NTNU – Trondheim
Norwegian University of
Science and Technology

Study of the Natural Frequencies of a Disc

Truls Edvardsen Aarønes

Master of Science in Mechanical Engineering
Submission date: Januar 2015
Supervisor: Pål Tore Selbo Storli, EPT
Co-supervisor: Torbjørn K. Nielsen, EPT

Norwegian University of Science and Technology
Department of Energy and Process Engineering

NTNU
Norges teknisk-
naturvitenskapelige universitet

Institutt for energi- og prosessteknikk

EPT-M-2014-157

MASTEROPPGAVE

for

Student
Truls Edvardsen Aarønes
Høsten 2014

Simulering av egenfrekvens til roterende disk og forberedelse til eksperimentelle målinger

Simulation of Eigen frequencies for a rotating disk and preparation for experimental measurements

Bakgrunn og målsetting

Egenfrekvensen til turbinhjul er en viktig egenskap ved hjulet da det er viktig ikke å eksitere resonans i hjulet under drift av turbinen. Egenfrekvensen til konstruksjoner er derimot avhengig av fluidet som omringer konstruksjonen, og derfor er det vanskelig å forutsi hvilke egenfrekvenser et turbinhjul i vann vil ha. I tillegg vil sannsynligvis viskøse effekter pga strømmende fluid også påvirke hvilke egenfrekvenser hjulet vil ha.

Det er ønskelig at man undersøker egenfrekvenser for en disk (forenklet turbin) i luft og vann både ved hjelp av analytiske metoder og simuleringer for å finne hvilke frekvenser som vil være de viktigste i for en slik forenklet turbin. I tillegg ønsker man i fremtiden å kunne gjøre eksperimentelle målinger av egenfrekvenser i Francisriggen ved Vannkraftlaboratoriet ved NTNU. Målingene ligger frem i tid, men måleoppsettet er det viktig at man får etablert innen den tid.

Oppgaven bearbeides ut fra følgende punkter

1 Litteraturstudie av
   a. Roterende konstruksjoners egenfrekvenser og nåværende forskningsstatus.
   b. Målinger av egenfrekvenser til en roterende konstruksjon

2 Simulere egenfrekvensene til en sirkulær disk ved hjelp av passende programvare med FSI-muligheter

3 Identifisere et måleoppsett som muliggjør frekvensmålinger på Francisriggen og lage et LabView-program for å logge og post prosessere målesignal fra måleoppsettet

4 Om det er mulig skal det utføres frekvensmålinger på Francisriggen, hvis ikke det er mulig skal det søkes etter alternative måter å få målt på

5 Sammenligne målinger og simuleringer

" - "

Senest 14 dager etter utlevering av oppgaven skal kandidaten levere/sende instituttet en detaljert fremdrift- og eventuelt forsøksplan for oppgaven til evaluering og eventuelt diskusjon med faglig ansvarlig/veiledere. Detaljer ved eventuell utførelse av dataprogrammer skal avtales nærmere i samråd med faglig ansvarlig.

Besvarelsen redigeres mest mulig som en forskningsrapport med et sammendrag både på norsk og engelsk, konklusjon, litteraturliste, innholdsfortegnelse etc. Ved utarbeidelsen av teksten skal kandidaten legge vekt på å gjøre teksten oversiktlig og velskrevet. Med henblikk på lesning av besvarelsen er det viktig at de nødvendige henvisninger for korresponderende steder i tekst, tabeller og figurer anføres på begge steder. Ved bedømmelsen legges det stor vekt på at resultatene er grundig bearbeidet, at de oppstilles tabellarisk og/eller grafisk på en oversiktlig måte, og at de er diskutert utførlig.

Alle benyttede kilder, også muntlige opplysninger, skal oppgis på fullstendig måte. For tidsskrifter og bøker oppgis forfatter, tittel, årgang, sidetall og eventuelt figurnummer.

Det forutsettes at kandidaten tar initiativ til og holder nødvendig kontakt med faglærer og veileder(e). Kandidaten skal rette seg etter de reglementer og retningslinjer som gjelder ved alle (andre) fagmiljøer som kandidaten har kontakt med gjennom sin utførelse av oppgaven, samt etter eventuelle pålegg fra Institutt for energi- og prosessteknikk.

Risikovurdering av kandidatens arbeid skal gjennomføres i henhold til instituttets prosedyrer. Risikovurderingen skal dokumenteres og inngå som del av besvarelsen. Hendelser relatert til kandidatens arbeid med uheldig innvirkning på helse, miljø eller sikkerhet, skal dokumenteres og inngå som en del av besvarelsen. Hvis dokumentasjonen på risikovurderingen utgjør veldig mange sider, leveres den fulle versjonen elektronisk til veileder og et utdrag inkluderes i besvarelsen.

I henhold til "Utfyllende regler til studieforskriften for teknologistudiet/sivilingeniørstudiet" ved NTNU § 20, forbeholder instituttet seg retten til å benytte alle resultater og data til undervisnings- og forskningsformål, samt til fremtidige publikasjoner.

Besvarelsen leveres digitalt i DAIM. Et faglig sammendrag med oppgavens tittel, kandidatens navn, veileders navn, årstall, instituttnavn, og NTNUs logo og navn, leveres til instituttet som en separat pdf-fil. Etter avtale leveres besvarelse og evt. annet materiale til veileder i digitalt format.

☒ Arbeid i laboratorium (vannkraftlaboratoriet, strømningsteknisk, varmeteknisk)
☐ Feltarbeid

NTNU, Institutt for energi- og prosessteknikk, 8. september 2014

Olav Bolland
Instituttleder

Pål-Tore Storli
Faglig ansvarlig/veileder

Medveileder(e): Torbjørn K. Nielsen

ABSTRACT

This thesis presents two experimental methods used to determine the natural frequencies of a disc in air and submerged in water, with the goal of showing the change in natural frequencies when the disc is submerged. In addition, analytical calculations and numerical simulations of the natural frequencies of the same disc are presented.

The *roving hammer experiment* measures the disc' vibration response to impacts on various positions. The *frequency response experiment* measures the vibration response to excitation from a piezoelectric patch glued to the disc. From these experiments the natural frequencies and the mode shapes are identified. The analytical calculations are based on table values of eigenvalues for an annular plate with clamped inside and free outside. The numerical simulations are performed in ANSYS with material properties of standard steel to simulate the disc in air, and with an increased density to simulate the disc submerged in water. The increased density is calculated based on results from the frequency response experiment.

All methods gave similar results for the natural frequencies of the disc, with only minor differences. The roving hammer experiment found the natural frequencies of the disc in air, and indicated their mode shape. The frequency response experiment gave the frequency response function of the disc in air and submerged in water, and by comparing the functions the change in natural frequencies was shown. The analytical calculations and numerical simulations gave satisfactory results, but with slight over/under prediction of the natural frequencies.

SAMMENDRAG

Denne avhandlingen presenterer to eksperimentelle metoder benyttet for å bestemme egenfrekvensene av en disk i luft og nedsenket i vann, med mål om å vise endringene i egenfrekvenser når disken er nedsenket i vann. I tillegg er det analytiske beregninger og numeriske simuleringer av egenfrekvenser av samme disk presentert.

Streifende hammer forsøket måler diskens vibrasjonsresponsen til slag på ulike posisjoner. Frekvensrespons forsøket måler vibrasjonsresponsen til eksitasjon fra en piezoelektrisklapp limt fast til platen. Fra disse forsøkene blir egenfrekvensene og normalmodene identifisert. Den analytiske beregningen er basert på tabellverdier av egenverdier for en ringformet plate med fastklemt innside og fri uteside. De numeriske simuleringer er utført i ANSYS med materialegenskapene til standard stål for å simulere platen i luft, og med en økt tetthet for å simulere platen nedsenket i vann. Den økte tettheten beregnes basert på resultatene fra de frekvensrespons forsøket.

Alle metodene ga lignende resultater for egenfrekvensene til disken, med bare mindre forskjeller. Streifende hammer forsøket fant egenfrekvensene til disken i luft, og indikerte normalmodene deres. Frekvensrespons forsøket ga frekvensrespons funksjonen til disken i luft og nedsenket i vann, og ved å sammenligne funksjonene ble endringen i egnefrekvensene vist. De analytiske beregninger og numeriske simuleringer ga tilfredsstillende resultater, men med svak over / under prediksjon av egenfrekvensene.

PREFACE

This master thesis is written for the Waterpower Laboratory, Department of Energy and Process Engineering (EPT), at Norwegian University of Science and Technology (NTNU) during the fall of 2014. The problem description was set so I could work in the laboratory. The thesis includes the use of measuring techniques, structural dynamics, programing in LabVIEW and some basic use of ANSYS. None of which I had much experience with. This has motivated me and made the thesis very interesting.

I would like to thank my supervisor Pål-Tore Storli for always taking the time to answer my questions, and my co-supervisor Torbjørn K. Nielsen for sharing his knowledge and experience. I am grateful for the patience and help of the laboratory staff, especially Joar Grillstad. I would also like to thank Øyvind Moen Fjeld for his cooperation. In addition, big thanks to everyone that has made the environment at the Waterpower Laboratory so great.

Trondheim January 26, 2015

Truls Edvardsen Aarønes

CONTENTS

NOMENCLATURE

A_{ND}	Amplitude of the mode shape with ND nodal diameters	m
c	Damper	$\text{N}\cdot s/m$
c_A	Added dampening	$\text{N}\cdot s/m$
d_H	Disc support high (pipe)	m
d_L	Disc support low (shaft)	m
E	Elasticity, Young's modulus	Pa
E_{steel}	Elasticity of steel	Pa
f_{air}	Natural frequency, in air	Hz
f_{calc}	Natural frequency, analytically calculated	Hz
f_f	Natural frequency, in fluid (theory)	Hz
f_g	Guide vane frequency	Hz
f_{ham}	Natural frequency, roving hammer experiment	Hz
f_n	Rotational speed	Hz
f_{ND}	Vibration frequency of mode shape with ND nodal diameters	Hz
f_{pred}	Natural frequency, predicted simulation	Hz
f_{den}	Natural frequency, simulated with increased density	Hz

f_s	Sample rate	Hz
f_{sim}	Natural frequency, simulated with standard steel	Hz
f_v	Natural frequency, in vacuum (theory)	Hz
f_{water}	Natural frequency, in water	Hz
h	Disc thickness	m
h_1	Disc submersion	m
h_2	Disc elevation	m
k	Spring constant	N/m
κ	Parameter for prediction of natural frequencies	1/m
λ^2	Eigenvalues	-
m	Mass	kg
m_A	Added mass	kg
ND	Number of nodal diamteres	-
ν	Poisson's ratio	-
R	Outer radius	m
r	Inner radius	m
ρ	Density	kg/m^3
ρ_{steel}	Density of steel	kg/m^3
ρ_w	Increased density	kg/m^3
t	Time	s
θ	Angular position	°
x	Position	m
ξ	Transverse deflection	m
Z_g	Number of guide vanes	-

INTRODUCTION

BACKGROUND

Francis runners have experienced breakdowns in recent years [1-3]. Changes in grid operation has changed the loads turbines experience. New manufacturing techniques and more powerful numerical tools, has made it possible to make turbines more efficient and with less steel. With more off design point operation, more power, and less steel, the Francis runners can experience serious vibrations that could cause failure. To avoid such damaging vibrations, it is important to determine accurately the natural frequencies and the dynamic behavior of the runner. However, the natural frequencies and the dynamic behavior of structures change when submerged in water and with its nearby structures.

The dynamical behavior and natural frequencies of submerged structures has been studied by many. Lamb [4] studied analytically a disc clamped along its circumference and in contact with water on one side. He was able to make estimations of the natural frequencies using Rayleigh's method [5] (§90), and found that the natural frequencies would be considerably reduced. Powell and Roberts [6] studied the case experimentally and found the results to be in good agreement with Lambs work.

Later, Kubota et al. [7] modeled a Francis runner using a bladed disc. With this model, Kubota studied analytically and experimentally the vibration of the disc' diametrical modes, when rotated and subjected to excitation simulating Rotor-Stator Interaction (RSI). Kubota et al. formulated a condition that had to be satisfied for a diametrical mode shape to be excited. Kubota and Suzuki [8] studied analytically and experimentally the added mass effect on disc' vibrating in fluid. They found an formulation to approximate the natural frequencies in fluid. Tanaka [3] built on Kubota's studies, and investigated high head Francis runners analytically and experimentally. Tanaka found that the reduction of natural frequencies of a runner due to added mass effect was considerable, and further explored the dynamic behavior and stress in the runner.

More recently, *Center of Industrial Diagnostics and Fluid Dynamics (CDIF)* in Barcelona (with Eduard Egusquiza as director) has studied the added mass effect on structures experimentally and numerically. Rodriguez et al. [9] studied experimentally the added mass effect on a Francis runner in still water. Rodriguez et al. [10] used *structural-acoustical FSI* model to accurately determine the natural frequencies of a cantilever plate submerged in water. Valentin et al. [11] studied experimentally and numerically the added mass on a disc. The study also showed the importance of nearby rigid surfaces on the natural frequencies of submerged structures.

THESIS STRUCTURE

The work put into this thesis has mainly been focused on preparation and execution of experiments designed to determine the natural frequencies of a disc (simplified Francis runner). Yet, analytical calculations and numerical simulations has also been performed, and are presented in the thesis.

The rotation of structures has been left out of this thesis, despite the problem description. Experimentally it would demand a more advanced setup. Meaning more time would be needed to setup the experiments. Therefore, to be sure results could be achieved given the limitations of this study, a decision was made to omit rotation from the thesis.

Chapter 1 Theory introduces concepts used in the thesis, and some vibration theory. The theoretical basis for the analytical calculations of the natural frequencies are described in this chapter. In chapter 2 Methods, descriptions of the experimental setup, the different methods used to excite the disc to vibrate, how vibration is measured, and the post processing methods are presented. The setup of the numerical simulations are also presented in this chapter. The results from all methods are presented in chapter 3 Results, with some interpretations. The methods of determining the natural frequencies are then evaluated in chapter 4 Discussion, and the major points are presented in the Conclusion chapter. Last, in the Further work chapter some thought of how the work in this thesis can be continued are presented.

1 THEORY

1.1 ROTOR-STATOR INTERACTION AND FATIGUE

Rotor-Stator Interaction (RSI) in Francis runners cause pressure pulsations. They occur each time a runner blade pass a guide vane, and will cause deflection and increased stress in the runner. This will force the runner to vibrate at the *guide vane frequency* f_g given by equation (1). Where Z_g is the number of guide vanes, and f_n is the rotational speed (rps) of the runner.

$$f_g = Z_g \cdot f_n \qquad \text{[Hz]} \quad (1)$$

For a turbine with 28 guide vanes and rotating at 375 rpm the frequency is 175 Hz. This will cause a very high number of load cycles ($N > 10^9$) during the lifetime of the turbine. To avoid fatigue it is therefore important to keep the amplitudes of the load cycles low.

The amplitudes of the load cycles could be mitigated by either reducing the amplitude of the pressure pulsation, or by reducing the runners response to the pressure pulsations. To keep the runners response low, it is important to avoid resonance [2]. However, accurately predicting at which frequencies resonance occur is not trivial since a runners natural frequencies change when submerged and with its surrounding structures [3, 11].

1.2 NATURAL FREQUENCIES AND ADDED MASS EFFECT

A single degree of freedom system with a mass (m), a damper (c) and a spring (k) in vacuum is described by equation (2). The natural frequency in vacuum f_v of this system is given equation by (3) (given $k \gg c^2/4m$). A system with multiple degree of freedom system will have as many natural frequencies as it have degrees of freedom.

$$m\ddot{x} + c\dot{x} + kx = 0 \qquad \text{[N] (2)}$$

$$f_v = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad \text{[Hz] (3)}$$

If the system is then put into a fluid, some surrounding mass m_A has to be moved when mass m is moved. In addition, when mass m is moving some additional energy will be lost due to the viscosity of the fluid (c_A). The system can now be described by equation (4) with a natural frequency in fluid f_f given by equation (5) (given $k \gg (c+c_A)^2/4(\mathrm{m}+\mathrm{m}_A)$).

$$(m+m_A)\ddot{x} + (c+c_A)\dot{x} + kx = 0 \qquad \text{[N] (4)}$$

$$f_f = \frac{1}{2\pi}\sqrt{\frac{k}{m+m_A}} \qquad \text{[Hz] (5)}$$

As we can see, the natural frequency is lower when the system is submerged in a fluid. This is called *added mass effect*, and it can cause natural frequencies of a Francis runner to decrease by 40 % [3, 9].

1.3 DIAMETRICAL MODE SHAPES ON A DISC

Discs *diametrical mode shapes* are described along the edge by cosines with maxima at the point of excitation, amplitude A_{ND} and ND periods around perimeter of the disc [3, 7]. Nodal diameters (ND) are lines intersecting the disc center where the mode shapes deflection amplitude is always zero. Each mode shape oscillate only transversely, meaning that the nodal diameters does not move relative to an excitation.

The transverse deflection ξ is a sum of the vibration of all mode shapes. Then, when each mode shape oscillate with its particular frequency f_{ND}, the transverse deflection of the perimeter of the disc is a function of time t and position θ given by (6) [7].

$$\xi(\theta,t) = \sum_{ND=0}^{\infty} A_{ND} \cos(ND\cdot\theta)\cdot\sin(2\pi\cdot f_{ND}\cdot t) \qquad [\text{m}] \quad (6)$$

In this thesis the name of the mode shapes will be the number of nodal diameter, e.g. a mode shape with two nodal diameters will be named ND2. The mode shapes studied in this thesis is presented in Figure 1.

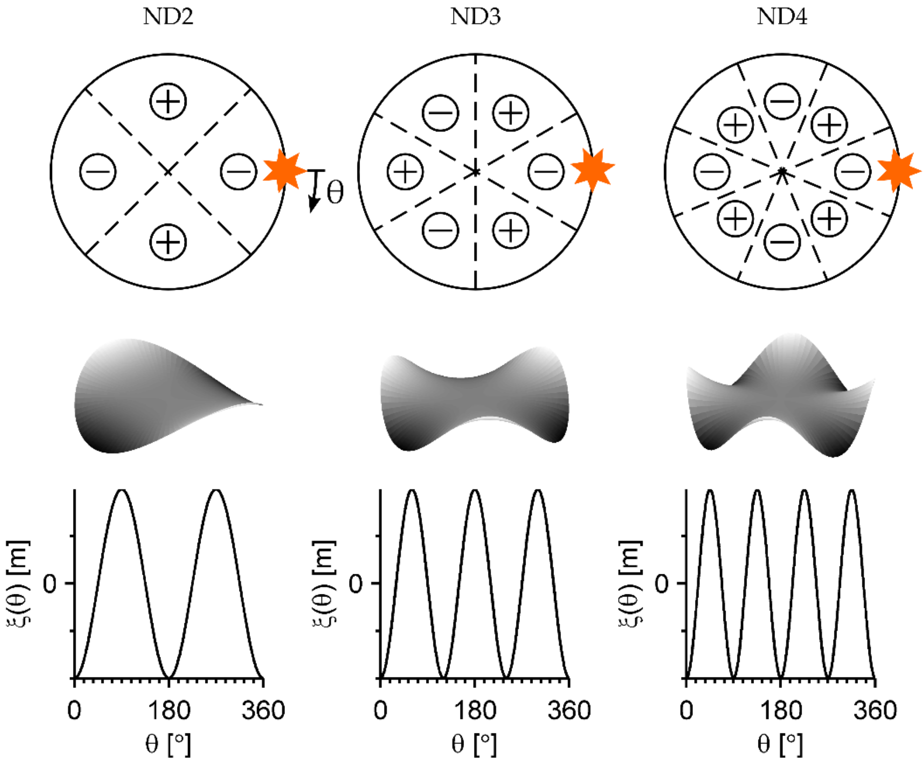

Figure 1: Diametrical mode shapes, ND2, ND3 and ND4. At the top, nodal diameters in dashed lines given impact/excitation at orange star. In the middle, 3D plot of the mode shapes. At the bottom, equation (6) plotted for each mode shape.

Natural frequencies of mode shapes

Each mode shape has its own natural frequency [12]. In [12] some general solutions for natural frequencies in vacuum are presented. The case studied in this thesis is described as an annular plate free on outside and clamped on inside. In Table 1 eigenvalues λ for ND2 and ND3 is given for the ratio between inner and outer radius $r/R = 0.1$.

ND	λ^2 [-]
2	5.62
3	12.4

Table 1: Eigenvalues for an annular plate, free on outside and clamped on inside, for two mode shapes given the ratio between inner and outer radius $r/R = 0.1$.

From the eigenvalues, the natural frequencies is calculated from equation (7) [13]. Here, R is the outer radius, h is the thickness of the plate, E is the elastic modulus, ν is Poisson's ratio and ρ is the density of the plate.

$$f_{calc} = \frac{1}{2\pi}\frac{\lambda^2}{R^2}\sqrt{\frac{Eh^2}{12(1-\nu^2)\rho}} \quad \text{[Hz]} \quad (7)$$

1.4 EXCITATION OF VIBRATION

In this thesis, three methods are used to excite vibration. *Impact excitation* is an impulse applied to the system. It excites instantaneously all frequencies, and will cause all mode shapes to vibrate with their natural frequency. Since the excitation is instantaneous, the vibration will be dampened with time. *Noise excitation* continuously excite all frequencies, by applying a white noise signal. A white noise signal is a random signal with a constant power spectral density, meaning in practical terms that it will excite all frequencies with the same amplitude. *Sweep excitation* start to excite at one frequency, and continuously increase the frequency over time until an end frequency.

The response of the excited system will always have the same frequency as the excitation. However, it will have a phase shift depending on the system properties, and the response amplitude depending on the excitation amplitude and frequency, and system properties.

1.5 MEASUREMENTS AND PROCESSING

This thesis presents measurements of natural frequencies of a disc.

Sampling rate, filtering and aliasing

When measured, a continuous signal is reduced to a discrete signal of f_s samples per second. If periodic signals are measured with a too low sample rate f_s, the measurements might show unreal peaks when analyzed with spectral analyses (i.e. FFT). This is called aliasing. To avoid aliasing, the sample rate has to be more than two times the maximum frequency occurring in the measured signal [14].

Often the high frequent part of the measured signal is not of interest. Still, to avoid aliasing effects the sample rate has to be larger than two times the highest frequency occurring. This can make the data set acquired from measurements much larger than required for its intended purpose. To avoid this, filters can be applied in post processing to remove frequencies higher than the highest frequency of interest from the measurements. Then, the sample rate only need to be larger than two times the highest frequency of interest (highest unfiltered frequency) [14].

Fast Fourier transform (FFT)

Fast Fourier transform (FFT) is a method of decomposing a discrete signal into sine waves. The method finds the amplitude and phase of the sine waves for frequencies up to half the sampling rate. Plotting the amplitudes versus the frequencies will reveal the significant frequencies occurring in the signal. This can be very useful to identify natural frequencies [14, 15].

Frequency response function (FRF)

The frequency response function (FRF) describes how a system will respond to an excitation. If a system is subjected to an excitation with a certain amplitude and frequency, the system will start to vibrate at the same frequency as the excitation, but with a phase shift. The amplitude of the response depend on the amplitude and frequency of the excitation. The FRF give the ratio between the response and excitation amplitude, and the phase shift of the response for any frequency. The FRF is usually presented in a Bode plot. Natural frequencies will be clearly visible as peak in the Bode magnitude plot [14, 16].

2 METHODS

2.1 OVERVIEW

Table 2 is a list of the methods used in this thesis to determine the natural frequencies of a disc attached to a shaft, in air and submerged in water.

A. Analytical calculations
B. Roving hammer, experimental identification of mode shapes
C. Frequency response, experimental determination of natural frequencies
    1. In air
        1.a. Noise excitation
        1.b. Sweep excitation
    2. In water
        2.a. Noise excitation
        2.b. Sweep excitation
D. Numerical simulations
    1. Simulation with standard steel
    2. Analytical prediction of simulation with increased density
    3. Simulation with increased density

Table 2: Methods applied in this thesis

2.2 DESCRIPTION OF EXPERIMENTAL RIG

The rig used in this thesis consist of an annular plate (disc) fitted on a shaft, both made of steel[1]. The disc is tightly fitted to the shaft, and is constrained between a flange on the shaft and a pipe. The pipe is tightened by a nut that is screwed on the shaft. The shaft is fixed vertically by wooden beams in a large tank (Figure 2). Table 3 lists the describing parameters of the rig.

Parameter	Variable	Value [mm]
Disc outer diameter		500
Disc inner diameter		40
Disc thickness		20
Tank diameter		800
Tank height		490
Disc support low (shaft)	d_L	60
Disc support high (pipe)	d_H	48
Disc submersion (when in water)	h_1	~100
Disc elevation	h_2	280

Table 3: Rig parameters. The four variables refer to Figure 2.

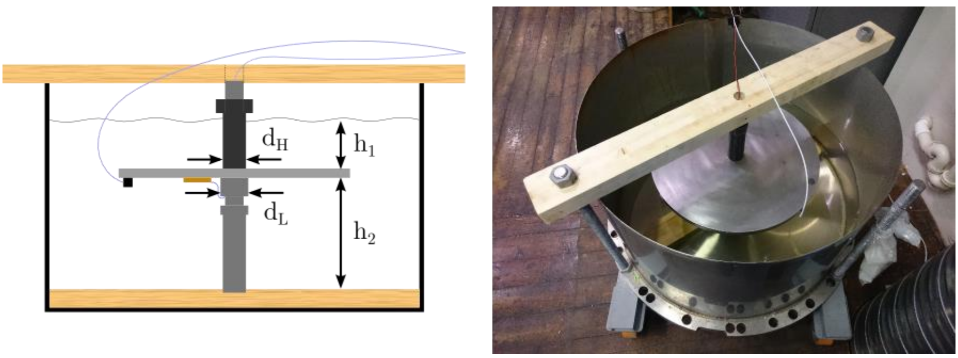

Figure 2: Test rig showing shaft and disc inside the tank. Accelerometer (black cube) and piezoelectric patch (brown rectangle) is shown in the drawing.

[1] Originally, the plate and shaft was part of a rig used to measure friction losses between a rotating disc and stationary discs in water.

Instrumentation

A Dytran 3006A accelerometer is installed on the disc to measure vibration (Figure 2). The accelerometer is connected to a National Instruments (NI) 9233 module in a NI cDAQ-9172 chassis that is connected to a computer with the software LabVIEW installed.

To excite vibration with specific frequencies, a piezoelectric patch (P-876.A15 DuraAct Patch Transducer) is installed on the disc (Figure 2). A piezoelectric patch works as an actuator, contracting or expanding proportionally to the voltage signal it receives. The patch is operated using a NI 9263 module to give a voltage signal (± 10 V), and a voltage amplifier (E-835 DuraAct Piezo Driver Module, voltage gain: 25). A NI 9239 module monitors the voltage output from the voltage amplifier. Both NI modules are connected to the NI chassis mentioned above.

2.3 ROVING HAMMER

The mode shapes of the disc are identified (connected to the disc' natural frequencies) using the *roving hammer* method. Measuring and analyzing (FFT analysis) the response of the disc to an impact, will reveal the natural frequencies of the disc. However, the natural frequencies' corresponding mode shape will not be clear. To identify the corresponding mode shapes, the disc is impacted on its edge by dropping a hammer from a constant height on various positions relative to the accelerometer. This will excite all mode shapes to vibrate at their natural frequencies. Since the mode shapes are defined by the impact position, the mode shapes' amplitude measured by the accelerometer will vary with the impact position. The impact positions are set so that a particular mode shape' amplitude is zero at the position of the accelerometer. This is done by setting the impact position so that the accelerometer is on a nodal diameter. In theory, one natural frequency will not appear in the analysis (FFT) of the measurements acquired from such an impact position. That natural frequency can then be identified as the natural frequency of the mode shape the impact position was set for. However, because of inaccuracies in the impacts, the natural frequencies will not disappear completely from the analysis', but they will be clearly lower than they are for other impact positions.

For the mode shapes studied in this thesis (ND2, ND3 and ND4), the impact position are calculated using an equation derived from equation (6)[2], and are shown in Table 4 and in Figure 3. In addition to these positions, the disc is impacted at 180° from the accelerometer for reference. Measurements from this impact will make a good reference since all mode shapes will have amplitude maxima (or minima) at the position of the accelerometer. All impacts are repeated five times. The measurements (4 impacts, 5 repetitions) are made by the accelerometer with a sample rate of 25000 Hz. When this experiment was performed, the disc was detached from the shaft and laying on a small cylinder. The piezoelectric patch was not glued on yet, and another accelerometer was used (Brül & Kjær DeltaTron 4397).

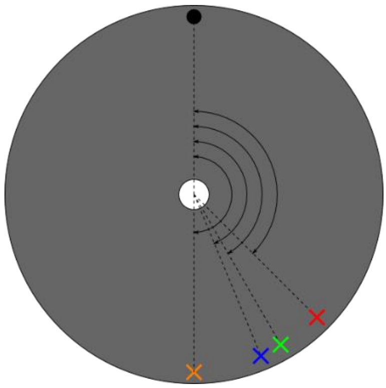

Figure 3: Impact positions in roving hammer experiment. Accelerometer (black), 180° (orange), 135° (red), 150° (green) and 157.5° (blue).

Mode shape	**Impact position, θ**
ND2	135°
ND3	150°
ND4	157.5°

Table 4: Impact position. Angular distance from hammer impact to accelerometer

In post processing, all measurements are analyzed using Fast Fourier Transform (FFT). For each impact point, an average FFT result is calculated from the FFT results of the repetitions. The FFT results will show peaks at the natural frequencies of the disc. The magnitudes (amplitude) of these peaks will vary with impact position. For each of the three impact positions in Table 4 there should be one natural frequency that is significantly lower than it is for the other impact positions. This will indicate that the

[2] Equation derived from equation (6): $\theta = 180° - \arccos(0)/ND = 180° - 90°/ND$.

frequency is the natural frequency of the mode shape the impact position was set for, see Table 4.

2.4 FREQUENCY RESPONSE

To determine the disc' natural frequencies in both air and submerged in water, and showing the added mass effect, frequency response experiments are performed in both air and submerged in water.

During the experiments, the disc is excited by the piezoelectric patch, and both the excitation signal (output from voltage amplifier) and the vibration (measured by the accelerometer) is measured. The excitation signal is created in LabVIEW as an analog output voltage signal. When noise excitation is used, the signal is a uniform white noise waveform with amplitude of 4 V[3]. When sweep excitation is used, the signal is an up-chirp waveform with amplitude of 4 V[3], start frequency of 100 Hz and a final frequency of 5000 Hz. In both cases, the sample rate is 12500 Hz. Both of the measurements are also made with sample rate of 12500 Hz. To get FRFs with proper resolution, the experiments using noise excitation is run for 60 seconds, while the experiments using sweep excitation is run for 600 seconds. The disc is set up as described in chapter 2.2.

In post processing, both the excitation measurements and the vibration measurements are filtered and normalized. To create a FRF, excitation (stimulus) and vibration (response) measurements are used as input in a *Frequency Response Function (Mag-Phase) VI* palette in LabVIEW. This palette give magnitude and phase of the FRF as output. Averaging is used to get a smooth FRF, without distortion. This is done by dividing both the excitation and the vibration measurements into segments. Then FRFs are created for each segment pair (i.e. first excitation segment and first vibration segment), and an average FRF is calculated from these[4].

2.5 NUMERICAL SIMULATIONS

To determine numerically the natural frequencies of the disc, and their corresponding mode shapes, Finite Element Analysis' are performed in ANSYS. A model of the disc

---

[3] This was later found to be outside the operating limits of the voltage amplifier, and was the probable cause of failures of the amplifier. 1 V should be sufficient, and be inside the operating limits.

[4] This method of calculating the average FRF has later been found not suitable when sweep excitation is used, see chapter 4.3 for further discussion.

and shaft is drawn in ANSYS after parameters given in Table 3. It is made of two parts, the disc and the shaft. The shaft is drawn so it is clamping the disc as in the experimental rig. The disc is meshed using *Sizing*[5] on the edges, and *Mapped face meshing*[5] on the faces. All four edges (inner/outer diameter, upper/lower face) is divided into 76 segments (θ -direction). The edge and center faces are dived into 3 segments (z-direction), and the upper and lower faces into 58 segments, creating 13224 elements (see Figure 4). This was chosen as results changed little with more elements. The shaft is meshed with default ANSYS settings. All faces of the shaft are fixed with *Fixed support*. The analysis is performed with *Modal* analysis in ANSYS. This will finds both the mode shapes and their natural frequencies.

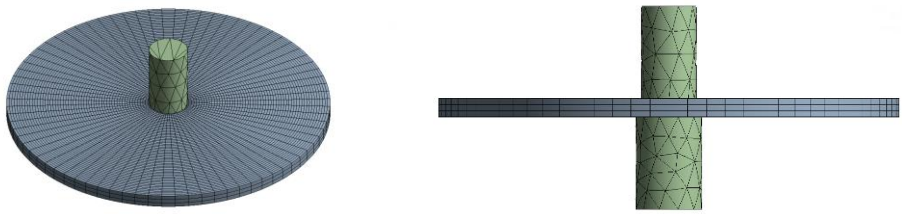

Figure 4: Model used for numerical simulations

Simulation with standard steel

To determine numerically the natural frequencies of the disc in air, a simulation using the material properties of standard steel is performed in ANSYS. Table 5 shows the material properties of standard steel.

Parameter	Value	Unit
ρ	7850	kg/m^3
E	200	GPa
ν	0.3	–

Table 5: Material properties of standard steel used to simulate the disc in air.

Analytical prediction of simulation with increased density

Based on results acquired from the simulation of the disc in air, a parameter κ can be calculated for each mode shape from equation (8), which is derived from equation (7). Using the parameters κ predictions of simulations using increased density are calculated from equation (9). Where f_{pred} is the predicted natural frequency, and ρ_w is the increased density used to simulate the disc submerged in water.

[5] Meshing method in ANSYS

$$\kappa = f_{sim} \cdot \sqrt{\frac{\rho_{steel}}{E_{steel}}} \qquad [1/\text{m}] \ (8)$$

$$f_{pred} = \kappa \cdot \sqrt{\frac{E_{steel}}{\rho_w}} \qquad [\text{Hz}] \ (9)$$

Simulating with increased density

The added mass effect of water is simulated using an increased density. This increased density is determined based on equation (9) and results from the frequency response experiment performed in water. The increased density is set to the value that minimize the root mean square (RMS) of the relative differences between experimental results and predictions, given by equation (10).

$$RMS = \sqrt{\left(\frac{f_{water} - f_{pred}}{f_{water}}\right)^2_{ND2} + \left(\frac{f_{water} - f_{pred}}{f_{water}}\right)^2_{ND3} + \left(\frac{f_{water} - f_{pred}}{f_{water}}\right)^2_{ND4}} \qquad [\text{-}] \ (10)$$

3 RESULTS

3.1 ANALYTICAL CALCULATIONS

Natural frequencies of ND2 and ND3 calculated from equation (7) are shown in Table 7. The eigenvalues used are given for a ratio between inner and outer radius of $r/R = 0.1$ [12]. Material properties of standard steel are used, and are given in Table 6 together with the outer radius and thickness of the disc.

Parameter	Value	Unit
ρ	7850	kg/m³
E	200	GPa
ν	0.3	–
R	0.25	m
h	0.02	m

Table 6: Parameters used to calculate natural frequencies

	ND2	ND3
λ^2 [-]	5.62	12.4
f_{calc} [Hz]	437	965

Table 7: Analytical calculation of natural frequencies using (7) and values from Table 6.

3.2 ROVING HAMMER

The results from the Roving Hammer experiment are presented in Figure 5. As we can see, there are three peaks occurring at the same frequencies for all impact positions.

Further we can see that the magnitude at each of the three frequencies change with the impact position.

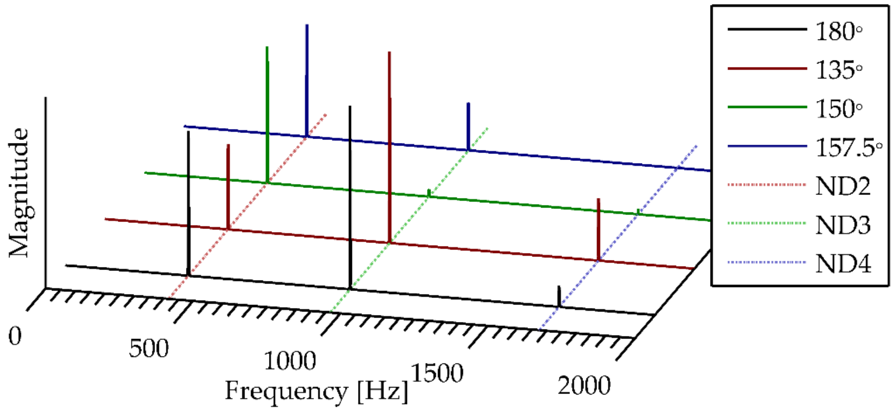

Figure 5: Averaged FFT of Roving Hammer measurements. Impact at 130°

Identifying mode shapes

The three frequencies and the magnitudes of these are given, for each impact position, in Table 8. We can see in Figure 5, and from the data in Table 8, that the magnitude of the lowest of the three frequencies (417 Hz) has a clearly lower value when impacted at 135° . This indicates that this frequency is the natural frequency of mode shape ND2, since the impact position was set to achieve low magnitude for ND2 (see Table 4). Further, the middle frequency (966 Hz) has a clearly lower magnitude when impacted at 150°, and the highest (1675 Hz) at 157.5°, indicating that they are the natural frequencies of mode shapes ND3 and ND4 (see Table 4).

	Magnitude when impacted at:				
f_{ham} [Hz]	180°	135°	150°	157.5°	Mode shape
417	3.24	**2.00**	3.68	3.04	ND2
966	4.38	4.46	**0.20**	1.28	ND3
1675	0.48	1.28	0.14	**0.00**	ND4

Table 8: Magnitude of averaged FFT of Roving Hammer measurements for the three peaks, and their corresponding mode shape.

3.3 FREQUENCY RESPONSE

The results from the frequency response experiments in air and water, using both noise and sweep methods to excite vibration, are presented as a FRF in a Bode magnitude plot in Figure 6.

Identifying natural frequencies of the mode shapes

We can see that both methods of excitation give similar response in air. We also see clear peaks around the frequencies found in the roving hammer experiment, indicating the natural frequency of ND2, ND3 and ND4 (red, green, blue vertical lines in Figure 6).

In water, the two methods of excitation give response that is more different. The peaks are not as clear either. However, some features of the curves can be recognized from the response in air. The "camelback" between 400 Hz and 600 Hz in air, can be seen between 300 Hz and 500 Hz in water. ND2 is identified as the first maxima of the "camelback" (red vertical line).

Looking at the magnitude in air, to the right of the "camelback" there is one minor maxima before a steep rise to the peak of ND3. In water, these features are not as clear, and the rise occur at different frequencies for the two methods. The natural frequency of ND3 is thought to be at the top of the steep rise of the noise method (green vertical line)[6].

Further to the right a "pulse" can be seen at the natural frequency of ND4 in air. The "pulse" can not be seen on the water curves, but there is a steep drop on the noise curve and a maxima on the sweep curve at the same frequency in water, indicating were on the curve the "pulse" should have been. The natural frequency of ND4 is then interpreted to be just before the drop in the noise curve (blue vertical line).

[6] Deviation between the excitation methods are discussed in chapter 4.3.

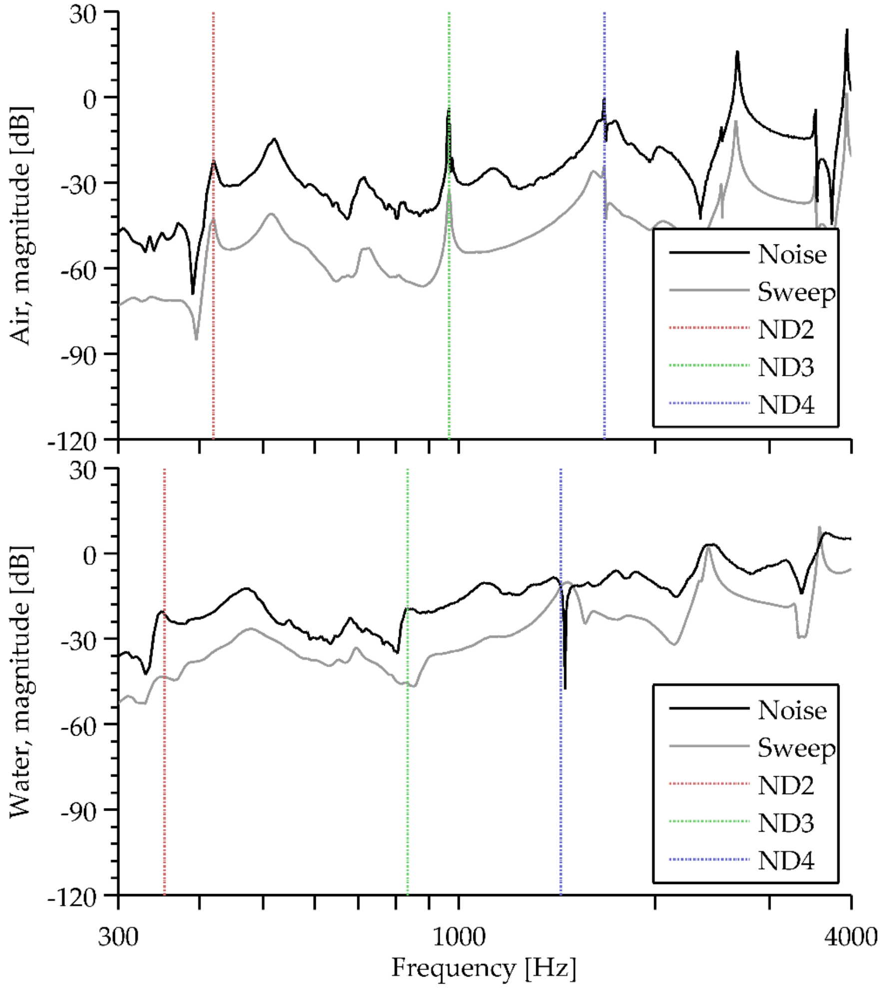

Figure 6: Frequency Response Function (FRF) in air and water. Both excitation methods are shown. Vertical lines show the natural frequency of ND2, ND3 and ND4.

The natural frequencies of ND2, ND3 and ND4 in air and water, as defined above are given in Table 9.

	ND2	ND3	ND4
f_{air} [Hz]	420	965	1671
f_{water} [Hz]	353	834	1432

Table 9: Natural frequencies of ND2, ND3 and ND4 in air and water.

3.4 NUMERICAL SIMULATIONS

Simulation with standard steel

Results from modal analysis in ANSYS using material properties of standard steel to simulate the disc in vacuum are presented in Table 10.

	ND2	ND3	ND4
f_{sim} [Hz]	434	949	1644

Table 10: Results from modal analysis in ANSYS, simulating the disc in vacuum.

Analytical prediction of simulation with increased density

The κ parameters is calculated from simulation using standard steel and shown in Table 11. The increased density that gave the lowest RMS of the relative difference between predictions and experimental results is given in Table 12. The analytical predictions of simulations with increased density are shown in Table 11.

	ND2	ND3	ND4
κ [1/m]	0.0859	0.1879	0.3257
f_{pred} [Hz]	370	809	1402

Table 11: Predictions of natural frequencies using increased density, given k parameters based on simulated results using standard steel.

Parameter	Value	Unit
ρ_w	10800	kg/m^3
E	200	GPa
ν	0.3	–

Table 12: Material properties used for modal analysis in ANSYS, simulating the disc submerged in water.

Simulation with increased density

The results from the modal analysis in ANSYS using increased density (see Table 12) is shown in Table 13.

	ND2	ND3	ND4
f_{den} [Hz]	370	809	1402

Table 13: Results from modal analysis in ANSYS, simulating the disc submerged in water.

3.5 SUMMARY OF RESULTS

Table 14 show the values of the natural frequencies of the mode shapes studied from all methods (except frequency response with sweep excitation).

Method		Natural frequencies [Hz] ND2	ND3	ND4
Analytical calculations	f_{calc}	437	965	-
Roving hammer	f_{ham}	417	966	1675
Frequency response				
In air	f_{air}	420	965	1671
In water	f_{water}	353	834	1432
Numerical simulations				
Simulation with standard steel	f_{sim}	434	949	1644
Analytical prediction of increased density	f_{pred}	370	809	1402
Simulation with increased density	f_{den}	370	809	1402

Table 14: Summary of results. Yellow background indicates in water.

Table 15 show the relative value of the methods compared to the values gotten from the frequency response experiments.

Method	Relative value [%] ND2	ND3	ND4
Analytical calculations	104.0	100.0	-
Roving hammer	99.3	100.1	100.2
Frequency response			
In air	**100.0**	**100.0**	**100.0**
In water	84.0	86.4	85.7
	100.0	***100.0***	***100.0***
Numerical simulations			
Simulation with standard steel	103.3	98.3	98.4
Analytical prediction of increased density	*104.8*	*97.0*	*97.9*
Simulation with increased density	*104.8*	*97.0*	*97.9*

Table 15: Relative values of results. Normal numbers are compared to frequency response in air. Italic numbers are compared to frequency response in water. Yellow background indicates in water.

4 DISCUSSION

4.1 EVALUATION OF ANALYTICAL CALCULATIONS

The results from the analytical calculations are quite similar to the results gotten from the other methods. Indicating that the analytical calculations are a good method of finding the natural frequencies of a disc. However, while the natural frequency of ND3 is the same as the measurements from the frequency response experiment gave, the natural frequency of ND2 is notably higher than the experiment gave. This might indicate an inaccuracy in the analytical model.

Material properties

The material properties used in the analytical calculations were chosen independently of the properties of the disc. However, this cannot be cause of the observed inaccuracies in the analytical model. Higher density would in general give lower analytical results for the natural frequencies. The density would have to be 8500 kg/m^3 for the natural frequency of ND2 to be the same as for the experimental results, but this would make the natural frequency of ND3 notably lower than the experimental results. Lowering the elasticity will have the same effect.

Radius ratio

In the experimental setup in this thesis, the disc is clamped from one side at $r = 0.03\text{ m}$ and at $r = 0.024\text{ m}$ on the other, and the diameter of the disc is $R = 0.25\text{ m}$. The radius ratio that would model this setup best is therefore not clear, but it should be between $0.096 \le r/R \le 0.12$. The ratio used in this thesis is $r/R = 0.1$. Using a lower ratio would give lower eigenvalues, and therefore lower natural frequencies [12].

Eigenvalues

In [12] there are three tables of eigenvalues for cases with clamped on the inside and free on the outside. The values are given for different mode shapes and for different radius rations. The eigenvalues vary somewhat from table to table, given a mode shape and a radius ratio. For ND2, two tables give values for the radios ratio used in this thesis. One of them also have eigenvalue for ND3 at this radius ratio, the values used in this thesis are taken from this table. If the eigenvalue for ND2 from the other table were used, the natural frequency of ND2 would be 428 Hz (101.9 %). This indicates that the eigenvalues might be the cause of the inaccuracies in analytical model.

4.2 EVALUATION OF ROVING HAMMER

The results from the roving hammer experiment is very similar to the results of the frequency response experiment. The small differences could be caused by roughness in the post processing of the frequency response measurements, or by the differences in setup.

The roving hammer method worked well to identify the three mode shapes studied in this thesis. However, the measurements using roving hammer in this thesis was not very clear by them self. Only when analytical and numerical results showed the same natural frequencies for the mode shapes a conclusion could be given. By making more accurate impacts with the hammer, increasing both location and force precision, the results might be more conclusive by themselves.

4.3 EVALUATION OF FREQUENCY RESPONSE

Frequency response using noise excitation

Acquiring a Frequency Response Function (FRF) using noise to excite vibration worked very well. The method was able to get results fast, acquiring adequate results from experiments run only for a few seconds. The results presented in Figure 6 are taken from a test run for 60 seconds, and are post processed to get a resolution of 5 Hz. To get better resolution, the experiment can be run longer. Alternately, a method could be created to include separate response measurements, performed in the frequency range where better resolution is wanted.

Unsuitable averaging of measurements using sweep excitation

The FRFs acquired from experiments performed with noise and sweep methods in air, quite similar. However, the functions acquired from measurements done in water have clear differences (see Figure 6). The curve of the sweep method does not have the same features as the curve of the noise method. The cause of this is probably the averaging method used in post processing.

The averaging method used in post processing for both noise and sweep excitation takes the first N (i.e. 2500) measurements of the excitation and response, and creates a FRF. Next, it takes the N measurements after and create a new FRF, and so on. Then it takes the average of the FRFs. This averaging method is suitable for noise excitation since the excitation is similar over time, therefore the response will be similar over time and an average can be taken to create a smooth FRF. However, this is not the case for sweep excitation. The excitation change with time, and therefore also the response. When excited at low frequencies, a FRF is created for all frequencies, but it will be inaccurate at high frequencies since these frequencies have not been properly excited. Similarly, when excited at high frequencies, the FRF will be inaccurate a low frequencies. Averaging the FRFs will therefore give an inaccurate final FRF.

Proposed new method: stepwise sweep excitation

Instead of changing the excitation frequency continuously and taking averages of FRFs, the excitation can be swept "stepwise" and the FRF can be made up by the response at each of these steps. Measurements made with constant excitation frequency for a time give the response at that frequency. The frequency is then changed and kept constant to give the response at that frequency. And so on. Together the responses at each step give a FRF curve. This should also make it possible to supplement the swept measurements with additional measurements done independently at specific frequencies, increasing resolution in frequency ranges of particular interest.

4.4 EVALUATION OF NUMERICAL SIMULATIONS

The numerical simulation performed for this thesis to determine the natural frequencies worked well. However, as with the analytical calculations, ND2 is notably higher than the experimental results and ND3 and ND4 is notably lower. There seems therefore to be something in the model that is not completely correct. Perhaps a more accurate model were the weight and placement of the accelerometer and other details are included could give even more accurate results. However, as mentioned in the discussion on the analytical calculations, when using other eigenvalues to calculate the natural frequencies analytically, they became more accurate. It could then be that the numerical model has the similar problem.

It should also be noted that the analytical calculation has performed just as well as the numerical simulations. The usefulness of the simulations on simple geometries like the one studied in this thesis, is therefore low.

Increased density and predictions

The simulation using increased density to fit experimental results in water worked satisfactory for all three mode shapes. However, the over prediction of the natural frequency of ND2, and the under prediction of the natural frequencies of ND3 and ND4 has become larger. This can indicate that each mode shape has a different added mass, and should therefore have different increased densities. This has been noted by [9]. In addition, the usefulness of simulation with increased density is low. To calculate the increased density, experimental results has to be known already. Moreover, the analytical prediction of the simulation gave exactly the same results.

CONCLUSION

Experimental, analytical and numerical methods to determine the natural frequencies of a disc in air and submerged in water has been presented. The natural frequencies of the disc studied have been found in air and submerged in water. The added mass effect has been shown by comparing the natural frequencies found in air to the ones found when the disc is submerged in water.

The roving hammer experiment identified which mode shape corresponded to which natural frequency successfully. However, because of inaccuracies in the impacts position and force the results were not clear.

The frequency response experiments performed with noise to excite vibration gave good results, both in air and submerged in water. The method gave smooth Frequency Response Functions (FRF) after running the experiment for only 60 seconds. The natural frequencies could then be identified from the FRF. However, the natural frequencies of the disc submerged in water was somewhat difficult to identify. The FRF did not have clear peaks, and conjectures had to be made. The post processing procedure used on experiments performed with both noise and sweep excitation, was unsuitable for experiment with sweep excitation and the results from that experiment were therefore inaccurate.

Analytical calculation for the natural frequencies in air gave results similar to the experimental results. The natural frequency of ND2 was a bit higher than found by the experiments. The cause of this seems to be inaccurate eigenvalues, since eigenvalues from other sources gave better results.

Numerical simulations gave good results for the natural frequencies. The simulation of the disc in air, using standard steel, slightly over predicted the natural frequency of ND2 and slightly under predicted the natural frequency of ND3 and ND4. When simulating the disc submerged in water, by increasing the density of the disc, the errors increased. This is probably because each mode shape has its own added mass, and can therefore not be modeled well with a globally increased density.

FURTHER WORK

This thesis has been a small step towards estimation of dynamic behavior and natural frequencies of Francis runners in operation. Methods to find the natural frequencies of a disc has been presented. The next step would be to improve these methods, as discussed in this thesis, and then apply them on model runners.

The experiment using sweep excitation presented in this thesis, should be replaced by an experiment using stepwise sweep excitation, as described in this thesis. A method of creating frequency response functions (FRF) from such an experiment would have to be developed, but there are examples of this in LabVIEW.

After making the improvements to the experimental methods, they can be applied on model runners. Then the natural frequencies of the mode shapes that are excited by Rotor-Stator Interaction (RSI) should be studied. The model runners are very stiff, measurements will therefore be challenging, making accuracy important.

The Structural-acoustical numerical model used by Rodriguez et al. [10], should be studied, and applied on a disc and Francis runners. This has been shown to be capable of modeling the added mass effect of water.

There are analytical methods of estimating the natural frequencies and dynamic behavior of structures not studied in this thesis. A study of such methods would enable a further understanding of the important factors describing the dynamic behavior of structures in air and submerged in water.

REFERENCES

1. Bjørndal, H., A. Reynaud, and L. Anders, *Mechanical robustness of Francis runners, requirements to reduce the risk of cracks in blades.* Hydro 2011 (Prague, Czech Republic, August 17, 2011. **19**: p. 2011.

2. Egusquiza, E., et al., *Failure investigation of a large pump-turbine runner.* Engineering Failure Analysis, 2012. **23**: p. 27-34.

3. Tanaka, H. *Vibration behavior and dynamic stress of runners of very high head reversible pump-turbines*. in *15th IAHR Symposium*. 1990. Belgrade.

4. Lamb, H., *On the vibrations of an elastic plate in contact with water.* Proceedings of the Royal Society of London. Series A, 1920. **98**(690): p. 205-216.

5. Rayleigh, J.W.S.B., *The theory of sound*. 1877.

6. Powell, J. and J. Roberts, *On the frequency of vibration of circular diaphragms.* Proceedings of the physical society of London, 1922. **35**(1): p. 170.

7. Kubota, Y., et al., *Vibration of rotating bladed disc excited by stationary distributed forces.* Bulletin of the JSME, 1983. **26**(221): p. 1952-1957.

8. Kubota, Y. and T. Suzuki, *Added mass effect on disc vibrating in fluid.* TRANS. JAPAN SOC. MECH. ENG., 1984. **50**(449): p. 243-248.

9. Rodriguez, C., et al., *Experimental investigation of added mass effects on a Francis turbine runner in still water.* Journal of Fluids and Structures, 2006. **22**(5): p. 699-712.

10. Rodriguez, C., et al., *Capability of structural–acoustical FSI numerical model to predict natural frequencies of submerged structures with nearby rigid surfaces.* Computers & Fluids, 2012. **64**: p. 117-126.

11. Valentín, D., et al., *Experimental study on the added mass and damping of a disk submerged in a partially fluid-filled tank with small radial confinement.* Journal of Fluids and Structures, 2014.

12. Leissa, A.W., *Vibration of plates*. 1969, DTIC Document.

13. Southwell, R., *On the free transverse vibrations of a uniform circular disc clamped at its centre; and on the effects of rotation.* Proceedings of the Royal Society of London. Series A, 1922. **101**(709): p. 133-153.

14. Wheeler, A., A. Ganji, and V. Krishnan, *Introduction to engineering experimentation*. 2004, Upper Saddle River, NJ: Pearson/Prentice Hall. XI.

15. Kreyszig, E., *Advanced engineering mathematics*. 2007: John Wiley & Sons.

16. Balchen, J., T. Andresen, and B. Foss, *Reguleringsteknikk.* Tapir, Trondheim, Norge, 2003.