

## AN OPTIMIZATION TECHNIQUE FOR RADIAL COMPRESSOR IMPELLERS

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### ABSTRACT

A software tool has been created to aid in automated impeller design within an integrated design system for radial flow impellers. The design tool takes the results from the 1D preliminary design process and uses these to define a parameterized blade geometry, which incorporates features that are required for low mechanical stresses and simple manufacturing. This geometry is then adjusted to minimize a global objective function using a throughflow computation. The adjustment is based on selection with a breeder genetic algorithm. The initial population includes “elite” designs from a database of earlier well-proven experience, and the final design is honed to perfection with a hill-climbing method.

With the help of a suitable global objective function incorporating mechanical and aerodynamic criteria, and taking into account wide experience with the design of impellers, the tool provides a fast screening of various design possibilities to produce a geometrical input for more advanced computational fluid dynamic and mechanical analysis. This is demonstrated through the redesign of an impeller previously designed by conventional methods. Comparisons of the results of the CFD analysis of the new impeller with that of the earlier design demonstrate that the tool can rapidly produce nearly optimal designs as an excellent basis for further refinement by the more complex analysis methods.

### NOMENCLATURE

d	=	recombination parameter
DH	=	De Haller number ( $W_2/W_1$ )
M	=	Mach number
P	=	penalty function
$P_0$	=	Start population
p	=	number of individual
u	=	Bezier parameter
w	=	weighting factor
x,y	=	Free parameters

X,Y = Vector of free parameters

### Greek Symbols

$\alpha$	=	parameter in optimizer
$\beta$	=	blade angle
$\delta$	=	blade thickness
$\lambda$	=	work coefficient

### Subscripts

c	=	casing
h	=	hub
mean	=	area-averaged
ref	=	reference value
req	=	required value
peak	=	maximum value
ss	=	suction surface

### INTRODUCTION

Centrifugal compressors for small gas turbines, for turbochargers and for compression of industrial gases are increasingly pushing the limits of efficiency, weight, inertia, compactness and cost effectiveness. These technologies require efficient impellers and the competitive nature of the business requires the design process to be as short as possible. The goal of this project was to develop a design tool for radial impellers with the capability of screening and selection of the key design variables before further refinement with computational fluid dynamic (CFD) and finite element mechanical (FEM) methods takes place. The work described in this paper was carried out during a period of practical training of the second author, who is a student with specialization in turbomachinery in Stuttgart University.

Most turbomachinery design systems use extensive RANS 3D CFD and FEM mechanical analysis for the detailed design in an iterative manner. The designer repeatedly adjusts the shape of the blades and flow channels (in a virtual sense) until

he finds a suitable geometry that combines acceptable aerodynamic performance, good matching to the associated system, low stress levels, low noise, no resonant frequencies in the operating range and is economic to manufacture. This process of continual refinement can be expensive, tedious and time-consuming since at each stage the geometrical data and grids for computational analysis must be prepared and the results of the simulations analyzed. Any effort made early in the design process to eliminate unsuitable designs by an effective screening process upstream of the more complex analysis tools, is rewarded by a quicker design. This provides more time for the engineer to examine real design issues rather than wasting time analyzing in considerable detail what prove to be totally unsuitable designs.

As part of a new radial turbomachinery design system, a number of earlier tools (Came and Robinson (1999)) have now been further improved and merged to form a fully integrated impeller design system, see Casey and Robinson (2007). The preliminary design tools produce a 1D design to match the prescribed duty of the machine and allow the designer to move seamlessly to a 3D geometry in the ANSYS BladeModeler environment for more detailed 2D and 3D analysis.

This paper concerns the extension of this design system with an optimizing tool which allows parts of the process to be automated. The goal is not to replace CFD in the final refinement of the design, merely to ensure that the time invested in CFD is wisely spent. Specific aspects of the design system are described in this paper, followed by a validation of the method. For this, an impellers previously designed “by hand” for an industrial air compressor has been redesigned by the new optimization tool. The results show that the tool is able to closely achieve the performance levels of an experienced designer, but in considerably less time.

### AUTOMATED DESIGN SYSTEM

The elements of this automated design system are as follows, and are described in the sections below (see figure 1):

- A correlation based preliminary design system for radial impellers (known as Vista CCD and Vista CCP).
- A new parameter-based geometry definition system for radial impellers (based on Bezier curves and similar to that published by Casey (1983)).
- An interface between the geometry system to ANSYS BladeModeler software allowing data transfer to other software systems (for example, ANSYS CFX for CFD and ANSYS Mechanical for mechanical stress and vibration).
- An interface between the geometry system and a streamline curvature through-flow code, (known as Vista TF and described by Casey and Robinson (2008)).
- An optimization method for determining the best impeller, which includes a breeder genetic algorithm (BGA) coupled with a hill-climbing technique and a process to take into account experience from earlier “elite” designs to initiate the optimization.

- Considerations with regard to the definition of the most suitable global objective function based on extensive experience of impeller design.

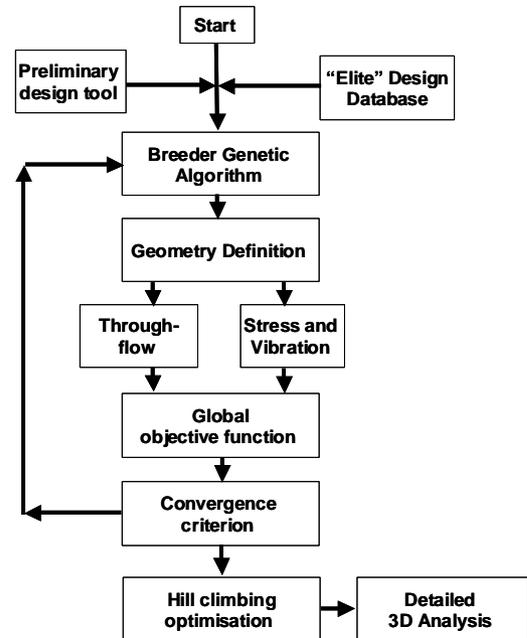


Fig 1 Components of the design system

Software development has been made to link the elements to the preliminary design tools to produce an integrated software-package with a Visual Fortran control unit. Note that the diagrammatic representation of the design system above includes aspects of stress and vibration optimization which are currently under development.

### GEOMETRY DEFINITION METHOD

The geometry definition method described here is closely based on that published by Casey (1983), where full details of the basic theory can be found. The new implementation of this theory takes into account the author’s experience and feedback from other users with the original software system over twenty-five years.

There are two key differences in the current implementation compared to that described by Casey (1983). Firstly, in the new code, all of the key input parameters of the blade meridional channel geometry and the blade and thickness distributions are based on Bezier polynomial approximations, whereas in the earlier method this was not the case for the blade angle and blade thickness distributions. Secondly, no attempt has been made to generate a Bezier representation of the blade surfaces, see appendix to Casey (1983), as this possibility is nowadays available within other CAD systems. The meridional channel is represented as a series of Bezier patches, and the blade is represented as a fine mesh of points in space, which in the examples in this paper lie along the straight line generators of the blade surface.

The parameters used to define the precise shape are based on experience with the earlier method. The actual choice of parameters for such a system is largely a matter of taste, but a selection has been made which allows the geometry to be defined with a minimum number of non-dimensional parameters and at the same time to provide maximum flexibility. 52 free parameters are used to define the whole geometry of an impeller with splitter vanes, as shown in figure 2. Many of these may remain fixed during the optimization of the impeller, such as the length of the inlet channel or the diffuser outlet radius ratio. In addition, it has been found that many other parameters retain almost constant values from impeller to impeller. This allows experience on a particular impeller type to be easily cloned into a new design.

### Meridional flow channel

The meridional geometry of the impeller is defined by a template (based on a single subroutine) which can be adapted to represent the different types of meridional channels that can be found, such as a radial impeller with axial inlet, radial impeller with radial inlet, radial turbine and so on.

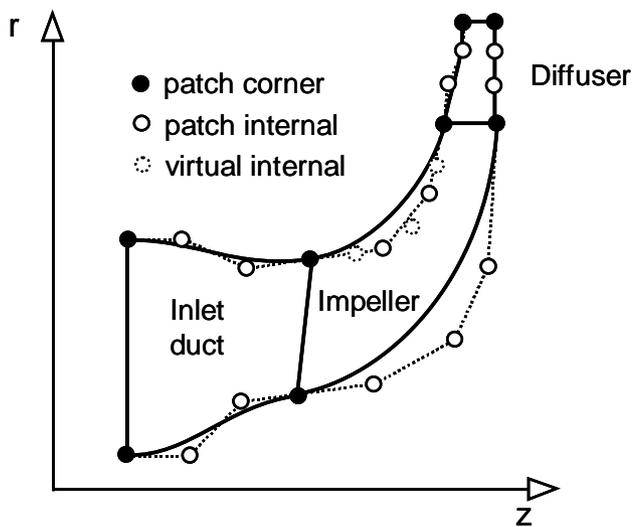


Fig 2. Template for a radial impeller meridional channel defined by Bezier points, following Casey (1983)

The template for a typical centrifugal impeller with an axial inlet and a radial outlet used in this paper comprises a series of three Bezier patches or segments, see figure 2. The first patch is the inlet channel of the impeller, the second is the impeller itself, whereby the leading and the trailing edges of the impeller are coincident with the patch boundaries on the hub and the casing, and the third represents a vaneless diffuser channel with pinch. Further templates are available which can be used to define a typical radial turbine impeller, and others can be envisaged to define a mixed flow pump impeller, a radial stage with return channel and other types of blade rows. The software has been developed to be as flexible as possible to allow these changes at some point in the future.

The meridional channel geometry definition takes into account the fact that the impeller design is usually carried out in a step-wise process, beginning with a preliminary 1D design method. This means that some key skeletal parameters are already fixed by the preliminary design optimization process or by other constraints, and no longer need to be changed during the subsequent optimization. For example, the choice of the impeller diameter generally results from the preliminary design process, and in some cases a fixed value of the backsweep angle might be imposed based on that needed for the required operating range.

The key geometry parameters for the meridional channel are split into four groups. The first group is dimensional and provides the impeller diameter and the axial location of the hub at the leading edge. This allows a change in size and a translation along the axis to be incorporated in different impellers.

All subsequent parameters are non-dimensional and are defined as a ratio to the impeller outlet radius or as a ratio to some other relevant dimension, such as the axial length of the impeller hub. The skeletal dimensionless geometry terms below can then be considered as parameters to define a family of impellers of a particular diameter. This allows easy transfer of experience gained on a certain design with a certain diameter to be simply scaled into another design at a different size for a similar application.

The second group of parameters define the location of the patch corner points in the meridional channel. The following parameters are needed:

- Inlet radius ratio on casing at duct inlet
- Inlet radius ratio on hub at duct inlet
- Axial length of the inlet duct
- Impeller inlet eye radius ratio
- Impeller inlet hub radius ratio
- Axial lean of leading edge at inlet
- Axial length of impeller hub
- Impeller outlet width ratio
- Diffuser outlet radius ratio
- Diffuser outlet width ratio

The third group of parameters for the meridional geometry determine the curvature of the meridional channel contours. Various shape factors along the hub and casing wall are used to determine the location of the internal Bezier patch polygon points. Each shape factor determines the location of the associated internal point as a fraction of an associated length in the channel. Roughly half of these parameters can be determined in advance and fixed for the design optimization process so in fact typically only 6 free shape parameters are generally used for the shape of the meridional walls in the impeller region of the channel during the design process. As these parameters are based on fractions of the length of the meridional channel experience shows that many of these remain sensibly constant across a range of designs of impellers. This eases the task of a new design in that parameters optimized in an earlier design can be used as the starting values.

The fourth group of parameters for the meridional

geometry determine the slope of the meridional channel contours at the patch corner defining points at impeller inlet and outlet. Separate slope angles for each of the casing and hub walls at the leading and trailing edges are defined. Note again that these angles may also be set by the preliminary design process or some other constraints, or one or more of them may be set to zero, so that in fact typically only three free slope parameters are free to be optimized.

Note that based on experience with the system described by Casey (1983) three internal polygon points are used to define the hub contour within the impeller, requiring four parameter values, whereas on the shroud only two free parameters are required. The shroud impeller contour is defined initially as a Bezier curve with two internal points and this is converted to become one with three internal points to be consistent with the hub, using general rules related to Bezier curves which increase the order of a curve whilst maintaining its shape. It would of course also be possible to define the shroud contour with three internal points but this would require two additional free parameters.

### Blade shape

The impeller blade is defined as a ruled surface of straight lines joining points on the hub and the shroud contours which are equidistant along the meridional channel of the impeller, between the leading edge and the trailing edge. Other orientations of the ruled surface can also be selected but are not used here. The use of ruled surfaces is a standard technique for impeller design leading to simpler manufacture (through flank milling). This is not considered to be a severe limitation from the aerodynamics point of view.

In the case of an impeller with a splitter, the splitter leading edge position is defined by the axial location of the leading edge on the hub and on the casing. The splitter leading edge is also a straight line and the orientation of the ruled surface of the impeller is adapted to make this line one of the blade generating lines of the main blade. Currently the splitter is considered to be a shortened version of the main blade with no leading edge re-camber (see Came and Robinson (1999)). This would be relatively easy to take into account but currently constitutes a part of the detailed design process.

Another group of parameters define the blade shape and the number of blades. The hub and shroud blade sections are defined as distributions of camber line and thickness specified as Bezier functions along the normalized meridional length, whereby the leading edge and trailing edge ellipses are defined as separate parameters. The angle distribution along the hub (and a similar equation is used for the casing) is defined as a Bezier polynomial with three internal points, see figure 3, as follows

$$\beta_h = \beta_{h0}(1-u)^4 + 4\beta_{h1}u(1-u)^3 + 6\beta_{h2}u^2(1-u)^2 + 4\beta_{h3}u^3(1-u) + \beta_{h4}u^4 \quad 1$$

whereby the Bezier parameter  $u$  in this distribution is the normalized meridional length, which varies linearly from 0 to 1 along the meridional walls of the impeller from the leading to the trailing edge. The blade outlet angle will sometimes not be a free parameter as the 1D design will determine this so

there remain 8 free parameters to determine the shape of the blade. These are also not completely free as the mechanical constraints on the design may also require no radial blade lean at the leading edge and a certain rake at the trailing edge. These constraints mean that actually only 7 completely free parameters remain for the blade shape.

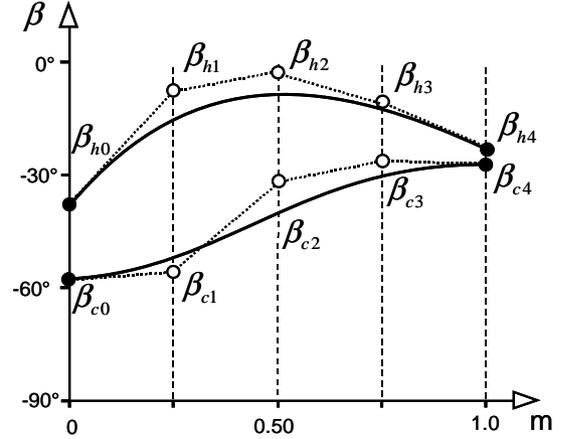


Fig 3. Blade hub and casing angle distributions using Bezier curves

A useful option has been included which allows the hub blade angle distribution to be modified internally by the code to achieve a specified lean (typically 0°) of the leading edge and a specified rake angle at the trailing edge, as these values are usually known in advance from mechanical considerations. This description of the blade with lean and rake parameters and the hub and shroud blade angle distribution according to equation 1 is over-determined, so that the code has to modify some of the information specified by the user. The code takes into account the relative importance of the shroud streamlines in the diffusion process of the impeller and modifies the user-specified hub angle distribution, while keeping the casing angle distribution the same as specified. The modification is made by applying a small correction  $\delta\beta$  to each internal Bezier point along the hub, whereby this correction is iteratively determined to match the specified lean and rake angles. The modification makes use of the user-defined hub angle distribution as a guide but overrules this within the blade row, keeping the inlet and the outlet blade angles the same so that incidence and work input are not affected.

The hub and casing thickness distribution is defined as a Bezier polynomial with two internal points, whereby the roundness of the leading edge and trailing edge need to be defined by separate parameters:

$$\delta_h = \delta_{h0}(1-u)^3 + 3\delta_{h1}u(1-u)^2 + 3\delta_{h2}u^2(1-u) + \delta_{h3}u^3 \quad 2$$

The hub and casing thickness distribution is defined as a Bezier polynomial with two internal points, whereby the roundness of the elliptical leading and trailing edges need to be defined by separate parameters.

## STREAMLINE CURVATURE THROUGHFLOW

The new streamline curvature throughflow code used in this design tool is described in detail in a companion paper, Casey and Robinson (2008), so only the features that are particularly relevant to its use in this optimization procedure are briefly described here. The code has a long pedigree and is derived from the throughflow method of Denton (1978) and its adaptation to radial machinery by Casey and Roth (1984). As with the geometry definition method, experience with the original version of the throughflow code over 25 years has strongly influenced the newly written version that is used here.

Casey and Roth (1984) provided several validation examples that amply demonstrate that the throughflow method is a useful tool for impeller design. It allows incidence and Mach number levels at the leading edge to be examined and gives a good estimate of blade loading in the first half of the impeller. It cannot properly take into account the viscous effects in the last part of the impeller, such as the secondary flows and strong jet-wake flows, but well-designed backward-swept impellers tend to be less affected by these features than earlier radial impellers, and the throughflow method is then highly suitable for preliminary design optimization.

Key features of the code of relevance to this application are listed below:

- Highly curved annulus walls are allowed providing a simple definition of axial and radial wall geometries and any combination of these required for radial impellers.
- Any combination of blade row calculating stations, together with duct flow regions, can be used in the domain, so that in this application the domain includes the axial inlet duct, the impeller and the diffuser channel.
- Internal blade row calculating stations are used, not just leading and trailing edges and blade force terms are included to take into account the lean of the blades, whereby the body force is assumed to act normal to the blade camber surface.
- Compressible and incompressible fluids are possible, including limited amounts of supersonic relative flow in blade rows, such that transonic impellers may be calculated.
- The presence of blade row choking is not just included as an additional loss, but the effect of choking of individual stream tubes on the redistribution of the meridional flow distribution is taken into account.
- In impeller blade rows 15 internal planes are typically used and this allows an approximation for the blade-to-blade flow field to be calculated estimating the suction and pressure surface velocity distributions, which includes the effect of splitter vanes in an approximate way.
- The code includes the automatic selection of the Wiesner slip factor for radial impellers and allows losses to be taken into account through a small-scale polytropic efficiency, which can be selected to be consistent with the correlations used in the 1D preliminary design process.

The code also includes an option that allows a restart from a previously converged solution. This considerably reduces the

effort for a new calculation with slightly changed geometry, which is particularly useful in combination with the BGA optimization method. The flow field information of a converged iteration is stored on the basis of non-dimensional span-wise and meridional coordinates. These can be used to start a new simulation, even if the geometry has been changed, by mapping the values onto the new geometry.

The restart file used is that for the current best impeller of the optimization process so that, as the optimization proceeds and smaller geometrical changes are made, less effort is needed for the throughflow simulations. This also brings enormous benefit in combination with the hill-climbing procedure where only small changes in geometry parameters are examined to obtain the gradients.

## OPTIMIZATION METHOD

The use of the type of breeder genetic algorithm (BGA) described here was biased by the good experience of the first author in using this approach for other applications, involving hydraulic turbines (Sallaberger et al (2000)) and axial compressors (Sieverding et al. (2004)). In the present application, the method is based on the optimization of a single objective function, rather than a multiple objective function, as the method is designed to provide a single initial design for subsequent detailed analysis. The objective function is described in more detail below.

Experience showed that the BGA had two principle weaknesses. It was unable to take into account experience from earlier designs that are known to be good and, although it converged on a reasonable design relatively quickly, it required extremely long calculating times to achieve the ultimate best design. A survey of the technical literature on optimization methods identified four different ways in which an acceleration of such an optimization can be achieved, Giannakoglou (2000):

- Improvement of the genetic operators by various technical features of the genetic algorithm (binary / real coding, asexual / multisexual reproduction, one point / multipoint / uniform reproduction, adaptive techniques).
- Multiprocessing with simultaneous evaluation of candidate solutions or separate processors for genetic operations and evaluation of a defined objective function.
- Reduction of exact solutions through the use of tools of lower accuracy to increase the speed, or the use of artificial neural networks (ANN) which are dynamically trained during the genetic evolution.
- Hybridization with numerical optimization methods including improved random search to provide a good starting position for the numerical optimization or coupling with a numerical gradient method for hill-climbing.

The first issue has been dealt with in that the breeder genetic algorithm BGA used here is one of a class which is among the best. Multiprocessing has not been attempted as in the cases considered here computational times of less than one

hour on a laptop can be achieved without this. The method makes use of a low-fidelity throughflow code rather than a fully 3D viscous RANS CFD code for the analysis so that very short computational times are possible. By reducing the number of exact solutions neural networks can accelerate a genetic algorithm optimization. The process described here already uses a rapid low-fidelity flow solver, so the effort of implementing a neural network seems not to be worthwhile.

The examination of these options have led to the selection of an optimization software used in this design tool with three different parts:

- The first population of impellers is generated at random, but includes additional impellers from a database of “elite” designs, to seed the second generation with good parameters.
- A breeder genetic algorithm to survey the available global design space in the search for better designs
- A final hill-climbing optimizer to ensure that the final design is at the local optimum of its design space

#### Elite designs adapted from preliminary design process

The preliminary design tool determines the approximate skeletal geometry and flow parameters for the design, with the help of well-established correlations. On this basis a start population  $P_0$  with  $p$  individuals is formed. Each of the individuals is defined as a vector  $X = (x_1, \dots, x_N)$  with  $x_i$  being the different free geometry parameters needed to define the impeller, whereby bounds can be imposed so that  $x \in [x_{\min,i}, x_{\max,i}]$ . The individual geometry parameters of each individual are chosen at random within the allowable bounds.

The start population is then extended by the addition of a number of well-proven “elite” impeller designs. No special additional interpolations are made, as the “elite” set of impellers includes designs covering the whole range of specific speeds and pressure ratios to be expected. The best impellers in the “elite” population seed the second generation with their own geometrical properties, which then become automatically adjusted to the bounds set for the parameters. As many radial compressor applications are relatively similar this usually means that one of the “elite” designs is fully retained during the first few generations of the genetic algorithm, until a better design has been found.

In the early development of the BGA, very large ranges were chosen for the bounds of all parameters. This leads to designs which fulfill the aerodynamic targets very well (according to the defined objective function, see below) but in which some features of the geometries look very unusual and would certainly not be accepted by an experienced designer. For example, in some cases the meridional channel had a very high hub line to avoid a low velocity at the hub on the pressure side of the blade. This has clearly unacceptable disadvantages regarding the rotor hub stresses, which has not been specified as a part of the objective function. The use of a set of elite designs to guide the choice of parameters and a sensible constriction of the bounds on the parameters leads to good improvement of the BGA optimised designs. It seems as if the BGA needs a little hint from the user, where it should look for a quasi-global optimum.

#### Breeder genetic algorithm

The breeder genetic algorithm (BGA) makes use of evolutionary computation, which is a sub-field of artificial intelligence that involves combinatorial optimization problems, Mitchell (1996). The basic principles of the genetic algorithm in optimization processes are similar to the theory of natural selection of Darwin, whereby a population of individuals changes over several generations following laws of natural or artificial selection, involving reproduction and mutation of the fittest surviving individuals. In this case each individual is a different impeller design. Its chance of survival into the next generation (fitness) is related to how well it meets the user defined design objectives.

Details of these methods can be found in standard text books, whereby the algorithm used here has been described in more detail by Sieverding et al (2004).

The BGA is the optimization algorithm that searches for the best individual according to the defined objective function. This search extends over several generations, whereby each new generation is formed using rules of selection, reproduction (or recombination) and mutation. Selection is simply the process of choosing the best  $\text{Tr} \%$  of the individuals and eliminating the rest. Recombination involves generating a new population of  $p$  individuals by combining geometry information from two randomly chosen parent individuals ( $X = (x_1, \dots, x_N)$  and  $Y = (y_1, \dots, y_N)$ ) to give a new individual  $W = (w_1, \dots, w_N)$ . Different recombination strategies are possible but in this case an extended recombination strategy on each individual parameter is used such that

$$w_i = x_i + \alpha(y_i - x_i) \quad 3$$

where  $\alpha$  is a random number between  $-d, 1+d$  and  $d$  (roughly 0.25) is the recombination parameter. Note that the value of  $\alpha$  can be greater and less than unity so that this process includes an element of extrapolation. Following the recombination all individuals are slightly changed (mutation) by the following algorithm

$$w_i = w_i \pm m(w_{\max,i} - w_{\min,i}) \quad 4$$

with  $m \in [0,1]$  the random mutation parameter.

After a new individual is created, the bounds represented by a lower and upper limit of each geometrical parameter have to be checked. If a parameter is out of this range its value is set to the lower or upper bound. In order to ensure that the absolute best individual of each generation is not lost through mutation or recombination, this individual is copied unchanged into the next generation (elitism).

After the BGA has created a new generation of individuals the geometry definition program uses the parameters of each individual to generate a virtual geometry of the appropriate impeller. The throughflow code is then used to calculate the flow in the impeller. The fitness of each impeller is determined by means of a user-defined objective function (see below). This evaluates its effectiveness to decide whether it may participate in the genetic process that produces the next generation.

### Hill-climbing algorithm

The breeder genetic algorithm (BGA) is good at searching the design space for a global optimum but is less effective at searching for the local optimum when the design is nearly complete. To finish off the optimization process a hill-climbing approach is used. The routine uses a quasi-Newton method and an active set strategy to solve minimization problems subject to simple bounds on the variables. A finite-difference method is used to estimate the gradients through repeated evaluations of the fitness function with the throughflow code.

### GLOBAL OBJECTIVE FUNCTION

Ideally it would be best to optimize the efficiency or minimize the losses in the impeller to derive the optimal aerodynamic performance, but this would require the use of a more complex 3D viscous RANS CFD simulation, and is prohibited because of the time required. The throughflow code is extremely fast, but cannot calculate the losses accurately enough for this process. The losses in the impeller throughflow simulation are chosen to be compatible with the overall stage performance correlations of the preliminary design process so cannot form the basis of the optimization. Because of this, other results and features of the flow-field from the throughflow analysis have to be included in the evaluation of the impeller.

The parameters selected in the objective function have been selected from a list of the most important features that may influence impeller performance. This was done by discussion with several experienced impeller designers (with a total of nearly 100 years of impeller design experience). Although there was general agreement on most parameters of relevance there was heated discussion and no general agreement on others. Some parameters were clearly felt by all designers to be important, and others were almost a matter of taste. This aspect is taken into account with the weighting of the individual parameters, see below. In this sense, the objective function provides a framework to quantify the experience and skill of several designers. So that this know-how and experience can be used by novice designers.

The following parameters were considered to have an effect on the efficiency and were included in the fitness function:

1. Suction surface peak Mach number
2. Suction surface average Mach number
3. Minimum Mach number on pressure surface hub
4. Incidence at hub
5. Incidence at tip
6. Loading limit of inducer
7. Loading limit of rear part of impeller
8. Loading limit of the middle part of the impeller
9. Loading limit hub to shroud
10. Shape of mean shroud velocity distribution
11. De Haller number
12. Work coefficient
13. Flow angle into diffuser

14. Rake angle at trailing edge
15. Lean angle at leading edge
16. Throat choke margin

In the first instance mechanical parameters have not been taken into account (other than through the specification of blade thickness and the lean and rake angles as outlined above). The aerodynamic analysis is carried out with a specified thickness distribution for the hub and the shroud taken from an earlier similar example which is not changed during the optimization. The following stress parameters could be taken into account at a later stage:

17. Bore stress parameter
18. Blade root stress
19. Blade natural frequencies

The problem we are faced with is the minimization of a function of several variables with different units and with different importance. Some are constraints that have to be attained (such as the desired work coefficient), others are known to have a strong effect on the efficiency, and others are “nice to have” as they are believed to have a small effect on the efficiency. The fitness function needs to take each of these into account and a penalty has to be derived that increases when the requirement for each parameter is not attained.

Different approaches to this problem are possible but the following fitness function is used based on various publications on this subject (see, for example, Verstraete et al (2007a) and (2007b))

$$F = \sum_{penalties} wP$$

The fitness function contains a sum of all individual penalties  $P$ , each of which is related to the list of parameters outlined above. Each of these is weighted by a weighting factor to allow the user to reflect the importance of the individual penalty factors in the optimization. The penalties can be split into different types. In some cases, such as the work input factor, the user is interested in meeting a design constraint of a required value, and the design has to achieve this objective more or less exactly. Similarly, other parameters have very little freedom for variation, such as the lean of the blade at the impeller inlet, which for mechanical reasons is always close to  $0^\circ$  in an open impeller. In others, the user is interested in achieving a low value or a high value of the parameter but small deviations from the required value should be allowed but penalized, such as incidence. In other cases a positive deviation of a parameter may be penalized but a negative deviation is of no consequence, such as the diffusion level on the impeller shroud contour which should not go below a limit defined by the De Haller number.

It is important that the individual penalty functions are non-dimensional so that the units of the different penalties have no consequence, and that the individual penalties are all of the same order of magnitude. This can be achieved by defining each penalty as follows:

$$P = \left( \frac{V - V_{req}}{V_{ref}} \right)^a$$

where the difference between each value  $V$  and the required value  $V_{req}$  is made non-dimensional with a reference value  $V_{ref}$ , and deviations are penalized by an exponential function with an exponent of  $a$ . In all the work reported here the exponent was held fixed at  $a = 2$ . In order to take into account the different types of penalties the follow modifications to this approach have been considered:

#### Category (A)

The fitness parameter should be allowed to vary within a small amount, but otherwise penalized strongly, for example the work input coefficient  $\lambda$  (that is the enthalpy rise made non-dimensional with the square of the blade tip velocity)

$$P = \left\{ \max \left[ \left( \frac{|\lambda - \lambda_{req}|}{\lambda_{req}} - \delta\lambda \right), 0.0 \right], 0.0 \right\}^a$$

If the value of  $\delta\lambda$  was specified to be 0.01 (that is 1%) then the penalty has no effect on an impeller in the range of  $0.99\lambda_{req} < \lambda < 1.01\lambda_{req}$  but increases when the work input coefficient differs from the desired value by more than 1%. This, together with a high weighting for this parameter, will effectively limit designs to give only those that have acceptable work input. It will kill off designs with totally wrong work input coefficient but will not penalize those that are relatively close to the required objective as the penalty then becomes zero.

#### Category (B)

Parameters that need to be within a certain range, but where the range is more flexible, may also be specified in this way and also may use a lower weighting function than the real constraints. The same equation is used as above but a wider range. An example of this type of parameter would be the hub incidence at the design point. Designs should achieve a sensible hub flow incidence but as flow in the hub streamtube generally accelerates through the impeller a fairly wide tolerance on the hub incidence may be allowed. The incidence at the shroud, however needs to be closely controlled and falls into the category (A) above.

#### Category (C)

Parameters that need to be minimized or maximized, for example the peak suction surface Mach number which needs to be as low as possible.

$$P = \left\{ \max \left[ \left( \frac{M_{ss,peak} - M_{ss,peak,req}}{M_{ss,peak,ref}} \right), 0.0 \right], 0.0 \right\}^a$$

If a low value of  $M_{ss,peak,req}$  is specified then this will penalise values of the suction surface Mach number which are higher than this. Note that the penalty function never falls below a value of zero so that when all other penalties are already reduced to a minimum value the peak suction surface Mach number is further reduced.

#### Category (D)

Parameters that need to be maximized or minimized but provided they are above or below a certain threshold, then no penalty should be incurred, such as the De Haller number

$$P = \left\{ \max \left[ \left( \frac{DH_{req} - DH}{DH_{ref}} \right), 0.0 \right], 0.0 \right\}^a$$

If the de Haller number is above the required value then no penalty is incurred.

The section above outlines the ideas of the fitness function. The actual implementation of this in the software system has been organized via a control unit which allows some flexibility in the setting up and adjustment of the parameters. This allows users with specific ideas about what constitutes a good design to adapt the fitness function to meet their own taste and style, although clearly defined default values are provided. Intermediate results of the optimization process can also be examined and the control unit allows the user to change the optimization parameters during the optimization process.

Experience with the penalty function during the validation process has caused some of the initial features to be changed, and experience is still being gathered on this. For example, the blade loading from hub to shroud is specified as the difference in meridional velocity between the hub and shroud divided by that on the mean stream line, following the blade loading criterion of Morris and Kenny (1971). A tendency of all optimizations aimed at reducing the hub to shroud loading was found to be that the optimizer tends to do this by increasing the meridional velocity on the mean streamline. There is no direct penalty associated with this, but this of course also affects the rate of deceleration and the loss production in the impeller and tended to produce heavy impellers with a large hub diameter. This problem and other similar problems has been dealt with by allowing the user to set the range of the free parameters to be within certain bounds that are considered acceptable.

## VALIDATION AND VERIFICATION

To verify the use of the optimizer several test cases have been chosen, ranging from small gas turbine impellers, turbocharger impellers and impellers for industrial compressors. The original successful designs were carried out by hand using state-of-the-art fluid dynamic and mechanical analysis design tools. The impeller have now been redesigned using the new optimization tool. The results show that the new optimization tool is able to closely achieve the performance levels of an experienced designer, but in considerably less time.

### High pressure ratio industrial compressor

This case represents a compressor impeller designed for an inter-cooled two-stage industrial air compressor. The application involved a direct drive electric motor and was speed limited so a high diameter impeller of medium specific speed resulted from the preliminary design process. A particularly difficult aspect of this design was the ambitious efficiency level that was required, and this was achieved in the initial design through extensive CFD optimization and so represents a particularly challenging case for the automatic optimizer coupled to a throughflow code. The final design incorporated an impeller of diameter 160 mm, a rotational speed of 60,125 rpm for a mass flow of approximately 1 kg/s and a total to total pressure ratio of the impeller of 4.7.

The BGA control parameters for the optimization of the test case are shown in Table 1 below.

Start population size	60
Population size	40
Number of generations	50
Selection parameter	0.30
Recombination extending factor	0.25
Mutation range	0.10

Table 1 BGA control parameters

The fitness function was established using the following key optimization targets as listed in Table 2 below:

Target	Value
Minimum Mach number on pressure surface along hub streamline	0.1
Work coefficient	0.7
De Haller number	0.6
Flow angle into diffuser	72°
Incidence angle at the hub	10°
Incidence angle at the tip	0°
Choke margin	15 %

Table 2 Optimization targets

The plots of the results from the throughflow analysis shown in figure 4 and figure 5 demonstrate that the use of the BGA has been successful in removing some features of the original design that are not considered to be optimal. Note that the hub line increases radius more slowly in the optimised design and this leads to a general lowering of the velocity levels in the impeller. In particular the optimised design has a lower shroud suction surface Mach number as seen in figure 5, which is an important feature for a transonic impeller. In addition the reduced velocity level in the impeller has very beneficial effects on the uniform blade loading diagrams at the tip as can be seen from figure 5. The optimised design also has

a more uniform distribution of deceleration through the impeller, as shown in the Mach number plots for the tip sections. The optimised design has a lower Mach number on the hub which leads to a higher blade loading in this region, which is probably not entirely beneficial

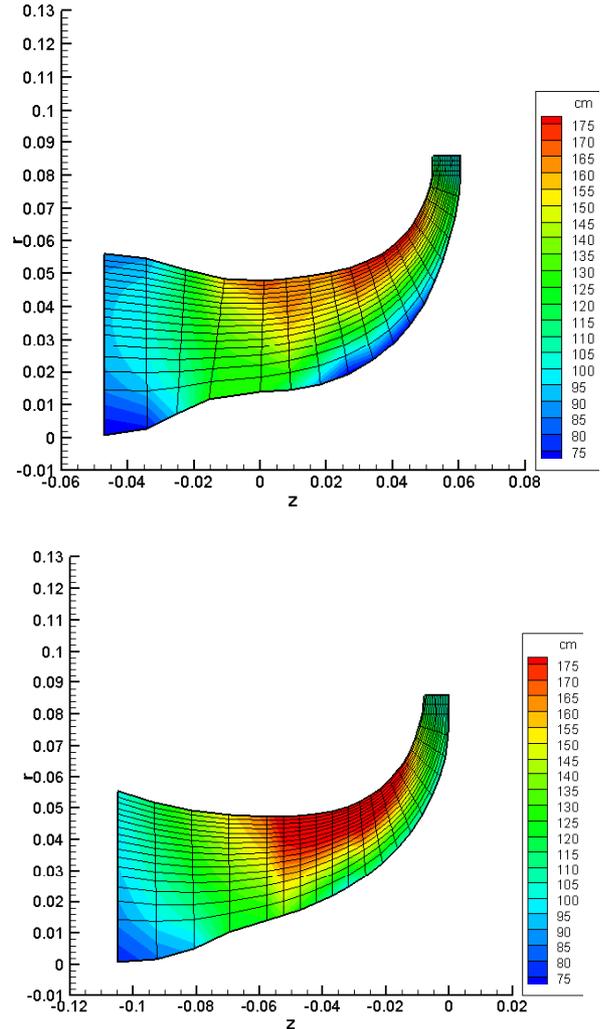


Fig 4. Meridional velocity flowfield of the BGA optimized design (above) and the original design (below)

Both designs have been analysed using 3D CFD simulations using ANSYS CFX11 and the global results are summarised in table 3 below. The CFD computations of the characteristic curves of the stage (figure 6) demonstrate that the optimised design (using the BGA and a simple throughflow code) nearly achieves the performance levels of the design optimised by hand using CFD. The optimizer produces a design that is nearly, but not quite, as good as that produced by an experienced designer with more complex tools. It certainly is an excellent starting point for more detailed analysis.

Design	Original	BGA Optimized
Pressure ratio (t-t)	4.77	4.61
Mass flow (kg/s)	1.013	1.015
Efficiency(s,t-t)	92.6	92.2%
Work coefficient	0.71	0.69
Throat area (mm <sup>2</sup> )	4044	4053
Speed (rpm)	60125	60125

Table 3 CFD determined performance parameters

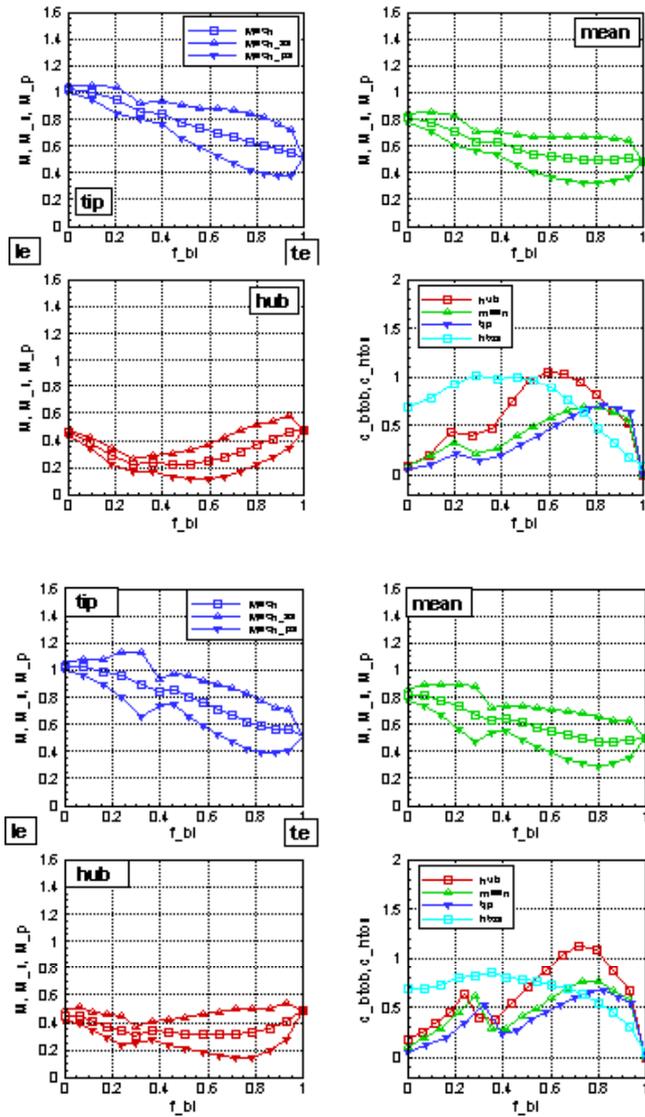


Fig 5 Mach number and blade loading distributions of the BGA optimized design (above) and the original design (below)

## OUTLOOK

The method described here includes the mechanical aspects only in the sense that the blade thickness distribution and the lean and rake angle are chosen from experience with other designs to meet the specified requirements. Work is continuing on the integration of a simple FEM mechanical calculation to allow the optimizer to take this into account during the optimization of the impeller. When the detailed mechanical analysis is included then a multi-objective optimization is planned so that the trade-off between the mechanical and aerodynamic aspects can be assessed.

The optimization system has been described with regard to the optimization of centrifugal compressor impellers, but there is no fundamental limitation of any parts of the system that limit its application to compressors alone. Care has been taken to allow subsequent development for application to pumps and for radial turbine design. Here appropriate templates in the parameterized geometry definition system are needed, together with a clear formulation of the objectives that should be achieved.

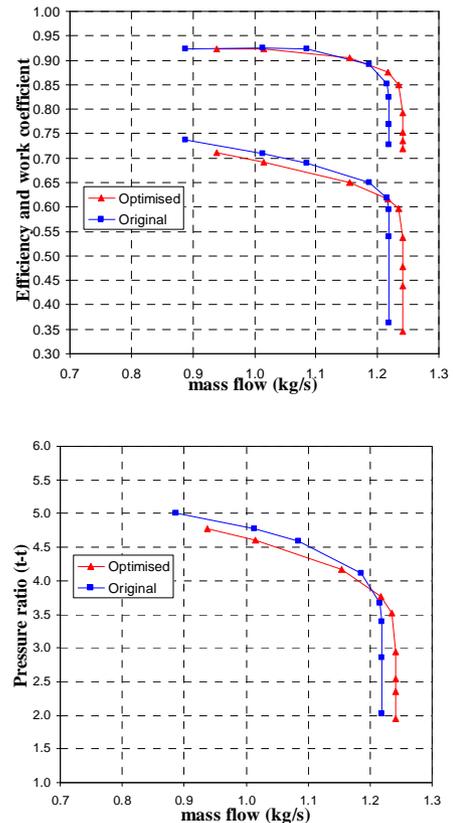


Fig 6. Characteristic curves of original and optimized impeller calculated with ANSYS CFX11, Isentropic efficiency and work coefficient (above) and total to total pressure ratio (below)

## CONCLUSIONS

The method described in this paper provides an exciting practical use of optimization techniques in the early stages of aerodynamic and mechanical design of centrifugal impellers. The method provides the designer with an optimized preliminary design for subsequent detailed fluid dynamic and mechanical analysis within an hour. This frees time for more complex engineering analysis. Real design issues can then be examined with the higher level analysis tools, rather than wasting time to examine designs that are totally unsuitable.

A key aspect is the flexibility and reliability of the throughflow code which provides both meridional and blade-to-blade information with empiricism that is consistent with the 1D mean-line design process. The example given shows that the design using the simple throughflow code nearly achieves the performance levels of designs optimized "by hand" using more complex design tools, but in a much shorter time.

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