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# Centrifugal compressor blade optimization based on uniform design and genetic algorithms

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**Abstract** An optimization approach to centrifugal compressor blade design, incorporating uniform design method (UDM), computational fluid dynamics (CFD) analysis technique, regression analysis method and genetic algorithms (GA), is presented. UDM is employed to generate the geometric information of trial samples whose performance is evaluated by CFD technique. Then, function approximation of sample information is performed by regression analysis method. Finally, global optimization of the approximative function is obtained by genetic algorithms. Taking maximum isentropic efficiency as objective function, this optimization approach has been applied to the optimum design of a certain centrifugal compressor blades. The results, compared with those of the original one, show that isentropic efficiency of the optimized impeller has been improved which indicates the effectiveness of the proposed optimization approach.

**Keywords** energy and power engineering, compressor, optimum design, blade, uniform design, genetic algorithms

## 1 Introduction

The design of high-efficiency compressor blades is a complex and challenging task. Better and faster design methods have to be developed to increase the performance and efficiency and to reduce the development and manufacture costs of blades. The traditional way to speed up the design procedure is to rely on experience gained during previous designs. However, the experienced designer may not always be available and/or they may not be omnipotent in all types of compressor blades. Fortunately, great progress in simulation technologies in several fields

such as computational fluid dynamics (CFD), structural dynamics, and thermal analysis combined with the emergence of improved optimization algorithms makes it possible to perform blade shape automatic optimization process.

At present, several optimization algorithms are available such as gradient-based method [1] or the recently developed adjoint equation approaches based on sensitivity [2], simulated annealing [3], artificial neural network (ANN) [4], response surface [5], inverse design [6], and genetic algorithms (GA) [7], etc. Generally speaking, it is hard to state the superiority of one method over the others for every type of design problems. For example, the gradient-based method is efficient in terms of convergence rate, but it may not guarantee to produce the global optimum [8]; while GA offers the advantage of enhancing the probability of reaching the global optimum, but it may require thousands of iterations. In other words, each method has its advantages and disadvantages and can, therefore, only provide solutions to a certain class of problems.

It is well known that CFD computations for turbomachinery are expensive in terms of computational and operator time. To decrease cost and duration of the design process, the optimization approach must be efficient and can obtain a global optimum within tolerable steps or considerable improvement over the original value.

In this paper, an optimization approach to centrifugal compressor blade design, incorporating UDM, regression analysis method, CFD technique and a genetic optimizer, has been presented. UDM selects from the  $n$ -dimensional space experimental samples that are uniformly scattered in the space. The number of samples produced by uniform design is substantially less than that by orthogonal design. CFD technique is employed to evaluate the aerodynamic performance of the samples generated by UDM. Then regression analysis method is adopted to approximate the relationships between the geometric parameters and aerodynamic performance of the samples. Finally, a global optimization is conducted by GA, and the information of the best sample predicted by GA is added to the sample

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database. The optimization process is to repeat the last two steps, which does not stop until the loop design is satisfied.

## 2 UDM and regression analysis

The main objective of UDM, an important experimental design technique, proposed by Fang et al. [9] is to sample a small set of points from a given set of points so that the sampled points are uniformly scattered. Suppose that there are  $n$  factors and  $q$  levels per factor. When  $n$  and  $q$  are given, the uniform design selects  $q$  combinations out of  $q^n$  possible combinations, such that these  $q$  combinations are scattered uniformly over the space of all possible combinations. The selected  $q$  combinations are expressed in terms of a uniform array  $U(n,q) = (U_{ij})_{q \times n}$ , where  $U_{ij}$  is the level of the  $j$ th factor in the  $i$ th combination. If  $q$  is a prime and  $q > n$ , then  $U_{ij}$  is given by

$$U_{ij} = (i\sigma^{j-1} \bmod q) + 1, \tag{1}$$

where  $\sigma$  is a parameter whose value is chosen by the user.

After the performance evaluation of the points generated by UDM, the information of the points contains the geometric parameters, boundary conditions and aerodynamic performance. In this work, the regression analysis method is adopted as the data analysis method, and the regression model mapping the relationships between factors and responses is shown in Eq. (2)

$$Y = \beta_0 + \sum_{i=1}^m \beta_i X_i + \sum_{i=1}^m \beta_{ii} X_i^2 + \varepsilon, \tag{2}$$

where  $\beta_0, \beta_i, \beta_{ii}$  ( $i = 1, 2, \dots, m$ ) are the regression coefficients, and  $\varepsilon$  is random error. It can be found that there are  $2m + 1$  terms in Eq. (2), and the number of coefficient is large when  $m$  is greater than 10. To simplify the model, the terms, whose contributions to the response of the model are relatively little, are filtered by Significance Index.

## 3 HFCDN-GA

Inspired by stratified competition in human society and in biology, a new genetic algorithm, HFCDN-GA, is developed to overcome the slow and premature convergence disadvantages of the simple genetic algorithm (SGA) in application.

In HFCDN-GA, the hierarchical fair competition with dynamic niche model was adopted to mimic the competitions occurring in populations and between populations in nature and society, and the extinction of inferior populations. In the algorithms, subpopulations are stratified by fitness at the beginning of evolution. That is, the population is divided into subpopulations so that fair

competitions occur in the subpopulations to ensure the development of excellent gene. The subpopulations merge into one population in which the individuals compete with each other. As a result, the superior individuals are preserved and the inferior ones are eliminated. Finally the optimal is obtained after certain evolution generations.

The flowchart of HFCDN-GA could be found in Ref. [10]. At the beginning of evolution, an initial population whose size is  $N$  is generated randomly. Then each individual's fitness is evaluated, and the population is divided into  $N_g$  subpopulations by fitness. The subpopulations merge into one population after  $N_e$  generations. After crossover and mutation operations are implemented, the population is selected and mutated by the dynamic niche. In this way, the current offsprings evolve into the next generation's parents. The global optimization ability of HFCDN-GA was evaluated with "benchmark" test functions.

## 4 Test case and its geometric parameterization

The present optimization approach is applied to redesign the blades of a test centrifugal compressor impeller, which is three-dimensional and has 13 blades and splitters, for the purpose of improving the aerodynamic performance. The entropic efficiency of the original impeller is 81.2%, which is comparatively low. Generally, the impeller blade is a ruled surface generated by blade root and blade tip curves; straight-lines connecting the points of the two curves compose this ruled surface, which makes it possible to manufacture the blades by flank milling. The conformal mapping blade sections on the hub and shroud were parameterized by means of a camber line and a thickness distribution mode. The camber line of the splitter is the same as that of the main blade except for circumferential rotation and fore part truncation. The 6th-order Bezier curve was used to represent the camber line, as shown in Fig. 1. The corresponding formula can be expressed as follows:

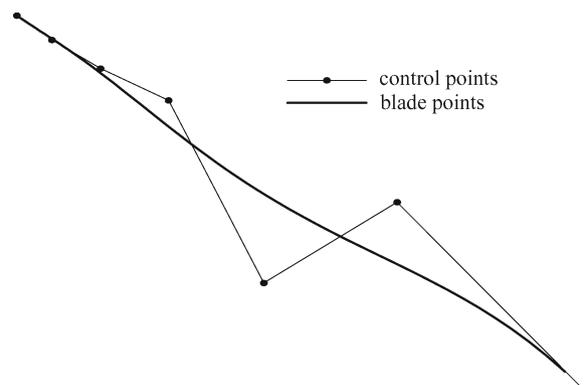


Fig. 1 Parametric representation camber line (hub section)

$$\mathbf{Q} = \sum_{j=0}^6 \mathbf{P}_j B_{6,j}(t), \quad (3)$$

where the vector  $\mathbf{Q} = (x(t), y(t))^T$  is the points on the camber line; the vector  $\mathbf{P}_j = (\xi_j, \eta_j)^T$  is the characteristic polygon's  $n + 1$  vertices, which are varied to generate different Bezier curves; and the superscript T denotes a transposition. The 6th-order Bernstein polynomials are given by

$$B_{6,j} = C_6^j t^j (1-t)^{6-j} \quad (j=0,1, \dots, 6), \quad (4)$$

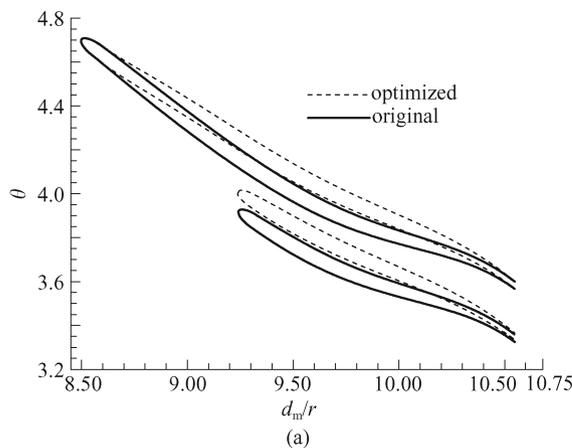
where the  $t$  value of interpolation points lies between 0 and 1.

The free variables contained the ordinates of the characteristic polygon's vertices of Bezier curve, while the radii of leading edge and trailing edge as well as the thickness distribution along the camber line were fixed. Six free variables were used to define each section of the blade. The total twelve design variables were applied as the design variables for the optimization design of the three-dimensional impeller blade shape.

### 5 Optimization design and results analysis

The preset ranges of the ordinates of the characteristic polygon's vertices, and the conditions that the flowrate and pressure ratio are no less than those of the original one are selected as constraint conditions. The treatment method of geometric constraints is the penalty function, which transforms the original constrained maximization problem into an unconstrained one by converting the constraints into penalty terms. The final objective function expression of this optimum design can be given as follows:

$$\max \text{OF} = \eta_{\text{isen}} + P_{\text{geom}} + P_{\text{perf}}, \quad (5)$$



where  $\eta_{\text{isen}}$  is isentropic efficiency,  $P_{\text{geom}}$  and  $P_{\text{perf}}$  are penalty terms of the geometric constraints and performance constraints, respectively.

After optimization, an optimized impeller is obtained. A comparison of the profiles between the original blade and the optimized one is shown in Fig. 2, where  $\theta$  refers to tangential angle, and  $d_m/r$  stands for relative meridional distance. It can be noticed that the camber angle of the optimized blade becomes narrower, and in the shroud side the optimized blade moves slightly toward the suction side. The optimized blade becomes straighter, and the throat area becomes larger and more suitable for the transonic flow. Figure 3 is a comparison of the entropy contours at mid-span between the original impeller and the optimized one. The maximum entropy locates on the rear suction side of the splitter. The value of the entropy in the optimized channel is mostly less than that of the original one, especially in the rear of the channel. Therefore, the flow loss of the optimized blade is less than that of the original one. The isentropic efficiency of the optimized impeller is 83.7%, which is 2.5% greater than that of the original one, while the flowrate and pressure ratio are no less than those of the original one.

### 6 Conclusions

An efficient optimization approach to centrifugal impeller blades is developed and applied to the design of a three-dimensional impeller. The comparatively short design cycle is due to the approximative function constructed by regression method, which allows for the use of HFCDN-GA in an efficient way. The developed approach can take into account different geometric, aerodynamic and mechanical objectives while satisfying the corresponding constraints. After the optimization, a considerable improvement of the objective function is obtained. The aerodynamic performance optimization

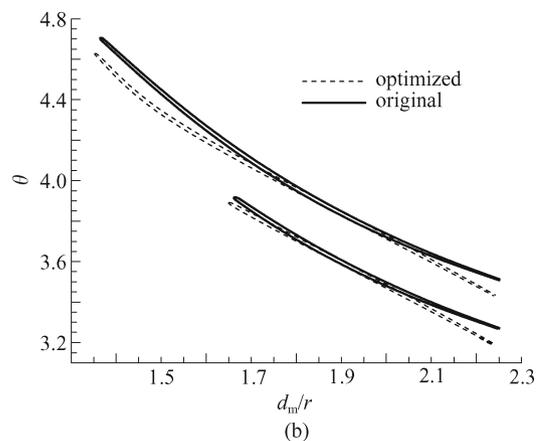


Fig. 2 Comparison of original and optimized blades shapes (a) Hub section; (b) shroud section

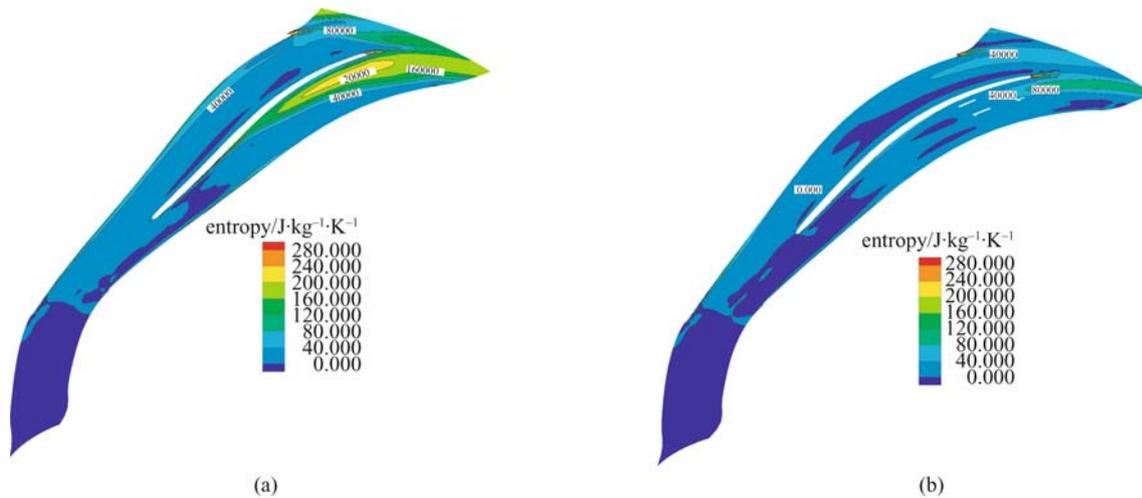


Fig. 3 Comparison of entropy contours at mid-span of channel (blade to blade view)  
(a) Original blade; (b) optimized blade

results and relatively small Navier-Stokes computations also prove that the present approach is effective and promising.

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