

Ten Years of Industrial Experience with the SST Turbulence Model

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Abstract – This document describes the current formulation of the SST turbulence models, as well as a number of model enhancements. The model enhancements cover a modified near wall treatment of the equations, which allows for a more flexible grid generation process and a zonal DES formulation, which reduces the problem of grid induced separation for industrial flow simulations. Results for a complete aircraft configuration with and without engine nacelles will be shown.

1. Introduction

The starting point for the development of the SST [1,2] model was the need for the accurate prediction of aeronautics flows with strong adverse pressure gradients and separation. Over decades, the available turbulence models had consistently failed to compute these flows. In particular, the otherwise popular $k-\epsilon$ [3] model was not able to capture the proper behaviour of turbulent boundary layers up to separation [4]. The Johnson-King model [5] was the first formulation, which allowed the accurate prediction of separated airfoil flows. Unfortunately, the model was not easily extensible to modern three-dimensional Navier-Stokes codes due to its algebraic formulation.

The $k-\omega$ model is substantially more accurate than $k-\epsilon$ in the near wall layers, and has therefore been successful for flows with moderate adverse pressure gradients, but fails for flows with pressure induced separation [1]. In addition the ω -equation shows a strong sensitivity to the values of ω in the freestream outside the boundary layer [6]. The freestream sensitivity has largely prevented the ω -equation from replacing the ϵ -equation as the standard scale-equation in turbulence modelling, despite its superior performance in the near wall region. This was one of the main motivations for the development of the zonal BSL and SST models.

The zonal formulation is based on blending functions, which ensure a proper selection of the $k-\omega$ and $k-\epsilon$ zones without user interaction. The main additional complexity in the model formulation compared to standard models lies in the necessity to compute the distance from the wall, which is required in the blending functions. This is achieved by the solution of a Poisson equation and is therefore compatible with modern CFD codes.

The SST model was originally used for aeronautics applications, but has since made its way into most industrial, commercial and many research codes. This is in agreement with the present authors experience that the need for accurate computations of flows with pressure-induced separation goes far beyond aerodynamics. The SST model has greatly benefited from the strength of the underlying turbulence models. In particular, the accurate and robust near wall formulation of the Wilcox model has substantially contributed to its industrial

usefulness. As well, all the model additions developed by Wilcox for rough walls and surface mass injection etc. can be used with minor modifications [7].

While the original model formulation has largely stayed unchanged from the formulation given in [1] (small modifications see bellow), there have been several areas of improvement carried out within the CFX codes. Robustness optimisation have brought the model to the same level of convergence as the standard k - ϵ model with wall functions. An improved near-wall formulation has reduced the near wall grid resolution requirements, which has resulted in a substantial improvement for industrial heat transfer predictions [8]. Finally, the zonal formulation of the model has been beneficial in the formulation of an industrial Detached Eddy Simulation (DES) model. A large number of model validation studies and applications can be found on the internet.

2. SST Model Formulation

In this section, the complete formulation of the SST model is given, with the limited number of modifications highlighted.

$$\frac{\partial(\rho k)}{\partial t} + \frac{\partial(\rho U_i k)}{\partial x_i} = \tilde{P}_k - \beta^* \rho k \omega + \frac{\partial}{\partial x_i} \left[(\mu + \sigma_k \mu_t) \frac{\partial k}{\partial x_i} \right] \quad (1)$$

$$\frac{\partial(\rho \omega)}{\partial t} + \frac{\partial(\rho U_i \omega)}{\partial x_i} = \alpha \rho S^2 - \beta \rho \omega^2 + \frac{\partial}{\partial x_i} \left[(\mu + \sigma_\omega \mu_t) \frac{\partial \omega}{\partial x_i} \right] + 2(1 - F_1) \rho \sigma_{\omega 2} \frac{1}{\omega} \frac{\partial k}{\partial x_i} \frac{\partial \omega}{\partial x_i}$$

Where the blending function F_1 is defined by:

$$F_1 = \tanh \left\{ \left[\min \left[\max \left(\frac{\sqrt{k}}{\beta^* \omega y}, \frac{500\nu}{y^2 \omega} \right), \frac{4\rho\sigma_{\omega 2} k}{CD_{kw} y^2} \right] \right]^4 \right\} \quad (2)$$

with $CD_{kw} = \max \left(2\rho\sigma_{\omega 2} \frac{1}{\omega} \frac{\partial k}{\partial x_i} \frac{\partial \omega}{\partial x_i}, 10^{-10} \right)$ and y is the distance to the nearest wall.

F_1 is equal to zero away from the surface (k - ϵ model), and switches over to one inside the boundary layer (k - ω model).

The turbulent eddy viscosity is defined as follows:

$$\nu_t = \frac{a_1 k}{\max(a_1 \omega, S F_2)} \quad (3)$$

Where S is the invariant measure of the strain rate and F_2 is a second blending function defined by:

$$F_2 = \tanh \left[\left[\max \left(\frac{2\sqrt{k}}{\beta^* \omega y}, \frac{500\nu}{y^2 \omega} \right) \right]^2 \right] \quad (4)$$

A production limiter is used in the SST model to prevent the build-up of turbulence in stagnation regions:

$$P_k = \mu_t \frac{\partial U_i}{\partial x_j} \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) \rightarrow \tilde{P}_k = \min(P_k, 10 \cdot \beta^* \rho k \omega) \quad (5)$$

All constants are computed by a blend from the corresponding constants of the $k-\varepsilon$ and the $k-\omega$ model via $\alpha = \alpha_1 F + \alpha_2 (1 - F)$ etc. The constants for this model are: $\beta^* = 0.09$, $\alpha_1 = 5/9$, $\beta_1 = 3/40$, $\sigma_{k1} = 0.85$, $\sigma_{\omega1} = 0.5$, $\alpha_2 = 0.44$, $\beta_2 = 0.0828$, $\sigma_{k2} = 1$, $\sigma_{\omega2} = 0.856$.

The only modifications from the original formulation are the use of the strain rate, S , instead of the vorticity in Equation 3 and the use of the factor 10 in the production limiter, instead of 20 as proposed in [1,2].

3. Near Wall Treatment

One of the essential features of a useful industrial turbulence model is an accurate and robust near wall treatment. In addition, the solutions should be largely insensitive to the near wall grid resolution. For complex industrial flows the requirement $y^+ < 2$ is excessive and can in most cases not be satisfied for all walls. On the other hand, the strict use of wall functions, which allow the use of coarser grids, limits the model accuracy on fine grids. A new near wall treatment was therefore developed [8], which automatically shifts from the standard low-Re formulation to wall functions, based on the grid spacing of the near-wall cell.

Figure 1 shows velocity profiles for Couette flow simulations on three vastly different grids ($y^+ \sim 0.2$; $y^+ \sim 9$; $y^+ \sim 100$). Despite the large differences in the near wall spacing, the computed wall shear-stress varies by less than 2% and all solutions follow the logarithmic profile. As a result, the new wall formulation has significantly improved the predictive accuracy of general industrial applications, as the user influence via the grid generation is drastically reduced.

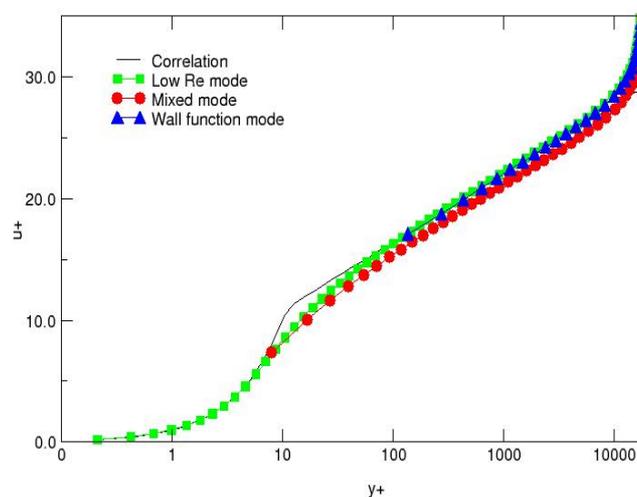


Figure 1 Velocity profiles for three different grids using the automatic wall treatment of CFX-5

4. Application of the SST Model to Aerodynamic Flows

The SST model was selected by CFX for its contribution to the testcases of the 2nd AIAA drag prediction workshop (<http://aaac.larc.nasa.gov/tsab/cfdlarc/aiaa-dpw/>). Two geometries have been selected by AIAA and the grids have been provided by the organizers. Figure 2 shows the geometries simulated by the participants.

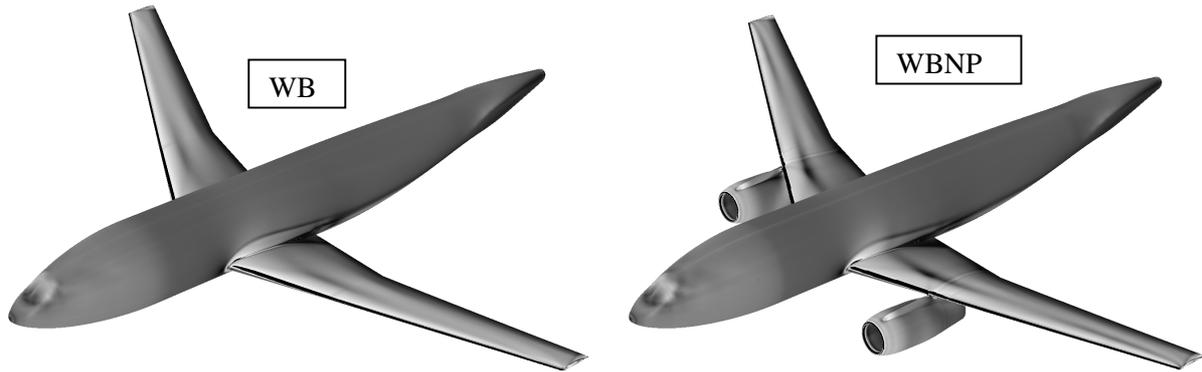


Figure 2 Geometries selected for AIAA drag prediction workshop

The low-Re grids had 5.83m (WB) and 8.43m (WBNP) hexahedral cells and have been provided by ICEM. Convergence for the drag (most sensitive variable) has been achieved typically with around 120-150 time steps.

Figure 3 shows the drag polar for the mandatory runs against the experimental data, as well as the convergence history. The simulated results are in very good agreement with the experimental data.

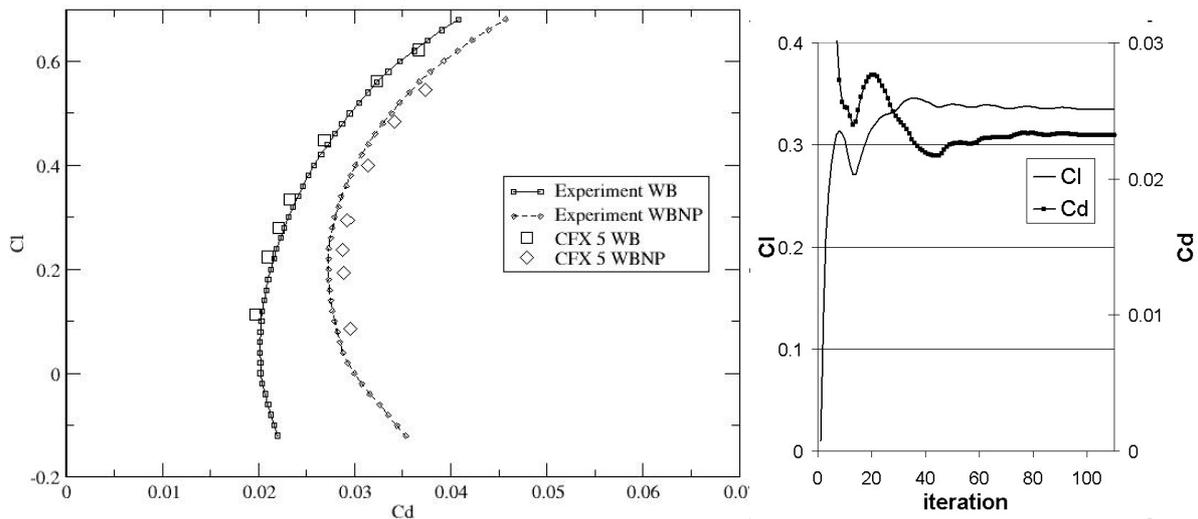


Figure 3 Drag polar for AIAA drag prediction testcases (left). Lift and Drag convergence (right)

This is a strong indication that optimized RANS models/codes can accurately simulate complete aircraft configurations. More information can be found on the web-page of the workshop and follow-up AIAA publications.

5. Zonal SST-DES Formulation

Recently, Spalart [9] has proposed a hybrid model formulation that utilises the RANS equations inside the boundary layer and an LES-like formulation for free shear flows. The model is termed Detached Eddy Simulation (DES) and is currently used in combination with the Spalart-Allmaras and the SST turbulence model [10]. The main reason why these models have been selected as the underlying RANS models lies in their improved separation prediction capability. The DES modification in the SST model is applied to the dissipation term in the k -equation as follows:

$$\rho \varepsilon = \beta^* \rho k \omega \rightarrow \beta^* \rho k \omega \cdot F_{DES} \quad \text{with} \quad F_{DES} = \max\left(\frac{L_t}{C_{DES} \Delta}, 1\right) \quad (9)$$

where ε is the dissipation rate, Δ is the maximum local grid spacing ($\Delta = \max(\Delta x, \Delta y, \Delta z)$ in case of a Cartesian grid), β^* is a constant of the SST model, $L_t = \frac{\sqrt{k}}{\beta^* \omega}$ is the turbulent length scale and $C_{DES} = 0.61$ is a calibration constant of the DES formulation.

For fine grids, the switch from RANS to DES can take place somewhere inside the boundary layer and produce a premature (grid-induced) separation [11]. Figure 4 shows the effect for a 2-dimensional airfoil simulation. In this case the grid spacing in the spanwise direction is assumed to be of the same order as the chordwise spacing (this is usually the case for unstructured meshes or for flows where the flow direction is unknown during the grid generation phase). It can be seen that the original DES limiter affects the RANS model and moves the separation point upstream relative to the original SST model, which was in good agreement with the data (upper right picture).

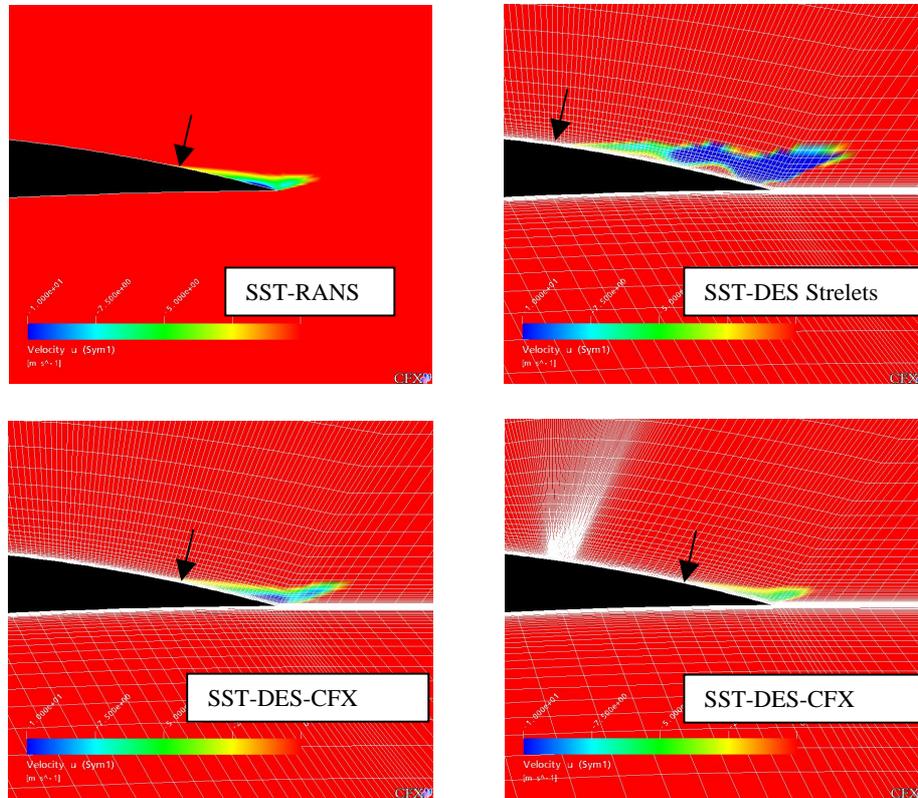


Figure 4 Region of flow separation on airfoil for different models. Lower right refined grid. Separation indicated by arrow.

In order to reduce the grid influence of the DES-limiter on the RANS part of the boundary layer, the SST model offers the option to “protect” the boundary layer from the limiter. This is achieved again with the help of the zonal formulation underlying the SST model [11]. The following modification significantly reduces the influence of the DES limiter on the boundary layer portion of the flow:

$$F_{DES-CFX} = \max\left(\frac{L_t}{C_{DES} \Delta} (1 - F_{SST}), 1\right) \quad \text{with} \quad F_{SST} = 0, F_1, F_2 \quad (10)$$

In this equation, F_{SST} can be selected from the blending functions of the SST model. For $F_{SST}=0$, the model of Strelets [10] is recovered. Figure 4 shows also the effect for the same 2D airfoil, using $F_{SST}=F_2$. It can be seen that with this modification, the boundary layer is not affected and the separation point predicted with the SST model is unchanged, even under more severe grid refinement (lower right picture).

Note that the zonal DES formulation does not completely eliminate the problem of grid sensitivity in the RANS region, as the F_2 function does not cover 100% of the boundary layer. It does however reduce the critical limit by one order of magnitude.

Another interesting effect of the zonal DES formulation can be seen in Figure 5 for the flow around a cube mounted inside a 2D channel. At the inlet, a fully developed channel flow enters the computational domain. For this type of flow, the maximum grid spacing is smaller than the turbulent length-scale over most of the domain. For the original SST-DES model, this would mean that the DES limiter is activated over most of the domain, which would essentially require a simulation carried out in LES modus. For the zonal SST-DES model, the inlet part is covered by the F_2 limiter and can be treated by the RANS model. The DES limiter is only activated downstream of the cube, where the large turbulent structures are resolved.

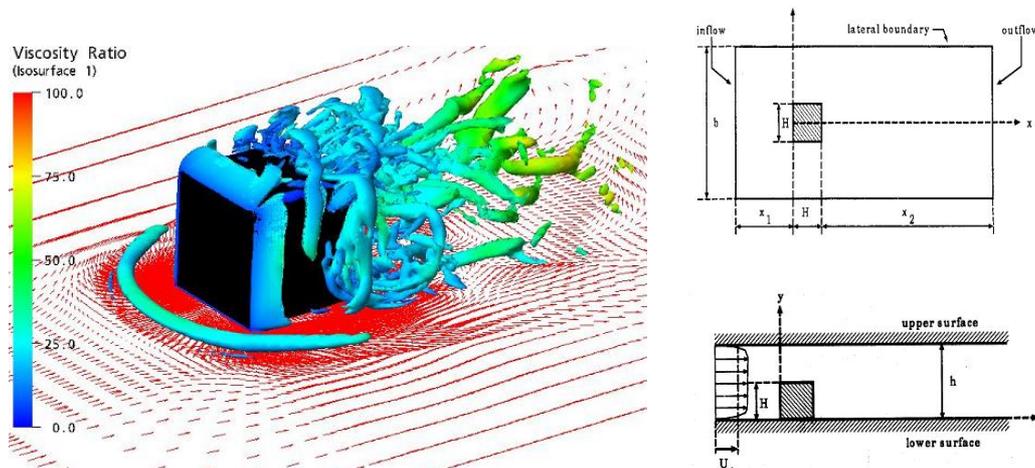


Figure 5 Flow around cube in channel flow. Solution SST-DES-CFX model.

Figure 5 shows the flowfield using iso-surfaces of the invariant $(\partial U_i / \partial x_j) \cdot (\partial U_j / \partial x_i)$ coloured by the ratio μ_t / μ . The flowfield upstream is covered by the SST model and is close to steady-state (except for pressure disturbances from the separated zone) and the flow downstream is covered by the DES formulation.

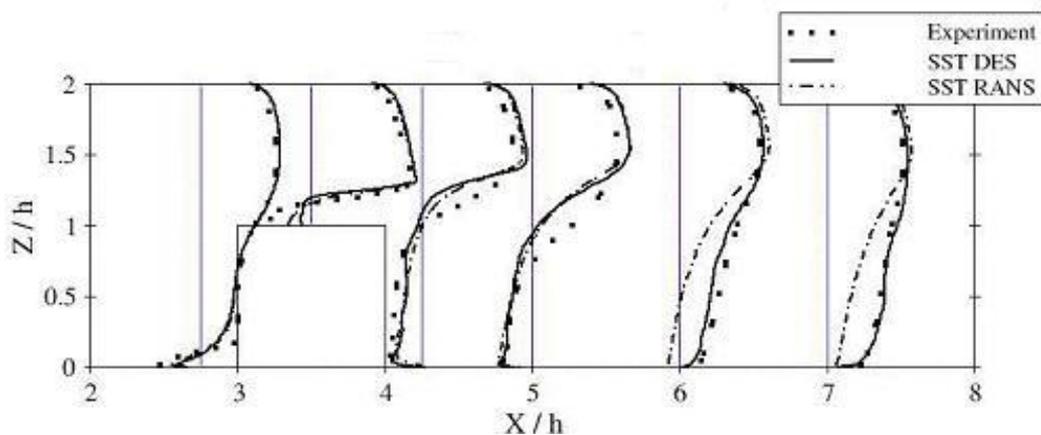


Figure 6 Velocity profiles in symmetry plane of cube. Comparison of SST and SST-DES-CFX model with experimental data.

Figure 6 shows a comparison of the velocity profiles computed with the SST and the SST-DES-CFX zonal model. The main difference is that the DES formulation captures the flow recovery downstream of the separation zone in good agreement with the experimental data.

6. Future Directions

It has been observed for a long time that RANS turbulence models underpredict the level of the turbulent stresses in the detached shear layer emanating from the separation line [13]. This in turn seems to be one of the main reasons for the incorrect flow recovery predicted by the models downstream of reattachment. It was found at the 9th ERCOFTAC/IAHR/COST Workshop on Refined Turbulence Modelling for the flow over a periodic hill, that models with improved separation prediction capabilities, like the SST and the SA model did overpredict the extent of the separated region. This is a matter of concern and is an area of current research. The problem is shown for the asymmetric diffuser testcase of Obi [14]. The SST model gives a significantly improved separation compared to the k- ϵ model, but predicts a flow recovery that is slower than observed in the experiments. Note that the better comparison of the k- ϵ model in this region is an artefact of the underpredicted separation.

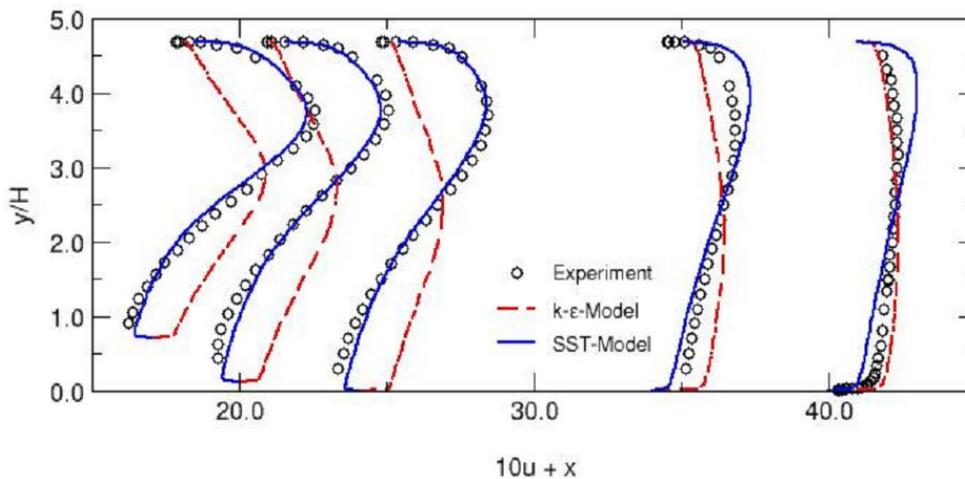


Figure 7 Velocity profiles for asymmetric diffuser flow

While an improved flow recovery could be computed with the DES formulation, as shown in Figure 6 this is not always possible. For pressure induced separation bubbles from smooth surfaces, the original DES model cannot be applied due to the danger of grid-induced separation. Alternatively, the zonal DES formulation would stay in RANS mode and would have no influence on the results. A possible alternative to current DES formulations is the extension of the Scale-Adaptive Simulation (SAS) approach [15] to the SST model.

Another interesting future development is the combination of the BSL model (underlying the SST model) with explicit algebraic stress models (EASM) as proposed by Helsten and Laine [16]. This allows the inclusion of secondary motions and the effects of streamline curvature and system rotation.

7. Summary

This paper gave an overview of the current state and direction of development of the SST turbulence model. The standard model formulation has been repeated and extensions for improved wall treatment and a zonal DES formulation have been presented. Simulations for a complete aircraft without and with engine nacelle have been briefly discussed. Directions for future developments have been outlined.

Acknowledgement

Part of this work was supported by research grants from the European Union under the FLOMANIA project (Flow Physics Modelling - An Integrated Approach) is a collaboration between Alenia, Ansys-CFX, Bombardier, Dassault, EADS-CASA, EADS-Military Aircraft, EDF, NUMECA, DLR, FOI, IMFT, ONERA, Chalmers University, Imperial College, TU Berlin, UMIST and St. Petersburg State University. The project is funded by the European Union and administrated by the CEC, Research Directorate-General, Growth Programme, under Contract No. G4RD-CT2001-00613.

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