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The Use of ExPAnD in Cyclic Low Temperature Processes

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By
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MASTER THESIS

for

Stud.techn. Seid Ehsan Marashi
Spring 2011

The Use of ExPAnD in Cyclic Low Temperature Processes

Bruk av ExPAnD i sykliske lavtemperatur-prosesser

Background

A design methodology for Low Temperature Processes (LTPs) has been under development in our research group for the last few years. This methodology is called ExPAnD (Extended Pinch Analysis and Design), and as the name indicates, it is an extension of basic Pinch Analysis with special focus on sub-ambient processes. In such processes, pressure plays an essential role since it affects phase and temperature and is closely related to the operation of compressors and expanders. Below ambient, external cooling is obtained by a sequence of compression (pump or compressor), condensation, expansion (valve or expander) and evaporation. In contrast, basic Pinch Analysis, which has been successfully applied in the design and retrofit of heat exchanger networks, only considers temperature as the main operating variable in the design. The ExPAnD methodology includes pressure and temperature levels as the main variables for example by integrating Exergy Analysis into the Pinch Analysis.

ExPAnD started its development in parallel with the invention of a new process for offshore natural gas liquefaction being part of what has been referred to as the Liquefied Energy Chain (LEC). Rather than using conventional refrigerants, where some working fluid is involved in a cyclic process between expansion and compression, the offshore part of the LEC avoids such refrigerants by using liquid CO_2 and liquid N_2 being shipped from offshore and being released to the atmosphere (N_2) or injected into the reservoir (CO_2) for EOR. Even though CO_2 and N_2 are involved in compression and expansion, they cannot be regarded as cyclic utilities, since they enter and exit the process at fixed flowrates.

Another recent development in our group is better defined measures for exergy efficiency based on new understanding about exergy transfer, and the decomposition of thermo-mechanical exergy into a temperature based component and a pressure based component. This enables the establishment of a new Exergy Transfer Effectiveness (ETE) focusing on exergy sources and exergy sinks. The decomposition of exergy is particularly important below ambient, due to the discontinuity in the definition of temperature based exergy at ambient temperature T_0 typically chosen to be 25°C or 298 K.

Motivation

The main motivation for this Master thesis is to further develop the ExPAnD methodology by considering cyclic processes and utilities, where the quality of the design is measured by the Exergy Transfer Effectiveness (ETE). In cyclic refrigeration processes, the mass flowrate of the refrigerant becomes an important design variable.

Objective

The main objective of this Master thesis is to evaluate recent enhancements of the ExPAnD design methodology such as insight about different exergy components and an improved exergy efficiency measure referred to as the Exergy Transfer Effectiveness (ETE), by considering expansion based processed for the liquefaction of natural gas to produce LNG.

The following issues should be considered in the Master thesis:

1. A literature study should be conducted, focusing on different exergy based measures for process quality (related to energy consumption) that have been proposed in the literature. As part of this, the different components of exergy should be studied, discussed and illustrated.

2. The Dual-Expander process for liquefaction of natural gas should be briefly described and presented and then analyzed on the basis of a simulation model that was developed in the last year Project during the fall 2010. In this analysis, the previously proposed Heuristic Rules that are part of ExPAnD should be attempted applied, keeping in mind the special features of cyclic utilities and fixed inlet and outlet conditions for natural gas.

3. The exergy sinks and sources for the equipment in the Dual-Expander process should be identified in order to apply the new Exergy Transfer Effectiveness for individual equipment and the entire process.

4. By combining points 2 and 3 above, a sensitivity analysis should be performed for the available degrees of freedom (flowrates of refrigerants, pressure levels, and splits ratios) in order to identify the scope for improving the base case Dual-Expander process. Special attention should be given to changes in the exergy sinks and sources of the process.

5. Propose constructive modifications or additions the ExPAnD heuristics. Is it possible to aggregate the insight of these heuristics at a higher level based on the experience of this Master thesis project?

6. If time allows and it is found to be of advantage for the results of this project, a few additional expander type LNG processes could be studied, such as the basic and a modified Niche liquefaction process.

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Within 14 days of receiving the written text on the Master thesis, the candidate shall submit a research plan for his project to the department.

When the thesis is evaluated, emphasis is put on processing of the results, and that they are presented in tabular and/or graphic form in a clear manner, and that they are analyzed carefully.

The thesis should be formulated as a research report with summary both in English and Norwegian, conclusion, literature references, table of contents etc. During the preparation of the text, the candidate should make an effort to produce a well-structured and easily readable report. In order to ease the evaluation of the thesis, it is important that the cross-references are correct. In the making of the report, strong emphasis should be placed on both a thorough discussion of the results and an orderly presentation.

The candidate is requested to initiate and keep close contact with his/her academic supervisor(s) throughout the working period. The candidate must follow the rules and regulations of NTNU as well as passive directions given by the Department of Energy and Process Engineering.

Pursuant to "Regulations concerning the supplementary provisions to the technology study program/Master of Science" at NTNU §20, the Department reserves the permission to utilize all the results and data for teaching and research purposes as well as in future publications.

<u>One – 1</u> complete original of the thesis shall be submitted to the authority that handed out the set subject. (A short summary including the author's name and the title of the thesis should also be submitted, for use as reference in journals (max. 1 page with double spacing)).
Two – 2 – copies of the thesis shall be submitted to the Department. Upon request, additional copies shall be submitted directly to research advisors/companies. A CD-ROM (Word format or corresponding) containing the thesis, and including the short summary, must also be submitted to the Department of Energy and Process Engineering

Department of Energy and Process Engineering, 14 February 2011

Olav Bolland
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Preface

This master thesis has done in the department of Energy and Process engineering at NTNU in spring 2011.

I would like to express my sincere gratitude to my supervisor, Professor Truls Gundersen whose support and guidance helped me to finish the thesis.

Also, I would like to thank my co-supervisor Danahe Marmolejo Correa whom I will always appreciate for her creativity, who gave me new ideas and motivated me for further work.

Finally, I am immensely grateful to my wife, my parents and friends who never left me alone in all kind of difficulties. I dedicate my thesis to them for their endless love.

Signature ....Seid...Ehsan...Marashi

Trondheim 25.07.2011

Abstract

The pinch analysis method has been applied to different processes successfully for heat integration. Since it does not include pressure as a factor in low temperature processes and it has important role in those processes, Extended Pinch Analysis and Design (ExPAnD) method has recently developed for heat integration in low temperature processes.

The main motivation for this Master thesis is to further develop the ExPAnD methodology by considering cyclic processes and utilities, where the quality of the design is measured by the Exergy Transfer Effectiveness (ETE).

In this project, the exergy concept is studied. A review among literatures has been done and different definitions for exergy efficiency are identified. The pinch analysis method and ExPAnD method are also reviewed briefly. Exergy analysis based on Exergy Transfer Effectiveness (ETE) has been applied to operation units which are used in LNG production processes. Moreover, dual independent expander refrigeration cycles process for LNG production has been analyzed from the exergy point of view. The related heuristic rules of ExPAnD method are studied. This method has been applied to the process of dual independent expander refrigeration cycles successfully. Finally, a new approach is proposed which improves the exergy efficiency of the cyclic low temperature processes.

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1. Introduction

Most scientists and engineers consider a process that work below -235°F (-148°C) as a low temperature process. This is due to the fact that the normal boiling point of most permanent gases (e.g. helium, hydrogen, nitrogen, air etc.) is below this temperature (ASHRAE handbook of refrigeration, 2006). According to the given definition, low temperature processes include LNG production, air separation, hydrogen liquefaction, nitrogen liquefaction processes and etc. However, the temperature range of a low temperature process has never been defined exactly.

Basic Pinch Analysis has been successfully applied in the design and retrofit of heat exchanger networks where the temperature is the only operating variable used in the design. However, in low temperature processes temperature levels are obtained through one or more compressions and expansions. Such a combination implies pressure manipulations. These adjustments make the pressure of a process, the most important variable in its design.

A design methodology for low temperature processes has been under development in the department of energy and process engineering at NTNU for last few years. This methodology is called ExPAnD (Extended Pinch Analysis and Design), that is an extension of the basic Pinch Analysis applied to processes where pressure plays an inevitable role.

The development of ExPAnD has been started in parallel with the invention of a new process for offshore natural gas liquefaction. Throughout this invention several heuristics were identified. This design method and the corresponding heuristics are innovative and include new approaches that have not been used in the design of other processes so far. Therefore, in order to turn this methodology into a more general and robust one, it is crucial applying it to a variety of low temperature processes such as cyclic one.

In addition, the quality evaluation of design by thermodynamic measures is a demanding requirement. Exergy efficiency is a suitable tool. However, researchers have not agreed on a specific definition and realization of this measure. Thus, this can cause confusion while comparing exergy efficiency of two processes. Additionally, most of these definitions are not applicable to a low temperature process.

To such an extent, our group has started to develop a new definition for exergy efficiency. The definition is based on a new understanding about exergy transfer, and the decomposition of thermo-mechanical exergy into a temperature based component and a pressure based component. This enables the establishment of a new Exergy Transfer Effectiveness (ETE)**,** focusing on exergy sources and exergy sinks. The decomposition of exergy is particularly important below ambient temperature due to the discontinuity in the definition of temperature based exergy at ambient temperature.

In chapter 2, I have studied the exergy concept and its classification. Temperature based exergy, pressure based exergy, and mechanical exergy are reviewed. Exergy balance equation for a closed system and control volume are derived, destruction exergy due to heat transfer and pressure drop is explained. A review among literatures has been done and different definitions for exergy efficiency are identified. The pinch analysis method and ExPAnD method are also reviewed briefly.

In the third chapter, I have applied exergy analysis based on Exergy Transfer Effectiveness (ETE) to operation units which are used in LNG production processes. Effect of pressure and temperature on exergy efficiency of these units are identified by analytical and sensitivity analysis techniques. Moreover, dual independent expander refrigeration cycles process for LNG production has been analyzed from the exergy point of view.

In the fourth chapter, the ExPAnD heuristic rules for streams with a target pressure that differs from the supply pressure are examined. The application procedure has been successful and improved the exergy efficiency of a given process. Additionally, I have proposed a new approach which significantly improves the exergy efficiency of the cyclic low temperature processes.

2. Theory

In this chapter, exergy concept and exergy efficiency will be introduced. Moreover, pinch analysis and Expand method will be introduced briefly.

2.1. Exergy

Energy is conserved in a process or equipment. It can transform from one state to another, but it will not be destroyed. Although quantity of energy is conserved, quality of energy may be changed. For instance, consider a system which consists of a room and a container of fuel. If we burn the fuel to warm up the room, our system has the same amount of energy as it has initially. So, the quantity of energy will remain constant but the quality of energy will change since the energy of fuel is more valuable economically than warm air.

‘’ Energy conservation idea alone is inadequate for depicting some important aspects of resource utilization (Moran & Shapiro, 2006)’’. As a result, we should define a parameter which explains the quality of energy. We can define exergy as potential for use.

For explaining exergy in a precise way, we can consider two different systems at different states. One of the systems is environment and another one is our system of interest. Exergy is maximum theoretical obtainable work as these systems interact to be in equilibrium with each other.

Now it is necessary to define the environment. ‘’Environment is a very large body or medium in the state of perfect thermodynamic equilibrium. Thus, this conceptual environment has no gradients or differences involving pressure, temperature, chemical potential, kinetic or potential energy. Therefore, there is no possibility of producing work from any form of interaction between parts of environment (Kotas, 1995)’’. So, any system outside of environment which has any different parameters compared to environment, has a potential for producing work when it interact by environment.

In conclusion, ‘’exergy is a measure of the departure of the state of a system from that of the environment (Moran & Shapiro, 2006)’’. The value of exergy is always positive or zero when the system condition changes to dead state. Exergy is not conserved and destroyed by irreversibility.

2.1.1. Classification of exergy

Figure (2-1) shows the classification of exergy.

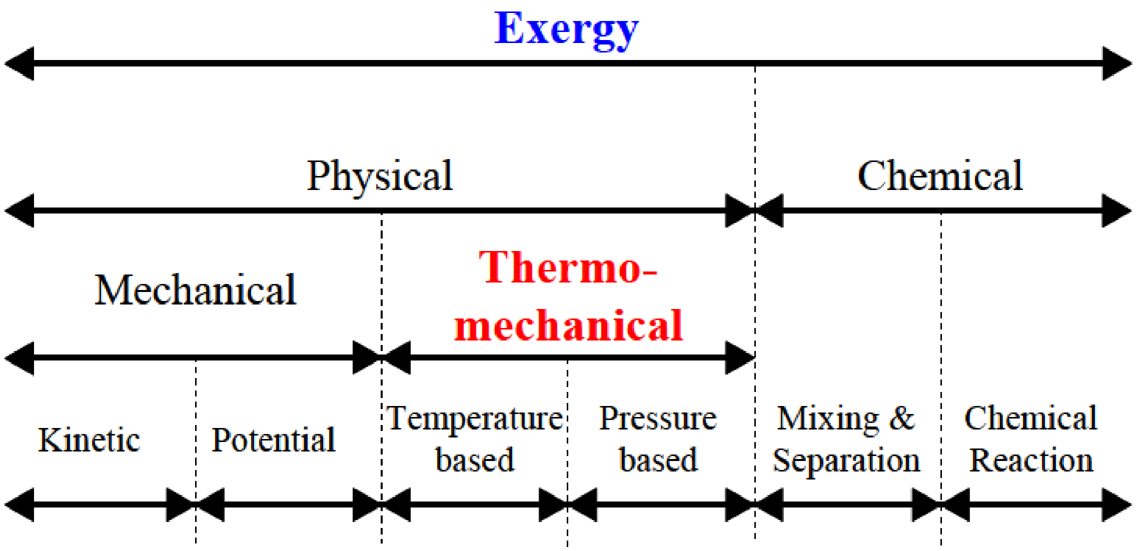


Figure 2.1 Classification of exergy (Gundersen, 2011)

As we can see in figure (2-1), total exergy of a stream of matter can be obtained with equation (2-1).

$$E = E_k + E_p + E_{TM} + E_c \qquad (2\text{-}1)$$

Where E_k is kinetic exergy, E_p is potential exergy, E_{TM} is thermo-mechanical exergy, and E_c is chemical exergy. By introducing specific exergy as $\varepsilon = E/\dot{m}$, we can write this equation in specific form.

$$\varepsilon = \varepsilon_k + \varepsilon_p + \varepsilon_{TM} + \varepsilon_c \qquad (2\text{-}2)$$

Chemical exergy is important for processes with separation, mixing, or chemical reaction. Since in natural gas liquefaction none of these processes is used, we will skip chemical exergy and discuss about other exergy components in detail.

2.1.2. Mechanical exergy

The kinetic and potential energy of a stream can be fully converted to work in an ideal device by 100% efficiency. Figure (2-2) shows different devices which can transfer kinetic and potential energy to shaft work.

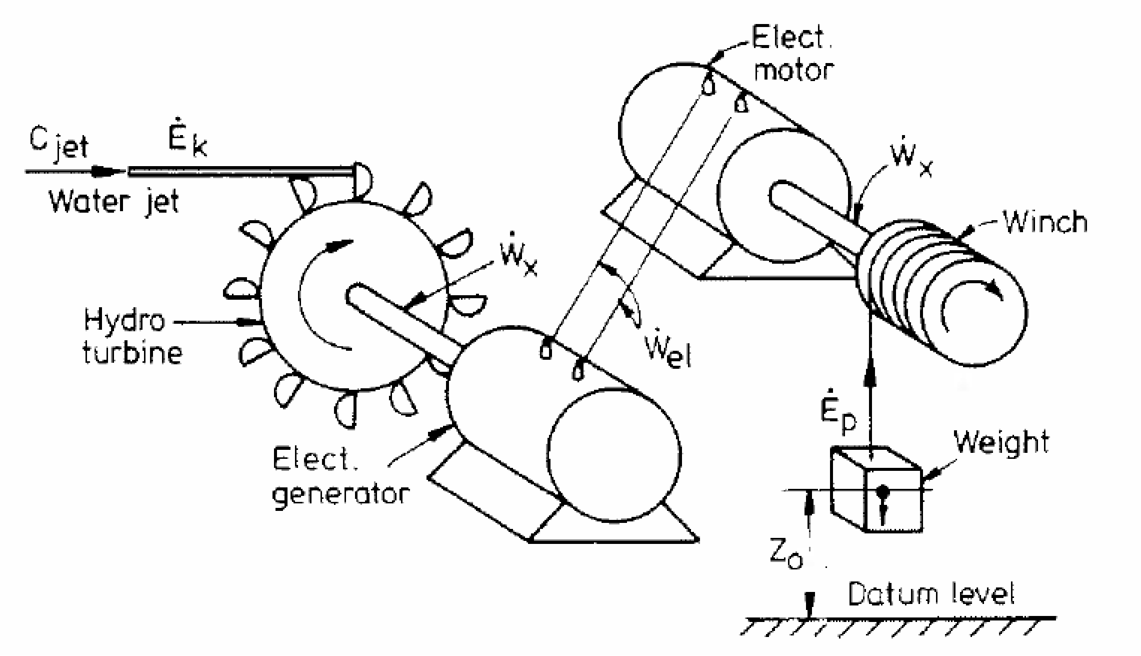


Figure 2.2 Example of transformation of kinetic and potential energy to work (Kotas, 1995)

As we can see the kinetic energy of water jet can rotate the turbine and produce work. In another case, the potential energy of box which is stored in gravitational form can drive the winch and produce work. As a result, by considering the definition of exergy, kinetic and potential exergy are equal to kinetic and potential exergy. Thus:

$$E_k = \dot{m}\frac{C^2}{2} \tag{2-3}$$

$$E_p = \dot{m}g_E Z \tag{2-4}$$

Where $\dot{m}$ the mass flow rate of stream, C is the bulk velocity of stream, Z is altitude of the stream above the sea level and g_E is constant of gravitational acceleration (Kotas, 1995).

2.1.3. Thermo-mechanical exergy

I should mention that Kotas categorized exergy in different way. In his category, physical exergy is same as thermo-mechanical exergy in the category which is introduced sooner. Kotas define thermo-mechanical (physical) exergy by ‘‘Physical exergy is equal to the maximum amount of work obtainable when the stream of substance is brought from its initial state to the environmental state defined by P_0 and T_0, by physical processes involving only thermal interaction with the environment (Kotas, 1995)’’.

We can derive an expression for thermo-mechanical exergy by using the definition of exergy in connection with an ideal device in which stream undergo a reversible process as shown in figure (2-3).

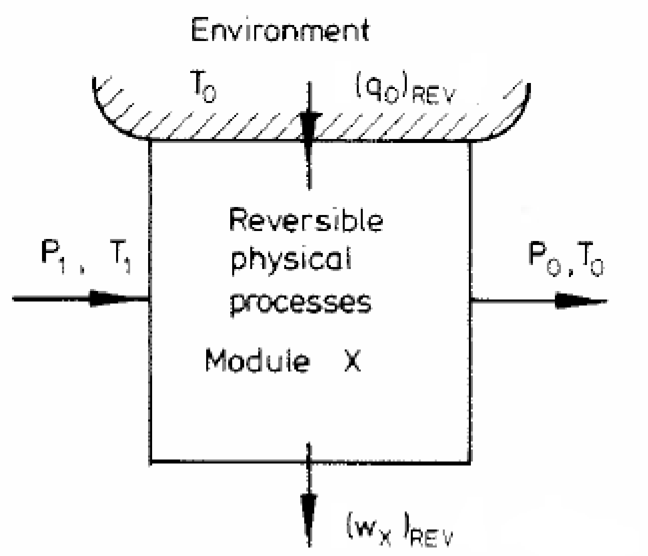


Figure 2.3 A reversible device for determining thermo-mechanical exergy (Kotas, 1995)

The inlet stream potential and kinetic energy is negligible. The pressure and temperature of inlet stream is P_1 and T_1. The inlet stream brought into equilibrium with environment in a reversible device, so the produced work is maximum. Heat transfer takes place at ambient temperature.

We can write energy balance for this control volume by neglecting the changes in kinetic and potential energy:

$$\frac{dE_{cv}}{dt} = \dot{m}.\left[\,(q_0)_{REV} - (w_X)_{REV} + (h_1 - h_0)\right] \tag{2-5}$$

By assuming steady state condition energy balance will be simplified into:

$$(q_0)_{REV} - (w_X)_{REV} + (h_1 - h_0) = 0 \tag{2-6}$$

Equation of entropy balance for reversible devise is:

$$\frac{dS_{cv}}{dt} = \dot{m}.\left[\frac{(q_0)_{REV}}{T_0} + (s_1 - s_0)\right] + \dot{\sigma_{cv}} \tag{2-7}$$

In which $\frac{dS_{cv}}{dt}$ and $\dot{E}_d$ are zero by assuming steady state condition and no irreversibility in process. So, the entropy balance equation will simplify to:

$$\frac{(q_0)_{REV}}{T_0} + (s_1 - s_0) = 0 \tag{2-8}$$

By combining equation (2-6) and (2-8) we will have:

$$(w_X)_{REV} = (h_1 - h_0) - T_0(s_1 - s_0) \tag{2-9}$$

By referring to thermo-mechanical exergy definition, specific thermo-mechanical exergy is equal to specific reversible work which is delivered by reversible devise.

$$\Rightarrow \varepsilon_{TM} = (h_1 - h_0) - T_0(s_1 - s_0) \tag{2-10}$$

As shown in figure (2-1) we can decompose the thermo-mechanical exergy to pressure based and temperature based. The accuracy of exergy analysis is improved by splitting the thermo-mechanical exergy to pressure base and temperature based exergy. To find pressure based and temperature based exergy we can consider either adiabatic and isothermal processes or isobaric and isothermal processes (Morosuk et al., 2005). Figure (2-4) shows the decomposition of thermo-mechanical exergy based on isothermal and isobaric processes.

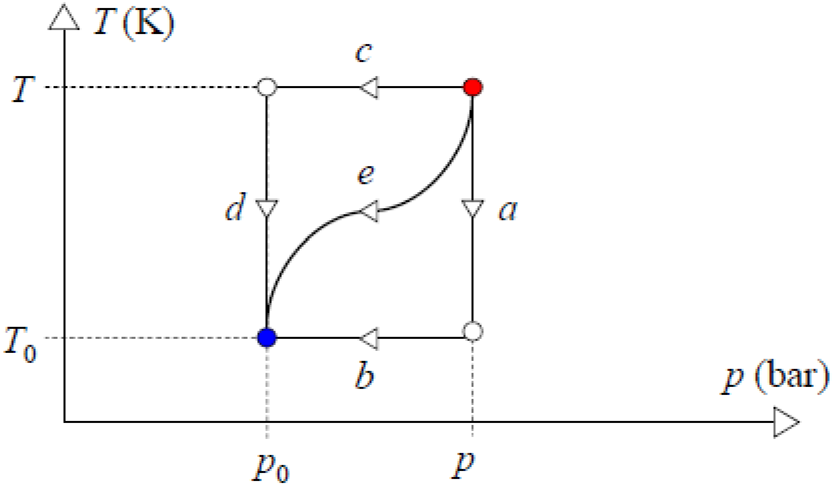


Figure 2.4 Decomposition of thermo-mechanical exergy (Gundersen,2011)

We can decompose thermo-mechanical exergy in two ways by using isobaric heat transfer and isothermal expansion or compression according to figure (2-4): (a+b) and (c+d). Calculating temperature based and pressure based exergy from each of these ways leads to same result for ideal gas. But for none ideal gas, each path gives different result.

In general, there is an agreement in the literatures to calculate temperature based and pressure based exergy. The temperature based exergy relates to changing the temperature of stream from T to T_0 when the pressure is kept at constant pressure P. The pressure based exergy relates to changing the pressure of stream from P to P_0 when the temperature is kept at constant temperature T_0 (Gundersen, 2011).

By using the above definition for temperature and pressure based exergy, following equations describe them.

$$\varepsilon_{TM}{}^{T} = [h(T,p) - h(T_0,p)] - T_0.[s(T,p) - s(T_0,p)] \tag{2-11}$$

$$\varepsilon_{TM}{}^{p} = [h(T_0,p) - h(T_0,p_0)] - T_0.[s(T_0,p) - s(T_0,p_0)] \tag{2-12}$$

‘’ It is interesting that within the total thermo-mechanical exergy, the two component can be trade-off against each other (Gundersen, 2011)’’. In the cryogenic process, a pressurized stream can be expanded and provide cooling and power. This is done by increasing temperature based exergy at the cost of pressure based exergy (Aspelund et al, 2007).

2.1.4. Exergy balance for a closed system

We can find the exergy balance equation for a closed system by combining energy and entropy balance. For a closed system, energy balance equation is:

$$(U_2 - U_1) + (KE_2 - KE_1) + (PE_2 - PE_1) = \int_1^2 \delta Q - W \tag{2-13}$$

And, entropy balance is:

$$S_2 - S_1 = \int_1^2 (\frac{\delta Q}{T})_b + S_{gen} \tag{2-14}$$

where b denotes the boundary and S_{gen} is the entropy generation in system due to internal irreversibility. By multiplying equation (2-14) by atmosphere temperature and subtracting from equation (2-13), we can find exergy balance for a closed system:

$$E_2 - E_1 = \int_1^2 (1 - \frac{T_0}{T_b}) \delta Q - [W - P_0(V_2 - V_1)] - T_0 S_{gen} \tag{2-15}$$

We can see from equation (2-15) that exergy change in the system is caused by exergy transfer by heat and work and exergy destruction because of internal irreversibility.

2.1.5. Exergy balance for a control volume

Since the control volume is most useful for engineering analysis, an equation for exergy balance in control volume is driven. The exergy of a system is given by equation (2-16).

$$E = m\varepsilon = m(e - e_0) + mP_0(v - v_0) - mT_0(s - s_0) \tag{2-16}$$

Where subscript ‘’0’’ refers to environment. Since environment properties are constant, rate of changes for Ebecomes:

$$\frac{dE}{dt} = \frac{d(me)}{dt} - e_0 \frac{dm}{dt} + P_0 \frac{dV}{dt} - P_0 v_0 \frac{dm}{dt} - T_0 \frac{d(ms)}{dt} + T_0 s_0 \frac{dm}{dt} \tag{2-17}$$

By using $h_0 = e_0 + P_0 v_0$ we can simplify equation (2-17).

$$\frac{dE}{dt} = \frac{d(me)}{dt} + P_0 \frac{dV}{dt} - T_0 \frac{d(ms)}{dt} - (h_0 - T_0 s_0) \frac{dm}{dt} \tag{2-18}$$

Now we can write mass balance, energy balance, and entropy balance for control volume:

$$\frac{dm}{dt} = \sum \dot{m}_i - \sum \dot{m}_o \tag{2-19}$$

$$\frac{dE}{dt} = \frac{dme}{dt} = \sum \dot{Q_{cv}} - \dot{W}_{cv} + \sum \dot{m}_i h_i - \sum \dot{m}_o h_o \tag{2-20}$$

$$\frac{dS}{dt} = \frac{dms}{dt} = \sum \frac{\dot{Q}_{cv}}{T} + \sum \dot{m}_i s_i - \sum \dot{m}_o s_o + \dot{\sigma}_{cv} \tag{2-21}$$

Where subscript ‘’i’’ refers to inlet stream and ‘’o’’ refers to outlet stream. By substituting equations (2-19)- (2-21into equation (2-18) we will have:

$$\frac{dE}{dt} = \sum \dot{Q_{cv}} - \dot{W}_{cv} + \sum \dot{m}_i h_i - \sum \dot{m}_o h_o + P_0 \frac{dV}{dt}$$

$$-T_0 \left[\sum \frac{\dot{Q}_{cv}}{T} + \sum \dot{m}_i s_i - \sum \dot{m}_o s_o + \dot{\sigma}_{cv}\right] - (h_0 - T_0 s_0) \frac{dm}{dt} \tag{2-22}$$

$$\begin{gathered}\Rightarrow \frac{dE}{dt} = \sum \dot{Q}_{cv}\left(1 - \frac{T_0}{T}\right) \\ -\dot{W}_{cv} + P_0\frac{dV}{dt} \\ +\sum \dot{m}_i\varepsilon_i - \sum \dot{m}_o\varepsilon_o \\ -T_0\dot{\sigma}_{cv} \end{gathered} \tag{2-23}$$

The first right hand term of equation (2-23) is exergy transfer accompanying heat, the second one is exergy transfer accompanying shaft/boundary work, the third one is exergy transfer accompanying with flow, and the last one is exergy destruction due to irreversibilities within the system. $E_d = T_0\dot{\sigma}_{cv}$ is positive when irreversibility is present in the system and it is zero for reversible process.

By referring to equation (2-23), the rate equation for exergy could be stated verbally as:

Rate of exergy storage = Transfer by heat + Transfer by shaft/ boundary work + Transfer by flow – Exergy destruction (Sonntag, 2003).

Since exergy destruction is always positive or zero, for steady state condition and fixed boundary control volume, we can conclude that:

$$\sum \dot{Q}_{cv}\left(1 - \frac{T_0}{T}\right) - \dot{W}_{cv} + \sum \dot{m}_i\varepsilon_i - \sum \dot{m}_o\varepsilon_o \geq 0 \tag{2-24}$$

Heat transfer and friction are two important factors in exergy destruction. To find effect of heat transfer and friction on exergy destruction, we can write the exergy balance for the counterflow heat exchanger which is shown in figure (2-5).

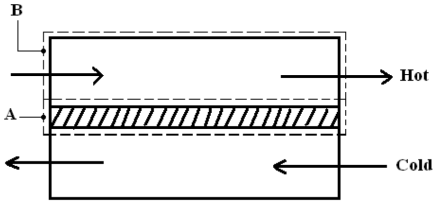


Figure 2.5 Counter flow heat exchanger to discuss the effect of heat transfer and friction on exergy destruction (Bejan et al., 1996)

Before writing this equation, it is useful to define thermodynamic average temperature. For defining that, consider a control volume with one inlet and one outlet which experience heat transfer. We can write entropy balance. $(\frac{\dot{Q}_{cv}}{\dot{m}})_{int\,rev} = T_a(s_o - s_i)$. Where T$_a$ is the thermodynamic average temperature (Bejan et al, 1996).

$$\Rightarrow T_a = \frac{\int_i^o Tds}{s_o - s_i} \tag{2-25}$$

By considering $dh = Tds + vdP$ we can rewrite above equation.

$$\Rightarrow T_a = \frac{(h_o - h_i) - \int_i^o vdP}{s_o - s_i} \tag{2-26}$$

If we assume there is not any change in pressure, for example in case of ideal heat exchanger with no pressure drop, we can simplify equation (2-26).

$$\Rightarrow T_a = \frac{(h_o - h_i)}{s_o - s_i} \text{ (constant pressure)} \tag{2-27}$$

Then, we can define exergy transfer due to heat transfer by equation (2-28).

$$\dot{E}_q = (1 - \frac{T_0}{T_a})\dot{Q} \tag{2-28}$$

We can write exergy balance for subsystem A by using thermodynamic average temperature term. Subsystem A is a wall which separates hot and cold stream. Heat transfer through the wall is$\dot{Q}$. Equation (2-29) is exergy balance for this subsystem.

$$0 = \left(1 - \frac{T_0}{T_{ha}}\right)\dot{Q} - \left(1 - \frac{T_0}{T_{ca}}\right)\dot{Q} - \dot{E}_D \tag{2-29}$$

where $\dot{E}_D$ is exergy destruction. T$_{ha}$ and T$_{ca}$ are thermodynamic average temperature for hot stream and cold stream.

$$\Rightarrow \dot{E}_D = T_0\dot{Q}\frac{T_{ha} - T_{ca}}{T_{ha}T_{ca}} \tag{2-30}$$

By considering heat transfer is proportional to difference temperature between hot and cold stream, we have, $\dot{E}_D \propto T_0 \frac{(T_{ha}-T_{ca})^2}{T_{ha}T_{ca}}$. This equation shows that exergy destruction due to heat transfer will increase by increasing the temperature difference between cold and hot stream.

For finding the exergy destruction due to friction we can consider subsystem B which is a control volume including the channel where hot stream is flowing at steady state. Friction is the only irreversibility in this control volume (Bejan et al, 1996).

We can write the exergy balance for this control volume by assuming kinetic and potential exergy terms are equal in the inlet and outlet,

$$\dot{E}_D = \left(1 - \frac{T_0}{T_{ha}}\right)\dot{Q} + \dot{m}[(h_i - h_o) - T_0(s_i - s_o)] \tag{2-31}$$

By considering $\dot{Q} = \dot{m}(h_o - h_i)$ we can simplify equation (2-31).

$$\dot{E}_D = T_0\dot{m}[(s_o - s_i) - \frac{h_o-h_i}{T_{ha}}]$$

By using equation (2-26) in above equation, we can write it in format of equation (2-32).

$$\dot{E}_D = -T_0\dot{m}\frac{\int_i^o vdP}{T_{ha}} \tag{2-32}$$

We can simplify above equation by using Bernoulli equation:

$$\int_i^o vdP + \frac{1}{2}\left(V_o{}^2 - V_i{}^2\right) + g(z_o - z_i) + \frac{\dot{W}_{cv}}{\dot{m}} + h_l = 0 \tag{2-33}$$

Where $\dot{W}_{cv}$ represents the work which is developed by control volume and hl is the head loss which shows the rate of irreversibility changes of mechanical energy to internal energy. h_l is zero when there is no irreversibility such as friction. For our case of study we can simplify equation (2-23) to$h_l = -\int_i^o vdP$. So, the exergy destruction equation due to friction is

$$\dot{E}_D = \frac{T_0\dot{m}h_l}{T_{ha}} \tag{2-34}$$

The term head loss is involving two terms, the wall friction due to passing of gas or liquid through the pipe and flow thorough resistances like valve or elbow. So, we can formulate head loss as equation (2-35).

$$h_l = \sum_p[(f\frac{4L}{D})\frac{V^2}{2}]_p + \sum_l[K\frac{V^2}{2}]_l \tag{2-35}$$

Where the first term of right hand is pipe friction and the second term is resistance. f is friction factor which depends on relative roughness of the wall and Reynolds number. K is loss coefficient for resistance. (Bejan et al, 1996).

So, by using equation (2-35) we can find the exergy losses due to friction in the case of study.

$$\dot{E}_D = \frac{T_0\dot{m}(f\frac{4L}{D})\frac{V^2}{2}}{T_{ha}} \tag{2-36}$$

As we can see in equation (2-36), exergy destruction varies directly by mass velocity and friction factor and inversely by temperature. So, by decreasing the temperature, effect of friction is more on exergy destruction. As a result, for low temperature processes its effect is significant.

Exergy destruction in heat transfer are important in LNG processes, since it needs to be compensated by higher power input, and heat transfer at low temperature gives higher destruction. (Pettersen, 2009) Figure (2-6) shows the heat transfer between hot stream and cold stream below ambient temperature (T_0).

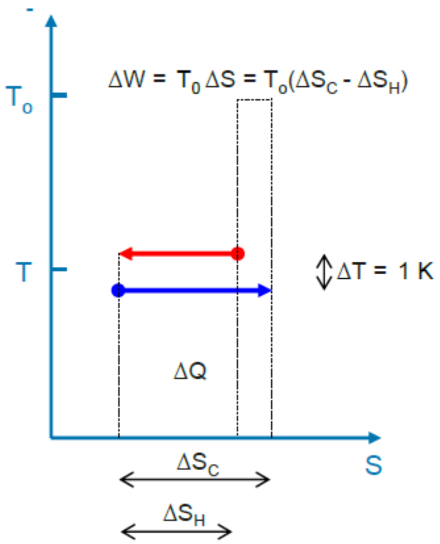


Figure 2.6 Temperature entropy diagram for heat transfer at sub-ambient temperature (Pettersen, 2009)

Heat transferred from hot stream to cold stream. So, area below hot stream and cold stream which corresponds to heat transfer should be equal. The exergy is transferred from cold stream to hot stream in opposite direction of heat transfer. The exergy which is transferred from cold stream to hot stream is more than which is received by cold stream. As a result, there is exergy loss in heat transfer which is shown by ΔW. It is clear that by increasing ΔT, exergy loss will be increased. The exergy loss or loss work should be compensated by additional work input. So, in designing heat exchanger it is favorable to reduce *ΔT*. However, there is a limitation in reducing *ΔT* which increases the capital cost due to increase in area of heat exchanger.

2.1.6. Exergy efficiency

Efficiency is a fundamental concept which describes the performance of a system or process. For assessment of a system or process, the relation between output which obtained the effect and input which supplied the effect are considered. So, definition of output and input will influence on the value of efficiency. Since there is not any scientifically unique definition for the values of input and output; there are many different treatments for describing efficiency. Some of them are incorrect and leads to senseless values. For instance, in some definitions, the efficiency of an imperfect process is more than 100%. On the other hand, the efficiency of an ideal process is less than 100% (Brodyansky, 1994).

To evaluate the performance of a process or system, it is necessary to consider that from economy point of view. Other factors such as impact on society and environment are important too. Since these parameters are instable, thermodynamic can be a good guide to evaluate the performance of a process.

In modern thermodynamic exergy efficiency developed to measure the thermodynamic performance of a process with respect to irreversibilities and losses. (Marmolejo-Correa & Gundersen, 2011) The less irreversibility in process, the greater work can be done and the decrease in availability reserves will be less. Moreover, ‘’ it is desirable to accomplish a given objective with the smallest irreversibility is an economic one (Sonntag, 2003)’’. However, we should consider that many factors have influence on economical design. For example, by decreasing ΔT in heat exchanger, irreversibility will be less but the capital cost will increase due to increase in area of heat exchanger.

In general, exergy efficiency is described by equation (2-37).

$$\eta_e = \frac{\sum E_{eff}}{\sum E_{exp}} = \frac{\sum E_{exp} - \sum D}{\sum E_{exp}} = \frac{A_{eff}}{A_{exp}} \qquad (2\text{-}37)$$

Where $\sum D$ is the sum of exergy losses in the process, A_{eff} is produced useful effect in a system, and A_{exp} is system’s expenditures. There are different approaches to define useful effect and system’s expenditures in literatures which lead to different definition for exergy efficiency and confusion when they are used to describe the performance of a process. The best known one is introduced in equation (2-38):

$$\eta_e = \frac{\sum E_{out}}{\sum E_{in}} \qquad (2\text{-}38)$$

This definition is simple and expenditure and useful effect are well defined but it cannot evaluate the performance of a process well since it usually calculates the efficiency of process close to one. The reason is due to the fact that part of exergy which enters to the process and remains unchanged dilutes the weight of irreversibilities (Marmolejo et al., 2011).

Another approach is based on subtraction. Kotas categorized exergy transfers into desired output and necessary input. The exergy input and output take different format of exergy such as work, exergy associated to heat transfer, exergy of flow of matter and change of exergy of stream which passes through a control volume like heat exchanger. By definition of exergy input and output we can write exergy balance (Kotas,1995).

$$\sum \Delta\dot{E}_{in} = \sum \Delta\dot{E}_{out} + \dot{I} \tag{2-39}$$

Where $\sum \Delta\dot{E}_{in}$ is the sum of all input exergy transfers, $\sum \Delta\dot{E}_{out}$ is the sum of all output exergy transfers, and $\dot{I}$ corresponds to irrevesibilities. Since $\dot{I} \geq 0$, $\frac{\sum \Delta\dot{E}_{out}}{\sum \Delta\dot{E}_{in}} \leq 1$. As a result, the ration of exergy output and exergy input is suitable to evaluate the performance of a process. Kotas called that ration rational efficiency (Kotas,1995):

$$\Psi = \frac{\sum \Delta\dot{E}_{out}}{\sum \Delta\dot{E}_{in}} \tag{2-40}$$

The difference between input exergy transfers and output exergy transfers depends on degree of irreversibility of process.

To define exergy efficiency Bejane defined product and fuel. Fuel is the resource to produce product. Product is the desired result of process. By this definition, we can write the exergy balance for a process by equation (2-41).

$$\dot{E}_F = \dot{E}_P + \dot{E}_D + \dot{E}_L \tag{2-41}$$

where, $\dot{E}_F$ is the exergy of fuel, $\dot{E}_P$is the exergy of product, $\dot{E}_D$ is destruction exergy of process, and $\dot{E}_L$ is exergy losses from process. The exergy efficiency will be the ratio between exergy of product and exergy of fuel.

$$\varepsilon = \frac{\dot{E}_P}{\dot{E}_F} \tag{2-42}$$

‘’ The exergetic efficiency shows the percentage of the fuel exergy provided to a system that is found in the product exergy’’ (Bejan et al, 1996). The difference between exergy of product and fuel shows how much exergy is wasted in the process. For using this definition, it is necessary to introduce fuel and product properly. The purpose of the system determines the fuel and product of the system. For example, the purpose of pump or compressor is increasing the outlet pressure by means of input power. As a result, fuel is input power and product is the difference between inlet and outlet exergy. On the other hand, since the purpose of turbine is to generate power, the product is generated power and fuel is the difference exergy of inlet and outlet stream. (Bejan et al, 1996)

Since, heat exchangers can have different purposes in different situations, the definition of fuel and product is different. The purpose of a heat exchanger which works above ambient temperature is increasing the exery of cold stream at expense of decreasing the exergy of hot stream. So, the difference between exergy of inlet and outlet of the cold streams is product and the exergy difference between inlet and outlet hot streams is fuel. In the other hand, for a heat exchanger which works below temperature, the definition of fuel and product is different since its purpose is to transfer exergy from cold stream to hot stream. (Bejan et al, 1996)

Since it is possible to define fuel and product or exergy input and exergy output differently in these definitions, there is not any unique result for exergy efficiency. For example, exergy increasing due to increasing concentration of components which are separated in separation process can be considered as useful effect. However exergy increasing can be result of an increase in their partial pressures. Generally, expenditure can be interpreted differently.

The third approach is based on subtracting of exergy which has not changed qualitatively. By excluding the unchanged exergy flows, it is possible to separate and evaluate those transformations of exergy which are essential to the purpose of system. This idea leads to evaluate the transformations of exergy which are important to the system's purpose. Brodyansky suggest equation (2-43) to calculate exergy efficiency.

$$\eta_e = \frac{\sum E_{out} - E^{tr}}{\sum E_{in} - E^{tr}} \qquad (2\text{-}43)$$

Where, E^{tr} is transient exergy. ‘‘ Transient exergy is a constant part of the exergy flow passing through a system's control surface which, within the limits of the the considered system, is not transferred into exergy of a new compound, new kind or component of exergy and is determind by the least magnitudes of the same kind of exergy flows at the system's input and output (Brodyansky, 1994)’’. Since, transient exergy can be understood in different ways, it is necessary to define that uniquely according to purpose of system. Table (2-1) shows how we can calculate transit exergy for different exergy carrier. I should mention that chemical exergy is not discussed in this table since it is not involved in the exergy analysis in the thesis.

Table 2-1 Formulas for calculating different kinds of transit exergy (Brodyansky, 1994)

Exergy carrier	Transit in an overall exergy stream	Transit in each exergy substream (stream)
Work flow	min $[\sum_w E_{in,w}, \sum_w E_{out,w}]$	
Heat flow	min $[\sum_q E_{in,q}, \sum_w E_{out,q}]$	a. $T_{in} > T_0, T_{out} > T_0$ $Q^{tr}\frac{T^{min} - T_0}{T^{min}}$ b. $T_{in} < T_0, T_{out} < T_0$ $Q^{tr}\frac{T^{max} - T_0}{T^{max}}$ c. $T_{in} < T_0, T_{out} > T_0$ or$T_{in} > T_0, T_{out} < T_0$ $Q^{tr}\frac{T_0 - T_0}{T_0} = 0$
Thermo-mechanical exergy of material flow	min $[\sum_b E_{in,p,T}, \sum_b E_{out,p,T}]$	min$[E_{in,p,T}, E_{out,p,T}]$
Sum of temperature based and pressure based exergy components of material flow		a. $T_{in} > T_0, T_{out} > T_0$ ${E_{p,T}}^{tr} = \min\,[E_{in}(T^{min}, p_{in}), E_{in}(T^{min}, p_{out}),$ $E_{out}(T^{min}, p_{in}), E_{out}(T^{min}, p_{out})]$ b. $T_{in} < T_0, T_{out} < T_0$ ${E_{p,T}}^{tr} = \min\,[E_{in}(T^{max}, p_{in}), E_{in}(T^{max}, p_{out}),$ $E_{out}(T^{max}, p_{in}), E_{out}(T^{max}, p_{out})]$ c. $T_{in} < T_0, T_{out} > T_0$ or$T_{in} > T_0, T_{out} < T_0$ ${E_{p,T}}^{tr} = \min\,[E_{in}(T_0, p_{in}), E_{in}(T_0, p_{out}),$ $E_{out}(T_0, p_{in}), E_{out}(T_0, p_{out})]$

Some points should be explained to use this table. b is index of material flows at inlet and outlet, w is index for work, and q is index for heat flow. Temperature based and pressure based exergy can be calculated only for each stream not for overall exergy flow. Since, it is not possible to sum up the temperature based exergy of streams at different pressure. As we can see in table (2-1), the transit exergy of heat and temperature based component of material flow depends on temperature of inlet and outlet. This is due to the fact that at temperature above ambient, minimum temperature based exergy is for the stream with minimum temperature. On the other hand, at temperature below ambient, the least temperature based exergy is for the stream with maximum temperature.

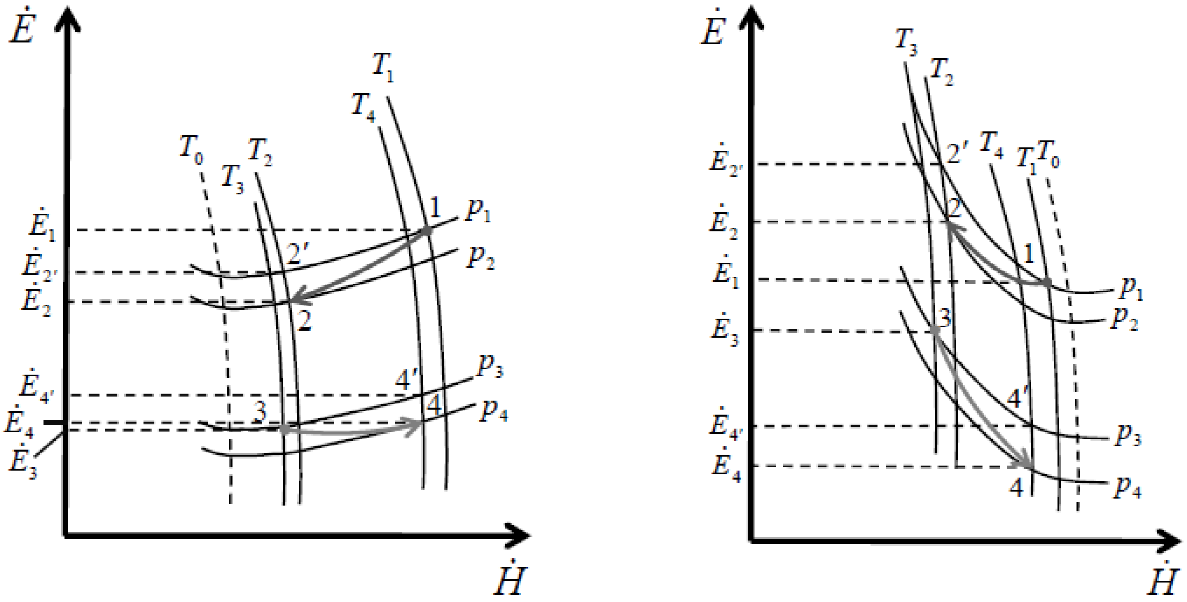
Brodyansky suggested general formula for calculating exergy efficiency. (Brodyansky, 1994)

$$\eta_e = \frac{\sum_q (E_{out,q} - {E_q}^{tr}) + \sum_w (E_{out,w} - {E_w}^{tr}) + \sum_b (E_{out,p,T} - {E_{p,T}}^{tr})}{\sum_q (E_{in,q} - {E_q}^{tr}) + \sum_w (E_{in,w} - {E_w}^{tr}) + \sum_b (E_{in,p,T} - {E_{p,T}}^{tr})} \tag{2-44}$$

Theses definitions are not suitable to calculate the performance of low temperature processes. ‘’ This incompatibility is due to the lack of a proper treatment of exergy flows inside the process unit (Marmolejo-Correa and Gundersen, 2011)’’. Since calculation of exergy efficiency is more important for these processes because of work requirement in refrigeration cycles, Marmolejo defines proposes a new expersion to calculate Exergy Transfer Effectiveness (ETE). ‘’The ETE is focused on measuring the exergy transfer within processes and rate the goodness of exergy transformations from sources to sinks (Marmolejo-Correa & Gundersen, 2011)’’. ETE can be calculated by equation (2-45).

$$\varepsilon = \frac{Exergy\ Sink}{Exergy\ Source} \tag{2-45}$$

To define exergy sink and source we can use a general rule for units which work above ambient temperature or below ambient temperature. All the positive changes in exergy of a process stream or its component are categorized as exergy sink while the negative changes represent exergy sources. There is only one exception for changes in exergy due to pressure drop. Pressure drop always has positive effect on exergy source and negative effect on exergy sink. We can explain that by referring to figure (2-7).



(a) (b)

Figure 2.7 Pressure drop effects in the exergy transfer for the system (a) above T0 (b) below T0 (Marmolejo-Correa & Gundersen, 2011)

Stream 1-2 is hot stream and 3-4 is cold stream. Points 2’ and 4’ are outlet stream by assuming no pressure drop. In case (a) the change in exergy of hot stream is source and the change in exergy of cold stream represents the exergy sink since (E2-E1)<0 and (E4-E3)>0. On the other hand, for case (b) change of exergy of hot stream act as sink and exergy change of cold stream act as exergy source. So, As we can see in figure (2-7) pressure drop causes reducing of sink and enlarging of source. As a result, exergy destruction will increase by pressure drop.

Table (2-2) shows the sink and source for different operation units which work below and above atmosphere temperature.

Table 2-2 Sources and sinks for selected unit operations above and below T_0 (Marmolejo-Correa & Gundersen, 2011)

		Above T_0	Below T_0
Heat Exchange	Source	$\left(\dot{E}_{H,in}{}^{T} - \dot{E}_{H,out}{}^{T}\right) + (\dot{E}_{H,in}{}^{P} - \dot{E}_{H,out}{}^{P})$	$\left(\dot{E}_{C,in}{}^{T} - \dot{E}_{C,out}{}^{T}\right) + (\dot{E}_{C,in}{}^{P} - \dot{E}_{C,out}{}^{P})$
	Sinke	$\left(\dot{E}_{C,out}{}^{T} - \dot{E}_{C,in}{}^{T}\right) - (\dot{E}_{C,in}{}^{P} - \dot{E}_{C,out}{}^{P})$	$\left(\dot{E}_{H,out}{}^{T} - \dot{E}_{H,in}{}^{T}\right) - (\dot{E}_{H,in}{}^{P} - \dot{E}_{H,out}{}^{P})$
Compression	Source	$\dot{W}$	$\dot{E}_{in}{}^{T} - \dot{E}_{out}{}^{T} + \dot{W}$
	Sinke	$\dot{E}_{out}{}^{TM} - \dot{E}_{in}{}^{TM}$	$\dot{E}_{out}{}^{P} - \dot{E}_{in}{}^{P}$
Expansion(expander)	Source	$\dot{E}_{in}{}^{TM} - \dot{E}_{out}{}^{TM}$	$\dot{E}_{in}{}^{P} - \dot{E}_{out}{}^{P}$
	Sinke	W	$\dot{E}_{out}{}^{T} - \dot{E}_{in}{}^{T} + \dot{W}$
Expansion(valve)	Source	$\dot{E}_{in}{}^{TM} - \dot{E}_{out}{}^{TM}$	$\dot{E}_{in}{}^{P} - \dot{E}_{out}{}^{P}$
	Sinke	Dissipative losses	$\dot{E}_{out}{}^{T} - \dot{E}_{in}{}^{T}$

For the operation units with heat exchange which work acroos theatmospheric temperature, we should introduce an intermediate state i (T_0, p_i). Then, the exergy source is the sum of temperature based exergy for cold and hot stream from supply temperature to T_0 and exergy sink is the sum of temperature based exergy change from T_0 to target temperature. By considering that temperature based exergy at T_0 is zero and effect of pressure drop on sink and source, the source and sink definition for heat exchange operation across T_0 are shown in table (2-3).

For compression across the ambient temperature source is equal to sum of the work and change in temperature based exergy from supply temperature to T_0 and exergy sink is sum of the change in pressure based exergy and the change in temperature based exergy from T_0 to target temperature. (Marmolejo-Correa & Gundersen, 2011)

The sink of expansion across the atmosphere temperature is sum of the produced work and the temperature based exergy from T_0 to the target temperature and exergy source is the sum of the change in pressure based exergy and change in temperature based exergy from supply temperature to T_0. (Marmolejo-Correa & Gundersen, 2011)

The source and sink of operation unit across atmospheric temperature is shown in table (2-3).

Table 2-3 Sources and sinks for selected unit operations across T_0 (Marmolejo-Correa & Gundersen, 2011)

Above T_0

Heat Exchange	Source	$\left(\dot{E}_{H,in}{}^{T} + \dot{E}_{C,in}{}^{T}\right) + (\dot{E}_{H,in}{}^{P} - \dot{E}_{H,T0}{}^{P} + \dot{E}_{C,in}{}^{P} - \dot{E}_{C,T0}{}^{P})$
	Sinke	$\left(\dot{E}_{C,out}{}^{T} + \dot{E}_{H,out}{}^{T}\right) - (\dot{E}_{C,T0}{}^{P} - \dot{E}_{C,out}{}^{P} + \dot{E}_{H,T0}{}^{P} - \dot{E}_{H,out}{}^{P})$
Compression	Source	$\dot{W} + \dot{E}_{in}{}^{T}$
	Sinke	$\dot{E}_{out}{}^{p} - \dot{E}_{in}{}^{p} + \dot{E}_{out}{}^{T}$
Expansion(expander)	Source	$\dot{E}_{in}{}^{p} - \dot{E}_{out}{}^{p} + \dot{E}_{in}{}^{T}$
	Sinke	$W + \dot{E}_{out}{}^{T}$
Expansion(valve)	Source	$\dot{E}_{in}{}^{p} - \dot{E}_{out}{}^{p} + \dot{E}_{in}{}^{T}$
	Sinke	$\dot{E}_{out}{}^{T}$

2.2. Pinch analysis method

Setting the amount of ΔT_{min} in the heat exchangers is important in heat recovery. This value adjusts the minimum amount of external cooling and heating in entire network of heat exchangers. If $\Delta T_{min} = 0$, there will be a point in heat exchanger where the driving force is zero for heat transfer. As a result, the area of heat exchanger must be infinite. It leads the capital cost to be infinite in return. Although by increasing ΔT_{min} the capital cost reduces, the cost of energy will be increased due to increased external cooling and heating duty. So, there is a trade-off between capital cost and energy cost.

ΔT_{min} is normally observed in one point between hot stream and cold stream called the pinch point. Now, it is possible to divide the process into two regions, above and below the pinch point. Above the pinch process acts as a heat sink since it is received heat from hot utility. Heat is rejected from below the pinch point to the cold utility and it acts as a heat source.

For maximising heat recovery and minimizing external duty, designer should avoid heat transfer across the pinch, using the cold utility above the pinch, and using the heating utility below the pinch.

If XP amount of heat transfers across pinch, it will create a deficit of XP above the pinch and surplus of XP below the pinch. So, for removing these effects we should add the external duty for heating and cooling utility by XP.

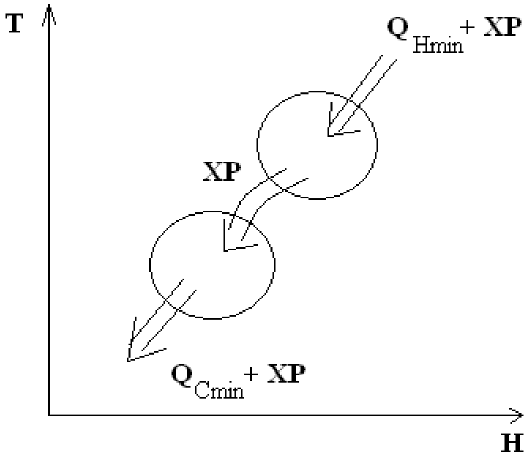


Figure 2.8 Process- process heat transfer across the pinch (Smith, 2008)

If the external cooling XP is used to cool the hot stream above the pinch, energy balance in that region will be imbalanced. So, additional amount of external heating (Q_{Hmin}+XP) is needed to balance the energy.

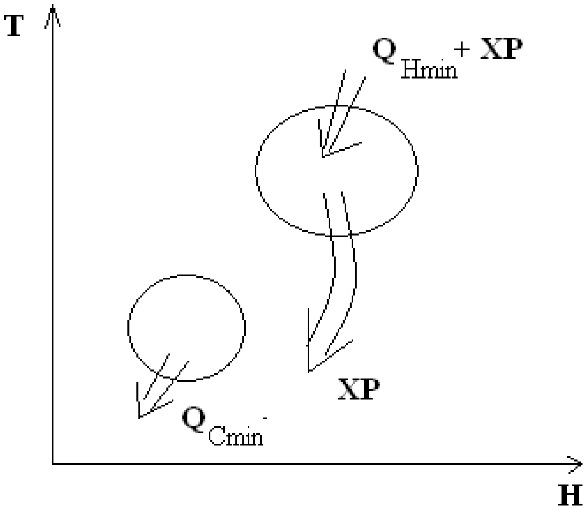


Figure 2.9 Cold utility above pinch (Smith, 2008)

If XP additional amount of hot utility is used below the pinch, then XP additional amount of cold utility is needed to balance the imbalanced enthalpy below the pinch region.

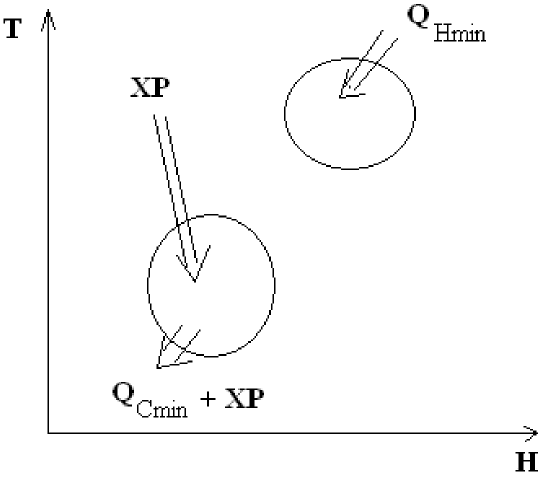


Figure 2.10 Hot utility below pinch (Smith, 2008)

Avoiding heat transfer across the pinch and appropriate use of utility are sufficient rules to ensure maximum heat recovery is achieved.

Pinch design method is a strategy which is adopted for heat recovery design based on the rules which are explained earlier (Smith, 2008).

1. The design should be started at the pinch point to avoid temperature difference less than ΔT_{min} or using excessive utility.
2. As ΔT_{min} occurs at the pinch, by moving away from pinch the difference in temperature of cold and hot streams must increase. So, CP of hot stream must be greater or equal to CP of cold stream below the pinch and CP of cold stream must be greater or equal to CP of hot stream above the pinch.
3. By listing the values of CP in descending order, it will clear the matches of stream in the region of the pinch. As using cooling utility above pinch is inappropriate, hot streams must be cooled to the pinch point by heat recovery. It is inappropriate to use heating utility below the pinch, so cold stream duty must be matched with hot stream duty below pinch to be warmed to pinch point.
4. Designers should minimise the capital cost after matching around pinch by minimising the number of units.
5. Designer can discriminate the region away from the pinch on the basis of operability, plant layout and etc.

2.3. ExPAnD method

The design of heat recovery is an important issue due to cost of energy. Pinch analysis is thus a strong method for reducing external cooling and heating, integration of separation system, and evaluation of using heat pump in an industrial plant. There is an important limitation in pinch analysis that it does not consider pressure and composition as a quality parameter for stream (Aspelund et al, 2007).

Pressure is an important design variable. In the processes above ambient temperature, changing the pressure of distillation columns and evaporators leads to changes in the temperature of heat sinks and heat sources. In the sub-ambient processes, pressure is more important since pressurized stream is a potential cooling resource. If the pressurized stream expands, its temperature will decrease and it can be used for cooling. As a result, pressure should be considered as an important parameter like does the temperature in low temperature processes (Gundersen et al, 2009). In exergy analysis, temperature, pressure, and composition of streams are considered as stream parameters but it focuses on equipments rather than the entire process.

By considering the pressure as stream data beside temperature, task definition for heat recovery will change. In traditional heat recovery task, the path between supply and target stream is unique and defined by temperature. In new definition the path from the supply state to the target state is not fixed, and can be changed by expansion and compression (Gundersen et al, 2009). Extended Pinch Analysis and Design (ExPAnD) is a method for designing heat recovery in extended definition as bellow:

‘’ Given a set of process streams with a supply state (temperature, pressure, and resulting phase) and target state, as well as utilities for external heating and cooling; design a system of heat exchangers, expanders and compressors in such a way that irreversibilities are minimized (Aspelund et al, 2007)’’.

The extended method is more complex than classical one due to:

- Pressure of stream will change in addition to changes in temperature.
- The cold stream often acts like cooling utility.
- Hot stream may act as cold stream and vice versa.

- The phase of stream may be changed by changing the pressure (Gundersen et al, 2009).

A set of heuristic rules are proposed for sub-ambient process by Aspelund et al (2007) to reduce external heating and cooling:

By expanding the pressurized stream, pressure based exergy can transform to temperature based exergy and produce cooling and work. So, the cold utility duty will be reduced. From pinch analysis point of view, expansion should be done below the pinch. If expansion is needed above the pinch, it should be done through the valve to minimize the increase in utility consumption.

On the other hand, in the case that target pressure is greater than supply pressure, power is required and heat is added to the system which may reduce hot utility requirements. So, compression should be done above the pinch point. Furthermore, fluid should be pumped to save compressor work.

Temperature difference greater than ΔT_{min} (between cold stream and hot stream) results in unnecessary irreversibility in the heat exchanger. In this case, the stream pressure should be manipulated to decrease irreversibility and the need for heating and cooling utility.

In the case that target pressure is equal to supply pressure, if compressed stream above the pinch be cooled to near pinch point and then expanded, the need for cold utilities and hot utilities will be reduced. Moreover, phase transition is necessary in a liquid stream because the effect of changing the pressure in the liquid phase alone is marginal.

3. Exergy analysis on LNG production processes

‘’ The method of exergy analysis aims at quantitive evaluation of the exergy destructions and losses associated with a system’’ (Bejan et al, 1996). In this chapter author will study effects of different parameters on exergy efficiency of operation units. Moreover, exergy efficiency of LNG production process with dual independent refrigeration cycles will be discussed. Exergy transfer effectiveness is used for analysing the exergy efficiency of the units and process. The study will focus on exergy efficiency of compressor which works above or across T_0, expander which works below T_0, and heat exchanger in all the temperature ranges, since these units are operating in the mentioned level of temperature in LNG processing.

3.1. Exergy analysis of operation units in LNG production processes

Since, compressor, expander, and heat exchanger have the most important role in LNG processing, it is important to find effect of different parameters on exergy efficiency of these units.

3.1.2. Exergy analysis of compressor

We should identify the parameters which have effects on efficiency of the compressors. In general, a compressor has seven parameters which define its characteristic: mass velocity, inlet pressure and temperature, outlet pressure and temperature, work, and isentropic efficiency. By assuming constant mass velocity, we have 6 variables. By considering two equations: energy balance and relation between isentropic work and real work, we should set four parameters to define characteristic of compressor. So, we can define exergy efficiency of compressor as a function of inlet temperature, inlet and outlet presser, and the isentropic efficiency of compressor.

$$\varepsilon_C = f(P_{out}, P_{in}, T_{in}, \eta_{is}) \tag{3-1}$$

$$\Longrightarrow d\varepsilon_C = \frac{\partial \varepsilon_C}{\partial P_{out}} dP_{out} + \frac{\partial \varepsilon_C}{\partial P_{in}} dP_{in} + \frac{\partial \varepsilon_C}{\partial T_{in}} dT_{in} + \frac{\partial \varepsilon_C}{\partial \eta_{is}} d\eta_{is} \tag{3-2}$$

The isentropic efficiency of rotary machines depends on different factors like losses due to heat exchange to cylinder wall, losses due to gas flow through valves and ports and tightness losses type of machines. So, η_{is} depends on design parameters of compressor, as a result the last term of equation (3-2) is zero. Typical compressor isentropic efficiency is 70-88% (Sonntag et al. 2003).

The exergy efficiency for a compressor which works above ambient temperature is calculated by equation (3-3):

$$\varepsilon_C = \frac{\dot{E}^{TM}{}_{out} - \dot{E}^{TM}{}_{in}}{\dot{W}_C} \tag{3-3}$$

Where, $\dot{E}^{TM}{}_{out} - \dot{E}^{TM}{}_{in} = \dot{m}.\left[h_{out} - h_{in} - T_0.\left[s_{out} - s_{in}\right]\right]$

and, $\dot{W}_C = \dot{m}.\left[h_{out} - h_{in}\right]$

$$\Longrightarrow \varepsilon_C = \frac{h_{out} - h_{in} - T_0.\left[s_{out} - s_{in}\right]}{h_{out} - h_{in}} = 1 - T_0.\frac{s_{out} - s_{in}}{h_{out} - h_{in}} \tag{3-4}$$

Figure (3-1) shows how exergy efficiency of compressor will change by changing isentropic efficiency. In this case study, the working fluid is methane and its mass velocity is 1 (kg/s). Inlet pressure and outlet pressure are set at 30 and 70 bar and inlet temperature is 30°C. SRK is used as equation of state.

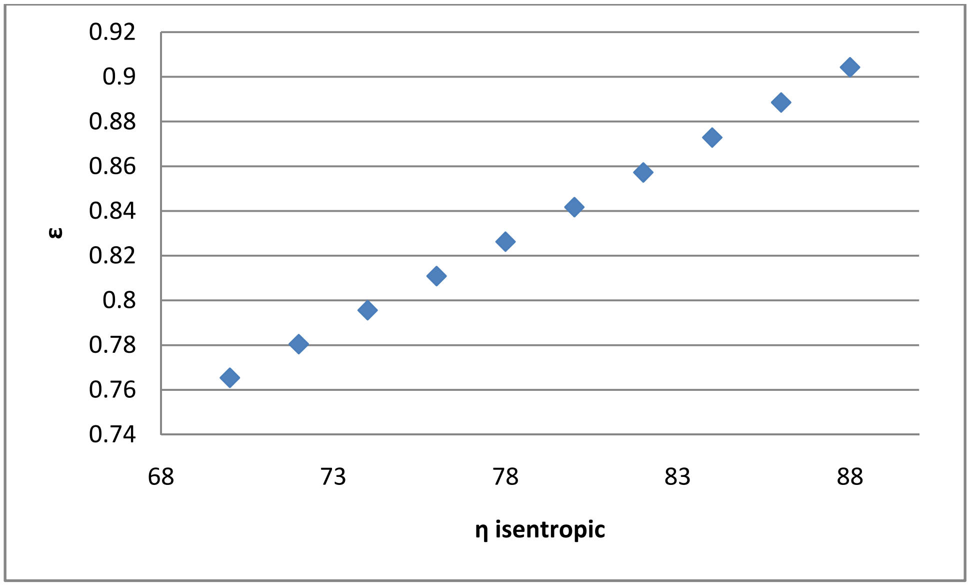


Figure 3.1 Effect of isentropic efficiency on exergy efficiency of compressor

As we can see in figure (3-1), by increasing isentropic efficiency of compressor, its exergy efficiency will increase. Equation (3-5) is an expression which calculates isentropic efficiency for a compressor where $h_{out,s}$ is the enthalpy of outlet stream when the isentropic efficiency of the compressor is 100%.

$$\eta_{is,com} = \frac{h_{out,s} - h_{in}}{h_{out} - h_{in}} \tag{3-5}$$

By increasing isentropic efficiency, exergy destruction will decrease. So, the exergy efficiency will increase. If the isentropic efficiency is 100%, there is no exergy destruction in compressor. As a result $E^{th}{}_{out} - E^{th}{}_{in} = W_s$. So, the exergy efficiency is 100%.

By considering h=h(P,T) and s=s(P,T), we can find $\frac{\partial \varepsilon_C}{\partial P_{out}}$, $\frac{\partial \varepsilon_C}{\partial P_{in}}$, and $\frac{\partial \varepsilon_C}{\partial T_{in}}$ to find an extensive expression for equation (3-2):

$$\frac{\partial \varepsilon_C}{\partial P_{out}} = \frac{\partial \varepsilon_C}{\partial h_{out}} . (\frac{\partial h_{out}}{\partial P_{out}})_T + \frac{\partial \varepsilon_C}{\partial s_{out}} . (\frac{\partial s_{out}}{\partial P_{out}})_T \tag{3-6}$$

where $\frac{\partial h_{out}}{\partial P_{out}}$ and $\frac{\partial s_{out}}{\partial P_{out}}$ depend on the temperature and working fluid. We can find $\frac{\partial \varepsilon_C}{\partial h_{out}}$ and $\frac{\partial \varepsilon_C}{\partial s_{out}}$ from equation (3-4):

$$\Rightarrow \frac{\partial \varepsilon_C}{\partial h_{out}} = T_0 \frac{s_{out} - s_{in}}{(h_{out} - h_{in})^2} \tag{3-7}$$

$$\Rightarrow \frac{\partial \varepsilon_C}{\partial s_{out}} = -T_0 \frac{1}{h_{out} - h_{in}} \tag{3-8}$$

By using equations (3-7) and (3-8) in equation (3-6) we will have:

$$\frac{\partial \varepsilon_C}{\partial P_{out}} = T_0 \frac{s_{out} - s_{in}}{(h_{out} - h_{in})^2} (\frac{\partial h_{out}}{\partial P_{out}})_{T_{out}} - T_0 \frac{1}{h_{out} - h_{in}} (\frac{\partial s_{out}}{\partial P_{out}})_{T_{out}}$$

$$\Rightarrow \frac{\partial \varepsilon_C}{\partial P_{out}} = \frac{T0}{h_{out} - h_{in}} [\frac{s_{out} - s_{in}}{h_{out} - h_{in}} (\frac{\partial h_{out}}{\partial P_{out}})_{T_{out}} - (\frac{\partial s_{out}}{\partial P_{out}})_{T_{out}}]$$

$$\Rightarrow \frac{\partial \varepsilon_C}{\partial P_{out}} = \frac{T0}{h_{out} - h_{in}} [(\frac{1 - \varepsilon_C}{T_0})(\frac{\partial h_{out}}{\partial P_{out}})_{T_{out}} - (\frac{\partial s_{out}}{\partial P_{out}})_{T_{out}}] \tag{3-9}$$

Equation (3-10) shows the changes of exergy efficiency of compressor by changing the inlet pressure.

$$\frac{\partial \varepsilon_C}{\partial P_{in}} = \frac{\partial \varepsilon_C}{\partial h_{in}} . (\frac{\partial h_{in}}{\partial P_{in}})_{T_{in}} + \frac{\partial \varepsilon_C}{\partial s_{in}} . (\frac{\partial s_{in}}{\partial P_{in}})_{T_{in}} \tag{3-10}$$

We can find $\frac{\partial \varepsilon_C}{\partial h_{in}}$ and $\frac{\partial \varepsilon_C}{\partial s_{in}}$ from equation (3-4).

$$\Rightarrow \frac{\partial \varepsilon_C}{\partial h_{in}} = -T_0 \frac{s_{out}-s_{in}}{(h_{out}-h_{in})^2} \tag{3-11}$$

$$\Rightarrow \frac{\partial \varepsilon_C}{\partial s_{in}} = T_0 \frac{1}{h_{out}-h_{in}} \tag{3-12}$$

By using equations (3-11) and (3-12) in equation (3-10) we will have:

$$\Rightarrow \frac{\partial \varepsilon_C}{\partial P_{in}} = -T_0 \frac{s_{out}-s_{in}}{(h_{out}-h_{in})^2} . (\frac{\partial h_{in}}{\partial P_{in}})_{T_{in}} + T_0 \frac{1}{h_{out}-h_{in}} . (\frac{\partial s_{in}}{\partial P_{in}})_{T_{in}}$$

$$\Rightarrow \frac{\partial \varepsilon_C}{\partial P_{in}} = \frac{-T0}{h_{out}-h_{in}} [\frac{s_{out}-s_{in}}{h_{out}-h_{in}} (\frac{\partial h_{in}}{\partial P_{in}})_{T_{in}} - (\frac{\partial s_{in}}{\partial P_{in}})_{T_{in}}]$$

$$\Rightarrow \frac{\partial \varepsilon_C}{\partial P_{in}} = \frac{-T0}{h_{out}-h_{in}} [(\frac{1-\varepsilon_C}{T_0})(\frac{\partial h_{in}}{\partial P_{in}})_{T_{in}} - (\frac{\partial s_{in}}{\partial P_{in}})_{T_{in}}] \tag{3-13}$$

By the same procedure, we can find $\frac{\partial \varepsilon_C}{\partial T_{in}}$.

$$\frac{\partial \varepsilon_C}{\partial T_{in}} = \frac{\partial \varepsilon_C}{\partial h_{in}} . (\frac{\partial h_{in}}{\partial T_{in}})_{P_{in}} + \frac{\partial \varepsilon_C}{\partial s_{in}} . (\frac{\partial s_{in}}{\partial T_{in}})_{P_{in}} \tag{3-14}$$

By using equations (3-11) and (3-12):

$$\Rightarrow \frac{\partial \varepsilon_C}{\partial T_{in}} = -T_0 \frac{s_{out}-s_{in}}{(h_{out}-h_{in})^2} . (\frac{\partial h_{in}}{\partial T_{in}})_{P_{in}} + T_0 \frac{1}{h_{out}-h_{in}} . (\frac{\partial s_{in}}{\partial T_{in}})_{P_{in}}$$

$$\Rightarrow \frac{\partial \varepsilon_C}{\partial T_{in}} = \frac{-T0}{h_{out}-h_{in}} [\frac{s_{out}-s_{in}}{h_{out}-h_{in}} (\frac{\partial h_{in}}{\partial T_{in}})_{P_{in}} - (\frac{\partial s_{in}}{\partial T_{in}})_{P_{in}}]$$

$$\Rightarrow \frac{\partial \varepsilon_C}{\partial T_{in}} = \frac{-T0}{h_{out}-h_{in}} [(\frac{1-\varepsilon_C}{T_0})(\frac{\partial h_{in}}{\partial T_{in}})_{P_{in}} - (\frac{\partial s_{in}}{\partial T_{in}})_{P_{in}}] \tag{3-15}$$

By using equations (3-9), (3-13), and (3-15) in equation (3-2):

$$\begin{aligned} \Rightarrow \qquad d\varepsilon_C = {} & dP_{out} \cdot \left[\frac{T0}{h_{out}-h_{in}}\left(\left(\frac{1-\varepsilon_C}{T_0}\right)\left(\frac{\partial h_{out}}{\partial P_{out}}\right)_{T_{out}} - \left(\frac{\partial s_{out}}{\partial P_{out}}\right)_{T_{out}}\right)\right] \\ & + dP_{in} \cdot \left[\frac{-T0}{h_{out}-h_{in}}\left(\left(\frac{1-\varepsilon_C}{T_0}\right)\left(\frac{\partial h_{in}}{\partial P_{in}}\right)_{T_{in}} - \left(\frac{\partial s_{in}}{\partial P_{in}}\right)_{T_{in}}\right)\right] \\ & + dT_{in} \cdot [\frac{-T0}{h_{out}-h_{in}}\left(\left(\frac{1-\varepsilon_C}{T_0}\right)\left(\frac{\partial h_{in}}{\partial T_{in}}\right)_{P_{in}} - \left(\frac{\partial s_{in}}{\partial T_{in}}\right)_{P_{in}}\right)] \end{aligned}$$

$$\Rightarrow d\varepsilon_C = \frac{T0}{h_{out}-h_{in}}.\,(\,dP_{out}\left(\left(\frac{1-\varepsilon_C}{T_0}\right)\left(\frac{\partial h_{out}}{\partial P_{out}}\right)_{T_{out}} - \left(\frac{\partial s_{out}}{\partial P_{out}}\right)_{T_{out}}\right) - dP_{in}\left(\left(\frac{1-\varepsilon_C}{T_0}\right)\left(\frac{\partial h_{in}}{\partial P_{in}}\right)_{T_{in}} - \left(\frac{\partial s_{in}}{\partial P_{in}}\right)_{T_{in}}\right)$$

$$-dT_{in}\left(\left(\frac{1-\varepsilon_C}{T_0}\right)\left(\frac{\partial h_{in}}{\partial T_{in}}\right)_{P_{in}} - \left(\frac{\partial s_{in}}{\partial T_{in}}\right)_{P_{in}}\right)) \tag{3-16}$$

By using thermodynamic relations we can rewrite equation (3-16).

$$(\frac{\partial h}{\partial P})_T = \vartheta + T(\frac{\partial s}{\partial P})_T \tag{3-17}$$

$$(\frac{\partial s}{\partial P})_T = -(\frac{\partial \vartheta}{\partial T})_P \tag{3-18}$$

$$C_P = (\frac{\partial h}{\partial T})_P = T(\frac{\partial s}{\partial T})_P \tag{3-19}$$

By using equations (3-17)-(3-19) in equation (3-16) we will find:

$$\Rightarrow d\varepsilon_C = \frac{T0}{h_{out}-h_{in}}(\,dP_{out}\left(\vartheta_{out}\left(\frac{1-\varepsilon_C}{T_0}\right) + (1 - T_{out}\left(\frac{1-\varepsilon_C}{T_0}\right))\left(\frac{\partial \vartheta_{out}}{\partial T_{out}}\right)_{P_{out}}\right)$$

$$-dP_{in}\left(\vartheta_{in}\left(\frac{1-\varepsilon_C}{T_0}\right) + (1 - T_{in}\left(\frac{1-\varepsilon_C}{T_0}\right))\left(\frac{\partial \vartheta_{in}}{\partial T_{in}}\right)_{P_{in}}\right)$$

$$-dT_{in}\left(\left(\frac{1-\varepsilon_C}{T_0}\right)C_P - \frac{C_P}{T_{in}}\right)) \tag{3-20}$$

We can simplify equation (3-20) for ideal gas by considering $P\vartheta = RT \Rightarrow \frac{\partial \vartheta}{\partial T} = \frac{R}{P} = \frac{\vartheta}{T}$

$$\Rightarrow d\varepsilon_C = \frac{T0}{h_{out}-h_{in}}(\,dP_{out}\left(\vartheta_{out}\left(\frac{1-\varepsilon_C}{T_0}\right) + (1 - T_{out}\left(\frac{1-\varepsilon_C}{T_0}\right))\frac{\vartheta_{out}}{T_{out}}\right)$$

$$-dP_{in}\left(\vartheta_{in}\left(\frac{1-\varepsilon_C}{T_0}\right) + (1 - T_{in}\left(\frac{1-\varepsilon_C}{T_0}\right))\frac{\vartheta_{in}}{T_{in}}\right)$$

$$-dT_{in}\left(\left(\frac{1-\varepsilon_C}{T_0}\right)C_P - \frac{C_P}{T_{in}}\right))$$

$$\Rightarrow d\varepsilon_C = \frac{T0}{h_{out}-h_{in}}(dP_{out}\frac{\vartheta_{out}}{T_{out}} - dP_{in}\frac{\vartheta_{in}}{T_{in}} - dT_{in}\left(\left(\frac{1-\varepsilon_C}{T_0}\right)C_P - \frac{C_P}{T_{in}}\right))$$

$$\Rightarrow d\varepsilon_C = \frac{T0}{h_{out}-h_{in}}(R\frac{dP_{out}}{P_{out}} - R\frac{dP_{in}}{P_{in}} - C_P.\,dT_{in}\left(\left(\frac{1-\varepsilon_C}{T_0}\right) - \frac{1}{T_{in}}\right)) \tag{3-21}$$

As we can see from equation (3-21), exergy efficiency of compressor will increase by increasing outlet pressure and decreasing inlet pressure (increasing pressure ratio) for ideal gas when the inlet temperature and isentropic efficiency are constant.

It is difficult to decide about the sign of $\frac{d\varepsilon_C}{dP_{out}}$ and $\frac{d\varepsilon_C}{dP_{out}}$ by referring to above equations. By using sensitivity analysis technique, we can find the effects of inlet and outlet pressure and inlet temperature on efficiency of compressor in a specified range. In these study cases, the working fluid is methane and its mass velocity is 1 (kg/s). The isentropic efficiency of compressor is equal to 82% and SRK is used as equation of state.

Figure (3-2) shows effect of outlet pressure on exergy efficiency of compressor. In this case inlet pressure is 30 bar and inlet temperature is 30°C.

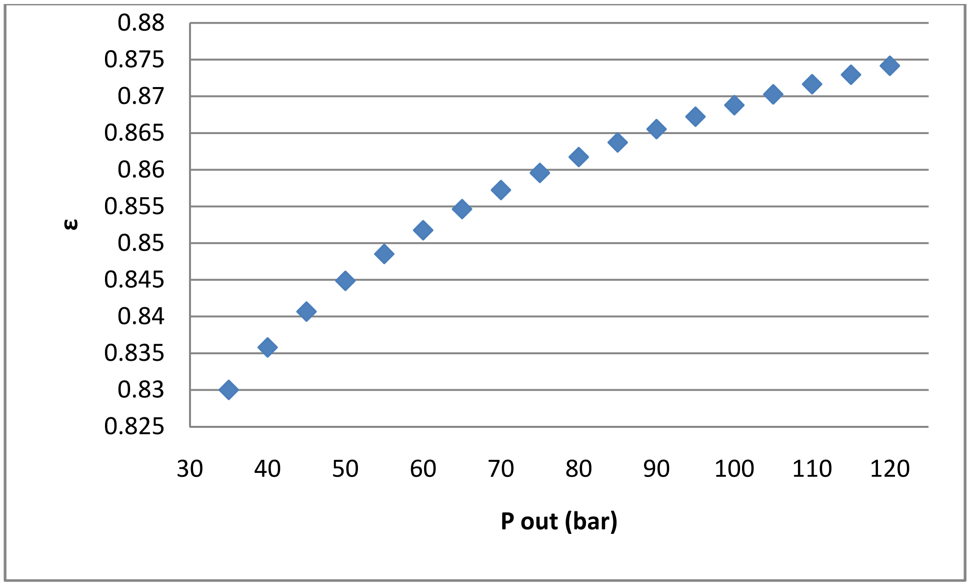


Figure 3.2 Effect of outlet pressure on exergy efficiency of a compressor

Figure (3-3) shows effect of inlet pressure on exergy efficiency of compressor. In this case outlet pressure is 100 bar and inlet temperature is 30°C.

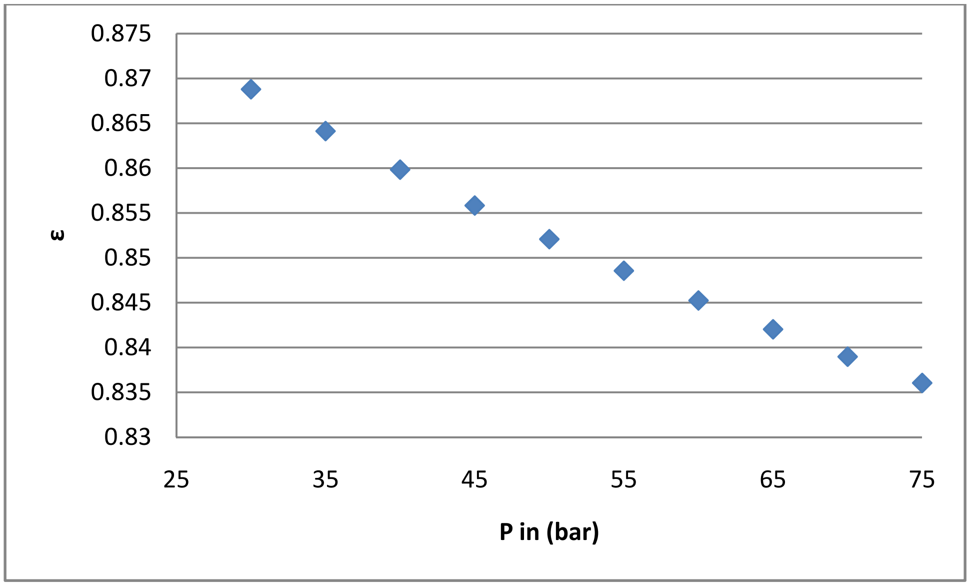


Figure 3.3 Effect of inlet pressure on exergy efficiency of a compressor

As we can see in figures (3-2) and (3-3), by increasing pressure ratio exergy efficiency of compressor will increase. For describing effect of pressure ratio on exergy efficiency of compressor we can refer to figure (3-4).

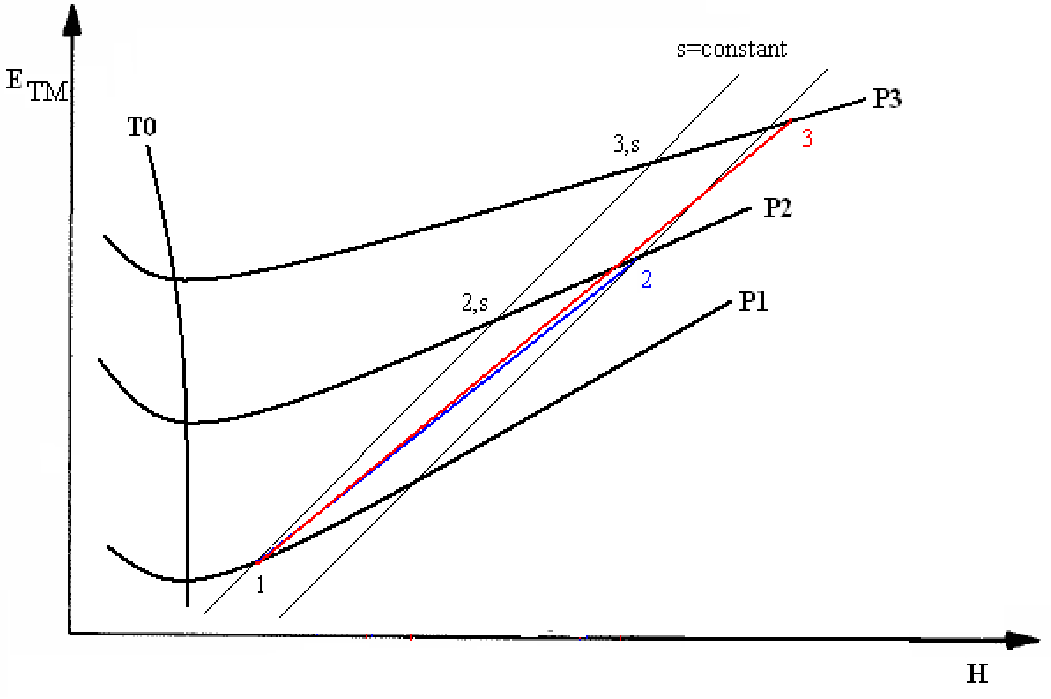


Figure 3.4 Exergy-enthalpy diagram for a compressor which works above ambient temperature

The slop of line between point 3 and point 1 is the exergy efficiency of compressor which increase the pressure from P1 to P3, since $slope(1-3)=\frac{\dot{E}^{TM}{}_{3}-\dot{E}^{TM}{}_{1}}{(H_3-H_1)}=\varepsilon_C$. With the same discussion the slop of line between point 2 and point 1 is the exergy efficiency of compressor which works between P1 and P2. If we can prove that the slope (1-3) is larger than slope (1-2), we can conclude that by increasing pressure ratio, exergy efficiency increases. For proving this fact we can use the below approach.

From isentropic efficiency definition:$\eta_{is,com}=\frac{H_{2,s}-H_1}{H_2-H_1}=\frac{H_{3,s}-H_1}{H_3-H_1}$

$$\Rightarrow\frac{H_{2,s}-H_1}{H_{3,s}-H_1}=\frac{H_2-H_1}{H_3-H_1} \qquad (*)$$

Moreover, for an isentropic compressor we have: $E^{TM}{}_{out,s}-E^{TM}{}_{in}=H_{out,s}-H_{in}$ since entropy is constant.

$$\Rightarrow\frac{E^{TM}{}_{2,s}-E^{TM}{}_{1}}{E^{TM}{}_{3,s}-E^{TM}{}_{1}}=\frac{H_{2,s}-H_1}{H_{3,3}-H_1} \qquad (**)$$

(*) and (**)$\Rightarrow\frac{E^{TM}{}_{2,s}-E^{TM}{}_{1}}{H_2-H_1}=\frac{E^{TM}{}_{3,s}-E^{TM}{}_{1}}{H_3-H_1}$

$$\Rightarrow\frac{E^{TM}{}_{2}-(E^{TM}{}_{2}-E^{TM}{}_{2,s})-E^{TM}{}_{1}}{H_2-H_1}=\frac{E^{TM}{}_{3}-(E^{TM}{}_{3}-E^{TM}{}_{3,s})-E^{TM}{}_{1}}{H_3-H_1}$$

$$\Rightarrow\frac{E^{TM}{}_{2}-E^{TM}{}_{1}}{H_2-H_1}-\frac{E^{TM}{}_{2}-E^{TM}{}_{2,s}}{H_2-H_1}=\frac{E^{TM}{}_{3}-E^{TM}{}_{1}}{H_3-H_1}-\frac{E^{TM}{}_{3}-E^{TM}{}_{3,s}}{H_3-H_1} \qquad (***)$$

$E^{TM}{}_{3}-E^{TM}{}_{3,s}<E^{TM}{}_{2}-E^{TM}{}_{2,s}$since slope of exergy-enthalpy line at constant pressure will increase by increasing pressure. We can see that in figure (3-5) And $H_3-H_1>H_2-H_1$ we can conclude that:

$$\frac{E^{TM}{}_{2}-E^{TM}{}_{2,s}}{H_2-H_1}>\frac{E^{TM}{}_{3}-E^{TM}{}_{3,s}}{H_3-H_1} \qquad (****)$$

(***) and (****)$\Rightarrow\frac{E^{TM}{}_{3}-E^{TM}{}_{1}}{H_3-H_1}>\frac{E^{TM}{}_{2}-E^{TM}{}_{1}}{H_2-H_1}\Rightarrow\varepsilon_{3-1}>\varepsilon_{2-1}$

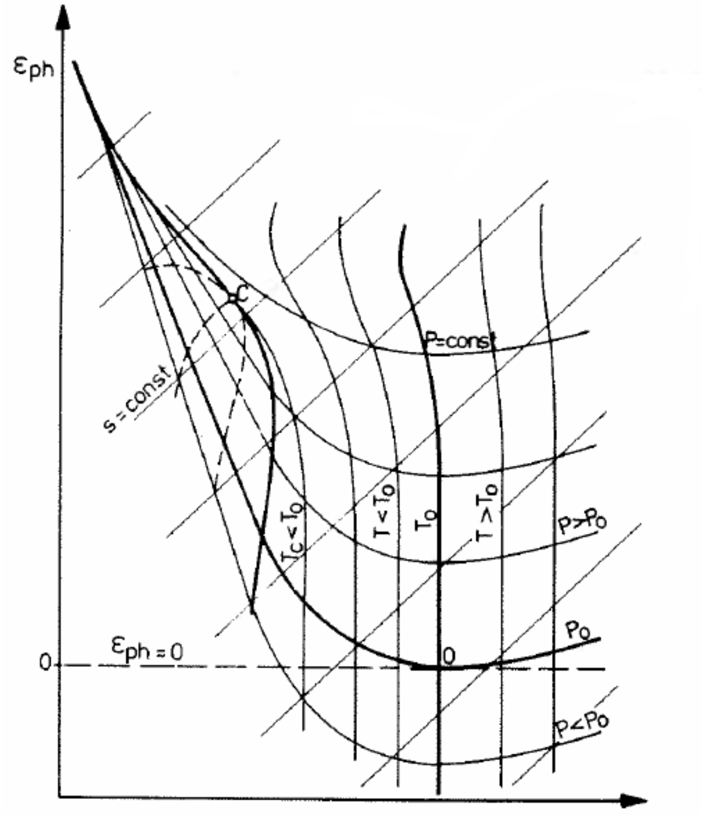


Figure 3.5 Exergy-enthalpy diagram (Kotas, 1995)

The effect of inlet temperature on compressor's exergy efficiency depends on magnitude of exergy efficiency and inlet temperature. As we can see in equation (3-20) $\frac{d\varepsilon_C}{dT_{in}} = -C_P\left(\left(\frac{1-\varepsilon_C}{T_0}\right) - \frac{1}{T_{in}}\right)$. If $T_{in}(1-\varepsilon_C) > T_0$, exergy efficiency of compressor will decrease by increasing inlet temperature for ideal gas. Otherwise, it will increase. Since $T_{in}(1-\varepsilon_C) > T_0$ occurs in very high temperature, we can simply say that exergy efficiency of compressor will increase by increasing inlet temperature.

Figure (3-6) shows effect of inlet temperature on exergy efficiency of compressor. In this case outlet pressure is 70 bar and inlet pressure is 30 bar.

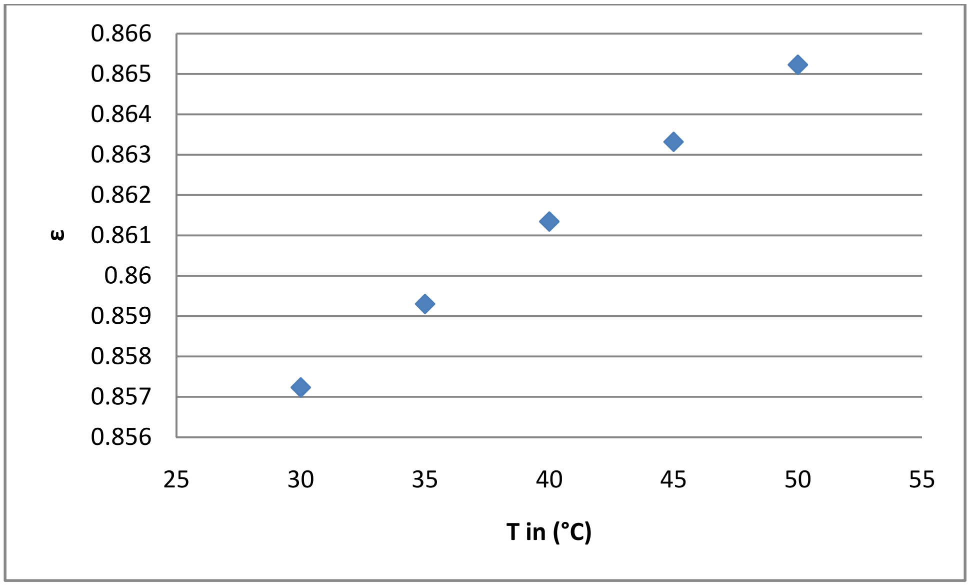


Figure 3.6 Effect of inlet temperature on exergy efficiency of a compressor

The effect of pressure and temperature on compressor which works across ambient temperature are found analytically only, since all the compressor which works in the process study case (Dual independent refrigeration cycles) are working above ambient temperature. The exergy efficiency for a compressor which works across ambient temperature is calculated by equation (3-22):

$$\varepsilon_C = \frac{\dot{E}^p{}_{out} - \dot{E}^p{}_{in} + \dot{E}^T{}_{out}}{\dot{W}_C + \dot{E}^T{}_{in}} \tag{3-22}$$

We can rewrite equation (3-22) in format of equation (3-23).

$$\varepsilon_C = 1 - \dot{m} T_0 \frac{s_{out} - s_{in}}{\dot{W}_C + \dot{E}^T{}_{in}} \tag{3-23}$$

$$\Rightarrow \varepsilon_C = 1 - T_0 \frac{s_{out} - s_{in}}{(h_{out} - h_{in}) + (h_{in} - h(T_0, p_{in})) - T_0(s_{in} - s(T_0, p_{in}))}$$

$$\Rightarrow \varepsilon_C = 1 - T_0 \frac{s_{out} - s_{in}}{(h_{out} - h(T_0, p_{in})) - T_0(s_{in} - s(T_0, p_{in}))} \tag{3-24}$$

We can calculate effect of different parameters on exergy efficiency of compressor which works across ambient temperature with the same approach as we have calculated equation (3-20).

$$\frac{d\varepsilon_C}{dP_{out}} = \frac{\partial \varepsilon_C}{\partial h_{out}} . \left(\frac{\partial h_{out}}{\partial P_{out}}\right)_T + \frac{\partial \varepsilon_C}{\partial s_{out}} . \left(\frac{\partial s_{out}}{\partial P_{out}}\right)_T \tag{3-25}$$

We can find $\frac{\partial \varepsilon_C}{\partial h_{out}}$ and $\frac{\partial \varepsilon_C}{\partial s_{out}}$ from equation (3-24):

$$\Rightarrow \frac{\partial \varepsilon_C}{\partial h_{out}} = T_0 \frac{s_{out} - s_{in}}{(h_{out} - h(T_0, p_{in})) - T_0(s_{in} - s(T_0, p_{in}))^2} \tag{3-26}$$

$$\Rightarrow \frac{\partial \varepsilon_C}{\partial s_{out}} = -T_0 \frac{1}{(h_{out} - h(T_0, p_{in})) - T_0(s_{in} - s(T_0, p_{in}))} \tag{3-27}$$

By using equations (3-26) and (3-27) in equation (3-25) we will have:

$$\frac{d\varepsilon_C}{dP_{out}} = T_0 \frac{s_{out} - s_{in}}{\left(h_{out} - h(T_0, p_{in})\right) - T_0(s_{in} - s(T_0, p_{in}))^2} \left(\frac{\partial h_{out}}{\partial P_{out}}\right)_{T_{out}} - T_0 \frac{1}{\left(h_{out} - h(T_0, p_{in})\right) - T_0(s_{in} - s(T_0, p_{in}))} \left(\frac{\partial s_{out}}{\partial P_{out}}\right)_{T_{out}}$$

$$\Rightarrow \frac{d\varepsilon_C}{dP_{out}} = \frac{T0}{(h_{out} - h(T_0, p_{in})) - T_0(s_{in} - s(T_0, p_{in}))} \left[\frac{s_{out} - s_{in}}{(h_{out} - h(T_0, p_{in})) - T_0(s_{in} - s(T_0, p_{in}))} \left(\frac{\partial h_{out}}{\partial P_{out}}\right)_{T_{out}} - \left(\frac{\partial s_{out}}{\partial P_{out}}\right)_{T_{out}}\right]$$

$$\Rightarrow \frac{d\varepsilon_C}{dP_{out}} = \frac{T0}{\dot{W}_C + \dot{E}^T{}_{in}} \left[\left(\frac{1 - \varepsilon_C}{T_0}\right)\left(\frac{\partial h_{out}}{\partial P_{out}}\right)_{T_{out}} - \left(\frac{\partial s_{out}}{\partial P_{out}}\right)_{T_{out}}\right] \tag{3-28}$$

$$\frac{d\varepsilon_C}{dP_{in}} = \frac{\partial \varepsilon_C}{\partial h(T_0, p_{in})} \cdot \left(\frac{\partial h(T_0, p_{in})}{\partial P_{in}}\right)_{T_{in}} + \frac{\partial \varepsilon_C}{\partial s_{in}} \cdot \left(\frac{\partial s_{in}}{\partial P_{in}}\right)_{T_{in}} \tag{3-29}$$

We can find $\frac{\partial \varepsilon_C}{\partial h(T_0, p_{in})}$ and $\frac{\partial \varepsilon_C}{\partial s_{in}}$ from equation (3-24).

$$\Rightarrow \frac{\partial \varepsilon_C}{\partial h(T_0, p_{in})} = -T_0 \frac{s_{out} - s_{in}}{(h_{out} - h(T_0, p_{in})) - T_0(s_{in} - s(T_0, p_{in}))^2} \tag{3-30}$$

$$\Rightarrow \frac{\partial \varepsilon_C}{\partial s_{in}} = T_0\left(\frac{1}{(h_{out} - h(T_0, p_{in})) - T_0\left(s_{in} - s(T_0, p_{in})\right)} - \frac{T_0(s_{out} - s_{in})}{(h_{out} - h(T_0, p_{in})) - T_0\left(s_{in} - s(T_0, p_{in})\right)^2}\right) \tag{3-31}$$

By using equations (3-30) and (3-31) in equation (3-29) we will have:

$$\Rightarrow \frac{d\varepsilon_C}{dP_{in}} = -T_0 \frac{s_{out} - s_{in}}{(h_{out} - h(T_0, p_{in})) - T_0\left(s_{in} - s(T_0, p_{in})\right)^2} \cdot \left(\frac{\partial h(T_0, p_{in})}{\partial P_{in}}\right)_{T_{in}}$$

$$+ \frac{T_0}{(h_{out} - h(T_0, p_{in})) - T_0\left(s_{in} - s(T_0, p_{in})\right)} \left(1 - \frac{T_0(s_{out} - s_{in})}{(h_{out} - h(T_0, p_{in})) - T_0\left(s_{in} - s(T_0, p_{in})\right)}\right) \cdot \left(\frac{\partial s_{in}}{\partial P_{in}}\right)_{T_{in}}$$

$$\Rightarrow \frac{d\varepsilon_C}{dP_{in}} = \frac{-T0}{(h_{out} - h(T_0, p_{in})) - T_0\left(s_{in} - s(T_0, p_{in})\right)} \left[\frac{(s_{out} - s_{in})}{(h_{out} - h(T_0, p_{in})) - T_0\left(s_{in} - s(T_0, p_{in})\right)} \left(\frac{\partial h(T_0, p_{in})}{\partial P_{in}}\right)_{T_{in}}\right.$$

$$\left. -\left(1 - \frac{T_0(s_{out} - s_{in})}{(h_{out} - h(T_0, p_{in})) - T_0\left(s_{in} - s(T_0, p_{in})\right)}\right)\left(\frac{\partial s_{in}}{\partial P_{in}}\right)_{T_{in}}\right]$$

$$\Rightarrow \frac{d\varepsilon_C}{dP_{in}} = \frac{-T0}{\dot{W}_C + \dot{E}^T{}_{in}} \left[\left(\frac{1 - \varepsilon_C}{T_0}\right)\left(\frac{\partial h(T_0, p_{in})}{\partial P_{in}}\right)_{T_{in}} - \varepsilon_C\left(\frac{\partial s_{in}}{\partial P_{in}}\right)_{T_{in}}\right] \tag{3-32}$$

By the same procedure, we can find $\frac{\partial \varepsilon_C}{\partial T_{in}}$.

$$\frac{d\varepsilon_C}{dT_{in}} = \frac{\partial \varepsilon_C}{\partial s_{in}} \cdot \left(\frac{\partial s_{in}}{\partial T_{in}}\right)_{P_{in}} \tag{3-33}$$

By using equations (3-31):

$$\Rightarrow \frac{d\varepsilon_C}{dT_{in}} = T_0\left(\frac{1}{(h_{out}-h(T_0,p_{in}))-T_0(s_{in}-s(T_0,p_{in}))} - \frac{T_0(s_{out}-s_{in})}{\left((h_{out}-h(T_0,p_{in}))-T_0(s_{in}-s(T_0,p_{in}))\right)^2}\right).\left(\frac{\partial s_{in}}{\partial T_{in}}\right)_{P_{in}}$$

$$\Rightarrow \frac{d\varepsilon_C}{dT_{in}} = \frac{T0}{(h_{out}-h(T_0,p_{in}))-T_0(s_{in}-s(T_0,p_{in}))}\left(1 - \frac{T_0(s_{out}-s_{in})}{(h_{out}-h(T_0,p_{in}))-T_0(s_{in}-s(T_0,p_{in}))}\right)\left(\frac{\partial s_{in}}{\partial T_{in}}\right)_{P_{in}}$$

$$\Rightarrow \frac{d\varepsilon_C}{dT_{in}} = \frac{T0\varepsilon_C}{\dot{W}_C+\dot{E}^T{}_{in}}\left(\frac{\partial s_{in}}{\partial T_{in}}\right)_{P_{in}} \tag{3-34}$$

By using equations (3-28), (3-32), and (3-34) in equation (3-2):

$$\begin{aligned}\Rightarrow \quad d\varepsilon_C = {} & dP_{out}\cdot\left[\frac{T0}{\dot{W}_C+\dot{E}^T{}_{in}}\left(\left(\frac{1-\varepsilon_C}{T_0}\right)\left(\frac{\partial h_{out}}{\partial P_{out}}\right)_{T_{out}} - \left(\frac{\partial s_{out}}{\partial P_{out}}\right)_{T_{out}}\right)\right] \\ & +dP_{in}\cdot\left[\frac{-T0}{\dot{W}_C+\dot{E}^T{}_{in}}\left(\left(\frac{1-\varepsilon_C}{T_0}\right)\left(\frac{\partial h(T_0,p_{in})}{\partial P_{in}}\right)_{T_{in}} - \varepsilon_C\left(\frac{\partial s_{in}}{\partial P_{in}}\right)_{T_{in}}\right)\right] \\ & +dT_{in}\cdot\left[\frac{T0\varepsilon_C}{\dot{W}_C+\dot{E}^T{}_{in}}\left(\frac{\partial s_{in}}{\partial T_{in}}\right)_{P_{in}}\right]\end{aligned}$$

$$\begin{aligned}\Rightarrow d\varepsilon_C = {} & \frac{T0}{\dot{W}_C+\dot{E}^T{}_{in}}.\Bigg(dP_{out}\left(\left(\frac{1-\varepsilon_C}{T_0}\right)\left(\frac{\partial h_{out}}{\partial P_{out}}\right)_{T_{out}} - \left(\frac{\partial s_{out}}{\partial P_{out}}\right)_{T_{out}}\right) \\ & -dP_{in}\left(\left(\frac{1-\varepsilon_C}{T_0}\right)\left(\frac{\partial h(T_0,p_{in})}{\partial P_{in}}\right)_{T_{in}} - \varepsilon_C\left(\frac{\partial s_{in}}{\partial P_{in}}\right)_{T_{in}}\right) + dT_{in}\left(\varepsilon_C\left(\frac{\partial s_{in}}{\partial T_{in}}\right)_{P_{in}}\right)\Bigg)\end{aligned} \tag{3-35}$$

By using thermodynamic relations (3-17)-(3-19) we can rewrite equation (3-35).

$$\begin{aligned}\Rightarrow d\varepsilon_C = {} & \frac{T0}{\dot{W}_C+\dot{E}^T{}_{in}}.\Bigg(dP_{out}\left(\left(\frac{1-\varepsilon_C}{T_0}\right)\left(\vartheta_{out} + T_{out}\left(\frac{\partial s_{out}}{\partial P_{out}}\right)_{T_{out}}\right) - \left(\frac{\partial s_{out}}{\partial P_{out}}\right)_{T_{out}}\right) \\ & -dP_{in}\left(\left(\frac{1-\varepsilon_C}{T_0}\right)\left(\vartheta(T_0,p_{in}) + T_0\left(\frac{\partial s(T_0,p_{in})}{\partial P_{in}}\right)_{T_0}\right) - \varepsilon_C\left(\frac{\partial s_{in}}{\partial P_{in}}\right)_{T_{in}}\right) + dT_{in}\left(\varepsilon_C\left(\frac{\partial s_{in}}{\partial T_{in}}\right)_{P_{in}}\right)\Bigg)\end{aligned}$$

$$\begin{aligned}\Rightarrow d\varepsilon_C = {} & \frac{T0}{\dot{W}_C+\dot{E}^T{}_{in}}\Bigg(dP_{out}\left(\vartheta_{out}\left(\frac{1-\varepsilon_C}{T_0}\right) + \left(1 - T_{out}\left(\frac{1-\varepsilon_C}{T_0}\right)\right)\left(\frac{\partial \vartheta_{out}}{\partial T_{out}}\right)_{P_{out}}\right) \\ & -dP_{in}\left(\left(\frac{1-\varepsilon_C}{T_0}\right)\left(\vartheta(T_0,p_{in}) - T_0\left(\frac{\partial \vartheta(T_0,p_{in})}{\partial T}\right)_{p_{in}}\right) + \varepsilon_C\left(\frac{\partial \vartheta_{in}}{\partial T_{in}}\right)_{p_{in}}\right) \\ & +dT_{in}\left(\varepsilon_C\frac{C_p}{T_{in}}\right)\Bigg)\end{aligned} \tag{3-36}$$

We can simplify equation (3-36) for ideal gas by considering $P\vartheta = RT \Rightarrow \frac{\partial\vartheta}{\partial T} = \frac{R}{P} = \frac{\vartheta}{T}$

$$\Rightarrow d\varepsilon_C = \frac{T0}{\dot{W}_C+\dot{E}^T{}_{in}}\Big(dP_{out}\left(\vartheta_{out}\left(\frac{1-\varepsilon_C}{T_0}\right) + (1 - T_{out}\left(\frac{1-\varepsilon_C}{T_0}\right))\frac{\vartheta_{out}}{T_{out}}\right)$$

$$-dP_{in}\left(\left(\frac{1-\varepsilon_C}{T_0}\right)\left(\vartheta(T_0, p_{in}) - T_0\frac{\vartheta(T_0,p_{in})}{T_0}\right) + \varepsilon_C\frac{\vartheta_{in}}{T_{in}}\right)$$

$$+dT_{in}\left(\varepsilon_C\frac{C_p}{T_{in}}\right)\Big)$$

$$\Rightarrow d\varepsilon_C = \frac{T0}{\dot{W}_C+\dot{E}^T{}_{in}}\left(dP_{out}\frac{\vartheta_{out}}{T_{out}} - dP_{in}\varepsilon_C\frac{\vartheta_{in}}{T_{in}} + dT_{in}\left(\varepsilon_C\frac{C_p}{T_{in}}\right)\right) \tag{3-37}$$

As we can found from equation (3-37), the exergy efficiency of a compressor which works across ambient temperature is increased by pressure ratio and inlet temperature.

3.1.3. Exergy analysis of turbine

With the same method as we analysis the exergy efficiency of compressor, we can define the exergy efficiency of turbine as a function of four parameters which is shown in equation (3-38).

$$\varepsilon_T = f(P_{out}, P_{out}, T_{in}, \eta_{is}) \tag{3-38}$$

$$\Rightarrow d\varepsilon_T = \left(\frac{\partial\varepsilon_T}{\partial P_{out}}\right)dP_{out} + \left(\frac{\partial\varepsilon_T}{\partial P_{in}}\right)dP_{in} + \left(\frac{\partial\varepsilon_T}{\partial T_{in}}\right)dT_{in} + \left(\frac{\partial\varepsilon_T}{\partial\eta_{is}}\right)d\eta_{is} \tag{3-39}$$

Since the isentropic efficiency of turbine is related to design parameters, it is constant and the last term of equation (3-40) is zero. It is necessary to find the effects of other parameter on exergy efficiency of turbine. Exergy efficiency of turbine which works below ambient temperature is calculated by equation (3-40).

$$\varepsilon_T = \frac{\dot{W}_C+(\dot{E}^T{}_{out}-\dot{E}^T{}_{in})}{\dot{E}^P{}_{in}-\dot{E}^P{}_{out}} \tag{3-40}$$

where, $\dot{W}_T = \dot{m}.[h_{in}(T,P) - h_{out}(T,P)]$

and $\dot{E}^T{}_{out} - \dot{E}^T{}_{in} = \dot{m}.[\big(h_{out}(T,P) - h_{in}(T,P)\big) - \big(h_{out}(T_0,P) - h_{in}(T_0,P)\big)]$

$$-\dot{m}T_0[\big(s_{out}(T,P) - s_{in}(T,P)\big) - \big(s_{out}(T_0,P) - s_{in}(T_0,P)\big)]$$

and $\dot{E}^P{}_{in} - \dot{E}^P{}_{out} = \dot{m}.[\big(h_{in}(T_0,P) - h_{out}(T_0,P)\big) - T_0\big(s_{in}(T_0,P) - s_{out}(T_0,P)\big)]$

$$\Rightarrow \varepsilon_T = 1 - T_0\frac{\big(s_{out}(T,P)-s_{in}(T,P)\big)}{\big(h_{in}(T_0,P)-h_{out}(T_0,P)\big)-T_0\big(s_{in}(T_0,P)-s_{out}(T_0,P)\big)} \tag{3-41}$$

We can find the three first terms of right hand of equation (3-39) by referring to equation (3-41). Equation (3-42) calculates the changes in exergy efficiency of turbine by changing the outlet pressure.

$$\Longrightarrow \frac{d\varepsilon_T}{dP_{out}} = \frac{\partial \varepsilon_T}{\partial h_{out}(T_0,P)} . \frac{\partial h_{out}(T_0,P)}{\partial P_{out}} + \frac{\partial \varepsilon_T}{\partial s_{out}(T_0,P)} . \frac{\partial s_{out}(T_0,P)}{\partial P_{out}} + \frac{\partial \varepsilon_T}{\partial s_{out}(T,P)} . \frac{\partial s_{out}(T,P)}{\partial P_{out}} \qquad (3\text{-}42)$$

We can find $\frac{\partial \varepsilon_T}{\partial h_{out}(T_0,P)}$, $\frac{\partial \varepsilon_T}{\partial s_{out}(T_0,P)}$,and $\frac{\partial \varepsilon_T}{\partial s_{out}(T,P)}$ from equation (3-41):

$$\frac{\partial \varepsilon_T}{\partial h_{out}(T_0,P)} = -T_0 \frac{\left(s_{out}(T,P) - s_{in}(T,P)\right)}{[\left(h_{in}(T_0,P) - h_{out}(T_0,P)\right) - T_0\left(s_{in}(T_0,P) - s_{out}(T_0,P)\right)]^2} \qquad (3\text{-}43)$$

$$\frac{\partial \varepsilon_T}{\partial s_{out}(T_0,P)} = {T_0}^2 \frac{\left(s_{out}(T,P) - s_{in}(T,P)\right)}{[\left(h_{in}(T_0,P) - h_{out}(T_0,P)\right) - T_0\left(s_{in}(T_0,P) - s_{out}(T_0,P)\right)]^2} \qquad (3\text{-}44)$$

$$\frac{\partial \varepsilon_T}{\partial s_{out}(T,P)} = -\frac{T_0}{\left(h_{in}(T_0,P) - h_{out}(T_0,P)\right) - T_0\left(s_{in}(T_0,P) - s_{out}(T_0,P)\right)} \qquad (3\text{-}45)$$

By using equations (3-43)-(3-45) in equation (3-42) we will find:

$$\frac{d\varepsilon_T}{dP_{out}} = \frac{T_0}{\left(h_{in}(T_0,P) - h_{out}(T_0,P)\right) - T_0\left(s_{in}(T_0,P) - s_{out}(T_0,P)\right)} .$$

$$[-\frac{\left(s_{out}(T,P) - s_{in}(T,P)\right)}{\left(h_{in}(T_0,P) - h_{out}(T_0,P)\right) - T_0\left(s_{in}(T_0,P) - s_{out}(T_0,P)\right)} . \frac{\partial h_{out}(T_0,P)}{\partial P_{out}} +$$
$$\frac{T_0 . \left(s_{out}(T,P) - s_{in}(T,P)\right)}{\left(h_{in}(T_0,P) - h_{out}(T_0,P)\right) - T_0\left(s_{in}(T_0,P) - s_{out}(T_0,P)\right)} . \frac{\partial s_{out}(T_0,P)}{\partial P_{out}} - \frac{\partial s_{out}(T,P)}{\partial P_{out}}] \qquad (3\text{-}46)$$

By considering $\frac{\left(s_{out}(T,P) - s_{in}(T,P)\right)}{\left(h_{in}(T_0,P) - h_{out}(T_0,P)\right) - T_0\left(s_{in}(T_0,P) - s_{out}(T_0,P)\right)} = \frac{1-\varepsilon_T}{T_0}$ from equation (3-41) and $\frac{\dot{E}^P{}_{in} - \dot{E}^P{}_{out}}{\dot{m}} = \left(h_{in}(T_0,P) - h_{out}(T_0,P)\right) - T_0\left(s_{in}(T_0,P) - s_{out}(T_0,P)\right)$ we can simplify the above equation.

$$\frac{d\varepsilon_T}{dP_{out}} = \dot{m} \frac{T_0}{\dot{E}^P{}_{in} - \dot{E}^P{}_{out}} [-(\frac{1-\varepsilon_T}{T_0}) \frac{\partial h_{out}(T_0,P)}{\partial P_{out}} + (1-\varepsilon_T) \frac{\partial s_{out}(T_0,P)}{\partial P_{out}} - \frac{\partial s_{out}(T,P)}{\partial P_{out}}] \qquad (3\text{-}47)$$

By using equations (3-17) and (3-18) we can simplify equation (3-47).

$$\Longrightarrow \frac{d\varepsilon_T}{dP_{out}} = \dot{m} \frac{T_0}{\dot{E}^P{}_{in} - \dot{E}^P{}_{out}} [(\frac{\partial \vartheta_{out}}{\partial T_{out}})_{P_{out}} - \left(\frac{1-\varepsilon_T}{T_0}\right) \vartheta(P_{out}, T_0)] \qquad (3\text{-}48)$$

For ideal gas $(\frac{\partial \vartheta_{out}}{\partial T_{out}})_{P_{out}} = \frac{R}{P_{out}}$ and $\vartheta(P_{out}, T_0) = \frac{R.T_0}{P_{out}}$

$$\Longrightarrow \frac{d\varepsilon_T}{dP_{out}} = \frac{\dot{m} R T_0}{P_{out}(\dot{E}^P{}_{in} - \dot{E}^P{}_{out})} \varepsilon_T \qquad (3\text{-}49)$$

With the same procedure we can find effect of inlet pressure and inlet temperature on exergy efficiency of turbine. Equation (3-50) calculates the changes in efficiency of turbine by changing the inlet pressure.

$$\frac{d\varepsilon_T}{dP_{in}} = \frac{\partial\varepsilon_T}{\partial h_{in}(T_0,P)} . \frac{\partial h_{in}(T_0,P)}{\partial P_{in}} + \frac{\partial\varepsilon_T}{\partial s_{in}(T_0,P)} . \frac{\partial s_{in}(T_0,P)}{\partial P_{in}} + \frac{\partial\varepsilon_T}{\partial s_{in}(T,P)} . \frac{\partial s_{in}(T,P)}{\partial P_{in}} \tag{3-50}$$

We can find $\frac{\partial\varepsilon_T}{\partial h_{in}(T_0,P)}$, $\frac{\partial\varepsilon_T}{\partial s_{in}(T_0,P)}$,and $\frac{\partial\varepsilon_T}{\partial s_{in}(T,P)}$ from equation (3-41):

$$\frac{\partial\varepsilon_T}{\partial h_{in}(T_0,P)} = T_0 \frac{\left(s_{out}(T,P)-s_{in}(T,P)\right)}{[\left(h_{in}(T_0,P)-h_{out}(T_0,P)\right)-T_0\left(s_{in}(T_0,P)-s_{out}(T_0,P)\right)]^2} \tag{3-51}$$

$$\frac{\partial\varepsilon_T}{\partial s_{in}(T_0,P)} = -{T_0}^2 \frac{\left(s_{out}(T,P)-s_{in}(T,P)\right)}{[\left(h_{in}(T_0,P)-h_{out}(T_0,P)\right)-T_0\left(s_{in}(T_0,P)-s_{out}(T_0,P)\right)]^2} \tag{3-52}$$

$$\frac{\partial\varepsilon_T}{\partial s_{in}(T,P)} = \frac{T_0}{\left(h_{in}(T_0,P)-h_{out}(T_0,P)\right)-T_0\left(s_{in}(T_0,P)-s_{out}(T_0,P)\right)} \tag{3-53}$$

By using equations (3-51)-(3-53) in equation (3-50), we will find:

$$\frac{d\varepsilon_T}{dP_{in}} = \frac{T_0}{\left(h_{in}(T_0,P)-h_{out}(T_0,P)\right)-T_0\left(s_{in}(T_0,P)-s_{out}(T_0,P)\right)} .$$
$$[\frac{\left(s_{out}(T,P)-s_{in}(T,P)\right)}{\left(h_{in}(T_0,P)-h_{out}(T_0,P)\right)-T_0\left(s_{in}(T_0,P)-s_{out}(T_0,P)\right)} . \frac{\partial h_{in}(T_0,P)}{\partial P_{in}}$$
$$- \frac{T_0 . \left(s_{out}(T,P)-s_{in}(T,P)\right)}{\left(h_{in}(T_0,P)-h_{out}(T_0,P)\right)-T_0\left(s_{in}(T_0,P)-s_{out}(T_0,P)\right)} . \frac{\partial s_{in}(T_0,P)}{\partial P_{in}} + \frac{\partial s_{in}(T,P)}{\partial P_{in}}]$$

By considering $\frac{\left(s_{out}(T,P)-s_{in}(T,P)\right)}{\left(h_{in}(T_0,P)-h_{out}(T_0,P)\right)-T_0\left(s_{in}(T_0,P)-s_{out}(T_0,P)\right)} = \frac{1-\varepsilon_T}{T_0}$ from equation (3-41) and $\frac{\dot{E}^P{}_{in}-\dot{E}^P{}_{out}}{\dot{m}} = \left(h_{in}(T_0,P)-h_{out}(T_0,P)\right)-T_0\left(s_{in}(T_0,P)-s_{out}(T_0,P)\right)$ we can simplify the above equation.

$$\frac{d\varepsilon_T}{dP_{in}} = \dot{m}\frac{T_0}{\dot{E}^P{}_{in}-\dot{E}^P{}_{out}}[\frac{1-\varepsilon_T}{T_0}\frac{\partial h_{in}(T_0,P)}{\partial P_{in}} - (1-\varepsilon_T)\frac{\partial s_{in}(T_0,P)}{\partial P_{in}} + \frac{\partial s_{in}(T,P)}{\partial P_{in}}] \tag{3-54}$$

By using equations (3-17) and (3-18) in equation (3-54) we can simplify it.

$$\Rightarrow \frac{d\varepsilon_T}{dP_{in}} = \dot{m}\frac{T_0}{\dot{E}^P{}_{in}-\dot{E}^P{}_{out}}[(\frac{1-\varepsilon_T}{T_0})\vartheta(T_0,P_{in}) - \frac{\partial\vartheta_{in}(T,P)}{\partial T_{in}}] \tag{3-55}$$

For ideal gas $\frac{\partial\vartheta_{in}}{\partial T_{in}} = \frac{R}{P_{in}}$ and $\vartheta(P_{in},T_0) = \frac{R.T_0}{P_{in}}$.

$$\Longrightarrow \frac{d\varepsilon_T}{dP_{in}} = -\frac{\dot{m}RT_0}{P_{in}(\dot{E}^P{}_{in} - \dot{E}^P{}_{out})}\varepsilon_T \qquad (3\text{-}56)$$

As we can see in equations (3-49) and (3-56) by increasing outlet pressure and decreasing inlet pressure (decreasing pressure ratio) exergy efficiency of turbine which works below T_0 will increase for ideal gas.

We can show effect of the discussed parameters on exergy efficiency of turbine by using sensitivity analysis technique for non ideal gas. In these study cases, the working fluid is methane and its mass velocity is 1 (kg/s). The isentropic efficiency of turbine is equal to 85% and SRK is used as equation of state.

Figure (3-7) shows the changes in exergy efficiency of turbine by changing in outlet pressure. In this case, inlet pressure and inlet temperature are set at 60 bar and -10°C.

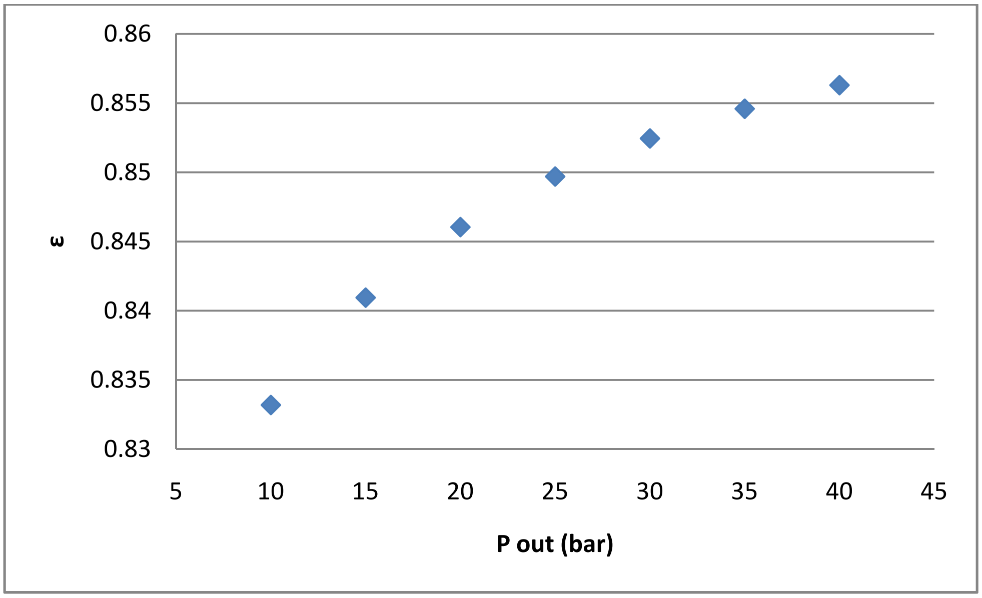


Figure 3.7 Effect of outlet pressure on exergy efficiency of turbine

Figure (3-8) shows the changes in exergy efficiency of turbine by changing inlet pressure. In this case, outlet pressure and inlet temperature are set at 20 bar and -10°C.

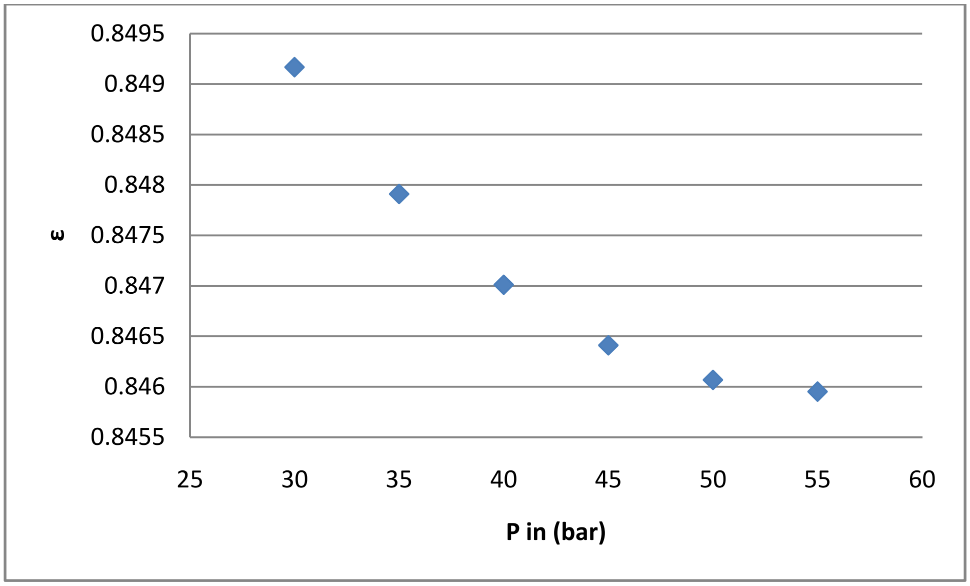


Figure 3.8 Effect of inlet pressure on exergy efficiency of turbine

We can see in figures (3-7) and (3-8), the exergy efficiency of turbine will increase by decreasing pressure ratio.

By increasing inlet temperature, the produced work increases but the diferrence between temperature based exergy at inlet and outlet will decrease more than the increase in produced work. So, the exergy efficiency of turbine will decrease. Figure (3-9) shows the effect of inlet temperature on exergy efficiency of turbine. In this case, inlet and outlet pressure are set at 60 bar and 15 bar.

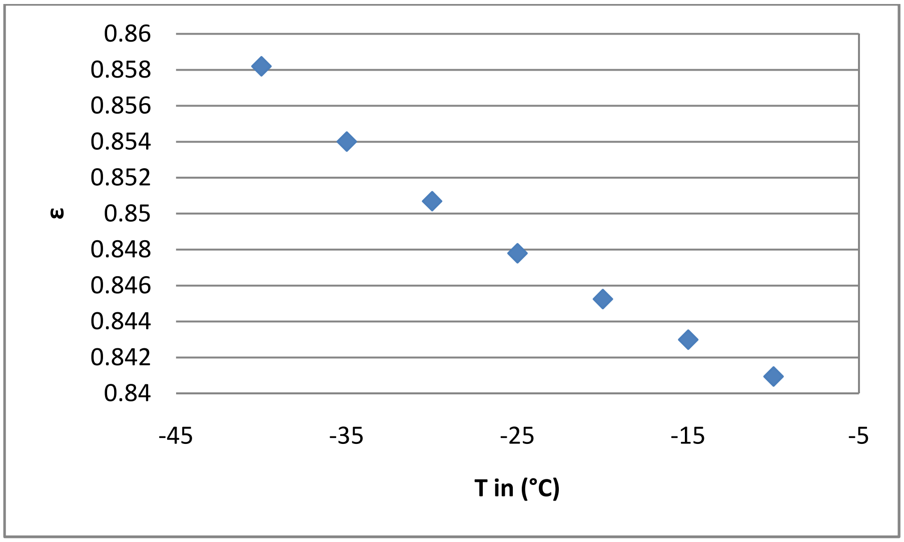


Figure 3.9 Effect of inlet temperature on exergy efficiency of turbine

Finally, we should find effect of isentropic efficiency of turbine on its exergy efficiency. Typical turbine isentropic efficiency is 70-88% (Sonntag et al. 2003). Figure (3-10) shows how exergy efficiency of turbine will change by changing isentropic efficiency. In this case study, the working fluid is methane and its mass velocity is 1 (kg/s). Inlet pressure and outlet pressure are set at 60 and 20 bar and inlet temperature is -10°C. SRK is used as equation of state.

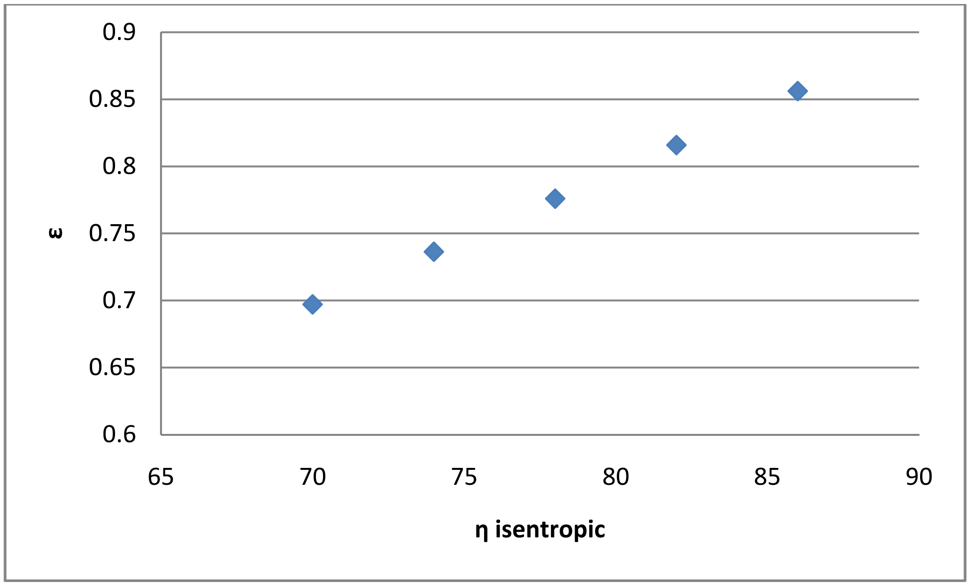


Figure 3.10 Effect of isentropic efficiency on exergy efficiency of turbine

As we can see in figure (3-10), by increasing isentropic efficiency of turbine, its exergy efficiency will increase. Equation (3-57) represents the definition of isentropic efficiency for turbine where $h_{out,s}$ is the enthalpy of outlet stream when the isentropic efficiency of the turbine is 100%.

$$\eta_{is,com} = \frac{h_{in}-h_{out}}{h_{in}-h_{out,s}} \tag{3-57}$$

By increasing isentropic efficiency of turbine, exergy destruction will reduce and exergy efficiency will increase. Moreover, increasing isentropic efficiency of turbine will increase exergy efficiency of process, too, since it does not effect on source of turbine. As a result, relative source will be constant but exergy efficiency will increase.

3.1.2. Exergy analysis of heat exchanger

As I discussed in chapter (2-1-6), pressure drop has positive effect on source and negative effect on sink. Since it increases the irreversibility, it causes reducing exergy efficiency. Moreover, the effect of pressure drop in exergy efficiency is significantly smaller than the effect of temperature except for two phase flow or highly viscous flow (Marmolejo-Correa and Gundersen, 2011).

By increasing the pressure level, the difference between pressure based exergy due to constant pressure drop in heat exchanger will be decreased. So, we can conclude that increasing the pressure decrease the magnitude of source and increase the magnitude of sink which leads to increasing the exergy efficiency in heat exchanger.

For further studying of heat exchanger to find effect of temperature level on its exrgy efficiency, author assumes no pressure drop. Analytical analysis of exergy efficiency for heat exchanger is complicated. As an example, author has done this technique for heat exchanger which works above ambient temperature.

For a heat exchanger which works above ambient temperature exergy efficiency is calculated by equation (3-58).

$$\varepsilon_{HX} = \frac{\dot{E}^T{}_{C,out} - \dot{E}^T{}_{C,in}}{\dot{E}^T{}_{H,in} - \dot{E}^T{}_{H,out}} = \frac{\dot{m}_C[(h_{C,out} - h_{C,in}) - T_0(s_{C,out} - s_{C,in})]}{\dot{m}_H[(h_{H,in} - h_{H,out}) - T_0(s_{H,in} - s_{H,out})]} \tag{3-58}$$

For increasing exergy efficiency, it seems that we can increase mass velocity of cold stream, decrease mass velocity of hot stream, increase the temperature difference between outlet and inlet of cold stream, and decrease the temperature difference between inlet and outlet of hot stream. But we should mention that all of these parameters are related to each other by energy balance in heat exchanger. For instance, if we increase the mass velocity of cold stream, the mass velocity of heat stream will increase too. So, we cannot consider effect of one parameter on exergy efficiency of heat exchanger without considering others.

Energy balance for heat exchanger with one cold stream and one hot stream is:

$$\dot{m}_C(h_{C,out} - h_{C,in}) = \dot{m}_H(h_{H,in} - h_{H,out}) \tag{3-59}$$

By assuming ideal gas we can rewrite equation (3-58) and (3-59).

$$\varepsilon_{HX} = \frac{(\dot{m}C_p)_C[(T_{C,out}-T_{C,in})-T_0\left(ln\frac{T_{C,out}}{T_{C,in}}\right)]}{(\dot{m}C_p)_H[(T_{H,in}-T_{H,out})-T_0\left(ln\frac{T_{H,in}}{T_{H,out}}\right)]} \tag{3-60}$$

$$(\dot{m}C_p)_C(T_{C,out} - T_{C,in}) = (\dot{m}C_p)_H(T_{H,in} - T_{H,out}) \tag{3-61}$$

$T_{C,out} \leq T_{H,in} - \Delta T$ and $T_{C,in} \leq T_{H,out} - \Delta T$. Since by increasing ΔT the exergy destruction will increase in heat exchanger, we assume $T_{C,out} = T_{H,in} - \Delta T$ and $T_{C,in} = T_{H,out} - \Delta T$ to decrease exergy destruction. So, for ideal gas with above assumption, $(\dot{m}C_p)_C = (\dot{m}C_p)_H$. Moreover, author assumed constant temperature difference between inlet and outlet of hot stream $T_{H,in} - T_{H,out} = \nabla$. Then we can rewrite equation (3-60).

$$\varepsilon_{HX} = \frac{\nabla - T_0\left(ln\frac{T_{H,in}-\Delta T}{T_{H,in}-\nabla-\Delta T}\right)}{\nabla - T_0\left(ln\frac{T_{H,in}}{T_{H,in}-\nabla}\right)} \tag{3-62}$$

$$\Longrightarrow \frac{d\varepsilon_{HX}}{dT_{H,in}} = \frac{\nabla.T_0[\frac{\left(\nabla-T_0\left(ln\frac{T_{H,in}}{T_{H,in}-\nabla}\right)\right)}{(T_{H,in}-\Delta T)(T_{H,in}-\Delta T-\nabla)} - \frac{\left(\nabla-T_0\left(ln\frac{T_{H,in}-\Delta T}{T_{H,in}-\nabla-\Delta T}\right)\right)}{T_{H,in}(T_{H,in}-\nabla)}]}{(\nabla-T_0\left(ln\frac{T_{H,in}}{T_{H,in}-\nabla}\right))^2} \tag{3-63}$$

As we can see, analysing equation (3-63) without numerical calculation is very difficult. Moreover, we should mention that this equation was calculated by some assumption. Considering exergy-enthalpy diagram can be used to find effect of temperature level and magnitude of difference between inlet and outlet on exergy efficiency. But, it is not useful since we cannot generalize the result to all heat exchanger because of different scenarios which can happen. Figure (3-11) shows the exergy-enthalpy diagram for the heat exchanger which works above ambient temperature and the pressure of cold stream and hot stream is the same.

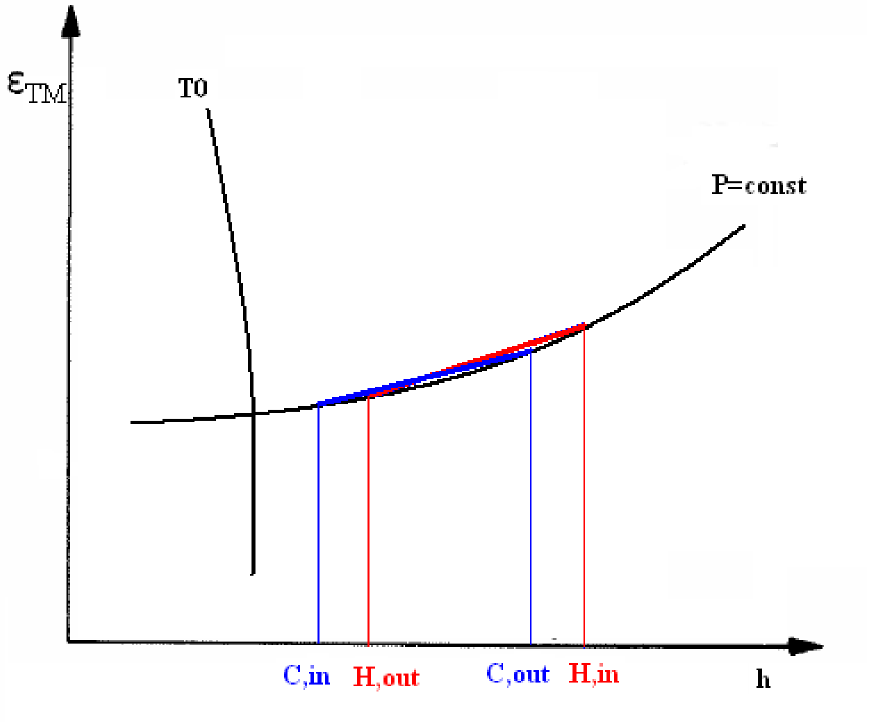


Figure 3.11 Exergy-enthalpy diagram for the heat exchanger which works above ambient temperature and the pressure of cold stream and hot stream is the same

As we can see by increasing the temperature level at the constant pressure, the difference between temperature based exergy of inlet and outlet of hot stream is increase more than the cold stream because the slop of exergy-enthalpy line at the constant pressure increases with increasing the temperature. As a result, the exergy efficiency decreases.

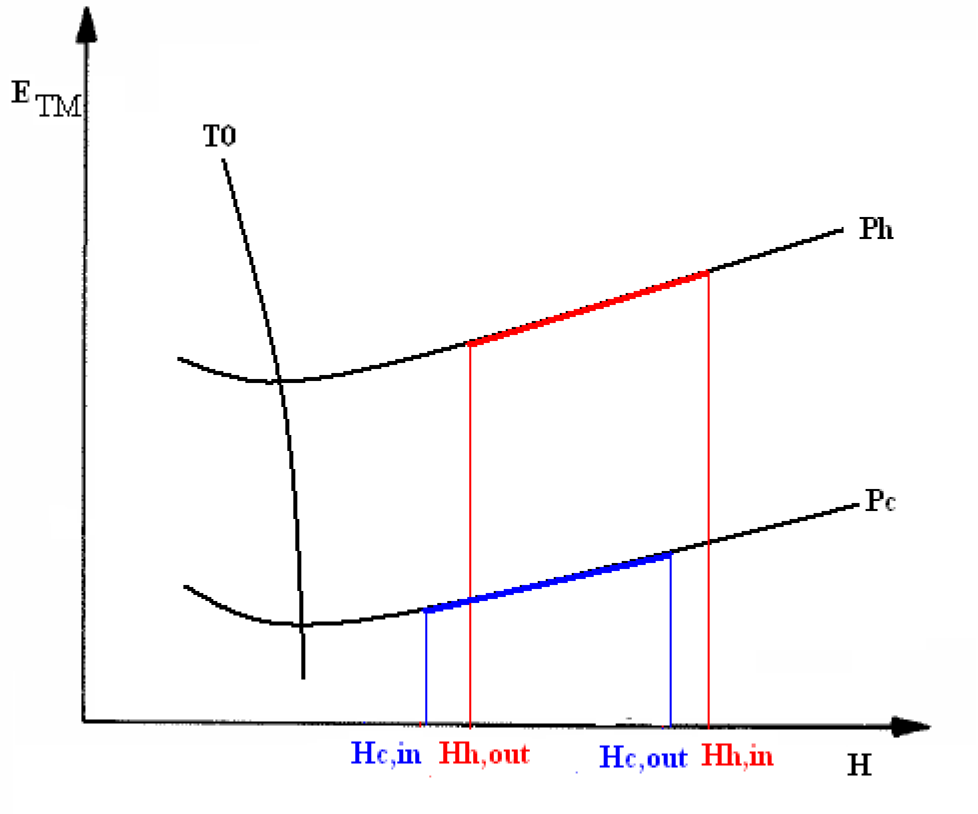


Figure 3.12 Exergy-enthalpy diagram for the heat exchanger which works above ambient temperature and the pressure of cold stream is less than hot stream pressure

Figure (3-12) shows the exergy-enthalpy diagram for the heat exchanger which works above ambient temperature and the pressure of cold stream is less than hot stream pressure. In this case by increasing the temperature level, the difference between inlet and outlet temperature based exergy of cold stream is increased more than hot stream since the slope of exergy enthalpy line at constant pressure increase more by decreasing pressure(figure (3-5)). So, the exergy efficiency will increase.

In conclusion, it is difficult to predict the behaviour of exergy efficiency of heat exchanger by changing the temperature level generally. The best way is using sensitivity analysis technique by considering working condition.

3.2. Exergy analysis of dual independent expander refrigeration cycles for LNG production

3.2.1. Brief description of process

This process is using for liquefaction natural gas using dual independent expander refrigeration cycles with at least two different refrigerants (Foglietta, 2002). This process includes two independent cycles in which natural gas exchange heat with expanded refrigerants at the same time or independently. The first refrigerant is selected from methane, ethane, or treated inlet gas. The second refrigerant is nitrogen. The schematic flow diagram of this process is shown in figure (3-13).

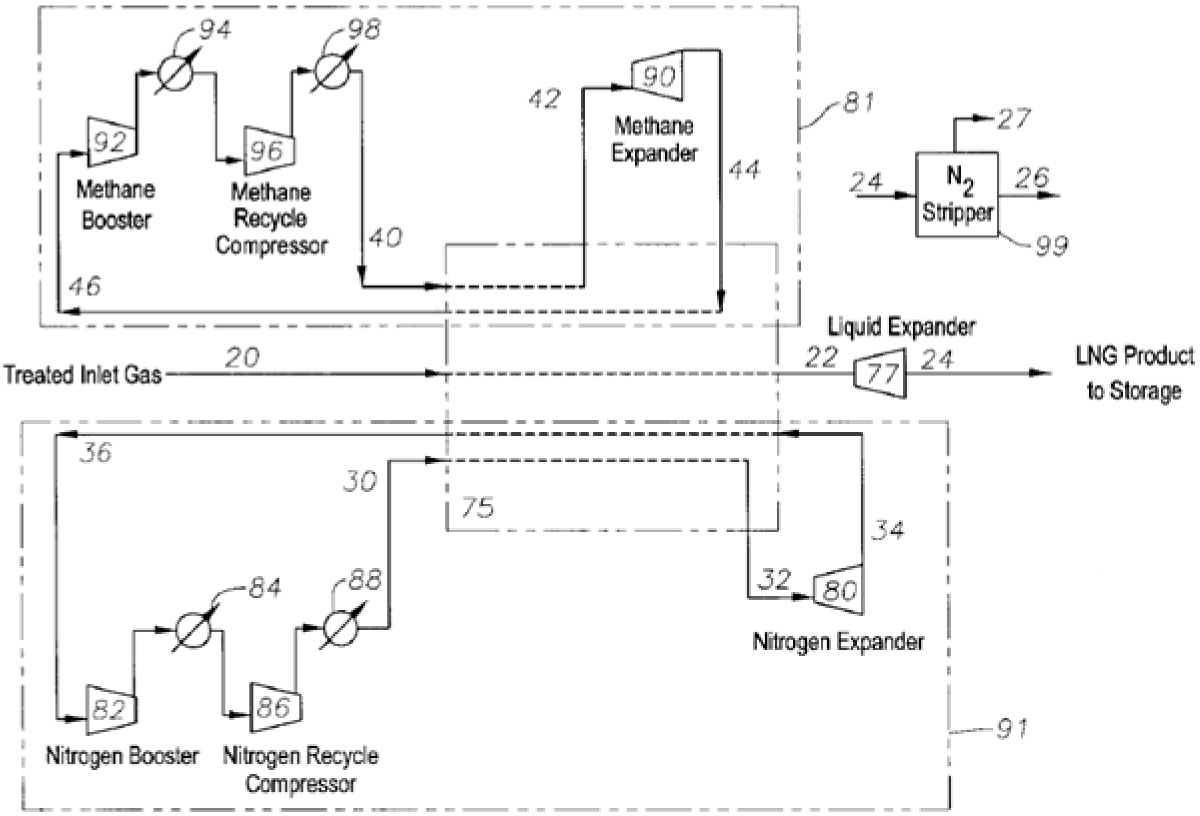


Figure 3.13 Dual independent expander refrigeration cycle (Foglietta, 2002)

This process has some advantages. First, it is adoptable for liquefaction of natural gas with different quality. By adjusting the flow rate of nitrogen and/or methane cycle we can make the process efficient. Since, changes in the flow rate of nitrogen and methane leads to changes in temperature difference between hot stream and cold stream in main heat exchanger. Moreover, refrigerants are in gaseous phase which eliminates the need for phase separator and makes design and construction of heat exchanger easier.

High pressure inlet gas stream 11 enters to treatment unit 71 to removed acid gas like H_2S and CO_2 which can cause corrosion in piping and equipments. Furthermore, the inlet gas will be dehydrated as water freezes in low temperature heat exchanger and cause plugging. The treated stream 12 is precooled with water in cooler 12 and conventional mechanical refrigeration unit 73. The treated precooled stream 20 enters to heat exchanger 75 at ambient temperature (25°C) and 62 bar to be cooled and liquefied with heat exchange by first refrigeration cycle 81 and second refrigeration cycle 91.

The stream 20 is liquefied and subcooled to -155°C in heat exchanger 75. The subcooled stream 22 is expanded in liquid expander 77 to 1 bar. Expanded stream 24 finally enters to nitrogen removal unit 99 to produce LNG product 26. Stream 27 which is enriched with nitrogen is usually used for low pressure fuel gas.

In the refrigeration cycle 81, the expanded cold methane stream 44 enters to heat exchanger 75. This stream absorbs heat from natural gas and high pressure methane stream and leaves heat exchanger at 25°C. Stream 46 is compressed at two stages. It is further cooled to 30°C in water cooler 98. High pressure stream 40 enters to the main heat exchanger and cooled. Stream 42 is expanded in methane expander 90 to provide cold stream for cooling of natural gas and high pressure methane stream.

As the minimum temperature approach in the heat exchanger is 3°C, the inlet temperature of cold nitrogen must be -158°C by considering the outlet temperature of subcooled natural gas from the main heat exchanger is -155°C where the pinch point is at the cold end of heat exchanger. In the refrigeration cycle 91, the cold expanded nitrogen stream 34 enters at -158°C to the main heat exchanger and is warmed in the heat exchanger. Stream 36 leaves the heat exchanger at 25°C. The stream 36 is compressed in two stages compression. It is cooled by water cooler 88 to 30°C. High pressure stream 30 enters to the main heat exchanger, cooled and leaves the main heat exchanger and expanded in nitrogen expander 80 to provide nitrogen cold stream.

The input data for streams and composition of natural gas are introduced in table (3-1) and (3-2).

Table 3-1 Data for input streams in dual independent expander refrigeration cycle

stream	20
Component	Natural gas
Total Flow (Kg/s)	1
Temperature(°C)	25
Pressure (bar)	62

Table 3-2 Composition of natural gas (U.S. patent 6412302)

Nitrogen	0.05816286
Methane	0.83433614
Ethane	0.07119936
Propane	0.02256318
iso-Butane	0.00401123
n-Butane	0.00601685
iso-Pentane	0.00120337
n-Pentane	0.00150421
n-hexane	0.0010028

Degree of freedom is the number of independent variable which can be varied and specify the system or process. It can be calculated by equation (3-64).

$$DOF = number\ of\ variable - number\ of\ equation \tag{3-64}$$

There are 17 streams, which mean there are 17*3=51 unknowns for the streams (mass velocity, pressure, temperature). In the process, four compressors and 3 turbines are used which add 7 more unknowns for their power. Furthermore, the duties of the cooling waters are unknown which add 4 more to the numbers of variables.

Mass velocity, temperature, and pressure of feed are constant. Also, temperature of stream no.22 and pressure of stream no.24 are fixed.

As a result, we have 51+7+4-5=**57 variables**.

We have 14 equations for mass balance:

(1) $\dot{m}_{46} = \dot{m}_{46-2}$

(2) $\dot{m}_{46-2} = \dot{m}_{46-3}$

(3) $\dot{m}_{46-3} = \dot{m}_{46-4}$

(4) $\dot{m}_{46-4} = \dot{m}_{40}$

(5) $\dot{m}_{40} = \dot{m}_{42}$

(6) $\dot{m}_{42} = \dot{m}_{44}$

(7) $\dot{m}_{36} = \dot{m}_{36-2}$

(8) $\dot{m}_{36-2} = \dot{m}_{36-3}$

(9) $\dot{m}_{36-3} = \dot{m}_{36-4}$

(10) $\dot{m}_{36-4} = \dot{m}_{30}$

(11) $\dot{m}_{30} = \dot{m}_{32}$

(12) $\dot{m}_{32} = \dot{m}_{34}$

(13) $\dot{m}_{20} = \dot{m}_{22}$

(14) $\dot{m}_{22} = \dot{m}_{24}$

In addition to 8 energy and entropy balance equations for compressors, 6 energy and entropy balance equations for turbines, 4 energy balance equations for cooling waters, and 1 energy balance equations for main heat exchanger (75). In general, we have **19** energy balance equations.

The outlet pressure of the first compressors in two stages compression is defined by:

(34) $P_{42-3} = \sqrt{P_{46}.P_{46-4}}$

(35) $P_{36-3} = \sqrt{P_{36}.P_{36-4}}$

There are **9** equations for pressure drop in each side of heat exchanger no.75 and cooling waters.

The minimum approach temperature in heat exchanger should be designed by trade off between capital cost and energy cost. By assuming ΔT_{min} in cooling waters is 5°C, the temperature of streams no. 36-3, 30, 46-3, and 40 is 30°C. This is chosen to decrease the exergy losses in cooling water heat exchangers.

By assuming ΔT_{min} in hot side of heat exchanger no.75 is 5°C:

(49) $T_{46} = T_{40} - 5 = 25$

(50) $T_{36} = T_{30} - 5 = 25$

By assuming ΔT_{min} in cold side of heat exchanger no.75 is 3°C:

$$(51) \qquad \boldsymbol{T_{34} = T_{22} - 3 = 158}$$

We should consider that equations number 49-51 are not constant and are chosen to reduce the temperature difference between hot streams and cold streams in heat exchanger. Moreover, by accepting the equation 51, we have specified the pinch point in the LNG heat exchanger. Although this point can be varied in the length of heat exchanger, setting that in the cold end makes study of improving the performance of the process easier. In general, we have **51 equations**.

Degree of freedom:

$$\boldsymbol{DOF} = n_{var} - n_{eqs} = 57 - 51 = \boldsymbol{6}$$

3.2.2. Exergy analysis of process

By considering a boundary around the process, the exergy efficiency of this process is defined by equation (3-65):

$$\varepsilon_p = \frac{W_{T_{77}}+W_{T_{80}}+W_{T_{90}}+\left(Ex_{24}{}^{T}-Ex_{20}{}^{T}\right)}{W_{C_{82}}+W_{C_{86}}+W_{C_{92}}+W_{C_{96}}+\left(Ex_{20}{}^{P}-Ex_{24}{}^{P}\right)} \tag{3-65}$$

For improving the exergy efficiency of the process, one should study the performance of not only external characteristic but also internal structure. For optimisation of exergy efficiency, although the qualitative characteristics of each element may change, the internal structure of process will remain constant. Evaluation of effects of these characteristics on performance of the process results in improving the efficiency (Brodyansky, 1994).

For finding the effect of each element on exergy efficiency of process, we should define an equation which shows the exergy efficiency of process as a function of characteristic of each element. We can use the following approach to find this equation. The efficiency of the process is defined by equation (3-66):

$$\varepsilon_p = \frac{sink_p}{source_p} = \frac{source_p - D - L}{source_p} = 1 - \frac{D+L}{source_p} \tag{3-66}$$

where D is the exergy destruction and L is exergy losses in the process. By defining D_i as the destruction and L_i as the losses in element i-th, we can write equation (3-67). The losses occurred at coolers since water transfer exergy to atmosphere.

$$\Rightarrow D + L = \sum D_i + \sum L_i \tag{3-67}$$

where $D_i + L_i = source_i.\left(1-\varepsilon_i\right)$ (3-68)

$$\Rightarrow source_p.\left(1-\varepsilon_p\right) = \sum source_i.\left(1-\varepsilon_i\right)$$

$$\Rightarrow \varepsilon_p = 1 - \sum\nolimits_{all\ blocks\ exept\ CWs} \frac{source_i}{source_p}.\left(1-\varepsilon_i\right) \tag{3-69}$$

where efficiency of the whole process and the efficiency of i-th element are denoted by ε_p and ε_i..By defining $\lambda_i = \frac{source_i}{source_p}$ as relative source of each element to source of process we can write equation (3-69) in format of equation (3-70).

$$\Longrightarrow \varepsilon_p = 1 - \textstyle\sum_i \lambda_i (1 - \varepsilon_i) \tag{3-70}$$

As we can see from equation (3-70), the exergy efficiency of process is function of exergy efficiency of each element and relative source of each element. Equation (3-71) shows the effect of efficiency of elements on exergy efficiency of process.

$$z_i = \left(\frac{\partial \varepsilon_p}{\partial \varepsilon_i}\right) = \lambda_i \tag{3-71}$$

z_i can vary to from a number close to zero to hundreds and more. The z_i is greater; the efficiency of i-th element has more influence on the efficiency of process (Brodyansky, 1994).

Since the input and output parameters of turbine 77 are fixed, the efficiency of that is constant. So we should study the effects of efficiency and relative source of other elements on improving the efficiency of whole process.

The parameters which affect on the performance of the elements should be discussed. For identification of these parameters, it is necessary to take a look on the definition of efficiency for each of these equipments.

Table 3-3 Sink and source for each unit in LNG processing by dual independent expander refrigeration cycle

Equipment	**Position according to T0**	**Sink**	**Source**
Compressor 92	above	$\dot{E}^{TM}{}_{46-2} - \dot{E}^{TM}{}_{46}$	$\dot{W}_{C92}$
Compressor 96	above	$\dot{E}^{TM}{}_{46-4} - \dot{E}^{TM}{}_{46-3}$	$\dot{W}_{C96}$
Compressor 82	above	$\dot{E}^{TM}{}_{36-2} - \dot{E}^{TM}{}_{36}$	$\dot{W}_{C82}$
Compressor 86	above	$\dot{E}^{TM}{}_{36-4} - \dot{E}^{TM}{}_{36-3}$	$\dot{W}_{C86}$
Turbine 90	below	$\dot{W}_{C90} + (\dot{E}^{T}{}_{44} - \dot{E}^{T}{}_{42})$	$\dot{E}^{P}{}_{42} - \dot{E}^{P}{}_{44}$
Turbine 80	below	$\dot{W}_{C80} + (\dot{E}^{T}{}_{34} - \dot{E}^{T}{}_{32})$	$\dot{E}^{P}{}_{32} - \dot{E}^{P}{}_{34}$
LNG HEX	across	$(\dot{E}^{TM}{}_{22} - \dot{E}^{TM}{}_{20})$ $+\dot{E}^{T}{}_{42} + \dot{E}^{T}{}_{32}$ $+(\dot{E}^{P}{}_{42} - \dot{E}^{P}{}_{40\rightarrow 42,T0})$ $+(\dot{E}^{P}{}_{32} - \dot{E}^{P}{}_{30\rightarrow 32,T0})$	$(\dot{E}^{TM}{}_{44} - \dot{E}^{TM}{}_{46}) + (\dot{E}^{TM}{}_{34} - \dot{E}^{TM}{}_{36})$ $+\dot{E}^{T}{}_{40} + \dot{E}^{T}{}_{30}$ $+(\dot{E}^{P}{}_{40} - \dot{E}^{P}{}_{40\rightarrow 42,T0})$ $+(\dot{E}^{P}{}_{30} - \dot{E}^{P}{}_{30\rightarrow 32,T0})$

Now, we should study the parameters which have effects on performance of units and as a result on performance of process:

- **Compressors**

All the compressors which are used in the process are working above ambient temperature.

All of the studies which have done in chapter (3-1) showed the effect of different parameters on exergy efficiency of compressor. When we want to find their effects on exergy efficiency of process, we should consider the effects of these parameters on product of λ and $(1-\varepsilon)$ since changing these parameters will change relative source and efficiency of compressor simultaneously.

For compressor which works above ambient temperature,

$$\lambda_{comp}\left(1-\varepsilon_{comp}\right)=\frac{source_i}{source_p}\left(1-\frac{\dot{E}^{TM}{}_{out}-\dot{E}^{TM}{}_{in}}{\dot{W}_C}\right) \tag{3-72}$$

By considering source of process in our study case, we can find (1-ε) for a compressor.

$$source_p=\dot{W}_C+\left(Ex_{20}{}^{P}-Ex_{24}{}^{P}\right)+\textstyle\sum_{other\ comps}\dot{W}_c$$

$$\Rightarrow\lambda_{comp}\left(1-\varepsilon_{comp}\right)=\frac{\dot{W}_C}{\dot{W}_C+\left(Ex_{20}{}^{P}-Ex_{24}{}^{P}\right)+\sum_{other\ comps}\dot{W}_c}\left(\frac{\dot{W}_C-\dot{E}^{TM}{}_{out}+\dot{E}^{TM}{}_{in}}{\dot{W}_C}\right)$$

$$\Rightarrow\lambda_{comp}\left(1-\varepsilon_{comp}\right)=\frac{\dot{W}_C-\dot{E}^{TM}{}_{out}+\dot{E}^{TM}{}_{in}}{\dot{W}_C+\left(Ex_{20}{}^{P}-Ex_{24}{}^{P}\right)+\sum_{other\ comps}\dot{W}_c} \tag{3-73}$$

$\left(Ex_{20}{}^{P}-Ex_{24}{}^{P}\right)+\sum_{other\ comps}\dot{W}_c$ is not function of parameters of compressor, so it is constant and we assume its value is equal to X.

$$\Rightarrow\lambda_{comp}\left(1-\varepsilon_{comp}\right)=\frac{\dot{W}_C-\dot{E}^{TM}{}_{out}+\dot{E}^{TM}{}_{in}}{\dot{W}_C+X}$$

$$\Rightarrow\lambda_{comp}\left(1-\varepsilon_{comp}\right)=\dot{m}T_o\frac{s_{out}-s_{in}}{\dot{W}_C+X} \tag{3-74}$$

By referring to equation (3-72):

$$\Rightarrow\frac{\partial\varepsilon_p}{\partial P_{out,comp}}=-\frac{\partial\lambda_{comp}\left(1-\varepsilon_{comp}\right)}{\partial P_{out,comp}}=-\dot{m}T_o\frac{\frac{\partial s_{out}}{\partial P_{out,comp}}(\dot{W}_C+X)-\frac{\partial\dot{W}_C}{\partial P_{out,comp}}(s_{out}-s_{in})}{(\dot{W}_C+X)^2} \tag{3-75}$$

We can find $\frac{\partial \dot{W}_C}{\partial P_{out,comp}}$ from definition of compressor work:

$$\dot{W}_C = \dot{m}.\left[h_{out} - h_{in}\right] \Longrightarrow \frac{\partial \dot{W}_C}{\partial P_{out,comp}} = \dot{m}\frac{\partial h_{out}}{\partial P_{out}} \tag{3-76}$$

By using equations (1-34) we can simplify equation (1-33):

$$\Longrightarrow \frac{\partial \varepsilon_p}{\partial P_{out,comp}} = -\dot{m}T_o \frac{\frac{\partial s_{out}}{\partial P_{out,comp}}(\dot{W}_C+X)-\dot{m}\frac{\partial h_{out}}{\partial P_{out}}(s_{out}-s_{in})}{(\dot{W}_C+X)^2} \tag{3-77}$$

For finding effect of outlet pressure on exergy efficiency of process, sensitivity analysis is used. In this case we assumed X is equal to 1754.

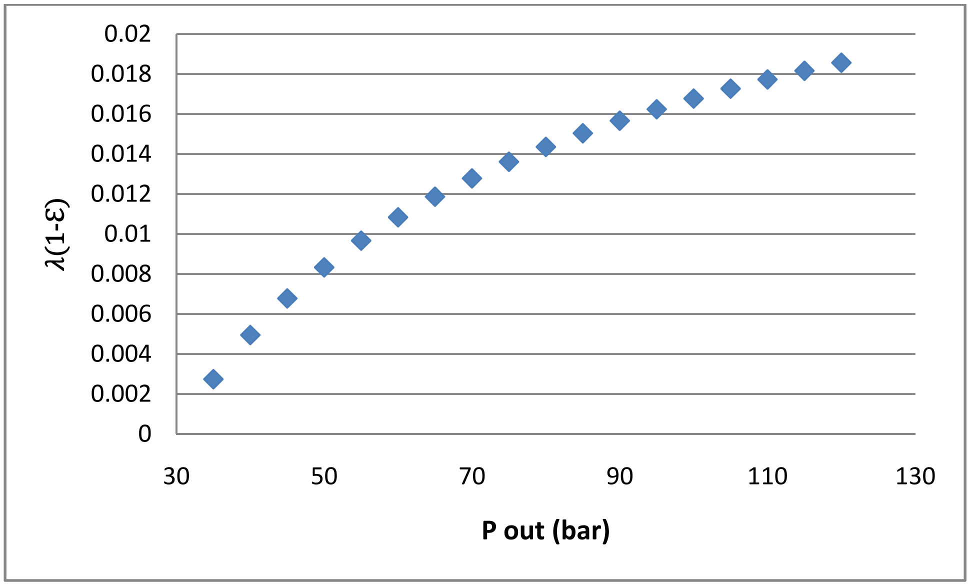


Figure 3.14 Effect of outlet pressure on (1-ε) for a compressor

By increasing the outlet pressure, λ.(1-ε) will increase. By referring to equation (3-70) we can see that exergy efficiency of process will decrease.

Equation (3-78) shows how the process efficiency changes by changing the inlet pressure of compressor.

$$\frac{\partial \varepsilon_p}{\partial P_{in,comp}} = -\frac{\partial \lambda_{comp}\left(1-\varepsilon_{comp}\right)}{\partial P_{in,comp}} = -\dot{m}T_0 \frac{-\frac{\partial s_{in}}{\partial P_{in,comp}}(\dot{W}_C+X)-\frac{\partial \dot{W}_C}{\partial P_{in,comp}}(s_{out}-s_{in})}{(\dot{W}_C+X)^2} \tag{3-78}$$

$$\dot{W}_C = \dot{m}.[h_{out} - h_{in}] \Rightarrow \frac{\partial \dot{W}_C}{\partial P_{in,comp}} = -\dot{m}\frac{\partial h_{in}}{\partial P_{in}} \qquad (3\text{-}79)$$

By using equation (3-79) we can simplify equation (3-78).

$$\Rightarrow \frac{\partial \varepsilon_p}{\partial P_{in,comp}} = \dot{m}T_0 \frac{\frac{\partial s_{in}}{\partial P_{in,comp}}(\dot{W}_C+X) - \dot{m}\frac{\partial h_{in}}{\partial P_{in}}(s_{out}-s_{in})}{(\dot{W}_C+X)^2} \qquad (3\text{-}80)$$

We can use sensitivity analysis to find effect of inlet pressure on exergy efficiency of process.

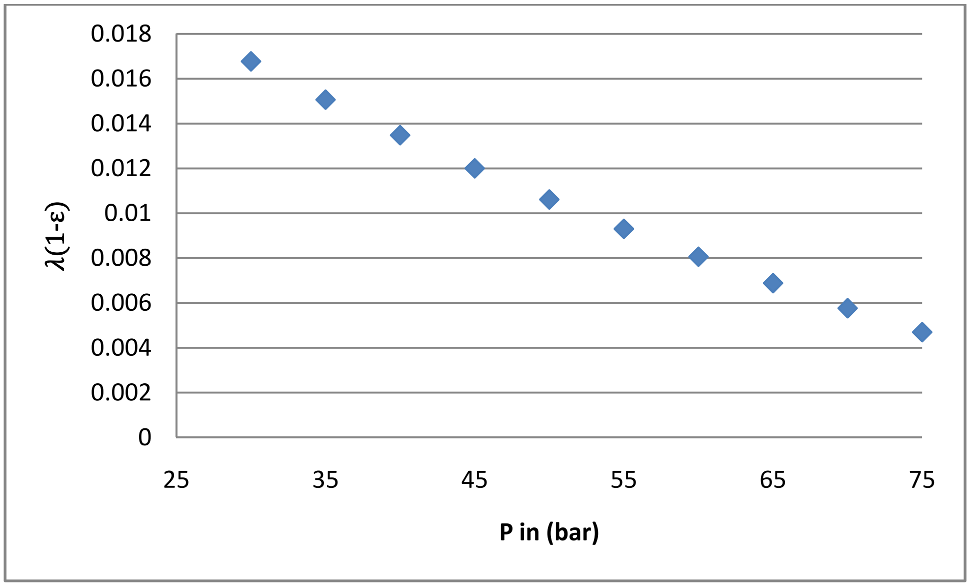


Figure 3.15 Effect of inlet pressure on λ.(1-ε) for a compressor

As we can see in figure (3-15) by increasing the inlet pressure, the value of λ.(1-ε) will decrease, so the exergy efficiency of process will increase. In general, these studies show that by decreasing pressure ratio of compressor, the exergy efficiency of process will increase

For finding effect of inlet temperature of compressor on exergy efficiency of process we can use sensitivity analysis technique.

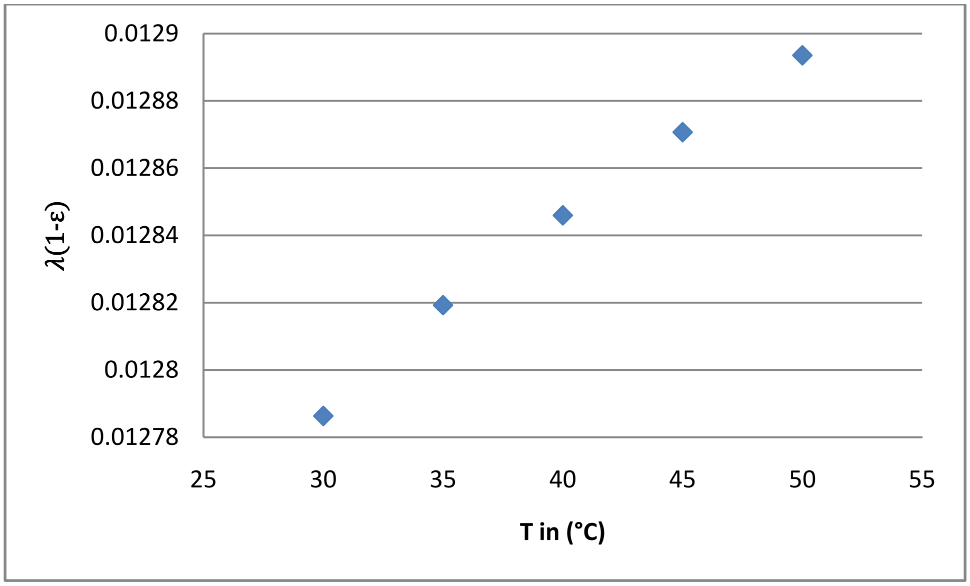


Figure 3.16 Effect of inlet temperature on λ.(1-ε) for a compressor

Figure (3-16) shows effect of inlet temperature on λ.(1-ε) for a compressor. By increasing inlet temperature of compressor, λ.(1-ε) will increase. So, we can find that by increasing inlet temperature of compressor, the exergy efficiency of process will decrease.

In general, increasing inlet temperature and pressure ratio leads to improvement of exergy efficiency of compressor, but decrease the exergy efficiency of process. On the other hand, increasing isentropic efficiency of compressor increases exergy efficiency of compressor and process. This is because of that by increasing isentropic efficiency, exergy destruction will reduce. We can see that in figure (3-17). By increasing isentropic efficiency λ.(1-ε) will decrease. As a result, exergy efficiency of process will increase.

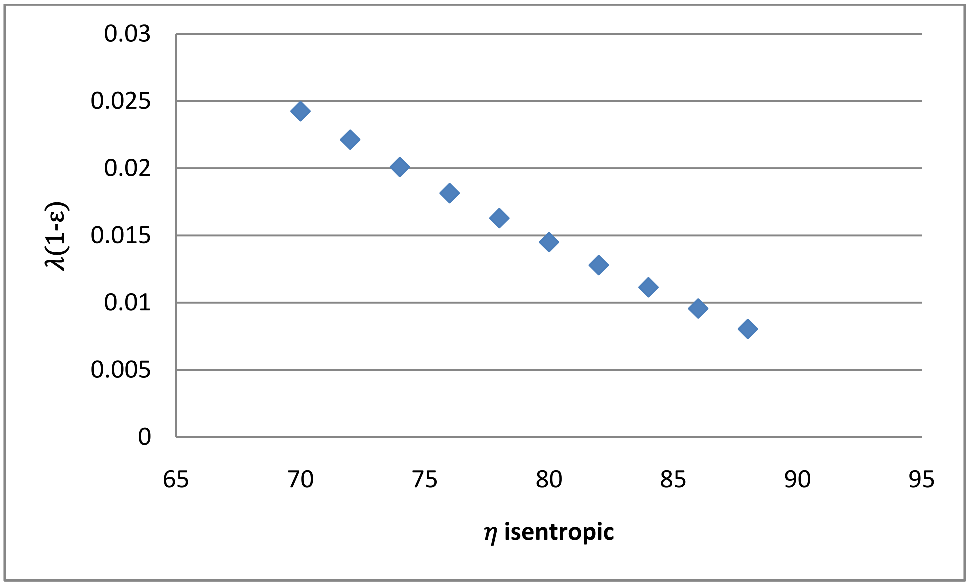


Figure 3.17 Effect of isentropic efficiency on λ.(1-ε) for a compressor

- **Turbine**

For finding the effects of these parameters on exergy efficiency of process we should refer to equation (3-70). With the same procedure as we have done for compressor, we should study effect of inlet pressure and temperature and outlet pressure of turbine on product of relative source and exergy efficiency of turbine. Source of turbine which works below ambient temperature is equal to $\dot{E}^P{}_{in} - \dot{E}^P{}_{out}$. So, it does not included in the source of process directly. For turbine which works below temperature

$$\lambda_{turb}(1-\varepsilon_{turb}) = \frac{\dot{E}^P{}_{in} - \dot{E}^P{}_{out}}{\left(Ex_{20}{}^P - Ex_{24}{}^P\right) + \sum_{comps} \dot{W}_c}(1 - \frac{\dot{W}_C + \left(\dot{E}^T{}_{out} - \dot{E}^T{}_{in}\right)}{\dot{E}^P{}_{in} - \dot{E}^P{}_{out}})$$

$$\Longrightarrow \lambda_{turb}(1-\varepsilon_{turb}) = \frac{\dot{E}^{TM}{}_{in} - \dot{E}^{TM}{}_{out} - \dot{W}_C}{\left(Ex_{20}{}^P - Ex_{24}{}^P\right) + \sum_{comps} \dot{W}_c}$$

$$\Longrightarrow \lambda_{turb}(1-\varepsilon_{turb}) = \dot{m}T_0 \frac{s_{out} - s_{in}}{\left(Ex_{20}{}^P - Ex_{24}{}^P\right) + \sum_{comps} \dot{W}_c} \qquad (3\text{-}81)$$

Denominator of the right side in equation (3-81) is not function of inlet pressure and temperature and outlet pressure of turbine, So we assumed it is constant and equal to X'.

$$\Longrightarrow \lambda_{turb}(1-\varepsilon_{turb}) = \dot{m}T_0 \frac{s_{out} - s_{in}}{X'} \qquad (3\text{-}82)$$

Now by using equations (3-70) and (3-82) we can find $\frac{\partial \varepsilon_P}{\partial P_{out,turb}}$, $\frac{\partial \varepsilon_P}{\partial P_{in,turb}}$, and $\frac{\partial \varepsilon_P}{\partial T_{in,turb}}$.

$$\frac{\partial \varepsilon_P}{\partial P_{out,turb}} = -\frac{\lambda_{turb}(1-\varepsilon_{turb})}{\partial P_{out,turb}} = -\frac{\dot{m}T_0}{X\prime}\frac{\partial s_{out}}{\partial P_{out,turb}} \qquad (3\text{-}83)$$

By increasing the pressure at the constant temperature, entropy will decrease, so $\frac{\partial s_{out}}{\partial P_{out,turb}}$ has negative value. As a result $\frac{\partial \varepsilon_P}{\partial P_{out,turb}}$ is positive which shows that by increasing outlet pressure of turbine, exergy efficiency of process will increase. We can see this trend in figure (3-18).

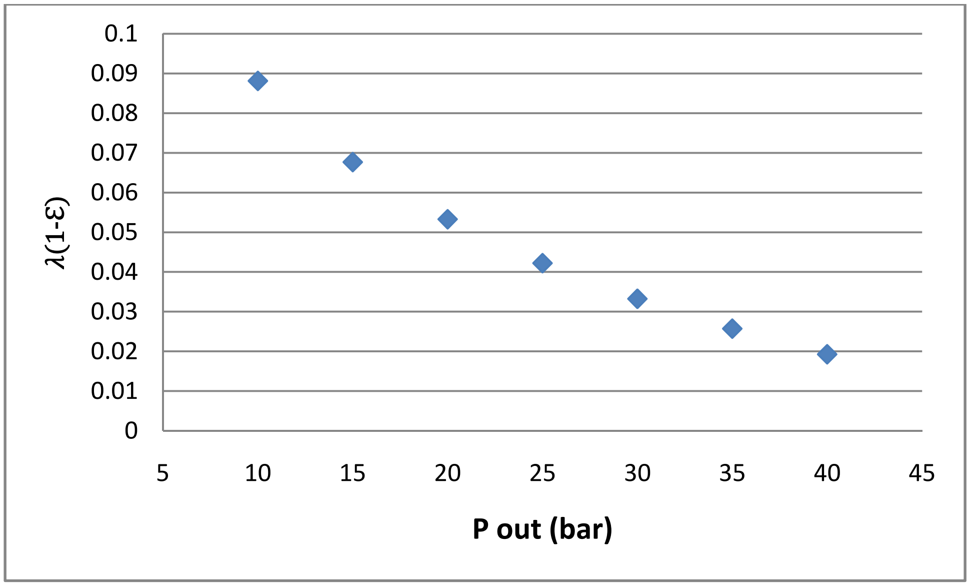


Figure 3.18 Effect of outlet pressure on λ.(1-ε) for turbine

In this case study, inlet temperature and pressure are set at -10°C and 60 bar and X' is 3000 (KJ/s). As it is clear by increasing outlet pressure, (1-ε) will decrease. So, the exergy efficiency of process will increase.

$$\frac{\partial \varepsilon_P}{\partial P_{in,turb}} = -\frac{\lambda_{turb}(1-\varepsilon_{turb})}{\partial P_{in,turb}} = \frac{\dot{m}T_0}{X\prime}\frac{\partial s_{in}}{\partial P_{in,turb}} \qquad (3\text{-}84)$$

By increasing inlet pressure of turbine, entropy will decrease. As a result $\frac{\partial \varepsilon_P}{\partial P_{in,turb}}$ is negative which show that by increasing inlet pressure of turbine, exergy efficiency of process will decrease. We can see this trend in figure (3-19). In this case study inlet temperature and outlet pressure are set at -10°C and 20 bar and X' is 3000 (KJ/s).

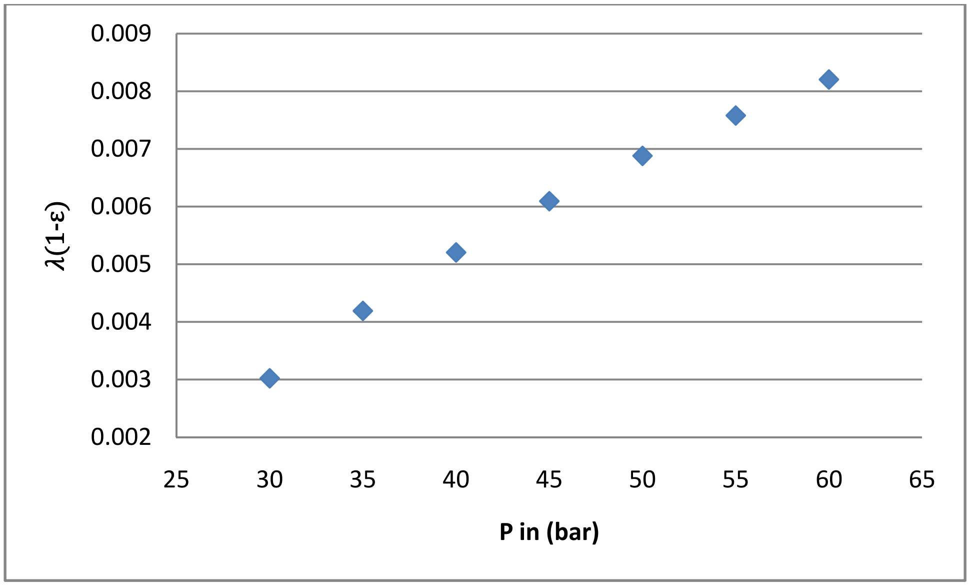


Figure 3.19 Effect of inlet pressure on 𝜆.(1-ε) for a turbine

We can see in figure (3-20) by increasing inlet temperature of turbine, exergy efficiency of process In this case study, inlet and outlet pressure are set at 60 and 15 bar. X’ is equal to 3000 (KJ/s).

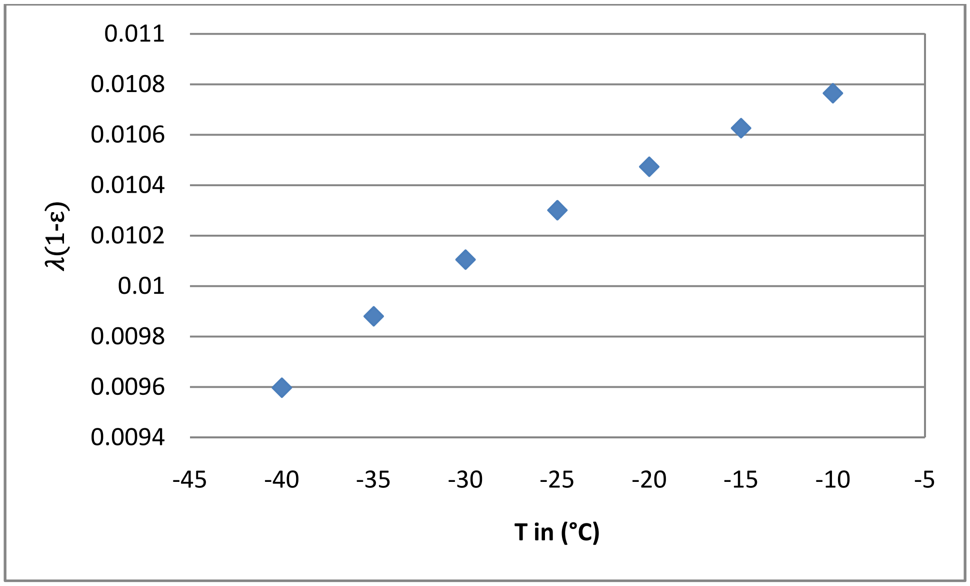


Figure 3.20 Effect of inlet temperature on 𝜆.(1-ε) for a turbine

In general, by increasing pressure ratio and inlet temperature of turbine exergy efficiency of process will decrease.

- **LNG heat exchanger**

For finding the effect of mass velocity of methane and nitrogen on efficiency of LNG heat exchanger we should use sensitivity analysis technique since analytical analysis is very complicated because of different parameters related to LNG heat exchanger. The pressure and temperature of all the streams which are connected to the LNG heat exchanger are fixed as given in table (3-4).

Table 3-4 The stream data for analyzing LNG heat exchanger efficiency

Name of stream	20	22	40	42	44	46	30	32	34	36
Temperature	25	-155	30	-7.27896	-84	25	30	-82	-158	25
Pressure	62	61.8	57.8	57.6	14	13.8	83.8	83.6	14	13.8

When the mass velocity of methane changes, the mass velocity of nitrogen should be changed simultaneously to balance the energy in LNG heat exchanger. Figures (3-21) and (3-22) show the effect of mass velocity of methane and nitrogen on exergy efficiency of LNG heat exchanger.

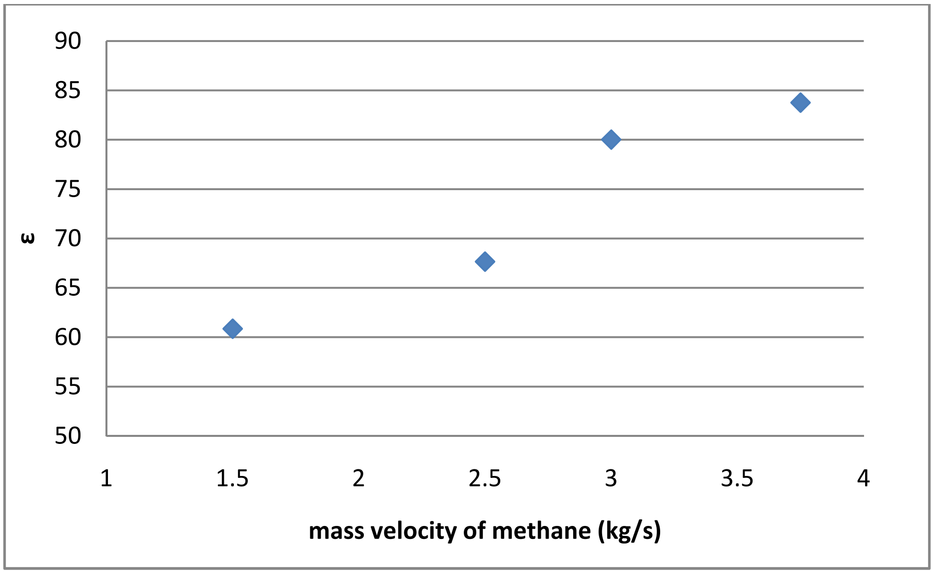


Figure 3.21 effect of mass velocity of methane on LNG heat exchanger

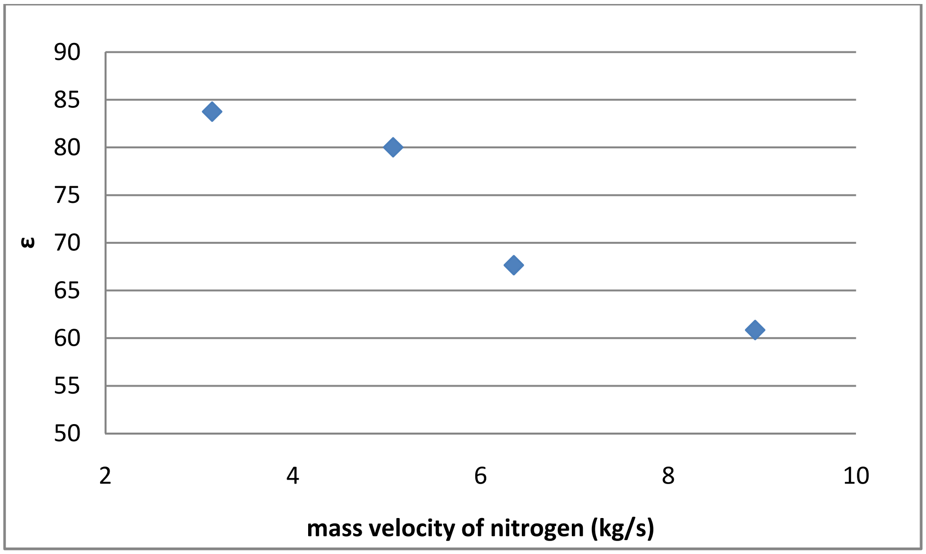


Figure 3.22 effect of mass velocity of nitrogen on LNG heat exchanger

It should be mentioned that by increasing the mass velocity of methane more, the exergy efficiency of LNG heat exchanger increases more, but the difference temperature between cold stream and hot stream will be less than 3°C.

- **Main process**

The degree of freedom of process is calculated and it is 6. By set the mass velocity of methane and nitrogen, the temperature of stream no.44 at -84°C, pressure of stream no.40, no. 30 and no.34 we can simulate the process. Figure (3-23) shows the flow diagram of simulation.

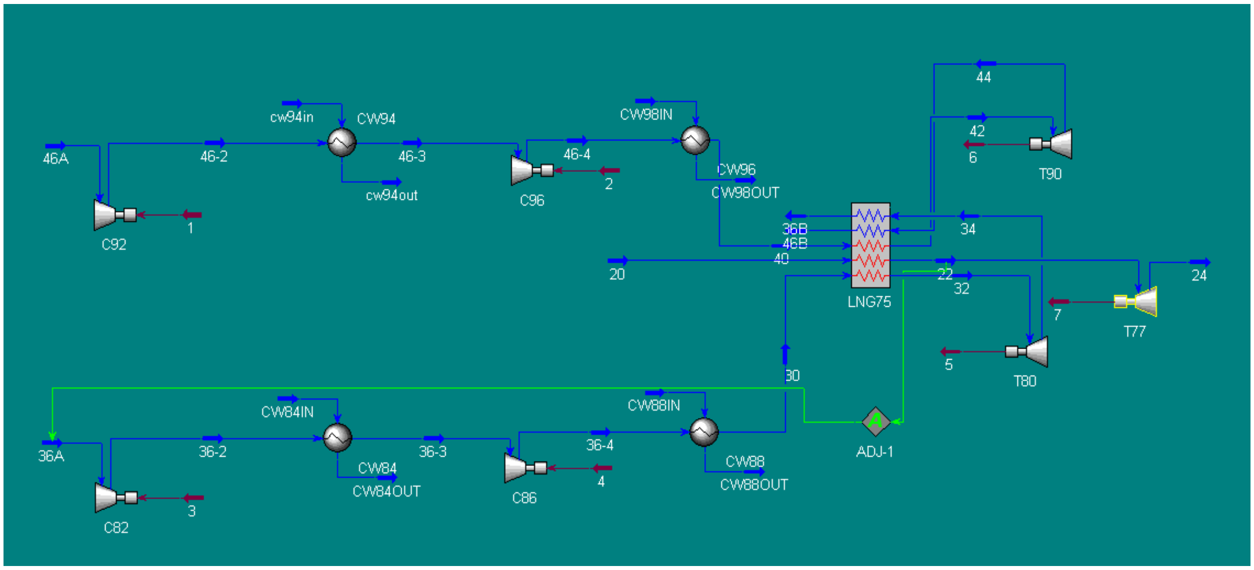


Figure 3.23 Flow sheet of simulation

Figure (3-24) shows the effect of pressure on methane cycle on efficiency of process. By increasing the pressure of methane after the compressor (stream no.46-4), the outlet pressure of turbine will increase to set the temperature of stream no.24 equal to -155°C. As we can see, dependency of efficiency of process on pressure in methane cycle is low. By increasing the pressure in refrigeration cycle, the efficiency of process will decrease because of negative effect of expander and compressor on exergy efficiency of process by increasing the pressure ratio of them.

Moreover, changing the pressure can result in changing the exergy efficiency of LNG heat exchanger if the temperature of streams which enter and leave the heat exchanger can change. Since in this master thesis changing of temperature is not considered, all the temperature of streams is fixed and as a result, the exergy efficiency of LNG heat exchanger will not change significantly.

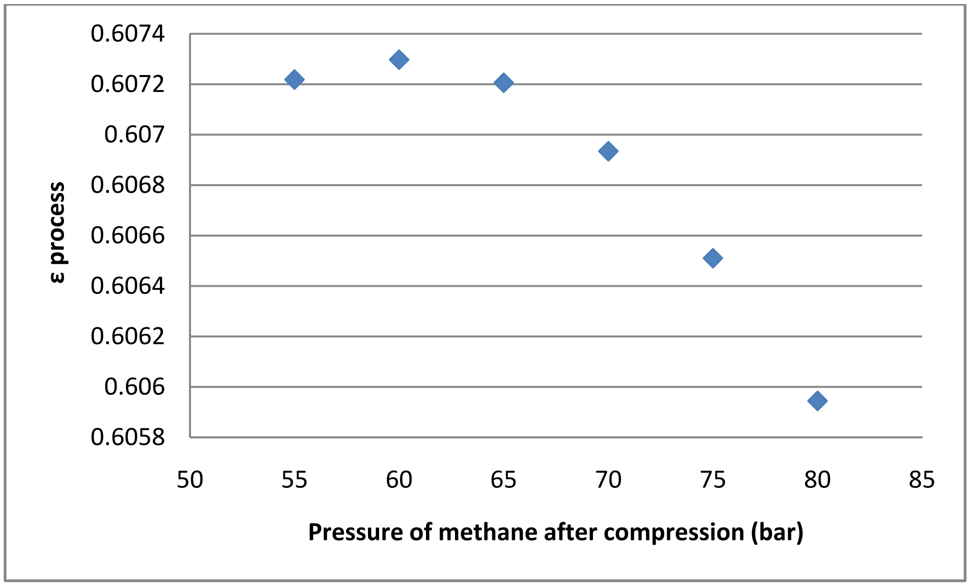


Figure 3.24 effect of pressure of methane cycle on exergy efficiency of process

Figure (3-25) shows dependency of the exergy efficiency of process by pressure in nitrogen cycle.

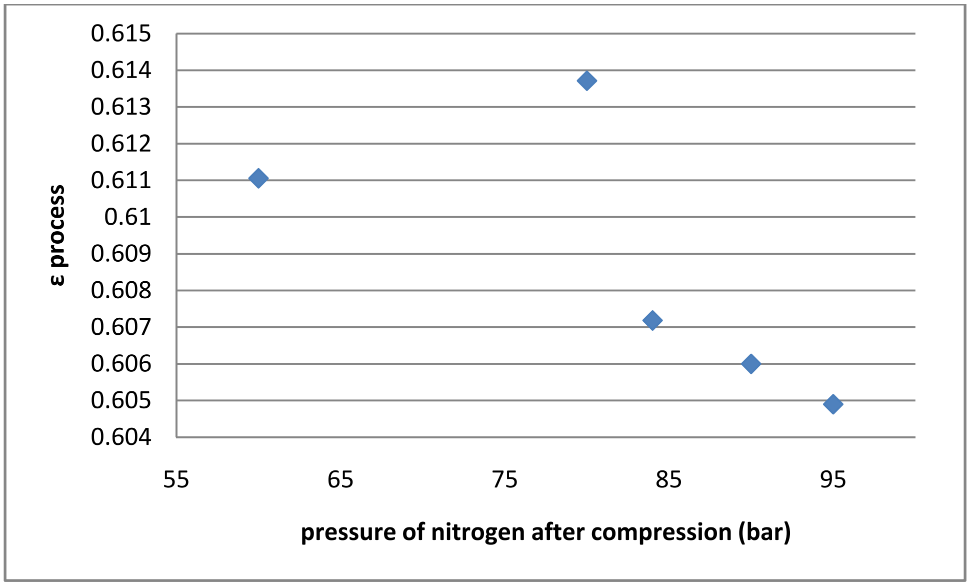


Figure 3.25 effect of pressure of nitrogen cycle on exergy efficiency of process

As we can see in figure (3-25), the efficiency of process is a little higher when the high pressure in nitrogen cycle is at 60 and 80 bar. In these cases, the difference temperature in the LNG heat exchanger is less than 3°C. But we can see that, even in this situation, the pressure of nitrogen cycle does not have significant effect on efficiency of process although the exergy efficiency in heat exchanger increase due to closer temperature difference between cold and hot stream.

Figure (3-26) shows the effect of mass velocity of methane on efficiency of process.

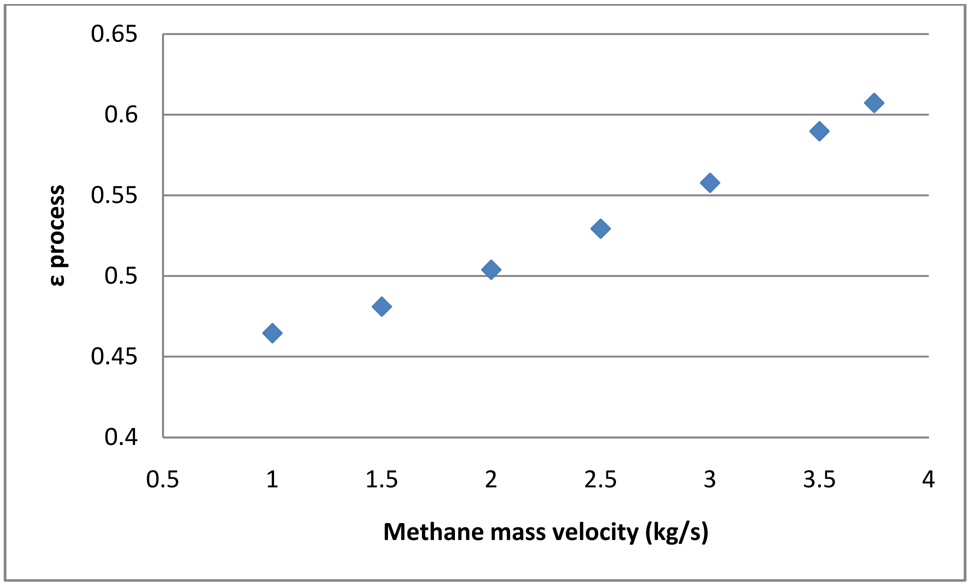


Figure 3.26 effect of mass velocity of methane on exergy efficiency of process

Table (3-5) shows the exergy efficiency of unit operations at different mass velocity of methane and nitrogen.

Table 3-5 Exergy efficiency of operation units and process at different refrigerant mass velocity

Methane mass velocity (kg/s)	1.5	2.5	3.75
Nitrogen mass velocity (kg/s)	8.9	6.35	3.13
Exergy efficiency of C92	84.9%	84.9%	84.9%
Exergy efficiency of C96	85.2%	85.2%	85.2%
Exergy efficiency of C82	86.4%	86.4%	86.4%
Exergy efficiency of C86	86.6%	86.6%	86.6%
Exergy efficiency of T90	83.8%	83.8%	83.8%
Exergy efficiency of T80	84.8%	84.8%	84.8%
Exergy efficiency of T77	98.7%	98.7%	98.7%
Exergy efficiency of CW94	6.4%	6.4%	6.4%
Exergy efficiency of CW98	6.2%	6.2%	6.2%
Exergy efficiency of CW84	5.1%	5.1%	5.1%
Exergy efficiency of CW88	4.4%	4.4%	4.4%
Exergy efficiency of LNG HX	60.8%	67.6%	83.7%
Exergy efficiency of process	51.3%	56.5%	65.18%

As we can see in table (3-5), although the exergy efficiency of coolers are low, it does not have much effect on exergy efficiency of process since its relative source is smal. Moreover, exergy efficiency of process increase significantly by increasing exergy efficiency of LNG heat exchanger.

In conclusion, by referring to above explanation we can see that mass velocity of nitrogen and methane has more effect on exergy efficiency of process since it can affect on efficiency of LNG heat exchanger significantly. Changing the pressure in nitrogen and methane cycles cannot improve the efficiency of process as it has low effect on efficiency of compressor, turbine, and LNG heat exchanger. Finally, LNG heat exchanger has the most important role in the process since increasing its exergy efficiency leads to increase in exergy efficiency of process significantly.

4. Using of ExPAnD method in dual expander cycle

It is useful to study very simple case to find out how the ExPAnD method can be used to heat integration in a process. In this case, methane should be cooled form -10°C to -85°C at 2 bar. The cold stream carries nitrogen. Its supply temperature is -55°C and target temperature is 10°C. The pressure of nitrogen should be changed from 4 to 1 bar. Mass velocity of both cold stream and hot stream is 1 kg/s. ΔT_{min} in heat exchanger is 5°C. The total duty of hot stream is 160 kw and toal duty of cold stream is 68.5 kw. So, there is not enough temperature driving force and duty in cold stream to cool down the hot stream. The total change in temperature based exergy of hot stream is 53.2 kw. Since the pressure of hot stream does not change, the change in pressure based exergy of hot stream is zero. The change in temperature based exergy of cold stream is 13.3 kw and in pressure based exergy is 122.6 kw.

There are different scenarios to reach the target points. The first scenario is to expand nitrogen before entering to LNG heat exchanger as it is shown in figure (4-1).

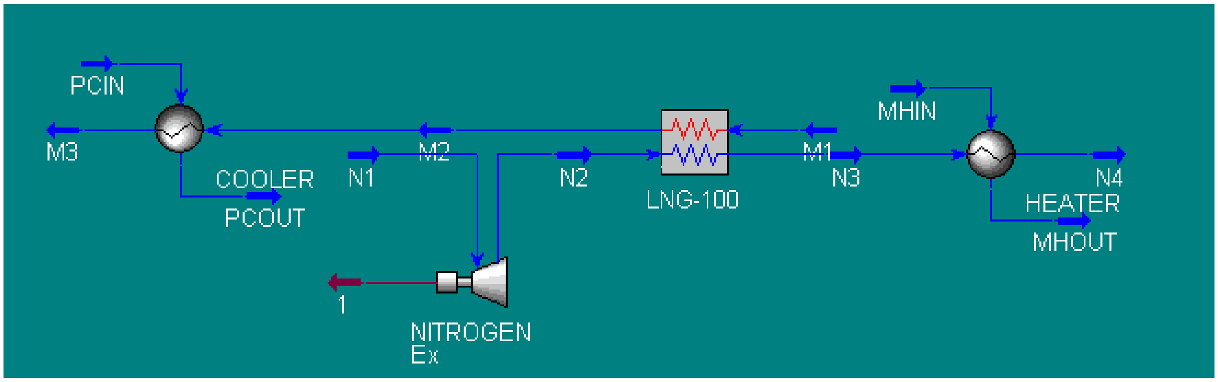


Figure 4.1 The first scenario flow sheet

The result of exergy efficiency and duty of operation units are given in table (4-1).

Table 4-1 Results of first scenario

Component	Duty (KJ/s)
Expander	62,31
Heater	25,93
Cooler	55,16

	Exergy efficiency
Expander	82%
LNG HX	54%
Heater	80%
Cooler	93%
Process	70%

The second scenario is to expand nitrogen after warming in the LNG heat exchanger as it is shown in figure (4-2).

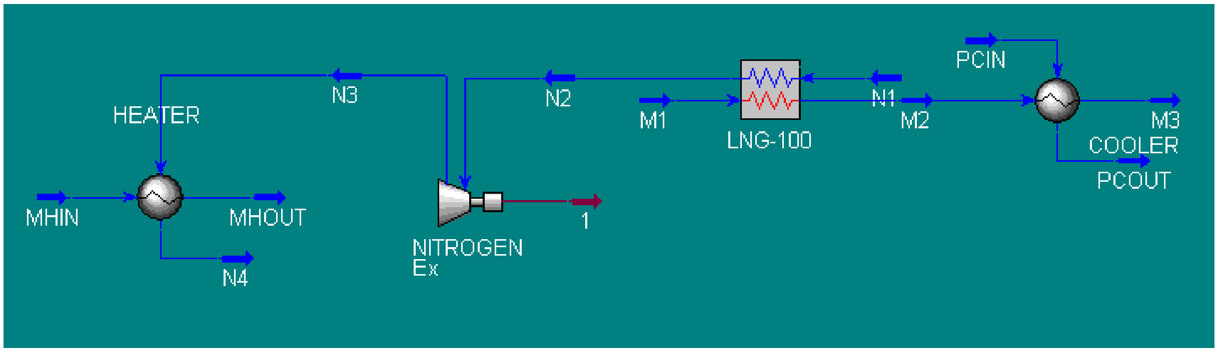


Figure 4.2 The second scenario flow sheet

Table 4-2 Result of second scenario

Component	Duty (KJ/s)
Expander	74.1
Heater	100.97
Cooler	118.41

	Exergy efficiency
Expander	82%
LNG HX	69%
Heater	90%
Cooler	93%
Process	68.7%

By comparing the result of the first and second scenario we can find that nitrogen expander produces more work when it works at higher temperature. Moreover, in the first case pressure based exergy of inlet nitrogen convert to temperature based exergy before nitrogen enters to LNG heat exchanger. So, the duty of cold stream in LNG heat exchanger is higher than the second scenario. As a result, duty of heater is less in the first scenario than the second one. The duty of heater and cooler in the second scenario is higher than the first one which means that heat integration in the first has done better in the first one. So, the exergy efficiency of the first process is more than the second one.

The third scenario is to heat up nitrogen, expand it, and heat it up again as we can see in figure (4-3).

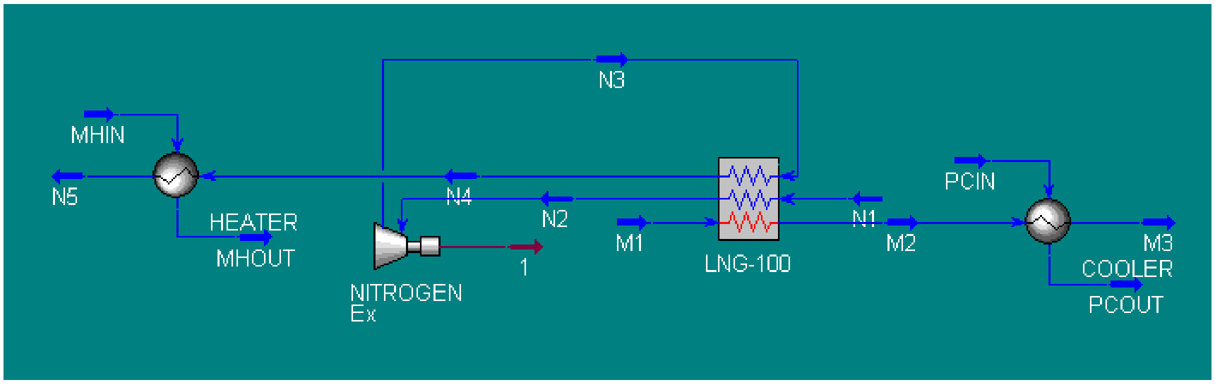


Figure 4.3 The third scenario flow sheet

In this scenario, the expanded stream (N3) enters to the LNG heat exchanger again. Since the temeparure of N3 is less than N1 after expansion, there is enough driving force in the cold end of LNG heat exchanger. As a result the duty of cooler will reduce. Since the maximum temperature of N4 must be -15°C because of driving force in the hot end of LNG heat exchanger, there is always constant temperature difference between N4 and N5. So, the duty of heater will remain constant.

Table 4-3 Result of third scenario

N3 Temp	-110°C	-100°C	-90°C
N2 Temp	-45.2°C	-31.5°C	-17.9°C
Component	Duty (KJ/s)	Duty (KJ/s)	Duty (KJ/s)
Expander	65.2	69.22	73.25
Heater	25.93	25.93	25.93
Cooler	52.3	48.25	44.22

Component	Exergy efficiency	Exergy efficiency	Exergy efficiency
Expander	82%	82%	82%
LNG HX	60%	69%	81%
Heater	80%	80%	80%
Cooler	93%	93%	93%
Process	72.4%	75.7%	79.1%

By decreasing temperature of N3, the temperature of N2 will decrease since there is constant pressure ratio between N2 and N3. So, the duty of cold stream will decrease because the enthalpy difference between stream N1 and N2 decreases. As a result, the temperature of outlet hot stream (M2) will increase and higher cooler duty is need to reach the target temperature of methane. Furthermore, the difference temperature between cold and hot stream will increase by decreasing temperature of cold stream. So, the exergy efficiency of LNG heat exchanger decreases which decreases the exergy efficiency of process.

In general, since the duty of cooler is less than the first case because of enough temperature driving force in LNG heat exchanger and the produced work of expander is more in the third scenario since it works in higher temperature (The nitrogen stream heated up first and then expanded), the exergy efficiency of the third scenario is more than the first one. We should also mention that in this scenario one stream is added to the LNG heat exchanger which can decrease the temperature difference between cold and hot stream by manipulating the pressure and temperature.

In the fourth case, the nitrogen expanded first to 2 bar, it heated up in LNG heat exchanger and after leaving the LNG heat exchanger, it expanded to 1 bar (figure (4-4)).

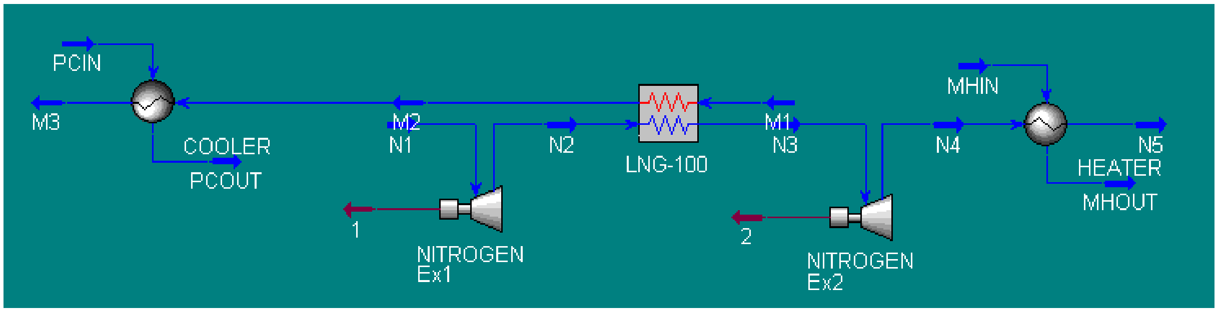


Figure 4.4 The fourth scenario flow sheet

Table 4-4 Result of fourth scenario

Component	Duty (KJ/s)
Expander1	34.32
Expander2	40.85
Heater	67.1
Cooler	83.47

	Exergy efficiency
Expander1	83.73%
Expander2	83.68%
LNG HX	60%
Heater	87%
Cooler	93%
Process	73.2%

In this case duty of heater increases compare to third case. This is because of producing extra cold stream by expansion of N3 which is not used to integrate with the hot stream. The duty of cooler is more than the first and third case. The temperature of nitrogen stream which enters to the LNG heat exchanger (N2) decreases by expansion of inlet nitrogen (N1). The temperature of methane which leaves the LNG heat exchanger decrease to provide enough temperature driving force in the cold end of LNG heat exchanger. So, we need more duty in cooler to reach the target temperature of methane. On the other hand, the produced work in two expanders is more than in one expander. Also, the exergy efficiency of LNG heat exchanger in the fourth case is more than the first case since the temperature difference in the cold end of LNG heat exchanger is less. Because the outlet temperature of expander decreases by decreasing the outlet pressure of expander.

In the fifth case two expanders are used as it shown in figure (4-5).

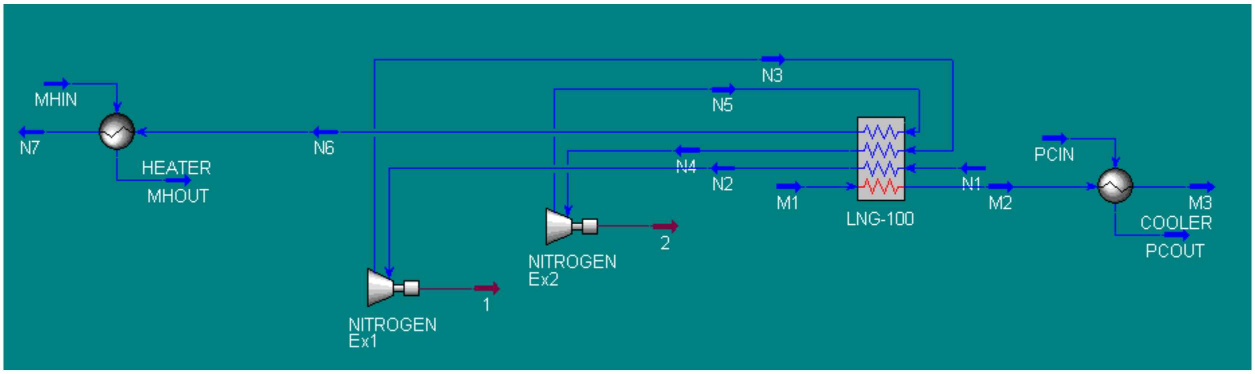


Figure 4.5 The fifth scenario flow sheet

Table 4-5 Result of fifth scenario

Component	Duty (KJ/s)
Expander1	39.84
Expander2	37.07
Heater	25.93
Cooler	40.56

	Exergy efficiency
Expander1	83.69%
Expander2	83.68%
LNG HX	89%
Heater	80.3%
Cooler	93%
Process	82.2%

This case has all the benefits which described for case 3. Moreover, it has more cold stream which results in closer temperature difference between cold and hot stream in LNG heat exchanger. So, the irreversibility in heat exchanger decrease and its exergy efficiency will increase. Also, the duty of cooler is less than other cases and produced work in expanders is more. So the exergy efficiency of this case is more than the others.

In conclusion, pressurized stream can be expanded and provide cooling duty to cool down the hot stream and reduce the duty of cooler. Expander produced more work at higher temperature. Additional expansion will add more cold stream to the LNG heat exchanger. So, by manipulating the pressure and temperature the difference temperature between cold steam and hot stream can be decreased which leads to reducing irreversibility in LNG heat exchanger and increasing the exergy efficiency of process. Also, it is desirable to heat up the cold stream before expansion to produce enough temperature driving force and produce more cold duty.

In cyclic processes for LNG production expander is used to produce cold stream by transforming pressure based exergy to temperature based exergy. There are some options to improve the design based on ExPAnD method.

First, high pressure stream no.40 enters to the LNG heat exchanger after expansion as it shown in figure (4-6).

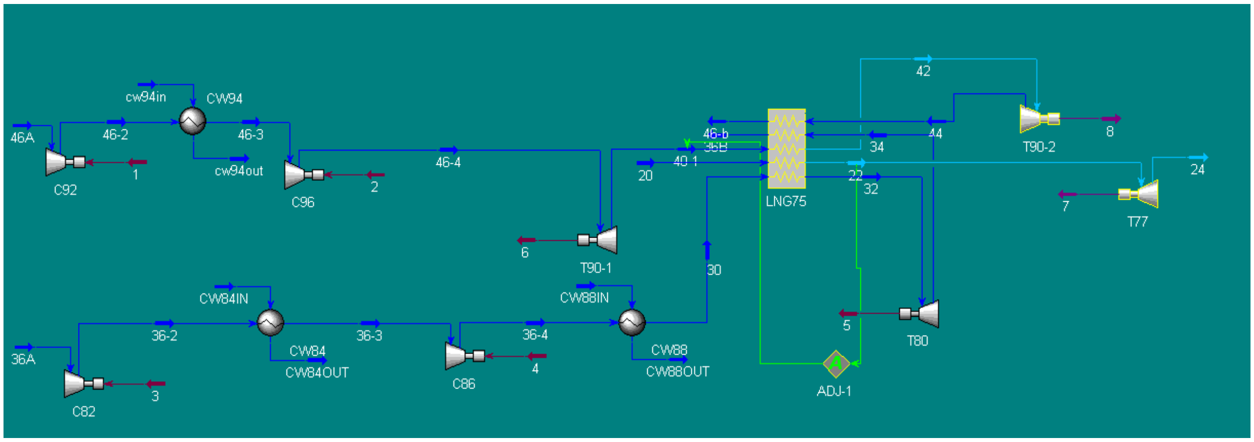


Figure 4.6 The first option to improve design of process with ExPAnD method

Although by using this option we can reduce cooling utility by omitting CW96 and produced, it is not useful, since there will not be enough pressure based exergy in stream no.42 to produce cooling duty to cool down the hot stream.

Another option is to expand high pressure stream no.42 two times after cooling in the LNG heat exchanger like the fifth scenario in the study case as it shown in figure (4-7).

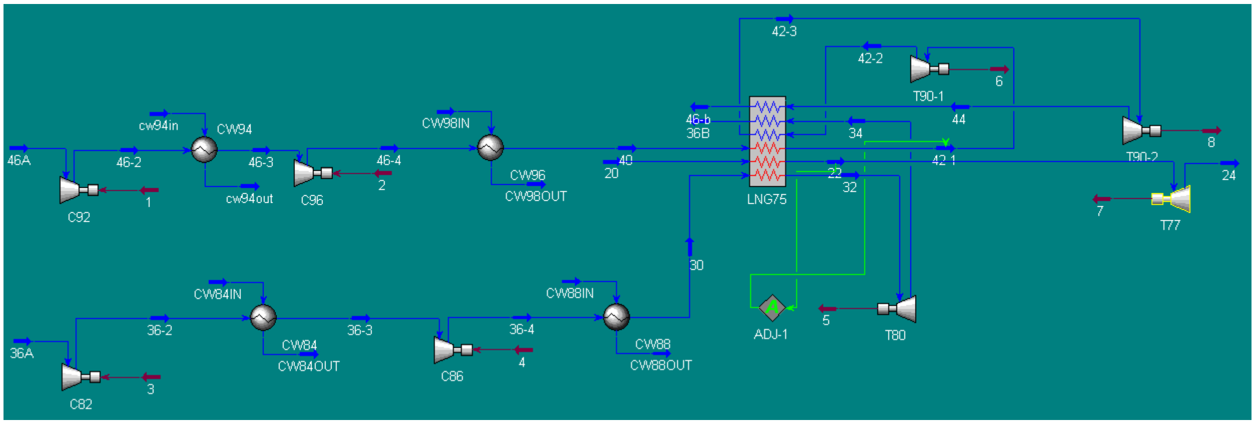


Figure 4.7 Flow sheet of the second option to improve design of process with ExPAnD method

In this method all the parameters set with the same value of base case (stream data of base case is given in appendix A). When we changed the pressure stream no.42-2, the temperature of stream 42-1 is changed to temperature of stream no.22 reach to -155°C. We can see the result in table (4-6).

Table 4-6 Result of second option to improve design of process with ExPAnD method

Pressure of stream no.42-2 (bar)	25	35	Based case
Exergy efficiency of process	56.5224%	56.5218%	56.5205%

If we set pressure of stream no,42-2 at 40 bar, we can find the effect of temperature stream no.42-1 on exergy efficiency of process when temperature stream no.44 is not fixed. In this study temperature stream no.42-3 is adjusted to temperature stream no.22 reach to -155°C.

Table 4-7 Result of second option to improve design of process with ExPAnD method

Temperature of stream no.42-1 (°C)	0	-5	-10
Produced work in expanders(T90-1,2) (kw)	294.2149	294.19	294.1779
Exergy efficiency of process	56.5227%	56.5229%	56.5216%

In all of these studies the mass velocity of methane has not been changed. It means that we did not use the additional cold utility which produced by two expansion for heat integration in the process. So, the exergy efficiency has not been changed significantly.

Third option is to expand cold stream two times when we changed the mass velocity of refrigerant to use the additional cold duty in the process as it shown in figure (4-8).

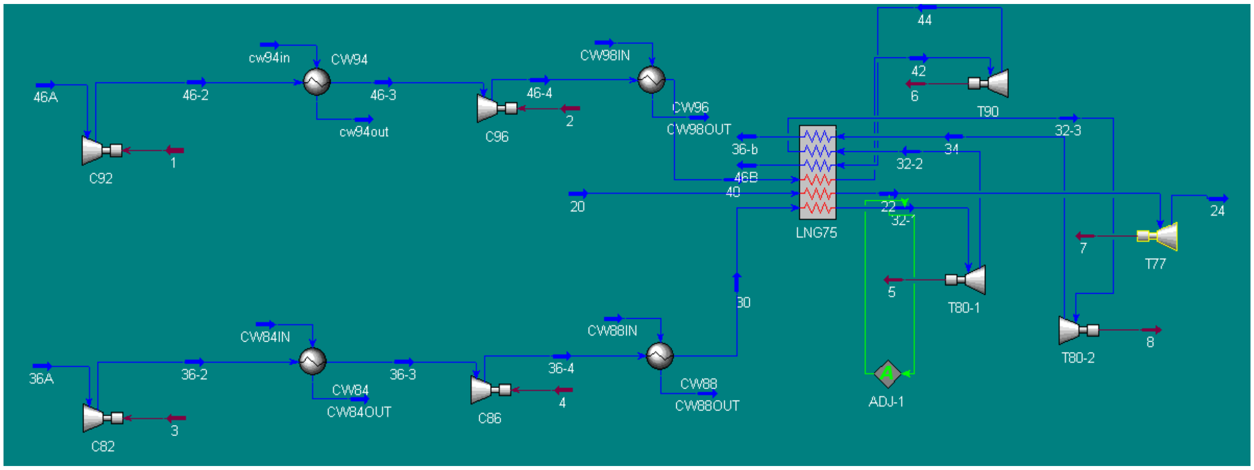


Figure 4.8 Flow sheet of the third option to improve design of process with ExPAnD method

In this option, the pressurized nitrogen expanded in T80-1, heated up in LNG heat exchanger, and again expanded in the T80-2. In the base case the pressure of high pressure stream of nitrogen is 84 bar and the low pressure is 14 bar. But the mass velocity of nitrogen is changed. When the pressure of stream no.32-3 was changed, the temperature stream no. 31-1 was changed to temperature stream no.22 reach to -155°C. We can see the result in table (4-9).

Table 4-8 Result of third option to improve design of process with ExPAnD method

Methane flowrate (kg/s)	2.5	2.5	2.5	2.5
Nitrogen flowrate (kg/s)	6.3525(base case)	6	6	6
Temperature of 32-1		-51.12	-55.64	-59.08
Pressure of 32-3		40	35	30
Exergy efficiency of process	56.5205%	61.1655%	61.1662%	61.1669%

By reviewing the results, we can find that heat up the cold stream and expand it again has positive effect on exergy efficiency of process. But the magnitude of improvement is not significant if we do not change the mass velocity of refrigerant. It seems that this amount of improvement in exergy efficiency is not economical if we consider external capital cost because of increasing in piping in LNG heat exchanger and using two expanders instead of one. If we expand the pressurised stream in two level and heat the first expanded stream in LNG heat exchanger we can produce more cold duty. By decreasing the flow rate of refrigerant we can utlise the extra cold duty and increase the exergy efficiency of process. Moreover, we can see in the results that, the pressure of stream after first expansion does not have important influence on exergy efficiency of process.

5. Conclusion and suggestions for future work

Basic Pinch Analysis is a well established method for the heat integration in low temperature processes. But in this method the temperature is only considered as a quality parameter. Thus, it effective enough in low temperature processes where pressure, has a significant role. A new methodology has developed in the department of energy and process engineering at NTNU which is called. It is an extension of the basic pinch method which is applied to processes where pressure plays an essential role.

ExPAnD has been used to design a new process for offshore natural gas liquefaction. To generalize this methodology it is necessary to use this method in other processes. In this thesis, ExPAnD method has successfully applied to the process of dual independent refrigeration cycles for LNG production which is a cyclic process.

Moreover, Exergy Transfer Effectiveness (ETE) introduced to measure the performance of a process from the thermodynamic points of view. Although the previous proposed definitions are not suitable, our definition for exergy efficiency can evaluate the performance of a low temperature process effectively.

The result from the study signifies that increasing the efficiency of one unit operation may increase or decrease the efficiency of the process; since, this is not the only factor which has an effect on the exergy efficiency of a process. Another factor which should be mentioned is the relative source of operation unit to the source of the main process. As an example, increasing the pressure ratio of a compressor which works above the ambient temperature results in a higher exergy efficiency of the corresponding compressor; while it decreases the exergy efficiency of the main process. On the other hand, decreasing the pressure ratio of a turbine which works below ambient temperature has a positive effect on both the exergy efficiency of turbine and the main process.

Additionally, we found out that the LNG heat exchanger is the most important unit operation in the main process. That is because of the significant effect of its exergy efficiency on the exergy efficiency of the main process.

Another result is about the trivial effect of the pressure variation of refrigeration cycles on the exergy efficiency of the main process; due to the fact that pressure variations have not influence on the exergy efficiency of LNG heat exchanger. However, changing the mass velocity of a refrigerant can change the exergy efficiency of process significantly by improving the exergy efficiency of LNG heat exchanger. Changing the flow rate of refrigerants results in closer temperature difference between cold and hot stream. So, the irreversibility, due to the heat transfer at limited temperature difference, decreases. As a result the exergy efficiency of LNG heat exchanger and the main process increase.

Expanding a pressurized stream results in producing cold duty. This fact has been used in the design of cyclic processes for LNG production. We can improve the exergy efficiency of the main process using the following procedure. First the pressurized stream should be expanded. Then it should be heated up in the LNG heat exchanger. Finally, it should be expanded again to reach the target pressure and provide enough temperature driving force in the cold end of LNG heat exchanger. By applying this procedure, we can produce more cold duty in the main process.

But this is not enough to improve the exergy efficiency, thus we continue to reduce the mass velocity of the refrigerant to increase the exergy efficiency of the main process. The pressure after the first expansion and flow rate of refrigerant should be changed as long as energy balance in LNG heat exchanger is not violated.

The ExPAnD method is not widely used yet. As an example, the compression effect is not examined nor explained in our project. Analyzing the compression effect to reduce the hot utility is a good opportunity for future work. In addition, ExPAnD method has ten heuristic rules. In this thesis the rules for streams with target pressure different from the supply pressure are used. It is important to study other rules by considering appropriate processes. Another research topic is to check the possibility of insight aggregation of these heuristics at a higher level.

References

Aspelund A., Berstad D.O., and Gundersen T., *An extended pinch analysis and design procedure utilizing pressure based exergy for subambient cooling*, Applied Thermal Engineering 27 (2007) 2633-2649.

Aspelund A. and Gundersen T., *The liquefied energy chain*, Proceeding of the 1st Annual Gas Processing Symposium, Doha, Qatar, January 2009.

Bejan A., Tsatsaronis G., and Moran M., *Thermal Design & Optimization*, First edition. John Wiley & Sons, New York, 1996.

Brodyansky M.V., Sorin M.V., and LE Goff P., *THE EFFICINCY OF INDUSTRIAL PROCESSES: EXERGY ANALYSIS AND OPTIMIZATION*, Elsevier, Amsterdam, 1994.

Foglietta J. H., *LNG production using dual independent expander refrigeration cycles*, U.S. patent 6412302 B1, 2002.

Gundersen T., *An introduction to the concept of exergy and energy quality*, version 4, 2011.

Gundersen T., Aspelund A., and Barton P.I., *An overview of new methodologies for the design of cryogenic processes with an emphasis on LNG*, Proceeding of the 1st Annual Gas Processing Symposium, Doha, Qatar, January 2009.

Kotas T.J., *The exergy method of thermal plant analysis*, Krieger Publishing Company, 1995.

Marmolejo-Correa D. And Gundersen T., *EXERGY EFFICIENCIES FOR LOW TEMPERATURE PROCESSES,* XXXII National Meeting and First International Congress AMIDIQ, Riviera Maya, Mexico, May 2011.

Moran M.J. and Shapiro H.N., *Fundamentals of engineering thermodynamics*, John Wiley & Sons, 5th edition, 2006.

Morosuk T. and Tsatsaroinis G., *GRAPHICAL MODELS FOR SPLITTING PHYSICAL EXERGY*, Proceeding of ECOS 2005, Trondheim, Norway, June 2005.

Smith R., *Chemical process design and integration*, John Wiley & Sons, 2008.

Sonntag R.E., Borgnakke C., and Van Wylen G.J, *Fundamentals of thermodynamic*, John Wiley & Sons, 6th edition, 2003.

Venkatarathnam G., *Cryogenic mixed refrigerant processes*, Springer, 2008.

ASHRAE handbook of refrigeration, Inch-Pound edition, 2006.

Appendix A. Data of base case

Name	20	22	24	40	42	44	46	46-2	46-3	46-4
Composition	NG	NG	NG	methane	methane	methane	methane	methane	methane	methane
Vapour fraction	1	0	9.16E-02	1	1	1	1	1	1	1
Temperature	25	-154.999	-166.946	30	-7.27896	-84	25	89.372	30	96.53028
Pressure	62	61.8	1	57.8	57.6	14	13.8	28.29134	28.09134	58
Mass flow (Kg/s)	1	1	1	1.5	1.5	1.5	1.5	1.5	1.5	1.5
Mass Enthalpy (KJ/Kg)	-3894.45	-4641.96	-4655.15	-4715.73	-4816.92	-4934.58	-4682.97	-4538.99	-4685.72	-4540.12
Mass Entropy (KJ/Kg.K)	7.85219	4.041998	4.063914	9.234317	8.879519	8.991857	10.05013	10.12266	9.684463	9.75632
Ex T (KJ/s)	0	388.865	889.8376	0.165504	7.788274	92.61491	0	22.70592	0.151279	29.40436
Ex P (KJ/s)	521.0704	520.7022	0	918.8721	918.1401	606.5866	603.3292	764.1581	762.5896	919.6014
Ex Th (KJ/s)	521.0704	909.5673	889.8376	919.0376	925.9283	699.2016	603.3292	786.864	762.7409	949.0058

Name	30	32	34	36	36-2	36-3	36-4
Composition	nitrogen	nitrogen	nitrogen	nitrogen	nitrogen	nitrogen	nitrogen
Vapour fraction	1	1	1	1	1	1	1
Temperature	30	-82.0431	-158	25	131.1095	30	138.7153
Pressure	83.8	83.6	14	13.8	34.04703	33.84703	84
Mass flow (Kg/s)	8.924199	8.924199	8.924199	8.924199	8.924199	8.924199	8.924199
Mass Enthalpy (KJ/Kg)	-10.406	-157.679	-208.783	-3.02997	108.8131	-1.69168	114.3347
Mass Entropy (KJ/Kg.K)	3.939875	3.327042	3.407455	4.501238	4.552214	4.239326	4.291209
Ex T (KJ/s)	0.43807	318.6442	1062.748	0	149.4024	0.407647	176.9324
Ex P (KJ/s)	3499.663	3497.755	2083.637	2072.279	2785.351	2780.695	3501.567
Ex Th (KJ/s)	3500.101	3816.399	3146.385	2072.279	2934.753	2781.103	3678.5

Name	CW84IN	CW84OUT	CW88IN	CW88OUT	cw94in	cw94out	CW98IN	CW98OUT
composition	water	water	water	water	water	water	water	water
Vapour fraction	0	0	0	0	0	0	0	0
Temperature	25	30	25	30	25	30	25	30
Pressure	2	1.8	2	1.8	2.00E+00	1.8	2	1.8
Mass flow (Kg/s)	44.50295	44.50295	50.23608	50.23608	9.932205	9.932205	11.88746	11.88746
Mass Enthalpy (KJ/Kg)	-15895.2	-15873.1	-15895.2	-15873.1	-15895.2	-15873.1	-1.59E+04	-15873.1
Mass Entropy (KJ/Kg.K)	2.958447	3.032242	2.958447	3.032242	2.958447	3.032242	2.958447	3.032242
Ex T (KJ/s)	0	8.185916	0	9.240475	0	1.82694	0	2.18659
Ex P (KJ/s)	5.894809	4.715854	6.654213	5.323379	1.315608	1.052488	1.574599	1.259681
Ex Th (KJ/s)	5.894809	12.90177	6.654213	14.56385	1.315608	2.879428	1.574599	3.446271

Block	C92	C96	C82	C86	T90	T80	T77	CW94	CW98	CW84	CW88	LNG HX	process
Position	above	above	above	above	below	below	below	above	above	above	above	across	
Sink	305.8913	310.4414	613.878	638.7342	435.5216	854.23	514.1681	2.606366	3.119455	5.629795	5.629795	625.9163	1521.775
Source	359.9473	364.0017	710.4183	736.9903	519.2557	1006.518	520.7022	40.20516	49.94694	109.3627	126.9774	925.1087	2692.428
ε	0.849822	0.852857	0.864108	0.866679	0.838742	0.848698	0.987451	0.064827	0.062455	0.051478	0.044337	0.676587	0.565205