Analysis and Optimization of Transonic Centrifugal Compressor Impellers Using the Design of Experiments Technique

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The turbomachine industry is increasingly interested in developing automated design procedures that are able to summarize current design experience, to take into account manufacturing limitations and to define new rules for improving machine performance. In this paper, a strategy for the parametric analysis and optimization of transonic centrifugal impellers was developed, using the technique of the design of experiments coupled with a three dimensional fluid-dynamic solver. The geometrical parameterization was conducted using Bezier curves and a few geometrical parameters, which were chosen after a screening analysis in order to determine the most significant ones. The range of variation of the parameters was defined taking into account the manufacturing requirements. The analysis of the influence of such parameters on the main impeller performance was subdivided into two steps: first, the effect of the parameters acting on the blade shape was investigated and an optimum configuration was chosen, then the influence of three functional parameters was analyzed, fixing the already optimized variables. The whole strategy aimed at an industrial design approach, and attention was focused on the time required in the design process. From the present analysis it was possible not only to define an optimum geometry, but also to understand the influence of the input parameters on the main machine performance. [DOI: 10.1115/1.1579507]

Introduction

Numerical optimization techniques seem to be a promising tool for the aerodynamic design of new generation turbomachine components. The development of the flow inside turbomachines has a complex three-dimensional nature and depends on the interaction between the various parameters that define the machine geometry.

Numerical optimization methods provide an efficient tool for exploiting the great amount of information provided by computational fluid dynamics (CFD) calculations, for analyzing the complex correlations between the geometrical parameters and the machine performance, and for finding their optimal combination. Another interesting feature of such techniques is that they are based on the parameterization of the machine geometry and can be easily linked to CAD systems.

Optimization methods can be grouped into three categories: gradient based optimizers, exploratory techniques and methods based on the concept of function approximation. The choice of the best technique depends on the nature of the problem under investigation and on the kind of analysis to be performed.

The first methods calculate the gradient of an objective function and move the solution toward the closest local optimum. They are very efficient if the parameter space is unimodal, convex and continuous.

Exploratory techniques, as Genetic Algorithm and Simulating Annealing, are based on statistical hypotheses such as the evolutionary theory (Goldberg [1]), and are able to seek the optimum solution in the whole design space. Their use is then necessary when dealing with a multi-peak problem. On the other hand, the number of calculations required to reach an optimal configuration increases.

The last techniques, such as design of experiments and neural networks, are based on the definition of approximated functions which correlate the input parameters with the objective functions, using statistical considerations. The drawback of looking for approximated functions is counterbalanced by the reduced number of calculations required.

All these techniques have been successfully applied for the optimization of axial machine blade profiles (Burgreen and Baysal [2], Trigg [3], Obayashi [4], Pierret and Van den Braembussche [5], Shahpar [6], Manna and Tucillo [7], Glas and Jaberg [8]).

In the last few years, these techniques have been used also for the design of centrifugal machine components.

Wahba and Tourlidakis [9] used a single-objective and a multi-objective Genetic Algorithm coupled with a two dimensional fluid-dynamic solver for the optimization of the blade shape of a centrifugal pump impeller.

Ashimara and Goto [10] used an inverse method together with both gradient-based techniques and exploratory techniques to optimize the performance of a pump impeller, and showed how only the latter are able to find the optimum value of a multi-peak problem.

Van den Braembussche et al. [11] combined a genetic algorithm with an artificial neural network to improve the efficiency of a centrifugal compressor impeller.

Bonaiuti and Pediroda [12] performed a constrained optimization of a radial centrifugal compressor, analyzing the influence on impeller peak efficiency of the parameters that are usually investigated through CFD in an industrial design process.

Benini and Tourlidakis [13] conducted the optimization of a diffuser using a multi-objective Genetic Algorithm and a CFD code. As a result, they found the Pareto optimal set regarding the pressure recovery and the efficiency of the component.

In the present paper the parametric analysis and optimization of transonic centrifugal impellers was conducted using the design of experiments technique (DOE). The DOE strategy is part of a quality production system, the 6 *Sigma*, and the designers of companies that adopt such system are encouraged to use tools belonging

Contributed by the International Gas Turbine Institute of ASME for publication in the JOURNAL OF TURBOMACHINERY. Manuscript received February 13, 2002; final manuscript received May 1, 2003. Paper No. 2002-GT-30619. Review Chair: E. Benvenuti.

to the DFSS (design for 6 *Sigma*). With this in mind, the University of Florence and GE Nuovo Pignone started a long-term project to develop a design procedure able to couple the DOE technique with a CFD solver for the aerodynamic optimization of centrifugal compressors.

The optimization of the impeller is a crucial point in the design of a centrifugal compressor, as the flow development inside that component not only determines the aerodynamic efficiency of the impeller itself, but also strongly affects the efficiency of the downstream diffuser. This aspect is particularly important when dealing with high flow rate and high Mach number impellers where the aerodynamic losses cause the greatest reduction in efficiency. This kind of impeller is usually employed in large centrifugal compressors for natural gas liquefaction or ethylene synthesis. Such applications are characterized by high molecular weight gases working at very low temperatures, which determine transonic conditions at the impeller inlet. The extreme values of flow coefficients (from 0.04 to 0.16) and peripheral Mach numbers (from 1.0 to 1.3) make these impellers critical both in terms of aerodynamic efficiency and in terms of achievable operating envelope.

The goal of the present paper is to develop a design strategy that, beginning with the analysis of the transonic impellers produced, will be able to summarize the industrial design experience, to determine the most critical parameters and to analyze their impact on machine performance. The aim is not only to define an optimal configuration, but also to understand the correlations between the input geometrical parameters and the main impeller performance in order to define new design rules. Parameters with an immediate physical meaning were selected to make the analysis easily understood.

The whole strategy was aimed at an industrial application, and attention was focused on both the accuracy of the result and the time required for the design.

Table 1 List of geometrical parameters

NB	Number of blades
D_{1H}	Inlet hub diameter
D_{1S}	Inlet shroud diameter
B_2	Outlet blade height
Δx	Impeller axial length
α_{LEAN}	Blade inlet leaning angle

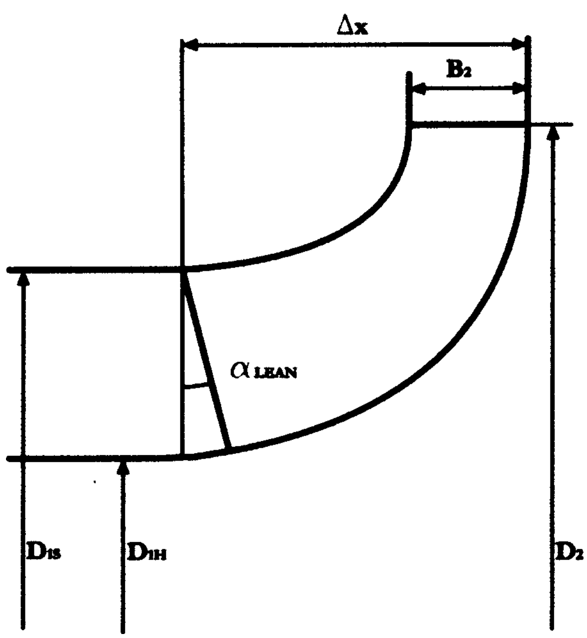

Fig. 1 Geometrical parameterization of the impeller

The Design Strategy

The DOE Approach. A design process can be considered as the optimization of the functions that correlate the machine performance U_j (responses) to the design parameters X_i (factors):

$$U_j = F_j(X_i) \quad j = 1,M \quad i = 1,N. \tag{1}$$

The exact value of these functions is given by an analysis code.

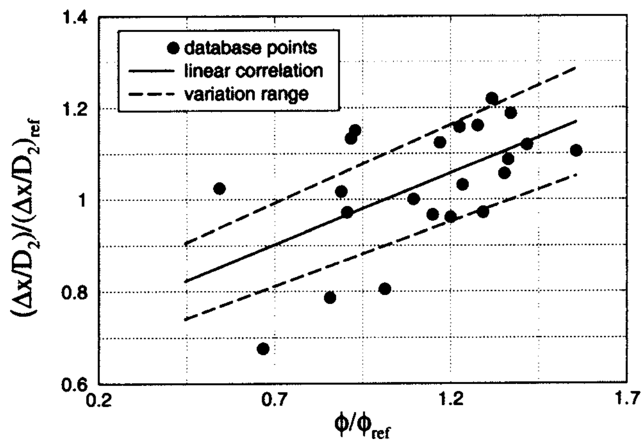

Fig. 2 Correlation between the axial length and the flow coefficient

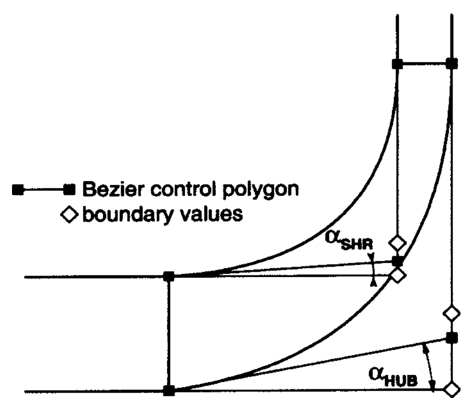

Fig. 3 Meridional channel parameterization

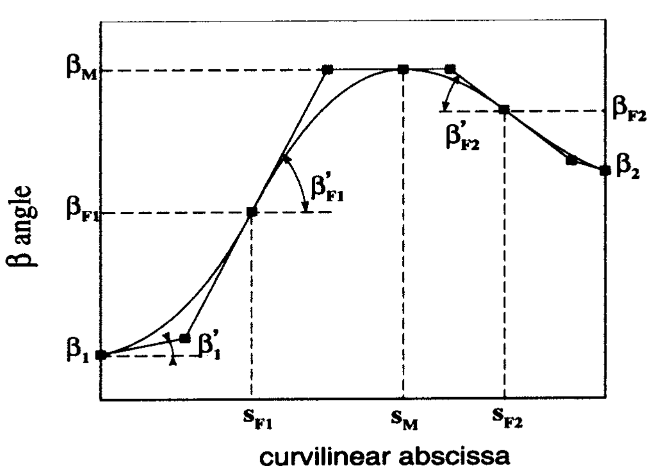

Fig. 4 Blade turning angle parameterization

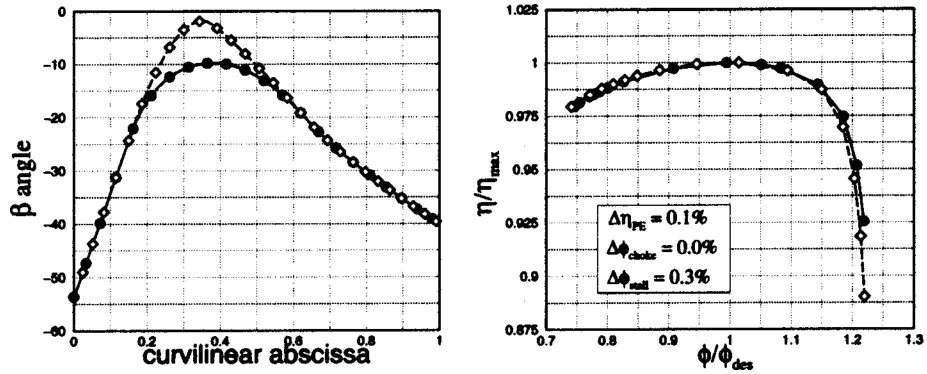

Fig. 5 Effect of the maximum value of the hub β distribution

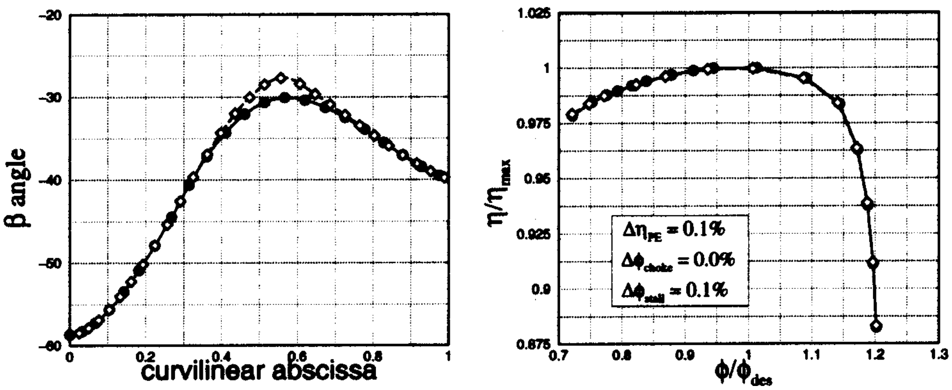

Fig. 6 Effect of the maximum value of the shroud β distribution

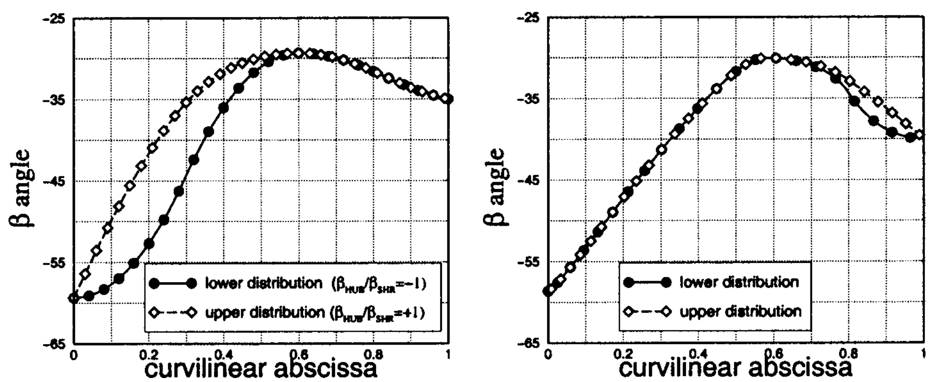

Fig. 7 Limit distributions of the inlet and outlet β shape

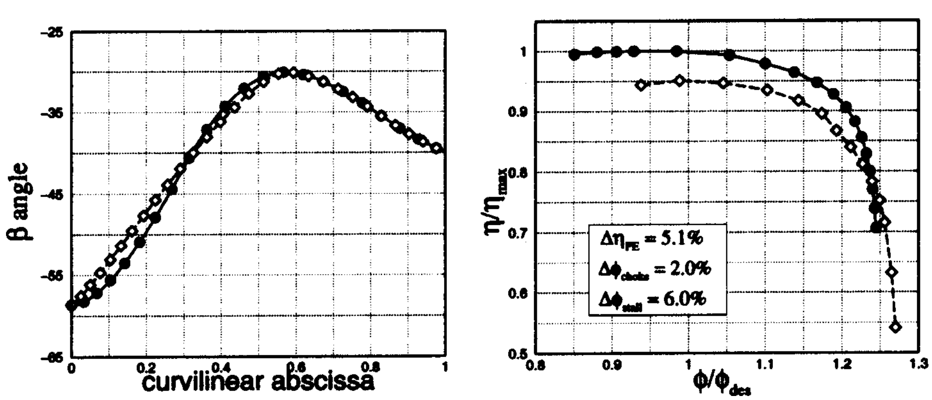

Fig. 8 Effect of the inlet shape of the shroud β distribution

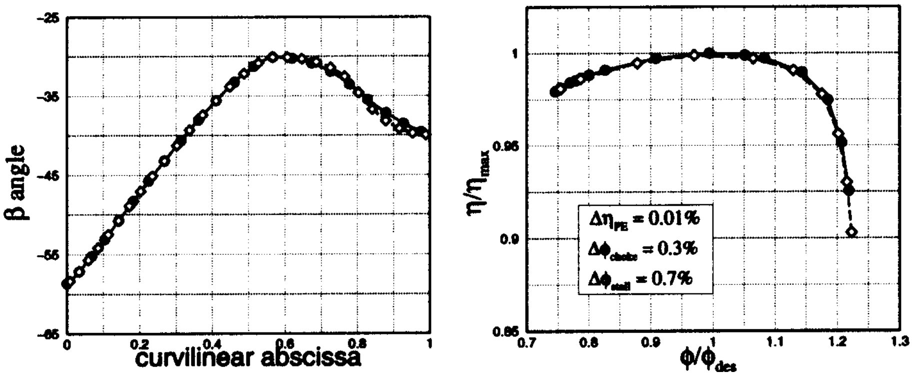

Fig. 9 Effect of the outlet shape of the shroud β distribution

Statistical techniques are widely used in engineering design to build *approximations* of these functions, using data from a set of analyses:

$$\tilde{U}_j=\tilde{F}_j(X_i) \quad j=1,M \quad i=1,N. \tag{2}$$

A common method for building approximations of computer analyses consists of the design of experiments technique (DOE) with the response surface methodology (RSM). RSM approximates the initial function with a polynomial, whose order must be defined a priori. In the case of a second-order polynomial:

$$\tilde{U}_j=\beta_0^j+\sum_{i=0}^{n}\beta_i^j X_i+\sum_{i=0}^{n}\beta_{ii}^j X_i^2+\sum_{i<k}^{n}\beta_{ik}^j X_i X_k. \tag{3}$$

The coefficients β_0, β_i, β_{ii}, and β_{ij} of the polynomials are determined through the least squares regression which minimizes the sum of the squares of the deviations of the predicted values $\tilde{U}_j$ from the actual values U_j, for a set of points.

The DOE theory gives the sampling points (experimental design) at which the tests have to be performed, in order to maximize the amount of information generated with the minimum number of simulations.

The main advantages of such a technique are the immediate intelligibility and robustness of the approach, the better understanding of the relationship between the input variables X_i and the responses U_j, and the possibility of an easy and fast sensitivity analysis and optimization. Optimization techniques like genetic algorithms can be quickly applied to the approximation function.

The main drawback is that the exact function is replaced by an approximated model and only afterwards it is possible to understand if the approximation is acceptable. It is possible to avoid this problem by using a limited number of parameters with a direct impact on the responses. It is well known that this technique is much less reliable when dealing with more than ten variables.

The strategy can be divided into four steps: definition of the problem, parameterization and screening analysis, characterization and experimental setup, analysis of the results and optimization.

Definition of the Problem. The input data of the problem are the design specifications for centrifugal impellers: the outlet diameter, the design mass flow, the rotational Mach number and the minimum value of the work input at design point. Given those specifications, the aim is to highlight the most critical design parameters and analyze their influence on the main machine performance.

In industrial practice, the performance parameters used to

Table 2 DOE factors

Shape parameters	
α_{HUB}	Inlet slope of the hub meridional contour
α_{SHR}	Inlet slope of the shroud meridional contour
β_{HUB}	Inlet shape of the hub β angle distribution
β_{SHR}	Inlet shape of the shroud β angle distribution
Δx	Impeller axial length
α_{LEAN}	Blade inlet leaning angle
Functional parameters	
NB	Number of blades
β_2	Outlet blade angle
DR	Deceleration ratio

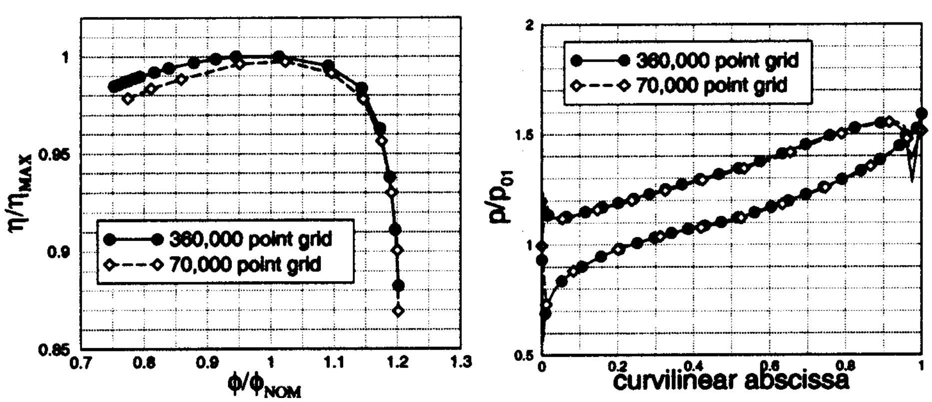

Fig. 10 Effect of the grid dimension on the efficiency curve and on the midspan blade pressure distribution at design point, for a reference impeller

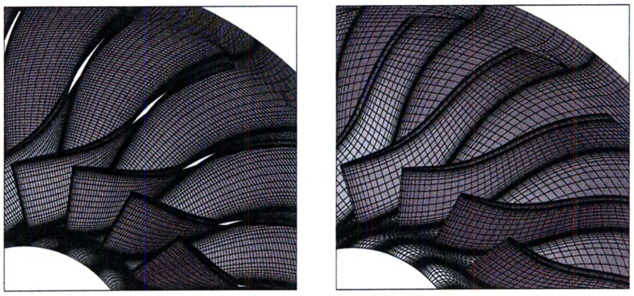

Fig. 11 Computational grids: 360,000 point grid (left) and 70,000 point grid (right)

Table 3 Impeller specifications

	ϕ_{des}	M_U	D_2 (mm)	τ_{des}
Impeller 1	0.1217	1.049	1280	0.60
Impeller 2	0.0488	1.275	1220	0.70
Impeller 3	0.1600	1.050	1100	0.57

Table 4 Validation analysis

	First analysis		Second analysis	
Variable	R^2	R^2_{adj}	R^2	R^2_{adj}
Impeller 1				
SL	98.2%	97.5%	95.7%	93.9%
CL	99.9%	99.2%	99.5%	99.0%
PE	97.2%	96.3%	99.0%	98.8%
TAU	87.2%	83.3%	99.9%	99.7%
Impeller 2				
SL	98.1%	96.9%	98.9%	97.8%
CL	99.9%	99.7%	99.9%	99.9%
PE	96.2%	94.3%	99.9%	99.9%
TAU	85.2%	82.7%	99.9%	99.9%
Impeller 3				
SL	98.4%	97.2%	98.7%	96.6%
CL	99.8%	99.7%	99.8%	99.7%
PE	97.3%	95.2%	99.8%	99.7%
TAU	90.4%	89.3%	99.9%	99.9%

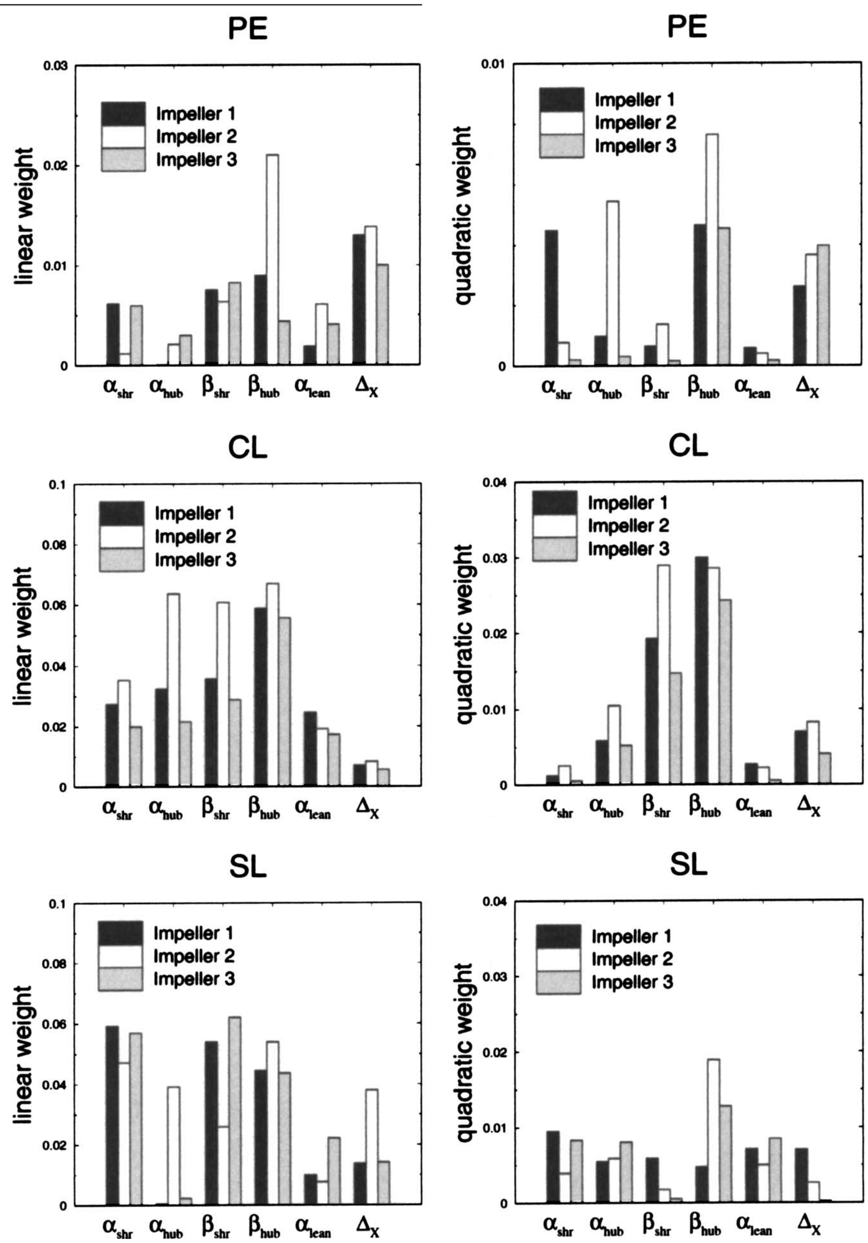

Fig. 12 First analysis: linear and quadratic weights of the single factors in the response functions

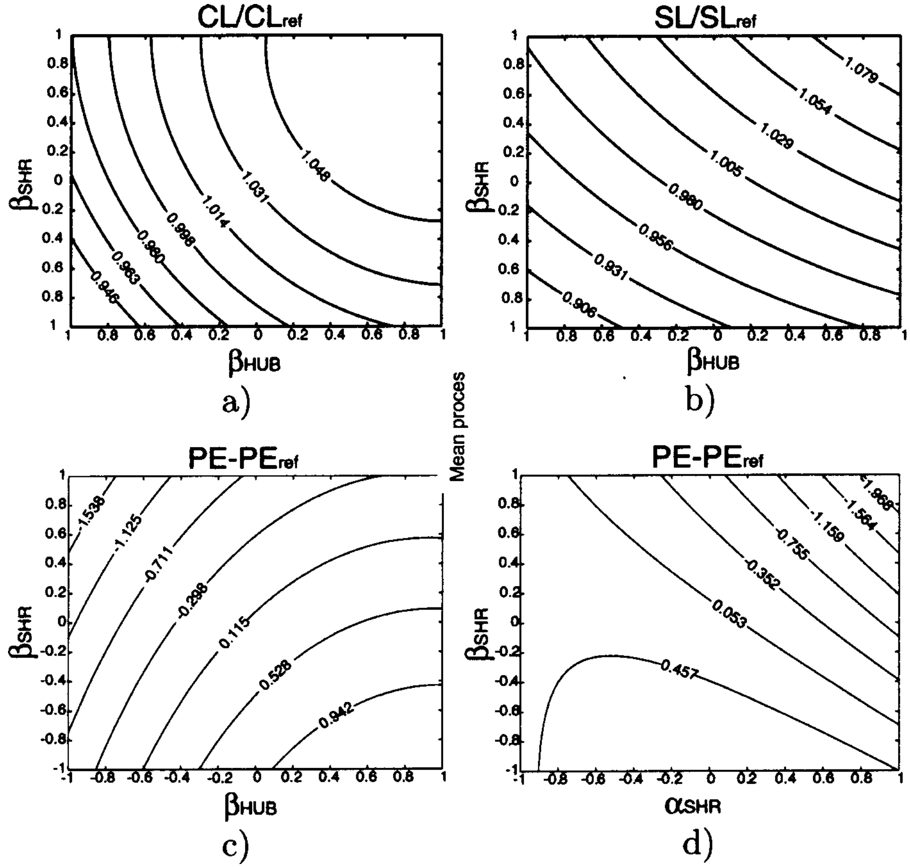

Fig. 13 First analysis: examples of interaction plots

qualify impellers are the peak efficiency, the stall and the choke limits. The following objective functions were considered:

$$PE=\eta_{pe} \tag{4}$$

$$CL=\frac{\phi_{\text{choke}}}{\phi_{\text{des}}} \tag{5}$$

$$SL=\frac{\phi_{\text{stall}}}{\phi_{\text{des}}}. \tag{6}$$

The *peak efficiency* was chosen instead of the efficiency at design point as it is more directly correlated to the design parameters.

As it is impossible to define beforehand if a specific configuration is able to guarantee the minimum work input required, the approximated function of the nondimensional work input coefficient at design point was also investigated:

$$TAU=\tau_{\text{des}}. \tag{7}$$

In the optimization process this function was considered a constraint and not an objective function.

In an industrial design process, the designer has to deal with two kinds of parameters: the *functional parameters* which are usually chosen through correlations or one-dimensional analyses, and the *shape parameters* which define the impeller blade shape and that are usually optimized using CFD calculations.

The present analysis was divided into two steps. First, the standard design tools were used to determine the *functional parameters*, and the effect of the *shape parameters* was investigated. After that, an optimal configuration was fixed and the influence of three *functional parameters* was studied.

The two analyses were conducted separately as their aim is different. The goal of the first one is to define an optimal blade shape, which will be kept constant for all the impeller applications. The aim of the second investigation is to define the effect of the *functional parameters* on the impeller performance, in order to give designers the rules to adjust the impeller geometry to the specific application performance requirements.

Geometric Parameterization and Screening Analysis. The geometrical parameterization was conducted using the GEPAC code, developed at the University of Florence. This code is able to handle axial-radial turbomachine geometries using few design parameters and Bezier curves or patches of Bezier curves.

The impeller geometry is defined when the parameters, shown in Table 1 and explained in Fig. 1, are set and the blade shape is prescribed.

Centrifugal impellers are commonly built using ruled surfaces. The use of ruled surfaces simplifies the design process and reduces manufacturing costs as the blades can be simply generated in only one continuous sweep of a 5-axis flank milling machine. The shape of this kind of blades is completely set when the coordinates of the hub and of the shroud airfoils are fixed. In order to do this, the following three curves are employed:

- The camberline meridional contour in the plane x-r.
- The distribution of the blade turning angle β. This angle is correlated to the azimuthal coordinate of the camberline θ:

$$\tan\beta=r\frac{d\theta}{ds} \tag{8}$$

where s is the curvilinear abscissa and r is the local radius. This angle is given as a function of the meridional curvilinear abscissa.

- The normal thickness distribution, usually given as a function of the meridional curvilinear abscissa.

A crucial point in the DOE strategy is to perform a screening analysis aimed at identifying the most critical parameters (*vital few*) and reducing the number of design factors to investigate in order to simplify the analysis and shorten the design time. This analysis was conducted trying to exploit the industrial design experience, and using standardized tools and correlations. First, a parameterization of the existing transonic impellers was performed to create a database of the already optimized and produced impellers, then each parameter was analyzed separately.

- The number of blades was considered a *functional parameter* and the standard value was imposed in the first analysis and investigated in the second one.
- The inlet hub diameter D_{1H} is standardized through a correlation with the external diameter while the value of D_{1S} was optimized using one-dimensional correlations, in order to minimize the inlet relative Mach number at the shroud.
- The outlet blade height B_2 was correlated to a one-dimensional definition of the deceleration ratio:

$$\mathrm{DR} = \frac{w_2}{w_{1S}} \tag{9}$$

  which was considered a *functional parameter*.
- The axial length Δx was treated as a *shape parameter*. Analyzing the database, a linear correlation was set between ΔX, adimensionalized with D_2, and the flow coefficient, as shown in Fig. 2. As most of the impeller axial lengths fall within $\pm 10\%$ of that value, that range of variation was imposed in the analysis.
- The hub and shroud meridional contours were parameterized using Bezier curves. From the analysis of the database, most of the impellers were well described by a simple second-order Bezier polynomial with a radial slope at the blade exit. Therefore, the curve slope at the inlet section was considered the only parameter acting on these distributions. With similar considerations as for Δx, the range of variation of the slopes was set for the hub and for the shroud curves, as shown in Fig. 3.
- Analyzing the database, the shape of the β angle distributions was always characterized by a curve with a maximum and two flex points. This kind of distribution can be described by four patches of second-order Bezier curves constrained to have the same slope at the conjunction points. This type of parameterization allows for the use of parameters with an immediate physical meaning, as shown in Fig. 4. The inlet blade angle β_1 was imposed optimizing the incidence angle for the hub and shroud sections by means of a through-flow analysis, while the outlet blade angle β_2, which is the same for the hub and shroud distributions, was considered a *functional parameter*. The position of the flex points S_{F1} and S_{F2} and of the maximum S_M were optimized after analyzing their effect on several characteristic impellers. The maximum value of the blade angle β_M was also investigated. Once an optimum value is defined, variations of less than 10% from the standard value may change the shape of the performance curves but do not have a relevant impact on the performance parameters. Figure 5 and Fig. 6 show the effect of changing the maximum of the hub and of the shroud distributions for a reference impeller. The other parameters were grouped into two functions in order to vary the inlet and outlet curve shapes, using only two variables. The limit distributions obtained varying such variables are shown in Fig. 7.

The curve shape in the inducer region has a much greater influence on the performance than the one in the rear part of the impeller, and only that parameter was introduced in the variable set. As an example, Figs. 8 and 9 show the effect of similar variations in the two shapes of the shroud distribution on the performance of a reference impeller.

- The blade normal thickness is imposed by structural limita-

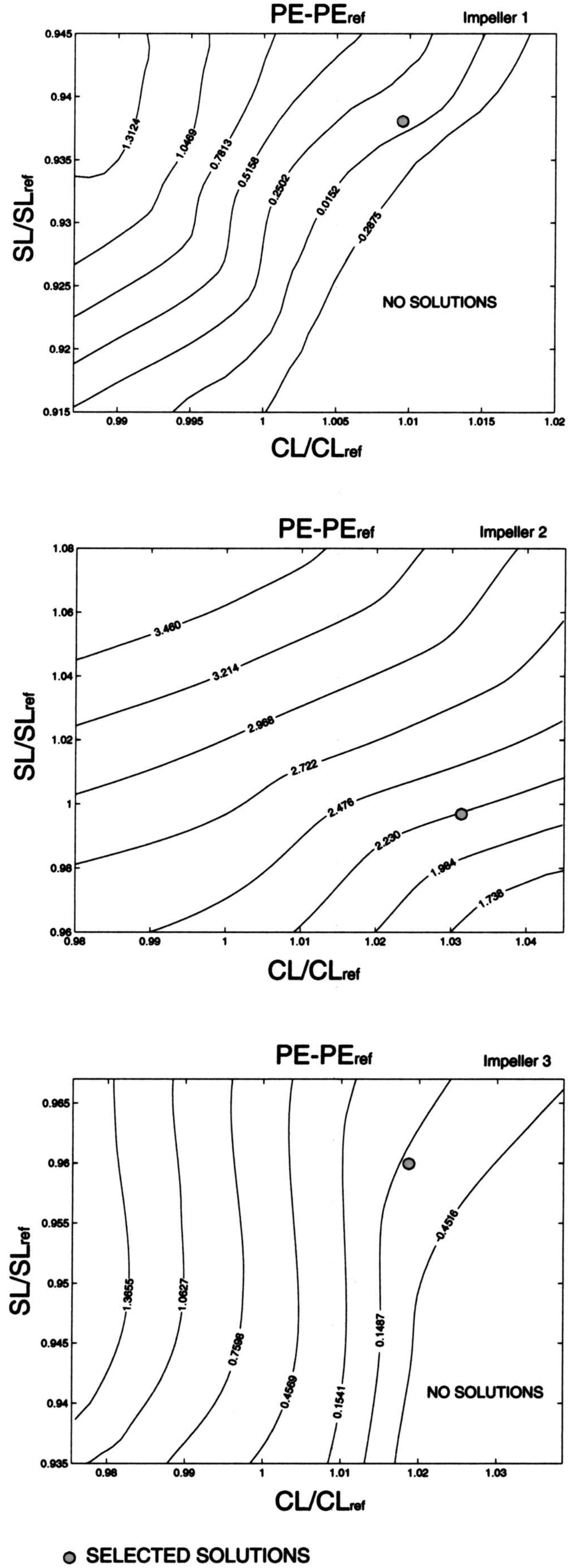

Fig. 14 First analysis: Pareto front

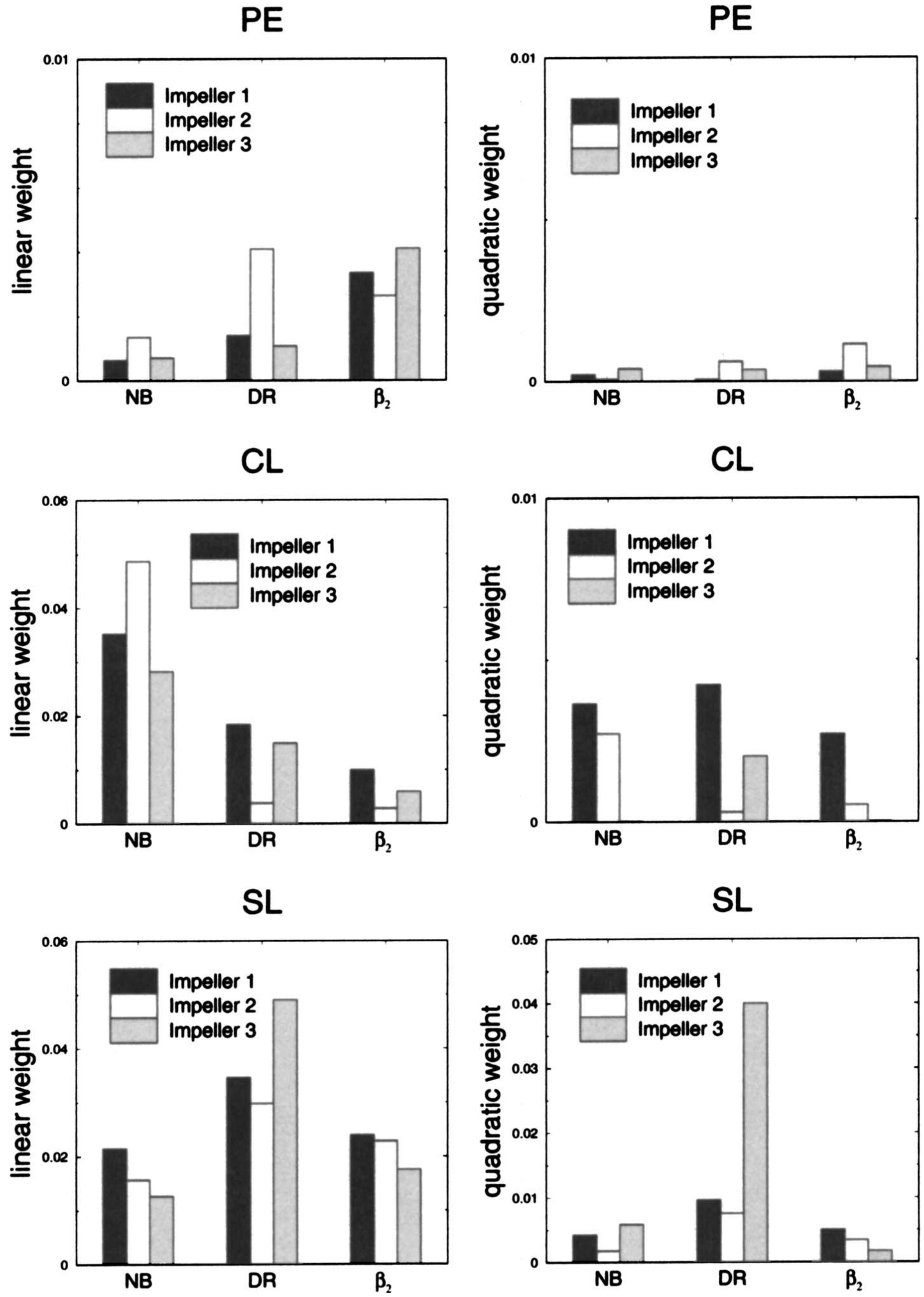

Fig. 15 Second analysis: linear and quadratic weights of the single factors in the response functions

tions. The effect of varying the thickness distribution near the leading edge (L.E.) was not considered and a circular rounded L.E. was imposed.

- The leaning angle α_{LEAN} was considered a *shape parameter*.

The parameters, selected after the screening analysis, are summarized in Table 2. The ranges of variation of all the variables were fixed, taking into account the manufacturing limitations.

Characterization and Experimental Setup. A second-order response surface was selected and coupled with the *face centered central composite* DOE model (FCCD) to set the test plane. This model varies the factors on three levels to estimate the quadratic effects, and leads to an experimental table with 45 tests for the first analysis and 15 tests for the second one.

All the tests were performed coupling the impellers with a straight inlet duct and with a short diffuser with 10% area reduction in order to avoid diffuser stall.

Computation results were obtained using the TRAF code (Arnone et al. [14]), a fully viscous, multigrid code, developed at the University of Florence.

In order to estimate the value of the four performance parameters, it was necessary to calculate the whole operating range of the impeller. To do this, the points on the functional curve were sequentially obtained, from the choke to the stall, imposing a low back pressure and gradually increasing its value. Fifteen to 20 points were calculated to describe the whole curve.

The peak efficiency and the nondimensional work input coeffi-

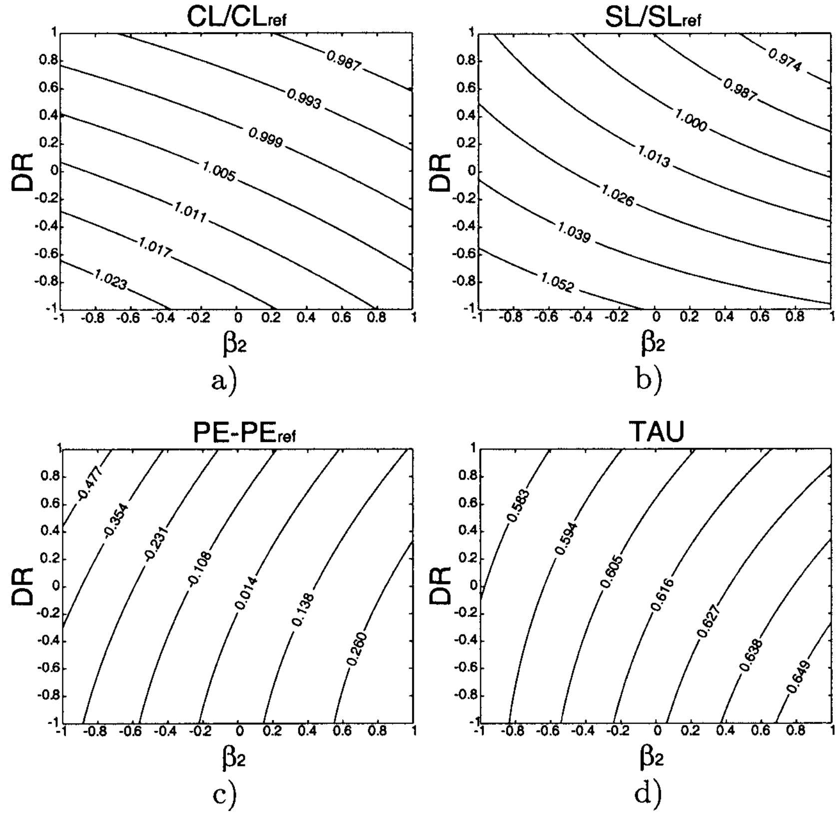

Fig. 16 Second analysis: example of interaction plots between β_2 and *DR* for a fixed number of blades

cient at design point were calculated using a cubic interpolation on the dataset. Such values were evaluated considering the impeller inlet section and a section at a diameter $D=1.08D_2$, where D_2 is the impeller exit diameter. Therefore, this evaluation allowed one to take into account the mixing losses, which occur in the first part of a vaneless diffuser in the estimate of the efficiency. Configurations with a smooth total pressure profile at the impeller exit were therefore favored.

As it is impossible to find the stall limit with a steady calculation, a conventional definition was used. The stall was defined as the last point for which the code reached convergence, when increasing the back pressure by a fixed step. Using such an approach, the stall identification may depend on the CFD code used and on its turbulence model, but the aim of this investigation was to compare different design configurations and not to give an exact estimate of the stall point. The assumption was made that the bias error in defining the stall point was the same for all the tested configurations. Furthermore, after analyzing the existing database, the numerical stall was correlated to the one found in the experimental tests.

All the calculations were performed on an nonperiodic H-type grid, with the same grid parameters. A sensitivity analysis of the grid dimension on the results was conducted. A good grid independence was obtained with a 360,000 point grid.

In order to reduce the design time, the effect of using a coarser grid was investigated. Calculations on coarser grids lead to a change in the performance curve shape, and in an underestimation of the peak efficiency and of the stall limit, but the main flow physics seems to be well described, as shown in Fig. 10. The effect of the grid reduction is the same for all the geometries tested, and it is still possible to make a comparison between different configurations. In terms of RSM, there is a shift of the response surface, but the surface shape is the same.

As the aim of the first analysis is to find the best blade shape and not to give quantitative information about the effect of the changes, a 70,000 point grid was chosen. A sensitivity analysis of the result is still possible, which does not consider the absolute value of the responses, but the difference from the performance of a reference geometry.

A 360,000 point grid was used for the second analysis. A three-dimensional view of the two computational grids is shown in Fig. 11.

Analysis of the Results. The data fitting was performed using a least squares regression. For numerical stability, the range of each variable was scaled to span $[-1,1]$.

The validation of the model was checked through the analysis of R^2 (the ratio of the model sum of squares to the total sum of squares), of the R^2_{adj} (R^2 adjusted to the number of parameters in the model) and other statistical tests.

A sensitivity analysis was conducted on the model to investigate the influence of the single parameters on the responses and the effect of their interaction.

A genetic algorithm optimization was performed using the approximation functions in order to define the Pareto front of the solutions.

Optimization of Three Impellers

The strategy was applied for the redesign of three impellers, whose specifications are shown in Table 3.

The two analyses were performed sequentially, as mentioned above, and their model validation is reported in Table 4.

The high value of R^2 indicates that the choice of the a quadratic response model was correct and that the terms of an order greater than the quadratic one can be neglected in the correlation functions. The only exception seems to be the τ_{des} function in the first

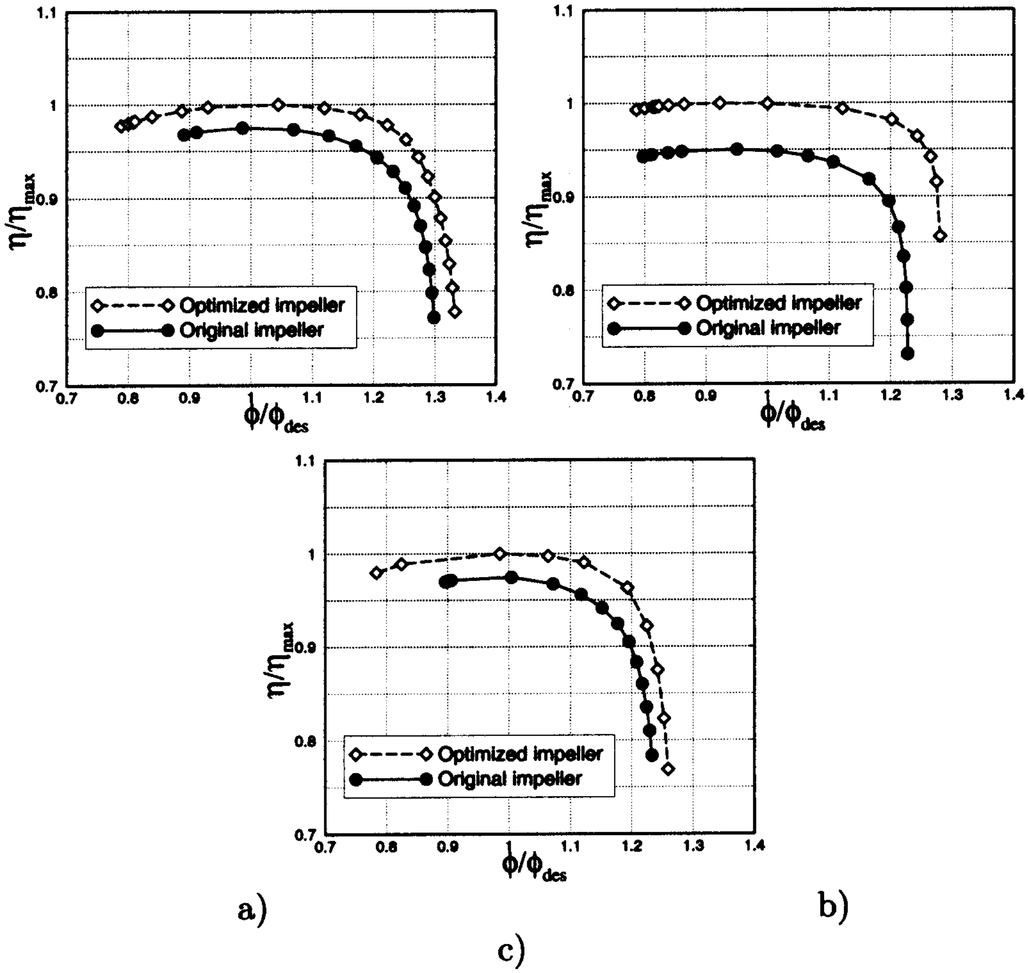

Fig. 17 Comparison of the efficiency curve between the original and optimized configuration, for the first (*a*), the second (*b*), and the third (*c*) impeller

analysis. For this reason, an increased value of the minimum τ_{des} was imposed as constraint in the optimization process.

The high value of R^2_{adj} indicates that parameters with relevant impact on the impeller performance were selected.

From the analysis of the model, it is possible to extract information about the influence of each parameter on the performance and the effect of their interactions. This analysis is very important for determining physical correlations and defining new design rules.

The coefficients β_i and β_{ii} of Eq. (3) are representative of the linear and quadratic weights of the single factors in the response function, while the coefficients β_{ij} are representative of their interactions. The values of the first two coefficients from the first analysis of the three impellers are plotted in Fig. 12.

In order to understand the effect of the interactions between two parameters, it is a common practice to set the value of all the other factors at a predetermined value and to plot the response surface only as a function of the two parameters under investigation. Figure 13 reports significant examples from the analysis of the first impeller. All the variables are scaled to span $[-1,1]$, while the performance parameters are adimensionalized with respect to the reference values. Figure 13(*a*) shows that hub and shroud β distributions close to the $+1$ one (Fig. 7) broaden the throat area and then extend the choke limit. Blade angle distributions close to the -1 one minimize the inducer curvature which determines the detrimental flow acceleration at low mass flow rates, and then improve the stall limit, as shown in Fig. 13(*b*). The combination of β_{SHR} close to the -1 distribution, and β_{HUB} close to the $+1$ one, leads to a configuration with high peak efficiency, as shown in Fig. 13(*c*). This is also the best compromise in order to have a large operating range: the hub distribution guarantees an acceptable throat area while the shroud one reduces the local flow acceleration which is more harmful going toward the shroud, because of the higher tangential velocity component. Figure 13(*d*) indicates that an axial inducer ($\alpha_{SHR} \simeq -1$) has to be coupled with such a β distribution at the shroud in order to keep a high value of the efficiency.

Similar results were obtained in the analysis of the other two impellers.

A genetic algorithm solver was applied to the approximation functions to find the Pareto front of the solutions. Then, a configuration belonging to the Pareto front was chosen for each impeller, as shown in Fig. 14.

The optimized *shape parameters* were fixed and kept constant in the following analysis.

The linear and quadratic weights of the three *functional parameters* investigated in the second analysis are reported in Fig. 15.

For all the impellers, the interaction plots between the diffusion ratio DR and the discharge angle β_2 were drawn for each number

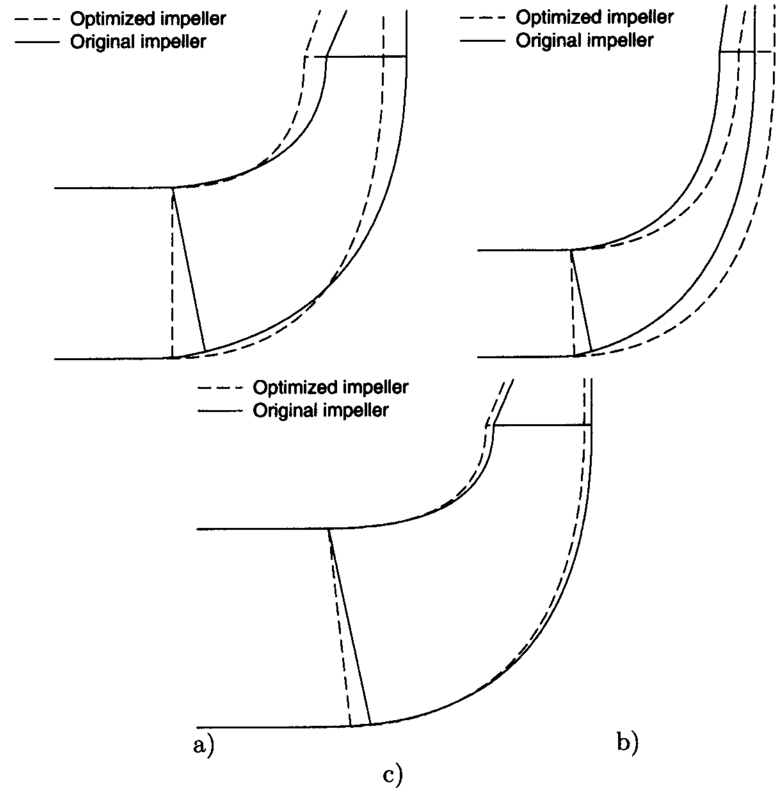

Fig. 18 Comparison of the meridional contour curve between the original and optimized configuration, for the first (*a*), the second (*b*), and the third (*c*) impeller

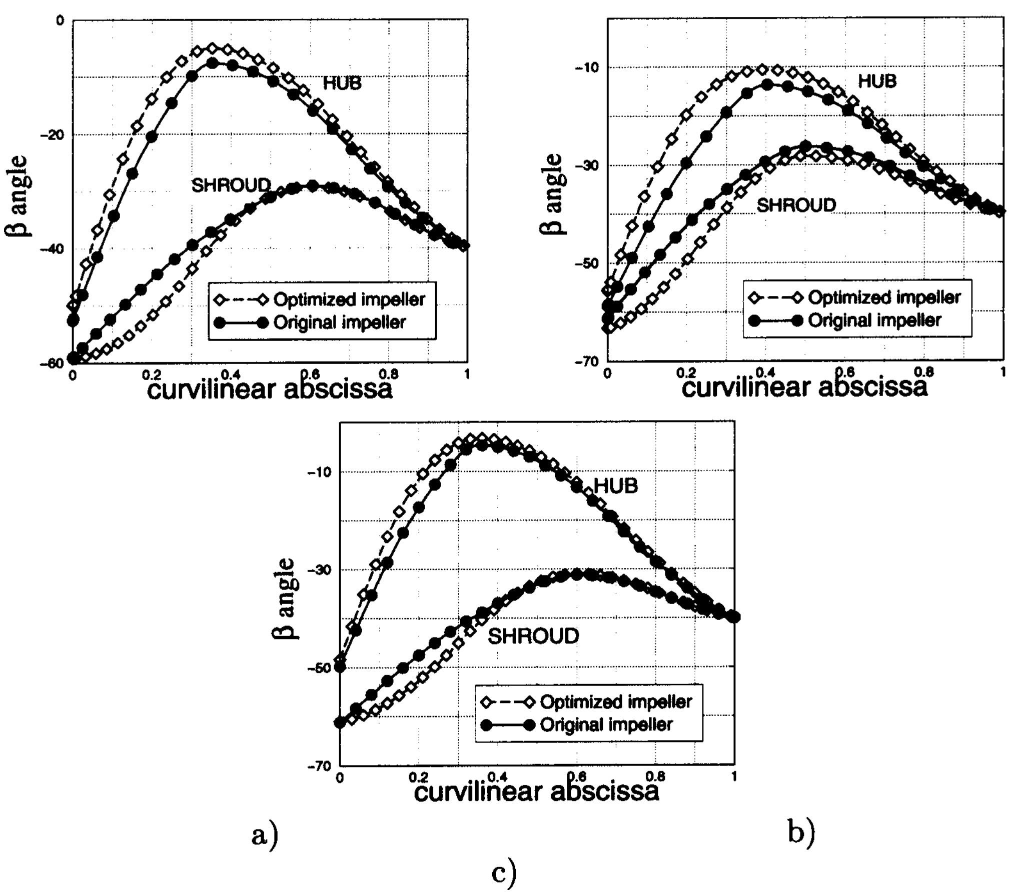

Fig. 19 Comparison of the β distribution curve between the original and optimized configuration, for the first (*a*), the second (*b*), and the third (*c*) impeller

of blades. An example is shown in Fig. 16. These graphs constitute a tool for the designer for choosing the best parameter combination when approaching to a new impeller design. In the present case, the new configurations were chosen in order to reach a balanced improvement of the three performance parameters with respect to the original impellers.

The efficiency curves relative to the original and to the optimized configurations are shown in Fig. 17, while Figs. 18 and 19 report the differences in the meridional channel and in the blade angle distributions.

Conclusions

In this paper, the design of experiments (DOE) technique was coupled with a fluid-dynamic solver for the analysis and optimization of centrifugal compressor impellers.

Starting from the analysis of the existing industrial impellers, and after conducting a screening analysis, the most influential geometrical parameters were isolated and their effect on machine performance was investigated.

The reasonable validation of the obtained model allowed us to use this tool for the design of new transonic impellers.

The strategy was applied for the redesign of three impellers. As a result, it was not only possible to define an optimized configuration but also to determine the correlations between the input geometrical parameters and the impeller performance in order to introduce new design rules.

Computer cost is consistent with industrial standards, as the optimization of each impeller required 900 CFD runs, corresponding to 600 h using a Personal Computer Pentium III 750 MHz.

Acknowledgment

The authors would like to thank Prof. Ennio Carnevale of the University of Florence for encouraging this work and Dr. Alberto Milani for his cooperation.

Nomenclature

D_2 = impeller exit diameter
h_0 = total enthalpy
$M_u \equiv \frac{U_2}{\sqrt{\gamma R T_{01}}}$ = impeller rotational Mach number
p_0 = total pressure
Q_1 = inlet volume flow
R = perfect gas constant
s = curvilinear abscissa
T_0 = total temperature
U_2 = impeller exit peripheral velocity
x,r,θ = cylindrical coordinates
β = blade turning angle
γ = gas specific heat ratio
$\eta \equiv \frac{\gamma-1}{\gamma}\left[\frac{\ln\left(\frac{p_{02}}{p_{01}}\right)}{\ln\left(\frac{T_{02}}{T_{01}}\right)}\right]$ = impeller polytropic efficiency
$\phi \equiv \frac{4Q_1}{\pi D_2^2 U_2}$ = mass flow coefficient
$\tau \equiv \frac{h_{02}-h_{01}}{U_2^2}$ = work input coefficient

Subscripts

1 = impeller inlet
2 = impeller exit
des = design point
H = hub
S = shroud
pe = peak efficiency

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