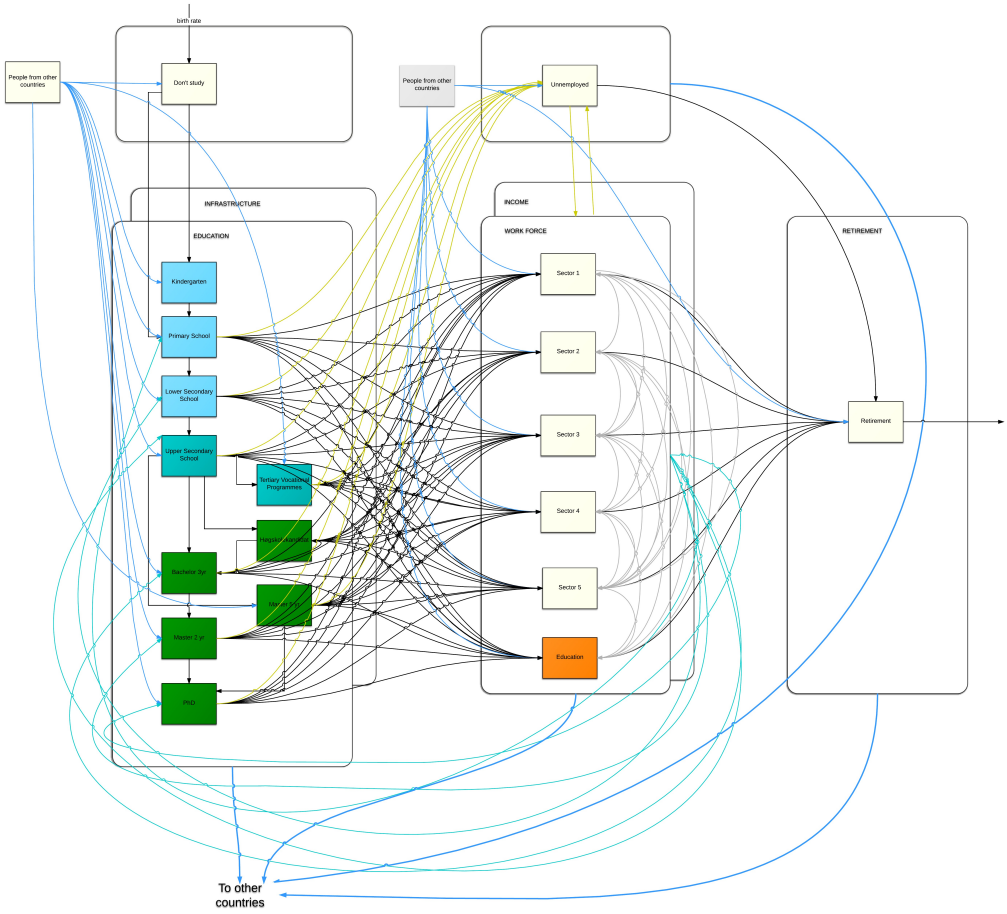


Figure 14. System concept 4.

This system has the education system on the left side, including higher education in separate processes. In the middle, the white boxes are the stocks of employed people in different sectors. The orange box aggregates all the teachers that work in education. And on top left, top middle, and right, are people that do not study, unemployed, and retired.

Appendix III. System Variables

Flows

Variable count	Symbol	Known/ Unkown	Description
1	a 1,2	U	Pupils enrolling in kindergarten
2	a 2,3	U	Pupils leaving kindergarten and enrolling primary and lower secondary school
3	a 1,3	U	Pupils enrolling primary and lower secondary school that do not were in kindergarten the same year
4	a 3,1	U	Pupils and students that finished primary and lower secondary education but did not enroll upper secondary school
5	b 3,1	U	Pupils and students that dropped out primary and lower secondary school
6	a 3,4	K	Students that finished primary and lower secondary and enrolled upper secondary education the same year
7	a 1,4	U	Students that enrolled upper secondary education that were not in primary and lower secondary school the same year they enrolled
8	a 4,1	U	Students that finished upper secondary education and did not enroll higher education
9	b 4,1	U	Students that dropped out upper secondary education
10	a 4,6	U	Students that finished upper secondary education and enrolled in higher education the same year
11	a 5,1	U	Students finishing or dropping tertiary vocational programmes

Variable count	Symbol	Known/ Unkown	Description
12	a 1,5	U	Students that enrolled in tertiary vocational programmes
13	a 1,6	U	Students that enrolled higher education that were not in upper secondary education the year of enrollment
14	c 6,1	U	Students that leave higher education without completing a degree
15	a 6,1	U	Graduations of higher education programmes other than education-related programmes and PhDs
16	b 6,1	U	Students that drop out higher education
17	a 6,8	K	Graduations of higher education programmes that give qualifications to teach in pre-school
18	a 6,9	K	Graduations of higher education programmes that give qualifications to teach in primary and lower secondary education (general teachers)
19	a 6,10	K	Graduations of higher education programmes that give qualifications to teach in higher education (subject teachers and PPU)
20	a 6,11	K	Graduations of PhDs
21	a 1,8	K*	People from the rest of the population entering the market for people with qualifications to teach in kindergartens
22	a 1,9	K*	People from the rest of the population entering the market for people with qualifications to teach in primary and lower secondary education
23	a 1,10	K*	People from the rest of the population entering the market for people with qualifications to teach in upper secondary schools
24	a 1,11	K*	People from the rest of the population entering the market for people with qualifications to teach in higher education
25	a 8,12	U	Preschool/kindergarten teachers hired in kindergarten education
26	a 8,13	U	Preschool/kindergarten teachers hired in primary and lower secondary education
27	a 8,14	U	Preschool/kindergarten teachers hired in upper secondary education

Variable count	Symbol	Known/ Unkown	Description
28	a 8,15	U	Preschool/kindergarten teachers hired in tertiary vocational education education
29	a 8,16	U	Preschool/kindergarten teachers hired in higher education
30	a 9,12	U	Primary and lower secondary teachers hired in kindergarten education
31	a 9,13	U	Primary and lower secondary teachers hired in primary and lower secondary education
32	a 9,14	U	Primary and lower secondary teachers hired in upper secondary education
33	a 9,15	U	Primary and lower secondary teachers hired in tertiary vocational education
34	a 9,16	U	Primary and lower secondary teachers hired in higher education
35	a 10,12	U	Subject teachers and teachers with PPU hired in kindergarten education
36	a 10,13	U	Subject teachers and teachers with PPU hired in primary and lower secondary education
37	a 10,14	U	Subject teachers and teachers with PPU hired in upper secondary education
38	a 10,15	U	Subject teachers and teachers with PPU hired in tertiary vocational education
39	a 10,16	U	Subject teachers and teachers with PPU hired in higher education
40	a 11,16	U	PhDs hired as professors and teachers in higher education
41	a 0,1	U	Births
42	b 0,1	U	Immigration
43	a 1,0	U	Deaths of the rest of the population
44	b 1,0	U	Emigration
45	a 7,0	U	Deaths of retired people
46	a 1,7	U	People from the rest of the population that retired
47	a 12,7	U	Teachers of kindergarten that retired
48	a 13,7	U	Teachers of primary and lower secondary education that retired
49	a 14,7	U	Teachers of upper secondary education that retired
50	a 15,7	U	Teachers of tertiary vocational education that retired

Variable count	Symbol	Known/ Unkown	Description
51	a 16,7	U	Teachers of higher education that retired
52	a 12,1	U	Teachers of kindergarten that leave work (other than retirement, early retirement, and disability)
53	a 13,1	U	Teachers of primary and lower secondary education that leave work (other than retirement, early retirement, and disability)
54	a 14,1	U	Teachers of upper secondary education that leave work (other than retirement, early retirement, and disability)
55	a 15,1	U	Teachers of tertiary vocational education that leave work (other than retirement, early retirement, and disability)
56	a 16,1	U	Professors and teachers of higher education that leave work (other than retirement, early retirement, and disability)
57	a 2,0	U	Deaths of pupils in kindergarten
58	a 3,0	U	Deaths of pupils and students in primary and lower secondary education
59	a 4,0	U	Deaths of students in upper secondary education
60	a 5,0	U	Deaths of students in tertiary vocational education
61	a 6,0	U	Deaths of students in higher education
62	a 8,0	K*	Emigration of teachers of kindergarten
63	a 9,0	K*	Emigration of teachers of primary and lower secondary education
64	a 10,0	K*	Emigration of teachers of upper secondary education
65	a 11,0	K*	Emigration of professors and teachers of higher education
66	a 0,8	K*	Immigration of teachers of pre-school
67	a 0,9	K*	Immigration of teachers of primary and lower secondary education
68	a 0,10	K*	Immigration of teachers of upper secondary education
69	a 0,11	K*	Immigration of professors and teachers of higher education
70	a 12,0	U	Deaths of teachers in kindergartens
71	a 13,0	U	Deaths of teachers in primary and lower secondary schools
72	a 14,0	U	Deaths of teachers in upper secondary schools

Variable count	Symbol	Known/ Unkown	Description
73	a 15,0	U	Deaths of teachers in post-secondary non-tertiary schools
74	a 16,0	U	Deaths of teachers in higher education

* Assumed as zero or inexistent

Stock changes

Variable count	Symbol	Known/ Unknown	Description
75	ΔS1	K	Stock change of the rest of the population
76	ΔS2	K	Stock change of pupils in kindergarten
77	ΔS3	K	Stock change of pupils in primary and lower secondary education
78	ΔS4	K	Stock change of pupils in upper secondary education
79	ΔS5	K	Stock change of students in tertiary vocational education
80	ΔS6	K	Stock change of students in higher education
81	ΔS7	K	Stock change of retired people
82	ΔS8	U	Stock change of the market of qualified teachers for pre-school
83	ΔS9	U	Stock change of the market of qualified teachers for primary and lower secondary education
84	ΔS10	U	Stock change of the market of qualified teachers qualified for upper secondary education
85	ΔS11	U	Stock change of the market of teachers and professors qualified for higher education (PhDs)
86	ΔS12	K	Stock change of teachers in kindergarten
87	ΔS13	K	Stock change of teachers in primary and lower secondary education
88	ΔS14	K	Stock change of teachers in upper secondary education
89	ΔS15	K	Stock change of teachers in tertiary vocational education
90	ΔS16	K	Stock change of professors and teachers in higher education
91	ΔS17	K	Stock change of students in higher education abroad

Stocks

Variable count	Symbol	Known/ Unknown	Description
92	S1	K	Rest of the population
93	S2	K	Pupils in Kindergarten
94	S3	K	Pupils in Primary and Lower Secondary Education
95	S4	K	Students in Upper Secondary Education
96	S5	K	Students in Post-Secondary Non-Tertiary and other types of Upper Secondary Education
97	S6	K	Students in Higher Education
98	S7	K	Persons retired, early retired, and disabled
99	S8	K*	Market of labor force qualified to teach at kindergarten
100	S9	K*	Market of labor force qualified to teach in primary and lower secondary education
101	S10	K*	Market of labor force qualified to teach in upper secondary education
102	S11	K*	Market of labor force qualified to teach in higher education
103	S12	K	Teachers in Kindergartens
104	S13	K	Teachers in Primary and Lower Secondary schools
105	S14	U	Teachers in Upper Secondary schools
106	S15	U	Teachers in Post-secondary Non-Tertiary education
107	S16	K	Professors and teachers in Higher education
108	S17	K	Students in Higher Education abroad

* Assumed to be zero or inexistent

Total variables: 108

Variables assumed as zero or inexistent: 16

Known variables: 27

Number of processes: 16

Total number of unknowns: 49

Minimum number of model approach equations needed: 49

Mass balance equations needed: 16

Appendix IV. System Equations

Mass balance equations

Equation count	Equation	Description
1	$a_{0,1} + b_{0,1} + a_{3,1} + b_{3,1} + a_{4,1} + b_{4,1} + a_{12,1} + a_{13,1} + a_{14,1} + a_{15,1} + a_{16,1} - \left(a_{1,2_t} + a_{1,3} + a_{1,4} + a_{1,5} + a_{1,6} + a_{1,7} + a_{1,8_t} + a_{1,9} + a_{1,10} + a_{1,11} + a_{1,0} + b_{1,0}\right) - \Delta S1 = 0$	Balance equation for stock change of process 1
2	$S2 = S_{2_{t-1}} + a_{1,2} - a_{2,3} - a_{2,0}$	Balance equation for process 2
	$\Delta S2 = a_{1,2} - a_{2,3} - a_{2,0}$	Balance equation for stock change of process 2
3	$S3 = S3_{t-1} + a_{1,3} + a_{2,3} - a_{3,0} - b_{3,1} - a_{3,1} - a_{3,4}$	Balance equation for process 3
	$\Delta S3 = a_{1,3} + a_{2,3} - a_{3,0} - b_{3,1} - a_{3,1} - a_{3,4}$	Balance equation for stock change of process 3
4	$S4_t = S4_{t-1} + a_{3,4} + a_{1,4} - a_{4,1} - b_{4,1} - a_{4,6} - a_{4,0}$	Balance equation for process 4
	$\Delta S4 = a_{1,4} + a_{3,4} - b_{4,1} - a_{4,6} - a_{4,1} - a_{4,0}$	Balance equation for stock change of process 4

Equation count	Equation	Description
5	$S5_t = S5_{t-1} + a_{1,5} - a_{5,1} - a_{5,0}$	Balance equation for process 5
	$\Delta S5 = a_{1,5} - a_{5,1} - a_{5,0}$	Balance equation for stock change of process 5
6	$S6_t = I6 - \left[H_k - H_g - H_v - H_h\right]_t - b_{6,1_t} + a_{1,6_t} + a_{4,6_t} - a_{6,0} - c_{6,1}$	Balance equation for process 6
	$\Delta S6 = a_{1,6} + a_{4,6} - a_{6,8} - a_{6,9} - a_{6,10} - a_{6,11} - b_{6,1} - a_{6,0} - c_{6,1}$	Balance equation for stock change of process 6
7	$S7 = S7_{t-1} + a_{1,7} + a_{12,7} + 1_{13,7} + a_{14,7} + a_{15,7} + a_{16,7} - a_{7,0}$	Balance equation for process 7
	$\Delta S7 = a_{1,7} + a_{12,7} + 1_{13,7} + a_{14,7} + a_{15,7} + a_{16,7} - a_{7,0}$	Balance equation for stock change of process 7
8	$\Delta S8 = a_{1,8} + a_{6,8} - a_{8,12} - a_{9,12} - a_{10,12} + a_{0,8} - a_{8,0}$	Balance equation for stock change of process 8
9	$\Delta S9 = a_{1,9} + a_{6,9} - a_{8,13} - a_{9,13} - a_{10,13} + a_{0,9} - a_{9,0}$	Balance equation for stock change of process 9
10	$\Delta S10 = a_{1,10} + a_{6,10} - a_{10,12} - a_{10,13} - a_{10,14} - a_{10,15} + a_{0,10} - a_{10,0}$	Balance equation for stock change of process 10
11	$\Delta S11 = a_{1,11} + a_{6,11} - a_{11,16} + a_{0,11} - a_{11,0}$	Balance equation for stock change of process 11
12	$\Delta S12 = a_{8,12} + a_{9,12} + a_{10,12} - a_{12,1} - a_{12,7} - a_{12,0}$	Balance equation for stock change of process 12
13	$\Delta S13 = a_{8,13} + a_{9,13_t} + a_{10,13} - a_{13,1} - a_{13,7} - a_{13,0}$	Balance equation for stock change of process 13
15	$\Delta S14 = a_{8,14} + a_{9,13} + a_{10,14} - a_{14,1} - a_{14,7} - a_{14,0}$	Balance equation for stock change of process 14

Equation count	Equation	Description
15	$\Delta S15 = a_{8,15} + a_{9,15} + a_{10,15} - a_{15,1} - a_{15,7} - a_{15,0}$	Balance equation for stock change of process 15
16	$\Delta S16 = a_{8,16} + a_{9,16} + a_{10,16} + a_{11,16} - a_{16,1} - a_{16,7} - a_{16,0}$	Balance equation for stock change of process 16

Model approach equations

Equation count	Equation
1	$a_{1,2} = C_{k_t} P_{0-6_t} - C_{k_{t-1}} P_{0-6_{t-1}} + S2_{5-6_{t-1}} + a_{2,0}$
2	$a_{3,1} + a_{3,4} = G_{10}$
3	$a_{1,3} = \frac{G_v}{10} + P_6 - a_{2,3} + a_{3,0} + b_{3,1} + a_{3,1}$
4	$a_{1,3} = G_1 - a_{2,3} + a_{3,0} + b_{3,1} + a_{3,1}$
5	$a_{1,4} = V_1 - a_{3,4} + a_{4,0}$
6	$a_{1,6_t} = I6_t - a_{4,6_t}$
7	$\boldsymbol{a_{4,6}} + \boldsymbol{a_{4,1}} = \frac{\left(V_{ac3_t} + V_{\neq ac3_t}\right) b_{4,1_t}}{S4} + V_{ac3} + V_{\neq ac3}$
8	$a_{3,4} = \theta_t G_{10}$
9	$a_{4,6} = \eta V_{ac3}$
10	$a_{4,6} = I6_{0-20} + \eta V_{21} V_{ac3}$
11	$a_{5,1} = \frac{S5}{\rho}$
12	$a_{6,1} = O6 - a_{6,8} - a_{6,9} - a_{6,10} - a_{6,11}$
13	$a_{6,8} = H_k$
14	$a_{6,8} = H_k$
15	$a_{6,8} = H_k$
16	$a_{6,8} = H_k$
17	$a_{6,8} = H_k$
18	$a_{6,8_t} = h_k O6$

Equation count	Equation
19	$a_{6,9_t} = h_g\ O6$
20	$a_{6,10_t} = h_v\ O6$
21	$a_{6,11_t} = h_h\ O6$
22	$b_{3,1} = \frac{\mu_3}{\sigma_3}\sum I3\ (t)\ Y3\ (t)$
23	$b_{4,1} = \frac{\mu_4}{\sigma_4}\sum I4\ (t)\ Y4\ (t)$
24	$b_{6,1} = \frac{\mu_6}{\sigma_6}\sum I6\ (t)\ Y6\ (t)$
25	$a_{12,7} = \frac{A_{g_{60}}}{67-60}\ S12 + \vartheta\left(S12 - \frac{A_{g_{60}}}{67-60}\right)$
26	$a_{13,7} = \frac{A_{g_{60}}}{67-60}\ S13 + \vartheta\left(S13 - \frac{A_{g_{60}}}{67-60}\right)$
27	$a_{14,7} = \frac{A_{v_{60}}}{67-60}\ S14 + \vartheta\left(S14 - \frac{A_{v_{60}}}{67-60}\right)$
28	$a_{15,7} = \frac{A_{v_{60}}}{67-60}\ S15 + \vartheta\left(S15 - \frac{A_{v_{60}}}{67-60}\right)$
29	$a_{16,7} = \frac{A_{h_{60}}}{67-60}\ S16 + \vartheta\left(S16 - \frac{A_{h_{60}}}{67-60}\right)$
30	$a_{12,1} = \Omega\left(a_{8,12_t} + a_{9,12} + a_{10,12}\right) + \zeta\ S12$
31	$a_{13,1} = \Omega\left(a_{8,13_t} + a_{9,13} + a_{10,13}\right) + \zeta\ S13$
32	$a_{14,1} = \Omega\left(a_{8,14} + a_{9,13} + a_{10,14}\right) + \zeta\ S14$
33	$a_{15,1} = \Omega\left(a_{8,15} + a_{9,15} + a_{10,15}\right) + \zeta\ S15$
34	$a_{16,1} = \Omega\left(a_{8,16} + a_{9,16} + a_{10,16} + a_{11,16}\right) + \zeta\ S16$
35	$a_{8,12} = \frac{l_{8,12}\left(\zeta\ S12 + a_{12,7} + a_{12,0} + \Delta S12\right)}{1-\Omega}$
36	$a_{9,12} = \frac{l_{9,12}\left(\zeta\ S12 + a_{12,7} + a_{12,0} + \Delta S12\right)}{1-\Omega}$
37	$a_{10,12} = \frac{l_{10,12}\left(\zeta\ S12 + a_{12,7} + a_{12,0} + \Delta S12\right)}{1-\Omega}$
38	$a_{8,13} = \frac{l_{8,13}\left(\zeta\ S13 + a_{13,7} + a_{13,0} + \Delta S13\right)}{1-\Omega}$
39	$a_{9,13} = \frac{l_{9,13}\left(\zeta\ S13 + a_{13,7} + a_{13,0} + \Delta S13\right)}{1-\Omega}$
40	$a_{10,13} = \frac{l_{10,13}\left(\zeta\ S13 + a_{13,7} + a_{13,0} + \Delta S13\right)}{1-\Omega}$
41	$a_{8,14} = \frac{l_{8,14}\left(\zeta\ S14 + a_{14,7} + a_{14,0} + \Delta S14\right)}{1-\Omega}$

Equation count	Equation
42	$a_{9,14} = \frac{l_{9,14}\left(\zeta\, S14 + a_{14,7} + a_{14,0} + \Delta S14\right)}{1-\Omega}$
43	$a_{10,14} = \frac{l_{10,14}\left(\zeta\, S14 + a_{14,7} + a_{14,0} + \Delta S14\right)}{1-\Omega}$
44	$a_{8,15} = \frac{l_{8,14}\left(\zeta\, S15 + a_{15,7} + a_{15,0} + \Delta S15\right)}{1-\Omega}$
45	$a_{9,15} = \frac{l_{9,15}\left(\zeta\, S15 + a_{15,7} + a_{15,0} + \Delta S15\right)}{1-\Omega}$
46	$a_{10,15} = \frac{l_{10,14}\left(\zeta\, S15 + a_{15,7} + a_{15,0} + \Delta S15\right)}{1-\Omega}$
47	$a_{8,16} = \frac{l_{8,16}\left(\zeta\, S16 + a_{16,7} + a_{16,0} + \Delta S16\right)}{1-\Omega}$
48	$a_{9,16} = \frac{l_{9,16}\left(\zeta\, S16 + a_{16,7} + a_{16,0} + \Delta S16\right)}{1-\Omega}$
49	$a_{10,16} = \frac{l_{10,16}\left(\zeta\, S16 + a_{16,7} + a_{16,0} + \Delta S16\right)}{1-\Omega}$
50	$a_{11,16} = \frac{l_{11,16}\left(\zeta\, S16 + a_{16,7} + a_{16,0} + \Delta S16\right)}{1-\Omega}$
51	$a_{1,0} = D - a_{7,0} - a_{2,0} - a_{3,0} - a_{4,0} - a_{5,0} - a_{6,0} - a_{12,0} - a_{13,0} - a_{14,0} - a_{15,0} - a_{16,0}$
52	$a_{2,0} = d_{1-5}\, S2$
53	$a_{3,0} = d_{6-15}\left(1 - C_{gv}\right) S3 + d_{20-59}\, C_{gv}\, S3$
54	$a_{4,0} = d_{16-19}(1 - V_{21})\, S4 + d_{20-59}\, V_{21}\, S4$
55	$a_{5,0} = d_{16-19}(1 - V_{21})\, S4 + d_{20-59}\, V_{21}\, S5$
56	$a_{6,0} = d_{20-59}\, S6$
57	$a_{7,0} = \left(d_{15-19}\, A_{r_{15-24}} + d_{20-59}\ A_{r_{25-54}} + d_{60}\, A_{r_{55}}\right) S7$
58	$a_{12,0} = \left(A_{g29}\, d_{20-29} + A_{g30-39}\, d_{30-39} + A_{g40-49}\, d_{40-49} + A_{g50-59}\, d_{50-59} + A_{g60}\, d_{60}\right) S12$
Equation count	**Equation**
59	$a_{13,0} = \left(A_{g29}\, d_{20-29} + A_{g30-39}\, d_{30-39} + A_{g40-49}\, d_{40-49} + A_{g50-59}\, d_{50-59} + A_{g60}\, d_{60}\right) S13$
60	$a_{14,0} = (A_{v29}\, d_{20-29} + A_{v30-39}\, d_{30-39} + A_{v40-49}\, d_{40-49} + A_{v50-59}\, d_{50-59} + A_{v60}\, d_{60})\, S14$
61	$a_{15,0} = (A_{v29}\, d_{20-29} + A_{v30-39}\, d_{30-39} + A_{v40-49}\, d_{40-49} + A_{v50-59}\, d_{50-59} + A_{v60}\, d_{60})\, S15$
62	$a_{16,0} = (A_{h29}\, d_{20-29} + A_{h30-39}\, d_{30-39} + A_{h40-49}\, d_{40-49} + A_{h50-59}\, d_{50-59} + A_{h60}\, d_{60})\, S16$

Equation count	Equation
63	$S1 = P - S3 - S4 - S5 - S6 - S7 - S13 - S14 - S15 - S16 - S17$
64	$S14 = \varepsilon\, S4$
65	$S15 = \varepsilon\, S5$
66	$\Delta S14 = \varepsilon(S4 - S4_{t-1})$
67	$\Delta S15 = \varepsilon(S5 - S5_{t-1})$
68	$a_{0,1} = P_b$
69	$b_{0,1} = P_i$
70	$b_{1,0} = P_e$

Analytical solutions

Equation count	Equation
1	$a_{1,2} = C_{k_t} P_{0-6_t} - C_{k_{t-1}} P_{0-6_{t-1}} + S2_{5-6_{t-1}} + d_{0-5}\, S2$
2	$a_{1,3} = P_{6_t} - a_{2,3} + P_{\neq 6} C_{g_{\neq 6}} + \left(d_{6-15}(1 - C_{gv})\, S3 + d_{20-59}\, C_{gv}\, S3\right) + \frac{\mu_3}{\sigma_3} \sum I3\,(t)\, Y3\,(t)$
3	$a_{1,4} = \Delta S4 - \theta G_{10} + \left(\left(V_{ac3} + V_{\neq ac3}\right) - \eta_t V_{ac3} - \frac{\left(V_{ac3} + V_{\neq ac3}\right) \mu \sum I4\,(t)\, Y4\,(t)}{S4} \right) + \mu \sum I4\,(t)\, Y4\,(t) + d_{16-19}(1 - V_{21})\, S4 + d_{20-59}\, V_{21}\, S4 + \eta\, V_{ac3}$
4	$a_{1,5} = S5 + S5_{t-1} + \frac{S5}{\rho} + \left(d_{16-19}(1 - V_{21})\, S4 + d_{20-59}\, V_{21}\, S5\right)$
5	$a_{1,6} = I6_t - \eta_t V_{ac3_t}$

Equation count	Equation
6	$$a_{1,7} = \Delta S7 - \left(\frac{A_{g_{60}}}{67-60} S12 + \vartheta\left(S12 - \frac{A_{g_{60}}}{67-60}\right)\right) - \left(\frac{A_{g_{60}}}{67-60} S13 + \vartheta\left(S13 - \frac{A_{g_{60}}}{67-60}\right)\right) - \left(\frac{A_{v_{60}}}{67-60} S14 + \vartheta\left(S14 - \frac{A_{v_{60}}}{67-60}\right)\right) - \left(\frac{A_{v_{60}}}{67-60} S15 + \vartheta\left(S15 - \frac{A_{v_{60}}}{67-60}\right)\right) - \left(\frac{A_{h_{60}}}{67-60} S16 + \vartheta\left(S16 - \frac{A_{h_{60}}}{67-60}\right)\right)$$
7	$a_{1,8} = 0$
8	$a_{1,9} = 0$
9	$a_{1,10} = 0$
10	$a_{1,11} = 0$
11	$a_{2,3} = S2_{5-6_{t-1}}$
12	$a_{3,4} = \theta\, G_{10}$
13	$a_{4,6} = \eta\, V_{ac3}$
14	$\eta_t = \frac{I6_0^{20}}{V_{ac3} - V_{ac3} V_{21}}$
15	$a_{3,1} = (1-\theta)G_{10}$
16	$a_{4,1} = (V_{ac3} + V_{\neq ac3}) - \eta_t V_{ac3} + \frac{(V_{ac3} + V_{\neq ac3})\,\mu \sum I4\,(t)\, Y4\,(t)}{S4}$
17	$a_{5,1} = \frac{S5}{\rho}$
18	$a_{6,1} = O6 - H_k - H_g - H_v - H_h$
19	$c_{6,1} = I6 - O6 - \kappa \sum I3\,(t)\, Y3\,(t) - d_{20-59}\, S6 - \Delta S6$
20	$b_{3,1} = \frac{\mu_3}{\sigma_3} \sum I3\,(t)\, Y3\,(t)$
21	$b_{4,1} = \frac{\mu_4}{\sigma_4} \sum I4\,(t)\, Y4\,(t)$
22	$b_{6,1} = \frac{\mu_6}{\sigma_6} \sum I6\,(t)\, Y6\,(t)$
23	$a_{6,8_t} = H_k$
24	$a_{6,9_t} = H_g$
25	$a_{6,10_t} = H_v$
26	$a_{6,11_t} = H_h$

Equation count	Equation
27	$h_k = \frac{H_k}{O6}$
28	$h_g = \frac{H_g}{O6}$
29	$h_v = \frac{H_v}{O6}$
40	$h_h = \frac{H_h}{O6}$
31	$a_{0,8} = 0$
32	$a_{0,9} = 0$
33	$a_{0,10} = 0$
34	$a_{0,11} = 0$
35	$a_{8,0} = 0$
36	$a_{9,0} = 0$
37	$a_{10,0} = 0$
38	$a_{11,0} = 0$
39	$a_{12,7} = \frac{A_{g_{60}}}{67-60}\, S12 + \vartheta\left(S12 - \frac{A_{g_{60}}}{67-60}\right)$
40	$a_{13,7} = \frac{A_{g_{60}}}{67-60}\, S13 + \vartheta\left(S13 - \frac{A_{g_{60}}}{67-60}\right)$
41	$a_{14,7} = \frac{A_{v_{60}}}{67-60}\, S14 + \vartheta\left(S14 - \frac{A_{v_{60}}}{67-60}\right)$
42	$a_{15,7} = \frac{A_{v_{60}}}{67-60}\, S15 + \vartheta\left(S15 - \frac{A_{v_{60}}}{67-60}\right)$
43	$a_{16,7} = \frac{A_{h_{60}}}{67-60}\, S16 + \vartheta\left(S16 - \frac{A_{h_{60}}}{67-60}\right)$
44	$a_{1,0} = D - a_{7,0} - a_{2,0} - a_{3,0} - a_{4,0} - a_{5,0} - a_{6,0} - a_{12,0} - a_{13,0} - a_{14,0} - a_{15,0} - a_{16,0}$
45	$a_{2,0} = d_{1-5}\, S2$
46	$a_{3,0} = d_{6-15}(1 - C_{gv})\, S3 + d_{20-59}\, C_{gv}\, S3$
47	$a_{4,0} = d_{16-19}(1 - V_{21})\, S4 + d_{20-59}\, V_{21}\, S4$
48	$a_{5,0} = d_{16-19}(1 - V_{21})\, S4 + d_{20-59}\, V_{21}\, S5$
49	$a_{6,0} = d_{20-59}\, S6$
50	$a_{7,0} = \left(d_{15-19}\, A_{r_{15-24}} + d_{20-59}\ A_{r_{25-54}} + d_{60}\, A_{r_{55}}\right) S7$
51	$a_{12,0} = \left(A_{g29}\, d_{20-29} + A_{g30-39}\, d_{30-39} + A_{g40-49}\, d_{40-49} + A_{g50-59}\, d_{50-59} + A_{g60}\, d_{60}\right) S12$

Equation count	Equation
52	$a_{13,0} = \left(A_{g29}\, d_{20-29} + A_{g30-39}\, d_{30-39} + A_{g40-49}\, d_{40-49} + A_{g50-59}\, d_{50-59} + A_{g60}\, d_{60}\right) S13$
53	$a_{14,0} = \left(A_{v29}\, d_{20-29} + A_{v30-39}\, d_{30-39} + A_{v40-49}\, d_{40-49} + A_{v50-59}\, d_{50-59} + A_{v60}\, d_{60}\right) S14$
54	$a_{15,0} = \left(A_{v29}\, d_{20-29} + A_{v30-39}\, d_{30-39} + A_{v40-49}\, d_{40-49} + A_{v50-59}\, d_{50-59} + A_{v60}\, d_{60}\right) S15$
55	$a_{16,0} = \left(A_{h29}\, d_{20-29} + A_{h30-39}\, d_{30-39} + A_{h40-49}\, d_{40-49} + A_{h50-59}\, d_{50-59} + A_{h60}\, d_{60}\right) S16$
56	$S1 = P - S3 - S4 - S5 - S6 - S7 - S13 - S14 - S15 - S16 - S17$
57	$S2 = C_k\, P_{0-6}$
58	$S3 = S3$
59	$S4 = S4$
60	$S5 = S5$
61	$S6 = S6$
62	$S7 = S7$
63	$S12 = S12$
Equation count	**Equation**
64	$S13 = S13$
65	$S14 = \varepsilon\, S4$
66	$S15 = \varepsilon\, S5$
67	$S16 = S16$
68	$S17 = S17$
69	$\Delta S2 = S\Delta 2$
70	$\Delta S3 = \Delta S2$
71	$\Delta S4 = \Delta S4$
72	$\Delta S5 = \Delta S5$
73	$\Delta S6 = \Delta S6$
74	$\Delta S7 = \Delta S7$
75	$\Delta S8 = \Delta S7$
76	$\Delta S9 = \Delta S7$
77	$\Delta S10 = \Delta S7$
78	$\Delta S11 = \Delta S7$
79	$\Delta S12 = \Delta S7$
80	$\Delta S13 = \Delta S7$
81	$\Delta S14 = \varepsilon(S4 - S4_{t-1})$

Equation count	Equation
82	$\Delta S15 = \varepsilon(S5 - S5_{t-1})$
83	$\Delta S16 = \Delta S16$
84	$\Delta S17 = \Delta S17$
85	$a_{0,1} = P_b$
86	$b_{0,1} = P_i$
87	$b_{1,0} = P_e$

(continues in next page)

Equation count	Equation
88	$a_{8,12} = \frac{l_{8,12}\left(\zeta\, S12 + \frac{A_{g_{60}}}{67-60}\, S12 + \vartheta\left(S12 - \frac{A_{g_{60}}}{67-60}\right) + \left(A_{g29}\, d_{20-29} + A_{g30-39}\, d_{30-39} + A_{g40-49}\, d_{40-49} + A_{g50-59}\, d_{50-59} + A_{g60}\, d_{60}\right) S12 + \Delta S12\right)}{1-\Omega}$
89	$a_{9,12} = \frac{l_{9,12}\left(\zeta\, S12 + \frac{A_{g_{60}}}{67-60}\, S12 + \vartheta\left(S12 - \frac{A_{g_{60}}}{67-60}\right) + \left(A_{g29}\, d_{20-29} + A_{g30-39}\, d_{30-39} + A_{g40-49}\, d_{40-49} + A_{g50-59}\, d_{50-59} + A_{g60}\, d_{60}\right) S12 + \Delta S12\right)}{1-\Omega}$
90	$a_{10,12} = \frac{l_{10,12}\left(\zeta\, S12 + \frac{A_{g_{60}}}{67-60}\, S12 + \vartheta\left(S12 - \frac{A_{g_{60}}}{67-60}\right) + \left(A_{g29}\, d_{20-29} + A_{g30-39}\, d_{30-39} + A_{g40-49}\, d_{40-49} + A_{g50-59}\, d_{50-59} + A_{g60}\, d_{60}\right) S12 + \Delta S12\right)}{1-\Omega}$
91	$a_{8,13} = \frac{l_{8,13}\left(\zeta\, S13 + \left(\frac{A_{g_{60}}}{67-60}\, S13 + \vartheta\left(S13 - \frac{A_{g_{60}}}{67-60}\right)\right) + \left(A_{g29}\, d_{20-29} + A_{g30-39}\, d_{30-39} + A_{g40-49}\, d_{40-49} + A_{g50-59}\, d_{50-59} + A_{g60}\, d_{60}\right) S13 + \Delta S13\right)}{1-\Omega}$
92	$a_{9,13} = \frac{l_{9,13}\left(\zeta\, S13 + \left(\frac{A_{g_{60}}}{67-60}\, S13 + \vartheta\left(S13 - \frac{A_{g_{60}}}{67-60}\right)\right) + \left(A_{g29}\, d_{20-29} + A_{g30-39}\, d_{30-39} + A_{g40-49}\, d_{40-49} + A_{g50-59}\, d_{50-59} + A_{g60}\, d_{60}\right) S13 + \Delta S13\right)}{1-\Omega}$
93	$a_{10,13} = \frac{l_{10,13}\left(\zeta\, S13 + \left(\frac{A_{g_{60}}}{67-60}\, S13 + \vartheta\left(S13 - \frac{A_{g_{60}}}{67-60}\right)\right) + \left(A_{g29}\, d_{20-29} + A_{g30-39}\, d_{30-39} + A_{g40-49}\, d_{40-49} + A_{g50-59}\, d_{50-59} + A_{g60}\, d_{60}\right) S13 + \Delta S13\right)}{1-\Omega}$
94	$a_{8,14} = \frac{l_{8,14}\left(\zeta\, S14 + \left(\frac{A_{v_{60}}}{67-60}\, S14 + \vartheta\left(S14 - \frac{A_{v_{60}}}{67-60}\right)\right) + \left(A_{v29}\, d_{20-29} + A_{v30-39}\, d_{30-39} + A_{v40-49}\, d_{40-49} + A_{v50-59}\, d_{50-59} + A_{v60}\, d_{60}\right) S14 + \Delta S14\right)}{1-\Omega}$
95	$a_{9,14} = \frac{l_{9,14}\left(\zeta\, S14 + \left(\frac{A_{v_{60}}}{67-60}\, S14 + \vartheta\left(S14 - \frac{A_{v_{60}}}{67-60}\right)\right) + \left(A_{v29}\, d_{20-29} + A_{v30-39}\, d_{30-39} + A_{v40-49}\, d_{40-49} + A_{v50-59}\, d_{50-59} + A_{v60}\, d_{60}\right) S14 + \Delta S14\right)}{1-\Omega}$
96	$a_{10,14} = \frac{l_{10,14}\left(\zeta\, S14 + \left(\frac{A_{v_{60}}}{67-60}\, S14 + \vartheta\left(S14 - \frac{A_{v_{60}}}{67-60}\right)\right) + \left(A_{v29}\, d_{20-29} + A_{v30-39}\, d_{30-39} + A_{v40-49}\, d_{40-49} + A_{v50-59}\, d_{50-59} + A_{v60}\, d_{60}\right) S14 + \Delta S14\right)}{1-\Omega}$
97	$a_{8,15} = \frac{l_{8,14}\left(\zeta\, S15 + \left(\frac{A_{v_{60}}}{67-60}\, S15 + \vartheta\left(S15 - \frac{A_{v_{60}}}{67-60}\right)\right) + \left(A_{v29}\, d_{20-29} + A_{v30-39}\, d_{30-39} + A_{v40-49}\, d_{40-49} + A_{v50-59}\, d_{50-59} + A_{v60}\, d_{60}\right) S15 + \Delta S15\right)}{1-\Omega}$

Equation count	Equation
98	$a_{9,15} = \frac{l_{9,15}\left(\zeta\, S15 + \left(\frac{A_{v60}}{67-60}\, S15 + \vartheta\left(S15 - \frac{A_{v60}}{67-60}\right)\right) + \left(A_{v29}\, d_{20-29} + A_{v30-39}\, d_{30-39} + A_{v40-49}\, d_{40-49} + A_{v50-59}\, d_{50-59} + A_{v60}\, d_{60}\right) S15 + \Delta S15\right)}{1-\Omega}$
99	$a_{10,15} = \frac{l_{10,14}\left(\zeta\, S15 + \left(\frac{A_{v60}}{67-60}\, S15 + \vartheta\left(S15 - \frac{A_{v60}}{67-60}\right)\right) + \left(A_{v29}\, d_{20-29} + A_{v30-39}\, d_{30-39} + A_{v40-49}\, d_{40-49} + A_{v50-59}\, d_{50-59} + A_{v60}\, d_{60}\right) S15 + \Delta S15\right)}{1-\Omega}$
100	$a_{8,16} = \frac{l_{8,16}\left(\zeta\, S16 + \left(\frac{A_{h60}}{67-60}\, S16 + \vartheta\left(S16 - \frac{A_{h60}}{67-60}\right)\right) + \left(A_{h29}\, d_{20-29} + A_{h30-39}\, d_{30-39} + A_{h40-49}\, d_{40-49} + A_{h50-59}\, d_{50-59} + A_{h60}\, d_{60}\right) S16 + \Delta S16\right)}{1-\Omega}$
101	$a_{9,16} = \frac{l_{9,16}\left(\zeta\, S16 + \left(\frac{A_{h60}}{67-60}\, S16 + \vartheta\left(S16 - \frac{A_{h60}}{67-60}\right)\right) + \left(A_{h29}\, d_{20-29} + A_{h30-39}\, d_{30-39} + A_{h40-49}\, d_{40-49} + A_{h50-59}\, d_{50-59} + A_{h60}\, d_{60}\right) S16 + \Delta S16\right)}{1-\Omega}$
102	$a_{10,16} = \frac{l_{10,16}\left(\zeta\, S16 + \left(\frac{A_{h60}}{67-60}\, S16 + \vartheta\left(S16 - \frac{A_{h60}}{67-60}\right)\right) + \left(A_{h29}\, d_{20-29} + A_{h30-39}\, d_{30-39} + A_{h40-49}\, d_{40-49} + A_{h50-59}\, d_{50-59} + A_{h60}\, d_{60}\right) S16 + \Delta S16\right)}{1-\Omega}$
103	$a_{11,16} = \frac{l_{11,16}\left(\zeta\, S16 + \left(\frac{A_{h60}}{67-60}\, S16 + \vartheta\left(S16 - \frac{A_{h60}}{67-60}\right)\right) + \left(A_{h29}\, d_{20-29} + A_{h30-39}\, d_{30-39} + A_{h40-49}\, d_{40-49} + A_{h50-59}\, d_{50-59} + A_{h60}\, d_{60}\right) S16 + \Delta S16\right)}{1-\Omega}$
104	$a_{12,1} = \Omega\, \frac{\left(\zeta\, S12 + \frac{A_{g60}}{67-60}\, S12 + \vartheta\left(S12 - \frac{A_{g60}}{67-60}\right)\right) + \left(A_{g29}\, d_{20-29} + A_{g30-39}\, d_{30-39} + A_{g40-49}\, d_{40-49} + A_{g50-59}\, d_{50-59} + A_{g60}\, d_{60}\right) S12 + \Delta S12}{1-\Omega} + \zeta\, S12$
105	$a_{13,1} = \Omega\, \frac{\left(\zeta\, S13 + \left(\frac{A_{g60}}{67-60}\, S13 + \vartheta\left(S13 - \frac{A_{g60}}{67-60}\right)\right) + \left(A_{g29}\, d_{20-29} + A_{g30-39}\, d_{30-39} + A_{g40-49}\, d_{40-49} + A_{g50-59}\, d_{50-59} + A_{g60}\, d_{60}\right) S13 + \Delta S13\right)}{1-\Omega} + \zeta\, S13$
106	$a_{14,1} = \Omega\, \frac{\left(\zeta\, S14 + \left(\frac{A_{v60}}{67-60}\, S14 + \vartheta\left(S14 - \frac{A_{v60}}{67-60}\right)\right) + \left(A_{v29}\, d_{20-29} + A_{v30-39}\, d_{30-39} + A_{v40-49}\, d_{40-49} + A_{v50-59}\, d_{50-59} + A_{v60}\, d_{60}\right) S14 + \Delta S14\right)}{1-\Omega} + \zeta\, S14$
107	$a_{15,1} = \Omega\, \frac{\left(\zeta\, S15 + \left(\frac{A_{v60}}{67-60}\, S15 + \vartheta\left(S15 - \frac{A_{v60}}{67-60}\right)\right) + \left(A_{v29}\, d_{20-29} + A_{v30-39}\, d_{30-39} + A_{v40-49}\, d_{40-49} + A_{v50-59}\, d_{50-59} + A_{v60}\, d_{60}\right) S15 + \Delta S15\right)}{1-\Omega} + \zeta\, S15$
108	$a_{16,1} = \Omega\, \frac{\left(\zeta\, S16 + \left(\frac{A_{h60}}{67-60}\, S16 + \vartheta\left(S16 - \frac{A_{h60}}{67-60}\right)\right) + \left(A_{h29}\, d_{20-29} + A_{h30-39}\, d_{30-39} + A_{h40-49}\, d_{40-49} + A_{h50-59}\, d_{50-59} + A_{h60}\, d_{60}\right) S16 + \Delta S16\right)}{1-\Omega} + \zeta\, S16$

Appendix V. System Parameters

List	Symbol	Units	Value	Description	Source
1	P	Cap.	5051275	Total population	SSB
2	P_{0-5}	Cap.	372438	Population ages 0 to 5	SSB
3	P_{0-6}	Cap.	437122	Population ages 0 to 6	SSB
4	P_{6-15}	Cap.	616773	Population ages 6 to 15	SSB
5	$P6$	Cap.	62108	Population of age 6	SSB
6	$P_{\neq 6}$	Cap.	4614153	Population 7 years old or older	SSB
7	$P16$	Cap.	65791	Population age 16	SSB
8	$P19$	Cap.	65464	Population age 19	SSB
12	d_{1-5}	-	0,000114473	Fractional death rate of persons ages 1-5	Calculated from data from SSB
13	d_{6-15}	-	5,83683E-05	Fractional death rate persons 6 to 15	Calculated from data from SSB
14	d_{16-19}	-	0,000324842	Fractional death rate of persons age 16 to 19	Calculated from data from SSB
15	d_{20-59}	-	0,001497119	Fractional death rate of persons ages 20 to 59	Calculated from data from SSB
16	d_{20-67}	-	0,002502797	Fractional death rate of persons ages 20-67	Calculated from data from SSB
17	d_{50}	-	0,022940762	Fractional death rate of persons 50 or older	Calculated from data from SSB
18	d_{20-29}	-	0,000450424	Fractional death rate of persons 20 to 29	Calculated from data from SSB
19	d_{30-39}	-	0,000661243	Fractional death rate of persons 30 to 29	Calculated from data from SSB

List	Symbol	Units	Value	Description	Source
20	d_{40-49}	-	0,001337824	Fractional death rate of persons 40 to 39	Calculated from data from SSB
21	d_{50-59}	-	0,003681864	Fractional death rate of persons 50 to 59	Calculated from data from SSB
22	d_{60}	-	0,034319141	Fractional death rate of persons 60 or older	Calculated from data from SSB
23	P_b	Cap./Yr	58995	Births	SSB
24	P_i	Cap./Yr	75789	Immigrants	SSB
25	P_e	Cap./Yr	35716	Emigrants	SSB
26	D	Cap.	41282	Total deaths	SSB
27	C_k	%	65,70%	Coverage of the pre-school service for the population ages 0 to 6	Calculated from data from SSB
28	$C_{g\neq6}$	%	0,02%	Share of persons ages different than 6 enrolling primary and lower secondary school	Calculated from data from SSB
29	C_{gv}	%	1,60%	Share of the population 16 or older participating in primary and lower secondary school	Calculated from data from SSB
30	$S2_6$	Cap.	286	Pupils age 6 in pre-school	SSB
31	$S2_{5-6}$	Cap.	62652	Pupils in pre-school ages 5 and 6	SSB
32	$G1$	Cap.	61853	Persons in primary and lower secondary school grade 1	SSB
33	$G10$	Cap.	62836	Persons in primary and lower secondary school grade 10	SSB
34	Gv	Cap.	9867	Adult students in primary and lower secondary school (older than 25)	SSB
35	θ	-	99,06%	Share of persons who finished primary and lower secondary school (grade 10) and enrolled upper secondary school the same year	Calculated from data from SSB

List	Symbol	Units	Value	Description	Source
36	$\mu_{\#}$	-	0,01	Fractional dropout rate from primary and lower secondary school (all primary and lower secondary school)	Calculated from data from SSB
	σ_3	Yr.	10	Average length of studies in primary and lower secondary education	Calculated from data from SSB
37	$V1$	Cap.	77043	Number of students in upper secondary school grade 1	SSB
38	V_{ac3}	Cap.	48764	Number of students in the third year of the academic strain of upper secondary education	SSB
39	$V_{\neq ac3}$	Cap.	46721	Number of students in last year of upper secondary school different than the academic strain of upper secondary education	SSB
41	ε	Teachers/student	0,11319	Ratio of teachers/students in upper secondary school (2003)	Calculated from data from SSB
42	$V21$	%	7,4%	Share of students 21 or older in upper secondary school	Calculated from data from SSB
43	η	%	59,5%	Share of persons in Vac3 that passd exams and enrolled h.e. (the same year)	System of equations
44	μ_4	-	0,3	Fractional dropout rate from upper secondary school	Calculated from data from the SSB
45	σ_5	-	1,25	Average length of studies of post-secondary non tertiary education	Calculated from literature from the SSB
	σ_4	Yr.	4	Average length of studies of upper secondary education	Calculated from data from the SSB

List	Symbol	Units	Value	Description	Source
48	$O6$	Cap./Yr	42132	Total graduations from higher education	SSB
49	$I6$	Cap./Yr	104456	Total enrollments	DBH
50	$I6_{0-20}$	Cap./Yr	26869	Total new enrollments age 20 or less	SSB
51	$I6_n$	Cap./Yr	52372	Total new enrollments all ages	SSB
52	H_k	Cap./Yr	2066	Total graduations persons qualified for teaching in pre-school	Calculated from data from the SSB
53	H_g	Cap./Yr	2833	Total graduations persons qualified for teaching in primary and lower secondary education	Calculated from data from the SSB
54	H_v	Cap./Yr	2512	Total graduations persons qualified for teaching in upper secondary education and primary and lower secondary school	Calculated from data from the SSB
55	H_h	Cap./Yr	1549	Total graduations PhDs	Calculated from data from the SSB
56	h_k	-	0,049026134	Fraction of graduates qualified to teach in pre-school	Model approach equations
57	h_g	-	0,067245083	Fraction of graduates qualified to teach in primary and lower secondary education	Model approach equations
58	h_v	-	0,059628337	Fraction of graduates qualified to teach in upper secondary education	Model approach equations
59	h_h	-	0,036765404	Fraction of graduates that attain PhD degree	Model approach equations
60	μ_6	-	0,17	Fractional dropout rate from higher education	Model approach equations
61	σ_6	Yr.	5,64	Average length of studies in higher education	Calculated from data from the SSB

List	Symbol	Units	Value	Description	Source
69	Ah_{29}	%	2,3%	Share of professors 29 or younger	Normal regression with data from European University Institute
70	Ah_{30-39}	%	7,2%	Share of professors ages 30 to 39	Normal regression with data from European University Institute
71	Ah_{40-49}	%	35,0%	Share of professors ages 40 to 49	Normal regression with data from European University Institute
72	Ah_{50-59}	%	43,1%	Share of professors ages 50 to 59	Normal regression with data from European University Institute
73	Ah_{60}	%	12,4%	Share of professors in higher education 60 years old or older	Normal regression with data from European University Institute
74	Ω	-	0,9	Fractional drop-out rate of newly enrolled teachers	SSB, NRK, UiB
75	ζ	-	0,02	Fractional drop-out rate of teachers in the work force	
76	$l_{8,12}$	%	96,6%	Share of pre-school/pre-school qualified teachers in pre-school	Model approach and data from SSB
77	$l_{9,12}$	%	1,5%	Share of primary and lower secondary teachers in pre-school	Model approach and data from SSB
78	$l_{10,12}$	%	1,9%	Share of upper secondary school/PPU teachers in pre-schools	Model approach and data from SSB
79	$l_{8,13}$	%	13,9%	Share of pre-school/pre-school qualified teachers in primary and lower secondary school	Model approach and data from SSB

List	Symbol	Units	Value	Description	Source
80	$l_{9,13}$	%	67,6%	Share of primary and lower secondary teachers in primary and lower secondary	Model approach and data from SSB
81	$l_{10,13}$	%	18,5%	Share of upper secondary school/PPU teachers in primary and lower secondary	Model approach and data from SSB
82	$l_{8,1}4$	%	2,0%	Share of pre-school/pre-school qualified teachers in upper secondary school	Model approach and data from SSB
83	$l_{9,14}$	%	12,0%	Share of primary and lower secondary teachers in upper secondary school	Model approach and data from SSB
84	$l_{10,14}$	%	86,0%	Share of upper secondary school/PPU teachers in upper secondary	Model approach and data from SSB
85	l_8_{16}	%	1,5%	Share of pre-school/pre-school qualified teachers in higher education	Model approach and data from SSB
86	l_9_{16}	%	4,7%	Share of primary and lower secondary teachers in higher education	Model approach and data from SSB
87	$l_{10,16}$	%	15,3%	Share of upper secondary school/PPU teachers in higher education	Model approach and data from SSB
88	$l_{11,16}$	%	78,5%	Share of PhDs in higher education (as teachers)	Model approach and data from SSB
89	Ag_{29}	%	9,0%	Share of teachers in primary and lower secondary 29 or younger	Oxford Research and Aarhus University
90	Ag_{30-39}	%	29,0%	Share of teachers in primary and lower secondary 30 to 30	Oxford Research and Aarhus University
91	Ag_{40-49}	%	28,0%	Share of teachers primary and lower secondary 40 to 49	Oxford Research and Aarhus University
92	Ag_{50-59}	%	24,0%	Share of teachers in primary and lower secondary 50-59	Oxford Research and Aarhus University

List	Symbol	Units	Value	Description	Source
93	Ag_{60}	%	10,0%	Share of teachers in primary and lower secondary 60 or older	Oxford Research and Aarhus University
94	Av_{29}	%	5,0%	Share of teachers in upper secondary 29 or younger	Oxford Research and Aarhus University
95	Av_{30-39}	%	19,0%	Share of teachers in upper secondary 30 to 30	Oxford Research and Aarhus University
96	Av_{40-49}	%	30,0%	Share of teachers upper secondary 40 to 49	Oxford Research and Aarhus University
97	Av_{50-59}	%	32,0%	Share of teachers in upper secondary 50-59	Oxford Research and Aarhus University
98	Av_{60}	%	14,0%	Share of teachers in upper secondary 60 or older	Oxford Research and Aarhus University
99	ϑ	%	2,7%	Relative fractional change of the number of early retired persons 25-66 of age	Calculated from data from the SSB
100	Ar_{15-24}	%	2,0%	Share of retired people ages 15 to 24	Calculated from data from the SSB
101	Ar_{125-54}	%	21,0%	Share of retired people ages 25 to 54	Calculated from data from the SSB
102	Ar_{55}	%	77,0%	Share of retired people 55 years of age or older	Calculated from data from the SSB

Variables as parameters

Parameter	Description	2012 (t-1) Cap.	2013 Cap.	Source
S2	Pupils in Kindergarten	286153	287177	SSB
S3	Pupils in Primary and Lower Secondary Education	614402	615327	SSB
S4	Students in Upper Secondary Education	239650	239758	SSB

Parameter	Description	2012 (t-1) Cap.	2013 Cap.	Source
S5	Students in Post-Secondary Non-Tertiary and other types of Upper Secondary Education	14116	15495	SSB
S6	Students in Higher Education	261164	269063	SSB
S7	Persons retired, early retired and disabled	682000	705000	SSB
S8	Market of labor force qualified to teach at kindergarten level	-	-	-
S9	Market of labor force qualified to teach in primary and lower secondary education	-	-	-
S10	Market of labor force qualified to teach in upper secondary education	-	-	-
S11	Market of labor force qualified to teach in higher education	-	-	-
S12	Teachers (academic staff) in Kindergartens	50022	51346	SSB
S13	Teachers (academic staff) in Primary and Lower Secondary schools	72626	72427	SSB
S14	Teachers (academic staff) in Upper Secondary schools	27126	27138	SSB
S15	Teachers (academic staff) in Post-secondary Non-Tertiary education	1598	1754	Calculated by same teacher/student ratio of upper secondary education
S16	Teachers (academic staff) in Higher education	21457	21799	SSB man-year data divided by 0,89man-year/cap*
S17	Students in Higher Education abroad	15592	15746	SSB**

* 0,89 man years of work are considered for general teachers (SSB 2014)

** Included in parameter S16. Only for visualization purposes.

Parameter	Description		Value (Cap.)
ΔS1	Stock change in the rest of the population		27 806
ΔS2	Stock change in the number of pupils in kindergarten		1 024
ΔS3	Stock change in the number of pupils in primary and lower secondary school		925
ΔS4	Stock change in the number of pupils in upper secondary school		108
ΔS5	Stock change in the number of students in post-secondary non tertiary education		1 379
ΔS6	Stock change in the number of students in higher education		7 899
ΔS7	Stock change in the number of retired people		23 000
ΔS12	Stock change of teachers in kindergarten		1 324
ΔS13	Stock change of teachers in primary and lower secondary schools		-199
ΔS14	Stock change of teachers in upper secondary schools		12
ΔS15	Stock change of teachers in post-secondary non tertiary education		156
ΔS16	Stock change of professors in higher education		342
ΔS17	Stock change in students in higher education abroad		154

Table 15. Parameter ϑ Relative change of early retired people

Age group	Early retired population		All population		Shares		Relative change of the share of the population early retired
	2012	**2013**	**2012**	**2013**	**2012**	**2013**	
25-66	354 000	368 000	2 752 109	2 785 563	12,86%	13,21%	**2,71%**

Source: Statistics Norway and own calculations

The following tables contain the parameters I3, I4, I6, and Y3, Y4, and Y6.

Table 16. Enrollments in higher education and weight.

	Enrollments (Cap.) SSB	Residence time Weight (Yr.)
(t)	**I6**	**Y6**
2008	69359	0,65
2009	85822	1
2010	87756	1
2011	99916	1
2012	98724	1
2013	104456	1
Total weight		**5,65**

Table 17. Enrollments in upper secondary education and weight.

	Enrollments (Cap.) SSB	**Residence time Weight (Yr.)**
(t)	**I4**	**Y4**
2010	76514	1
2011	76514	1
2012	79279	1
2013	77043	1
Total weight		**4**

Table 18. Enrollments in primary and lower secondary education and weight

	Enrollments (Cap.) SSB	**Residence time Weight (Yr.)**
(t)	**I3**	**Y3**
2002	62280	1
2003	60988	1
2004	59425	1
2005	60345	1
2006	60486	1
2007	58366	1
2008	57586	1
2009	59137	1
2010	59976	1
2011	59997	1
2012	61946	1
2013	61853	1
Total weight		**10**

Appendix VI. Residence times in education: complementary information

To find the average residence time of students that graduate from higher education, data on the duration of higher education programmes and the time of completion since registration of students from the SSB were used.

The average percentages in the following tables were then weighted by the number of years (duration) and summed to find the average number of years that students take to complete a degree of a theoretical duration of four years or less, or five years or more.

The two average residence times were then weighted again with the total number of graduates of 2009-2010 (period with data available for the two theoretical duration of the programmes) and an average residence time to graduate from higher education was found to be 5,64 years. In Table 19 and Table 20, the calculation of the two single average times is shown.

Table 19. Tertiary qualifications (five years or more) and average residence time

Tertiary qualifications (lasting five years or more) in Norway, by tertiary qualification (long), time and years since first-time registered (relative)										
Year	**Total graduations**	**5 years and less**	**6 years**	**7 years**	**8 years**	**9 years**	**10 years**	**11 years**	**12 years**	**13 years and more**
2006-2007	**10727**	22%	16%	13%	10%	6%	5%	4%	3%	22%
2007-2008	**8980**	24%	20%	14%	10%	7%	4%	3%	3%	16%
2008-2009	**9365**	26%	19%	14%	10%	6%	4%	2%	2%	17%
2009-2010	**10104**	28%	19%	13%	10%	6%	4%	3%	2%	17%
Average		26%	19%	14%	10%	6%	4%	3%	2%	17%
Weight (years)		5	6	7	8	9	10	11	12	13
Weighted duration		**1,29**	**1,14**	**0,99**	**0,78**	**0,55**	**0,39**	**0,28**	**0,26**	**2,15**
Weighted average duration		**7,84**								

Source: Statistics Norway 2015 and own calculations

Table 20. Tertiary qualifications (four years or less) and aerage residence time

Tertiary qualifications (lasting four years or less) in Norway, by tertiary qualification (short), time and years since first-time registered (relative)							
	Total graduations	**3 years and less**	**4 years**	**5 years**	**6 years**	**7 years**	**8 years and more**
2009-2010	**25730**	36%	21%	12%	8%	5%	18%
2010-2011	**27001**	38%	21%	11%	8%	5%	18%
2011-2012	**27028**	39%	22%	11%	6%	4%	18%
2012-2013	**28368**	40%	22%	10%	6%	4%	18%
Average		38%	21%	11%	7%	4%	18%
Weight (years)		3	4	5	6	7	8
Weighted duration		**1,15**	**0,85**	**0,55**	**0,42**	**0,31**	**1,43**
Weighted average duration		**4,72**					

Source: Statistics Norway 2015 and own calculations

$$Residence\ time = \frac{\left(10\ 104\ (7{,}84) + 25\ 730\ (4{,}72)\right)\ student\ years}{(10\ 104 + 25\ 730)\ students} = 5{,}65\ years$$

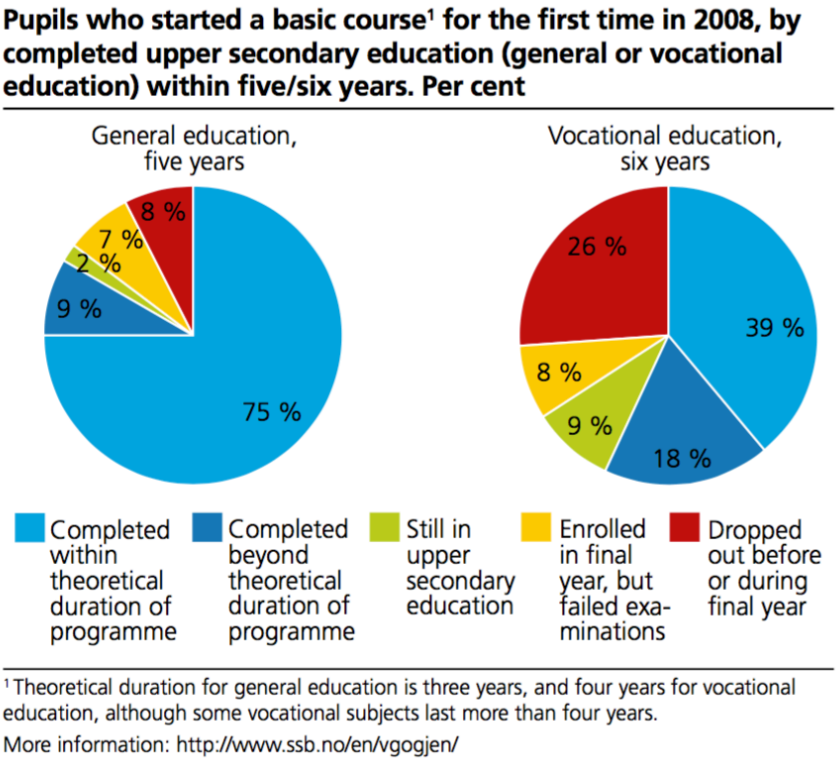


Figure 15. Student throughput in upper secondary education

Source: Statistics Norway 2014.

To find the residence time of students in upper secondary education, a similar approach was used using the shares of figure 7. The share of students that completed within theoretical duration of the programme and those that dropped out before or during final year was weighted according to the theoretical duration of the programme (3 and 4 years for general and vocational education respectively). The rest was weighed 5 and 6 years according to each general and vocational education, respectively. Then, each weight in years was re-weighted with the number of students that enrolled in 2008: 32 154 for general and 31 102 for vocational education (Statistics Norway, 2015). The following formula summarizes this procedure.

$$Residence\ time = \frac{(32154(0{,}83 * 3 + 0{,}17 * 5) + 31102(0{,}65 * 4 + 0{,}35 * 6)\ student\ years}{(63\ 256)\ students}$$

$$= 4{,}0\ years$$

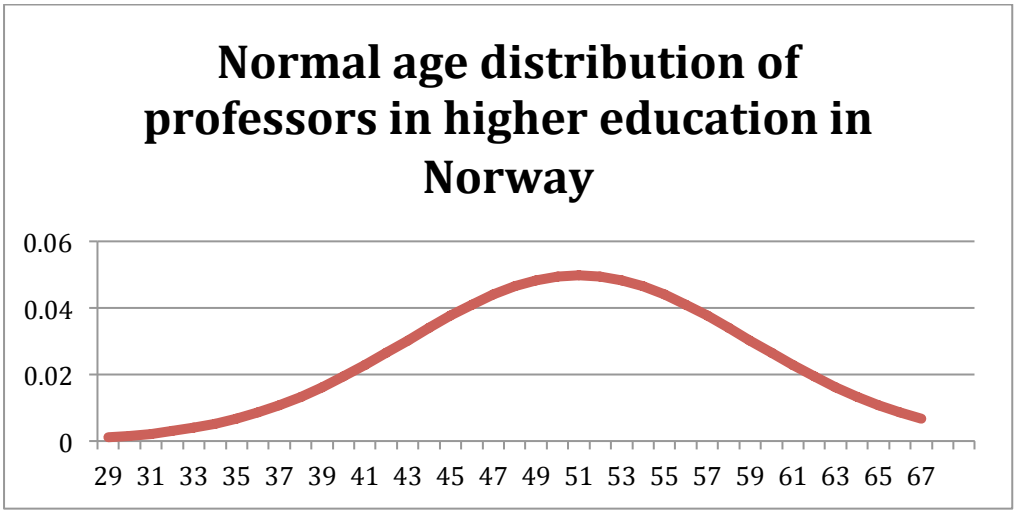
Appendix VII. Age composition of teacher stocks

Tabell 8: Alder fordelt på lærere i grunnskolen og lærere i videregående opplæring. Prosentverdier

	Grunnskolen	Videregående opplæring
Under 30 år	9	5
30 – 39 år	29	19
40 – 49 år	28	30
50 – 59 år	24	32
60 år og eldre	10	15
Totalt antall	3157	3103
Kilde: Oxford Research AS og Aarhus Universitet		

Source:

Figure 16. Normal age distribution of professors in higher education



The only data found for the age of professors was taken from the European University Institute (EUI, 2015) and is an average of 47 years old and the mean age of finishing a PhD is 38. With the average age, a normal data regression was performed

with the following parameters: average 48 (closest value to validate result), mean 51, and standard deviation 16. The data obtained was then listed as fractions and aggregated by age groups. The results are the following:

Table 21. Age composition of professors in higher education in Norway. 2015

Share of professors 29 or younger	2,3%
Share of professors ages 30 to 39	7,2%
Share of professors ages 40 to 49	35,0%
Share of professors ages 50 to 59	43,1%
Share of professors in higher education 60 years old or older	12,4%

Table 22. Age composition of teachers. Statistics and Results.

Age composition of teachers			
Age group	**Primary and lower secondary school**	**Upper secondary school**	**Higher education**
29 or younger	9,00%	5,00%	2,3%
30 to 39	29,00%	19,00%	7,2%
40 to 49	28,00%	30,00%	35,0%
50 to 59	24,00%	32,00%	43,1%
60 or older	10,00%	14,00%	12,4%
Total	100,00%	100,00%	100%

Source: Oxford Resarch and Aarhus University 2012, and results of normal regression (for higher education values only)

Appendix VIII. Teacher mix

The teacher mix in *Arbeidsmarkedet for lærere og førskolelæarere fram mot år 2035* (Statistics Norway 2012), shows the composition of man-years for different schools: kindergartens, compulsory education schools, upper secondary schools, universities and higher education schools, adult education and those outside teaching (columns).

Some of the data of Figure 17 in this Appendix was used to adapt a teacher mix usable with the model of this thesis. First, special education teachers were discarded and subtracted from the total teachers in each level of education (the last two rows). Second, only the compositions of teachers in kindergarten, compulsory education, upper secondary education and higher education were used (the first three columns[12]). Third, *faglæarere, and* both *PPU* rows were grouped and assumed to be upper secondary teachers from our model, *allmennlærere* were interpreted as teachers for primary and lower secondary education, and *førskolelærere* were interpreted as teachers for pre-school. After the absolute sums of teachers of the first three columns were made, the new shares of the new groups of teachers were calculated.

For the mix of teachers of higher education, in addition to the previous procedure, a fourth group was introduced: PhDs. The total sum was then re-balanced to match the 18 984 man-years reported by the SSB in 2014 for the year 2010 in *Facts about education in Norway 2015: key figures 2013* (Statistics Norway 2014), since we

[12] The other two columns were neglected, because these types of education are embedded in the stocks of our model.

account PhDs as an important inflow of teachers to the markets of labor force for higher education.

Then, the new teacher mix is the following:

Table 23. New teacher mix.

	Composition of teachers in pre-school	Composition of teachers in primary and lower secondary school	Composition of teachers in upper secondary education	Composition of teachers in higher education
Pre-school teachers	21 394	7 680	404	329
Primary and lower secondary teachers	336	37 343	2 330	1 001
Upper secondary teachers	415	10 242	16 624	3 331
PhDs	-	-	-	17323*
Total	22 145	55 265	19 358	18 984
	Percent			
Pre-school teachers	96,60%	13,90%	2%	1,53%
Primary and lower secondary teachers	1,50%	67,60%	12%	4,67%
Upper secondary teachers	1,90%	18,50%	86%	15,28%
PhDs	-	-	-	78,50%
Total	100%	100%	100%	100%

* By mass balance.

Tabell A.1. Antall og andel årsverk i 2010 fra ulike grupper av undervisningspersonell i de ulike aktivitetsområdene i LÆRERMOD

	Barne-hager	Førskoler og grunnnskole-undervisning	Videre-gående skole	Universitets- og høgskolenivå	Voksenopplæring og annen undervisning	Utenfor under-visningen	Sum
Allmennlærere	336	37 343	2 330	1 001	2 261	9 146	**52 416**
Førskolelærere	21 394	7 680	404	329	869	5 264	**35 940**
Faglærere	260	2 835	2 600	575	1 127	3 947	**11 344**
PPU for universitets- og høgskolekandidater	110	6 364	8 733	2 294	1 152	8 000	**26 653**
PPU for yrkesfag	45	1 043	5 291	462	307	3 966	**11 112**
Spesialpedagogisk utdanning	84	370	75	54	158	419	**1 161**
Annen pedagogisk utdanning	2 268	7 833	2 614	1 701	1 910	19 546	**35 871**
SUM alle grupper	**24 497**	**63 468**	**22 047**	**6 415**	**7 783**	**50 288**	**174 498**
				Prosent			
Allmennlærere	0,6	71,2	4,4	1,9	4,3	17,4	**100,0**
Førskolelærere	59,5	21,4	1,1	0,9	2,4	14,6	**100,0**
Faglærere	2,3	25,0	22,9	5,1	9,9	34,8	**100,0**
PPU for universitets- og høgskolekandidater	0,4	23,9	32,8	8,6	4,3	30,0	**100,0**
PPU for yrkesfag	0,4	9,4	47,6	4,2	2,8	35,7	**100,0**
Spesialpedagogisk utdanning	7,2	31,9	6,5	4,7	13,6	36,1	**100,0**
Annen pedagogisk utdanning	6,3	21,8	7,3	4,7	5,3	54,5	**100,0**
SUM alle grupper	**14,0**	**36,4**	**12,6**	**3,7**	**4,5**	**28,8**	**100,0**

Figure 17. Teacher composition LÆRERMOD

Source: Statistics Norway 2012

Man-years worked and students per man-year in higher education, by type of institution

	Teacher man-years	Students	Students per teacher man-year
2008	17 947	206 063	11,5
2009	18 255	213 702	11,7
2010	18 757	218 264	11,6
2011	18 984	229 593	12,1
2012	19 097	239 268	12,5
Total 2013	**19 401**	**245 004**	**12,6**
Universities	12 043	106 349	8,8
State specialised university institutions	893	8 838	9,9
Private specialised university institutions	435	23 031	52,9
State university colleges	5 289	93 827	17,7
Private university colleges	617	12 099	19,6
University colleges of the arts	123	860	7,0

Source: Statistics on Higher Education (DBH) at Norwegian Social Science Data Services.
More information: http://dbh.nsd.uib.no

Source: Statistics Norway 2014

Figure 18. Man-years worked in higher education.

Appendix IX. Age composition of student stocks

The next tables show the age composition of students of upper secondary school and higher education. Note that the table for higher education is only for new students.

Table 24. Students in upper secondary education by age. Several years.

Age group	Year			
	2011	**2012**	**2013**	**2014**
0-15 år	241	256	290	239
16 år	61387	62426	61519	60213
17 år	59377	60060	60910	60286
18 år	48194	49143	49068	49511
19 år	9653	9403	8816	8633
20 år	4550	4607	4853	4684
21 år	2584	2636	2772	2863
22 år	1558	1622	1710	1697
23 år	924	1089	1158	1155
24 år	677	808	883	843
25 år eller eldre	6219	7504	8077	8094

Source: Statistics Norway

Table 25. New students in higher education by age. Several years.

	Year					
	2008	**2009**	**2010**	**2011**	**2012**	**2013**
0-18 år	183	148	130	167	264	313
19 år	10332	11765	11570	11803	11867	13186
20 år	10625	11528	12088	12537	12705	13370
21 år	5013	5735	5931	6165	6349	6772
22 år	2489	2975	3151	3165	3364	3749
23 år	1550	2014	2117	2313	2377	2535
24 år	1174	1469	1511	1563	1688	1686
25 år	957	1235	1226	1275	1381	1413
26 år	822	967	990	1088	1068	1106
27 år	742	815	813	920	902	859
28 år	638	689	730	757	724	704
29 år	552	592	611	666	572	593
30-34 år	2128	2111	2201	2353	2166	1946
35 år eller eldre	6102	6089	5819	5211	5544	4140

Source: Statistics Norway

Table 26. Pupils in kindergarten by age. 2012 and 2013.

	Age							
	Total	0 years	1 years	2 years	3 years	4 years	5 years	6 years
2012	286153	2318	42754	57384	61409	61556	60338	394
2013	287177	1894	42336	56365	60949	62981	62266	386

Source: Statistics Norway 2015

Appendix X. Fractional death rates

The fractional death rates were calculated dividing the number of deaths by the number of living persons (population) of selected age groups. The data was extracted from population data of the SSB.

	Deaths		Population		Fractional death rates	
Age group	**2012**	**2013**	**2012**	**2013**	**2012**	**2013**
0-5	193	183	372438	375014	0,000518207	0,000487982
1-5	43	36	311972	314484	0,000137833	0,000114473
6-15	64	36	616409	616773	0,000103827	5,83683E-05
16-19	64	85	259718	261666	0,000246421	0,000324842
20-29	291	302	652787	670480	0,000445781	0,000450424
30-39	448	450	677174	680536	0,000661573	0,000661243
40-49	956	985	725007	736270	0,001318608	0,001337824
50-59	2262	2339	628176	635276	0,003600902	0,003681864
50 and older	39976	39241	1682337	1710536	0,023762183	0,022940762
20-59	3957	4076	2683144	2722562	0,001474762	0,001497119
60 and older	37714	36902	1054161	1075260	0,035776319	0,034319141
20-67	7917	7952	3133419	3177245	0,002526633	0,002502797

Source: Statistics Norway and own calculations.

Appendix XI. Model results

The next tables show the results of the variables of the model; unknown flows, stocks, stock changes, and unknown parameters.

Table 27. Model results

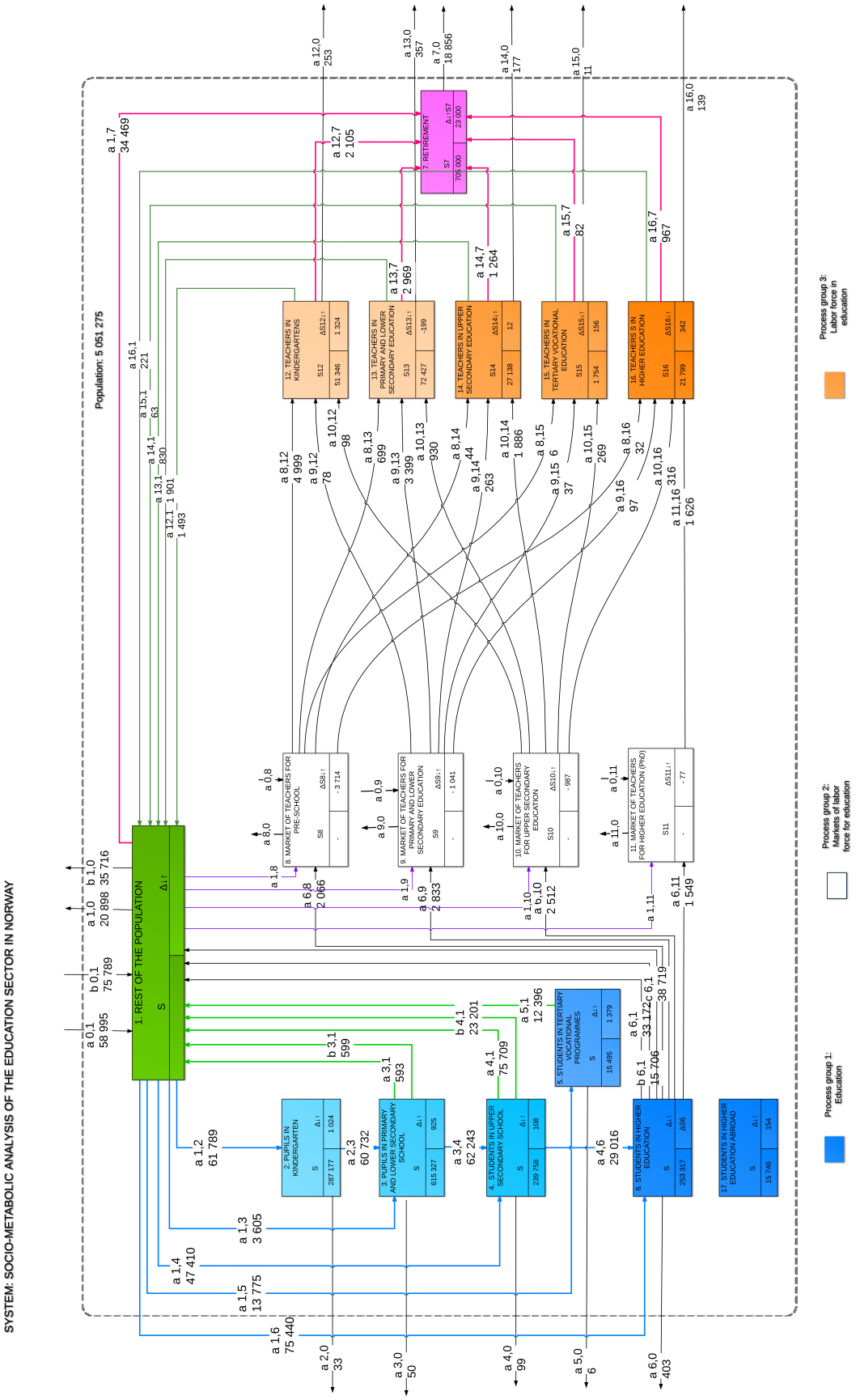
Variable count	Symbol	Result (Cap./yr)	Description
1	a 1,2	61 789	Pupils enrolling in kindergarten
2	a 2,3	60 732	Pupils leaving kindergarten and enrolling primary and lower secondary school
3	a 1,3	3 605	Pupils enrolling primary and lower secondary school that do not were in kindergarten the same year
4	a 3,1	593	Pupils and students that finished primary and lower secondary education but did not enroll upper secondary school
5	b 3,1	599	Pupils and students that dropped out primary and lower secondary school
6	a 3,4	62 243	Students that finished primary and lower secondary and enrolled upper secondary education the same year
7	a 1,4	65 890	Students that enrolled upper secondary education that were not in primary and lower secondary school the same year they enrolled
8	a 4,1	75 709	Students that finished upper secondary education and did not enroll higher education
9	b 4,1	23 201	Students that dropped out upper secondary education
10	a 4,6	29 016	Students that finished upper secondary education and enrolled in higher education the same year
11	a 5,1	12 396	Students finishing or dropping tertiary vocational programmes

Variable count	Symbol	Result (Cap./yr)	Description
12	a 1,5	13 775	Students that enrolled in tertiary vocational programmes
13	a 1,6	75 440	Students that enrolled higher education that were not in upper secondary education the year of enrollment
14	c 6,1	38 719	Students that leave higher education without completing a degree
15	a 6,1	33 172	Graduations of higher education programmes other than education-related programmes and PhDs
16	b 6,1	15 706	Students that drop out higher education
17	a 6,8	2 066	Graduations of higher education programmes that give qualifications to teach in pre-school
18	a 6,9	2 833	Graduations of higher education programmes that give qualifications to teach in primary and lower secondary education (general teachers)
19	a 6,10	2 512	Graduations of higher education programmes that give qualifications to teach in higher education (subject teachers and PPU)
20	a 6,11	1 549	Graduations of PhDs
21	a 1,8	-	People from the rest of the population entering the market for people with qualifications to teach in kindergartens
22	a 1,9	-	People from the rest of the population entering the market for people with qualifications to teach in primary and lower secondary education
23	a 1,10	-	People from the rest of the population entering the market for people with qualifications to teach in upper secondary schools
24	a 1,11	-	People from the rest of the population entering the market for people with qualifications to teach in higher education
25	a 8,12	4 999	Preschool/kindergarten teachers hired in kindergarten education
26	a 8,13	699	Preschool/kindergarten teachers hired in primary and lower secondary education
27	a 8,14	44	Preschool/kindergarten teachers hired in upper secondary education

Variable count	Symbol	Result (Cap./yr)	Description
28	a 8,15	6	Preschool/kindergarten teachers hired in tertiary vocational education education
29	a 8,16	32	Preschool/kindergarten teachers hired in higher education
30	a 9,12	78	Primary and lower secondary teachers hired in kindergarten education
31	a 9,13	3 399	Primary and lower secondary teachers hired in primary and lower secondary education
32	a 9,14	263	Primary and lower secondary teachers hired in upper secondary education
33	a 9,15	37	Primary and lower secondary teachers hired in tertiary vocational education
34	a 9,16	97	Primary and lower secondary teachers hired in higher education
35	a 10,12	98	Subject teachers and teachers with PPU hired in kindergarten education
36	a 10,13	930	Subject teachers and teachers with PPU hired in primary and lower secondary education
37	a 10,14	1 886	Subject teachers and teachers with PPU hired in upper secondary education
38	a 10,15	269	Subject teachers and teachers with PPU hired in tertiary vocational education
39	a 10,16	316	Subject teachers and teachers with PPU hired in higher education
40	a 11,16	1 626	PhDs hired as professors and teachers in higher education
41	a 0,1	58 995	Births
42	b 0,1	75 789	Immigration
43	a 1,0	20 898	Deaths of the rest of the population
44	b 1,0	35 716	Emigration
45	a 7,0	18 856	Deaths of retired people
46	a 1,7	34 469	People from the rest of the population that retired
47	a 12,7	2 105	Teachers of kindergarten that retired
48	a 13,7	2 969	Teachers of primary and lower secondary education that retired
49	a 14,7	1 264	Teachers of upper secondary education that retired
50	a 15,7	82	Teachers of tertiary vocational education that retired

Variable count	Symbol	Result (Cap./yr)	Description
51	a 16,7	967	Teachers of higher education that retired
52	a 12,1	1 493	Teachers of kindergarten that leave work (other than retirement, early retirement, and disability)
53	a 13,1	1 901	Teachers of primary and lower secondary education that leave work (other than retirement, early retirement, and disability)
54	a 14,1	830	Teachers of upper secondary education that leave work (other than retirement, early retirement, and disability)
55	a 15,1	63	Teachers of tertiary vocational education that leave work (other than retirement, early retirement, and disability)
56	a 16,1	221	Professors and teachers of higher education that leave work (other than retirement, early retirement, and disability)
57	a 2,0	33	Deaths of pupils in kindergarten
58	a 3,0	50	Deaths of pupils and students in primary and lower secondary education
59	a 4,0	99	Deaths of students in upper secondary education
60	a 5,0	6	Deaths of students in tertiary vocational education
61	a 6,0	403	Deaths of students in higher education
62	a 8,0	-	Emigration of teachers of kindergarten
63	a 9,0	-	Emigration of teachers of primary and lower secondary education
64	a 10,0	-	Emigration of teachers of upper secondary education
65	a 11,0	-	Emigration of professors and teachers of higher education
66	a 0,8	-	Immigration of teachers of pre-school
67	a 0,9	-	Immigration of teachers of primary and lower secondary education
68	a 0,10	-	Immigration of teachers of upper secondary education
69	a 0,11	-	Immigration of professors and teachers of higher education
70	a 12,0	253	Deaths of teachers in kindergartens
71	a 13,0	357	Deaths of teachers in primary and lower secondary schools
72	a 14,0	177	Deaths of teachers in upper secondary schools

Variable count	Symbol	Result (Cap.)	Description
73	a 15,0	11	Deaths of teachers in post-secondary non-tertiary schools
74	a 16,0	139	Deaths of teachers in higher education
75	S1	2 744 991	Rest of the population
76	S8	-	Market of labour force qualified to teach at kindergarten level
77	S9	-	Market of labor force qualified to teach in primary and lower secondary education
78	S10	-	Market of labor force qualified to teach in upper secondary education
79	S11	-	Market of labor force qualified to teach in higher education
80	S14	27138	Teachers (academic staff) in Upper Secondary schools
81	S16	1754	Teachers (academic staff) in Post-secondary Non-Tertiary education
82	ΔS8	-3 714	Stock change in the market of teachers for kindergarten
83	ΔS9	-1 041	Stock change in the market of teachers for primary and lower secondary education
84	ΔS10	-987	Stock change in the market of teachers for upper secondary education
85	ΔS11	-77	Stock change in the market of professors (PhDs) for higher education
Para-meter	**Symbol**	**Result (%)**	**Description**
86	θ	99%	Percentage of students that finished primary and lower secondary education and enrolled upper secondary education the same year
87	η	60%	Percentage of students that finished the academic strain of upper secondary education and enrolled higher education the same year
88	h_k	4,90%	Fraction of graduates qualified to teach in pre-school
89	h_g	6,72%	Fraction of graduates qualified to teach in primary and lower secondary education
90	h_v	5,96%	Fraction of graduates qualified to teach in upper secondary education
91	h_h	3,68%	Fraction of graduates that attain PhD degree


SYSTEM: SOCIO-METABOLIC ANALYSIS OF THE EDUCATION SECTOR IN NORWAY
Population: 5 051 275
1. REST OF THE POPULATION
2. PUPILS IN KINDERGARTEN
3. PUPILS IN PRIMARY AND LOWER SECONDARY SCHOOL
4. STUDENTS IN UPPER SECONDARY SCHOOL
5. STUDENTS IN TERTIARY VOCATIONAL PROGRAMMES
6. STUDENTS IN HIGHER EDUCATION
7. RETIREMENT
8. MARKET OF TEACHERS FOR PRE-SCHOOL
9. MARKET OF TEACHERS FOR PRIMARY AND LOWER SECONDARY EDUCATION
10. MARKET OF TEACHERS FOR UPPER SECONDARY EDUCATION
11. MARKET OF TEACHERS FOR HIGHER EDUCATION (PhD)
12. TEACHERS IN KINDERGARTENS
13. TEACHERS IN PRIMARY AND LOWER SECONDARY EDUCATION
14. TEACHERS IN UPPER SECONDARY EDUCATION
15. TEACHERS IN TERTIARY VOCATIONAL EDUCATION
16. TEACHERS IN HIGHER EDUCATION
17. STUDENTS IN HIGHER EDUCATION ABROAD
Process group 1: Education
Process group 2: Markets of labor force for education
Process group 3: Labor force in education

Appendix XII.- Student and teacher ratios-

The next two tables show the data on the number of students and teachers available from the SSB for several years. Note that data is very fragmented. The teacher/student ratios were calculated manually. In the case of teachers for higher education, data was only available in man-years. In the model, however, these units were converted to capita.

Table 28. Students, teachers, and student teacher ratios in kindergarten, primary and lower secondary school. Data and own calculations.

	Kindergarten			Primary and lower secondary		
Year	**Students**	**Teachers**	**Student/teacher ratio**	**Students**	**Teachers**	**Student/teacher ratio**
1991	150566	40061	3,76	469482	53109	8,84
2000	n.a.	20742	n.a.	590471	n.a.	n.a.
2001	n.a.	21915	n.a.	599468	n.a.	n.a.
2002	n.a.	23003	n.a.	610297	n.a.	n.a.
2003	205172	24863	8,25	617577	n.a.	n.a.
2004	213097	26191	8,14	618250	n.a.	n.a.
2005	223501	28381	7,88	619640	n.a.	n.a.
2006	234948	32644	7,20	619038	n.a.	n.a.
2007	249815	36276	6,89	616388	n.a.	n.a.
2008	261886	42409	6,18	612854	n.a.	n.a.
2009	270174	43824	6,16	612721	n.a.	n.a.
2010	277139	45547	6,08	612798	72806	8,42
2011	282737	47391	5,97	612627	73425	8,34
2012	286153	50022	5,72	614402	72626	8,46
2013	287177	51346	5,59	615327	72427	8,50

Source: Statistics Norway and own calculations

Table 29. Students, teachers, and student teacher ratios. Data and own calculations.

	Upper secondary			Higher education		
Year	**Students**	**Teachers**	**Student/teacher ratio**	**Students**	**Teachers**	**Student/teacher ratio**
1991	259477	28016	9,26	148865	10259	14,555
2000	220816	n.a.	n.a.	190671	n.a.	n.a.
2001	215760	n.a.	n.a.	197613	n.a.	n.a.
2002	220067	n.a.	n.a.	208693	n.a.	n.a.
2003	235160*	26618*	8,83	209770	n.a.	n.a.
2004	226952	n.a.	n.a.	211001	n.a.	n.a.
2005	237437	n.a.	n.a.	211264	n.a.	n.a.
2006	248335	n.a.	n.a.	211229	n.a.	n.a.
2007	250801	n.a.	n.a.	208238	n.a.	n.a.
2008	250530	n.a.	n.a.	214183	17947**	n.a.
2009	n.a.	n.a.	n.a.	n.a.	18255**	n.a.
2010	n.a.	n.a.	n.a.	n.a.	18757**	n.a.
2011	232516	n.a.	n.a.	n.a.	18984**	n.a.
2012	239650	n.a.	n.a.	261164	19097**	13,67
2013	239758	n.a.	n.a.	269063	19401**	13,86

* With these values the parameter ε was calculated.

** Units in man-years. For that reason, the ratio student/teacher is man-years/student.

Source: Statistics Norway and own calculations