A Computational Geometry for the Blades and Internal Flow Channels of Centrifugal Compressors

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A new computational geometry for the blades and flow passages of centrifugal compressors is described and examples of its use in the design of industrial compressors are given. The method makes use of Bernstein-Bezier polynomial patches to define the geometrical shape of the flow channels. This has the following main advantages: the surfaces are defined by analytic functions which allow systematic and controlled variation of the shape and give continuous derivatives up to any required order: and the parametric form of the equations allows the blade and channel coordinates to be very simply obtained at any number of points and in any suitable distribution for use in subsequent aerodynamic and stress calculations and for manufacture. The method is particularly suitable for incorporation into a computer-aided design procedure.

Introduction

The design of centrifugal compressor impellers usually assumes two distinct stages. A preliminary design, making use of one-dimensional flow analysis based on previous experience, is sketched out to specify the inlet and outlet blade angles and the "skeletal" dimensions (such as inlet and outlet diameters, axial length, and impeller tip outlet width). This is followed by a detailed design in which the complete blade and channel geometry is specified and then subsequently refined by means of successive aerodynamic and stress analyses.

During the detailed design stage the designer repeatedly adjusts the shape of the blades and flow channels until he finds a suitable geometry that combines acceptable aerodynamic performance with low stress levels and is economic to manufacture. This process of continual refinement of the shape can be expensive, tedious, and time-consuming, since at each stage the geometrical data for the necessary aerodynamic or stress analysis must be prepared. The task is greatly simplified when a simple flexible system of geometry definition for the components of a compressor is available.

The definition of compressor geometry is especially complicated for industrial centrifugal compressors, where three different types of impeller are to be found (Fig. 1):

- High flow coefficient impellers ($\Phi > 0.1$) with three-dimensional twisted blade surfaces, axial inlet flow, and a radial leading edge (similar to modern supercharger impellers)
- Intermediate flow coefficient impellers ($0.1 > \Phi > 0.04$) with three-dimensional twisted blade surfaces, radial inlet flow, and an inclined leading edge
- Low flow coefficient impellers ($\Phi < 0.04$) with two-dimensional curved blades, comprised of purely axial elements, radial inlet flow and an axial leading edge

It is the purpose of this paper to describe a method of computational geometry that has proved extremely useful for the definition of component geometry in industrial centrifugal compressors. The method can be used to specify each of the impeller types mentioned above and, in addition, is capable of defining the complete flow passage through the compressor. For example, the geometry of the hub and shroud contours, including inlet, diffusor, crossover channel and return channel, as well as all of the vaned cascades of the compressor, such as inlet guide vanes, diffusor vanes and return channel de-swirl vanes, can be defined using the present method.

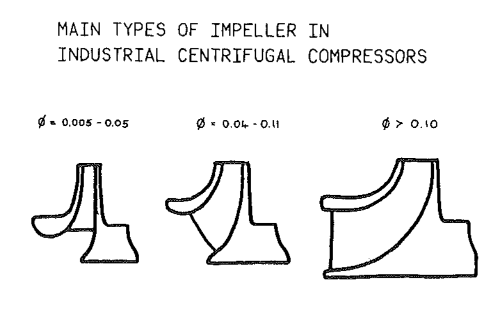

Fig. 1 Sketch of main types of impeller in industrial centrifugal compressors

Contributed by the Gas Turbine Division of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS and presented at the 27th International Gas Turbine Conference and Exhibit, London, England, April 18–22, 1982. Manuscript received at ASME Headquarters December 14, 1981. Paper No. 82-GT-155.

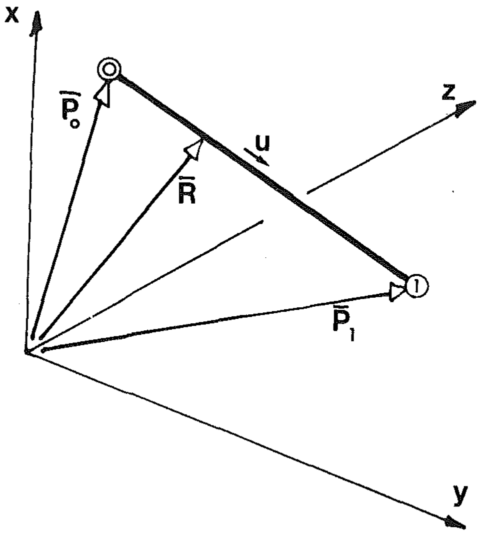

Fig. 2(a) Example of Bezier curve, degree 1

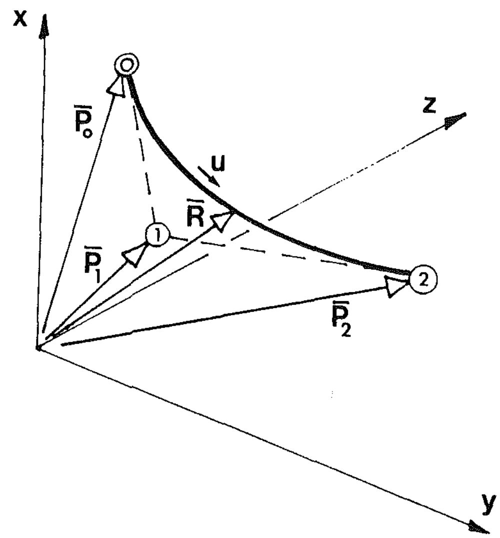

Fig. 2(b) Example of Bezier curve, degree 2

Geometry Definition Methods

Several methods for the definition of the shape of impeller blades have been previously published. These are briefly reviewed below.

An early choice of many designers for the definition of impeller geometry was conic sections (for example, circular arc, ellipse, etc.), for both the hub and shroud contours and the impeller blades. A general description of the blade surface geometry using conic sections is, for example,

$$r\vartheta = ar^2 + 2brz + cz^2 + 2dr + 2ez + f \tag{1}$$

where the parameters $a,b,c,d,e,$ and f determine the inlet and outlet angles and the blade curvature. Examples of elliptical bladed impellers which fall into this class are quoted by Moore [1] and by Eckardt [2]. An alternative procedure was suggested by Whitfield, Atkey, and Wallace [3]. This made use of Lamé ovals of the general form

$$\left\{\frac{\varphi + a}{b}\right\}^e + \left\{\frac{z + c}{d}\right\}^f = 1 \tag{2}$$

where φ is either r or ϑ to define the blade surface shape. A more complex equation was suggested by Krain [4], who proposed the following formula to define the mean blade surface for "through flow" calculations

$$\vartheta = (a + be^{-r} + c(\pi/2 - \tan^{-1}(r))\cdot$$
$$(1 + e\tan^{-1}(z) + f\tanh(z) + g\sinh(z)) \tag{3}$$

All of these methods allow considerable freedom of shape through adjustment of the parameters a,b,c, etc., but are clearly unsuitable for a general method.

An early general method was described by Jansen and Kirschner [5]. The blade shape was specified by straight-line elements from hub to shroud which were distributed from the inlet to the outlet of the impeller to provide a specified blade aerodynamic loading or a specified blade camber angle (β) distribution. No equations, however, were developed for the blade surface. Smith and Merryweather [6] and Came [7] described a similar computer-aided design method in which the impeller blade is represented by a system of three-dimensional analytic equations, following the method of Coons [8] for surface geometry definition. Fister and Eikelmann [9] described another method that was also based on the interpolation formulae of Coons [8].

These last two methods are quite similar to the procedure adopted here. The fundamental difference in the present work, however, is that the interpolation formulae due to Bezier [10] are used. The methods of both Coons and Bezier fall into a new class of geometries that have been recently developed for numerical controlled machining and manufacturing. Both methods are given in some detail by Faux and Pratt [11]. The particularly elegant technique developed by Bezier has been adopted in the present work for the following reasons:

- The definition of the geometry of any surface is relatively simple.

Nomenclature

a,b,c,d,e,f,g = parameters in various equations
$B_k^n(u)$ = Bernstein Polynomial
D_2 = impeller tip diameter (m)
j,k = indices
m = meridional distance (m)
$\dot{m}$ = mass flow rate (kg/s)
n = degree of Bernstein Polynomial
n = number of Bezier patches
$\bar{P}_k$ = vector of point P_k
r = radius (m)
$\bar{R}$ = vector of point R
s = camberline distance (m)
u,v = parameters to define Bezier patches
U_2 = impeller tip speed (m/s)
x,y,z = rectangular Cartesian coordinates
r,ϑ,z = cylindrical coordinates
β = blade camber angle (deg)
δ = blade thickness (m)
ϵ = meridional pitch angle (deg)
Φ = flow coefficient ($\Phi = \dot{m}/\rho U_2 D_2{}^2$)
ρ = inlet total density (kg/m^3)

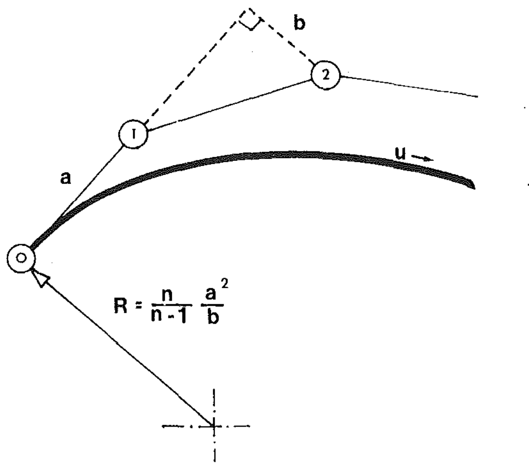

Fig. 3(*a*) Properties of Bezier curves, radius of curvature end point

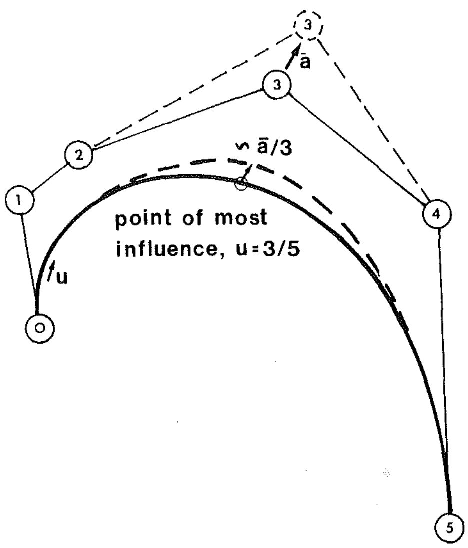

Fig. 3(*b*) Properties of Bezier curves, point of most influence

- Both the flow passages and the blade surfaces can be defined by equations of the same type.
- The shapes produced are general enough to be used in the design of new compressors and in approximation of the geometry of existing ones.
- The defining equations are particularly suitable for incorporation into a computer-aided design (CAD) procedure for compressors.
- The surfaces are defined by algebraic functions which allow systematic and controlled variation of the shape and which provide continuous derivatives up to any required degree.
- The parametric form of the equations allows the coordinate geometry of blades and flow passages to be very simply obtained at any number of points and in any suitable distribution for aerodynamic and stress calculations and for manufacture.

A brief and simple description of the method follows.

Bernstein-Bezier Surfaces

Bezier Curves. A Bezier curve is a parametric representation of a space curve. The curve is specified by the coordinates of a series of points in space of which only the first and last lie on the curve they define. The points are known as the polygon points of the curve, and the figure constructed by connecting these polygon points with straight lines is known as the Bezier polygon of the curve.

The simplest example is a Bezier curve of degree 1 which can be written as

$$\bar{R} = (1-u)\bar{P}_0 + u\bar{P}_1 \tag{4}$$

where R is a vector of a point on the curve with coordinates (x,y,z), and $\bar{P}_0$ and $\bar{P}_1$ are vectors of the polygon points 0 and 1 with coordinates (x_0, y_0, z_0) and (x_1, y_1, z_1), u is a parameter that is constrained to vary from 0 to 1 along the curve. Note that when $u = 0$, $\bar{R} = \bar{P}_0$ and when $u = 1$, $\bar{R} = \bar{P}_1$. For values of u between 0 and 1, the vector $\bar{R}$ describes a straight line from point 0 to point 1, as shown in Fig. 2(*a*). The Bezier curve of degree 2 requires three polygon points to define it and can be written as follows

$$\bar{R} = (1-u)^2\bar{P}_0 + 2u(1-u)\bar{P}_1 + u^2\bar{P}_2 \tag{5}$$

An example is given in Fig. 2(*b*). It can be seen that the second polygon point, P_1, does not lie on the curve, but that the tangents to the curve at the starting point ($u = 0$) and end point ($u = 1$) are in the direction of this point.

The Bezier curve of degree 3 requires four polygon points to define it and can be written as follows

$$\bar{R} = (1-u)^3\bar{P}_0 + 3u(1-u)^2\bar{P}_1 + 3u^2(1-u)\bar{P}_2 + u^3\bar{P}_3 \tag{6}$$

From the form of these equations it can be seen that the Bezier curve is, in some sense, the weighted average of the polygon points defining the curve. The weighting functions are the Bernstein polynomials $\mathrm{B}_k^n(u)$ where

$$\mathrm{B}_k^n(u) = \binom{n}{k} u^k (1-u)^{n-k} \tag{7}$$

The general form of the Bezier polynomial of degree n is given by

$$\begin{aligned}\bar{R} &= (1-u)^n\bar{P}_0 + n(1-u)^{n-1}u\bar{P}_1 + \\ &\frac{n(n-1)}{2}(1-u)^{n-2}u^2\bar{P}_2 + \dots \\ &+ \binom{n}{k}u^k(1-u)^{n-k}\bar{P}_k + u^n\bar{P}_n \\ &= \sum_{k=0}^{n}\bar{P}_k\mathrm{B}_k^n(u)\end{aligned} \tag{8}$$

Some important properties of Bezier curves are listed below, without proof:

(*a*) Degree of polynomial. A polynomial of degree n is defined by $n + 1$ points in space.

(*b*) Tangents at end points of curve. The tangent at the beginning ($u = 0$) of a Bezier polynomial is in the direction of the second polygon point. Similarly, at the end point ($u = 1$), the tangent is in the direction of the last but one point. This can be easily proved by differentiating equation (8) with respect to u and setting $u = 0$ or 1.

(*c*) Second derivatives at end point. The second derivative of the curve at the end points depends only on the first three points P_0, P_1, and P_2. The radius of curvature at the end point is given by the construction shown in Fig. 3(*a*). This simple construction can be used to ensure that a polynomial will have a certain radius of curvature at its ends. For example, if a curve is needed which has zero curvature at the end then the first three points must form a straight line ($b = 0$ in Fig. 3(*a*)).

(*d*) Points of most influence. If the point, P_k, of an nth degree Bezier polynomial is moved by a vector, $\bar{a}$, then this movement has most influence at the point on the curve, where $u = k/n$. The point of most influence moves by approximately $\bar{a}/3$. This is demonstrated in Fig. 3(*b*). This

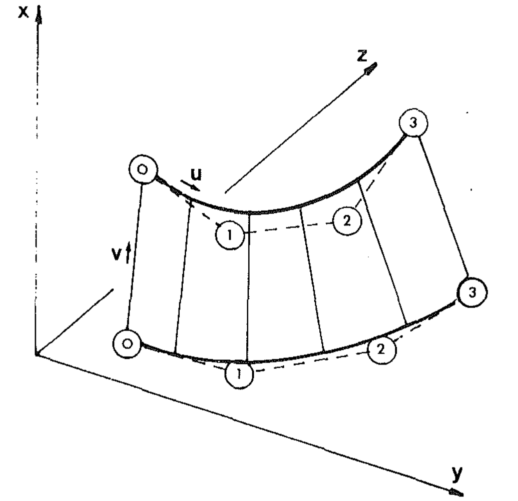

Fig. 4(*a*) Examples of Bezier surfaces, a cubic linear patch

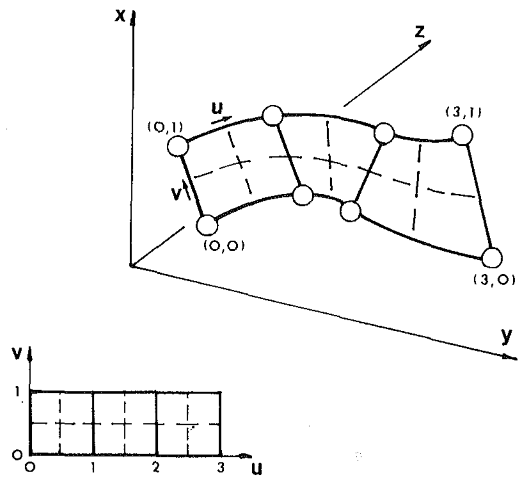

Fig. 4(*b*) Examples of Bezier surfaces, a string of patches

property can be used to produce controlled modifications to the shape of the curve.

(*e*) Increasing the degree of a Bezier curve. It is usually possible to obtain a Bezier curve of low degree that gives approximately the required shape. Sometimes, however, it may be necessary to increase the degree of the curve in order to obtain more freedom of shape. The degree, n, of a curve can be increased in the first instance without changing the shape by following some elementary geometrical rules (see Forrest [12]. This enables the degree of a curve to be systematically increased until it can represent the appropriate shape.

(*f*) Derivative with respect to u. The following identities can be used to calculate the derivatives with respect to u

$$\frac{d}{du}\{B_k^n(u)\} = n\{B_{k-1}^{n-1}(u) - B_k^{n-1}(u)\} \tag{9}$$

where

$$B_{-1}^{n-1}(u) = 0 \quad \text{and} \quad B_n^{n-1}(u) = 0$$

Bezier Surface. A simple three-dimensional curved surface can be defined by a series of straight lines joining points of constant, u, of two adjacent Bezier curves, as shown in Fig. 4(*a*). A second parameter, v, can be introduced which varies from 0 to 1 as one proceeds along the straight lines between the two curves. Thus any point on this surface can be referred to by the parametric coordinates (u,v). The three-dimensional surface (x,y,z) is mapped onto a two-dimensional plane (u,v) by the equations

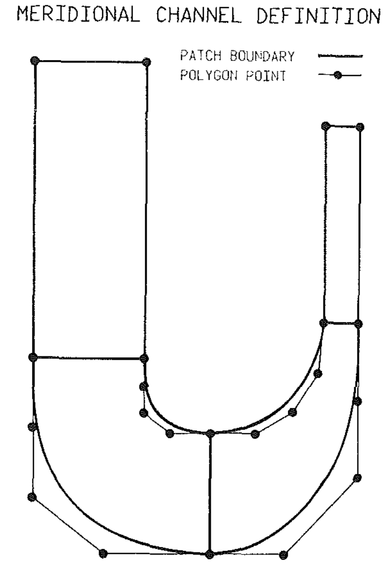

Fig. 5 Definition of meridional channel by means of Bezier surfaces

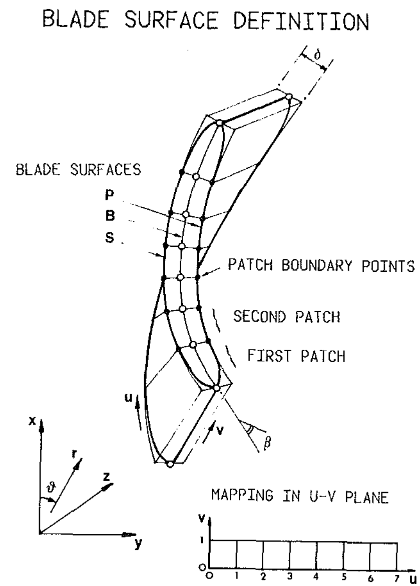

Fig. 6 Definition of blade surfaces

$$\begin{aligned} \bar{R}(x,y,z) &= \bar{R}(u,v) \\ &= \sum_{k=0}^{n} \{(1-v)\bar{P}_k^0 + (v)\bar{P}_k^1\} B_k^n(u) \end{aligned} \tag{10}$$

where $\bar{P}_k^0$ are the vectors of the polygon points along the $v = 0$ curve and the $\bar{P}_k^1$ are those along the $v = 1$ curve.

In the present method, it is only possible to construct surfaces which are linear in the v-direction. Such surfaces are considered to be aerodynamically acceptable and clearly greatly simplify the manufacture of the surface.

Bezier Patches. The segment of the surface shown in fig. 4(*a*) is known as a cubic-linear patch as it is third degree in the u-direction and first degree in the v-direction. More complicated surfaces can be obtaned by increasing the degree of the Bezier curves in the u-direction, or, alternatively by connecting several curved patches together. An example of a surface comprising three patches and its mapping onto the (u,v) plane is shown in Fig. 4(*b*).

The curves u = constant and v = constant can be mapped onto the surface to form a net, as shown in Fig. 4(b). Thus the parametric representation of the surface provides a simple method of referring to any point on the curved surface.

Implementation of Bezier Patches to Describe Geometry of a Compressor

The Meridional Channel (M). The meridional channel is defined by a string of Bezier patches in the $x-z$-plane, thus giving a plane surface that is a meridional section through the compressor. The mathematical rotation of this plane surface around the z-axis defines the internal flow channel of the compressor. The blades are defined separately, and there is no distinction between moving and stationary parts of the channel.

A representation of a meridional channel defined in this way is shown in Fig. 5. The channel includes an inlet section, an impeller section, and a diffusor section. It comprises four patches and the polygon points of these patches are shown. The consecutive patches are of degree 1,4,4, and 1. The line $v = 0$ represents the hub side contour, and the line $v = 1$ represents the shroud side contour.

The meridional channel can be specified by expressly defining the polygon points themselves. This procedure is, however, rather inconvenient, and in the present method various subprograms are used to determine the polygon points from data that is more readily available to the designer. For example, the channel shown in Fig. 5 has been specified with the following data:

(a) Radii of patch corner defining points
(b) Axial spacing between patch corner defining points, that is inlet and outlet channel widths, axial length of inlet, and axial length of impeller section
(c) Additional data to position the internal polygon points of each patch, in general expressed as fractions of the length of the sides of the polygon, so that the shape of the shape of the channel walls can be simply adjusted

Various subprograms have been developed for different types of compressor channels. These subprograms are usually written to provide:

(a) Continuity of derivatives across patch boundaries
(b) Sufficient free parameters that the designer can readily specify the "skeletal" dimensions of the channel and easily change the shape of the curved portions of the channel

The use of these subprograms allows the designer to specify the hub and shroud contours as one of a family of related shapes.

Blade Surfaces (B, S and P). The method of this paper can be used to define all the vaned cascades of a compressor, that is, inlet guide vanes, impeller vanes, diffusor vanes and return channel deswirl vanes. For each vane three separate strings of patches are used to define three blade surfaces, as in the method of Smith and Merryweather [6].

The first surface, known as the blade camber surface (B), is the three-dimensional equivalent of the two-dimensional camber line used in the traditional aerofoil design methods. The other surfaces are the suction surface (S) and the pressure surface (P) of the vane. These are spaced equal distances on either side of the camber surface, as shown in Fig. 6.

In the present method the three blade surfaces (B, S, and P) can be defined as single patches of high polynomial degree or as strings of patches of lower degree. Each surface must, however, have the same number of patches of the same degree, and the variation of the parameters (u, v) on adjacent points must be the same. With this formulation only one coordinate value of the parametric variables (u, v) refers to equivalent points on these three surfaces.

The most convenient method for determining the polygon points of the blade is by means of a subprogram which calculates the polygon points from data provided by the designer, such as blade camber angle (β) and thickness (δ) distributions. Various subprograms have been developed which, in general, utilize the following procedure:

(a) A meridional channel (M) is defined within which the blade is to be specified.

(b) The position of the blade within the channel M is defined by specifying four values of parameter u corresponding to the leading and trailing edge on the hub and shroud.

(c) The distribution of camber line angle (β) is specified as a function of the meridional distance (m) along the hub and shroud. The shape of the camber line along the hub and shroud is then calculated by integration along the meridional channel.

$$\vartheta = \int \frac{\tan \beta}{r} dm \qquad (11)$$

(d) The blade thickness distribution (δ) is specified as a function of the meridional distance (m) along the hub and shroud. The suction and pressure surfaces are placed an equal distance either side of the camber line to give the required thickness distribution.

(e) The blade shape along the hub and shroud contours is fully specified by the operations a, b, c, and d, outlined above, but the coordinates of the Bezier patches for this shape have yet to be found. The first step is to select a certain spacing of the patch boundary points along the hub and shroud contours. The patch boundaries can, for example, be spaced equidistant along the blade surface or equidistant along the projection of the blade surface onto the meridional channel. The choice of position for patch boundary points influences the final shape of the blade as it determines the orientation of the straight line elements from hub to shroud. Having selected the required number of patches and the required patch distribution along the blade, the polygon points at the junctions between the patches are then suitably interpolated.

(f) It now remains to calculate the position of the internal polygon points within each patch, such that the shape produced is a close approximation to that specified by operations a, b, c, and d above. Experience has shown that sufficient accuracy is obtained if each patch is chosen to be of degree 3 in the u-direction, giving rise to cubic-linear patches that resemble the patch shown in Fig. 4(a). For each cubic-linear patch there are four internal polygon points so that if the surface has n patches there are $4n$ points whose coordinates must be determined. Conditions of continuity of slope and curvature across n-1 patch junctions provide $4(n-1)$ equations for the unknown points. The remaining four equations are obtained by the specification of the blade angles at the leading and trailing edges on the hub and shroud. This algebraic problem is exactly equivalent to fitting parametric cubic splines through the patch corner defining points on the hub and shroud, and is explained in detail in the Appendix.

(g) In order to improve the aerodynamics of the blade a final modification to the end patches at the leading and trailing edges may be made in order to produce rounded shapes, much in the same way suggested by Smith and Merryweather [6].

This procedure produces blade shapes that are defined by means of a string of cubic-linear patches for each of the blade surfaces (B, S, and P). The cubic curves lie in streamwise direction, and the straight line elements lie roughly normal to

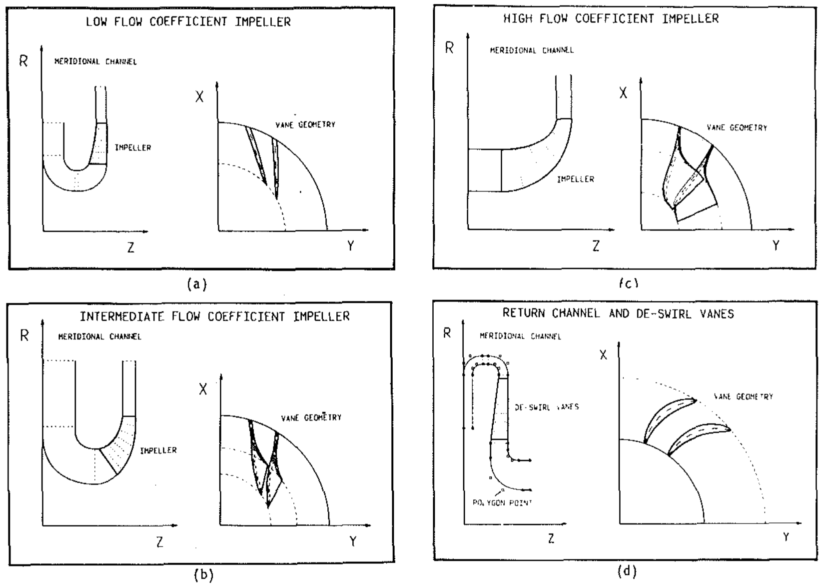

Fig. 7 Examples of channels and blades defined by the present method (a) low flow coefficient impeller, (b) intermediate flow coefficient impeller, (c) high flow coefficient impeller, (d) return channel and deswirl vanes

the stream lines from hub to shroud. The generated blade shapes are restricted to having trapezoidal thickness distributions, i.e., constant taper along any line element between hub and shroud. The orientation of the straight line blade elements can be controlled to give rise to the following blade surfaces:

(*a*) An arbitrary blade surface in which the elements have no particular preferred orientation, but lie roughly normal to the channel walls

(*b*) An axial element blade surface in which all blade elements are straight lines in the axial directions

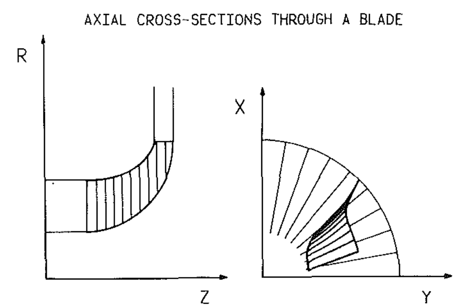

Fig. 8 Axial cross-sections through an impeller blade

Examples of Use

Definition of Compressor Geometry. Impellers and blades of almost any geometrical form can be defined by this method. Some examples of impellers for high, intermediate and low flow coefficients are shown in Fig. 7(*a*), 7(*b*), and 7(*c*), respectively. A further example of the flexibility, is shown in Fig. 7(*d*), which represents a return channel and deswirl vane. In each of these diagrams the patch boundaries are shown, and in Fig. 7(*d*) the channel polygon points are also given. Typical computer run times for these problems are of the order 1–3 s on an IBM 3033 computer, depending on the amount of output and the number of plots.

Blade Sections. The use of analytic equations for the blade definition makes geometrical operations, such as rotation or intersection with a specified plane, relatively straightforward. An example is shown in Fig. 8, where cross sections of a blade with planes normal to the axis are delineated. In this example, the sections through the blade are very nearly radial lines, which produces low bending stresses at the blade root. If these lines were not radial, the designer might wish to modify the blade by introducing a rake angle at the trailing edge or by changing the blade angle distribution along the hub or shroud.

Automatic Net Generation for Flow and Stress Calculations. The great advantage of using a parametric description for all of the surfaces is that the coordinates of points on the surface can then be very simply obtained by specifying the values of the parametric coordinates (u, v). By this means the geometric data of the channel walls and blade surfaces can be generated at any number of points and in any suitable distribution for subsequent aerodynamic and stress calculations or for manufacture.

Any point on a Bezier surface can be referred to by its appropriate (u,v) coordinate. Normally, however, the coordinates of many points on the surface are required and it would be extremely tedious to have to specify all of the individual u, v coordinates. In the present method, this problem is solved by using supplementary parameters to set up an array of points in the u, v plane at which the geometrical details are to be calculated. An example is shown in Fig. 9, where the blade and channel parametric coordinates (u, v) are specified to set up a suitable grid for a streamline curvature "throughflow" calculation. A further example is given in Fig. 10, where a grid for a three-dimensional stress calculation is generated.

This aspect of the method has proved extremely useful in the computer-aided procedure, since the interface between the geometry definition and the subsequent aerodynamic or stress calculations can be made extremely general. The differing distributions of coordinate data needed by different flow and stress programs can be very simply obtained. The net generation method has shown itself to be extremely well adapted for flow calculations, as it automatically produces a grid of "body-fitted" coordinates.

Standarization of Impellers. In the standarization of industrial centrifugal compressor, it is often the case that a

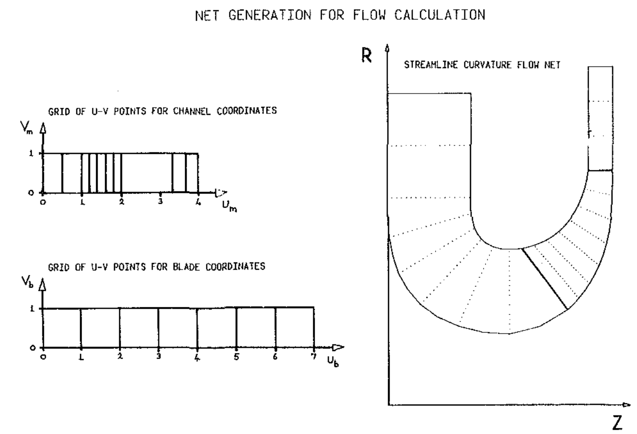

Fig. 9 Automatic net generation for streamline curvature flow calculation

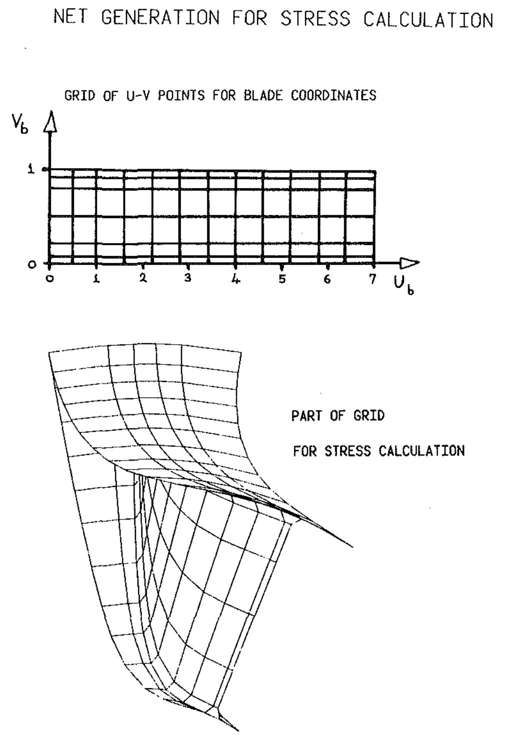

Fig. 10 Automatic net generation for a finite element stress calculation

particular blade shape is used in combination with different meridional channels in order to adapt the blade to different aerodynamic conditions. The original blade may be modified by removing (or adding) a section on the shroud side (shroud-cut) or a section on the hub side (hub-cut). The present formulation allows this standardization to be carried out in the following way:

(*a*) The impeller blade surfaces (B, S, and P) are originally defined to lie within a meridional channel (M′).

(*b*) A new meridional channel (M) is then defined which may be narrower or wider than the original channel (M′).

(*c*) The new flow channel between the blades is then redefined by the intersection of the new meridional surface (M) with the blade surfaces (B, S, and P).

(*d*) The net of u, v-coordinates that refers to the blade surfaces (u_b, v_b) is then suitably modified so that the values are truly confined to that part of the blade within the flow channel. The use of algebraic equations makes the mathematics of the rotation and intersection relatively simple.

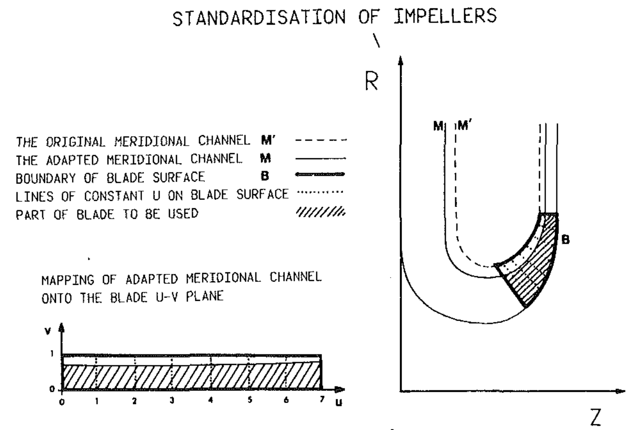

Fig. 11 Use of Bezier surfaces in the standarization of impellers

The definition of the blade surface is extrapolated linearly outside the range $0 < v < 1$, if the new meridional channel (M) is wider than the blade in question.

An example of this procedure is given in Fig. 11.

Manufacture of Impellers. The use of equations to define the shape of impeller blades enables the coordinates required for manufacture to be easily generated. The impeller can then be machined by any conventional method. This method of geometry description is particularly well adapted for manufacture by a five-axis numerically controlled milling machine. The straight line generators of the blade surface focus approximately on a common point and are nearly normal to the hub wall and this allows the impeller to be flank milled on a numerically controlled machine.

Concluding Remarks

An extremely flexible computational geometry for the shape definition of various components in centrifugal compressors has been put forward. In addition to its use in the definition of impeller geometry, the method can be used to define the complete flow passages in a compressor. The method has been including in a computer-aided design system and offers the following advantages over earlier methods:

- Blade and flow channels of almost any required shape can be designed.
- The coordinate data for flow or stress calculations or for manufacture can be readily obtained.

By using this method the designer is able to get a better "feel" for the geometry during the design process, he spends less time on mundane data preparation, and he has more freedom to make full use of his creativity to design a better machine.

Acknowledgments

The author would like to thank Sulzer-Escher Wyss Brothers Ltd., Switzerland, for permission to publish this paper, and Dr. M. Engeli of Fides, Zurich, for introducing him to the theory of Bezier surfaces.

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APPENDIX

In this appendix we illustrate some aspects of the algebra required to determine the polygon points of the blade surface Bezier patches. For the sake of clarity we describe only the equations for the x-coordinates of the polygon points in detail.

On both the hub and shroud of each blade surface (B, S, and P) we determine a set of $n+1$ patch corner defining points with x-coordinates, $x_1, x_2 \ldots . x_{n+1}$. A curve in space between these points is interpolated with a cubic parametric spline of the form

$$x = a_j(1-u)^3 + b_j\, 3u(1-u)^2 + c_j\, 3u^2(1-u) + d_j\, u^3 \tag{A1}$$

where u varies from 0 to 1 along each curve between two adjacent defining points and j taken values from 1 to n for the n intervals between the points. If we now compare equation (A1) with the Bezier curve of degree 3 as given in equation (6) we see that the two equations are identical, whereby the parameters a_j, b_j, c_j and d_j are the x-coordinates of the polygon points and j is the number of the patch.

If we differentiate equation (A1) with respect to u and apply the conditions for continuity of x, dx/du and d^2x/du^2 at the junction between patch j and patch $j+1$ we obtain the following relationships

$$\begin{aligned} a_{j+1} &= x_j \\ b_{j+1} &= 2\,d_j - c_j \\ c_{j+1} &= b_j + 4\,d_j - 4\,c_j \\ d_{j+1} &= x_{j+1} \end{aligned} \tag{A2}$$

In addition we can also derive that at the leading edge (where $u=0$ and $j=1$),

$$\frac{dx}{du} = -3a_1 + 3b_1 \tag{A3}$$

and at the trailing edge (where $u=1$ and $j=n$),

$$\frac{dx}{du} = -3c_n + 3d_n \tag{A4}$$

The values of parameters a_j, b_j, c_j, and d_j are fully defined by the recurrence relationship given in equation (A2), the set of defining points $x_1, x_2, \ldots x_{n+1}$ and two additional items of information. In the present method it is convenient to specify the derivatives at the leading and trailing edges as additional information, as this ensures that the inlet and outlet angles of the blading are exactly as specified. The required derivatives are determined from the known geometry of the blade by the following equations

$$\begin{aligned} \frac{dx}{du} &= \frac{ds}{du}\cdot(\sin\beta.\ \sin\vartheta + \cos\beta.\ \sin\epsilon.\ \cos\vartheta) \\ \frac{dy}{du} &= \frac{ds}{du}\cdot(-\sin\beta.\ \cos\vartheta + \cos\beta.\ \sin\epsilon.\ \sin\vartheta) \\ \frac{dz}{du} &= \frac{ds}{du}\cdot(\cos\beta.\ \cos\epsilon) \end{aligned} \tag{A5}$$

where s is the length along the camber line of the blade, β is the blade camber angle, ϵ is the meridional pitch angle of the hub or shroud

$\left(\epsilon = \tan^{-1}\left(\frac{dr}{dm}\right)\right)$ and ϑ is the circumferential coordinate.

The parameters a_j and d_j are easily obtained from equation (A2) as these are identical to the defining point coordinates, but parameters c_j and b_j must be calculated. This is done in the following way. Parameter b_1 is determined from equations (A3) and (A5). The value of c_1 is then taken as $c_1 = 0$, and the recurrence relations (A2) are successively applied at each patch junction until a value of c_n is determined. This precedure is repeated for $c_1 = 1$, and another value for c_n is established. The correct value of c_1 is then chosen by linear extrapolation such that equation (A4) gives the correct slope at impeller outlet. Equations (A2) are then applied once more to each patch successively to fully determine the polygon point coordinates.